The Union Threat*

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Abstract

This paper studies the impact of labor unions on wage inequality, output and unemployment. To do so, it proposes a search and matching model of union formation in which unions arise endogenously through a voting process within firms. In a union firm, workers bargain their wages collectively. In a nonunion firm, each worker bargains individually with the firm. Because of this wage setting asymmetry, a union lowers the profit of a firm and compresses the wage distribution of the workers. Furthermore, to prevent unionization, nonunion firms distort their hiring decisions in a way that also lowers the dispersion of wages. After being calibrated on the United States, the model shows that, even though a standard empirical estimate would predict a small impact of unions on wage inequality, removing the threat of unionization increases the variance of wages substantially. It also increases output and reduces unemployment. Completely outlawing unions increases wage inequality further while forcing all firms to be unionized lowers inequality considerably. These results suggest that, even with a small membership, unions might have a significant impact on the economy through general equilibrium mechanisms and the way they distort firms' decisions.

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1 Introduction

Labor unions are commonly viewed as rent seeking entities. By organizing the bargaining effort of the workers, they hope to extract a higher share of any production surplus. This view is broadly consistent with the data: unionized firms tend to be less profitable (Hirsch, 2004) and the wage of union workers is on average higher than that of their nonunion counterparts (Card et al., 2004). Unions also seem to reduce wage dispersion.[1] Yet, since only about 8% of U.S. private sector workers are now covered by a union agreement, it is hard to believe that the *direct* impact that unions have on their members' wage influences the aggregate economy substantially.[2] For instance, the classical estimator introduced by Freeman (1980) suggests that unions lower the variance of log wages by only 0.4%.[3] But unions might also affect the economy through other channels. Among them are general equilibrium linkages. For instance, by raising the wage associated with certain jobs, unions might be responsible for an increase in the reservation wage which would them spill over to nonunion firms.[4] Furthermore, the *threat* of a possible unionization alone might influence firms that are *not* unionized. Indeed, if unionization lowers profits, nonunion firms might distort their behavior to prevent their own unionization. Through this channel, union laws have the potential to influence wages and hiring policies in *all* firms which, in turn, might have an important impact on macroeconomic aggregates.

This paper analyzes the impact of this union threat on the economy and finds that its effects can be substantial. To do so, it proposes a general equilibrium theory of endogenous union formation in which each firm hires a set of workers who differ in their productivity. If a simple majority of the workers vote in favor of unionization, a union is created and wages are collectively bargained between the firm and all of its employees. If the vote fails to gather enough support, the firm remains union free and wages are bargained individually between each worker and the firm. Because collective bargaining tends to compress the distribution of wages, the possibility of unionization creates a conflict among workers. Those with high productivity vote against the creation of a union while low productivity workers vote in favor. Moreover, the interaction between the two bargaining mechanisms and the production technology implies that the average wage among workers is higher when a firm is unionized. The profit of the firm is, however, smaller. This creates an incentive for firms to hire more high-skill and fewer low-skill workers in order to prevent unionization. This distortion influences the workers marginal products which naturally compresses the wage distribution. Furthermore, the threat of unionization, by constraining the problem of the firm, creates an inefficiency which reduces employment.

The theory is consistent with stylized facts associated with unions: union wages have a smaller variance, and are on average higher, than nonunion wages (Card et al., 2004), the preference for unionization and the union wage gap decrease with skill (Farber and Saks, 1980), and unionized firms are on average less profitable (Hirsch, 2004). The theory also suggests an explicit mechanisms to explain why regression discontinuity studies find little impact of unionization on firms (DiNardo

[1] Card et al. (2004) provide a summary of the large empirical literature on this topic along with their own estimates.

[2] See Hirsch and Macpherson (2003) and the database they created at `http://www.unionstats.com/`

[3] The Freeman estimator can be written (Card et al., 2004) as $V - V^N = U\Delta_v + U(1-U)\Delta_w^2$ where V is the observed variance of log wages, V^N is the variance of log wages without unions, U is the unionization rate, Δ_v is the difference in the variance of log union and nonunion wages and Δ_w is the difference between the mean log of union and nonunion wages. Using it on the CPS data for male private sector workers in 2005, the year used in the calibration, gives 0.4%.

[4] See for instance Dickens and Katz (1987); Neumark and Wachter (1995); Rosen (1969).

and Lee, 2004).[5]

To quantify the impact of unions on the economy, I calibrate the model on the private sector of the United States in 2005. I then perform three policy simulations in general equilibrium. The first one consists in removing the threat of unionization. In other words, nonunion firms do not have to worry about the vote on the formation of a union anymore. Firms that are unionized remain unionized and vice versa. In the new equilibrium, the variance of log wages goes up by 8.1% when compared to the calibrated economy. This shows that the threat of unionization alone might have an important impact on inequality. Output also goes up by 1.1% while the unemployment rate decreases considerably. The second simulation consists in eliminating unions completely. All wages are then negotiated on a one-on-one basis with the firms. In this scenario, the variance of log wages goes up by 12.3% with respect to the calibrated economy which, as we noted earlier, a much larger number than standard empirical estimators would predict. Total production also goes up by 1.2% and the unemployment rate goes down by 2.6 percentage points. Finally, the third simulation finds that forcing all firms to be unionized would lower the variance of low wages by 27% while also improving output and unemployment, although less so than when the economy is union free. These results suggest that, even with low membership, unions seem to have an important impact on wage inequality, output and unemployment through the threat they exert and through general equilibrium mechanisms.

There is some evidence to suggest that the possibility of unionization distorts the behavior of firms. For instance, Holmes (1998) shows that firms prefer to locate their establishments in states with union-weakening laws. Firms also employ a wide array of techniques, legal and illegal, to prevent their own unionization (Bronfenbrenner, 1994; Dickens, 1983; Freeman and Kleiner, 1990). Matsa (2010) shows that union bargaining power influences corporate financing decisions.

