Politically feasible reforms of non-linear tax systems*

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Abstract

We develop a framework that allows us to study the political economy and the welfare-implications of tax reforms. Given a predetermined non-linear income tax schedule we characterize a set of reforms that are preferred by a majority of voters and a set of welfare-improving reforms. We clarify the conditions under which there exist reforms that belong to both sets. We call such reforms welfare-improving and politically feasible. We also characterize conditions under which welfare-improvements are politically infeasible and conditions under which politically feasible reforms are detrimental for welfare.

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1 Introduction

Beginning with the seminal work of Mirrlees (1971), the normative analysis of non-linear tax systems has seen major breakthroughs. The mathematical tools that are needed to characterize welfare-maximizing tax systems are by now well understood. The more recent line of research that begins with Diamond (1998) and Saez (2001) has managed to bring the theory to data and to trace out the quantitative implications of this approach.

By contrast, there is no established conceptual framework for a political economy analysis of non-linear tax systems. A major difficulty is that there is typically no Condorcet winner in the set of non-linear tax policies. A tax policy that is particular attractive to middle incomes will not win a majority against an alternative tax policy that makes both individuals with incomes in the bottom 30 percent and individuals with incomes in the top 30 percent better off. The latter tax policy will in turn be defeated by one that makes all individuals that belong to the bottom 60 percent better off and so on.

Our first contribution is a theoretical result that makes it possible to relate political economy forces to the analysis of non-linear tax systems. We assume that there is a predetermined non-linear income tax schedule and consider reforms of this income tax schedule that satisfy a monotonicity property so that the reform-induced change in tax payments is either (weakly) increasing or decreasing in income. By Theorem 1, such a reform is preferred by a majority of voters if and only if it is preferred by the voter with median income. For the sake of brevity, we refer to such reforms as being politically feasible.[1]

Second, we use the perturbation method to study whether under, a given status quo tax policy, there are reforms that would be supported by a majority of taxpayers. We also ask whether it is possible to reform the tax schedule in a way that is Pareto- or welfare-improving. Ultimately, we provide conditions under which a tax schedule can be improved even if political economy constraints are taken into account. All these conditions can be expressed in terms of what has become known as sufficient statistics, see e.g. Chetty (2009) for a review. With data on the distributions of earnings and wages and estimates of labor supply elasticities one can check what types of reforms exist under a given status quo schedule.

We establish our main results using a generic version of the model of income taxation by Mirrlees (1971). This implies, in particular, that there is only one source of heterogeneity among individuals, the variable costs of productive effort. We also discuss in what sense Theorem 1 extends to richer models of taxation.

[1]This terminology is convenient but requires a qualification. Many, but not all political economy models predict that a Condorcet winner will be the outcome of the political process. Forces that may push political outcomes away from a Condorcet winner include special interest politics, legislative bargaining, abstention form voting, or the presence of swing voters. While "sufficient" political support is certainly essential for the implementability of reforms, one may debate whether the support of a majority of the population is "sufficient" in all circumstances.

Specifically we consider a setup akin to Atkinson and Stiglitz (1976) in which individuals can split their after-tax income between two consumption goods and possibly differ in their preferences for these consumption goods. These two consumption goods can be interpreted as current consumption and savings. We clarify the conditions under which a revenue-neutral tax reform that lowers income tax rates and increases tax rates on savings is politically feasible. Under a separability and homogeneity assumption on preferences efficient tax policies do not involve distortionary savings taxes. We show that the introduction of such taxes may, however, be supported by a majority of tax-payers.

We also discuss extensions of the basic Mirrleesian setup in which individuals have different fixed costs of entering the labor market or different valuations of public goods. We finally look at a model in which income can be due to luck and in which voting behavior is motivated by a desire to reduce the contribution of luck to after-tax income.

Our formal analysis proceeds as follows: Individuals are confronted with a status quo income tax schedule T_0 so that their after-tax-income becomes a function of their earnings y via $C_0(y) = c_0 + y - T_0(y)$, where c_0 is the transfer to an individual with no earnings. Most of our results are derived form considering reforms of the status quo schedule that take the following form: For all incomes in a range $[y_a, y_b]$ marginal tax rates are increased (or decreased) by an amount τ. The change in tax revenue due to the reform, denoted by $\Delta^R(\tau, y_a, y_b)$, is used to increase (or decrease) the intercept of the tax schedule. After the reform the individuals' after-tax-income is therefore determined by $C_1(y) = c_0 + \Delta^R(\tau, y_a, y_b) + y - T_1(y)$, where T_1 is the reformed income tax schedule.

Our characterization of politically feasible reforms is based on the observation that preferences over such reforms satisfy a single-crossing property. Consider indifference curves in a τ-Δ^R-diagram. The slope of such an indifference curve gives the increase in the intercept of the consumption schedule that an individual requires as a compensation for increased marginal tax rates. We show that more productive individuals have steeper indifference curves. Intuitively, they generate higher earnings and are therefore hit harder by an increase of marginal tax rates. Consequently, to compensate them for this utility loss, a larger increase of c_0 is required.

The single crossing property enables us to characterize reforms that are supported by a majority of voters. A reform makes a majority of tax-payers better off if and only if the individual with median income prefers the reformed schedule over the status quo schedule. If the median voter thinks that an increase of c_0 offsets the utility loss from higher marginal tax rates, then any individual with a lower income will agree. Alternatively, if the median voter thinks that the increase of c_0 is not worth the sacrifice, then any individual with a higher income will agree.

A preliminary step in our characterization of politically feasible reforms is the analysis of Pareto-improving reforms. By the single crossing property, a reform that involves a decrease of marginal tax rates is Pareto-improving only if even those who do not benefit from decreased tax rates are made better off. This requires that tax revenue increases which is possible only if marginal tax rates in the status quo exceed a Laffer bound.

We provide a characterization of this Laffer bound. The Laffer bound is a non-linear function of income. We show that it depends on the status quo tax system if there are income effects so that indivdiual adjust their earnings in response to a change of the intercept of the consumption schedule. Whether, say, it is possible to raise revenue by an increase of marginal tax rates for incomes between 70,000 and 75,000 dollars depends not only on the behavioral responses of taxpayers but also on the whole schedule of marginal tax rates in the status quo.

The single-crossing property also implies that a reform that involves an increase of marginal tax rates is Pareto-improving only if revenue increases so strongly that even those who are confronted with higher marginal tax rates are made better off. We show that this is possible only if marginal tax rates fall below a lower Pareto bound in the status quo. The characterization of this lower bound is interesting in itself. At low levels of income, it is negative and can therefore be interpreted as a Pareto-bound for the earnings subsidies of low income earners.[2] Such earnings subsidies are, for instance, part of the US tax system via the Earned Income Tax Credit.

What do the reforms look like that are attractive to the median voter and would therefore be supported by a majority of taxpayers? Consider first reforms (τ, y_a, y_b) where y_b is smaller than the median income. An increase of marginal taxes at lower levels of income implies that the median voter's tax payment goes up. We show that the median voter will support such a reform only if it is Pareto-improving, i.e. only if under the status quo schedule marginal tax rates for incomes in $[y_a, y_b]$ are inefficiently low.

By contrast, a reform that lowers marginal tax rates reduces the median voter's tax burden and is welcome unless it brings tax rates into the range that is no longer Pareto-efficient. Now consider reforms (τ, y_a, y_b) where y_a exceeds the median voter's income. A tax increase does not affect the median voter's tax burden so that he will support such a reform as long as it raises revenue, i.e. as long as marginal tax rates are below the Laffer bound. By the same logic, a tax decrease will be supported only if it raises revenue which is the case if marginal tax rates are above the Laffer bound.

In sum, the median voter wants to have the lowest marginal tax rates that are consistent with the requirement of Pareto-efficiency for incomes below the median and she wants to have the highest marginal tax rates consistent with Pareto-efficiency for incomes above the median. Politically feasible reforms therefore exhibits a discontinuity at the

[2]Possibly, it is also negative for higher levels of income, but this depends on both on the status quo schedule and the strength of income effects. If there are no income effects, then it is negative at all levels of income.

median level of income. The Laffer bound is the tool to diagnose politically feasible reforms that affect incomes above the median. The lower Pareto bound is relevant for incomes below the median.

This characterization has interesting implications: First, since the lower Pareto bound is negative for low levels of income, moving from a laissez-faire status quo to a system that involves earnings subsidies for low income earners is politically feasible. Second, if under the status quo, tax rates are above the lower Pareto bound, then there is no politically feasible reform that involves higher tax rates for the poor. Third, for high levels of income, an increase of tax rates for the rich is politically feasible for any status quo schedule under which the rich are not yet taxed at the revenue-maximizing rate. By the same logic, if tax rates under the status quo are below the Laffer bound, then lowering taxes for the rich is politically infeasible.

We complement the analysis of politically feasible reforms by looking at welfare-improving reforms. What is welfare-improving depends both on the status quo schedule and the welfare function that is used. We do not restrict these welfare functions a priori, but allow for any welfare function under which welfare weights are a non-increasing function of income. We can then, for any status quo schedule, look at the set of welfare weights under which a tax reform (τ, y_a, y_b) would be welfare improving.[3]

Looking at the intersection of politically feasible and welfare-improving reforms generates various insights. Remember that tax increases for the rich are politically feasible unless that status quo has tax rates that exceed the Laffer bound. Such tax increases are welfare-improving only if the weights on the rich are sufficiently low. However, if the status quo schedule is taken to be the laissez-faire schedule, then tax increases for the rich are welfare-improving for any welfare function. Likewise, tax decreases for the poor are typically politically feasible. However, if the status quo schedule is the laissez-faire schedule, then there is no specification of welfare weights under which such reforms would be welfare-improving.[4] The introduction of earnings subsidies is then diagnosed as being politically feasible but as being detrimental to welfare.

To diagnose whether there are politically feasible or welfare-improving reforms we focus on a particular class of perturbations that can be described by three parameters, τ, y_a and y_b, where τ is the change in marginal tax rates and $[y_a, y_b]$ is the range of incomes over which this change applies. In analyzing these perturbations, we do not make explicit

[3]This exercise can be related to the work that seeks to identify society's social welfare function from observed tax schedules. In particular, we can say that a tax increase for incomes in $[y_a, y_b]$ is welfare-improving only for welfare weights that assign less mass to taxpayers with incomes in $[y_a, y_b]$ than the empirically identified social welfare function.

[4]This observation is reminiscent of the finding that, in a Mirrleesian model with labor supply responses only at the intensive margin, optimal marginal tax rates are always non-negative.

use of optimal control theory or the calculus of variations. In principle, we could use these methods. However, this would imply that we start from an abstract class of perturbations which has been proven useful for the purpose of developing a general theory of dynamic optimization and optimal control. Our approach is different for two reasons. First, we focus on perturbations which represent a natural thought experiment in the analysis of tax systems: Raise marginal tax rates for a subset of taxpayers and use the proceeds to finance additional transfers to those with no income. Second, our main objective is not to characterize an optimal tax system. Instead we seek to identify conditions under which a typically sub-optimal status quo can be reformed. This is different from optimal control theory and the calculus of variations also for a third reasons: We have various measures of performance (tax revenue, Pareto-efficiency, welfare, political support), and not just one objective function.

In the theory of optimal control or the calculus of variations, looking at small perturbations has proven useful for the purpose of deriving optimality conditions. We use small perturbations to derive auxiliary tax schedules that enable us to diagnose whether there are Pareto-improving reforms, welfare-improving reforms, or politically feasible reforms. To illustrate this, consider the reform-induced change in tax revenue $\Delta^R(\tau, y_a, y_b)$. If $\Delta^R_\tau(0, y_a, y_b) > 0$, then starting from the status quo with $\tau = 0$, an increase of marginal tax rates applied to incomes in $[y_a, y_b]$ generates additional tax revenue. Obviously, $\Delta^R_\tau(0, y_a, y_a) = 0$, as a reform that applies only to a null set of agents will not generate additional revenue. However, if the cross-derivative $\Delta^R_{\tau y_b}$ evaluated at $(0, y_a, y_a)$ is positive, this means that Δ^R_τ turns positive if, starting from $y_b = y_a$ we increase y_b. Hence, if $\Delta^R_{\tau y_b}(0, y_a, y_a) > 0$, then tax revenue can be increased by raising marginal tax rates in a small neighborhood of income level y_a. We derive the Laffer curve by looking at the conditions under which $\Delta^R_{\tau y_b}(0, y_a, y_a) > 0$. We use this line of reasoning for all measures that we use to evaluate tax reforms. Any such measure will be a function of τ, y_a and y_b and to diagnose whether there are reforms with particular properties we always look at the cross derivative of this measure, first with respect to τ, then with respect to y_b and finally evaluate the cross-derivative at $(0, y_a, y_a)$.

The schedules for the identification of reforms that raise revenue, yield Pareto- or welfare-improvements or make a majority better off are easily interpretable. They depend on the status quo schedule and the behavioral responses of the taxpayers. The latter are measured by (uncompensated) elasticities of earnings with respect to changes in net wages and non-labour income.[5]

[5] We also clarify the conditions under which the diagnosis schedules admit a representation akin to an ABC-formula in the sense of Diamond (1998). Diamond's ABC-formula provides a characterization of welfare-maximizing tax rates so that, ceteris paribus, tax rates at income level y are higher if A) the mass of people with an income above y is larger, B) their contribution to social welfare is lower, and C) people with an income close to y respond less elastically to changes in marginal tax rates.

The reminder is structured as follows. The next section discusses related literature. The formal framework is introduced in Section 3. Section 4 contains the result that a reform is preferred by a majority of citizens if and only if it is preferred by the median voter. We then characterize Pareto-improving reforms in Section 5. The characterization of politically feasible reforms is in Section 6 and the characterization of welfare-improving reforms in Section 7. The conditions under which a reform is both politically feasible and welfare-improving can be found in Section 8. Section 9 contains various extensions. Concluding remarks are in the last section. Unless stated otherwise, proofs are relegated to the Appendix.

2 Related literature

Our analysis is based on the model of income taxation due to Mirrlees (1971). The Mirrleesian framework is the workhorse for the normative analysis of non-linear tax systems, see Hellwig (2007) and Scheuer and Werning (2016) for more recent analyses of this model and Piketty and Saez (2013) for a review.

Well-known political economy approaches to redistributive income taxation have used the model of linear income taxation due to Sheshinski (1972). In this model, marginal tax rates are the same for all levels of income and the resulting tax revenue is paid out as uniform lump sum transfer. As has been shown by Roberts (1977), the median voter's preferred tax rate is a Condorcet winner in the set of all linear income tax systems. This median voter theorem has been widely used. A prominent example is the prediction due to Meltzer and Richard (1981) that tax rates are an increasing function of the difference between median and average income. Another example is the analysis of Alesina and Angeletos (2005) that includes a motive to correct income differences that are due to luck – as opposed to effort – into the behavior of voters. Our finding that the set of monotonic tax reform contains a Condorcet winner generalizes results in this literature in particular the one by Roberts (1977) and Gans and Smart (1996) who restrict attention to affine tax schedules. Gans and Smart (1996) adapted earlier findings by Rothstein (1990; 1991) who shows that the median voter's preferred policy is a Condorcet winner if preferences over policies satisfy a single-crossing condition.[6]

The focus on the conditions under which a set of reforms contains a Condorcet winner distinguishes our work from papers that explicitly analyze political competition as a strategic game and then characterize equilibrium tax policies. Recent papers that characterize non-linear income tax schedules that emerge in a political equilibrium include Acemoglu, Golosov and Tsyvinski (2010), Bierbrauer and Boyer (2013; 2016), and Brett and Weymark (2016a; 2016b).

We derive an auxiliary tax schedule that identifies reforms that are in the median

[6]Single-peaked preferences also imply the existence of Condorcet winner. The single crossing-condition neither implies nor is implied by single-peakedness of preferences over policies.

voter's interest and would therefore be supported by a majority of voters. This schedule has some properties which are familiar from the work of Röell (2012) and Brett and Weymark (2016a) who characterize the non-linear income tax schedule that the median voter would pick if she could dictate tax policy. Both schedules reveal that the median voter wants to have low taxes on the poor and high taxes on the rich. For the special case of quasi-linear in consumption preferences, the auxiliary schedule indeed coincides with the median voter's preferred schedule whenever the latter does not give to rise to bunching.

We are ultimately interested in reforms that are both welfare-improving and politically feasible. This links our analysis to a literature that characterizes optimal policies subject to political economy constraints: Acemoglu et al. (2010) relate dynamic problems of optimal taxation to problems of political agency as in Barro (1973) and Ferejohn (1986). Farhi, Sleet, Werning and Yeltekin (2012) study optimal capital taxation subject to the constraints from probabilistic voting, see Lindbeck and Weibull (1987). Battaglini and Coate (2008) study optimal taxation and debt financing in a federal system using the model of legislative bargaining due to Baron and Ferejohn (1989). Saez and Stantcheva (2016) study generalized welfare functions with weights that need not be consistent with the maximization of an additive utilitarian welfare function. The generalized weights may as well reflect alternative, non-utilitarian value judgments or political economy forces.

Our analysis of welfare-improving reforms is related to a literature that seeks to identify society's social welfare function empirically.[7] Through the lens of our model, this literature can alternatively be interpreted as identifying the set of social welfare functions for which, say, an increase of marginal tax rates for incomes close to the sixtieth-percentile of the income distribution would be welfare-improving.

Our formal analysis makes use of what has become known as the perturbation method. Piketty (1997) and Saez (2001) introduce it as a heuristic and more intuitive alternative to optimal control theory. More recently, Golosov, Tsyvinski and Werquin (2014) and Jacquet and Lehmann (2016) provide a rigorous theoretical analysis of the perturbation method. The main difference between these predecessors and our work is that we focus on a class of perturbations that contains a Condorcet winner.[8]

[7]See, for instance, Christiansen and Jansen (1978), Blundell, Brewer, Haan and Shephard (2009), Bourguignon and Spadaro (2012), Bargain, Dolls, Neumann, Peichl and Siegloch (2011), Zoutman, Jacobs and Jongen (2014), Lockwood and Weinzierl (2016).

[8]A minor difference to Golosov et al. (2014) is that we also consider perturbations that involve discontinuous jumps of marginal tax rates: An increase of marginal tax rates over an interval $[y_a, y_b]$ give rise to an upward jump of marginal tax rates at the income level y_a and hence induces bunching of taxpayers at y_a. Golosov et al. (2014) look at perturbations which avoid such discontinuous behavioral responses.

3 The model

3.1 Preferences

There is a continuum of individuals of measure 1. Individuals are confronted with a predetermined income tax schedule T_0 that assigns a (possibly negative) tax payment $T_0(y)$ to every level of pre-tax income $y \in \mathbb{R}_+$. Under the initial tax system individuals with no income receive a transfer equal to $c_0 \geq 0$. We assume that T_0 is everywhere differentiable so that marginal tax rates are well-defined for all levels of income. We also assume that $y - T_0(y)$ is a non-decreasing function of y and that $T_0(0) = 0$.

Individuals have a utility function u that is increasing in private goods consumption, or after-tax income, c, and decreasing in earnings or pre-tax income y. Utility also depends on a measure of the individual's productive ability, referred to as the individual's type. The set of possible types is denoted by Ω and taken to be a compact subset of the positive reals, $\Omega = [\underline{\omega}, \overline{\omega}] \subset \mathbb{R}_+$. A typical element of Ω will be denoted by ω. The utility that an individual with type ω derives from c and y is denoted by $u(c, y, \omega)$. The cross-section distribution of types in the population is represented by a cumulative distribution function F with density f. We write ω^x for the skill type with $F(\omega^x) = x$. For $x = \frac{1}{2}$, this yields the median skill type, also denoted by ω^M.

The slope of an individual's indifference curve in a y-c-diagram $-\frac{u_y(c,y,\omega)}{u_c(c,y,\omega)}$ measures how much extra consumption an individual requires as a compensation for a marginally increased level of pre-tax income. We assume that this quantity is decreasing in the individual's type, i.e. for any pair (c, y) and any pair (ω, ω') with $\omega' > \omega$,

$$-\frac{u_y(c, y, \omega')}{u_c(c, y, \omega')} \leq -\frac{u_y(c, y, \omega)}{u_c(c, y, \omega)} \; .$$

This assumption is commonly referred to as the *Spence-Mirrlees single crossing property.*

Occasionally, we will illustrate our results by looking at more specific utility functions. One case of interest is that there is a utility function $U : \mathbb{R}^2_+ \to \mathbb{R}$ so that $u(c, y, \omega) = U\left(c, \frac{y}{\omega}\right)$. We can then interpret ω as an hourly wage and $l = \frac{y}{\omega}$ as the time that an individual needs to generate a pre-tax-income of y. Another case of interest is that the utility function is quasi-linear in private goods consumption so that $u(c, y, \omega) = c - k(y, \omega)$. The function k then gives the cost of productive effort. The Spence-Mirrlees single crossing property holds if more productive individuals experience lower effort costs both in absolute terms and at the margin: For every y, and every ω, $k_2(y, \omega) < 0$ and $k_{12}(y, \omega) < 0$. If we combine both cases utility can be written as $c - \tilde{k}\left(\frac{y}{\omega}\right)$, where the function $\tilde{k}$ is taken to be increasing and convex. The cost function $\tilde{k}$ is said to be iso-elastic if it takes the form $\tilde{k}\left(\frac{y}{\omega}\right) = \left(\frac{y}{\omega}\right)^{1+\frac{1}{\epsilon}}$, for some parameter $\epsilon > 0$.

We assume that leisure is a non-inferior good. If individuals experience an increase in an exogenous source of income e, they do not become more eager to work. More formally, we assume that for any pair (c, y) any ω and any $e' > e$,

$$-\frac{u_y(c+e, y, \omega')}{u_c(c+e, y, \omega')} \leq -\frac{u_y(c+e', y, \omega)}{u_c(c+e', y, \omega)} .$$

We can also express this condition by requiring that, for any combination of c, y, e and ω, the derivative of $-\frac{u_y(c+e,y,\omega)}{u_c(c+e,y,\omega)}$ with respect to e is non-negative. This yields the following condition: For all c, y, e and ω,

$$-u_{cc}(c+e, y, \omega) \frac{u_y(c+e, y, \omega)}{u_c(c+e, y, \omega)} + u_{cy}(c+e, y, \omega) \leq 0 . \tag{1}$$

Finally, we assume that an individual's marginal utility of consumption $u_c(c, y, \omega)$ is both non-increasing in c and non-increasing in ω, i.e. $u_{cc}(c, y, \omega) \leq 0$ and $u_{c\omega}(c, y, \omega) \leq 0$. These assumptions hold for any utility function $u(c, y, w) = v(c) - k(y, \omega)$ that is additively separable between private goods consumption c on the one hand and the pair (y, ω) on the other, where v is a (weakly) concave function. With $u(c, y, \omega) = U\left(c, \frac{y}{\omega}\right)$, $u_{c\omega}(c, y, \omega) \leq 0$ holds provided that $U_{cl}\left(c, \frac{y}{\omega}\right) \geq 0$ so that working harder makes you more hungry.[9]

3.2 Reforms

A reform induces a new tax schedule T_1 that is derived from T_0 so that, for any level of pre-tax income y, $T_1(y) = T_0(y) + \tau\ h(y)$, where τ is a scalar and h is a function. We represent a reform by the pair (τ, h). Throughout we restrict attention to monotonic reforms, i.e. to reforms under which h is a non-decreasing function of y.[10] For instance, if $\tau > 0$ and h is strictly increasing, then the reform involves higher taxes for all levels of income and, moreover, the tax increase $T_1(y) - T_0(y)$ is larger for higher levels of income. Without loss of generality, we focus on reforms so that $y - T_1(y)$ is non-decreasing. The reform induces a change in tax revenue denoted by $\Delta^R(\tau, h)$. This additional tax revenue is used to increase the basic consumption level c_0.

Many results below follow from looking at a special class of reforms. For this class, there exists a first threshold level of income y_a, so that the new and the old tax schedule coincide for all income levels below the threshold, $T_0(y) = T_1(y)$ for all $y \leq y_a$. There exists a second threshold $y_b > y_a$ so that for all incomes between y_a and y_b marginal tax rates are increased by τ, $T_0'(y) + \tau = T_1'(y)$ for all $y \in (y_a, y_b)$. For all incomes above y_b, marginal tax rates coincide so that $T_0'(y) = T_1'(y)$ for all $y \geq y_b$. Hence, the function h is such that

$$h(y) = \begin{cases} 0, & \text{if } \ y \leq y_a , \\ y - y_a, & \text{if } \ y_a < y < y_b , \\ y_b - y_a, & \text{if } \ y \geq y_b . \end{cases}$$

[9] Seade (1982) refers to $U_{cl}\left(c, \frac{y}{\omega}\right) \geq 0$ as non-Edgeworth complementarity of leisure and consumption.

[10] The logic of the analysis would be the same if we instead considered non-increasing h-functions.

For reforms of this type we will write (τ, y_a, y_b) rather than (τ, h).

To study the implications of a reform it proves useful to introduce the following optimization problem: Choose y so as to maximize

$$u\left(c_0 + e + y - T_0(y) - \tau h(y), y, \omega\right) \ , \tag{2}$$

where e is a source of income that is exogenous from the individual's perspective. We assume that this optimization problem has, for each type ω, a unique solution that we denote by $y^*(e, \tau, \omega)$. The corresponding indirect utility function V is defined as

$$V(e, \tau, \omega) := u\left(c_0 + e + y^*(e, \tau, \omega) - T_0(y^*(e, \tau, \omega)) - \tau h(y^*(e, \tau, \omega)), y^*(e, \tau, \omega), \omega\right) \ .$$

Armed with this notation we can express the reform-induced change in tax revenue as

$$\Delta^R(\tau, h) = \int_{\underline{\omega}}^{\overline{\omega}} \{T_1(y^*(\Delta^R(\tau, h), \tau, \omega)) - T_0(y^*(0, 0, \omega))\} f(\omega) \, d\omega \ .$$

The reform-induced change in indirect utility for a type ω individual is given by

$$\Delta^V(\omega \mid \tau, h) := V(\Delta^R(\tau, h), \tau, \omega) - V(0, 0, \omega) \ .$$

For ease of exposition, we ignore the non-negativity constraint $y \leq 0$ in the body of the text, but relegate this extension to part B in the Appendix. There we clarify how the analysis has to be modified if there is a set of unemployed individuals whose labor market participation might be affected by a reform.

Pareto-improving reforms. A reform (τ, h) is said to be Pareto-improving if, for all $\omega \in \Omega$, $\Delta^V(\omega \mid \tau, h) \geq 0$, and if this inequality is strict for some $\omega \in \Omega$.

Welfare-improving reforms. We consider a class of social welfare functions. Members of this class differ with respect to the specification of welfare weights. Admissible welfare weights are represented by a non-increasing function $g : \Omega \to \mathbb{R}_+$ with the property that the average welfare weight equals 1, $\int_{\underline{\omega}}^{\overline{\omega}} g(\omega) f(\omega) d\omega = 1$. We denote by $G(\omega) := \int_{\omega}^{\overline{\omega}} g(s) \frac{f(s)}{1-F(\omega)} ds$ the average welfare weight among individuals with types above ω. Note that, if g is strictly decreasing, then $G(\omega) < 1$, for all $\omega > \underline{\omega}$. For a given function g, the welfare change that is induced by a reform is given by

$$\Delta^W(g \mid \tau, h) := \int_{\underline{\omega}}^{\overline{\omega}} g(\omega) \Delta^V(\omega \mid \tau, h) f(\omega) \, d\omega \ .$$

A reform (τ, h) is said to be welfare-improving if $\Delta^W(g \mid \tau, h) > 0$.

Political support for reforms. Political support for the reform is measured by the mass of individuals who are made better if the initial tax schedule T_0 is replaced by T_1,

$$S(\tau, h) := \int_{\underline{\omega}}^{\overline{\omega}} \mathbf{1}\{\Delta^V(\omega \mid \tau, h) > 0\} f(\omega) \, d\omega \ ,$$

where $\mathbf{1}\{\cdot\}$ is the indicator function. A reform (τ, h) is said to be supported by a majority of the population if $S(\tau, h) \geq \frac{1}{2}$.

3.3 Behavioral responses to reforms in the (τ, y_a, y_b)-class

Below we provide a characterization of a given (but otherwise arbitrary) status quo tax schedule T_0 by looking at the ways in which this status quo can be reformed. As will become clear, reforms in the (τ, y_a, y_b)-class are particularly useful for that purpose. These reforms, do however, involve discontinuous jumps of marginal tax rates at y_a and y_b. Lemmas 1-3 below clarify the behavioral responses to these discontinuities.

The Spence-Mirrlees single crossing property implies that, under any tax schedule, more productive individuals choose a higher level of pre-tax income. Thus, $\omega' > \omega$ implies that

$$y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega') \geq y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) ,$$

for $\tau' \in \{0, \tau\}$. We can therefore define threshold types $\omega_a(\tau')$ and $\omega_b(\tau')$ so that

$$\omega \leq \omega_a(\tau') \quad \text{implies} \quad y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) \leq y_a ,$$

$$\omega \in (\omega_a(\tau'), \omega_b(\tau')) \quad \text{implies} \quad y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) \in (y_a, y_b) ,$$

and

$$\omega \geq \omega_b(\tau') \quad \text{implies} \quad y^*(\Delta^R(\tau', y_a, y_b), \tau', \omega) \geq y_b .$$

The following Lemma asserts that, for a reform that involves an increase of the marginal tax rate, $\tau > 0$, type $\omega_a(0)$ who chooses an income of y_a before the reform does not choose a level of income above y_a after the reform. Analogously, if marginal taxes go down, type $\omega_b(0)$ does not choose an income above y_b after the reform. For a reform with $\tau > 0$, the logic is as follows: After the reform, because of the transfer Δ^R, a type $\omega_a(0)$-individual is, at income level y_a, less eager to work more. Working more also is less attractive after the reform because of the increased marginal tax rates for incomes above y_a. Thus, both the income and the substitution effect associated with the reform make it less attractive for a type $\omega_a(0)$-individual to increase her income above y_a. The individual will therefore either stay at y_a or decrease her income after the reform. Only higher types will end up with an income of y_a after the reform, which implies $\omega_a(0) \leq \omega_a(\tau)$.

