Searching For Information and the Diffusion of Knowledge

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Abstract

We study a dynamic learning model in which heterogeneously connected Bayesian players choose between two activities: learning from one's own experience (*work*) or learning from the experience of others (*search*). Players who work produce an inflow of information which is local and dispersed across the society. Players who search aggregate the information produced by others and facilitate its diffusion, thereby transforming what is inherently a private good into information that everyone can access more easily. The structure of social connections affects the interaction between equilibrium information production and its social diffusion in ways that are complex and subtle. We show that increasing the connectivity of the society can lead to a strict decrease in the quality of social information. We link these inefficiencies to frictions in peer-to-peer communications. Moreover, we find that the socially optimal allocation of learning activities can differ dramatically from the equilibrium one: under certain conditions, the planner would reverse the equilibrium allocation, forcing highly connected players to work, and moderately connected ones to search. We conclude with an application that studies how resilient a society is to external manipulation of public opinion through changes in the meeting technology.

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1. Introduction

This paper studies learning in large connected societies. The novelty of our approach is to explicitly capture the interactions between the *creation* of information and its social *diffusion*. When a society is effective at diffusing information, this reduces individual incentives to create new information. However, when those who create information are few and peripheral, this hinders the diffusion of information. Our goal is to study the effects of these interactions on learning and their dependence on the structure of social connections. We identify a novel externality that we call the *noise-amplification* effect. This is an equilibrium mechanism by which noise reverberates and amplifies throughout the society, a phenomenon akin to the telephone game. We show the implications of this externality on different aspects of social learning: Do societies allocate learning tasks among differently connected players in ways that promote learning? Are more connected societies necessarily better informed? Are they more resilient to the external manipulation of public opinions?

We introduce a dynamic learning model for heterogeneously connected societies. A population of Bayesian players choose how to allocate time between two activities: learning directly from information sources (*work*) and learning indirectly from others (*search*). Players who work produce an inflow of information, which is initially local, available only to the worker. Indirect learning is frictional, in two distinct ways. First, in order to learn from others a player needs to meet them–and this takes time. The rate at which a player can meet others is heterogeneous and determined by her *type*, representing how connected she is to the rest of society. Second, searcher-to-searcher communication is potentially subject to frictions. Namely, we allow for some information to be lost in these exchanges. A distinguishing feature of our model is that it captures the important social role played by those who search for information. While not producing new information, searchers aggregate the information produced by others and facilitate its diffusion. By doing so, they transform what is inherently a private good, information produced by a worker for herself, into a *more public* one, information that can be accessed more easily by everyone else. This increases the value of search and attracts a group of marginally connected players into this activity. Their diffusion ability, however, is no better than their ability to create information. This introduces a distortion that reverberates through the rest of the searching population, thereby causing its amplification.

Our main contribution is to identify novel inefficiencies in social learning, generated by the interaction between information production and its diffusion. First, we highlight the critical role played by those who search for information in the aggregation and diffusion of information. A society needs searchers to achieve some degree of informational efficiency. However, not all searchers are alike and the structure of social connections can affect efficiency in ways that are complex and subtle. To explore this, we study the equilibrium consequences of increasing the connectivity of a society. We show that increasing connectivity can lead to a strict decrease in the quality of social information. Our results reveal how this inefficiency is directly linked to communication frictions. Second, we analyze how players of different connectivity levels choose their learning activity in equilibrium, and compare this to the planner's solution where each type is allocated to an activity to maximize social welfare. This allows us to study more generally the inefficiencies associated with social learning. We show that allocation of activities in the planner's solution can be in direct contrast with the one in equilibrium, especially when communication frictions are severe. In such cases, the planner's solution requires players with high connectivity to be the producers of information in the society, whereas in equilibrium, this role is necessarily taken on by players with low connectivity. Third, we apply our model to study how resilient a society is to external manipulation of public opinion and how this depends on the connectivity of the society. An important implication of our analysis is that societies that are very effective in aggregating and diffusing information are also particularly susceptible to manipulations.

This paper combines ideas and modeling tools from literatures such as strategic experimentation, social networks, and search. First, in order to study *both* the equilibrium production of information and its social diffusion, we introduce heterogeneity across players in a parsimonious way. We focus on a single dimension of heterogeneity: how connected each player is to these society. By doing so, we assume that the connectivity of a player determines the *rate* at which she meets others, not *who* she meets. This affects the frequency with which she encounters opportunities to receive, as well as to transfer information to others. While this formulation abstracts from other interesting ways in which the network structure can affect learning dynamics, we argue that it captures its most prominent feature, namely heterogeneity in the *number* of connections. Second, we allow for frictions in searcher-to-searcher communication. These frictions are modeled as a garbling, a decrease in the quality of a signal as it travels among searchers. These frictions create a gap between *first-hand* information, the information created by players who work, and *second-hand* information, the information collected by anybody else. When these frictions are absent, our model reduces to the special case in which players perfectly observe each other's beliefs upon meeting.

The stationary equilibrium of this game is unique and has a remarkably simple structure. Specifically, the allocation into activities is fully characterized by a threshold connectivity level, below which players work and, above which players search. Intuitively, more connected players meet others at a higher rate and, rather than working, they prefer to learn from them. Due to their higher connectivity level, searchers are also easier to meet. Therefore, the information they carry is made *more accessible* to everybody else in the society, further attracting players away from work. Frictions in peer-to-peer communication drive a wedge between a searcher's ability to *aggregate* information and her ability to *diffuse* it to others. In equilibrium, there exists a group of moderately connected players who decide to search, but doing so imposes a negative externality on the rest of the society. Due to communication frictions, these players' diffusion abilities as searchers are inferior to their diffusion abilities as workers. This introduces an inefficiency that goes beyond the fact that meeting these players is now less informative: it impacts *all* social meetings. These moderately connected players are effectively responsible for injecting extra noise in the society. As it travels from one searcher to another, this noise accumulates and amplifies through the structure of social connections. We call this effect the *noise-amplification* externality.

We study whether this externality is fueled or dampened as the society becomes more connected. The net effect of increasing connectivity ultimately arises from the conflict of two opposing forces. A highly connected society provides more opportunities for players to learn from others and aggregate information at a faster rate. But at the same time, it can tilt incentives away from producing information, thereby enlarging the pool of searchers and, thus, increasing the noise-amplification externality. We model increased connectivity with a class of stochastic transformations of the type-distribution. In particular, we focus on sequences of stochastic transformations satisfying single-crossing property and a version of the monotone likelihood ratio property. We show that the equilibrium quality of social information is *quasi-convex*. This property implies that, along any sequence of increasingly connected societies, the equilibrium quality of social information undergoes two phases. During the first one, it declines because increased connectivity comes at the cost of amplifying these social distortions. In the second phase, increased diffusion ability overcomes noise, and social encounters become more informative. What determines the relative magnitude of these two phases is the communication technology.

To better understand these inefficiencies, we study how the planner allocates types into learning activities to maximize welfare. We show that the socially optimal allocation can diverge substantially from the equilibrium one. In particular, players with a higher connectivity do not necessarily spend more time searching, a feature that is necessarily true in equilibrium. Depending on the constraints the social planner faces, the socially optimal allocation can differ from equilibrium in two distinct ways: *reversal*, a situation in which highly connected players work, whereas lower types search, in stark contrast with equilibrium; or *time-switching*, a situation in which a region of players is constantly switched back and forth between work and search as a function of the actual information they carry. Both deviations highlight how the planner's allocation can differ from equilibrium in a qualitative sense. These deviations are caused by the interplay of connectivity and frictions in communication. When these frictions are severe, a searcher's contribution to social information is curtailed. Although she accumulates information at a high rate, her diffusion ability is limited. From a social perspective, she free-rides more than she can diffuse information. By forcing a highly connected searcher to work, the planner trades-off her individual gains with the fact that every signal she produces as a worker will be easier to find by those who search. This is because her type determines *both* the rate at which she meets others and the rate at which others meet her.

Finally, we apply our framework to study how resilient a society is to external manipulations of public opinions and how this depends on the connectivity of the society. We imagine altering the meeting technology in a way that consistently exposes a small group of players to biased information.[1] We evaluate the overall impact of this manipulation on the distribution of beliefs in the society. To do so, we construct a measure of *influence* for players in our society. It combines all the different forces that are at play in equilibrium. Our analysis shows that searchers become more influential as the share of the population producing information declines and connectivity increases. This result is line with the role of the amplification mechanism described above. An important implication of our analysis is that that societies that are very effective in aggregating and diffusing information also correspond to those that are highly susceptible to manipulations. As the influence of each type increases in this society –possibly due to the communication channel becoming more efficient or the society becoming more connected– it becomes easier to shift public opinion by manipulating the learning process for an increasingly small share of agents in the population.

The rest of the paper is structured as follows. Section 2 gives a comprehensive account of the related literature. Our model is introduced in Section 3 and we discuss its main assumptions in Section 3.4. We proceed by characterizing the equilibrium in Section 4 and derive our main results in Section 5. Section 6 is dedicated to normative solutions and efficiency benchmarks. In Section 7, we introduce our measure of influence and study the resilience of the society to external manipulations. Finally, Section 8 provides a discussion of our results in relation to possible extensions, while Section 9 concludes.

2. Related Literature

This paper is at the intersection of two principal strands of the learning literature. The basic trade-off between learning-by-doing (creating information) and learning-from-others

[1]Recently, a number of controlled large-scale social media experiments have shown the power of altering the news-feed in affecting users' beliefs and behavior. This corresponds to tweaking the probability that a given content will be shown to (in the language of our model, "will be found by") a given user. Aral (2012) study the impact of manipulation on the decision to vote, Muchnik et al. (2013) study the likelihood of informational cascades and Bakshy et al. (2012) study product adoption decisions.

(free-riding) is focal in the literature on strategic experimentation. There, however, how efficiently information is diffused is of no particular interest. In fact, the observable information generated by players, either via their payoffs or their actions, is usually public and, therefore, instantaneously diffused to all other players. Information diffusion is, to the contrary, the distinguishing feature of the literature on social networks, herding and word-of-mouth learning. There, it is the problem of information production that is assumed away, as players are usually endowed with a set of exogenously given private signals.

The fundamental trade-off between producing information and learning from others is a classic feature of strategic experimentation problems, as in Bolton and Harris (1999), Keller et al. (2005), Rosenberg et al. (2007), Bonatti and Hörner (2011) and many others. In bandit problems, players learn via costly experimentation or the observation of other players' experimentation. Pulling the *safe* arm effectively consists in free-riding on the information produced by others. However, experimentation is public, as these models do not accommodate heterogeneity in connections. Therefore, the problem of diffusing information is not really an issue. Relative to bandit problems, we reduce players' strategic interactions to their minimal components. In particular, we work with a continuum of players in a stationary environment. Moreover, the absence of a "safe" allows us to abstract from the classic experimentation-exploration trade-off, which is of interest in that literature. This makes the player's problem simple, allowing us to enrich social interactions in novel directions that are of fundamental importance for our questions. Recently, Che and Horner (2015) and Frick and Ishii (2016) have analyzed bandit-like environments with a continuum of players to study optimal information design and technological adoption. In a spirit similar to ours, Sadler (2015) studies a model of strategic experimentation on incomplete networks, in which non-Bayesian players best respond to naïve expectations about their neighbors' beliefs.

A fundamentally new feature of our model is the introduction of heterogeneity in players' connections. Empirical evidence shows how vastly different people are when it comes to how connected they are to the rest of the society (Newman (2010)). Of course, this heterogeneity affects players' ability to learn from others as well as the influence they exert on others (Ballester et al. (2006)). One of our motivations is to understand how the structure of social connections affects equilibrium outcomes. From this point of view, we borrowed a lot from the social networks literature (Jackson (2008) and Golub and Sadler (2016)). In particular, our paper somewhat relates to the games studied by Bramoullé and Kranton (2007) and Bramoullé et al. (2014) and to the questions in Acemoglu et al. (2010). We substantially deviate from most of this literature as we model connections probabilistically in the context of a search framework. We do so by tweaking the standard search theory set up to conveniently account for degree-heterogeneity. In our model, players are characterized by a *type* that determines the rate at which they meet other players. These meetings are random and their nature is a function of the type-distribution itself. With this, we are able to capture the idea that more connected players are easier to meet. The idea of learning from others by sampling opinions from the society is a feature that comes from the word-of-mouth learning literature, e.g. Banerjee (1993), Ellison and Fudenberg (1995) and Banerjee and Fudenberg (2004). In a different context, it is also a feature of Duffie et al. (2009) and Duffie et al. (2014) and, to some extent, of Callander (2011). Caplin and Leahy (1998) and Caplin and Leahy (2000) also use search tools to model learning in an economy with a continuum of agents. Our paper differs substantially from these ones as we explicitly account for heterogeneity in the rate at which players meet. Farboodi et al. (2016) have independently developed a similar meeting technology to model this heterogeneity, although they have applied it to a markedly distinct environment. Moreover, unlike these authors, we are explicitly interested in the study of equilibrium outcomes as the underlying society becomes *increasingly connected*. To this purpose, the tools we develop are orthogonal to the literature above. Our main Theorem reduces to an integral aggregation of the single-crossing property. This problem closely relates to a tradition in economics that studies comparative statics under uncertainty. Seminal examples are Milgrom and Shannon (1994), Athey (2001), Athey (2002) and, specifically, Quah and Strulovici (2012) from which our definition of "regular" sequence is inspired.

A second key innovation in our paper is to explicitly model frictions in the communication technology. The idea is that communications between players take place through a communication channel with *finite capacity* that inevitably distorts the message. This idea is common in the information theory literature, especially in computer science. In economics, finite capacity channels have been used to model rational inattentive agents, Sims (2003), Steiner et al. (2016) and Jung et al. (2016). We abstract away from the problem of strategic information transmission that has been amply studied in the communication literature, see for example, Milgrom (1981), Grossman (1981), Jovanovic (1982), Crawford and Sobel (1982), Okuno-Fujiwara et al. (1990) and more recently by Kamenica and Gentzkow (2011). Frictions in communication are an implicit feature also in most of the herding literature, Banerjee (1992) and Bikhchandani et al. (1992), Smith and Sorensen (2000), Acemoglu et al. (2011), but also Gale and Kariv (2003). In this paper, we create a flexible environment in which these communication frictions interact with changes in the social structure of the underlying society. Ali (2016) contributes to the herding literature by studying social learning when information can be endogenously produced by the society: a countable set of myopic players act sequentially after gathering information from the observation of past actions (learning from others) and from costly signals (production of information). In this sense, our paper share a very similar motivation, although differ substantially in both the question of interest and the methodology used.

Finally, our paper also relates to some recent work done in growth theory. In particular, Perla and Tonetti (2014) and Lucas and Moll (2014) study growth models in which firms, by search among other firms, can "update" and improve their own technologies. Jovanovic (2015) studies a dynamic learning problem in which an agent chooses between production or investment in information. Fogli and Veldkamp (2014) analyze, both theoretically and empirically, the dual aspects of diffusion: encouraging the spread of good versus bad behavior. In political science, Larson and Lewis (2016) model an information diffusion process that accounts for the fact that people may trust some of their contacts more than others. In such context, higher network density potentially impedes the wide reach of information to diverse communities. Relatedly, Grossman et al. (2014) study empirically how the access to information communication technology affects who gets heard and what gets communicated to politicians.

3. Model

In this section, we introduce the model. We begin by describing players' characteristics and objectives, and the learning activities available to them. In Section 3.2, we define and solve the players' dynamic choice problem. In Section 3.3, we model information exchanges and introduce the communication technology. We postpone the discussion of our model to Section 3.4, in which we motivate our main assumptions.

3.1. Types and Meetings

Time runs continuously and uncertainty is characterized by a persistent and unknown state of nature $\theta \in \Theta := \{-1, 1\}$. A continuum of Bayesian and forward looking players enter and leave the economy at a fixed Poisson rate $\delta > 0$. We index their age with $t \geq 0$ and denote $\tau(t) := \delta e^{-\delta t}$ the exit distribution. Players discount the future at common rate $r > 0$ and, when entering the game, have a common prior belief $p_0 := \Pr(\theta = 1)$.[2] Each player is born with a *type* $x \in \mathbb{R}_+$, distributed according to a density f with support X. Denote $\mathcal{F}$ the set of all such densities. We refer to the distribution of types $f \in \mathcal{F}$ as a *society*.

Each player is endowed with a type-dependent search technology. In particular, a player's type describes how *connected* she is to the rest of the society, a measure of how easily she can meet others and, potentially, learn from them. More specifically, a player's type x denotes the rate at which meetings take place. The nature of these meetings is random and their

[2]These two assumptions are not necessary for most of our results, but make the exposition more straightforward.

distribution is given by the conditional density function $h(z) := zf(z)/\int_X z'f(z')dz'$, which depends on the primitive $f \in \mathcal{F}$. Notice that while a player's type x determines the rate at which she meets others, it does not affect the conditional distribution of these meetings. Moreover, higher types are *ceteris paribus* also those that are more likely to be met by others.

Player's flow (indirect) utility is given by $u(p(x,t)) := \max\{p(x,t), 1-p(x,t)\}$, with $p(x,t)$ denoting the player's posterior belief at age t.[3] To learn about θ, each player continuously chooses between two costless *activities*: working and searching. These activities provide players with private information about the state θ. Signals that originate from the *work* activity are exogenous as they do not depend on the activities chosen by others. They are also independent of a player's type, meaning that everyone has equal access to this technology. When a player chooses to work for a dt-interval, she receives a signal π_w, distributed normally with mean $\eta_w\theta dt$ and variance dt.[4] These signals are conditionally independent across time and players.

Signals originating from the *search* activity are our main equilibrium object. While searching, type x meets other players at rate x. From each player she meets, she receives a signal, whose nature will depend on the type of the player she met, her age, the activity she chose, her particular experience etc. Yet, what matters for her decision to work or search in a dt-interval of time is the information the player *expects* to receive. This can be represented as a signal that aggregates the features listed above. We denote this instantaneous information flow with $\pi_s(x)$ and posit that it is normally distributed with mean $x\eta_s\theta dt$ and variance dt. Signal $\pi_s(x)$ depends on searcher's type x only insofar as it scales the mean-to-variance ratio η_s. This is consistent with our meeting technology: a player's type only affects the *rate* at which she meets others, not *who* she meets. Ultimately, the equilibrium object in the information structure $\pi_s(x)$ is η_s. It captures the "per-meeting" expected quality of social information, a measure of the informativeness of a random meeting in the society. In a non-stationary environment, this variable evolves as the society learns about the state θ. For the time being, however, we will focus on stationary environments, those in which η_s is a stationary equilibrium object.[5] In these cases, the aggregate social distribution of beliefs is stationary, even if individual beliefs are not. Players learn as they grow older, but the fact that they eventually leave the economy – being replaced with new players with prior belief p_0 – guarantees stationarity.

