Implementation and Orderings of Public Information *

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Abstract

We explore the relationship between public information and implementable outcomes in an environment characterized by random endowments and private information. We show that if public signals carry no information about private types, then an exact relationship holds: a more informative public signal structure, in the sense of Blackwell, induces a smaller set of ex-ante implementable outcomes. This holds for a large set of implementation standards, including Nash implementation, and Bayesian incentive compatibility. The result extends the notion, dating to Hirshleifer (1971), that public information can have negative value to an endowment economy under uncertainty.

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1. INTRODUCTION

When does information have negative value? This paper explores this question for a particular environment, a neoclassical economy with uncertain endowments in which state-contingent allocations of goods may be agreed to by agents before the resolution of uncertainty. There is a long-standing notion that in such an environment, public information about the realized state of the world, revealed before agents exchange or agree to state-contingent claims, may have a negative value to the agents. An early source of this notion can be found in the following example, adapted from one constructed by Hirshleifer (1971). Assume a pure exchange economy in which there is one state-contingent commodity, and consumers are von Neumann-Morgenstern expected utility maximizers. Each consumer's utility is strictly increasing in the amount of final good she consumes, so no Pareto-improving ex-post exchange is possible; however, consumers may exchange state-contingent claims before the uncertainty is resolved to their mutual benefit. Now suppose that the consumers have common access to a signal about the final state, which they observe before any exchange takes place. Under the maximum possible information, the final state is fully revealed; however, this precludes mutually beneficial exchange, since all consumers now only value the commodity in the state that is certain to be realized. Thus, among all information that the economy might learn about the final state in advance of trade, fully revealing information is the worst possible.

This observation began a substantial literature on the relationship between welfare and the amount of public information in an economy under uncertainty. Important contributions include Marshall (1974), Green (1981), Eckwert and Zilcha (2001), and Schlee (2001). A primary question of interest has been whether Hirshleifer's result, in which the maximum possible amount of information is compared to an arbitrary instance of less information, generalizes to all possible pairs of information structures that might be ordered by their informative content. In addressing this question, the above papers, and the literature in general, has used competitive price equilibrium as the solution concept to attach an outcome to a particular instance of an economy and information, and Blackwell's ordering to rank signal structures by their informative content. The conclusion drawn is that Hirshleifer's example does not generalize: an improvement in public information, in the sense of Blackwell's ordering, may or may not result in an ex-ante Pareto regression for agents from competitive equilibrium outcomes. Schlee (2001) explores the boundaries of when more information necessarily has negative value in the Pareto sense. He identifies three sufficient conditions on an exchange economy for which a negative value of public information is guaranteed. While these conditions permit an interesting class of environments, it is a fairly narrow one, and the idea that public information has negative value must be said generally to fail for competitive equilibrium outcomes.

This paper, along with Campbell (2002), is an attempt to argue that public information intuitively *should* have negative value to a neoclassical economy under uncertainty, and in fact necessarily will under solution concepts different than competitive equilibrium. Campbell (2002) considers the same environment as in the cited literature, in which agents are risk-averse and have no private information. That paper invokes the set-valued solution concept of allocations that are feasible and individually rational, in the sense that for any realized signal, the associated state-dependent allocation gives each agent at least her expected endowment utility. The welfare ordering on sets of allocations used is that A is ordered ahead of B if for any allocation in B, there is some allocation in A that ex-ante Pareto dominates it. Thus, if we think of a social planner as having the ability to implement any feasible, individually rational allocation, the social planner can do at least as well, under the Pareto ordering, with set A as with set B. Under this ranking, the following exact relationship between public information and welfare is found: for any two signal structures ζ and ζ', the solution set under ζ is better than the solution set under ζ' for all possible endowment economies if and only if ζ' is more informative than ζ in the sense of Blackwell. Campbell (2002) also shows that the relationship holds for the set-valued solution concept of allocations that are feasible, individually rational and Pareto efficient, and for the solution concept that maximizes a linearly additive social welfare function with fixed welfare weights.

The current paper expands on, and in fact generalizes, the results of Campbell (2002) by considering environments in which agents have private information. While use of solution concepts other than competitive equilibrium is possibly controversial in a neo-classical economy, where existence of equilibrium in Arrow-Debreu securities under weak conditions is well established, when agents have private information it is common practice to view implementable social choice functions as the feasible set of what can obtain in the economy. Such implementable sets are simply the extension of the solution concept considered in Campbell (2002), where, in the absence of private information, there are no incentive constraints for the agents that the social planner must satisfy. When agents do have private information, these incentive constraints may take many forms, depending on the standard of implementation used. For instance, Bayesian incentive compatible implementation would require that the desired social choice function be a Bayesian-Nash equilibrium when the social choice function is employed as a direct revelation mechanism, while Nash implementation would require that a mechanism can be constructed for which all Nash equilibria induce the desired social choice function. In particular, social choice functions in the implementable set will depend on the standard of implementation that is invoked.

