

One in a Million: A Field Experiment on Belief Formation and Pivotal Voting*

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February 5, 2013

Abstract

Instrumental voting models predict that turnout depends on the chance of casting a pivotal vote, which is typically extremely low in large elections. Evidence from psychology and behavioral economics suggests that misperceptions of extremely unlikely events are common and subject to systematic biases, sometimes called the non-belief in the law of large numbers. We provide a model of voting when voters suffer from these biases and show that they inflate the perceived pivot probabilities, and hence turnout. Moreover, voters do not fully account for new information of pivot probabilities in this model. We then test the model in a large-scale field experiment during the 2010 U.S. gubernatorial elections where we elicited voter beliefs about a very close election before and after showing different polls. We find that voters massively inflate pivot probabilities and this inflation is most pronounced among subjects measured to have the highest non-belief in the law of large numbers. Furthermore, subjects respond to but fail to fully incorporate new information presented in the experiment. However, the decision to vote is not affected by beliefs about pivotality. Even in a controlled setting and in response to an experimental manipulation that significantly changed the perceived probability of being pivotal, voting behavior is unaffected.

Preliminary.

*We thank Jason Abaluck, Stefano DellaVigna, Fred Finan, Sean Gailmard, Don Green, Jennifer Green, Gianmarco Leon, Matthew Rabin, Rob Van Houweling, and seminar participants at the Stanford Institute on Theoretical Economics for helpful comments. David Arnold, Christina Chew, Sandrena Frischer, Will Kuffel, Elena Litvinova, Melina Mattos, Kevin Rapp, Nick Roth, and Irina Titova provided outstanding research assistance. Financial support from the National Science Foundation Dissertation Completion Fellowship, the Haas School of Business, the Center for Equitable Growth, and the Burch Center is gratefully acknowledged.

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1 Introduction

Why do people vote? The process by which individuals decide whether and for whom to vote is one of the most enduring and important questions in political economy. Despite a plethora of hypotheses and a wealth of data, many puzzles persist, particularly with regard to turnout. For instance, even across similar wealthy countries, differences in voting behavior are stark. The U.S. exhibits low voter turnout as well as large disparities in turnout by socioeconomic status. The canonical theory of voting, dating back to [Downs \(1957\)](#), postulates that voters are mainly concerned with electoral outcomes; that is, voting is an instrumental process to achieve the rational end of obtaining one's preferred alternative from among those on offer. Thus, in deciding whether and for whom to vote, a voter weighs the net benefit from obtaining her most preferred outcome multiplied by the chance that her vote is decisive, or pivotal in the parlance of political economy. This expected benefit is weighed against the cost of voting and, only if the benefit outweighs the cost will a voter turn out at the poll.

This theory is often criticized for its implications on turnout. The inexorable arithmetic of pivotal probabilities imply that the chance of casting a decisive vote falls precipitously as turnout increases, and so, unless the benefits from obtaining one's preferred candidate are exceptionally high or costs exceptionally low, the voting calculus favors staying at home, leading to unreasonably low turnout percentages. Indeed, so long as the benefits are finite and the costs non-negative, the theory predicts that the turnout percentage will approach zero in a large election. This is obviously grossly at odds with the data. The conundrum, often called "the paradox of voting," has led researchers to look for ways to amend the voting calculus; thereby obtaining a more satisfactory solution. Downs speculated that the pang of conscience, the duty as citizen to vote, could reconcile the paradox to the data. Under this theory, first formalized by [Riker and Ordeshook \(1968\)](#), citizens experience a loss from shame or guilt in staying away from the polls. These voters have, in effect, negative voting costs and turn up at the polls even if the expected benefit from voting is approximately zero. Subsequent research, including [Fiorina \(1976\)](#), [Feddersen and Sandroni \(2006\)](#), [Benabou and Tirole \(2003\)](#), [Funk \(2010\)](#), and [Morgan and Vardy \(2013\)](#) explore the implications of this insight on turnout and voting outcomes.

In this paper, we examine, both theoretically and empirically, another aspect of the calculus of voting: the misperception of pivotal probabilities. There is by now a large literature in psychology and behavioral economics showing that beliefs of individuals about the likelihood of certain events often diverge from the true mathematical properties in systematic ways. This divergence is particularly acute in calculating extremely unlikely events, such as the probability of casting a decisive vote in a large election. We explore what [Kahneman and Tversky \(1972\)](#) dubbed the *non-belief in the law of large numbers* (NBLLN). This phenomenon consists of two types of cognitive errors: Individuals tend to misperceive the nature of the underlying distribution of probabilities, ascribing fatter tail probabilities than are warranted. In addition, individuals tend to underweight the precision of a large sample; in effect, inflating the sample standard deviation as the number of draws becomes large.

Our contribution is to show that the non-belief in the law of large numbers leads to a systematic

misperception and inflation of pivot probabilities, particularly in large elections. As a consequence, individuals view the expected benefit from voting in a large election as non-negligible and turn out to vote even in the absence of motives such as civic duty. This is not to say that duty has no role to play, but rather to point out a complementary effect from the misperception of pivotal probabilities. Both effects work in the direction of increasing turnout relative to the canonical model. Building on [Benjamin et al. \(2012b\)](#) formalizing the non-belief in the law of large numbers, we apply their framework to the study of pivotal probabilities. The two main implications of the theory are: (1) Individuals subject to NBLLN inflate pivotal probabilities such that they remain bounded away from zero, even in the limit as the electorate grows large; and (2) individuals subject to NBLLN under-react to new information, even if it arises from a large and representative poll, instead clinging to prior beliefs.

We then test these implications in a large-scale field experiment consisting of 16,000 voters during the 2010 gubernatorial elections. Using computer surveys with potential voters in 13 U.S. states, we asked voters their beliefs about the chance that the state gubernatorial election would be very close. We then exposed these voters to different informational treatments to assess the impact of new information on their beliefs and behavior. Exploiting the variation in poll results prior to the election, we divided subjects into three groups. The control group received no polling information. We informed the “not close” group of the results of a poll indicating the greatest gap between the two candidates. We informed the “close” group of the results of a poll indicating the narrowest margin between the candidates. Following this, we then observed the impact of this information by subsequently asking them the chance the election would be very close. Finally, we used administrative data to determine whether our survey respondents did, in fact, turn up to vote. To gauge subjects susceptibility for the non-belief in the law of large numbers, we also conducted a “lab experiment in the field” using a design introduced by [Benjamin et al. \(2012a\)](#), which provided a measure of NBLLN.

We obtained two main findings from these experiments. First, both belief formation and updating are strongly consistent with a non-belief in the law of large numbers. Subjects dramatically overestimate the probability of a very close election. The median probabilities that the gubernatorial election would be decided by less than 100 or less than 1,000 votes were 10% and 20%, respectively. Even among the 1400 voters with Masters and PhD degrees, the median perceived probabilities of less than 100 and less than 1,000 votes were 5% and 10%, respectively. Moreover, subjects who scored high in our NBLLN measure also exhibited a stronger tendency to overestimate the chance of being pivotal and to update less when presented with poll results. In addition to our directional findings, we estimate a structural model of NBLLN to see how well the model can explain the data and to summarize the degree of NBLLN in a single parameter that can be compared across domains.

Second, although beliefs imply a high and changeable expected benefit from voting, this does not translate into behavior as predicted by the instrumental models of voting, even with the amendment of the misperception of pivotal probabilities. Indeed, voter turnout was, statistically, independent of differences in beliefs about the chance of casting a decisive vote. This suggests that non-instrumental considerations, such as expressive voting, loom larger in the minds of voters determining whether

and for whom to vote. We are pains to stress that we have no direct evidence of these alternative considerations; nonetheless, the results are simply inconsistent with an electoral calculus whereby voter compute the expected benefit of voting (perhaps incorrectly) and then adjust turnout and voting behavior accordingly.

To summarize, the two contributions of the paper are (1) to postulate and then demonstrate that non-belief in the law of large numbers plays a significant role in pivotality calculations of voters, and (2) to show that differences in beliefs about pivotality seem to play no role in actual turnout. To the best of our knowledge, we are the first to study the relationship between beliefs about pivotality and voting in the field, as well as the first to provide field evidence on the non-belief in the law of large numbers.

Before proceeding, it is helpful to place the work in context. It is well-known that turnout tends to rise in close elections, which is broadly consistent with an instrumental theory of voting and turnout.¹ But there are many confounds in simply comparing turnout across elections. Close elections tend to attract more advertising and news coverage leading up to the vote compared with landslides. Close elections, like sporting events, are more interesting to monitor and discuss than walkovers. Coate et al. (2008) overcome these confounds by examining turnout across towns in Texas voting on whether to legalize the sale of liquor. Since the towns varied in size, so did the chance of being pivotal. Coate et al. (2008) found higher turnout in smaller towns than in larger. Even this study is not immune to confounding factors, most notably variations in the sense of “duty” across towns.. Turnout is more readily monitored in small towns than in large cities, and this may alter the costs of remaining at home. Indeed, in a study of the introduction of voting by mail in Swiss cantons, Funk (2010) noted that turnout fell by a greater amount in smaller communities than in large, presumably because the social enforcement of voting was greater.

Rather than relying on natural experiments, we experimentally manipulate individual beliefs about the chance of casting a decisive vote. This has the advantage of eliminating confounds, but the disadvantage that the manipulation lacks power. For instance, if subjects were voracious consumers of polling data, then our informational treatment should have no effect, as no new information was, in fact, presented. The belief response of subjects plus introspection about how interesting gubernatorial polling data is to an average person, suggests this was not the case.

Many laboratory experiments of voting have manipulated the chance of being pivotal and found effects on voting decisions (e.g. Tyran, 2004). Mainly, these papers are concerned with expressive voting, which should manifest itself more when pivot probabilities are small rather than large. There are several important differences between laboratory studies and our setting. First, the size of the elections is considerably smaller and, indeed, the chance of being pivotal in any of these studies are orders of magnitude larger than in statewide gubernatorial races. Second, and perhaps more importantly, pivotal probabilities in laboratory experiments represent objective probabilities rather than subjective. Indeed, subjects are often informed about their precise chance of being pivotal. Finally, none of these studies concerns itself with the non-belief in the law of large numbers.²

¹Foster (1984) and Matsusaka and Palda (1993) contain extensive surveys of the literature on turnout.

²Shayo and Harel (2012) simulate somewhat large elections by using a lottery whereby, with probability p the

We are aware of no other field experiments that randomly assign polls to voters so as to examine the impact on turnover. Since our field experiment was conducted in 2010, [Agranov et al. \(2012\)](#) conducted a lab experiment where voters were assigned different polling information. In addition, we are aware of very few studies that seek to measure or influence voter beliefs about electoral closeness. [Delavande and Manski \(2010\)](#) measure voter beliefs about the probability of a very close election in the RAND Life Panel, but do not experimentally manipulate them.

Our paper also links with the small literature on non-Bayesian beliefs and updating. In doing so it builds on the model of [Benjamin et al. \(2012b\)](#). While there is an extensive literature estimating models of beliefs and learning in games (examples of this literature include [Camerer et al. \(2002\)](#) and [Crawford et al. \(2012\)](#)), the literature on structurally estimating models of beliefs and updating using field data is much smaller.³ Third, because the paper also experimentally elicits a measure of NBLLN it touches on a literature that links experimental measures of parameters to real-world beliefs and behavior, discussed in detail in [Camerer \(2011\)](#).

The rest of the paper is as follows. In [Section 2](#) we develop our theory of NBLLN and voting. [Section 3](#) describes the design of our field experiment, as well as the lab experiment designed to measure NBLLN. [Section 4](#) describes the reduced-form results of the field experiment. [Section 5](#) structurally estimate the degree of NBLLN implied by voter beliefs. [Section 6](#) concludes.

2 Model

In order to understand the relationship between non-Bayesian beliefs and voting, we will put the model of non-belief in the Law of Large Numbers (NBLLN) developed by [Benjamin et al. \(2012b\)](#), hereafter BRR, into a simple model of elections and voting.⁴

There are two candidates A and B . Individuals can vote for one or the other. For the moment, we will assume that there are states of the world, $\theta \in (0, 1)$. Given state θ , then any individual has a probability θ of voting for candidate A .⁵

outcome chosen by the subject is picked and with probability $1 - p$ the computer picks the outcome. When the computer picks, the weights between the two outcomes are determined according to an equilibrium of some voting model. This design has the important advantage that it does not require a large number of subjects interacting with one another to produce the pivot probabilities of a large election. Moreover, the design allows the experimenter to directly control beliefs about pivotality. A key drawback of this approach is that the probability of being pivotal is no longer subjective, as it is in the field. Many studies, dating back to [Allais \(1953\)](#), demonstrated substantial differences in choice behavior when individuals face events with subjective versus objective probabilities. An important difference between our study versus [Shayo and Harel \(2012\)](#) is that our experimental manipulations are on subjective beliefs rather than objective.

³Examples include recent papers by [Malmendier and Nagel \(2012\)](#), [Barseghyan et al. \(2012\)](#), [Goettler and Clay \(2011\)](#), [Grubb and Osborne \(2011\)](#), and [Hoffman and Burks \(2012\)](#). [DellaVigna \(2009\)](#) provides a survey of earlier work.

⁴Although NBLLN seems likely to have strong impacts on beliefs [Benjamin et al. \(2012a\)](#) provide strong evidence that the impact can be very large), other factors are likely also at play. For example, the probability weighting function of [Prelec \(1998\)](#) also can change small probabilities into probabilities of moderate size. However, this would have trouble explaining the lack of updating we find. For an alternative model that rationalizes turnout using leaders exerting more get-out-the-vote effort in close elections, see [Shachar and Nalebuff \(1999\)](#).

⁵This setup might seem excessively simply, since it implies that individuals randomize in voting. However, more realistic alternative specifications, such as individual taste shocks, or idiosyncratic costs of voting, will generate equivalent results, but at the cost of added complexity. See [Appendix A](#) for an illustration.

We will assume that individuals suffer from non-belief in the Law of Large Numbers (NLLN). Abusing notation slightly, we will also refer to a non-believer in the Law of Large Number using the abbreviation NLLN. In contrast we will refer to an agent who has the correct understanding of statistics and the data generating process as a true Bayesian (TB). Recall that if a NLLN knows the rate is θ , he believes that signals are generated with a binomial process, where the probability of an outcome of type a is equal to β . $\beta \in [0, 1]$ is drawn from a density $f_{\beta|\theta}^{\psi}(\beta|\theta)$, where $f_{\beta|\theta}^{\psi}(\beta|\theta)$ satisfies assumptions the mild assumptions of A1 through A5 in BRR.

2.1 Beliefs about Election Outcomes

Using this framework we can now discuss individuals' beliefs about election outcomes. For the moment, we will assume that there is a single known state of the world with rate θ . We will call an election ϵ -percent close if the margin of victory (in terms of percent) is less than ϵ . We will call an election ι -vote close if the margin of victory (in terms of number of votes) is less than ι .

