Financial Innovation, Collateral and Investment.

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Abstract

We show that financial innovations that change the collateral capacity of assets in the economy can affect investment even in the absence of any shift in utilities, productivity, or asset payoffs. First we show that the ability to leverage an asset by selling non-contingent promises can generate over-investment compared to the Arrow-Debreu level. Second, we show that the introduction of naked CDS can generate under-investment with respect to the Arrow-Debreu level. Finally, we show that the introduction of naked CDS can robustly destroy competitive equilibrium.

Keywords: Financial Innovation, Collateral Capacity, Investment, Leverage, Naked CDS, Collateral Equilibrium, Non-Existence.

JEL Codes: D52, D53, E44, G01, G10, G12.

1 Introduction

After the recent subprime crisis and the sovereign debt crisis in the euro zone, many observers have placed financial innovations such as leverage and credit default swaps (CDS) at the root of the problem.[1] The crisis was preceded by years in which the

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[1]See for example Brunneimeier (2009), Geanakoplos (2010), Gorton (2009) and Stultz (2009). Geanakoplos (2003) and Fostel and Geanakoplos (2008) wrote before the crisis.

amount of leverage in the financial system, both institutionally and at the individual asset level, increased dramatically. CDS began to be traded in large quantities after the run up in prices, just before the crash.

The goal of this paper is to study the effect of financial innovation on investment and production. The main result is that financial innovation can affect investment decisions, even in the absence of any changes in fundamentals such as preferences, production technologies or asset payoffs. We show that financial innovations such as leverage and CDS can change the collateral capacity of durable assets, which in turn can alter investment decisions.

First we consider an economy in which heterogenous agents have access to an intra-period production technology that can transform a riskless asset into an asset with uncertain future payoffs. Agents can issue non-contingent promises using the risky asset as collateral. That is, they can leverage using the risky asset. But we suppose that insurance markets for low production are absent. We show that equilibrium aggregate investment and production of the risky asset can be above the first best level. In other words, the ability to leverage an asset by selling non-contingent promises can generate over-investment compared to the Arrow-Debreu level.

Second, into the previous leverage economy we introduce naked CDS on the risky asset collateralized by the durable consumption good.[2] In this case, equilibrium aggregate investment and production can dramatically fall not only below the initial leverage level but beneath the first best level. In short, the introduction of naked CDS can generate under-investment compared to the Arrow-Debreu investment level.

Finally, we show how the introduction of naked CDS into an economy with production can robustly destroy competitive collateral equilibrium. CDS is a derivative, whose payoff depends on some underlying instrument. The quantity of CDS that can be traded is not limited by the market size of the underlying instrument.[3] If the value of the underlying security diminishes, the derivative CDS trading may continue at the same high levels. But when the value of the underlying instrument falls to zero, CDS trading must come to an end by definition. This discontinuity can cause robust non-existence.

[2] As we explain later in the paper, adding covered CDS would not alter the equilibrium.

[3] Currently the outstanding notional value of CDS in the United States if far in excess of $50 trillion, more than three times the value of their underlying asset.

In order to prove unambiguous theorems about the effect of financial innovation on asset prices and production, we restrict attention to the simplest economies in which uncertainty and heterogeneity are important, and leverage is not only endogenous but easy to compute. We suppose that the economy lasts for two periods, with two states of nature, and that all future consumption is generated by dividends from current assets. These economies are complex enough to allow for the possibility that financial innovation can have a big effect on production. But they are simple enough to be tractable and provide an intuition for why leverage can increase production and why CDS can decrease (or even destroy) production.

Leverage is a primitive way of tranching the risky asset. It allows the purchase of the asset to be divided between two kinds of buyers, the optimists who hold the residual, which pays off exclusively in the good state, and the general public who holds the riskless piece that pays the same in both states. By dividing up the risky asset payoffs into two different kinds of assets, attractive to two different clienteles, demand is increased and the price is raised. The optimistic buyers do not have to spend their wealth on the whole risky asset, but can concentrate on the "junior piece" which pays off only in the good state. The concentration of demand for the "Arrow Up" security tends to raise its price. Since all the agents agree on the value of the riskless senior piece, its price is undiminished. Thus the total price of the risky asset tends to go up, and thus so does its production.[4]

CDS can be thought of as a sophisticated tranching of the riskless asset, since cash is generally used as collateral for sellers of CDS. This tends to raise the relative price of the riskless asset, thereby reducing the production of risky assets. The seller of CDS is effectively making the same kind of investment as the buyer of the leveraged risky asset: he obtains a portfolio of the riskless asset as collateral and the CDS obligation, which on net pays off precisely when the asset does very well, just like the leveraged purchase. The creation of CDS thus lures away many potential leveraged purchasers of the risky asset, reducing the price of the "Arrow Up" security, and thus the price of the whole risky asset and its production.

Finally, taking our logic to the extreme, we show that the creation CDS may in fact destroy equilibrium by choking off all production. When production is reduced to

[4]An even more sophisticated tranching of the risky asset into Arrow securities would have raised its price and production even more. We do not analyze this case in this paper. For more details see Fostel and Geanakoplos (2012b).

zero, CDS is ill-defined. But if CDS were to disappear, production would reappear, making CDS reappear, etc. As we show in the paper, it is very easy to construct a robust set of parameters for which production would be zero with CDS, and positive without CDS.

We make all this intuition rigorous both algebraically and by way of a diagram. One novelty in the paper is an Edgeworth Box diagram for trade with a continuum of agents with heterogeneous but linear preferences. This diagram is used to illustrate the different equilibrium outcomes as well as most of our algebraic proofs.

In this paper we follow the modeling strategy of collateral equilibrium as in Geanakoplos (1997, 2003, 2010) and Fostel-Geanakoplos (2008, 2012a and 2012b). Geanakoplos (2003) showed that leverage can raise asset prices. Geanakoplos (2010) and Che and Sethi (2011) showed that in the kind of models studied by Geanakoplos (2003), CDS can lower risky asset prices. Fostel-Geanakoplos (2012b) showed more generally how different kinds of financial innovations that alter collateral capacities can have big effects on asset prices. In this paper we move a step forward and show that these changes in collateral capacities due to financial innovation affect investment and production as well as asset prices.

Our paper is related to a macro literature that connects leverage to investment as in Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). However, in these early papers the possibility of leverage generated an under-investment with respect to the first best outcome. One would expect that the need for collateral would prevent some investors from borrowing the money to invest because they could not pledge their future output, thus reducing production. The reason for the discrepancy indeed is that in these early macro models, it was assumed that only some agents are capable of making the most productive investments, and that it is impossible for them to pledge the whole future value of the assets they produce (the bruised fruit in Kiyotaki-Moore), despite the fact that there was no uncertainty. It is important to note that if the whole future value of an investment can be pledged, then with no uncertainty a first best investment level should be achieved, absent other frictions. If pledging the whole future value does not generate enough cash to pay for the inputs, then the investment would not be worth doing in an Arrow-Debreu world either.

In our model, we assume that all future value of investment can be pledged, and also

that all agents are equally productive and have the same endowments.[5] We have in mind the production of durable goods, like houses, that can be seized, capturing by definition their whole future value (assuming no transactions costs in seizing them). The most important difference is that we have uncertainty, and heterogeneity in how agents rank uncertain payoffs. Moreover, we have in mind other streams of future consumption, perhaps generated by other assets, that cannot be tranched or leveraged. An asset whose future payoffs can be leveraged, or more generally be tranched, is worth more because its purchase can be divided up among agents who are heterogeneous in how they value risky payoffs, though not in their productivity. This induces more production of the tranchable asset. In short, with our modeling strategy we expose a countervailing force (absent in the early models) in the incentives to produce: when only some assets can be used as collateral, they become relatively more valuable, and are therefore produced more.

Our model is related to a literature on financial innovation as in Allen and Gale (1994), though in our paper financial innovation is taken as given. Finally, our paper is related to literature on the effect of derivatives on the existence of equilibrium as in Polemarchakis and Ku (1990). They provide a robust example of non-existence in a general equilibrium model with incomplete markets due to the presence of derivatives. Existence was proved to be generic in the canonical general equilibrium model with incomplete markets and no derivatives by Duffie and Shaffer (86). Geanakoplos and Zame (1997) proved that equilibrium always exists in pure exchange economies even with derivatives if there is a finite number of potential contracts, with each requiring collateral. Our paper gives a robust example of non-existence in a general equilibrium model with incomplete markets with collateral, production, and derivatives. Thus the need for collateral to enforce deliveries on promises eliminates the non-existence problem in pure exchange economies with derivatives. But the non-existence problem emerges again with derivatives and production, despite the collateral.

The paper is organized as follows. Section 2 presents the model with incomplete markets and collateral. Section 3 defines different economies and shows how to cast financial innovation within the model of Section 2. Section 4 presents the over and under-investment results. Section 5 shows how the previous results translate into CDS and the non-existence result. The Appendix presents all the proofs.

[5] Allowing for future endowments or output that cannot be pledged makes it impossible to prove that one kind of financial innovation must lead to higher output than another.

2 General Equilibrium Model with Collateral

2.1 Time and Assets

The model is a two-period general equilibrium model, with time $t = 0, 1$. Uncertainty is represented by a tree $S = \{0, U, D\}$ with a root $s = 0$ at time 0 and two states of nature $s = U, D$ at time 1.

There are two assets in the economy which produce dividends of the consumption good at time 1. The riskless asset X produces $d_U^X = d_D^X = 1$ unit of the consumption good in each state, and the risky asset Y produces d_U^Y units in state U and $0 < d_D^Y < d_U^Y$ units of the consumption good in state D.

We take the price of X in state 0 and the price of consumption in each state U, D to be 1. Thus X is both riskless and the numeraire, hence it is in some ways analogous to a durable consumption good like gold or to money in our one commodity model. We denote the price of the risky asset Yat time 0 by p.

2.2 Agents

There is a continuum of agents $h \in H = [0, 1]$.[6] Each agent is risk neutral and characterized by a linear utility for consumption of the single consumption good x at time 1, and subjective probabilities, $(\gamma_U^h, \gamma_D^h = 1 - \gamma_U^h)$. The expected utility to agent h is $U^h(x_U, x_D) = \gamma_U^h x_U + \gamma_D^h x_D$. We assume that γ_U^h is strictly increasing and continuous in h. Notice that since only the output of Y depends on the state and $d_U^Y = 1 > d_D^Y$, higher h denotes more optimism. Finally, each agent $h \in H$ has an endowment x_{0^*} of asset X at time 0, and no other endowment.

