

Keeping the Little Guy Down: A Debt Trap for Informal Lending

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Abstract

Microcredit and other forms of informal finance have so far failed to catalyze business growth among small scale entrepreneurs in the developing world, despite their high return to capital. This prompts a re-examination of the special features of informal credit markets that cause them to operate inefficiently. We present a theory of informal lending that highlights two of these features. First, borrowers and lenders bargain not only over division of surplus but also over contractual flexibility (the ease with which the borrower can invest to grow her business). Second, when the borrower's business becomes sufficiently large she exits the informal lending relationship and enters the formal sector – an undesirable event for her informal lender. We show that in Stationary Markov Perfect Equilibrium these two features lead to a poverty trap and study its properties. The theory facilitates reinterpretation of a number of empirical facts about microcredit: business growth resulting from microfinance is low on average but high for businesses that are already relatively large, and microlenders have experienced low demand for credit. The theory features nuanced comparative statics which provide a testable prediction and for which we establish novel empirical support. Using the Townsend Thai data and plausibly exogenous variation to the level of competition Thai money lenders face, we show that as predicted by our theory, money lenders in high competition environments impose fewer contractual restrictions on their borrowers. We discuss robustness and policy implications.

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1 Introduction

Microcredit was long celebrated for its promise to lift the developing world out of poverty.¹ Its proponents argued that, by offering a sustainable source of capital, microcredit would enable small scale entrepreneurs to leverage profitable investment opportunities and begin a path to a more prosperous future. These hopes were bolstered by a number of experimental studies that found that many microfirms in the developing world enjoy extremely high marginal returns to capital (on the order of five to ten percent *per month*; see [De Mel, McKenzie, and Woodruff \(2008\)](#), [Fafchamps, McKenzie, and Woodruff \(2014\)](#), and [McKenzie and Woodruff \(2008\)](#)). However, a variety of recent experimental and non-experimental evidence suggests that the impact of microcredit falls far short of previous expectations: on average, firms that randomly receive microcredit are no more profitable than those that do not (see [Banerjee, Karlan, and Zinman \(2015b\)](#) and [Meager \(2016\)](#) for a summary of the recent experimental evidence). A similar puzzle presents itself when considering other forms of informal finance available to these small firms: why haven't informal financiers such as money lenders enabled these small scale entrepreneurs to leverage their high return to capital opportunities?

The proximate answer for why microcredit, and more broadly informal finance, has so far failed to empower these entrepreneurs may lie in the various contractual features, other than the transfer of working capital, that are common in informal loans.² Many of these features seem to stymie the borrower's ability to invest her loan into high growth opportunities. One prominent example is that many microfinance institutions (MFIs) require that repayment begin immediately after the initial disbursement of funds and take place in frequent installments. The need to have cash on hand may restrict the borrower's ability to undertake long term investments that serve to grow her business at the expense of short term output. [Field, Pande, Papp, and Rigol \(2013\)](#) describe a field experiment in which this restriction was relaxed for a random set of borrowers. Three years after the study took place, borrowers who received two-month grace periods had roughly 80% more business capital and enjoyed 41% higher profits than their counterparts who received standard contracts.

¹See e.g. the 2006 Nobel Peace Prize awarded to Muhammad Yunus and the Grameen Bank for the innovation and practice of microcredit.

²There may be disagreement about whether microfinance should be classified as a form of informal finance. In this paper, we refer to any form of lending as informal if relatively wealthy borrowers are likely to terminate the relationship in favor of more attractive sources of financing and if the lender has the capacity to influence the borrower's project selection. We argue below that both of these criteria are satisfied by microfinance.

While money lenders are known to allow flexible repayment schedules, they may utilize other means to deter some forms of investment. For instance money lenders commonly require that borrowers work on their land (such as in tenancy arrangements) or that borrowers must forfeit their own land for the money lender to use for the duration of the loan (see e.g. [Sainath \(1996\)](#)). In the former case the lender ties up the borrower's labor, preventing him from focusing on projects to expand his own productive capacity and in the latter case the lender ties up an asset that the borrower could otherwise put to productive use. A final contractual restriction common to both money lenders and microfinance is the use of guarantors (or joint liability, in the case of microfinance). [Banerjee, Besley, and Guinnane \(1994\)](#) theorized that guarantors might pressure borrowers to eschew profitable but potentially risky investments in favor of safer uses of the loan to ensure its repayment. And [Fischer \(2013\)](#) provides evidence from a lab in the field experiment that such pressures indeed exist.

The question then becomes why is the rigid enforcement of these contractual features commonplace? Often these restrictive features are attributed to ensuring the repayment of loans, since formal recourse is unavailable to many informal lenders. That they also restrict investment is largely seen as an unintended consequence. However, with the exception of immediate and frequent repayments, there is little conclusive evidence that these contractual provisions actually serve to reduce default.³ And in Section 6 we suggest that even in the case of immediate and frequent repayments the story may not be clear.

We address both questions raised above by invoking an old explanation: informal lenders may benefit from keeping their borrowers in a debt trap, discouraging them from taking profitable investments to ensure they will continue to borrow for as long as possible (see e.g. [Bhaduri \(1973\)](#), and [Bhaduri \(1977\)](#)). However [Braverman and Srinivasan \(1981\)](#) and [Braverman and Stiglitz \(1982\)](#) argue profit maximizing lenders would not discourage such investments on the grounds that they should allow the efficient level of investment so long as they can extract the surplus. In this paper we argue that relatively wealthy borrowers may leave the informal lending relationship when they become eligible for cheaper, more formal sources of credit, providing a natural source of non-transferable utility. For money lenders it may be that their relatively wealthy borrowers become eligible for microloans, and for MFIs it may be that their borrowers become eli-

³[Gine and Karlan \(2014\)](#) and [Attanasio, Augsburg, De Haas, Fitzsimons, and Harmgart \(2015\)](#) each provide experimental evidence that joint liability does not affect the likelihood of default. In contrast, using non-experimental variation, [Carpena, Cole, Shapiro, and Zia \(2013\)](#) finds that joint liability loans are more likely to be repaid.

gible for collateralized bank loans. In either case, relatively poor borrowers may not be able to commit to share the benefits of “formal sector” lending (should they ever reach it) with their informal lender, and thus informal lenders may not be able to extract the surplus of investment. This reopens the possibility that a profit maximizing lender would deliberately impose a debt trap.⁴

Specifically, our model rests on three critical assumptions which are characteristic of much of the informal lending sector. First, borrowers who save and become sufficiently wealthy cease interaction with their informal lenders and enter the formal sector. While borrowers who engage in the formal lending sector enjoy its benefits, their informal lenders may regret losing customers. Second, borrowers and their informal lenders bargain not only over the division of surplus (i.e. the interest rate of the loan) but also over contractual restrictions which govern the ease with which the borrower can invest her loan to grow her business. Finally, neither the borrower nor the lender can commit to long term contracts. The borrower cannot commit to share the benefits of the formal sector (should she ever reach it) with her informal lender, and the lender cannot commit to provide favorable financing to the borrower in the future in exchange for the borrower’s cooperation in the short term.

For a stylized example, consider a fruit vendor. At low levels of wealth she operates a mobile cart. Each year she receives an endowment of cash and decides how to allocate it between two projects: a working capital project (e.g. buying fruits to sell throughout the week) and a fixed capital project (e.g. buying wood to expand from a mobile cart to a permanent stall from which to sell her fruits). If she invests in working capital, she begins to realize its returns immediately and consumes the proceeds from investment, as for the duration of the period she may not again have the minimum investment required to undertake the fixed capital project. In contrast, if she invests in the fixed capital project, she will forgo consumption for a period but have expanded her capacity for production in future periods. After several more business expansions she will gain access to the formal lending sector, and reap the corresponding benefits.

Each year she is also offered a loan from an MFI (her informal lender). Not only does the loan contract stipulate an initial cash transfer and an interest rate, but it also specifies whether she will be subjected to a variety of measures that make it difficult for her to undertake fixed capital investment and grow her business. For concreteness, assume it specifies whether the borrower must begin repayment immediately, or whether she

⁴Ray (1998) discusses another explanation for debt traps: borrowers who become sufficiently wealthy may default on their loans, causing lenders to discourage investments that help them grow their business.

can wait until the end of the period and repay in one lump installment. By controlling this additional feature, the lender may guide the borrower's project selection. If the borrower can wait until the end of the year to repay her loan, she may be able to choose her project flexibly. In contrast, if the borrower must have cash on hand to repay her initial installments, she may need to invest in her working capital project at the beginning of the period. And if she has trouble saving cash and unutilized assets (for instance, because she feels pressured to share unutilized assets with family members) then by the time she has earned enough to be able to cover her initial installments she may no longer have the necessary cash to undertake her fixed capital project. That is, the moment at which the lender makes the initial cash transfer is special; this is the moment when the borrower has enough cash to undertake her fixed investment, and if she cannot undertake it immediately she may not be able to for the remainder of the period. Of course, accepting the contract is voluntary, so if the borrower does not find the initial cash transfer and interest rate sufficiently attractive to offset the additional contractual restrictions, she can reject it and allocate her own, smaller endowment flexibly among her projects.

We show that subject to a plausible contracting friction, an asymmetry arises between contracts that restrict the borrower's ability to grow her business and those that do not. If the lender is unable to set an interest rate which leaves the borrower with exactly the level of output she would have had in his absence, she will retain more utility from unrestrictive contracts in equilibrium. This asymmetry arises because she cannot commit to share the proceeds of business growth, and therefore values the investment of her residual income into fixed capital more highly than its investment into working capital. If this asymmetry is sufficiently large the borrower may get stuck in a debt trap; despite large welfare gains from growth, the lender imposes repeated contractual restrictions on the borrower and she remains in poverty (and borrowing from her informal lender) forever. We show such a debt trap occurs if and only if the additional surplus the borrower gains from unrestrictive contracts exceeds the additional social welfare generated from business growth.

Beyond establishing that firms offered access to credit often fail to reach their efficient size, the theory organizes a number of other well established empirical facts about microcredit. In our equilibrium, sufficiently wealthy borrowers always receive unrestrictive contracts. These are borrowers for whom business expansion is especially valuable due to their proximity to the formal sector, and thus a lender seeking to restrict their investment would need to compensate them with prohibitively low interest rates. This is consistent with many experimental estimates of the impact of microcredit which find that,

while the marginal return to microcredit for *the average* firm is indistinguishable from zero, relatively richer business owners do exhibit high marginal returns to microcredit (see Angelucci, Karlan, and Zinman (2015), Augsburg, De Haas, Harmgart, and Meghir (2015), Banerjee, Duflo, Glennerster, and Kinnan (2015a), and Crepon, Devoto, Duflo, and Pariente (2015)).

A further empirical regularity noted in the above experiments is that demand for microcredit contracts is substantially lower than previously expected. This too emerges as a prediction of our theory. Because the lender transfers less surplus to borrowers via restrictive contracts than unrestrictive ones, borrowers who receive restrictive contracts may be nearer their indifference condition and their demand for credit may be low.

The model also sheds light on a number of nuanced comparative statics. Improving the attractiveness of the formal sector improves welfare of relatively richer borrowers because they anticipate eventually entering it. On the other hand, this improvement in the formal sector may *harm* the welfare of poorer borrowers and cause them to be trapped at even *lower* levels of wealth. Intuitively this is because of a “trickle down” effect whereby lenders anticipate that richer borrowers become more demanding, and restrict the investment of poor borrowers to ameliorate their increased bargaining power. This is especially striking given that fixing *any lender behavior*, an improvement in the formal sector unambiguously increases the borrower’s welfare. It is because of the lender’s endogenous response that this improvement harms the borrower.

Our comparative static on the borrower’s patience offers a counterpoint to a standard intuition that poverty traps are driven by impatience. In our model, the effect of increasing the borrower’s patience is ambiguous. Increasing the borrower’s patience increases her value of investment, and thus relatively richer borrowers who anticipate eventually entering the formal sector are made better off. However, similar to the comparative static on the attractiveness of the formal sector, increasing the borrower’s patience can make poor borrowers worse off. Anticipating that rich borrowers have improved bargaining power, the lender may restrict the investment of poor borrowers, *tightening* the debt trap.

Finally we provide two novel empirical facts in support of our theory. The first observation supports one of our key modeling assumptions. Relatively richer borrowers of a large Indian MFI terminate the informal borrowing relationship with higher frequency than their poorer counterparts. This creates potential for the MFI to desire to restrict the business growth of its borrowers to ensure a continued relationship.

Our second observation supports one of the model's key testable predictions. We provide evidence of our comparative static on competition. During the 2002 Million Baht Program, the Thai government established village funds endowed with one million Baht in each of many villages. Importantly the size of these lending institutions was constant even across villages of varying population size, inducing plausibly exogenous variation in the per capita credit shock. [Kaboski and Townsend \(2012\)](#) leverage this variation to estimate the return to credit for the customers of these village funds. In contrast, we study the variation in contracts offered by money lenders, and argue that Thai money lenders resemble the informal lenders in our model. In particular we show that there is a steep decline in the likelihood a villager borrows from a money lender as a function of his household's income. Thus we treat the Million Baht Program as exogenous variation in the level of competition faced by these informal money lenders.

We find that in villages with larger per capita credit shocks there is a decline in the incidence of restrictive contracts offered by money lenders, and argue that *given the contractual patterns observed in these villages* this is the unambiguous prediction of our theory. In essence borrowers in villages with more attractive village funds have better outside options, and the most efficient way for informal lenders to transfer surplus to their borrowers is to relax contractual restrictions. Thus lenders who previously offered them restrictive contracts at relatively low interest rates now find it unprofitable to do so, and switch to loans which do not restrict the nature of investment. We invoke a number of placebo tests to argue that other theories are unlikely to explain the observed patterns.

A number of other papers offer explanations for the fact that credit markets operate inefficiently. Classic explanations include adverse selection (see e.g. [Stiglitz and Weiss \(1981\)](#)), moral hazard in project selection ([Jensen and Meckling \(1976\)](#)) and moral hazard in repayment (see e.g. [Banerjee and Duflo \(2010\)](#)). [Bizer and DeMarzo \(1992\)](#) suggest that credit markets may operate inefficiently when borrowers cannot commit to exclusive lending relationships and [Green and Liu \(2016\)](#) apply this logic in a development setting to argue that informal lenders may lend the least to borrowers with the most productive investment opportunities. While each of these theories offers an explanation for why credit may not allow firms to fully realize their growth potential, they struggle to match the other empirical regularities we note. Most notably, each of these theories predicts that firms will be credit constrained, and will demand as much credit as they are offered. In contrast our model offers an explanation for the empirical regularity that the demand for microcredit is low.

Finally, several papers offer theories that yield comparative statics of a similar flavor to our own. [Petersen and Rajan \(1995\)](#) argue that credit markets in which there is a high degree of competition for rich borrowers may feature more constrained lending to poor borrowers, as lenders in high competition environments are less able to reap the rewards of investment in poor borrowers. [Jensen and Miller \(2015\)](#) provide a theoretical model of a farmer choosing a level of education for his child. Highly educated children may opt to migrate to the city rather than assisting their parents with farm work, and therefore as the urban returns to education increase the parent may decrease the level of education he allows his child to reach. In both of these models, the comparative static unambiguously harms one of the parties. In contrast, in our model improving the attractiveness of the formal sector only harms poor borrowers by virtue of helping richer borrowers. We expand on this point in section 4.3.

The rest of the paper proceeds as follows. In Section 2 we describe the model. Section 3 characterizes the equilibrium of our game. Section 4 describes comparative statics. Section 5 discusses some extensions of the model where we relax some of our stylized assumptions. Section 6 documents our novel empirical facts. Section 7 concludes. All proofs are relegated to the appendix.

2 The Model

Players, Actions, and Timing: We study a dynamic game of complete information and perfectly observable actions. There are two players, a borrower (she) and a lender (he). Each period lasts length dt and players discount the future at rate ρ . For analytical convenience we study the continuous time limit as dt converges to 0. The borrower's business is indexed with a state variable $w \in \{1, \dots, n + 1\}$ referred to as her business size.

At the beginning of each period the borrower has an endowment E_w , which may be augmented by a loan from her lender. She can invest her endowment into two projects: a working capital project C and a fixed capital project I . The working capital project C produces consumption goods which she uses to repay her lender and to eat, and the fixed capital project I governs the rate at which her business size increases. The allocation of her endowment between these two projects may be influenced by contractual restrictions imposed by the lender. We defer detailed explanation of financial contracts, and transition between states to the discussion of timing below, after which we map the modeling assumptions to our earlier anecdote about a fruit vendor.

The timing within each period is as follows:

- a) The lender offers a contract $\tilde{c} = \langle R, a \rangle \in \mathcal{C} \equiv \mathbb{R}^+ \times \{I, C\}$, where R represents the (contractable) repayment from the borrower to the lender, and a represents the contractual restrictiveness.
- b) The borrower chooses to accept or reject the contract. Formally she chooses a decision $d \in \{\text{Accept}, \text{Reject}\}$.
 - i. If she rejects the contract:
 - i. She receives an endowment $E_w > 0$ to flexibly allocate between two projects.
 - ii. She chooses an amount $c \leq E_w$ to invest into her *working capital project* C .
 - A. We assume this project has linear return: $C(w, c) = q_w c$, for some $q_w > 1$ and the borrower consumes this output.
 - iii. She chooses an amount $i = E_w - c$ to invest into her *fixed capital project* $I(w, i)$, the output of which is specified below.
 - iv. The lender receives a flow payoff of 0
 - ii. If she accepts the contract
 - i. The lender transfers $T_w > 0$ working capital to the borrower, making her endowment $E_w + T_w$.⁵
 - ii. If $a = C$, the borrower must invest everything in the working capital project. That is, she invests $i = 0$ in the fixed capital project and $c = E_w + T_w$ in the working capital project.
 - iii. If $a = I$, the borrower must invest $i = E_w + T_w - \frac{R}{q_w}$ in the fixed capital project and $c = \frac{R}{q_w}$ in the working capital project.
 - iv. The borrower repays R to the lender who receives a flow payoff of $R - T_w$.
- c) If the borrower invests i into her fixed capital project I , her business size moves from state w to $w + 1$ according to a Poisson process with arrival rate $\frac{i}{\phi_w} dt$ with $\phi_w > 0$ and remains constant otherwise.⁶

⁵Note, we assume that T_w is fixed, and therefore do not study the lender's decision of loan size in this paper.

⁶This assumption may be generalized to allow for any transition process in which the probability of transition from w to any other state scales linearly with investment.

- d) If the game ever reaches state $n + 1$ both players cease acting.
 - i. The borrower receives a continuation payoff $U \equiv \frac{u}{\rho}$
 - ii. The lender receives a continuation payoff 0
- e) Else the period concludes and after discounting the next one begins.

The timing above can be understood through the lens of the example in our introduction. The borrower is a fruit vendor, and at state w she operates a mobile cart. At the beginning of the year she has a cash endowment E_w . If she rejects the lender's contract then she flexibly allocates her endowment between her two projects: a working capital project C which can be understood as purchasing fruits to sell during the week, and a fixed capital project I which can be understood as buying raw materials to expand to a market stall from which she may have access to a broader market, improving her productivity. For every unit she invests in the working capital project, she produces $q_w > 1$ units of output. So q_w may be thought of as the markup she enjoys from selling fruits, and ϕ_w may be thought of as the cost of fixed investment. The more she invests in fixed capital, the more likely she is to succeed in expanding her productive capacity by moving to state $w + 1$.

If instead she accepts the contract $\langle R, a \rangle$, the lender transfers T_w working capital to the borrower and her endowment is $E_w + T_w$. The borrower's subsequent investment decision is determined exclusively by the contractual restriction $a \in \{I, C\}$. If the contract specifies that the borrower should invest in the fixed capital project, that is $a = I$, then she invests her entire endowment into fixed capital save for just enough which she invests into working capital to repay her debt. If instead $a = C$ she invests her entire endowment into the working capital project. At the end of the period she repays her debt R to the lender, and they begin anew in the next period.

Though the stylized model above does not include an detailed description of the timing of output within a period, the contractual restriction $a = C$ can be understood as the requirement of early and frequent repayments. If the lender demands that the borrower has cash on hand each day to repay a small fraction of her loan, she may not be able to initially invest in the long term, fixed capital project which may not return output for weeks. By the time she has generated enough income through her working capital project to ensure she can repay each installment, she may no longer have enough cash on hand to meet the minimum required investment in her fixed capital project, as would be the case if she has trouble saving cash from day to day (for instance because she faces pressure from her family to share underutilized assets). In contrast, a borrower uninhibited by a

restrictive repayment plan ($a = I$) may invest freely.⁷ Therefore we refer to a contract that specifies $a = I$ as an *unrestrictive* contract and a contract that specifies $a = C$ as a *restrictive* contract.

The borrower's business expansion is represented by the discrete state space $\{1, \dots, n + 1\}$. Each state w represents a different business size (i.e. $w = 1$ may be a mobile cart, $w = 2$ a fixed stall, $w = 3$ a small store and so on). State $n + 1$ is a reduced form representation of the formal sector. The borrower enjoys (unmodeled) benefits of formal loans, and the lender receives a 0 continuation payoff having lost his customer. Notably, because the theorems below hold for any fixed investment cost and any number of states, it is straightforward to extend the model to accommodate a continuous state space. We discuss this further in Section 5.

Parametric Assumptions:

Arguably our most important parametric assumption is on the range of feasible repayment rates R .

Assumption 1. *We assume that the feasible range of repayment rates satisfies $R \in [T_w, q_w T_w - h_w]$ with $h_w > 0$ for all w .*

This assumption guarantees that if the borrower accepts the lender's contract and sets aside $\frac{R}{q_w}$ of her endowment to invest in her working capital project for repayment, the residual endowment she can invest in either project is at least $E_w + \frac{h_w}{q_w}$ which necessarily exceeds the endowment E_w she could have invested on her own. This can be motivated in a number of ways. Most straightforwardly, the borrower might be able to hide h_w from her lender every period, and thus the repayment rate he sets is bounded above by the residual output resulting from the loan, $q_w T_w - h_w$. Alternatively one could assume that the borrower can renege on her debt in any period, in which case she must find a new lender at cost $\nu_w = q_w T_w - h_w$. Then the borrower would never repay a debt in excess of this cost.⁸

⁷For technical convenience we assume that upon receiving an "unrestrictive" contract, the borrower *must* invest her endowment into the fixed capital project. Because we assume below that the borrower values investment in fixed capital more highly than she values investment in working capital, this assumption could be replaced by allowing her to invest flexibly upon receiving an unrestrictive contract with minimal consequence.

⁸In the equilibrium of our model the borrower may not extract positive rents from the lending relationship. Thus, to take this microfoundation seriously, one can ensure that she always finds it profitable to find a new lender in the event of renegeing on the first by assuming she receives an additional positive flow utility from interacting with any lender, that is unaffected by which loan she is offered. This can be motivated by an insurance benefit she receives from knowing her lender, that operates independently from the loans she receives every period.

The repayment ceiling is critical to many of our results below. Because the borrower cannot commit to share the proceeds from business expansion with her lender, she values investment in fixed capital more highly than she values investment in working capital. This in turn implies that the extra endowment $E_w + \frac{h_w}{q_w}$ the borrower retains induces an asymmetry between the *utility* she derives from restrictive and unrestrictive contracts. When this wedge is sufficiently large, the lender will impose inefficient contractual restrictions on the borrower, trapping her in poverty.⁹

We next assume that both the borrower and lender are risk neutral.

Assumption 2. *Both the borrower and lender enjoy a linear utility of consumption.*

Assumption 2 implies that if either player receives a sequence of flow consumptions $\{u_t\}$, their lifetime utility is

$$\int_0^{\infty} e^{-\rho t} u_t dt$$

Histories and Strategies: A history \tilde{h}_t is a sequence $\{\tilde{c}_{\tilde{t}}, d_{\tilde{t}}, i_{\tilde{t}}, w_{\tilde{t}}\}_{\tilde{t} < t}$ of contracts, accept/reject decisions, investment allocations and business states at all periods prior to t . We define \tilde{H}_t to be the set of histories up to time t .

The lender's strategy is a sequence of (potentially mixed) contractual offers $\vec{c} = \{\vec{c}(\tilde{h}_t)\}_{\tilde{h}_t \in \tilde{H}_t}$ where $\vec{c}(\tilde{h}_t) \in \Delta(\mathcal{C})$ is the probability weighting of contracts he offers the borrower following history \tilde{h}_t . The borrower's strategy is a sequence of accept/reject decisions $\mathbf{d} = \{d(\tilde{h}_t, \tilde{c})\}_{\tilde{h}_t \in \tilde{H}_t, \tilde{c} \in \mathcal{C}}$ and investment decisions in the event of rejection $\mathbf{i} = \{i(\tilde{h}_t, \tilde{c})\}_{\tilde{h}_t \in \tilde{H}_t, \tilde{c} \in \mathcal{C}}$. Here $d(\tilde{h}_t, \tilde{c})$ denotes the probability the borrower accepts the contract \tilde{c} following history \tilde{h}_t , and $i(\tilde{h}_t, \tilde{c})$ denotes the investment allocation the borrower undertakes following history \tilde{h}_t and rejecting contract \tilde{c} .