The main result of the paper is that for any fixed implementation standard from among a large selection (that includes Bayesian incentive compatible, Nash, and several others), if public signal structure ζ' is more informative than public signal structure ζ in the sense of Blackwell, then the set of ex-ante social choice functions implementable under ζ' is a subset of those that are implementable under ζ. This naturally implies that any ex-ante utility profile achievable under ζ' can also be achieved under ζ, and thus we may properly conclude that public information has negative value in an exchange economy under uncertainty in which agents have private information. This result holds under a particular restriction on the nature of public and private information. Specifically, it depends on the state of the world being decomposable into two components, one comprised of agents' private information, and the other an unobserved common component; these components must be statistically independent. Furthermore, the public signal can contain information *only* about the common component. Under these conditions, the realization of the public signal does not alter agents' beliefs about each other's information. If the public signal could contain information about agents' private types, it would be simple to construct an example in which more information would permit new social choice functions to be implemented. Note that when agents have no private information, this independence assumption is trivially satisfied.

The body of the paper contains two sections. The first describes the environments that are considered and presents the formal standards of implementability for which the main result is proved. The second reviews Blackwell's ordering of information structures and states the main result, followed by discussion. A final section concludes.

2. THE ENVIRONMENT

There is a finite set Ω of M contractable random states containing elements ω. This set is held fixed throughout the analysis. An environment E is described by the following fundamentals. There is a set of I agents indexed by i. In addition to ω, nature chooses a vector of types for the agents $\theta \equiv (\theta_1, \theta_2, \ldots, \theta_I)$, where θ_i lies in a finite set Θ_i and $\theta \in \Theta \equiv \Pi_{i=1}^{I} \Theta_i$. Thus, a proper state of the world is a pair (ω, θ). Agent i observes the realization of θ_i, but no other information about θ. No agent observes the realization of ω. Agent i is characterized by: an ω-contingent endowment $e_i(\omega)$, which for each ω is a point in the consumption set X, a vector space; and a state-contingent von Neumann-Morgenstern utility function $u_i(x_i, \omega, \theta)$ over sure-thing consumption x_i in state (ω, θ). State-contingent production is possible and is summarized by ω-contingent production possibility sets $X(\omega)$. It is assumed that for every ω the grand endowment $\sum_i e_i(\omega)$ is in $X(\omega)$, so that null production is possible. This nests the special case of an exchange economy with no disposal, in which $X(\omega)$ is the singleton set $\{\sum_i e_i(\omega)\}$ for all ω. Finally, the realized state is generated according to a probability function on $\Omega \times \Theta$. A crucial condition for our results to hold will be that ω and θ are statistically independent, under which condition the probability of a given state (ω, θ) may be expressed $\lambda(\omega)\rho(\theta)$, where $\lambda(\cdot)$ is the marginal probability of ω and $\rho(\cdot)$ the marginal probability of θ. Note that given this independence, agents have identical information about ω. The set of all environments satisfying these criteria is denoted $\mathcal{E}$.

Agents have access to a public signal containing information about the realization of ω. The process governing this signal is an information structure ζ, comprised of elements (Y, Π). Y is a finite set of N possible signals with elements y. While the set Ω is held fixed, the set of signals Y, and its size N, may vary across different information structures. Π is an $M \times N$ matrix of M probability distributions on the N signals. Specifically, π_{mn} is the probability that signal y_n is observed given that ω_m is realized, written $\pi(y_n|\omega_m)$. Thus, Π contains only nonnegative elements, and its rows each sum to 1; we refer to the class of matrices having these characteristics as Markov matrices. The signal Y and the vector of types θ are statistically independent. The elements of ζ are common knowledge among the agents. Agents are assumed to observe, before any activity takes place, a single realization of a signal generated from the distribution $\lambda(\cdot)$ combined with the conditional distribution π, i.e., the probability that they observe signal y is $\sum_{\omega\in\Omega}\lambda(\omega)\pi(y|\omega)$.

Given an environment E, an ex-post allocation is defined as an array of consumptions for each consumer $x \equiv \left(x_i\right)_{i=1}^{I}$. A *finite lottery* over ex-post allocations is defined as a pair $(\tilde{X}, \delta)$, where $\tilde{X}$ is a finite set of ex-post allocations x, and δ is a probability distribution on X, where $\delta(x)$ is the probability that ex-post allocation x obtains, and $\delta(x) = 0$ for all $x \notin \tilde{X}$. Let X_Δ be the set of all finite lotteries over ex-post allocations, and x_Δ a typical element. An *ω-contingent allocation* is defined to be a mapping from Ω to X_Δ. For the purposes of analyzing implementation, it will be appropriate to define an *outcome* as an ω-contingent allocation; thus, the set of all ω-contingent allocations is called O, with typical element o.

We now describe the implementation problem. Take as given that the agents have learned the realization of the public signal y. A *direct revelation mechanism* is a function $g : \Theta \to O$. By definition of the set of outcomes O, such a mechanism can be written $(\tilde{X}(\omega,\theta), \delta_{\omega,\theta})$. The mechanism is *feasible* if $\delta_{\omega,\theta}(x) > 0$ implies $\sum_i x_i \in X(\omega)$, i.e., any ex-post allocation that is made with positive probability given (ω, θ) is feasible given ω. Let $\rho(\theta|\theta_i)$ denote the conditional probability that θ is the realized vector of types conditional on θ_i being agent i's realized type. The mechanism is *individually rational* if every agent obtains a higher expected utility from the truth-telling play of the mechanism than from consuming her endowment, under her beliefs given y:

$$\sum_{\omega\in\Omega}\lambda(\omega)\pi(y|\omega)\sum_{\theta\in\Theta}\rho(\theta|\theta_i)\sum_{x\in\tilde{X}(\omega,\theta)}\delta_{\omega,\theta}(x)(u_i(x_i,\omega,\theta)-u_i(e_i(\omega),\omega,\theta)) \geq 0,$$

for all agents i and types θ_i.