2.3 Production

Agents have access to an intra-period production technology at $t = 0$ that allows them to invest the riskless asset X and produce the risky asset Y. Let $Z_0^h \subset \mathbb{R}^2$ denote the set of feasible intra-period production plans for agent $h \in H$ in state

[6] We suppose that agents are uniformly distributed in $(0, 1)$, that is they are described by Lebesgue measure.

0. We assume that Z_0^h is convex, compact and that $(0,0) \in Z_0^h$. Inputs appear as negative components, $z_x < 0$ of $z \in Z_0^h$, and outputs as positive components, $z_y > 0$ of $z \in Z_0^h$. We will assume in the remainder of the paper that $Z_0^h = Z_0, \forall h$.

In our model X is instantaneously transformed into Y, and then Y produces dividends next period. An alternative model would instead collapse the two steps into one longer step, assuming that X could be used as input into the production of goods at U and D. This alternative model would be equivalent to our model provided that we allowed agents to trade the firm in the middle, after committing to its production plan but before buying its inputs.

2.4 Arrow Debreu Equilibrium

Arrow-Debreu equilibrium is given by present value consumption prices (q_U, q_D), which without loss of generality we can normalize to add up to 1, and by consumption $(x_U^h, x_D^h)_{h \in H}$ and production $(z_x^h, z_y^h)_{h \in H}$ satisfying

1. $\int_0^1 x_s^h dh = \int_0^1 (x_{0^*} + z_x^h + d_s^Y z_y^h) dh, \ s = U, D.$
2. $(x_U^h, x_D^h) \in B_W^h(q_U, q_D, \Pi^h) \equiv \{(x_U^h, x_D^h) \in R_+^2 : q_U x_U^h + q_D x_D^h \leq (q_U + q_D) x_{0^*} + \Pi^h\}$.
3. $(x_U, x_D) \in B_W^h(q_U, q_D, \Pi^h) \Rightarrow U^h(x_U, x_D) \leq U^h(x_U^h, x_D^h), \forall h.$
4. $\Pi^h \equiv q_U(z_x^h + z_y^h d_U^Y) + q_D(z_x^h + z_y^h d_D^Y) \geq q_U(z_x + z_y d_U^Y) + q_D(z_x + z_y d_D^Y), \forall (z_x, z_y) \in Z_0^h$.

Condition (1) says that supply equals demand for the consumption good at U and D. Conditions (2) and (3) state that each agent optimizes in his budget set, taking into account the profit from his firm. Condition (4) says that each firm maximizes profits, where the price of X and Y are implicitly defined by state prices q_U and q_D as $q_X = q_U + q_D$ and $q_Y = q_U d_U^Y + q_D d_D^Y$.

We can easily compute Arrow-Debreu equilibrium. Since $Z_0^h = Z_0$, then $\Pi^h = \Pi$. Because Z_0 is convex, without loss of generality we may suppose that every agent chooses the same production plan (z_x, z_y). Since we have normalized the mass of agents to be 1, (z_x, z_y) is also the aggregate production. In Arrow-Debreu equilibrium

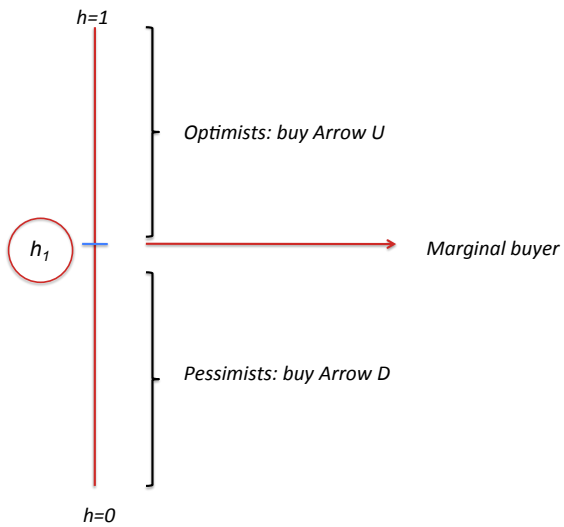

Figure 1: Equilibrium regime in the Arrow-Debreu Economy with Production.

there is a marginal buyer h_1.[7] All agents $h > h_1$ use all their endowment and profits from production $(q_U + q_D)x_{0^*} + \Pi = (x_{0^*} + \Pi)$ and buy all the Arrow U securities in the economy. Agents $h < h_1$ instead buy all the Arrow D securities in the economy. Figure 1 describes the equilibrium regime.

It is also clarifying to describe the equilibrium using the Edgeworth box diagram in Figure 2. The axes are defined by the potential total amounts of x_U and x_D available as dividends from the stock of assets emerging at the end of period 0. Point P represents the dividends flowing from the actual equilibrium choice of aggregate production (z_x, z_y), so $P = (z_y d_U^Y + x_{0^*} + z_x, z_y d_D^Y + x_{0^*} + z_x)$.

The 45-degree dotted line in the diagram is the set of consumption vectors that are collinear with the dividends of the aggregate endowment x_{0^*}. The steeper dotted line includes all consumption vectors collinear with the dividends of Y. The curve connecting the two dotted lines is the aggregate production possibility frontier, describing how the aggregate endowment of the riskless asset, x_{0^*}, can be transformed into Y. As more and more X is transformed into Y, the total amount of dividends in U and

[7] This is because of the linear utilities, the continuity of utility in h and the connectedness of the set of agents H at state $s = 0$.

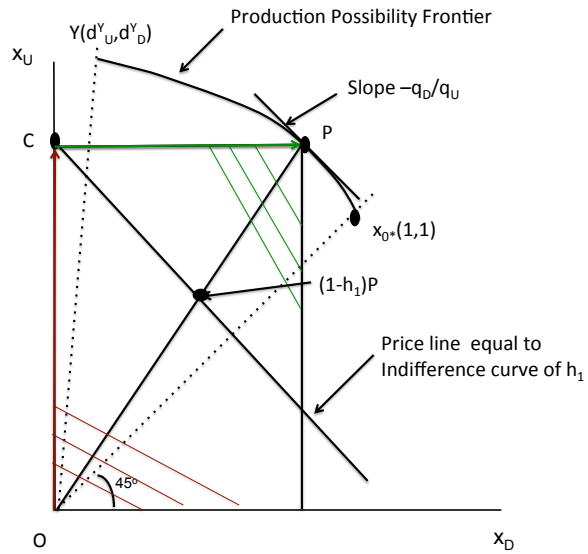

Figure 2: Equilibrium in the Arrow Debreu Economy with Production. Edgeworth Box.

D gets closer and closer to the Y dotted line.

The equilibrium prices $q = (q_U, q_D)$ determine parallel price lines orthogonal to q. One of these price lines is tangent to the production possibility frontier at P.

In the classical Edgeworth Box there is room for only two agents. One agent takes the origin as his origin, while the second agent looks at the diagram in reverse from the point of view of the aggregate point P, because he will end up consuming what is left from the aggregate production after the first agent consumes. The question is, how to put a whole continuum of heterogeneous agents into the same diagram? When the agents have linear preferences and the heterogeneity is one-dimensional and monotonic, this can be done. Suppose we put the origin of agent $h = 0$ at P. We can mark the aggregate endowment of all the agents between 0 and any arbitrary h_1 by its distance from P. Since endowments are identical, and each agent makes the same profit, it is clear that this point will lie h_1 of the way on the straight line from P to the origin at 0, namely at $(1 - h_1)P = P - h_1 P$. The aggregate budget line of these agents is then simply the price line determined by q through their aggregate endowment, (their budget set is everything below the line). We have drawn the

aggregate budget set for the agents between 0 and a particular h_1. Of course looked at from the point of view of $h = 1$, with origin at 0, the same point represents the aggregate endowment of the agents between h_1 and 1. Since every agent has the same endowment, the fraction $(1 - h_1)$ of the agents can afford to buy the fraction $(1 - h_1)$ of P. Therefore the same price line represents the aggregate budget line of the agents between h_1 and 1, as seen from their origin at 0, (and their budget set is everything below the budget line).

At this point we invoke the assumption that all agents have linear utilities, and that they are monotonic in the probability assigned to the U state. Suppose the prices q are equal to the probabilities $(\gamma_U^{h_1}, \gamma_D^{h_1})$ of agent h_1. Agents $h > h_1$, who are more optimistic than h_1, have flatter indifference curves, illustrated in the diagram by the indifference curves near the origin 0. Agents $h < h_1$, who are more pessimistic than h_1, have indifference curves that are steeper, as shown by the steep indifference curves near the origin P. The agents more optimistic than h_1 collectively will buy at the point C where the budget line crosses the x_U axis above the origin, consuming exclusively at U. The pessimists $h < h_1$, will collectively choose to consume at the point where the budget line crosses the x_D axis through their origin at P, the same point C, consuming exclusively at D. Clearly, total consumption of optimists and pessimists equals P, i.e. $(z_y d_U^Y + x_{0^*} + z_x, 0) + (0, z_y d_D^Y + x_{0^*} + z_x) = P$.

From the previous analysis it is clear that the equilibrium marginal buyer h_1 must have two properties: (i) one of his indifference curves is tangent to the production possibility frontier at P, and (ii) his indifference curve through the collective endowment point $(1 - h_1)P$ cuts the top left point of the Edgeworth Box whose top right point is determined by P.

Finally, the system of equations that characterizes the Arrow Debreu equilibrium is given by

$$(z_x, z_y) \in Z_0 \tag{1}$$

$$\Pi = z_x + q z_y \geq \tilde{z_x} + q\tilde{z_y}, \forall(\tilde{z_x}, \tilde{z_y}) \in Z_0. \tag{2}$$

$$q_U d_U^Y + q_D d_D^Y = q_Y \tag{3}$$

$$\gamma_U^{h_1} = q_U \tag{4}$$

$$\gamma_D^{h_1} = q_D \tag{5}$$

$$(1 - h_1)(x_{0^*} + \Pi) = q_U((x_{0^*} + z_x) + z_y d_U^Y) \tag{6}$$

Equations (1) and (2) state that production plans should be feasible and should maximize profits. Equation (3) uses state prices to price the risky asset Y. Equations (4) and (5) state that the price of the Arrow U and Arrow D are given by the marginal buyer's state probabilities. Equation (6) states that all the money spent on buying the total amount of Arrow U securities in the economy should equal the income of buyers in equilibrium.