Proposition 1 shows that although a TB believes that for appropriately large elections in states where voters are sufficiently partisan, there is a negligible chance of the election being close. However, a NLLN, regardless of the size of the election, or the partisanship of the electorate, always believes that probability of a close elections will not be below some strictly positive value. This of course, immediately implies that for large elections, and sufficiently partisan electorates, a NLLN will always believe a close election is more likely than a TB.⁶

Proposition 1⁷

1. *For a TB, for all $0 < \epsilon < .5$, $0 < \zeta < 1$, and $\theta \in (0, 1)$ such that $|\theta - .5| > \epsilon$, there exists an N' such that for all elections of size $N > N'$, then the probability of an 2ϵ -percent close election is less than ζ . For a NLLN, for all $0 < \epsilon < .5$, and all $\theta \in (0, 1)$ such that $|\theta - .5| > \epsilon$, there exists a $0 < \zeta^* < 1$, such that for elections of all size N , the probability of an 2ϵ -percent close election is greater than ζ^* .*
2. *For all $0 < \epsilon < .5$, all $0 < \zeta < 1$, and all $\theta \in (0, 1)$ such that $|\theta - .5| > \epsilon$, there exists an N' such that for all elections of size $N > N'$, then a NLLN thinks there is a higher probability of an 2ϵ -percent close election than a TB.*

However, in contrast to the previous proposition, if the electorate is not very partisan (e.g. $\theta = .5$) then a NLLN can underestimate the probability of a close election. This is because if the electorate is not very partisan then close elections are the expected mean outcome. A NLLN overestimates extreme outcomes, and underestimates the probability of the mean outcome.

Although a NLLN believes that ϵ -percent close elections are always possible, he, like a TB, does not believe this is true for ι -vote close elections. This is simply because as the electorate gets larger and larger, the same *iota*-vote close elections requires a closer and closer percentage ϵ to achieve.

⁶Mandler (2012) generates similar outcomes in a model with aggregate uncertainty. However, that model will fail to explain the lack of updating from large polls that we discuss in the following sub-section.

⁷Proofs are in Appendix B.

However, a NBLLN still believes ι -voter close elections are more likely than a TB for large partisan elections. This is because a NBLLN overestimates the likelihood of any ϵ -percentage election.

Proposition 2

1. For a TB and a NBLLN, for all $\theta \neq .5$, all $0 < \zeta < 1$, and all $0 < \iota < N$, there exists an N' such that for all elections of size $N > N'$, then the probability of an ι -vote close election is less than ζ .
2. For all $0 < \epsilon < .5$, all $0 < \zeta < 1$, and all $\theta \in (0, 1)$ such that $|\theta - .5| > \epsilon$, there exists an N' such that for all elections of size $N > N'$, then a NBLLN thinks there is a higher probability of an $2N\epsilon$ -vote close election than a TB.

Despite their difference in the level of probability they assign to close elections, both a NBLLN and a TB respond in similar ways to changes in the partisanship of the electorate.

Proposition 3 *If $\theta > \theta' > .5$ then for all $0 < \epsilon < 1$ and $0 < \iota < N$, both a TB and a NBLLN believe that the probability of either an ϵ or ι close election is higher under θ' than θ .*

Obviously the reverse of Proposition 3, where the states are less than .5, is also true.

Of course, the assumption of a single, known state is extreme. It is more natural to assume that the agent has a set of possible states (i.e. values of θ), including a countably infinite number. Not all of the results naturally extend. A TB will now believe, given some sets of priors, that even from with an infinite voting population, that a close election is always possible. However, given large enough polls that are extreme enough, a TB will cease to believe in the possibility of close elections. This will not happen with a NBLLN. So long a sufficient weight (in terms of prior probabilities) are assigned to extreme states of the world (i.e. states sufficiently far from .5) a NBLLN will continue to overestimate the probability of a close election relative to a TB.

2.2 Polls and Updating

We will now assume there are two possible states θ_1, θ_2 .⁸ The individual has some prior over them $0 < f_\Theta(\theta_1), f_\Theta(\theta_2) < 1$ with $f_\Theta(\theta_1) + f_\Theta(\theta_2) = 1$. Individuals will observe the results of a poll, and then update their beliefs. For the moment, suppose that a poll is a random sample of individuals, who when asked the poll question, at that moment decide who they will vote for (with the probability being determined by either θ_1 or θ_2 , whichever is the true state). Assume furthermore, that these individuals are then thrown out of the population for the actual vote, but replaced by voters with the correct probability of voting for candidate A , (either θ_1 or θ_2). We will assume that the poll has a sample size of N .⁹

A TB, given a large enough poll, regardless of the size of the state, will learn the true state of the world arbitrarily well. On the other hand, given a poll that asymptotically reflects the underlying

⁸The results easily generalize to more than just two states.

⁹This setup allows us to ignore the issues of sampling from a finite pool without replacement, which are important when considering our limit results. One could again frame the polls in a more realistic way, with individuals knowing their tastes-ex ante, but there being uncertainty about who votes, which would give the same results.

population, a NBLLN has an upper bound on the amount he can learn from any poll.¹⁰ Therefore, with a sufficiently large poll, a NBLLN will infer less than a TB from large enough polls.¹¹

Proposition 4

1. *Suppose θ_1 is the true state, and assume that polls are random samples from a population. For a TB, for all $\theta_1, \theta_2 \neq .5$, all $0 < f_{\Theta}(\theta_1), f_{\Theta}(\theta_2) < 1$, all $0 < \zeta < 1$ then there exists an N' such that for all polls of size $N > N'$, then almost surely the posterior probability that a TB assigns to θ_2 being the true state is less than ζ . For a NBLLN, for all $\theta_1, \theta_2 \neq .5$, all $\zeta < f_{\Theta}(\theta_2) < 1$, and polls of all sizes N , then there exists a ζ and τ such that with probability τ the posterior probability that a TB assigns to θ_2 being the true state is greater than ζ for all N .*
2. *Suppose θ_1 is the true state, and assume that polls are random samples from a population. For all $\theta_1, \theta_2 \neq .5$, all $0 < f_{\Theta}(\theta_1), f_{\Theta}(\theta_2) < 1$, $1 > \tau > 0$, there exists an N' and such that for all polls of size $N > N'$, then a NBLLN places a higher posterior on θ_2 being the true state compared to a TB with probability greater than τ .*

Of course, a NBLLN and a TB are both sensitive to the sample size and population size — fixing a sample size N , they both infer less about a larger finite population, and fixing a population size, they infer more about it as the sample size grows.^{12,13} In many situations observing a poll that predicts a close margin of victory, instead of a poll that predicts a large margin, will lead to the decision-maker’s posterior ratio will place a higher weight on a close margin of victory relative to a large margin of victory. However, this only occurs in situations where both possible states, and both poll results, all agree on which candidate is favored. It is a direct implication of the following proposition.

Proposition 5 *Fix $\theta_1 > \theta_2$, and a prior $0 < f_{\Theta}(\theta_1), f_{\Theta}(\theta_2) < 1$, and two polls, P_1 and P_2 . Assume that the proportion of individuals voting for A in P_1 is larger than in P_2 . If an individual sees P_1 they will place a higher posterior probability on θ_1 than if they see P_2 .*

Of course, if the states or the poll results are on either side of .5, then it could be the case that observing a more extreme poll leads to a higher posterior on the closer state. For example, imagine that priors are equal, $\theta_1 = .7, \theta_2 = .48$, the proportion of A voters in P_1 is .48 and the proportion

¹⁰Where the polls samples from the population without replacement, then obviously a NBLLN will learn the true state for sure if he observes the entire finite population.

¹¹Of course, there could be alternative behavioral explanations for why individual’s do not update much based on a poll. For example, they could believe in the Law of Large Numbers, but not be sure whether he has seen the information before. However, this type of a model would still fail to predict that individuals would over-estimate the probability of a close election.

¹²We do not formalize these intuitions as the subjects in our experiment were not directly informed of the survey size, so we have no way to control for that in a regression. Furthermore, it would require us to consider a slightly different framework, sampling without replacement, from a finite sample.

¹³Furthermore, if we instead assume that a poll is a sample without replacement of voters, there are additional comparisons that can be done. For small polls, a NBLLN believes that the marginal benefit of an additional observation is smaller than a TB. But for large polls (i.e. those close enough to the full population), then the marginal benefit of an additional observation is bigger for a NBLLN than for a TB.

of A voters in P_2 is .2. In this case, if an individual observes P_2 they find state 2 (the closer state) more likely relative to if they had observed P_1 , even though P_2 is the more extreme poll.

One way to pull apart non-belief in the law of large numbers from competing explanations (such as aggregate uncertainty) is by looking at beliefs in close elections after large polls. After large polls, a TB should be almost certain about the state, and so for a sufficiently partisan population, should believe that in a large election, the probability of a close election is negligible. Even after a large poll, a NBLLN will still be uncertain about the state, and always believe that there is a non-negligible chance of a ϵ -percentage close election.

Proposition 6 *Suppose θ_1 is the true state, and that polls are random samples from a population. For all $0 < \epsilon < .5$, all $\theta_1 \in (0, 1)$ such that $|\theta - .5| > \epsilon$, all $\theta_2 \in (0, 1)$, all $0 < f_\Theta(\theta_1), f_\Theta(\theta_2) < 1$:*

- *There exists M' and N' such that for all elections of size $N > N'$, and polls of size $M > M'$ so that $N > M$, a NBLLN believes a 2ϵ percent close election and $2N\epsilon$ -vote close election is more likely than a TB.*
- *For all $1 < \zeta_1, \zeta_2 < 0$, there exists M' and N' such that for all elections of size $N > N'$, and polls of size $M > M'$ so that $N > M$, a TB believes that the probability of a 2ϵ percent close election and $2N\epsilon$ -vote close election are less than ζ_2 with probability greater than ζ_1 .*
- *There exists $1 < \zeta_1, \zeta_2 < 0$, such that for all elections of size N , and polls of size M so that $N > M$, a NBLLN believes that the probability of a 2ϵ percent close election is greater than ζ_2 with probability greater than ζ_2 .*
- *For all $1 < \zeta_1, \zeta_2 < 0$, there exists M' and N' such that for all elections of size $N > N'$, and polls of size $M > M'$ so that $N > M$, a NBLLN believes that the probability of a $2N\epsilon$ -vote close election are less than ζ_2 with probability greater than ζ_1 .*

2.3 Voting

The previous propositions have examined beliefs about elections. This section will consider behavior — how NBLLN affects voting. We will begin by considering the seminal model of [Riker and Ordeshook \(1968\)](#). A voter should decide to vote if

$$pB - C + D > 0$$

where p is the probability of being pivotal, B is the benefit of being pivotal, C is the cost of voting and D is the non-instrumental benefit of voting.

To make voting probabilistic, we add an error term $-\varepsilon$ reflecting idiosyncratic factors affecting. The variable ε is assumed to have a cumulative distribution function $F(\varepsilon)$ and density function $f(\varepsilon)$ centered at zero. Thus, the probability of voting is given by $\Pr(pB - C + D - \varepsilon) = F(pB - C + D)$.

Proposition 7 *An individual with a higher probability of being pivotal will have a higher probability of voting.*

Of course, all else being equal, we would expect individuals who were shown a polls with a smaller margin of victory to have beliefs that they are more likely to be pivotal, and so they should be more likely to vote. The change in the probability of voting caused by the treatment can be written as:

$$\begin{aligned}\Delta \Pr(\textit{Vote}) &\approx \frac{\partial \Pr(\textit{Vote})}{\partial p} * \Delta p \\ &= B * f(pB - C + D) * \Delta p\end{aligned}$$

Proposition 8 *An individual shown a poll with a smaller margin of victory will have a higher probability of voting.*

2.4 Summary of Claims

This sub-section summarizes the implications of non-belief in the Law of Large Numbers in the our simple model of elections that we will test in the data set.

1. Individuals will overestimate the probability of close elections in sufficiently partisan elections (Propositions 1 and 2)
2. Individuals who believe the mean electoral outcome is farther from .5 believe that close elections are less likely (Proposition 3)
3. Individuals will not update sufficiently from poll outcomes (Proposition 4)
4. Individuals who will overestimate the probability of close elections in sufficiently partisan elections should also not update sufficiently from poll outcomes (Conjunction of Propositions 1,2 and 4)
5. Individuals who observe a poll more in favor of candidate *A* will update their beliefs more in the direction of candidate *A* winning the election than if they observe a poll less in favor of candidate *A* (Proposition 5)
6. Individuals who observe sufficiently partisan large poll results will still overestimate the probability of a close election (Proposition 6)
7. Individuals who believe in a closer mean outcome, or place a higher probability on a close election, should be more likely to vote (Propositions 7)
8. Individuals who are shown a poll with a closer margin of victory should be more likely to vote (Proposition 8)

3 Methods and Data

We test the theory using our field experiment with 16,000 voters.

3.1 Sample Design and Experiment Logistics

The first survey was sent out to subjects on October 19th, 2012, which was two weeks before the election. The survey was administered by Knowledge Networks, a large online survey company. The subjects were participants in the Knowledge Networks KnowledgePanel.

We focused on states with gubernatorial races. In each state selected, we used all the respondents in the Knowledge Networks KnowledgePanel who were registered voters. We obtained poll information from the websites FiveThirtyEight.com and RealClearPolitics.com.

In choosing our sample of states, we excluded Colorado, Massachusetts, Maine, Minnesota, and Rhode Island, as these were states where there was a major third party candidate. In addition, we restricted our sample to states (1) where there was a poll within the last 30 days indicating a vote margin between the Democrat and Republican candidates of 6 percentage points or less and (2) where there were two polls that differed between each other by 4 percentage points or more. This left us with 13 states: California, Connecticut, Florida, Georgia, Illinois, Maryland, New Hampshire, New York, Ohio, Oregon, Pennsylvania, Texas, and Wisconsin.

During the survey, we asked subjects about the chance that they would vote, their chance of voting for the different candidates, their prediction of the vote margin, and the chance that the election would be decided by less than 100 or 1,000 votes.¹⁴ Before the probability questions, subjects received the standard “explanation of probabilities” developed in the pioneering working of Charles Manski and used in [Delavande and Manski \(2010\)](#). All belief questions were administered without any incentives for accuracy.¹⁵ We then provided the information treatment, described below. Immediately after the information treatment, subjects were asked the same questions again. We decided to ask the same questions immediately after treatment so as to detect if there was any immediate impact on voting intentions. Given that the time before an election is often filled with significant information in the news media, we wanted to give our treatment the best possible chance of having an impact on voting intentions. Screen shots with exact question wording are given in the Appendix.

On the Friday before the election (October 29th), we went out a reminder email to subjects that contained the same information from the first survey.

The post-election survey was sent out on November 19th, 2010, 17 days after the election, and subjects completed the survey until November 30th, 2010.

¹⁴To avoid any issues of anchoring or voters trying to make their answers consistent across questions, voters were randomly assigned to be asked about *either* the chance the election would be decided by less than 100 *or* less than 1,000 votes.

¹⁵We decided not to use incentives for accuracy after a political scientist colleague informed us that doing so may be illegal, possibly constituting either gambling on elections or potentially even being a form of paying people to vote (for the question where we ask people about their intended voting probability). Field experiments that have randomized incentives for accuracy often find little impact of using incentives on beliefs ([Hoffman and Burks, 2012](#)). Given the wide range of backgrounds, ages, and education levels in our sample, we suspect that adding financial incentives for accuracy via a quadratic scoring rule might have actually increased elicitation error due to additional complexity instead of decreasing it.