As is customary, we will consider mechanisms that satisfy feasibility and individual rationality, as well as some form of incentive compatibility. Many standards for incentive compatibility may be posited, and our results will hold for several such standards. We provide the formal requirements of these constraints in our setting below. To do so, we define a pure strategy for agent i in a direct revelation mechanism as a mapping $s_i : \Theta_i \to \Theta_i$. A generic profile of pure strategies for players other than i is written $s_{-i}(\theta_{-i})$.

Dominant strategy incentive compatibility:

$$(IC1) \quad \sum_{\omega\in\Omega} \lambda(\omega)\pi(y|\omega) \sum_{\theta\in\Theta} \rho(\theta|\theta_i) \sum_{x\in\tilde{X}(\omega,\theta_i,s_{-i}(\theta_{-i}))} \delta_{\omega,\theta_i,s_{-i}(\theta_{-i})}(x) u_i(x_i,\omega,\theta)$$
$$\geq \sum_{\omega\in\Omega} \lambda(\omega)\pi(y|\omega) \sum_{\theta\in\Theta} \rho(\theta|\theta_i) \sum_{x\in\tilde{X}(\omega,\theta_i',s_{-i}(\theta_{-i}))} \delta_{\omega,\theta_i',s_{-i}(\theta_{-i})}(x) u_i(x_i,\omega,\theta),$$

for all i, θ_i, θ_i', and $s_{-i}(\theta_{-i})$.

Ex-post incentive compatibility:

$$(IC2) \quad \sum_{\omega\in\Omega} \lambda(\omega)\pi(y|\omega) \sum_{x\in\tilde{X}(\omega,\theta)} \delta_{\omega,\theta}(x) u_i(x_i,\omega,\theta)$$
$$\geq \sum_{\omega\in\Omega} \lambda(\omega)\pi(y|\omega) \sum_{x\in\tilde{X}(\omega,\theta_i',\theta_{-i})} \delta_{\omega,\theta_i',\theta_{-i}}(x) u_i(x_i,\omega,\theta),$$

for all i, θ, and θ_i'.

Bayesian incentive compatibility:

$$(IC3) \quad \sum_{\omega\in\Omega} \lambda(\omega)\pi(y|\omega) \sum_{\theta\in\Theta} \rho(\theta|\theta_i) \sum_{x\in\tilde{X}(\omega,\theta)} \delta_{\omega,\theta}(x) u_i(x_i,\omega,\theta)$$
$$\geq \sum_{\omega\in\Omega} \lambda(\omega)\pi(y|\omega) \sum_{\theta\in\Theta} \rho(\theta|\theta_i) \sum_{x\in\tilde{X}(\omega,\theta_i',\theta_{-i})} \delta_{\omega,\theta_i',\theta_{-i}}(x) u_i(x_i,\omega,\theta),$$

for all i, θ_i, and θ_i'.

We also consider the stricter requirement of implementation in which all equilibria of a mechanism must yield the same outcome. To this end, we consider more general mechanisms, in which each agent has access to an abstract message space $\mathcal{M}$. A mechanism is then a mapping $g : \mathcal{M}^I \to O$. A strategy for agent i is a mapping $s_i : \Theta_i \to \mathcal{M}$; we assume that $\mathcal{M}$ is large enough to make mixed strategies redundant, in the sense that any mixture over elements of $\mathcal{M}$ is equivalent to some element of $\mathcal{M}$. A strategy profile $s(\theta) \equiv (s_i(\theta_i))_{i=1}^I$ induces a social choice function $f : \Theta \to O$, where $f(\theta) \equiv g(s(\theta))$. For convenience we adopt the notation $\tilde{X}(\omega, s(\theta))$ and $\delta_{\omega,s(\theta)}$ to denote the induced mapping from Θ to outcomes when agents use strategy profile s.

Let $s_{-i}(\theta_{-i})$ denote a strategy profile for agents other than i. Strategy profile $s(\theta)$ is a Nash equilibrium of mechanism g if for every agent i and strategy $s_i'(\theta_i)$,

$$\sum_{\omega\in\Omega}\lambda(\omega)\pi(y|\omega)\sum_{\theta\in\Theta}\rho(\theta)\sum_{x\in\tilde{X}(\omega,s(\theta))}\delta_{\omega,s(\theta)}(x)u_i(x_i,\omega,\theta)$$
$$\geq\sum_{\omega\in\Omega}\lambda(\omega)\pi(y|\omega)\sum_{\theta\in\Theta}\rho(\theta)\sum_{x\in\tilde{X}(\omega,s_i'(\theta_i),s_{-i}(\theta_{-i}))}\delta_{\omega,s_i'(\theta_i),s_{-i}(\theta_{-i})}(x)u_i(x_i,\omega,\theta).$$

Mechanism g implements social choice function f if the range of g is feasible, a Nash equilibrium of g exists, and for every Nash equilibrium $s^*(\theta)$ of g, $g(s^*(\theta)) = f(\theta)$.

