1) Consider the following matrix.

\[
\begin{pmatrix}
1 & 9 & 8 & -2 & 1 & 4 & 2 & 2 \\
6 & 3 & -2 & 4 & 7 \\
7 & 2 & 2 & 6 & 8 \\
1 & -1 & 4 & 2 & 1 \\
4 & 5 & -10 & 0 & 5
\end{pmatrix}
\]

a) Put this matrix in row reduced echelon form.

b) How many linearly independent columns does this matrix have?

c) How many linearly independent rows does this matrix have?

2) The prices of goods 1 and 2 are \( p_1 \) and \( p_2 \), respectively. The demand functions for these goods are \( D_1(p_1, p_2) \) and \( D_2(p_1, p_2) \), respectively, and the supply functions are \( S_1(p_1) \) and \( S_2(p_2) \), respectively. These functions are of the form

\[
\begin{align*}
D_1(p_1, p_2) &= 23 - 6p_1 + p_2 \\
S_1(p_1) &= 8p_1 \\
D_2(p_1, p_2) &= 36 + 2p_1 - 5p_2 \\
S_2(p_2) &= 3p_2
\end{align*}
\]

The prices \( p_1 \) and \( p_2 \) are at equilibrium values when for each good supply equals demand. Find the equilibrium values for the prices.

3) Now suppose that the functions of the previous problem take the values

\[
\begin{align*}
D_1(p_1, p_2) &= A_1 - 6p_1 + p_2 \\
S_1(p_1) &= 8p_1 \\
D_2(p_1, p_2) &= A_2 + 2p_1 - 5p_2 \\
S_2(p_2) &= 3p_2
\end{align*}
\]

where \( A_1 \) and \( A_2 \) are positive parameters. The function carrying these parameters to the equilibrium prices is linear. Find its matrix representation with respect to the standard bases of \( \mathbb{R}^2 \).