3.2 Selection of Polls and Information Treatment

Poll choices were finalized on October 17th, 2010. To select the polls, we located the poll over the 40 days prior to the start of the experiment (which started October 19th) with the greatest margin between the Democrat and Republican candidates. This served as our not close poll. We then selected the poll that was most close, conditional on the same candidates being ahead and behind. If two polls were tied for being least close or most close, we selected the poll that was most recent.

In the experiment, the language we used to present the poll was as follows:

Below are the results of a recent poll about the race for governor. The poll was conducted over-the-phone by a leading professional polling organization. People were interviewed from all over the state, and the poll was designed to be both non-partisan and representative of the voting population. Polls such as these are often used in forecasting election results. Of people supporting either the Democratic or Republican candidates, the percent supporting each of the candidates were:

Jerry Brown (Democrat): 50%

Meg Whitman (Republican): 50%

That is, poll numbers were calculated using the share of poll respondents favoring the Democratic or Republican candidates. The number we gave for the Democrat was equal to $100 * \text{Percent Dem} / (\text{Percent Dem} + \text{Percent Reb})$.

3.3 Coins Experiment

The coins experiment is from [Benjamin et al. \(2012a\)](#), which is based on [Kahneman and Tversky \(1972\)](#). Subjects were asked the following question: “Imagine you had a fair coin that was flipped 1,000 times. What do you think is the percent chance that you would get the following number of heads.” Subjects typed in a number corresponding to a percentage in each of the following bins: 0-200 heads, 201-400 heads, 401-480 heads, 481-519 heads, 520-599 heads, 600-799 heads, 800-1,000 heads.

3.4 Voting Data

We obtained administrative voting data on the voters in the sample for the last 10 years. We worked with a “voting validation firm” that collects administrative voting records from the Secretaries of State in different U.S. states.

4 Experimental Results

Before testing the theory, we verify in [Table 2](#) that the randomization was successful. Across most variables, the Close Poll group, No Close Poll group and Control group have similar characteristics. Exceptions are that voters in the Not Close group had a slightly higher *ex ante* belief that the election would be decided by less than 100 votes (but not for less than 1,000 votes or Predicted Margin) and that voters in the Control group were more likely to vote in previous elections than voters in the

Close or Not Close groups. We thus control for ex ante perceived probability of less than 100 votes or shared voted in previous elections in the analysis.

4.1 Beliefs and Updating

We first examine the subjects beliefs about close elections. In Figure 2, we show that voters systematically overpredict the probability of a very close election. There is a large amount of mass around 0, 1, or 2 percent, with many voters predicting that a very close election is unlikely. However, there is also a large mass of voters who are not 2 percent or less. Similar patterns are observed in Figure 3, which is restricted to voters with Master’s or PhD degrees. Here, the median belief is smaller, but still quite high on average. These results support Claim 1, as the probability of an election being decided by less than 1000 votes even in the smallest state in our sample (New Hampshire) is less than 1%.

While beliefs are very high, Table 3 shows that beliefs vary with expected predictors. In particular, the actual vote margin in a state is a strong predictor of the perceived vote margin, as well as the perceived probability the election will be decided by less than 100 or 1,000 votes.

In addition, Table 3 shows that voters with greater NBLLN are more likely to overestimate the probability of a very close election. Voters who believed that the probability of getting outside of 481-519 Heads on 1,000 coin flips is higher are much more likely to report high perceived probabilities of being pivotal. This result holds controlling for educational level, income, and other controls.

Turning next to the updating of beliefs by our subjects, Table 4 provides simple non-parametric evidence that voters updated in response to the experimental poll information. It tabulates whether votes increase, decrease, or did not change their beliefs, showing impacts on predicted vote margin, probability decided by less than 100 votes, and probability decided by less than 1,000 votes. The poll information was given to them in terms of vote margin, so it is perhaps unsurprising that voters would update on this metric, but there is also clear updating on less than 100 or 1,000 vote margins. Consider, for example, the probability the election would be decided by less than 1,000 votes. Of voters in the Close Poll treatment, 10% decrease their beliefs, 65% did not change them, and 24% increase their beliefs, whereas for voters in the Not Close Poll treatment, 18% decrease their beliefs, 67% do not change them, and 14% increase their beliefs. Many voters are not changing their beliefs at all. However, for the share that do, more do so in the expected direction. The lack of updating is in line with Claim 3. Furthermore, the broad patterns of updating are also consistent with Claim 5.

Tables 5 and 6 confirms the same result as in Table 4 using a regression with controls:

$$b_{i,post} = \alpha_0 + \alpha_1 T_i + X_i \beta + \epsilon_i \tag{1}$$

where $b_{i,post}$ is person i ’s post-treatment belief about the closeness, T_i is their randomized treatment status, X_i is controls, and ϵ_i is an error. Tables 5 uses predicted vote margin for b whereas Table 6 analyzes probability election decided by less than 100 or 1,000 votes. In addition to $b_{i,post}$ as the dependent variable, we can also look at changes in beliefs across people, estimating:

$$\Delta b_i = \alpha_0 + \alpha_1 T_i + \Delta \epsilon_i \quad (2)$$

Panel A of Table 5 shows that receiving the close treatment leads the average voter to decrease their predicted vote margin by 2.8 percentage points, with similar estimates using $b_{i,post}$ and Δb_i . In addition, consistent with theory, we see that voters who are less informed update more. We measure how informed voters are using their self-expressed interest in politics (1-5 scale), whether they could correctly identify Nancy Pelosi as the Speaker of the House, and the share of the time they voted in the previous 5 elections. For example, a voter who identifies as having very low interest in politics would update 4.7 percentage points, whereas a voter with a very high interest in politics updated only 1.8 percentage points. In Panel B, we repeat the regressions using a continuous version of the treatment variable, the vote margin in the poll they were assigned (e.g. a 57-43 poll would have a margin of 14 points). Voters update their margin 0.22 points for every 1 point change they see in the polls, with large updates for less informed voters.¹⁶

Table 6 repeats the analysis for the impact on probability that the vote margin is less than 100 or 1,000 votes. In Panel A, we see that both probabilities increased by about 2.5 percentage points after receiving the close poll treatment.¹⁷ Panel B shows that each additional percentage point drop in the margin in the randomly assigned poll led to 0.14 percentage point increase in the probability of less than 100 or 1,000 votes.

An alternative explanation for our result is that voters do not actually update their beliefs at all, but rather appear to change their beliefs as a result of a Hawthorne Effect, changing their beliefs to please the experimenter. While we cannot fully rule out this possibility, we provide several reasons why we believe it to be unlikely to explain our results. First, we note we that subjects updated strongly both on vote margins, on which they were provided information, and the probability of a very close election, on which they were not provided information. If voters were simply telling the researchers what “they wanted to hear,” it is not clear that they would update on both. Second, as noted earlier, the amount of updating is strongly negatively correlated with political information, that is, less informed people update more. A pure Hawthorne effect seems unlikely to deliver this result (unless, of course, for some reason the people who are less informed, controlling for observable characteristics, are also the ones who are more prone to Hawthorne effects).

4.2 Pivotality and Voting

Table 7 regresses turnout on voter beliefs and various characteristics, showing that many standard predictors of turnout are operative in our setting. Older, more educated, and richer people are all more likely to vote. Although our sample is not a random sample from the U.S. population, these

¹⁶In Panel B, column 1 has a coefficient of 0.42 whereas once controls are added in column 2, the coefficient shrinks to 0.22. This occurs because states with actual wider vote margins tend to have polls with wider vote margins. Even though our treatment is randomly assigned within state, the level of the poll vote margins is not randomly assigned across states.

¹⁷Column 1 of Panel A shows an insignificant effect because as discussed earlier in Table 2, it so happened during our randomization that people assigned to the Not Close Poll group happened to have higher initial beliefs of the margin less than 100 votes.

basic voting trends suggest that our sample is not especially atypical. In addition, belief measures do not predict turnout, though we emphasize that the belief measures here are endogenous in these regressions for the reasons we discussed in the Introduction (closer elections are often accompanied by other factors such as increased media coverage, campaign spending, and overall discussion).

Table 8 shows IV regressions of turnout on beliefs instrumenting with our experiment, showing that exogenously affected beliefs do not affect turnout:

$$\begin{aligned} TURNOUT_i &= \alpha_0 + \alpha_1 b_{i,post} + X_i \beta + \epsilon_i \\ b_{i,post} &= a_0 + a_1 T_i + X_i \beta + u_i \end{aligned} \tag{3}$$

where $TURNOUT_i$ is a dummy for turnout, $b_{i,post}$ is post-treatment beliefs, and T_i is the treatment dummy. We estimate by 2SLS. In column 1, the coefficient of 0.17 means that for every 1 point increase in the believed vote margin (that is, the election becomes *less close*), turnout *increases* by 0.17 percentage points. The standard error of 0.42 leads to a 95% confidence interval of $(-0.67, 1.01)$, meaning we can rule out an effect size smaller than -0.67 . Thus, we can rule out that moving from a perceived moderately sized vote margin of 4 percentage points to a perceived close election with a margin of 0 percentage points would increase turnout by more than 2.68 percentage points. As a point of comparison, being in the 35-44 year old age bracket increases turnout by about 20 percentage points relative to the 25-34 year old age bracket. We can thus rule out that the impact of beliefs is anywhere near as important as that of other predictors like age, education, and income. The F-statistic on the excluded instrument varies by specification, but is often around 40, well over the mark of 10 often used to designate weak instruments (Stock et al., 2002).¹⁸

Tables 9 and 10 show in addition that changes in belief have no impact on intended turnout or on information acquisition. An alternative explanation of our zero impact on turnout result is that while the experiment may have affected voting tendencies, other events may have occurred in the several days before the actual election that would have over-ridden our impact. Since voting intention was asked immediately after treatment, we can observe whether our experiment had any short-run impacts. Using the IV strategy from Table 8, we see that the experiment had no impact on voting intention. Another alternative explanation is that the experiment may have spurred voters in the close poll treatment to acquire more information about the election, for example, to start following all the polls and reading the polls. Such voters might have discovered that we provided them with the most close recent poll, and might discard the information content once they learn this. However, we see no impact of the experiment on voters' self-reported tendency to pay greater attention to the election.¹⁹

¹⁸There are 3 cases where the F-statistic is below 10, columns 4, 7, and 10. Column 4 and 10 involve the post-treatment probability the election is less by 100 votes; as discussed earlier, it is important to control for the pre-treatment belief the election is decided by less than 100 votes because the close and not close groups were slightly uneven in the randomization. In addition, the F-statistic is 6.95 for column 7 looking at the post-election belief the election is decided by less than 1,000 votes.

¹⁹Self-reported information acquisition was reported by subjects after the election.

Therefore, although we find support for Claims 1-6, in that individuals beliefs and updating are consistent with a model of NBLLN, we do not find support that they vote in accordance with their (incorrect) beliefs about being pivotal — Claims 7 and 8. This means that even if we accommodate individuals’ true beliefs about being pivotal into models of voting, this still does not generate the behavior predicted by pivotal voting models.

5 Structural Estimation

In this section we estimate a structural model of NBLLN. We use the parameterized form of NBLLN introduced by BRR. The subjective rate distribution $f_{B|\Theta}^\psi(\beta|\theta)$ is a beta-distribution:

$$f_{B|\Theta}^\psi(\beta|\theta) = \beta^{\theta\psi-1} (1-\beta)^{(1-\theta)\psi-1} \frac{\Gamma(\psi)}{\Gamma(\theta\psi)\Gamma((1-\theta)\psi)},$$

The one free parameter $0 < \psi < \infty$ is the exogenous parameter that governs the degree of NBLLN ($\psi = \infty$ is simply a TB).²⁰ From Lemma β -1 of BRR we know that a NBLLN believes the probability of observing A_s votes for candidate A in a total number of N votes is:

$$f_{S_N|\Theta}^\psi(s|\theta_A) = \frac{\Gamma(\psi)}{\Gamma(\psi+N)} \frac{\Gamma(\theta\psi+A_s)}{\Gamma(\theta\psi)} \frac{\Gamma((1-\theta)\psi+N-A_s)}{\Gamma((1-\theta)\psi)} \frac{\Gamma(N+1)}{\Gamma(A_s+1)\Gamma(N-A_s+1)}.$$

By applying Bayes’ Rule, a NBLLN’s likelihood ratio after observing a poll of N voters, with A_s individuals who will votes for candidate A is:

$$\Pi_{S_N|\Theta \times \Theta}^\psi(s|\theta_1, \theta_2) = \frac{\Gamma(\theta_1\psi+A_s)}{\Gamma(\theta_2\psi+A_s)} \frac{\Gamma((1-\theta_1)\psi+N-A_s)}{\Gamma((1-\theta_2)\psi+N-A_s)} \frac{\Gamma(\theta_2\psi)}{\Gamma(\theta_1\psi)} \frac{\Gamma((1-\theta_2)\psi)}{\Gamma((1-\theta_1)\psi)}.$$

One potential concern, which we defer discussion of until the final sub-section, is the issue of response bias in the survey. In particular individuals tend to give responses that are focal more often — individuals may round their answers to the nearest salient number, such as those ending in 5 or 0, or in extreme cases, either 0, 50 or 100.

5.1 Data used in Structural Estimation.

Individuals in the survey vary by state. Within each state individuals are assigned to one of 3 possible treatments. Furthermore, we have measurements of beliefs before the treatment and after the treatment. Denote individual i in state s with treatment t (where $t = m$ denotes the most close poll, $t = l$ denotes the least close poll and $t = \emptyset$ denotes the control treatment), at time τ (where $\tau = b$ for before and a for after the treatment).

For each individual we have reported beliefs about the probability of an election being decided by less than 100 and 1000 votes. Furthermore, we have both these probabilities for each individual

²⁰BRR note that this particular functional form has some additional, stronger implications beyond those discussed in the body of their paper. Furthermore, although the beta distribution always has a single interior maximum, it can be u -shaped for particular small values of ψ .

before the treatment and after the treatment. Denote individual i 's belief in state s , with treatment t at time τ that the election will be decided by less than 100 votes the treatment be $\phi_{i,s,t,\tau}^{100}$. Denote the same belief, but for the election being decided by less than 1000 votes before the treatment be $\phi_{i,s,t,\tau}^{1000}$.

We also have data on each individual's belief about the mean margin of victory in the election. Denote the mean margin of victory, by individual i , in state s , with treatment t at time τ as $\rho_{i,s,t,\tau}$.

For each state s we have the margin of victory predicted by each of the two polls m and l — denoted $\omega_{s,m}$ and $\omega_{s,l}$. We also have realized turnout in state s denoted N_s .

For individual level statistics $z_{i,s,t,\tau}$, we denote the average value across individuals at state, treatment, and time levels as $\bar{z}_{s,t,\tau}$. We denote similar averages across other variables in an equivalent fashion.