Lemma 1

1. *Consider a reform so that $\tau > 0$ and $\Delta^R(\tau, y_a, y_b) > 0$. Then $\omega_a(\tau) \geq \omega_a(0)$.*
2. *Consider a reform so that $\tau < 0$ and $\Delta^R(\tau, y_a, y_b) < 0$. Then $\omega_b(\tau) \geq \omega_b(0)$.*

According to the next lemma a reform induces bunching of individuals who face an upward jump of marginal tax rates after the reform. Specifically, a reform with $\tau > 0$ will induce bunching at y_a because marginal tax rates jump upwards at income level y_a. A reform with $\tau < 0$ will induce bunching at y_b because marginal tax rates jump upwards at income level y_b.

Lemma 2

1. *Consider a reform so that* $\tau > 0$ *and* $\Delta^R(\tau, y_a, y_b) > 0$. *Then there is a set of types* $[\underline{\omega}_a(\tau), \overline{\omega}_a(\tau)]$ *who bunch at* y_a *after the reform.*

2. *Consider a reform so that* $\tau < 0$ *and* $\Delta^R(\tau, y_a, y_b) < 0$. *Then there is a set of types* $[\underline{\omega}_b(\tau), \overline{\omega}_b(\tau)]$ *who bunch at* y_b *after the reform.*

Individuals who bunch at y_a after a reform with $\tau > 0$ neither have an incentive to increase their earnings above y_a since

$$u_c(\cdot)(1 - T_1'(y_a)) + u_y(\cdot) = u_c(\cdot)(1 - T_0'(y_a) - \tau) + u_y(\cdot) \leq 0$$

nor an incentive to lower their earnings since

$$u_c(\cdot)(1 - T_0'(y_a)) + u_y(\cdot) \geq 0 \ .$$

By contrast, there are no individuals who bunch at y_b after a reform that involves increased marginal tax rates. Individuals who do not have an incentive to increase their income at y_b since

$$u_c(\cdot)(1 - T_1'(y_b)) + u_y(\cdot) = u_c(\cdot)(1 - T_0'(y_b)) + u_y(\cdot) \leq 0$$

definitely have an incentive to lower their income as

$$u_c(\cdot)(1 - T_0'(y_b) - \tau) + u_y(\cdot) < 0 \ .$$

An analogous argument implies that no one will bunch at y_a after a reform that involves a decrease of marginal tax rates.

The next Lemma establishes that, in the absence of income effects, for a reform with $\tau > 0$, individuals who choose an income above y_b after the reform also chose an income above y_b before the reform. This ordering is reversed for a reform with $\tau < 0$. We will subsequently discuss why these statements need no longer be true if there are income effects.

Lemma 3 *Suppose that there are no income effects, i.e. for all* (c, y, ω) *and any pair* (e, e') *with* $e' > e$,

$$-\frac{u_y(c + e, y, \omega')}{u_c(c + e, y, \omega')} = -\frac{u_y(c + e', y, \omega)}{u_c(c + e', y, \omega)} \ .$$

1. *Consider a reform so that* $\tau > 0$ *and* $\Delta^R(\tau, y_a, y_b) > 0$. *Then* $\omega_b(\tau) \geq \omega_b(0)$.

2. *Consider a reform so that* $\tau < 0$ *and* $\Delta^R(\tau, y_a, y_b) < 0$. *Then* $\omega_a(\tau) \geq \omega_a(0)$.

If there are no income effects, then, after a reform with $\tau > 0$, an individual with type $\omega_b(0)$ prefers an income level of y_b over any income above y_b before and after the reform since (i) the indifference curve through $(c_0 + y_b - T_0(y_b), y_b)$ has the same slope as the indifference curve through $(c_0 + \Delta^R + y_b - T_1(y_b), y_b)$ and (ii) for $y > y_b$, $T_0'(y) = T_1'(y)$ so that the incentives to increase income above y_b are unaffected by the reform. The individual has, however, an incentive to lower y since, because of the increased marginal tax rate, working less has become cheaper; i.e. it is no longer associated with as big a reduction of consumption. Thus, $\omega_b(0) \leq \omega_b(\tau)$ if there are no income effects. Figure 1 provides an illustration. With income effects there is also an opposing force since the individual also has to pay additional taxes $\tau(y_b - y_a) - \Delta^R$ which tends to flatten the indifference curve through y_b. Thus, there may both be income levels below y_b and income levels above y_b that the individual prefers over y_b. If the indifference curve flattens a lot, the individual will end up choosing $y > y_b$ after the reform which implies that $\omega_b(\tau) < \omega_b(0)$.

3.4 Types and earnings

In our theoretical framework, earnings depend inter alia on the individuals' types. Observed tax policies, however, express marginal tax rates as a function of income. To relate the results from our theoretical analysis to observed tax policies we repeatedly invoke the function $\tilde{y}^0 : \Omega \to \mathbb{R}_+$ where $\tilde{y}^0(\omega) = y^*(0, 0, \omega)$ gives earnings as a function of type in the status quo. We denote the inverse of this function by $\tilde{\omega}^0$ so that $\tilde{\omega}^0(y)$ is the type who earns an income of y in the status quo.

By the Spence-Mirrlees single crossing property the function $\tilde{y}^0$ is weakly increasing. The existence of its inverse $\tilde{\omega}^0$ requires in addition that, under the status quo schedule T_0, there is no bunching so that different types choose different levels earnings.

Assumption 1 *The function $\tilde{y}^0$ is strictly increasing.*

It is not difficult to relax this assumption and requires only a minor adaptation of the analysis: In all that follows, for reforms (τ, y_a, y_b)-class where y_a is, under the status quo schedule T_0, a point of bunching, the function $\tilde{\omega}^0$ has to be replaced by the function $\tilde{\omega}^{0b}$ with $\tilde{\omega}^{0b}(y) = \max\{\omega \mid y^*(0, 0, \omega) = y\}$. However, taking account of bunching requires additional steps in the formal analysis that we relegate to part B of the Appendix. This extension is relevant because empirically observed tax schedules frequently have kinks and hence give rise to bunching, see e.g. Saez (2010) and Kleven (2016). It is therefore important to clarify whether reforms that modify such kink points yield Pareto- or welfare improvements and whether they are politically feasible. That said, in the body of the text, we impose Assumption 1 without further mention.

In principle, the functions $\tilde{y}^0$ and $\tilde{\omega}^0$ could be estimated from any data set that contains both data on individual earnings and productive abilities. In the original analysis of Mirrlees (1971) hourly wages are taken to be the measure of productive abilities. Our framework is consistent with this approach, but it would also be compatible with other measures of ability.

When we illustrate our formal results be means of examples, we will occasionally assume that both the distributions of earnings and the distribution of productive abilities are Pareto-distributions.[11] Under these assumptions, earnings are an iso-elastic function of types.

Lemma 4 *Suppose that the type distribution is a Pareto distribution that is represented by the cdf* $F(\omega) = 1 - \left(\frac{\omega_{min}}{\omega}\right)^a$*. Also suppose that, under the status quo tax schedule* T_0*, the earnings distribution is a Pareto-distribution with cdf* $\tilde{F}^0(y) = 1 - \left(\frac{y_{min}}{y}\right)^b$*, where* $y_{\min} > 0$ *and* $\omega_{min} > 0$*. Then, for some multiplicative constant* α*,*

$$\tilde{y}^0(\omega) = \alpha\, \omega^\gamma \quad \textit{and} \quad \tilde{\omega}^0(y) = \left(\frac{y}{\alpha}\right)^{\frac{1}{\gamma}} , \quad \textit{where} \quad \gamma = \frac{a}{b} .$$

Lemma 4 gives a relation between types and earnings that holds only in the status quo. A sizable tax reform might affect this relation. Even if we were prepared to assume that the post-reform earnings distribution is also a Pareto distribution, the coefficients b and α might change with τ and Δ^R. More formally, $\tilde{y}^0(\omega) := y^*(0, 0, \omega) = \alpha\, \omega^\gamma$, does not imply that for any reform (τ, y_a, y_b), $y^*(\Delta^R(\tau, y_a, y_b), \tau, \omega) = \alpha\, \omega^\gamma$.

4 Preferences over reforms: A median voter theorem

In this section we establish a first main result: Under an ancillary condition, a reform is preferred by a majority of tax payers if and only if it is preferred by the median voter. The ancillary condition is that the reform belongs to the (τ, y_a, y_b)-class or that the reform (τ, h) is such that post-reform-earnings satisfy, for all types ω, the first order condition of the optimization problem in (2).

Assumption 2 *The reform* (τ, h) *belongs to the* (τ, y_a, y_b)*-class or, for all* ω*,* $y^*(e, \tau, \omega)$ *solves*

$$u_c(c_0 + e + y - T_1(y), y, \omega)(1 - T_1'(y)) + u_y(c_0 + e + y - T_1(y), y, \omega) = 0 .$$

for $e = \Delta^R(\tau, h)$*.*

We impose this assumption in the remainder of this section with no further mention.

Lemma 5 below is the key to the proof of Theorem 1. Consider a τ-Δ^R diagram and let $s(\tau, \Delta^R, \omega)$ be the slope of a type ω individuals' indifference curve through point (τ, Δ^R). This slope is a measure of how much of additional transfers an individual requires to compensate for increased tax rates. The following Lemma provides a characterization.

[11] Diamond (1998) takes hourly wages as a measure of productive abilities and shows that relevant parts of the wage distribution are well approximated by a Pareto-distribution. Saez (2001) argues that relevant parts of the earnings distribution are also well approximated by a Pareto-distribution.

Lemma 5 *For any ω and any point (τ, Δ^R), $s(\tau, \Delta^R, \omega) = h(\tilde{y}^1(\omega))$.*

To illustrate this result consider a reform in the (τ, y_a, y_b)-class. Individuals who choose earnings below y_a are not affected by the increase of tax rates. As a consequence, $s(\tau, \Delta^R, \omega) = h(\tilde{y}^1(\omega)) = 0$ which means that they are indifferent between a tax increase $\tau > 0$ and increased transfers $\Delta^R \geq 0$ only if $\Delta^R = 0$. Put differently, as soon as $\tau > 0$ and $\Delta^R > 0$, they are no longer indifferent, but benefit from the reform. Individuals with higher levels of income are affected by the increase of the marginal tax rate, and would be made worse off by any reform with $\tau > 0$ and $\Delta^R = 0$. Keeping them indifferent requires $\Delta^R > 0$ as reflected by the observation that $s(\tau, \Delta^R, \omega) = h(\tilde{y}^1(\omega)) > 0$. Moreover, since h is an increasing function of y, the higher an individual's income the larger is the increase in Δ^R that is needed in order to compensate the individual for an increase of marginal tax rates.

Under the Spence-Mirrlees single crossing property $\tilde{y}^1(\omega)$ is a non-decreasing function of ω. We therefore obtain the following Corollary to Lemma 5.

Corollary 1 *For any pair ω, ω' with $\omega' > \omega$, and any pair (Δ^R, τ),*

$$\tilde{y}^1(\omega) \leq \tilde{y}^1(\omega') \quad \textit{and} \quad s(\tau, \Delta^R, \omega) \leq s(\tau, \Delta^R, \omega') \; .$$

Corollary 1 establishes a single-crossing property for indifference curves in a τ-Δ^R-space. At any point in this space, the indifference curve of a richer individual is steeper than the indifference curve of a poorer individual. Consequently, any such pair of indifference curves crosses at most once.

Roberts (1977) has used a more specific single-crossing property to show that the median voter's preferred tax policy is a Condorcet winner in the set of affine tax policies, see the discussion in Gans and Smart (1996). Our analysis generalizes this observation in two ways. First, we show that a single crossing-property holds if we consider reforms that move away from some predetermined non-linear tax schedule, and not only to reforms that modify a predetermined affine tax schedule. Second, we do not require that marginal tax rates are increased for all levels of income. For instance, the single-crossing property also holds for reforms where the increase applies only to a subset $[y_a, y_b]$ of all possible income levels. This enables us to prove the following Theorem.

Theorem 1

1. *If the median voter prefers a reform over the status quo, then there is a majority of voters who strictly prefer the reform over the status quo.*

2. *If the median voter weakly prefers the status quo over a reform, then there is a majority of voters who weakly prefer the status quo over the reform.*

Proof Consider a reform that involves $\tau > 0$ and $\Delta^R > 0$ and moreover is such that the individual with the median skill type is made strictly better off, so that

$$V_e(\Delta^R, \tau, \omega^M)\ d\,\Delta^R + V_\tau(\Delta^R, \tau, \omega^M)\ d\,\tau = u_c(\cdot)\left(d\,\Delta^R - h(y^*(\Delta^R, \tau, \omega^M))\ d\,\tau\right) > 0,$$

or, equivalently,

$$s(\tau, \Delta^R, \omega^M) = h(y^*(\Delta^R, \tau, \omega^M)) < \frac{d\,\Delta^R}{d\,\tau}\ ,$$

then, by Corollary 1,

$$s(\tau, \Delta^R, \omega) = h(y^*(\Delta^R, \tau, \omega)) < \frac{d\,\Delta^R}{d\,\tau}\ ,$$

for all $\omega < \omega^M$ which implies that the reform is preferred by a majority of individuals, $S(\tau, h) \geq \frac{1}{2}$. Conversely, if

$$s(\tau, \Delta^R, \omega^M) = h(y^*(\Delta^R, \tau, \omega^M)) \geq \frac{d\,\Delta^R}{d\,\tau}\ ,$$

so the median voter prefers the status quo schedule weakly over the reformed schedule, then

$$s(\tau, \Delta^R, \omega) = h(y^*(\Delta^R, \tau, \omega)) \geq \frac{d\,\Delta^R}{d\,\tau}\ ,$$

for all $\omega > \omega^M$ which implies that the status quo is preferred weakly by a majority of individuals, i.e. $S(\tau, h) \leq \frac{1}{2}$. Analogously, one can show that a reform that involves $\tau < 0$ and $\Delta^R < 0$ is supported by a majority of tax payers if and only if it is preferred by the median voter. □

The next two Corollaries also follow from the observation that individual preferences over reforms are monotonic in incomes and therefore, by the Spence-Mirrlees single crossing property, monotonic in types.

Corollary 2

1. *A reform (τ, h) with $\tau > 0$ is Pareto-improving if and only if the most-productive or richest individual is not made worse off, i.e. if and only if $\Delta^V(\overline{\omega} \mid \tau, h) \geq 0$.*

2. *A reform (τ, h) with $\tau < 0$ is Pareto-improving if and only if the least-productive or poorest individual is not made worse off, i.e. if and only if $\Delta^V(\underline{\omega} \mid \tau, h) \geq 0$.*

Corollary 2 will be useful for answering the question whether a given status quo tax policy admits Pareto-improving reforms.

The next Corollary clarifies the conditions under which a reform is beneficial for the poorest individuals in society, say the bottom x percent, or redistributes at the cost of the richest individuals in society, e.g. the top x' per cent.

Corollary 3

1. *A reform (τ, h) with $\tau > 0$ benefits the bottom x per cent of the income distribution if and only if $\Delta^V(\omega^x \mid \tau, h) \geq 0$.*

2. *A reform (τ, h) with $\tau > 0$ harms the top x' per cent of the income distribution if and only if $\Delta^V(\omega^{1-x'} \mid \tau, h) \leq 0$.*

3. *A reform (τ, h) with $\tau > 0$ benefits the bottom x per cent at the cost of the top $1-x$ per cent if and only if $\Delta^V(\omega^x \mid \tau, h) = 0$.*

The preceding results involved no interpersonal comparison of utilities. They involved only marginal rates of substitution which remain unaffected by monotone transformations of utilities. The following Lemma, by contrast, involves an interpersonal comparison of utilities. Under the assumption that any one individuals' well-being can be measured by the utility function u, it shows that the reform-induced utility gains are a monotonic function of the individuals' types. For a reform with $\tau > 0$, the marginal utility gain of a type ω' is a lower bound for the utility gain of any lower type $\omega < \omega'$.

The Lemma looks at how a marginal change of τ affects individuals while taking into account that a change in τ affects $\Delta^R(\tau, h)$ via its derivative with respect to the first argument, $\Delta^R_\tau(\tau, h)$. It asserts that the partial derivative of $\Delta^V(\omega \mid \tau, h)$ with respect to τ is a decreasing function of ω.

Lemma 6 *For any pair ω and $\omega' > \omega$, $\Delta^V_\tau(\omega \mid \tau, h) \geq \Delta^V_\tau(\omega' \mid \tau, h)$.*

The proof of the Lemma involves two steps. The first is to show that, for all ω,

$$\Delta^V_\tau(\omega \mid \tau, h) = \tilde{u}^1_c(\omega) \left(\Delta^R_\tau(\tau, h) - h(\tilde{y}^1(\omega)) \right) , \tag{3}$$

where $\tilde{u}^1_c(\omega)$ is a shorthand for the marginal utility of consumption that a type ω individual realizes after the reform. By our previous arguments,

$$\Delta^R_\tau(\tau, h) - h(\tilde{y}^1(\omega))$$

is a non-increasing function of ω. The second step, then is to show that $\tilde{u}^1_c(\omega)$ is a non-increasing function of ω. It follows from the assumption that leisure is a non-inferior good and that $u_{c\omega}(c, y, \omega) \leq 0$.

5 Pareto-improving reforms

From now on, we focus on reforms that belong to the (τ, y_a, y_b)-class. As will become clear, this class of reforms is particularly useful for the derivation of conditions under which a given initial tax schedule T_0 admits Pareto-improving reforms, welfare-improving reforms, or reforms that are politically feasible. In this section, we focus on Pareto-improving reforms.

Recall equation (3),

$$\Delta_\tau^V(\omega \mid \tau, y_a, y_b) = \tilde{u}_c^1(\omega) \left(\Delta_\tau^R(\tau, y_a, y_b) - h(\tilde{y}^1(\omega)) \right) \ .$$

Consequently, a reform (τ, y_a, y_b) with $\tau > 0$ is Pareto-improving if and only if

$$\Delta_\tau^R(\tau, y_a, y_a) - h(\tilde{y}^1(\omega)) \geq 0 \ ,$$

for all ω, with a strict inequality for some types ω. For a reform in the (τ, y_a, y_b)-class, the function h is increasing with a minimal value of 0 and a maximal value of $y_b - y_a$. Such a reform is therefore Pareto-improving if and only if

$$\Delta_\tau^R(\tau, y_a, y_b) - (y_b - y_a) \geq 0 \ . \tag{4}$$

By contrast, a reform (τ, y_a, y_b) with $\tau < 0$ is Pareto-improving if and only if

$$\Delta_\tau^R(\tau, y_a, y_a) - h(\tilde{y}^1(\omega)) \leq 0 \ ,$$

for all ω with a strict inequality for some ω, or, equivalently, if

$$\Delta_\tau^R(\tau, y_a, y_b) \leq 0 \ . \tag{5}$$

According to (4) a tax-increasing reform is Pareto-improving if the increase in tax revenue is so strong that even those with an income above y_b are compensated for the additional taxes they have to pay. According to (5) a tax-decreasing reform is Pareto-improving if and only it raises tax revenue, i.e. if and only if the status quo tax rates exceed a Laffer bound. We first provide a characterization of this Laffer bound by looking at the implications of a reform for tax revenue. Subsequently, we turn to the possibility of Pareto-improving tax increases.

5.1 Revenue-increasing reforms and Pareto-improving tax cuts

Proposition 1 below provides conditions for the existence of revenue increasing reforms.

To be able to state the Proposition in a concise way, we introduce some notation. We define

$$D^R(\omega_0) := -\frac{1 - F(\omega_0)}{f(\omega_0)} \left(1 - \tilde{I}_0(\omega_0)\right) \frac{y_\omega^*(0, 0, \omega_0)}{y_\tau^*(0, 0, \omega_0)},$$

where

$$\tilde{I}_0(\omega_0) := \int_{\omega_0}^{\overline{\omega}} T_0'(y^*(0,0,\omega))\; y_e^*(0,0,\omega) \frac{f(\omega)}{1-F(\omega_0)}\; d\omega \;, \tag{6}$$

and

$$\mathcal{D}^R(y) := D^R(\tilde{\omega}^0(y)) \;.$$

We provide a more detailed interpretation of these expressions below. For now, we simply note that the function $D^R : \Omega \rightarrow \mathbb{R}$ is shaped by the hazard rate $\frac{1-F}{f}$ of the type distribution and by behavioral responses as reflected by the partial derivatives of the function y^*. In particular, $\tilde{I}_0(\omega_0)$ is a measure of the behavioral response that can be attributed to the income effect $y_e^*(0,0,\omega_0)$. In the absence of income effects this expression would be equal to 0. It would also be equal to zero if the status quo was the laissez-faire schedule with marginal tax rates of zero at all level of income. The function $\mathcal{D}^R = D^R \circ \tilde{\omega}^0$ translates D^R into a function of earnings, rather than types.

Proposition 1

1. *Suppose that there is an income level y_0 so that $T_0'(y_0) < \mathcal{D}^R(y_0)$. Then there exists a revenue-increasing reform with $\tau > 0$, and $y_a < y_0 < y_b$.*

2. *Suppose that there is an income level y_0 so that $T_0'(y_0) > \mathcal{D}^R(y_0)$. Then there exists a revenue-increasing reform with $\tau < 0$, and $y_a < y_0 < y_b$.*

In Section 5.1.2 we provide a sketch of the derivation of Proposition 1. We first provide an interpretation.

Proposition 1 shows that the function $\mathcal{D}^R$ is a tool for checking whether under a given status quo tax schedule T_0 there is scope for a revenue-increasing reform. If $\mathcal{D}^R(y_0) > T_0'(y_0)$, then tax-revenue can be increased by raising marginal tax rates in a neighborhood of y_0. Note that $\mathcal{D}^R$ is not itself the tax revenue-maximizing tax schedule; i.e. it is only useful for the purpose of learning something about a given status quo tax schedule, and not itself an admissible tax policy, see Section 5.1.3 below for a further discussion of this point.

Using elasticities to derive diagnosis schedules. All partial derivatives of the function y^* shape D^R. The effect of increased transfers Δ^R on pre-tax incomes enters via $\tilde{I}_0(\omega_0)$, the effect of an increased marginal tax rate and the effect of increased productive abilities via the ratio $\frac{y_\omega^*(0,0,\omega_0)}{y_\tau^*(0,0,\omega_0)}$. This ratio admits an interpretation as an elasticity of labor supply with respect to the net wage rate. To see this, suppose that pre-tax income can be written as $y = \omega l$, where l is labor supply in hours and ω is the hourly gross wage. These assumptions appear particularly natural for a utility function of the form $u(c,y,\omega) = U\left(c, \frac{y}{\omega}\right)$. For a given tax system, after-tax income can then be written as a function of l and is given by $C(l) := c_0 + \omega l - T_0(\omega l) - \tau h(\omega l)$. An increase in hours then translates into an increase of consumption according to $C'(l) = \omega(1 - T_0'(y) - \tau h'(y))$, where we used once more that $y = \omega l$. Thus, we can think of the expression $\omega_n(\tau) := \omega(1 - T_0'(y) - \tau h'(y))$ as a net wage that gives the increase in after-tax-income that is made possible by working one more hour. We now look at the utility-maximization problem of an individual that faces a fixed net wage given by $\omega_n(\tau)$ so that $C(l) = e + \omega_n(\tau)\, l$. The individual chooses l so as to maximize $U(C(l), l)$. The utility-maximizing labour supply l^* is then a function of only two arguments, the lump-sum transfer e and the net wage $\omega_n(\tau)$. Thus, we can write $y^*(e, \tau, \omega) = \omega\ l^*(e, \omega_n(\tau)) = \omega\ l^*(e, \omega(1 - T_0' - \tau h'))$, where T_0' and h' are kept constant as labor supply varies. Suppose that the individual's income falls in the range that is affected by the change in marginal tax rates, hence $h' = 1$ and $y^*(e, \tau, \omega) = \omega\ l^*(e, \omega(1 - T_0' - \tau))$. Straightforward computations yield

$$y^*_\omega(e, \tau, \omega) = l^*(e, \omega(1 - T_0' - \tau)) + \omega_n(\tau)\ l^*_{\omega_n}(e, \omega(1 - T_0' - \tau)) ,$$

$$y^*_\tau(e, \tau, \omega) = -\omega^2\ l^*_{\omega_n}(e, \omega(1 - T_0' - \tau)) ,$$

and, therefore,

$$\frac{y^*_\omega(e, \tau, \omega)}{y^*_\tau(e, \tau, \omega)} = -\frac{1 - T_0' - \tau}{\omega} \left(1 + \frac{1}{\varepsilon(e, \omega_n(\tau))}\right) ,$$

where $\varepsilon(e, \omega_n(\tau)) = \frac{\omega_n(\tau)}{l^*(e, \omega_n(\tau))} l^*_{\omega_n}(e, \omega_n(\tau)))$ is the elasticity of hours with respect to the net wage. If we evaluate these expressions for $e = 0$ and $\tau = 0$, and moreover assume that T_0' has been fixed at $T_0'(y^*(0, 0, \omega))$ we obtain

$$\frac{y^*_\omega(0, 0, \omega)}{y^*_\tau(0, 0, \omega)} = -\frac{1 - T_0'(y^*(0, 0, \omega))}{\omega} \left(1 + \frac{1}{\varepsilon(0, \omega_n(0))}\right) . \tag{7}$$

Using this equation and the definitions

$$\tilde{D}^R(\omega) := \frac{1 - F(\omega)}{f(\omega)\ \omega} \left(1 - \tilde{I}_0(\omega)\right) \left(1 + \frac{1}{\varepsilon(0, \omega_n(0))}\right)$$

and

$$\tilde{\mathcal{D}}^R(y) := \tilde{D}^R(\tilde{\omega}^0(y))$$

we obtain the following Corollary to Proposition 1.

Corollary 4 *Suppose that the function u is such that equation (7) holds for all ω.*

1. *Suppose there is an income level y_0 so that $\frac{T_0'(y_0)}{1 - T_0'(y_0)} < \tilde{\mathcal{D}}^R(y_0)$. Then there exists a tax-revenue-increasing reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.*

2. *Suppose there is an income level y_0 so that $\frac{T_0'(y_0)}{1 - T_0'(y_0)} > \tilde{\mathcal{D}}^R(y_0)$. Then there exists a tax-revenue-increasing reform (τ, y_a, y_b) with $\tau < 0$, and $y_a < y_0 < y_b$.*

Corollary 4 is a particular version of what has become known as an ABC-formula à la Diamond (1998). The expression $\tilde{D}^R(\omega)$ is a product of an inverse hazard rate, usually referred to as A, an inverse elasticities term, C, and the third expression in the middle, B. Without income effects this middle term would simply be equal to 1. With income effects, however, we have to correct for the fact that a change of the intercept of the schedule $c_0 + y - T(y)$ affects the individuals' choices. In the presence of income effects, and with non-negative marginal tax rates in the status quo, $\tilde{I}_0(\omega_0) < 0$ because individuals become less eager to generate income if the intercept moves up. As a consequence, B exceeds 1 in the presence of income effects. Thus, everything else being equal, the right hand side becomes larger, so that there is more scope for revenue-increasing reforms if there are income effects. This effect can be shown on Figure 1. After the reform, the behavior of individuals with an income above y_b is as if they were facing a new schedule that differs from the old schedule only in the level of the intercept. The intercept is c_0 initially and $c_0 + \Delta^R - \tau(y_b - y_a) < c_0$ after the reform. Thus, for high income earners the intercept becomes smaller and they respond to this by increasing their earnings. These increased earnings translate into additional tax revenue. The term $-\tilde{I}_0(\omega_0) > 0$ takes account of this fiscal externality.

We can also write $\tilde{I}_0$ as an expression that depends on the elasticity of earnings with respect to the intercept of the schedule $c_0 + y - T(y)$. If we define this elasticity for a type ω-individual under the status quo schedule T_0 as

$$\eta_0(\omega) = y_e^*(0,0,\omega)\ \frac{c_0}{y^*(0,0,\omega)}\ ,$$

we can write

$$\tilde{I}_0(\omega_0) = \int_{\omega_0}^{\overline{\omega}} T_0'(y^*(0,0,\omega))\ \frac{y^*(0,0,\omega)}{c_0}\ \eta_0(\omega)\frac{f(\omega)}{1-F(\omega_0)}\ d\omega\ ,$$

an expression that can be computed as soon as estimates for the elasticities of earnings $\{\eta_0(\omega)\}_{\omega\in\Omega}$ with respect to the basic consumption level c_0 are available. The expressions $\tilde{D}^R(\omega)$ and $\tilde{\mathcal{D}}^R(y)$ can therefore be viewed as sufficient statistics that can be used to check whether revenue-increasing tax reforms are possible under a given status quo tax schedule. They are sufficient statistics in the sense that they not depend on a specific formulation of preferences, but only on the intensity of behavioral responses as expressed by the elasticities in $\{\eta_0(\omega)\}_{\omega\in\Omega}$ and $\{\varepsilon(0,\omega_n(0))\}_{\omega\in\Omega}$.

5.1.1 An example

We will repeatedly use a specific example of a status quo tax schedule for purposes of illustration. In this example, marginal tax rates equal to 0 for incomes below an exemption threshold y^e, and constant for incomes above a top threshold y^t. In between, marginal tax rates are a linearly increasing function of income. Consequently,

$$T_0'(y) = \begin{cases} 0, & \text{for} \quad y \leq y^e \ , \\ \beta(y - y^e), & \text{for} \quad y^e \leq y \leq y^t \ , \\ \beta(y^t - y^e), & \text{for} \quad y \geq y^t \ ; \end{cases} \tag{8}$$

where $\beta > 0$ determines how quickly marginal tax rates rise with income. Also note that

$$\frac{T_0'(y)}{1 - T_0'(y)} = \begin{cases} 0, & \text{for} \quad y \leq y^e \ , \\ \frac{\beta(y-y^e)}{1-\beta(y-y^e)}, & \text{for} \quad y^e \leq y \leq y^t \ , \\ \frac{\beta(y^t-y^e)}{1-\beta(y^t-y^e)}, & \text{for} \quad y \geq y^t \ . \end{cases}$$

Hence, for $y^e \leq y \leq y^t$, $\frac{T_0'}{1-T_0'}$ is an increasing and convex function of income. This status quo schedule is represented in Figure 2.