2.5 Financial Contracts and Collateral

The heart of our analysis involves contracts and collateral. In Arrow Debreu equilibrium the question of why agents repay their loans is ignored. We suppose from now on that the only enforcement mechanism is collateral. We regard the use of new kinds of collateral, or new kinds of promises that can be backed by collateral, as financial innovations.

At time 0 agents can trade any of a fixed set J of financial contracts. A financial contract (A, C) consists of both a promise, $A = (A_U, A_D)$, and an asset $C \in \{X, Y\}$ acting as collateral backing it. The lender has the right to seize as much of the collateral as will make him whole once the promise comes due, but no more: the contract therefore delivers $(\min(A_U, d_U^C), \min(A_D, d_D^C))$.

We shall suppose every contract is collateralized either by one unit of X or by one unit of Y. The set of contracts j backed by one unit of X is denoted by J^X and the set of contracts backed by one unit of Y is denoted by J^Y. Each contract $j \in J = J^X \bigcup J^Y$ will trade for a price π_j. An investor can borrow π_j today by selling contract j in exchange for a promise of A_j tomorrow, provided he owns C_j. We shall denote the sale of contract j by $\varphi_j > 0$ and the purchase of the same contract by $\varphi_j < 0$. Notice

that the sale of $\varphi_j > 0$ units of a contract $j \in J^C$ requires the ownership of φ_j units of $C_j \in \{X, Y\}$, whereas the purchase of the same number of contracts does not require any ownership of C_j.

Notice that using Y as collateral in our model is tantamount to using the firm as collateral in the interpretation discussed in Section 2.3 where production takes two steps.

2.6 Budget Set

Now we are ready to characterize each agent's budget set. Given the prices $(p, (\pi_j)_{j \in J})$ of asset Y and contracts at time 0, each agent $h \in H$ decides his asset holdings x of X and y of Y, production plans $z = (z_x, z_y) \in R_- \times R_+$ and contract trades φ_j in state 0 in order to maximize their utility subject to the budget set defined by

$$B^h(p, \pi) = \{(x, y, z_x, z_y, \varphi, x_U, x_D) \in R_+ \times R_+ \times R_- \times R_+ \times R^J \times R_+ \times R_+ :$$

$$(x - z_x - x_{0^*}) + p(y - z_y) \leq \textstyle\sum_{j \in J} \varphi_j \pi_j,$$

$$\textstyle\sum_{j \in J^X} max(0, \varphi_j) \leq x, \sum_{j \in J^Y} max(0, \varphi_j) \leq y,$$

$$z = (z_x, z_y) \in Z_0^h,$$

$$x_s = x + d_s^Y y - \textstyle\sum_{j \in J^X} \varphi_j min(A_s^j, 1) - \sum_{j \in J^Y} \varphi_j min(A_s^j, d_s^Y), s \in \{U, D\}\}$$

Notice that because production is instantaneous, agents will always choose z_x and z_y to maximize profits $z_x + pz_y$, as is clear from the first constraint in the budget set.

2.7 Collateral Equilibrium

A *Collateral Equilibrium* in this economy is a price of asset Y, contract prices, asset allocations, production plans and contract trade decisions by all agents, $((p, \pi), (x^h, y^h, z_x^h, z_x^h, \varphi^h, x_U^h, x_D^h)_{h \in H}) \in (R_+ \times R_+^J) \times (R_+ \times R_+ \times R_- \times R_+ \times R^J \times R_+ \times R_+)^H$, such that

1. $\int_0^1 x^h dh = \int_0^1 (x_{0^*}^h + z_x^h) dh$
2. $\int_0^1 y^h dh = \int_0^1 z_y^h dh$
3. $\int_0^1 \varphi_j^h dh = 0, \forall j \in J$
4. $(x^h, y^h, z_x^h, z_x^h, \varphi^h, x_U^h, x_D^h) \in B^h(p, \pi), \forall h$
   $(x, y, z_x, z_y, \varphi, x_U, x_D) \in B^h(p, \pi) \Rightarrow U^h(x_U, x_D) \leq U^h(x_U^h, x_D^h), \forall h.$

Markets for the consumption good, asset and promises clear, and agents maximize their utility on their budget sets. Geanakoplos and Zame (1997) show that collateral equilibrium always exists.

Finally, like in the Arrow Debreu economy, since $Z_0^h = Z_0$ is convex, without loss of generality we may suppose that every agent chooses the same production plan (z_x, z_y) in equilibrium and $\Pi^h = \Pi$. Since we have normalized the mass of agents to be 1, (z_x, z_y) is also the aggregate production. We will use this fact extensively in the next section.

3 Financial Innovation and Collateral

A vitally important source of financial innovation involves the possibility of using assets and firms as collateral to back promises. Financial innovation in our model is described by a different set J.

In this section we study the effect of financial innovation on equilibrium by considering three different versions of the collateral economy introduced in the last section, each defined by a different set of feasible contracts J. We describe each variation and the system of equations that characterizes the equilibrium.

3.1 The Leverage Y Economy (L-economy)

The first type of financial innovation we focus on is leverage. Consider an economy in which agents can leverage asset Y. That is, agents can issue non-contingent promises of the consumption good using the risky asset as collateral. In this case $J = J^Y$, and each $A_j = (j, j)$ for all $j \in J = J^Y$. Fostel and Geanakoplos (2012a) proved that in

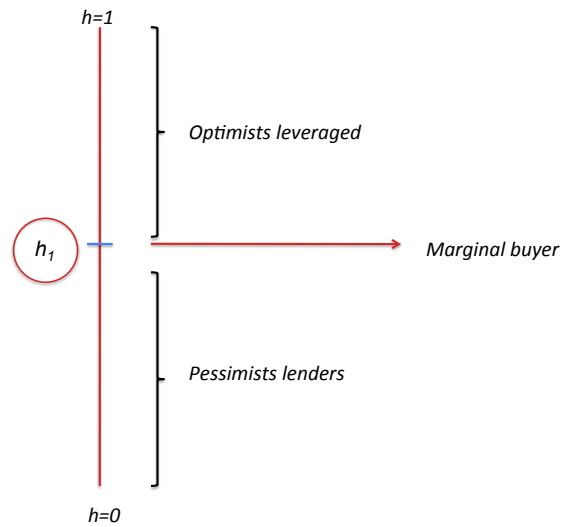

Figure 3: Equilibrium regime in the L-economy.

this model, there is a unique equilibrium in which the only contract actively traded is $j^* = d_D^Y$ (provided that $j^* \in J$) and that the riskless interest rate equals zero. Hence, $\pi_{j^*} = j^* = d_D^Y$ and there is no default in equilibrium.

In equilibrium there is a marginal buyer h_1 at state $s = 0$ whose valuation $\gamma_U^{h_1} d_U^Y + \gamma_D^{h_1} d_D^Y$ of the risky asset Y is equal to its price p. The optimistic agents $h > h_1$ collectively buy all the risky asset z_y produced in the economy. The pessimistic agents $h < h_1$ buy all the remaining safe asset. Figure 3 shows the equilibrium regime.

The endogenous variables to solve for are the price of the risky asset p, the marginal buyer h_1 and production plans (z_x, z_y). The system of equations that characterizes the equilibrium in the L-economy is given by

$$(z_x, z_y) \in Z_0 \tag{7}$$

$$\Pi = z_x + pz_y \geq \tilde{z}_x + p\tilde{z}_y, \forall(\tilde{z}_x, \tilde{z}_y) \in Z_0. \tag{8}$$

$$(1-h_1)(x_{0^*}+\Pi)+d_D^Y z_y = pz_y \tag{9}$$

$$\gamma_U^{h_1} d_U^Y + \gamma_D^{h_1} d_D^Y = p \tag{10}$$

Equations (7) and (8) describe profit maximization. Equation (9) equates money pz_y spent on the asset, with total income from optimistic buyers in equilibrium: all their endowment $(1-h_1)x_{0^*}$ and profits from production $(1-h_1)\Pi$, plus all they can borrow $d_D^Y z_y$ from pessimists using the risky asset as collateral. Equation (10) states that the marginal buyer prices the asset.

We can also describe the equilibrium using the Edgeworth box diagram in Figure 4. As in Figure 2, the axes are defined by the potential total amounts of x_U and x_D available as dividends from the stock of assets emerging at the end of period 0. The probabilities $\gamma^{h_1} = (\gamma_U^{h_1}, \gamma_D^{h_1})$ of the marginal buyer h_1 define state prices that are used to price x_U and x_D, and to determine the price lines orthogonal to γ^{h_1}. One of those price lines is tangent to the production possibility frontier at P, which as before represents the dividends flowing from the actual equilibrium choice of aggregate production, so $P = (z_y d_U^Y + x_{0^*} + z_x, z_y d_D^Y + x_{0^*} + z_x)$.

We can place in the box the individual dividends for each equilibrium choice of asset. The dividend coming for the equilibrium choice of X, $x_{0^*} + z_x$, lies in the intersection of the "X-dotted" line and the "Y-dotted" line starting at P. The dividends coming for the equilibrium production choice of Y, $z_y(d_U^Y, d_D^Y)$, lies in the intersection of the "Y-dotted" line and the "X-dotted" line starting at P.

Again we put the origin of agent $h = 0$ at P. We can mark the aggregate endowment of all the agents between 0 and any arbitrary h_1 by its distance from P. Since endowments are identical, and each agent makes the same profit, it is clear that this point will lie h_1 of the way on the line from P to the origin, namely at $(1-h_1)P = P - h_1 P$. Similarly the same point describes the aggregate endowment of all the optimistic agents $h > h_1$ looked at from the point of view of the origin at 0.

In equilibrium optimists $h > h_1$ consume at point C. As in the Arrow Debreu equilibrium they only consume on the U state. They consume the total amount of Arrow

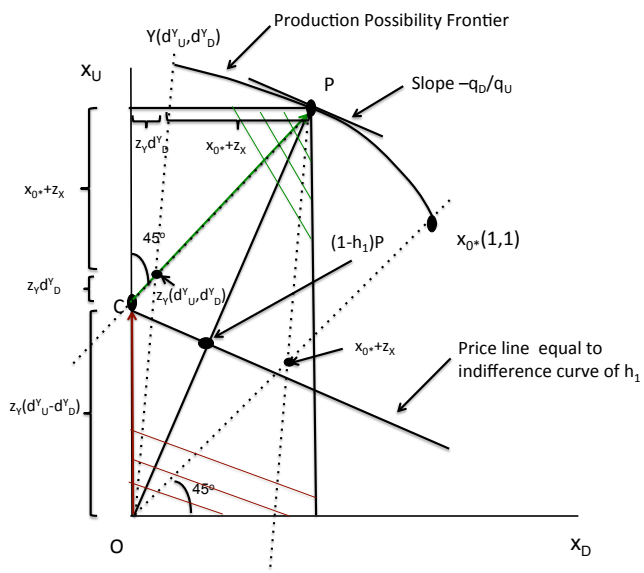

Figure 4: Equilibrium regime in the L-economy. Edgeworth Box.