We also consider the criterion of implementation in equilibria in weakly undominated strategies. Strategy profile $s(\theta)$ is a Nash equilibrium in weakly undominated strategies of mechanism g if $s(\theta)$ is a Nash equilibrium of g, and if there is no agent i and strategy $s_i'(\theta_i)$ such that

$$\sum_{\omega\in\Omega}\lambda(\omega)\pi(y|\omega)\sum_{\theta\in\Theta}\rho(\theta)\sum_{x\in\tilde{X}(\omega,s_i(\theta_i),s'_{-i}(\theta_{-i}))}\delta_{\omega,s_i(\theta_i),s'_{-i}(\theta_{-i})}(x)u_i(x_i,\omega,\theta)$$
$$\leq\sum_{\omega\in\Omega}\lambda(\omega)\pi(y|\omega)\sum_{\theta\in\Theta}\rho(\theta)\sum_{x\in\tilde{X}(\omega,s_i'(\theta_i),s'_{-i}(\theta_{-i}))}\delta_{\omega,s_i'(\theta_i),s'_{-i}(\theta_{-i})}(x)u_i(x_i,\omega,\theta)$$

for all $s'_{-i}(\theta_{-i})$, with the inequality strict for some $s'_{-i}(\theta_{-i})$. Mechanism g implements social choice function f in weakly undominated strategies if the range of g is feasible, a Nash equilibrium of g in weakly undominated strategies exists, and for every Nash equilibrium in weakly undominated strategies $s^*(\theta)$ of g, $g(s^*(\theta)) = f(\theta)$.

We now establish a system for referring to the set of implementation problems under the different standards defined above. For index $j \in \{1, 2, 3, 4, 5\}$, we shall define the statement "social choice function f is implementable under standard j" to mean the following. For $j \in \{1, 2, 3\}$, the statement means that direct revelation mechanism $g(\theta) = f(\theta)$ is feasible, individually rational, and satisfies incentive compatibility constraint ICj. For $j = 4$, it means that there exists a mechanism g that implements social choice function f, and that every agent obtains at least her expected endowment utility under f. For $j = 5$, it means that there exists a mechanism g that implements social choice function

f in weakly undominated strategies, and that every agent obtains at least her expected endowment utility under f.

The above definitions apply to environments in which a public signal y has been realized. Thus, whether a particular social choice function f is implementable under standard j may depend on the realized signal y. For this reason, the set of all social choice functions implementable under standard j is contingent on y; we therefore denote this set for a given signal $F_j(y)$. Recall that a social choice function $f(\theta)$ may be represented as a pair $(\tilde{X}(\omega,\theta),\delta_{\omega,\theta})$. Any array of implementable social choice functions $(f_y)_{y\in Y}$, where $f_y \in F_j(y)$ for all y, induces an *ex-ante implementable social choice function* $h : \Theta \to O$, represented by $\tilde{X}(\omega,\theta) \equiv \cup_{y\in Y}\tilde{X}_y(\omega,\theta)$, and $\delta_{\omega,\theta}(x) \equiv \sum_{y\in Y}\pi(y|\omega)\delta_{\omega,\theta_y}(x)$ for $x \in \tilde{X}(\omega,\theta)$. That is, h is the ex-ante mapping from states of the world into finite lotteries over allocations, taking account of the randomness of the realized signal y. Note that even if, for instance, $\tilde{X}_y(\omega,\theta)$ is a singleton for every signal and state, so that there is no randomness in the state-contingent allocation for a given signal, there is nevertheless ex-ante state-contingent randomness if any $\tilde{X}_y(\omega,\theta)$ differs from $\tilde{X}_{y'}(\omega,\theta)$ for some pair of signals y and y'. H_j denotes the set of all ex-ante social choice functions implementable under standard j. Because this set is determined by the environment E and the information structure ζ, we write it $H_j(E,\zeta)$.

3. IMPLEMENTATION AND BLACKWELL'S ORDERING

We now review Blackwell's criterion for ordering information structures. It is a partial ordering with numerous equivalent characterizations (see, e.g., Blackwell (1951) for an original exposition, Blackwell and Girshick (1954) or Kihlstrom (1984) for summaries). The definition most useful for our purposes is a mathematical condition on the matrix of posterior distributions of ω conditional on the signals y. This matrix, which we will call P, is an $N \times M$ Markov matrix. Recall that the marginal probability of signal y_n is $\sum_{\omega\in\Omega}\lambda(\omega)\pi(y_n|\omega) \equiv \gamma_n$. If $\gamma_n > 0$, then element P_{nm} of matrix P is $\lambda_m\Pi_{mn}/\gamma_n$. If $\gamma_n = 0$ then row n of matrix P may be specified arbitrarily; for convenience it is assumed to equal the marginal distribution λ, i.e., $P_{nm} = \lambda_m$. Under these definitions and given λ, any information structure (Y,Π) maps uniquely to a pair (P,γ).