All our figures, with the exceptions of Figures 1 and 13, are drawn under the assumption of a status quo schedule as in (8). The figures also assume that both distribution of types and the distribution of incomes follow Pareto-distributions so that we can use Lemma 4 to connect types and earnings. In addition, the figures assume preferences of the form $u(c, y, \omega) = U\left(c, \frac{y}{\omega}\right)$ so that we can use $\tilde{\mathcal{D}}^R$ as a sufficient statistic for revenue-increasing tax reforms. Finally, the figures are drawn under the assumption that the elasticities in $\{\eta_0(\omega)\}_{\omega \in \Omega}$ and $\{\varepsilon(0, \omega_n(0))\}_{\omega \in \Omega}$ are constant across types, i.e., that there exist numbers η and ϵ so that, for all ω, $\eta_0(\omega) = \eta$ and $\varepsilon(0, \omega_n(0)) = \epsilon$.[12]

Figure 3 (resp. Figure 4) illustrate our approach without (resp. with) income effects. If there are no income effects the auxiliary schedule $\tilde{\mathcal{D}}^R$ is independent of the status quo tax schedule T_0. With income effects, $\tilde{\mathcal{D}}^R$ depends on the status quo schedule T_0 via the expression

$$\tilde{I}_0(\omega_0) = \int_{\omega_0}^{\overline{\omega}} T_0'(y^*(0, 0, \omega)) \ \frac{y^*(0, 0, \omega)}{c_0} \ \eta_0(\omega) \frac{f(\omega)}{1 - F(\omega_0)} \ d\omega \ .$$

A status quo schedule of particular interest is the laissez-faire-schedule with $c_0 = 0$ and $T_0'(y) = 0$, for all y. As a consequence, even if there are income effects ($\eta_0(\omega) < 0$ for a positive mass of agents), $\tilde{I}_0(\omega_0) = 0$ and hence $\tilde{\mathcal{D}}^R$ is as in Figure 3.

5.1.2 Sketch of the proof of Proposition 1

Proposition 1 follows from looking at a cross-derivative of the function $\Delta^R(\tau, y_a, y_b)$. The basic idea is as follows: if we choose $y_a = y_b$, then, obviously $\Delta^R(\tau, y_a, y_a) = 0$ and also $\Delta^R_\tau(\tau, y_a, y_a) = 0$ since the increased marginal tax rate applies only to a null set of incomes. However, suppose that the derivative of $\Delta^R_\tau(\tau, y_a, y_b)$ with respected to y_b is strictly positive if evaluated at $(\tau, y_a, y_b) = (0, y_a, y_a)$. This implies that $\Delta^R_\tau(0, y_a, y_a)$ becomes strictly positive as we slightly raise y_b above y_a. This in turn implies that there exists a pair y_b and $y_a < y_b$ and a tax rate τ so that $\Delta^R(\tau, y_a, y_b) > 0$.

[12]The figures are drawn for following parameters: $\beta = 0.08$, $y^e = 5$ and $y^t = \frac{15}{2}$; $c_0 = 1$; $\omega_{min} = y_{min} = 1$, $\alpha = 1$, $a = 2$ and $b = 0.5$; $\eta = -0.02$ and $\epsilon = 0.8$.

The formal proof of the proposition in the Appendix proceeds as follows: We first derive an expression for $\Delta^R_\tau(\tau, y_a, y_b)$ without making any assumption on τ, y_a and y_b. In particular, we do not assume a priori that τ is close to 0 or that y_b is close to y_a. We then evaluate this expression at $\tau = 0$ and obtain

$$\Delta^R_\tau(0, y_a, y_b) = \frac{1}{1 - I_0}\ \mathcal{R}(y_a, y_b),$$

where $I_0 := \tilde{I}(\underline{\omega})$ is our measure of income effects applied to the population at large and

$$\begin{array}{rcl} \mathcal{R}(y_a, y_b) & = & \int_{\omega_a}^{\omega_b} T_0'(y^*(0,0,\omega))\ y^*_\tau(0,0,\omega)\ f(\omega)\ d\omega \\ & & + \int_{\omega_a}^{\omega_b} \{y^*(0,0,\omega)) - y_a\}\ f(\omega)\ d\omega \\ & & +(y_b - y_a)(1 - F(w_b)) \\ & & -(y_b - y_a) \int_{\omega_b}^{\overline{\omega}} T_0'(y^*(0,0,\omega))\ y^*_e(0,0,\omega)\ f(\omega)\ d\omega\ . \end{array}$$

The first entry in this expression is a measure of how people with incomes in (y_a, y_b) respond to the increased marginal tax rate and the second entry is the mechanical effect according to which these individuals pay more taxes for a given level of income. The third entry is the analogous mechanical effect for people with incomes above y_b and the last entry gives their behavioral response due to the fact that they now operate on an income tax schedule with an intercept lower than c_0.

We now exploit the assumption that there is no bunching under the initial schedule, so that $y^*(0, 0, \omega)$ is strictly increasing and hence invertible over $[\omega_a, \omega_b]$. We can therefore, without loss of generality, view a reform also as being defined by τ, ω_a and ω_b. With a slight abuse of notation, we can therefore write

$$\Delta^R_\tau(0, \omega_a, \omega_b) = \frac{1}{1 - I_0}\ \mathcal{R}(\omega_a, \omega_b),$$

where

$$\begin{array}{rcl} \mathcal{R}(\omega_a, \omega_b) & = & \int_{\omega_a}^{\omega_b} T_0'(y^*(0,0,\omega))\ y^*_\tau(0,0,\omega)\ f(\omega)\ d\omega \\ & & + \int_{\omega_a}^{\omega_b} \{y^*(0,0,\omega) - y^*(0,0,\omega_a)\}\ f(\omega)\ d\omega \\ & & +(y^*(0,0,\omega_b) - y^*(0,0,\omega_a))(1 - F(w_b)) \\ & & -(y^*(0,0,\omega_b) - y^*(0,0,\omega_a)) \int_{\omega_b}^{\overline{\omega}} T_0'(y^*(0,0,\omega))\ y^*_e(0,0,\omega)\ f(\omega)\ d\omega. \end{array}$$

Note that $\Delta^R_\tau(0, \omega_a, \omega_a) = \frac{1}{1-I_0}\ \mathcal{R}(\omega_a, \omega_a) = 0$. If $\Delta^R_{\tau\omega_b}(0, \omega_a, \omega_a) = \frac{1}{1-I_0}\ \mathcal{R}_{\omega_b}(\omega_a, \omega_a) > 0$, then $\Delta^R_\tau(0, \omega_a, \omega_b)$ turns positive, if starting from $\omega_a = \omega_b$, we marginally increase ω_b. Straightforward computations yield:

$$\mathcal{R}_{\omega_b}(\omega_a, \omega_a) = T_0'(y^*(0,0,\omega_a))\ y^*_\tau(0,0,\omega_a)\ f(\omega_a) + (1 - \tilde{I}_0(\omega_a))\ y^*_\omega(0,0,\omega_a)\ (1 - F(\omega_a))\ .$$

Hence, if this expression is positive we can increase tax revenue by increasing marginal tax rates in a neighborhood of y_a. Using $y^*(0, 0, \omega_a) = y_a$, $\omega_a = \tilde{\omega}^0(y_a)$ and $y^*_\tau(0, 0, \omega_a) < 0$ (which is formally shown in the Appendix), the statement $\mathcal{R}_{\omega_b}(\omega_a, \omega_a) > 0$ is easily seen to be equivalent to $T_0'(y_a) < D^R(\omega_a) = \mathcal{D}^R(y_a)$, as claimed in the first part of Proposition 1.

5.1.3 Revenue-increasing reforms vs. revenue-maximizing income taxation

The schedule $\tilde{D}^R$ that allows us to identify revenue-increasing reforms does not generally coincide with the revenue-maximizing income tax schedule T^R.[13] One reason is that $\tilde{D}^R$ depends on the status quo schedule T_0 via the function $\tilde{I}_0$ with

$$\tilde{I}_0(\omega_0) = \int_{\omega_0}^{\overline{\omega}} T_0'(y^*(0,0,\omega))\ y_e^*(0,0,\omega)\frac{f(\omega)}{1-F(\omega_0)}\ d\omega\ .$$

The status quo schedule will typically differ from the revenue-maximizing schedule.

The dependence on the status quo schedule disappears if there are no income effects so that preferences can be described by a utility function that is quasi-linear in private goods consumption. In this case, $\tilde{D}^R$ coincides with the revenue-maximizing income tax schedule for all levels of income where the latter does not give rise to bunching. This can be shown formally by looking at the problem to maximize tax revenue from a mechanism design perspective. The requirement that individuals choose earnings in a utility-maximizing way is then captured by incentive compatibility conditions. As is well known, incentive compatibility holds if and only if two conditions are met: First, a condition of local incentive-compatibility according to which no type of individual could reach a higher utility level by generating the earnings of a neighboring type, typically formalized as an envelope condition on an indirect utility function. Second, a monotonicity requirement on earnings.

A typical approach is to first solve a relaxed problem that ignores this monotonicity constraint and then to check whether or not the solution to this relaxed problem is monotonic or not. If it is, then the solution to the relaxed problem is also a problem of the full problem. If it is not, one has to deal with the complication that the monotonicity constraint is binding for certain subsets of types. The first-order conditions of the relaxed problem can be shown to imply

$$\frac{T^{R'}(\bar{y}^*(0,0,\omega))}{1-T^{R'}(\bar{y}^*(0,0,\omega))} = \frac{1-F(\omega)}{f(\omega)\ \omega}\left(1+\frac{1}{\epsilon}\right) = \tilde{D}^R(\omega)\ ,$$

with $\bar{y}^*(0,0,\omega) \quad := \quad \text{argmax}_{y'} \quad u\left(y' - T^R(y'), y', \omega\right)$. Hence, the sufficient statistic $\tilde{D}^R$ coincides with a solution to the relaxed problem of revenue-maximizing income taxation, but not necessarily with the solution to the full problem of revenue-maximizing income taxation. It coincides with the full problem only if the solution to the relaxed problem does not violate the monotonicity constraint on the function y. This observation also

[13] Our previous results lend themselves, however, to a characterization of the revenue-maximizing schedule T^R via the following condition: Let $\bar{y}^*(0,0,\omega)$ be the earnings level that a type ω individual chooses under a status quo equal to the revenue-maximizing tax schedule T^R. Then for all ω_0, so that $\bar{y}$ is a strictly increasing function of ω at ω_0, T^R has to satisfy

$$\frac{T^{R'}(\bar{y}^*(0,0,\omega_0))}{1-T^{R'}(\bar{y}^*(0,0,\omega_0))} = \tilde{D}^R(\omega_0) = \frac{1-F(\omega_0)}{f(\omega_0)\ \omega_0}\ \left(1-\tilde{I}_0(\omega_0)\right)\ \left(1+\frac{1}{\varepsilon(0,\omega_n(0))}\right)\ .$$

provides a justification for an analysis of the relaxed problem in a model with quasi-linear in consumption preferences. Even though the solution to the relaxed problem is not necessarily a tax schedule that can be implemented, it yields a sufficient statistic that can be used to check whether a given status quo schedule can be modified in such a way that tax revenue goes up.

5.2 Pareto-improving tax increases

The following Proposition derives conditions under which reforms that involve tax increases of marginal tax rates are Pareto-improving. It is the counterpart to Proposition 1 that characterizes Pareto-improving tax decreases. Proposition 2 also relies on an auxiliary tax schedule $\mathcal{D}^P$ which is defined by

$$D^P(\omega) := \frac{1}{f(\omega)} \left\{ \left(1 - \tilde{I}_0(\omega)\right) F(\omega) + \left(\tilde{I}_0(\omega) - I_0\right) \right\} \frac{y^*_\omega(0,0,\omega)}{y^*_\tau(0,0,\omega)} ,$$

and

$$\mathcal{D}^P(y) = D^P(\tilde{\omega}^0(y)) .$$

Proposition 2 *Suppose that there is an income level y_0 so that $T_0'(y_0) < \mathcal{D}^P(y_0)$. Then there exists a Pareto-improving reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.*

The proof of Proposition 2 can be found in the Appendix. It is based on adaptation of the arguments in Section 5.1.2 with the difference that now the revenue-increase must not only exceed a threshold of 0, but a larger threshold that is equal to $y_b - y_a$.

The auxiliary tax schedule $\mathcal{D}^P$ provides a lower bound for marginal tax rates. If marginal tax rates fall below this lower bound, then an increase of marginal tax rates is Pareto-improving.[14] Note that $\mathcal{D}^P(y)$ is often negative. To see this, fix an arbitrary type ω_0. If there are no income effects we have

$$D^P(\omega_0) = \frac{F(\omega_0)}{f(\omega_0)} \frac{y^*_\omega(0,0,\omega_0)}{y^*_\tau(0,0,\omega_0)} < 0 ,$$

since $y^*_\tau(0,0,\omega_0) < 0$. Alternatively, for low types, $\tilde{I}_0(\omega_0)$ is close to I_0 so that $D^P(\omega_0)$ is approximately equal to

$$\frac{F(\omega_0)}{f(\omega_0)}(1 - I_0)\frac{y^*_\omega(0,0,\omega_0)}{y^*_\tau(0,0,\omega_0)} ,$$

which is again negative. Thus, $\mathcal{D}^P$ can be interpreted as a Pareto-bound on earnings subsidies. Such subsidies are, for instance, part of the earned income tax credit in the

[14]Under the assumption of quasi-linear in consumption preferences the $\mathcal{D}^P$-schedule admits an alternative interpretation as the solution of relaxed problem with the objective to maximize the utility of the most productive individuals subject to the requirements of physical feasibility and an envelope condition which is necessary for incentive compatibility. Brett and Weymark (2016a) refer to $\tilde{D}^P$ as a maximax-schedule as it is derived from maximizing the utility of most privileged individual.

US. If those subsidies imply marginal tax rates lower than $D^P(\omega_0)$, then a reduction of these subsidies is Pareto-improving.

The following Corollary is the counterpart to Corollary 4. It introduces the sufficient statistic $\tilde{\mathcal{D}}^P$ that corresponds to the auxiliary schedule $\mathcal{D}^P$ which is defined by

$$\tilde{D}^P(\omega) := -\frac{1}{f(\omega)\omega}\left\{\left(1-\tilde{I}_0(\omega)\right)F(\omega)+\left(\tilde{I}_0(\omega)-I_0\right)\right\}\left(1+\frac{1}{\varepsilon(0,\omega_n(0))}\right),$$

and

$$\tilde{\mathcal{D}}^P(y) = \tilde{D}^P(\tilde{\omega}^0(y)) .$$

Corollary 5 *Suppose that the function u is such that equation (7) holds for all ω. Suppose that there is an income level y_0 so that $\frac{T_0'(y_0)}{1-T_0'(y_0)} < \tilde{\mathcal{D}}^P(y_0)$. Then there exists a Pareto-improving reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.*

Figure 5 extends Figure 3 so as to also include $\tilde{D}^P$. Note that with a Pareto-distribution of types the ratio $\frac{1-F(\omega)}{f(\omega)\omega}$ is the same for all types ω but the ratio $\frac{F(\omega)}{f(\omega)\omega}$ is not. This explains why in Figure 5 the schedule $\tilde{\mathcal{D}}^R$ is a horizontal line whereas $\tilde{\mathcal{D}}^P$ is not. Figure 4 assumes that there are no income effects. Figure 6, by contrast, illustrates a case with income effects.

6 Politically feasible reforms

By Corollary 1 a reform (τ, y_a, y_b) is politically feasible if and only if it is preferred by the median type ω^M or, equivalently, by the individual with the median income. By equation (3), a reform (τ, y_a, y_b) with $\tau > 0$ is politically feasible if and only if

$$\Delta_\tau^R(\tau, y_a, y_a) - h(\tilde{y}^1(\omega^M)) \geq 0 .$$

Consider first reforms so that under the status quo schedule $\tilde{y}^0(\omega^M) \leq y_a < y_b$, i.e. so that the change of marginal tax rates affects only incomes above the median. Consequently, $h(\tilde{y}^1(\omega^M)) = 0$ and the median voter prefers such a reform if and only if it increases tax revenue, $\Delta_\tau^R(\tau, y_a, y_a) > 0$. Recall that the schedule $\mathcal{D}^R$ is the tool that can be used to identify such situations. Consequently, the median voter prefers a small tax increase for incomes in a small neighborhood of y_a if $T_0'(y_a) < \mathcal{D}^R(y_a)$. Analogously, the median voter prefers a tax cut if $T_0'(y_a) > \mathcal{D}^R(y_a)$.

Now consider reforms so that $y_a < y_b \leq \tilde{y}^0(\omega^M)$. This implies that $h(\tilde{y}^1(\omega^M)) = y_b - y_a$ so that the median voter prefers such a reform if and only if $\Delta_\tau^R(\tau, y_a, y_a) > y_b - y_a$. The schedule $\mathcal{D}^P$ can be used to identify such constellations. The median voter supports reforms that involve a small tax increase for incomes in a small neighborhood of y_a if $T_0'(y_a) < \mathcal{D}^P(y_a)$. The median voter prefers a tax cut if $T_0'(y_a) > \mathcal{D}^P(y_a)$. We summarize these observations in the following Proposition that we state without proof.

Proposition 3 *Let*

$$\mathcal{D}^M(y) := \begin{cases} \mathcal{D}^P(y), & \text{if} \quad y < \tilde{y}^0(\omega^M) \ , \\ \mathcal{D}^R(y), & \text{if} \quad y \geq \tilde{y}^0(\omega^M) \ . \end{cases} \tag{9}$$

1. *Suppose there exists* $y_0 \neq \tilde{y}^0(\omega^M)$ *so that* $T_0'(y_0) < \mathcal{D}^M(y_0)$*. Then there is a politically feasible reform* (τ, y_a, y_b) *with* $\tau > 0$*, and* $y_a < y_0 < y_b$*.*

2. *Suppose there exists* $y_0 \neq \tilde{y}^0(\omega^M)$ *so that* $T_0'(y_0) > \mathcal{D}^M(y_0)$*. Then there is a politically feasible reform* (τ, y_a, y_b) *with* $\tau < 0$*, and* $y_a < y_0 < y_b$*.*

In plain words, the median voter wants tax increases for those who are richer as long as their tax rates are below the Laffer bound $\mathcal{D}^R$. The median voter wants tax cuts for those who are poorer as long as their tax rates are above the lower Pareto-bound $\mathcal{D}^P$.

If the function u is such that equation (7) holds for all ω, we can again define a sufficient statistic for politically feasible tax reforms,

$$\tilde{\mathcal{D}}^M(y) := \begin{cases} \tilde{\mathcal{D}}^P(y), & \text{if} \quad y < \tilde{y}^0(\omega^M) \ , \\ \tilde{\mathcal{D}}^R(y), & \text{if} \quad y \geq \tilde{y}^0(\omega^M) \ . \end{cases} \tag{10}$$

If $\frac{T_0'(y)}{1-T_0'(y)}$ exceeds $\tilde{\mathcal{D}}^M(y)$ there are politically feasible tax cuts, if $\tilde{\mathcal{D}}^M(y)$ exceeds $\frac{T_0'(y)}{1-T_0'(y)}$ there are politically feasible tax increases.

Figures 6 and 8 illustrate the relation between the sufficient statistics $\tilde{\mathcal{D}}^R$, $\tilde{\mathcal{D}}^P$ and $\tilde{\mathcal{D}}^M$. The Figures show that the schedule $\tilde{\mathcal{D}}^M$ is discontinuous at the median voter's type. It jumps from $\tilde{\mathcal{D}}^P$ to $\tilde{\mathcal{D}}^R$. This jump reflects that the median voter is easy to convince that reforms that raise marginal tax rates only for incomes above the median are worthwhile, while she is more hesitant to support reforms that also imply a higher tax burden for her.

The discontinuity in Figures 6 and 8 invites a speculative remark about a force that may shape real world tax schedules. A common complaint is that real world tax schedules exhibit too much progressivity for middle incomes (e.g. Zoutman, Jacobs and Jongen (2016)). A look at these figures suggests that the median voter would appreciate any attempt to move quickly from $\tilde{\mathcal{D}}^P$ to $\tilde{\mathcal{D}}^R$ as one approaches median income. Any attempt to do so in a continuous fashion will imply a strong increase of marginal rates for incomes in a neighborhood of the median. This increase in marginal tax rates is the price to be paid for having marginal tax rates close to $\tilde{D}^P$ for incomes below the median and for having marginal tax rates close to $\tilde{D}^P$ for incomes above the median.

The approach that yields a characterization of politically feasible reforms can be adapted so as to characterize reforms that are, say, appealing to the bottom x percent of the income distribution. The relevant auxiliary schedule for this purpose is

$$\mathcal{D}^x(y) := \begin{cases} \mathcal{D}^P(y), & \text{if} \quad y < \tilde{y}^0(\omega^x) \ , \\ \mathcal{D}^R(y), & \text{if} \quad y \geq \tilde{y}^0(\omega^x) \ . \end{cases} \tag{11}$$

We can also define the corresponding sufficient statistic $\tilde{\mathcal{D}}^x$ in the obvious way. Figure 8 below provides an illustration. The discontinuity shifts from the median to a different percentile of the income distribution if we do not seek to characterize the reforms that are appealing to a majority of the population but that, are, say, appealing to people with incomes in the bottom 70 percent.

Political competition. The schedule $\mathcal{D}^M$ allows us to identify reforms that are politically feasible in the sense that the reformed tax schedule is preferred over the status quo tax schedule by a majority of the population. Now suppose that we enrich the analysis by a strategic game of political competition between two vote-share maximizing policy-makers as in the Downsian model of political competition, Downs (1957). To what extent can $\mathcal{D}^M$ be interpreted as a prediction for the outcome of such strategic interaction? Different assumptions about the set of admissible reforms i.e. about the strategy space in such a strategic game, give rise to different answers to this question.

Let us first look at an extreme case where the set of admissible reforms is heavily restricted. There is a status quo tax schedule T_0 in place, and admissible reforms must belong to the (τ, y_a, y_b)-class. Moreover, assume for the sake of the argument, that political campaigns focus on the taxation of the rich. Specifically, suppose that reforms need to satisfy $y_a \geq y^{\cdot 9}$, where $y^{\cdot 9}$ is the highest income in the bottom 90 percent. Hence, competition is only over reforms that affect the tax rates of the top 10 percent in the income distribution. Then, by Theorem 1, we would predict that both candidates propose the median voter's preferred perturbation and the equilibrium prediction would indeed be that marginal tax rates are set in a revenue-maximizing way, i.e. according to $\mathcal{D}^R$. This is exactly the force that is driving in the analysis of Roberts (1977), except that the increase in marginal tax rates is not applied uniformly to all taxpayers but only to the top 10 percent.

The other extreme case is that reforms are entirely unrestricted. Then a politician who proposes a reform that is attractive for the median type, can still be defeated by an alternative proposal that combines a reform that is attractive to all voters with an income below $y^{\cdot 4}$ with a reform that is attractive to all voters with an income above $y^{\cdot 6}$ and thereby gain the support of a majority of the population. This is the familiar observation that, with a multi-dimensional policy space, there is typically no Condorcet winner, i.e. there is no reform that wins a majority against any other reform.

By Theorem 1, a Condorcet winner exists, however, if attention is restricted to reforms under which the change in marginal tax rates is monotonic in income. This has the following implication: Suppose that the status quo schedule T_0 is such that the median voter's preferred tax schedule, denoted by T^M, can be reached with a reform that is monotonic in income.[15] Then there is a Condorcet winner, namely the reform that is

[15]The schedule T^M solves $\max_{c_0', T'} \; u\left(c_0' + y - T'(y), y, \omega^M\right)$ subject to the constraints that, for all ω, $y(\omega) \in \operatorname{argmax}_{y'} \; u\left(c_0' + y' - T(y'), y', \omega\right)$, and $\int_{\underline{\omega}}^{\overline{\omega}} T(y(\omega)) \; f(\omega) \; d\omega \geq c_0'$. Assuming quasi-linear in

most preferred by the median voter and which yields tax schedule T^M.

It seems clear that the support of the majority is no the only force shaping empirically observed tax schedules. Otherwise, we would have to observe tax rates close to the revenue-maximizing one for incomes above the median and earnings subsidies for incomes below the median. There are number of candidate explanations for why we do not seem to observe these phenomena in their purest form. The model of political competition may be different from the Downsian model in many ways: Voters may not be exclusively motivated by self-interest, but also by concerns for social welfare, or a party may not be able to attract all voters that would benefit from its proposal due to party loyalties as in a model of probabilistic competition. Also, the economic environment may be different from the one we considered so far: There may be fixed costs of labor market participation, or the rich may decide to migrate to another country if taxes are set in a revenue-maximizing way. In the following, we explore one of these possibilities. We first characterize welfare-improving reforms and then look at tax reforms that are both politically feasible and welfare-improving.

7 Welfare-improving reforms

The following Proposition, which is proven in the Appendix, clarifies the conditions under which, for a given specification of welfare weights g, a small tax reform yields an increase in welfare. The following notation enables us to state the Proposition in a concise way. We define

$$\gamma_0 := \int_{\underline{\omega}}^{\overline{\omega}} g(\omega)\tilde{u}_c^0(\omega)f(\omega)\, d\omega \ ,$$

where $\tilde{u}_c^0(\omega)$ is a shorthand for the marginal utility of consumption that a type ω individual realizes under the status quo, and

$$\Gamma_0(\omega') := \int_{\omega'}^{\overline{\omega}} g(\omega)\tilde{u}_c^0(\omega)\frac{f(\omega)}{1-F(\omega')}\, d\omega \ .$$

Thus, γ_0 can be viewed as an average welfare weight in the status quo. It is obtained by multiplying each type's exogenous weight $g(\omega)$ with the marginal utility of consumption in the status quo $\tilde{u}_c^0(\omega)$ and then computing a population average. By contrast, $\Gamma_0(\omega')$ gives the average welfare weight of individuals with types above ω'. Note that $\gamma_0 = \Gamma_0(\underline{\omega})$ and that Γ_0 is a non-increasing function. In addition, we define

$$D_g^W(\omega) := -\frac{1-F(\omega)}{f(\omega)}\ \Phi(\omega)\ \frac{y_\omega^*(0,0,\omega)}{y_\tau^*(0,0,\omega)} \ ,$$

where

$$\Phi_0(\omega) := 1 - \tilde{I}_0(\omega) - (1-I_0)\frac{\Gamma_0(\omega)}{\gamma_0} \ ,$$

consumption preferences Brett and Weymark (2016a) provide a complete characterization of T^M with a particular focus on the bunching region that surrounds the median voter's level of income.

and

$$\mathcal{D}_g^W(y) := D_g^W(\tilde{\omega}^0(y)) \; .$$

Proposition 4

1. *Suppose there is an income level y_0 so that $T_0'(y_0) < \mathcal{D}_g^W(y)$. Then there exists a welfare-increasing reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.*

2. *Suppose there is an income level y_0 so that $T_0'(y_0) > \mathcal{D}_g^W(y)$. Then there exists a welfare-increasing reform (τ, y_a, y_b) with $\tau < 0$, and $y_a < y_0 < y_b$.*

The corresponding sufficient statistic $\tilde{\mathcal{D}}_g^W$ admits an easy interpretation if there are no income effects and the utility function is quasi-linear in private goods consumption. In this case, we have $I_0 = 0$, $\tilde{I}_0(\omega) = 0$, and $\tilde{u}_c^0(\omega) = 1$ for all ω. This implies, in particular, that $\gamma_0 = 1$, $\Gamma_0(\omega) = G(\omega)$, and $\Phi_0(\omega) = 1 - G(\omega)$, for all ω. Consequently,

$$\tilde{D}_g^W(\omega) = \frac{1 - F(\omega)}{f(\omega)\,\omega}\;(1 - G(\omega))\;\left(1 + \frac{1}{\varepsilon(0, \omega_n(0))}\right) \; ,$$

where the right-hand side of this equation is the ABC-formula due to Diamond (1998). Again, in the absence of income effects, the sufficient statistic does not depend on the status quo schedule and coincides with the solution to a (relaxed) problem of welfare-maximizing income taxation. The characterization of $\tilde{D}_g^W$ looks slightly more complicated if we allow for income effects, but take the status quo schedule T_0 to be the laissez-faire situation with marginal tax rates of zero at all levels of income. In this case, we still have $I_0 = 0$, and $\tilde{I}_0(\omega) = 0$, for all ω, which implies that

$$\tilde{D}_g^W(\omega) = \frac{1 - F(\omega)}{f(\omega)\,\omega}\;\left(1 - \frac{\Gamma_0(\omega)}{\gamma_0}\right)\;\left(1 + \frac{1}{\varepsilon(0, \omega_n(0))}\right) \; .$$

Note in particular that this expression will never turn negative, so that a welfare-maximizer would never want to move away from laissez faire in the direction of earnings subsidies, or, equivalently negative marginal tax rates.

Figure 10 below relates the schedules $\tilde{D}^P$, $\tilde{D}^R$ and $\tilde{D}_g^W$ to each other. The figure assumes that welfare weights take the form $g(\omega) = \frac{1}{1+\omega^2}$, for all ω. Figure 11, adapts Figure 10 to a case with income effects. Both figures illustrate a status quo schedule that has inefficiently high tax rates at the top. At low levels of income tax increases, while not mandated by Pareto-efficiency, would yield-welfare improvements. For an intermediate range of incomes, status quo tax rates are again within the Pareto bounds, but tax cuts would be welfare-improving.