U securities available in the economy, $z_y(d_U^Y - d_D^Y)$. Notice that when agents leverage asset Y, they are effectively creating and buying a "synthetic" Arrow U security that pays $d_U^Y - d_D^Y$ and costs $p - d_D^Y$, namely at price $\gamma_U^{h_1} = (d_U^Y - d_D^Y)/(p - d_D^Y)$.

The total income of the pessimists between 0 and h_1 is equal to $h_1 P$. Hence looked at from the origin P, the pessimists must also be consuming on the same budget line as the optimists. However, unlike the Arrow-Debreu economy, pessimists now must consume in the cone generated by the 45-degree line from P and the vertical axis starting at P. Since their indifference curves are steeper than the budget line, they will also choose consumption at C. However at C, unlike in the Arrow Debreu equilibrium, they consume the same amount, $x_{0^*} + z_x + z_y d_D^Y$, in both states. Clearly, total consumption of optimists and pessimists equals P, i.e. $(z_y(d_U^Y - d_D^Y), 0) + (x_{0^*} + z_x + z_y d_D^Y, x_{0^*} + z_x + z_y d_D^Y) = P$.

From the previous analysis we deduce that the marginal buyer h_1 must satisfy two properties: (i) one of his indifference curves must be tangent to the production possibility frontier at P, and (ii) his indifference curve through the point $(1 - h_1)P$ must intersect the vertical axis at the level $z_y(d_U^Y - d_D^Y)$, which corresponds to point C and the total amount of Arrow U securities in equilibrium in the L-economy.

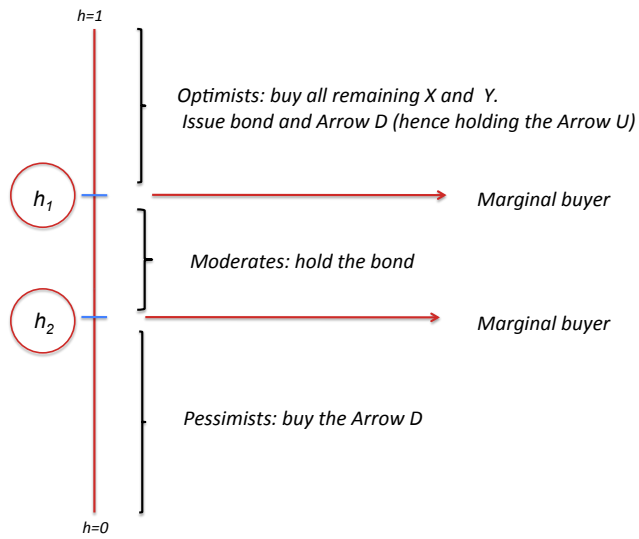

Figure 5: Equilibrium in the LT-economy.

3.2 The Leverage Y and Tranche X Economy (LT-economy)

Now we introduce into the previous L-economy an "Arrow Down" contract, from now on Arrow D, which pays off in the bad state D. Just as importantly, we suppose the Arrow D contract is collateralized by X. Thus we take $J = J^X \bigcup J^Y$ where J^X consists of the single contract promising $(0, 1)$ and J^Y consists of contracts $A_j = (j, j)$ as described in the leverage economy above. Selling the Arrow D using X as collateral is like "tranching" the riskless asset into contingent securities. The holder of X can get the Arrow U security by selling the Arrow D using X as collateral.

As in the L-economy, we know that the only contract in J^Y that will be traded is $j^* = d_D^Y$. The equilibrium, however, is more subtle in this case. There are two marginal buyers $h_1 > h_2$. Optimistic agents $h > h_1$ hold all the X and all the Y produced in the economy leveraging on Y and tranching X. Hence, they are effectively buying the Arrow U security. Moderate agents $h_2 < h < h_1$ buy the riskless bonds sold by more optimistic agents. Finally, agents $h < h_2$ buy the Arrow D security from the most optimistic investors. This regime is described in Figure 5.

The variables to solve for are the two marginal buyers, h_1 and h_2, the asset price, p, the

price of the riskless bond π_{j^*}, the price of the Arrow D security π_D, and production plans, (z_x, z_y). The system of equations that characterizes the equilibrium in the LT-economy with positive production of Y is given by

$$(z_x, z_y) \in Z_0 \tag{11}$$

$$\Pi = z_x + pz_y \geq \tilde{z}_x + p\tilde{z}_y, \forall(\tilde{z}_x, \tilde{z}_y) \in Z_0. \tag{12}$$

$$\frac{d_U^Y - d_D^Y}{p - \pi_{j^*}} = \frac{1}{1 - \pi_D} \tag{13}$$

$$\frac{\gamma_U^{h_1}}{1 - \pi_D} = \frac{d_D^Y}{\pi_{j^*}} \tag{14}$$

$$\frac{\gamma_D^{h_2}}{\pi_D} = \frac{d_D^Y}{\pi_{j^*}} \tag{15}$$

$$(1 - h_1)(x_{0^*} + \Pi) + (x_{0^*} + z_x)\pi_D + \pi_{j^*}z_y = x_{0^*} + z_x + pz_y \tag{16}$$

$$h_2(x_{0^*} + \Pi) = \pi_D(x_{0^*} + z_x) \tag{17}$$

Equations (11) and (12) describe profit maximization. Equation (13) rules away arbitrage between buying the Arrow U through asset Y and asset X. Equation (14) states that h_1 is indifferent between holding the Arrow U security (through asset X) and holding the riskless bond. Equation (15) states that h_2 is indifferent between holding the Arrow D security and the riskless bond. Equation (16) states that total money spent on buying the total available collateral in the economy should equal the optimistic buyers' income in equilibrium, which equals all their endowments and profits $(1 - h_1)(x_{0^*} + \Pi)$, plus all the revenues $(x_{0^*} + z_x)\pi_D$ from selling Arrow D promises backed by their holdings $(x_{0^*} + z_x)$ of X, plus all they can borrow $\pi_{j^*}z_y$ using their holdings z_y of Y as collateral. Finally, equation (17) states the analogous condition for the market of the Arrow D security, that is the total money spent on

buying all the Arrow D in the economy, $\pi_D(x_{0^*} + z_x)$, should equal the income of the pessimistic buyers, $h_2(x_{0^*} + \Pi)$.

It will be useful to define prices by equations (18)-(20) below.

$$p = \frac{\gamma_U^{h_1} d_U^Y + \gamma_D^{h_1} d_D^Y}{\gamma_U^{h_1} + \gamma_D^{h_2}} \tag{18}$$

$$\pi_{j^*} = \frac{j^*}{\gamma_U^{h_1} + \gamma_D^{h_2}} \tag{19}$$

$$\pi_D = \frac{\gamma_D^{h_2}}{\gamma_U^{h_1} + \gamma_D^{h_2}} \tag{20}$$

$$\pi_U \equiv \frac{p - \pi_{j^*}}{d_U^Y - d_D^Y} = \frac{\gamma_U^{h_1}}{\gamma_U^{h_1} + \gamma_D^{h_2}} \tag{21}$$

One can immediately verify that with definitions (18)-(20), equations (13)-(15) are satisfied. Equation (21) follows by plugging in (18) and (19).

As before, we can describe the equilibrium using the Edgeworth box diagram in Figure 6. Before starting the description, it will prove useful to define state prices q_U, q_D. The equilibrium price p of Y and the price 1 of X give two equations that uniquely determine these state prices.

$$p = q_U d_U^Y + q_D d_D^Y \tag{22}$$

$$p_X = 1 = q_U + q_D \tag{23}$$

Equations (22) and (23) define state prices that can be used to price X and Y, but not the other securities. These state prices determine orthogonal price lines, one of which must be tangent to the production possibility frontier at P. It is immediately apparent by looking at equation (18) that

$$\frac{\gamma_U^{h_1}}{\gamma_D^{h_2}} > \frac{q_U}{q_D} \tag{24}$$

otherwise plugging in $q_U = \gamma_U^{h_1}$ and $q_D = \gamma_D^{h_2}$ would give too high a ratio of the value of Y to the value of X (since $\gamma_U^{h_1} + \gamma_D^{h_2} > 1$).

The complication with respect to the previous diagrams in Figures 2 and 4 is that now there are three classes of consumers, and state pricing does not hold for all securities. The optimistic agents $h > h_1$ collectively own $(1 - h_1)P$, indicated in the diagram. Consider the point x_1 where the orthogonal price line with slope $-q_D/q_U$ through $(1 - h_1)P$ intersects the X line. That is the amount of X the optimists could own by selling all their Y. Scale up x_1 by the factor $\gamma_U^{h_1} + \gamma_D^{h_2} > 1$, giving the point x_1^*. That is how much riskless consumption those agents could afford by selling X (at a unit price) and buying the cheaper bond (at the price $\pi_{j^*} < 1$). Now draw the indifference curve of agent h_1 with slope $-\gamma_D^{h_1}/\gamma_U^{h_1}$ from x_1^* until it hits the vertical axis. By equations (19) and (21), that is the budget trade-off between j^* and x_U. Similarly, draw the indifference curve of agent h_2 with slope $-\gamma_D^{h_2}/\gamma_U^{h_2}$ from x_1^* until it hits the horizontal axis of the optimistic agents. By equations (19) and (20), that is the budget trade-off between j^* and x_D. These two lines together form the collective budget constraint of the optimists. It is convex, but kinked at x_1^*. Notice that unlike before, the aggregate endowment is at the interior of the budget set (and not on the budget line). This is a consequence of lack of state prices that can price all securities. Because they have such flat indifference curves, optimists collectively will choose to consume at C_0, which gives $x_U = (x_{0^*} + z_x) + z_y(d_U^Y - d_D^Y)$.[8]

The pessimistic agents $h < h_2$ collectively own h_2P, which looked at from P is indicated in the diagram by the point $P - h_2P$. Consider the point x_2 where the orthogonal price line with slope $-q_D/q_U$ through $(1 - h_2)P$ intersects the X line drawn from P. Scale up that point by the factor $\gamma_U^{h_1} + \gamma_D^{h_2} > 1$, giving the point x_2^*. This represents how much riskless consumption those agents could afford by selling all their Y for X, and then selling X and buying the cheaper bond. The budget set for the pessimists can now be constructed as it was for the optimists, kinked at x_2^*. Their aggregate endowment is at the interior of their budget set for the same reason given above. Pessimists collectively will consume at C_P, which gives $x_D = (x_{0^*} + z_x)$.[9] Finally,

[8] If we were to connect the point x_1 with C_0, this new line would describe the budget trade-off between x_U and x_D, obtained via tranching X, and would have a slope $-\pi_D/\pi_U$. By (24) the line would be flatter than the orthogonal price lines with slope $-q_D/q_U$.