There is a large and sophisticated empirical literature that studies the effects of unions on wages.[6] In the search literature, Pissarides (1986) was perhaps the first to introduce a monopoly union into a search framework. Alvarez and Veracierto (2000) study the impact of many labor market policies in a search model. They find that unions have negative effects on unemployment and welfare. Ebell and Haefke (2006) and Delacroix (2006) investigate the interaction between union formation and product market regulations. Alvarez and Shimer (2008) show that including a union in a search model can lead to rest unemployment. Açikgöz and Kaymak (2008) builds a model of union formation to estimate the impact of a rising skill premium on the decline of union membership in the United States. Boeri and Burda (2009) look into the impact of endogenous bargaining regime on economic activity. Recently, Krusell and Rudanko (2012) have studied the dynamic problem of a monopoly union that sets wages with or without commitment. They find substantial impact of unions on unemployment and wage stickiness.

In the next section, I introduce the model and explain how firms behave in an environment with unions. In particular, I highlight the distortion created by the union threat. A discussion of the link between a firm's technology and its union status follows. I then calibrate the model on the US economy and do counterfactual policy experiments to see how unions affect the economy. The last section contains concluding remarks.

[5] DiNardo and Lee (2004) discuss the possibility that the union threat might explain part of their results but dismiss it as improbable since wages are relatively stable in firms before the unionization vote. Frandsen (2011) uses a regression discontinuity approach to estimate the impact of unionization on the full wage distribution.

[6] See, for instance, Card (1996); Card et al. (2004); DiNardo et al. (1996); DiNardo and Lee (2004); Freeman (1980); Freeman and Medoff (1984); Lemieux (1993).

2 The Model

2.1 Preferences and technology

There is a single good and time is discrete. I focus on steady state equilibria. The economy is populated by a continuum of heterogeneous agents, each endowed with a specific type of labor $s \in \{1, \ldots, S\} \equiv \mathcal{S}$. I refer to s as the skill. The exogenous density of skills in the economy is N_s with $N_s > 0$ for all s. An agent's skill is constant over time and agents live forever. They are risk neutral and maximize a linear utility function

$$\mathcal{U}(c) = \mathrm{E}_0 \sum_{t=0}^{\infty} \gamma^t c_t$$

where c_t denotes consumption in period t and $0 < \gamma < 1$ is the discount factor.

Firms combine the labor provided by workers of different skills to produce goods. To do so, they use heterogeneous production technologies, indexed by $j \in \{1, 2\}$. There is a mass 1 of firms endowed with each technology. A firm of type j employing a (non-normalized) distribution of workers g_s produces goods according to the production function

$$F_j(g) = A_j \left(\sum_{s=1}^{S} z_{j,s} g_s^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\alpha_j}$$

where $A_j > 0$ and the parameter $\sigma > 0$ is the elasticity of substitution between different skills. The vector $z_{j,s} > 0$ represents the relative intensity of skill utilization in firm j and is therefore normalized such that $\sum_s z_{j,s} = 1$. The parameter $0 < \alpha_j < 1$ describes the returns to scale of the production function. To avoid cluttering the notation, I often omit the subscript j when referring to a single firm.[7]

2.2 Labor markets

There is a continuum of labor markets in which unemployed agents look for jobs and firms post vacancies. Each unit of vacancy has a cost of κ. Each market is indexed by the skill s of agents searching in it. Agents can only search in the labor market corresponding to their skill.[8] Firms, on the other hand, are free to post a continuum of vacancies that covers all the markets. Figure 1 represents this structure. In each market, matches happen randomly at a rate determined by aggregate conditions. If, in a given market, U agents are searching and V vacancies have been posted, $m(U, V)$ matches are creared. The matching function $m(\cdot, \cdot)$ is identical across labor markets and is homogenous of degree one. By defining the labor market tightness $\theta \equiv V/U$, the probability that a vacancy is filled in a given period is $q(\theta) \equiv m(U, V)/V$. Similarly, the probability that an unemployed agent finds a job is $\theta q(\theta)$. Search is free and requires no effort. Every unemployed agent is therefore searching.

Since each type of firm is free to post vacancies in each submarket, a searching worker can be matched with firms using different technologies and with different union status.

[7] The link between technological changes and labor unions has been investigated by Acemoglu et al. (2001), Açikgöz and Kaymak (2008) and Dinlersoz and Greenwood (2012).

[8] In the calibrated model, the job finding probability as well as the expected wage are increasing with s. Therefore, even if agents were allowed to search in markets with a lower s than their own, they would choose not to do so.

This segmentation of the labor market allows the firm to control precisely the skill composition of its workforce and, through this channel, influence the unionization vote. It also allows us to study the effects of unionization on unemployment rates across skill groups in a convenient way.

In the theoretical part of this paper, the index $s \in \mathcal{S}$ is only used to characterize some form of worker heterogeneity. Later, in the empirical part of the paper, I calibrate this index such that it could intuitively be defined as *skill*. I use this name right away to make the interpretation of the theory consistent with the calibration.

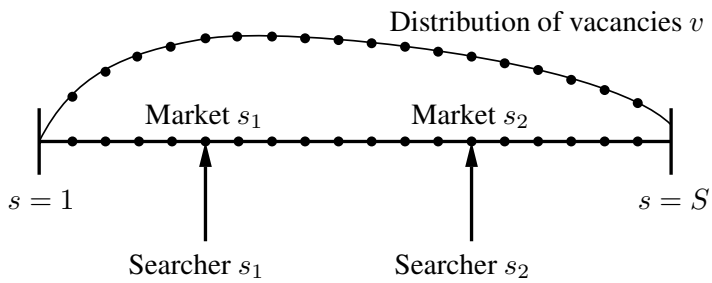


Figure 1: Continuum of labor markets

2.3 Agents

Agents provide labor to firms in exchange for a wage. In each period, an agent is either employed or unemployed. An employed worker loses his job with exogenous probability $\delta > 0$, in which case he goes to search in the labor market corresponding to his type. With probability $1 - \delta$, the agent remains employed in his current job. The lifetime discounted expected utility of a worker of type s who has been matched with a firm of type j and who is earning a wage w can be written as

$$W^e_{j,s}(w) = w + \gamma \left[(1-\delta)W^e_{j,s}(w_{j,s}) + \delta W^u_s\right]$$

where W^u_s is the lifetime utility of being unemployed and $w_{j,s}$ is the wage that a worker of type s expects to receive next period if he remains with firm j. Since, wages are bargained every period, the negotiations with the firm are over w only. Both parties consider that $w_{j,s}$ is determined in equilibrium with rational expectations. They have no direct influence over it.