On the identification of critical welfare weights. Instead of taking the specification of welfare weights as exogenously given, we can ask the following question: Given a status quo schedule and a level of income y_0, what is the set of welfare weights for which increased marginal tax rates in a neighborhood of y_0 would be welfare-improving? To formalize this question recall that welfare weights enter the auxiliary schedule $\mathcal{D}_g^W$ via

$$\Phi_0(\omega_0) := 1 - \tilde{I}_0(\omega_0) - (1 - I_0)\frac{\Gamma_0(\omega_0)}{\gamma_0} .$$

In this expression, $\tilde{I}_0(\omega_0)$ and I_0 measure behaviorial responses due to income effects. Value judgments, by contrast, shape the ratio $\frac{\Gamma_0(\omega_0)}{\gamma_0}$; i.e. the social value of (marginally) increased consumption for people with incomes above $y_0 = \tilde{y}^0(\omega_0)$ relative to increased consumption for the average taxpayer.

We now treat $\frac{\Gamma_0(\omega_0)}{\gamma_0}$ as a variable and write

$$\Phi_0(\omega_0, x) := 1 - \tilde{I}_0(\omega_0) - (1 - I_0)x .$$

for the value of $\Phi_0(\omega_0)$ that results if $\frac{\Gamma_0(\omega_0)}{\gamma_0}$ is equal to x. Analogously, we write $\mathcal{D}^W(y_0, x)$ and $\tilde{\mathcal{D}}^W(y_0, x)$ for the expressions that we obtain upon replacing $\Phi_0(\omega)$ by $\Phi_0(\omega, x)$ in the equations that define, respectively, $\mathcal{D}_g^W(y_0)$ and $\tilde{\mathcal{D}}_g^W(y_0)$. Finally, for any level of income y, we denote by $\tilde{G}_0(y)$ the level of x that solves $\tilde{\mathcal{D}}^W(y, x) = \frac{T_0'(y)}{1-T_0'(y)}$.[16]

The resulting level of $\tilde{G}_0(y)$ is a cutoff that separates welfare functions under which an increase of marginal tax rates for incomes close to y would be welfare-increasing from welfare functions under which such an increase would be welfare-damaging. Specifically, if $\frac{\Gamma_0(\tilde{\omega}^0(y))}{\gamma_0}$ falls below $\tilde{G}_0(y)$, then individuals with an income larger than or equal to y have little weight and tax increases would be welfare-improving. If, by contrast, $\frac{\Gamma_0(\tilde{\omega}^0(y))}{\gamma_0}$ exceeds $\tilde{G}_0(y)$ than such tax increases would lower welfare.

Figure 12 provides an illustration. The figure shows the cutoff function $\tilde{G}_0$ that is implied by a given status quo schedule. It also shows how, for a given welfare function, $\frac{\Gamma_0(\tilde{\omega}^0(y))}{\gamma_0}$ varies with y. For the situation illustrated in the Figure, tax increases are welfare-improving for low levels of income, and are welfare-decreasing for high levels of income.

8 Politically feasible welfare-improvements

We are now in a position to state Theorems 2 and 3. These theorems provide conditions for the possibility and impossibility of reforms that are both politically feasible and welfare-improving. These Theorems follows directly from our previous results in Propositions 1 - 4 and therefore do not require a separate proof.

Theorem 2 *Given a status quo schedule T_0 and a specification of welfare weights g.*

[16]Note that a solution to this equation exists provided that $T_0'(y)$ is neither inefficiently low nor inefficiently high, i.e. $T_0'(y) \in [\mathcal{D}^P(y), \mathcal{D}^R(y)]$.

1. *Suppose that there is an income level y_0 so that*

$$T_0'(y_0) < \mathcal{D}_g^W(y_0) \quad \text{and} \quad T_0'(y_0)) < \mathcal{D}^M(y_0)\ . \tag{12}$$

   Then there exists a politically feasible and welfare-increasing reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.

2. *Suppose that there is an income level y_0 and a type ω_0 with $y^*(0,0,\omega_0) = y_0$ so that*

$$T_0'(y_0) > \mathcal{D}_g^W(y_0) \quad \text{and} \quad T_0'(y_0)) > \mathcal{D}^M(y_0)\ . \tag{13}$$

   Then there exists a politically feasible and welfare-increasing reform (τ, y_a, y_b) with $\tau < 0$, and $y_a < y_0 < y_b$.

Theorem 2 states sufficient conditions for the existence of welfare-improving and politically feasible reforms. This raises the question of necessary conditions. The theorem has been derived from focussing on "small" reforms, i.e., one small increases of marginal tax rates applied to a small range of incomes. The arguments in the proofs of Propositions 1 - 4 imply that if either condition (12) or condition (13) is violated at y_0, then there is no "small" reform applied to incomes in a small neighborhood of y_0 that is both welfare-improving and politically feasible.

Theorem 3 *Given a status quo schedule T_0 and a specification of welfare weights g. Consider an income level y_0 so that*

$$T_0'(y_0) \geq \min\{\mathcal{D}_g^W(y_0), \mathcal{D}^M(y_0)\}$$

and

$$T_0'(y_0) \leq \max\{\mathcal{D}_g^W(y_0), \mathcal{D}^M(y_0)\}\ .$$

Then there exist $\delta > 0$ and $\epsilon > 0$, so that any reform (τ, y_a, y_b) with $-\delta \leq \tau \leq \delta$, $y_0 - y_a \leq \epsilon$ and $y_b - y_0 \leq \epsilon$ is politically infeasible or welfare-damaging.

Figure 13 provides an illustration of Theorems 2 and 3 under the assumptions that there are no income effect and that the status quo schedule is the laissez-faire-schedule. The figure shows that for incomes above the median, an increase in marginal tax rates is both welfare-improving and politically feasible. This holds for any welfare function under which the function g is strictly decreasing.[17] By contrast, for incomes below the median, there is no reform that is both politically feasible and welfare-improving: Welfare improvements require to raise marginal taxes relative to laissez faire, whereas political feasibility requires to introduce negative marginal tax rates.

Figure 14 modifies the status quo tax schedule so that T_0 is defined as in (8) and keeps the assumption that there are no income effects. In this figure, for very low incomes, there

[17]If $g(\omega) = 1$, for all ω, then such tax increases are politically feasible, but not welfare improving.

is no reform that is both politically feasible and welfare-improving. For incomes close to but below the median, tax cuts are politically feasible and welfare-improving according to some but not all social welfare functions. For incomes above the median, taxes are inefficiently high so that tax cuts are both politically feasible and welfare-improving. Figure 15 modifies Figure 14 by assuming a utility function that gives rise to income effects with the consequence that the schedules $\tilde{D}_g^W$ and $\tilde{D}^M$ are no longer independent of the status quo. There is now a range of incomes close to but above the median, where tax increases are politically feasible but welfare-damaging according to some social welfare function.

Our analysis suggests that existing tax schedules can be viewed as a resulting from compromise between concerns for welfare-maximization on the one hand, and concerns for political support on the other. If the maximization of political support was the only force in the determination of tax policy, we would expect to see tax rates close to the revenue-maximizing rate $\tilde{\mathcal{D}}^R$ for incomes above the median and negative rates close to $\tilde{\mathcal{D}}^P$ for incomes below the median. Concerns for welfare dampen these effects. A welfare-maximizing approach will generally yield higher marginal tax rates for incomes below the median and lower marginal tax rates for incomes above the median.

Our analysis also raises a question. Diamond (1998) and Saez (2001) have argued that, for plausible specifications of welfare weights, existing tax schedules have marginal tax rates for high incomes that are too low. Our analysis would suggest that an increase of these tax rates would not only be welfare-improving but also politically feasible. Why don't we see more reforms that involve higher tax rates for the rich?

9 Extensions

In this section, we show that a weakened version of the median voter theorem (Theorem 1) applies to models with more than one source of heterogeneity among individuals. Specifically, we show that a small tax reform is preferred by a majority of taxpayers if and only if it is preferred by the taxpayer with median income. This is similar to Theorem 1, except that Theorem 1 applies to any reform and not just to small ones.

Throughout we stick to the assumption that individuals differ in their productive abilities ω. We introduce a second consumption good and a possibility of heterogeneity in preferences over consumption goods in Section 9.1. We use this framework to discuss whether the introduction of distortionary taxes on savings is politically feasible. In Section 9.2 we consider fixed costs of labor market participation as an additional source of heterogeneity.[18] In Section 9.3 we assume that individuals differ in their valuation of increased public spending.[19] Finally, in Section 9.4, individuals differ by how much of their income is due to luck as in Alesina and Angeletos (2005).

[18]See Saez (2002), Choné and Laroque (2011), and Jacquet, Lehmann and Van der Linden (2013).

[19]See Boadway and Keen (1993), Hellwig (2004), Bierbrauer and Sahm (2010) or Bierbrauer (2014).

9.1 Political support for taxes on savings

We now suppose that there are two consumption goods. We refer to them as food and savings, respectively. An individual's budget constraint now reads as

$$c_f + c_s + T_{0s}(c_s) + \tau_s h_s(c_s) \leq c_0 + y - T_0(y) - \tau h(y) \; . \tag{14}$$

The variables on the right-hand side of the budget constraint have been defined before. On the left-hand side, c_f denotes food consumption and c_s savings. In the status quo savings are taxed according to a possibly non-linear savings-tax function T_{0s}. A reform replaces both the status quo income tax schedule T_0 by $T_1 = T_0 + \tau h$ and the status quo savings tax schedule T_{0s} by $T_{1s} = T_{0s} + \tau_s h_s$. We maintain the assumption that the functions h and h_s are non-decreasing and focus on revenue neutral reforms so that either $\tau > 0$ and $\tau_s < 0$ or $\tau < 0$ and $\tau_s > 0$.

Preferences of individuals are given by a utility function $u(v(c_f, c_s, \beta), y, \omega)$, where v is a sub-utility function that assigns consumption utility to any consumption bundle (c_f, c_s). The marginal rate of substitution between food and savings depends on a parameter β. We do not assume a priori that β is the same for all individuals. Under this assumption, however, the utility function u has the properties under which an efficient tax system does not involve distortionary commodity taxes, see Atkinson-Stiglitz (1976), or Laroque (2005) for a more elementary proof. In particular, distortionary taxes on savings are then undesirable from a welfare-perspective.

Individuals choose c_f, c_s and y to maximize utility subject to the budget constraint above. We denote the utility maximizing choices by $c_f^*(\tau_s, \tau, \beta, \omega)$, $c_s^*(\tau_s, \tau, \beta, \omega)$ and $y^*(\tau_s, \tau, \beta, \omega)$ and the corresponding level of indirect utility by $V(\tau_s, \tau, \beta, \omega)$. The slope of an indifference curve in a τ-τ_s diagram determines the individuals' willingness to accept higher savings taxes in return for lower taxes on current earnings. The following Lemma provides a characterization of this marginal rate of substitution in a neighborhood of the status quo.

Before we can state the Lemma, we introduce some notation. Let

$$s(\tau, \tau^s, \beta, \omega) = -\frac{V_\tau(\tau_s, \tau, \beta, \omega)}{V_{\tau_s}(\tau_s, \tau, \beta, \omega)}$$

be the slope of an individual's indifference curve in a τ-τ_s diagram. The status quo has $\tau = \tau_s = 0$. The slope in the status quo is denoted by $s^0(\omega, \beta)$. We denote the individuals food consumption, savings and earnings in the status quo by $\tilde{c}_f^0(\omega, \beta)$, $\tilde{c}_s^0(\omega, \beta)$ and $\tilde{y}^0(\omega, \beta)$, respectively.

Lemma 7 *In the status quo the slope of a type (ω, β)-individual's indifference curve in a τ-τ_s diagram is given by*

$$s^0(\omega, \beta) = -\frac{h(\tilde{y}^0(\omega, \beta))}{h_s(\tilde{c}_s^0(\omega, \beta))} \; .$$

The proof of Lemma 7 can be found in the Appendix. The Lemma provides a generalization of Roy's identity that is useful for an analysis of non-linear tax systems. As is well known, with linear tax systems, the marginal effect of, say, an increased savings tax on indirect utility is equal to $-\lambda^* c_s^*(\cdot)$, where λ^* is the multiplier on the individual's budget constraint, also referred to as the marginal utility of income. Analogously, the increase of a linear income tax affects indirect utility via $-\lambda^* y^*(\cdot)$ so that the slope of an indifference curves in a $\tau_s - \tau$-diagram would be equal to the earnings-savings-ratio $-\frac{y^*(\cdot)}{c_s^*(\cdot)}$. Allowing for non-linear tax systems and allowing for more general perturbations of those implies that the simple earnings-savings-ratio is replaced by $-\frac{h(y^*(\cdot))}{h_s(c_s^*(\cdot))}$. That said, even with non-linear taxes in the status quo, the simple earnings-savings-ratio is relevant if the reform is such that $h(y) = y$, for all y, and $h_s(c_s) = c_s$, for all c_s, i.e. if all marginal income tax rates change by τ and all marginal savings tax rates change by τ_s.

Consider a reform that involves an increase in the savings tax rate $d\tau_s > 0$ and a reduction of taxes on income $d\tau < 0$. We say that a type (ω, β)-individual strictly prefers a small reform with increased savings taxes over the status quo if

$$V_{\tau_s}(0, 0, \beta, \omega) d\tau_s + V_\tau(0, 0, \beta, \omega) d\tau > 0 ,$$

or, equivalently, if

$$\frac{d\tau_s}{d\tau} > s^0(\omega, \beta) = -\frac{h(\tilde{y}^0(\omega, \beta))}{h_s(\tilde{c}_s^0(\omega, \beta))} . \tag{15}$$

Since h_s is an increasing function, this condition is, ceteris paribus, easier to satisfy if the individual has little savings in the status quo.

The ratio $\frac{d\tau_s}{d\tau}$ on the left-hand side of inequality (15) follows from the behavioral responses of individuals to a small revenue-neutral tax reform. Let

$$\Delta^{R_s}(\tau_s, \tau) = \int_{\underline{\omega}}^{\overline{\omega}} \{T_{0s}(c_s^*(\cdot)) + \tau_s h_s(c_s^*(\cdot))\} dF(\omega)$$

be the tax revenue from savings taxes and

$$\Delta^R(\tau_s, \tau) = \int_{\underline{\omega}}^{\overline{\omega}} \{T_0(y^*(\cdot)) + \tau h(y^*(\cdot))\} dF(\omega)$$

be the tax revenue from income taxation. Revenue-neutrality requires that

$$\Delta_{\tau_s}^{R_s}(\tau_s, \tau) d\tau_s + \Delta_\tau^{R_s}(\tau_s, \tau) d\tau + \Delta_{\tau_s}^R(\tau_s, \tau) d\tau_s + \Delta_\tau^R(\tau_s, \tau) d\tau = 0 ,$$

or, equivalently, that

$$\frac{d\tau_s}{d\tau} = -\frac{\Delta_\tau^R(\tau_s, \tau) + \Delta_\tau^{R_s}(\tau_s, \tau)}{\Delta_{\tau_s}^{R_s}(\tau_s, \tau) + \Delta_{\tau_s}^R(\tau_s, \tau)} ,$$

which has to be evaluated for $(\tau_s, \tau) = (0, 0)$. A more detailed characterization of this expression would lead us astray. We simply assume that, for the reforms that we consider, this expression is well-defined and takes a finite negative value.

Different types will typically differ in their generalized earnings-savings-ratio $s^0(\omega,\beta)$ and we can order types according to this one-dimensional index. Let $(\omega,\beta)^{0M}$ be the type with the median value of $s^0(\omega,\beta)$. The following proposition extends Theorem 1. It asserts that a small reform is politically feasible if and only if it is supported by the median type $(\omega,\beta)^{0M}$.

Proposition 5 *For a given status quo tax policy and a given functions h and h_s,*

1. *If type $(\omega,\beta)^{0M}$ strictly prefers a small reform with increased savings taxes over the status quo, then there is a majority of individuals who strictly prefer such a reform over the status quo.*

2. *If type $(\omega,\beta)^{0M}$ weakly prefers the status quo over a small reform with increased savings taxes, then there is a majority of individuals who weakly prefer the status quo over such a reform.*

Proof We first show that a small reform is strictly supported by a majority of the population if it is strictly preferred by the median voter. Suppose that $\frac{d\tau_s}{d\tau} > s^0\left((\omega,\beta)^{0M}\right)$. This also implies $\frac{d\tau_s}{d\tau} > s^0(\omega,\beta)$ for all individuals with $s^0\left((\omega,\beta)^{0M}\right) \geq s^0(\omega,\beta)$. By the definition of the status quo median voter $(\omega,\beta)^{0M}$ the mass of taxpayers with this property is equal to $\frac{1}{2}$. Hence, the reform is supported by a majority of the population.

Second, we show that the status quo is weakly preferred by a majority of individuals if it is weakly preferred by the status quo median voter. Suppose that the status quo is weakly preferred by the median voter so that $\frac{d\tau_s}{d\tau} \leq s^0\left((\omega,\beta)^{0M}\right)$. This also implies $\frac{d\tau_s}{d\tau} \leq s^0(\omega,\beta)$, for all types (ω,β) so that $s^0\left((\omega,\beta)^{0M}\right) \leq s^0(\omega,\beta)$. By the definition of $(\omega,\beta)^{0M}$ the mass of taxpayers with this property is equal to $\frac{1}{2}$. Hence, the status quo is weakly preferred by a majority of individuals. □

As Theorem 1, Proposition 5 exploits the observation that individuals can be ordered according to a one-dimensional statistic that pins down whether or not they benefit from a tax reform. This makes it possible to prove a median-voter theorem for reforms that remain in a neighborhood of the status quo.

Theorem 1, by contrast, did not require the assumption that reforms are small. With only one-dimensional heterogeneity, there is a monotonic relation between types and earnings so that the identity of the type with median income does not depend on the status quo. Whatever the tax system, the person with the median income is the person with the median type ω^M. Here, by contrast, we allow for heterogeneity both in productive abilities and in preferences over consumption goods. The type with the median value of the generalized earnings-savings-ratio $s^0(\omega,\beta)$ will then typically depend on the status quo tax system.

This does not pose a problem if we focus on small reforms, i.e. $(\tau_s,\tau) = (0,0)$. In this case, preferences over reforms follow from the generalized earnings-savings-ratios in the status quo, and a small reform is preferred by a majority of individuals if and only if it is preferred by the individual with the median ratio. The sufficient statistics for Pareto-improving, welfare-improving and politically feasible reforms in previous sections were derived by an analysis of small reforms. Proposition 5 suggests that the extension of this approach to richer models of taxation should face no conceptual difficulty.

If we impose the assumption that the marginal rate of substitution between food and consumption is the same for all individuals, or equivalently, that the parameter β is the same for all, then, by the arguments of Atkinson and Stiglitz (1976), a welfare-maximizing tax system does not involve distortionary savings taxes. Proposition 5 suggests, however, that the introduction of such taxes may well be politically feasible. To see this, consider a reform so that $h(y) = y$, for all y, and $h_s(c_s) = c_s$, for all c_s. Hence $s^0(\omega)$ is simply the ratio of earnings to savings times -1. If the person with median income has positive earnings and hardly any savings, i.e. lives hand to mouth, then $s^0\left(\omega^M\right)$ will be close to $-\infty$ and the median voter will support such a tax reform.[20] Alternatively, we may consider a perturbation h_s that that takes values close to zero for the median voter's savings level but involves substantial tax increases for higher savings levels. Then, again

$$s^0(\theta^M) = -\frac{h(\tilde{y}^0(\omega^M))}{h_s(\tilde{c}_s^0(\omega^M))}$$

is close to $-\infty$ so that such a reform is politically feasible.

9.2 Fixed costs of labor market participation

With fixed costs of labor market participation individuals derive utility $u(c - \theta\ \mathbf{1}_{y>0}, y, \omega)$ from a (c, y)-pair. Fixed costs θ absorb some of the individuals after-tax income if the individual becomes active on the labor market, e.g. because of additional child care expenses. As before, there is an initial status quo tax schedule under which earnings are transformed into after-tax income according to the schedule C_0 with $C_0(y) = c_0 + y - T_0(y)$. After a reform, the schedule is

$$C_1(y) = c_0 + \Delta^R + y - T_0(y) - \tau\ h(y)\ ,$$

where h is a non-decreasing function of y. We denote by $y^*(\Delta^R, \tau, \omega, \theta)$ the solution to

$$\max_y\ u(C_1(y) - \theta\ \mathbf{1}_{y>0}, y, \omega),$$

and the corresponding level of indirect utility by $V(\Delta^R, \tau, \omega, \theta)$. We proceed analogously for other variables: What has been a function of ω in previous sections is now a function of ω and θ.

[20] Also note that if all individuals had the same earnings-to-savings-ratio in the status quo, then a reform would be politically feasible if and only if it was Pareto-improving.

For a given function h, the marginal gain that is realized by an individual with type (ω, θ) if the tax rate τ is increased, is given by the following analogue to equation (3),

$$\Delta_\tau^V(\omega, \theta \mid \tau, h) = \tilde{u}_c^1(\omega, \theta)\left(\Delta_\tau^R(\tau, h) - h(\tilde{y}^1(\omega, \theta))\right) \; , \tag{16}$$

where $\tilde{u}_c^1(\omega, \theta)$ is the marginal utility of consumption realized by a type (ω, θ)-individual after the reform, and $\tilde{y}^1(\omega, \theta)$ are the individual's post-reform earnings. At $\tau = 0$, we can also write

$$\Delta_\tau^V(\omega, \theta \mid 0, h) = \tilde{u}_c^0(\omega, \theta)\left(\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega, \theta))\right) \; , \tag{17}$$

where $\tilde{u}_c^0(\omega, \theta)$ and $\tilde{y}^0(\omega, \theta)$ are, respectively, marginal utility of consumption and earnings in the status quo.

For a given status quo tax policy and a given function h we say that type (ω, θ) strictly prefers a small tax reform over the status quo if $\Delta_\tau^V(\omega, \theta \mid 0, h) > 0$. The status quo median voter strictly prefers a small reform if $\Delta_\tau^V\left((\omega, \theta)^{0M} \mid 0, h\right) > 0$, where y^{0M} is the median of the distribution of earnings in the status quo and $(\omega, \theta)^{0M}$ is the corresponding type; i.e. $\tilde{y}^0\left((\omega, \theta)^{0M}\right) = y^{0M}$.

Proposition 6 *For a given status quo tax policy and a given function h,*

1. *If the status quo median voter strictly prefers a small reform over the status quo, then there is a majority of individuals who strictly prefer a small reform over the status quo.*

2. *If the status quo median voter weakly prefers the status quo over a small reform, then there is a majority of individuals who weakly prefer the status quo over a small reform.*

Proof We focus without loss of generality on tax increases, i.e. $\tau > 0$.

We first show that a small reform is strictly supported by a majority of the population if it is strictly preferred by the median voter. Suppose that $\Delta_\tau^V\left((\omega, \theta)^{0M} \mid 0, h\right) > 0$. Since $\tilde{u}_c^0(\cdot) > 0$, this implies

$$\Delta_\tau^R(0, h) - h(y^{0M}) > 0 \; .$$

Since h is a non-decreasing function, this also implies

$$\Delta_\tau^R(0, h) - h(\tilde{y}^0(\omega, \theta)) > 0 \; ,$$

for all (ω, θ) so that $\tilde{y}^0\,(\omega, \theta) \leq y^{0M}$. By definition of the status quo median voter the mass of taxpayers with $\tilde{y}^0\,(\omega, \theta) \leq y^{0M}$ is equal to $\frac{1}{2}$. Hence, the reform is supported by a majority of the population.

Second, we show that the status quo is weakly preferred by a majority of individuals if it is weakly preferred by the status quo median voter. Suppose that the status quo is weakly preferred by the median voter so that

$$\Delta^R_\tau(0,h) - h(y^{0M}) \leq 0 \; .$$

Since h is a non-decreasing function, this also implies

$$\Delta^R_\tau(0,h) - h(\tilde{y}^0(\omega,\theta)) \leq 0 \; ,$$

for all (ω,θ) so that $\tilde{y}^0\left(\omega,\theta\right) \leq y^{0M}$. By definition of the status quo median voter the mass of taxpayers with $\tilde{y}^0\left(\omega,\theta\right) \leq y^{0M}$ is equal to $\frac{1}{2}$. Hence, the status quo is weakly preferred by a majority of individuals.

□

Proposition 6 exploits that the slope of a type (ω,θ) individual's indifference curve through a point (τ,Δ^R),

$$s(\tau,\Delta^R,\omega,\theta) = h(y^*(\Delta^R,\tau,\omega,\theta)) \; .$$

is a function of the individual's income. As in the basic Mirrleesian setup, the interpretation is that individuals with a higher income are more difficult to convince that a reform that involves tax increases ($\tau > 0$) is worthwhile. A difference to the Mirrleesian setup is, however, that there is no monotonic relation between types and earnings. In the presence of income effects, and for a given level of ω, y^* will increase in θ as long as θ is below a threshold $\hat{\theta}(\omega)$ and be equal to 0 for θ above the threshold. This also implies that there is no longer a fixed type whose income is equal to the median income whatever the tax schedule. As for Proposition 5 this does not pose a problem if we focus on small reforms, i.e. on small deviations from $(\tau,\Delta^R) = (0,0)$. In this case, preferences over reforms follow from the relation between types and earnings in the status quo, and a small reform is preferred by a majority of individuals if and only if it is preferred by the individual with the median level of income in the status quo.

The sufficient statistics for Pareto-improving, welfare-improving and politically feasible reforms in previous sections were derived by an analysis of small reforms. Proposition 6 suggests that the extension of this approach to richer models of taxation should face no conceptual difficulty.

9.3 Public-goods preferences

Suppose that the change in revenue Δ^R is used to increase or decrease spending on publicly provided goods. The post-reform consumption schedule is then given by

$$C_1(y) = c_0 + y - T_0(y) - \tau \, h(y) \; ,$$

We assume that individuals differ with respect to their public-goods preferences. Now the parameter θ is a measure of an individual's willingness to give up private goods consumption in exchange for more public goods. More specifically, we assume that individual utility is

$$u(\theta(R^0 + \Delta^R) + C_1(y), y, \omega) \ ,$$

where R^0 is spending on publicly provided goods in the status quo. Again, we denote by $y^*(\Delta^R, \tau, \omega, \theta)$ the solution to

$$\max_y \ u(\theta(R^0 + \Delta^R) + C_1(y), y, \omega)$$

and indirect utility by $V(\Delta^R, \tau, \omega, \theta)$. By the envelope theorem, the slope of a type (ω, θ) individual's indifference curve through point (τ, Δ^R) is now given by

$$s(\tau, \Delta^R, \omega, \theta) = \frac{h(y^*(\Delta^R, \tau, \omega, \theta))}{\theta} \ .$$

This marginal rate of substitution gives the increase in public-goods provision that an individual requires as a compensation for an increase of marginal tax rates. Ceteris paribus, individuals with a lower income and individuals with a higher public-goods preference require less of a compensation, i.e. they have a higher willingness to pay higher taxes for increased public-goods provision. If we focus on small reforms we observe, again, that if a type (ω, θ)-individual benefits from a small tax-increase, then the same is true for any type (ω', θ') with

$$\frac{h(\tilde{y}^0(\omega, \theta))}{\theta} \geq \frac{h(\tilde{y}^0(\omega', \theta'))}{\theta'} \ .$$

By the arguments in the proof of Proposition 6, a small reform with $\tau > 0$ is preferred by a majority of individuals if and only if

$$\left(\frac{h(\tilde{y}^0(\omega, \theta))}{\theta}\right)^{0M} < \frac{d\Delta^R}{d\tau} \ ,$$

where $\left(\frac{h(\tilde{y}^0(\omega,\theta))}{\theta}\right)^{0M}$ is the median willingness to pay higher taxes for increased public spending in the status quo. These observations are summarized in the following Proposition that we state without proof.

Proposition 7 *For a given status quo tax policy and a given function h,*

1. *If* $\left(\frac{h(\tilde{y}^0(\omega,\theta))}{\theta}\right)^{0M} < \frac{d\Delta^R}{d\tau}$*, then there is a majority of individuals who strictly prefer a small reform over the status quo.*
2. *If* $\left(\frac{h(\tilde{y}^0(\omega,\theta))}{\theta}\right)^{0M} \geq \frac{d\Delta^R}{d\tau}$*, then there is a majority of individuals who weakly prefer the status quo over a small reform.*

Like Proposition 6, Proposition 7 shows that a small reform is supported by a majority of individuals if and only if it is supported by a median type. For Proposition 6 the relevant median type is simply the one who has the median income in the status quo. For Proposition 7, by contrast, we have to look at the median of the distribution of $\frac{h(\tilde{y}^0(\omega,\theta))}{\theta}$.

9.4 Fairness and politically feasible reforms

We adapt the model of Alesina and Angeletos (2005) to our setting. Their main result is that a demand for fair taxes can give rise to multiple equilibria. An important building block of their analysis is the validity of the median voter theorem. This allows them to pin down equilibrium tax rates. However, they focus on affine tax schedules as in the original analysis of Roberts (1977). We show that a median voter theorem for small reforms of non-linear tax systems applies to their setting. Hence, a derivation of sufficient statistics for Pareto-improving, welfare-improving or politically feasible reforms for a model that includes a demand for fair taxes should face no conceptual difficulty.

There are two periods. When young in $t = 0$ individuals choose a level of human capital k. When old in $t = 1$ individuals choose productive effort or labor supply l. Pre-tax income is determined by

$$y = \pi(l, k) + \eta \ ,$$

where π is a production function that is increasing in both arguments and η is a random source of income, also referred to as luck. An individual's life-time utility is written as $u(c, l, k, \omega)$. Utility is increasing in the first argument. It is decreasing in the second and third argument to capture the effort costs of labor supply and human capital investments, respectively. Effort costs are decreasing in ω. More formally, lower types have steeper indifference curves both in a (c, l)-space and in a (c, k)-space. We consider reforms that lead to a consumption schedule

$$C_1(y) = c_0 + \Delta^R + y - T_0(y) - \tau \ h(y) \ .$$

To be specific, we assume that individuals first observe how lucky they are and then choose how hard they work, i.e. given a realization of η and given the predetermined level of k, individuals choose l so as to maximize

$$u(C_1(\pi(l, k) + \eta), l, k, \omega) \ .$$

We denote the solution to this problem by $l^*(\Delta^R, \tau, \omega, \eta, k)$. Indirect utility is denoted by $V(\Delta^R, \tau, \omega, \eta, k)$. As of $t = 1$, there is multi-dimensional heterogeneity among individuals: They differ in their type ω, in their realization of luck η and possibly also in their human capital k.