[9] If we were to connect the point x_2 with C_P, this new line would describe the budget trade-off between x_D and x_U, obtained via selling X and buying the down tranche, and would have a slope $-\pi_D/\pi_U$. By (24) the line would be flatter than the orthogonal price lines with slope $-q_D/q_U$.

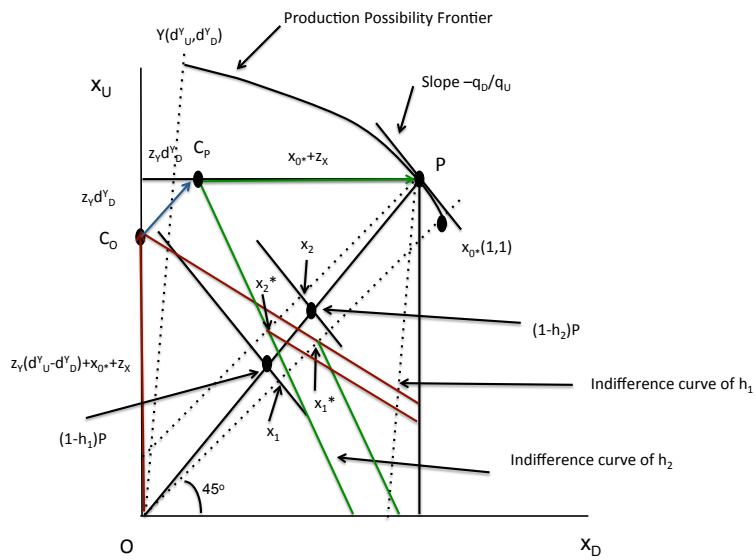

Figure 6: Equilibrium in the LT-economy. Edgeworth Box.

the moderate agents $h_1 < h < h_2$ collectively must consume $z_y d_D^Y$, which collectively gives them the 45-degree line between C_0 and C_P.

Let us conclude this section with the following observation. If there is no production of Y, then the equilibrium in the LT-economy is an Arrow-Debreu equilibrium in which agents choose not to produce. Note that without production, $h_1 = h_2$. Define the Arrow Debreu prices (p_U, p_D) by taking the $p_D = \pi_D = q_D$ and $p_U = 1 - \pi_D = q_U$. Then the price of Y in the LT-economy $p = q_U d_U^Y + q_D d_D^Y = p_U d_U^Y + p_D d_D^Y = p^A$. Since nobody wanted to produce in the LT-economy, it must be case that there is no $(z_x, z_y) \in Z_0$ such that $p^A z_y + z_x > 0$.

3.3 The Tranching Y and X Economy (TT-Economy)

Finally, consider the economy defined by the set of available financial contracts as follows. We take $J = J^X \bigcup J^Y$ where J^X consists of the single contract promising $(0, 1)$ and J^Y consists of a single contract $A = (0, d_D^Y)$. In this case both assets in the economy can be used as collateral to issue the Arrow D promise. Hence, in this economy both assets X and Y can be tranched into contingent securities.

Table 1: Equilibrium for $k = 1.5$.

Arrow-Debreu Economy		L-economy		LT-economy	
q_Y	0.6667	p	0.6667	p	0.6667
q_U	0.5833	h_1	0.3545	π_{j^*}	0.1904
q_D	0.4167	z_x	-0.92	π_D	0.4046
h_1	0.3545	z_y	1.38	h_1	0.3880
z_x	-0.2131			h_2	0.3480
z_y	0.3197			z_x	-0.14
				z_y	0.2

It is easy to see that the equilibrium in the TT-economy is equivalent to the Arrow-Debreu equilibrium analyzed in Section 2.4.[10]

4 Over-Investment and Under-Investment

In this section we show how the financial innovations described in Section 3 affect equilibrium investment decisions. We first present numerical examples in order to illustrate the theorems that follow.

4.1 Numerical Results

Consider a constant returns to scale technology, in which $Z_0 = \{z = (z_x, z_y) \in R_- \times R_+ : z_y = -kz_x\}$, where $k \geq 0$. Beliefs are given by $\gamma_U^h = 1 - (1-h)^2$, and parameter values are $x_{0^*} = 1$, $d_U^Y = 1, d_D^Y = .2$ and $k = 1.5$. Table 1 presents the equilibrium in the three economies we just described.

There are two main results coming out from the Table 1. First, investment and production are the highest in the L-economy. Second, investment and production are the lowest in the LT-economy. Figure 7 reinforces the results showing total output in each economy for different values of k.

The most important lesson coming from these numerical examples is that financial innovation affects investment decisions, even without any change in fundamentals.

[10]Notice that, in the absence of endowment at the terminal states, all the cash flows in the economy get tranched into Arrow U and D securities and hence the Arrow-Debreu equilibrium can be implemented.

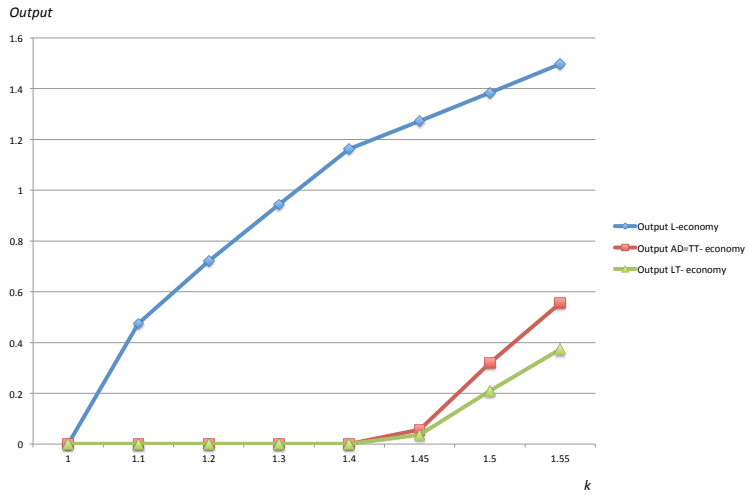

Figure 7: Total output in different economies for varying k.

Notice that across the three economies we do not change fundamentals such as asset payoffs or productivity parameters or utilities. The only variation is in the type of financial contracts available for trade using the assets as collateral, as described by the different sets J. In other words, financial innovation changes the collateral capacities of the assets X and Y, and these changes in asset collateral capacities drive investment variations. We now discuss these results in more detail.

4.2 Over-Investment

First we show that when agents can leverage the risky asset in the L-economy, investment levels are above those of the Arrow Debreu level. Hence, leverage generates *over*-investment with respect to the first best allocation. Our numerical example is consistent with a general property of our model as the following proposition shows.

Proposition 1: Over-Investment compared to First Best.

Suppose that $d_U^Y > d_D^Y > 0$, $x_{0^*} > 0$, *and that* $\gamma_U(h)$ *is strictly increasing and continuous. Let* $(p^L, (z_x^L, z_y^L)$, and $(p^A, (z_x^A, z_y^A))$ *denote the asset price and aggregate outputs for any equilibria in the L-economy and the Arrow Debreu Economy respec-*

tively. Then $(p^L, z_y^L) \geq (p^A, z_y^A)$ *and at least one of the two inequalities is strict, except possibly when* $z_x^L = -x_{0^*}$*, in which case all that can be said is that* $z_y^L \geq z_y^A$*.*

Proof: *See appendix.*

The Edgeworth Box diagrams in Figures 2 and 4 allow us to see why production is higher in the L-economy than in the Arrow-Debreu economy. In the L-economy, optimists collectively consume $z_y^L(d_U^Y - d_D^Y)$ in state U while in the Arrow Debreu economy they consume $z_y^A d_U^Y + (x_0^* + z_x^A)$. The latter is evidently much bigger, at least as long as $z_y^A \geq z_y^L$. So suppose, contrary to what we want to prove, that Arrow-Debreu output were at least as high, $z_y^A \geq z_y^L$. Since total output P^L maximizes profits at the leverage equilibrium prices, at those leverage prices $(1-h_1^L)P^A$ is worth no more than $(1-h_1^L)P^L$. Thus $(1-h_1^L)P^A$ must lie on the origin side of the h_1^L indifference curve through $(1-h_1^L)P^L$. Suppose also that the Arrow Debreu price is higher than the leverage price: $p^A \geq p^L$. Then the Arrow Debreu marginal buyer is at least as optimistic, $h_1^A \geq h_1^L$. Then $(1-h_1^A)P^A$ would also lie on the origin side of the h_1^L indifference curve through $(1-h_1^L)P^L$. Moreover, the indifference curve of h_1^A would be flatter than the indifference curve of h_1^L and hence cut the vertical axis at a lower point. By property (ii) of the marginal buyer in both economies, this means that optimists would collectively consume no more in the Arrow Debreu economy than they would in the leverage economy, a contradiction. It follows that either $z_y^A < z_y^L$ or $p^A < p^L$. But a routine algebraic argument from profit maximization (given in the appendix) proves that if one of these strict inequalities holds, the other must also hold weakly in the same direction. (If the price of output is strictly higher, it cannot be optimal to produce strictly less.) This geometrical proof shows that in the Arrow Debreu economy there is more of the Arrow U security available (coming from the tranching of X as well as better tranching of Y) and this extra supply lowers the price of the Arrow U security, and hence lowers the marginal buyer and therefore the production of Y.

The basic intuition of the result is the following. In the L-economy the collateral capacity of Y is high, since Y is the only way in this economy of buying the Arrow U security. In the Arrow Debreu economy, the collateral capacity of Y is even a little bit higher, since by holding Y as collateral, one can do in the Arrow Debreu economy anything that could be done in the L-economy. But in the Arrow Debreu economy, the collateral capacity of X is also very high. In contrast, in the L-economy X cannot

be used as collateral at all. This gives agents the incentive to use more X to produce Y.