Every period, an unemployed agent s receives b^0_s from home production. He finds a job with probability $\theta_s q(\theta_s)$. His lifetime discounted utility is therefore

$$W^u_s = b^0_s + \gamma \left[\theta_s q(\theta_s)\mathrm{E}(W^e_{j,s}) + (1 - \theta_s q(\theta_s))W^u_s\right]$$

where $\mathrm{E}(W^e_{j,s})$ is the expected discounted value of a match. The expectation is therefore taken over all the vacancies, which might have been posted by different types of firms, in submarket s.

An agent will accept to work only if the utility provided by employment exceeds the utility of continuing the search for a job. In equilibrium, an agreement is always reached. By combining the last two equations we can characterize the utility gain provided by employment:

$$W^e_{j,s}(w) - W^u_s = w + \frac{\gamma(1-\delta)w_{j,s} - (1-\gamma)W^u_s}{1-\gamma(1-\delta)}. \tag{1}$$

It is useful to define the flow utility of being unemployed:

$$b_s \equiv (1-\gamma)W_s^u = \frac{(1-\gamma(1-\delta))b_s^0 + \gamma\theta_s q(\theta_s)\mathrm{E}(w_s)}{1-\gamma(1-\delta)+\gamma\theta_s q(\theta_s)}. \tag{2}$$

The utility of an unemployed worker takes into account the fact that this worker will spend a part of his time employed in the future. It is therefore a weighted average of b_s^0 and of the wage this agent expects to receive in future jobs.

To simplify the notation, it is also convenient to define the equilibrium quantity

$$c_{j,s} \equiv \frac{b_s - \gamma(1-\delta)w_{j,s}}{1-\gamma(1-\delta)} \tag{3}$$

which is the *net* outside option of a worker of type s who has been matched with a firm of type j. This notation allows us to write the gain from employment at a wage w as

$$W_{j,s}^e(w) - W_s^u = w - c_{j,s}.$$

2.4 Firms

A firm that employed a distribution of workers g_{-1} during the previous period loses a fraction δ of all of its workers and therefore starts the current period with the distribution $(1-\delta)g_{-1}$. It then posts a schedule of vacancies v to maximize its expected discounted profits. Since the firm is posting a continuum of vacancies in each labor market, a law of large numbers implies that the number of successful matches is deterministic.

Once the new hires have joined the firm, the workers vote on the formation of a union and the firm's optimal behavior will depend on the specifics of the unionization process as well as on how the union and nonunion wages are set. These will be described shortly. For now, it is sufficient to use an abstract function $w_s(g)$ to denote the wages that the firm pays as a function of its current distribution of workers.

By defining the current period profit of a firm as $\pi(g) \equiv F(g) - \sum_s w_s(g)\cdot g_s$, we can write the problem of a firm as

$$\tilde{J}(g_{-1}) = \max_v \pi(g) - \kappa\sum_{s=1}^{S} v_s + \gamma\tilde{J}(g) \tag{4}$$

subject to, for all $s \in \mathcal{S}$,

$$\begin{cases} g_s = g_{-1,s}(1-\delta) + v_s q(\theta_s) \\ v_s \geq 0 \end{cases}$$

where $\tilde{J}(g_{-1})$ is the value function of a firm that ended the previous period with workers g_{-1}. The first constraint is simply the law of motion of the stock of workers; current workers were either with the firm last period or are new recruits. The second constraint states that all job separations are exogenous; firms cannot post negative vacancies.

In a steady state equilibrium in which the aggregate variables remain constant, it is possible to simplify the firm's problem substantially. To see why, suppose that in such an equilibrium, a firm's optimal distribution of workers is given by g_s^*. Two events might move the firm away from

g_s^*. First, every period, it loses a fraction δ of its workers. Second, if one of the wage bargaining sessions breaks down without an agreement, the firm loses additional workers.[9] In both of these cases, the firm has to hire a positive number of workers in the next period to replace those that have been lost. Therefore, $v_s > 0$ in all markets s such that $g_s^* > 0$ and $v_s = 0$ elsewhere. We can therefore ignore the second constraint and substitute v from the law of motion of the workers directly into the objective function. The problem of the firm can be simplified as

$$J\left(\sum_{s=1}^{S} \frac{g_{-1,s}}{q(\theta_s)}\right) = \max_g \pi(g) - \kappa \sum_{s=1}^{S} \frac{g_s - g_{-1,s}(1-\delta)}{q(\theta_s)} + \gamma J\left(\sum_{s=1}^{S} \frac{g_s}{q(\theta_s)}\right) \tag{5}$$

where $\sum \frac{g_{-1,s}}{q(\theta_s)}$ is a new state variable that represents the value of the stock of workers with which the firm enters the period.[10]

This last value function has two additively separable pieces: one that depends on the distribution of previous period g_{-1} and a second one that depends on the firm's decision in the current period. This implies that, in a steady state, the firm's current period decision is independent of its state variable.

Lemma 1. *In a steady-state equilibrium, the firm's dynamic problem can be written as*

$$\max_g \pi(g) - \kappa(1-(1-\delta)\gamma) \sum_{s=1}^{S} \frac{g_s}{q(\theta_s)}. \tag{6}$$

Proof. All proofs are relegated to the appendix. ■

This result comes directly from the linearity of the hiring costs, the constant value of θ and the fact that, at the steady state, a firm never wants to fire workers.