In Alesina and Angeletos (2005) preferences over reforms have a selfish and fairness component. The indirect utility function V shapes the individuals' selfish preferences over reforms, for predetermined levels of human capital. The analysis of these selfish preferences can proceed along similar lines as the extension that considered fixed costs of labor market participation. Selfish preferences over small reforms follow from the relation between types and earnings in the status quo, and a small reform makes a majority better off if and only if it is beneficial for the individual with the median level of income in the status quo. More formally, let $\tilde{y}^0(\omega, \eta, k) := y^*(0, 0, \omega, \eta, k)$ be a shorthand for the earnings of a type (ω, η, k)-individual in the status quo and recall that the sign of

$$s(0, 0, \omega, \eta, k) = h(\tilde{y}^0(\omega, \eta, k)) \ .$$

determines whether an individual benefits from a small tax reform.Since h is a non-decreasing function of income, if

$$s^0\left((\omega, \eta, k)^{0M}\right) = h\left(\tilde{y}^{0M}\right) < \frac{d\Delta^R}{d\tau} \ ,$$

where y^{0M} is the median level of income in the status quo and $(\omega, \eta, k)^{0M}$ is the corresponding type, then a majority of individuals is, according to their selfish preference component made better off. The following Proposition that we state without proof summarizes.

Proposition 8 *For a given status quo tax policy and a given function h,*

1. *If, according to her selfish preferences, the status quo median voter strictly prefers a small reform over the status quo, then there is a majority of individuals who, according to their selfish preferences strictly prefer a small reform over the status quo.*

2. *If, according to her selfish preferences, the status quo median voter weakly prefers the status quo over a small reform, then there is a majority of individuals who, according to their selfish preferences, weakly prefer the status quo over a small reform.*

In their formalization of social preferences, Alesina and Angeletos (2005) view $\pi(l, k)$ as a reference income. It is the part of income that is due to effort as opposed to luck. A tax reform affects the share of $y = \pi(l, k) + \eta$ that individuals can keep for themselves. After the reform, the difference between disposable income and the reference income is given by[21]

$$C_1(y) - \pi(l, k) = \eta - T_0(\pi(l, k) + \eta) - \tau h(\pi(l, k) + \eta) \ .$$

A social preferences for fair taxes is then equated with a desire to minimize the variance of $\eta - T_0(\pi(l, k) + \eta) - \tau h(\pi(l, k) + \eta)$ taking into account that k and l are endogenous variables.[22] Denote this variance henceforth by $\Sigma(\Delta^R, \tau)$.

[21] The analysis in Alesina and Angeletos (2005) looks at special case of this. They focus on a status quo equal to the laissez-faire schedule so that $T_0(y) = 0$, for all y, and a reform that introduces a linear tax schedule, i.e. $h(y) = y$, for all y. Under these assumptions,

$$\eta - T_0(\pi(l, k) + \eta) - \tau h(\pi(l, k) + \eta) = (1 - \tau)\eta + \tau \pi(l, k) \ .$$

[22] Human capital investment is a function of effort costs ω and the expectations (Δ^{Re}, τ^e) of the young on the tax reforms that will be adopted when they are old.

Any one individual is assumed to evaluate a tax reform according to

$$V(\Delta^R, \tau, \omega, \eta, k) - \rho\, \Sigma(\Delta^R, \tau)$$

where ρ is the weight on fairness considerations which is assumed to be the same for all individuals. Therefore, heterogeneity in preferences over reforms is entirely due to heterogeneity in selfish preferences. Consequently, the finding that a small reform is preferred by a majority of taxpayers if and only if it is preferred by the voter with median income in the status quo is not affected by the inclusion of a demand for fair taxes.

10 Concluding remarks

This paper develops a framework for an analysis of tax reforms. This framework can be applied to any given income tax system. It makes it possible to identify reforms that are politically feasible in the sense that they would be supported by a majority of taxpayers or to identify welfare-improving reforms. One can also study the intersection of politically feasible and welfare-improving reforms. If this set is empty, the status quo is constrained efficient because the scope for politically feasible welfare-improvements has been exhausted. Any reform that is politically feasible, and therefore has benefits for a majority of taxpayers, then imposes a burden on a minority of tax-payers that is so large that a welfare-maximizer would reject the reform. Reforms with benefits that outweigh the costs lack political support because the set of beneficiaries is too small. By contrast, if one identifies reforms that are both politically feasible and welfare-improving, one might expect that those reforms will be taken up in the political process.

With non-linear tax systems, the policy-space is multi-dimensional with the implication that political economy forces are difficult to characterize. A main result in our paper is that this difficulty can be overcome by looking at tax reforms that are monotonic in the sense that marginal tax rates for the poor do not increase more than marginal tax rates for the rich. We show that, with such a policy space, a reform is preferred by a majority of taxpayers if and only if it is preferred by the taxpayer with median income. We show that this implies that tax decreases for incomes below the median and tax increases for incomes above the median are politically feasible, provided that tax rates stay within Pareto bounds. Concerns for welfare tend to dampen these effects. Very low taxes for the poor can be supported only by a welfare function that assigns more weight to middle incomes than to low incomes. Very high taxes at the top require a welfare function that assigns little weight to the rich.

We develop the argument in the context of a Mirrleesian model of income taxation. However, what is really driving the analysis is a link between two different single crossing properties. The Spence-Mirrlees single crossing property holds provided that more productive individuals are more willing to work harder in exchange for additional consumption. By Theorem 1, the Spence-Mirrlees single-crossing property implies that political preferences over tax reforms also satisfy a single crossing property. We therefore conjecture that a similar analysis is possible for any model of taxation in which a version of the Spence-Mirrlees single crossing property holds. Essentially, this is any model in which taxes affect the decisions of individuals and in which those decisions are monotonic in the individuals' types. We show that this conjecture holds for models with fixed costs of labor market participation as in Saez (2002), models that include diverse preferences over public goods, or models that include an investment in human capital as in Alesina and Angeletos (2005).

Our analysis identifies, for each level of income, Pareto-bounds for marginal tax rates. It also identifies the welfare weights that are needed to justify tax cuts or tax increases from a welfare perspective. Finally, it identifies reforms that would be preferred by a majority of voters. Moreover, we derive sufficient statistics that make it possible to bring this framework to the data. This makes it possible to see what types of reforms are possible under a given status quo tax policy. Future research might use this framework to complement existing studies on the history of income taxation. For instance, Scheve and Stasavage (2016) study, among other questions, whether tax systems have become more progressive in response to increases in inequality or in response to extensions of the franchise. Their analysis compares tax policies that have been adopted at different points in time, or by countries with different institutions. It does not include an analysis of the reforms that appear to have been politically feasible or welfare-improving in a given year for a given country and a given status quo tax schedule.

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Appendix

A Proofs

Proof of Lemma 1

We only prove the first statement. The proof of the second statement follows from an analogous argument. Under the initial tax schedule T_0 an individual with type $\omega_a(0)$ prefers y_a over all income levels $y \geq y_a$. We argue that the same is true under the new tax schedule T_1. This proves that $\omega_a(\tau) \geq \omega_a(0)$. Consider a y-c-diagram and the indifference curve of a type $w_a(0)$-type through the point $(y_a, c_0 + y_a - T_0(y_a))$. By definition of type $w_a(0)$, all points $(y, y - T_0(y))$ with $y > y_a$ lie below this indifference curve under the initial tax schedule. Under the new schedule T_1, this individual receives a lump-sum transfer $\Delta^R > 0$. Hence, the indifference curve through $(y_a, c_0 + \Delta^R + y_a - T_0(y_a))$ is at least as steep as the indifference curve through $(y_a, c_0 + y_a - T_0(y_a))$. Thus, the individual prefers $(y_a, c_0 + \Delta^R + y_a - T_0(y_a))$ over all points $(y, c_0 + \Delta^R + y - T_0(y))$ with $y > y_a$. Since for all $y > y_a$, $T_1(y) > T_0(y)$, the individual also prefers $(y_a, c_0 + \Delta^R + y_a - T_0(y_a))$ over all points $(y, c_0 + \Delta^R + y - T_1(y))$ with $y > y_a$.

Proof of Lemma 2

Consider a reform that involves an increase of marginal tax rates $\tau > 0$. If before the reform, all types where choosing an income level $y^*(0, 0, \omega)$ that satisfies the first order condition

$$u_c(\cdot)(1 - T_0'(\cdot)) + u_y(\cdot) = 0 \ ,$$

then, after the reform there will be a set of types $(\omega_a(\tau), \overline{\omega}(\tau)]$ who will now bunch at y_a. These individuals chose an income level above y_a before the reform. After the reform they will find that

$$u_c(\cdot)(1 - T_0'(\cdot) - \tau) + u_y(\cdot) < 0 \ ,$$

for all $y \in (y_a, y^*(0, 0, \omega)]$ and therefore prefer y_a over any income in this range. At the same time, they will find that

$$u_c(\cdot)(1 - T_0'(\cdot)) + u_y(\cdot) > 0 \ ,$$

so that there is also no incentive to choose an income level lower than y_a. Hence, the types who bunch are those for which at y_a,

$$1 - T_0'(y_a) > -\frac{u_y(\cdot)}{u_c(\cdot)} > 1 - T_0'(y_a) - \tau \ .$$

An analogous argument implies that for a reform that involves a decrease in marginal tax rates $\tau > 0$, there will be a set of types $[\omega_b(\tau), \overline{\omega}_b(\tau)]$ who bunch at y_b after the reform.

Proof of Lemma 3

We only prove the first statement. The proof of the second statement follows from an analogous argument. Under the initial tax schedule T_0 an individual with type $\omega_b(0)$ prefers y_b over all income levels $y \geq y_b$. We argue that the same is true under the new tax schedule T_1. This proves that $\omega_b(\tau) \geq \omega_b(0)$. Consider a y-c-diagram and the indifference curve of a type $w_b(0)$-type through the point $(y_b, c_0 + y_b - T_0(y_b))$. By definition of type $w_b(0)$, all points $(y, c_0 + y - T_0(y))$ with $y > y_b$ lie below this indifference curve under the initial tax schedule. Under the new schedule T_1, this individual receives a lump-sum transfer $\Delta^R - \tau(y_b - y_a) < 0$. Without income effects, the indifference curve through $(y_b, c_0 + \Delta^R - \tau(y_b - y_a) + y_b - T_0(y_b))$ has the same slope as the indifference curve through $(y_b, c_0 + y_b - T_0(y_b))$. Thus, the individual prefers $(y_b, c_0 + \Delta^R - \tau(y_b - y_a) + y_b - T_0(y_b))$ over all points $(y, c_0 + \Delta^R - \tau(y_b - y_a) + y - T_0(y))$ with $y > y_b$, or equivalently over all points $(y, c_0 + \Delta^R + y - T_1(y))$ with $y > y_b$.

Proof of Lemma 4

By definition of the function $\tilde{y}^0$, for any $\omega' \in \Omega$,

$$\tilde{F}^0(\tilde{y}^0(\omega')) = F(\omega'), \tag{18}$$

and, hence,

$$\tilde{f}^0(\tilde{y}^0(\omega'))\tilde{y}^{0'}(\omega') = f(\omega') \;, \tag{19}$$

where f is the derivative of F, $\tilde{f}^0$ is the derivative of $\tilde{F}^0$ and $\tilde{y}^{0'}$ is the derivative of $\tilde{y}^0$. Equations (18) and (19) imply that, for any given ω',

$$\frac{1 - \tilde{F}^0(\tilde{y}^0(\omega'))}{\tilde{f}^0(\tilde{y}^0(\omega'))\tilde{y}^{0'}(\omega')} = \frac{1 - F(\omega')}{f(\omega')} \;. \tag{20}$$

The assumptions that $F(\omega') = 1 - \left(\frac{\omega_{min}}{\omega'}\right)^a$ and $\tilde{F}^0(\tilde{y}^0(\omega')) = 1 - \left(\frac{y_{min}}{\tilde{y}^0(\omega')}\right)^b$ imply that

$$\frac{1 - F(\omega')}{f(\omega')} = \frac{\omega'}{a}, \tag{21}$$

and

$$\frac{1 - \tilde{F}^0(\tilde{y}^0(\omega'))}{\tilde{f}^0(\tilde{y}^0(\omega'))} = \frac{\tilde{y}^0(\omega')}{b} \;. \tag{22}$$

Substituting (21) and (22) into (20) yields

$$\frac{\tilde{y}^0(\omega')}{b} \frac{1}{\tilde{y}^{0'}(\omega')} = \frac{\omega'}{a} \;,$$

or, equivalently,

$$\tilde{y}^{0'}(\omega') \frac{\omega'}{\tilde{y}^0(\omega')} = \frac{a}{b} = \gamma \;.$$

The solution to this differential equation is defined up to a multiplicative constant α, and given by $\tilde{y}^0(\omega) = \alpha\omega^\gamma$.

Proof of Lemma 5

For individuals whose behavior is characterized by a first-order condition, the utility realized under a reform that involves a change in the lump-sum-transfer by Δ^R and a change of marginal taxes by τ for incomes in $[y_a, y_b]$ is given by $V(\Delta^R, \tau, \omega)$. The marginal rate of substitution between τ and Δ^R is therefore given by

$$\left(\frac{d\Delta^R}{d\tau}\right)_{|dV=0} = -\frac{V_\tau(\Delta^R, \tau, \omega)}{V_e(\Delta^R, \tau, \omega)}.$$

By the envelope theorem,

$$V_\tau(\Delta^R, \tau, \omega) = -u_c(\cdot)\ h(y^*(\Delta^R, \tau, \omega)) \quad \text{and} \quad V_e(\Delta^R, \tau, \omega) = u_c(\cdot)\ .$$

Thus,

$$\left(\frac{d\Delta^R}{d\tau}\right)_{|dV=0} = h(y^*(\Delta^R, \tau, \omega))\ .$$

If we consider reforms in the (τ, y_a, y_b)-class we have to take account of the possibility of bunching. We only consider the case with $\tau > 0$. The case $\tau < 0$ is analogous. With $\tau > 0$, there will be individuals who bunch at y_a. For these individuals, marginal changes of Δ^R and τ do not trigger an adjustment of the chosen level of earnings. Hence, a marginal change of Δ^R yields a change in utility equal to $u_c(\cdot)$. The change in utility due to a change in the marginal tax rate is given by $-u_c(\cdot)\ h(y_a) = 0$. Again, the marginal rate of substitution equals $h(y_a) = 0$.

Proof of Lemma 6

The proof follows two steps: First we show that, for all ω,

$$\Delta^V_\tau(\omega \mid \tau, h) = \tilde{u}^1_c(\omega)\left(\Delta^R_\tau(\tau, h) - h(\tilde{y}^1(\omega))\right).$$

Second, we show that $\tilde{u}^1_c(\omega)$ is a non-increasing function of ω.

Step 1. We show that for any ω,

$$\Delta^V_\tau(\omega \mid \tau, h) = \tilde{u}^1_c(\omega)\left(\Delta^R_\tau(\tau, h) - h(\tilde{y}^1(\omega))\right)\ .$$

We only look at reforms with $\tau > 0$. The case $\tau < 0$ is analogous.

We first consider reforms that belongs to the (τ, y_a, y_b)-class and verify our claim separately for types with $\omega \leq \omega_a(\tau)$, $\omega \in [\omega_a(\tau), \overline{\omega}_a(\tau)]$, $\omega \in [\overline{\omega}_a(\tau), \omega_b(\tau)]$ and $\omega \geq \omega_b(\tau)$.

Consider individuals with $\omega \leq \omega_a(\tau)$. For these individuals $\Delta^V_\tau(\omega \mid \tau, y_a, y_b)$ is given by the derivative of $V(\Delta^R(\cdot), \tau, \omega)$ with respect to τ, and, moreover, $h(y^*(\Delta^R(\cdot), \tau, \omega)) = 0$. Hence, the envelope theorem implies

$$\Delta^V_\tau(\omega \mid \tau, y_a, y_b) = u_c(\cdot)\, \Delta^R_\tau(\tau, y_a, y_b)\ ,$$

where $u_c(\cdot)$ is marginal utility evaluated at $c = \Delta^R(\cdot) + y - T_0(y)$ and $y = y^*(\Delta^R(\cdot), \tau, \omega))$.

Individuals with types in $[\omega_a(\tau), \overline{\omega}_a(\tau)]$ bunch at income level y_a after the reform so that

$$\Delta^V_\tau(\omega \mid \tau, y_a, y_b) = u(\Delta^R(\cdot) + y_a - T_0(y_a), y_a, \omega) - V(0, 0, \tau)\ .$$

Hence, again,

$$\Delta^V_\tau(\omega \mid \tau, y_a, y_b) = u_c\left(\cdot\right) \Delta^R_\tau(\tau, y_a, y_b) \ ,$$

where $u_c(\cdot)$ is marginal utility evaluated at $c = \Delta^R(\cdot) + y - T_0(y)$ and $y = y_a$.

For individuals with $\omega \in [\overline{\omega}_a(\tau), \omega_b(\tau)]$, $\Delta^V_\tau(\omega \mid \tau, y_a, y_b)$ is given by the derivative of $V(\Delta^R(\cdot), \tau, \omega)$ with respect to τ. For these individuals $h(y^*(\Delta^R(\cdot), \tau, \omega)) = y^*(\Delta^R(\cdot), \tau, \omega) - y_a$, so that the envelope theorem implies

$$\Delta^V_\tau(\omega \mid \tau, y_a, y_b) = u_c\left(\cdot\right) \left(\Delta^R_\tau(\tau, y_a, y_b) - (y^*(\Delta^R(\cdot), \tau, \omega) - y_a)\right) \ ,$$

where $u_c(\cdot)$ is marginal utility evaluated at $c = \Delta^R(\cdot) + y - T_1(y)$ and $y = y^*(\Delta^R(\cdot), \tau, \omega))$.

For individuals with $\omega \geq \omega_b(\tau)$, $\Delta^V_\tau(\omega \mid \tau, y_a, y_b)$ is, again, given by the derivative of $V(\Delta^R(\cdot), \tau, \omega)$ with respect to τ. For these individuals $h(y^*(\Delta^R(\cdot), \tau, \omega)) = y_b - y_a$, so that the envelope theorem implies

$$\Delta^V_\tau(\omega \mid \tau, y_a, y_b) = u_c\left(\cdot\right) \left(\Delta^R_\tau(\tau, y_a, y_b) - (y_b - y_a)\right) \ ,$$

where $u_c(\cdot)$ is marginal utility evaluated at $c = \Delta^R(\cdot) + y - T_1(y)$ and $y = y^*(\Delta^R(\cdot), \tau, \omega))$.

The analysis is analogous for more general reforms (τ, h) under which individual earnings are characterized by a first order condition. In this case, the envelope theorem implies

$$\Delta^V_\tau(\omega \mid \tau, h) = u_c\left(\cdot\right) \left(\Delta^R_\tau(\tau, h) - h(\tilde{y}^1(\omega))\right) \ .$$

Step 2. By the Spence-Mirrlees single crossing property, $\tilde{y}^1$ is a non-decreasing function. In addition, h is a nondecreasing function. Consequently, $\Delta^R_\tau(\tau, h) - h(\tilde{y}^1(\omega))$ is a non-increasing function of ω. To complete the proof of the Lemma we show that $\tilde{u}^1_c(\omega)$ is also a non-increasing function of ω. To this end, we show that, for all ω,

$$\tilde{u}^{1'}_c(\omega) := \frac{\partial}{\partial \omega} \tilde{u}^1_c(\omega) \leq 0 \ .$$

We also define

$$\tilde{y}^{1'}(\omega) := \frac{\partial}{\partial \omega} \tilde{y}^1(\omega) \ .$$

By definition,

$$\tilde{u}^1_c(\omega) = u_c(\Delta^R + \tilde{y}^1(\omega) - T_1(\tilde{y}^1(\omega)), \tilde{y}^1(\omega), \omega) \ ,$$

where $\tilde{y}^1(\omega)$ equals y_a for a reform in the (τ, y_a, y_b)-class and $\omega \in [\omega_a(\tau), \overline{\omega}_a(\tau)]$ and equals $y^*(\Delta^R(\cdot), \tau, \omega)$ otherwise. Hence,

$$\tilde{u}^{1'}_c(\omega) = \left(u_{cc}(\cdot)(1 - T_1'(\cdot)) + u_{cy}\right) \tilde{y}^{1'}(\omega) + u_{c\omega}(\cdot)$$

If the individual is bunching at y_a, then $\tilde{y}^{1'}(\omega) = 0$, which implies

$$\tilde{u}^{1'}_c(\omega) = u_{c\omega}(\cdot) \leq 0 \ .$$

It remains to be shown that $\tilde{u}^{1'}_c(\omega) \leq 0$ also holds if the individual is not bunching at y_a. If the individual is not bunching at y_a, $\tilde{y}^1(\omega)$ satisfies the first order condition

$$1 - T_1'(\cdot) = -\frac{u_y(\cdot)}{u_c(\cdot)} \ .$$

Hence,

$$\tilde{u}_c^{1'}(\omega) = \left(-u_{cc}(\cdot)\,\frac{u_y(\cdot)}{u_c(\cdot)} + u_{cy}(\cdot)\right)\tilde{y}^{1'}(\omega) + u_{c\omega}(\cdot)\ ,$$

where, because of (1),

$$-u_{cc}(\cdot)\,\frac{u_y(\cdot)}{u_c(\cdot)} + u_{cy}(\cdot) \leq 0\ .$$

Proof of Proposition 1

We provide a proof of the first statement in the Proposition by considering a reform with $\tau > 0$. The second statement follows from an analogous argument.

Notation. We look at different subsets of the population separately. The change in tax revenue that is due to individuals with types $\omega \leq \omega_a(\tau)$ who chose an income level smaller or equal to y_a after the reform is given by

$$\begin{aligned}\Delta^{R1}(\tau, y_a, y_b) &:= \int_{\underline{\omega}}^{\omega_a(\tau)}\{T_1(y^*(\Delta^R(\cdot),\tau,\omega)) - T_0(y^*(0,0,\omega))\}f(\omega)d\omega \\ &= \int_{\underline{\omega}}^{\omega_a(\tau)}\{T_0(y^*(\Delta^R(\cdot),\tau,\omega)) - T_0(y^*(0,0,\omega))\}f(\omega)d\omega \\ &= \int_{\underline{\omega}}^{\omega_a(\tau)}\{T_0(y^*(\Delta^R(\cdot),0,\omega)) - T_0(y^*(0,0,\omega))\}f(\omega)d\omega\ ,\end{aligned}$$

where the last equality uses the fact that the behavior of individuals with types in $\omega_a(\tau)$ is affected only by $\Delta^R(\cdot)$ but not by the increased marginal tax rates for individuals with incomes in $[y_a, y_b]$.

The change in tax revenue that comes from individuals with types in $[\omega_a(\tau), \overline{\omega}_a(\tau)]$ who bunch at an income level of y_a after the reform equals

$$\begin{aligned}\Delta^{R2}(\tau, y_a, y_b) &:= \int_{\omega_a(\tau)}^{\overline{\omega}_a(\tau)}\{T_1(y_a) - T_0(y^*(0,0,\omega))\}f(\omega)d\omega \\ &= \int_{\omega_a(\tau)}^{\overline{\omega}_a(\tau)}\{T_0(y_a) - T_0(y^*(0,0,\omega))\}f(\omega)d\omega\ .\end{aligned}$$

Individuals with types in $(\overline{\omega}_a(\tau), \omega_b(\tau))$ choose an income level between y_a and y_b after the reform. The change in tax revenue that can be attributed to them equals

$$\begin{aligned}\Delta^{R3}(\tau, y_a, y_b) &:= \int_{\overline{\omega}_a(\tau)}^{\omega_b(\tau)}\{T_1(y^*(\Delta^R(\cdot),\tau,\omega)) - T_0(y^*(0,0,\omega))\}f(\omega)d\omega \\ &= \int_{\overline{\omega}_a(\tau)}^{\omega_b(\tau)}\{T_0(y^*(\Delta^R(\cdot),\tau,\omega)) + \tau(y^*(\Delta^R(\cdot),\tau,\omega) - y_a) - T_0(y^*(0,0,\omega))\}f(\omega)d\omega.\end{aligned}$$

Finally, the change in tax revenue that comes from individuals with types above $\omega_b(\tau)$ who choose an income larger than y_b after the reform is given by

$$\begin{aligned}\Delta^{R4}(\tau, y_a, y_b) &:= \int_{\omega_b(\tau)}^{\overline{\omega}}\{T_1(y^*(\Delta^R(\cdot),\tau,\omega)) - T_0(y^*(0,0,\omega))\}f(\omega)d\omega \\ &= \int_{\omega_b(\tau)}^{\overline{\omega}}\{T_0(y^*(\Delta^R(\cdot),\tau,\omega)) + \tau(y_b - y_a) - T_0(y^*(0,0,\omega))\}f(\omega)d\omega \\ &= \int_{\omega_b(\tau)}^{\overline{\omega}}\{T_0(y^*(\Delta^R(\cdot) - \tau(y_b - y_a),0,\omega)) + \tau(y_b - y_a) - T_0(y^*(0,0,\omega))\}f(\omega)d\omega,\end{aligned}$$

where the last equality uses the fact that the behavior of individuals with types above $\omega_b(\tau)$ is affected only by the transfer $\Delta^R(\cdot) - \tau(y_b - y_a)$ but not by the increased marginal tax rates for individuals with incomes in $[y_a, y_b]$, see Figure 1. To sum up,

$$\Delta^R(\tau, y_a, y_b) = \Delta^{R1}(\tau, y_a, y_b) + \Delta^{R2}(\tau, y_a, y_b) + \Delta^{R3}(\tau, y_a, y_b) + \Delta^{R4}(\tau, y_a, y_b)\ .$$

How does a marginal change of τ affect tax revenue? In the following we will provide a characterization of $\Delta_\tau^R(\tau, y_a, y_b)$, i.e. of the change in tax revenue that is implied by a marginal change of the tax rate τ. We will be particularly interested in evaluating this expression at $\tau = 0$ since $\Delta_\tau^R(0, y_a, y_b)$ gives the increase in tax revenue from a small increase of marginal tax rates for incomes in the interval $[y_a, y_b]$. Since

$$\Delta_\tau^R(\tau, y_a, y_b) = \Delta_\tau^{R1}(\tau, y_a, y_b) + \Delta_\tau^{R2}(\tau, y_a, y_b) + \Delta_\tau^{R3}(\tau, y_a, y_b) + \Delta_\tau^{R4}(\tau, y_a, y_b) \ ,$$

we can characterize $\Delta_\tau^R(\tau, y_a, y_b)$ by looking at each subset of types separately.

For instance, it is straightforward to verify that

$$\begin{aligned}\Delta_\tau^{R1}(\tau, y_a, y_b) \quad &= \quad \Delta_\tau^R(\cdot) \int_{\underline{\omega}}^{\omega_a(\tau)} T_0'(y^*(\Delta^R(\cdot), 0, \omega)) \ y_e^*(\Delta^R(\cdot), 0, \omega) \ f(\omega) \ d\omega \\ &\quad +\{T_0(y_a) - T_0(y^*(0, 0, \omega_a(\tau)))\} f(\omega_a(\tau))\omega_a'(\tau)\end{aligned}$$

where $\omega_a'(\tau)$ is the derivative of $\omega_a(\tau)$. Analogously we derive expressions for $\Delta_\tau^{R2}(\tau, y_a, y_b)$, $\Delta_\tau^{R3}(\tau, y_a, y_b)$, $\Delta_\tau^{R4}(\tau, y_a, y_b)$. This is a tedious, but straightforward. We provide the details in Appendix C. It yields an expression for $\Delta_\tau^R(\tau, y_a, y_b)$. If we evaluate this expression at $\tau = 0$ and use Assumption 1, i.e. the assumption that there is no bunching at y_a under the initial tax schedule T_0, so that $\omega_a(0) = \hat{\omega}_a(0)$, we obtain

$$\begin{aligned}\Delta_\tau^R(0, y_a, y_b) \quad &= \quad \Delta_\tau^R(0, y_a, y_b) I_0 \\ &\quad + \int_{\omega_a(0)}^{\omega_b(0)} T_0'(y^*(0, 0, \omega)) \ y_\tau^*(0, 0, \omega) \ f(\omega) \ d\omega \\ &\quad + \int_{\omega_a(0)}^{\omega_b(0)} \{y^*(0, 0, \omega) - y_a\} \ f(\omega) \ d\omega \\ &\quad +(y_b - y_a)(1 - F(w_b(0))) \\ &\quad -(y_b - y_a) \int_{\omega_b(0)}^{\overline{\omega}} T_0'(y^*(0, 0, \omega)) \ y_e^*(0, 0, \omega) \ f(\omega) \ d\omega,\end{aligned}$$

where $I_0 := \tilde{I}(\underline{\omega})$ is the income effect measure $\tilde{I}$ defined in the body of the text, see equation (6), applied to the population at large.