Equilibrium: Our solution concept is the standard notion of *Stationary Markov Perfect Equilibrium* (henceforth *equilibrium*) which imposes that at every period agents are best responding to one another and that they only condition their strategies on payoff relevant state variables (in this case, business size). In particular, neither agent has the ability to commit to a long term contract.

Formally, a strategy profile $(\vec{c}, \mathbf{d}, \mathbf{i})$ is an *equilibrium* if

- a) $\vec{c}(\tilde{h}_t)$ is optimal for the lender at every \tilde{h}_t given the borrower's strategy (\mathbf{d}, \mathbf{i}) .

⁹For technical convenience we also require that the repayment level is bounded below, but that it must be larger than the principle transfer T_w is unimportant.

- b) $d(\tilde{h}_t, \tilde{c})$ and $i(\tilde{h}_t, \tilde{c})$ are optimal for the borrower at every \tilde{h}_t and for every contract \tilde{c} given the lender's strategy \tilde{c} .
- c) At any two histories \tilde{h}_t and \tilde{h}'_t , for which w is the same, we have $\vec{c}(\tilde{h}_t) = \vec{c}(\tilde{h}'_t)$, $d(\tilde{h}_t, \tilde{c}) = d(\tilde{h}'_t, \tilde{c})$, and $i(\tilde{h}_t, \tilde{c}) = i(\tilde{h}'_t, \tilde{c})$.

By studying Stationary Markov Perfect Equilibria, we impose that the lender uses an impersonal strategy: any borrower with the same business size must be offered the same contract. This may be an especially plausible restriction in the context of large informal lenders such as microfinance institutions whose policy makers may be far removed from the recipients of their loans, rendering overly personalized contract offers infeasible.

3 Equilibrium Structure

We now describe the borrower and lender's equilibrium behavior and our main results about the structure of the equilibrium. Section 3.1 describes the borrower's autarky problem and sets forth an assumption that guarantees the borrower will eventually reach the formal sector (state $n + 1$) in autarky. Section 3.2 describes the key incentives of the borrower and lender necessary to understand the structure of the equilibrium. Section 3.3 provides our main results: The equilibrium is unique, and under additional assumptions specified below the probability that the lender offers a restrictive contract is single peaked in the state. Thus, the lender's poorest and richest clients may receive unrestrictive contracts and grow faster than they would have in his absence. But borrowers with intermediate levels of wealth receive restrictive contracts every period and find themselves in a poverty trap. Notably, this poverty trap may exist even if the borrower would have reached the formal sector in autarky and even if the discounted utility from expanding to the formal sector is greater than the total surplus generated from investing the total endowment in working capital in every state. In Section 3.4 we argue that several well established empirical facts about microfinance can be contextualized through the lens of this equilibrium.

3.1 The Borrower's Autarky Problem

First consider the borrower's autarky problem. That is, the economic environment is as specified in Section 2, but the borrower is forced to reject the lender's contract at all times

(i.e. she must choose $d(\tilde{h}_t, \tilde{c}) = 0$ for all histories \tilde{h}_t and contracts \tilde{c}).¹⁰

Let B_w^{aut} be the borrower's continuation value in autarky in state w . This can be decomposed into a weighted average of her flow payoff in the time interval $[t, t + dt]$, and her expected continuation utility at time $t + dt$. We have

$$B_w^{aut} = \max_i q_w (E_w - i) dt + (1 - \rho dt) \left(\frac{i}{\phi_w} dt B_{w+1}^{aut} + \left(1 - \frac{i}{\phi_w} dt \right) B_w^{aut} \right) \quad (1)$$

Fixing the optimal level of investment i in state w , rearranging, and ignoring higher order terms we have

$$B_w^{aut} = \frac{q_w (E_w - i)}{\rho + \frac{i}{\phi_w}} + \frac{\frac{i}{\phi_w}}{\rho + \frac{i}{\phi_w}} B_{w+1}^{aut} \quad (2)$$

That is, the borrower's autarky continuation value in state w is a weighted sum of her flow consumption $q_w (E_w - i)$ and her continuation value upon increasing business size, B_{w+1}^{aut} . Because equation 1 is linear in i (and equation 2 is monotone in i), the borrower will choose an extremal level of investment. From here on we will use the notation $\kappa_w \equiv \frac{E_w}{\phi_w}$, which is the maximum speed the borrower can invest in fixed capital and grow in autarky. We have the following proposition about the borrower's autarky behavior.

Proposition 1. *The borrower invests her entire income in every state iff*

$$\frac{q_w E_w}{\rho} \leq \left(\prod_{w'=w}^n \alpha_{w'} \right) \frac{u}{\rho} \text{ for all } w,$$

where $\alpha_w \equiv \frac{\kappa_w}{\rho + \kappa_w}$.

Proof. See appendix. □

The borrower's autarky problem has an attractive structure. If she chooses to invest in fixed capital in state w at every period then her continuation utility in state w is $B_w^{aut} = \alpha_w B_{w+1}^{aut}$. That is, she spends a fraction $(1 - \alpha_w)$ of her expected, discounted lifetime in the current state, and a fraction α_w of her expected, discounted lifetime in all future states $w + 1$ and onwards. Likewise in state w she anticipates spending a fraction $\prod_{w'=w}^m \alpha_{w'}$ of her expected, discounted lifetime in state $w + m$ and onwards if she invests in fixed capital at every period until reaching state $w + m$.

¹⁰Alternatively, one can imagine that the borrower simply does not have access to a lender.

This property is closely related to the Poisson arrival of jumps. Letting t denote the time of the jump and κ be the arrival intensity, v_1 be the flow utility the borrower enjoys prior to a jump and v_2 the flow utility she enjoys post jump, the borrowers utility is represented by:

$$\begin{aligned}\mathbb{E}_t \left[\int_0^t v_1 e^{-\rho s} ds + \int_t^\infty v_2 e^{-\rho s} ds \right] &= \int_0^\infty \left[\int_0^t v_1 e^{-\rho s} ds + \int_t^\infty v_2 e^{-\rho s} ds \right] \kappa e^{-\kappa t} dt \\ &= (1 - \alpha) \left(\frac{v_1}{\rho} \right) + \alpha \left(\frac{v_2}{\rho} \right)\end{aligned}$$

where $\alpha \equiv \frac{\kappa}{\rho + \kappa}$. Thus the borrower's utility from this process can be represented as the convex combination of her lifetime utility from staying in the initial state forever and her lifetime utility from staying in the post-jump state forever, where the weights on each are a function of the intensity of the arrival process. Having established that the borrower invests in fixed capital in all states in autarky if and only if $\frac{q_w E_w}{\rho} \leq \left(\prod_{w'=w}^n \alpha_{w'} \right) \frac{u}{\rho}$ for all w , we make the following, stronger assumption and maintain it throughout the subsequent analysis.

Assumption 3. $\frac{q_w E_w + h_w}{\rho} \leq \left(\prod_{w'=w}^n \alpha_{w'} \right) \frac{u}{\rho}$ for all w .

Assumption 3 guarantees that the borrower would prefer to invest her income into fixed capital rather than invest it into working capital for any flow income stream weakly less than $q_w E_w + h_w$. In addition to ruling out an uninteresting case in the analysis, Assumption 3 serves to highlight that the introduction of a lender may cause a poverty trap to emerge despite the autarkic borrower's eventual entry into the formal sector. That is, business growth among borrowers with access to credit may be *lower* than growth among their counterparts without access to credit.

3.2 Relationship Value Functions

We now outline the borrower and lender's relationship maximization problems and describe their value functions. Let B_w be the borrower's equilibrium continuation utility at the beginning of a period in state w , and let $B_w(\langle R, a \rangle)$ be her equilibrium continuation utility upon receiving the contract $\langle R, a \rangle$ in state w . Further, define

$$B_w^{Rej} \equiv \max_i q_w (E_w - i) dt + (1 - \rho dt) \left(\frac{i}{\phi_w} dt B_{w+1} + \left(1 - \frac{i}{\phi_w} dt \right) B_w \right)$$

to be her equilibrium continuation utility upon rejecting a contract. These functions satisfy

$$B_w(\langle R, C \rangle) = \max \left\{ (q_w (E_w + T_w) - R) dt + (1 - \rho dt) B_w, B_w^{Rej} \right\}$$

and

$$B_w(\langle R, I \rangle) = \max \left\{ (1 - \rho dt) \left(\frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt B_{w+1} + \left(1 - \frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt \right) B_w \right), B_w^{Rej} \right\}$$

where for both value functions above, the first expression in the brackets corresponds to the borrower's continuation utility if she accepts the contract $\langle R, a \rangle$ and the second term corresponds to her continuation utility if she rejects the contract, she is left with her smaller endowment E_w and chooses her own allocation of investment.

The lender's value function L_w in state w satisfies

$$L_w = \max_{\langle R, a \rangle} (R - T_w) dt + (1 - \rho dt) \left(L_w + \mathbb{I}_{a=I} \frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt (L_{w+1} - L_w) \right)$$

such that

$$\begin{aligned} q_w (E_w + T_w) - R &\geq \kappa_w (B_{w+1} - B_w) \quad \text{if } a = C \\ T_w &\leq R \leq q_w T_w - h_w \end{aligned}$$

Note that the lender's maximization problem and constraints assume the lender never finds it optimal to offer the borrower a contract she will reject.¹¹ The lender's maximization problem also assumes that the borrower accepts any unrestrictive contract. This is the case so long as the borrower would invest her entire income in fixed capital if she were to reject the contract.¹² The borrower accepts a restrictive contract $\langle R, C \rangle$ if and only

¹¹It is straightforward to show that in any Stationary Markov perfect equilibrium, either offering the borrower a restrictive contract with the highest acceptable repayment rate or offering her an unrestrictive contract with the highest feasible repayment rate will dominate offering the borrower a contract she would reject.

¹²The borrower may invest in working capital in her outside option in equilibrium if she expects a high rate of investment in fixed capital from the lender. This case is dealt with in the appendix, but the analysis does not substantively differ from the above.

if her value of consuming what the lender offers is weakly higher than that of rejecting the contract and choosing her own allocation of investment, i.e.

$$q_w (E_w + T_w) - R \geq \kappa_w (B_{w+1} - B_w).$$

We refer to the above inequality as the borrower's *individual rationality constraint*. In equilibrium the lender always offers one of three contracts:

- $\langle T_w, I \rangle$ in states w where the lender's value function satisfies $L_{w+1} - L_w \geq \phi_w$ and thus he wants the borrower to expand as quickly as possible. In such states, the lender charges the lowest possible interest rate, T_w .
- $\langle q_w T_w - h_w, I \rangle$ in states where the lender prefers unrestrictive contracts but where $L_{w+1} - L_w < \phi_w$ so that the lender's preference for expansion is not so strong so as to drive the him to offer the borrower a higher than necessary flow payoff. In such states the lender offers the highest possible interest rate, $q_w T_w - h_w$.
- $\langle q_w (E_w + T_w) - \kappa_w (B_{w+1} - B_w), C \rangle$ in states where the lender prefers a restrictive contract, and therefore charges the highest *acceptable* interest rate.

Expansion rents

The lender's maximization problem illuminates an important force in our model. If the lender offers the borrower a restrictive contract, he optimally offers her the most extractive repayment rate she finds acceptable, denoted by \bar{R}_w . This repayment rate is determined by the borrower's indifference condition between accepting the restrictive contract or investing in fixed capital at her autarkic rate. Receiving this contract at every period the borrower's continuation utility would be

$$B_w = (q_w (E_w + T_w) - \bar{R}_w) dt + (1 - \rho dt) B_w = (1 - \rho dt) (\kappa_w dt B_{w+1} + (1 - \kappa_w dt) B_w)$$

Rearranging and ignoring higher order terms we have

$$B_w = \frac{\kappa_w}{\rho + \kappa_w} B_{w+1} = \alpha_w B_{w+1}$$

That is, if the lender offers the borrower the least generous acceptable restrictive contract the borrower's continuation utility is exactly what it would be if she invested in fixed

capital at her autarkic rate.

On the other hand, if the lender offers a maximally extractive unrestrictive contract in every period, the borrower's continuation value will satisfy

$$B_w = (1 - \rho dt) \left(\frac{E_w + \frac{h_w}{q_w}}{\phi_w} dt B_{w+1} + \left(1 - \frac{E_w + \frac{h_w}{q_w}}{\phi_w} dt \right) B_w \right)$$

Rearranging and ignoring higher order terms we have

$$B_w = \frac{\gamma_w}{\rho + \gamma_w} B_{w+1} = \beta_w B_{w+1}$$

where $\gamma_w \equiv \frac{E_w + \frac{h_w}{q_w}}{\phi_w}$ is the rate of expansion the borrower enjoys when she receives a maximally extractive unrestrictive contract, and $\beta_w \equiv \frac{\gamma_w}{\rho + \gamma_w}$ is the fraction of her discounted lifetime she expects to spend in state $w + 1$ and onwards if she invests in fixed capital at rate γ_w in state w . Note that if the lender offers the borrower an unrestrictive contract, the borrower's continuation utility is strictly higher than it would be in autarky, because she is allowed to invest strictly more into fixed capital than she would in autarky.

The difference between the borrower's continuation value upon receiving an unrestrictive contract and upon receiving a restrictive one is $(\beta_w - \alpha_w) B_{w+1}$. We refer to this term as the *expansion rent* in state w . This asymmetry arises because of the ceiling on feasible repayment rates the lender may set. Recall, after transferring T_w endowment to the borrower, the lender must set a repayment weakly less than $q_w T_w - h_w$ with $h_w > 0$. Thus upon accepting a loan and allocating $\frac{R}{q_w}$ to the working capital project for repayment, the borrower necessarily has a larger residual endowment to allocate to either project than she would have had on her own. Because she values investment in fixed capital more highly than she values investment in working capital, she values this extra income more highly when receiving unrestrictive contracts than she does when receiving restrictive ones.

As will be clear in the following sections, this expansion rent is critical for our main results. The lender may prohibit efficient growth by offering the borrower restrictive contracts, and in equilibrium will do so if and only if the expansion rent highlighted above exceeds the change in joint surplus resulting from expansion.

3.3 Results

We are now in a position to state our first result.

Proposition 2. *An equilibrium exists and is generically unique.*

Proof. See Appendix. □

The result follows by backward induction on the state. In any state w the borrower's accept/reject decision is pinned down by her state w continuation value B_w and her state $w + 1$ continuation value B_{w+1} . The primary subtlety arises from the fact that the borrower's welfare in state w is increasing in the probability the lender offers an unrestrictive contract in w . The more frequently the borrower anticipates unrestrictive contracts in w the *less demanding* she will be of restrictive contracts. Formally, we define $\delta_w(p_w) \equiv p_w \kappa_w + (1 - p_w) \gamma_w$. It is straightforward to show that a borrower who expects a restrictive contract with probability p_w in state w will have a continuation utility of $B_w(p_w) = \frac{\delta_w(p_w)}{\rho + \delta_w(p_w)} B_{w+1}$ which is decreasing in p_w . The lender determines the interest rate associated with restrictive contracts, $R_w(p_w)$, to solve

$$\kappa_w (B_{w+1} - B_w(p_w)) = q_w (E_w + T_w) - R_w(p_w)$$

from which it is immediate that $R_w(p_w)$ is decreasing in p_w . Thus it may be that when the borrower expects a restrictive contract with certainty the lender strictly prefers to offer an unrestrictive contract, and when the borrower expects an unrestrictive contract with certainty the lender strictly prefers to offer a restrictive contract. In such a case the unique equilibrium involves a strictly interior p_w and the expansion rent is $\left(\beta_w - \frac{\kappa_w}{\rho + \delta_w(p_w)} \right) B_{w+1}$.

A second subtlety is due to the possibility that in equilibrium the borrower invests her autarkic endowment in working capital after rejecting the lender's contract in some state w . Despite Assumption 3, the borrower may invest her autarkic endowment in working capital in state w if she expects to receive sufficiently attractive unrestrictive contracts in state w , which causes B_w to be near to B_{w+1} and depresses the value of business expansion. We show that if there is an equilibrium in which the borrower invests her autarkic flow endowment in working capital in state w , this can only be due to the fact that the lender offers her an attractive unrestrictive contract, which is feasible irrespective of the borrower's autarkic action and therefore occurs across all equilibria. Thus after rejecting the lender's contract in state w , she invests her autarkic flow endowment in working capital in any equilibrium.

Equilibrium Contract Structure

For the remainder of this section and the next we make the following parametric assumptions. We do so for simplicity and ease of exposition but argue in Section 5 that their complete relaxation does not change the qualitative lessons to be drawn from the model.

First we assume that the flow working capital output within the relationship is increasing and weakly concave in the state. Let $y_w \equiv q_w (E_w + T_w) - T_w$.

Assumption 4. $y_w > y_{w-1}$ for all w and $y_w - y_{w-1} \geq y_{w+1} - y_w$ for all w .

Second we assume that the borrower's autarky endowment E_w , the amount she can hide h_w , and the cost of investment in fixed capital ϕ_w are constant in w .

Assumption 5. $E_w = E_{w'} \equiv E$ for all w, w' , $h_w = h_{w'} \equiv h$ for all w, w' , and $\phi_w = \phi_{w'} \equiv \phi$ for all w, w' .

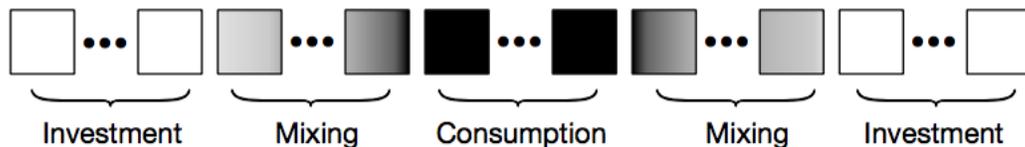
Note that Assumption 5 allows us to omit the subscripts on κ, γ, α , and β .

Our next result regards the equilibrium organization of restrictive states and unrestrictive states under the parametric assumptions above.

Proposition 3. *In equilibrium, the probability the lender offers a restrictive contract p_w is single peaked in w .*

Proof. See Appendix. □

This result implies that in equilibrium the states can be partitioned into three regions of consecutive states: An initial region with only unrestrictive contracts, an intermediate region in which both kinds of contracts are possible, and a final region in which only unrestrictive contracts are offered. In the intermediate region, the probability a restrictive contract is offered is increasing (potentially reaching 1) and then decreasing. This is depicted in the figure below where white states denote unrestrictive states, black states denote restrictive states and grey states denote mixing states.



Borrowers who arrive at a state in which only restrictive contracts are offered never

grow beyond it. The next natural question, therefore, is when do such states arise? We have the following result.

Proposition 4. *In equilibrium, the probability the lender offers a restrictive contract $p_w = 1$ if and only if*

$$\beta \left((L_{w+1} + B_{w+1}) - \frac{q_w(E + T_w) - T_w}{\rho} - \phi \right) \leq (\beta - \alpha) B_{w+1}$$

The left hand side of the above inequality may be loosely understood as the social gain from investing in fixed capital at rate $E + \frac{h}{q}$ rather than investing everything into working capital. If the borrower invests at rate $E + \frac{h}{q}$, then she and the lender expect to spend a fraction β of their lifetime in state $w + 1$ and onwards. Once in $w + 1$ they jointly enjoy continuation values of $L_{w+1} + B_{w+1}$ but forgo the consumption they could have enjoyed in state w , $\frac{q_w(E+T_w)-T_w}{\rho}$, and the cost they incur from expansion is $\beta\phi$. In contrast, the right hand side of the inequality is the borrower's expansion rent: the additional surplus she commands from unrestrictive contracts relative to restrictive ones. Thus if this expansion rent exceeds the social gain of business expansion, the lender will offer only restrictive contracts, pinning the borrower to the current state.

Note that because of the restrictions on feasible repayment rates, this is not a model of transferrable utility. Thus the left hand side of the above inequality should not literally be interpreted as a change in social welfare. Nevertheless we will sometimes abuse terminology and say that it is socially efficient to invest in fixed capital when the left hand side of the above inequality is positive.

We are now ready to discuss the intuition behind Proposition 3. When the borrower is near the formal sector, it is extremely costly to offer her a restrictive contract. For concreteness, consider a borrower in state n . A lender who offers this borrower a restrictive contract in every period needs to compensate her with $\alpha \frac{u}{\rho}$ consumption over the life of the relationship. For u sufficiently high this is prohibitively costly. However, as the borrower becomes poorer it becomes cheaper to offer her a restrictive contract. Consider a borrower who is at state w and who expects unrestrictive contracts in all future states. A lender who offers this borrower a restrictive contract in every period needs to transfer her only $\alpha \beta^{n-w} \frac{u}{\rho}$ consumption over the lifetime of the relationship. Thus as the borrower becomes poorer it becomes exponentially cheaper to offer her the restrictive contract.

In this intermediate region the expansion rent may become important. As discussed above, when the borrower's expansion rent exceeds the social gain from business expan-

sion, the lender offers only restrictive contracts, keeping her inefficiently small. Note that this poverty trap is created by the presence of the lender. Assumption 3 guarantees that in autarky the borrower would have grown to her efficient size.

Last, as the borrower becomes sufficiently poor her expansion rent $(\beta - \alpha) B_{w+1}$ decreases, as it is tied to her continuation value in the next state. Moreover, because of the concavity of the output from working capital investment, the joint surplus increase from expansion becomes increasingly large as the borrower becomes poorer. Thus sufficiently poor borrowers receive unrestrictive contracts.

We close this section with a discussion of the source of this poverty trap. One crucial feature is that the minimum endowment residual of repayment the borrower enjoys when contracting with the lender, $E + \frac{h}{q_w}$, is strictly larger than the endowment she would have had on her own, E . We encode this fact in the following proposition.

Proposition 5. *If $h = 0$, then $p_w = 0$ for any w in which it is socially efficient to invest in fixed capital (i.e. whenever $\beta \left((L_{w+1} + B_{w+1}) - \frac{q_w(E+T_w)-T_w}{\rho} - \phi \right) > 0$).*

When the lender can choose interest rates flexibly enough such that the borrower can be left with exactly the same amount of income that she would have produced alone, he offers unrestrictive contracts in any state in which the social gain from business expansion is positive. When the lender offers a restrictive contract, he gives the borrower just enough consumption to make her indifferent between accepting the contract and rejecting it and investing E in fixed capital. But if the lender instead offers the borrower a maximally extractive unrestrictive contract, the borrower remains indifferent, because the endowment she can invest into business expansion is exactly what she could have invested on her own. Since the total social surplus increases and the residual surplus accrues to the lender, he prefers unrestrictive contracts.

While the poverty trap disappears when $h = 0$ it is important to note that the unique equilibrium still features inefficiently slow business expansion relative to the social optimum. A natural question then, is what contractual flexibility is required to reach the first best level of investment in fixed capital. It is straightforward to verify that *equity contracts* – contracts that allow the borrower to commit a fraction of her formal sector flow payoff to the lender in exchange for favorable unrestrictive contracts – are sufficient to guarantee first best investment. However this is primarily a theoretical exercise, as the participants of informal financial markets rarely have the capacity to write equity contracts.

3.4 Connection to Empirical Evidence

At this point the model already starts to organize much of the empirical evidence on microcredit cited in our introduction. That firms fail to grow from being offered access to microcredit can be understood through the fact that in our model, firms who enter a state where the lender offers restrictive contracts (the black region in the figure above) never leave it, despite the fact that they would have continued to grow in autarky. That is, this is a model in which having access to an informal lender can *reduce* business growth.

While the microcredit studies listed above find low marginal returns to credit on average, a number of them find considerable heterogeneity in observed returns to credit. In particular they consistently find a long right tail in returns to credit – the largest businesses in areas that randomly received access to microcredit are substantially larger than the largest businesses in areas that did not. Our model sheds light on this heterogeneity to returns as well. Firms at very low and very high business sizes grow faster in the presence of a lender than in his absence (they grow at least at rate γ rather than κ). Whereas firms at intermediate business sizes may not grow at all in the presence of a lender.

In contrast to many other models with credit constrained borrowers, this model offers a novel explanation for the regular finding that demand for microcredit contracts is low. Borrowers in the restrictive region are pushed exactly to their individual rationality constraint – they are indifferent between taking loans and not. While the exact indifference of these borrowers may seem an artifact of the model, the intuition that the lender can push the borrower nearer to her outside option when preventing her from investing in business expansion seems robust. Thus these borrowers may be expected to waver on their decision to accept a loan. In the appendix we discuss an extension to the model in which the lender is incompletely informed about the borrower’s outside option. In equilibrium borrowers in the restrictive region sometimes reject his offer, whereas those in the unrestrictive region never do.