2.5 Wages

Before defining the wage schedule $w_s(g)$, it is useful to detail the sequence of events that occurs once a firm has recruited its new workers, as represented on Figure 2. First, the workers vote to decide whether to form a union or not.[11] Then, if a union is established, wages are bargained collectively. The outcome of this bargaining is a wage schedule $w_s^u(g)$ and a profit function $\pi^u(g)$. If the union is rejected, wages are bargained individually. This generates the wage schedule $w_s^n(g)$ and the profit function $\pi^n(g)$. Notice that when the vote takes place and when wages are bargained, the distribution of workers g is fixed. Also, when the workers cast their vote, they know exactly what wages they will get if the union is created or not. I first describe the two bargaining procedures and then come back to the voting process.[12]

[9] This does not happen in equilibrium but the value function needs to be defined along these paths to correctly specify the bargaining problems.

[10] Notice that J and $\tilde{J}$ are two different objects but they give the same first order conditions in a steady state equilibrium.

[11] Modeling unionization as a firm-level process is consistent with evidence presented by Traxler (1994) and Nickell and Layard (1999) that suggest that the coverage of union contracts is mostly at the enterprise level in the US, Canada and the United Kingdom. This assumption is not appropriate for some European countries in which the bargaining occurs at the industry or country level.

[12] As does most of the literature on search and large firms, we restrict our attention to a specific set of equilibria. In particular, we rule out any reputation considerations. See Wolinsky (2000) for a more general approach.

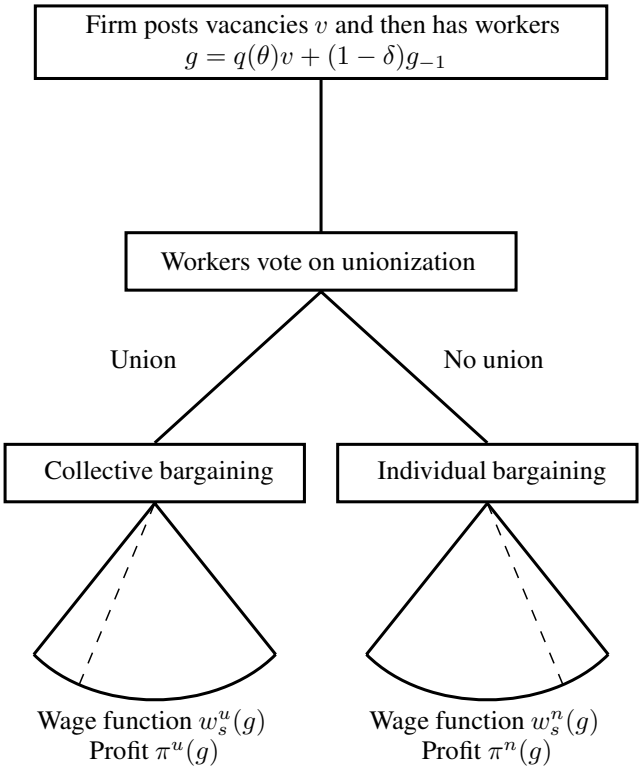


Figure 2: Sequence of events

In both a union and a nonunion firm, wages are set using Nash bargaining to share the surplus generated by the match. The surplus that is bargained over is, however, different in both cases. If the firm is unionized and an agreement on wages cannot be reached, the whole workforce quits the firm and no production takes place. In a non unionized firm, if the bargaining with a single worker breaks down, this specific worker goes back to unemployment but the firm can still produce with the other workers. In a nonunion firm, the bargaining therefore takes place over the marginal surplus generated by each worker. In a union firm, the workers and the firm bargain over the total surplus generated by the whole workforce. This asymmetry between the two surpluses interacts with the decreasing returns of the production function and has important consequences for the firm's profits.

Collective bargaining

If the workers vote in favor of unionization, all wages are bargained collectively. In particular, no worker can decide to break ranks to negotiate directly with the firm.[13] The total surplus of the match is split using Nash bargaining with multiple agents.[14] Abstracting from possible bargaining

[13] In the U.S. the wages of all unionized workers must be set through an agreement between the union and the firm.

[14] Nash's axiomatic theory of bilateral bargaining extends unchanged to a context with numerous players. Krishna and Serrano (1996) provides a strategic approach to multilateral bargaining.

powers, it has the following structure:

$$\underbrace{\text{Surplus of worker } 1 \times ... \times \text{Surplus of worker } i \times ...}_{\text{Union surplus}} \times \text{Surplus of the firm.}$$

Consider first the firm's surplus from agreeing on a wage schedule w with the union. In a steady-state, the difference in discounted profits for the firm, denoted by $\Delta^u(w)$, is

$$\Delta^u(w) = \left[\pi(g,w) + \gamma J\left(\sum_{s=1}^{S} \frac{g_s}{q(\theta_s)}\right)\right] - [\pi(0) + \gamma J(0)] \tag{7}$$

where the first term between brackets is discounted profit if an agreement is reached and $\pi(0)+\gamma J(0)$ is the firm's discounted profit if negotiations break down. In such a scenario, the firm has no worker; it produces nothing and pays no wage. Therefore, $\pi(0) = 0$. $J(0)$ is the value function of a firm that starts the period with no workers. Because the firm's employment decision is independent of the distribution of its workers, the firm hires back to its steady-state optimal level g^* right away. After simplification,[15] we find that the firm's surplus from reaching an agreement is

$$\Delta^u(w) = \pi(g,w) + (1-\delta)\gamma\kappa \sum_{s=1}^{S} \frac{g_s}{q(\theta_s)}. \tag{8}$$

The intuition is for this last equation is straightforward; if negotiations break down, the firm loses the current period profit π and pays a higher hiring cost tomorrow to compensate for the loss of the fraction $1-\delta$ of its current workforce that would have remained with the firm next period.