5.2 Estimation Strategy and Results

Our basic estimation result estimates a population level mean of ψ . We currently use only a subset of the belief data to do this.²¹ In particular, we will use their initial belief about the probability of an election being decided by less than 1000 votes: $\phi_{i,s,t,\tau}^{1000}$. We assume that individuals have a degenerate prior on the state, and that these priors are represented by their belief about the mean margin of victory $\rho_{i,s,t,\tau}$.²² We furthermore assume that individuals know, ex-ante, the realized turnout in their state N_s . Of course, this leaves only a single data point per individual. Therefore, we aggregate across individuals at the state level. Since we are taking initial beliefs, we restrict $\tau = b$ and also aggregate across treatments (since individuals were randomly assigned). We obtain state level values $\bar{\phi}_{s,b}^{1000}$ and $\rho_{s,b}$.

Given our model of NLLN, a rate θ , and a number of voters of size N , the probability of of an election being closer than 1000 votes is the sums of $f_{S_N|\Theta}^\psi(s|\theta_A)$ between the values of $0.5N - 1000$ and $0.5N + 1000$, or

$$F_{S_N|\Theta}^\psi(0.5N + 1000|\theta_A) - F_{S_N|\Theta}^\psi(0.5N - 1000|\theta_A)$$

We can substitute in for the CDF of the outcomes using

²¹Please see last subsection for details of how we plan to extend our analysis to individual level estimates using the full set of available data.

²²This must be false in the sense that it would not allow the subjects to update at all given new information, but we will use it as a simplifying assumption. In ongoing work we let the prior be a distribution.

$$\begin{aligned}
F_{S_N|\Theta}^\psi (K|\theta_A) &= \sum_{i=0}^K f_{S_N|\Theta}^\psi (i|\theta_A) \\
&= \sum_{i=0}^K \int_0^1 \frac{N!}{i!(N-i)!} \beta^i (1-\beta)^{N-i} \beta^{\theta\psi-1} (1-\beta)^{(1-\theta)\psi-1} \frac{\Gamma(\psi)}{\Gamma(\theta\psi)\Gamma((1-\theta)\psi)} d\beta \\
&= \int_0^1 \beta^{\theta\psi-1} (1-\beta)^{(1-\theta)\psi-1} \frac{\Gamma(\psi)}{\Gamma(\theta\psi)\Gamma((1-\theta)\psi)} \sum_{i=0}^K \frac{N!}{i!(N-i)!} \beta^i (1-\beta)^{N-i} d\beta
\end{aligned}$$

Therefore

$$\begin{aligned}
&F_{S_N|\Theta}^\psi (0.5N + 1000|\theta_A) - F_{S_N|\Theta}^\psi (0.5N - 1000|\theta_A) \\
&= \int_0^1 \beta^{\theta\psi-1} (1-\beta)^{(1-\theta)\psi-1} \frac{\Gamma(\psi)}{\Gamma(\theta\psi)\Gamma((1-\theta)\psi)} \sum_{i=0}^{0.5N+1000} \frac{N!}{i!(N-i)!} \beta^i (1-\beta)^{N-i} d\beta \\
&\quad - \int_0^1 \beta^{\theta\psi-1} (1-\beta)^{(1-\theta)\psi-1} \frac{\Gamma(\psi)}{\Gamma(\theta\psi)\Gamma((1-\theta)\psi)} \sum_{i=0}^{0.5N-1000} \frac{N!}{i!(N-i)!} \beta^i (1-\beta)^{N-i} d\beta \\
&= \int_0^1 \beta^{\theta\psi-1} (1-\beta)^{(1-\theta)\psi-1} \frac{\Gamma(\psi)}{\Gamma(\theta\psi)\Gamma((1-\theta)\psi)} \\
&\quad \left(\sum_{i=0}^{0.5N+1000} \frac{N!}{i!(N-i)!} \beta^i (1-\beta)^{N-i} - \sum_{i=0}^{0.5N-1000} \frac{N!}{i!(N-i)!} \beta^i (1-\beta)^{N-i} \right) d\beta
\end{aligned}$$

We will use the normal approximation of the Binomial distribution

$$\begin{aligned}
&\int_0^1 \beta^{\theta\psi-1} (1-\beta)^{(1-\theta)\psi-1} \frac{\Gamma(\psi)}{\Gamma(\theta\psi)\Gamma((1-\theta)\psi)} \\
&\quad \left(\sum_{i=0}^{0.5N+1000} \frac{N!}{i!(N-i)!} \beta^i (1-\beta)^{N-i} - \sum_{i=0}^{0.5N-1000} \frac{N!}{i!(N-i)!} \beta^i (1-\beta)^{N-i} \right) d\beta \\
&\approx \int_0^1 \beta^{\theta\psi-1} (1-\beta)^{(1-\theta)\psi-1} \frac{\Gamma(\psi)}{\Gamma(\theta\psi)\Gamma((1-\theta)\psi)} \\
&\quad (\Phi_{N\beta, N\beta(1-\beta)}(0.5N + 1000) - \Phi_{N\beta, N\beta(1-\beta)}(0.5N - 1000)) d\beta
\end{aligned}$$

Where $\Phi_{N\beta, N\beta(1-\beta)}$ is the normal pdf for a normal distribution with parameters $N\beta$ and $N\beta(1-\beta)$. Therefore we can estimate the below equation using non-linear least squares or, using overall averages instead of state-level statistics, solve it using a non-linear solver:

$$\bar{\phi}_{s,b}^{1000} = \int_0^1 \beta^{\rho_{s,b}\psi-1} (1-\beta)^{(1-\rho_{s,b})\psi-1} \frac{\Gamma(\psi)}{\Gamma(\rho_{s,b}\psi)\Gamma((1-\rho_{s,b})\psi)} (\Phi_{N_s\beta, N_s\beta(1-\beta)}(0.5N_s + 1000) - \Phi_{N_s\beta, N_s\beta(1-\beta)}(0.5N_s - 1000)) d\beta$$

Solving the above equation using a non-linear solver, we obtain a preliminary estimate of ψ of 45. This is larger than the estimates from the experimental literature. However, several caveats are in order. We neglect to accommodate additional known biases (such as extremeness aversion and fat-tailed distributions) which are documented in the literature (see BRR Appendix A). This leads to difficulty in model being able to accurately match the high level of belief individuals place in close elections. Taking the reported beliefs at face value, this would require individuals' subjective sampling distribution $f_{\beta|\Theta}^{\psi}(\beta|\theta)$ to place an extremely large mass of probability quite close to $\beta = .5$ — i.e. nearly 20 percent of the probability mass. This seems calibrationally implausible. Additional biases, such as extremeness aversion and fat-tailed distributions can help alleviate this problem, and also will lead to lower estimates of ψ . That said, there remain some individuals in the sample whose beliefs about close elections will remain difficult to rationalize under any reasonable model — for example those that place upwards of 30 percent likelihood on close elections while still believing the average margin of victory to be greater than 1 percent. This is because those subjects' subjective sampling distribution will not be single peaked, and have interior intervals where the support is negligible; distributions which are possible, but seem at odds with the experimental evidence.

5.3 Ongoing Extensions

Our current structural estimation is limited in that it estimates population averages using a single statistic (out of the five we have available). Furthermore, it imposes extremely strong assumptions about the nature of individuals' beliefs about the state of the world (i.e. the underlying probability of support for candidate A), in that beliefs are degenerate.

We can once again estimate a parameter ψ_i for each individual. In order to both beliefs about close elections and updating in our estimation using a single theoretical framework, we will have to assume that individual's prior beliefs about the margin of victory in a state are not a point mass (as we currently do) but are rather a distribution — $G(\theta)$. We will parameterize G for each individual by assuming that it is a β distribution with mean $\rho_{i,s,t,\tau}$, and an unknown variance ν_i (which will be the second parameter estimated in the model). Using Bayes' Rule, the poll result, along with ψ and the prior distribution, will fully characterize the posterior distribution of states for any individual. We will estimate ψ_i and ν_i jointly for each individual using generalized methods of moments. We will match five moments. The first two are simply each individual's probability of the state gubernatorial election being decided by less than 100 votes or less than 1000 votes before the treatment. The second two are each individual's probability of the state gubernatorial election being decided by less than 100 votes or less than 1000 votes after the treatment. The final moment is the change in the

mean margin of victory given by the individual from before the treatment to after the treatment.²³

6 Conclusion

Our results shed light on two important areas of research in economics — voting behavior and biases in probabilistic judgement. We find that NBLLN provides a tractable model that better captures beliefs and learning from polls than a neo-classical model. However, we find that even individual’s overestimation of their likelihood of being pivotal cannot save the pivotal voting model – individuals’ beliefs about their likelihood of being pivotal do not influence their likelihood of voting.

Our results for two reasons. First because they provide important field evidence on the existence of biases in probabilistic judgement. The data allow us to structurally estimate a model of NBLLN, and support the contention that experimentally elicited parameters can predict outcomes in similar real-world situations. Although other possible reasons could explain either the belief in close elections, or the lack of updating, our model can explain both at the same time, as well as the correlation between them. Furthermore, our model parsimoniously captures the relationship between the experimental measures of NBLLN and the observed field data.

Second, our data are an important field test of the pivotal voting model. Our analysis indicates that the pivotal voting model cannot be saved by accurately modeling individuals (incorrect) beliefs. We find that even though individuals overestimate their probability of being pivotal, differences in beliefs have no effect on whether individuals vote or not. This suggests that attempts to rationalize voting must focus on alternative explanations, such as social norms and intrinsic rewards.

²³Estimating parameters for each individual separately cannot allow us to address issues of focal point responses. In order to attempt to correct for this, we first need to develop a structural model of changing responses to focal points. This also requires an assumption regarding the form of the underlying distribution of ψ in the population, e.g. log-normal. We then estimate the parameters of both the model of focal points and the parameters of the distribution of ψ using the observed distribution of beliefs.

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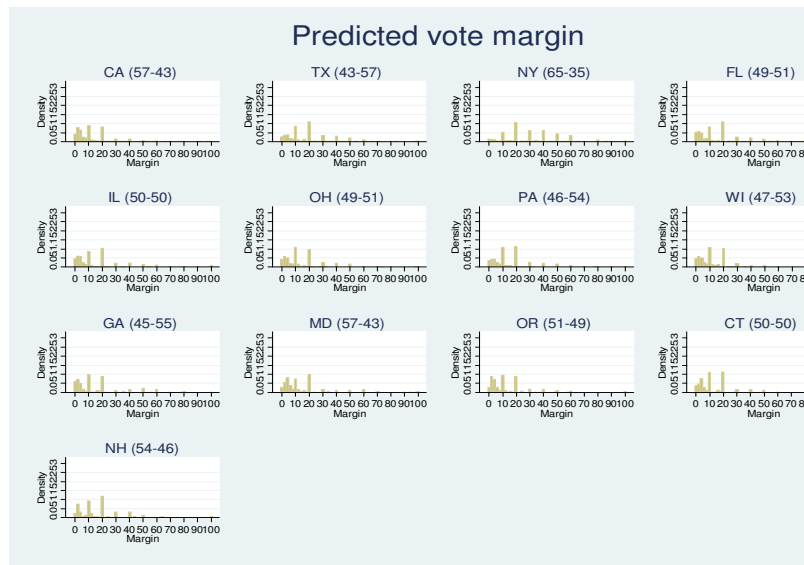
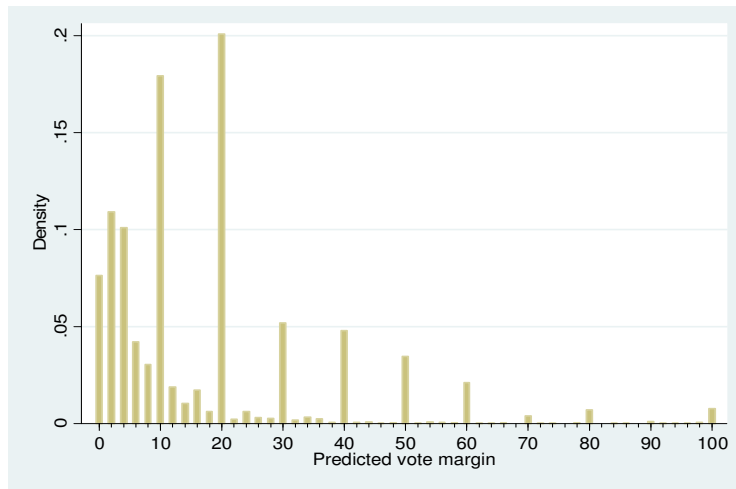
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Shayo, Moses and Alon Harel, “Non-consequentialist voting,” *Journal of Economic Behavior & Organization*, 2012, 81 (1), 299 – 313.

Stock, James H, Jonathan H Wright, and Motohiro Yogo, “A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments,” *Journal of Business & Economic Statistics*, 2002, 20 (4), 518–29.

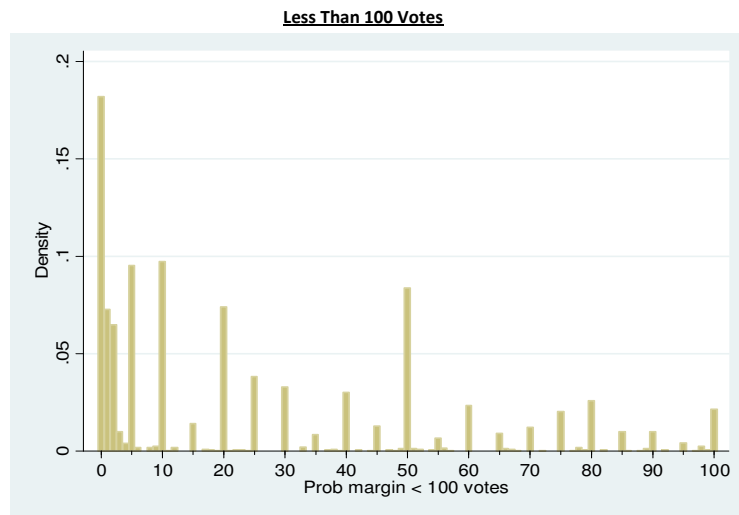
Tyran, Jean-Robert, “Voting when money and morals conflict: an experimental test of expressive voting,” *Journal of Public Economics*, 2004, 88 (7-8), 1645–1664.

Figure 1: Distribution of the Predicted Margin of Victory

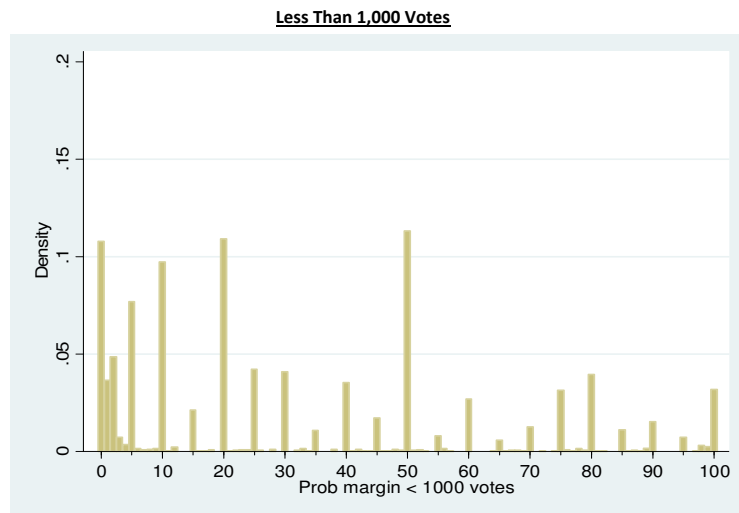


Notes: This graph plots the distribution of subjects' predicted margin of victory.

