The Power of Forward Guidance Revisited

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Abstract

In recent years, central banks have increasingly turned to "forward guidance" as a central tool of monetary policy, especially as interest rates around the world have hit the zero lower bound. Standard models imply that far future forward guidance is extremely powerful: promises about far future interest rates have huge effects on current economic outcomes, and these effects grow with the horizon of the forward guidance. We show that the power of forward guidance is highly sensitive to the assumption of complete markets. If agents face uninsurable income risk and borrowing constraints, a precautionary savings effect tempers their responses to far future promises about interest rates. As a consequence, the ability of central banks to combat recessions using small changes in interest rates far in the future, is greatly reduced relative to the complete markets benchmark. We show that the incomplete markets model induces behavior analogous to a complete markets model with "discounting" in the consumption Euler equation.

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1 Introduction

Forward guidance has become an increasingly important tool of monetary policy in recent years. Gurkaynak, Sack, and Swanson (2005) show that much of the surprise news about monetary policy at the time of FOMC announcements arises from signals about the central banks intentions about future monetary policy. In many cases, changes in the current Federal Funds rate are fully expected, and all of the *news* about monetary policy has to do with how the central bank is expected to set interest rates in the future.[1]

Promises about future interest rates have been shown to have a powerful effect on the economy in standard macroeconomic models. Eggertsson and Woodford (2003) show that a shock to the natural rate of interest that causes the economy to hit the zero lower bound on nominal interest rates induces a powerful deflationary spiral and a crippling recession. However, the recession can be entirely abated if the central bank commits from the outset to holding interest rates at the zero lower bound for a few additional quarters beyond what is justified by contemporaneous economic conditions.

Recent work argues that the magnitude of the effects of forward guidance in New Keynesian models stretches the limits of credibility. Carlstrom, Fuerst, and Paustian (2012) show that a promise by the central bank to peg interest rates below the natural rate of interest for a finite number of quarters generates explosive dynamics for inflation and output in a workhorse New Keynesian model (the Smets and Wouters (2007) model).[2] Del Negro, Giannoni, and Patterson (2013) refer to this phenomenon as the *forward guidance puzzle*. Along the same lines, consider an experiment whereby the central bank promises a 1 percentage point lower real interest rate *for a single quarter* at some point in the future. We show that in the plain vanilla New Keynesian model, this promise has an eighteen times greater impact on inflation when the promise pertains to interest rates 5 years in the future than when it pertains to the current interest rate.

It may seem unintuitive that an interest rate cut far in the future has a greater effect than a near-term one. To see why this arises in standard models, consider the response of consumption

[1] Campbell et al. (2012) reinforce these results using a longer sample period spanning the Great Recession.

[2] Carlstrom, Fuerst, and Paustian (2012) show that the responses of output and inflation experience a sign reversal when the interest rate peg is extended from eight quarters to nine quarters in the Smets and Wouters (2007) model. We interpret this as the model "blowing up" at this point, i.e., the monetary stimulus becomes so large that the response of output and inflation is infinite. This manifests itself as sign reversals in the set of linear equations used to describe the model. Intuitively, if you divide by a smaller and smaller number, the number you divide by can eventually switch sign. See the discussion of the "deflationary death spiral" in section 5 for an example of this.

to a decrease in the real interest rate for a single quarter 5 years in the future. The consumption Euler equation dictates that consumption will rise immediately to a higher level and stay constant at that higher level for 5 years before returning to its normal level.[3] The cumulative response of consumption to the shock is therefore quite large and gets larger the further in the future the interest rate shocks occurs. It is the cumulative response of consumption (with some discounting) that determines the response of current inflation in the basic New Keynesian model. So, the further in the future is the interest rate that the monetary authority announces it will change, the larger is the current response of inflation. At the zero lower bound, this large effect on inflation will affect real rates and thus create a powerful feedback loop on output.

But is it a realistic prediction of the standard model that agents increase their consumption by the same amount in response to an interest rate change 5 years in the future as they do to a change in the current interest rate? An individual who raises his consumption 5 years in advance of an anticipated interest rate cut will need to run down his savings for 5 years. For many agents, this is not feasible since it would entail hitting a borrowing constraint. More generally, as an agent's assets fall, the marginal value of wealth increases for precautionary reasons, raising the motive to save. This precautionary savings effect counteracts the intertemporal substitution motive to dissave in anticipation of the low interest rate. As the low interest rate is further in the future, the change in assets needed to take full advantage of intertemporal substitution grows and the precautionary savings effect therefore grows stronger, tempering the effects of forward guidance.

To investigate the quantitative magnitude of these effects, we consider a general equilibrium model in which agents face uninsurable, idiosyncratic income risk and borrowing constraints. In this model, the effect of forward guidance about future interest rates on current output falls the further out in the future the interest rate change is. For forward guidance about the interest rate 5 years in the future, the effect on output and inflation is roughly 40% as large as in the standard model. For forward guidance about the interest rate 10 years in the future, the effect on current output is essentially zero.

Our results indicate that forward guidance is a much less effective policy tool at the zero lower

[3] The response is a step function because consumption growth only deviates from normal when the real interest rate deviates from normal and this only occurs in the single quarter that the forward guidance pertains to. Another way to see this is that the the forward guidance does not change the relative price of consumption for any two dates before the date of the interest rate change. All these dates must therefore have the same level of consumption. The end-point of consumption is pinned down by the fact that the economy will eventually return to its steady state.

bound in a model with a realistic degree of precautionary savings than it is in standard macro models. We consider a shock that lowers the natural rate of interest enough that the zero lower bound binds for 5 years and the initial fall in output is -4% in the absence of forward guidance. If we assume markets are complete (and precautionary savings thus absent), a policy of maintaining interest rates at zero for a little more than three quarters beyond what a strict inflation targeting central bank would do completely eliminates the fall in output. In contrast, in our incomplete markets model with idiosynctratic risk and borrowing constraints, the effect of this amount of forward guidance is substantially smaller and a significant recession remains.

Returning to the intuition for the forward guidance puzzle, a key reason why the puzzle occurs is that the sensitivity of current output to far future interest rates is the same as the sensitivity to current interest rates. The key property of our incomplete markets model that reduces the effects of far future forward guidance is that this is no longer the case. We show that the responsiveness of output in the incomplete markets model can be approximated by a consumption Euler equation with "discounting."[4] Furthermore, we show that this "discounted Euler equation" can be micro-founded with a simplified version of our incomplete markets model.[5] This formulation has the advantage of being highly tractable, and easy to incorporate into the DSGE models used for policy analysis at central banks around the world.

We use the "discounted Euler equation model" to revisit the question of how severe are the deflationary effects of shocks that lead the natural real rate of interest to become negative. In particular, we consider a shock that lowers the natural rate of interest to -2% and keeps it their on average for 10 quarters (this is the shock considered in Eggertsson and Woodford, 2003). In the plain vanilla New Keynesian model, this shock leads output to fall by roughly 14% and inflation to fall by roughly 10%. In contrast, in our "discounted Euler equation model" this same shock generates a much more modest drop in output of 3% and inflation of 2%. It is well known that shocks like these can lead the economy into a "deflationary death spiral" (i.e., log output and inflation go to negative infinity) if they are persistent enough and monetary policy is unable to commit to time-inconsistent forward guidance. In the plain vanilla New Keynesian model, the

[4] Interestingly, such a formulation of the consumption Euler equation has actually been used in policy calculations by the Central Bank of Norway, to combat the forward guidance puzzle. We thank Oistein Roisland for pointing this out to us.

[5] The main simplification is that instead of facing a rich distribution of idiosyncratic shocks we assume that a fraction of agents are hit by an uninsurable expenditure shock that is large enough that they hit their borrowing constraint for sure.

deflationary death spiral occurs even for shocks that are expected to last only roughly 3 years on average. In our discounted Euler equation model, however, the deflationary death spiral occurs only for shocks that are considerably more persistent.

Our work builds on recent papers that incorporate market incompleteness and idiosyncratic uncertainty into New Keynesian models starting with Guerrieri and Lorenzoni (2011) and Oh and Reis (2012). The papers closest in spirit to ours are McKay and Reis (2014) who investigate the power of automatic stabilizers at the zero lower bound and Gornemann, Kuester, and Nakajima (2014) who investigate the distributional implications of monetary policy shocks. Several other recent papers suggest "solutions" to the forward guidance puzzle. Del Negro, Giannoni, and Patterson (2013) argue that the experiment that gives rise to the puzzle is, itself, unreasonable. They argue that it is unreasonable to assume that the central bank really can engender substantial changes in long-term interest rates, which are at the core of why the forward guidance puzzle arises. Carlstrom, Fuerst, and Paustian (2012) and Kiley (2014) show that the magnitude of the forward guidance puzzle is substantially reduced in a sticky information (as opposed to a sticky price) model.

The paper proceeds as follows. Section 2 explains why forward guidance is so powerful in standard New Keynesian models. Section 3 presents our incomplete markets model featuring uninsurable idiosyncratic income risk and borrowing constraints. Section 4 describes our results about the reduced power of forward guidance in our incomplete markets model relative to the standard complete markets models. Section 5 show how the behavior of the incomplete markets model can be approximated by a model with discounting in the consumption Euler equation. Section 6 concludes.

2 Why Is Forward Guidance So Powerful?

It is useful to start with an explanation for why forward guidance is so powerful in the standard model. Consider the basic New Keynesian model as developed, e.g., in Woodford (2003) and Gali (2008). The implications of private sector behavior for output and inflation in this model can be described by an intertemporal "IS" equation of the form

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n), \tag{1}$$

and a Phillips curve of the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \tag{2}$$

Here, x_t denotes the output gap—i.e., the percentage difference between actual output and the natural rate of output that would prevail if prices are fully flexible—π_t denotes inflation, i_t denotes the nominal, short-term, risk-free interest rate, r_t^n denotes the natural real rate of interest—i.e., the real interest rate that would prevail if prices were fully flexible—σ denotes the intertemporal elasticity of substitution, β denotes the subjective discount factor of households, and κ is determined by the degree of nominal and real rigidities in the economy.

Suppose for simplicity that the monetary policy of the central bank is given by an exogenous rule for the real interest rate where the real interest rate tracks the natural real rate with some error: $r_t = i_t - E_t\pi_{t+1} = r_t^n + \epsilon_{t,t-j}$, where $\epsilon_{t,t-j}$ denotes the shock to the short term real rate in period t that becomes known in period $t-j$.[6] Absent any monetary shocks, the real interest rate will perfectly track the natural real rate and both the output gap and inflation will be zero. Suppose we start in such a state, but then the monetary authority announces that the real interest rate will be lower by 1% for 1 quarter 5 years in the future, but maintained at the natural real rate of interest in all other quarters (i.e., $\epsilon_{t+20,t} = 0.01$).