On the workers' side, each of them receives $W_s^e(w) - W_s^u$ if an agreement is reached. By assuming that all workers have the same bargaining power, the surplus of the union is given by

$$\prod_{s=1}^{S} (W_s^e(w) - W_s^u)^{\frac{g_s}{n}}$$

where $n = \sum g_s$ is the number of workers employed by the firm.[16] The union simply aggregates the individual surplus of each worker.[17]

The bargaining problem between the firm and the union is then simply

$$\max_w \left[\prod_{s=1}^{S} (W_s^e(w) - W_s^u)^{\frac{g_s}{n}}\right]^{\beta_u} \left[F(g) - \sum_{s=1}^{S} w_s g_s + (1-\delta)\kappa\gamma \sum_{s=1}^{S} \frac{g_s}{q(\theta_s)}\right]^{1-\beta_u} \tag{9}$$

[15] A detailed derivation of the firm's surplus can be found in the proof of Lemma 2.

[16] To see this, consider the discrete case in which there are $h_s \in \mathrm{N}$ workers of type s who all have mass $\epsilon > 0$ such that $h_s \times \epsilon \to g_s$ as we move to the continuum. The surplus of the workers can be written as $(W_1^e - W_1^u)^{h_1} \times \cdots \times (W_i^e - W_i^u)^{h_i} \times \cdots \times (W_S^e - W_S^u)^{h_S}$. By normalizing the bargaining powers such that their sum equals 1, we get

$$(W_1^e - W_1^u)^{\frac{\epsilon h_1}{\epsilon H}} \times \cdots \times (W_i^e - W_i^u)^{\frac{\epsilon h_i}{\epsilon H}} \times \cdots \times (W_S^e - W_S^u)^{\frac{\epsilon h_S}{\epsilon H}}$$

where $H = \sum_s h_s$. By taking the limit to continuity, $\frac{\epsilon h_i}{\epsilon H} \to \frac{g_i}{n}$ for all i and the result follows.

Normalizing the union bargaining power to 1 is required to prevent the bargaining weight of the firm from going to zero in the limit (the firm is only one player). Alternatively, we could assume that the bargaining power of the firm increases with the number of employees. Both assumptions are equivalent.

[17] Other authors model the collective bargaining problem differently. For instance, Açikgöz and Kaymak (2008) and Bauer and Lingens (2010) assume that the union maximizes the sum of the workers' surplus. In a model with heterogeneous agents, this approach only pins down the *total* share of the surplus going to the workers, not how it is shared among them. This approach and the one used in the current paper give the same union surplus.

where $0 < \beta_u < 1$ denotes the bargaining power of the union. This coefficient is exogenous to the model and could possibly be influenced by labor market policies.[18]

Lemma 2. *If the joint surplus of the match is positive and if $g_s > 0$ for all $s \in \mathcal{S}$, then the function*

$$w_s^u(g) - c_s = \frac{\beta_u}{n}\left(F(g) - \sum_{s=1}^{S} c_s g_s + \gamma(1-\delta)\kappa \sum_{s=1}^{S} \frac{g_s}{q(\theta_s)}\right) \tag{10}$$

is the unique solution to the collective bargaining problem described by Equation 9.

Equation 10 implies that the group of workers get a fraction β_u of the joint surplus generated by the match, which is the term between parentheses. The equation also indicates that all the workers are paid the same amount over their net outside option c_s. The union is basically mixing together the characteristics of all its members. As a result, the variance of wages comes from c_s only . The macroeconomic conditions, through b_s, have a direct influence on the dispersion of wages in a unionized firm.

It is straightforward to show that the one-period profit of a union firm employing the distribution of workers g is given by

$$\pi^u(g) = (1-\beta_u)F(g) - (1-\beta_u)\sum_{s=1}^{S} c_s g_s - \beta_u(1-\delta)\kappa\gamma \sum_{s=1}^{S} \frac{g_s}{q(\theta_s)}. \tag{11}$$

so that the firm gets a fraction β_u of the joint surplus.

Individual bargaining

If the workers vote against unionization, they each bargain individually with the firm. In this case, the workers cannot interact with each other. Once again, the worker and the firm use Nash bargaining to split the surplus created by the match. This surplus is, however, not identical across all workers. Because of the decreasing returns to scale, the surplus generated by the first worker with whom the firm bargains is higher than the one generated by the last one. Stole and Zwiebel (1996a,b) introduce a framework to solve bargaining problems such as this. In their setup, the firm negotiates with each of its workers in turn. If any of the one-on-one negotiation sessions breaks down, wages are renegotiated with all the workers remaining in the firm. Therefore, when considering the marginal surplus generated by a worker, the firm is aware that if no agreement is reached, the other workers will want to rebargain their wages differently. For instance, with decreasing returns to labor, the firm knows that by losing a worker the marginal product of each other worker will go up which, through renegotiation, will lead to higher wages.[19]

In this context, the marginal surplus of the firm from hiring a worker of type s is given by[20]

$$\Delta_s^n(w) = \frac{\partial F(g)}{\partial g_s} - \sum_{k=1}^{S} g_k \frac{\partial w_k(g)}{\partial g_s} - w_s(g) + \gamma(1-\delta)\frac{\kappa}{q(\theta_s)}.$$

[18] It is straightforward to generalize the model such that β_u is a function of skill s.

[19] Cahuc and Wasmer (2001) were perhaps the first to build a search and matching model with firms with decreasing returns to scale in which wages are set through individual bargaining.

[20] See the proof of Lemma 3 in the appendix for a derivation of this equation.

The first term is the extra output produced by the additional worker. The next one represents the marginal effect of this worker on the wages of other members of the workforce. The third term is simply the wage paid to the worker and the fourth term is the expected vacancy costs saved from retaining, with probability $1-\delta$, this worker next period.

Defining $0 < \beta_n < 1$ as the bargaining power of a worker, Nash bargaining implies that the nonunion wage must solve the following system of partial differential equations:

$$\Delta_s^n(w) = \frac{1-\beta_n}{\beta_n}\left(W_s^e(w) - W_s^u\right), \tag{12}$$

for all $s \in \mathcal{S}$.