4 Comparative Statics

In this section we discuss how the equilibrium changes with respect to a number of comparative statics. Each of them emphasizes an important “trickle down” nature of our model. Namely, changes to the fundamentals of the contracting environment can have nuanced impacts on equilibrium contracts and welfare that vary depending on the bor-

rower's business size. We close this section with comparison to other theories that leverage similar comparative statics.

4.1 Comparative Statics on the Borrower's Continuation Utility u from Entering the Formal Sector.

Increasing the borrower's continuation value u from entering the formal sector shifts the entire restrictive region leftward. The poverty trap is relaxed for rich borrowers but *tightened* for poor borrowers. The intuition behind this observation relies on the fact that, when his borrower is rich enough to be in the final unrestrictive region, increasing the attractiveness of the formal sector makes restrictive contracts more expensive for the lender because it increases the borrower's desire to expand. So the lender shifts towards unrestrictive contracts, relaxing the poverty trap for rich borrowers.

On the other hand, it is precisely this force that causes the lender to tighten the reins on poorer borrowers, increasing the likelihood he offers them restrictive contracts and trapping them at lower levels of wealth. Holding the lender's strategy fixed, increasing the attractiveness of the formal sector improves the borrower's bargaining power in all states. However, the richer the borrower is, the more her bargaining power improves because of her proximity to the formal sector. Thus the lender shifts towards restrictive contracts for poorer borrowers, to prevent them from reaching higher levels of wealth where they can exercise their additional bargaining power. This is encoded in the propositions below.

Let $\bar{w} \equiv \underset{w}{\operatorname{argmax}} \{w : p_w = 1\}$, and $\underline{w} \equiv \underset{w}{\operatorname{argmin}} \{w : p_w = 1\}$.

Proposition 6. *Increasing the attractiveness of the formal sector relaxes the poverty trap for relatively rich borrowers, but tightens it for poorer borrowers.*

That is, $\frac{dp_w}{du} \leq 0$ for $w \geq \bar{w}$ with strict inequality for $0 < p_w < 1$. $\frac{dp_w}{du} \geq 0$ for $w < \underline{w}$ with strict inequality for $0 < p_w < 1$.

Proof. See appendix. □

The intuition for the above proposition is inextricably linked to the equilibrium effects on welfare, codified in the next proposition.

Proposition 7. *Increasing u weakly decreases the lender's continuation value in all states, and strictly so for $w \leq \bar{w}$. Increasing u strictly increases the borrower's continuation utility in all states $w \geq \underline{w}$, but can decrease it in states $w < \underline{w}$.*

That is, $\frac{dL_w}{du} \leq 0$ for all w with strict inequality for $w \leq \bar{w}$. $\frac{dB_w}{du} > 0$ for $w \geq \underline{w}$. For $\rho > \frac{\kappa\gamma}{\kappa+\gamma}$, if $p_{\underline{w}-1} > 0$ then $\frac{dB_w}{du} < 0$ for states $w < \underline{w}$.

Proof. See appendix. □

Though some of the details are cumbersome, the intuition behind these results is instructive. The comparative static for states $w > \bar{w}$ is most easily understood. Consider the largest state n at which the borrower remains in the informal sector. Increasing u makes it more expensive to offer the borrower a restrictive contract, because she finds investment in fixed capital more valuable. On the other hand, the borrower accepts any unrestrictive contract due to her expansion rent. So the lender finds unrestrictive contracts relatively more attractive and shifts towards them if he previously chose an interior solution.

The borrower's continuation utility increases for two reasons. She benefits from the increased prevalence of unrestrictive contracts, and conditional on entering the formal sector her utility increases. By assumption, the lender at least weakly prefers to offer unrestrictive contracts and since the utility he derives from doing so is unaffected, so is his equilibrium continuation value. This logic extends straightforwardly by backward induction to all states weakly larger than \bar{w} .

The story changes at or prior to \bar{w} . By definition of \bar{w} , the lender offers a restrictive contract with certainty (i.e. $p_{\bar{w}} = 1$), and therefore transfers $\alpha B_{\bar{w}+1}$ utility to the borrower over the lifetime of the relationship. Having already established that the borrower's continuation utility in state $\bar{w} + 1$ increases with u , we can now see that the borrower's utility also increases in state \bar{w} . However, now the increase in her utility results from a direct transfer from the lender, so his continuation utility in state \bar{w} decreases. A similar conclusion is reached for all states $w \in \{\underline{w}, \bar{w}\}$ by backward induction.

Finally, consider state $\underline{w} - 1$, in which, by definition, the lender at least weakly prefers to offer an unrestrictive contract. Further, suppose that the preference is indeed weak, so that $p_{\underline{w}-1} \in (0, 1)$ (i.e. the lender offers a restrictive contract with positive probability). First, note that since the borrower never grows beyond state \underline{w} , increasing u has no effect on social welfare in state $\underline{w} - 1$. However, since the borrower's equilibrium continuation utility in state \underline{w} , $B_{\underline{w}}$ increases, so does her expansion rent in state $\underline{w} - 1$. Recall that her state $\underline{w} - 1$ expansion rent $\left(\beta - \frac{\kappa}{\rho + \delta(p_{\underline{w}-1})} \right) B_{\underline{w}}$ is a fraction of her continuation utility in state \underline{w} . Because the borrower's share of surplus from business expansion increases but the change in social surplus accruing from business expansion does not, the lender shifts

towards restrictive contracts, slowing down the borrower's growth.

Another way to understand this is the increase in the attractiveness of the formal sector trickles down and increases the borrower's bargaining power in state \underline{w} . Markov Perfection prevents her from committing not to exercise this additional bargaining power and, because state $\underline{w} - 1$ borrowers are less affected, they become relatively more attractive and the lender shifts towards offering them restrictive contracts.

How this affects the borrower's state $\underline{w} - 1$ equilibrium continuation utility $B_{\underline{w}-1}$ is in general ambiguous. That $B_{\underline{w}}$ increases is a force towards increasing $B_{\underline{w}-1}$. However, the rate at which she grows to state \underline{w} slows, which is a force towards reducing $B_{\underline{w}-1}$. For sufficiently impatient borrowers the latter force dominates, as impatience amplifies the difference between slow and fast rates of expansion, and $B_{\underline{w}-1}$ decreases in the attractiveness of the formal sector. The lender is made unambiguously worse off from the increase in u , because he weakly prefers unrestrictive contracts and his state \underline{w} continuation utility is decreasing in u . Thus an increase in the attractiveness of the formal sector causes a Pareto disimprovement. Because the borrower cannot commit to forgo her improved bargaining power in state \underline{w} , the lender traps her in state $\underline{w} - 1$ to both of their detriments. And the story continues in much the same way for all states prior to $\underline{w} - 1$.

That increasing u can make the borrower worse off at some business state w is especially striking in light of the following consideration. Fix any (potentially non-equilibrium) lender behavior characterized by $\{p_w\}$, such that in state w the lender offers a restrictive contract with probability p_w and an unrestrictive contract with probability $1 - p_w$. Then increasing u unambiguously makes the borrower strictly better off in all states. The restrictive contract in state w becomes more generous when B_{w+1} improves, and the borrower's utility from receiving an unrestrictive contract in state w improves when B_{w+1} increases. This logic is codified in the following proposition.

Proposition 8. *Fixing the lender's behavior characterized by $\{p_w\}$ defined above, increasing u strictly improves the borrower's continuation utility in all states.*

Proof. See appendix. □

So fixing the lender's behavior, *regardless* of what that behavior is, increasing u unambiguously improves the borrower's continuation utility. It is because of an equilibrium adjustment to the lender's behavior, namely that he shifts towards restrictive contracts, that impatient borrowers are made worse off at all business states $w < \underline{w}$.

4.2 Comparative Statics on the Borrower's Level of Patience

A standard intuition about poverty traps is that they are driven by impatience. However in this model increasing patience has a very similar effect to increasing the attractiveness of the formal sector, and hence can tighten the poverty trap and make the borrower *worse off* at some levels of wealth. Let ρ^B be the borrower's level of patience and ρ^L be the lender's level of patience (and note that decreasing ρ^B is equivalent to increasing patience).

Proposition 9. *Increasing the borrower's patience relaxes the poverty trap for relatively rich borrowers, but may tighten it for poorer borrowers.*

That is, $\frac{dp_w}{d\rho^B} \geq 0$ for $w > \bar{w}$ with strict inequality for $p_w > 0$. For $w < \underline{w}$, the sign of $\frac{dp_w}{d\rho^B}$ is ambiguous.

Proof. See appendix. □

For rich borrowers above the highest pure restrictive state ($w > \bar{w}$), the comparative static on ρ^B works in exactly the same way as the comparative static on u . Increasing the borrower's patience increases how much she values business expansion. This causes her to be more demanding of restrictive contracts, but leaves the lender's payoff from offering unrestrictive contracts unchanged. Thus, in all such states the lender shifts towards unrestrictive contracts, increasing the rate that these rich borrowers reach the formal sector.

For borrowers in pure restrictive states ($w \in \{\underline{w}, \dots, \bar{w}\}$), the comparative static on the borrower's patience again works as it did for changes in the attractiveness of the formal sector. The borrower's continuation utility in state $w + 1$, B_{w+1} , increases so the amount of consumption she demands in return for contractual restrictions increases. This increases her welfare at the direct expense of the lender's.

Finally, consider state $\underline{w} - 1$. Recall the borrower's expansion rent in this state is $\left(\beta - \frac{\kappa}{\rho^{B+\delta}(p_{\underline{w}-1})} \right) B_{\underline{w}}$. That her utility in state \underline{w} , $B_{\underline{w}}$, increases is a force towards increasing her expansion rent. However, as she becomes more patient, the difference she perceives between slow and fast rates of expansion is muted. That is $\frac{d \left(\beta - \frac{\kappa}{\rho^{B+\delta}(p_{\underline{w}-1})} \right)}{d\rho^B} > 0$, which is a force towards decreasing the expansion rent. Which of these two forces dominates is in general ambiguous, but we show in the appendix that these forces can resolve

in favor of increasing the expansion rent. Thus, in contrast to standard models of poverty traps, increasing the borrower's patience can make this poverty trap worse.

4.3 Comparison to **Petersen and Rajan (1995)** and to **Jensen and Miller (2015)**

There are a number of principal agent models that feature a comparative static similar to ours. Two such theories are those of **Petersen and Rajan (1995)** and **Jensen and Miller (2015)**. Petersen and Rajan study a credit market, and argue that if competition for rich borrowers becomes more fierce, lenders may invest less in their current borrowers. Interpreting this theory through the lens of our model, this is akin to reducing the attractiveness of the formal sector for the lender without changing the attractiveness of the formal sector for the borrower. Doing so reduces the lender's payoff from unrestrictive contracts and weakly increases the equilibrium probability of restrictive contracts in all states, making the borrower weakly worse off.

Jensen and Miller (2015) study Indian agricultural households in which parents decide a level of education for their children, and then children decide whether to stay at home and work on the farm or migrate to the city, leaving their parents behind. While education has positive returns in both locations, it has higher returns in the city. They show, both theoretically and empirically, that reducing the cost of migrating to the city causes parents to reduce educational investment in their children. Interpreting their insight in the language of this paper, theirs is an exercise in reducing the wealth level required to enter the formal sector in a model where the lender doesn't need to respect the borrower's individual rationality constraint. This is eminently sensible for a parent who chooses a child's level of education, but may be less so for a lender offering a borrower a loan. Like Petersen and Rajan, the result of their comparative static is to make the borrower weakly worse off in all states.

In contrast to both theories, the welfare of rich borrowers in our model is unambiguously improved by improvements in the formal sector. The sources of improvement are twofold: borrowers who reach the formal sector receive higher utility, and borrowers enjoy more frequent unrestrictive contracts because restrictive contracts become relatively more expensive for the lender. This improvement in the rich borrower's welfare (and her inability to commit not to exercise her improved bargaining power) is absent in both theories highlighted above, yet is critical to our result that poor borrowers can be made

worse off. The lender ameliorates this improvement in bargaining power by transitioning from away from unrestrictive contracts, harming the borrower's equilibrium welfare.

5 Extensions and Robustness

In this section we argue that the key intuitions highlighted above survive a number of extensions to the model. We begin by completely relaxing parametric Assumptions 4 and 5 and arguing that the qualitative lessons are unchanged.

We then extend the model to study direct competition. We show that if the incumbent lender has a sufficiently large lending advantage, the results are unchanged relative to the monopolist case above. If instead the incumbent has no advantage, the borrower necessarily reaches the formal sector in finite time in equilibrium. We further derive a sufficient condition for a monotone comparative static in the incumbent's lending advantage which provides a testable prediction for the empirical exercise in Section 6.

In the appendix we explore several other extensions. First we allow for the lender to be incompletely informed about the borrower's outside option and show that in equilibrium the borrower sometimes rejects the lender's offer of a restrictive contract, providing an explanation for the low demand of microcredit. We then discuss an extension in which we allow the borrower to flexibly allocate a fraction of her income irrespective of contractual restrictions, and show that the lender may still restrict the rate at which the borrower grows relative to autarky.

5.1 Arbitrary production functions

In this section we relax Assumptions 4 and 5 and discuss how it affects our results. In particular we make no assumptions about q_w, E_w, T_w, ϕ_w or h_w other than that $h_w > 0$ for all w and Assumption 3 above, which guarantees that in autarky the borrower reaches the formal sector in finite time.

5.1.1 Structure of the Equilibrium

First, recall that Proposition 2 which states that the equilibrium is unique was shown without Assumptions 4 and 5 and thus continues to hold. In this section we discuss

the structure of the unique equilibrium. A typical equilibrium is depicted below, with each circle representing a state and shaded circles representing states in which restrictive contracts are offered.



Even though in general we cannot say anything about the organization of restrictive and unrestrictive states, we argue that many of the empirical facts discussed in Section 4 can still be understood through the equilibrium above. In fact, with the exception of heterogeneity in returns to credit, our explanation of the facts in that discussion only depended on the potential for each type of contract to coexist in a single equilibrium. As such we focus our attention for the remainder of this discussion on the prediction that wealthy borrowers will receive unrestrictive contracts and thus will enjoy high returns to credit.

To do so, we first outline how to transfer the insights in the above model to one with a countably infinite number of states. Given that our results do not depend on the number of states n , or the cost of investment ϕ_w , this is a straightforward task. We define a sequence of games satisfying the above assumptions, each with successively more states.

Let Γ^1 be an arbitrary game satisfying the assumptions, with n business states.

For $m > 1$, let Γ^m be constructed in the following way:

- Γ^m has $2^{m-1}n$ business states, and let $q_w^m, E_w^m, T_w^m, h_w^m$, and ϕ_w^m be the corresponding parameters for game Γ^m .

- If w is even, set $q_w^m = q_{w/2}^{m-1}, E_w^m = E_{w/2}^{m-1}, T_w^m = T_{w/2}^{m-1}$, and $h_w^m = h_{w/2}^{m-1}$

- If w is odd, set

$$q_w^m \in [\min \{q_{w-1}^m, q_{w+1}^m\}, \max \{q_{w-1}^m, q_{w+1}^m\}]$$

$$E_w^m \in [\min \{E_{w-1}^m, E_{w+1}^m\}, \max \{E_{w-1}^m, E_{w+1}^m\}]$$

$$T_w^m \in [\min \{T_{w-1}^m, T_{w+1}^m\}, \max \{T_{w-1}^m, T_{w+1}^m\}]$$

and

$$h_w^m \in [\min \{h_{w-1}^m, h_{w+1}^m\}, \max \{h_{w-1}^m, h_{w+1}^m\}]$$

- If w is even, set $\phi_w^m = \frac{\phi_{w/2}^{m-1}}{2}$ and if w is odd, set $\phi_w^m = \frac{\phi_{(w-1)/2}^{m-1}}{2}$

Thus Γ^m has twice as many states at Γ^{m-1} , and even states in Γ^m correspond to states in Γ^{m-1} . The parameters in odd states take values intermediate to those in the surrounding states. Because the cost of investment in Γ^m is only half that in Γ^{m-1} , a borrower investing in fixed capital at the same rate in either game would reach the formal sector in the same expected time. One way to understand Γ^m relative to Γ^{m-1} is that the borrower and lender appreciate more nuanced differences in the borrower's business size. Holding investment rate fixed, it takes the same amount of time to get from w to $w + 2$ in Γ^{m+1} as it does to get from $\frac{w}{2}$ to $\frac{w}{2} + 1$ in Γ^m , but along the way in Γ^m the borrower and lender realize an intermediate production function change. For $m' > m$, we say $\Gamma^{m'}$ is *descended from* Γ^m if there is a sequence of games $\Gamma^m, \dots, \Gamma^{m'}$ that can be derived in this manner. We have the following result.

Proposition 10. *For any Γ^m , there is an \bar{m} such that for all $m' > \bar{m}$, the equilibrium in any $\Gamma^{m'}$ descended from Γ^m features a \tilde{w} such that for $w \geq \tilde{w}$ the borrower reaches the formal sector in finite time starting from state w if it is socially efficient to do so.*

Proof. See appendix. □

The above result says that for any game with sufficiently fine discrimination between states, all sufficiently wealthy borrowers receive unrestrictive contracts in equilibrium, and thus realize high returns to credit. The intuition is simple. Because entering the formal sector is efficient, the lender is unable to offer a sufficiently wealthy borrower (one who is sufficiently near to the formal sector) a restrictive contract she will accept. As the borrower and lender become arbitrarily discerning of different states, there will eventually be business states where the borrower is indeed sufficiently wealthy.

5.1.2 Comparative statics

As before, we can be fairly precise in describing how the equilibrium changes with respect to various fundamentals of the game. In this section we focus on the comparative static with respect to u .

Note that without loss of generality we can identify m disjoint, contiguous sets of states $\{\underline{w}_1, \dots, \bar{w}_1\}, \dots, \{\underline{w}_m, \dots, \bar{w}_m\}$ such that $\bar{w}_m = \max\{w : p_w = 1\}$, $\underline{w}_m = \max\{w : p_w = 1, p_{w-1} < 1\}$, and in general for $k \geq 1$, $\bar{w}_k = \max\{w < \underline{w}_{k+1} : p_w = 1\}$, $\underline{w}_k = \max\{w \leq \bar{w}_k : p_w = 1, p_{w-1} < 1\}$. An arbitrary set $\{\underline{w}_k, \dots, \bar{w}_k\}$ is a contiguous set of states where restrictive contracts are offered with probability 1, and each pure restrictive state is contained in one of these sets.

We consider an impatient borrower and establish the following result.

Proposition 11. *For impatient borrowers, the regions of contiguous restrictive states merge together as the formal sector becomes more attractive.*

That is, for $\rho > \max_w \frac{\kappa_w \gamma_w}{\kappa_w + \gamma_w}$, $\frac{dp_w}{du} < 0$ for $w \in \{\bar{w}_m + 1, n\}$, $\frac{dp_w}{du} > 0$ for $w \in \{\bar{w}_{m-1} + 1, \underline{w}_m - 1\}$, $\frac{dp_w}{du} < 0$ for $w \in \{\bar{w}_{m-2} + 1, \underline{w}_{m-1} - 1\}$ and so on.

Proof. See appendix. □

Proposition 11 states that the highest region of pure restrictive states moves leftward, the second highest region moves rightward and so on. This is depicted in the following figure.



The intuition is as follows. For $(\underline{w}_m \bar{w}_m)$, the analysis exactly follows that of Section 4.1, and hence it shifts leftward as u increases. But recall that for the impatient borrowers to the left of a restrictive state, the leftward shift *lowers* their utility. This is akin to lowering the utility of entering the formal sector, and hence for the next set of restrictive states $(\underline{w}_{m-1} \bar{w}_{m-1})$ the analysis reverses and \bar{w}_{m-1} and \underline{w}_{m-1} shift rightward. The rest follows by backward induction.

5.2 Direct Competition

Throughout the above analysis we have assumed the lender is a monopolist. In this section we introduce the possibility of a second lender who can make offers to the borrower. We label one lender the incumbent and one lender the entrant. The timing and technologies are the same as above, however now each period both lenders offer the borrower a

contract. If the borrower accepts the incumbent's contract everything proceeds as above. If the borrower accepts the entrant's contract, everything proceeds as above except that the borrower incurs a non-pecuniary penalty of $\psi dt > 0$. This penalty can be understood as a lending disadvantage the entrant suffers relative to the incumbent, perhaps because the incumbent is better equipped to screen or monitor its borrowers and thus borrowers interacting with the entrant undergo more costly screening processes.

We are now ready to state our first result.

Proposition 12. *There exists a unique equilibrium in which the borrower accepts the incumbent's loan offer in all periods.*

Proof. See Appendix. □

The proof of the above result proceeds much in the same way as the proof of Proposition 2, except that rather than the borrower's outside option being that she can flexibly invest her endowment E_w , her outside option may now be to receive an attractive loan from the entrant and incur the non-pecuniary cost of ψdt . Specifically, because the entrant never expects the borrower to accept his loan in the future, he is willing to offer the borrower any loan she would accept in the current period. Thus, in effect the borrower's outside option is whichever she prefers between flexibly allocating E_w , and flexibly allocating $E_w + T_w$ but incurring the non-pecuniary cost ψdt .

Our next proposition aims to demonstrate that the intuitions derived under the monopolist case survive to the case with two lenders so long as ψ is large enough. In contrast, as ψ becomes sufficiently small, the equilibrium approaches the first best level of business expansion.

Proposition 13. *There exists a $\bar{\psi} > 0$ such that for $\psi > \bar{\psi}$, the equilibrium probability p_w that the incumbent lender offers the borrower a restrictive contract in state w is the same as in the model with one lender.*

So long as it is efficient to invest in business growth, there exists a $\underline{\psi} > 0$ such that for $\psi < \underline{\psi}$, the equilibrium probability p_w that the incumbent lender offers the borrower a restrictive contract in state w is 0. And as $\psi \rightarrow 0$, the equilibrium repayment rate R charged by the incumbent lender converges to T_w in each state w .

For a sufficiently strong lending advantage, the incumbent lender behaves as a monopolist. While this is intuitive, it may nevertheless be important to highlight that *de facto* monopoly power need not arise from being the only lender available. It may also arise

from having low screening and monitoring costs relative to one's competitors. Proposition 13 also asserts that as the incumbent's lending advantage vanishes, he offers increasingly generous unrestrictive contracts and the borrower's rate of business expansion approaches first best. This serves to highlight that in the limit of perfect competition, the inefficiencies highlighted in the model vanish.

Finally, we discuss comparative statics with respect to the incumbent's lending advantage. While the model is suitable to study the effect of a localized increase in competition across many scenarios, we highlight only one, which motivates the empirical exercise to follow.

Proposition 14. *If the equilibrium is characterized by a \bar{w} such that $p_w = 1$ for $w \leq \bar{w}$, and $p_w \in (0, 1)$ for $w > \bar{w}$, $\frac{dp_w}{d\psi} \geq 0$ for $w > \bar{w}$.*

Proposition 14 states that if the equilibrium structure is such that there are a set of states in which the incumbent lender offers restrictive contracts with probability 1 and mixes between the two types of contracts with strictly interior probability at all higher states, then the comparative static with respect to the level of competition is unambiguous. As the incumbent's lending advantage diminishes, the probability he offers restrictive contracts weakly declines in all states, and strictly so in states in which he had previously been offering a restrictive contract with positive probability. As competition increases, the incumbent is forced to transfer more surplus to his borrower, and the most efficient way to do so is to relax the contractual restrictions he imposes. In section 6.2, we provide empirical evidence for this testable prediction.¹³

6 Novel Empirical Support

In this section we present two novel empirical observations in support of our theory. The first observation supports one of our key modeling assumptions; relatively richer borrowers are more likely to cease interaction with an MFI than are their poorer counterparts. Second we provide empirical evidence for a comparative static of our model. Leveraging exogenous variation in the level of competition facing money lenders in Thai villages, we show that competition causes a relaxation of the contractual restrictions they impose. We further argue that this is the unambiguous prediction of the model given the equilibrium

¹³The comparative static in the empirical exercise may be more closely modeled by increasing the amount of money the entrant has to lend to the borrower. This would have exactly the same impact on the lending environment as reducing ψ .

contract structure we observe in the data. As discussed above, this comparative static prediction arises from the fact that money lenders in competitive markets face pressure to transfer additional surplus to their borrowers, and the primary method by which they do so is to ease contractual restrictions.

6.1 Rich Borrowers Cease Interaction With an MFI

The data from this exercise are from [Field et al. \(2013\)](#). Their paper reports on an experiment conducted in partnership with Village Financial Services, a large Indian MFI. As discussed in the introduction, the authors randomly relaxed the contractual requirement that borrowers begin repayment immediately after loan disbursement; a random subset of borrowers received a two month grace period during which they did not need to meet any repayment obligation, and after which repayment took place in standard installments. The authors found that three years after the initial loan disbursement, borrowers in the treatment group reported weekly profits between Rs. 450 and Rs. 900 more than those in the control group. However they also report that borrowers in the treatment group default significantly more on average. While the probability that any amount of money remains in default at the end of the loan cycle more than quadruples (from about 2% in the control group to 9% in the treatment group), the increase in average amount in default is much more modest. On average borrowers in the treatment group default on an additional Rs. 150. Put another way, for a one time additional cost of Rs. 150, the grace period increased *weekly* business profits by Rs. 450 - Rs. 900. Why then, have MFIs (including Village Financial Services) maintained their strict repayment schedules that begin immediately after loan disbursement?