Armed with this notation, we can write

$$\Delta_\tau^R(0, y_a, y_b) = \frac{1}{1 - I_0} \ \mathcal{R}(y_a, y_b),$$

where

$$\begin{aligned}\mathcal{R}(y_a, y_b) \quad &= \quad \int_{\omega_a(0)}^{\omega_b(0)} T_0'(y^*(0, 0, \omega)) \ y_\tau^*(0, 0, \omega) \ f(\omega) \ d\omega \\ &\quad + \int_{\omega_a(0)}^{\omega_b(0)} \{y^*(0, 0, \omega)) - y_a\} \ f(\omega) \ d\omega \\ &\quad +(y_b - y_a)(1 - F(w_b)) \\ &\quad -(y_b - y_a) \int_{\omega_b(0)}^{\overline{\omega}} T_0'(y^*(0, 0, \omega)) \ y_e^*(0, 0, \omega) \ f(\omega) \ d\omega.\end{aligned}$$

In the following, to simplify notation, we suppress the dependence of $\omega_a(0)$, $\omega_b(0)$ on $\tau = 0$ and simply write ω_a, and ω_b. By assumption, there is no bunching under the initial schedule, so that $y^*(0, 0, \omega)$ is strictly increasing and hence invertible over $[\omega_a, \omega_b]$. We can therefore, without loss of generality, view a reform also as being defined by τ, ω_a and ω_b. With a slight abuse of notation, we will therefore write

$$\Delta_\tau^R(0, \omega_a, \omega_b) = \frac{1}{1 - I_0} \ \mathcal{R}(\omega_a, \omega_b) \tag{23}$$

where

$$\begin{aligned}\mathcal{R}(\omega_a,\omega_b) &= \int_{\omega_a}^{\omega_b} T_0'(y^*(0,0,\omega))\, y^*_\tau(0,0,\omega)\, f(\omega)\, d\omega \\ &\quad + \int_{\omega_a}^{\omega_b}\{y^*(0,0,\omega)-y^*(0,0,\omega_a)\}\, f(\omega)\, d\omega \\ &\quad +(y^*(0,0,\omega_b)-y^*(0,0,\omega_a))(1-F(w_b(0))) \\ &\quad -(y^*(0,0,\omega)-y^*(0,0,\omega_a))\int_{\omega_b}^{\overline{\omega}} T_0'(y^*(0,0,\omega))\, y^*_e(0,0,\omega)\, f(\omega)\, d\omega\end{aligned}$$

We now investigate under which conditions a marginal tax increase of τ over a small interval of types increases tax revenue. Note that

$$\Delta^R_\tau(0,\omega_a,\omega_a) = \frac{1}{1-I_0}\,\mathcal{R}(\omega_a,\omega_a) = 0\ .$$

If

$$\Delta^R_{\tau\omega_b}(0,\omega_a,\omega_a) = \frac{1}{1-I_0}\,\mathcal{R}_{\omega_b}(\omega_a,\omega_a) > 0\ ,$$

then $\Delta^R_\tau(0,\omega_a,\omega_b)$ turns positive, if starting from $\omega_a=\omega_b$, we marginally increase ω_b.

Straightforward computations yield:

$$\mathcal{R}_{\omega_b}(\omega_a,\omega_a) = T_0'(y^*(0,0,\omega_a))\, y^*_\tau(0,0,\omega_a)\, f(\omega_a) + (1-\tilde{I}_0(\omega_a))\, y^*_\omega(0,0,\omega_a)\, (1-F(\omega_a))\ .$$

We summarize these observations in the following Lemma:

Lemma A.1 *Suppose that, under tax schedule T_0, there is an income level y_0 and a type ω_0 with $y^*(0,0,\omega_0)=y_0$ so that*

$$T_0'(y^*(0,0,\omega_0))\, y^*_\tau(0,0,\omega_0)\, f(\omega_0) + (1-\tilde{I}_0(\omega_0))\, y^*_\omega(0,0,\omega_0)\, (1-F(\omega_0)) > 0\ .$$

Then there exists a revenue-increasing tax reform (τ, y_a, y_b) with $\tau>0$, $y_a<y_0<y_b$.

On the sign of $y^*_\tau(0,0,\omega_0)$. In the following, we argue that in Lemma A.1 above, we have $y^*_\tau(0,0,\omega_0)<0$. To see this, consider the optimization problem

$$\max_y\ u(c_0+y-T_0(y)-\tau(y-y_a), y, \omega_0)$$

for type ω_0. By assumption $y^*(0,0,\omega_0)\in(y_a,y_b)$. We argue that for any τ so that $y^*(0,\tau,\omega_0)\in(y_a,y_b)$ and y_b-y_a sufficiently small, we have $y^*_\tau(0,0,\omega_0)<0$. The first order condition of the optimization problem is

$$u_c(\cdot)(1-T_0'(\cdot)-\tau)+u_y(\cdot)=0.$$

The second order condition is, assuming a unique optimum,

$$B := u_{cc}(\cdot)(1-T_0'(\cdot)-\tau)^2 + 2u_{cy}(\cdot)(1-T_0'(\cdot)-\tau) + u_{yy}(\cdot) - u_c(\cdot)T_0''(\cdot) < 0\ .$$

From totally differentiating the first order condition with respect to c_0, we obtain

$$y^*_e(0,\tau,\omega_0) = -\frac{u_{cc}(\cdot)(1-T_0'(\cdot)-\tau)+u_{cy}(\cdot)}{B} \le 0\ .$$

This expression is non-positive by our assumptions on the utility function that ensure that leisure is a non-inferior good. From totally differentiating the first order condition with respect to τ, and upon collecting terms, we obtain

$$y^*_\tau(0,\tau,\omega_0) = \frac{u_c(\cdot)}{B} - (y^*(0,\tau,\omega_0) - y_a) y^*_e(0,\tau,\omega_0) \ ,$$

and hence

$$y^*_\tau(0,0,\omega_0) = \frac{u_c(\cdot)}{B} - (y^*(0,0,\omega_0) - y_a) y^*_e(0,0,\omega_0) \ ,$$

which is the familiar decomposition of a behavioral response into a substitution and an income effect. As ω_0 approaches ω_a, $y^*(0,0,\omega_0)$ approaches $y^*(0,0,\omega_a) = y_a$ so that the income effect vanishes. Again, this is a familiar result: For small price changes, observed behavioral responses are well approximated by compensated or Hicksian behavioral responses.

The observation that for y_a close to y_b, we may, without loss of generality, assume that $y^*_\tau(0,0,\omega_0) < 0$, enables us to rewrite Lemma A.1.

Lemma A.2 *Suppose that, under tax schedule T_0, there is an income level y_0 and a type ω_0 with $y^*(0,0,\omega_0) = y_0$ so that*

$$T'_0(y_0) < -\frac{1-F(\omega_0)}{f(\omega_0)} \left(1 - \tilde{I}_0(\omega_0)\right) \frac{y^*_\omega(0,0,\omega_0)}{y^*_\tau(0,0,\omega_0)} \ .$$

Then there exists a tax-revenue-increasing reform (τ, y_a, y_b) with $\tau > 0$, and $y_a < y_0 < y_b$.

The right-hand-side of this inequality equals $D^R(\omega_0) = D^R(\tilde{\omega}^0(y_0)) = \mathcal{D}^R(y_0)$. This proves the first statement in Proposition 1.

Proof of Proposition 2

Starting from $\tau = 0$ a tax increase is Pareto-improving if $\Delta^R_\tau(0, y_a, y_b) - (y_b - y_a) \geq 0$, where we recall that

$$\Delta^R_\tau(0, y_a, y_b) = \frac{1}{1 - I_0} \, \mathcal{R}(y_a, y_b),$$

and

$$\begin{aligned} \mathcal{R}(y_a, y_b) \quad = \quad & \textstyle\int_{\omega_a}^{\omega_b} T'_0(y^*(0,0,\omega)) \, y^*_\tau(0,0,\omega) \, f(\omega) \, d\omega \\ & + \textstyle\int_{\omega_a}^{\omega_b} \{y^*(0,0,\omega)) - y_a\} \, f(\omega) \, d\omega \\ & + (y_b - y_a)(1 - F(w_b)) \\ & - (y_b - y_a) \textstyle\int_{\omega_b}^{\overline{\omega}} T'_0(y^*(0,0,\omega)) \, y^*_e(0,0,\omega) \, f(\omega) \, d\omega \ . \end{aligned}$$

Again, we exploit the assumption that there is no bunching under the initial schedule, so that $y^*(0,0,\omega)$ is strictly increasing and hence invertible over $[\omega_a, \omega_b]$. We can therefore, without loss of generality, view a reform also as being defined by τ, ω_a and ω_b. With a slight abuse of notation, we therefore need to check whether

$$\frac{1}{1 - I_0} \, \mathcal{R}(\omega_a, \omega_b) - (y^*(0,0,\omega_b) - y^*(0,0,\omega_a)) \geq 0 \ ,$$

where

$$\begin{array}{rcl}\mathcal{R}(\omega_a,\omega_b) & = & \int_{\omega_a}^{\omega_b} T_0'(y^*(0,0,\omega))\ y_\tau^*(0,0,\omega)\ f(\omega)\ d\omega \\ & & +\int_{\omega_a}^{\omega_b}\{y^*(0,0,\omega)-y^*(0,0,\omega_a)\}\ f(\omega)\ d\omega \\ & & +(y^*(0,0,\omega_b)-y^*(0,0,\omega_a))(1-F(w_b)) \\ & & -(y^*(0,0,\omega_b)-y^*(0,0,\omega_a))\int_{\omega_b}^{\overline{\omega}} T_0'(y^*(0,0,\omega))\ y_e^*(0,0,\omega)\ f(\omega)\ d\omega\ .\end{array}$$

Note that

$$\begin{array}{l}\Delta_\tau^R(0,\omega_a,\omega_a)-(y^*(0,0,\omega_a)-y^*(0,0,\omega_a)) \\ =\frac{1}{1-I_0}\ \mathcal{R}(\omega_a,\omega_a)-(y^*(0,0,\omega_a)-y^*(0,0,\omega_a)) \\ =0\ .\end{array}$$

Thus, if

$$\frac{1}{1-I_0}\ \mathcal{R}_{\omega_b}(\omega_a,\omega_a)-y_\omega^*(0,0,\omega_a)>0\ ,$$

then $\Delta_\tau^R(0,\omega_a,\omega_b)-(y^*(0,0,\omega_b)-y^*(0,0,\omega_a))$ turns positive, if starting from $\omega_a=\omega_b$, we marginally increase ω_b.

Straightforward computations yield:

$$\begin{array}{l}\frac{1}{1-I_0}\mathcal{R}_{\omega_b}(\omega_a,\omega_a)-y_\omega^*(0,0,\omega_a) \\ =\frac{1}{1-I_0}\left\{T_0'(y^*(0,0,\omega_a))\ y_\tau^*(0,0,\omega_a)\ f(\omega_a)+(1-\tilde{I}_0(\omega_a))\ y_\omega^*(0,0,\omega_a)\ (1-F(\omega_a))\right\}-y_\omega^*(0,0,\omega_a)\ .\end{array}$$

Hence, if this expression is positive we can Pareto-improve by increasing marginal tax rates in a neighborhood of y_a. Upon noting that $y_\tau^*(0,0,\omega_a)<0$, the statement $\frac{1}{1-I_0}\mathcal{R}_{\omega_b}(\omega_a,\omega_a)-y_\omega^*(0,0,\omega_a)>0$ is easily seen to be equivalent to the claim $T_0'(y_a)<\mathcal{D}^P(y_a)$ in Proposition 2.

Proof of Proposition 4

We only prove the first statement in Proposition 4, the second follows from an analogous argument.

Recall that by equation (3)

$$\Delta_\tau^V(\omega\mid\tau,y_a,y_b)=\tilde{u}_c^1(\omega)\left(\Delta_\tau^R(\tau,y_a,y_b)-h(\tilde{y}^1(\omega))\right)\ ,$$

where $\tilde{u}_c^1(\omega)$ is a shorthand for the marginal utility of consumption that a type ω individual realizes after the reform $\tilde{y}^1(\omega)$ is the income level chosen after the reform. Note that $\tilde{y}^1(\omega)$ equals y_a for types who bunch at y_a, and $y^*(\Delta^R(\tau,y_a,y_b),\tau,\omega)$ otherwise. Consequently,

$$\begin{array}{rcl}\Delta_\tau^W(\tau,y_a,y_b\mid g) & := & \int_{\underline{\omega}}^{\overline{\omega}} g(\omega)\tilde{u}_c^1(\omega)\left(\Delta_\tau^R(\tau,y_a,y_b)-h(\tilde{y}^1(\omega))\right)f(\omega)\ d\omega\ . \\ & = & \gamma(\tau,y_a,y_b)\Delta_\tau^R(\tau,y_a,y_b) \\ & & -\int_{\omega_a(\tau)}^{\omega_b(\tau)} g(\omega)\tilde{u}_c^1(\cdot)\{y^*(\Delta^R(\cdot),\tau,\omega)-y_a\}f(\omega)d\omega \\ & & -(1-F(w_b(\tau)))(y_b-y_a)\Gamma(w_b(\tau)\mid\tau,y_a,y_b)\ ,\end{array}$$

where

$$\gamma(\tau,y_a,y_b):=\int_{\underline{\omega}}^{\overline{\omega}} g(\omega)\tilde{u}_c^1(\omega)f(\omega)\ d\omega,$$

and

$$\Gamma(w_b(\tau) \mid \tau, y_a, y_b) := \int_{\omega_b(\tau)}^{\overline{\omega}} g(\omega)\tilde{u}_c^1(\omega)\frac{f(\omega)}{1-F(\omega_b(\tau))}\, d\omega \; .$$

If we evaluate these welfare weights for $\tau = 0$, we obtain the welfare weights under the initial allocation. It will be convenient to have a more concise notation available. In the following, we will simply write γ_0 for the average welfare weight under the initial allocation, and $\Gamma_0(w_b)$ for the average welfare weight among individuals with types larger or equal to $\omega_b(0)$. Thus,

$$\begin{aligned} \Delta_\tau^W(0, y_a, y_b \mid g) \;&=\; \gamma_0 \Delta_\tau^R(0, y_a, y_b) \\ &\quad - \textstyle\int_{\omega_a(0)}^{\omega_b(0)} g(\omega)\tilde{u}_c^0(\cdot)\{y^*(0,0,\omega) - y_a\} f(\omega) d\omega \\ &\quad -(1 - F(w_b(0)))(y_b - y_a)\Gamma_0(w_b) \; , \end{aligned}$$

In the following, to simplify notation, we suppress the dependence of $\omega_a(0)$, $\omega_b(0)$ on $\tau = 0$ and simply write ω_a and ω_b. By assumption, there is no bunching under the initial schedule, so that $y^*(0,0,\omega)$ is strictly increasing and hence invertible over $[\omega_a, \omega_b]$. We can therefore, without loss of generality, view a reform also as being defined by τ, ω_a and ω_b. With a slight abuse of notation, we will therefore write

$$\begin{aligned} \Delta_\tau^W(0, \omega_a, \omega_b \mid g) \;&=\; \gamma_0 \Delta_\tau^R(0, \omega_a, \omega_b) \\ &\quad - \textstyle\int_{\omega_a}^{\omega_b} g(\omega)\tilde{u}_c^0(\cdot)\{y^*(0,0,\omega) - y^*(0,0,\omega_a)\} f(\omega) d\omega \\ &\quad -(1 - F(w_b))\{y^*(0,0,\omega_b) - y^*(0,0,\omega_a)\}\Gamma_0(w_b) \; , \end{aligned}$$

where, by equation (23)

$$\Delta_\tau^R(0, \omega_a, \omega_b) = \frac{1}{1 - I_0}\, \mathcal{R}(\omega_a, \omega_b) \; .$$

Hence,

$$\begin{aligned} \Delta_\tau^W(0, \omega_a, \omega_b \mid g) \;&=\; \tfrac{\gamma_0}{1-I_0} \mathcal{R}(\omega_a, \omega_b) \\ &\quad - \textstyle\int_{\omega_a}^{\omega_b} g(\omega)\tilde{u}_c^0(\cdot)\{y^*(0,0,\omega) - y^*(0,0,\omega_a)\} f(\omega) d\omega \\ &\quad -(1 - F(w_b))\{y^*(0,0,\omega_b) - y^*(0,0,\omega_a)\}\Gamma_0(w_b) \; , \end{aligned}$$

We now investigate under which conditions a marginal tax increase of τ over a small interval of types increases welfare. Note that

$$\Delta_\tau^W(0, \omega_a, \omega_a \mid g) = 0 \; .$$

If $\Delta_{\tau\omega_b}^W(0, \omega_a, \omega_a \mid g) > 0$ then $\Delta_\tau^W(0, \omega_a, \omega_b \mid g)$ turns positive, if starting from $\omega_a = \omega_b$, we marginally increase ω_b.

Straightforward computations yield

$$\Delta_{\tau\omega_b}^W(0, \omega_a, \omega_a \mid g) = \frac{\gamma_0}{1 - I_0} \mathcal{R}_{\omega_b}(\omega_a, \omega_a) - (1 - F(\omega_a))\Gamma_0(\omega_a) y_\omega^*(0, 0, \omega_a).$$

Recall that

$$\mathcal{R}_{\omega_b}(\omega_a, \omega_a) = T_0'(y^*(0,0,\omega_a))\; y_\tau^*(0,0,\omega_a)\; f(\omega_a) + (1 - \tilde{I}_0(\omega_a))\; y_\omega^*(0,0,\omega_a)\; (1 - F(\omega_a)) \; ,$$

so that we can write

$$\Delta_{\tau\omega_b}^W(0, \omega_a, \omega_a \mid g) = \frac{\gamma_0}{1 - I_0}\left\{T_0'(y^*(0,0,\omega_a))\; y_\tau^*(0,0,\omega_a)\; f(\omega_a) + (1 - F(\omega_a))\Phi_0(\omega_a) y_\omega^*(0,0,\omega_a)\right\} \; ,$$

where

$$\Phi_0(\omega_a) = 1 - \tilde{I}_0(\omega_a) - (1 - I_0)\frac{\Gamma_0(\omega_a)}{\gamma_0}.$$

Using this expression and the fact that $y^*_\tau(0,0,\omega_0) < 0$ for $\omega_0 \in (\omega_a, \omega_b)$ if ω_b is close to ω_a we obtain the characterization of welfare-increasing reforms in the first statement of Proposition 4.

Proof of Lemma 7

By the definition of an indirect utility function

$$V_\tau(\tau_s, \tau, \beta, \omega) = u_c(\cdot)(v_f(\cdot)c^*_{f\tau}(\cdot) + v_s(\cdot)c^*_{s\tau}(\cdot)) + u_y(\cdot)y^*_\tau(\cdot), \tag{24}$$

and

$$V_{\tau_s}(\tau_s, \tau, \beta, \omega) = u_c(\cdot)(v_f(\cdot)c^*_{f\tau_s}(\cdot) + v_s(\cdot)c^*_{s\tau_s}(\cdot)) + u_y(\cdot)y^*_{\tau_s}(\cdot)\ , \tag{25}$$

where $v_f(\cdot)$ and $v_s(\cdot)$ denote, respectively, the derivative of v with respect to c_f and c_s.

We use the first order conditions of the problem to maximize $u(v(c_f, c_s, \beta), y, \omega)$ subject to the budget constraint in (14) to rewrite these expressions. The first order conditions are

$$u_c(\cdot)v_f(\cdot) = \lambda^*\ , \tag{26}$$

$$u_c(\cdot)v_s(\cdot) = \lambda^*(1 + T'_{0s}(\cdot) + \tau_s h'_s(\cdot))\ , \tag{27}$$

and

$$-u_y(\cdot) = \lambda^*(1 - T'_0(\cdot) - \tau h'(\cdot))\ , \tag{28}$$

where λ^* is the value of the Lagrangian multiplier on the budget constraint.

Substituting these first order conditions into (24) and (25) yields

$$V_\tau(\tau_s, \tau, \beta, \omega) = \lambda^*\Big(c^*_{f\tau}(\cdot) + (1 + T'_{0s}(\cdot) + \tau_s h'_s(\cdot))c^*_{s\tau}(\cdot) - (1 - T'_0(\cdot) - \tau h'(\cdot))y^*_\tau(\cdot)\Big), \tag{29}$$

and

$$V_{\tau_s}(\tau_s, \tau, \beta, \omega) = \lambda^*\Big(c^*_{f\tau_s}(\cdot) + (1 + T'_{0s}(\cdot) + \tau_s h'_s(\cdot))c^*_{s\tau_s}(\cdot) - (1 - T'_0(\cdot) - \tau h'(\cdot))y^*_{\tau_s}(\cdot)\Big). \tag{30}$$

At a solution of the utility-maximization problem the budget constraint holds with equality. Differentiating both sides with respect to τ yields

$$c^*_{f\tau}(\cdot) + (1 - T'_{0s}(\cdot) - \tau_s h'_s(\cdot))c^*_{s\tau}(\cdot) - (1 - T'_0(\cdot) - \tau h'(\cdot))y^*_\tau(\cdot) = -h(y^*(\cdot))\ . \tag{31}$$

Analogously, differentiating both sides with respect to τ_s yields

$$c^*_{f\tau_s}(\cdot) + (1 - T'_{0s}(\cdot) - \tau_s h'_s(\cdot))c^*_{s\tau_s}(\cdot) - (1 - T'_0(\cdot) - \tau h'(\cdot))y^*_{\tau_s}(\cdot) = -h_s(c^*_s(\cdot))\ . \tag{32}$$

Substituting (31) and (32) into (29) and (30) yields

$$V_\tau(\tau_s, \tau, \beta, \omega) = -\lambda^* h(y^*(\cdot)), \tag{33}$$

and

$$V_{\tau_s}(\tau_s, \tau, \beta, \omega) = -\lambda^* h_s(c^*_s(\cdot)), \tag{34}$$

and hence

$$s(\tau, \tau^s, \beta, \omega) = -\frac{V_\tau(\tau_s, \tau, \beta, \omega)}{V_{\tau_s}(\tau_s, \tau, \beta, \omega)} = -\frac{h(y^*(\cdot))}{h_s(c^*_s(\cdot))}\ .$$

Lemma 7 follows from evaluating this equation for $\tau = \tau_s = 0$. □

B Bunching and non-negativity constraints

B.1 Bunching

Proposition 9 below extends Proposition 1 in the body of the text so as to allow for bunching in the characterization of revenue-increasing reforms. We also provide a formal proof of Proposition 9. We leave the extensions of Propositions 2, 3 and 4 to the reader. These extensions simply require to replace the function $\tilde{\omega}^0$ by its analog $\tilde{\omega}^{0b} : y \mapsto \max \omega \mid y^*(0,0,\omega) = y$ that takes account of the possibility bunching.

Proposition 9 *Define* $\mathcal{D}^{Rb}(y) := D^R(\tilde{\omega}^{0b}(y))$.

1. *Suppose that there is an income level* y_0 *so that* $T_0'(y_0) < \mathcal{D}^{Rb}(y_0)$. *Then there exists a revenue-increasing reform with* $\tau > 0$, *and* $y_a < y_0 < y_b$.
2. *Suppose that there is an income level* y_0 *so that* $T_0'(y_0) > \mathcal{D}^{Rb}(y_0)$. *Then there exists a revenue-increasing reform with* $\tau < 0$, *and* $y_a < y_0 < y_b$.

Proof. We prove only the first statement in the Proposition. The proof of the second statement is analogous.

We consider a reform in the (τ, y_a, y_b)-class with $\tau > 0$ and assume that, prior to the reform, there is a set of types $[\underline{\omega}_a(0), \overline{\omega}_a(0)]$ who bunch at y_a. More formally, for all $\omega \in [\underline{\omega}_a(0), \overline{\omega}_a(0)]$, $y^*(0,0,\omega) = \tilde{y}^0(\omega) = y_a$. We also assume, without loss of generality, that there is no further bunching between y_a and y_b so that the function $\tilde{y}^0$ is strictly increasing for $\omega \in (\overline{\omega}_a(0), \omega_b(0)]$. The reform will affect the set of types who bunch at y_a and we denote by $\underline{\omega}_a(\tau)$ and $\overline{\omega}_a(\tau)$ the minimal and the maximal type, respectively, who chose an earnings level of y_a after the reform.

We first look at the implication of such a reform for tax revenue. Again, we decompose the change in tax revenue.

$$\Delta^R(\tau, y_a, y_b) = \Delta^{R1}(\tau, y_a, y_b) + \Delta^{R2}(\tau, y_a, y_b) + \Delta^{R3}(\tau, y_a, y_b) + \Delta^{R4}(\tau, y_a, y_b) ,$$

where

$$\Delta^{R1}(\tau, y_a, y_b) = \int_{\underline{\omega}}^{\underline{\omega}_a(\tau)} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega ,$$

$$\Delta^{R2}(\tau, y_a, y_b) = \int_{\underline{\omega}_a(\tau)}^{\overline{\omega}_a(\tau)} \{T_1(y_a) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega ,$$

$$\Delta^{R3}(\tau, y_a, y_b) = \int_{\overline{\omega}_a(\tau)}^{\omega_b(\tau)} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega ,$$

and

$$\Delta^{R4}(\tau, y_a, y_b) = \int_{\omega_b(\tau)}^{\overline{\omega}} \{T_1(y^*(\Delta^R(\cdot), \tau, \omega)) - T_0(\tilde{y}^0(\omega))\} f(\omega) d\omega .$$

Following the same steps as in the proof of Proposition 1, we obtain a characterization of $\Delta^R_\tau(0, y_a, y_b)$,

$$\Delta^R_\tau(0, y_a, y_b) = \frac{1}{1 - I_0^b(y_a, y_b)} \mathcal{R}^b(y_a, y_b) ,$$

where

$$I_0^b(y_a, y_b) = \int_{\underline{\omega}}^{\underline{\omega}_a(0)} T_0'(\cdot) y_e^*(0,0,\omega) f(\omega) d\omega + \int_{\overline{\omega}_a(0)}^{\overline{\omega}} T_0'(\cdot) y_e^*(0,0,\omega) f(\omega) d\omega \; ,$$

and

$$\begin{aligned} \mathcal{R}^b(y_a, y_b) &= \int_{\overline{\omega}_a(0)}^{\omega_b(0)} T_0'(\cdot) y_\tau^*(0,0,\omega) f(\omega) d\omega + \int_{\overline{\omega}_a(0)}^{\omega_b(0)} \{y^*(0,0,\omega) - y_a\} f(\omega) d\omega \\ &\quad + (y_b - y_a)(1 - F(\omega_b(0))) - (y_b - y_a) \int_{\omega_b(0)}^{\overline{\omega}} T_0'(\cdot) y_e^*(0,0,\omega) f(\omega) d\omega \; . \end{aligned}$$

The superscripts b indicate that these expressions are the analogs to I_0 and $\mathcal{R}$ in the proof of Proposition 1 that take account of taxes.

In the following to save on notation, we write $\underline{\omega}_a$, $\overline{\omega}_a$ and ω_b rather than $\underline{\omega}_a(0)$, $\overline{\omega}_a(0)$ and $\omega_b(0)$. Again, we perform a change in variables and interpret $I_0^b(y_a, y_b)$ as a function of the parameters $\underline{\omega}_a$ and $overline \omega_a$ and $\mathcal{R}^b(y_a, y_b)$ as a function of $\overline{\omega}_a$ and ω_b and write, with some abuse of notation,

$$I_0^b(\underline{\omega}_a, \overline{\omega}_a) = \int_{\underline{\omega}}^{\underline{\omega}_a} T_0'(\cdot) y_e^*(0,0,\omega) f(\omega) d\omega + \int_{\overline{\omega}_a}^{\overline{\omega}} T_0'(\cdot) y_e^*(0,0,\omega) f(\omega) d\omega \; ,$$

and

$$\begin{aligned} \mathcal{R}^b(\overline{\omega}_a, \omega_b) &= \int_{\overline{\omega}_a}^{\omega_b} T_0'(\cdot) y_\tau^*(0,0,\omega) f(\omega) d\omega + \int_{\overline{\omega}_a}^{\omega_b} \{y^*(0,0,\omega) - y^*(0,0,\overline{\omega}_a)\} f(\omega) d\omega \\ &\quad + (y^*(0,0,\omega_b) - y^*(0,0,\overline{\omega}_a))(1 - F(\omega_b)) \\ &\quad - (y^*(0,0,\omega_b) - y^*(0,0,\overline{\omega}_a)) \int_{\omega_b}^{\overline{\omega}} T_0'(\cdot) y_e^*(0,0,\omega) f(\omega) d\omega \; . \end{aligned}$$

Hence,

$$\Delta_\tau^R(0, \underline{\omega}_a, \overline{\omega}_a, \omega_b) = \frac{1}{1 - I_0^b(\underline{\omega}_a, \overline{\omega}_a)} \mathcal{R}^b(\overline{\omega}_a, \omega_b) \; ,$$

Moreover, note that since there is, by assumption, no further bunching between y_a and y_b,

$$\mathcal{R}^b(\overline{\omega}_a, \omega_b) = \mathcal{R}(\overline{\omega}_a, \omega_b) \; .$$

We now investigate the conditions under which

$$\Delta_{\tau\omega_b}^R(0, \underline{\omega}_a, \overline{\omega}_a, \omega_a) > 0 \; .$$

If this inequality holds then there exists a reform (τ, y_a, y_b) for some $y_b > y_a$ that leads to an increase of tax revenue. Also note that $\Delta_{\tau\omega_b}^R(0, \underline{\omega}_a, \overline{\omega}_a, \omega_a) > 0$ holds if and only if

$$\mathcal{R}_{\omega_b}(\overline{\omega}_a, \overline{\omega}_a) > 0 \; .$$

The condition under which this last inequality holds have been characterized in the proof of Proposition 1.

If

$$T_0'(y_a) < -\frac{1 - F(\overline{\omega}_a)}{f(\overline{\omega}_a)} \left(1 - \tilde{I}_0(\overline{\omega}_a)\right) \frac{y_\omega^*(0,0,\overline{\omega}_a)}{y_\tau^*(0,0,\overline{\omega}_a)} \; ,$$

or, equivalently,

$$T_0'(y_a) < D^R(\overline{\omega}_a)$$

Using the function $\tilde{\omega}^{0b} : y \mapsto \max \omega \mid y^*(0,0,\omega) = y$ we can express this also as

$$T_0'(y_a) < D^R(\tilde{\omega}^{0b}(y_a)) \; ,$$

or as

$$T_0'(y_a) < D^R(\tilde{\omega}^{0b}(y_a)) \; ,$$

which proves statement 1. in Proposition 9.