Figure 2: Subjective Probabilities that Gubernatorial Election Will be Decided by Less than 100 Votes or 1,000 Votes



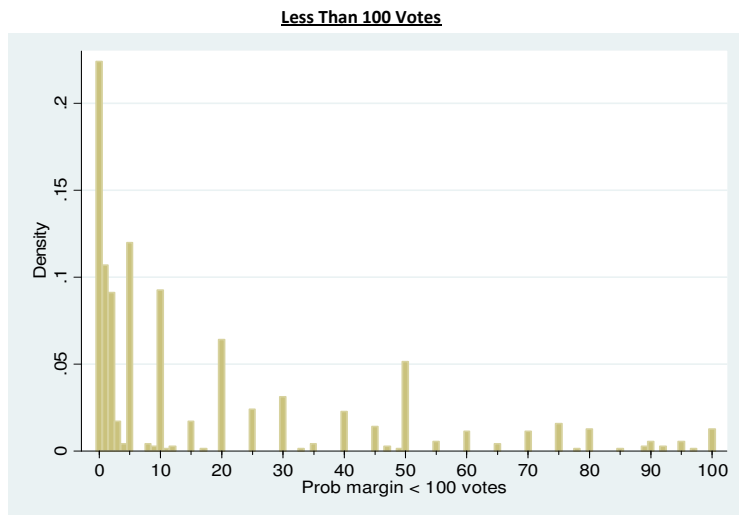
Median = 10, 25th Percentile = 1, 75th Percentile = 45



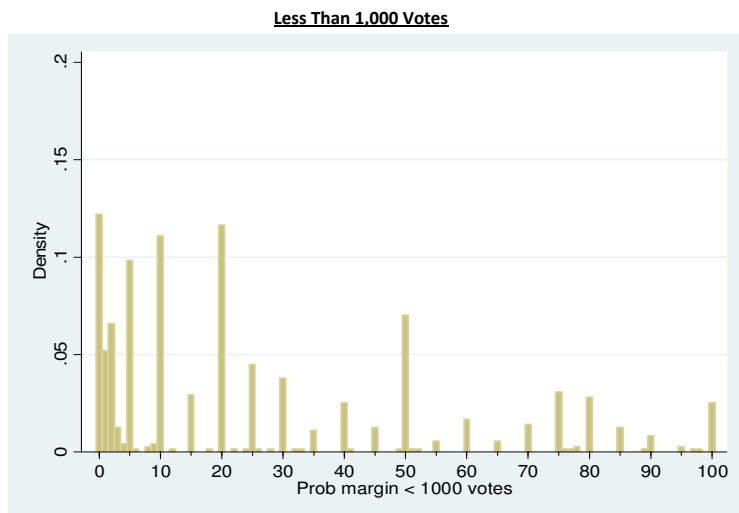
Median = 20, 25th Percentile = 5, 75th Percentile = 50

Notes: These graphs plot the distribution of answers to the question asking for the probability the election in the respondent's state would be decided by less than 100 votes or less than 1,000 votes.

Figure 3: Subjective Probabilities that Gubernatorial Election Will be Decided by Less than 100 Votes or 1,000 Votes—Voters with Master’s or PhD



Median = 5, 25th Percentile = 1, 75th Percentile = 20



Median = 15, 25th Percentile = 3, 75th Percentile = 40

Notes: These graphs plot the distribution of answers to the question asking for the probability the election in the respondent’s state would be decided by less than 100 votes or less than 1,000 votes.

Table 1: Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
<u>Panel A: Demographics</u>					
Male	0.39	0.49	0	1	6705
Black	0.08	0.27	0	1	6705
Hispanic	0.06	0.24	0	1	6705
Other	0.03	0.18	0	1	6705
Mixed race	0.02	0.15	0	1	6705
Age	53.33	14.2	18	93	6705
Less than high school	0.03	0.16	0	1	6705
High school degree	0.13	0.34	0	1	6705
Some college or associate degree	0.34	0.47	0	1	6705
Bachelor's degree	0.29	0.45	0	1	6705
Master's or PhD	0.21	0.41	0	1	6705
Income \$25k-\$50k	0.23	0.42	0	1	6705
Income \$50k-\$75k	0.23	0.42	0	1	6705
Income \$75k-\$100k	0.18	0.38	0	1	6705
Income \$100k +	0.24	0.43	0	1	6705
Catholic	0.3	0.46	0	1	11452
Protestant	0.48	0.5	0	1	11452
Other Christian	0.16	0.37	0	1	11452
Jewish	0.05	0.21	0	1	11452
<u>Panel B: Politics</u>					
Registered Democrat	0.48	0.5	0	1	9640
Registered Republican	0.36	0.48	0	1	9640
No party affil/decline to state/indep	0.13	0.34	0	1	9640
Other party registration	0.03	0.16	0	1	9640
Identify Nancy Pelosi as Speaker	0.82	0.38	0	1	6595
Interest in politics (1-5 scale)	3.71	1.06	1	5	6684
Affiliate w/ Democrat party (1-7)	4.24	2.15	1	7	16098
Ideology (1=Extremely Conserv, 7=Extremely Liberal)	3.88	1.52	1	7	15960
<u>Panel C: Beliefs</u>					
Predicted vote margin, pre-treatment	17.08	17.78	0	100	6652
Predicted vote margin, post-treatment	14.76	15.83	0	100	6650
Prob margin < 100 votes, pre-treatment	24.42	28.3	0	100	3284
Prob margin < 100 votes, post-treatment	24.95	28.97	0	100	3286
Prob margin < 1,000 votes, pre-treatment	31.69	29.7	0	100	3409
Prob margin < 1,000 votes, post-treatment	33.22	30.51	0	100	3407
Prob voting, pre-treatment	87.06	27.79	0	100	6698
Prob voting, post-treatment	87.91	27.08	0	100	6700
Prob vote Dem, pre-treatment	49.94	43.77	0	100	6705
Prob vote Republican, pre-treatment	41.5	43.08	0	100	6705
Prob vote Dem, post-treatment	50.14	43.68	0	100	6705
Prob vote Republican, post-treatment	41.72	43.03	0	100	6705
Prob vote underdog, pre-treatment	41.16	43.07	0	100	6705
Prob vote underdog, post-treatment	41.14	42.98	0	100	6705
<u>Panel D: Voting</u>					
Voted (self-reported)	0.84	0.36	0	1	5867
Voted (administrative)	0.66	0.47	0	1	16628
Share voted previous 5 elections (administrative)	0.59	0.39	0	1	16628

Notes: This table presents summary statistics. The ‘Share voted previous 5 elections’ refers to voting in the general elections of 2000, 2002, 2004, 2006, and 2008. As can be seen, our sample is more white, female, and older than the general population.

Table 2: Randomization Check

	Close	Not Close	t-test of (1) vs (2)	Assigned Close	Assigned Not Close	Assigned Control	t-test of (4) vs (5)	t-test of (4) vs (6)	t-test of (5) vs (6)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<u>Panel A: Demographics</u>									
Male	0.39	0.39	0.91						
Black	0.08	0.08	0.87						
Hispanic	0.06	0.06	0.67						
Other	0.03	0.03	0.96						
Mixed race	0.02	0.02	0.34						
Age	53.21	53.45	0.49						
Less than high school	0.03	0.03	0.56						
High school degree	0.14	0.13	0.54						
Some college or associate degree	0.34	0.34	0.96						
Bachelor's degree	0.29	0.29	0.76						
Master's or PhD	0.21	0.21	0.91						
Income \$25k-\$50k	0.22	0.24	0.09						
Income \$50k-\$75k	0.24	0.23	0.14						
Income \$75k-\$100k	0.18	0.17	0.31						
Income \$100k +	0.24	0.25	0.32						
Catholic	0.30	0.30	0.82	0.31	0.30	0.29	0.36	0.09	0.46
Protestant	0.48	0.48	0.80	0.48	0.48	0.47	0.73	0.71	0.48
Other Christian	0.16	0.16	0.79	0.16	0.16	0.17	0.43	0.06	0.28
Jewish	0.05	0.05	0.76	0.05	0.05	0.05	0.91	0.29	0.34
<u>Panel B: Politics</u>									
Registered Democrat	0.49	0.50	0.46	0.49	0.49	0.47	0.80	0.11	0.07
Registered Republican	0.36	0.35	0.77	0.35	0.35	0.37	0.56	0.04	0.15
No party affil/decline state/indep	0.13	0.12	0.58	0.14	0.12	0.13	0.14	0.41	0.50
Other party registration	0.03	0.02	0.75	0.03	0.03	0.03	0.54	0.52	0.98
Identify Nancy Pelosi as Speaker	0.82	0.83	0.23						
Interest in politics (1-5 scale)	3.73	3.70	0.31						
Affiliate w/ Democrat party (1-7)	4.23	4.24	0.87	4.26	4.25	4.20	0.80	0.14	0.23
Ideology (1-7 Scale, 7=Ext Liberal)	3.89	3.87	0.65	3.89	3.87	3.87	0.50	0.47	0.96
<u>Panel C: Beliefs</u>									
Predicted vote margin, pre-treat	17.05	17.10	0.91						
Prob margin < 100 votes, pre-treat	23.44	25.44	0.04						
Prob margin < 1,000 votes, pre-treat	31.93	31.46	0.65						
Prob voting, pre-treatment	87.08	87.04	0.95						
Prob vote Dem, pre-treatment	49.71	50.17	0.67						
Prob vote Republican, pre-treat	41.46	41.53	0.95						
Prob vote underdog, pre-treat	40.79	41.52	0.49						
<u>Panel D: Voting</u>									
Voted (self-reported)	0.84	0.85	0.62						
Voted (administrative)	0.66	0.66	0.90	0.64	0.63	0.71	0.70	0.00	0.00
Share voted previous 5 election	0.59	0.59	0.96	0.57	0.57	0.63	0.83	0.00	0.00
Number of observations	3,348	3,357		5,413	5,387	5,543			

Notes: This table presents averages across the different treatments. Columns (1) and (2) are for subjects assigned to the Close or Not Close treatments who answer the survey. (4), (5), and (6) are averages for voters assigned to the Close, Not Close, and Control treatments. The 'Share voted previous 5 elections' refers to voting in the general elections of 2000, 2002, 2004, 2006, and 2008.

Table 3: Predicting Pre-treatment Beliefs

Dep var:	Prob < 100 votes		Prob < 1,000 votes		Margin of victory		Democrat vote share	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Actual vote margin in state	-0.13 (0.06)**		-0.39 (0.06)***		0.47 (0.03)***		0.43 (0.02)***	
Log size of electorate	-0.65 (0.88)		-0.43 (0.87)		-0.81 (0.34)**		-1.27 (0.23)***	
Affiliate w/ Democrat party (1-7)	0.18 (0.24)	0.16 (0.24)	0.64 (0.26)**	0.60 (0.27)**	-0.19 (0.11)*	-0.11 (0.11)	1.64 (0.07)***	1.51 (0.07)***
Interest in politics (1-5)	-1.46 (0.54)***	-1.50 (0.54)***	-0.35 (0.55)	-0.33 (0.55)	-0.07 (0.25)	-0.01 (0.25)	-0.34 (0.17)**	-0.36 (0.16)**
Prob in middle, coins experiment	-0.08 (0.01)***	-0.08 (0.01)***	-0.04 (0.02)**	-0.04 (0.02)**	-0.04 (0.01)***	-0.04 (0.01)***	-0.00 (0.00)	-0.00 (0.00)
Male	-11.38 (0.99)***	-11.36 (0.99)***	-14.28 (1.06)***	-14.31 (1.06)***	-2.90 (0.45)***	-2.89 (0.44)***	0.30 (0.30)	0.22 (0.29)
Black	14.62 (2.46)***	14.45 (2.50)***	3.74 (2.31)	3.27 (2.36)	4.32 (1.16)***	4.46 (1.15)***	4.57 (0.75)***	5.68 (0.75)***
Hispanic	10.23 (2.42)***	9.71 (2.45)***	6.91 (2.47)***	6.84 (2.51)***	1.86 (1.13)	2.05 (1.14)*	0.45 (0.79)	1.30 (0.78)*
Other	8.29 (3.16)***	7.94 (3.17)**	0.80 (2.74)	0.36 (2.71)	0.35 (1.37)	1.57 (1.35)	0.19 (0.92)	-0.14 (0.92)
Mixed race	6.19 (3.93)	6.60 (3.90)*	1.19 (3.96)	0.76 (3.91)	0.10 (1.42)	0.47 (1.43)	-0.15 (1.17)	-0.52 (1.13)
Age 25-34	4.17 (3.96)	4.29 (3.96)	-0.41 (3.78)	-0.24 (3.77)	-4.32 (2.49)*	-4.60 (2.53)*	2.21 (1.72)	1.91 (1.69)
Age 35-44	2.32 (3.72)	2.43 (3.70)	1.88 (3.60)	2.18 (3.59)	-4.84 (2.43)**	-5.06 (2.47)**	2.35 (1.66)	2.03 (1.63)
Age 45-54	3.22 (3.67)	3.26 (3.66)	-0.19 (3.53)	0.05 (3.50)	-5.01 (2.42)**	-5.26 (2.46)**	2.92 (1.65)*	2.57 (1.62)
Age 55-64	2.27 (3.70)	2.32 (3.69)	0.93 (3.48)	1.35 (3.46)	-6.29 (2.41)***	-6.71 (2.45)***	2.61 (1.64)	2.02 (1.61)
Age 65-74	1.19 (3.76)	1.02 (3.74)	-0.25 (3.57)	-0.09 (3.55)	-7.88 (2.42)***	-8.05 (2.47)***	3.13 (1.64)*	2.33 (1.61)
Age 75 or more	8.08 (4.26)*	7.96 (4.25)*	2.43 (3.97)	2.90 (3.96)	-9.12 (2.48)***	-9.43 (2.52)***	4.15 (1.67)**	2.97 (1.64)*
Less than high school	8.35 (4.12)**	8.42 (4.13)**	-4.99 (4.05)	-5.00 (4.07)	-1.18 (2.34)	-1.10 (2.33)	0.81 (1.56)	0.59 (1.55)
Some college or associate degree	-1.82 (1.87)	-2.04 (1.87)	-3.79 (1.85)**	-4.13 (1.85)**	-2.98 (0.87)***	-2.34 (0.87)***	-0.58 (0.58)	-0.72 (0.57)
Bachelor's degree	-7.11 (1.89)***	-7.33 (1.90)***	-6.99 (1.90)***	-7.36 (1.89)***	-5.45 (0.85)***	-4.80 (0.85)***	0.63 (0.57)	0.56 (0.56)
Master's or PhD	-9.13 (1.98)***	-9.22 (1.98)***	-9.04 (2.00)***	-9.41 (2.00)***	-6.35 (0.88)***	-5.94 (0.87)***	1.18 (0.58)**	0.93 (0.57)
Income \$25k-\$50k	0.95 (2.01)	1.10 (2.02)	0.53 (1.96)	0.23 (1.95)	-0.70 (0.93)	-0.83 (0.92)	-0.31 (0.63)	-0.07 (0.62)
Income \$50k-\$75k	-2.16 (1.93)	-2.19 (1.93)	-1.27 (1.96)	-1.63 (1.97)	-1.30 (0.91)	-1.32 (0.90)	-0.56 (0.62)	-0.33 (0.61)
Income \$75k-\$100k	-2.62 (2.00)	-2.44 (2.01)	-2.87 (2.06)	-3.45 (2.06)*	-2.09 (0.91)**	-2.15 (0.89)**	-0.58 (0.62)	-0.46 (0.60)
Income \$100k +	-5.17 (1.92)***	-5.20 (1.92)***	-8.62 (1.93)***	-9.38 (1.93)***	-1.40 (0.91)	-1.10 (0.90)	-0.37 (0.62)	-0.44 (0.60)
State FE	No	Yes	No	Yes	No	Yes	No	Yes
Observations	2717	2717	2773	2773	5462	5462	5479	5479
R-squared	0.16	0.16	0.13	0.14	0.11	0.14	0.21	0.27