4.3 Under-Investment

Second we show that investment in the LT-economy falls below the investment level in the L-economy. Hence, introducing Arrow D securities through cash tranching generates *under*-investment with respect to the investment level in the L-economy. The result coming out of our numerical example is a general property of our model as the following proposition shows.

Proposition 2: Under-Investment compared to leverage.

Suppose that $d_U^Y > d_D^Y > 0$, $x_{0^*} > 0$, *and that* $\gamma_U(h)$ *is strictly increasing and continuous. Let* $(p^L, (z_x^L, z_y^L)$, and $(p^{LT}, (z_x^{LT}, z_y^{LT}))$ *denote the asset price and aggregate outputs for the L-economy and the LT-economy respectively. Then* $(p^L, z_y^L) \geq (p^{LT}, z_y^{LT})$ *and at least one of the two inequalities is strict, except possibly when* $z_x^L = -x_{0^*}$, *in which case all that can be said is that* $z_y^L \geq z_y^{LT}$.

Proof: *See appendix.*

The geometrical proof of Proposition 2 using the Edgeworth boxes in Figures 4 and 6 is almost identical to that of Proposition 1. The optimists in the LT-economy consume $z_y^{LT}(d_U^Y - d_D^Y) + (x_0^* + z_x^{LT})$ which is strictly more than in the L-economy as long as production is at least as high in the LT-economy, and not all of X is used in production. So suppose $z_y^{LT} \geq z_y^L$ and $p^{LT} \geq p^L$. Then by (18), $h_1^{LT} \geq h_1^L$. By the argument given in the geometrical proof of Proposition 1 above, consumption of the optimists in the LT-economy cannot be higher than in the L-economy, which is a contradiction. Thus either $z_y^{LT} < z_y^L$ or $p^{LT} < p^L$. But as we show in the appendix, profit maximization implies that if one inequality is strict, the other holds weakly in the same direction.

The basic intuition behind the result is the same as before, but even clearer. In the LT-economy, the collateral capacity of Y is just as high as in the leverage economy (and less than in the Arrow Debreu economy). But the collateral capacity of X is also high, unlike in the leverage economy. This gives agents less incentive in the LT-economy to use X to produce Y.

Finally, we show that investment in the LT-economy falls even below the investment level in the Arrow-Debreu economy, provided that we make the additional assumption that $\gamma_U(h)$ is concave. Hence, introducing Arrow D through cash tranching generates *under*-investment with respect to the first best level of investment. The result coming out of our numerical example is a general property of our model as proposition 3 below shows.

Proposition 3: Under-Investment compared to First Best.

Suppose that $d_U^Y > d_D^Y > 0$, $x_{0^*} > 0$, *and that* $\gamma_U(h)$ *is strictly increasing and continuous. If* $\gamma_U(h)$ *is concave on* h, *then* $(p^A, z_y^A) \geq (p^{LT}, z_y^{LT})$ *and at least one of the two inequalities is strict, except possibly when* $z_x^A = -x_{0^*}$, *in which case all that can be said is that* $z_y^A \geq z_y^{LT}$, *and when* $(z_y^{LT}) = 0$, *in which case the Arrow Down equilibrium is an Arrow-Debreu equilibrium, and* $(p^A, z_y^A) = (p^{LT}, z_y^{LT})$.

Proof: *See appendix.*[11]

The intuition for our last proposition is as follows. The collateral capacity of Y in the LT-economy is lower than in the TT-eonomy (which equals the Arrow Debreu economy), since Y cannot be used to sell the Arrow D in the LT-economy. However, the collateral capacity of X is the same in the LT and TT (Arrow Debreu) economies. This gives agents even less incentive to use X to produce Y in the LT than in Arrow Debreu.

5 Naked CDS and Investment

In this section we show that our result concerning under-investment tells us something important about Credit Default Swaps.

5.1 Naked CDS and Under-Investment

A Credit Default Swap (CDS) on the asset Y is a contract that promises to pay 0 when Y pays $d_U^Y = 1$, and promises $d_U^Y - d_D^Y$ when Y pays only d_D^Y. CDS is a derivative, since its payoffs depend on the payoff of the underlying asset Y. Figure 8 shows the payoff of a CDS and the underlying security.

[11] The proof of Proposition 3 involves some irreducible algebra, so we do not try to give a purely geometric proof. But the diagram is helpful in following the algebra.

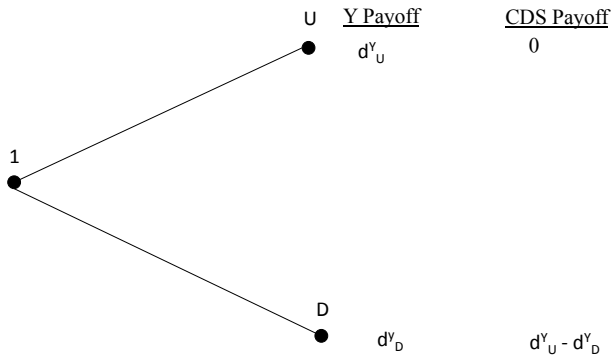

Figure 8: CDS and underlying Y payoffs.

A CDS can be seen as an insurance policy for Y. A seller of a CDS must post collateral, typically in the form of money. In a two-period model, buyers of the insurance would insist on $d^Y_U - d^Y_D$ of X as collateral. Thus, for every one unit of payment, one unit of X must be posted as collateral. We can therefore incorporate CDS into our economy by taking J^X to consist of one contract promising $(0, 1)$. A very important real world example is CDS on sovereign bonds or on corporate debt. The bonds themselves give a risky payoff and can be leveraged, but not tranched. The collateral for their CDS is generally cash, and not the bonds themselves.

A CDS can be "covered" or "naked" depending on whether the buyer of the CDS needs to hold the underlying asset Y. Notice that holding the asset and buying a CDS is equivalent to holding the riskless bond, which was already available without CDS in the L-economy analyzed in Section 3.1. Hence, introducing "Covered" CDS has no effect on the equilibrium above. For this reason in what follows we will focus on the case of "Naked" CDS.

The reader may already have noticed that with this definition, a "naked CDS" is very similar to what in Section 3.2 we called an Arrow D security. When Y exists, they both promise $(0, 1)$ and both use X as collateral.

Hence, Propositions 2 and 3 above show that the introduction of naked CDS can reduce equilibrium investment all the way below the first best level. Naked CDS can generate *under*-investment with respect to the Arrow-Debreu level.

The only difference between naked CDS and Arrow D is that when Y ceases to be produced the naked CDS is no longer well-defined. By definition, a derivative does not deliver when the underlying asset does not exist. It is precisely this difference that can bring about interesting non-existence properties as we now show.

5.2 Naked CDS and Non-Existence

In this last section we show how introducing CDS can robustly destroy competitive equilibrium in economies with production. In order to fix ideas, consider the same constant returns to scale production and parameter values as in Section 4.1 except that now $k = 1.2$.

Consider first the L-economy analyzed in Section 3.1. The equilibrium with $k = 1.2$ is given by $p = 0.833$, $h_1 = 0.5436$ and $(z_x, z_y) = (-0.6006, 0.7207)$.

Now suppose we introduce into the L-economy a naked CDS. Let us call this economy the LC-economy. Thus we take $J = J^X \bigcup J^Y$ where J^X consists of the single contract promising $(0, 1)$, as long as Y is produced, and (as we saw in Section 3.1) J^Y might just as well consist of the single contract promising $(d^Y_{D,} d^Y_D)$. Hence, the LT- economy described in Section 3.2 is exactly the same as the LC-economy, except that J^X consists of the single contract promising $(0, 1)$ independent of the production of Y.

The LT-economy always has an equilibrium. When $k = 1.2$, there is a unique equilibrium in which Y is not produced. (The system of equations (11)-(17) in Section 3.2 has no solution.) The agents use all the X as collateral to back Arrow D contracts. There is a marginal buyer h_1 and agents $h > h_1$ buy all the X and sell all the Arrow D, whereas agents $h < h_1$ buy only the Arrow D security. The agents above h_1 are effectively holding an "Arrow U" security. Thus, this equilibrium with no production is equivalent to the Arrow-Debreu equilibrium in which agents choose not to produce (as described at the end of Section 3.2).

We solve for the two Arrow prices q_U and q_D and the marginal buyer h_1. The system of equations that characterizes the equilibrium in the Arrow-Debreu economy with no production is given by

$$\gamma_U^{h_1} = q_U \tag{25}$$

$$\gamma_D^{h_1} = q_D \tag{26}$$

$$(1 - h_1)x_{0^*} = q_U x_{0^*} \tag{27}$$

The equilibrium is given by $h_1 = 0.3820$, $q_U = 0.6180$ and $q_D = 0.3820$. Hence, the equilibrium in the LT-economy is given by $h_1 = 0.3820$, $\pi_D = q_D = 0.3820$, $1 - \pi_D = p_U = 0.6180$ and $p = 0.6180(1) + 0.3820(.2) = 0.6944$. Observe that at a cost of $1/k = 0.8333$ no agent would like to produce the risky asset Y in equilibrium.

But if there is no production of Y, naked CDS are not well-defined and cannot be traded. But if CDS are not traded, we would be back in the equilibrium in the L-economy studied above with a price of $p = 0.8333$ and positive production. In short, competitive equilibrium does not exist.

More generally the argument is the following. Equilibrium in the LC-economy is equal to the equilibrium in the LT-economy if Y is produced, and is equal to the equilibrium in the L-economy if Y is not produced.[12] Thus, if all LT equilibria involve no production of Y and all L equilibria involve positive production of Y, then there cannot exist a LC equilibrium. This is precisely the situation for a big range of the production parameter k (for given asset Y payoffs and beliefs). In fact, in this example equilibrium does not exist for all k such that $k \in (1, 1.4)$, as shown in Figure 9.

CDS is a derivative, whose payoff depends on some underlying instrument. The quantity of CDS that can be traded is not limited by the market size of the underlying instrument. If the value of the underlying security diminishes, the derivative CDS trading may continue at the same high levels, as shown in the figure. But when the value of the underlying instrument falls to zero, CDS trading must come to an end by definition. This discontinuity can cause robust non-existence. The classical non-existence observed in Hart (1975), Radner (1979) and Polemarchakis-Ku (1990)

[12] This corresponds to an autarky equilibrium.