To see how this forward guidance announcement affects the output gap, it is useful to solve the intertemporal IS equation forward to get

$$x_t = -\sigma \sum_{j=0}^{\infty} E_t(i_{t+j} - E_{t+j}\pi_{t+j+1} - r_{t+j}^n). \tag{3}$$

Notice, that there is no discounting in the sum on the right hand side of this equation. This implies that the output gap will rise immediately by 1% (if we assume for simplicity that $\sigma = 1$) and will stay at that higher level for the next five years and then fall back to zero all at once when the low interest rate period passes. The same is true of forward guidance for any other horizon. So, the further out in the future the forward guidance is, the larger is the *cumulative* response of output.

To see how the announcement affects inflation, it is useful to solve the Phillips curve forward to get

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j E_t x_{t+j}. \tag{4}$$

[6] Given this specification of monetary policy, the model has a unique solution for which $\lim_{j\to\infty} E_t x_{t+j} = 0$ and inflation is bounded. We could alternatively assume that the monetary authority sets the nominal rate according to the following rule $i_t = r_t^n + \phi\pi_t + \tilde{\epsilon}_{t,t-j}$ and $\phi > 1$. In this case, the model has a unique bounded solution (without the additional restriction that $\lim_{j\to\infty} E_t x_{t+j} = 0$) and there exists a path for $\tilde{\epsilon}_{t,t-j}$ that gives the same solution as the model with monetary policy given be the exogenous path for the real rate we assume. We prefer to describe the monetary policy as an exogenous rule for the real interest rate because this simplifies our exposition substantially.

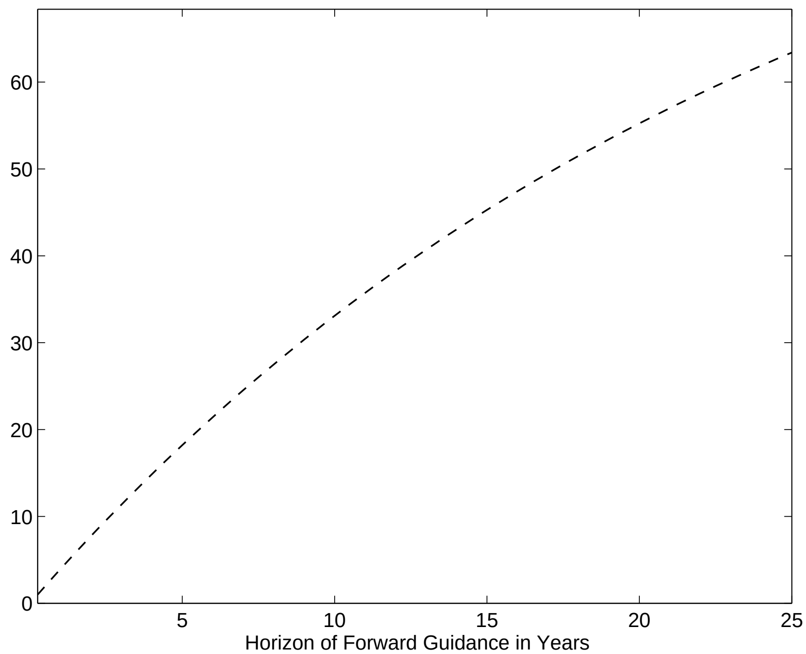

Figure 1: Response of current inflation to forward guidance about interest rates at different horizons relative to response to equally large change in current real interest rate.

This shows that it is the entire cumulative response of the output gap (albeit with some discounting) that determines the current response of inflation to forward guidance. The further in the future is the interest rate that the monetary authority announces it will change, the larger is the current response of inflation. While the response of inflation to a 1% change in the current real rate is $\kappa\sigma$, the response of inflation to a 1% change in the real rate for one quarter in the infinite future is $\kappa\sigma/(1-\beta)$. If $\beta = 0.99$, the current response of inflation to forward guidance about a single quarter in the infinite future is 100 times larger than the response of inflation to an equally large change in the current real interest rate. Figure 1 plots the response of inflation to forward guidance about interest rates at different horizons relative to the response of inflation to an equally large change in the current real interest rate. We see that the response of inflation to forward guidance about interest rates five years in the future is roughly 18 times larger than the response of inflation to an equally sized change in the current real interest rate.

Above, we have assumed that the monetary authority can perfectly control the real interest rate period-by-period. A prominent example of a situation where this is not the case is when the short-term nominal interest rate is at its zero lower bound. In this case, if the monetary authority is able to lower expectations about future real rates, the resulting increase in current inflation will lower current real interest rates and this will in turn raise current output. At the zero lower bound, forward guidance about future real interest rates will therefore—through this feedback loop—lead to potentially very large increases in current output. These effects on current output become larger the further in the future the interest rate is that the monetary authority provides forward guidance about. In other words, the standard model implies that forward guidance about far future real interest rates is a very powerful tool to stabilize the economy at the zero lower bound.

3 An Incomplete Markets Model with Nominal Rigidities

Section 2 shows that the huge power of far future forward guidance in the standard model depends crucially on the prediction of the model that the current response of output to an expected change in real interest rates in the far future (say 20 years in the future) is equally large as the response of output to a change in the current real interest rate. But is this realistic? Increasing consumption today by 1% in anticipation of a 1% change in real interest rates 20 years from today would entail a large run down of assets over the 20 years until the interest rate changes. Agents that face uninsurable idiosyncratic income risk and borrowing constraints will trade off the benefits of intertemporal substitution and the costs in terms of reduced ability to smooth consumption over time of having lower buffer stock savings. To analyze this trade-off we develop a model with uninsurable idiosyncratic shocks to household productivity, borrowing constraints, and nominal rigidities.

3.1 The Environment

The economy is populated by a unit continuum of ex ante identical households with preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{\ell_t^{1+\psi}}{1+\psi} \right],$$

where c_t is consumption and ℓ_t is labor supply. Households are endowed with stochastic idiosyncratic productivity z_t that generates pre-tax labor income $w_t z_t \ell_t$, where w_t is the aggregate real wage. Each household's productivity z_t follows a Markov chain with transition probabilities $\Pr(z_{t+1}|z_t)$. We assume that the initial cross-sectional distribution of idiosyncratic productivities is equal to the ergodic distribution of this Markov chain. As the the Markov chain transition matrix is constant over time, it follows that the cross-sectional distribution of productivities is constant. We use $\Gamma^z(z)$ to denote this distribution.

In this economy, a final good is produced from intermediate inputs according to the production function

$$y_t = \left(\int_0^1 y_t(j)^{1/\mu} dj \right)^\mu ,$$

and the intermediate goods are produced using labor as an input according to the production function

$$y_t(j) = N_t(j).$$

The market structure of this model economy combines elements that are familiar from the standard New Keynesian model with elements that are familiar from the standard incomplete markets model (Bewley, undated; Huggett, 1993; Aiyagari, 1994). While the final good is produced by a representative competitive firm, the intermediate goods are produced by monopolistically competitive firms. The intermediate goods firms face frictions in adjusting their prices that imply that they can only update their prices with probability θ per period as in Calvo (1983). These firms are controlled by a risk-neutral manager who discounts future profits at rate β. Whatever profits are produced are paid out immediately to the households with each household receiving an equal share d_t. Households cannot trade their stakes in the firms.

Households trade a risk-free real bond with real interest r_t between periods t and $t+1$. Borrowing constraints prevent these households from taking negative bond positions. There is a stock of government debt outstanding with real face value B. The government raises tax revenue to finance interest payments on this debt. These taxes are collected by taxing households according to their labor productivity z. Let $\tau_t \bar{\tau}(z)$ be the tax paid by a household with productivity z at date t. By levying the taxes on labor productivity, which is exogenous, the tax does not distort household decisions in the same way that a lump-sum tax does not. At the same time, the dependence of the tax on z allows us to manipulate the cross-sectional correlation of tax payments and earnings.

We assume that the government runs a balanced budget so as to maintain a stable level of debt in each period. The government budget constraint is

$$\frac{B}{1+r_t} + \sum_z \Gamma^z(z) \tau_t \bar{\tau}(z) = B. \tag{5}$$

To illustrate our main results about the power of forward guidance, we will consider several monetary policy experiments involving somewhat different specifications of monetary policy. These are described in Section 4. The relationship between the real interest rate, the nominal interest rate, and inflation is given by the Fisher relation in the usual way

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \tag{6}$$

where $\pi_{t+1} \equiv p_{t+1}/p_t - 1$ and p_t is the aggregate price level.

3.2 Decision Problems

The decision problem faced by the households in the economy is

$$V_t(b_t, z_t) = \max_{c_t, b_{t+1}, \ell_t} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} - \frac{\ell_t^{1+\psi}}{1+\psi} + \beta \sum_{z_{t+1}} \Pr(z_{t+1}|z_t) V_{t+1}(b_{t+1}, z_{t+1}) \right\}$$

subject to

$$c_t + \frac{b_{t+1}}{1 + r_t} = b_t + w_t z_t \ell_t - \tau_t \bar{\tau}(z_t) + d_t$$

$$b_{t+1} \geq 0.$$

Let $c_t(b, z)$, $g_t(b, z)$, and $\ell_t(b, z)$ be the decision rules for consumption, savings (i.e., b_{t+1}), and labor supply, respectively. These policy rules vary over time in response to aggregate events that affect current or future prices, taxes, or dividends.

The final goods producer's cost minimization problem implies that

$$y_t(j) = \left(\frac{p_t(j)}{p_t} \right)^{\mu/(1-\mu)} y_t, \tag{7}$$

where the aggregate price index is given by

$$p_t = \left(\int_0^1 p_t(j)^{1/(1-\mu)} dj \right)^{1-\mu}.$$

The intermediate goods producer's problem is

$$\max_{p_t^*, \{y_s(j), N_s(j)\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^t (1-\theta)^t \left(\frac{p_t^*}{p_s} y_s(j) - w_s N_s(j) \right)$$

subject to

$$y_s(j) = \left(\frac{p_t^*}{p_s} \right)^{\mu/(1-\mu)} y_s,$$

$$y_s(j) = N_s(j).$$

The solution to this problem satisfies

$$\frac{p_t^*}{p_t} = \frac{\sum_{s=t}^{\infty} \beta^{s-t}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} y_s \mu w_s}{\sum_{s=t}^{\infty} \beta^{s-t}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} y_s}. \tag{8}$$

3.3 Equilibrium

Let $\Gamma_t(b, z)$ be the distribution of households over idiosyncratic states at date t. This distribution evolves according to

$$\Gamma_{t+1}(\mathcal{B}, z') = \int_{\{(b,z):g_t(b,z)\in\mathcal{B}\}} \Pr(z'|z) d\Gamma_t(b, z) \tag{9}$$

for all sets $\mathcal{B} \subset \mathbb{R}$.

