1) This problem has to do with the seeming paradox that in some types of businesses, such as fast food restaurants, an increase in the minimum wage can increase employment. I start with a numerical example. Assume that a firm's revenue from using \( L \) units of labor is

\[
R(L) = 3L - \frac{L^2}{100}.
\]

Assume that the function describing the supply of labor to the firm is \( S(w) = 100w \), where \( w \) is the wage.

Assume for the moment that the firm chooses the amount of labor, \( L \), that it uses and believes that its demand for labor does not influence the wage.

a) Calculate firm's demand function for labor, \( D(w) \).

b) State the firm's profit maximization problem.

c) Calculate the equilibrium wage.

Assume now that the firm sets the wage \( w \), knowing the its supply of labor will be 100\( w \).

d) State the firm's profit maximization problem.

e) Calculate the profit maximizing wage and the amount of labor used by the firm.

Assume in addition that the firm is required by law to pay a wage at least as great as a minimum wage of 9/10.

f) State the firm's profit maximization problem.

g) Calculate the profit maximizing wage and the amount of labor used by the firm.

I now generalize this example slightly by letting the supply function for labor be \( S(w) = \sigma w \), where the parameter \( \sigma \) is positive. Let \( R(L) \) be the firm's revenue net of all costs but labor, where \( R \) is twice differentiable and \( dR(L)/dL > 0 \) and \( d^2R(L)/dL^2 < 0 \). The firm chooses the wage at which it hires labor.

h) Describe formally the profit maximization problem for the firm.
Let \( w \) be the profit maximizing wage and let \( L_e = \sigma w_e \) be the corresponding level of labor input. Now suppose that a government imposes a minimum wage, \( w_m \), where \( w_m > w_e \).

i) Describe formally the profit maximization problem for the firm subject to the constraint that it must pay at least the minimum wage, \( w_m \).

Let \( L_m \) be the profit maximizing employment level of the firm when it chooses the wage and labor input so as to maximize profit subject to the minimum wage constraint.

j) Show that there is a wage \( w_m \) exceeding \( w_e \) and such that when \( w_m < w_e < w_m \), then \( L_m > L_e = \sigma w_e \), i.e., imposition of the minimum wage increases employment. (Hint: It might be helpful to draw the demand and supply curves for labor in a scissors diagram.)