Notes: This table presents OLS regressions of voters' pre-treatment beliefs on various covariates. It covers voters' perception the election is decided by less than 100 or 1,000 votes, as well as voters' predictions of the vote margin and vote share for the Democrat. Robust standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 4: Changes in Beliefs and Voting Intentions After the Treatment

	N	Decrease	Same	Increase	N	Decrease	Same	Increase
	Not Close Treatment				Close Treatment			
Predicted margin of victory	3311	19.0%	61.8%	19.2%	3301	30.1%	62.8%	7.1%
Prob margin < 100 votes	1601	18.2%	69.3%	12.6%	1681	11.3%	68.2%	20.5%
Prob margin < 1000 votes	1749	18.4%	67.3%	14.3%	1657	10.4%	65.3%	24.3%
Intended prob of voting	3350	3.4%	88.3%	8.3%	3347	3.7%	88.0%	8.4%
Intended prob of voting for underdog	3357	6.1%	87.7%	6.3%	3348	5.7%	88.2%	6.1%
	Treatment That's Less Favorable for Democrat				Treatment That's More Favorable for Democrat			
Predicted Dem vote share	3,364	25.6%	61.3%	13.1%	3,301	14.3%	59.0%	26.7%
Pred Dem vote share, affil w/ Dem party	1,798	29.5%	59.0%	11.5%	1,765	17.6%	56.7%	25.8%
Pred Dem vote share, don't affiliate w/ Dem party	1,552	20.8%	64.2%	15.0%	1,519	10.3%	61.8%	27.9%
Intended prob of voting for Democrat	3384	5.3%	88.1%	6.6%	3321	5.9%	87.3%	6.7%

Notes: This table describes how voters' perception of the vote margin, their perception the election is decided by less than 100 or 1,000 votes, their predicted probability of voting, and their intended probability of voting for the underdog candidate (the candidate behind in the polls) change under the two information treatments (close poll and not close poll). In addition, it shows how the intended probability of voting for the Democrat changes under the poll that is less favorable to the Democrat and the poll that is more favorable to the Democrat.

Table 5: The Effect of the Close Poll Treatment on Vote Margin Predictions

Panel A: Treatment Var is Discrete							
Dep. var = Predicted vote margin	b_{post} (1)	b_{post} (2)	Δb (3)	b_{post} (4)	b_{post} (5)	b_{post} (6)	b_{post} (7)
Close poll treatment	-2.80*** (0.39)	-2.79*** (0.36)	-2.62*** (0.34)	-2.72*** (0.36)	-5.45*** (1.44)	-3.83*** (1.00)	-4.66*** (0.78)
Close poll*Interest in politics (1-5 scale)					0.73** (0.36)		
Close poll*Identify Nancy Pelosi as Speaker						1.35 (1.07)	
Close poll*Share voted previous 5 elections							2.98*** (1.01)
Interest in politics (1-5 scale)				-0.03 (0.21)	-0.38 (0.27)	-0.03 (0.21)	-0.01 (0.21)
Identify Nancy Pelosi as Speaker				-1.59*** (0.54)	-1.60*** (0.54)	-2.27*** (0.78)	-1.60*** (0.54)
Share voted previous 5 elections (admin)				-1.16** (0.56)	-1.15** (0.56)	-1.17** (0.56)	-2.66*** (0.77)
State FE	No	Yes	Yes	Yes	Yes	Yes	Yes
Demog Controls	No	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6650	6650	6612	6529	6529	6529	6529
R-squared	0.01	0.10	0.02	0.14	0.14	0.14	0.14
Panel B: Treatment Var is Continuous							
Dep. var = Predicted vote margin	b_{post} (1)	b_{post} (2)	Δb (3)	b_{post} (4)	b_{post} (5)	b_{post} (6)	b_{post} (7)
Margin in viewed poll	0.42*** (0.02)	0.22*** (0.03)	0.21*** (0.02)	0.22*** (0.03)	0.35*** (0.09)	0.24*** (0.06)	0.30*** (0.05)
Viewed margin*Interest in politics (1-5 scale)					-0.03 (0.02)		
Viewed margin*Identify Nancy Pelosi as Speaker						-0.02 (0.06)	
Viewed margin*Share voted previous 5 elections							-0.13* (0.06)
Interest in politics (1-5 scale)				-0.02 (0.21)	0.30 (0.28)	-0.02 (0.21)	-0.01 (0.21)
Identify Nancy Pelosi as Speaker				-1.53*** (0.54)	-1.53*** (0.54)	-1.33* (0.77)	-1.53*** (0.54)
Share voted previous 5 elections (administrative)				-1.13** (0.56)	-1.13** (0.56)	-1.14** (0.56)	0.05 (0.77)
State FE	No	Yes	Yes	Yes	Yes	Yes	Yes
Demographic controls	No	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6650	6650	6612	6529	6529	6529	6529
R-squared	0.06	0.14	0.03	0.15	0.15	0.15	0.15

Notes: Robust standard errors in parentheses. Demographic controls include gender, race, 10-year age bins, education dummies, and \$25k income bins. When the treatment variable is used in discrete form, it is a dummy for getting the close poll (versus getting the not close poll). When the treatment variable is continuous, it is equal to the vote margin in the viewed poll (e.g. if the voter was shown a 55-45 poll, the margin in viewed poll is equal to 10). * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 6: The Effect of the Close Poll Treatment on the Perceived Likelihood of the Election Being Decided by Less than 100 or Less than 1,000 Votes

Panel A: Treatment Var is Discrete									
	Prob < 100 votes			Prob < 1,000 votes			< 100 or 1,000 votes		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Close poll treatment	0.80 (1.01)	2.47 (0.53)***	2.54 (0.53)***	2.93 (1.04)***	2.55 (0.53)***	2.33 (0.52)***	1.67 (0.73)**	2.47 (0.38)***	2.43 (0.37)***
Prob <100 votes, pre-treat		0.87 (0.01)***	0.85 (0.01)***						
Prob <1,000 votes, pre-treat					0.88 (0.01)***	0.86 (0.01)***			
Prob <100 or 1,000 votes, pre-treat								0.88 (0.01)***	0.86 (0.01)***
Demog Controls	No	No	Yes	No	No	Yes	No	No	Yes
State FE	No	No	Yes	No	No	Yes	No	No	Yes
Observations	3286	3282	3282	3407	3406	3406	6693	6688	6688
R-squared	0.00	0.73	0.73	0.00	0.74	0.75	0.00	0.74	0.75

Panel B: Treatment Var is Continuous									
	Prob < 100 votes			Prob < 1,000 votes			< 100 or 1,000 votes		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Margin in viewed poll	-0.10 (0.06)*	-0.13 (0.03)***	-0.14 (0.04)***	-0.39 (0.05)***	-0.19 (0.03)***	-0.14 (0.04)***	-0.24 (0.04)***	-0.16 (0.02)***	-0.14 (0.02)***
Prob <100 votes, pre-treat		0.87 (0.01)***	0.85 (0.01)***						
Prob <1,000 votes, pre-treat					0.88 (0.01)***	0.86 (0.01)***			
Prob <100 or 1,000 votes, pre-treat								0.88 (0.01)***	0.86 (0.01)***
Demog Controls	No	No	Yes	No	No	Yes	No	No	Yes
State FE	No	No	Yes	No	No	Yes	No	No	Yes
Observations	3286	3282	3282	3407	3406	3406	6693	6688	6688
R-squared	0.00	0.72	0.73	0.01	0.75	0.75	0.01	0.74	0.75

Notes: The dependent variable is a voter's post-treatment belief that the election will be decided by less than 100 votes or less than 1,000 votes. Voters were either asked about 100 votes or about 1,000 votes. The data is pooled in columns 7-9. Robust standard errors in parentheses. Demographic controls include gender, race, 10-year age bins, education dummies, and \$25k income bins. When the treatment variable is used in discrete form, it is a dummy for getting the close poll (versus getting the not close poll). When the treatment variable is continuous, it is equal to the vote margin in the viewed poll (e.g. if the voter was shown a 55-45 poll, the margin in viewed poll is equal to 10). * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 7: Correlation Between Beliefs About the Closeness of the Election and Voter Turnout, OLS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Pred vote margin, post-treat	-0.056 (0.039)	-0.012 (0.051)						
Pred vote margin, pre-treat		-0.068 (0.046)						
Pr(Marg <100 votes), post			-0.036 (0.029)	0.064 (0.052)				
Pr(Marg <100 votes), pre				-0.123 (0.054)**				
Pr(Marg <1,000 votes), post					0.021 (0.027)	0.017 (0.051)		
Pr(Marg <1,000 votes), pre						0.006 (0.052)		
<100 or 1,000 votes, post							-0.008 (0.020)	0.037 (0.036)
<100 or 1,000 votes, pre								-0.054 (0.038)
Male	2.782 (1.107)**	2.691 (1.111)**	0.746 (1.624)	0.504 (1.627)	4.366 (1.638)***	4.369 (1.641)***	2.607 (1.147)**	2.503 (1.149)**
Black	0.083 (2.229)	0.317 (2.254)	1.157 (3.134)	1.731 (3.154)	-1.073 (3.112)	-1.089 (3.113)	-0.078 (2.210)	0.098 (2.215)
Hispanic	-3.249 (2.449)	-3.087 (2.448)	-1.165 (3.753)	-0.502 (3.767)	-5.721 (3.194)*	-5.727 (3.196)*	-3.515 (2.438)	-3.324 (2.443)
Other	-4.026 (3.123)	-4.073 (3.121)	1.793 (4.296)	1.943 (4.298)	-8.957 (4.406)**	-8.955 (4.407)**	-3.992 (3.105)	-3.977 (3.106)
Mixed race	4.509 (3.601)	3.858 (3.631)	11.282 (5.044)**	11.668 (5.019)**	-1.047 (5.072)	-1.069 (5.083)	4.620 (3.611)	4.832 (3.621)
Age 25-34	2.622 (4.463)	2.908 (4.487)	6.263 (6.540)	6.934 (6.561)	0.732 (5.990)	0.724 (5.993)	3.090 (4.442)	3.354 (4.445)
Age 35-44	22.838 (4.216)***	22.803 (4.239)***	23.928 (6.207)***	24.187 (6.218)***	22.864 (5.641)***	22.849 (5.646)***	23.428 (4.193)***	23.538 (4.195)***
Age 45-54	29.460 (4.116)***	29.437 (4.139)***	30.508 (6.038)***	31.025 (6.047)***	29.142 (5.526)***	29.127 (5.531)***	29.821 (4.093)***	30.021 (4.094)***
Age 55-64	34.154 (4.095)***	34.239 (4.118)***	34.939 (6.053)***	35.326 (6.063)***	34.770 (5.458)***	34.736 (5.464)***	34.765 (4.074)***	34.975 (4.075)***
Age 65-74	42.136 (4.127)***	41.956 (4.151)***	43.978 (6.065)***	44.431 (6.075)***	42.091 (5.527)***	42.069 (5.537)***	42.942 (4.108)***	43.156 (4.111)***
Age 75 or more	44.959 (4.352)***	44.733 (4.381)***	46.896 (6.398)***	47.696 (6.411)***	44.784 (5.855)***	44.766 (5.862)***	45.673 (4.335)***	45.939 (4.340)***
Less than high school	-9.762 (4.307)**	-10.629 (4.360)**	-13.542 (6.184)**	-13.045 (6.229)**	-6.704 (5.991)	-6.712 (5.992)	-10.086 (4.276)**	-10.007 (4.283)**
Some college or assoc deg	2.219 (1.876)	2.506 (1.881)	2.546 (2.645)	2.455 (2.638)	2.996 (2.655)	2.991 (2.656)	2.628 (1.868)	2.539 (1.869)
Bachelor's degree	8.599 (1.935)***	8.579 (1.939)***	11.204 (2.752)***	10.872 (2.746)***	7.370 (2.721)***	7.375 (2.722)***	9.151 (1.930)***	9.010 (1.931)***
Master's or PhD	10.902 (2.025)***	10.780 (2.030)***	12.675 (2.883)***	12.539 (2.875)***	9.761 (2.868)***	9.772 (2.874)***	11.153 (2.023)***	11.047 (2.023)***
Income \$25k-\$50k	9.414 (2.105)***	9.578 (2.107)***	8.638 (2.976)***	8.561 (2.976)***	10.273 (2.966)***	10.271 (2.967)***	9.647 (2.095)***	9.640 (2.095)***
Income \$50k-\$75k	11.952 (2.107)***	11.810 (2.108)***	12.170 (2.934)***	12.177 (2.931)***	12.023 (3.002)***	12.003 (3.004)***	12.233 (2.094)***	12.303 (2.093)***
Income \$75k-\$100k	12.925 (2.222)***	12.668 (2.223)***	14.050 (3.074)***	13.948 (3.069)***	11.469 (3.201)***	11.466 (3.201)***	12.946 (2.217)***	12.937 (2.215)***
Income \$100k +	14.753 (2.154)***	14.690 (2.155)***	14.737 (3.027)***	14.776 (3.023)***	15.234 (3.045)***	15.238 (3.047)***	15.048 (2.148)***	15.097 (2.147)***
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6125	6090	3047	3043	3119	3118	6166	6161
R-squared	0.14	0.14	0.15	0.15	0.14	0.14	0.14	0.14