As a result of nominal rigidities, price dispersion will result in some loss of efficiency. Integrating both sides of (7) across firms and using $y_t(j) = N_t(j)$ yields an aggregate production function

$$S_t y_t = \int N_t(j) dj \equiv N_t, \tag{10}$$

where N_t is aggregate labor demand and $S_t \equiv \int_0^1 \left(\frac{p_t(j)}{p_t}\right)^{\mu/(1-\mu)} dj$ reflects the efficiency loss due to price dispersion. S_t evolves according to

$$S_{t+1} = (1-\theta) S_t \left(1 + \pi_{t+1}\right)^{-\mu/(1-\mu)} + \theta \left(\frac{p_{t+1}^*}{p_{t+1}}\right)^{\mu/(1-\mu)}. \tag{11}$$

Inflation can be written as a function of the relative price selected by firms that update their prices

$$1 + \pi_t = \left(\frac{1-\theta}{1 - \theta \left(\frac{p_t^*}{p_t}\right)^{1/(1-\mu)}}\right)^{1-\mu}. \tag{12}$$

Aggregate labor supply is given by

$$L_t \equiv \int z \ell_t(b, z) d\Gamma(b, z). \tag{13}$$

and labor market clearing requires

$$L_t = N_t. \tag{14}$$

Bond market clearing requires

$$B = \int g_t(b, z) d\Gamma_t(b, z). \tag{15}$$

Table 1: Baseline calibration

Parameter	Description	Value	Target
β	discount factor	0.986	2% annual interest rate
γ	risk aversion	2	
$1/\psi$	Frisch elasticity	1/2	Chetty (2012)
B	supply of assets	1.4 × annual GDP	aggregate liquid assets (see text)
μ	markup	1.2	Christiano, Eichenbaum, and Rebelo (2011)
θ	price revision rate	0.15	Christiano, Eichenbaum, and Rebelo (2011)

The aggregate dividend paid by the intermediate goods firms is

$$d_t = y_t - w_t N_t. \tag{16}$$

Finally, integrating across the household budget constraints and using the government budget constraint and equation (16) gives

$$C_t = y_t \tag{17}$$

as the aggregate resource constraint, where $C_t \equiv \int c_t(b,z) d\Gamma_t(b,z)$.

An equilibrium of this economy consists of decision rules and value functions $\{g_t(b,z),\ \ell_t(b,z),\ V_t(b,z)\}_{t=0}^{\infty}$ that solve the household's problem, distributions $\{\Gamma_t(b,z)\}_{t=0}^{\infty}$ that evolve according to (9). In addition, an equilibrium involves sequences $\{C_t,\ L_t,\ N_t,\ y_t,\ d_t,\ i_t,\ w_t,\ \pi_t,\ r_t,\ p_t^*/p_t,\ S_t,\ \tau_t\}_{t=0}^{\infty}$, that satisfy the definitions of C_t and L_t and equations (5), (6), (8), (10), (11), (12), (14), (16), (17), and a monetary policy rule as described in section 4.

3.4 Calibration

Our model period is one quarter and our calibration is summarized in Table 1. We fix the steady state real interest rate at 2% annually and adjust the discount factor to match this.[7] We set the coefficient of risk aversion to 2. We set the Frisch elasticity of labor supply to 1/2, which is in line

[7] We use the term "steady state" to refer to the stationary equilibrium in which aggregate quantities and prices are constant and inflation is zero.

with the findings of Chetty (2012). In our baseline calibration we set the supply of government bonds, B, to match the ratio of aggregate liquid assets to GDP. We calculate liquid assets from aggregate household balance sheets reported in the Flow of Funds Accounts and take the average ratio over the period 1970 to 2013.[8] Our choice to calibrate the aggregate supply of assets to match liquid assets is motivated by the view that much of household net worth is illiquid and therefore not easily used for consumption smoothing and intertemporal substitution.[9] In a sensitivity analysis we also consider a calibration in which we match aggregate household net worth, which we also calculate from the Flow of Funds Accounts (described below).

For our choices of the desired markup of intermediate firms, μ, and probability of maintaining a fixed price, θ, we follow Christiano, Eichenbaum, and Rebelo (2011) and set $\mu = 1.2$ and $\theta = 0.15$. These parameters values are conservative for the power of forward guidance as the elasticity of substitution of 6 implied by our value of μ is on the low side and the degree of price stickiness is on high side of values used in the business cycle literature, both of which make inflation less sensitive to changes in current marginal costs. With a lower elasticity of substitution between varieties, the aggregate price index responds by less when firms set low relative prices as shown by equation (12).[10]

We calibrate the idiosyncratic wage risk to the persistent component of the estimated wage process in Floden and Lindé (2001).[11] The estimates of Floden and Lindé are for an AR(1) with annual observations of log residual wages after the effects of age, education, and occupation have been removed. Floden and Lindé find an autoregressive coefficient of 0.9136 and an innovation variance of 0.0426. We convert these estimates to parameters of a quarterly AR(1) process for log wages by simulating the quarterly process and aggregating to annual observations. We find the parameters of the quarterly process such that estimating an AR(1) on the simulated annual data reproduces the Floden and Lindé estimates, which results in an autoregressive coefficient of 0.966

[8] We use the same definition of liquid assets as Guerrieri and Lorenzoni (2011). Flow of Funds Table B.100 Lines 10 (deposits), 17 (treasury securities), 18 (agency and GSE securities), 19 (municipal securities), 20 (corporate and foreign bonds), 24 (corporate equities), 25 (mutual fund shares).

[9] Kaplan and Violante (2014) present a lifecycle savings model with liquid and illiquid assets and show that the illiquidity of household net worth leads to stronger and more realistic consumption responses to transitory income fluctuations.

[10] That the sensitivity of inflation to p_t^*/p_t depends on μ is a feature of the non-linear model that is lost when equation (12) is linearized.

[11] While it is common to include a transitory income shock in empirical models of wage dynamics we do not include such transitory shocks in our analysis because their impact on the quantitative results will be small as these shocks are easily smoothed be virtue of being transitory.

and an innovation variance of 0.017. We discretize the resulting AR(1) process for log wages to a three-point Markov chain using the Rouwenhorst (1995) method.[12] Finally, to capture the fact that the bulk of tax payments are made by those with high earnings we set $\bar{\tau}(z)$ to be positive only for the highest z.

3.5 Alternative Calibrations

Our baseline calibration potentially implies too little volatility in household earnings. Guvenen, Ozkan, and Song (2014) report the standard deviation of the distribution of five-year earnings growth rates to be 0.73.[13] Our model calibrated as described above implies this standard deviation is only 0.53.

We therefore consider an alternative calibration in which we raise the variance of the productivity shocks in the model so that our model matches this moment of the five-year earnings growth rate distribution. Doing so requires raising the variance of the idiosyncratic productivity innovation from 0.017 to 0.033. With more risk, the larger precautionary savings motive raises the total demand for assets by households. In this calibration we, thus, reduce the discount factor so that the model is again consistent with the total supply of assets and a 2% annual interest rate. This requires a discount factor of 0.978. We refer to this as the High Risk calibration.

We also explore the extent to which our results depend on the average level of assets in the economy. With more assets, households will generally have more self-insurance and therefore will be less concerned with running down their assets. To explore this possibility we consider an alternative calibration in which we raise the supply of government debt, B, so that the average wealth in the economy is equal to the aggregate net worth of the household sector from the Flow of Funds, including both liquid and illiquid wealth.[14] This yields a ratio of assets to annual GDP of 3.79. With a larger supply of assets, bond market clearing requires that households are more patient so as to increase the demand for assets at a given interest rate. In this calibration, we set the discount factor to 0.992 to be able to match a 2% annual interest rate. We refer to this as the High Asset calibration.

[12]Kopecky and Suen (2010) prove that the Rouwenhorst method can match the conditional and unconditional mean and variance, and the first-order autocorrelation of any stationary AR(1) process.

[13]This value is the average across years of the values reported in Table A8 of Guvenen, Ozkan, and Song (2014).

[14]Here we use the ratio of household net worth to GDP averaged over 1970 to 2013. Household net worth is taken from Table B.100 Line 42.

Finally we consider a case where we increase both the supply of assets and the extent of risk that households face. In this case we match a ratio of assets to GDP of 3.79 and the standard deviation of five-year earnings growth rates of 0.73. The discount factor needed to match a 2% annual interest rate is 0.990. We refer to this as the High Risk and Asset calibration.

3.6 Computation

In Section 4, we compute the perfect foresight transition paths of the economy in response to monetary policy and demand shocks. We assume that the economy begins in the steady state and returns to steady state after 250 quarters. We begin by guessing paths for all aggregate quantities and prices. We can then verify whether this guess is an equilibrium by checking that the definition of an equilibrium given above is satisfied. Part of this step involves solving and simulating the households problem at the guessed prices. We solve the household's problem by iterating on the Euler equation backwards through time using the endogenous gridpoint method of Carroll (2006) to compute the policy rules for each period of the transition. We then simulate the population of households forwards through time using a non-stochastic simulation algorithm to compute the distribution of wealth at each date. We can then compute aggregate choices using the policy rules and distribution of wealth for each date. If our guess is not an equilibrium we update it to a new guess that is closer to an equilibrium. We generate the new guess of prices and aggregate quantities by making use of an auxiliary model that approximates the aggregate behavior of the incomplete markets households and then solving for an equilibrium under this approximating model. We perform this step using a version of Newton's method. We provide additional details of the computational methods in Appendix A.

3.7 Model Intuition

Our incomplete markets model does not admit a simple two-equation representation in terms of a dynamic "IS" equation and a Phillips curve as does the standard new Keynesian model analyzed in Section 2. For this reason, it is harder to describe the forces at work in this model and, therefore, useful to review the main forces to aid intuition about the results presented in Section 4.

The behavior of firms is identical in the incomplete markets model to the standard New Keynesian model. Hence, one can derive a Phillips curve like relationship between inflation and current and expected future marginal costs that is identical to the standard New Keynesian Phillips curve, except that it is non-linear. The fact that we are working with the non-linear model implies that price dispersion due to price rigidity affects the relationship between aggregate hours and aggregate output (equation (10)). Price dispersion, effectively, reduces productivity in the economy. This effect is ignored in linearized models since it is a second order effect. In certain cases, it can however become quite large. A second reason why the relationship between marginal costs and output differs from the standard model is that workers differ in their labor supply (due to the uninsurable idiosyncratic productivity shocks), but this does not play an important role in our results.