Moments η_w and η_s, together with a player's type x, fully characterize the information

[3] We postpone to Section 3.4 the discussion and microfoundation of $u(p_t)$.

[4] We normalize the variance of signals to 1, letting η_w capture the mean-to-standard deviation ratio. This normalization is without loss of generality with respect to the problem solved by each player.

[5] In Section 8 and in Appendix C, we will lose our focus on stationary environments.

structures associated with the two activities. In particular, these variables pin down how valuable each activity is. How to optimally allocate time between these two turns out to be particularly simple, for two reasons. First, by construction, π_w is more informative (in a Blackwell sense) than $\pi_s(x)$ if and only if $\eta_w \geq x\eta_s$. Second, each player has zero-mass and therefore her choice does not affect equilibrium η_s. We formalize this intuition in the next section.

3.2. Optimal Learning Activities

Players continuously allocate time between work and search to learn most effectively about the uncertain state θ. Let $p(x,t)$ be the Bayesian posterior beliefs of type x at age t and $v(p(x,t))$ be the value of the player's problem at that particular belief. Denote $\alpha_t \in [0,1]$ the instantaneous probability that a player searches at age t. Her dynamic problem can be expressed recursively as follows:

$$v(p(x,t)) = \max_{\alpha(x,t)\in[0,1]} (r+\delta)u(p(x,t))dt + e^{-(r+\delta)dt}\mathbb{E}\Big(v\big(p(x,t+dt)\big) \;\Big|\; \alpha(x,t)\Big), \tag{1}$$

where the expectation is taken with respect to the future posterior beliefs $p(x,t+dt)$ given the choice of α_t. The choice of the activity only affects future information sets and therefore does not enter the flow payoff. Moreover, we focus on a stationary equilibrium and, since players are strategically small, their choice of the activity does not affect the variable η_s either. Her objective boils down to the maximization of the variance of her posterior beliefs. In Lemma A2, relegated to the Appendix, we show that the recursive equation above can be written as

$$v(p(x,t)) = \max_{\alpha(x,t)\in[0,1]} u\big(p(x,t)\big) + \frac{2}{r+\delta}p(x,t)^2\big(1-p(x,t)\big)^2 v''\big(p(x,t)\big)Q\big(\alpha(x,t)\big).$$

In the equation above, the term $Q(\alpha(x,t)) := \big((1-\alpha(x,t))\eta_w\big)^2 + \big(\alpha(x,t)x\eta_s\big)^2$ captures how the choice of $\alpha(x,t)$ affects beliefs' variance. In the next result, we establish that the choice of the activity does not depend on current beliefs $p(x,t)$, but just on x.

Lemma 1. *Given η_w and η_s, there exists a unique threshold type $x^\star = \frac{\eta_w}{\eta_s}$, such that all types above $x^\star$ search $(\alpha(x,t)=1)$ and all types below $x^\star$ work $(\alpha(x,t)=0)$.*

In a stationary environment, namely when η_s is time-independent, the result above implies that players never switch between activities during their lives. More specifically, there is a unique threshold-type $x^\star$ such that a player searches if and only if she is more connected than $x^\star$. In the following, we will refer to the map

$$\eta_s(x) = \frac{\eta_w}{x} \tag{2}$$

as the *individual rationality* condition (IR). Given any η_s, we call $x^\star$ individually rational if it satisfies Equation 2.

3.3. Information Exchanges and Communication Technology

When a searcher meets a type x, be it a worker or another searcher, a transfer of information occurs *from* x *to* the searcher. The informativeness of this meeting depends on two factors: how informed type x is and how efficiently she can communicate. Of course, a player can never transfer more information than she possesses. Let $\Gamma(x,t)$ denote the *stock of information* a player of type x has gathered up to time t. From Lemma 1, we know that players do not switch between activities. It is therefore particularly simple to model the stochastic process $\Gamma(x,t)$, which takes the form of a Brownian motion with an endogenous drift-to-variance ratio. We have that:

$$\Gamma(x,t) := \begin{cases} \eta_w t\theta + B(t) \sim \mathcal{N}(\eta_w t\theta, t) & \text{if } x \text{ works,} \\ \eta_s x t\theta + B(t) \sim \mathcal{N}(\eta_s x t\theta, t) & \text{if } x \text{ searches,} \end{cases} \tag{3}$$

where $B(t)$ is the standard Weiner process. Intuitively, when $\Gamma(x,t)$ is positive (resp. negative), the player has accumulated evidence in favor of hypothesis $\theta = 1$ (resp. $\theta = -1$). A straightforward application of Bayes' law shows that the stock of information $\Gamma(x,t)$ and the posterior belief $p(x,t)$ are in a one-to-one relationship (Lemma A3). Therefore, observing the current state of $\Gamma(x,t)$ is equivalent to observing player's x posterior belief $p(x,t)$. Both contain information about θ. Observing a player's posterior belief corresponds to a situation in which all information she has ever collected in her life is be instantaneously extracted. More realistically, imperfect communication technologies could introduce distortions in the communications among players. In such case, a noisy version of $\Gamma(x,t)$ will be observed by the searcher. In the following, we will assume that players who gathered their own information through work are (weakly) better suited at conveying this information to others. We implement this idea in the following way. First, we normalize $\Gamma(x,t)$ with $\pi(x,t) := t^{-\frac{1}{2}}\Gamma(x,t)$, which consequently becomes normally distributed with some mean – depending on time and activity – and unit variance.[6] Second, the communication between a *searcher* of type x and any other player happens through a communication channel with possibly finite capacity.[7] This imperfect technology accounts for potential distortions applied to the input signal $\pi(x,t)$. The output signal, denoted $\tilde{\pi}(x,t)$, is a garbling of the input one. Formally, we define a communication technology as follows:

[6] This normalization is without loss of generality: since both x and t are observable, and activities are persistent, the normalized signal induces the same belief updating, which is ultimately what matters for the equilibrium.

[7] Sims (2003) popularized the notion of a finite capacity channel in economics. See Steiner et al. (2016) and Jung et al. (2016) for recent applications.

	Input		Output
If x works	$\pi(x,t) \sim \mathcal{N}\Big(\sqrt{t}\theta\eta_w, 1\Big)$	$\rightarrow$	$\tilde{\pi}(x,t) \sim \mathcal{N}\Big(\sqrt{t}\theta\eta_w, 1\Big)$
If x searches	$\pi(x,t) \sim \mathcal{N}\Big(x\sqrt{t}\theta\eta_s, 1\Big)$	$\rightsquigarrow$	$\tilde{\pi}(x,t) \sim \mathcal{N}\Big(g(x\sqrt{t})\theta\eta_s, 1\Big)$

TABLE 1: Communication Frictions

Definition 1. *A **communication technology** is a map*

$$g \in \mathcal{G} := \{g \in C(\mathbb{R}_+) \mid \ g \textit{ non-decreasing and } g(y) \le y\}.$$

Let $(\mathcal{G}, \ge)$ be the partially ordered set of communication technologies.[8] *When $g' \ge g$, we write that g' is* more informative *than g.*

A communication technology $g \in \mathcal{G}$ satisfies two natural requirements: first, g is non-decreasing, i.e. players with more information are weakly more informative; second, $g(y) \le y$, i.e. no player can transfer more than what she gathered. Frictions in communication are implemented as illustrated in Table 1. When $g = \mathrm{id}_{\mathbb{R}_+}$,[9] there are no communication frictions and players can read each other's beliefs. For any other $g \in \mathcal{G}$, signals that are *intermediated* by searchers depreciates; they are garbled by g. This introduces an inherent difference between *first-hand* information, the one coming directly from the source, i.e. a worker, and *second-hand* information, the one coming from a searcher, someone who has herself learned indirectly from either a worker or another searcher. We postpone further discussions on the communication technology to Section 3.4.

Upon meeting a player of type x at age t, a searcher receives a signal distributed according to $\tilde{\pi}(x,t)$. Yet, meetings are random. Therefore, in order to assess the value of the search activity, a searcher needs to evaluate the information she can *expect* to receive – something that, in turn, depends on η_s. This creates a consistency problem: in equilibrium the value of η_s needs to be *consistent* with the beliefs of every other player in the society. More specifically:

$$\mathbb{E}(\pi_s|\eta_s) = \mathbb{E}\Big(\int_X \Big(\int_0^\infty \tilde{\pi}(x,t|\eta_s)\ \tau(t)\ dt\Big) h(x)dx\Big). \tag{4}$$

[8] A partially ordered set $(\mathcal{Y}, \trianglerighteq)$ is a set $\mathcal{Y}$ endowed with some partial order $\trianglerighteq$. We write $\max(\mathcal{Y}, \trianglerighteq)$ and $\min(\mathcal{Y}, \trianglerighteq)$ to identify the maximum and the minimum elements of such poset whenever exists.

[9] We denote $\mathrm{id}_{\mathbb{R}_+}$ the identity function on $\mathbb{R}_+$, that is $\mathrm{id}_{\mathbb{R}_+}(x) = x$ for all $x \ge 0$.

The condition in Equation 4 directly links the informativeness of the search activity η_s with the expected "amount" of information communicated in a random meeting. We refer to such condition as *Bayes consistency* and, in Section 4, it will play an important role in our equilibrium concept. This condition ensures that the information a player expects to receive from others is *consistent* with what others actually believe, under the assumption they are *Bayesian*. In a stationary environment, Equation 4 boils down to a fixed-point problem. To determine the behavior of a given player – which activity she will choose and, consequently, how much information she will gather – we need to pin down η_s. However, η_s itself depends on the amount of information that can be communicated by a type x at age t, which is given by $\tilde{\pi}(x, t)$, also depending on η_s.

3.4. Discussion of the Model

Before moving to the equilibrium definition and properties, it is useful to discuss our main assumptions, their motivation and robustness.

The Meeting Technology. A fundamentally new feature in our model is the introduction of heterogeneity in players' connections. The basic assumption is that more connected players meet others more easily. This assumption has an obvious implication: *ceteris paribus*, the more connected a player is, the more likely it is to meet her. In this framework, the meeting rate x only affects the *rate* at which meetings take place, not their nature. For this reason, $f \in \mathcal{F}$ does not represent a generic *social network*.[10] By reducing all heterogeneity in the society to heterogeneity in the type x, we abstract from other potentially interesting features that a generic network has, such as the heterogeneity in players' neighborhoods, homophily, etc. Yet, we capture an aspect of this heterogeneity that is of first order in real-world networks, namely degree heterogeneity: different people have different "levels of access" to the general society, because they have more or less connections. From this perspective, f is a convenient way to introduce a fundamental level of heterogeneity in the model without explicitly having to account for a full-scale network. This assumption makes our model particularly tractable and it also allows us to study *global* and general shifts in the distribution of connections f, rather than *local* and particular changes, such as the deletion of one particular node or the other.

Information Exchanges and Communication Frictions. The second key component in our analysis is the way in which information is exchanged from one player to another. We do explicitly model the *motives* that lead players to share their information with others. The problem of strategic information transmission has been amply studied in the literature and

[10] It is indeed a very particular type of network: an infinite, complete, weighted graph on X, with weights proportional to the *type* of a node.

goes beyond the scope of this paper.[11] Moreover, since in our model players have zero mass, the strategic consequences of peer-to-peer information transmission are immaterial. From an individual point of view, players are indifferent as their communication choices do not affect aggregate equilibrium variables. The novelty of our approach is to introduce frictions in the communication technology that transfers information from one player to the other. Frictions only apply to information that is accumulated via the *search* activity. The idea is that workers, having literally produced the information themselves, are better at conveying it to others. Searchers, instead, having received information indirectly – from workers or from other searchers – are not quite as effective as workers at reproducing it without altering its content. This introduces a wedge between *first-hand* information, stemming directly from workers, and *second-hand* information, signals that have been intermediated by searchers. This assumption makes the main tensions in our model particularly transparent and straightforward. However, it can be relaxed. In Section 8, we discuss an extension of our model in which the communication technology is frictional for all players, independent of their activity. In Appendix B, we show that, when $g \in \mathcal{G}$ is concave, many of our results extend to this more general case.[12]

Stationarity and Payoffs. Most of our analysis focuses on a stationary environment. In Section 8, we discuss the dynamic version of our model. In such case, the equilibrium variable η_s becomes a time-dependent equilibrium object. Consistently, players can switch from work to search when the society has learned enough and the search activity has become sufficiently attractive. In Appendix C, we show that dynamic equilibria, although difficult to characterize, have information paths $\eta_s(t)$ that necessarily converge to the stationary equilibrium that will be introduced in the next section. This dynamic stability makes the stationary equilibrium a natural benchmark to study, on top of being one that is very tractable from an analytical point of view. Moreover, we associate with the stationary environment the following interpretation; We think of players becoming aware of (or interested in) a given *issue* θ, about which they have some prior information p_0. At a random future time $\tau(t)$, a player takes an irreversible action, a guess of θ, that will determine her material payoff. We interpret the random time $\tau(t)$ at which the player has to cast her final guess as the time at which issue θ becomes subjectively obsolete for the player. A new issue θ' will become of interest for her and, therefore, she leaves the game. Since the arrival of such date is random, it is optimal for the player to continuously update her guess, as new information comes along. The flow (indirect) utility $u(p_t)$ represents the value of this guessing problem, that is

[11]See for example Crawford and Sobel (1982), Grossman (1981), Milgrom (1981), Jovanovic (1982) and, more recently, Kamenica and Gentzkow (2011).

[12]A concave $g \in \mathcal{G}$ captures a natural idea. Under a "finite capacity" communication channel, the higher the number of signals that needs to be transferred, the harder it is to transfer an additional one. See Section 8 for a more detailed discussion.

$u(p_t) := \max_{b_t \in \Theta} \mathbb{E}_{p_t}\left(\mathbb{1}(b_t = \theta)\right) = \max\{p_t, 1 - p_t\}$.[13] While we focus on stationary equilibria mostly for analytical tractability, there are many environments in which it is a natural solution concept. Consider a population in which each player sequentially faces different issues about which she has to form an opinion. When a player decides to learn about this issue from other players, she acquires information mostly from those that are also currently interested in that specific issue. This is natural in social networks that are issue-specific. The same is true even in a general-interest social network, such as Facebook or Twitter, as long as active players only post information about the issues they are currently interested in, while older posts become obsolete or harder to access. From this perspective, a stationary solution concept can be interpreted as applying to those environments in which the share of the population interested in a specific issue is evolving over time and yet its size is relatively stable.

4. Equilibrium

In this section, we define and establish the existence and uniqueness of stationary equilibria in this game. We show how to reduce the equilibrium to a relatively simple fixed-point map, thus condensing all the complexity introduced in the previous sections into one simple equation, from which most of our results will later be derived.

Definition 2. *A* **Stationary Equilibrium** *is a pair $(x^\star, \eta_s)$, composed by a threshold type $x^\star$ and a social information quality η_s, such that:*

(IR) Given η_s, $x^\star$ is individually rational. That is, type x searches if and only if $x \geq x^\star$.

(BC) Given $x^\star$, η_s is Bayes consistent. That is, η_s is a fixed-point of Equation 4.

In the definition above, the first requirement is that no player, at any point in her life, wants to deviate from the activity she is engaged in. For this to be true, type $x^\star$ has to be indifferent between work and search. All types with higher connectivity will search and, vice versa, all types with lower connectivity will work. The second requirement, instead, can be seen as a particular kind of *market clearing* for information. As in a production economy, players cannot "consume" more than the economy is producing. Similarly, Bayes consistency requires that information shall not be created from nowhere or erased with no

[13] The results we present in this paper do not hinge on the particular choice of $u(p_t)$. Any flow utility u generating a convex value function v will do. Convexity implies that players are information lovers. This particular choice of u is motivated by the fact that it provides us with a simple analytical solution for the second-order ODE that describes the player's problem.

reason. Information must be produced and diffused according to the "rules of the game" that we have outlined in the Section 3.

The fixed point implicit in Bayes consistency has a unique solution (Lemma A4), which is given by the following expression:

$$\eta_s = \eta_w \frac{cH(x)}{1 - \int_x \tilde{g}(z)h(z)dz}, \tag{5}$$

where $\tilde{g}(z) := \mathbb{E}_\tau\big(g(z\sqrt{t})\big)$ and $c := \mathbb{E}_\tau(\sqrt{t})$.[14] The value of $\tilde{g}(z)$ represents type z's life-long expected contribution to η_s, filtered through the communication technology g. Similarly, c is the expected contribution to η_s of a worker. To rule out explosive dynamics, we need to guarantee η_s is a positive real number. A sufficient condition is given by the following assumption.

Assumption 1. *We assume $f \in \mathcal{F}$ satisfies $\mathbb{E}_h(x)c \leq 1$.*

Assumption 1 imposes restrictions on how thick the right tail of f is and on how long players are expected to remain in the game.[15] If this condition fails to apply, the society is able to multiply information unboundedly, precluding the possibility for a stationary environment. We will maintain Assumption 1 throughout the paper. In equilibrium, according to Definition 2, both individual rationality and Bayes consistency (Equations 2 and 5 respectively) are satisfied simultaneously. This provides us with a single fixed-point equation that fully characterizes the equilibrium:

$$x = \frac{1}{c} + \int_x m(z)h(z)dz, \tag{6}$$

where we denoted $m(z) := \frac{1}{c}\big(cx - \tilde{g}(z)\big)$. The following result establishes existence and uniqueness of our stationary equilibrium. Figure 1 provides a graphical representation of the interactions between individual rationality and Bayes consistency.

Proposition 1. *Fix a society $f \in \mathcal{F}$ and a communication technology $g \in \mathcal{G}$. A stationary equilibrium exists and is unique.*

In equilibrium, we observe rich interactions among types, activities and information. From Figure 1, the equilibrium features two important thresholds in players' type. The first one, $x^\star$, we have already encountered. It marks the type above which it is *individually* optimal to search. The latter instead, $\hat{x}$, will play a crucial role in the next section. This threshold marks

[14] In the following, we use notation $\mathbb{E}_\tau(q(t)) := \int_{\mathbb{R}_+} q(t)\tau(t)dt$ and $\mathbb{E}_f(q'(x)) := \int_X q'(x)f(x)dx$. Moreover, we write $H(x) := \int_0^x h(z)dz$.

[15] Equivalently, by the definition of h, the requirement in Assumption 1 can be expressed as $c\mathbb{E}_f(x^2) \leq \mathbb{E}_f(x)$.