We propose that the answer lies in the pattern of default as a function of business size. The red line in Figure 1 below plots the expected amount in default a year after loan origination as a function of business profits three years after loan origination.¹⁴ As may be expected, there is a clear decreasing relationship between a borrower's level of wealth and the amount of money she is expected to have in default.

However the blue line in Figure 1 plots the *likelihood* a borrower defaulted on any amount of money as a function of business profits three years after loan origination. In this case there is U-shaped relationship between profits and default; those with relatively low and relatively high business profits are the most likely to default. Table 1 presents

¹⁴This is the publicly available measure of profits.

this pattern in regression form. Specifically we regress

$$Default_i = \alpha + \beta_1 \ln prof_i + \beta_2 \ln profsq_i + \gamma X_i + \epsilon_i$$

where $Default_i$ is an indicator taking a value of 1 if borrower i has not completed repayment of her loan a year after origination (and substantially after the final tranche was due), $\ln prof$ is the log of borrower i 's business profits three years after loan origination, $\ln profsq$ is the square of her profits three years after origination and X_i is the vector of controls used in [Field et al. \(2013\)](#). As can be seen, β_1 is negative, β_2 is positive, and both are statistically significant once the vector of controls is included.

Taken together these findings paint a clear picture. Borrowers with large businesses are substantially less likely to finish repaying their loans but the amount of money they take in default is low. One interpretation of these findings is that these richer customers no longer found it worthwhile to attend repayment meetings, either because they found the biweekly meetings too time consuming or because they no longer saw the value in maintaining the option for further microloans. While the borrowers who do not completely repay their loans are only a subset of those who do not continue to borrow, the pattern is highly suggestive. The primary loss to the MFI may come from losing these customers rather than the default itself, and presents the possibility that the MFI may want to restrict the business growth of their customers.

6.2 Comparative Statics on Likelihood of Restrictive Contracts

Among the most distinctive features of our model is the comparative static of contractual restrictions with respect to the level of competition. Specifically, [Proposition 14](#) makes an unambiguous, testable prediction and in this section we present evidence in its support in the context of informal borrowing in Thai villages.

To do so we leverage variation induced by the 2002 Million Baht Program, which endowed each village with a village fund with one million baht to lend. Importantly, the amount of money endowed to each village fund is invariant to the village size - thus smaller villages received more credit per capita than larger villages. [Kaboski and Townsend \(2012\)](#) leverage this variation to estimate the return to credit offered by village funds. In contrast, we focus on informal money lenders and their borrowers, and study how this change in competition affected the contracts they offered. As we show below, there is a steep decline in the likelihood a villager borrows from a money lender as a

function of the villager's wealth, and thus we think of informal money lenders and their borrowers as an empirical analogue of the informal sector in our model.

In what follows, we present an empirical definition of contractual restrictions, and show that borrowers subjected to these restrictions indeed anticipate lower income the following year. Further, we show there is a negative correlation between the imposition of contractual restrictions and the interest rate, suggesting that money lenders may need to compensate borrowers whose investments they restrict. We next verify that the condition assumed by Proposition 14 is satisfied, and that as it predicts, money lenders in villages with larger increases in competition impose fewer contractual restrictions. Finally we invoke a variety of robustness and placebo tests to argue that this comparative static is not likely to be the result of other theories.

Data

The data used for this exercise were gathered as part of the ongoing Townsend Thai Project to track the financial lives of members of 64 Thai villages across 4 provinces: Chachoengsao, Lopburi, Sisaket, and Buriram. Specifically we utilize data from the household survey that tracked a representative sample of 15 households in each village on an annual basis. The dataset is extremely detailed; of particular relevance to this exercise it contains information about all loans received by study households (both formal and informal loans) including information on loan size, interest rates, collateral, consequence of default, and loan originator. The data also contain demographic information such as village size and composition, occupation, businesses operated, and a detailed breakdown of household income, and expectation about future income. For most of our regressions we utilize the unbalanced panel of loan level observations from survey rounds collected between 1997 (at the inception of the Townsend Thai Project) to 2007 (6 years after the Million Baht program was initiated).

Validity of the Natural Experiment

As described in [Kaboski and Townsend \(2012\)](#), two important elements make the Million Baht Program suitable for research. First, it was proposed during Prime Minister Thaksin Shinawatra's election campaign following the dissolution of the Thai Parliament in November of 2000, and the program was then rapidly implemented between 2001 and

2002. So households were unlikely to anticipate the program in earlier years. Second the amount of credit endowed to each village was constant across villages of varying size, inducing variation in the per capita level of credit injected by the program. This motivates a continuous difference in differences strategy examining outcomes before and after the implementation of the program and across villages of varying sizes. That is, our principle regressions of interest will be of the form

$$outcome_{i,t} = \alpha + \beta_1 invsize_v + \beta_2 post_{i,t} + \beta_3 invsize_v * post_{i,t} + \gamma X_{i,t} + \theta_t + \theta_h + \epsilon_{i,t}$$

where $invsize_v$ is the inverse size of the village, $post_{i,t}$ is an indicator variable that is 1 if year t is after 2001, $X_{i,t}$ is a vector of loan and household characteristics for the borrower of loan i in year t , θ_t is a year fixed effect, and θ_h is a household fixed effect. All standard errors are clustered at the village level.

The validity of this regression rests on the orthogonality assumption

$$post_{i,t}, invsize_v \perp \epsilon_{i,t} | X_{i,t}, \theta_t, \theta_h \quad (3)$$

Under the above assumption, β_3 captures the effect of treating small villages with a higher per capita credit shock than that of larger villages. To lend credibility to our identification assumption, we verify parallel trends in our pre-periods in the regressions to follow.

Further, in Table 2 we regress a number of village characteristics on inverse village size. We examine characteristics related to the village head, demographics, financial penetration, technology adoption, distance to a main road, and occupational distribution. The variables that have a significant relationship with inverse size are the number of agricultural cooperatives with a presence in the village, the education of the village head, and whether the village has common land that is shared among the villagers. In the regressions to follow we control for each of these characteristics as well as their interaction with $post_{i,t}$. Finally, as in [Kaboski and Townsend \(2012\)](#), our main analysis restricts attention to villages with between 50 and 250 households, but we show it is robust to including all villages.

Money Lenders are the Informal Sector

We focus on borrowers in these villages who receive loans from money lenders. The two defining features of informal lenders in the model are that richer villagers are less likely to

borrow from them, and that they have some capacity to influence the project selection of their borrowers. In this section we argue that rich villagers stop borrowing from money lenders. Figure 2 plots the likelihood a household borrows from a money lender as a function of its wealth. As can be seen, the poorest households are about three times as likely to borrow from money lenders as are the richest households, creating the potential for money lenders to desire limiting the business growth of their clients.

Contractual Restrictions

Next we set forth two empirical definitions of contractual restrictiveness, one theoretically driven and one empirically driven. In line with our motivating examples our theoretical and primary measure of contractual restrictiveness is a binary indicator which takes a value of 1 if the borrower must forfeit productive capital for the duration of the loan or if the borrower must use a guarantor. In particular, in about 21% of cases in which land is used to collateralize a loan, the money lender is also given rights of use during the loan's tenure. Because 60% of employed respondents report that farming is their primary means of income generation, and many more report it as a secondary means of income generation, we view this contractual feature as a restriction on the borrower's ability to put a productive asset to use. Additionally, we code contracts which require a guarantor as restrictive, as guarantors may pressure borrowers to eschew profitable yet risky investments in favor of safer and less profitable ones to ensure repayment (see e.g. [Banerjee et al. \(1994\)](#) and [Fischer \(2013\)](#)).

All other forms of collateral, and the absence of a collateral requirement, are coded as unrestrictive. Other forms of collateral include the deed to the borrower's land (but allow the borrower to retain the rights of use for the duration of the loan), jewelry, the deed to their house, and the right to proceeds from future crop production. Importantly, none of these forms of collateral involve the transfer of productive capital from the borrower to the lender.

The defining feature of restrictive contracts in our model is that they restrict growth of the borrower's business. Table 3 offers suggestive evidence that this may be the case in practice. Columns 1 and 2 present a regression of the borrower's expected income in the following year on whether her money lender asks for a restrictive form of collateral. That

is, we present regressions of the following form

$$\log(y)_{i,t+1} = \alpha + \beta \text{restrictive}_{i,t} + X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}$$

where $\log(y)_{i,t+1}$ is the log of the borrower's year t expectation of her year $t + 1$ income, and all other variables are as defined above. In addition to household and year fixed effects, the controls $X_{i,t}$ include the borrower's income and her loan size. As can be seen from the table, there is a negative and significant correlation between the borrower's expected income and the imposition of restrictive contractual features. In the next two columns we present a regression of the form

$$\log(y)_{i,t+1} = \alpha + \sum_j \beta_j \text{collateral}_{j,i,t} + X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}$$

where $\text{collateral}_{j,i,t}$ is 1 if the borrower of loan i in year t was asked for a collateral of type collateral_j . Several forms of collateral have large negative relationships with the borrower's expected income. Consistent with the theoretically motivated definition of contractual restrictiveness, these are that the borrower forfeits her land, that the lender asks for multiple guarantors, and a catch-all category labeled "other." Collateral in "other" are typically other forms of third party guarantees, such as using a third party's assets to guarantee the loan. This motivates a data driven definition of contractual restrictiveness that takes a value of 1 if the collateral takes any of the preceding three forms and 0 otherwise. In Table 15 we show that our main regressions are robust to using this empirically driven notion of restrictiveness rather than our primary one.

Finally we verify that, as predicted by our theory, restrictive forms of collateral are correlated with a reduction in the interest rate. Specifically we regress

$$\text{MonthlyInterest}_{i,t} = \alpha + \beta \text{restrictive}_{i,t} + X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}$$

where $\text{MonthlyInterest}_{i,t}$ is the monthly interest rate that the borrower is charged for the loan, and all other variables are as defined above. We present this regression in Table 4.

While one could imagine stories for the negative relationship between contractual restrictions and future expected income in which the arrow of causality points in the opposite direction (e.g. borrowers with low expectations about their future income are more tolerant of restrictive forms of collateral), it is a little tougher to do the same for the negative relationship between contractual restrictions and the interest rate. In particular, if

contractual restrictions were a proxy for low quality borrowers, we would expect a positive relationship between the two variables. Thus, while acknowledging that the above relationships are merely correlational, we take them to be reassuringly consistent with the predictions of the model.

The Theory Predicts Increased Competition Corresponds to a Reduction in Restrictive Contracts

In general, the theory's prediction about how money lenders should react to increased competition is ambiguous. On the one hand, competition among lenders increases the borrower's bargaining power, which is a force increasing the frequency of unrestrictive contracts. On the other hand, if the lender anticipates a sufficiently large increase competition for relatively rich borrowers, the lender may increase the frequency of restrictive contracts for poorer borrowers. Which of these two forces dominates is in general sensitive to parametric assumptions.

Taking the theory literally, however, Proposition 14 asserts that if in equilibrium the lender offers all borrowers (above a certain wealth level) a restrictive contract with strictly interior probability, the comparative static is unambiguous. In such a case we should expect that the first force always dominates, and that after the program was introduced, villages with a larger increase in competition (i.e. smaller villages) should have a correspondingly larger decrease in the frequency of restrictive contracts. Figure 3 plots the likelihood a borrower is asked for a restrictive form of collateral as a function of his income and verifies that the condition of Proposition 14 is satisfied. Thus the theory makes an unambiguous prediction.¹⁵

Comparative Static on the Level of Competition

We now turn to our main regression:

$$restrictive_{i,t} = \alpha + \beta_1 invsize_v + \beta_2 post_{i,t} + \beta_3 invsize_v * post_{i,t} + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}$$

¹⁵We do note that Proposition 14 requires the lender is following a mixed strategy between offering restrictive and unrestrictive contracts. While it is difficult to verify whether money lenders are indifferent between the two contracts in practice, it is reassuring that the true distribution of restrictive contracts is far from either corner case for all sufficiently wealthy borrowers.

where all variables are as defined above. In addition to household and year fixed effects, the controls include the three village characteristics correlated with inverse size and their interactions with $post_{i,t}$, the borrower's income and loan size, and whether the household has a loan from the village fund. In our primary regressions we use data from 1997 to 2007, but also show in Table 14 that our results are robust to including only the three years before and after the program took effect. Note that our theoretical prediction is that $\beta_3 < 0$, as higher inverse village size corresponds to a larger credit shock, which should correspond to a greater relaxation in contractual restrictiveness.

Our main results are presented in Table 5. As can be seen β_3 has the desired sign and large magnitude. For instance, β_3 is approximately -10.8 in the OLS specification in column 1. The standard deviation of inverse village size is $.006$, so a one standard deviation increase in inverse village size corresponds to an approximately 6.5 percentage point decrease in the probability a restrictive form of collateral is demanded (off of a base of about 20 percentage points).

In Table 6, we verify parallel trends prior to 2002. Specifically, we regress

$$restrictive_{i,t} = \alpha + \beta_1 invsize_v + \beta_2 wave_t + \beta_3 invsize_v * wave_t + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}$$

and verify that we cannot reject $\beta_3 = 0$. We also regress

$$restrictive_{i,t} = \alpha + \beta_1 invsize_v + \sum_{t=1997}^{2001} \beta_t wave_t + \sum_{t=1997}^{2001} \tilde{\beta}_t invsize_v * wave_t + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}$$

and verify we cannot reject that $\tilde{\beta}_t$ are jointly 0.

Finally we regress

$$restrictive_{i,t} = \alpha + \beta_1 invsize_v + \sum_{t=1997}^{2007} \beta_t wave_t + \sum_{t=1997}^{2007} \tilde{\beta}_t invsize_v * wave_t + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}$$

and plot the coefficients $\tilde{\beta}_t$ in Figure 4. As can be seen, there does not seem to be a strong trend before or after 2002, but with the exception of 2005, all coefficients post 2002 are below 5 and all coefficients pre 2002 are above 5.

The Comparative Static is Not Driven by Borrower Selection

One threat to identification would be that the villagers who borrow from money lenders are selected differently in high and low competition environments. While our main regression would still capture the causal effect of competition on the incidence of restrictive contracts, it would be capturing an effect on the composition of borrowers rather than an effect on borrowers' bargaining power. Because we use an unbalanced panel (we use loan level observations), our inclusion of household fixed effects does not eliminate the threat of selection at the household level.

Nevertheless we argue selection is unlikely to be driving our results. First we examine whether fewer households borrow from money lenders in high competition environments. Specifically we regress

$$borrower_{i,t} = \alpha + \beta_1 invsize_v + \beta_2 post_{i,t} + \beta_3 invsize_v * post_{i,t} + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}$$

where our unit of observation i, t is a household \times wave, our outcome $borrower_{i,t}$ is an indicator taking a value of 1 if household i borrows from a money lender in wave t and 0 otherwise, and everything else is defined as above. These regressions are presented in Table 7. The estimates fluctuate in sign, are always small in economic terms, and are all far from statistically significant. For example, recalling that standard deviation on inverse village size is .006, the estimates in column 2 imply that a village with one standard deviation smaller size saw a .2% larger decline in the frequency with which villagers borrow from money lenders after the Million Baht Program.

Next we examine whether households borrow less money from money lenders in villages with higher competition. We regress

$$\log(amount)_{i,t} = \alpha + \beta_1 invsize_v + \beta_2 post_{i,t} + \beta_3 invsize_v * post_{i,t} + \gamma X_{i,t} + \theta_h + \theta_t + \epsilon_{i,t}$$

where $\log(amount)_{i,t}$ is the log of the total amount borrowed from any money lender by household i in wave t and everything else is as defined above. The results are presented in Table 8. Again, the estimates fluctuate in sign, are always economically small, and far from statistically significant.

Finally Table 9 presents the results of a Heckman selection model of our primary specification. The selection equation assumes that the error terms are jointly normal and conditions on a number of village and borrower characteristics including $post_{i,t}$, $invsize_v$ and

$post_{i,t} \times invsize_v$. Two observations are of note. First, the coefficient on interaction term $invsize_v * post_{i,t}$ in the selection equation is far from significant. Second, the estimates in Table 9 are strikingly similar to the corresponding estimates in Table 5. Taken together these offer further reassurance that selection is not driving our results.

Ruling Out Alternative Theories

We now present a number of tests to rule out alternative theories. First we aim to rule out a general trend away from restrictive forms of collateral. Specifically we rerun our main regression, but rather than focusing on the population of borrowers interacting with informal money lenders, we focus on the borrowers who take loans from their neighbors. These lenders should have no desire to keep their borrowers in poverty, and thus we would expect no relationship between the variation induced by the Million Baht Program and the likelihood that these lenders ask for restrictive forms of collateral. The regressions are presented in Table 10. The estimate on the interaction term fluctuates in sign and magnitude and is never statistically significant.

Next, we aim to separate our theory from any other one that makes no distinction between different forms of collateral. Again focusing on the population of borrowers who take loans from money lenders we regress

$$unrestrictive_{i,t} = \alpha + \beta_1 invsize_v + \beta_2 post_{i,t} + \beta_3 invsize_v * post_{i,t} + \gamma X_{i,t} + \theta_v + \theta_t + \epsilon_{i,t}$$

where $unrestrictive_{i,t}$ is an indicator taking a value of 1 if a money lender asks for any form of collateral that was not coded as restrictive and 0 otherwise. We present this regression in Table 11 from which it is apparent that there is a positive relationship between the interaction term and the likelihood a money lender asks for unrestrictive collateral. This is what one would expect if money lenders substitute away from restrictive forms of collateral toward unrestrictive forms of collateral in high competition environments. Thus any theory consistent with these results must draw a distinction between restrictive and unrestrictive forms of collateral, and must also predict the reduction in restrictive forms of collateral arising from increased competition from village funds.

Finally, we aim to rule out the possibility that the increased competition from village funds caused money lenders to screen their borrowers less carefully, and that this explains the shift away from restrictive forms of collateral in high competition environ-

ments. Though we do not observe screening efforts directly, we provide indirect evidence that this is not the case by examining how the interest rates charged by money lenders are influenced by competition. Specifically we regress

$$\text{MonthlyInterest}_{i,t} = \alpha + \beta_1 \text{invsize}_v + \beta_2 \text{post}_{i,t} + \beta_3 \text{invsize}_v * \text{post}_{i,t} + \gamma X_{i,t} + \theta_v + \theta_t + \epsilon_{i,t}$$

where all variables are as defined above. The results are in presented in Table 12. Though the estimates are noisy, and diminish as we include finer levels of fixed effects, it appears that if anything money lenders charge *higher* interest rates in higher competition environments.¹⁶ Thus, a theory based on screening effort should predict that money lenders increased their screening efforts in high competition environments to compensate for the more adversely selected population of borrowers, and would thus predict a corresponding increase in restrictive contracts, counter to what we observe.

7 Discussion

In this paper we explored a novel theory of informal lending, in which relatively rich borrowers cease lending from informal financiers and enter the formal sector. Each period the borrower and her informal lender bargain not only over the interest rate, but also over a contractual restriction that governs the borrower’s ability to invest in business expansion. In contrast with earlier theories (e.g. [Braverman and Srinivasan \(1981\)](#) and [Braverman and Stiglitz \(1982\)](#)) we show that the borrower may indeed be caught in a debt trap when she cannot commit to share the benefits of entering the formal sector.

Our theory therefore reconciles the seemingly disparate facts that small scale entrepreneurs enjoy very high return to capital yet are unable to leverage microcredit and other forms of informal finance to realize those high returns. Moreover it offers an explanation for the robust findings that relatively wealthier business owners do enjoy high returns from microcredit and that on average the demand for microcredit is low.

The theory also offers nuanced predictions on comparative statics of the lending environment. Increasing the attractiveness of the formal sector improves the bargaining power of rich borrowers and hence increases their welfare and relaxes the poverty trap.

¹⁶While it may at first seem unintuitive that an increase in competition is accompanied by an increase in the interest rate charged by money lenders, it is entirely consistent with our theory. We have documented that as competition rises money lenders offer more unrestrictive contracts, which tend to carry higher interest rates than restrictive ones.

However the same improvement may harm the welfare of poorer borrowers; anticipating that rich borrowers have improved bargaining power, the lender tightens contractual restrictions on poor borrowers to prevent them from reaching higher levels of wealth and exploiting their improved position. Similarly, and counter to standard intuitions, increasing the borrower's patience (and hence her value for business expansion) can make relatively poor borrowers worse off, and tighten the poverty trap. We finally showed that, while the model primarily studies a monopolist lender, similar effects can operate in contexts with imperfect competition, and derived intuitive comparative statics on the level of competition.

Then, studying money lenders in rural Thailand, we offered empirical evidence for the comparative static prediction on competition. Utilizing the Townsend Thai data and the plausibly exogenous shock to competition induced by the Million Baht Program, we found that Thai money lenders in environments with high competition reduced the frequency with which they imposed contractual restrictions more than money lenders in low competition environments. We argue that, because the same effects cannot be found for loans given by neighbors or for other forms of collateral, other theories are unlikely to explain this robust pattern.

In addition to the theories cited in the introduction, it is worth contrasting our theory with two other theories development economists seem to carry with them in the field. The first of these theories might sensibly be labeled as "blaming the borrower." These are theories that allude to the argument that many borrowers are not natural entrepreneurs, and are primarily self employed for lack of being able to find steady wage work (see e.g. [Schoar \(2010\)](#)). While these theories enjoy some empirical support, they are at best a partial explanation of the problem as they are inconsistent with the large impacts of cash grants, cited in our introduction.

Second are the theories that assign blame to the lender for not having figured out the right lending contract. These theories are implicit in each of the experiments that evaluate local modifications to standard contracts (see e.g. [Gine and Karlan \(2014\)](#), [Attanasio et al. \(2015\)](#), and [Carpena et al. \(2013\)](#) on joint liability, [Field et al. \(2013\)](#) on repayment flexibility, and [Feigenberg, Field, and Pande \(2013\)](#) on meeting frequency). While many of these papers contribute substantially to our understanding of the ways in which microfinance operates, none have so far generated a lasting impact on the models that MFIs employ.

In contrast, ours is a theory that assumes that borrowers have the competence to grow their business, and that lenders are well aware of the constraints imposed on borrowers

by the lending paradigm. Instead we focus on the rents that lenders enjoy from retaining customers, and the fact that customers who become sufficiently wealthy will find alternative forms of finance. Part of the value of this theory therefore may very well be its distance from the main lines of reasoning maintained by empirical researchers.

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8 Appendix

8.1 Additional Extensions to the Model

8.1.1 The Borrower is Privately Informed About Her Outside Option

In this section we explore an extension in which the borrower maintains some private information about her outside option. In particular we augment the model such that the borrower’s autarkic endowment is privately known. If she rejects the lender’s contract she receives an endowment of $E_w + \nu_t$. Let $\nu_t \stackrel{\text{iid}}{\sim} G$ be a random variable privately known to the borrower, and redrawn each period in an iid manner from some distribution G . Further, assume that if the borrower accepts the lender’s contract, her endowment is still

$E_w + T_w$. One way to understand this is that in the event that the borrower rejects the lender's contract, a relative will offer her a gift of size v_t , which she can allocate flexibly between her projects. We make the following additional assumption on the range of G to simplify the discussion.

Assumption 6. *Let G have bounded support with minimum 0 and maximum \bar{v} such that $\bar{v} < \frac{h_w}{q_w}$.*

The above assumption guarantees that the borrower will accept any unrestrictive contract. However, if the lender offers the borrower a restrictive contract, he will now face a standard screening problem. Because he would like to extract the maximum acceptable amount of income, borrowers with unusually good outside options will reject his offer. This is encoded in the following proposition.

Proposition 15. *The borrower never rejects an unrestrictive contract on the equilibrium path. However the borrower may reject restrictive contracts with positive probability.*

Proof. See Appendix. □

This intuitive result offers an explanation for the low takeup of microcredit contracts referenced in the introduction. Lenders who offer restrictive contracts to borrowers aim to extract all of the additional income generated by the loan, but in doing so sometimes are too demanding and therefore fail to attract the borrower. In contrast, lenders who offer unrestrictive contracts necessarily leave the borrower with excess surplus, and therefore demand for these contracts is high.

8.1.2 The Borrower Flexibly Allocates a Fraction of Her Income

In this section we explore an extension to the model in which, even when subjected to contractual restrictions, the borrower maintains flexible control over a fraction of her income. In doing so we aim to show that our main result is qualitatively robust. Rather than finding that the borrower may remain in inefficiently small forever, we now find that having access to a lender may *slow* the borrower's growth relative to her autarkic rate.