Definition : *Information structure* (Y', Π') *is more informative than information structure* (Y, Π) *in Blackwell's sense if and only if for every* λ *there exists an* $N \times N'$ *Markov matrix* T *such that* $P = TP'$ *and* $\gamma T = \gamma'$.

Remark: Note that for any fixed λ and any two information structures (Y, Π) and (Y', Π'), it is necessarily the case that the induced (P, γ) and (P', γ') satisfy $\gamma P = \gamma' P' = \lambda$. Thus, as λ varies, so to do (P, γ) and (P', γ'). However, if the conditions of Blackwell's criterion are satisfied for one such λ, they are satisfied for all λ, albeit with different T.

This characterization of Blackwell's ordering lends itself to the following interpretation. Suppose that each y_n is not a signal containing information about a predetermined but as yet unobserved state ω, but rather a process that will generate a realization of ω according to the distribution represented by row n of matrix P. When (Y', Π') is more informative than (Y, Π) in Blackwell's sense, it is as if a process y_n is determined according to distribution γ; a process $y'_{n'}$ is then determined from y_n according to row n of matrix T, and finally ω is determined from $y'_{n'}$ according to row n' of matrix P'. That is, the primitive elements of the environment are γ, T, and P'. Then information structure (Y, Π) may be interpreted as information that is known after the resolution of γ, while information structure (Y', Π') may be interpreted as information that is known after the resolution of γ and of T. Thus, (Y', Π') contains strictly more information about the ultimate realization of ω.

We are now ready to state the main result of the paper. Recall that the set of ex-ante social choice functions that are implementable under standard j, for environment E and information structure ζ, is written $H_j(E, \zeta)$.

Theorem 1: *For two information structures* ζ *and* ζ', $H_j(E, \zeta') \subseteq H_j(E, \zeta)$ *for all* $E \in \mathcal{E}$ *and all* $j \in \{1, 2, 3, 4, 5\}$ *if and only if* ζ' *is more informative than* ζ *in Blackwell's sense.*

Proof: See appendix.

The idea of the proof is as follows. Suppose that a particular array of signal-dependent social choice functions is implementable, according to a given standard, under more informative information structure ζ'. A social planner can essentially replicate those functions, and their properties, by "simulating" a realization of signal y' conditional on the realized

ω, according to the given Π'. This ex-post simulation is possible because allocations can be made ω-contingent. In particular, all incentive compatibility constraints will be met by this simulating mechanism, because it simply chooses one of the elements of y' and implements the associated social choice function. The importance of γ containing less information than γ' is that if the individual rationality constraints hold for the y'-conditional mechanisms, then they also necessarily do so for the constructed y-contingent mechanisms, which effectively pool the individual rationality constraints that are satisfied for each y'. However, there is no way to reverse the process: if the agents have strong information y', it is not possible to hold them to individual rationality constraints that would only be satisfied if they had less information. Thus, the set containment can be expected to hold strictly in most cases in which ζ' is ordered strictly ahead of ζ.

The importance of the individual rationality constraints mirrors that found in Campbell (2002) for the case of no private information. When state-contingent allocations are possible, information has no bearing whatever on which mechanisms are incentive compatible, because the information can effectively be ignored in the determination of the final allocation. In particular, if feasibility and incentive compatibility (in whatever form) were the *only* constraints, then $H_j(E, \zeta)$ would be *identical* for all ζ. But when individual rationality is required, having less information, or weaker signals, generally broadens the scope of what agents will agree to. This is seen most easily in the interpretation of ζ' as ζ with a resolution of some noise: whatever agents would be willing to do for each resolution of that noise, they will also be willing to do as a function of that noise. This relationship is similar to one from standard mechanism design: implementing some outcome ex-ante, before agents learn private types, is easier, in the sense of inclusion, than interim implementation, after agents have learned their types. Here, the distinction is quality of public information rather than timing of private information, but the conclusion is the same.

We note that the implementability standards we have considered are only a sampling of those we might have considered; for instance, one could impose dominant-strategy ex-post incentive compatibility, or subgame perfect implementation. We suspect that the result would hold for any such standard one would wish to impose. As described above, for any information structure ζ and signal-dependent mechanism, it is possible to duplicate all the properties of that mechanism that hold under ζ, even under a different information structure, simply by simulating a signal realization from ζ ex-post, and implementing the associated mechanism.

Finally, we comment on the restriction to the independence between public information and the private types θ. This restriction is critical to the result that public information always has negative value, in the sense of contracting the set of implementable social choice functions. However, an example may demonstrate that in the absence of this a restriction, such a result is doomed to fail for obvious reasons. Suppose that ω is degenerate, and that there are two public signal structures, one that contains no information, and one that fully reveals the agents' types. Then the social planner can clearly implement a larger set of social choice functions under the fully revealing information structure, as under said structure there are no relevant incentive constraints for the agents. In general, whenever the public signal contains any information about agents' private types, then the social planner faces a relaxed problem with respect to the agents' incentive constraints. Whether this outweighs any disadvantages of more information due to individual rationality constraints, for two given information structures, may vary depending on the structures being compared; however, it is transparent that a universal result such as Theorem 1 must fail without the independence restriction. We do note that independence of ω and θ may be a natural assumption if, for instance, ω does not directly affect utilities, so that ω strictly represents randomness in endowments and θ randomness in preferences; the public signal would then contain only information about endowments in the environments for which our results hold.