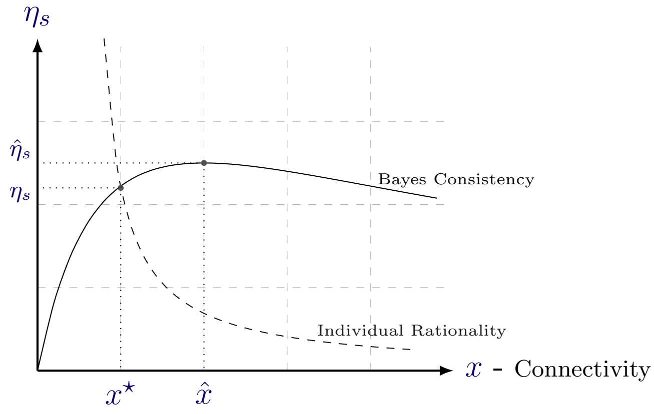

Figure 1: A graphical representation of the stationary equilibrium.

the type above which is *socially* optimal to search. Work and search are neither complements nor substitutes of each other. For example, when few players work, there is little information injected in the society and, therefore, little information to be searched for. The search activity becomes less attractive, especially for types whose connectivity is not particularly high. This creates incentives for these players to switch to work, therefore rebalancing the inflow of information. This tension stresses how work and search can be seen as substitutes. However, this tension does not apply uniformly across the population. Generically, the quality of social information η_s is non-monotone in the threshold type x (Figure 1). Past a critical value $\hat{x}$, at which social information quality is maximal, η_s starts declining as the working population becomes larger. This seemingly technical observation has the power to uncover the unique role that *some* searchers play in equilibrium. While searchers do not create "new" information, they nevertheless *(i)* aggregate information produced by others and *(ii)* enable it to be diffused to different parts of the society. Searchers transform what is inherently a *private good* – information produced by a worker for her own self – into a *public good*, information that everybody else can access more easily. Due to their higher connectivity, searchers meet (and are met by) others relatively more frequently. Therefore, they aggregate information better and, modulo the distortions introduced by the communication channel, make it more salient for others. For highly connected types, choosing the search activity could not only be individually optimal, but also beneficial from a social point of view. When nobody searches, information is scattered around the society in private "goods" that are for the sole use of those who have produced them. Searchers bring together these "goods" and make them more accessible to the rest of the society. By doing so, they can increase the value of search and, in principle, the welfare of the society. And yet, the now higher value of search can attract workers away from their activity, thus possibly reducing the value of search. The unique balance among these rich interactions is a feature of the equilibrium of this game, which we analyze in the next section.

5. Results

Our model is characterized by two primitives: the distribution of types $f \in \mathcal{F}$, describing how connected the society is, and the communication technology $g \in \mathcal{G}$, describing potential frictions in searcher-to-searcher communications. In this section, we study how g (Sections 5.1 and 5.2) and f (Section 5.3) affect the equilibrium of this game and how these two interact with each other.

5.1. Equilibrium and the Communication Technology

We start by uncovering the equilibrium role of the communication technology. In Definition 1, we have introduced a natural order on $\mathcal{G}$: g' is more informative than g if $g' \geq g$. Every meeting is, *ceteris paribus*, less informative (in a Blackwell sense) under g' than g. The partially ordered set $(\mathcal{G}, \geq)$ has two interesting extrema: $\max(\mathcal{G}, \geq) = \mathrm{id}_{\mathbb{R}_+}$ and $\min(\mathcal{G}, \geq) = 0$. When $g = \mathrm{id}_{\mathbb{R}_+}$, players can observe each other's posterior beliefs upon meeting. When $g = 0$, second-hand information is fully depreciated: searchers serve no social role and the strategic interactions reduce to the free-riding of the information created by workers.

Proposition 2. *Let $g, g' \in \mathcal{G}$ be such that g' is* more informative *than g. Then $x^\star(g') < x^\star(g)$ and $\eta_s(g') > \eta_s(g)$. That is, equilibrium information quality η_s increases even if the adjusted mass of workers $H(x^\star)$ shrinks.*

As the communication technology improves, we observe players shifting away from work towards search. This effect is intuitive. A better communication technology implies that, *ceteris paribus*, a player is able to extract more information from any given meeting. This implies that the indifferent type $x^\star(g)$ could strictly prefer to search under g'. This leads to a decrease in $x^\star$ and, therefore, in $H(x^\star)$, the mass of workers adjusted by their connectivity.[16] However, even though information production is reduced, the equilibrium information quality increases because information gets aggregated and diffused with higher efficiency under the new communication technology. It is no surprise that improvements in the communication technology are unambiguously beneficial for the society. In fact, not only η_s increases in equilibrium, but it is also the case that $\eta_s(x)$ increases at any $x \in X$, as depicted in Figure 2. This implies that, under the superior communication technology, any level of η_s can be achieved with a strictly smaller set of workers. In the next proposition, we will show what happens in the limit case when g is maximally informative, namely when players can

[16] The word *adjusted*, here, refers to the use of the cumulative distribution H instead of F. This controls for the connectivity of workers. For example, type $x = 0$, even if she is a worker, does not contribute to η_s as she is never met by anyone.

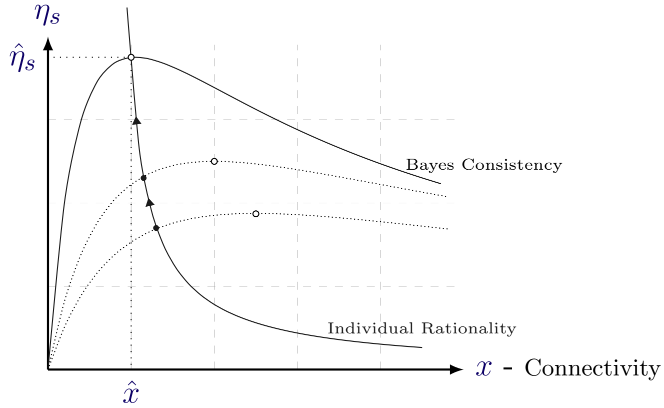

FIGURE 2: Equilibrium and Communication Technology

observe each other's posteriors. There, $x^\star = \hat{x}$ and therefore, individual and social objectives coincide. Under such g, players can effectively access each other's posterior beliefs with no friction and learn instantaneously all the information a player has ever collected in her life. This is a noiseless society in which no signal is ever lost.

Proposition 3 (Observing Posteriors Beliefs)**.** *The equilibrium threshold $x^\star$ maximizes the Bayes consistency map $\eta_s(x)$ if and only if g is maximally informative.*

Proposition 3 shows that, whenever frictions compromise players' ability to communicate, the society fails to allocate learning activities as to maximize the quality of social information η_s. Specifically, under any frictional g, there is a set $[x^\star, \hat{x})$ of players with intermediate connectivities who choose to search in equilibrium, without internalizing the fact that they would have better contributed to η_s had they worked. A frictional communication technology introduces a wedge between a player's own incentives and her social role in the determination of η_s. When searching, a player aggregates information at a rate proportional to her type x. However, her ability to diffuse information is depressed due to the friction imposed by the communication technology g. While players' individual incentives are entirely determined by the former, their social role is also linked to the latter. A discrepancy between the two creates a region characterized by players who are better than workers at aggregating information, but worse than them at its diffusion. Of course, this situation has implications for the efficiency of these equilibria and, more specifically, on how a social planner would redistribute players into activities, as we will see in Section 6. For the rest of this section, however, we focus on the effects that this region produces on equilibrium outcomes.

5.2. Searching and the Amplification of Noise

The previous section highlighted the crucial role of communication frictions in creating a region in the type-space X populated by searchers whose individual decisions are detrimental to η_s, the quality of social information. Now, we single out the implications of such decisions for the equilibrium. In particular, we show how these distortions amplify due to the activity of other searchers.

When the communication technology $g \in \mathcal{G}$ is frictional, Proposition 3 implies that $\eta_s(x)$ is strictly increasing for all $x \in [x^\star, \hat{x})$ (Figure 1). Rearranging the definition of η_s from Equation 5, we can write

$$\frac{d\eta_s(x)}{dx} > 0 \quad \Rightarrow \quad 1 - \int_x \tilde{g}(z)h(z)dz > \tilde{g}(x)H(x) \quad \Rightarrow \quad \tilde{g}(x)\eta_s < c\eta_w \quad \text{for all } x \in [x^\star, \hat{x}).$$

Fix any type $x \in X$. One can think of $c\eta_w$ (resp. $\tilde{g}(x)\eta_s$) as the extent to which type x is expected to contribute to η_s in her lifetime, if she works (resp. searches). Type x's contribution when she works is exogenous, it does not depend on her type and on frictions. It is only a function of τ, the age-distribution. On the contrary, type x's contribution when she searches is endogenous, it is type-dependent and affected by frictions. When $x \in [x^\star, \hat{x})$, type x's decision to search effectively reduces η_s. Equivalently, she introduces extra *noise* into the society. When someone meets such a player, the signal she receives is *noisier* than it should have been. However, the distortions for which type x is responsible go well beyond the fact that meeting such type is now less informative. The extra noise that type x, with her decision to search, seeds in the society is then collected by all searchers who meet her. Therefore, it spreads around the society and becomes part of everyone's information set. In this sense, a problem that was created locally, that is, in the region $[x^\star, \hat{x})$, becomes a *global* phenomenon affecting all searchers. This *noise-amplification mechanism* reduces the informativeness of *all* social meetings, not just those involving players from the compromised region.

Next, we formalize the idea of the amplification mechanism by computing a relative measure of "social information elasticity." To begin, fix a type $x \in X$ and an activity, w or s. We compute the elasticity of η_s with respect to a marginal increase in type x's diffusion abilities:

$$\varepsilon_w(x) := \frac{d\eta_s/dc_x}{\eta_s/c_x} \qquad \text{and} \qquad \varepsilon_s(x) := \frac{d\eta_s/d\tilde{g}(x)}{\eta_s/\tilde{g}(x)}.$$

These elasticities measure how much the equilibrium quality of social information η_s is affected by type x. We model this experiment by marginally increasing either c_x or $\tilde{g}(x)$, according to the chosen activity. This marginal increase affects type x specifically.[17] The

[17]We abused notation by writing c_x, instead of c, to stress the fact that we are increasing c uniquely for type x.

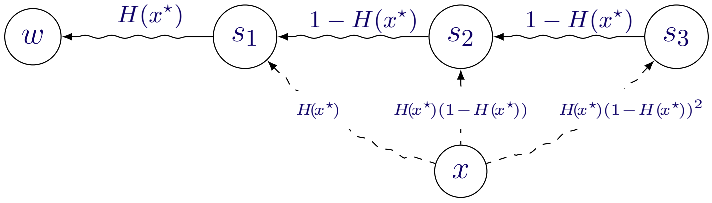

FIGURE 3: The amplification effect.

relative difference between these elasticities, namely $\varepsilon_s(x) - \varepsilon_w(x)$, is a measure of the total equilibrium impact that type x produces on η_s when she decides to deviate *from* work *to* search.

Proposition 4. *The work-to-search* relative elasticity *can be decomposed as:*

$$\varepsilon_s(x) - \varepsilon_w(x) = \underbrace{\Big(\tilde{g}(x)\eta_s - c\eta_w\Big)}_{\substack{\textit{social}\\ \textit{contribution}}} \underbrace{\frac{x}{H(x^\star)}}_{\substack{\textit{amplification}\\ \textit{effect}}} \kappa(x), \tag{7}$$

where $\kappa(x) = \frac{f(x)}{c\eta_w \mathbb{E}_f(z)}$ *only depends on primitives.*

Proposition 4 provides a decomposition of the negative externality that players in the compromised region $[x^\star, \hat{x})$ are exerting on η_s. For these players, we have just established that $\tilde{g}(x)\eta_s < c\eta_w$. The first term in Equation 7 represents the extra noise that their activity seeds in the system. This wedge, we said, is entirely due to frictions implied by g. Yet, these distortions are not meant to stay local. They are amplified by the rest of the searching population, as captured by the second term in Equation 7. This term has two factors:

Centrality. The amplification effect is increasing in x. The value of x measures how attractive, or "central," this player is. The higher x, the higher the rate at which other searchers will meet her, the stronger the social distortions that her individual decision creates.

Expected Path Length. The amplification effect is decreasing in $H(x^\star)$. In our model, all signals originate from some workers. These signals are then spread around the society by the activity of searchers. The term $\frac{1}{H(x^\star)}$ captures exactly the expected number of searchers each signal encounters before reaching a player $x \in X$. To see this, consider Figure 3. Player x, a searcher, meets another searcher called $s \in \{s_1, s_2, s_3\}$. Although x can observes s's type and age, she cannot reconstruct the path followed by the information she is about to relay. However, for each signal player s has collected, player x can speculate on the path it traveled. For example, if $s = s_1$, the signal that s is carrying comes directly from a worker w. Such path has length $k = 1$. This event has probability $H(x^\star)$, namely the probability player s meets a worker. If $s = s_2$, instead, the signal went through another searcher before reaching s_2. This path has length $k = 2$ and probability $H(x^\star)\big(1 - H(x^\star)\big)$. Player x can compute the probability of each k and, thus, the expected length of such paths:

$$\mathbb{E}\Big(\text{path of length } k \;\Big|\; x^\star\Big) = \sum_{k=1}^{\infty} kH(x^\star)\big(1 - H(x^\star)\big)^{k-1} = \frac{1}{H(x^\star)}$$

The smaller $H(x^\star)$, the higher the expected path length that each signal travels in this society, the higher the probability the signal ever went through the compromised region $[x^\star, \hat{x})$.[18]

The combination of the strength of attraction of type x and the amplification power of the society determine the overall impact of the negative externality that type x induces on the whole society.

5.3. The Pitfalls of Increasingly Connected Societies

We now turn the analysis to the effects of the social structure f on equilibrium outcomes. More precisely, we are interested in understanding how the equilibrium and, in particular, the ability to produce and diffuse information, is affected when the society becomes *more connected*. Our discussion in the previous sections shows how a frictional communication technology is bound to introduce a wedge between an agent's individual incentives and her social role. This introduces distortions that are spread and amplified throughout the society by the activity of searchers. Intuitively, this may suggest that increasing the connectivity of a society can have ambiguous effects on the quality of social information η_s. We begin this Section by making this intuition explicit. We show how different changes in the social structure f can systematically produce opposite effects on the quality of social information. This illustrates how the ambiguity mentioned above is an endemic feature of this model. In our Theorem 1, we put forward a positive resolution to this ambiguity. We show that, by organizing the changes in the social structure in a coherent and reasonable manner, the tension between negative and positive effects gains some order, providing a clean and general illustration of the equilibrium effects of increasing social connectivity.

To begin, let us introduce a partial order $\trianglerighteq$ on $\mathcal{F}$ that captures the idea of "more connected." Since f is a probability density function, it seems natural to use stochastic orderings to track the changes in the distribution f. The use of stochastic ordering is particularly convenient

[18] This is reminiscent of what, in the United States, is sometimes referred to as the *telephone game* effect. The telephone game consists of having a group of people arranged in a line with a message being whispered by one player to her immediate neighbor, until it reaches the last player, who then announces the message to the group. Errors typically accumulate and amplify in the re-tellings, so that the statement announced by the last player differs significantly from the one uttered by the first. A conceptually similar force is at play in our model.

in our model because it allows us to abstract from *local* changes to the social structure – like adding a connection between two particular players – and rather focus on *global* ones. We capture the idea of f' being more connected than f via first-order stochastic dominance.[19]

Definition 3. *Let $(\mathcal{F}, \trianglerighteq)$ be the poset of societies endowed with the first order stochastic order. We say that f' is* **more connected** *than f whenever $f' \trianglerighteq f$.*

Now consider any society $f \in \mathcal{F}$ with a *frictional* communication technology $g \in \mathcal{G}$. By Proposition 3, we know that $x^\star < \hat{x}$. Players whose type falls between $x^\star$ and $\hat{x}$ are contributing negatively to the equilibrium quality of information society. They find it individually optimal to be searcher, but due to frictions in the communication technology, they end up relaying less information than they would if they had worked. In the next Observation, we construct two examples of first-order stochastic shifts that, on the one hand, exacerbate the *influence* of these trouble-types and, on the other, alleviate it. Denote $\eta_s(f)$ the equilibrium quality of social information under a given society f.

Observation 1. *Fix a society $f \in \mathcal{F}$ and any* frictional *communication technology $g \in \mathcal{G}$. There exists two societies $f', f'' \in \mathcal{F}$ such that $f', f'' \trianglerighteq f$ such that $\eta_s(f') < \eta_s(f) < \eta_s(f'')$.*

Increasing connectivity can produce opposite effects on the quality of social information η_s, according to which types have increased their social influence. The way examples in Observation 1 are constructed is particularly instructive on the more general tensions that characterize the transitions from one society to a more connected one. To illustrate this clearly, fix f and let $(x^\star, \eta_s)$ be the respective stationary equilibrium. When the communication technology $g \in \mathcal{G}$ is frictional, we know from the previous section that $\eta_w c > \eta_s \tilde{g}(x)$ for all $x \in [x^\star, \hat{x})$ and $\eta_w c < \eta_s \tilde{g}(x)$ for all $x \in (\hat{x}, \infty)$. It is useful to consider the following decomposition of η_s (see Equation A.2):

$$\eta_s = \eta_w c + \int_{x^\star}^{\hat{x}} \underbrace{\big(\eta_s \tilde{g}(z) - \eta_w c\big)}_{\text{negative}} h(z)dz + \int_{\hat{x}}^{\infty} \underbrace{\big(\eta_s \tilde{g}(z) - \eta_w c\big)}_{\text{positive}} h(z)dz$$

From a social perspective, players in $x \in [x^\star, \hat{x})$ create a negative externality on the equilibrium allocation. Searchers in this region could have contributed to η_s more effectively if they had worked. The effects on η_s of increasing connectivity crucially depend on which types see their relative "weight" increased, whether it is $[x^\star, \hat{x})$ or $(\hat{x}, \infty)$. This provides intuition on how the examples in Observation 1 can be constructed. If f increases in a first-order stochastic sense so does h. If the implied change is such that under the new conditional density h', mass has been shifted from $[0, x^\star]$ to $[x^\star, \hat{x})$, the equilibrium adjustment of η_s will

[19] Recall that f' first-order stochastically dominates f if $F'(x) \leq F(x)$, where F' and F are the respective cumulative functions.

be negative. Vice versa, if mass has been shifted from $[0, x^\star]$ to $[\hat{x}, \infty)$, bypassing the region $[x^\star, \hat{x})$, the equilibrium adjustment of η_s will be positive.[20]

This discussion highlights how the potential negative effects associated with increased connectivity are closely related to the frictions in the communication channel. As shown in Proposition 3, both $x^\star$ and $\hat{x}$ are changing as the communication channel becomes more efficient and, in particular, one converges to the other, therefore making the region $[x^\star, \hat{x})$ increasingly small. We summarize the discussion above in the next result.

Proposition 5. *Fix a society $f \in \mathcal{F}$ and a communication technology $g \in \mathcal{G}$. The following are equivalent:*

(*i*) $\eta_s(f') > \eta_s(f)$ for all $f' \in \mathcal{F}$ *such that* $f' \trianglerighteq f$.