Formally, the model is as in Section 2 but after accepting a contract $\langle R, a \rangle$, the borrower is free to invest a fraction $s < 1$ of her endowment flexibly, irrespective of the contractual restriction a the lender imposes. Thus we have weakened the lender's ability to influence the borrower's project choice. We maintain all other assumptions from Sections 2 and 3, and make the following observation.

Proposition 16. For sufficiently small s , the lender may impose the contractual restriction C on the equilibrium path. In such an equilibrium the borrower reaches the formal sector in finite time, but will grow more slowly than she would have in autarky.

Proof. See Appendix. □

8.2 Omitted Proofs

Proof of Proposition 1

In state n the borrower chooses her investment allocation i to maximize

$$B_n^{aut} = \max_{i \leq E_n} q_n (E_n - i) dt + e^{-\rho dt} \left(\left(1 - e^{-\frac{i}{\phi_n} dt}\right) B_{n+1}^{aut} + e^{-\frac{i}{\phi_n} dt} B_n^{aut} \right)$$

At the optimal level of i , the borrower's continuation utility can be rewritten as

$$\begin{aligned} B_n^{aut} &= \frac{q_n (E_n - i) dt + e^{-\rho dt} \left(\left(1 - e^{-\frac{i}{\phi_n} dt}\right) B_{n+1}^{aut} \right)}{1 - e^{-\left(\frac{i}{\phi_n} + \rho\right) dt}} \\ &\rightarrow \frac{q_n (E_n - i)}{\frac{i}{\phi_n} + \rho} + \frac{\frac{i}{\phi_n}}{\frac{i}{\phi_n} + \rho} B_{n+1}^{aut} \end{aligned}$$

Because the problem is stationary, the borrower's maximization problem can equivalently be written as choosing i to maximize her continuation utility above.

We now take the derivative of B_n^{aut} with respect to i .

$$\frac{dB_n^{aut}}{di} = \frac{-q_n \rho - q_n \kappa_n}{\left(\frac{i}{\phi_n} + \rho\right)^2} + \frac{\frac{\rho}{\phi_n}}{\left(\frac{i}{\phi_n} + \rho\right)^2} B_{n+1}^{aut}$$

where recall $\kappa_w \equiv \frac{E_w}{\phi_w}$. The denominator is positive. The numerator is positive iff

$$\begin{aligned} -q_n \rho - q_n \kappa_n + \frac{\rho}{\phi_n} B_{n+1}^{aut} &> 0 \\ &\iff \\ \alpha_n B_{n+1}^{aut} &> \frac{q_n E_n}{\rho} \end{aligned}$$

We conclude that if $\frac{q_n E_n}{\rho} < \alpha_n B_{n+1}^{aut}$ then the borrower's value in state n B_n^{aut} is increasing in i and otherwise it is decreasing. The result for earlier states follows similarly by backward induction. This completes our proof.

Proof of Proposition 2

The existence of an equilibrium follows standard arguments (See [Maskin and Tirole \(2001\)](#)). In this section we prove that generically the equilibrium is unique. We do so by backward induction on the state. In each state we first argue that if there exists an equilibrium in which the borrower invests her autarkic endowment in the working capital project conditional on rejecting the offered contract, then she does so in all equilibria and the lender gives her an unrestrictive contract with lower than necessary repayment rate.

We then argue that in any equilibrium where the borrower invests her autarkic endowment in the fixed capital project conditional on rejecting the offered contract (which, by the above statement, can only exist in the absence of an equilibrium in which the borrower invests her autarkic flow payoff in the working capital project), the lender's behavior is uniquely determined.

We first consider equilibrium behavior in state n .

Lemma 1. *There is no equilibrium in which the borrower weakly prefers to invest her autarkic endowment in the working capital project in state n .*

Proof. Suppose toward contradiction that upon rejecting the contract, the borrower weakly prefers investing her autarkic endowment in the working capital project. Then the borrower would accept any restrictive contract (which necessarily grants her more consumption than her outside option), and in state n the lender would clearly choose to give the borrower a maximally extractive restrictive contract in every period. The borrower's equilibrium continuation value is $B_n = \frac{h_n + q_n E_n}{\rho}$ which by Assumption 3 is less than $\alpha_n \frac{u}{\rho}$. This contradicts that the borrower weakly prefers to consume her autarkic flow payoff. \square

We next establish that conditional on the borrower strictly preferring to invest her autarkic endowment in fixed capital in equilibrium, the lender's behavior is uniquely determined.

Lemma 2. *The probability the lender offers a restrictive contract in state n , p_n is generically uniquely determined across any equilibrium in which the borrower strictly prefers to invest her autarkic endowment in the fixed capital project in state n .*

Proof. Assume the borrower strictly prefers to invest her autarkic endowment in the fixed capital project. Then if the lender offers the borrower any unrestrictive contract she accepts it, and the lender never benefits in state n from offering an excessively generous unrestrictive contract. Further, the lender never offers the borrower an excessively generous restrictive contract. So in equilibrium, the lender either offers the borrower the contract $\langle q_n T_n - h_n, I \rangle$, or the contract $\langle R(p), C \rangle$ for some $R(p)$ that pushes the borrower to her outside option. Now conjecture that in equilibrium the lender offers the borrower a restrictive contract with probability p .

Noting that the borrower receives the maximum of the utility from investing her outside option in fixed capital and from consuming $q_n E_n + h_n$ upon receiving a restrictive contract, we have

$$B_n(p) = p \left(\max \left\{ e^{-\rho dt} \left(\left(1 - e^{-\kappa_n dt} \right) \frac{u}{\rho} + e^{-\kappa_n dt} B_n(p) \right), (q_n E_n + h_n) dt + e^{-\rho dt} B_n(p) \right\} \right) \\ + (1-p) \left(e^{-\rho dt} \left(\left(1 - e^{-\gamma_n dt} \right) \frac{u}{\rho} + e^{-\gamma_n dt} B_n(p) \right) \right)$$

$$B_n(p) = \max \left\{ \frac{p(1 - e^{-\kappa_n dt}) + (1-p)(1 - e^{-\gamma_n dt})}{1 - pe^{-(\rho+\kappa_n)dt} - (1-p)e^{-(\rho+\gamma_n)dt}} e^{-\rho dt} \frac{u}{\rho}, \frac{p(q_n E_n + h_n) dt + (1-p)e^{-\rho dt} (1 - e^{-\gamma_n dt}) \frac{u}{\rho}}{1 - pe^{-\rho dt} - (1-p)e^{-(\rho+\gamma_n)dt}} \right\}$$

It is straightforward to verify that $\frac{dB_n(p)}{dp} < 0$. This is intuitive as a restrictive contract pushes the borrower to her individual rationality constraint (if possible), whereas an investment contract does not.

The highest possible repayment rate $R(p)$ that can be required for a restrictive contract is pinned down by the borrower's individual rationality constraint

$$(q_n(E_n + T_n) - R(p)) dt + e^{-\rho dt} B_n(p) \geq e^{-\rho dt} \left(\left(1 - e^{-\kappa_n dt} \right) \frac{u}{\rho} + e^{-\kappa_n dt} B_n(p) \right) \\ \implies \\ q_n(E_n + T_n) - R(p) = \max \left\{ \frac{e^{-\rho dt}}{dt} \left(1 - e^{-\kappa_n dt} \right) \left(\frac{u}{\rho} - B_n(p) \right), q_n E_n + h_n \right\}$$

The maximal acceptable repayment rate is increasing in $B_n(p)$ - this is intuitive as the

higher is the borrower's continuation value in state n , the less she values investment.¹⁷

Now, consider the lender's decision of whether to offer an unrestrictive or restrictive contract. Fixing the borrower's expectation that the lender offers a restrictive contract with probability p , in any period in which the lender offers an unrestrictive contract his utility is

$$(q_n T_n - h_n - T_n) dt + e^{-\rho dt} (1 - e^{-\gamma_n dt}) L_n(p)$$

If he offers a restrictive contract his utility is

$$(R(p) - T_n) dt + e^{-\rho dt} L_n(p)$$

So he offers a restrictive contract if and only if the following incentive compatibility constraint holds

$$\begin{aligned} (R(p) - T_n) dt + e^{-\rho dt} L_n(p) &\geq (q_n T_n - h_n - T_n) dt + e^{-\rho dt} (1 - e^{-\gamma_n dt}) L_n(p) \\ &\iff \\ (q_n T_n - h_n - R(p)) dt &\leq e^{-(\rho + \gamma_n) dt} L_n(p) \end{aligned}$$

The left hand side is the additional consumption the lender must forgo to persuade the borrower to accept a restrictive contract, and the right hand side is the discounted expected loss the lender incurs from allowing the borrower to invest in fixed capital.

Note that the lender's continuation utility $L_n(p)$ is weakly decreasing in p . This is so because the set of restrictive contracts the borrower will accept is decreasing in p , while the set of unrestrictive contracts is unchanged.

Thus the left hand side of the above inequality is weakly increasing in p , and the right hand side is weakly decreasing in p . Given the lender's incentive compatibility constraint, we argue that generically there can only be one equilibrium level of p .

If at $p = 0$ (pure unrestrictive), the lender's incentive compatibility constraint for unrestrictive contracts is strictly satisfied, i.e.

$$(q_n T_n - h_n - R(0)) dt > e^{-(\rho + \gamma_n) dt} L_n(0) \quad (4)$$

¹⁷Note that by Assumption 3, $q_n(E_n + T_n) - R(1) > q_n E_n + h_n$ for sufficiently small dt , but in general $q_n(E_n + T_n) - R(p)$ may equal $q_n E_n + h_n$ for some $p < 1$.

then it will be strictly satisfied for all higher levels of p , contradicting that any $p > 0$ can be supported in equilibrium.

If at $p = 1$ (pure restrictive) the lender's incentive compatibility constraint for restrictive contracts is strictly satisfied, i.e.

$$(q_n T_n - h_n - R(1)) dt < e^{-(\rho + \gamma_n) dt} L_n(1) \quad (5)$$

then it will be strictly satisfied for all lower levels of p , contradicting that any $p < 1$ can be supported in equilibrium.

If neither of the above inequalities holds even weakly then by the intermediate value theorem there will be a \bar{p} at which

$$(q_n T_n - h_n - R(\bar{p})) dt = e^{-(\rho + \gamma_n) dt} L_n(\bar{p})$$

Note that when the borrower believes she will receive a restrictive contract with probability \bar{p} , the amount of consumption she demands when given a restrictive contract, $q_n(E_n + T_n) - R(\bar{p})$, is strictly larger than than $q_n E_n + h_n$ (the minimum feasible consumption the lender can leave the borrower). To see this, note that by assumption

$$(q_n T_n - h_n - R(0)) dt < e^{-(\rho + \gamma_n) dt} L_n(0).$$

Now, supposing that $q_n(E_n + T_n) - R(\bar{p}) = q_n E_n + h_n$, we'd have $L_n(\bar{p}) = L_n(0)$ (because the borrower is willing to accept all feasible contracts in both cases), which would imply that $(q_n T_n - h_n - R(\bar{p})) dt < e^{-(\rho + \gamma_n) dt} L_n(\bar{p})$ and would contradict that the lender is indifferent between restrictive and unrestrictive contracts. Therefore we know that $q_n(E_n + T_n) - R(\bar{p}) = q_n E_n + h_n$ and thus $\frac{dR(\bar{p})}{d\bar{p}} < 0$. At $p > \bar{p}$ the lender will strictly prefer investment loans and at $p < \bar{p}$ the lender will strictly prefer consumption loans, contradicting that any $p \neq \bar{p}$ can be supported in equilibrium.¹⁸ \square

So far we have argued that in state n the borrower and lender's behavior are uniquely determined across all equilibria. We now proceed to complete the proof by backward induction. Suppose in all states $\tilde{w} \geq w + 1$ it has been shown that equilibrium behavior is generically unique.

We split the proof for state w into the following two lemmas.

¹⁸Note that since both $R(p)$ and $L(p)$ can both be written in terms of exogenous parameters of the model, it will hold generically that neither 4 nor 5 holds with equality.

Lemma 3. *If there exists an equilibrium in which the borrower weakly prefers to invest her autarkic endowment in the working capital project in state w , she does so in all equilibria. Moreover, the lender offers her the same (unrestrictive) contract across all equilibria.*

Proof. Assume that in equilibrium the borrower weakly prefers to invest her autarkic endowment in working capital in the event of rejecting the lender's contract in state w .

Then the borrower would accept a maximally extractive restrictive contract. But if the lender were to offer one in equilibrium, this would contradict that the borrower would invest her autarkic endowment in working capital. To see this, note that Assumption 3 guarantees

$$\alpha_w B_{w+1} > \frac{q_w E_w + h_w}{\rho}$$

Thus the borrower would strictly prefer to invest her entire autarkic endowment E_w in fixed capital than to accept the maximally extractive contract $\langle q_w T_w - h_w, C \rangle$ if she expects that the lender will offer her that contract in all future periods in state w .

So in equilibrium the lender must weakly prefer to offer the contract $\langle q_w T_w - h_w, I \rangle$ to the contract $\langle q_w T_w - h_w, C \rangle$. Suppose in equilibrium the lender offers the borrower an unrestrictive contract for which the borrower's IR constraint binds. Then in every period the borrower is indifferent between consuming $q_w E_w$ and accepting the lender's contract. The borrower's continuation utility is $B_w = \frac{q_w E_w}{\rho}$, which, by Assumption 3, contradicts her desire to invest her autarkic endowment in working capital.

The final possibility, which we expand on below, is that in equilibrium the lender offers the borrower an unrestrictive contract $\langle R, I \rangle$ that is more generous than the borrower's IR constraint demands. But if the borrower's IR constraint doesn't bind when offered $\langle R, I \rangle$, then it must be that this is the lender's unconstrained optimal contract and will continue to be feasible in any conjectured equilibrium in which the borrower strictly prefers to invest her autarkic endowment in fixed capital. Noting that, by assumption she weakly prefers to invest her autarkic flow income in working capital when offered the contract $\langle R, I \rangle$, this contradicts the existence of an alternative equilibrium.

Formally, if the borrower weakly prefers to invest her autarkic endowment in working capital, then the lender solves

$$\max_R (R - T_w) dt + e^{-\rho dt} \left(\left(1 - e^{-\frac{E_w + T_w - R}{\phi_w} dt} \right) L_{w+1} - e^{-\frac{E_w + T_w - R}{\phi_w} dt} L_w \right)$$

s.t.

$$e^{-\rho dt} \left(\left(1 - e^{-\frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt} \right) B_{w+1} + e^{-\frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt} B_w \right) \geq q_w E_w dt + e^{-\rho dt} B_w$$

$$T_w \leq R \leq q_w T_w - h_w$$

Note that, because the problem is stationary, we could equivalently solve for the contract that maximizes the lender's continuation value in state w . That is we can solve

$$\max_R \frac{(R - T_w) dt + e^{-\rho dt} \left(1 - e^{-\frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt} \right) L_{w+1}}{1 - e^{-\left(\rho + \frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} \right) dt}}$$

s.t.

$$e^{-\rho dt} \left(\left(1 - e^{-\frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt} \right) B_{w+1} + e^{-\frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt} B_w \right) \geq q_w E_w dt + e^{-\rho dt} B_w$$

$$T_w \leq R \leq q_w T_w - h_w$$

It is straightforward to show that the maximand is increasing if and only if

$$L_{w+1} > \frac{\phi_w}{q_w e^{-\rho dt}} \frac{1 - e^{-\left(\rho + \frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} \right) dt}}{e^{-\frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt} - e^{-\left(\rho + \frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} \right) dt}}$$

and that $\frac{1 - e^{-\left(\rho + \frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} \right) dt}}{e^{-\frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt} - e^{-\left(\rho + \frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} \right) dt}}$ is increasing and always less than 1 for $R \leq q_w T_w - h_w$.

Thus the maximand is either monotonically decreasing, monotonically increasing, or increasing and then decreasing. If the borrower's IR constraint doesn't bind, the lender achieves a local maximum. And in any of these cases, the local maximum is also a global maximum in the range of repayments $R \in [T_w, q_w T_w - h_w]$. This uniquely pins down the repayment rate in an equilibrium in which the borrower invests her autarkic endowment in working capital.

Further, if such an equilibrium exists, there could not also be an equilibrium in which the borrower strictly preferred to invest her autarkic endowment in fixed capital. In such an equilibrium the lender would be solving

$$\max_R \frac{(R - T_w) dt + e^{-\rho dt} \left(1 - e^{-\frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt} \right) L_{w+1}}{1 - e^{-\left(\rho + \frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} \right) dt}}$$

s.t.

$$e^{-\rho dt} \left(\left(1 - e^{-\frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt} \right) B_{w+1} + e^{-\frac{E_w + T_w - \frac{R}{q_w}}{\phi_w} dt} B_w \right) \geq e^{-\rho dt} \left(\left(1 - e^{-\frac{E_w}{\phi_w} dt} \right) B_{w+1} + e^{-\frac{E_w}{\phi_w} dt} B_w \right)$$

$$T_w \leq R \leq q_w T_w - h_w$$

where the borrower's IR constraint clearly never binds. So in this case as well the lender achieves the global maximum within the range of repayments $R \in [T_w, q_w T_w - h_w]$, contradicting that the borrower would invest her autarkic endowment in fixed capital in the event that she has rejected the lender's contract. \square

Thus if there exists an equilibrium where the borrower weakly prefers to invest her autarkic endowment in working capital in state w , she does so in all equilibria. We complete the proof by noting that in any equilibrium in which the borrower strictly prefers invest her autarkic endowment in fixed capital (which by the Lemma 3 only occurs when there is no equilibrium in which the borrower weakly prefers to invest her autarkic endowment in working capital), the lender's behavior is uniquely pinned down.

Lemma 4. *The probability the lender offers a restrictive contract in state w , p_w is generically uniquely determined across any equilibrium in which the borrower strictly prefers to invest her autarkic endowment in the fixed capital project in state w .*

Proof. The proof proceeds exactly as in Lemma 2 and is thus omitted. \square

This completes the proof that the equilibrium is generically unique.

Proof of Proposition 3

We aim to show that in equilibrium the probability the lender offers the borrower a restrictive contract in state w , p_w , is single peaked in w . We split the proof into two steps. First we show that conditional on the borrower investing her autarkic endowment in fixed capital in a given region of states $w_k, \dots, w_{k'}$ with $k' > k$, the probability the lender offers a restrictive contract is single peaked in the state.

Second we show that if in equilibrium the lender offers the borrower an excessively generous unrestrictive contract in state w , then he also does so for all states $w' < w$, and hence $p_{w'} = 0$ in all states $w' < w$. Because we showed that in equilibrium the borrower only invests her outside option in working capital in states where she gets an excessively generous unrestrictive contract, this completes the argument.

Lemma 5. *Suppose that in equilibrium the borrower invests her autarkic endowment in fixed capital in states $\tilde{w} - 1, \tilde{w}$, and $\tilde{w} + 1$. Then if $p_{\tilde{w}} < p_{\tilde{w}+1} \implies p_{\tilde{w}-1} \leq p_{\tilde{w}}$.*

Proof. Assume that in equilibrium the borrower invests her outside option in fixed capital in states $\tilde{w} - 1, \tilde{w}$ and $\tilde{w} + 1$. We begin by defining a function that implicitly determines the equilibrium probability $p_{\tilde{w}}$ that the lender offers the borrower a restrictive contract. To do so we first determine the borrower's value in state \tilde{w} as a function of the probability p she expects a restrictive contract. This allows us to determine the maximal repayment rate $R(p)$ she is willing to accept for a restrictive contract given the probability p she expects the lender to offer a restrictive contract. Finally, $R(p)$ allows us to determine the lender's payoff from offering restrictive contracts, and by comparing this to his payoff from offering unrestrictive contracts we pin down the equilibrium probability $p_{\tilde{w}}$.

In state \tilde{w} , if in equilibrium the borrower receives a restrictive contract with probability $p_{\tilde{w}}$, her utility is

$$\begin{aligned}
 B_{\tilde{w}}(p_{\tilde{w}}) &= e^{-\rho dt} \left(p_{\tilde{w}} \left((1 - e^{-\kappa dt}) B_{\tilde{w}+1} + e^{-\kappa dt} B_{\tilde{w}} \right) + (1 - p_{\tilde{w}}) \left((1 - e^{-\gamma dt}) B_{\tilde{w}+1} + e^{-\gamma dt} B_{\tilde{w}} \right) \right) \\
 B_{\tilde{w}}(p_{\tilde{w}}) &= \frac{p_{\tilde{w}} \left(e^{-\rho dt} - e^{-(\kappa+\rho)dt} \right) B_{\tilde{w}+1} + (1 - p_{\tilde{w}}) \left(e^{-\rho dt} - e^{-(\gamma+\rho)dt} \right) B_{\tilde{w}+1}}{1 - p_{\tilde{w}} e^{-(\rho+\kappa)dt} - (1 - p_{\tilde{w}}) e^{-(\rho+\gamma)dt}} \\
 &\rightarrow \frac{p_{\tilde{w}} \kappa B_{\tilde{w}+1} + (1 - p_{\tilde{w}}) \gamma B_{\tilde{w}+1}}{\rho + p_{\tilde{w}} \kappa + (1 - p_{\tilde{w}}) \gamma} \\
 &= \frac{\delta(p_{\tilde{w}})}{\rho + \delta(p_{\tilde{w}})} B_{\tilde{w}+1}
 \end{aligned}$$

where $\delta(p_{\bar{w}}) \equiv p_{\bar{w}}\kappa + (1 - p_{\bar{w}})\gamma$.

Further recall that in equilibrium, the maximum repayment $R(p)$ that the borrower would accept is given by

$$\begin{aligned} (q_{\bar{w}}(E + T_{\bar{w}}) - R(p))dt + e^{-\rho dt}B_{\bar{w}} &= e^{-\rho dt} \left((1 - e^{-\kappa dt})B_{\bar{w}+1} + e^{-\kappa dt}B_{\bar{w}} \right) \\ R(p)dt &= q_{\bar{w}}(E + T_{\bar{w}})dt - e^{-\rho dt} \left((1 - e^{-\kappa dt})(B_{\bar{w}+1} - B_{\bar{w}}) \right) \end{aligned}$$

Now, given the borrower's equilibrium expectation $p_{\bar{w}}$, we can calculate the lender's payoff from offering a maximally extractive, acceptable restrictive or unrestrictive contract. Because the problem is stationary, we can determine which contract the lender prefers by comparing the lender's lifetime utility if he were to offer only restrictive contracts or only unrestrictive contracts in state \bar{w} . If the lender were to offer only restrictive contracts in state \bar{w} his utility would be

$$\begin{aligned} L_{\bar{w}}^R(p_{\bar{w}}) &= (R(p_{\bar{w}}) - T_{\bar{w}})dt + e^{-\rho dt}L_{\bar{w}}^R(p_{\bar{w}}) \\ &= (q_{\bar{w}}(E + T_{\bar{w}}) - T_{\bar{w}})dt - e^{-\rho dt} \left((1 - e^{-\kappa dt})(B_{\bar{w}+1} - B_{\bar{w}}) \right) + e^{-\rho dt}L_{\bar{w}}^R(p_{\bar{w}}) \\ &= \frac{(q_{\bar{w}}(E + T_{\bar{w}}) - T_{\bar{w}})dt - e^{-\rho dt} \left((1 - e^{-\kappa dt}) \left(\frac{\rho}{\rho + \delta(p_{\bar{w}})}B_{\bar{w}+1} \right) \right)}{1 - e^{-\rho dt}} \\ &\rightarrow \frac{(q_{\bar{w}}(E + T_{\bar{w}}) - T_{\bar{w}})}{\rho} - \frac{\kappa}{\rho + \delta(p_{\bar{w}})}B_{\bar{w}+1} \end{aligned}$$

On the other hand, if the lender were to offer only unrestrictive contracts in state \bar{w} his utility would be

$$\begin{aligned} L_{\bar{w}}^U(p_{\bar{w}}) &= (q_{\bar{w}}T_{\bar{w}} - h - T_{\bar{w}})dt + e^{-\rho dt} \left((1 - e^{-\gamma dt})L_{\bar{w}+1} + e^{-\gamma dt}L_{\bar{w}}^U(p_{\bar{w}}) \right) \\ &= \frac{(q_{\bar{w}}T_{\bar{w}} - h - T_{\bar{w}})dt + \left((e^{-\rho dt} - e^{-(\rho+\gamma)dt})L_{\bar{w}+1} \right)}{1 - e^{-(\rho+\gamma)dt}} \\ &\rightarrow \frac{(q_{\bar{w}}T_{\bar{w}} - h - T_{\bar{w}}) + \gamma L_{\bar{w}+1}}{\rho + \gamma} \\ &= (1 - \beta) \frac{(q_{\bar{w}}T_{\bar{w}} - h - T_{\bar{w}})}{\rho} + \beta L_{\bar{w}+1} \end{aligned}$$

Next, consider the function $g_{\bar{w}}(p) \equiv L_{\bar{w}}^U(p) - L_{\bar{w}}^R(p)$. Note that $L_{\bar{w}}^U(p)$ is independent

of p and $L_{\tilde{w}}^R(p)$ is decreasing in p , so $g_{\tilde{w}}(p)$ is increasing. If $g_{\tilde{w}}(1) < 0$, then the unique equilibrium is for the lender to offer a restrictive contract with probability 1. If $g_{\tilde{w}}(0) > 0$, then the unique equilibrium is for the lender to offer an unrestrictive contract with probability 1. Else, as shown in Proposition 2, generically there is a unique $p_{\tilde{w}} \in [0, 1]$ such that $g_{\tilde{w}}(p_{\tilde{w}}) = 0$, and the unique equilibrium is for the lender to offer a restrictive contract with probability $p_{\tilde{w}}$.