4. CONCLUSION

We have provided a concrete sense in which better public information about an unknown state of the world has negative value to an economy in which state-contingent claims can be enforced. The paper provides a generalization of the results of Campbell (2002) by allowing for agents with private information. The intuition for the result holds across both papers: when state-contingent allocations can be agreed to, better information about the final state has no instrumental value. However, agents' willingness to agree to any allocation depends on their expected welfare from rejecting the allocation and instead consuming their endowment. This individual rationality is easier to satisfy when there is less information, as less information may simply be thought of as a garbled mixture of better information: if individual rationality holds for each realization of the better information, it must hold for the mixture as well.

The result does depend critically on the restriction that the public information in question is unrelated to the agents' private information. Though this restriction leaves only a subclass of environments within all possible structures of public signals, it is nevertheless a legitimate extension of environments in which there is no private information, as independence between private and public information holds there trivially.

APPENDIX

Proof of Theorem 1. We first prove the "if" statement. We must do so for each of the five incentive compatibility criteria.

For $j \in \{1, 2, 3\}$, the proof structure is similar. Begin with an arbitrary environment E, and two information structures (Y', Π') and (Y, Π), with the former more informative than the latter in Blackwell's sense. If $H_j(E, \zeta')$ is empty, then the claim holds trivially. Else, choose an arbitrary array of signal-dependent direct revelation mechanisms implementable under standard j for ζ', represented by $(\tilde{X}'_{y'}(\omega, \theta), \delta'_{\omega,\theta_{y'}})$. It must be shown that their induced ex-ante social choice function is in $H_j(E, \zeta)$.

Under information structure ζ, when signal y_n with $\gamma_n > 0$ is observed, construct an array of signal-dependent direct revelation mechanisms $(\tilde{X}_y(\omega, \theta), \delta_{\omega,\theta_y})$ as follows:

$$\tilde{X}_{y_n}(\omega_m, \theta) = \cup_{y' \in Y'} \tilde{X}'_{y'}(\omega_m, \theta)$$
$$\delta_{\omega_m,\theta_{y_n}}(x) = \sum_{n'} \frac{T_{nn'} P'_{n'm}}{P_{nm}} \delta'_{\omega_m,\theta_{y'_{n'}}}(x).$$

We claim that the constructed mechanisms are necessarily implementable under standard j, for any $j \in \{1, 2, 3\}$. First, they are feasible: since for any ω_m they only put positive probability on allocations that are made with positive probability under $(\tilde{X}'_{y'}(\omega, \theta), \delta'_{\omega,\theta_{y'}})$, feasibility of the latter implies feasibility of the former. Next, they are individually rational. For convenience, write $u_i(x_i, \omega, \theta) - u_i(e_i(\omega), \omega, \theta)$ as $\tilde{u}_i(x_i, \omega, \theta)$.

Individual rationality of $(\tilde{X}'_{y'}(\omega,\theta), \delta'_{\omega,\theta_{y'}})$ means that for every n',

$$\sum_m \sum_{\theta\in\Theta} P'_{n'm}\rho(\theta|\theta_i) \sum_{x\in\tilde{X}'_{y'_{n'}}(\omega_m,\theta)} \delta'_{\omega_m,\theta_{y'_{n'}}}(x)\tilde{u}_i(x_i,\omega_m,\theta) \geq 0,$$

for all agents i and types θ_i. Consider now the relevant expression for the constructed $(\tilde{X}_{y_n}(\omega_m,\theta), \delta_{\omega_m,\theta_{y_n}})$:

$$\begin{aligned}
&\sum_m \sum_{\theta\in\Theta} P_{nm}\rho(\theta|\theta_i) \sum_{x\in\tilde{X}_{y_n}(\omega_m,\theta)} \delta_{\omega_m,\theta_{y_n}}(x)\tilde{u}_i(x_i,\omega_m,\theta)\\
=&\sum_m \sum_{\theta\in\Theta} P_{nm}\rho(\theta|\theta_i) \sum_{x\in\tilde{X}_{y_n}(\omega_m,\theta)} \sum_{n'} \frac{T_{n'n}P'_{n'm}}{P_{nm}}\delta'_{\omega_m,\theta_{y'_{n'}}}(x)\tilde{u}_i(x_i,\omega_m,\theta)\\
=&\sum_{n'} T_{n'n} \sum_m \sum_{\theta\in\Theta} P'_{n'm}\rho(\theta|\theta_i) \sum_{x\in\tilde{X}'_{y'_{n'}}(\omega_m,\theta)} \delta'_{\omega_m,\theta_{y'_{n'}}}(x)\tilde{u}_i(x_i,\omega_m,\theta).
\end{aligned}$$

Because $T_{n'n} \geq 0$ for all n' and n, and individual rationality is satisfied for every signal $y'_{n'}$, each element of the sum over n' is nonnegative. Thus, the sum is nonnegative, and individual rationality is satisfied for signal y_n.