(*ii*) $g \in \mathcal{G}$ *is maximally informative.*

Paraphrasing, the quality of social information unambiguously improves irrespectively of the shift if and only if players can observe each other's posteriors. In all other cases, that is when frictions are present, there always exists a more connected society in which η_s has decreased. Also note that a decrease in η_s does not directly imply a decline in social welfare. Social welfare is determined by the interaction between the new quality of social information $\eta_s(f')$ *and* the new distribution f'. When $\eta_s(f')$ decreases, the present discounted expected utility of any given type x decreases too. Nonetheless, since it is always the case that higher types are better off, it is possible that the changes in f' overturn the negative welfare effect. It is important to emphasize that an increase in welfare is *never* guaranteed whenever there are frictions in the communication technology. Particularly, it is always possible to construct examples where a decrease in η_s is accompanied by a decline in social welfare.

The result of Proposition 5 hinges on manipulating the distribution of connectivities in specific ways, while respecting the stochastic order $\trianglerighteq$. The ambiguous effects on η_s are induced by the fact that the set of first-order stochastic shifts are too general and poorly structured. Realistically, connectivity increases in ways that are more regular. Moreover, if f belongs to a known parametric family of distributions, it is unclear whether one can replicate the manipulations of Observation 1. For these reasons, we now introduce more structure to our problem. Specifically, we define a natural class of stochastic shifts that has the merit of introducing order in the way the negative and positive components highlighted in Observation 1 affect social information. A natural requirement to impose, for example, is the following: if a type x becomes less prevalent under f' than f, the same should hold true

[20]Formally, this shows that $\eta_s(f', x^\star(f)) < \eta_s(f, x^\star(f)) < \eta_s(f'', x^\star(f))$. However, since the equation that defines individual rationality is strictly decreasing in x, this will imply that $\eta_s(f', x^\star(f')) < \eta_s(f, x^\star(f)) < \eta_s(f'', x^\star(f''))$, as we wished to show.

for all other less connected types $z < x$. This regularity requirement is not guaranteed under $\trianglerighteq$. Since the structure of connectivity in the society influences the equilibrium only through the meeting technology h, we directly put structure on h.[21] In the following definition, let $N := \{1, 2, \ldots, \bar{n}\}$ be an index set with $\bar{n} \leq \infty$.

Definition 4. *A sequence $(h^n)_{n\in N} \subset \mathcal{H}$ is **increasing** if, for all $n < \bar{n}$, $h^n_\Delta := h^{n+1} - h^n$ has the single crossing property.[22] The sequence is **regular** if, whenever positive, $h^{n+1}_\Delta / h^n_\Delta$ is non-decreasing in z.*

A sequence is *increasing* if concentration of connectivity moves from low types to high types in a monotone way. For example, if a player with connectivity x becomes more prevalent in the new society h^n relative to the old one h^{n-1}, all types x' with connectivity higher than x become weakly more prevalent as well. Analogously, if type x is less prevalent, it must be that all lower types are less prevalent as well.

Regularity of a sequence, instead, imposes a condition on how h increases along the sequence. The condition closely resembles the Monotone Likelihood Ratio, but applies to the transformations of h, not to h directly.[23] One natural implication of this property is that, for $n'' > n' > n$, letting $\bar{z}$ and $\underline{z}$ being respectively defined as $h^{n''}(\bar{z}) = h^{n'}(\bar{z})$ and $h^{n'}(\underline{z}) = h^n(\underline{z})$, we have $\bar{z} \geq \underline{z}$, something that is not guaranteed by having an increasing sequence alone. When this is not the case, i.e. when $\bar{z} < \underline{z}$, all types $z \in [\bar{z}, \underline{z}]$ would be less prevalent in $h_{n'}$ relative to h_n, but more prevalent in $h_{n''}$ relative to $h_{n'}$. Definition 4 rules out these anomalies by imposing a form of regularity along the sequence. An alternative way to interpret the condition is to focus on a specific type x and see how $h^n(x)$ changes along the sequence with n. Regularity guarantees that the sequence can be divided into at most two parts, the first part where $h^n(x)$ is increasing and then the subsequent part where it is decreasing.

Theorem 1. *Fix a regular sequence $(h^n)_{n\in N} \subset \mathcal{H}$ of increasingly connected societies. Let $(\eta^n_s)_{n\in N}$ be the corresponding sequence of equilibrium social information qualities. The sequence $(\eta^n_s)_{n\in N}$ is* quasi-convex *in n.*

Formally, $\eta_s(h)$ is quasi-convex along any increasing and regular sequence. That is, the equilibrium evolution of the quality of social information η_s has two distinct phases. In the

[21] One can always consider the meeting technology h, instead of the distribution of connectivity f, as the primitive of our model. Starting with f is more natural for introducing the model.

[22] A function $q : X \to \mathbb{R}$ has the single crossing property if $q(x) \geq (>)\ 0$ implies $q(x') \geq (>)\ 0$ for all $x' > x$.

[23] This condition is a version of the Monotone Signed Ratio property introduced by Quah and Strulovici (2012). It is a regularity condition necessary for successfully aggregating the single crossing property.

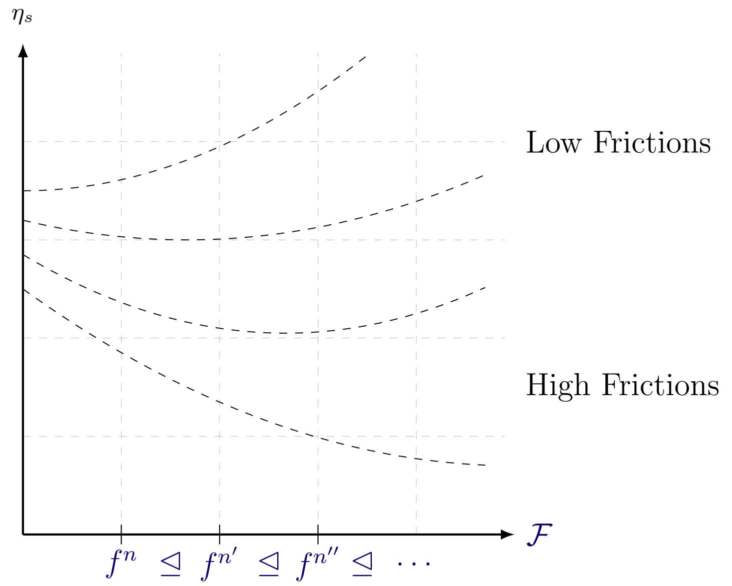

FIGURE 4: Social information quality as the society becomes increasingly connected.

first one, η_s *decreases* and the quality of social information deteriorates. In this phase, the increase in connectivity comes at the cost of amplifying the negative social role that the new searchers are exerting. Due to their increased connectivity, these players are more attracted to search. Yet, they don't internalize the social cost that their choice imposes on the rest of the society. In the second phase, η_s starts increasing. As the society becomes more connected, so do highly connected types. At some point the increased ability of diffusing information overcomes the negative impact of the additional noise that the marginal searchers are introducing. What determines the relative importance of these two phases is, once again, the communication technology g and, in particular, how severe communication frictions are. To pair this result with Proposition 5, when g is maximally efficient, the decreasing phase disappears along any increasing and regular sequence.

Figure 4 illustrates the result graphically. Theorem 1 sheds light on the relationship between how connected a society is and the quality of the information that it is able to produce and diffuse in equilibrium. Our result arises from the clash between two opposing forces. Increased connectivity improves the speed at which information is aggregated and diffused in a society. This contributes positively to the overall information quality. This is reflected in the fact that searchers, conditional on meeting another searcher, always prefer to meet a more connected type rather than a less connected one. However, this comparative static is only true *conditional* on meeting another searcher. When there are communication frictions, a searcher could be better off meeting a worker than a poorly connected searcher: although the latter is more connected than the worker, she is not connected enough to overcome such frictions. In this case, although the searcher is more informed than the worker, she is not quite as efficient at transferring her information. Moreover, increased connectivity also provides incentives for some players to quit their working activity, and to switch to search. This effect decreases the size of the working population, and thus the amount of "original" information that is injected into the system. The now larger searching population propagates noise at a higher magnitude, because the average path length connecting the signal fetched by a worker to its "final" user is now longer and thus signals are garbled more often.

We conclude this section with the analysis of the two extreme cases of no frictions and maximal frictions. In the first case, our model captures interactions in which players can observe each other's posteriors. In the second case, our model converges to a pure exploitation problem. Searchers benefit from their higher connectivity to learn from workers more effectively. However, they serve no social role as the information they collect cannot be re-used by anyone else. Consistently, in these two extreme cases, we obtain two opposite and monotonic results.

Corollary 1. *Fix a regular sequence $(h^n)_{n \in N} \subset \mathcal{H}$ of increasingly connected societies. The equilibrium quality of social information η_s is monotonically increasing if $g = \max(\mathcal{G}, \geq)$ and monotonically decreasing if $g = \min(\mathcal{G}, \geq)$.*

6. Normative Solutions

In this section, we analyze the efficiency of the equilibrium characterized in Section 5. In particular, we ask how *should* activities be allocated to maximize social welfare. We consider two distinct definitions of efficiency and conclude that, irrespective of the definition, equilibrium allocations are inefficient for almost all $g \in \mathcal{G}$. More interestingly, due to the richness of our type-space, the equilibrium allocation *vs* the optimal one may not just diverge at a quantitative level, but also at qualitative one. The first planner we consider allocates players into activities as a function of their type and maximizes the *ex ante* welfare of a generation of newborns. We show that this planner can find it optimal to entirely reverse the order of the society, by allocating lower types to search and higher types to work (non-monotone allocations). The second planner, instead, maximizes the *ad interim* welfare of the society by optimally allocating players into activities as a function of both their type and the information they have accumulated up to a given point in time. We show that in the optimal allocation a non-empty group of players is constantly swapped between activities, depending on the amount of information they have acquired.

6.1. The Optimal Time-Independent Allocation of Labor

Let's suppose the planner allocates players into activities at the beginning of their lives to maximize the present discounted value of a generation of newborn players. Formally, the planner chooses an allocation function $\alpha \in \mathcal{A} := \{\alpha : X \to [0,1]\}$. The planner is not bound to respect individual rationality, but it still needs to abide to Bayes consistency. In fact, the planner cannot affect how meetings take place, how information is collected, exchanged, and possibly garbled due to communication frictions. However, in this case, Bayes consistency can take a more general form:

$$\eta_s(\alpha) := \eta_w \, \frac{c \int_X (1-\alpha(x))h(x)dz}{1 - \int_X \alpha(x)\tilde{g}(x)h(x)dz}. \tag{8}$$

The consistency condition above differs from Equation 5 as the social planner is not bound to respect individual rationality and therefore she can choose allocations that are no longer characterized by a unique threshold-type.[24] The planner's problem can be expressed in the following way.

$$\begin{aligned} W^{\mathrm{SP}} \quad = \quad & \max_{\alpha \in \mathcal{A}} \quad \int_X \Big(\big(1 - \alpha(x)\big) v_w(\eta_w) + \alpha(x) v_s(x, \eta_s) \Big) \, f(x) dz dt, \\ & \text{sub to} \quad \eta_s = \eta_s(\alpha) \text{ as in Equation 8.} \end{aligned} \tag{9}$$

In Lemma A8, we show that the planner's trade-off relative to the allocation of type x can be described as follows:

$$\underbrace{v_s(x,\eta_s) - v_w(\eta_w)}_{\text{net individual gain}} \quad \geq \quad \underbrace{x\big(\eta_w c - \eta_s \tilde{g}(x)\big)K}_{\text{net social loss}}, \tag{10}$$

where K is a positive constant independent of x. The left-hand side in the inequality above represents the *individual* gain of allocating type x to search rather than work. The right-hand side, instead, represents the marginal *social* loss resulting from a similar decision. If type x searches, throughout her life she will contribute to η_s at rate $x\eta_s\tilde{g}(x)K$. The term x captures the fact that, the higher the type x, the more frequently she is met by others, and therefore, the bigger her contribution to η_s is, irrespective of her activity. Term $\tilde{g}(x)$, we have already encountered in Section 5: it captures the life-long expected contribution to η_s – filtered through the communication technology g – of a type x who searches. A similar intuition can be given to $x\eta_w cK$ when type x works.[25] We will say that the equilibrium allocation, which is fully characterized by the threshold $x^\star$, is *efficient* if it cannot be improved upon by any $\alpha \in \mathcal{A}$ in terms of *ex ante* welfare.

[24] Specifically, Equation 8 reduces to Equation 5 if there exists some threshold type $x^\star \in X$ such that $\alpha(x) \in \{0,1\}$, with $\alpha(x) = 1$ if and only if $x \geq x^\star$.

[25] Recall that $c := \mathbb{E}_\tau(\sqrt{t})$.

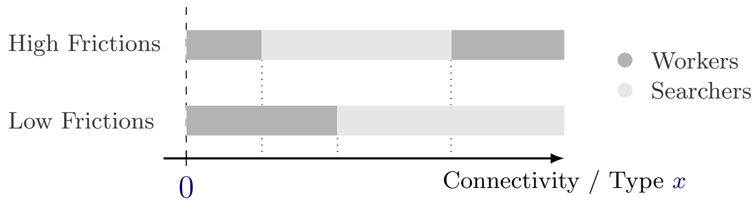

FIGURE 5: Planner's allocation for different $g \in \mathcal{G}$

Proposition 6 (*ex ante* efficiency)**.** *The equilibrium allocation is* (ex ante) *socially efficient* if and only if *the communication technology is maximally informative, i.e.* $g = \max(\mathcal{G}, \geq)$.

This means that *generically* – i.e. for almost all $g \in \mathcal{G}$ and, in particular, whenever there are frictions – the equilibrium allocation is inefficient. The result in Proposition 6 demonstrates that, whenever there are frictions in the communication technology, the equilibrium inefficiently supplies the public good produced by this economy, namely information. This result is in line with the literature on the competitive provision of public goods. From this literature we know that, when free-riding is at play, as it is in our case, the equilibrium usually underprovides the public good.[26] And yet, our model produces outcomes that go beyond the simple quantitative deviation from the social optimum. When the communication frictions are particularly severe, the equilibrium allocation is inefficient also from a qualitative point of view: the social planner finds it optimal to actually reverse the equilibrium allocation. For example, the planner can allocate intermediate types to search and higher types to work, in stark contrast with the equilibrium allocation (see Figure 5).

Corollary 2. *The optimal allocation can fail to have a monotone threshold type structure.*

The intuition is the following. Consider a highly connected type x such that $c\eta_w > \tilde{g}(x)\eta_s$. This situation is bound to happen when g is concave. In such a case, this player is contributing less than she could to the total amount of information in the society. However, since the player is highly connected, she will be met extremely often by others in the society. This effect is captured by the multiplicative term x appearing on the right hand side of Equation 10. Therefore, the negative social contribution of player x is amplified by the fact that others meets her very frequently (x is high). For this reason, the marginal social loss she generates by searching can offset her individual gain and the planner would rather have her work.

6.2. Optimal Time-Dependent Allocation of Labor

In this section, we consider a more demanding definition of a planner. Not only can she allocate people into activities based on their type, but she can also condition based on which information they have at any given point in their lives. Formally, the planner selects an allocation function $\alpha \in \mathcal{A}' := \{\alpha : X \times \mathbb{R} \to [0,1]\}$, a map that is adapted to the filtration it induces. The number $\alpha(x,t)$ denotes the instantaneous probability that type x will be assigned to search when she is of age t. By the choice of α, the planner decides what players do as a function of their type and conditional on every possible history they could experience. The planner maximizes total welfare subject to Bayes consistency:

$$\eta_s(\alpha) := \eta_w \frac{\int_{X\times\mathbb{R}_+} \big(1-\beta(z,t)\big)\sqrt{t}h(z)\tau(t)dzdt}{1-\int_{X\times\mathbb{R}_+} g(\beta(z)z\sqrt{t})h(z)\tau(t)dzdt}. \tag{11}$$

where $\beta(t) := \int_0^t \alpha(t)dt = \frac{1}{t}\int_0^t \beta(t)dt$ directly derives from α. The value of $\beta(t) \in [0,1]$ captures the proportion of time player x spent on the search activity until age t. The planner's problem can be expressed in the following way:

$$\begin{aligned} W^{\mathrm{SP}} \;=\; & \max_{\alpha\in\mathcal{A}'} \quad \mathbb{E}\Big(\int_X\int_{\mathbb{R}_+} u(p(x,t))\;\tau(t)dt\; f(z)dz \;\Big|\; \alpha\Big), \\ & \text{sub to} \quad \eta_s = \eta_s(\alpha) \text{ as in Equation 11.} \end{aligned} \tag{12}$$

To begin, notice that the welfare levels that the planner can achieve under $\mathcal{A}$, the feasible allocations from the previous section, are also achievable under $\mathcal{A}'$. Indeed, these allocations depend, not only on type, but also on time. This suggests that the *only if* part of Proposition 6 extends immediately to this case as well. In the next result, we show that this is also the case for the *if part.*

Corollary 3 (*ad interim* efficiency)**.** *The equilibrium allocation is* (ad interim) *socially efficient* if and only if *the communication technology is maximally informative, i.e.* $g = \max(\mathcal{G}, \geq)$.

This result stresses once again how special the extreme case of $g = \max(\mathcal{G}, \geq)$ is. Similar to the previous section, when there are communication frictions the equilibrium allocation can be qualitatively different from the socially optimal one. The planner would like to modify the equilibrium allocation in two different dimensions: types and time.

Corollary 4 (Characterization)**.** *There exist thresholds* $0 < x_1 \leq x_2 \leq \infty$ *such that, in the planner's solution, for* $x \in X$,

[26] This is also the case in the strategic experimentation literature.

- *If $x \leq x_1$ (resp. $x \geq x_2$), the player is allocated to work (search), irrespective of her beliefs.*
- *If $x_1 < x < x_2$, the player is switched between work and search as a function her posterior beliefs.*

The planner trades off two principal forces. On the one hand, she wants to allocate players to those activities that maximize their current individual gains. To do so, she chooses the activity capable of inducing the highest posterior variance. However, the variance itself depends on $p(x,t)$. When $p(x,t)$ is very close to 1 (or, equivalently, 0) the gains from learning a bit more are smaller and the differential across activities becomes negligible. Vice versa when $p(x,t)$ is very close to $\frac{1}{2}$, gains from learning are very high. On the other hand, the planner internalizes the net effect that every player induces on η_s and wants to maximize social welfare. There exists a group of types for which it is individually optimal to search but contribute to η_s negatively in relative terms. The planner finds it optimal to switch these types back and forth between work and search as a function of how informed they are. The idea is that, when a player is poorly informed, her own individual gains from learning are high. If, instead, she is very informed, her own individual gains are negligible. In the first case, this player would be allocated to search and she would be switched to work when she built a sufficiently high stock of information.