We now verify that p_w is single peaked in w by considering the following five exhaustive cases:

$$0 < p_{\tilde{w}} < p_{\tilde{w}+1} < 1$$

$$0 < p_{\tilde{w}} < p_{\tilde{w}+1} = 1$$

In these cases we aim to verify that $p_{\tilde{w}-1} < p_{\tilde{w}}$

$$0 = p_{\tilde{w}} < p_{\tilde{w}+1} < 1$$

$$0 = p_{\tilde{w}} < p_{\tilde{w}+1} = 1$$

In this case we aim to verify that $p_{\tilde{w}-1} = p_{\tilde{w}} = 0$ and $g_{\tilde{w}-1}(0) > g_{\tilde{w}}(0)$

$$0 = p_{\tilde{w}} = p_{\tilde{w}+1} \text{ and } g_{\tilde{w}}(0) > g_{\tilde{w}+1}(0)$$

In this case we aim to verify that $p_{\tilde{w}-1} = p_{\tilde{w}} = 0$ and $g_{\tilde{w}-1}(0) > g_{\tilde{w}}(0)$.

We will provide the proof for the case where $0 < p_{\tilde{w}} < p_{\tilde{w}+1} < 1$ and omit the others as they are all similar.

Because $p_{\tilde{w}+1}$ is interior, we have

$$L_{\tilde{w}+1} = L_{\tilde{w}+1}^R(p_{\tilde{w}+1}) = \frac{q_{\tilde{w}+1}(E + T_{\tilde{w}+1}) - T_{\tilde{w}+1}}{\rho} - \frac{\kappa}{\rho + \delta(p_{\tilde{w}+1})} B_{\tilde{w}+2}$$

and

$$B_{\tilde{w}+1} = \frac{\delta(p_{\tilde{w}+1})}{\rho + \delta(p_{\tilde{w}+1})} B_{\tilde{w}+2}$$

Thus in state w we have

$$\begin{aligned}
g_{\bar{w}}(p_{\bar{w}}) &\equiv L_{\bar{w}}^U - L_{\bar{w}}^R(p_{\bar{w}}) \\
&= (1 - \beta) \frac{(q_{\bar{w}}T_{\bar{w}} - h - T_{\bar{w}})}{\rho} + \beta \left(\frac{(q_{\bar{w}+1}(E + T_{\bar{w}+1}) - T_{\bar{w}+1})}{\rho} - \frac{\kappa}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \right) \\
&\quad - \left(\frac{(q_{\bar{w}}(E + T_{\bar{w}}) - T_{\bar{w}})}{\rho} - \frac{\kappa}{\rho + \delta(p_{\bar{w}})} \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \right) \\
&= \beta \frac{(q_{\bar{w}+1}(E + T_{\bar{w}+1}) - T_{\bar{w}+1}) - (q_{\bar{w}}(E + T_{\bar{w}}) - T_{\bar{w}})}{\rho} - (1 - \beta) \frac{h + q_{\bar{w}}E}{\rho} - \frac{\kappa}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \left(\beta - \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} \right) \\
&= 0
\end{aligned}$$

Similarly we have

$$\begin{aligned}
g_{\bar{w}-1}(p) &= L_{\bar{w}-1}^U - L_{\bar{w}-1}^R(p) \\
&= \beta \frac{(q_{\bar{w}}(E + T_{\bar{w}}) - T_{\bar{w}}) - (q_{\bar{w}-1}(E + T_{\bar{w}-1}) - T_{\bar{w}-1})}{\rho} - (1 - \beta) \frac{h + q_{\bar{w}-1}E}{\rho} \\
&\quad - \frac{\kappa}{\rho + \delta(p_{\bar{w}})} \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \left(\beta - \frac{\delta(p_{\bar{w}})}{\rho + \delta(p)} \right) \\
&= 0
\end{aligned}$$

Comparing the expression for $g_{\bar{w}-1}(p)$ to the that of $g_{\bar{w}}(p)$ we can see that the sum of first two terms is strictly larger in the expression for $g_{\bar{w}-1}(p)$ (by Assumption 4). That means that in order for p to set $g_{\bar{w}-1}(p) = 0$ (if possible), we need that the third term in $g_{\bar{w}-1}(p)$ is strictly smaller than the third term in $g_{\bar{w}}(p)$. That is

$$\begin{aligned}
-\frac{\kappa}{\rho + \delta(p_{\bar{w}})} \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \left(\beta - \frac{\delta(p_{\bar{w}})}{\rho + \delta(p)} \right) &< -\frac{\kappa}{\rho + \delta(p_{\bar{w}+1})} B_{\bar{w}+2} \left(\beta - \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} \right) \\
&\iff \\
\frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} \left(\beta - \frac{\delta(p_{\bar{w}})}{\rho + \delta(p)} \right) &> \left(\beta - \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} \right)
\end{aligned}$$

Recall that $p_{\bar{w}+1} > p_{\bar{w}}$ by assumption. So $\frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} < 1$. Thus

$$\begin{aligned}
\left(\beta - \frac{\delta(p_{\bar{w}})}{\rho + \delta(p)} \right) &> \left(\beta - \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} \right) \\
\implies -\frac{\delta(p_{\bar{w}})}{\rho + \delta(p)} &> -\frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} \\
\implies \frac{\delta(p_{\bar{w}})}{\rho + \delta(p)} &< \frac{\delta(p_{\bar{w}+1})}{\rho + \delta(p_{\bar{w}})} \\
\implies p &< p_{\bar{w}}
\end{aligned}$$

On the other hand, if there is no $p \geq 0$ such that $g_{\tilde{w}-1}(p) = 0$, then $g_{\tilde{w}-1}(0) > 0$, and the unique equilibrium includes $p_{\tilde{w}-1} = 0$. This completes the argument for this case. As the remaining cases are similar they are omitted. \square

To complete the proof we now only need to address the possibility that $p_{\tilde{w}} = 0$ and that the lender offers the borrower an unrestrictive contract with smaller than necessary repayment in state \tilde{w} in equilibrium. We do so with the following lemma.

Lemma 6. *Suppose $p_{\tilde{w}} = 0$ and the lender offers the borrower an unrestrictive contract with smaller than necessary repayment in state \tilde{w} in equilibrium. Then for all $w < \tilde{w}$, $p_w = 0$ and the lender gives the borrower an unrestrictive contract with a smaller than necessary repayment.*

Proof. Given that the lender offers an unrestrictive contract in state \tilde{w} , he optimally sets the repayment rate to maximize

$$L_{\tilde{w}}^U = \max_{T_{\tilde{w}} \leq R \leq q_{\tilde{w}} T_{\tilde{w}} - h} \frac{(R - T_{\tilde{w}}) + \frac{E + T_{\tilde{w}} - \frac{R}{q_{\tilde{w}}}}{\phi} L_{\tilde{w}+1}}{\rho + \frac{E + T_{\tilde{w}} - \frac{R}{q_{\tilde{w}}}}{\phi}}$$

It is easily verified that the above objective function is monotonically decreasing (increasing) in R if and only if $L_{\tilde{w}+1} \geq (\leq) \frac{q_{\tilde{w}}(E + T_{\tilde{w}}) - T_{\tilde{w}}}{\rho} + \phi$. That in state \tilde{w} the lender optimally offers a more generous than necessary repayment rate implies that $L_{\tilde{w}+1} \geq \frac{q_{\tilde{w}}(E + T_{\tilde{w}}) - T_{\tilde{w}}}{\rho} + \phi$.

Now consider state $\tilde{w} - 1$, and suppose toward contradiction that the lender does not offer an excessively generous unrestrictive contract in $\tilde{w} - 1$. By the discussion above, this implies that

$$L_{\tilde{w}} \leq \frac{q_{\tilde{w}-1}(E + T_{\tilde{w}-1}) - T_{\tilde{w}-1}}{\rho} + \phi \quad (6)$$

Define w_R to be the lowest state larger than \tilde{w} in which a restrictive contract is offered with positive probability (if no such state exists, the proposition holds trivially). We will show equation 6 implies that for all states $w \in \{\tilde{w} - 1, \dots, w_R - 1\}$ the lender offers the least generous unrestrictive contract, contradicting the premise of this lemma.

We know that $L_{\tilde{w}} \geq (1 - \beta) \left(\sum_{w'=\tilde{w}}^{w_R-1} \frac{q_{w'} T_{w'} - h - T_{w'}}{\rho} \beta^{w'-\tilde{w}} \right) + \beta^{w_R-\tilde{w}} L_{w_R}$, as the lender would derive this utility if he allowed the borrower to invest in fixed capital at the slowest possible rate in each state $w' > \tilde{w}$ until w_R (by assumption he optimally allows the borrower

to invest at a weakly higher rate).

Combining the above two inequalities we have that

$$(1 - \beta) \left(\sum_{w'=\bar{w}}^{w_R-1} \frac{q_{w'} T_{w'} - h - T_{w'} \beta^{w'-\bar{w}}}{\rho} \right) + \beta^{w_R-\bar{w}} L_{w_R} \leq \frac{q_{\bar{w}-1} (E + T_{\bar{w}-1}) - T_{\bar{w}-1}}{\rho} + \phi \quad (7)$$

We now show that the above inequality together with concavity of $\frac{q_w(E+T_w)-T_w}{\rho}$ in w (Assumption 4) implies

$$(1 - \beta) \left(\sum_{w'=\bar{w}+1}^{w_R-1} \frac{q_{w'} T_{w'} - h - T_{w'} \beta^{w'-(\bar{w}+1)}}{\rho} \right) + \beta^{w_R-(\bar{w}+1)} L_{w_R} \leq \frac{q_{\bar{w}} (E + T_{\bar{w}}) - T_{\bar{w}}}{\rho} + \phi \quad (8)$$

The right hand side of equation 8 is $\frac{q_{\bar{w}}(E+T_{\bar{w}})-T_{\bar{w}}}{\rho} - \frac{q_{\bar{w}-1}(E+T_{\bar{w}-1})-T_{\bar{w}-1}}{\rho}$ larger than that of 7. To evaluate the difference in left hand sides note

$$\begin{aligned} & \left((1 - \beta) \left(\sum_{w'=\bar{w}+1}^{w_R-1} \frac{q_{w'} T_{w'} - h - T_{w'} \beta^{w'-(\bar{w}+1)}}{\rho} \right) + \beta^{w_R-(\bar{w}+1)} L_{w_R} \right) \\ & - \left((1 - \beta) \left(\sum_{w'=\bar{w}}^{w_R-1} \frac{q_{w'} T_{w'} - h - T_{w'} \beta^{w'-\bar{w}}}{\rho} \right) + \beta^{w_R-\bar{w}} L_{w_R} \right) = \\ (1 - \beta) & \left(\sum_{w'=\bar{w}}^{w_R-2} \beta^{w'-\bar{w}} \left(\frac{q_{w'+1} (E + T_{w'+1}) - T_{w'+1}}{\rho} - \frac{q_{w'} (E + T_{w'}) - T_{w'}}{\rho} \right) - \frac{q_{w_R-1} T_{w_R-1} - h - T_{w_R-1}}{\rho} \beta^{w_R-1-\bar{w}} + \beta^{w_R-(\bar{w}+1)} L_{w_R} \right) = \\ & (1 - \beta) \sum_{w'=\bar{w}}^{w_R-2} \beta^{w'-\bar{w}} \left(\frac{q_{w'+1} (E + T_{w'+1}) - T_{w'+1}}{\rho} - \frac{q_{w'} (E + T_{w'}) - T_{w'}}{\rho} \right) - (1 - \beta) \frac{q_{w_R-1} T_{w_R-1} - h - T_{w_R-1}}{\rho} \beta^{w_R-1-\bar{w}} \\ & + (1 - \beta) \beta^{w_R-(\bar{w}+1)} \left(\frac{q_{w_R} T_{w_R} - h - T_{w_R}}{\rho} - \frac{\kappa}{\rho + \delta(p_{w_R})} B_{w_R+1} \right) = \\ (1 - \beta) & \left(\sum_{w'=\bar{w}}^{w_R-1} \beta^{w'-\bar{w}} \left(\frac{q_{w'+1} (E + T_{w'+1}) - T_{w'+1}}{\rho} - \frac{q_{w'} (E + T_{w'}) - T_{w'}}{\rho} \right) - \beta^{w_R-(\bar{w}+1)} \frac{\kappa}{\rho + \delta(p_{w_R})} B_{w_R+1} \right) < \\ (1 - \beta) & \left(\sum_{w'=\bar{w}}^{w_R-1} \beta^{w'-w} \left(\frac{q_{w'+1} (E + T_{w'+1}) - T_{w'+1}}{\rho} - \frac{q_{w'} (E + T_{w'}) - T_{w'}}{\rho} \right) \right) < \\ & \left(\frac{q_{\bar{w}+1} (E + T_{\bar{w}+1}) - T_{\bar{w}+1}}{\rho} - \frac{q_{\bar{w}} (E + T_{\bar{w}}) - T_{\bar{w}}}{\rho} \right) (1 - \beta) \left(\sum_{w'=\bar{w}}^{w_R-1} \beta^{w'-\bar{w}} \right) < \\ & \frac{q_{\bar{w}+1} (E + T_{\bar{w}+1}) - T_{\bar{w}+1}}{\rho} - \frac{q_{\bar{w}} (E + T_{\bar{w}}) - T_{\bar{w}}}{\rho} \end{aligned}$$

The first equality follows from rearrangement of terms, the second follows from substituting for L_{w_R} , the third follows from further rearrangement, the fourth inequality follows from deletion of a negative term, and the fifth inequality follows from the concavity of $\frac{q_w(E+T_w)-T_w}{\rho}$ in w . Thus the difference in the left hand side of equations 8 and 7 is smaller than that of the right hand sides. So we conclude that equation 8 holds.

Proceeding by forward induction we have that

$$(1 - \beta) \left(\sum_{w'=\tilde{w}+1}^{w_R-1} \frac{q_{w'} T_{w'} - h - T_{w'}}{\rho} \beta^{w'-(w+2)} \right) + \beta^{w_R-(w+1)} L_{w_R} \leq \frac{q_w (E + T_w) - T_w}{\rho} + \phi$$

for all \tilde{w} such that $\tilde{w} \leq w \leq w_R - 1$. Note that at $w = w_R - 1$ the above equation implies that $L_{w_R} \leq \frac{q_{w_R-1}(E+T_{w_R-1})-T_{w_R-1}}{\rho} + \phi$ and hence at $w_R - 1$ the lender invests at the lowest possible rate. Therefore $L_{w_R-1} = (1 - \beta) \frac{q_{w_R-1} T_{w_R-1} - h - T_{w_R-1}}{\rho} + \beta L_{w_R}$. Proceeding backward the argument extends for all w such that $\tilde{w} \leq w \leq w_R - 1$, so that $L_w = (1 - \beta) \left(\sum_{w'=\tilde{w}}^{w_R-1} \frac{q_{w'} T_{w'} - h - T_{w'}}{\rho} \beta^{w'-w} \right) + \beta^{w_R-w} L_{w_R}$ for $\tilde{w} \leq w \leq w_R - 1$ and hence the lender allows the borrower to invest at the lowest possible rate in all such states. This completes the contradiction \square

Together the above two lemmas complete the proof.

Proof of Proposition 4

Recall from the proof of Proposition 2 that in equilibrium the lender offers a restrictive contract in state w with probability 1 if and only if $L_w^R(1) \geq L_w^U$, where $L_w^R(p) \equiv \frac{q_w(E+T_w)-T_w}{\rho} - \frac{\kappa}{\rho+\delta(p)} B_{w+1}$ and $L_w^U \equiv (1 - \beta) \frac{q_w T_w - h - T_w}{\rho} + \beta L_{w+1}$. Now

$$\begin{aligned} L_w^R(1) &\geq L_w^U \\ &\iff \\ \frac{q_w(E+T_w)-T_w}{\rho} - \alpha B_{w+1} &> (1 - \beta) \frac{q_w T_w - h - T_w}{\rho} + \beta L_{w+1} \\ &\iff \\ -\alpha B_{w+1} &> -(1 - \beta) \left(\frac{q_w E + h}{\rho} \right) + \beta \left(L_{w+1} - \frac{q_w(E+T_w)-T_w}{\rho} \right) \\ &\iff \\ (\beta - \alpha) B_{w+1} &> \beta \left(B_{w+1} + L_{w+1} - \frac{q_w(E+T_w)-T_w}{\rho} - \phi \right) \end{aligned}$$

which completes the proof.

Proof of Proposition 5

Suppose $h = 0$ and consider the lender's behavior in state n . Fixing a probability p_n that the borrower anticipates a restrictive contract in equilibrium, (and recalling that we can consider the lender's continuation utility in state n from a fixed action due to the stationarity of the problem), the lender's utility from offering a restrictive contract is

$$\begin{aligned} L_n^R(p_n) &= \frac{q_n(E + T_n) - T_n}{\rho} - \frac{\kappa}{\rho + \delta(p_n)} \frac{u}{\rho} \\ &= \frac{q_n(E + T_n) - T_n}{\rho} - \alpha \frac{u}{\rho} \end{aligned}$$

where the equality follows from the fact that $h = 0$.

On the other hand, his utility from offering an unrestrictive contract is

$$\begin{aligned} L_n^U(p_n) &= \frac{q_n T_n - h - T_n}{\rho} (1 - \beta) \\ &= \frac{q_n T_n - h - T_n}{\rho} (1 - \alpha) \end{aligned}$$

The lender prefers offering an unrestrictive contract over a restrictive one if and only if

$$\begin{aligned} \frac{q_n T_n - h - T_n}{\rho} (1 - \alpha) &\geq \frac{q_n(E + T_n) - T_n}{\rho} - \alpha \frac{u}{\rho} \\ (1 - \alpha) \frac{q_n T_n - h - T_n}{\rho} + \alpha \frac{u}{\rho} &\geq \frac{q_n(E + T_n) - T_n}{\rho} \end{aligned} \quad (9)$$

The left hand side of inequality 9 is the sum of the borrower and lender's welfares if the borrower invests in fixed capital at the slowest possible rate in the relationship, and the right hand side is the sum of their welfares from if the borrower invests in working capital. Thus if it is socially efficient to invest in fixed capital the lender strictly prefers unrestrictive contracts, irrespective of the borrower's expectation p_n , and thus in equilibrium in state n the lender chooses unrestrictive contracts with probability 1.

Moving backwards, the proof proceeds similarly.

Proof of Propositions 6 and 7

Lemma 7. For any state $w > \bar{w}$, $\frac{dp_w}{du} \leq 0$ with strict inequality if $p_w > 0$.

Proof. By definition, in states $w > \bar{w}$, $p_w < 1$. Thus in equilibrium the lender at least weakly prefers offering the borrower an unrestrictive contract. We can thus write the lender's continuation utility in each such state as the utility he derives from offering an unrestrictive contract at every period (fixing the borrower's expectation at p_w).¹⁹ That is

$$L_w = L_w^U \equiv (1 - \beta) \frac{q_w T_w - h - T_w}{\rho} + \beta L_{w+1}$$

On the other hand, if the lender were to offer a minimally generous restrictive contract at every period (again, fixing the borrower's expectation at p_w) he would receive a continuation utility of

$$L_w = L_w^R(p_w) \equiv \frac{q_w(E + T_w) - T_w}{\rho} - \frac{\kappa}{\rho + \delta(p_w)} B_{w+1}$$

In state n $L_n^U = (1 - \beta) \frac{q_n T_n - h - T_n}{\rho}$ which is not a function of u . On the other hand $L_n^R(p_n) = \frac{q_n(E + T_n) - T_n}{\rho} - \frac{\kappa}{\rho + \delta(p_n)} \frac{u}{\rho}$ is decreasing in u . Hence if in state n $L_n^U > L_n^R(0)$, then $p_n = 0$ and $\frac{dp_n}{du} = 0$. The lender's continuation utility is $L_n = (1 - \beta) \frac{q_n T_n - h - T_n}{\rho}$ and $\frac{dL_n}{du} = 0$. The borrower's utility is $B_n = \beta \frac{u}{\rho}$ so $\frac{dB_n}{du} > 0$.

If $L_n^I = L_n^C(p_n)$ for some $p_n \in [0, 1]$ then p_n is the solution to

$$g(p_n) \equiv (1 - \beta) \frac{q_n T_n - h - T_n}{\rho} - \left(\frac{q_n(E + T_n) - T_n}{\rho} - \frac{\kappa}{\rho + \delta(p_n)} \frac{u}{\rho} \right) = 0$$

Since δ is a decreasing function, it is clear that $\frac{dp_n}{du} < 0$. But we still have $L_n = (1 - \beta) \frac{q_n T_n - h - T_n}{\rho}$ so that $\frac{dL_n}{du} = 0$. $B_n = \frac{\delta(p_n)}{\rho + \delta(p_n)} \frac{u}{\rho}$, so

$$\frac{dB_n}{du} = \frac{\left(\frac{d\delta(p_n)}{dp_n} \frac{dp_n}{du} \right) \rho u}{(\rho + \delta(p_n))^2 \rho} + \frac{\delta(p_n)}{\rho + \delta(p_n)} > 0$$

¹⁹Note that in full generality he may offer the borrower an unrestrictive contract with positive transfer in state w . If so his continuation utility is $L_w^U = \left(1 - \frac{q_w(E + T_w) - R}{\frac{\phi}{q_w(E + T_w) - R} + \rho} \right) \frac{R - T_w}{\rho} + \left(\frac{q_w(E + T_w) - R}{\frac{\phi}{q_w(E + T_w) - R} + \rho} \right) L_{w+1}$, but otherwise the argument goes through unchanged.

Proceeding backward to any state $w > \bar{w}$, suppose $\frac{dB_{w+1}}{du} > 0$, $\frac{dL_{w+1}}{du} = 0$. Then the proof proceeds exactly as above. This completes the proof of the lemma. \square

We next consider the comparative static in states $w \in \{\underline{w}, \dots, \bar{w}\}$.

Lemma 8. For $w \in \{\underline{w}, \dots, \bar{w}\}$, generically $\frac{dp_w}{du} = 0$, $\frac{dB_{\bar{w}+1}}{du} > 0$ and $\frac{dL_{\bar{w}}}{du} < 0$.

Proof. By definition $p_w = 1$ for $w \in \{\underline{w}, \dots, \bar{w}\}$. Generically this preference will be strict and thus $\frac{dp_{\bar{w}}}{du} = 0$.

Recall that in Lemma 7 we established $\frac{dB_{\bar{w}+1}}{du} > 0$. We also know that $L_{\bar{w}} = L_{\bar{w}}^R(1) \equiv \frac{q_{\bar{w}}(E+T_{\bar{w}})-T_{\bar{w}}}{\rho} - \alpha B_{\bar{w}+1}$. Hence generically $\frac{dL_{\bar{w}}}{du} < 0$. Further, $B_{\bar{w}} = \alpha B_{\bar{w}+1}$ so $\frac{dB_{\bar{w}}}{du} = \alpha \frac{dB_{\bar{w}+1}}{du} > 0$.

For the remainder of the states $w \in \{\underline{w}, \dots, \bar{w}\}$, the result follows from straightforward induction. \square

We now consider the comparative statics for $w < \underline{w}$ in the following three lemmas.

Lemma 9. Suppose $p_{\underline{w}-1} = 0$. Then generically $\frac{dp_w}{du} = 0$, $\frac{dL_w}{du} < 0$, and $\frac{dB_w}{du} > 0$ for all $w < \underline{w}$.

Proof. If $p_{\underline{w}-1} = 0$ and $L_{\underline{w}-1}^U > L_{\underline{w}-1}^R(0)$, then the lender's preference for unrestrictive contracts is strict so $\frac{dp_{\underline{w}-1}}{du} = 0$. Further, Lemma 8 established that $\frac{dL_{\underline{w}}}{du} < 0$ and $\frac{dB_{\underline{w}}}{du} > 0$. Therefore, because $L_{\underline{w}-1} = (1 - \beta) \frac{q_{\underline{w}-1}T_{\underline{w}-1}-h}{\rho} + \beta L_{\underline{w}}$, we know $\frac{dL_{\underline{w}-1}}{du} < 0$. And $B_{\underline{w}-1} = \beta B_{\underline{w}}$ so $\frac{dB_{\underline{w}-1}}{du} > 0$. Moving backwards proceeds by straightforward induction. \square

The remainder of the proof deals with the case for which $p_{\underline{w}-1} > 0$. We split the analysis into two cases based on the players' level of patience.