With regards to the dynamic IS equation (the consumption Euler equation), the model is fundamentally different. While the intertemporal substitution motive is the sole driver of consumption dynamics in the standard New Keynesian model, the incomplete markets model introduces a new motive arising from precautionary savings. Our main results regarding the reduced power of forward guidance stem from the precautionary savings motive. The precautionary savings motive raises households' marginal propensity to consume (MCP) out of additional income. However, it reduces the sensitivity of consumption to changes in interest rates and does so differentially for interest rates at different horizons.

Since households are hetergeneous in the incomplete markets model, their MPCs differ widely. Households with little wealth have high MPCs, while households with a great deal of wealth have much lower MPCs. As a consequence, wealth redistribution matters for aggregate consumption dynamics. For example, a reallocation of income from high to low net worth households leads to higher consumption demand, all else equal. One way in which this shows up in our model is that the government levies taxes to finance interest payments on debt. A fall in interest rates, therefore, may lead to a redistribution of wealth due to variation across households in holdings of government debt as well as tax obligations. Our (realistic) assumption that taxes are paid mostly by the rich leads these tax effects to be relatively small.

4 Results

Our main result is that the power of forward guidance is substantially muted in the incomplete markets model we present in Section 3 relative to the standard complete-markets New Keynesian model. To illustrate this, we first consider a simple policy experiment: suppose the central bank

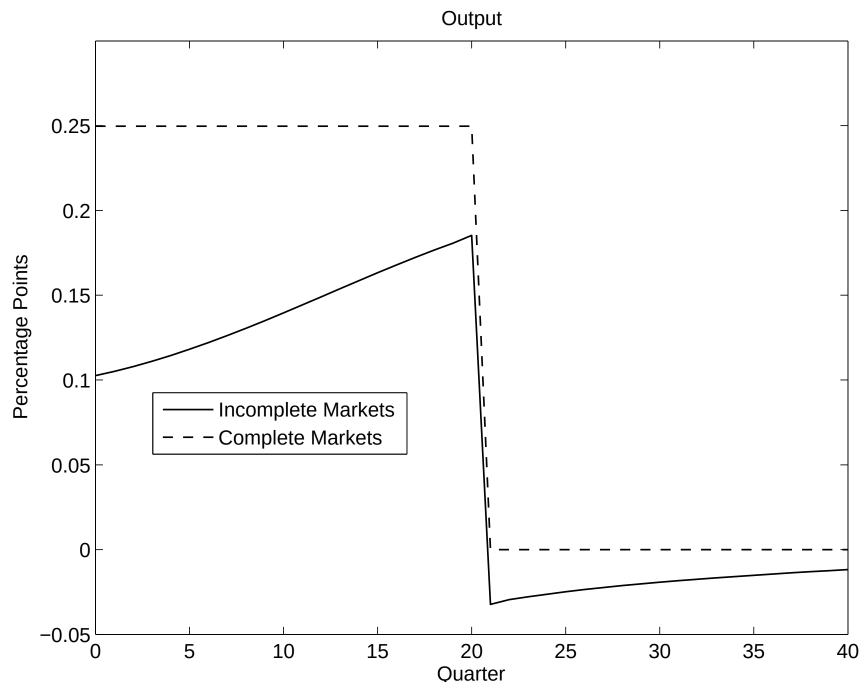

Figure 2: Response of output to 50 basis point forward guidance about the real interest rate in quarter 20 (with real interest rates in all other quarters unchanged).

promises a 50 basis point (i.e., 2% annualized) decrease in the real interest rate for a single quarter 5 years in the future.[15]

Figure 2 plots the response of output to this shock in our incomplete markets model and, for comparison, in the complete markets version of this model. With complete markets, output immediately jumps up by 25bp and remains at that elevated level for 20 quarters before returning to steady state. In contrast, in the incomplete markets model the initial increase in output is only about 40% as large. Output then gradually rises as the interest rate decrease gets closer. But even in the period right before the interest rate increase, the increase in output is substantially smaller than under complete markets.

Figure 3 plots the response of inflation to this same shock. The five year output boom induced by the forward guidance about real interest rates leads to a large inflation response in the complete markets case. Since the output boom is much smaller in the incomplete markets model, the rise in inflation is also much smaller. The initial response of inflation in the incomplete markets model is again only about 40% as large as in the complete markets model.

[15] As in Section 2, we assume here that the monetary authority sets an exogenous path for the real interest rate.

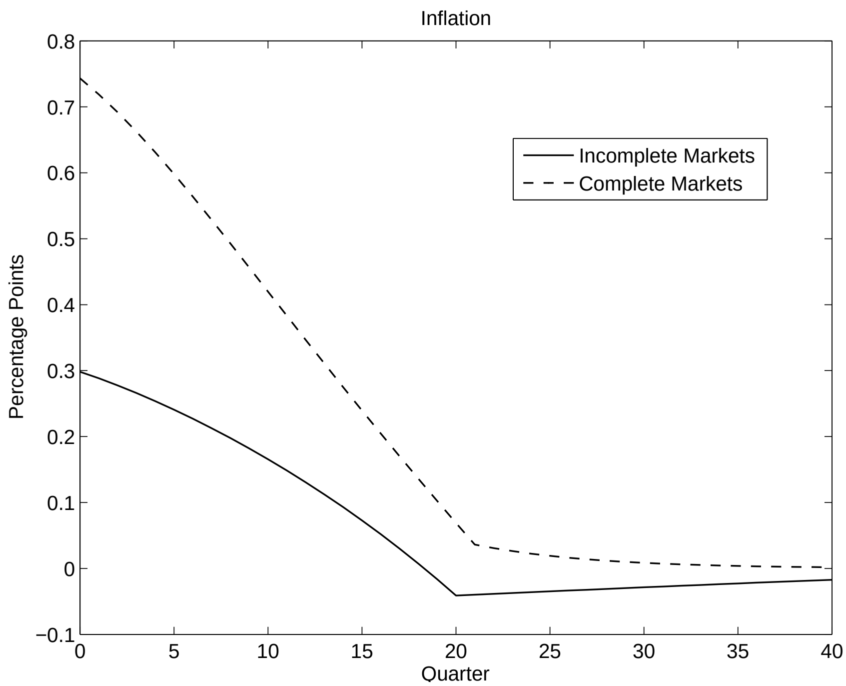

Figure 3: Response of inflation to 50 basis point forward guidance about the real interest rate in quarter 20 (with real interest rates in all other quarters unchanged).

4.1 Intuition

To build intuition for why the effects of forward guidance are greatly reduced in the incomplete markets model, it is useful to start by considering the response of a single household in partial equilibrium. Figure 4 plots the partial equilibrium response of consumption and assets for a household with median productivity and wealth. For comparison, Figure 4 also plots the responses of these variables for a household with this same amount of wealth in a model with complete markets.[16]

As in Figure 2, the response of consumption under complete markets is to jump up immediately and remain high for 5 years. The shape of the response under complete markets is the same whether the interest rate shock is expected to occur 1 quarter or 40 quarters in the future. The reason the response has this shape is that the interest rate change alters the price of consumption before the change relative to the price of consumption after the change, but does not alter the relative price of consumption between two dates before the change or between two dates after the change.

[16]Specifically, the household being plotted is a household with median productivity and median wealth among households with median productivity in the incomplete markets model. The consumption and asset responses plotted are the percentage change in the evolution of these variables relative to what they would be without the shock to interest rates. We assume a realization of idiosyncratic productivity that leaves the household's productivity unchanged.

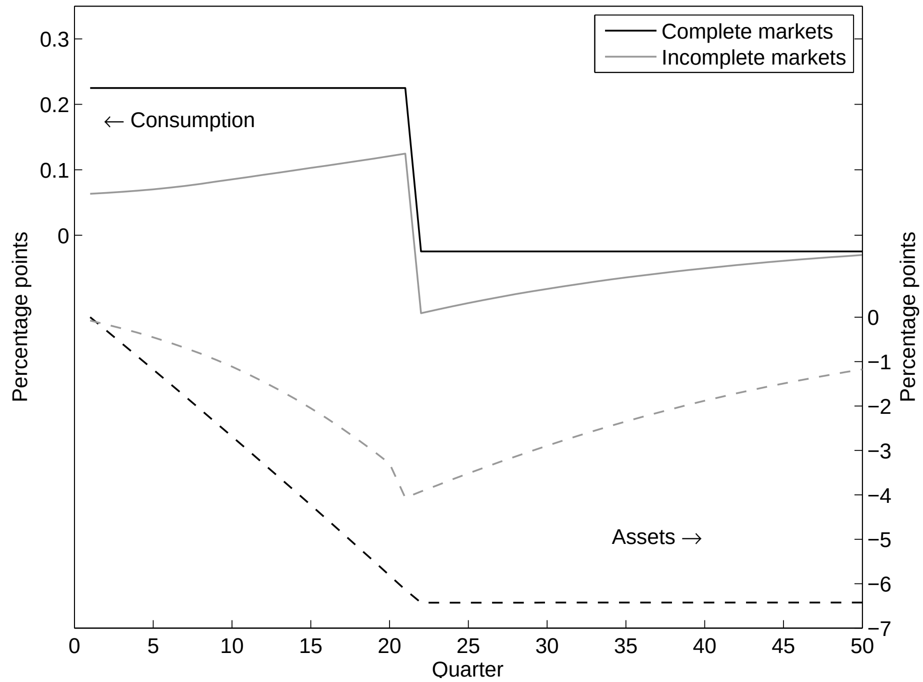

Figure 4: Partial equilibrium response of consumption and assets under complete and incomplete markets.

Under complete markets, consumption at all dates before the change must therefore be equal, and consumption at all dates after the change must also be equal. The result is a step function.

Why doesn't the incomplete markets household respond in the same way? Figure 4 shows that the complete markets response requires a substantial run-down in assets. This poses no concern for the complete markets household (since it is fully insured against all shocks). In contrast, the incomplete markets household needs to maintain a buffer stock of assets to help insulate itself from future shocks. Were the incomplete markets household to run-down its assets the way the complete markets household does, it would face permanently higher consumption volatility going forward due its reduced buffer-stock savings. This precautionary motive leads the incomplete markets household to choose a more conservative consumption path.

In general equilibrium, households can't all run down their wealth. The increase in demand resulting from the desire to increase consumption instead results in an increase in aggregate income (recall that in our New Keynesian model output is largely demand determined). Each household, however, takes the evolution of the aggregate economy as given and conditional on this faces a similar trade-off between precautionary savings and intertemporal substitution. It is this trade-off that ultimately reduces the power of forward guidance in our model.