7. Social Influence and Public Opinion

In this section, we use our model to assess how resilient a society is against external manipulations of the information arriving to a small share of players. In particular, we study how resilience depends on the structure of social connections, especially as the society becomes more connected. Ultimately, we are interested in understanding how difficult it is to influence public opinion by manipulating the information of a relatively small group of players, which group is optimal to target, and how this depends on the level of connectivity of such a society. We think of manipulations as "tweaks" in the meeting technology h. These tweaks are such that the targeted group of players is consistently exposed to biased information. These players are more likely to meet peers who have collected information that is more favorable to one or the other hypothesis. We will assume that these manipulations happen under a regime of unawareness. That is, the society commonly believes that h is unbiased, while it is not. We believe this assumption is particularly realistic when the society is large and its structure complex, as it is in our case. In such case, players can find it hard, if not impossible, to detect these small tweaks in their meeting algorithm. The exercise we do in this section is a normative one: the manipulator is a malevolent planner, constrained by individual rationality, but not by Bayes consistency – the opposite of Section 6.

Let $\delta \in (0, 1)$ be a parameter of the problem. It sets the mass of players that will be directly affected by the manipulation. The planner selects a *target* type $\bar{x} \in X$ and all types in the δ-neighborhood of target $\bar{x}$ will have their meeting technology affected in the following way:[27] when searching for a dt interval of time, these players receive an aggregate signal, $\pi_s(x)$, distributed normally with mean $(\eta_s + b)x\theta dt$ and variance dt. The parameter $b \in \mathbb{R}$ represents the planner's *bias*. Without loss of generality, we will assume $b = 1$. The planner filters the players that type x meets so as to bias the information she receives. Our model offers a natural way to evaluate the effects of these manipulations. In Section 3.3, we showed that the evolution of posterior beliefs for players who learn from others can be described in terms of the following Brownian motion:

$$\Gamma(x, t) = \eta_s xt\theta + B(t) \sim \mathcal{N}(\eta_s xt\theta, t)$$

This implies that the values of η_w and η_s, which are respectively associated with work and search, together with the distribution of types f, pin down the entire social distribution of posterior beliefs. Therefore, we can capture the effect of the planner's manipulation on public opinion just by tracking its impact on η_s. Then, we combine changes in η_s with their effects on the evolution of beliefs for the society, as by the equation above. As a simple example of this, we already know that η_s does not affect beliefs of the working population. Therefore, the larger the working population, the less disruptive these manipulations of h. Let $\tilde{\eta}_s(\bar{x})$ be the altered version of η_s, following from the manipulation described above. Denote $\tilde{\Gamma}(x, t) := \tilde{\eta}_s xt\theta + B(t)$ the corresponding altered information process for a type x who searches. Therefore, the impact on public opinions that follows from a manipulation of the meeting algorithm for a δ-neighbor of $\bar{x} \in X$ can be defined as the expected aggregate deviation between the manipulated $\tilde{\Gamma}(x, t)$ and the original $\Gamma(x, t)$:

$$I(\bar{x}, \delta) := \mathbb{E}\Big(\int_{X \times \mathbb{R}_+} \big(\tilde{\Gamma}(x, t) - \Gamma(x, t)\big) f(z)\tau(t) dz dt \,\Big|\, \theta \Big)$$

This discrepancy can be interpreted as the average distortion in beliefs that the manipulation has induced. Clearly, $I(\bar{x}, \delta)$ depends on how types in the δ-neighbor of player $\bar{x}$ can affect the opinion of other players. Therefore, it is natural to think of $I(\bar{x}, \delta)$ as a measure of their influence on the rest of the society.

Definition 5. *The* **social influence** *of type* $x \in X$ *is defined as* $\iota(x) := \lim_{\delta \to 0} \frac{1}{\delta} I(x, \delta)$.

The study of public opinion manipulation led us to a definition of social influence. This is a natural outcome. How susceptible a society is to external manipulation is a function of the relative influence of its members. The stronger the influence a single player exerts on the society, the easier its manipulation. The next result characterizes our measure of influence.

[27] The δ-neighbor of $\bar{x}$ is defined as the only interval of X that solves $F([\bar{x} - \epsilon, \bar{x} + \epsilon]) = \delta$.

Proposition 7. *Fix $f \in \mathcal{F}$ and $g \in \mathcal{G}$ and let $(x^\star, \eta_s)$ be stationary equilibrium. The social influence exerted by a player of type x is*

$$\iota(x) = \begin{cases} \dfrac{x\tilde{g}(x)\eta_s}{c\eta_w} \dfrac{1 - H(x^\star)}{H(x^\star)} & \text{if } x \geq x^\star \\ 0 & \text{else.} \end{cases}$$

Proposition 7 shows how the different components of our model jointly contribute to $\iota(x)$. The equilibrium, the social structure f, the communication frictions g, a player's type; all these ingredients affect $\iota(x)$ in different ways. To begin, notice that manipulating the meeting technology of a worker produces no consequences whatsoever. Workers do not learn from others, and therefore make no use of their meeting technology. Searchers, on the contrary, can be used by the planner as drivers of her manipulation. Indeed, the information they gather is affected by h, via η_s. The way h affects η_s is spelled out in the Proposition above. First, the higher a searcher's type is, the stronger her influence. This is because, by attracting others more strongly, her biased beliefs leak through the society in a more pervasive way. Second, the higher the searcher 's type, the higher $\frac{\tilde{g}(x)}{c}$. This searcher collects information at a faster rate, relative to workers. In a sense, she makes use of the tweaked meeting function h more than others do. Finally, and perhaps most importantly, the influence of a player depends on the equilibrium. In particular, $\iota(x)$ depends on two important factors:

The Relative Speed of Learning. The ratio $\frac{\eta_s}{\eta_w}$ represents the relative speed at which a searcher learns relative to a worker. The higher η_s, the faster her opinion spreads out, the stronger her influence on the society.

Amplification effect. As discussed in Section 5.2, the biased beliefs of the searchers are then spread out in the society by other searchers, even those outside the δ-neighborhood of $\bar{x}$. The strength of this effect depends on the relative size of the searcher population, adjusted by its connectivity. In particular, when $x^\star$ decreases, the expected length of a chain increases and so does the influence of player x. This intuition is clear. When $x^\star$ decreases, more players are searching. Moreover, the inflow of information produced by workers is smaller. Therefore, the feedbacks among the searcher population are stronger. In this context, manipulating the meetings of a player has amplified effects on the whole society. The biased information rebounds among searchers and become part of everyone's information set. An important implication of this analysis is that any change in g or f leading to an increase in η_s makes the society less resilient to external manipulations. This observation is captured in the next result.

Corollary 5. *Let $f, f' \in \mathcal{F}$ be such that $f' \trianglerighteq f$. The increase in connectivity either decreases the quality of social information η_s or makes the society less resilient to external manipulation.*

We observe that societies that are highly effective at aggregating and diffusing information also happen to be particularly susceptible to manipulations. These societies are less reliant on work precisely because they are efficient at the diffusion of information. However, this situation creates a weak spot: as the influence of each type increases in this society, it becomes easier to shift public opinion by manipulating the learning process for an increasingly small share of players in the population.

8. Discussion

In this section, we discuss some of our assumptions, their generalization and extensions in more depth.

8.1. Communication Technologies with Finite Capacity

In our model, the communication technology $g \in \mathcal{G}$ imposes frictions in searcher-to-searcher communications. These frictions do not apply to worker-to-searcher communications. This assumption captures the idea that *intermediated* signals lose part of their initial value. Searchers don't fetch the information directly from a primitive source. Rather, they receive signals from others, possibly searchers. For this reason, they are not as effective as workers are at conveying the information they collected. This particular assumption makes the equilibrium tensions particularly stark and transparent, but it is by no means essential. Most of our results extend to the more general case in which the communication frictions apply to everyone, independent of their activity (Appendix B). This requires an extra assumption on g, *concavity*, capturing the idea of a *finite capacity* communication channel: the more *bits* of information one tries to transfer, the more laborious the communication will be. This idea goes back to Shannon (1948) and became a standard tool in information theory. In recent years, Sims (2003), Steiner et al. (2016) and Jung et al. (2016) have used and discussed variations of this idea in the literature on rational inattention.

A finite capacity communication technology introduces distortions similar to those discussed so far. Players who search are more connected in equilibrium. Therefore, they collect signals of precision η_s at an intensity which is higher than it is for workers. Concavity of g imposes a wedge between the communication abilities of a player who owns fewer signals of higher precision η_w (a *worker*) relative to one who owns more signals of lower precision η_s (a *searcher*). This wedge is not internalized by players when making their individual choices. This establishes a primacy for original sources, a gap between first and second-hand information that is very similar to the one discussed in Section 3. More concretely, in the model presented so far, $c := \mathbb{E}_\tau(\sqrt{t})$ represents a measure of the expected number of signals a worker collects in her life. When frictions apply uniformly to all players, c becomes $\mathbb{E}_\tau(g(\sqrt{t})) =: \tilde{g}(1)$. The trade-off discussed in Equation 7, which is the core inefficiency in our model, becomes $\tilde{g}(x)\eta_s - \tilde{g}(1)\eta_w$.

8.2. Stationary and Non-Stationary Equilibria

A working assumption used in most of the paper is the stationarity of the dynamic environment. In our game, players enter and leave at stochastic times. The outflow of "knowledge" associated with players' departure, together with the inflow of "ignorance" associated with their arrival, both guarantee that the amount of information in the economy is steady across time. Focusing on the unique stationary equilibrium makes our model tractable and it allows us to study the comparative statics as the society becomes more connected. In Appendix C, we study dynamic equilibria to understand the stability of the unique stationary equilibrium. There is an initial time $t = 0$ at which a unit mass of players is born. As time unfolds, new players enter the economy, while older players may leave. Our analysis leads to three conclusions. First, a dynamic equilibrium can be reduced to a non-linear second-order differential equation. The solution of such ODE is an "information path" for $\eta_s(t)$, now a *time-dependent* equilibrium object.[28] Second, we show that for each dynamic equilibrium, the information path converges to the stationary equilibrium defined in Section 4, thereby establishing its stability. Third, we show that, when $\eta_s(t)$ increases, players transit *from* work *to* search, starting with the more connected ones. These dynamics are intuitive. At time zero, all players share the same (prior) beliefs. As they cannot learn anything from each other, they work. This builds a stock of information, which makes $\eta_s(t)$ increase. Highly connected types will then find it optimal to switch to search, and this effect will unravel to lower types as $\eta_s(t)$ increases.

9. Conclusion

In this paper, we introduced a dynamic learning model for large connected societies. The novelty in our approach is to capture *both* the creation of information, arising from players' decision to work, and its diffusion, arising from their decision to search. These two aspects of social learning are tightly related: when a society is effective at diffusing information, this reduces individual incentives to create new information. However, when those who

[28] The existence and uniqueness of such equilibria can be investigated by invoking Picard-Lindelöf theorem and transforming the equilibrium condition in a system of first-order ODEs, whenever possible.

create information are few and peripheral, this hinders the diffusion of information. The existing literature on social learning captures the components of these trade-offs, but not their joint force. For example, in the strategic experimentation literature, the problem of information diffusion is absent, as all "social" information, whether in payoffs or in the actions, is usually assumed to be public. The social networks literature explicitly tackles the problem of information aggregation and diffusion. However, information is not created endogenously, but rather by players endowed with a set of exogenous signals.

This paper combines ideas and modeling tools from these literatures. Players face a basic tension between learning by doing (*work*) and learning from others (*search*). Learning is *frictional* as we explicitly allow for both search and communication frictions. Under a frictional communication technology, we assume that any searcher-to-searcher information exchange could entail some loss of information. In the unique stationary equilibrium of our game, all existing information is originally produced by some worker. Players who search do not contribute to the production of new information. Nevertheless, these players exert a critical social role in enabling information that would otherwise remain local and private, to be aggregated and diffused to different parts of the society. This increases the value of search and attracts a group of marginally connected players into this activity. Their diffusion ability, however, is no better than their ability to create information. This introduces a distortion that reverberates through the rest of the searching population, thereby causing its amplification (Section 5.2).

Our main contribution is to formally identify new inefficiencies that characterize social learning in this richer environment. These inefficiencies go beyond the known free-riding effect. First, we highlighted the critical role played by those who search for information in the aggregation and diffusion of information, by showing that a society needs searchers to achieve some degree of informational efficiency (Section 4). However, not all searchers are alike and the structure of social connections affects efficiency in ways that are complex and subtle. To explore this, we studied the equilibrium consequences of increasing the connectivity of a society. We show that increasing connectivity can lead to a strict decrease in the quality of social information (Section 5.3). Our results reveal how this inefficiency is directly linked to communication frictions. Second, we analyze how players of different connectivity levels choose their learning activity in equilibrium, and compare this to the planner's solution where each type is allocated to an activity to maximize social welfare (Section 6). This allows us to study more generally the inefficiencies associated with social learning. We show that allocation of activities in the planner's solution can be in direct contrast with the equilibrium one, especially when communication frictions are severe. In such cases, the planner's solution requires players with high connectivity to be the producers of information in the society, whereas in equilibrium, this role is necessarily taken on by players with low connectivity. Third, we apply our model to study how resilient a society is to external manipulation of public opinion and how this depends on the connectivity of the society (Section 7). An important implication of our analysis is that societies that are very effective in aggregating and diffusing information are also particularly susceptible to manipulations.

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A. Proofs.

Lemma A1. *Following a choice of α_t, posterior belief p_t evolves according to:*

$$dp_t \sim \mathcal{N}\Big(0, 4dt\big(p_t(1-p_t)\big)^2\big((1-\alpha_t)^2\eta_w^2 + \alpha_t^2 x^2 \eta_s^2\big)\Big).$$

Proof. Fix a time t and a posterior belief p_t. To begin with, suppose we want to compute dp_t following a generic signal $ds \sim \mathcal{N}(\mu\theta dt, \sigma^2 dt)$ for some μ and σ. From Bayes' rule,

$$p_{t+dt} = \frac{p_t e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\bar{\theta}dt\right)^2}}{p_t e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\bar{\theta}dt\right)^2} + (1-p_t)e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\underline{\theta}dt\right)^2}}.$$

Therefore,

$$\begin{aligned} dp_t &= p_{t+dt} - p_t \\ &= \frac{p_t(1-p_t)\left(e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\bar{\theta}dt\right)^2} - e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\underline{\theta}dt\right)^2}\right)}{p_t e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\bar{\theta}dt\right)^2} + (1-p_t)e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\underline{\theta}dt\right)^2}} \end{aligned}$$

Taking the squares and using the fact that $ds_t^2 = \sigma^2 dt$, the exponential terms can can be simplified. For example,

$$e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\bar{\theta}dt\right)^2} = e^{-\frac{1}{2}} e^{\bar{\theta}\frac{\mu}{\sigma^2}ds_t - \frac{1}{2}\frac{\mu^2}{\sigma^2}dt}.$$

Moreover, notice that, by a Taylor expansion,

$$e^{\bar{\theta}\frac{\mu}{\sigma^2}ds_t - \frac{1}{2}\frac{\mu^2}{\sigma^2}dt} = 1 + \bar{\theta}\frac{\mu}{\sigma^2}ds_t$$

where we neglected all terms of order $dt^{\frac{3}{2}}$ and higher. Putting all this together in the expression for dp_t:

$$\begin{aligned} dp(x,t) &= \frac{2p_t(1-p_t)\frac{\mu}{\sigma^2}ds_t}{1 + (2p_t - 1)\frac{\mu}{\sigma^2}ds_t} \\ &= 2p_t(1-p_t)\frac{\mu}{\sigma^2}ds_t\big(1 - (2p_t-1)\frac{\mu}{\sigma^2}ds_t\big) \\ &= 2p_t(1-p_t)\big(\frac{\mu}{\sigma^2}ds_t - (2p_t-1)\frac{\mu^2}{\sigma^2}dt\big) \end{aligned}$$

where we used the approximation $(1+x)^{-1} \approx 1 - x$. Inside the expression in the last line, there is the random variable $\frac{\mu}{\sigma^2}ds_t - (2p_t-1)\frac{\mu^2}{\sigma^2}dt$. Unconditional on θ, its expectation is zero and its variance $\frac{\mu^2}{\sigma^2}dt$. It inherits its distribution from ds_t. Therefore,

$$dp_t \sim \mathcal{N}\Big(0, \frac{4\mu^2 dt}{\sigma^2}\big(p_t(1-p_t)\big)^2\Big).$$

Now suppose that in the interval $[t, t+dt)$, a player of type x chooses α_t. The evolution of p_t can be written as $dp_t = (1-\alpha_t)dp_{t,w} + \alpha_t dp_{t,S}$, where $dp_{t,w}$ and $dp_{t,s}$ are the evolutions of dp_t in case the signal received is $d\pi_w$ and $d\pi_s$, respectively. Notice that since $d\pi_w$ or $d\pi_s$ are independent, so are $dp_{t,w}$ and $dp_{t,s}$. Therefore, using the result proven above:

$$dp_t = (1-\alpha_t)dp_{t,w} + \alpha_t dp_{t,s} \sim \mathcal{N}\Big(0, 4dt\big(p_t(1-p_t)\big)^2\big((1-\alpha_t)^2\eta_w^2 + \alpha_t^2\eta_s^2x^2\big)\Big).$$

Rearranging gives us the result. □

Lemma A2. *The HBJ equation of the agent's problem is:*

$$v(p_t) = \max_{\alpha_t\in[0,1]} u(p_t) + \frac{2}{\delta+r}p_t^2(1-p_t)^2v''(p)Q(\alpha_t),$$

where $Q(\alpha_t) = (1-\alpha_t)^2\eta_w^2 + \alpha_t^2x^2\eta_s^2$.

Proof. We can approximate v with a second-order Taylor expansion:

$$\mathbb{E}(v(p_{t+dt})|\alpha_t) \approx \mathbb{E}\Big(v(p_t) + v'(p_t)dp_t + \frac{1}{2}v''(p_t)(dp_t)^2 \;\Big|\; \alpha_t\Big)$$

where dp_t is a random variable that depends on α_t. By Lemma A1, we know the distribution of dp_t and we can write

$$\mathbb{E}(v(p_{t+dt})|\alpha_t) \approx v(p_t) + v''(p)2p_t^2(1-p_t)^2Q(\alpha_t)dt,$$

since $\mathbb{E}(dp_t) = 0$, by Lemma A1. Therefore, plugging this back into Equation 1 gives:

$$v(p_t) = (r+\delta)u(p_t)dt + \big(1-(r+\delta)dt\big)\Big(v(p_t) + v''(p)2p_t^2(1-p_t)^2Q(\alpha_t)dt\Big)$$

where we used the approximation $e^{-(r+\delta)t} \approx 1-(r+\delta)dt$. Rearranging and ignoring terms dt^2, gives us the result. □

Proof of Lemma 1: From Lemmas A1 and A2, we have that

$$v(p_t) = \max_{\alpha_t\in[0,1]} u(p_t) + \frac{2}{r+\delta}p_t^2(1-p_t)^2v''(p_t)Q(\alpha_t) = u(p_t) + \frac{2}{r+\delta}p_t^2(1-p_t)^2v''(p_t)\max_{\alpha_t\in[0,1]} Q(\alpha_t),$$

where $Q(\alpha_t) := (1-\alpha_t)^2\eta_w^2 + \alpha_t^2x^2\eta_s$. If $v'' > 0$, the problem is maximized with $\alpha^\star = 1$, if $x^2\eta_s > \eta_w^2$, and with $\alpha_t^\star = 0$, otherwise. Rearranging gives $x^\star = \eta_w/\eta_s$. We are therefore left to show that v is convex. To do so, we solve the ODE.