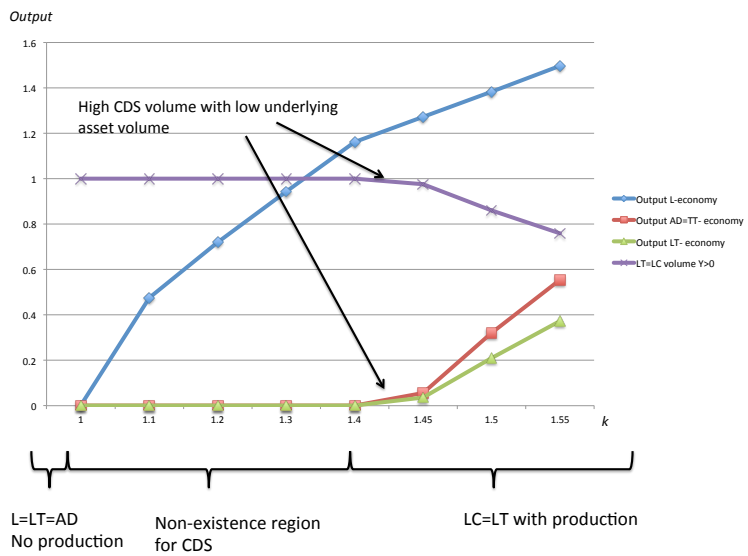

Figure 9: Naked CDS and Robust Non-Existence

stemmed from the possibility that asset trades might tend to infinity when the payoffs of the assets tended toward collinear. A discontinuity arose when they became actually collinear. Collateral restores existence by (endogenously) bounding the asset trades. In our model CDS trades stay bounded away from zero and infinity even as production disappears. Collateral does not affect this, since the bounded promises can be covered by the same collateral. But the moment production disappears, the discontinuity arises, since then CDS sales must become zero.

6 Conclusion

Assets that become usable as collateral for various promises become more valuable, and will be produced in greater quantity. We believe this is an important factor in the housing boom that followed the increase in leverage and tranching of mortgages on houses. The crucial driver of our results is the linkage between promises and the collateral needed to back them. Theoretically, the introduction of new promises, like derivatives, into the GEI model has an ambiguous effect on asset prices. In the capital asset pricing model, for example, the introduction of new promises leaves

relative asset prices unchanged. But if the promises deliver only insofar as they are backed by collateral, as in the collateral equilibrium described in this paper, and if they can only be backed by special kinds of collateral, then those collateral goods become more valuable, and more produced.

We have worked with a special model in order to illustrate the relative value creation of tranching as starkly as possible. We were able to prove theorems unambiguously ranking the amount of investment each kind of financial innovation would generate. More generally, countervailing considerations would muddy the conclusions. For example we assumed that all future output is generated by assets created today. Had we added to the model future endowments of the consumption good (that could not be pledged), the leverage equilibrium and the CDS equilibrium would remain absolutely unchanged, because of our assumption of constant marginal utility. Thus we could still conclude in that case that leverage output is higher than CDS output. But the Arrow Debreu equilibrium would be affected. Indeed the Arrow Debreu level of output could be increased or decreased, thus rendering comparisons of leveraged output or CDS output to Arrow Debreu output ambiguous.

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Appendix

Proof of Proposition 1

Step 1: In every equilibrium, each agent must be maximizing profit. Without loss of generality, we can suppose that every agent chooses the same production (z_x, z_y). Since by hypothesis the mass of agents is normalized to 1, total holdings in the economy are then $(x_{0^*} + z_x, z_y)$. Consider two asset prices p, q, and production plans $z^p = (z_x^p, z_y^p), z^q = (z_x^q, z_y^q)$ that maximize profits at the corresponding prices, so

$$\begin{aligned} z_x^p + pz_y^p &\geq z_x^q + pz_y^q \\ z_x^q + qz_y^q &\geq z_x^p + qz_y^p \end{aligned}$$

Adding the inequalities and rearranging,

$$(p - q)(z_y^p - z_y^q) \geq 0$$

so $p > q$ implies $z_y^p \geq z_y^q$, and $z_y^p > z_y^q$ implies $p \geq q$. Moreover, it is clear that maximizing profit $\Pi(p)$ also maximizes total wealth $W(p)$ (since wealth is profit plus x_{0^*}). It is more convenient to think of maximizing wealth. It is obvious that increasing either the price of X or Y gives rise to higher value of maximal wealth, since choosing the same production plan gives at least the same wealth when prices are higher. Hence $W(p)$ is weakly increasing in p, and strictly increasing if $z_y > 0$. Finally it is also clear that

$$\frac{W(p)}{p}$$

is weakly decreasing in p, and strictly decreasing if $x_{0^*} + z_x > 0$. The reason is that multiplying both prices by a common scalar does not change the profit maximizing production plan, so that wealth is therefore homogeneous of degree 1 in the price vector. Scaling up just the price of Y, holding the price of X fixed at 1, therefore does less than scale up the value of wealth.

Step 2: Step 1 shows that if the leverage asset price $p^L > p^A$, or if the leverage output $z_y^L > z_y^A$, we are done. Hence we only need to show that assuming $p^L \leq p^A$

and $z_y^L \leq z_y^A$ leads to a contradiction. With that assumption, individual profits Π^L in the leverage economy are no higher than in the Arrow-Debreu economy Π^A.

Step 3: Since $(d_U^X, d_D^X, d_X^Y, d_D^Y) >> 0$, there will always be positive aggregate consumption in both states U and D. Thus for the Arrow-Debreu marginal buyer, $0 < h^A < 1$. Suppose that $z_y^L = 0$. Then every agent in the leverage economy consumes his initial endowment x_{0^*}. And no agent, including $h = 1$, prefers Y to X at price p^L. Hence $p^L \geq \gamma_U(1)d_U^Y + \gamma_D(1)d_D^Y > \gamma_U(h^A)d_U^Y + \gamma_D(h^A)d_D^Y = p^A$ and we are done. Alternatively, suppose that $x_{0^*} + z_x^L = 0$. Then the leverage economy is producing the maximum possible y, so trivially $z_y^L \geq z_y^A$. Thus without loss of generality, we suppose that $x_{0^*} + z_x^L > 0$ and $z_y^L > 0$.That guarantees that there is a marginal buyer $0 < h^L < 1$.

Step 4: First, since the prices are set by the marginal buyer in both economies, under the maintained hypothesis $p^L \leq p^A$, we must have $h^L \leq h^A$. In equilibrium,

$$\frac{W(p^A)}{p^A}(1 - h^A) = z_y^A d_U^Y + (x_{0^*} + z_x^A)1$$

$$\frac{W(p^L)}{p^L}(1 - h^L) = z_y^L(d_U^Y - d_D^Y)$$

By our second maintained hypothesis, $z_y^A \geq z_y^L$. Hence RHS of the first equation above is strictly more than the RHS of the second equation above. But by the maintained price hypothesis and Step 1

$$\frac{W(p^A)}{p^A} \leq \frac{W(p^L)}{p^L}$$

It follows that $(1 - h^A) > (1 - h^L)$, and hence that $h^A < h^L$, a contradiction.

Proof of Proposition 2

Step 1: Reasoning as in the last proof, we need only reach a contradiction from the hypothesis that $p^L \leq p^{LT}$ and $z_y^L \leq z_y^{LT}$. From this hypothesis we deduce that $\Pi^L \leq \Pi^{LT}$ and $W^L \leq W^{LT}$.

Step 2: Since $(d_U^X, d_D^X, d_X^Y, d_D^Y) >> 0$, there will always be positive aggregate consumption in both states U and D. Thus for the Arrow-Down marginal buyer, $0 < h_2^{LT} \leq h_1^{LT} < 1$. Suppose that $z_y^L = 0$. Then every agent in the leverage economy consumes his initial endowment x_{0^*}. And no agent prefers Y to X at price p^L. Hence $p^L \geq \gamma_U(1)d_U^Y + \gamma_D(1)d_D^Y > \gamma_U(h^{LT})d_U^Y + \gamma_D(h^{LT})d_D^Y \geq p^{LT}$ and we are done. Alternatively, suppose that $x_{0^*} + z_x^L = 0$. Then the leverage economy is producing the maximum possible y, so trivially $z_y^L \geq z_y^{LT}$. Thus without loss of generality, we suppose that $x_{0^*} + z_x^L > 0$ and $z_y^L > 0$. That guarantees that there is a marginal buyer $0 < h^L < 1$.

Step 3: Under the maintained assumption that more resources are devoted to production in the LT-economy, the remaining X must be at least as high in the leverage economy:

$$(x_{0^*} + z_x^L) + (z_y^L)d_D^Y \geq (x_{0^*} + z_x^{LT}) + (z_y^{LT})d_D^Y$$

Recall that the wealth of each agent in the respective economies is

$$\begin{aligned} W^L &= x_{0^*} + \Pi^L = (x_{0^*} + z_x^L) + z_y^L p^L \\ W^{LT} &= x_{0^*} + \Pi^{LT} = (x_{0^*} + z_x^{LT}) + z_y^{LT} p^{LT} \end{aligned}$$

It then follows from $\Pi^{LT} \geq \Pi^L$ and $p^{LT} \geq p^L$ that $W^{LT} \geq W^L$ and therefore

$$\frac{(x_{0^*} + z_x^L) + z_y^L d_D^Y}{(x_{0^*} + z_x^L) + z_y^L p^L} \geq \frac{(x_{0^*} + z_x^{LT}) + z_y^{LT} d_D^Y}{(x_{0^*} + z_x^{LT}) + z_y^{LT} p^{LT}}$$

From the equilibrium conditions presented earlier for the LT-economy, we know that

$$\begin{aligned} p^{LT} &= \frac{\gamma_U(h_1^{LT})d_U^Y + (1 - \gamma_U(h_1^{LT}))d_D^Y}{\gamma_U(h_1^{LT}) + (1 - \gamma_U(h_2^{LT}))} \leq \gamma_U(h_1^{LT})d_U^Y + (1 - \gamma_U(h_1^{LT}))d_D^Y \\ \pi_{j^*} &= \frac{1}{\gamma_U(h_1^{LT}) + (1 - \gamma_U(h_2^{LT}))} d_D^Y \leq d_D^Y \end{aligned}$$

with a strict inequality in both cases if $z_y^{LT} > 0$, since then $h_1^{LT} > h_2^{LT}$. For the L-economy,

$$p^L = \gamma_U(h_1^L)d_U^Y + (1 - \gamma_U(h_1^L))d_D^Y$$

It follows from the strict monotonicity of $\gamma_U(h)$ and from $p^L \leq p^{LT}$ that

$$h_1^L \leq h_1^{LT}$$

with a strict inequality if $z_y^{LT} > 0$. In LT-equilibrium we must have that the agents above h_1^{LT} spend all their money to buy all the assets

$$\begin{aligned}
(x_{0^*} + z_x^{LT} + p^{LT} z_y^{LT})(1 - h_1^{LT}) &= (x_{0^*} + z_x^{LT} + p^{LT} z_y^{LT}) - z_y^{LT} \pi_{j^*} - (x_{0^*} + z_x^{LT}) p_D^{LT} \\
(x_{0^*} + z_x^{LT} + p^{LT} z_y^{LT}) h_1^{LT} &= z_y^{LT}) \pi_{j^*} + (x_{0^*} + z_x^{LT}) p_D^{LT} \\
&< z_y^{LT} d_D^Y + (x_{0^*} + z_x^{LT})
\end{aligned}$$