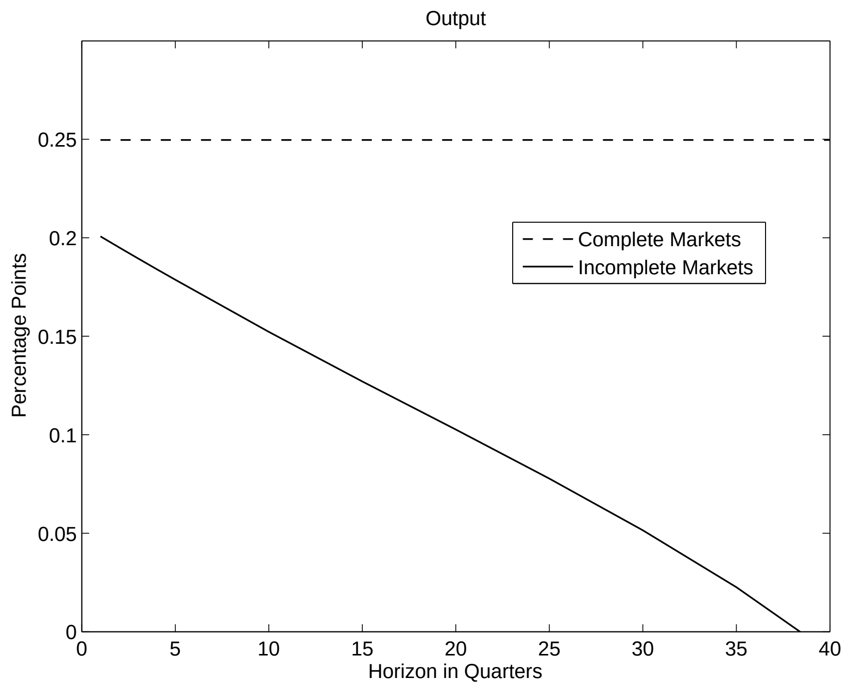

Figure 5: Initial response of output to 50 basis point forward guidance about the real interest rate for a single quarter at different horizons.

4.2 Dependence on Horizon of Forward Guidance

The difference between the complete and incomplete markets models grows with the horizon of the interest rate shock. Figure 5 plots the initial response of output to 50 basis point forward guidance about the real interest rate in a single quarter as the horizon of that single quarter changes from zero to 40 quarters.[17]

In the complete markets model, output always rises by 25 basis points, regardless of the horizon of the forward guidance. In contrast, in the incomplete markets model, the effect is only about 20 basis points for an announcement about the real interest rate next quarter and falls monotonically thereafter. It is roughly 10 basis points for an announcement about the real interest rate 5 years ahead; and essentially zero for an announcement about the real interest rate 10 years ahead.

Intuitively, the response of consumption is governed by two forces: an intertemporal substitution motive (as in the complete markets case) and a precautionary savings motive. For far-future interest rate changes, the run-down in assets needed to reap the benefits of intertemporal substitution are

[17] For example, the points at horizon 20 in Figure 5 are the first points on each line in Figure 2. And the points at horizon 10 in Figure 5 are the initial response of output in the two models if the central bank announces that it will lower the real interest rate by 50 basis point for a single quarter 10 quarters in the future.

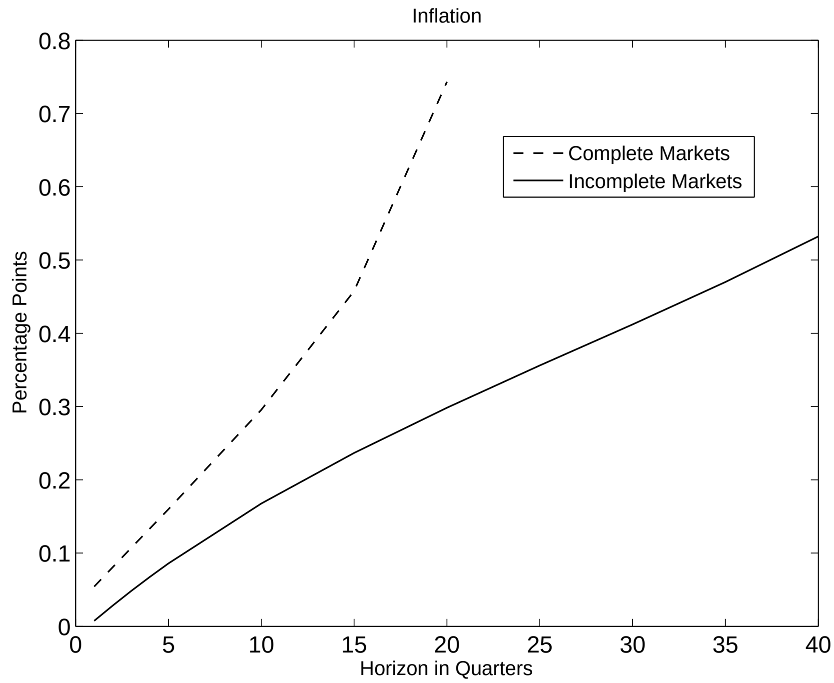

Figure 6: Initial response of inflation to 50 basis point forward guidance about the real interest rate for a single quarter at different horizons.

very large. This implies that eventually the benefits from intertemporal substitution are simply too small to make it worth it for households to incur the costs associated with running down their buffer-stock savings.

The results for inflation are even starker. Figure 6 plots the initial response of inflation to forward guidance about the real interest rate at different horizons. In the complete markets model, the response of inflation rises explosively with the horizon of the forward guidance. Above 20 quarters, the model "explodes": the inflation response grows so quickly that we can no longer compute it numerically. In the incomplete markets model, in contrast, the inflation response is smaller to start out with, grows more slowly, and therefore generates very different results at long horizons.

4.3 Results for Alternative Calibrations

Table 2 presents the results of the forward guidance experiment described above for our baseline incomplete markets model as well as for several alternative calibrations of our incomplete markets model. We also repose the results for the the complete markets version of our model, for comparison. In each case, we present the initial response of output and inflation to 50 basis point forward

Table 2: Power of 20 Quarter Ahead Forward Guidance

	Initial Responses of	
	Output	Inflation
Baseline	10.3	29.8
High Risk	4.8	23.8
High Asset	14.5	36.2
High Risk and Asset	11.6	33.8
Complete Markets	25.0	74.3

Initial response of output and inflation (in bases points) to forward guidance that reduces the expected real interest rate 20 quarters ahead by 50 basis point for four different calibrations of our incomplete markets model.

guidance about the real interest rate for a single quarter 5 years in the future.

The High Risk calibration features greater uninsurable risk than our baseline calibration. We roughly double the volatility of idiosyncratic productivity shocks relative to our baseline calibration, allowing us to match recent evidence on the volatility of earnings growth from Guvenen, Ozkan, and Song (2014). This boosts the precautionary savings motive and further reduces the power of forward guidance relative to the complete markets benchmark. The response of output in this case is only about 20% of the complete markets benchmark and the response of inflation only about 32% of the complete markets benchmark.

In the High Asset calibration, we set the ratio of assets to GDP in the model to be almost three times higher than in our baseline calibration (3.79 versus 1.4). We do this to match the ratio of total net worth in the economy to GDP (as opposed to total liquid assets as in our baseline calibration). Increasing the quantity of available assets in the economy increases the size of the precautionary savings buffers available to households and thus reduces their reluctance to engage in intertemporal substitution. This change therefore moves the incomplete markets model closer to the complete markets benchmark. The output response rises to 58% of the complete markets benchmark, while the inflation effect rises to 49%.

Finally, we consider a High Risk and Asset calibration with both of the above-mentioned alternative parameter values. These two modifications largely offset each other. As a consequence, the results lie between the two calibrations described above and close to the baseline calibration. The response of output in this calibration is 46% of the complete markets benchmark, while the response of inflation is 45% of the complete markets benchmark.

An alternative way of calibrating the model would be to choose parameters to fit empirical estimates of the marginal propensity to consume (MPC) out of additional wealth. The average MPC in our baseline calibration is only 12%, and even in our High Risk calibration it rises only to 14%. In contrast, a substantial amount of empirical evidence suggests larger values for the MPC, with many studies estimating values close to 20%.[18] On this basis, one might argue for calibrations in which households are more credit constrained than even our High Risk calibration implies. Such a calibration would further amplify the effects we emphasize regarding the differences between the complete and incomplete markets models.

4.4 Zero Lower Bound Analysis

In recent years, risk-free nominal interest rates around the world have hit zero. At the zero lower bound (ZLB), forward guidance has become an indispensable policy tool, since it is no longer possible to implement monetary policy via the current policy rate. Eggertsson and Woodford (2003) show how a persistent shock to the natural interest rate that causes the economy to hit the ZLB can provoke a massive recession if the central bank does not engage in unconventional monetary policy. They show, however, that the recession can be fully abated by a relatively modest amount of forward guidance about future interest rates.

Our conclusions above suggest that forward guidance may not be as powerful at the ZLB in our incomplete markets model. To investigate this question, we follow Eggertsson and Woodford (2003) in assuming that the ZLB is brought on by a temporary shock to the subjective discount factor of households in the economy that depresses the natural rate of interest below zero. In other words, we now consider a case were the discount factor can vary over time. The specific shock we consider is an increase in the discount factor that lasts for a known number of quarters and then reverts

[18] Parker (1999) estimates that household spend 20% of increases in disposable income when they hit the Social Security tax cap. Johnson, Parker, and Souleles (2006) estimate that households spent 20-40% of tax rebate checks they received in 2001, and Parker et al. (2013) estimate that households spent 12-30% of tax rebate checks they received in 2008. These studies as well as most others on this topic consider anticipated changes in income. They therefore provide a lower bound for responses to unanticipated changes in income. See Jappelli and Pistaferri (2010) for an excellent recent survey of the literature on the response of consumption to changes in income.

to normal.[19] We choose the size and persistence of the shock so that the initial output decline is 4% and the ZLB binds for 20 quarters under a naive monetary policy (described below).[20] This is meant to roughly match the magnitude and persistence of the Great Recession.

We consider two alternative monetary policies. First, we consider a policy where the central bank sets the nominal interest rate equal to a simplified Taylor rule whenever this yields an interest rate greater than zero, and, otherwise, sets the nominal rate to zero: $i_t = \max[0, \bar{r} + \phi\pi_t]$, where $\phi = 1.5$ and $\bar{r}$ is the steady state real interest rate. We refer to this policy as the "naive" policy. We also consider an "extended" policy whereby the central bank sets the nominal rate to zero for several additional quarters after the ZLB shock and then reverts back to the policy rule. We choose the length of the additional monetary stimulus to fully eliminate the initial fall in output in the complete markets model.