Rewrite the ODE can be written as $\bar{v}(p) = p + Kp^2(1-p)^2\bar{v}''(p)$, for $p \geq \frac{1}{2}$ and $\underline{v}(p) = 1-p+Kp^2(1-p)^2\underline{v}''(p)$ otherwise, where we set $K := \frac{2Q(\alpha_t^\star)}{r+\delta}$ and drop all t subscripts. Let $\zeta := \frac{\sqrt{1+\frac{4}{K}}}{2}$ and $a := \frac{1}{2} - \zeta < 0$ and $b := \frac{1}{2} + \zeta > 0$. It can be verified by substitution that equations

$$\begin{aligned}\bar{v}(p) &= \quad p + c_1p^a(1-p)^b + c_2p^b(1-p)^a \\ \underline{v}(p) &= 1-p+\tilde{c}_1p^a(1-p)^b + \tilde{c}_2p^b(1-p)^a\end{aligned} \tag{A.1}$$

are generic solutions of their respective ODE. To pin down the values of c_1, c_2, $\tilde{c}_1$, and $\tilde{c}_2$, we invoke three properties that v must posses: (1) symmetry around $p = \frac{1}{2}$, (2) smooth pasting at $\frac{1}{2}$, and (3) and boundaries conditions.

(1) The problem faced by the agent is symmetric in the sense that the flow payoff she optimally respond to beliefs symmetric around $\frac{1}{2}$, e.g. $\bar{p} = \frac{1}{2} - \varepsilon$ or $\underline{p} = \frac{1}{2} + \varepsilon$, when $\varepsilon \in [0, 1/2]$, are the same. Thus, also the corresponding values $\underline{v}(\bar{p})$ and $\bar{v}(\underline{p})$ need to match. We require that for all such ε, $\bar{v}(\underline{p}) = \underline{v}(\bar{p})$. Notice that $1 - \bar{p} = \underline{p}$. Thus, symmetry implies that

$$\underline{p}^a \bar{p}^b (c_1 - \tilde{c}_2) = \underline{p}^b \bar{p}^a (c_2 - \tilde{c}_1)$$

which is true for all $\varepsilon \in [0, 1/2]$ if and only if $c_1 = \tilde{c}_2$ and $c_2 = \tilde{c}_1$.

(2) Next, we impose smooth pasting at $p = \frac{1}{2}$. This requires that $\bar{v}'(p^\star) = \underline{v}'(p^\star)$. Computing the derivatives and evaluating them at p gives

$$\bar{v}'(p^\star) = \ \ 1 + c_1\big(a - p^\star\big) + c_2\big(b - p^\star\big) \ = \ \ 1 + c_1(-\zeta) + c_2\zeta$$

$$\underline{v}'(p^\star) = -1 + \tilde{c}_1\big(a - p^\star\big) + \tilde{c}_2\big(b - p^\star\big) = -1 + \tilde{c}_1(-\zeta) + \tilde{c}_2\zeta$$

hence

$$2 + \zeta(\tilde{c}_1 + c_2) = \zeta(c_1 + \tilde{c}_2).$$

(3) Finally, at the boundaries $p \in \{0, 1\}$, the agent is certain that the state is either 1 or -1. Thus, the value of the problem must necessarily be equal to 1. Let $\bar{v}(1) = 1$. Then, since $a < 0$,

$$\bar{v}(1) = 1 + c_1 1^a 0^b + c_2 1^b 0^a = 1 + c_1 0 + c_2 \infty = 1.$$

Thus, $c_2 = 0$ is the only constant that can guarantee $\bar{v}(1) = 1$. A similar reasoning at $p = 0$ gives us $\tilde{c}_1 = 0$.

Putting these three conditions together we get the system

$$\begin{cases} c_1 = \tilde{c}_2 \text{ and } c_2 = \tilde{c}_1 \\ 2 + \zeta(\tilde{c}_1 + c_2) = \zeta(c_1 + \tilde{c}_2) \\ \tilde{c}_1 = c_2 = 0 \end{cases} \quad \Rightarrow \quad \begin{array}{l} c_1 = \tilde{c}_2 = \frac{1}{\zeta} = 2\frac{\tilde{\mu}\sqrt{x}}{\sqrt{\tilde{\mu}^2 x + 2(r+\delta)}} \\ \tilde{c}_1 = c_2 = 0 \end{array}$$

To conclude, the value function is:

$$v(p) = \begin{cases} p + cp^a(1-p)^b & \text{if } p \geq \frac{1}{2} \\ 1 - p + cp^b(1-p)^a & \text{else.} \end{cases}$$

where $c > 0$, $a < 0$ and $b > 0$. For $p > \frac{1}{2}$, its second derivative is

$$v''(p) = cp^a(1-p)^b\Big(\frac{a(a-1)}{p^2} + \frac{b(b-1)}{(1-p)^2}\Big) = -cp^a(1-p)^b ab\Big(\frac{1}{p^2} + \frac{1}{(1-p)^2}\Big) > 0$$

where we used $a-1=-b$ and $b-1=-1$ and $ab<0$. In a specular way, one can show that $v''(p)>0$ for $p\leq\frac{1}{2}$. $\square$

Lemma A3. *Fix a player of type x and a time t. Let $\pi(x,t)$ be the stock of information and $p(x,t)$ the one for posterior beliefs. There exists a one-to map $\xi:\mathbb{R}\to\mathbb{R}$, independent of x and t, such that $p(x,t)=\xi(\pi(x,t))$.*

Proof. Define the log-likelihood ratio of posterior beliefs as follow:

$$z(x,t):=\ln\frac{p(x,t)}{1-p(x,t)}=\ln\frac{p_0\phi(\pi(x,t)|\theta=1)}{(1-p_0)\phi(\pi(x,t)|\theta=-1)}=z(x,0)+\ln\frac{\phi(\pi(x,t)|\theta=1)}{\phi(\pi(x,t)|\theta=-1)}$$

where $\phi(\pi(x,t)|\theta)$ is the probability density of finding $\pi(x,t)$ at the given level, conditional on the state being θ. Notice that,

$$z(x,t)=K+2\eta_a\pi(x,t)$$

where K is a constant and $\eta_a=\eta_w$ if x is a worker and η_s otherwise. We conclude that, the process for $p(x,t)$ is a one-to-one transformation of $z(x,t)$, which, in turn, is a linear one transformation of $\pi(x,t)$. $\square$

Lemma A4. *Bayes consistency $\mathbb{E}(\pi_s)=\eta_s\theta$ implies a unique positive solution given by*

$$\eta_s=\eta_w\frac{cH(x)}{1-\int_x\tilde{g}(z)h(z)dz}\geq 0.$$

Proof. From Equation 4, we have that

$$\eta_s=\eta_w H(x)\int\sqrt{t}\tau(t)dt+\eta_s\int_x\Big(\int g(z\sqrt{t})\tau(t)dt\Big)h(z)dz. \tag{A.2}$$

Denote $\tilde{g}(z)=\mathbb{E}_\tau(g(z\sqrt{t}))$ and $c=\mathbb{E}_\tau(\sqrt{t})$ and rearranging we get the result. We are left to show that $\eta_s\geq 0$ for all x. Notice that $1-\int_x\tilde{g}(z)h(z)dz$ is increasing in x. Moreover, for any $g\in\mathcal{G}$ such that $g\leq \mathrm{id}_{\mathbb{R}_+}$, we have

$$\tilde{g}(z):=\int g(z\sqrt{t})\tau(t)dt\leq\int z\sqrt{t}\,\tau(t)dt.$$

Therefore for all $x\in X$ and $g\in\mathcal{G}$, we have

$$1-\int_x\tilde{g}(z)h(z)dz\geq 1-\int_0\Big(\int z\sqrt{t}\,\tau(t)dt\Big)h(z)dz.$$

Therefore, in order to ensure $\eta_s\geq 0$, it is enough to show that the right hand side of this equation is positive. However, notice that

$$\int_0\Big(\int z\sqrt{t}\,\tau(t)dt\Big)h(z)dz=\mathbb{E}_h(z)\mathbb{E}_\tau(\sqrt{t}).$$

By definition of f, we have that $\mathbb{E}_h(z) := \mathbb{E}_f(z^2)/\mathbb{E}_f(z)$. Finally, Assumption 1 ensures that $\mathbb{E}_h(z)\mathbb{E}_\tau(\sqrt{t}) \leq 1$.

Proof of Proposition 1. (*Existence*) Equation (6) can be rewritten as

$$\Phi(x) := cxH(x) + \int_x \tilde{g}(z)h(z)dz = 1. \tag{A.3}$$

First we show that the $\Phi(x)$ crosses 1 at least once. Notice that at $x = 0$, $\int_0 \tilde{g}(z)h(z)dz \leq 1$, as shown in the proof of Lemma A4. Therefore $\Phi(0) \leq 1$. Vice versa, $\lim_{x\to\infty} Q(x) = \infty$. Next, we show that Φ is continuous, as being the sum and products of continuous functions. First, notice that H is absolutely continuous, as it admits a density h. Second, for any sequence (x_n) such that $x_n \to x$, the sequence $\int_{x_n} \tilde{g}(z)h(z)dz$ is a positive and non-increasing. Every monotonic and bounded sequence admits a limit point, from which we conclude also $\int_x \tilde{g}(z)h(z)dz$ is continuous. Continuity of Φ, via a straightforward application of Bolzano's Theorem, guarantees the existence of a crossing point $\Phi(x) = 1$.

(Uniqueness). To show the fixed-point is unique, we show that Φ is strictly increasing. Fix $x' > x$. We have

$$\begin{aligned}\Phi(x') - \Phi(x) &= c\big[x'H(x') - xH(x)\big] - \textstyle\int_x^{x'} \tilde{g}(z)h(z)dz \\ &\geq c\Big(\big[x'H(x') - xH(x)\big] - \textstyle\int_x^{x'} zh(z)dz\Big).\end{aligned}$$

Therefore, $\Phi(x') - \Phi(x) > 0$, as we wished to show. We conclude that the equilibrium is unique. $\square$

Proof of Proposition 2. Fix any $g, g' \in \mathcal{G}$ with $g' \geq g$. We need to show that equilibrium η_s under g is lower than under the equilibrium η_s under g'. To show this, we prove a stronger claim, which we will later use in the main text. Namely, let $\eta_s(x, g)$ be defined as in Equation 5, were we make explicit the dependence on x and g. We show next that $\eta_s(x, g) \leq \eta_s(x, g')$, for all $x \in X$. We have

$$\eta_s(x, g) = \eta_w \frac{cH(x)}{1 - \int_x \tilde{g}(z)h(z)dz} \leq \eta_w \frac{cH(x)}{1 - \int_x \tilde{g}'(z)h(z)dz} = \eta_s(x, g'),$$

since, for all x,

$$\tilde{g}'(x) := \int g'(x\sqrt{t})\tau(t)dt \geq \int g(x\sqrt{t})\tau(t)dt =: \tilde{g}(x).$$

Since the equation that defines individual rationality, $\eta_s = \eta_w/x$, is strictly decreasing in x, this proves our claim. $\square$

Proof of Proposition 3. First, notice that $\eta_s(x)$ as defined by Equation 5, has a maximum at

$$\frac{d\eta_s(x)}{dx} = 0 \quad \Rightarrow \quad 1 - \int_x \tilde{g}(z)h(z)dz = \tilde{g}(x)H(x).$$

From Equation A.3, the equilibrium $x^\star$ is pinned down by:

$$1-\int_{x^\star}\tilde{g}(z)h(z)dz = cx^\star H(x^\star).$$

Notice that $\tilde{g}(x)\le cx$, where the inequality is strict for all $g<\mathrm{id}_X$. Putting all together we conclude that, for all $g<\mathrm{id}_X$, at the respective equilibrium $x^\star$, $\frac{d\eta_s(x)}{dx}>0$. Vice versa, when $g=\mathrm{id}_X$, $\frac{d\eta_s(x)}{dx}=0$. □

Proof of Proposition 4 Let $(x^\star,\eta_s)$ be the equilibrium. Fix $\bar{x}\in X$ as suppose $\bar{x}<x^\star$. The information elasticity for this type can be computed as follows:

$$\varepsilon_w(\bar{x}) := \frac{d\eta_s/dc_{\bar{x}}}{\eta_s/c_{\bar{x}}} = \frac{\eta_w h(\bar{x})}{1-\int_{x^\star}\tilde{g}(z)h(z)dz}\frac{c_{\bar{x}}}{\eta_s} = \frac{h(\bar{x})}{H(x^\star)}\frac{\eta_w c_{\bar{x}}}{\eta_w c}.$$

Since $\bar{x}$ is a worker $c_{\bar{x}}=c$. Moreover, by definition of h, we have $h(\bar{x})=\bar{x}f(\bar{x})/\mathbb{E}_f(z)$. Therefore,

$$\varepsilon_w(\bar{x}) = c\eta_w\frac{\bar{x}}{H(x^\star)}\frac{f(\bar{x})}{c\eta_w\mathbb{E}_f(z)}.$$

Now suppose $\bar{x}\ge x^\star$. The information elasticity for this type can be computed as follows:

$$\varepsilon_s(\bar{x}) := \frac{d\eta_s/d\tilde{g}(\bar{x})}{\eta_s/\tilde{g}(\bar{x})} = \frac{c\eta_w H(x^\star)}{\left(1-\int_{x^\star}\tilde{g}(z)h(z)dz\right)^2}h(\bar{x})\frac{\tilde{g}(\bar{x})}{\eta_s} = \tilde{g}(\bar{x})\eta_s\frac{h(\bar{x})}{H(x^\star)}\frac{1}{c\eta_w}.$$

Using the definition of h,

$$\varepsilon_s(\bar{x}) = \tilde{g}(\bar{x})\eta_s\frac{\bar{x}}{H(x^\star)}\frac{f(\bar{x})}{c\eta_w\mathbb{E}_f(z)}.$$

From there, we can compute relative difference $\varepsilon_s(\bar{x})-\varepsilon_w(\bar{x})$ and rearrange. □

Proof of Proposition 5. $(ii)\Rightarrow(i)$. Let $g(y)=y$ for all $y\in\mathbb{R}_+$ and consider any $f'\trianglerighteq f$. It is straightforward to check that the latter implies $h'\trianglerighteq h$. The fixed-point map of Equation 6 can be rewritten as

$$x=\frac{1}{c}+\int_x(x-z)h(z)dz.$$

Moreover,

$$\int_x(x-z)h(z)dz = \int_X(x-z)\mathbb{1}_{\{z>x\}}(z)h(z)dz > \int_X(x-z)\mathbb{1}_{\{z>x\}}(z)h'(z)dz = \int_x(x-z)h'(z)dz,$$

by definition of FOSD and the fact that $(x-z)\mathbb{1}_{\{z>x\}}$ is non-increasing. Thus, letting $x^\star$ be the equilibrium under f,

$$x^\star=\frac{1}{c}+\int_{x^\star}(x^\star-z)h(z)dz > \frac{1}{c}+\int_{x^\star}(x^\star-z)h'(z)dz.$$

Notice that $\frac{d}{dx}\int_x(x-z)h(z)=1-H(x)>0$. Therefore, the equilibrium is re-established at f' by decreasing $x^\star$, therefore increasing η_s as we wished to prove.

$(i) \Rightarrow (ii)$. This direction is equivalent to $\neg(ii) \Rightarrow \neg(i)$ which, however is the content of Observation 1. $\square$

Proof of Theorem 1. For clarity, we divide the proof of these result in four Lemmas. To being, fix an increasing uniform sequence $(h_n)_{n\in N}$, fix $n' > n$ in N and some $x \in X$. Denote:

$$h_\Delta(z,n) := h_{n'}(z) - h_n(z) \qquad \text{and} \qquad D(z) := \frac{h_\Delta(z,n')}{\int_x h_\Delta(z,n')dz} - \frac{h_\Delta(z,n)}{\int_x h_\Delta(z,n)dz}.$$

Notice that, since $h_{n'} \trianglerighteq h_n$, the function $h_\Delta(z,n)$ is single-crossing (SC)[29] in z and integrate to 0. In fact,

$$h_\Delta(z,n) := h_{n'}(z) - h_n(z) = (\gamma(z) - 1)h_n(z),$$

where $\gamma(z) = h_{n'}(z)/h_n(z)$ is positive, non-decreasing and crosses 1, by definition of MLR. Since $h_n(z) > 0$, we have that $h_\Delta(z,n) \geq 0$ implies $h_\Delta(z',n) \geq 0$ for all $z' \geq z$. Moreover, $\int_0^\infty h_\Delta(z,n)dz = 1 - 1 = 0$. For this reason, together with the fact that $h_\Delta(z,n)$ is SC, we have that $\int_x h_\Delta(z,n)dz \geq 0$. Also, notice that $\int_x^\infty D(z)dz = 0$.

We begin by showing that $D(z)$ inherits the single-crossing property from $h_\Delta(z,n')$ and $h_\Delta(z,n)$.[30]

Lemma A5. *$D(z)$ is single-crossing in z in the interval $[x,\infty)$.*

Proof. Since the SC property is preserved under scalar transformation, we prove that the function

$$D'(z) := h_\Delta(z,n') - \beta h_\Delta(z,n) \qquad \text{with} \quad \beta := \frac{\int_x h_\Delta(z,n')dz}{\int_x h_\Delta(z,n)dz} \geq 0$$

is single-crossing in z in the interval $[x,\infty)$. Let $D'(z) \geq 0$. We want to show that $D'(z) \geq 0$ for all $z' \geq z$. Note that, since h_n and $h_{n'}$ belong to a uniform sequence, we have that $h_\Delta(z,n') \geq 0$ implies that $h_\Delta(z,n) \geq 0$. That is, $h_\Delta(z,n)$ crosses zero before $h_\Delta(z,n)$. Therefore, we have to consider only three cases: (1) when $h_\Delta(z,n'), h_\Delta(z,n) \geq 0$, (2) when $h_\Delta(z,n') \leq 0 \leq h_\Delta(z,n)$ and (3) $h_\Delta(z,n'), h_\Delta(z,n) \leq 0$.

(1) Suppose $h_\Delta(z,n'), h_\Delta(z,n) \geq 0$. Since $D'(z) \geq 0$, we have $h_\Delta(z,n') \geq \beta h_\Delta(z,n)$. Therefore,

$$\beta \leq \frac{h_\Delta(z,n')}{h_\Delta(z,n)} = \frac{h_{n''}(z) - h_{n'}(z)}{h_{n'}(z) - h_n(z)}.$$

which is increasing since the sequence is uniform (Definition 4). We conclude that for any $z' \geq z$, $D'(z') \geq 0$.