We now show that if ICj is satisfied for array $(\tilde{X}'_{y'}(\omega,\theta), \delta'_{\omega,\theta_{y'}})$, it is satisfied for array $(\tilde{X}_y(\omega,\theta), \delta_{\omega,\theta_y})$, $j \in \{1,2,3\}$. Let $\delta_{\omega_m,\theta_i,s_{-i}(\theta_{-i})_{y_n}}(x) - \delta_{\omega_m,\theta'_i,s_{-i}(\theta_{-i})_{y_n}}(x) \equiv \Delta_{mn}(\theta_i,\theta'_i,s_{-i}(\theta_{-i}),x)$. To satisfy $IC1$, it is required that

$$\sum_m \sum_{\theta\in\Theta} P_{nm}\rho(\theta|\theta_i) \sum_{x\in\tilde{X}_{y_n}(\omega,\theta)} \Delta_{mn}(\theta_i,\theta'_i,s_{-i}(\theta_{-i}),x)u_i(x_i,\omega,\theta)$$

be nonnegative for all i, θ_i, θ'_i and $s_{-i}(\theta_{-i})$. By construction of $\delta_{\omega_m,\theta_i,s_{-i}(\theta_{-i})_{y_n}}$, we have

$$\Delta_{mn}(\theta_i,\theta'_i,s_{-i}(\theta_{-i}),x) = \sum_{n'} \frac{T_{n'n}P'_{n'm}}{P_{nm}}\Delta'_{m,n'}(\theta_i,\theta'_i,s_{-i}(\theta_{-i}),x),$$

and the critical expression may be written

$$\sum_{n'} T_{n'n} \sum_m \sum_{\theta\in\Theta} P'_{n'm}\rho(\theta|\theta_i) \sum_{x\in\cup_{n'}\tilde{X}'_{y'_{n'}}(\omega_m,\theta)} \Delta'_{m,n'}(\theta_i,\theta'_i,s_{-i}(\theta_{-i}),x)u_i(x_i,\omega_m,\theta).$$

If $(\tilde{X}'_{y'}(\omega,\theta), \delta'_{\omega,\theta_{y'}})$ satisfies $(IC1)$, then the expression for each n' in the sum is nonnegative, and thus the sum is nonnegative and $IC1$ is satisfied for $(\tilde{X}_y(\omega,\theta), \delta_{\omega,\theta_y})$.

For $IC2$ to hold, it is required that $\sum_m P_{nm} \sum_{x \in \tilde{X}_{y_n}(\omega_m,\theta)} \Delta_{mn}(\theta_i, \theta_i', \theta_{-i}, x) u_i(x_i, \omega_m, \theta)$ be nonnegative for all n, θ_i, θ_i' and θ_{-i}. Substituting for $\Delta_{mn}(\theta_i, \theta_i', \theta_{-i}, x) u_i(x_i, \omega_m, \theta)$ as before, this expression equals

$$\sum_{n'} T_{n'n} \sum_m P'_{n'm} \sum_{x \in \cup_{n'} \tilde{X}'_{y'_{n'}}(\omega_m,\theta)} \Delta'_{m,n'}(\theta_i, \theta_i', \theta_{-i}) u_i(x_i, \omega_m, \theta).$$

Every term n' is nonnegative if $(\tilde{X}'_{y'}(\omega,\theta), \delta'_{\omega,\theta_{y'}})$ satisfies $IC2$, and thus $(\tilde{X}_y(\omega,\theta), \delta_{\omega,\theta_y})$ satisfies $IC2$. Similarly, expression

$$\sum_{n'} T_{n'n} \sum_m \sum_{\theta \in \Theta} P'_{n'm} \rho(\theta|\theta_i) \sum_{x \in \cup_{n'} \tilde{X}'_{y'_{n'}}(\omega_m,\theta)} \Delta'_{m,n'}(\theta_i, \theta_i', \theta_{-i}) u_i(x_i, \omega_m, \theta)$$

is nonnegative if $(\tilde{X}'_{y'}(\omega,\theta), \delta'_{\omega,\theta_{y'}})$ satisfies $IC3$, in which case $(\tilde{X}_y(\omega,\theta), \delta_{\omega,\theta_y})$ also satisfies $IC3$.

Finally, we must show that $(\tilde{X}'_{y'}(\omega,\theta), \delta'_{\omega,\theta_{y'}})$ and $(\tilde{X}_y(\omega,\theta), \delta_{\omega,\theta_y})$ induce the same ex-ante social choice function. The ex-ante probability that allocation x is chosen in state (ω_m, θ) under $(\tilde{X}_y(\omega,\theta), \delta_{\omega,\theta_y})$ is

$$\begin{aligned}
&\sum_n \pi_{mn} \delta_{\omega_m,\theta_{y_n}}(x) \\
=&\sum_n \pi_{mn} \sum_{n'} \frac{T_{n'n} P'_{n'm}}{P_{nm}} \delta'_{\omega_m,\theta_{y'_{n'}}}(x) \\
=&\sum_n \pi_{mn} \sum_{n'} \frac{T_{n'n} \pi'_{mn'} \gamma_n}{\pi_{mn} \gamma_{n'}} \delta'_{\omega_m,\theta_{y'_{n'}}}(x) \\
=&\sum_{n'} \pi_{mn'} \delta'_{\omega_m,\theta_{y'_{n'}}}(x) \sum_n \frac{T_{n'n} \gamma_n}{\gamma_{n'}} \\
=&\sum_{n'} \pi_{mn'} \delta'_{\omega_m,\theta_{y'_{n'}}}(x),
\end{aligned}$$

where the final equality follows from the fact that $\gamma' = \gamma T$, and thus that $\sum_n \frac{T_{n'n}\gamma_n}{\gamma_{n'}} = 1$ for all n'.