B.2 Non-negativity constraints

Binding non-negativity constraints on earnings are a particular type of bunching. The behavioral responses to a reform in the (τ, y_a, y_b)-class with $y_a > 0$ and $\tau > 0$ then look as follows: Individuals with $\omega \leq \hat{\omega}^P(\tau)$ choose earnings of zero after the reform, individuals with $\omega \in (\hat{\omega}^P(\tau), \omega_a(\tau))$ choose $y \in (0, y_a)$ after the reform, individuals with $\omega \in [\omega_a(\tau), \overline{\omega}_a(\tau)]$ choose $y = y_a$ after the reform, individuals with $\omega \in (\overline{\omega}_a(\tau), \omega_b(\tau))$ choose $y \in (y_a, y_b)$, and individuals with $\omega \geq \omega_b(\tau)$ choose $y \geq y_b$.

We leave it to the reader to verify that a small reform in the (τ, y_a, y_b)-class raises revenue if and only if the conditions in Proposition 1 are fulfilled. While the accounting of behavioral responses has to take account of individuals with no income, the analysis in the end boils down to an analysis of the conditions under which $\mathcal{R}_{\omega_b}(\omega_a, \omega_a) > 0$ holds, just as in the proof of Proposition 1. If we consider instead a reform in the (τ, y_a, y_b)-class with $y_a = 0$, we modify marginal tax rates at a point of bunching. In this case the conditions in Proposition 9 evaluated for $y_a = 0$ clarify whether such a reform raises tax revenue.

Once the revenue implications are clear, extensions of Propositions 2, 3 and 4 that allow for binding non-negativity constraints can be obtained along the same lines as in the body of the text.

C Online Appendix

The Online Appendix provides the details for one part of the Proof of Proposition 1 we omitted, see in Appendix A.

We provide a characterization of $\Delta_\tau^R(\tau, y_a, y_b)$ by looking at the different subsets of the population separately. We then evaluate $\Delta_\tau^R(\tau, y_a, y_b)$ at $\tau = 0$ since $\Delta_\tau^R(0, y_a, y_b)$ gives the increase in tax revenue if we start to increase marginal tax rates for incomes in the interval $[y_a, y_b]$. Since

$$\Delta_\tau^R(\tau, y_a, y_b) = \Delta_\tau^{R1}(\tau, y_a, y_b) + \Delta_\tau^{R2}(\tau, y_a, y_b) + \Delta_\tau^{R3}(\tau, y_a, y_b) + \Delta_\tau^{R4}(\tau, y_a, y_b) \;,$$

we can characterize $\Delta_\tau^R(\tau, y_a, y_b)$ by looking at each subset of types separately

For the first subset:

$$\begin{aligned} \Delta_\tau^{R1}(\tau, y_a, y_b) &= \Delta_\tau^R(\cdot) \int_{\underline{\omega}}^{\omega_a(\tau)} T_0'(y^*(\Delta^R(\cdot), 0, \omega))\, y_e^*(\Delta^R(\cdot), 0, \omega)\, f(\omega)\, d\omega \\ &\quad + \{T_0(y_a) - T_0(y^*(0, 0, \omega_a(\tau)))\} f(\omega_a(\tau)) \omega_a'(\tau) \;, \end{aligned}$$

where $\omega_a'(\tau)$ is the derivative of $\omega_a(\tau)$. Analogously we derive

$$\begin{aligned} \Delta_\tau^{R2}(\tau, y_a, y_b) &= \{T_0(y_a) - T_0(y^*(0, 0, \hat{\omega}_a(\tau)))\} f(\hat{\omega}_a(\tau)) \hat{\omega}_a'(\tau) \\ &\quad - \{T_0(y_a) - T_0(y^*(0, 0, \omega_a(\tau)))\} f(\omega_a(\tau)) \omega_a'(\tau) \;, \end{aligned}$$

$$\begin{aligned}
\Delta_\tau^{R3}(\tau, y_a, y_b) &= \textstyle\int_{\hat\omega_a(\tau)}^{\omega_b(\tau)} \{T_0'(y^*(\Delta^R(\cdot),\tau,\omega)) + \tau\}\ \{y_e^*(\Delta^R(\cdot),\tau,\omega)\Delta_\tau^R(\cdot) + y_\tau^*(\Delta^R(\cdot),\tau,\omega)\}\ f(\omega)\ d\omega \\
&\quad + \textstyle\int_{\hat\omega_a(\tau)}^{\omega_b(\tau)} \{y^*(\Delta^R(\cdot),\tau,\omega)) - y_a\}\ f(\omega)\ d\omega \\
&\quad - \{T_0(y_a) - T_0(y^*(0,0,\hat\omega_a(\tau)))\} f(\hat\omega_a(\tau))\hat\omega_a'(\tau) \\
&\quad + \{T_0(y_b) + \tau(y_b - y_a) - T_0(y^*(0,0,\omega_b(\tau)))\} f(\omega_b(\tau))\omega_b'(\tau) \\
&= \Delta_\tau^R(\cdot) \textstyle\int_{\hat\omega_a(\tau)}^{\omega_b(\tau)} T_0'(y^*(\Delta^R(\cdot),\tau,\omega)) y_e^*(\Delta^R(\cdot),\tau,\omega)\ f(\omega)\ d\omega \\
&\quad + \textstyle\int_{\hat\omega_a(\tau)}^{\omega_b(\tau)} T_0'(y^*(\Delta^R(\cdot),\tau,\omega))\ y_\tau^*(\Delta^R(\cdot),\tau,\omega)\ f(\omega)\ d\omega \\
&\quad + \tau \textstyle\int_{\hat\omega_a(\tau)}^{\omega_b(\tau)} \{y_e^*(\Delta^R(\cdot),\tau,\omega)\Delta_\tau^R(\cdot) + y_\tau^*(\Delta^R(\cdot),\tau,\omega)\}\ f(\omega)\ d\omega \\
&\quad + \textstyle\int_{\hat\omega_a(\tau)}^{\omega_b(\tau)} \{y^*(\Delta^R(\cdot),\tau,\omega)) - y_a\}\ f(\omega)\ d\omega \\
&\quad - \{T_0(y_a) - T_0(y^*(0,0,\hat\omega_a(\tau)))\} f(\hat\omega_a(\tau))\hat\omega_a'(\tau) \\
&\quad + \{T_0(y_b) + \tau(y_b - y_a) - T_0(y^*(0,0,\omega_b(\tau)))\} f(\omega_b(\tau))\omega_b'(\tau),
\end{aligned}$$

and

$$\begin{aligned}
\Delta_\tau^{R4}(\tau, y_a, y_b) &= (y_b - y_a)(1 - F(w_b(\tau))) \\
&\quad \Delta_\tau^R(\cdot) \textstyle\int_{\omega_b(\tau)}^{\overline{\omega}} T_0'((\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega))\ y_e^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega)\ f(\omega)\ d\omega \\
&\quad - (y_b - y_a) \textstyle\int_{\omega_b(\tau)}^{\overline{\omega}} T_0'((\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega))\ y_e^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega)\ f(\omega)\ d\omega \\
&\quad - \{T_0(y_b) + \tau(y_b - y_a) - T_0(y^*(0,0,\omega_b(\tau)))\} f(\omega_b(\tau))\omega_b'(\tau).
\end{aligned}$$

Define

$$\begin{aligned}
I(\tau, y_a, y_b) &:= \textstyle\int_{\underline{\omega}}^{\omega_a(\tau)} T_0'(y^*(\Delta^R(\cdot),0,\omega))\ y_e^*(\Delta^R(\cdot),0,\omega)\ f(\omega)\ d\omega \\
&\quad + \textstyle\int_{\hat\omega_a(\tau)}^{\omega_b(\tau)} T_0'(y^*(\Delta^R(\cdot),\tau,\omega)) y_e^*(\Delta^R(\cdot),\tau,\omega)\ f(\omega)\ d\omega \\
&\quad + \textstyle\int_{\omega_b(\tau)}^{\overline{\omega}} T_0'((\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega))\ y_e^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega)\ f(\omega)\ d\omega\ .
\end{aligned}$$

Upon collecting terms we find

$$\begin{aligned}
\Delta_\tau^R(\tau, y_a, y_b) &= \Delta_\tau^R(\tau, y_a, y_b) I(\tau, y_a, y_b) \\
&\quad + \textstyle\int_{\hat\omega_a(\tau)}^{\omega_b(\tau)} T_0'(y^*(\Delta^R(\cdot),\tau,\omega))\ y_\tau^*(\Delta^R(\cdot),\tau,\omega)\ f(\omega)\ d\omega \\
&\quad + \tau \textstyle\int_{\hat\omega_a(\tau)}^{\omega_b(\tau)} \{y_e^*(\Delta^R(\cdot),\tau,\omega)\Delta_\tau^R(\cdot) + y_\tau^*(\Delta^R(\cdot),\tau,\omega)\}\ f(\omega)\ d\omega \\
&\quad + \textstyle\int_{\hat\omega_a(\tau)}^{\omega_b(\tau)} \{y^*(\Delta^R(\cdot),\tau,\omega)) - y_a\}\ f(\omega)\ d\omega \\
&\quad + (y_b - y_a)(1 - F(w_b(\tau))) \\
&\quad - (y_b - y_a) \textstyle\int_{\omega_b(\tau)}^{\overline{\omega}} T_0'(y^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega))\ y_e^*(\Delta^R(\cdot) - \tau(y_b - y_a), 0, \omega)\ f(\omega)\ d\omega\ .
\end{aligned}$$

If we evaluate this expression at $\tau = 0$ and assume that there is no bunching at y_a under the initial tax schedule T_0 so that $\omega_a(0) = \hat\omega_a(0)$ we obtain

$$\begin{aligned}
\Delta_\tau^R(0, y_a, y_b) &= \Delta_\tau^R(0, y_a, y_b) I(0, y_a, y_b) \\
&\quad + \textstyle\int_{\omega_a(0)}^{\omega_b(0)} T_0'(y^*(0,0,\omega))\ y_\tau^*(0,0,\omega)\ f(\omega)\ d\omega \\
&\quad + \textstyle\int_{\omega_a(0)}^{\omega_b(0)} \{y^*(0,0,\omega)) - y_a\}\ f(\omega)\ d\omega \\
&\quad + (y_b - y_a)(1 - F(w_b(0))) \\
&\quad - (y_b - y_a) \textstyle\int_{\omega_b(0)}^{\overline{\omega}} T_0'(y^*(0,0,\omega))\ y_e^*(0,0,\omega)\ f(\omega)\ d\omega,
\end{aligned}$$

where

$$I_0 = \tilde{I}(\underline{\omega}) = I(0, y_a, y_b) = \int_{\underline{\omega}}^{\overline{\omega}} T_0'(y^*(0,0,\omega))\ y_e^*(0,0,\omega)\ f(\omega)\ d\omega$$

is an expression that no longer depends on the parameters y_a and y_b, thus, we use a shorter notation and simply write I_0. The rest of the Proof is in Appendix A.

D Figures

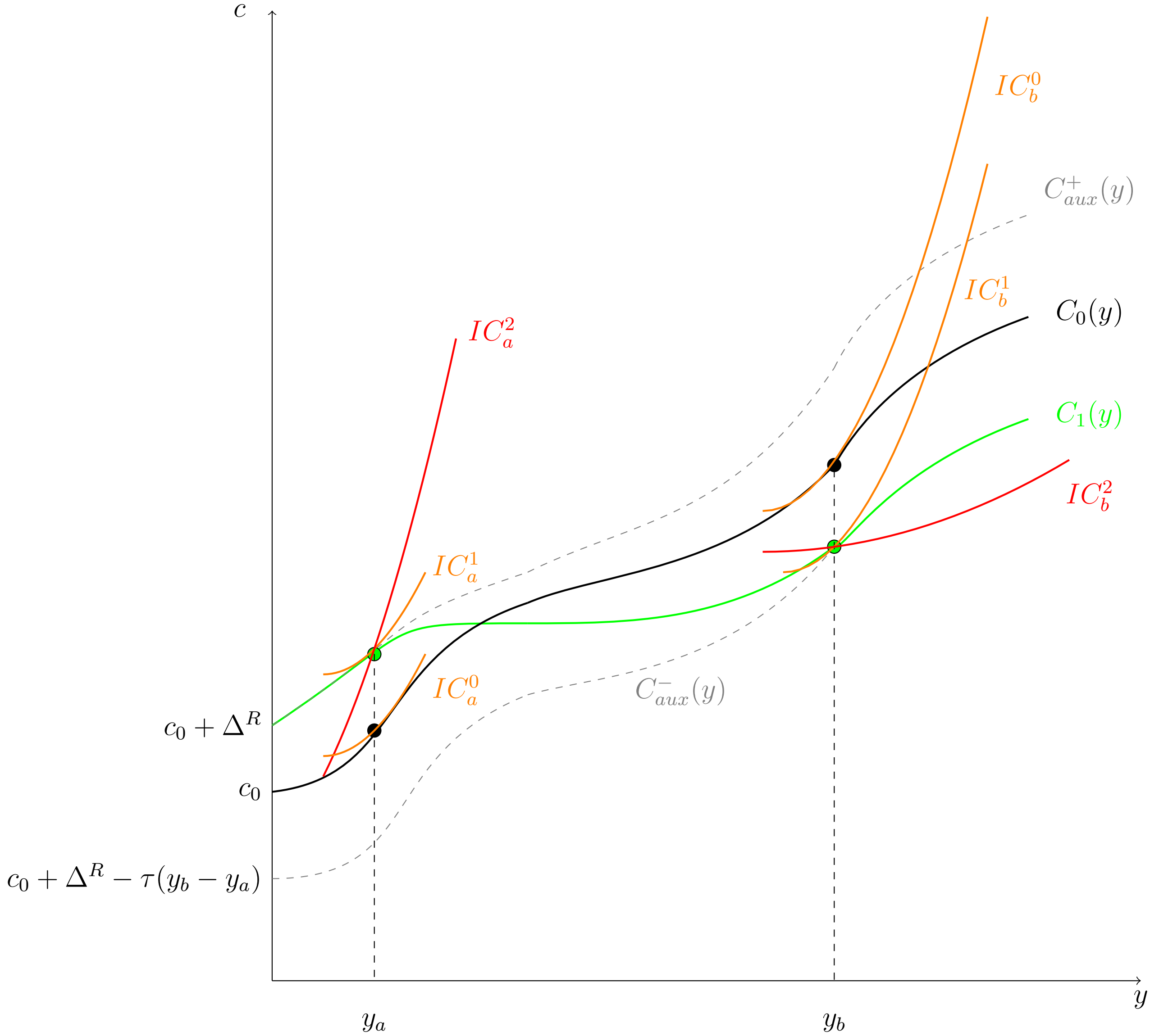

Figure 1:

The figure depicts the following curves:

(a) $C_0(y) = c_0 + y - T_0(y), \quad C_1(y) = c_0 + \Delta^R + y - T_0(y) - \tau h(y),$

$$C^+_{aux}(y) = c_0 + \Delta^R + y - T_0(y), \quad C^-_{aux}(y) = c_0 + \Delta^R - \tau(y_b - y_a) + y - T_0(y);$$

(b) IC_b^0 *indifference curve of type* $\omega_b(0)$ *through* y_b *under* $C_0(y)$, IC_b^1 *curve through* y_b *under* $C_1(y)$ *when there are no income effects,* IC_b^2 *curve through* y_b *under* $C_1(y)$ *with income effects;*

(c) IC_a^0 *indifference curve of type* $\omega_a(0)$ *through* y_a *under* $C_0(y)$, IC_a^1 *curve through* y_a *under* $C_1(y)$ *when there are no income effects,* IC_a^2 *curve through* y_a *under* $C_1(y)$ *if there are income effects.*

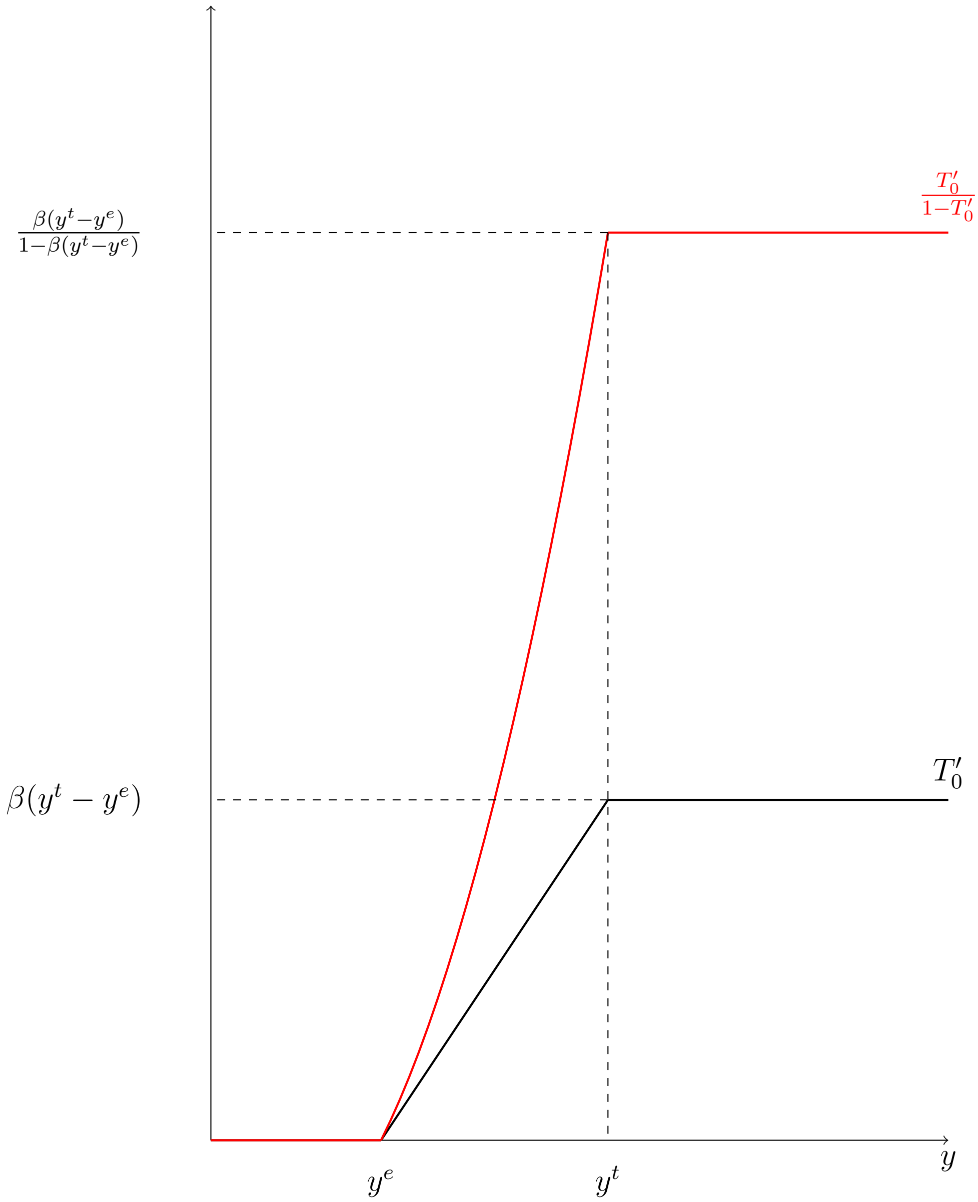

Figure 2: Status quo tax schedule defined by equation (8)

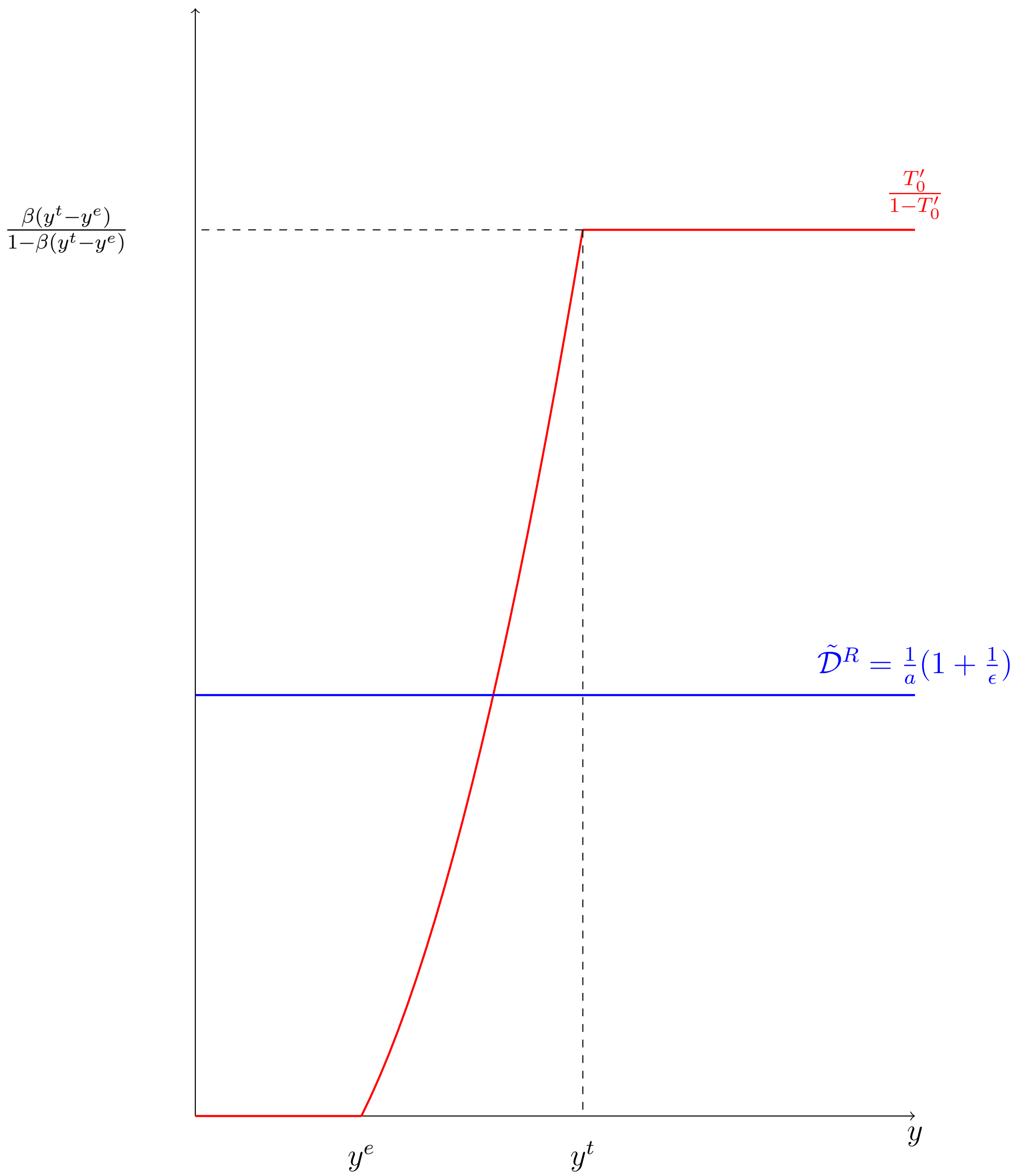

Figure 3: Sufficient statistics for revenue-increasing reforms without income effects

The absence of income effects implies that the sufficient statistic $\tilde{\mathcal{D}}^R$ does not depend on the level of income. The figure shows an example so that $\tilde{\mathcal{D}}^R$ lies above $\frac{T_0'}{1-T_0'}$ for low levels of income and below $\frac{T_0'}{1-T_0'}$ for high levels of income. Thus, for low levels on income, revenue can be raised by an increase of marginal tax rates. For high levels of income the status quo is inefficient: an increase of revenue requires lower marginal taxes.

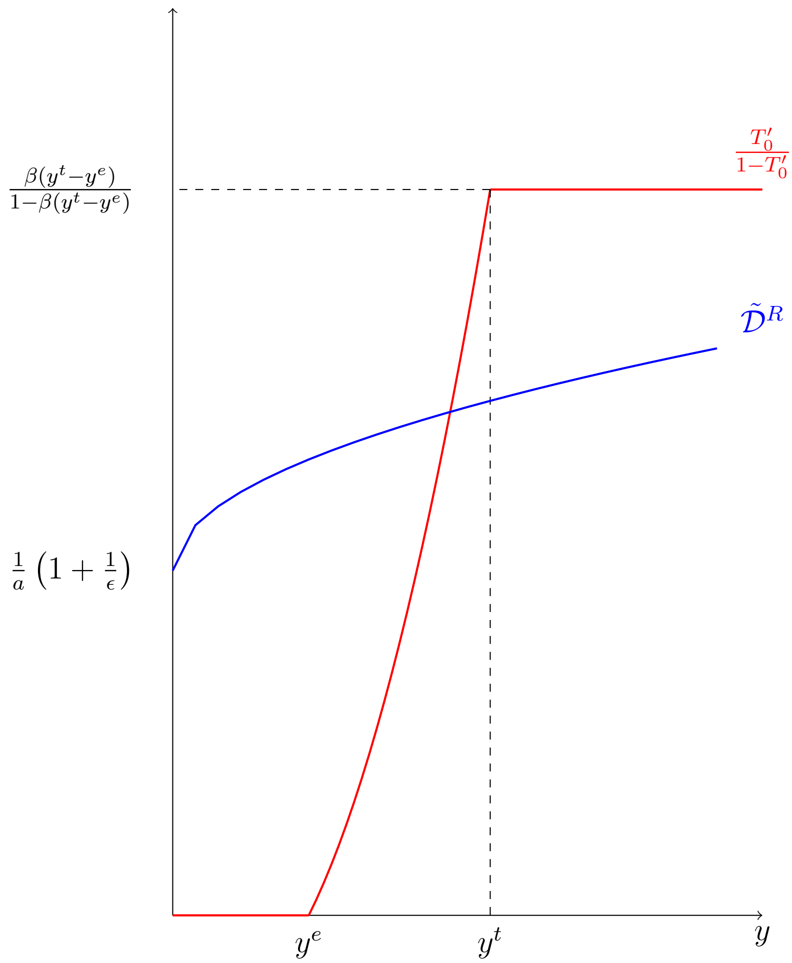

Figure 4: Sufficient statistics for revenue-increasing reforms with income effects

Figure 3 is obtained form Figure 2 by assuming $\eta < 0$ rather than $\eta = 0$. The sufficient statistic $\tilde{\mathcal{D}}^R$ is now an increasing and concave function of income.

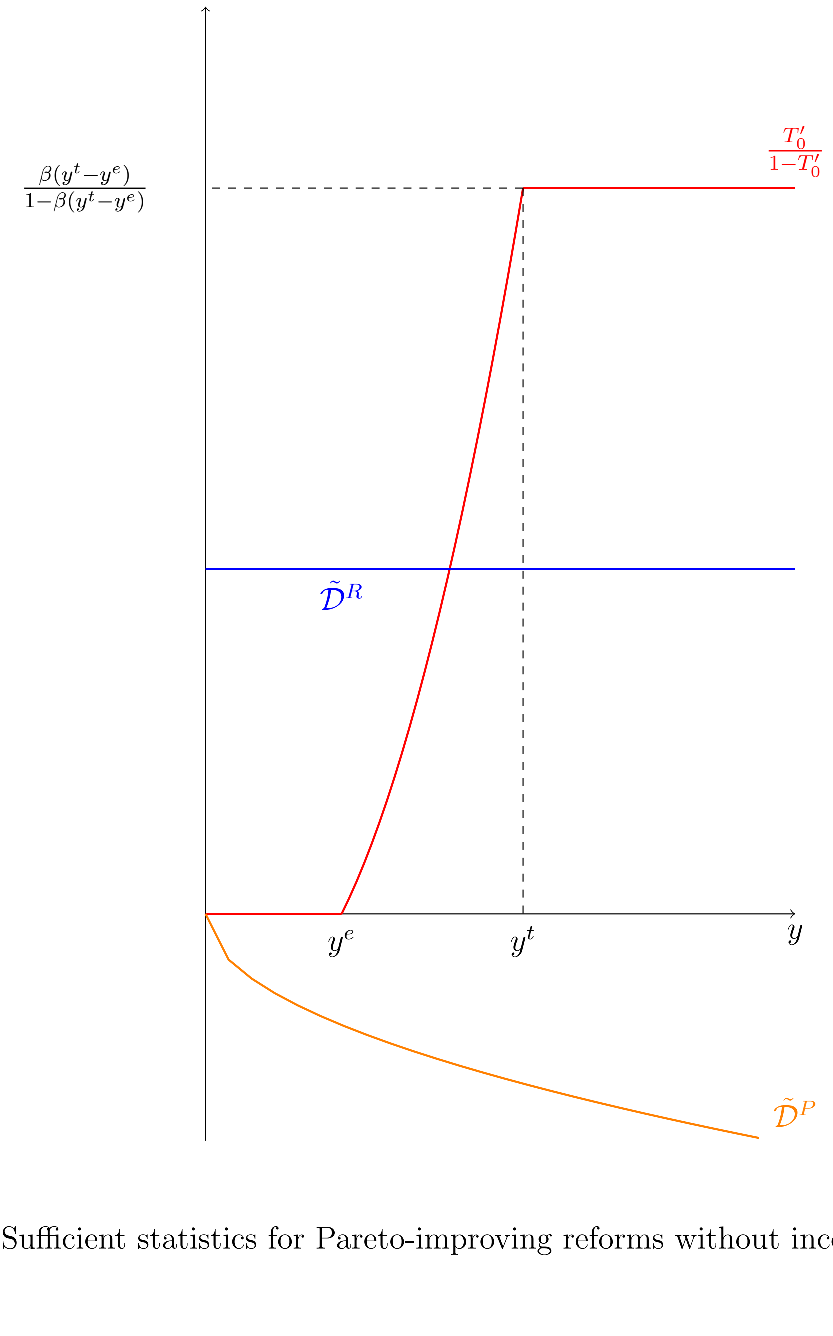

Figure 5: Sufficient statistics for Pareto-improving reforms without income effects

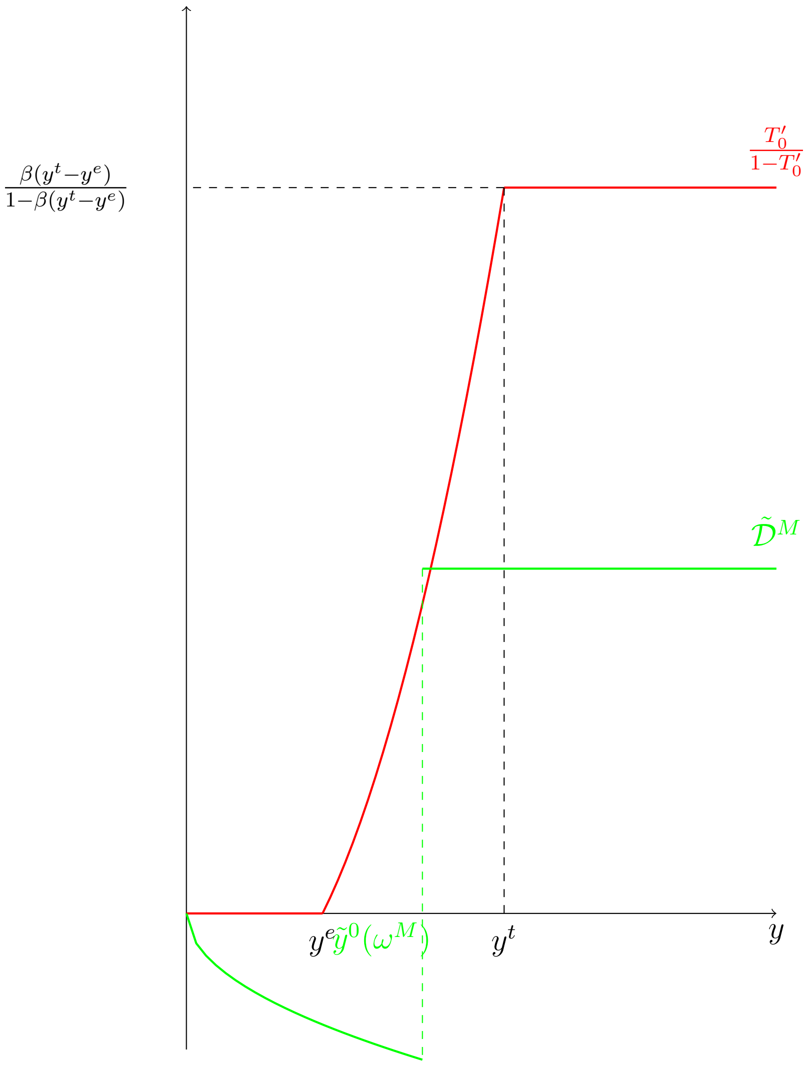

Figure 6: Sufficient statistics for politically feasible reforms without income effects

The figure provides an examples so that tax cuts are politically feasible both for incomes below the median and high incomes. For incomes slightly above the median, tax increases are politically feasible.

