Suppose that good 1 is produced from good 2 according to the production function
\[ y_{11} = f(y_{12}), \]
where \( y_{11} \) is the output of good 1 and \( y_{12} \) is the non-negative input of good 2 in the production of good 1. Similarly, suppose that good 2 is produced from good 1 according to the production function
\[ y_{22} = f(y_{21}), \]
where \( y_{22} \) is the output of good 2 and \( y_{21} \) is the non-negative input of good 1 in the production of good 2. The total net output vector for goods 1 and 2 is
\[ (y_{11} - y_{21}, y_{22} - y_{12}). \]

The set of feasible net output vectors is
\[ F = \{(y_{11} - y_{21}, y_{22} - y_{12}) \mid y_{11} - y_{21} \geq 0, y_{22} - y_{12} \geq 0, y_{11} \leq f(y_{21}), y_{22} \leq f(y_{21})\}. \]

1) Draw \( F \) in the case in which \( f_1(y) = f_2(y) = 2 + y/2 \).

Assume that \( f_1(0) \geq 0 \) and \( f_2(0) \geq 0 \).

2) Show that \( F \) is non-empty.

3) Give an example such that \( f_n \) is continuous and non-decreasing, for \( n = 1 \) and \( 2 \) and yet \( F \) is unbounded.

Assume that \( f_n \) is continuous, non-decreasing, and that \( 0 \leq f_n(y) \leq 2 + y/2 \), for \( n = 1 \) and \( 2 \).

4) Show that \( F \) is a compact and non-empty subset of \( \mathbb{R}^2 \).

Let \( u : \mathbb{R}^2 \to \mathbb{R} \) be a continuous utility function, where \( \mathbb{R}^2 = \{x \in \mathbb{R}^2 \mid x \geq 0\} \).

5) Show that under the given assumptions, \( u \) achieves a maximum on \( F \).