Notes: This table reports OLS regressions where the dependent variable is turnout (0-1) from administrative voting records. Coefficients are multiplied by 100 for ease of readability. Robust standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 8: Beliefs About the Closeness of the Election and Voter Turnout, IV Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Pred vote margin, post-treat	0.17 (0.42)	0.19 (0.43)	0.08 (0.41)									
Pred vote margin, pre-treat		-0.32 (0.24)	-0.12 (0.22)									
Pr(Marg <100 votes), post				-1.50 (1.85)	-0.70 (0.58)	-0.68 (0.56)						
Pr(Marg <100 votes), pre					0.45 (0.51)	0.50 (0.47)						
Pr(Marg <1,000 votes), post							0.17 (0.57)	0.22 (0.68)	0.63 (0.70)			
Pr(Marg <1,000 votes), pre								-0.24 (0.59)	-0.52 (0.60)			
<100 or 1,000 votes, post										-0.32 (0.63)	-0.25 (0.45)	-0.09 (0.43)
<100 or 1,000 votes, pre											0.12 (0.39)	0.05 (0.37)
F-stat on excl instrument	48.09	78.24	76.99	1.22	25.54	25.85	6.95	17.88	15.89	5.63	41.77	40.69
Demog Controls	No	No	Yes	No	No	Yes	No	No	Yes	No	No	Yes
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6125	6090	6090	3047	3043	3043	3119	3118	3118	6166	6161	6161

Notes: The dependent variable is turnout (0-1) from administrative voting records. In all specifications, post-treatment beliefs are instrumented with a dummy variable for receiving the close poll treatment. Coefficients are multiplied by 100 for ease of readability. Robust standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 9: Beliefs About the Closeness of the Election and Intended Probability of Voting, IV Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Pred vote margin, post-treat	0.07 (0.24)	0.10 (0.25)	0.05 (0.23)									
Pred vote margin, pre-treat		-0.15 (0.14)	-0.04 (0.13)									
Pr(Marg <100 votes), post				-1.42 (1.89)	-0.50 (0.37)	-0.51 (0.35)						
Pr(Marg <100 votes), pre					0.37 (0.32)	0.42 (0.30)						
Pr(Marg <1,000 votes), post							0.24 (0.35)	0.28 (0.38)	0.48 (0.39)			
Pr(Marg <1,000 votes), pre								-0.25 (0.34)	-0.37 (0.34)			
<100 or 1,000 votes, post										-0.16 (0.40)	-0.10 (0.27)	-0.03 (0.25)
<100 or 1,000 votes, pre											0.06 (0.23)	0.05 (0.22)
Demog Controls	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6645	6607	6607	3285	3281	3281	3406	3405	3405	6691	6686	6686

Notes: The dependent variable is the post-treatment intended probability of voting (ranging from 0%-100%). In all specifications, post-treatment beliefs are instrumented with a dummy variable for receiving the close poll treatment. Robust standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 10: Beliefs About the Closeness of the Election and Information Acquisition, IV Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Pred vote margin, post-treat	0.60 (0.44)	0.55 (0.42)	0.55 (0.42)									
Pred vote margin, pre-treat		-0.25 (0.23)	-0.26 (0.23)									
Pr(Marg <100 votes), post				-1.16 (1.90)	-0.44 (0.59)	-0.37 (0.60)						
Pr(Marg <100 votes), pre					0.35 (0.51)	0.29 (0.51)						
Pr(Marg <1,000 votes), post							-0.70 (0.61)	-0.86 (0.71)	-0.94 (0.75)			
Pr(Marg <1,000 votes), pre								0.76 (0.63)	0.79 (0.65)			
<100 or 1,000 votes, post										-0.86 (0.69)	-0.63 (0.46)	-0.65 (0.46)
<100 or 1,000 votes, pre											0.54 (0.40)	0.54 (0.40)
F-stat on excl instrument	47.52	85.70	86.16	0.98	25.77	24.89	7.34	18.69	17.28	6.24	43.53	42.64
Demog Controls	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	5790	5758	5758	2874	2871	2871	2957	2956	2956	5831	5827	5827

Notes: The dependent variable is whether an agent started to pay less attention (coded as -1), more attention (coded as +1), or the same amount of attention (coded as 0) after being exposed to a poll, as reported in the post-election survey. In all specifications, post-treatment beliefs are instrumented with a dummy variable for receiving the close poll treatment. Coefficients are multiplied by 100 for ease of readability. Robust standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table 11: Testing for the Bandwagon Effect: The Effect of Beliefs About Democrat Likely Vote Share on Voting for the Democratic Candidate, IV Results

	(1)	(2)	(3)	(4)
Predicted Dem share, post-treatment	1.12 (0.16)***	0.43 (0.80)	0.50 (0.72)	0.44 (0.71)
Predicted Dem share, pre-treatment			1.55 (0.44)***	1.41 (0.42)***
Constant	-2.74 (8.18)	39.71 (44.12)	-46.83 (17.52)***	-29.29 (20.05)
F-stat on excl instrument (Dem vote share in shown poll)	798.30	35.92	66.26	65.81
Demong Controls	No	No	No	Yes
State FE	No	Yes	Yes	Yes
Observations	4211	4211	4201	4201

Notes: The dependent variable is whether a voter voted for the Democratic candidate and is self-reported. In all specifications, the voters' beliefs about the likely Democratic vote share are instrumented with the Democratic vote share in the poll they were shown. Coefficients are multiplied by 100 for ease of readability. Demographic controls include gender, race, 10-year age bins, education dummies, and \$25k income bins. Robust standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Appendix:

A Model of Random Voting with NLLN

Imagine that a voter knows the state of the world θ , but that θ represents the proportion of individuals in the population that support. This is fixed (i.e. individuals preferences over candidates are not random). what is random is the probability of any individual voting. There are N individuals in the population. Denote the set of supporters as S_A and S_B for each candidate respectively.

Denote the reduced form probability of any member S_A voting as p_A , and the probability of a member of S_B of voting as p_B . This generates a distribution over possible turnouts of A supporters and B supporters T_A and T_B . A TB believes that as N gets large, $\frac{T_A}{N}$ and $\frac{T_B}{N}$ converge almost surely to θ and $1 - \theta$ respectively. Therefore, the probability of a percent close election closer than $\theta - (1 - \theta)$ converges to 0. Furthermore, the probability of a ι -vote close election for a fixed ι converges to 0.

On the other hand, a NLLN believes that $\frac{T_A}{N}$ and $\frac{T_B}{N}$ will converge to distributions to f_A and f_B .

The probability that an ϵ -close election occurs is the probability that the average difference in probabilities is less than epsilon $P(|x - y| < \epsilon)$, where x is distributed with probability f_A and y with probability f_B . Note that this is some positive number.

Therefore, for large elections a NLLN will overestimate the probability of an ϵ percent close election, and similarly an ι -vote close election, for sufficiently partisan electorates.

B Proofs

Proposition 1

1. For a TB, for all $0 < \epsilon < .5$, $0 < \zeta < 1$, and $\theta \in (0, 1)$ such that $|\theta - .5| > \epsilon$, there exists an N' such that for all elections of size $N > N'$, then the probability of an 2ϵ -percent close election is less than ζ . For a NLLN, for all $0 < \epsilon < .5$, and all $\theta \in (0, 1)$ such that $|\theta - .5| > \epsilon$, there exists a $0 < \zeta^* < 1$, such that for elections of all size N , the probability of an 2ϵ -percent close election is greater than ζ^* .
2. For all $0 < \epsilon < .5$, all $0 < \zeta < 1$, and all $\theta \in (0, 1)$ such that $|\theta - .5| > \epsilon$, there exists an N' such that for all elections of size $N > N'$, then a NLLN thinks there is a higher probability of an 2ϵ -percent close election than a TB.

Proof

1. Denote the sample mean of the vote from a vote with N individuals as V_N . The margin of victory given V_N is $2|V_N - .5|$. Denote $|\theta - .5| = \delta < \epsilon$. For a TB, note that by the weak law of large numbers the sample average converges almost surely to θ . Then for all $1 - \zeta$ there exists an N' such that for all $N > N'$, $|V_N - \theta| < \epsilon - \delta$ with probability greater than $1 - \zeta$, and so $|V_N - .5| > \epsilon$ with probability greater than $1 - \zeta$, and therefore the margin of victory is larger than $2|V_N - .5|$ with probability greater than $1 - \zeta$.

By Lemma 1 of BRR, a NLLN believes that V_N converges in distribution to $f_{\beta|\Theta}^{\psi}(\beta|\theta)$. This has strictly positive support on the range $[\cdot 5 - \epsilon, \cdot 5 + \epsilon]$, and so a NLLN places strictly positive probability on the margin of victory being weakly less than 2ϵ in the limit. **[[Extend to finite N by using the convergence argument. Need to set up multiple bounds as in Lemma C from BRR]]** \square

2. Consider the ζ^* from Proposition 1. Then by Proposition 1, there exists a N'_{ζ^*} , such that for all $N \geq N'_{\zeta^*}$, a TB believes that there is a smaller than ζ^* chance of the election being 2ϵ -percent close. \square

Proposition 2

1. For a TB and a NLLN, for all $\theta \neq .5$, all $0 < \zeta < 1$, and all $0 < \iota < N$, there exists an N' such that for all elections of size $N > N'$, then the probability of an ι -vote close election is less than ζ .
2. For all $0 < \epsilon < .5$, all $0 < \zeta < 1$, and all $\theta \in (0, 1)$ such that $|\theta - .5| > \epsilon$, there exists an N' such that for all elections of size $N > N'$, then a NLLN thinks there is a higher probability of an $2N\epsilon$ -vote close election than a TB.

Proof

1. Observe that given an N and an *iota*-close election, we can construct an $\epsilon(N, \iota)$ -percent close election which exactly corresponds. For a TB, Proposition 3 is an immediate implication of Proposition 1, since for a fixed ι , as N goes to infinity, the corresponding $\epsilon(N, \iota)$ goes to 0. For a NBLLN, note that by Proposition 1, the distribution of vote outcomes in percentage space converges to the distribution $f_{\mathbb{B}|\Theta}^{\psi}(\beta|\theta)$. For a fixed ι , note that as N goes to infinity, the measure of percentage close elections supporting an ι -vote close election goes to 0, and so the probability of any of them being realized goes to 0. [[[Need to deal with joint convergence issues]]]

2. □

Proposition 3 *If $\theta > \theta' > .5$ then for all $0 < \epsilon < 1$ and $0 < \iota < N$, both a TB and a NBLLN believe that the probability of either an ϵ or ι close election is higher under θ' than θ .*

Proof Note that fixing N an increase in θ causes a shift in the a TB's beliefs about the distribution of election outcomes that respects FOSD. Similarly, an increase in θ causes a shift in $f_{\mathbb{B}|\Theta}^{\psi}(\beta|\theta)$ that respects FOSD, and in the distribution of election outcomes that respects FOSD (see Proposition 1 of BRR).

Clearly if θ is outside the margin then clearly a shift in FOSD causes a decline in the distribution inside, because of the single-peakedness of both binomial and a NBLLN's distribution as well.

If θ inside margin then... □

Proposition 4

1. *Suppose θ_1 is the true state, and assume that polls are random samples from a population. For a TB, for all $\theta_1, \theta_2 \neq .5$, all $0 < f_{\Theta}(\theta_1), f_{\Theta}(\theta_2) < 1$, all $0 < \zeta < 1$ then there exists an N' such that for all polls of size $N > N'$, then almost surely the posterior probability that a TB assigns to θ_2 being the true state is less than ζ . For a NBLLN, for all $\theta_1, \theta_2 \neq .5$, all $\zeta < f_{\Theta}(\theta_2) < 1$, and polls of all sizes N , then there exists a ζ and τ such that with probability τ the posterior probability that a TB assigns to θ_2 being the true state is greater than ζ for all N .*
2. *Suppose θ_1 is the true state, and assume that polls are random samples from a population. For all $\theta_1, \theta_2 \neq .5$, all $0 < f_{\Theta}(\theta_1), f_{\Theta}(\theta_2) < 1$, $1 > \tau > 0$, there exists an N' and such that for all polls of size $N > N'$, then a NBLLN places a higher posterior on θ_2 being the true state compared to a TB with probability greater than τ .*

Proof

1. Without loss of generality assume $\theta_1 > \theta_2$. The likelihood ratio assigned to state θ_1 after N signals with a difference in A and B votes is $\frac{\theta_1^{N-A}}$.
For a NBLLN, multiple stages. First, pick out a ζ_1 that's equivalent with the limit posterior. Pick out a ζ_2 that the posterior ratio after N is guaranteed to exceed that. Pick out a τ such that with probability τ (using Chebyshev) the posterior is within some bound. Mark out ζ_3 . Then take minimum of ζ_3, ζ_2 .
2. Step 1: For two fixed rates and a ratio of A signals in the poll equal to the true rate, show that for large enough N , a NBLLN strictly under infers relative to a TB. Step 2: Show inference is 'continuous' in the ratio as N gets big. Step 3: Show that the probability being close to the correct ratio becomes arbitrarily close to 1 as N gets large.

□

Proposition 5 *Fix $\theta_1 > \theta_2$, and a prior $0 < f_{\Theta}(\theta_1), f_{\Theta}(\theta_2) < 1$, and two polls, P_1 and P_2 . Assume that the proportion of individuals voting for A in P_1 is larger than in P_2 . If an individual sees P_1 they will place a higher posterior probability on θ_1 than if they see P_2 .*

Proof This is directly implied by the monotone likelihood ratio property of both the binomial distribution and by A.4 of BRR (see Whitt 1979). □

Proposition 6 *Suppose θ_1 is the true state, and that polls are random samples from a population. For all $0 < \epsilon < .5$, all $\theta_1 \in (0, 1)$ such that $|\theta - .5| > \epsilon$, all $\theta_2 \in (0, 1)$, all $0 < f_{\Theta}(\theta_1), f_{\Theta}(\theta_2) < 1$:*

- *There exists M' and N' such that for all elections of size $N > N'$, and polls of size $M > M'$ so that $N > M$, a NBLLN believes a 2ϵ percent close election and $2N\epsilon$ -vote close election is more likely than a TB.*
- *For all $1 < \zeta_1, \zeta_2 < 0$, there exists M' and N' such that for all elections of size $N > N'$, and polls of size $M > M'$ so that $N > M$, a TB believes that the probability of a 2ϵ percent close election and $2N\epsilon$ -vote close election are less than ζ_2 with probability greater than ζ_1 .*

- There exists $1 < \zeta_1, \zeta_2 < 0$, such that for all elections of size N , and polls of size M so that $N > M$, a NLLN believes that the probability of a 2ϵ percent close election is greater than ζ_2 with probability greater than ζ_2 .
- For all $1 < \zeta_1, \zeta_2 < 0$, there exists M' and N' such that for all elections of size $N > N'$, and polls of size $M > M'$ so that $N > M$, a NLLN believes that the probability of a $2N\epsilon$ -vote close election are less than ζ_2 with probability greater than ζ_1 .