Lemma 3. *The wage schedule*

$$w_s^n(g) - c_s = \frac{\beta_n}{1-(1-\alpha)\beta_n} \underbrace{\frac{\alpha z_s}{g_s^{1/\sigma}} A \left(\sum_{k=1}^{S} z_k g_k^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\sigma(1-\alpha)}{\sigma-1}}}_{\text{Marginal product of worker } s} - \beta_n c_s + \beta_n \gamma (1-\delta)\frac{\kappa}{q(\theta_s)} \tag{13}$$

solves the bargaining problem of a nonunion firm employing the distribution of workers g.

It follows directly from the wage of nonunion workers that the one-period profit of the firm is

$$\pi^n(g) = \frac{1-\beta_n}{1-(1-\alpha)\beta_n} F(g) - (1-\beta_n)\sum_{s=1}^{S} c_s g_s - \beta_n(1-\delta)\kappa\gamma \sum_{s=1}^{S} \frac{g_s}{q(\theta_s)}. \tag{14}$$

Comparing collective and individual bargaining

The two wage equations (10 and 13) have a remarkably similar structure. They are both made of three terms: one related to production, one related to the outside option of workers and one related to the hiring costs. The main difference between the two equations resides in how these quantities influence wages. Indeed, notice that the union wage is mostly a function of the average characteristics of the firm's workforce while the nonunion wage is a function of the individual characteristics of each worker. In particular, the union wage depends on the average production $F(g)/n$ while the nonunion wage is a function of the marginal product of each worker. This implies that a worker with valuable characteristics, for instance a high marginal product, would rather bargain individually with the firm than to share his advantage with the other employees. Through this channel, the bargaining structures influence the variance of wages and which workers favor unionization.

The firm and the group of workers, are not indifferent between the two types of bargaining. To illustrate this point, consider the case with equal bargaining powers $\beta_n = \beta_u \equiv \beta$. Then, one can show that,

$$\mathrm{E}_g(w^n) - \mathrm{E}_g(w^u) = -\frac{\beta(1-\beta)(1-\alpha)}{1-(1-\alpha)\beta}\frac{F(g)}{n} < 0$$

where $\mathrm{E}_g(x) = \sum x \cdot g / \sum g$ is the expectation across skills. Similarly, with equal bargaining power, the difference in the value of the firm is

$$J^n(g) - J^u(g) = \frac{(1-\alpha)\beta}{1-(1-\alpha)\beta} F(g) > 0.$$

This shows that, for any distribution of workers, the firm prefers to bargain individually while the workers, on average, would rather be represented by a union. This conflict of preferences is a direct consequence of the decreasing returns to scale. Indeed, as $\alpha \to 1$, the differences in profits and in average wages go to zero. When bargaining individually, the firm considers producing with or without the *marginal* worker, who has a relatively small impact on the total production. On the other hand, when the firm bargains with the union, the surplus is a function of the *total* production, which includes the relatively high production generated by the infra-marginal workers. By forming a union, the workers can extract a bigger part of these high marginal products, which lowers the firm's profit.

This feature of the model is consistent with evidence presented by Kleiner (2001) which suggests that firms generally oppose unions. Bronfenbrenner (1994) and Freeman and Kleiner (1990) also detail various tactics used by firms to prevent unionization. Hirsch (2004) summarizes the literature on union and profitability and concludes that union firms are in general less profitable than firms that are not unionized.

2.6 Voting procedure

Now that the wage equations are defined, we can go up a step on the tree shown at Figure 2 and discuss the vote on unionization. When the vote takes place, the distribution of workers is fixed and the workers are fully aware of the wages they would get after either outcome of the vote. Each worker has random preferences on the union status of the firm. One can think that some workers have a negative or positive opinion of unions for reasons that are exogenous to the model. Specifically,

$$\text{Worker } s \text{ votes for a union} \Leftrightarrow w_s^u(g) - w_s^n(g) > \epsilon$$

where ϵ is a logistic random variable drawn independently across all workers. It has mean 0 and scale parameter $1/\rho$. Denote its CDF by $\phi(x)$ such that $\phi(x) \equiv P(\epsilon \leq x) = 1/(1 + \exp(-\rho x))$.

A law of large numbers applies when aggregating the workers of a given skill. Therefore, a fraction

$$\phi(w_s^u(g) - w_s^n(g)) \equiv \frac{1}{1 + \exp\{-\rho(w_s^u(g) - w_s^n(g))\}}$$

of workers of type s will vote in favor of unionization. By summing up the voters across skill, we can denote the excess number of workers in favor of unionization by

$$V(g) \equiv \sum_{s=1}^{S} g_s \phi(w_s^u(g) - w_s^n(g)) - \frac{1}{2}n. \tag{15}$$

With that notation, we get the following condition for unionization:

$$\text{Firm is unionized} \Leftrightarrow V(g) > 0. \tag{16}$$

A firm is unionized if a majority of its workers vote to form a union.

Notice that even though the preferences are random, the outcome of the vote is fully deterministic. Therefore, at the moment of posting vacancies, the firm knows whether the workers will form a union or not. In fact, the firm is effectively deciding to be unionized or not. Notice also that, as the curvature parameter ρ goes to infinity, the outcome of the vote is decided by the median voter.

Modeling the unionization process as a majority vote is broadly consistent with the procedures of the National Labor Relation Board to gain a union certification in the United States (DiNardo and Lee, 2004).

2.7 Full problem of a firm

Now that we have derived the wage schedules w^n and w^u, and that we have outlined the voting procedure, we can go back to the problem of a firm. As shown in Lemma 1, at a steady state, a firm solves

$$\max_g \mathcal{J}(g, w(g))$$

with

$$\mathcal{J}(g, w(g)) \equiv F(g) - \sum_{s=1}^{S} g_s w_s(g) - \kappa(1 - (1-\delta)\gamma) \sum_{s=1}^{S} \frac{g_s}{q(\theta_s)}$$

where

$$w(g) = \begin{cases} w^u(g) & \text{if } V(g) > 0 \\ w^n(g) & \text{if } V(g) \leq 0 \end{cases}.$$

2.8 Steady state equilibrium

In a steady-state equilibrium, the flows in and out of unemployment in all sub-markets must be equal:

$$[N_s - U_s]\delta = U_s \theta_s q(\theta_s). \tag{17}$$

where U_s is the number of unemployed agents searching in market s. This equation shows the direct mapping between the labor market tightness θ and the unemployment rate U/N.