Figure 7 shows that forward guidance is substantially less powerful at the ZLB in the incomplete markets model than in the standard New Keynesian model. While the extended monetary policy fully eliminates the recession in the complete markets case, a substantial recession remains in the incomplete markets model. Figure 8 shows the implications for inflation: the extended policy is much more successful in preventing deflation in the complete markets model versus the incomplete markets model. While the initial deflation is only about 30 basis points in the complete markets case, it is more than 100 basis points in the incomplete markets case. The fact that inflation is lower in the incomplete markets case implies that real interest rates are higher (since nominal rates are stuck at zero). This contributes to the larger fall in output.

Figure 9 plots the implications of the naive and extended monetary policies for the nominal interest rate. Under the naive policy the ZLB binds for 20 quarters and then rises gradually to its steady state value of 50 basis points. Under the extended policy, the nominal interest rate remains at zero 23 quarters (an additional 3 quarters), and interest rates are somewhat lower in quarter 24

[19] Our shock differs from the shock considered in Eggertsson and Woodford (2003) in that its persistence is known implying that agents have perfect foresight about the evolution of the aggregate economy. Eggertsson and Woodford (2003) consider a shock that reverts back to normal with constant probability each period. Clearly, both formulations are approximations. Eggertsson and Woodford's formulation abstracts from time-variation in the probability of the ZLB period ending, while our framework abstracts from uncertainty about when it will end. However, the incomplete markets model is more difficult to solve computationally without the assumption of perfect foresight for aggregate shocks. In Section 5, we consider Eggertsson and Woodford's formulation of the shock in a more tractable approximation of the incomplete markets model.

[20] Hitting these targets requires slightly different calibrations of the discount factor shock in the complete versus the incomplete markets model: it corresponds to a decline in the natural rate of 16.4 basis points in the complete markets model, but only 14.8 basis points in the incomplete markets model. In each case, the duration of the shock is 33 quarters.

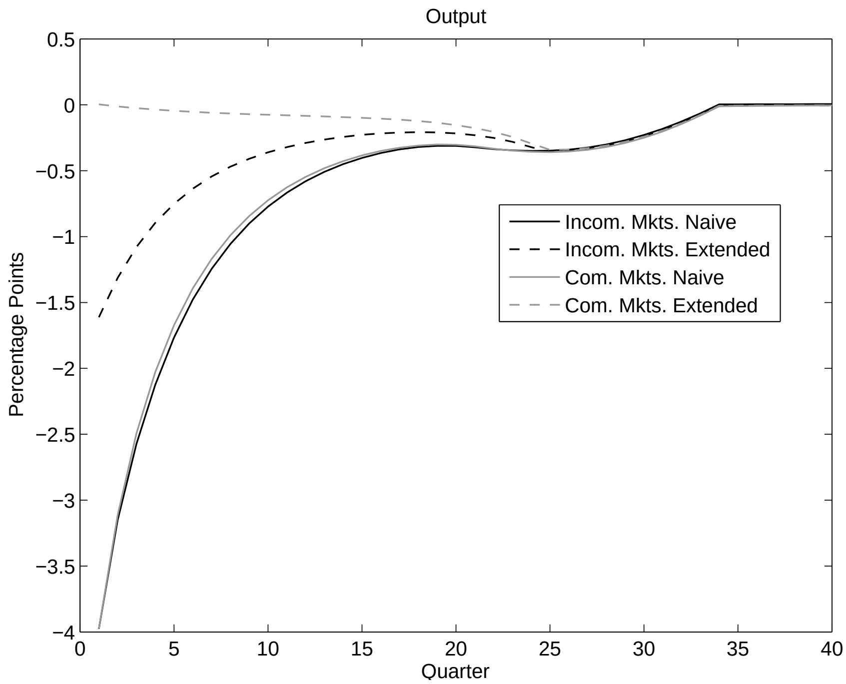

Figure 7: Response of output to the ZLB shock.

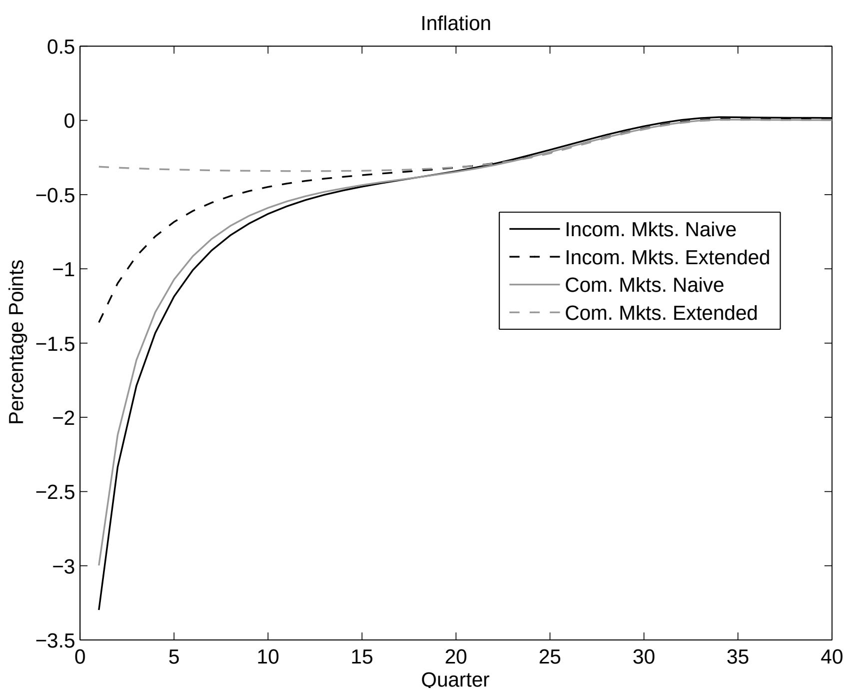

Figure 8: Response of inflation to the ZLB shock.

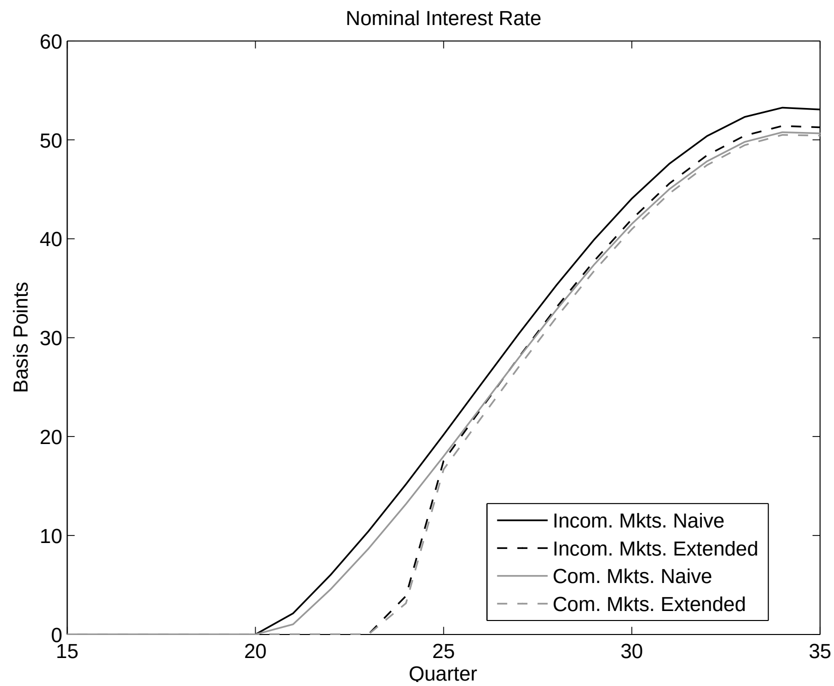

Figure 9: Response of nominal interest rate to the ZLB shock.

than the naive policy implies (this partial stimulus in the 24th quarter is what is needed to exactly eliminate the initial fall in output due to the shock). The difference between the dashed and solid lines, thus, indicates the amount of additional stimulus provided by the extended policy.

5 Discounted Euler Equation Model

We argued in Section 2 that the power of forward guidance in the standard model results from the fact that current output responds just as strongly to a change in the expected real interest rate 20 years in the future as it does to an equally large change in the current real interest rate (see equation (3)). Our analysis in Sections 3 and 4 indicates that this is no longer the case in a model where agents face uninsurable idiosyncratic income risk and borrowing constraints. In such a model, precautionary savings forces imply that output reacts much less to changes in far future expected real interest rates than current real interest rates (Figure 5).

However, our full model with uninsurable idiosyncratic income risk and borrowing constraints is more difficult to analyze than the linearized DSGE models commonly used in business cycle analysis. It is therefore useful to develop a modification to the standard linearized consumption

Euler equation that better approximates a model with uninsurable idiosyncratic income risk and borrowing constraints. Figures 2 and 5 suggest that what is needed is to "discount" the effect of future interest rates on current consumption.

We therefore consider the following simple modification to the standard linearized consumption Euler equation:

$$x_t = -\zeta\sigma E_t \sum_{j=0}^{\infty} \alpha^j (i_{t+j} - E_{t+j}\pi_{t+j+1} - r^n_{t+j}), \tag{18}$$

where $\alpha < 1$ causes future interest rates to be discounted exponentially, and $\zeta < 1$ is a factor that reduces the the response of output to all interest rates. Equation (18) can be first differenced to yield

$$x_t = \alpha E_t x_{t+1} - \zeta\sigma(i_t - E_t\pi_{t+1} - r^n_t). \tag{19}$$

We refer to this equation as the discounted Euler equation. In Appendix B, we show how the discounted Euler equation can be micro-founded with a simplified version of our incomplete markets model. In this simplified version, we replace the rich distribution of idiosyncratic shocks in the model in section 3 with a simpler specification where each period a fraction of agents are hit by an expenditure shock that is large enough that they hit their borrowing constraint for sure. We assume that this expenditure shock is uninsurable and that agents have some positive marginal utility of wealth in the period when they are hit by the expenditure shock. The combination of these features implies that agents discount future consumption in the Euler equation but their overall desire to save is not changed and thus the equilibrium interest rate remains low. Together these features yield the discounted Euler equation above.

Notice that the discount factor α applies equally to changes in expected real interest rates and changes in expected natural real rates in future periods (this is easy to see in equation (18)). This implies that the discounting mutes the effects of forward guidance shocks and the discounting *also* mutes the effect of shocks to future natural rates. It is only the difference between the realized rate and the natural rate at each horizon that matters for the consumption-savings decisions of households.