Lemma 10. Suppose $p_{\underline{w}-1} > 0$ and $\rho > \frac{\kappa\gamma}{\kappa+\gamma}$. Then $\frac{dp_w}{du} > 0$, $\frac{dL_w}{du} < 0$, and $\frac{dB_w}{du} < 0$ for all $w < \underline{w}$.

Proof. Consider first state $\underline{w} - 1$. We know $p_{\underline{w}-1}$ is the solution to $g(p_{\underline{w}-1}) = 0$. So

$$\beta \frac{(q_{\underline{w}}(E+T_{\underline{w}})-T_{\underline{w}}) - (q_{\underline{w}-1}(E+T_{\underline{w}-1})-T_{\underline{w}-1})}{\rho} - (1-\beta) \frac{h+q_{\underline{w}-1}E}{\rho} - \left(\beta - \frac{\delta(p_{\underline{w}})}{\rho + \delta(p_{\underline{w}-1})} \right) \frac{\kappa}{\rho + \delta(p_{\underline{w}})} B_{\underline{w}+1} = 0$$

\Leftrightarrow

$$\beta \frac{(q_{\underline{w}}(E + T_{\underline{w}}) - T_{\underline{w}}) - (q_{\underline{w}-1}(E + T_{\underline{w}-1}) - T_{\underline{w}-1})}{\rho} - (1 - \beta) \frac{h + q_{\underline{w}-1}E}{\rho} = \left(\beta - \frac{\delta(p_{\underline{w}})}{\rho + \delta(p_{\underline{w}-1})} \right) \frac{\kappa}{\rho + \delta(p_{\underline{w}})} B_{\underline{w}+1}$$

Note that the left hand side of the above equation is constant in u .²⁰ Thus

$$\begin{aligned} & \frac{d \left(\beta \frac{\kappa}{\rho + \delta(p_{\underline{w}})} - \frac{\kappa}{\rho + \delta(p_{\underline{w}-1})} \frac{\delta(p_{\underline{w}})}{\rho + \delta(p_{\underline{w}})} \right) B_{\underline{w}+1}}{du} = 0 \quad (10) \\ & \Rightarrow \\ & -\beta \frac{\kappa \frac{d\delta(p_{\underline{w}})}{dp_{\underline{w}}} \frac{dp_{\underline{w}}}{du}}{(\rho + \delta(p_{\underline{w}}))^2} B_{\underline{w}+1} + \frac{\kappa \frac{d\delta(p_{\underline{w}-1})}{dp_{\underline{w}-1}} \frac{dp_{\underline{w}-1}}{du}}{(\rho + \delta(p_{\underline{w}-1}))^2} \frac{\delta(p_{\underline{w}})}{\rho + \delta(p_{\underline{w}})} B_{\underline{w}+1} \\ & - \frac{\kappa}{\rho + \delta(p_{\underline{w}-1})} \frac{\left(\frac{d\delta(p_{\underline{w}})}{dp_{\underline{w}}} \frac{dp_{\underline{w}}}{du} \right) (\rho + \delta(p_{\underline{w}})) - \delta(p_{\underline{w}}) \frac{d\delta(p_{\underline{w}})}{dp_{\underline{w}}} \frac{dp_{\underline{w}}}{du}}{(\rho + \delta(p_{\underline{w}}))^2} B_{\underline{w}+1} \\ & + \left(\beta \frac{\kappa}{\rho + \delta(p_{\underline{w}})} - \frac{\kappa}{\rho + \delta(p_{\underline{w}-1})} \frac{\delta(p_{\underline{w}})}{\rho + \delta(p_{\underline{w}})} \right) \frac{dB_{\underline{w}+1}}{du} = 0 \\ & \Rightarrow \\ & \frac{\kappa \frac{d\delta(p_{\underline{w}-1})}{dp_{\underline{w}-1}} \frac{dp_{\underline{w}-1}}{du}}{(\rho + \delta(p_{\underline{w}-1}))} < 0 \\ & \Rightarrow \\ & \frac{dp_{\underline{w}-1}}{du} > 0 \end{aligned}$$

Where the second implication follows by removing positive terms from the right hand side and noting that $p_{\underline{w}} > p_{\underline{w}-1}$ which implies that $\frac{\delta(p_{\underline{w}})}{\rho + \delta(p_{\underline{w}-1})} < \frac{\delta(p_{\underline{w}-1})}{\rho + \delta(p_{\underline{w}-1})} \leq \beta$.

Next, note that

$$L_{\underline{w}-1} = L_{\underline{w}-1}^U = (1 - \beta) \frac{q_{\underline{w}-1}T_{\underline{w}-1} - h}{\rho} + \beta L_{\underline{w}} \quad (11)$$

so by Lemma 8 we know that $\frac{dL_{\underline{w}-1}}{du} < 0$.²¹

²⁰The right hand side of the above equation can be simplified by noting that $p_{\underline{w}} = 1$, but we leave it in this more general form to economize on notation in the backward induction step.

²¹Note that in full generality the lender may offer the borrower an investment loan with positive

To find the sign of $\frac{dB_{\underline{w}-1}}{du}$ recall that $\frac{\rho+\delta(p_{\underline{w}-1})}{\delta(p_{\underline{w}-1})}B_{\underline{w}-1} = B_{\underline{w}} = \frac{\delta(p_{\underline{w}})}{\rho+\delta(p_{\underline{w}})}B_{\underline{w}+1}$. Hence we know that

$$\begin{aligned} \frac{d\left(\beta\frac{\kappa}{\rho+\delta(p_{\underline{w}})} - \frac{\kappa}{\rho+\delta(p_{\underline{w}-1})}\frac{\delta(p_{\underline{w}})}{\rho+\delta(p_{\underline{w}})}\right)B_{\underline{w}+1}}{du} &= 0 \\ \frac{d\left(\beta\frac{\kappa}{\delta(p_{\underline{w}})} - \frac{\beta\frac{\kappa}{\delta(p_{\underline{w}})}\rho^{-\kappa}}{\delta(p_{\underline{w}-1})}\right)B_{\underline{w}-1}}{du} &= \\ \frac{d\left(\beta + \frac{\beta\rho-\kappa}{\delta(p_{\underline{w}-1})}\right)B_{\underline{w}-1}}{du} &= \\ -(\beta\rho-\kappa)\frac{\frac{d\delta(p_{\underline{w}-1})}{dp_{\underline{w}-1}}\frac{dp_{\underline{w}-1}}{dU}}{(\delta(p_{\underline{w}-1}))^2}B_{\underline{w}-1} + \frac{dB_{\underline{w}-1}}{du}\left(\beta + \frac{\beta\rho-\kappa}{\delta(p_{\underline{w}-1})}\right) & \end{aligned} \quad (12)$$

where the second equality follows from noting that $p_{\underline{w}} = 1$. Reducing we have

$$\frac{dB_{\underline{w}-1}}{du} = NEG(\beta\rho - \kappa) \quad (13)$$

where NEG is a negative constant. The one subtle algebraic reduction in this final step is that $\left(\beta + \frac{\beta\rho-\kappa}{\delta(p_{\underline{w}-1})}\right) = \frac{\rho+\delta(p_{\underline{w}-1})}{\delta(p_{\underline{w}-1})}\left[\beta - \frac{\kappa}{\rho+\delta(p_{\underline{w}-1})}\right] > 0$.

Since we have assumed $\rho > \frac{\kappa\gamma}{\kappa+\gamma}$, which is equivalent to $\rho\beta > \kappa$ we have $\frac{dB_{\underline{w}-1}}{du} < 0$.

Moving backward to state $\underline{w}-2$, suppose $p_{\underline{w}-2} > 0$ (or $p_{\underline{w}-2} = 0$, but $L_{\underline{w}-2}^I - L_{\underline{w}-2}^C(0) = 0$). Then $p_{\underline{w}-2}$ is the solution to $g_{\underline{w}-2}(p_{\underline{w}-2}) = 0$. That is

$$(1-\beta)\frac{q_{\underline{w}-2}T_{\underline{w}-2}-h}{\rho} + \beta L_{\underline{w}-1} - \left(\frac{q_{\underline{w}-2}(E+T_{\underline{w}-2})}{\rho} - \frac{\kappa}{\rho+\delta(p_{\underline{w}-2})}B_{\underline{w}-1}\right) = 0$$

transfer in state $\underline{w}-1$. If so his continuation utility is $L_{\underline{w}-1}^U = \left(1 - \frac{q_{\underline{w}-1}(E+T_{\underline{w}-1})^{-R}}{\phi}\right)\frac{R-T_{\underline{w}-1}}{\rho} +$

$\left(\frac{q_{\underline{w}-1}(E+T_{\underline{w}-1})^{-R}}{\phi}\right)L_{\underline{w}}$, and the interest rate becomes weakly higher but otherwise the argument to follow goes through unchanged.

Differentiating both sides with respect to u we see

$$\beta \frac{dL_{\underline{w}-1}}{du} + \left(\frac{-\kappa \frac{d\delta(p_{\underline{w}-2})}{dp_{\underline{w}-2}} \frac{dp_{\underline{w}-2}}{du}}{(\rho + \delta(p_{\underline{w}-2}))^2} B_{\underline{w}-1} + \frac{\kappa}{\rho + \delta(p_{\underline{w}-2})} \frac{dB_{\underline{w}-1}}{du} \right) = 0 \quad (14)$$

We know that $\frac{dL_{\underline{w}-1}}{du} < 0$ and $\frac{dB_{\underline{w}-1}}{du} < 0$. Hence $\frac{dp_{\underline{w}-2}}{du} > 0$. Further

$$L_{\underline{w}-2} = (1 - \beta) \frac{q_{\underline{w}-2} T_{\underline{w}-2} - h - T_{\underline{w}-2}}{\rho} + \beta L_{\underline{w}-1}$$

so $\frac{dL_{\underline{w}-2}}{du} < 0$. And $B_{\underline{w}-2} = \frac{\delta(p_{\underline{w}-2})}{\rho + \delta(p_{\underline{w}-2})} B_{\underline{w}-1}$ so

$$\frac{dB_{\underline{w}-2}}{du} = \frac{\frac{d\delta(p_{\underline{w}-2})}{dp_{\underline{w}-2}} \frac{dp_{\underline{w}-2}}{du} \rho}{(\rho + \delta(p_{\underline{w}-2}))^2} B_{\underline{w}-1} + \frac{\delta(p_{\underline{w}-2})}{\rho + \delta(p_{\underline{w}-2})} \frac{dB_{\underline{w}-1}}{du} < 0 \quad (15)$$

If instead we had $p_{\underline{w}-2} = 0$ and $L_{\underline{w}-2}^U > L_{\underline{w}-2}^R(0)$, then $\frac{dp_{\underline{w}-2}}{du} = 0$. $L_{\underline{w}-2} = (1 - \beta) \frac{q_{\underline{w}-2} T_{\underline{w}-2} - h}{\rho} + \beta L_{\underline{w}-1}$ so $\frac{dL_{\underline{w}-2}}{du} < 0$. $B_{\underline{w}-2} = \beta B_{\underline{w}-1}$ so $\frac{dB_{\underline{w}-2}}{du} < 0$.

Because we used only that $\frac{dB_{\underline{w}-1}}{du} < 0$ and $\frac{dL_{\underline{w}-1}}{du} < 0$, moving backwards from state $\underline{w} - 2$ to state 0 is straightforward induction. \square

We now complete the proof of Propositions 6 and 7 by considering a patient borrower.

Lemma 11. Suppose $p_{\underline{w}-1} > 0$ and $\rho < \frac{\kappa\gamma}{\kappa+\gamma}$. Then $\frac{dp_w}{du} > 0$ and $\frac{dL_w}{du} < 0$ for all $w < \underline{w}$.

Proof. In state $\underline{w} - 1$ everything follows as it did in Lemma 10 except that $\frac{dB_{\underline{w}-1}}{du}$, determined by equation (13) is positive. In state $\underline{w} - 2$, the considerations are similar. $\frac{dp_{\underline{w}-2}}{du} > 0$ is determined by equation (10) (reducing all indices by 1) and $\frac{dL_{\underline{w}-2}}{du} < 0$ is determined by (11) (reducing all indices by 1). However the sign of $\frac{dB_{\underline{w}-2}}{du}$, determined by (12) is now ambiguous.

Moving back to an arbitrary state $w < \underline{w}$ such that $\frac{dB_{w+1}}{du} > 0$, the considerations will be exactly the same as for $\underline{w} - 2$. In any state $w < \underline{w}$ for which $\frac{dB_{w+1}}{du} < 0$, $\frac{dp_w}{du} > 0$ is determined by equation (14), $\frac{dL_w}{du} < 0$ is determined by (11), and $\frac{dB_w}{du} < 0$ is determined by (15). This completes the proof. \square

Together Lemmas 7 through 11 complete the proof of Propositions 6 and 7.

Proof of Proposition 8

Fixing the lender's behavior, the borrower's continuation utility in state n is

$$B_n(p_n) = \frac{p_n(1 - e^{-\kappa dt}) + (1 - p_n)(1 - e^{-\gamma dt})}{1 - p_n e^{-(\rho + \kappa)dt} - (1 - p_n)e^{-(\rho + \gamma)dt}} e^{-\rho dt} \frac{u}{\rho}$$

which increases linearly in u . Moving backward, suppose B_{w+1} is increasing in u . Then, noting that

$$B_w(p_w) = \frac{p_w(1 - e^{-\kappa dt}) + (1 - p_w)(1 - e^{-\gamma dt})}{1 - p_w e^{-(\rho + \kappa)dt} - (1 - p_w)e^{-(\rho + \gamma)dt}} e^{-\rho dt} B_{w+1}$$

increases linearly in B_{w+1} completes the proof.

Proof of Proposition 9

The proof for states $w \geq \underline{w}$ proceeds exactly as in Proposition 6 and is thus omitted. In this section we provide an example in which $\frac{dp_{\underline{w}-1}}{d\rho^B} < 0$ so that making the borrower more patient can strengthen the poverty trap.

We prove this result with a three state model $w \in \{1, 2, 3\}$ where the game ends if the borrower ever reaches state 3. We take

$E = .15$, $q_1 = q_2 \equiv q = 2$, $\phi = \frac{1}{2}$, $h = 100$, $T_1 = 600$, $T_2 = 1000$, $\frac{u}{\rho^B} = 2000$ and $\rho^B = \rho^L = 1$. It is easily verified that Assumption 3 hold in states 1 and 2. That is,

$$\alpha^2 \frac{u}{\rho^B} = \left(\frac{.3}{1.3}\right)^2 2000 > \frac{qE + h}{\rho^B} = 100.3$$

Now we verify that in state 2 the lender offers the borrower a restrictive contract with probability 1. If the borrower expects a restrictive contract with probability 1 then the lender gets the following continuation utility if he offers the borrower a restrictive contract in state 2.

$$L_2^R(1) = \frac{q(E + T_2) - T_2}{\rho^L} - \alpha \frac{u}{\rho^B} = 1000.3 - \frac{.3}{1.3} 2000 \approx 539$$

If instead the lender offers the borrower an unrestrictive contract at every period in state 2, his continuation utility is

$$L_2^U = \frac{qT_2 - h - T_2}{\rho^L} (1 - \beta) \approx 9$$

Because the lender finds it *least* appealing to offer a restrictive contract when the borrower expects restrictive contracts with probability 1, we conclude that in the unique equilibrium the lender offers the borrower a restrictive contract with probability 1.

We next verify that in equilibrium, the lender mixes between restrictive and unrestrictive contracts in state 1.

First, consider the lender's continuation utility in state 1 from offering the borrower a restrictive contract with probability 1 *when she expects an restrictive contract with probability p* .

$$\begin{aligned} L_1^R(p) &= \frac{q(E + T_1) - T_1}{\rho^L} - \max \left\{ \frac{qE + h}{\rho^L}, \frac{\kappa}{\rho^B + \delta(p)} B_2 \right\} \\ &\approx 600.3 - \max \left\{ 100.3, \left(\frac{.3}{1 + .3p + 100(1 - p)} \right) \left(\frac{.3}{1.3} \right) 2000 \right\} \end{aligned}$$

Note that the repayment rate the lender must set is the larger of $qT - h$ and what the lender must set so that the borrower achieves the utility she would have received from investing E in fixed capital.

If instead the lender were to offer an unrestrictive contract with probability 1, her state 1 continuation utility would be

$$L_1^I = \frac{qT - h - T_1}{\rho^L} (1 - \beta) + \beta L_2 \approx \frac{500}{101} + \frac{100}{101} 539$$

It is easily verified that $L_1^R(0) > L_1^U > L_1^R(1)$ and hence the unique equilibrium in state 1 involves the lender mixing between restrictive and unrestrictive contracts. The probability p_1 that the lender offers a restrictive contract is determined by the following

equation.

$$\begin{aligned}
\frac{qT_1 - h - T_1}{\rho^L} (1 - \beta) + \beta L_2 &= \frac{q(E + T_1) - T_1}{\rho^L} - \frac{\kappa}{\rho^B + \delta(p_1)} B_2 \\
\implies \\
\frac{500}{101} + \frac{100}{101} \left(1000.3 - \frac{.3}{1.3} 2000 \right) &\approx 600.3 - \frac{.3}{1 + .3p_1 + 100(1 - p_1)} \left(\frac{.3}{1.3} 2000 \right) \\
\implies \\
p_1 &\approx .99
\end{aligned}$$

Now, recall the investment rent in state 1 is

$$\left(\beta - \frac{\kappa}{\rho^B + \delta(p_1)} \right) B_2 \approx \left(\frac{100}{101} - \frac{.3}{1 + .3p_1 + (1 - p_1) 100} \right) B_2$$

We have

$$\frac{d \left(\beta - \frac{\kappa}{\rho^B + \delta(p)} \right) B_2}{d\rho^B} = \frac{d \left(\beta - \frac{\kappa}{\rho^B + \delta(p)} \right)}{d\rho^B} B_2 + \left(\beta - \frac{\kappa}{\rho^B + \delta(p)} \right) \frac{d}{d\rho^B} B_2$$

Now,

$$\begin{aligned}
\frac{d}{d\rho^B} B_2 &= \frac{d \frac{.3}{\rho^B + .3} \frac{u}{\rho^B}}{d\rho^B} \\
&\approx -\frac{.3}{(\rho^B + .3)^2} \frac{u}{\rho^B} - \frac{.3}{\rho^B + .3} \frac{u}{(\rho^B)^2} \\
&\approx -\frac{.3}{(1.3)^2} 2000 - \frac{.3}{1.3} 2000 \\
&\approx -816.56
\end{aligned}$$

And,

$$\begin{aligned}
\frac{d}{d\rho^B} \left(\frac{100}{\rho^B + 100} - \frac{.3}{\rho^B + .3p + (1 - p) 100} \right) &\approx \left(-\frac{100}{(\rho^B + 100)^2} + \frac{.3}{(\rho^B + .3p + (1 - p) 100)^2} \right) \\
&\approx .05
\end{aligned}$$

So,

$$\frac{d}{d\rho^B} \left[\left(\beta - \frac{\kappa}{\rho^B + \delta} \right) B_2 \right] \approx -675.87 < 0$$

Therefore making the borrower more patient increases the investment rent and reduces p_1 .

Proof of Proposition 10

Fix a game Γ with n states, and a cost of fixed investment $\{\phi_w\}$. Then for game Γ^m with $m > 0$, a borrower investing in fixed capital rate i in state $2^m n$ will derive a state $2^m n$ continuation value of

$$\frac{\frac{i}{\phi_{2^m n}} u}{\rho + \frac{i}{\phi_{2^m n}} \rho}$$

which converges to $\frac{u}{\rho}$ as m becomes large. If the borrower's equilibrium expectation is that the lender will offer the restrictive contract with probability 1, then the lender's continuation utility in state $2^m n$ from doing so is

$$L_{2^m n}^R(1) = \frac{q_{2^m n} (E_{2^m n} + T_{2^m n}) - T_{2^m n}}{\rho} - \frac{\kappa_{2^m n} u}{\rho + \kappa_{2^m n} \rho}$$

which for sufficiently large m will be negative when it is socially efficient to invest.

On the other hand the lender's continuation utility in state n if he offers an unrestrictive contract in every period is

$$L_{2^m n}^U = \frac{q_{2^m n} T_{2^m n} - h - T_{2^m n}}{\rho} \left(1 - \frac{\gamma_{2^m n}}{\rho + \gamma_{2^m n}} \right)$$

which is positive for all $m > 0$. Thus for sufficiently high m , the lender will offer an unrestrictive contract with positive probability in state $2^m n$, completing the proof.

Proof of Proposition 11

For states $w > \bar{w}_{m-1}$, the proof closely follows that of Proposition 6. Specifically, for states $w > \bar{w}_m$, the proof follows that of Lemma 7. For states $w \in \{\underline{w}_m, \dots, \bar{w}_m\}$ the proof follows that of Lemma 8. And for states $w \in \{\bar{w}_{m-1} + 1, \dots, \underline{w}_m - 1\}$ the proof follows that of Lemma 10.

For $w \in \{\bar{w}_{m-2} + 1, \bar{w}_{m-1}\}$ the logic of Proposition 6 is reversed, as $\frac{dB_{\bar{w}_{m-1}+1}}{du} < 0$. That is, in the state directly beyond the pure restrictive state \bar{w}_{m-1} , the borrower's continuation

utility is decreasing in u . In contrast, in the state directly beyond the pure restrictive state \bar{w}_m , the borrower's continuation utility is *increasing* in u . So, the comparative static for $w \in \{\bar{w}_{m-2} + 1, \bar{w}_{m-1}\}$ comes directly from reversing the signs in Lemmas 8 and 10.

The proof proceeds similarly for all consumption regions $\{\underline{w}_{\tilde{m}} \dots, \bar{w}_{\tilde{m}}\}$ for $\tilde{m} \leq m - 2$.

Proof of Proposition 12

Lemma 12. *In any equilibrium in which the borrower never accepts the entrant's contract, the borrower must always be weakly better off than she would be from receiving the contract $\langle T_w, I \rangle$ or $\langle T_w, C \rangle$ from the entrant in state w .*

Proof. Consider any equilibrium in which the borrower never accepts the entrant's contract. In such an equilibrium the entrant's continuation value in any state is 0. Suppose toward contradiction that there is some state w in which with positive probability the incumbent offers the borrower a contract \tilde{c} that provides her with strictly less utility than she would receive from accepting the contract $\langle T_w, I \rangle$ or $\langle T_w, C \rangle$ from the entrant. Then there exists an $\epsilon > 0$ such that the borrower would strictly prefer the entrant's contract $\langle T_w + \epsilon, I \rangle$ or $\langle T_w + \epsilon, C \rangle$ to the incumbent's contract \tilde{c} . Therefore by deviating to offer one of these contracts, the entrant could guarantee himself a positive payoff, contradicting the premise that the borrower never accepts the entrant's contract in equilibrium. \square

Lemma 12 implies that in equilibrium the borrower's outside option is the maximum of the utility she would receive from accepting the entrant's contract $\langle T_w, I \rangle$ or $\langle T_w, C \rangle$, and the utility she would receive from allocating her autarkic endowment flexibly.

The remainder of the proof proceeds exactly as in Proposition 2, with this new individual rationality constraint in place of the borrower's autarkic constraint, and is thus omitted.

Proof of Proposition 13

We first identify a $\bar{\psi}$ such that for $\psi > \bar{\psi}$, the equilibrium is the same as in the monopolist case. We determine $\bar{\psi}$ as follows. Let B_{w+1}^{max} be the maximal feasible (potentially out of equilibrium) state $w + 1$ continuation utility that the borrower can achieve. Further suppose that in state w the borrower anticipates the incumbent will never make a loan offer,

inducing her to have maximal feasible value of investment in state w . Then if she opts not to borrow from the entrant, her state w continuation utility is

$$B_w = e^{-\rho dt} \left(\left(1 - e^{-\kappa_w dt}\right) B_{w+1}^{max} + e^{-\kappa_w dt} B_w \right)$$

Let $\bar{\psi}_w$ satisfy

$$e^{-\rho dt} \left(\left(1 - e^{-\frac{E_w + T_w - \frac{T_w}{q_w}}{\phi_w} dt}\right) B_{w+1}^{max} + e^{-\frac{E_w + T_w - \frac{T_w}{q_w}}{\phi_w} dt} B_w \right) - \bar{\psi}_w dt < e^{-\rho dt} \left(\left(1 - e^{-\kappa_w dt}\right) B_{w+1}^{max} + e^{-\kappa_w dt} B_w \right)$$

ensuring that the borrower would prefer to invest her autarkic endowment rather than borrow from the entrant even when her value from investment is as high as is feasible in state w . Now let $\bar{\psi} \equiv \max_w \bar{\psi}_w$. Clearly in equilibrium, if the borrower ever rejects the incumbent's contract then she will choose not to borrow from the entrant and her individual rationality constraint will be the same as in Proposition 2. Thus the equilibrium outcome will be the same as in the monopolist case.