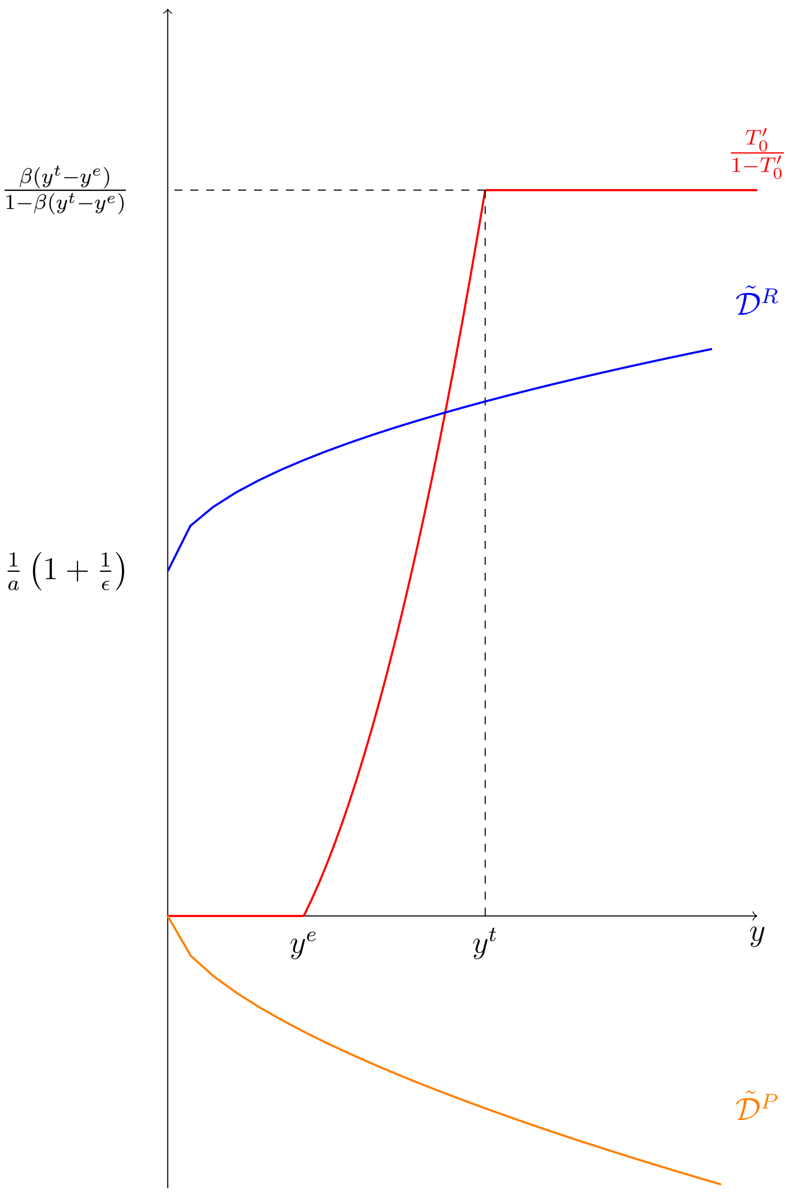

Figure 7: Sufficient statistics for Pareto-improving reforms with income effects

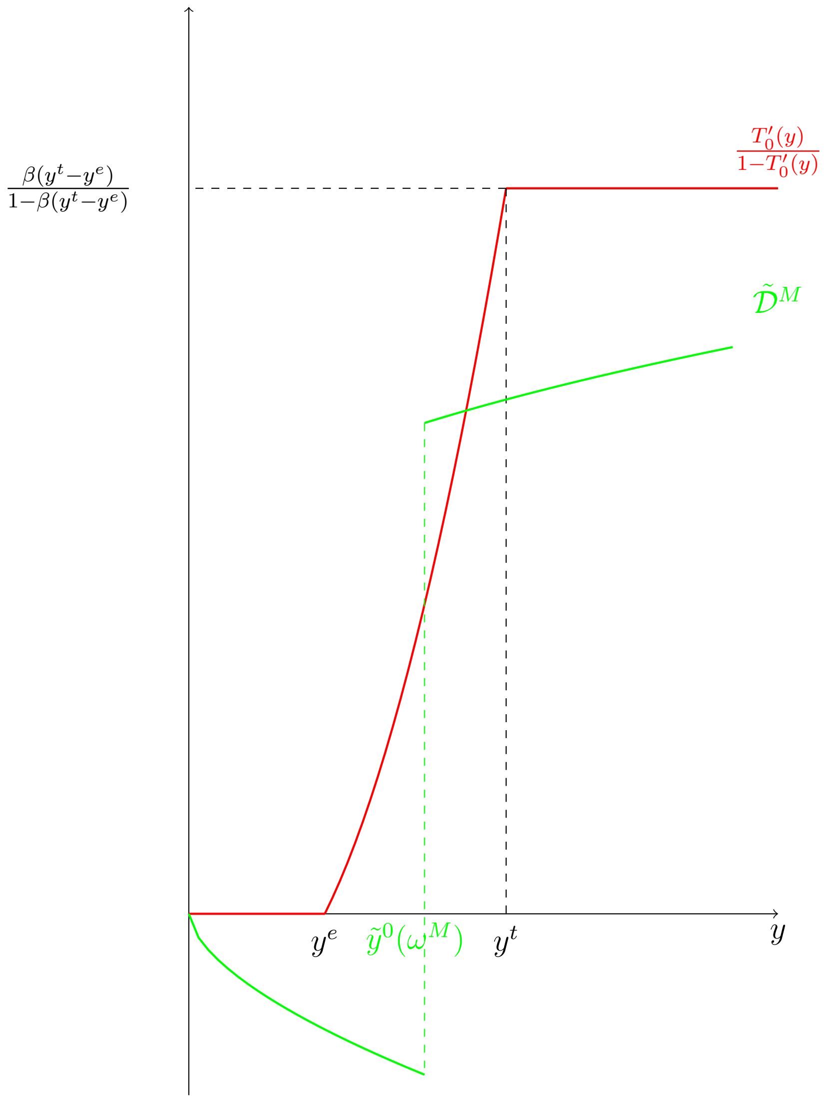

Figure 8: Sufficient statistics for politically feasible reforms with income effects

This figure is an adaptation of Figure 5.

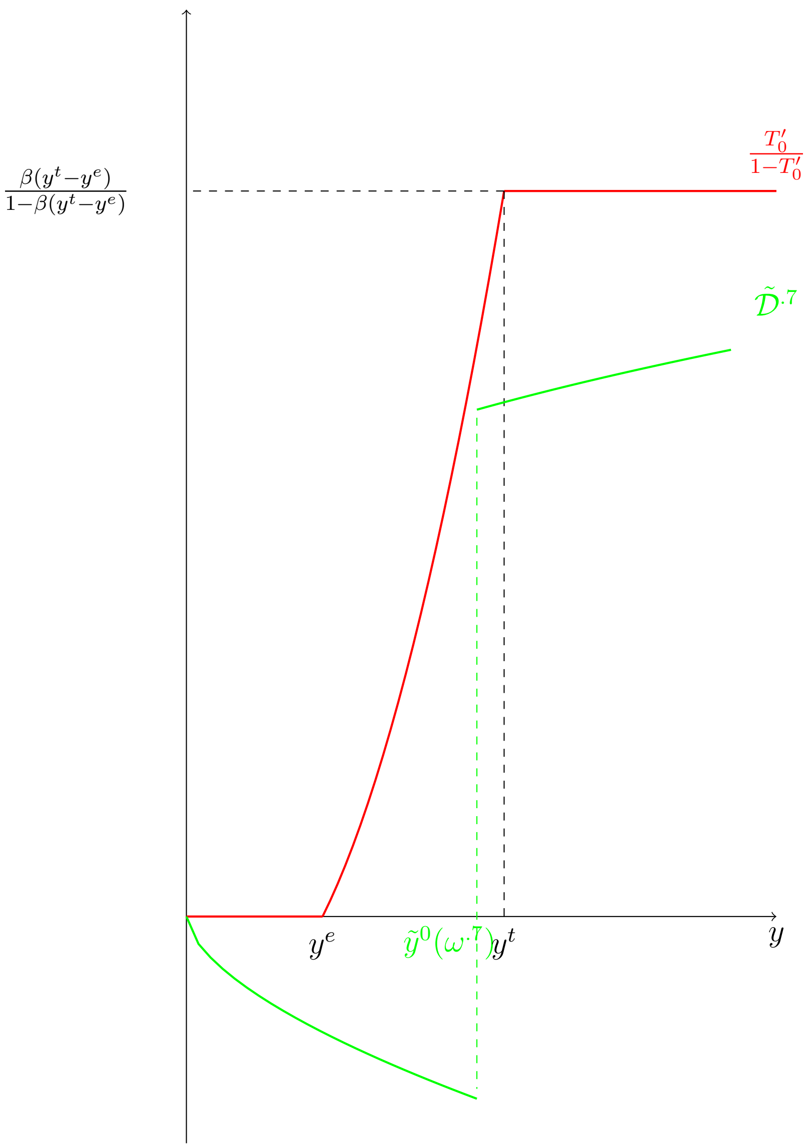

Figure 9: Sufficient statistics for reforms that benefit the bottom 70 percent

The figure illustrates a situation where people in the bottom 70 percent would benefit from tax cuts both for low incomes and for high incomes.

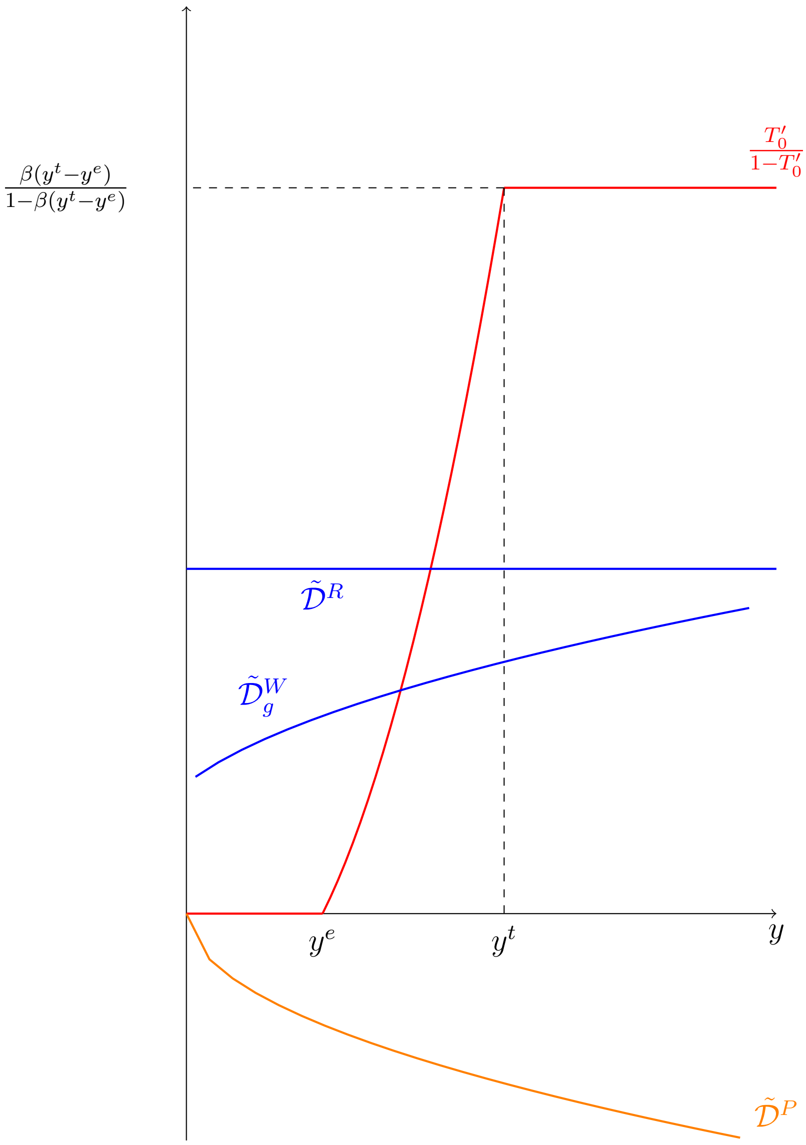

Figure 10: Sufficient statistics for Pareto-improving, and Welfare-improving tax reforms, no income effects

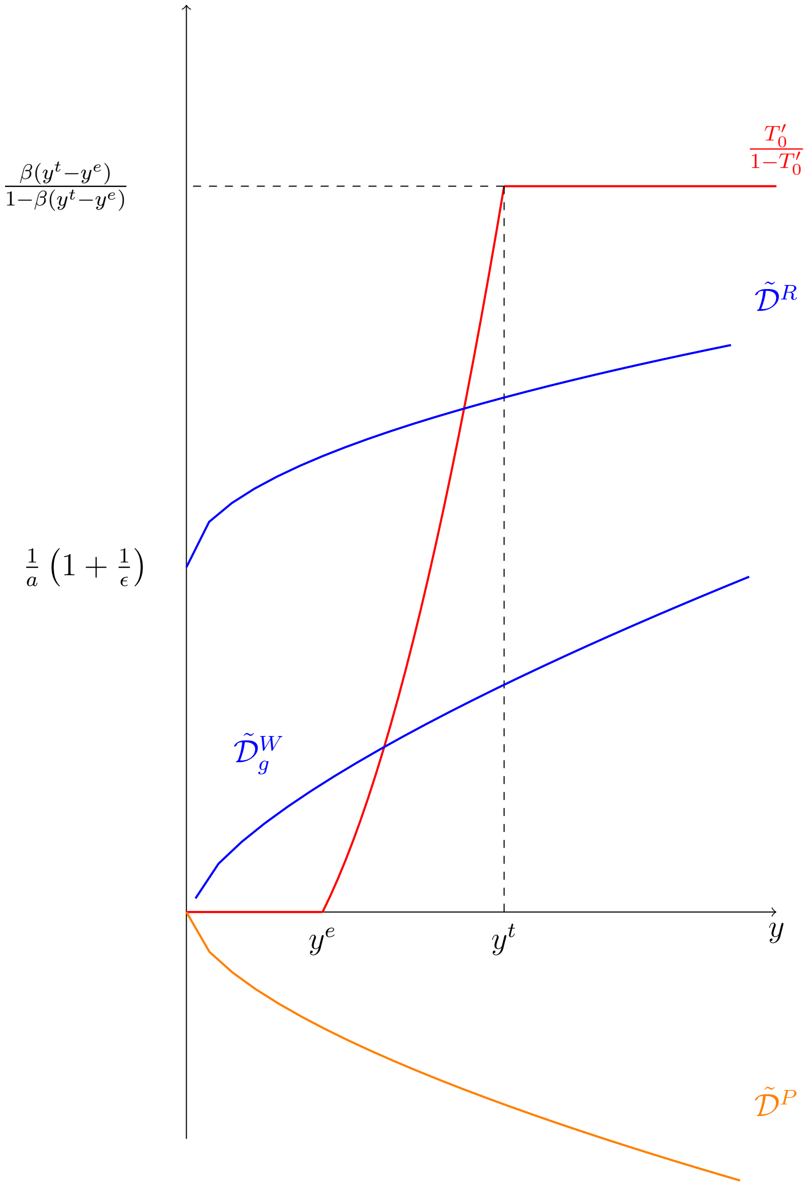

Figure 11: Sufficient statistics for Pareto-improving, and Welfare-improving tax reforms with income effects

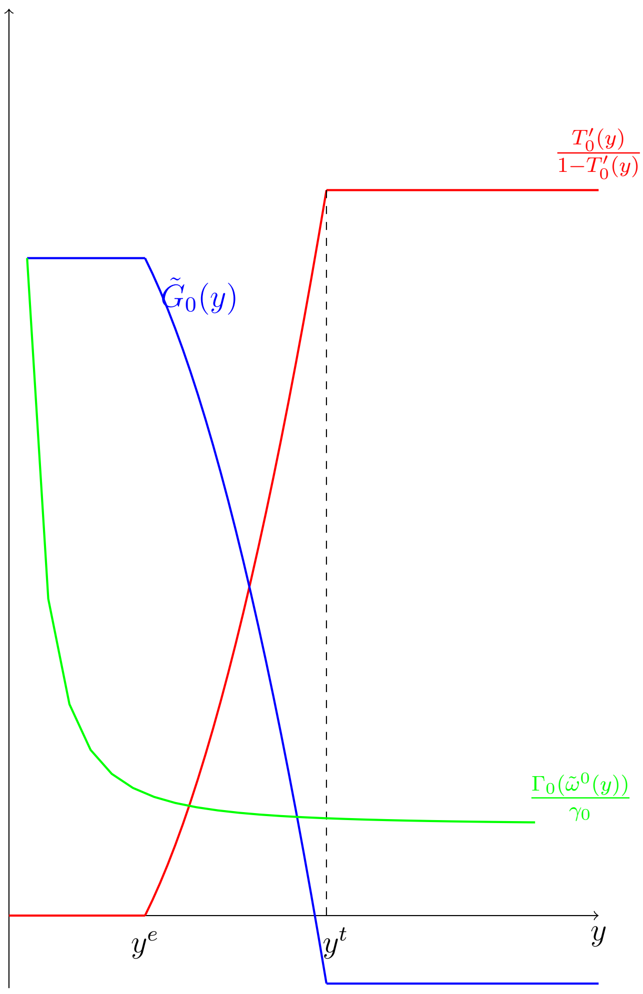

Figure 12: Critical welfare weights

We assume that the function g that is associated to $\frac{\Gamma_0(\tilde{\omega}^0(y))}{\gamma_0}$ take the form $g(\omega) = \frac{1}{1+\omega^2}$, for all ω. Otherwise parameters are as in Figure 7.

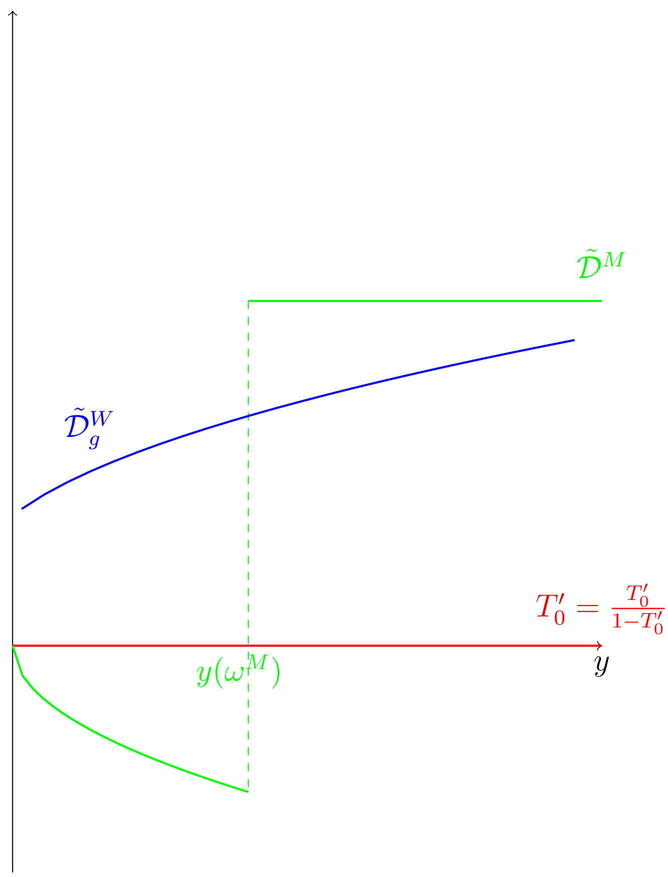

Figure 13: Sufficient statistics for politically feasible reforms and welfare-improving reforms under laissez-faire, no income effects

The welfare weights associated to $\tilde{\mathcal{D}}_g^W$ take the form $g(\omega) = \frac{1}{1+\omega^2}$, for all ω. Otherwise parameters are as in Figure 5.

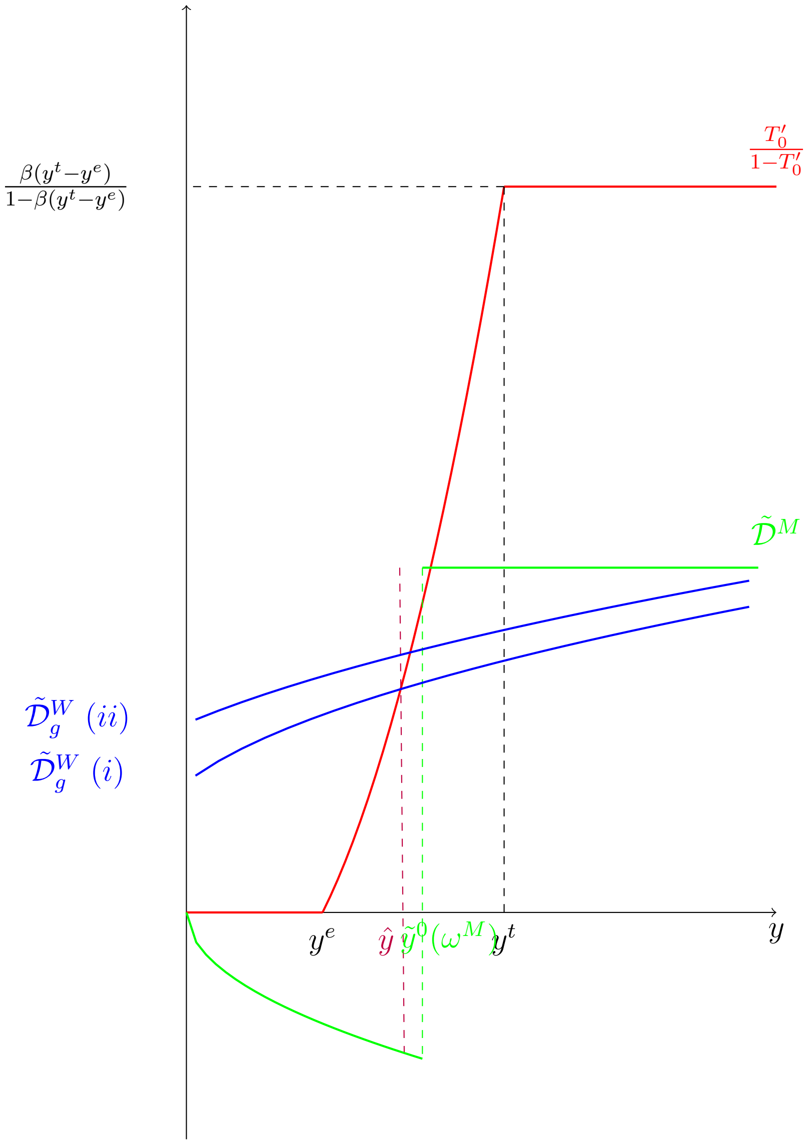

Figure 14: Sufficient statistics for politically feasible reforms and welfare-improving reforms, no income effects

We assume that the welfare weights associated to $\tilde{\mathcal{D}}_g^W$ *either take the form (i)* $g(\omega) = \frac{1}{1+\omega^2}$ *or (ii)* $\tilde{g}(\omega) = \frac{1}{1+\omega^4}$*. The constellation in the figures is such that taxes are inefficiently high for incomes above the median. Tax cuts are therefore welfare-improving and politically feasible. For incomes in* $[\hat{y}, \tilde{y}^0(\omega^M)[$*, tax cuts are not mandated by Pareto-efficiency, but they are politically feasible. Whether they are desirable depends on the welfare function. For low incomes, tax cuts are politically feasible, but welfare-damaging.*

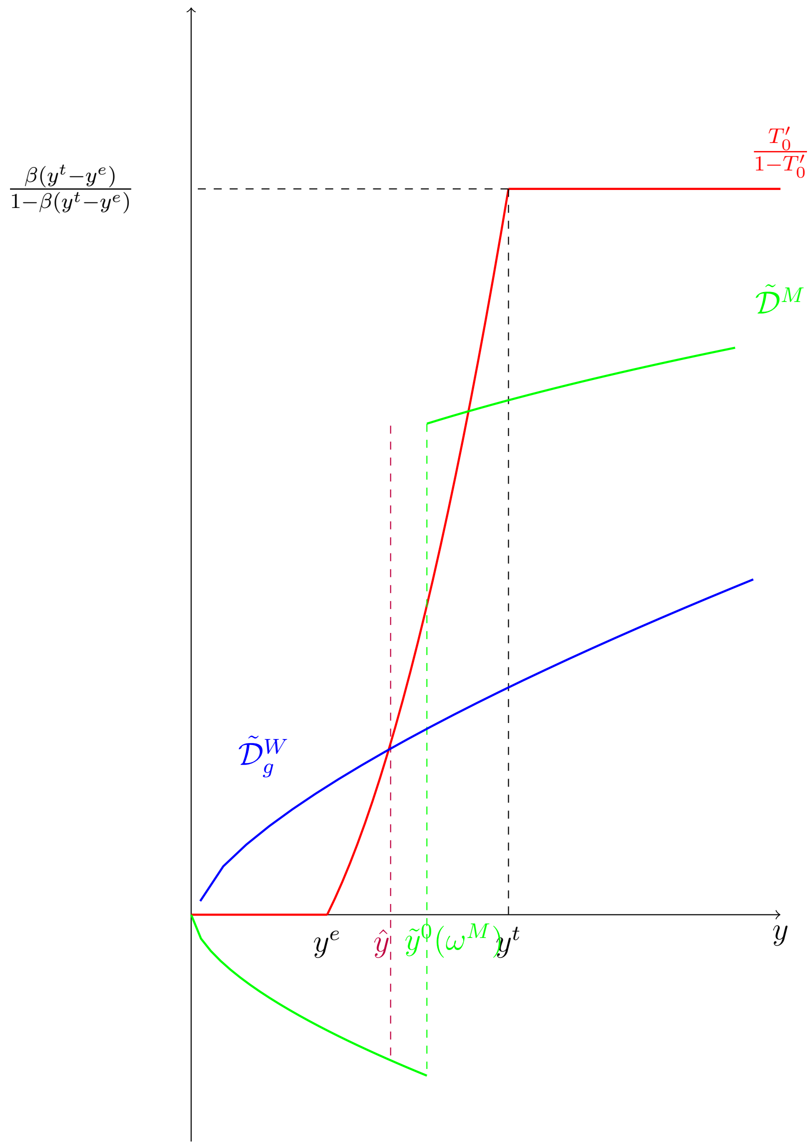

Figure 15: Sufficient statistics for politically feasible reforms and welfare-improving reforms with income effects

The welfare weights associated to $\tilde{\mathcal{D}}_g^W$ *take the form* $g(\omega) = \frac{1}{1+\omega^2}$*, for all* ω*. As in Figure 14 for high incomes, tax rates are inefficiently high so that tax cuts are both politically feasible and welfare-improving. There is a range of incomes above the median income where tax cuts are not mandated by Pareto-efficiency. In this region, tax increases are therefore politically feasible. They are, however, not desirable for the given welfare function. For a range of incomes below the median income, tax cuts are politically feasible and welfare-improving. For low incomes, tax cuts are politically feasible, but welfare-damaging.*