Figure 10 shows that with $\alpha = 0.97$ and $\zeta = 0.75$, the discounted Euler equation provides a good approximation to the response of output to a real interest rate shock 20 quarters in the future. The approximation is nearly perfect up until the time that the interest rate changes. What the discounted Euler equation misses is the fall in consumption after the interest rate shock. This fall is

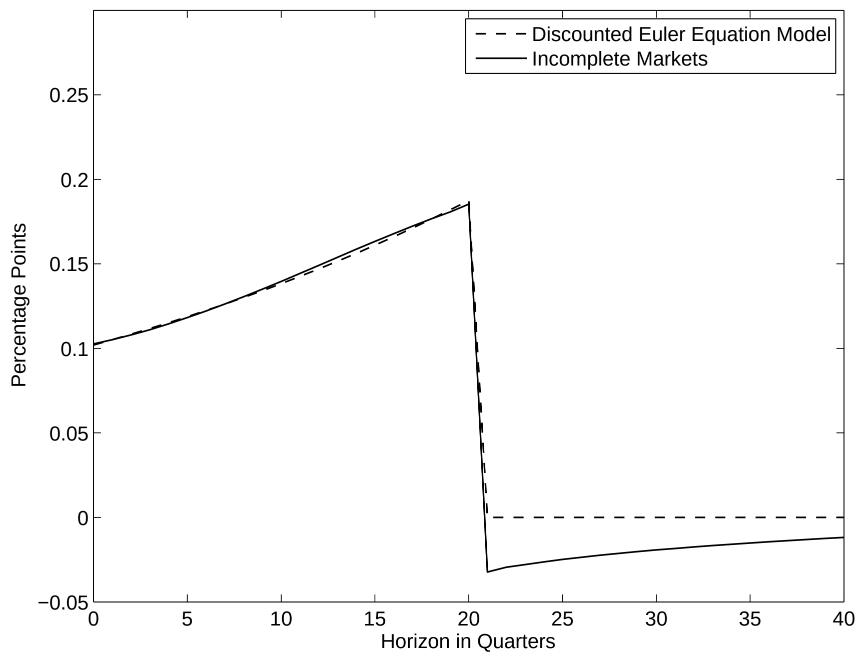

Figure 10: Response of output to a 50 bp change in the expected real interest rate in quarter 20 in the incomplete markets model and in the discounted Euler equation model.

due to redistribution of wealth in the incomplete markets model (from households with high MPCs to households with low MPCs), which the discounted Euler equation does not capture.

To illustrate the importance of allowing for discounting in the Euler equation, we revisit the question of monetary policy at the zero lower bound using the standard linearized New Keynesian model analyzed in Section 2 with the standard Euler equation replaced by the discounted Euler equation. An advantage of this approach to incorporating the precautionary savings effects we emphasize is that we are able to deviate from our previous setting of perfectly anticipated shocks. In particular, we assume that the ZLB binds due to a shock that lowers the natural rate below zero and persists at the same negative value with probability λ each quarter. With probability $1 - \lambda$, it reverts back to normal. For simplicity, we assume that once the natural rate reverts back to normal, the zero lower bound on nominal interest rates never binds again in the future. This is the same type of shock as Eggertsson and Woodford (2003) consider.

We assume that the central bank follows a "naive" monetary policy similar to the one we consider in Section 4

$$i_t = \max(0, r_t^n + \phi \pi_t), \tag{20}$$

where $\phi > 1$. The full model then consists of equations (2), (19), and (20). For comparability with

Eggertsson and Woodford (2003), we assume that $\beta = 0.99$, $\sigma = 0.5$, and $\kappa = 0.02$. To mimic the behavior of the incomplete markets model regarding the effects of future interest rates, we set $\alpha = 0.97$ and $\zeta = 0.75$ as in Figure 10. We also consider the standard case of $\alpha = 1$ and $\zeta = 1$.

We start by solving for the level of the output gap and inflation after the shock has dissipated. Since we have assumed that the natural rate will never go negative again, it is feasible for the monetary authority to set $i_t = r_t^n$ at all times after the shock dissipates. This implies that both the output gap and inflation will be zero at all times after the shock dissipates. Given this, it is easy to solve for the output gap and inflation while the shock persists. First, notice that all periods while the shock persists are identical since the probability of the shock reverting to normal does not change over time. This implies that output and inflation will be constant while the shock persists. We refer to the period during which the shock persists as the short run. Next, notice that in the short run $E_t x_{t+1} = \lambda x_t$ and $E_t \pi_{t+1} = \lambda \pi_t$ since with probability $1 - \lambda$ the economy will revert to normal. Using these facts and equations (2) and (19), a few steps of algebra (presented in Appendix C) yield

$$\pi_S = \frac{\kappa}{1 - \beta p} x_S, \tag{21}$$

$$x_S = \frac{\zeta\sigma}{1 - \alpha\lambda - \frac{\zeta\sigma\lambda\kappa}{1-\lambda\beta}} r_S^n, \tag{22}$$

where π_S and x_S denote inflation and the output gap in the short run, and r_S^n denotes the natural real rate of interest in the short run.

Eggertsson and Woodford (2003) present results for $\lambda = 0.1$ and $r_S^n = -0.02$ (annualized). They show that in the standard model, a shock of this size and persistence generates a very large recession—an output gap of -14.3%—accompanied by a large amount of deflation (-10.5%). In Table 3, we show that in the discounted Euler equation model, this same shock leads to a much more modest recession. The output gap is a mere -2.9%, and inflation falls by only 2.1%. Clearly, incorporating discounting of future interest rates radically alters the conclusions one comes to about the severity of the problem that the monetary authority faces with this type of shock.

The strength of the deflationary forces in the standard model are due to a feedback loop that gets stronger the more persistent is the shock to the natural rate. The basic feedback loop results from the following chain of logic: The negative natural rate leads to a positive interest rate gap—a real rate that is higher than the natural rate—because the nominal rate can't fall below zero. This leads output to fall, and if the shock is persistent it leads expectations of future output to fall,

Table 3: How Severe a Constraint Is the ZLB?

	Output	Inflation
Standard Model	-14.3%	-10.5%
($\alpha = 1$, $\zeta = 1$)		
Discounted Euler Equation Model	-2.9%	-2.1%
($\alpha = 0.97$, $\zeta = 0.75$)		

Response of output and inflation when the natural rate falls to -2% (annualized) with a 10% per quarter probability of returning to normal.

which in turn leads expected inflation to fall, which causes the current real rate to rise further, and current output to fall further, etc. The more persistent is the shock, the more it affects expected inflation and the stronger this feedback loop becomes.

It is well known that the strength of the deflationary forces associated with negative shocks to the natural real rate become infinitely strong—i.e. imply that the (log) output gap and inflation go to negative infinity—for even relatively modest levels of persistence. This can be seen by looking at the denominator of the expression for the short run output gap in equation (22). As this denominator goes to zero, the short run output gap goes to negative infinity. In the standard model (with $\alpha = 1$ and $\zeta = 1$), this occurs for a shock with an expected duration of 11 quarters.

In the discounted Euler equation model, the strength of the deflationary forces are muted and consequently the persistence of the ZLB shock needs to be greater for this "deflationary death spiral" to occur. This is depicted in Figure 11, which plots the drop in output for different levels of persistence of the ZLB shock. The solid line is the standard model, while the two broken lines are two calibrations of the discounted Euler equation model that match the baseline and high-risk calibrations of our incomplete markets model, respectively.[21] The deflationary death spiral occurs only for shocks that are considerably more persistent in the discounted Euler equation model than in the standard model.

[21] As we discuss in the text, $\alpha = 0.97$ and $\zeta = 0.75$ matches the baseline calibration of the incomplete markets model well. We use the same approach to match the high-risk calibration of the incomplete markets model with the discounted Euler equation model. This yields $\alpha = 0.94$ and $\zeta = 0.7$.

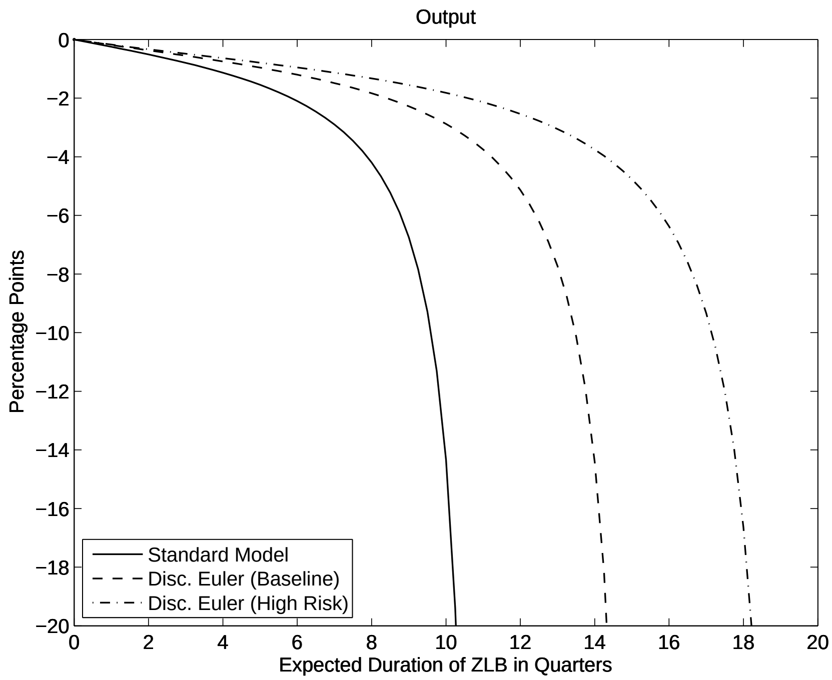

Figure 11: Response of output to shock that makes the natural real rate of interest -2% (annualized) for different levels of persistence of the shock (different values of λ).

6 Conclusion

We study the effects of forward guidance about monetary policy. We do this in a standard New Keynesian model augmented with uninsurable income risk and borrowing constraints. Our main finding is that allowing for uninsurable income risk and borrowing constraints substantially decrease the power of forward guidance relative to the standard New Keynesian model.

While an interest rate announcement in the standard New Keynesian model has the same effect on consumption whether it occurs 1 or 40 quarters in the future, the effect declines with the horizon of the announcement in the incomplete markets model. Forward guidance 10 years ahead has essentially zero effect on output in the incomplete markets model. Intuitively, in the incomplete markets model, precautionary savings effects work to offset the standard intertemporal substitution motive. Our results have important implications for monetary policy at the zero lower bound. They imply both that persistent shocks to the natural rate of interest have smaller effects on current output and also that far future forward guidance has substantially less power to stimulate the economy.

We show that the response of consumption to interest rates at different horizons in the incomplete markets model can be approximated by a consumption Euler equation with "discounting." Such an equation is a tractable way of incorporating the precautionary savings forces we emphasize into workhorse linearized macroeconomic models. We show that this discounted Euler equation can be micro-founded using a simple model of borrowing constraints.