(2) Suppose $h_\Delta(z,n') < 0 \leq h_\Delta(z,n)$. This implies $D(z) < 0$, a contradiction.

[29]A function $f : \mathbb{R} \to \mathbb{R}$ is single-crossing if $f(z) \geq 0$ implies $f(z') \geq 0$ for all $z' \geq z$.

[30]This is done in a very much similar spirit of Athey (2002) and, more closely, of Quah and Strulovici (2012).

(3) Finally, suppose $h_\Delta(z,n'), h_\Delta(z,n) \leq 0$. We will show that this is incompatible with $D'(z) \geq 0$. By way of contradiction, suppose that $D'(z) \geq 0$. Note that at the right-most boundary of this region, we have that $h_\Delta(z_0,n) = 0$ (since $h_\Delta(z,n)$ crosses zero first). Therefore, at z_0, we have that $D'(z_0) = h_\Delta(z_0,n') \leq 0$. By continuity, there must be a $z^\star \in [z, z_0]$, such that $D'(z^\star) = 0$. This implies that $\frac{h_\Delta(z^\star,n')}{h_\Delta(z^\star,n)} = \beta$. Since, the sequence is uniform, we have $\frac{h_\Delta(z,n')}{h_\Delta(z,n)} \leq \beta$. Now we use the definition of β.

$$\beta := \frac{\int_x h_\Delta(z,n')dz}{\int_x h_\Delta(z,n)dz} = \frac{\int^x h_\Delta(z,n')dz}{\int^x h_\Delta(z,n)dz} = \frac{\int^x h_\Delta(z,n')\frac{h_\Delta(z,n)}{h_\Delta(z,n)}dz}{\int^x h_\Delta(z,n)dz} < \frac{\int^x h_\Delta(z,n)\beta dz}{\int^x h_\Delta(z,n)dz} = \beta$$

This gives us the contradiction. In the second equality, we used that for any n and x, $\int h_\Delta(z,n)dz = 0$. For the inequality, we used the fact that $x < z^\star$. This is automatically the case since we are trying to show $\mathcal{SC}$ of D' on $[x,\infty)$.

This shows that $D'(z) \geq 0$ only in case (1), where we showed $D'(z') \geq 0$ for all $z' \geq z$. Therefore D' is $\mathcal{SC}$ in the interval $[x,\infty)$ and so is D. □

Now that we have established the SCP for $D(z)$, we move to a second instrumental result, which builds on Lemma A5.

Lemma A6. *We have that* $\displaystyle\int_x m(z)D(z)dz \leq 0$.

Proof. By definition of $D(z)$, notice that $\int_x^\infty D(z)dz = 0$. Since $D(z)$ is $\mathcal{SC}$ in the interval $[x,\infty)$ (Lemma A5), we have that $\int_x^y D(z)dz \leq 0$ for any $y < \infty$. Integrating by parts:

$$\begin{aligned}\int_x m(z)D(z)dz &= m(z)\int_x^z D(y)dy\Big|_{z=x}^{z=\infty} - \int_x \Big(m'(z)\int_x^z D(y)dy\Big)dz \\ &= -\int_x \Big(m'(z)\int_x^z D(y)dy\Big)dz \leq 0.\end{aligned}$$

The second equality comes from the fact that $\int_x^\infty D(z)dz = \int_x^x D(z)dz = 0$. The inequality comes from $\int_x^y D(z)dz \leq 0$ and the fact that $m'(z) \leq 0$. To confirm the latter, recall that $m(z) := c^{-1}\big(cx - \tilde{g}(z)\big)$. Therefore, $m'(z) = -c^{-1}\int \sqrt{t}g'(z\sqrt{t})\tau(t)dt \leq 0$, by the fact that $g' \geq 0$ is increasing. (Definition 1) □

Finally, the last and most important of these instrumental results.

Lemma A7. *Fix $x \in X$ arbitrarily and consider an $\trianglerighteq$-increasing uniform sequence in $\mathcal{H}$. The functional $L : N \to \mathbb{R}$, defined as*

$$L(n) = \int_x m(z)h_n(z)dz,$$

is quasi-concave in $n \in N$.

Proof. To show this, it is enough to prove that, for $n'' \geq n' \geq n$,

$$L(n'') - L(n') \geq 0 \quad \Rightarrow \quad L(n') - L(n) \geq 0.$$

Notice that

$$0 \leq L(n'') - L(n') = \int_x m(z)\big(h_{n''}(z) - h_{n'}(z)\big)dz = \int_x m(z)h_\Delta(z,n')dz$$

Therefore, we need to show that, for any $n' \geq n$

$$\int_x m(z)h_\Delta(z,n')dz \geq 0 \quad \Rightarrow \quad \int_x m(z)h_\Delta(z,n)dz \geq 0.$$

Fix $n' \geq n$ and assume $\int_x m(z)h_\Delta(z,n')dz \geq 0$. As argued above, $\int_x h_\Delta(z,n')dz \geq 0$ (for any n'). Therefore,

$$\frac{\int_x m(z)h_\Delta(z,n')dz}{\int_x h_\Delta(z,n')dz} \geq 0.$$

Moreover,

$$\frac{\int_x m(z)h_\Delta(z,n')dz}{\int_x h_\Delta(z,n')dz} - \frac{\int_x m(z)h_\Delta(z,n)dz}{\int_x h_\Delta(z,n)dz} = \int_x m(z)D(z)dz \leq 0.$$

Thus,

$$0 \leq \frac{\int_x m(z)h_\Delta(z,n')dz}{\int_x h_\Delta(z,n')dz} \leq \frac{\int_x m(z)h_\Delta(z,n)dz}{\int_x h_\Delta(z,n)dz} \quad \Rightarrow \quad \int_x m(z)h_\Delta(z,n)dz \geq 0,$$

concluding the proof of Lemma A7. □

With this last result, we can finally provide the proof for Theorem 1.

Let $(h_n)_n$ be a $\trianglerighteq$-increasing uniform sequence in $\mathcal{H}$ and let $h_{n''} \trianglerighteq h_{n'} \trianglerighteq h_n$. Call x_n, $x_{n'}$ and $x_{n''}$ the fixed points of Equation (6) for h_n, $h_{n'}$ and $h_{n''}$, respectively. To show quasi-concavity we need to show that $x_{n'} \geq \min\{x_n, x_{n''}\}$. That is, we need to show: (Case 1) if $x_{n'} \leq x_{n''}$, then $x_n \leq x_{n'}$, and (Case 2) if $x_n \geq x_{n'}$ then $x_{n'} \geq x_{n''}$. To begin, notice that, by Proposition 1, we know that the self-map in Equation (6) has a unique fixed point. Since $c > 0$, it must be the case that the function $\frac{1}{c} + L(x,h)$ crosses the function x from *above*. Indeed,

$$\frac{1}{c} + L(0,n) = \frac{1}{c} + \frac{1}{c}\int_0^\infty (0 - \tilde{g}(z))h_n(z)dz = \frac{1}{c}\Big(1 - \int_0^\infty \tilde{g}(z)h_n(z)dz\Big) > 0.$$

We discuss Case 1 and Case 2 separately.

> *Case 1.* Let $x_{n'} \leq x_{n''}$. Then, by the argument just made, it must be that $\frac{1}{c} + L(x_{n'}, n'') \geq \frac{1}{c} + L(x_{n'}, n') = x_{n'}$, otherwise we would contradict $x_{n'} \leq x_{n''}$. This implies that $L(x_{n'}, n'') \geq L(x_{n'}, n')$. By Lemma A7, we know L is quasi-concave in h. That is, $L(x_{n'}, n'') \geq L(x_{n'}, n')$ implies that $L(x_{n'}, n') \geq L(x_{n'}, n)$. Again, by the single crossing argument above, this implies that $x_n \leq x_{n'}$.

Case 2. This case mimics the previous one. Let $x_n \geq x_{n'}$. We know that this implies $L(x_{n'}, n) \geq L(x_{n'}, n')$. By Lemma A7, we get that $L(x_{n'}, n') \geq L(x_{n'}, n'')$ and conclude that $x_{n'} \geq x_{n''}$.

The two cases above showed that $x_{n'} \geq \min\{x_n, x_{n''}\}$. Since $n'' \geq n' \geq n$ were arbitrary, we conclude that the fixed point x of Equation 6 is quasi-concave in n. By the equation that defines individual rationality this implies that η_s is quasi-convex, concluding the proof of Theorem 1. $\square$

Lemma A8. *Fix a type $z \in X$. The derivative of $W^{SP_1}(\alpha)$ with respect to a marginal increase in $\alpha(z)$ is:*

$$W^{SP_1}_{\alpha(z)}(\alpha) = f(z)\Big(v_s(z,\eta_s) - v_w(\eta_w) + z\Big(\eta_s\tilde{g}(z) - \eta_w c\Big)K\Big),$$

where $K \geq 0$ is a positive constant.

Proof. We compute the derivative of W^{SP_1} with respect to a marginal increase in $\alpha(z)$, the probability that type z is allocated to search. In computing this derivative, we need to compute

$$\frac{dv_s(z,\eta_s)}{d\alpha(z)} = \frac{dv_s(z,\eta_s)}{d\eta_s}\frac{d\eta_s}{d\alpha(z)}.$$

We know that the first term is positive for all z. The second term instead is

$$\frac{d\eta_s}{d\alpha(z)} = zf(z)C\Big(\tilde{g}(z)\eta_s - c\eta_w\Big).$$

where we used the definition of h, of η_s and we denoted $C := \big(\mathbb{E}_f(z)\big(1 - \int_X \alpha(z)\tilde{g}(z)h(z)dz\big)\big)^{-1}$, a positive constant. Putting all together, we get

$$W^{\mathrm{SP}_1}_{\alpha(z)}(\alpha) = f(z)\Big(v_s(z,\eta_s) - v_w(\eta_w) + z\Big(\eta_s\tilde{g}(z) - \eta_w c\Big)K\Big),$$

where $K = C\int_X \alpha(y)\frac{dv_s(y,\eta_s)}{d\eta_s}f(y)dy \geq 0$. $\square$

Proof of Corollary 1. The first part follows from Proposition 5. We are left to show that for any regular sequence $(f^n(z))_{n\in N} \subset \mathcal{F}$, η_s is monotonically decreasing if $g = \min(\mathcal{G}, \geq)$. When the communication technology is completely uninformative, we we have that $\eta_s = \eta_w cH(x^\star)$, a strictly increasing function of $x^\star$. If $f' \trianglerighteq_{MLR} f$, the respective h' and h are also ranked by $\trianglerighteq_{MLR}$, i.e. $h' \trianglerighteq_{MLR} h$. Moreover, since $\trianglerighteq_{MLR} \subset \trianglerighteq$, $H'(x) \leq H(x)$, for all $x \in X$. Thus, fixing $x^\star$, η_s is decreasing in the shift. Since $\eta_s = \eta_w/x$ is strictly decreasing in x, we conclude that $x^\star$ is increasing in the shift, and therefore η_s is decreasing.

Proof of Proposition 6. (*If part*). Let $g = \max\{\mathcal{G}, \geq\}$. As by Lemma A8, the sign of $W^{\mathrm{SP}_1}_{\alpha(z)}(\alpha)$ is positive, meaning that the planner wants a type z to search, if and only if $v_s(z,\eta_s) - v_w(\eta_w) \geq z\big(\eta_w c - \eta_s\tilde{g}(z)\big)K$. In the particular case when g is maximal, we have that $\tilde{g}(z) = cz$. Therefore, the inequality becomes

$$v_s(z,\eta_s) - v_w(\eta_w) \geq zc\big(\eta_w - z\eta_s\big)K.$$

Now let $(x^\star, \eta_s)$ be the equilibrium under such g. Notice that by definition of $x^\star$, we have $\eta_w = x^\star \eta_s$ and $v_s(x^\star, \eta_s) = v_w(\eta_w)$. For all types above $x^\star$, the LHS of the inequality is strictly positive, while the RHS is strictly negative. For all types below $x^\star$, the RHS of the inequality is strictly negative, while the RHS is strictly positive. Therefore, $W^{\mathrm{SP}_1}_{\alpha(z)}(\alpha) \geq 0$ if and only if $x \geq x^\star$, showing that the allocation is indeed efficient.

(*Only if part*). Now take any $g < \max\{\mathcal{G}, \geq\}$. By continuity, we have that $\tilde{g}(z) < \mathbb{E}(\sqrt{t})z$. Now consider the equilibrium $(x^\star, \eta_s)$. We have that

$$\eta_w \mathbb{E}(\sqrt{t}) - \eta_s \tilde{g}(x^\star) > c\big(\eta_w - x^\star \eta_s\big) = 0 = v_s(x^\star, \eta_s) - v_w(\eta_w).$$

The planner would strictly prefer this type to work. This constitutes a deviation from the equilibrium allocation, thereby proving that it cannot be efficient. □

Proof of Corollary 2. Consider the extreme case $g(y) := 0$. In this case, meeting other searchers is completely unproductive. We have that $\tilde{g}(z) = 0$ for all $z \in X$ and η_s can be written as

$$\eta_s(\alpha) = \eta_w c \int_X (1 - \alpha(z)) h(z) dz.$$

The planner's incentive can be represented with the following inequality

$$v_s(z, \eta_s) - v_w(\eta_w) \geq z c \eta_w K.$$

The left hand side is increasing and strictly concave, starting at $-v_w(\eta_w)$ when $z = 0$. The right hand side is linear and increasing. For appropriate values of η_w and c, there are exactly two solutions for this equation, x_1 and x_2 with $x_1 \leq x_2$, such player z searches if and only if $z \in (x_1, x_2)$. □

Proof of Corollary 3. (*Only if part*). When g is not maximally informative, the equilibrium allocation is inefficient in the *ex ante* sense (Proposition 6) and, *a fortiori*, is inefficient in the *ad interim* sense.

(*If part*). Suppose g is maximally informative, i.e. $g = \mathrm{id}_{\mathbb{R}_+}$, and let $(x^\star, \eta_s)$ be the equilibrium. Consider a type x of arbitrary age t and suppose there exists a profitable deviation from the equilibrium plan. We consider a *simple* deviation that consists in switching type x's activity for a dt interval and then reverting back to the equilibrium allocation forever. Let x be a searcher. Switching x to work for a dt interval generates two effects. First, dp_t, namely the instantaneousness change in type x's posterior beliefs, has lower variance. On an individual basis, type x is worse off. Second, her social contribution is affected. In the dt interval she accumulated information at rate $\eta_w dt$ rather than $x\eta_s dt$. Since x searches in equilibrium, $x\eta_s dt > \eta_w dt$. Since $g = \mathrm{id}_{\mathbb{R}_+}$, this necessarily implies her social contribution is diminished. The deviation considered reduces both η_s *and* type x's present discounted value and therefore cannot improve social welfare. □

Proof of Corollary 4. When $g = \max\{\mathcal{G}, \geq\}$, Corollary 3 shows that $x_1 = x_2 = x^\star$ and there is nothing to prove. Let $g < \max\{\mathcal{G}, \geq\}$. There are three distinct cases to consider:

Case 1. Suppose there exists no such $x_1 > 0$. This implies that the social planner finds optimal to allocate $x = 0$ to search at some particular p_t. However, both type x and the society lose from this deviation. As a searcher, type x's contribution to the society is null and so is her personal gain. As a worker, these are both strictly positive. A contradiction. Therefore there must exists $x_1 > 0$. By monotonicity, all types below x_1 will be allocated in a similar way.

Case 2. Next, suppose $x_2 < \infty$. For any type $x > x_2$, the cost of reverting back to work is higher. In terms of social contribution, the most extreme case is when $g = 0$. In such case, x contributes exactly as x_2. Yet, the individual gains for x dominate those of x_2, while the implied social loss is the same. Therefore, x is allocated to search independently of time.

Case 3. Now consider a type $x_1 < x < x_2$. By Case 1 and 2, this type individually gains from search, at all p_t, but she contributes negatively to society due to the interaction between her type and the communication technology g. When p_t converges to 0 or 1, her individual gain for engaging in search relative to work goes to zero, whereas her social contribution does not. Therefore, the planner would want this type to p_t to work. Vice versa, when p_t goes to $\frac{1}{2}$, her individual gain are maximized, the planner would want this type to search. $\square$

Proof of Proposition 7. Fix a target type $\bar{x}$ a bias b and a basin of δ. The altered $\tilde{\eta}_s$ can be computed in ways similar to Lemma A4. We have that

$$\tilde{\eta}_s = c\eta_w H(x^\star) + \tilde{\eta}_s \int_{x^\star} \tilde{g}(z)h(z)dz + b\int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} \alpha(z)h(z)dz = \frac{c\eta_w H(x^\star) + b\int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} \alpha(z)h(z)dz}{1 - \int_{x^\star} \tilde{g}(z)h(z)dz}$$

where $\alpha(x) = \tilde{g}(x)$ if $x \geq x^\star$ and $\alpha(x) = c$ otherwise. The influence exerted by the δ-neighbor of $\bar{x}$ is then

$$\begin{aligned} I(\bar{x}, \delta) &= \int_{x^\star} z(\tilde{\eta}_s - \eta_s)f(z)dz \\ &= \mathbb{E}_f(x)(1 - H(x^\star))\tfrac{b\int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} \alpha(z)h(z)dz}{1-\int_{x^\star} \tilde{g}(z)h(z)dz} \\ &= \mathbb{E}_f(x)\tfrac{(1-H(x^\star))\eta_s}{cH(x^\star)\eta_w} b\int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} \alpha(z)h(z)dz. \end{aligned}$$

where we used that $h(z) = zf(z)/\mathbb{E}_f(x)$. Finally, setting $b = 1$,

$$\begin{aligned}
\iota(x) &:= \lim_{\delta\to 0} \tfrac{I(x,\delta)}{\delta} \\
&= \mathbb{E}_f(x) \tfrac{(1-H(x^\star))\eta_s}{cH(x^\star)\eta_w} \lim_{\delta\to 0} \tfrac{1}{\delta} \int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} \alpha(z)h(z)dz \\
&= \tfrac{\mathbb{E}_f(x)}{\mathbb{E}_f(x)} \tfrac{(1-H(x^\star))\eta_s}{cH(x^\star)\eta_w} \lim_{\delta\to 0} \tfrac{1}{\delta} \int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} z\alpha(z)f(z)dz \\
&= \tfrac{\eta_s}{c\eta_w} \tfrac{1-H(x^\star)}{H(x^\star)} x\alpha(x) \\
&= \tfrac{x\alpha(x)\eta_s}{c\eta_w} \tfrac{1-H(x^\star)}{H(x^\star)},
\end{aligned}$$

since $F([\bar{x} - \epsilon_\delta, \bar{x} + \epsilon_\delta]) = \delta$. □

Proof of Corollary 5. Take any two societies $f, f' \in \mathcal{F}$ with $f' \trianglerighteq f$. If $\eta_s(f') < \eta_s(f)$ there is nothing to prove. Suppose $\eta_s(f') > \eta_s(f)$. In such case, we have that $x^\star$ declined. Since $f' \trianglerighteq f$ implies $h' \trianglerighteq h$, we must have $H(x^\star)$ has also decreased under this transformation. From Proposition 7 we know that the influence of all searchers has strictly increased. The manipulative strategy can therefore produce higher distortions for any fixed δ, or the same amount of distortion for a strictly smaller δ. □

B. Finite Capacity Communication Technologies

In this Appendix, we extend our model to a communication technology g that equally applies to both workers and searchers.