We can now define an equilibrium.

Definition 1. *A steady-state equilibrium is a vector of unemployment utility b, a labor market tightness vector θ, workers distributions $\{g_j\}_{j=1}^2$ and wage schedules $\{w_j\}_{j=1}^2$ such that,*

1. *g_j solves the optimization problem of firm j,*
2. *w_j solves the collective bargaining problem (Equation 10) if firm j is unionized or solves the individual bargaining problem (Equation 13) if firm j is not unionized,*
3. *b_s satisfies Equation 2,*
4. *unemployment is stationary in each labor market: Equation 17 is satisfied.*

3 Economic forces at work

As we have seen in the previous section, a firm generally prefers to be union free while its workers extract a higher share of the surplus when they are unionized. But suppose, as a thought experiment, that a firm believes, without any doubt, that its workers will vote against unionization.

Then, this firm should hire a distribution of workers, denoted by g^{n*}, that maximizes discounted profit with wages given by w^n:

$$g^{n*} = \underset{g}{\text{argmax}} \ \mathcal{J}(g, w^n(g)).$$

But this distribution is only optimal if the workers actually vote against unionization ($V(g^{n*}) \leq 0$). Since workers are generally better off, on average, when the firm is unionized, chances are that they will form a union ($V(g^{n*}) > 0$). If this is so, the firm is *constrained* by the unionization vote and g^{n*} does not actually solve the firm's problem. In these cases, the union threat influences the behavior of the firm.

A constrained firm will attempt to prevent unionization by optimally distorting g^{n*}. By denoting this distorted distribution by g^n, we have that

$$\begin{aligned} g^n = \underset{g}{\text{argmax}} \ & \mathcal{J}(g, w^n(g)) \\ \text{subject to } \ & V(g) \leq 0. \end{aligned}$$

Obviously, by imposing the voting constraint on the firm's problem, the discounted profit goes down: $\mathcal{J}(g^n, w^n(g^n))) \leq \mathcal{J}(g^{n*}, w^n(g^{n*}))$. In particular, if the constraint is important enough, the firm will consider being unionized as an option. After all, since workers generally accept unions, its profit would then be $\mathcal{J}(g^{u*}, w^u(g^{u*}))$ where

$$g^{u*} = \underset{g}{\text{argmax}} \ \mathcal{J}(g, w^u(g)).$$

If $\mathcal{J}(g^{u*}, w^u(g^{u*})) > \mathcal{J}(g^n, w^n(g^n)))$ the firm is rationally choosing to be unionized. It is an optimal reaction to the threat imposed by the union.[21]

Studying the full problem of the firm must be done numerically. It is however useful to consider an equilibrium in which the union status of each firm is given exogenously, such that no union vote takes place. In this case, we can characterize the firm's behavior, the wages it pays as well as the workers' preference for unionization. The full problem of a firm, in which its union status depends endogenously on the workers' vote, can then be thought of as a deviation from the exogenous case.

3.1 Exogenous union status

In this section, we consider the problem of a firm whose union status is exogenously given, such that the union threat has no impact on its behavior. Such a firm solves:

$$\max_g \mathcal{J}(g, w^i(g))$$

where $i = u$ if the firm is unionized and $i = n$ otherwise. By using Equations 11 and 14, we can rewrite the problem of the firm as:

$$\max_g \Gamma^i F(g) - (1 - \beta_i) \sum_{s=1}^{S} c_s g_s - (1 - \gamma(1 - \delta)(1 - \beta_i))\kappa \sum_{s=1}^{S} \frac{g_s}{q(\theta_s)}$$

[21] It is possible to construct examples in which $V(g^{u*}) < 0$ but this usually require extreme parameter values.

where

$$\Gamma^i \equiv \begin{cases} 1-\beta_u & \text{if } i=u \\ \dfrac{1-\beta_n}{1-(1-\alpha)\beta_n} & \text{if } i=n \end{cases}$$

is the share of output retained by the firm. By defining,

$$\mathrm{MC}_s^i \equiv (1-\beta_i)c_s + (1-\gamma(1-\delta)(1-\beta_i))\frac{\kappa}{q(\theta_s)} \tag{18}$$

as a measure of the marginal cost paid to hire a worker s, the firm's optimal hiring decision, g^{i*} is given by[22]

$$\mathrm{MC}_s^i = \Gamma^i \frac{\alpha A z_s}{(g_s^{i*})^{1/\sigma}} \left(\sum_{k=1}^{S} z_k (g_k^{i*})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1-\sigma(1-\alpha)}{\sigma-1}}. \tag{19}$$

Notice that the firm has a different hiring strategy whether it expects its workers to be unionized or not. This last equation simply states that the marginal revenue from hiring and extra worker of type s must equal its marginal cost. Notice that MC^i depends on the firm's equilibrium wage, on aggregate variables and on the union status of the firm. Solving for g^{i*}, we get

$$g_s^{i*} = (\alpha A \Gamma_i)^{\frac{1}{1-\alpha}} \left(\frac{z_s}{\mathrm{MC}_s^i} \right)^{\sigma} \left(\sum_{k=1}^{S} z_k \left(\frac{z_k}{\mathrm{MC}_k^i} \right)^{\sigma-1} \right)^{\frac{1-\sigma(1-\alpha)}{(\sigma-1)(1-\alpha)}}. \tag{20}$$

We see that workers who search in tight labor markets (θ_s big) or who have relatively attractive outside options (b_s big) are expensive to hire (MC_s^i big) and the firm therefore relies less on them for production (g_s^{i*} small). The equilibrium wage schedule w_j also affects the marginal cost through c_j: a worker who knows he will get a high wage in the next period has more to lose if the bargaining breaks down. We also see that, all else equal, nonunion firms are larger than union firms (since $\Gamma_n > \Gamma_u$ if the bargaining powers are equal). Since nonunion firms bargains over the marginal surplus, they tend to increase the overall number of workers to lower the marginal product and therefore the wages they have to pay.[23]

The following proposition characterizes the wages paid by firms in equilibrium.