This completes the proof for $j \in \{1, 2, 3\}$; we proceed to $j \in \{4, 5\}$. Let $\hat{m}$ be a profile of messages for the agents, $\hat{m} \in \mathcal{M}^I$. A mechanism g may be written $(\tilde{X}(\omega, \hat{m}), \delta_{\omega,\hat{m}})$. $(\tilde{X}'_{y'_{n'}}(\omega, \hat{m}), \delta_{\omega,\hat{m}_{y'_{n'}}})$ is an array of mechanisms, one for each $y'_{n'} \in Y'$. Take as given that $(\tilde{X}'_{y'_{n'}}(\omega,\hat{m}), \delta_{\omega,\hat{m}_{y'_{n'}}})$ implements social choice functions $(\tilde{X}'_{y'_{n'}}(\omega,\theta), \delta_{\omega,\theta_{y'_{n'}}})$, and corresponding ex-ante social choice function $(\tilde{X}'(\omega,\theta), \delta'_{\omega,\theta})$, for information structure ζ'.

We now construct an array of signal-dependent mechanisms for information structure ζ that duplicate ex-ante social choice function $(\tilde{X}'(\omega,\theta), \delta'_{\omega,\theta})$. For each signal, the message space in the associated mechanism is $\mathcal{M}^{N'}$ with typical element $\tilde{m}$, where N' is the number of signals in Y'. Let $\tilde{m}_i^{n'}$ be the n'th component of agent i's message, and $\tilde{m}^{n'}$ be the profile of n'th components. The mechanism for realized signal y_n is $\tilde{X}_{y_n}(\omega,\tilde{m}) = \cup_{n'}\tilde{X}'_{y'_{n'}}(\omega,\tilde{m}^{n'})$ and

$$\delta_{\omega_m,\tilde{m}_{y_n}}(x) = \sum_{n'} \frac{T_{n'n}P_{n'm}}{P_{nm}} \delta'_{\omega_m,\tilde{m}^{n'}{}_{y'_{n'}}}(x).$$

As before, this array of mechanisms is feasible by feasibility of $(\tilde{X}'_{y'_{n'}}(\omega,\theta), \delta_{\omega,\theta_{y'_{n'}}})$. To show implementation, fix any realized signal y_n. The constructed mechanism ex post and randomly chooses a signal $y'_{n'}$ using probabilities $T_{n'n}P'_{n'm}/P_{nm}$, and implements the original mechanism associated with that signal, using the n'th message in each agent's vector as the message argument of the original mechanism. Thus, there is no interaction between the components of a given agent's message, and the sent messages do not affect the probabilities with which n' is chosen. In particular, a message profile is an equilibrium if and only if the profile of n'th components is an equilibrium conditional on the n'th signal being chosen. The probability that state m is realized given signal n, and given that the mechanism associated with n' is implemented, is exactly $P'_{n'm}$. Thus, the set of message profiles that are equilibrium under common posterior $P'_{n'm}$ are identical to the set of equilibria for the n'th component under the constructed mechanism. Furthermore, the ex-ante probability that the allocation associated with state m and signal n' obtains is exactly $\lambda_m \pi_{mn'}$. Thus, the identical ex-ante social choice rule is implemented. Finally, because the interim utility to each agent from the mechanism for signal n is a weighted mixture of the utilities from the signals n', each of which is sufficient for individual rationality, the constructed mechanism also satisfies individual rationality for each n.

The proof of "only if" may be found in Campbell (2002), in which an environment E is constructed in which containment of implementable ex-ante social choice functions fails for any γ' that is not ordered ahead of γ by Blackwell's ordering. ∎

REFERENCES

D. Blackwell, "Comparison of Experiments," in J. Neymann, ed., *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability.* Berkeley: University of California Press, 1951, 93-102.

D. Blackwell and M. A. Girshick *Theory of Games and Statistical Decisions.* New York: John Wiley and Sons, 1954.

C. M. Campbell, "Blackwell's Ordering and Public Information," mimeo, 2002.

B. Eckwert and I. Zilcha, Incomplete Risk-Sharing Arrangements and the Value of Information, *Journal of Economic Theory* 100 (2001), 172-186.

J. Green, Value of Information with Sequential Futures Markets, *Econometrica* 49 (1981), 335-358.

J. Hirshleifer, The Private and Social Value of Information and the Reward to Incentive Activity, *American Economic Review* 61 (1971), 561-574.

R. E. Kihlstrom, "A 'Bayesian' Exposition of Blackwell's Theorem on the Comparison of Experiments," in M. Boyer and R. E. Kihlstrom, eds., *Bayesian Models of Economic Theory.* Amsterdam: North-Holland, 1984.

J. M. Marshall, Private Incentives and Public Information, *American Economic Review* 64 (1974), 373-390.

E. E. Schlee, The Value of Information in Efficient Risk Sharing Arrangements, *American Economic Review* 91 (2001), 509-524.