Proof

□

Proposition 7 *An individual with a higher probability of being pivotal will have a higher probability of voting.*

Proof

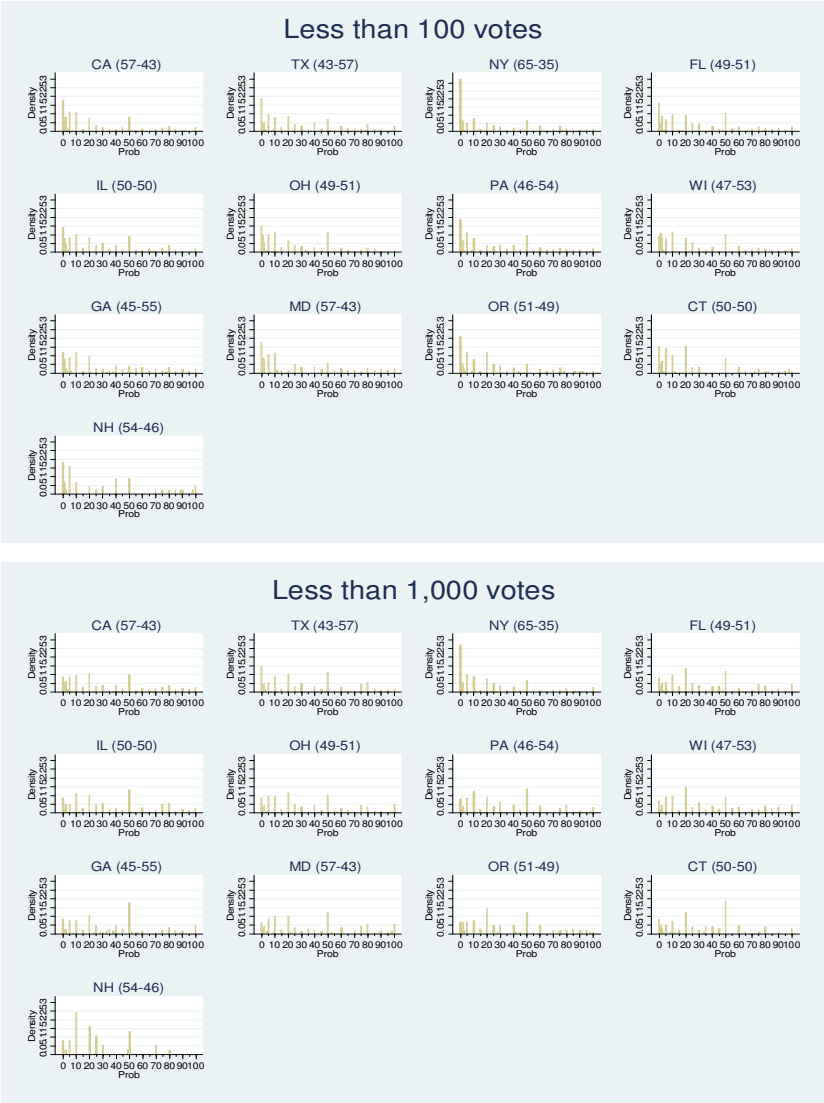
□

Proposition 8 *An individual shown a poll with a smaller margin of victory will have a higher probability of voting.*

Proof

□

Figure C1: Subjective Probabilities that Gubernatorial Election Will be Decided by Less than 100 Votes or 1,000 Votes in Different States



Notes: These graphs plot the distribution of answers to the question asking for the probability the election in the respondent's state would be decided by less than 100 votes or less than 1,000 votes in different states.

Table C1: Beliefs About the Closeness of the Election and Voter Turnout, IV Results: Sample Restricted to Voters with Middle Ideology

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Pred vote margin, post-treat	-0.13 (0.58)											
Pred vote margin, change		-0.15 (0.73)	0.11 (0.70)									
Pr(Marg <100 votes), post				-16.43 (213.56)								
Pr(Marg <100 votes), change					-0.64 (1.00)	-1.07 (0.94)						
Pr(Marg <1000 votes), post							0.60 (0.55)					
Pr(Marg <1000 votes), change								0.91 (0.80)	0.93 (0.76)			
<100 or 1,000 votes, post										0.17 (0.81)		
<100 or 1,000 votes, change											0.19 (0.62)	-0.04 (0.57)
Observations	2,473	2,466	2,466	1,221	1,218	1,218	1,266	1,266	1,266	2,487	2,484	2,484
R-squared	0.04	0.03	0.13	-96.69	-0.00	0.00	-0.15	-0.06	0.07	0.01	0.03	0.13

Notes: The dependent variable is turnout (0-1) from administrative voting records. In all specifications, post-treatment beliefs are instrumented with a dummy variable for receiving the close poll treatment. Robust standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table C2: Beliefs About the Closeness of the Election and Voter Turnout, IV Results: Sample Restricted to Voters with Low Interest in Government

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Pred vote margin, post-treat	-0.27 (0.54)											
Pred vote margin, change		-0.29 (0.59)	-0.29 (0.57)									
Pr(Marg <100 votes), post				-1.20 (2.66)								
Pr(Marg <100 votes), change					-0.41 (0.80)	-0.69 (0.75)						
Pr(Marg <1000 votes), post							1.19 (1.59)					
Pr(Marg <1000 votes), change								0.85 (0.96)	1.20 (1.10)			
<100 or 1,000 votes, post										0.27 (1.14)		
<100 or 1,000 votes, change											0.16 (0.60)	0.17 (0.58)
Observations	2,593	2,573	2,573	1,284	1,281	1,281	1,327	1,326	1,326	2,611	2,607	2,607
R-squared	0.05	0.04	0.13	-0.44	0.01	0.06	-0.51	-0.06	-0.06	0.02	0.05	0.14

Notes: The dependent variable is turnout (0-1) from administrative voting records. In all specifications, post-treatment beliefs are instrumented with a dummy variable for receiving the close poll treatment. Robust standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table C3: Beliefs About the Closeness of the Election and Voter Turnout, IV Results: Sample Restricted to Voters with Higher Interest in Government

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Pred vote margin, post-treat	0.94 (0.59)											
Pred vote margin, change		0.92 (0.57)	0.81 (0.57)									
Pr(Marg <100 votes), post				-1.79 (2.86)								
Pr(Marg <100 votes), change					-0.70 (0.76)	-0.56 (0.76)						
Pr(Marg <1000 votes), post							-0.66 (0.54)					
Pr(Marg <1000 votes), change								-1.07 (0.85)	-0.84 (0.84)			
<100 or 1,000 votes, post										-0.96 (0.74)		
<100 or 1,000 votes, change											-0.84 (0.56)	-0.71 (0.56)
Observations	4,037	4,019	4,019	1,993	1,992	1,992	2,071	2,071	2,071	4,064	4,063	4,063
R-squared	-0.13	-0.04	0.02	-1.41	-0.03	0.03	-0.21	-0.15	-0.03	-0.43	-0.07	-0.00

Notes: The dependent variable is turnout (0-1) from administrative voting records. In all specifications, post-treatment beliefs are instrumented with a dummy variable for receiving the close poll treatment. Robust standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table C4: Beliefs About the Closeness of the Election and Voter Turnout, IV Results: Sample Restricted to Voters who Don't Always Vote

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Pred vote margin, post-treat	-0.01 (0.47)											
Pred vote margin, change		-0.02 (0.54)	-0.04 (0.54)									
Pr(Marg <100 votes), post				10.47 (99.53)								
Pr(Marg <100 votes), change					-0.49 (0.79)	-0.46 (0.73)						
Pr(Marg <1000 votes), post							0.56 (1.01)					
Pr(Marg <1000 votes), change								0.43 (0.75)	0.55 (0.75)			
<100 or 1,000 votes, post										-0.11 (1.40)		
<100 or 1,000 votes, change											-0.02 (0.54)	0.02 (0.52)
Observations	4,086	4,061	4,061	1,991	1,987	1,987	2,120	2,119	2,119	4,111	4,106	4,106
R-squared	0.04	0.04	0.09	-39.71	0.01	0.07	-0.10	0.00	0.05	0.04	0.04	0.09

Notes: The dependent variable is turnout (0-1) from administrative voting records. In all specifications, post-treatment beliefs are instrumented with a dummy variable for receiving the close poll treatment. Robust standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table C5: The Effect of the Close Poll Treatment on Predicted Democratic Vote Share

Sample restriction:	OLS		Constrained Regression (Regression Coefficients Sum to 1)				
	(1)	(2)	Overall (3)	Low interest in govt (4)	Hi interest in govt (5)	Don't usually vote (6)	Usually vote (7)
Pred Dem share, pre-treatment	0.60 (0.02)***	0.59 (0.02)***	0.61 (0.02)***	0.56 (0.03)***	0.65 (0.02)***	0.59 (0.02)***	0.63 (0.02)***
Dem vote share in viewed poll	0.35 (0.02)***	0.26 (0.03)***	0.39 (0.02)***	0.44 (0.03)***	0.35 (0.02)***	0.41 (0.02)***	0.37 (0.02)***
Constant	2.78 (0.85)***	8.12 (1.48)***					
State FE	No	Yes	No	No	No	No	No
Observations	6665	6665	6665	2602	4042	3398	3267
R-squared	0.57	0.57					

Notes: The dependent variable is the post-treatment predicted Democratic vote share. Robust standard errors in parentheses. In the constrained regression, the regression coefficients on the pre-treatment Democratic vote share and on the Democratic vote share in the viewed poll are required to sum to 1. "Don't usually vote" is people voting less than 80% of the time in the past 5 general elections. "Usually vote" is people voting 80% of the time or more in the past 5 general elections. * significant at 10%; ** significant at 5%; *** significant at 1%.

We will now ask you questions about the upcoming November election for the governor of Oregon. The elections will be held on Tuesday, November 2nd, 2010.

As of today, have you already voted in the November elections, for example, by absentee ballot or early voting?

Select one answer only

- Yes
- No

Next

How interested are you in information about what's going on in government and politics?
Extremely interested, very interested, moderately interested, slightly interested, or not interested at all?

Select one answer only

- Extremely interested
- Very interested
- Moderately interested
- Slightly interested
- Not interested at all

Next

How often would you say you vote? Seldom, part of the time, nearly always, or always?

Select one answer only

- Seldom
- Part of the time
- Nearly always
- Always

Next

What job or political office is held by Nancy Pelosi?

Select one answer only

- U.S. Secretary of State
- U.S. Secretary of Labor
- U.S. Secretary of Homeland Security
- Speaker of the U.S. House of Representatives
- Majority Leader of the U.S. Senate

Next

DOV:SHOWFIRST

Select one answer only

- DEMOCRAT
- REPUBLICAN

Next

In the election for governor, of the people voting for either the Democratic or Republican candidates, what share do you predict will vote for the Democratic candidate and what share do you predict will vote for the Republican candidate?

Type in the answer into each cell in the grid

%

John Kitzhaber (Democrat)	<input type="text"/>
Chris Dudley (Republican)	<input type="text"/>
Total	<input type="text" value="0"/>

Please make sure these numbers add up to 100%.

Next

DOV: VERSION

Select one answer only

- Version 2
- Version 1

Next

Many of the next questions ask you to think about the **percent chance** that something will happen in the future.

The **percent chance** can be thought of as the number of chances out of 100. You can use any number between 0 and 100 (including 0 and 100).

For example, numbers like:

1 and 2 percent may be "almost no chance",

20 percent or so may mean "not much chance",

a 45 or 55 percent chance may be a "pretty even chance",

80 percent or so may mean a "very good chance",

and a 98 or 99 percent chance may be "almost certain"

Next

What do you think is the percent chance that you will vote in this year's election for governor?

Type in the number for the answer

 %

Next

If you do vote in this year's election for governor, what do you think is the percent chance that you will vote for the following candidates:

Type in the answer into each cell in the grid

%

John Kitzhaber (Democrat)	<input type="text"/>
Chris Dudley (Republican)	<input type="text"/>
Someone else	<input type="text"/>
Total	<input type="text" value="0"/>

Note: This question asks about your chances of voting for the different candidates; it is not the same question as the previous one on predicting vote shares.

Next

DOV: VOTES

Select one answer only

- 1000
- 100

Next

What do you think is the percent chance the election for governor will be decided by 1000 or fewer votes?

Type in the number for the answer

%

Next

Below are the results of a recent poll about the race for governor. The poll was conducted over-the-phone by a leading professional polling organization. People were interviewed from all over the state, and the poll was designed to be both non-partisan and representative of the voting population. Polls such as these are often used in forecasting election results.

Of people supporting either the Democratic or Republican candidates, the percent supporting each of the candidates were:

John Kitzhaber (Democrat):	51%
Chris Dudley (Republican):	49%

[Next](#)

We would like to again ask you some of the same questions we did above:

Next

In the election for governor, of the people voting for either the Democratic or Republican candidates, what share do you predict will vote for the Democratic candidate and what share do you predict will vote for the Republican candidate?

Type in the answer into each cell in the grid

%

John Kitzhaber (Democrat)	<input type="text"/>
Chris Dudley (Republican)	<input type="text"/>
Total	<input type="text" value="0"/>

Recent Poll Results:

John Kitzhaber (Democrat): 51%

Chris Dudley (Republican): 49%

Next

What do you think is the percent chance that you will vote in this year's election for governor?

Type in the number for the answer

%

Recent Poll Results:

John Kitzhaber (Democrat): 51%

Chris Dudley (Republican): 49%

Next

If you do vote in this year's election for governor, what do you think is the percent chance that you will vote for the following candidates:

Type in the answer into each cell in the grid

%

John Kitzhaber (Democrat)	<input type="text"/>
Chris Dudley (Republican)	<input type="text"/>
Someone else	<input type="text"/>
Total	<input type="text" value="0"/>

Recent Poll Results:

John Kitzhaber (Democrat): 51%

Chris Dudley (Republican): 49%

Next

What do you think is the percent chance the election for governor will be decided by 1000 or fewer votes?

Type in the number for the answer

%

Recent Poll Results:

John Kitzhaber (Democrat): 51%

Chris Dudley (Republican): 49%

Next

What do you think is the percent chance the election for governor will be decided by 1000 or fewer votes?

Type in the number for the answer

%

Recent Poll Results:

John Kitzhaber (Democrat): 51%

Chris Dudley (Republican): 49%

Next

Thinking about this topic, do you have any comments you would like to share?

Any comments welcome!

Next

The variables on this screen are select demographic and other data that will be imported into the questionnaire by the system. These questions will be removed prior to fielding and will NOT be visible to the respondents. They are shown here only for testing purposes. If this survey's functionality depends on some or all of these variables, please enter the appropriate values here.

State - numeric

Type in the number for the answer

XPIVOTAL

Select one answer only

- Treatment1
- Treatment2
- Control

XSHOW

Select one answer only

- Show Democrat first
- Show Republican first

[Next](#)

DOV: Stateside

Select one answer only

- California
- Texas
- New York
- Florida
- Illinois
- Ohio
- Pennsylvania
- Wisconsin
- Georgia
- Maryland
- Oregon
- Connecticut
- New Hampshire

Next

Imagine you had a fair coin that was flipped 1,000 times. What do you think is the percent chance that you would get the following number of heads:

Type in the answer into each cell in the grid

	%
Between 0 and 200 heads:	<input type="text"/>
Between 201 and 400 heads:	<input type="text"/>
Between 401 and 480 heads:	<input type="text"/>
Between 481 and 519 heads:	<input type="text"/>
Between 520 and 599 heads:	<input type="text"/>
Between 600 and 799 heads:	<input type="text"/>
Between 800 and 1,000 heads:	<input type="text"/>
Total	<input type="text" value="0"/>

Please make sure your answers add up to 100 percent. Also, please try not to spend more than 1 minute on this question.

Next