The last inequality is strict, because if $z_y^{LT} > 0$, then $z_y^{LT} \pi_{j^*} < z_y^{LT} d_U^Y$, while if $z_y^{LT} = 0$, then $(x_{0^*} + z_x^{LT}) = x_{0^*} > 0$ and $p_D^{LT} = \frac{(1-\gamma_U(h_2^{LT}))}{\gamma_U(h_1^{LT}) + (1-\gamma_U(h_2^{LT}))} < 1$ because $\gamma_U(h_1^{LT}) > 0$ since $h_1^{LT} > 0$. Similarly, in leverage equilibrium we must have that the agents above h_1^L spend all their money to buy all the Y assets

$$\begin{aligned}
(x_{0^*} + z_x^L + p^L z_y^L)(1 - h_1^L) &= (x_{0^*} + z_x^L + p^L z_y^L) - z_y^L d_D^Y - (x_{0^*} + z_x^L) \\
(x_{0^*} + z_x^L + p^L z_y^L) h_1^L &= z_y^L d_D^Y + (x_{0^*} + z_x^L)
\end{aligned}$$

Putting these last two conclusions together we get

$$\begin{aligned}
h_1^{LT} &< \frac{z_y^{LT} d_D^Y + (x_{0^*} + z_x^{LT})}{(x_{0^*} + z_x^{LT} + p^{LT} z_y^{LT})} \\
h_1^L &= \frac{z_y^L d_D^Y + (x_{0^*} + z_x^L)}{(x_{0^*} + z_x^L + p^L z_y^L)}
\end{aligned}$$

But we showed at the outset of the proof that the upper RHS is no bigger than the lower RHS. This implies that $h_1^{LT} < h_1^L$, which is the desired contradiction.

Proof of Proposition 3

Step 1: Reasoning as in the last proofs, we need only reach a contradiction from the hypothesis that $p^A \leq p^{LT}$ and $z_y^A \leq z_y^{LT}$. From this hypothesis we deduce that $\Pi^A \leq \Pi^{LT}$.

Step 2: Since $(d_U^X, d_D^X, d_U^Y, d_D^Y) >> 0$, there will always be positive aggregate consumption in both states U and D. Thus for the Arrow Debreu marginal buyer, $0 < h_1^A < 1$, and for the LT-economy marginal buyers, $0 < h_2^{LT} \leq h_1^{LT} < 1$. It is obvious that if there is no Y in a LT-equilibrium, then the equilibrium must also be an Arrow-Debreu equilibrium. Hence we may assume that $z_y^{LT} > 0$, and hence that $h_2^{LT} < h_1^{LT}$.

Step 3: We begin by deducing several properties of the implicit state prices (q_U, q_D) defined by the prices of$X$ and Y in the LT-economy

$$p^{LT} = q_U d_U^Y + q_D d_D^Y$$

$$p_X^{LT} = 1 = q_U + q_D$$

These state prices can be used to price X and Y, but not the other securities. From the hypothesis $p^{LT} \geq p^A$, it follows that $q_U \geq p_U^A$. It also follows from the equations defining LT-equilibrium that

$$\begin{aligned}
\frac{\gamma_U(h_1^{LT})(d_U^Y - d_D^Y) + d_D^Y}{\gamma_U(h_1^{LT}) + 1 - \gamma_U(h_2^{LT})} &= \frac{\gamma_U(h_1^{LT})d_U^Y + (1 - \gamma_U(h_1^{LT}))d_D^Y}{\gamma_U(h_1^{LT}) + 1 - \gamma_U(h_2^{LT})} = p^{LT} \\
&= \frac{q_U d_U^Y + (1 - q_U)d_D^Y}{1} = \frac{q_U(d_U^Y - d_D^Y) + d_D^Y}{1}
\end{aligned}$$

Comparing the last term and the first term

$$\frac{\gamma_U(h_1^{LT})(d_U^Y - d_D^Y) + d_D^Y}{\gamma_U(h_1^{LT}) + 1 - \gamma_U(h_2^{LT})} = \frac{q_U(d_U^Y - d_D^Y) + d_D^Y}{1}$$

Since $\gamma_U(h_1^{LT}) + 1 - \gamma_U(h_2^{LT}) > 1$, it follows immediately that

$$\begin{aligned}
\frac{\gamma_U(h_1^{LT})}{\gamma_U(h_1^{LT}) + 1 - \gamma_U(h_2^{LT})} &> q_U \\
\frac{1 - \gamma_U(h_2^{LT})}{\gamma_U(h_1^{LT}) + 1 - \gamma_U(h_2^{LT})} &< q_D
\end{aligned}$$

Cross multiplying the original equality gives

$$\begin{aligned}
\gamma_U(h_1^{LT})(d_U^Y - d_D^Y) + d_D^Y &= [1 + \gamma_U(h_1^{LT}) - \gamma_U(h_2^{LT})](q_U(d_U^Y - d_D^Y) + d_D^Y) \\
(\gamma_U(h_1^{LT}) - q_U)(d_U^Y - d_D^Y) &= (\gamma_U(h_1^{LT}) - \gamma_U(h_2^{LT}))(q_U(d_U^Y - d_D^Y) + d_D^Y) \\
\frac{\gamma_U(h_1^{LT}) - q_U}{\gamma_U(h_1^{LT}) - \gamma_U(h_2^{LT})} &= \frac{q_U(d_U^Y - d_D^Y) + d_D^Y}{(d_U^Y - d_D^Y)} \\
&> q_U
\end{aligned}$$

where we have used the strict inequalities $h_1^{LT} > h_2^{LT}$ and $d_U^Y > d_D^Y > 0$, and the strict monotonicity of $\gamma_U(h)$. It follows that

$$\frac{\gamma_U(h_1^{LT}) - q_U}{q_U - \gamma_U(h_2^{LT})} > \frac{q_U}{1 - q_U}$$

From the continuity and the strict monotonicity of $\gamma_U(h)$, we can define a unique h^* with $\gamma_U(h^*) = q_U$. From the concavity of $\gamma_U(h)$, we deduce that

$$\frac{h_1^{LT} - h^*}{h^* - h_2^{LT}} > \frac{q_U}{1 - q_U}$$

As usual, define the wealth of each agent by $W^{LT} = (x_{0^*} + \Pi^{LT})$ where

$$W^{LT} = p^{LT} z_y^{LT} + (x_{0^*} + z_x^{LT}) = q_U[z_y^{LT} d_U^Y + (x_{0^*} + z_x^{LT})] + q_D[z_y^{LT} d_D^Y + (x_{0^*} + z_x^{LT})]$$

It is now convenient to define the fictitious agents $h^{**}, h_1^{**}, h_2^{**}$ who act as if they could trade U and D goods at the state prices (q_U, q_D). In terms of the diagram in Figure 6, call the points where the orthogonal price lines through C_0 and C_P and the top left of the Edeworth box intersect the diagonal $(1 - h_1^{**})P$ and $(1 - h_2^{**})P$ and $(1 - h^{**})P$, respectively. It is obvious that $(1 - h_1^{**})P > (1 - h^{**})P$ and that $(1 - h_2^{**})P > (1 - h_2)P$.

More precisely, define h^{**} to solve

$$\begin{aligned}
W^{LT}(1 - h^{**}) &= q_U[z_y^{LT} d_U^Y + (x_{0^*} + z_x^{LT})] \\
W^{LT} h^{**} &= q_D[z_y^{LT} d_D^Y + (x_{0^*} + z_x^{LT})]
\end{aligned}$$

Observe that $h^{**} \leq h^A$. This follows from the fact that by hypothesis, $W^{LT} \geq W^A$ and $q_D \leq p_D^A$ and $[z_y^{LT} d_D^Y + (x_{0^*} + z_x^{LT})] \leq [z_y^A d_D^Y + (x_{0^*} + z_x^A)]$.

Now define h_1^{**}, h_2^{**} by the following equations

$$\begin{aligned} W^{LT}(1-h_1^{**}) &= q_U[z_y^{LT}(d_U^Y - d_D^Y) + (x_{0^*} + z_x^{LT})] \\ W^{LT}h_2^{**} &= q_D[(x_{0^*} + z_x^{LT})] \end{aligned}$$

Then

$$\begin{aligned} W^{LT}(h_1^{**} - h^{**}) &= q_U z_y^{LT} d_D^Y \\ W^{LT}(h^{**} - h_2^{**}) &= q_D z_y^{LT} d_D^Y \\ \frac{(h_1^{**} - h^{**})}{(h^{**} - h_2^{**})} &= \frac{q_U}{(1-q_U)} \end{aligned}$$

In LT-equilibrium,

$$\begin{aligned} W^{LT}(1-h_1^{LT}) &= [z_y^{LT}(d_U^Y - d_D^Y) + (x_{0^*} + z_x^{LT})]\gamma_U(h_1^{LT})/([\gamma_U(h_1^{LT}) + 1 - \gamma_U(h_2^{LT})] \\ W^{LT}h_2^{LT} &= [(x_{0^*} + z_x^{LT})](1-\gamma_U(h_2^{LT}))/([\gamma_U(h_1^{LT}) + 1 - \gamma_U(h_2^{LT})] \end{aligned}$$

From the inequalities on probabilies derived earlier, it is obvious that $h_2^{LT} < h_2^{**}$ and $(1-h_1^{LT}) > (1-h_1^{**})$, that is also $h_1^{LT} < h_1^{**}$.

Thus it follows that

$$h^* < h^{**} \leq h^A$$

giving us the desired contradiction to our previous findings that $q_U = \gamma_U(h^*) \geq p_U^A = \gamma_U(h^A)$.