We now prove the existence of a $\underline{\psi} > 0$ such that for $\psi < \underline{\psi}$ the incumbent offers an unrestrictive contract with probability 1 in every period. This is so because by Lemma 12, for $\psi = 0$ the borrower's outside option is to take whichever she prefers of the maximally generous restrictive and unrestrictive contracts from the entrant at no additional cost. So long as it is efficient to invest in business expansion, the borrower will always prefer the maximally unrestrictive contract. By continuity, there exists a $\underline{\psi} > 0$ such that the same will be true for any $\psi < \underline{\psi}$. So for $\psi < \underline{\psi}$ the incumbent must offer unrestrictive contracts in equilibrium. And by the same logic, as $\psi \rightarrow 0$, the incumbent's equilibrium contract offer must converge to the maximally generous unrestrictive contract. This completes the proof.

Proof of Proposition 14

Lemma 13. *If business growth is efficient and $p_n \in (0, 1)$, $\frac{dp_n}{d\psi} \geq 0$, $\frac{dB_n}{d\psi} \leq 0$, and $\frac{dL_n}{d\psi} = 0$.*

Proof. Suppose that in state n the equilibrium probability the incumbent lender offers a restrictive contract is $p_n \in (0, 1)$. If ψ is sufficiently high that the borrower's outside option is to invest her own autarkic endowment rather than borrow from the entrant, then $\frac{dp_n}{d\psi} = 0$ and the conclusion of the lemma is satisfied. Else the borrower's outside option is to borrow from the entrant and incur the non pecuniary cost of ψdt .

Now suppose that in equilibrium the borrower derives more utility from the maximally extractive unrestrictive contract than she does from the maximally extractive but acceptable restrictive contract (and hence her expansion rent is positive). Then p_n will be determined as it was in Lemma 7, respecting the borrower's modified individual rationality constraint. Decreasing ψ has the same impact as increasing U , it improves the borrower's outside option so that she becomes more demanding of restrictive contracts. As in Lemma 7 this causes an equilibrium shift towards unrestrictive contracts, so $\frac{dp_n}{d\psi_n} > 0$. The lender remains indifferent between the two types of contracts so his continuation utility L_n is unaffected.

Last suppose that in equilibrium the borrower derives the same utility from the maximally extractive unrestrictive contract as she does from the maximally extractive but acceptable restrictive contract (and hence her expansion rent is zero). Then by the logic is Proposition 5, it must be that $p_n = 0$, contradicting the premise of the lemma. \square

We complete the proof of the proposition by backward induction.

Lemma 14. *Consider state w for which $p_w \in (0, 1)$ for and for which $\frac{dL_{w+1}}{d\psi} = 0$ and $\frac{dB_{w+1}}{d\psi} \leq 0$. Then $\frac{dp_w}{d\psi} \geq 0$.*

Proof. The proof for this lemma proceeds in the same way as the proof for Lemma 13, however when considering how demanding the borrower is of restrictive contracts, her outside option now improves both due to the increase in competition in the current state and to her improved continuation value in state $w + 1$. Otherwise the proof is the same and is thus omitted. \square

Together Lemmas 13 and 14 complete the proof of the result.

Proof of Proposition 15

Lemma 15. *The borrower never rejects an unrestrictive contract on the equilibrium path.*

Proof. Assumption 6 guarantees that if the borrower receives an unrestrictive contract, she necessarily invests more in the fixed capital project than she could have in autarky. Thus by the logic in Lemma 3, the borrower would only ever reject the maximally extractive unrestrictive contract if in equilibrium she gets a more generous unrestrictive contract with certainty. \square

Lemma 16. *The borrower may reject a restrictive contract on the equilibrium path.*

Proof. To prove this lemma we need only find an example in which the borrower rejects a restrictive contract with positive probability. To do so we modify the example from the proof of Proposition 9. Specifically consider the one state example in which we take $E = .15$, $q = 2$, $\phi = \frac{1}{2}$, $h = 100$, $T = 1000$, $\frac{u}{\rho^B} = 2000$ and $\rho^B = \rho^L = 1$. We define the distribution G such that $\nu = 0$ with probability $1 - \varepsilon$, and $\nu = 45$ with probability ε .

We verified in the proof of Proposition 9 that this example satisfies Assumption 3 and that in equilibrium the lender offers the borrower a restrictive contract with probability 1. Clearly for sufficiently small ε , the lender would prefer to offer the least generous restrictive contract that borrowers of type $\nu = 0$ would accept. The loss the lender suffers from being rejected with probability ε is vanishing. In contrast, if the lender offers a contract that both types of borrowers would accept, he incurs a first order loss in order to compensate the high type borrower for the $\nu = 45$ additional forgone investment. \square

Proof of Proposition 16

Define $\beta_s \equiv \frac{\gamma - \frac{s}{\phi}}{\rho + \gamma - \frac{s}{\phi}}$. Now suppose in state w the borrower anticipates a restrictive contract with probability 1. It is straightforward to show that the borrower's expansion rent when she can invest a fraction of her endowment s flexibly no matter the contractual restriction is $(\beta_s - \alpha) B_{w+1}$. Further as in Proposition 4, in equilibrium the borrower receives a restrictive contract with certainty in state w if and only if

$$(\beta_s - \alpha) B_{w+1} \geq \beta \left(B_{w+1} + L_{w+1} - \frac{q_w (E + T_w) - T_w}{\rho} - \phi \right).$$

For $s < \frac{E_w}{T_w + E_w}$, the borrower will grow more slowly in equilibrium than in autarky in any state in which the above condition is satisfied.

8.3 Tables and Figures

Table 1
Default is U-Shaped in Income in Field et. al. Data

	(1) Doesn't Complete Repayment	(2) Doesn't Complete Repayment
Log Profits	-0.153 (0.0955)	-0.167* (0.0945)
Log Proits Sq.	0.0108 (0.00684)	0.0118* (0.00672)
N	660	660
Controls	No	Yes

Notes: Robust standard errors in parentheses. All columns are borrower level OLS regressions. The outcome variable is whether the borrower has completed repayment a year after disbursal. Controls are those included in Field et. al. (2013) and include borrower education, household size, religion, literacy, marital status, age, household shocks, business ownership at baseline, financial control, home ownership, and whether the household has a drain. Log Profits are measured three years after loan disbursal - this is the publicly available measure.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2
Village Covariates Correlated With Inverse Size

	(1) Village Head Age	(2) Village Head Education	(3) Number Agricultral Cooperatives	(4) Fraction with Electricity
Inv. Size	245.0 (248.7)	-532.3** (210.9)	-115.2** (51.99)	10.32 (7.317)
Constant	44.44*** (3.041)	20.37*** (2.909)	4.079*** (0.659)	0.778*** (0.112)
N	55	55	55	55

Notes: Robust standard errors in parentheses. All columns are village level OLS regressions.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2 Continued

	(1)	(2)	(3)	(4)
	Fraction with TV	Fraction with Phone	Fraction with Tractor	Fraction with Chemical Fertilizer
Inv. Size	-63.27 (62.63)	-67.78 (64.54)	1.515 (8.970)	7.406 (8.377)
Constant	1.704* (0.933)	1.032 (0.964)	0.273*** (0.0967)	0.721*** (0.114)
<i>N</i>	55	55	54	55

Notes: Robust standard errors in parentheses. All columns are village level OLS regressions.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2 Continued

	(1)	(2)	(3)	(4)
	Has Factory	Has Temple	Distance to Main Road	Has Common Land
Inv. Size	-25.92 (26.46)	-10.59 (31.76)	-57.89 (193.9)	-47.89* (24.91)
Constant	1.772*** (0.312)	1.762*** (0.351)	7.478* (3.949)	2.068*** (0.316)
<i>N</i>	55	55	55	55

Notes: Robust standard errors in parentheses. All columns are village level OLS regressions.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2 Continued

	(1)	(2)	(3)	(4)
	Soil Problems	Fraction Agricultural Workers	Irrigation	Fraction Wage Workers
Inv. Size	-1.867 (33.51)	11.45 (7.393)	6.571 (21.76)	24.43 (17.26)
Constant	1.819*** (0.366)	0.663*** (0.111)	2.642*** (0.249)	0.289 (0.199)
<i>N</i>	55	54	55	55

Notes: Robust standard errors in parentheses. All columns are village level OLS regressions.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3
Restrictive Loans are Correlated with Lower Income Expectation

	(1)	(2)	(3)	(4)
	Log Expected Income	Log Expected Income	Log Expected Income	Log Expected Income
Restrictive	-0.110** (0.0531)	-0.193*** (0.0528)		
Log Income	0.693*** (0.0422)	0.612*** (0.0651)	0.688*** (0.0422)	0.609*** (0.0648)
Log Loan Size	-0.00210 (0.0187)	-0.00310 (0.0177)	0.00559 (0.0180)	0.00917 (0.0160)
Land - borrower use			-0.0401 (0.0674)	-0.103 (0.0809)
Land - lender use			-0.275** (0.132)	-0.345*** (0.120)
Future Crop			0.0644 (0.182)	-0.0893 (0.127)
Other			-0.238 (0.163)	-0.199** (0.0935)
Single Guarantor			0.0705 (0.0609)	-0.0919 (0.0651)
Mult. Guarantor			-0.182** (0.0753)	-0.191** (0.0743)
N	1999	1783	1999	1783
Wave FEs	Yes	Yes	Yes	Yes
Village FEs	Yes	No	Yes	No
Household FEs	No	Yes	No	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is the log of expected income in the next year. Land - borrower use is an indicator taking the value 1 if land is used as collateral but the borrower uses it. Land - lender use is an indicator taking the value of 1 if the borrower forfeits his land to the lender. Income and loan size are trimmed at the 99th percentile.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4
Restrictive Loans Are Correlated with Lower Interest Rate

	(1)	(2)	(3)
	Monthly Interest	Monthly Interest	Monthly Interest
Restrictive	-0.0328*** (0.00910)	-0.0326*** (0.0101)	-0.0168 (0.0125)
Log Income	-0.00423 (0.00384)	0.000321 (0.00515)	0.000263 (0.00737)
<i>N</i>	1101	1068	878
Wave FEs	No	Yes	Yes
Village FEs	No	Yes	No
Household FEs	No	No	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable the monthly interest rate. We do not control for loan size as it is used to construct interest rates. Income and interest rate are trimmed at the 99th percentile.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5
Main Comparative Static on Village Fund Intensity

	(1)	(2)	(3)
	Restrictive	Restrictive	Restrictive
Inv. Size	8.893*		
	(5.126)		
Post	0.222***		
	(0.0815)		
Post*Inv. Size	-10.87*	-13.27**	-21.65***
	(6.364)	(5.351)	(5.572)
Log Loan Size		0.0613***	0.0421***
		(0.0142)	(0.0140)
Log Income		-0.00516	-0.00320
		(0.0126)	(0.00957)
Village Fund		0.0770	0.130***
		(0.0581)	(0.0296)
<i>N</i>	1710	1710	1620
Wave FEs	No	Yes	Yes
Village FEs	No	Yes	No
Household FEs	No	No	Yes
Controls	No	Yes	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6
Parallel Trends Prior to 2002

	(1)	(2)	(3)
	Restrictive	Restrictive	Restrictive
Wave	0.00787 (0.0321)	0.0115 (0.0298)	-0.0183 (0.0183)
Inv. Size	-625.4 (5916.8)		
Wave*Inv. Size	0.317 (2.960)	-0.121 (3.086)	2.841 (1.709)
Log Loan Size		0.0453*** (0.0159)	0.0146 (0.0157)
Log Income		-0.0282* (0.0150)	-0.0233 (0.0179)
Village Fund		-0.00676 (0.0411)	0.107 (0.0734)
<i>N</i>	916	914	826
Household FEs	No	No	Yes
Village FEs	No	Yes	No
Controls	No	Yes	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 7
Whether a Household Borrows from Money Lenders Regressed on post*invsize

	(1)	(2)	(3)
	Borrow From Money Lender	Borrow From Money Lender	Borrow From Money Lender
Inv. Size	3.421 (6.093)		
Post	-0.109*** (0.0378)	-0.101 (0.0729)	-0.0637 (0.0788)
Post*Inv. Size	1.810 (3.427)	-0.739 (3.471)	-0.528 (3.798)
Log Income		-0.0127 (0.00954)	-0.00263 (0.00848)
Village Fund		0.0238 (0.0156)	-0.0251 (0.0196)
<i>N</i>	6782	6782	6715
Wave FEs	No	Yes	Yes
Village FEs	No	Yes	No
Household FEs	No	No	Yes
Controls	No	Yes	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are villager x time level OLS regressions. The outcome variable is whether the villager borrows from a money lender in a given survey wave. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 8
Total Household Borrowing from Money Lenders Regressed on post*invsize

	(1)	(2)	(3)
	Log Total Borrowing From Money Lenders	Log Total Borrowing From Money Lenders	Log Total Borrowing From Money Lenders
Inv. Size	39.29 (60.64)		
Post	-0.964** (0.368)	-0.920 (0.688)	-0.586 (0.771)
Post*Inv. Size	13.60 (33.72)	-11.82 (34.36)	-5.858 (38.65)
Log Income		-0.0505 (0.0956)	0.00934 (0.0896)
Village Fund		0.308* (0.159)	-0.216 (0.195)
<i>N</i>	6782	6782	6715
Wave FEs	No	Yes	Yes
Village FEs	No	Yes	No
Household FEs	No	No	Yes
Controls	No	Yes	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are villager x time level OLS regressions. The outcome variable is the log of how much the villager borrows from a money lender in a given survey wave. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 9
Heckman Selection Model

	(1)	(2)	(3)
	Restrictive	Restrictive	Restrictive
Inv. Size	8.803*		
	(5.206)		
Post	0.231***		
	(0.0896)		
Post*Inv. Size	-10.96*	-13.10**	-37.79
	(6.355)	(5.353)	(70.73)
Log Loan Size		0.0613***	0.0414
		(0.0141)	(0.0587)
Log Income		-0.000382	-0.451
		(0.0136)	(1.661)
Village Fund		0.0771	0.124
		(0.0578)	(0.214)
Selection Equation			
Inv. Size	0.919	0.837	0.872
	(16.59)	(16.60)	(4.937)
Post	-0.244	-0.247	-0.247**
	(0.172)	(0.171)	(0.120)
Post*Inv. Size	-5.092	-5.015	-5.018
	(9.189)	(9.184)	(6.610)
Log Income	-0.134***	-0.135***	-0.135***
	(0.0310)	(0.0310)	(0.0136)
<i>N</i>	24916	24916	24916
Wave FEs	No	Yes	Yes
Village FEs	No	Yes	No
Household FEs	No	No	Yes
Controls	No	Yes	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level Heckman Selection models under the assumption that errors are jointly normally distributed. The outcome variable is whether the money lender demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Controls in the selection equation include all non fixed effect regressors except for loan size in main specification and whether the household's primary means of income generation is farm or nonfarm work. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 10
 Placebo Test: Main Regression with Population of Borrowers From Neighbors Rather
 Than Money Lenders

	(1)	(2)	(3)
	Restrictive	Restrictive	Restrictive
Inv. Size	-2.981 (5.829)		
Post	0.00409 (0.147)		
Post*Inv. Size	7.824 (12.56)	18.80 (11.48)	-6.507 (12.28)
Log Loan Size		0.0971*** (0.0207)	0.0890*** (0.0228)
Log Income		-0.0467* (0.0268)	-0.0151 (0.0230)
Village Fund		0.0439 (0.0693)	-0.0869 (0.0677)
<i>N</i>	746	745	649
Wave FEs	No	Yes	Yes
Village FEs	No	Yes	No
Household FEs	No	No	Yes
Controls	No	Yes	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the neighbor demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 11
 Placebo Test: Main Regression with Unrestrictive Collateral as Outcome Variable

	(1)	(2)	(3)
	Unrestrictive	Unrestrictive	Unrestrictive
Inv. Size	1.503 (6.379)		
Post	-0.0971 (0.0725)		
Post*Inv. Size	4.165 (5.664)	12.03** (5.869)	14.34*** (5.366)
Log Loan Size		0.0878*** (0.0147)	0.0665*** (0.0155)
Log Income		-0.0331** (0.0144)	-0.00913 (0.0135)
Village Fund		-0.128*** (0.0435)	-0.0177 (0.0421)
<i>N</i>	1710	1710	1620
Wave FEs	No	Yes	Yes
Village FEs	No	Yes	No
Household FEs	No	No	Yes
Controls	No	Yes	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands an unrestrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 12
Main Regression with Interest Rates as Outcome Variable

	(1)	(2)	(3)
	Monthly Interest	Monthly Interest	Monthly Interest
Inv. Size	-0.547 (1.186)		
Post	-0.0456** (0.0195)		
Post*Inv. Size	1.959 (1.751)	0.512 (2.104)	-0.197 (2.056)
Log Income		-0.00366 (0.00455)	-0.00254 (0.00697)
Village Fund		-0.0362* (0.0208)	-0.0395*** (0.0140)
N	860	859	757
Wave FEs	No	Yes	Yes
Village FEs	No	Yes	No
Household FEs	No	No	Yes
Controls	No	Yes	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is the monthly interest rate. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post, but not loan size as it is used to construct the outcome variable. Income and interest rate are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 13
Robustness Check: Main Comparative Static Without Restriction on Village Size

	(1)	(2)	(3)
	Restrictive	Restrictive	Restrictive
Inv. Size	6.259** (3.089)		
Post	0.205*** (0.0570)		
Post*Inv. Size	-9.173** (4.119)	-8.012** (3.969)	-9.837 (5.988)
Log Loan Size		0.0628*** (0.0137)	0.0408*** (0.0137)
Log Income		-0.00170 (0.0122)	-0.00207 (0.00916)
Village Fund		0.0821 (0.0558)	0.125*** (0.0290)
<i>N</i>	1826	1824	1721
Wave FEs	No	Yes	Yes
Village FEs	No	Yes	No
Household FEs	No		Yes
Controls	No	Yes	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 14
Robustness Check: Main Comparative Static Using Data Only From 1999-2004

	(1)	(2)	(3)
	Restrictive	Restrictive	Restrictive
Inv. Size	8.374 (5.995)		
Post	0.196** (0.0842)		
Post*Inv. Size	-10.35 (6.554)	-7.474 (4.812)	-11.51** (4.723)
Log Loan Size		0.0609*** (0.0160)	0.0543*** (0.0176)
Log Income		-0.00407 (0.0201)	-0.00658 (0.0130)
Village Fund		0.0635 (0.0646)	0.0888*** (0.0228)
<i>N</i>	1189	1188	1124
Wave FEs	No	Yes	Yes
Village FEs	No	Yes	No
Household FEs	No	No	Yes
Controls	No	Yes	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands a restrictive form of collateral. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households and only waves collected between 1999 and 2004 are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 15
Robustness Check: Main Comparative Static with Data Driven Definition of Restrictiveness

	(1) Restrictive	(2) Restrictive	(3) Restrictive
Inv. Size	0.0578 (5.149)		
Post	0.186** (0.0811)	0.224 (0.137)	0.320** (0.138)
Post*Inv. Size	-8.709 (6.490)	-7.608 (5.609)	-15.55*** (5.553)
Log Loan Size		0.0444*** (0.0119)	0.0148 (0.0115)
Log Income		-0.00607 (0.0112)	-0.00491 (0.00949)
Village Fund		0.0597* (0.0326)	0.0842*** (0.0290)
<i>N</i>	1710	1710	1620
Wave FEs	No	Yes	Yes
Village FEs	No	Yes	No
Household FEs	No	No	Yes
Controls	No	Yes	Yes

Notes: Robust standard errors clustered at the village level in parentheses. All columns are loan level OLS regressions. The outcome variable is whether the money lender demands a restrictive form of collateral, using the data driven definition of restrictive. Controls include number of agricultural cooperatives, whether the village has common land, and the education of the village head as well as their interaction with post. Income and loan size are trimmed at the 99th percentile. Only villages with between 50 and 250 households are included.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Figure 1
Patterns of Default by Wealth Level

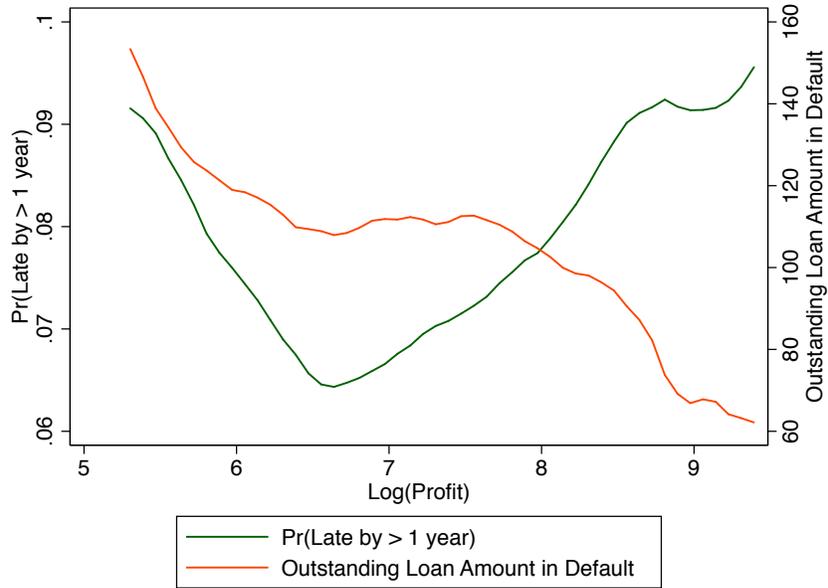


Figure 2
Probability Household Borrows from Money Lender by Wealth Level

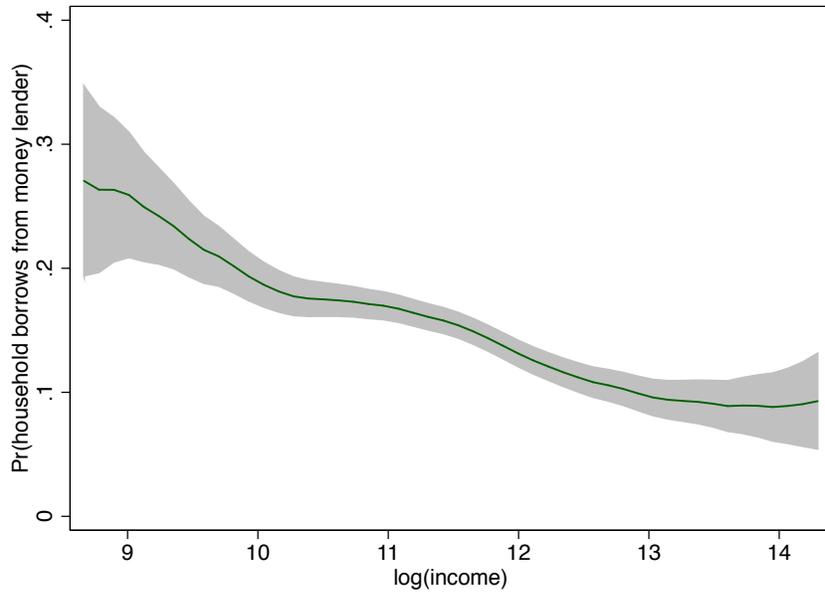


Figure 3
Probability Money Lender Uses Restrictive Collateral by Wealth Level

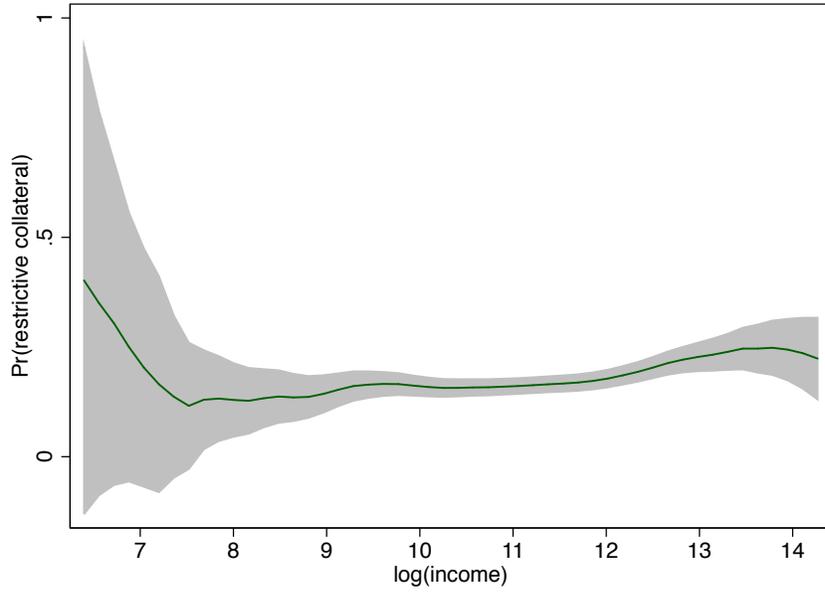


Figure 4
Coefficients on $invsize_v * year_t$