A Computational methods

Here we describe the procedure used to find an equilibrium path of the heterogeneous agent model along a perfect foresight transition for the zero-lower-bound episode considered in Section 4.4. The algorithm used to compute the results for a one-time change in the real interest rate is closely related to what we present here.

Writing the firm's first order condition recursively. For the numerical analysis it is convenient to rewrite equation (8) recursively. Define

$$p_t^A \equiv \sum_{s=t}^{\infty} \beta^{s-t}(1-\theta)^{s-t}\left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} y_s \mu w_s \tag{23}$$

$$p_t^B \equiv \sum_{s=t}^{\infty} \beta^{s-t}(1-\theta)^{s-t}\left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} y_s. \tag{24}$$

then equation (8) becomes

$$\frac{p_t^*}{p_t} = \frac{p_t^A}{p_t^B}. \tag{25}$$

Equations (23) and (24) can be written recursively

$$p_t^A = \mu w_t y_t + (1-\theta)\beta \mathbb{E}_t (1+\pi_{t+1})^{-\mu/(1-\mu)} p_{t+1}^A \tag{26}$$

$$p_t^B = y_t + \beta(1-\theta)\mathbb{E}_t(1+\pi_{t+1})^{-1/(1-\mu)} p_{t+1}^B. \tag{27}$$

Initial guess. We assume that the economy has returned to steady state after $T = 250$ periods and look for equilibrium values for endogenous variables between dates $t = 0$ to T. In this explanation of our methods we use variables without subscripts to represent sequences from 0 to T. Let X denote a path for all endogenous aggregate variables from date 0 to date T. These variables include aggregate quantities and prices

$$X \equiv \{C_t, L_t, N_t, y_t, d_t, i_t, w_t, \pi_t, r_t, p_t^*/p_t, S_t, \tau_t, p_t^A, p_t^B\}_{t=0}^T.$$

The dimension of X is given by 14 variables for each date and 251 dates. We require an initial guess X^0. In most cases we found it sufficient to guess that the economy remains in steady state.

Solving the household's problem. The household's decision problem depends on X through r, w, τ, and d. For a given X^i we solve the household's problem using the endogenous gridpoint method (Carroll, 2006). We approximate the household consumption function $c(b, z)$ with a shape-preserving cubic spline with 200 unequally-spaced knot points for each value of z with more knots placed at low asset levels where the consumption function exhibits more curvature. Given the consumption function we calculate labor supply from the household's intratemporal optimality condition and savings from the budget constraint.

Simulating the population of households. We simulate the population of households in order to compute aggregate consumption and aggregate labor supply. We use a non-stochastic simulation method. We approximate the distribution of wealth with a histogram with 1000 unequally-spaced bins for each value of z again placing more bins at low asset levels. We then update the distribution of wealth according to the household savings policies and the exogenous transitions across z. When households choose levels of savings between the center of two bins, we allocate these households to the adjacent bins in a way that preserves total savings. See Young (2010) for a description of non-stochastic simulation in this manner.

Checking the equilibrium conditions. An equilibrium value of X must satisfy equations (5), (6), (10), (11), (12), (14), (16), (17), (25), (26), and (27) and the monetary policy rule $i_t = \max[0, \bar{r} + \phi * \pi_t + \epsilon_t]$, where ϵ_t is the exogenous deviation from the Taylor rule that takes a negative value under our "extended" policy. Call these 12 equations the "analytical" equilibrium conditions. The remaining two equilibrium conditions that pin down X are that C and L are consistent with household optimization and the dynamics of the distribution of wealth given the prices. Call these the "computational" equilibrium conditions.

To check whether a given X represents an equilibrium of the model is straightforward. We can easily verify whether the analytical equilibrium conditions hold at X. In addition, we can solve the household problem and simulate the population of households to verify that aggregated choices for consumption and labor supply of the heterogeneous households match with the values of C and L that appear in X.

Updating X^i The difficult part of the solution method arises when X^i is not an equilibrium. In this case we need to find a new guess X^{i+1} that moves us towards an equilibrium. To do this, we construct an auxiliary model by replacing the computational equilibrium conditions with additional analytical equilibrium conditions that approximate the behavior of the population of heterogeneous households but are easier to analyze. Specifically we use the equations

$$C_t^{-\gamma} = \eta_t^1 \beta(1 + r_t) C_{t+1}^{-\gamma} \tag{28}$$

$$C_t^{-\gamma} w_t = \eta_t^2 L_t^{\psi}. \tag{29}$$

where η^1 and η^2 are treated as parameters of the auxiliary model. For a given X^i, we have computed C and L from the computational equilibrium conditions. We then calibrate η^1 and η^2 from (28) and (29). We then solve for a new value of X from the 12 analytical equilibrium conditions and (28) and (29). This is a problem of solving for 14 unknowns at each date from 14 non-linear equations at each date for a total of 3514 unknowns and 3514 non-linear equations. We solve this system using the method described by Juillard (1996) for computing perfect foresight transition paths for non-linear models. This method is a variant of Newton's method that exploits the sparsity of the Jacobian matrix. Call this solution $X^{i\prime}$. We then form X^{i+1} by updating partially from X^i towards $X^{i\prime}$. We iterate until X^i satisfies the equilibrium conditions within a tolerance of 5×10^{-6}.

B A Simple Model of Borrowing Constraints

The discounted Euler equation (equation (19)) can be micro-founded with a simple model of borrowing constraints that is similar in structure to Blanchard's (1985) perpetual youth model. Suppose that with probability ω a household is hit by an expenditure shock that requires that the household consume all available resources and hit a borrowing constraint. An example of such a shock could be a large hospital bill that drives the household to its borrowing constraint. We refer to the households that are hit by this shock as the constrained households and the other households as the unconstrained households. Suppose for simplicity that the marginal utility of consumption of the constrained households is constant at a value D. Suppose furthermore, that the institutions of the economy are such that it is not feasible for the household to transfer resources intertemporally across this event and as a result the household will continue from that point on with no assets. In this regard the model is akin to Blanchard's perpetual youth model—it is as if the household dies and is replaced by a new household.

In contrast to the standard perpetual youth model, we assume that the household does not have access to insurance against these idiosyncratic shocks. Instead households save in a zero-net-supply bond with real interest rate R_t. As the bond is in zero net supply, if all unconstrained households begin with zero bond positions they will have to choose zero bond positions for the bond market to clear. This follows because all unconstrained households are identical and therefore if there are no savings in the aggregate there must be no savings for each individual. Those households who are constrained also continue without any savings in the next period so the degenerate distribution of wealth is preserved.

For simplicity, we assume that constrained households do not supply labor nor do they receive dividends from firms. From these assumptions, it follows that the only resources the constrained households have to fund their consumption is their stock of assets, but in equilibrium they hold no assets so they do not consume in equilibrium. Each unconstrained household receives an equal share d of the profits of the firms.

The Bellman equation for the representative unconstrained household is

$$V(b, \Xi) = \max_{b', \ell} \left\{ \frac{c^{1-\gamma}}{1-\gamma} - \frac{\ell^{1+\psi}}{1+\psi} + \beta_t \mathbb{E}\left[(1-\omega)V(b', \Xi') + \omega D b' \right] \right\}$$

where b denotes the household's asset holdings, Ξ represents the aggregate state, the natural rate of interest in our application, and the expectation is taken over Ξ'. Maximization is subject to the budget constraint

$$c + \frac{b'}{R} = w\ell + b + d.$$

This problem generates a consumption Euler equation of

$$c_t^{-\gamma} = \beta_t R_t \mathbb{E}_t \left[(1-\omega) c_{t+1}^{-\gamma} + \omega D \right].$$

In equilibrium, goods market clearing implies that $c_t = y_t$.

Log-linearizing this equation yields

$$x_t = -\frac{1}{\gamma}(r_t - r_t^n) + \alpha \mathbb{E}_t \left[x_{t+1} \right]$$

where x_t is the output gap,

$$\alpha = \frac{(1-\omega)\bar{c}^{-\gamma}}{(1-\omega)\bar{c}^{-\gamma} + \omega D},$$

and r_t^n is γ times the log deviation of β_t from its steady state value. To match the functional form presented in the text, set $1/\gamma = \sigma\zeta$, set $D = \bar{c}^{-\gamma}$, and then the above expression becomes $\alpha = 1 - \omega$ so the parameter ω controls the discounting in the Euler equation.

In the standard model, there is no discounting in the Euler equation because the return on saving offsets the fact that the household discounts utility at future dates. This is also true in Blanchard's perpetual youth model. In that model households discount the future at a higher rate reflecting their mortality risk, but households hold assets that only pay off if they live and these assets have returns that are exactly high enough conditional on living to offset the households' increased discounting of the future. In contrast, in the model we lay out above, household discount the future at a higher rate than in the standard model but the return to savings remains low. Households discount the future at a higher rate because they completely discount any states of the world following expenditure shocks (since they can't influence their situation in these states). The return to savings remains low because households value wealth in the constrained states and this bids up the price of assets and drives down the interest rate. The result is that households discount future consumption more than is offset by the returns to savings that they have access to.

The other equations of the model, including the intratemporal labor supply condition, are unaffected by the possibility that the borrowing constraint binds.

C Algebra Behind Equations (21) and (22)

Consider first the Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$

Since the output gap and inflation are constant at x_S and π_S, respectively, and $E_t\pi_{t+1} = \lambda\pi_S$ in the short run, we have that

$$\pi_S = \beta\lambda\pi_S + \kappa x_S,$$

which implies

$$\pi_S = \frac{\kappa}{1-\beta\lambda} x_S \tag{30}$$

as long as x_S and π_S are finite.

Consider next the discounted Euler equation

$$x_t = \alpha E_t x_{t+1} - \zeta\sigma(i_t - E_t\pi_{t+1} - r_t^n).$$

Again, since the output gap and inflation are constant at x_S and π_S, respectively, and $E_t\pi_{t+1} = \lambda\pi_S$ and $E_t x_{t+1} = \lambda x_S$ in the short run, and, in addition, since the natural real rate is r_S^n in the short run, we have that

$$x_S = \alpha\lambda x_S + \zeta\sigma(\lambda\pi_S + r_S^n).$$

If we now use equation (30) to eliminate π_S from this equation we get that

$$x_S = \alpha\lambda x_S + \zeta\sigma(\lambda\frac{\kappa}{1-\beta\lambda}x_S + r_S^n),$$

which implies

$$x_S = \frac{\zeta\sigma}{1-\alpha\lambda-\frac{\zeta\sigma\lambda\kappa}{1-\lambda\beta}}r_S^n$$

as long as x_S is finite.

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