Definition B1. *A* communication technology with finite capacity *is a* concave *and* non-decreasing *self-map g on $\mathbb{R}_+$ with $g(y) \leq y$. Let $\mathcal{G}$ denote the set of such functions.*

Relative to the model presented in the paper, the coefficient c, which was defined as $c := \mathbb{E}_\tau(\sqrt{(t)})$ in Section 3, becomes a $\tilde{g}(1) := \mathbb{E}_\tau(\sqrt{(t)})$, an object that directly depends on the communication channel. Following Lemma A4, it is straightforward to show that Bayes consistency implies

$$\eta_s = \eta_w \frac{\tilde{g}(1)H(x)}{1 - \int_x \tilde{g}(z)h(z)dz} \geq 0. \tag{B.1}$$

The fixed-point map of 6, becomes

$$x = \frac{1}{\tilde{g}(1)} + \int_x m(z)h(z)dz, \tag{B.2}$$

where we denoted $m(z) := \frac{1}{\tilde{g}(1)}\big(\tilde{g}(1)x - \tilde{g}(z)\big)$. The following result establishes existence and uniqueness of our stationary equilibrium when g is a finite capacity channel.

Assumption B2. *Let δ be high enough so that $\mathbb{E}_{h,\tau}(x\sqrt{t} \mid x \geq 1) \leq 1$.*

Lemma B9. *Fix a society $f \in \mathcal{F}$ and a communication technology with finite capacity $g \in \mathcal{G}$. A stationary equilibrium exists and is unique.*

Proof. (*Existence*) Notice that Equation B.2 can be rewritten as

$$x\tilde{g}(1)H(x) + \int_x \tilde{g}(z)h(z)dz = 1.$$

First we show that the left hand side crosses 1 at least once. Notice that at $x = 0$, $\int_0 \tilde{g}(z)h(z)dz \leq 1$, by Assumption 1. When $x \to \infty$ the right hand side grows unboundedly. Since the right hand side is continuous in x, this proves that there exists at least one stationary equilibrium in this game. Next we show that $x \geq 1$.

(Uniqueness). To show that the left hand side crosses 1 exactly once, we start by computing the derivative of Equation B.1:

$$\frac{d\eta_s}{dx} = K\Big(1 - \int_x \tilde{g}(z)h(z)dz - \tilde{g}(x)H(x)\Big),$$

where $K > 0$. It is enough to show that $1 - \int_x \tilde{g}(z)h(z)dz \geq \tilde{g}(x)H(x)$ at any equilibrium point to ensure uniqueness.

To begin, suppose the equilibrium point is $x \geq 1$. Notice that, at the equilibrium, it must be that

$$1 - \int_x \tilde{g}(z)h(z)dz = \tilde{g}(1)H(x).$$

Therefore, $1 - \int_x \tilde{g}(z)h(z)dz \geq \tilde{g}(x)H(x)$ at the equilibrium if $\tilde{g}(x) \leq x\tilde{g}(1)$. This is true if $g(x\sqrt{t}) \leq xg(\sqrt{t})$, which is guaranteed by concavity of g (Definition B1), together with the fact that $x \geq 1$.[31]

To complete the proof, we analyze the $x < 1$ and conclude that there cannot be an equilibrium of this kind. Notice that at $x = 1$, $1 - \int_1 \tilde{g}(z)h(z)dz > \tilde{g}(1)H(1)$. Indeed,

$$\begin{aligned}
1 - \textstyle\int_1 \tilde{g}(z)h(z)dz - H(1)\tilde{g}(1) &> 1 - (1 - H(1))\mathbb{E}(\sqrt{xt}|x > 1) - H(1)\mathbb{E}(\sqrt{t}) \\
&> 1 - (1 - H(1))\mathbb{E}(\sqrt{xt}|x > 1) - H(1)\mathbb{E}(\sqrt{xt}|x > 1) \\
&> 1 - \mathbb{E}(\sqrt{xt}|x > 1) \\
&\geq 0
\end{aligned}$$

where the last inequality follows from Assumption B2. Since the derivative is single-crossing, $1 - \int_x \tilde{g}(z)h(z)dz > \tilde{g}(x)H(x)$ for any $x < 1$. Moreover, concavity of g in the interval $x \in [0, 1]$

[31] To see this, fix $x \geq 1$ and t arbitrarily. Let $\alpha := \frac{1}{x} < 1$ and notice that by concavity of g,

$$g(\alpha x\sqrt{t} + (1 - \alpha)0) \geq \alpha g(x\sqrt{t}) + (1 - \alpha)g(0),$$

and, since $g(0) = 0$, $g(\alpha x\sqrt{t}) \geq \alpha g(x\sqrt{t})$. Rearranging gives us $g(x\sqrt{t}) \leq xg(\sqrt{t})$.

implies $xg(1) < g(x)$. Therefore, $1 - \int_x \tilde{g}(z)h(z)dz > x\tilde{g}(1)H(x)$ and therefore $x < 1$ cannot be an equilibrium. □

After ensuring the existence and uniqueness of the stationary equilibrium, Theorem 1 follows immediately since the fixed-point problem of Equation B.2 has the same property than the one of Equation 6, in particular, function m is decreasing in z. Similarly, the proofs of Proposition 2, 3 and 5 can be readily applied to the new problem of Equation B.2 to reach the same conclusions.

C. Non-Stationary Equilibria and Convergence

In this section, we discuss the non-stationary version of our model. For concreteness, we will assume there is a unit mass of players alive a time $t = 0$ and that $\delta = 0$. Players are infinitely lived is and there is no inflow of newborn player in the society. Moreover, we restrict attention to a simpler class of communication technologies, linear functions $g(y) = \kappa g(y)$, $0 \leq \kappa \leq 1$. Let $\eta_s : \mathbb{R}_+ \to \mathbb{R}$ be a time-dependent precision for social information. We refer to $\eta_s(t)$ as a *social information path*. It describes the precision of the instantaneous signal coming from search. Due to the continuum of players, we can think of η_s as being deterministic. We will denote $E_s(t) := \int_0^t \eta_s(t')dt'$ and, therefore, $E_s'(t) = \eta_s(t)$. At time t, every players has two channel from which she can receive information, work and search:

$$\pi_w \sim \mathcal{N}\Big(\eta_w \theta dt, dt\Big) \qquad \pi_s(x) \sim \mathcal{N}\Big(xE_s'(t)\theta dt, dt\Big)$$

Players allocate their time optimally. Since they are of measure zero, they cannot affect the path of η_s. Their problem is in fact very similar to the of Section 3.2. A player at time t faces a decision problem that consists in choosing the signal, between π_w and $\pi(x)$, that maximizes the variance of her posterior beliefs. As in Lemma 1, her problem is solved by the following stopping function:

Lemma C10. *Fix a strictly increasing social information path $E_s'(t)$. There exists a stopping rule $\zeta : X \to \mathbb{R}_+$, such that, for all $x \in X$, player x searches at time t if and only if $t \geq \zeta(x)$. Moreover, $\zeta(x)$ is strictly decreasing in x.*

Proof. Fix a path $E_s(t)$. From Lemma 1, we know that at time t the *indifferent type* at is pinned down by Equation $\eta_w = x^\star(t)E_s'(t)$. Define $\zeta(x)$ the inverse function of $x^\star(t) = \eta_w / E_s'(t)$. Since $E_s'(t)$ is strictly increasing, ζ is well-defined. □

When $E_s'(t)$ is increasing, ζ is decreasing. This means that players unravel away from work, starting from highly connected types down to less connected types. In such case, players necessarily switch from *work to search*. The stock of information that each players collect is

$$\Gamma(x,t) := \begin{cases} \eta_w t\theta + B(t) & \text{if } t < \zeta(x), \\ \Big(\eta_w \zeta(x) + x\big(E(t) - E(\zeta(x))\big)\Big)\theta + B(t) & \text{else.} \end{cases} \tag{C.1}$$

We normalize to 1 the variance of the signal that type (x,t) relay onto others. Its conditional expectation becomes, $\mathbb{E}\big(\pi(x,t)|\theta\big) = \eta_w\sqrt{t}\theta$ if if $t < \zeta(x)$, and $\mathbb{E}\big(\pi(x,t)|\theta\big) = \eta_w\frac{\zeta(x)}{\sqrt{t}}\theta + \frac{\kappa x}{\sqrt{t}}\big(E(t) - E(\zeta(x)\big)\theta$ otherwise.

Definition C2. *A Dynamic Equilibrium is a* stopping rule $\zeta : X \to \mathbb{R}_+$ *and* social information path $E_s : \mathbb{R}_+ \to \mathbb{R}_+$ *such that*

1. *Given the social information path E_s, the stopping rule is adapted to E_s as by Lemma C10. That is, for all types x and times t, player x searches at t if and only if $t \geq \zeta(x)$.*
2. *Given the stopping rule ζ, E_s us Bayes consistent, that is*

$$E_s'(t) = \int_X \mathbb{E}(\pi(z,t)|E_s)h(z)dz \qquad \forall t \geq 0.$$

In the equilibrium definition, we impose the obvious consistency requirement, which replace its stationary counterpart of Equation 4. By replacing the values for $\mathbb{E}(\pi(z,t)|E_s)$ in the Bayes consistency requirement and rearranging, we get

$$\sqrt{t}E_s'(t) = \eta_w t H(x^\star(t)) + \int_{x^\star(t)} \Big(\eta_w\zeta(z) + \kappa x\big(E_s(t) - E_s(\zeta(z))\big)\Big)h(z)dz. \tag{C.2}$$

This condition can be expressed as a second-order non-linear ODE, as we show in the next result.

Lemma C11. *A pair (ζ, E_s) is a Dynamic Equilibrium if and only if E_s is a solution to the ODE*

$$tE_s''(t) = \eta_w\sqrt{t}H\Big(\frac{\eta_w}{E_s'(t)}\Big) + E_s'(t)\Big(\sqrt{t}\int_{\frac{\eta_w}{E_s'(t)}} zh(z)dz - \frac{1}{2}\Big)$$

and ζ is adapted to E_s.

Proof. We begin by deriving Equation C.2. Since by definition of ζ we have $\zeta(x^\star(t)) = t$, the derivative simplifies a lot. In particular, notice that

$$\begin{aligned}
\tfrac{d}{dt}\sqrt{t}E_s'(t) &= \eta_w H(x^\star(t)) + x_t^\star(t)h(x^\star(t))\Big(\eta_w t - \eta_w\zeta(x^\star(t)) + \kappa x^\star(t)\big(E_s(t) - E_s(\zeta(x^\star(t)))\big)\Big) \\
&\quad + \kappa \textstyle\int_{x^\star(t)} zE_s'(t)h(z)dz \\
&= \eta_w H(x^\star(t)) + x_t^\star(t)h(x^\star(t))\Big(\eta_w t - \eta_w t + \kappa x^\star(t)\big(E_s(t) - E_s(t)\big)\Big) + \kappa \textstyle\int_{x^\star(t)} zE_s'(t)h(z)dz \\
&= \eta_w H(x^\star(t)) + \kappa \textstyle\int_{x^\star(t)} zE_s'(t)h(z)dz.
\end{aligned}$$

Therefore,

$$\frac{1}{2\sqrt{t}}E_s'(t) + \sqrt{t}E_s''(t) = \eta_w H(x^\star(t)) + \kappa E_s'(t)\int_{x^\star(t)} zh(z)dz$$

and rearranging, with $x^\star(t) = \eta_w / E_s'(t)$,

$$tE_s''(t) = \eta_w \sqrt{t} H\Big(\frac{\eta_w}{E_s'(t)}\Big) + E_s'(t)\Big(\kappa\sqrt{t}\int_{\frac{\eta_w}{E_s'(t)}} zh(z)dz - \frac{1}{2}\Big)$$

which concludes the proof. □

C.1. Convergence Towards the Stationarity Equilibrium

The dynamic model introduced in the previous section differs from the on introduced in Section 3 because players are infinitely lived. In this section, we perform the same exercise of the previous one, but in a dynamic model in which players at different ages can coexist. Although equilibria cannot be readily expressed as we did in Lemma C11, we can conclude they has nice features. In particular we show that when $g(x) = x$, $\eta_s(t)$ is necessarily a strictly increasing path that converges to the stationary equilibrium of Section 4. To avoid confusion, we will refer to time with variable t and to age with variable m. Notice that $m \leq t$. As a consequence of Lemma C10, we still have that the activity choice for a player of type x only depends on t, not m. In particular, such choice is determined by $\tau(x)$ the stopping rule adapted to E_s. The stock of information is given by

$$\Gamma(x,m,t) := \begin{cases} \eta_w m\theta + B(m) & \text{if } t < \tau(x), \\ \big(\eta_w(\tau(x)-m) + x\big(E(t) - E(\tau(x))\big)\theta + B(m) & \text{if } m > t - \tau(x). \\ \big(x\big(E(t) - E(m)\big)\theta + B(m) & \text{if } m < t - \tau(x). \end{cases}$$

Similarly, we can define the normalized $\mathbb{E}\big(\pi(z,m,t)\big|E_s\big)$, as we did in the previous section. The consistency conditions now takes into account the fact that a type can be a multiple different ages at the same time. In particular, at time t, the probability that type x is of age $m \leq t$ is given by the truncated exponential distribution $\tau(m|t)$. Therefore, Bayes consistency becomes:

$$E_s'(t) = \int_X \Big(\int_0^t \mathbb{E}\big(\pi(z,m,t)\big|E_s\big)\tau(m|t)dm\Big)h(z)dz \qquad \forall t \geq 0. \tag{C.3}$$

The equilibrium is then defined as follows:

Definition C3. *A Dynamic Equilibrium with overlapping generations is composed by a* stopping rule $\zeta : X \to \mathbb{R}_+$ *and a* social information path $E_s : \mathbb{R}_+ \to \mathbb{R}_+$ *such that*

1. *Given the social information path E_s, the stopping rule is adapted to E_s as by Lemma C10. That is, for all types x and times t, player x searches at t if and only if $t \geq \zeta(x)$.*
2. *Given the stopping rule ζ, E_s is a solution to Equation C.3.*

In the next result we argue that, when an equilibrium exists, its information path $\eta_s(t)$ it has two important property. continuous solution will satisfy the property of being monotonically increasing and converging to the stationary equilibrium analyzed in Section 4. For that, we prove first two other results about $\eta_s(t)$.

Lemma C12. *Let $(x^\star, \eta_s^\star)$ be the unique stationary equilibrium. Suppose (τ, E_s) is a dynamic equilibrium with overlapping generations and that $g(y) = y$. Then $\eta_s(t)$ is strictly increasing.*

Proof. Suppose not. Let $\bar{t}$ be the first time at which $\eta'(t) \leq 0$. Fix a type z and an age m. We want to show $\mathbb{E}(\pi(z, m, \bar{t} + dt)|\eta_s) \geq \mathbb{E}(\pi(z, m, \bar{t})|\eta_s)$. This compares the social contribution of two identical players that where born dt-apart from each others. If $\bar{t} < \zeta(z)$, then both players have worked all their lives, which are of length m. Therefore they accumulated the same amount of information in expectation, or $\mathbb{E}(\pi(z, m, \bar{t} + dt)|\eta_s) = \mathbb{E}(\pi(z, m, \bar{t})|\eta_s)$. If, instead, $\bar{t} \geq \zeta(z)$, the younger player, the one who is of age m at time $\bar{t} + dt$, has collected more information in expectation, $\mathbb{E}(\pi(z, m, \bar{t} + dt)|\eta_s) > \mathbb{E}(\pi(z, m, \bar{t})|\eta_s)$. This is because, by assumption on $\bar{t}$, in the interval $[0, \bar{t})$ the information path $\eta_s(t)$ was strictly increasing. Therefore, keeping $\tau(m|\bar{t})$ constant, the integral in Equation C.3 is strictly increasing. The change in the distribution only reinforce this effect. In fact, the distribution of ages $\tau(m|\bar{t})$ is first-order stochastically dominated by $\tau(m|\bar{t} + dt)$. Since $\mathbb{E}(\pi(z, m, \bar{t})|\eta_s)$ is trivially increasing in m (older players have more information in expectation), we conclude that $\eta_s(\bar{t}) < \eta_s(\bar{t} + dt)$ and therefore $\eta_s'(\bar{t}) > 0$. A contradiction. □

Lemma C13. *Let $(x^\star, \eta_s^\star)$ be the unique stationary. Suppose (τ, E_s) is a non-stationary equilibrium with overlapping generations and that $g(y) = y$. Then $\eta_s(t)$ is bounded above by η_s^**

Proof. Suppose not. Then, by continuity of $\eta_s(t)$, there must be a $\bar{t}$ such that $\eta_s(\bar{t}) = \eta_s^\star$. By Claim C12, we know that $\eta_s(t) < \eta^\star$ for all $t \in [0, \bar{t})$. As before, fix any type x and any age m. Under the dynamic equilibrium, this player cannot have accumulated more information than the same player of the same age under the stationary equilibrium. If (z, m) works at $\bar{t}$, she has been working since $\bar{t} - m$ and has the expected amount of information under the two regimes. If she ever searched, then by $\eta_s(t) < \eta^\star$, she must have strictly less information under the dynamic equilibrium. Therefore, keeping $\tau(m|\bar{t})$ constant, at $\bar{t}$ the integral in Equation C.3 is strictly smaller than $\eta_s^\star$, or $\eta_s(\bar{t}) < \eta_s^\star$, a contradiction. □

Lemma C14. *Let $(x^\star, \eta_s^\star)$ be the unique stationary equilibrium. Suppose (τ, E_s) is a non-stationary equilibrium with overlapping generations and that $g(y) = y$. Then $\lim \eta_s(t) = \eta^\star$.*

Proof. We know that $\eta_s(t)$ is an monotone increasing (Claim C12) and bounded (Claim C13) sequence. As such, it necessarily converges to some real limit point $\lim \eta_s(t) \leq \eta^\star$. Suppose $\lim \eta_s(t) < \eta^\star$. In such case, $\lim \eta_s(t)$, together with the implied threshold x, must satisfies Definition 2. A contradiction on the uniqueness result of Proposition 1. □