Proposition 1. *If, in an equilibrium in which the union status of firms is given exogenously, b_s and θ_s are increasing functions of s, then the wages $w_s^u(g^{u*})$ and $w_s^n(g^{n*})$ are increasing in s and the union wage gap $w_s^u(g^{u*}) - w_s^n(g^{n*})$ is decreasing in s.*[24]

This proposition is consistent with a large empirical literature that finds that the union wage gap in the U.S. declines with income (Card et al., 2004). It characterizes the *observed* wages that are paid in equilibrium but not the workers' preferences about unionization. To do so, we need to consider the counterfactual wages that the workers would receive if they would vote in favor or against unionization in a given firm. The following proposition describes how the workers would vote, in a given firm, if a vote were to take place.

[22] To be concise, I only present the equations for $\sigma \neq 1$. The case with $\sigma = 1$ can be obtained by taking the limit.
[23] This is a standard consequence of using the Stole and Zwiebel bargaining framework.
[24] In the calibrated model, θ_s and b_s are increasing in s.

Proposition 2. *If, in an equilibrium in which the union status of firms is given exogenously, b_s and θ_s are increasing functions of s, then the union wage gap $w_s^u(g^{i*}) - w_s^n(g^{i*})$ is decreasing in s for $i \in \{u, n\}$.*

Proposition 2 is consistent with the empirical work of Farber and Saks (1980) who show that the desire to be unionized goes down with the position of the worker in the intrafirm earnings distribution.

Finally, it is straightforward to compare the discounted profit in both the union and nonunion scenarios:

Proposition 3. *An unconstrained firm strictly prefers to be union free if and only if*

$$\Gamma_n^{\frac{1}{\alpha}} \left(\sum_{s=1}^{S} z_s^{\sigma} \left(\frac{1}{\mathrm{MC}_s^n} \right)^{\sigma-1} \right)^{\frac{1}{\sigma-1}} > \Gamma_u^{\frac{1}{\alpha}} \left(\sum_{s=1}^{S} z_s^{\sigma} \left(\frac{1}{\mathrm{MC}_s^u} \right)^{\sigma-1} \right)^{\frac{1}{\sigma-1}}$$

The left-hand side of this equation is the ratio of a measure of the relative share of output that the firm retains to the marginal cost of hiring workers when the firm is union free. The term on the right-hand side represent the same quantity when the firm is unionized. If $\beta_n = \beta_u$, then $\mathrm{MC}^n = \mathrm{MC}^u$ and this condition is automatically satisfied. Also, the firm prefers to be union free when a union would be very strong ($\beta_u \to 1$) and it would gladly welcome a union if individual workers have a strong bargaining power ($\beta_n \to 1$), as the intuition would predict. As we will see, the condition stated in proposition 3 holds for all firms in the calibrated economy.

The numerical example presented at Figure 3 is useful to understand the behavior of the firm. Panel (a) presents the two optimal distributions, g^{n*} and g^{u*}, that are chosen by firms with exogenous union status.[25] Panel (b) and (c) show the wages that voters would be considering if a vote were to take place. Panel (b) depicts the wage functions when the firm hires according to g^{n*} while Panel (c) shows the same functions when the firm employs g^{u*}. As the previous propositions predicted, the wage schedules are increasing functions of s while the union wage gap $w^u - w^n$ decreases with the skill, indicating that low-skill workers have a higher preference for unions than high-skill ones.

In this example, if a vote were to take place in a firm hiring according to g^{n*}, 66% of the workers would vote for unionization and a union would be created. The distribution g^{n*} is therefore not an optimal decision if, instead of being exogenous, the union status of the firm was decided by the workers' vote.

We therefore need to understand how the firm behaves when this threat of unionization is binding.

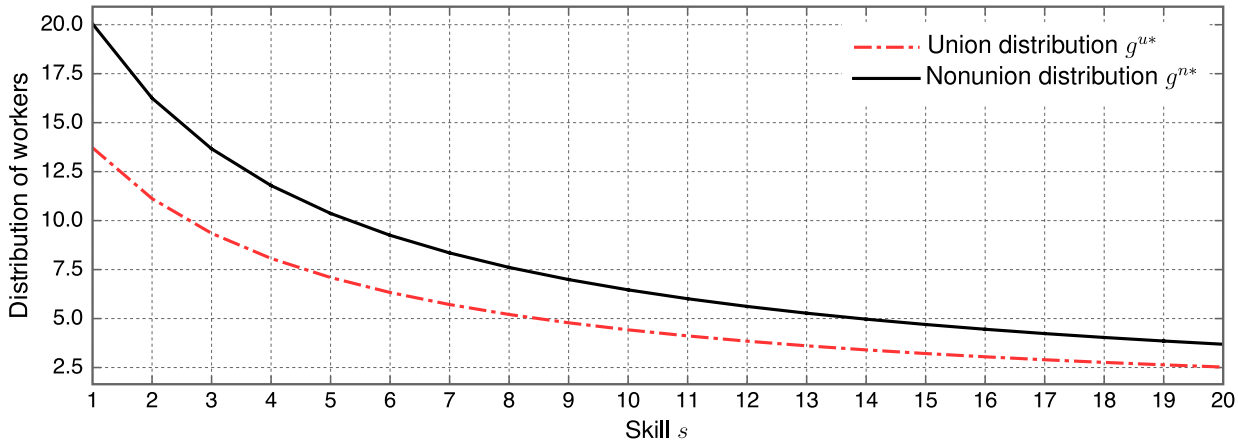
3.2 Preventing unionization

When a firm is constrained by the union vote, the first order condition given by Equation 19 must be modified to include the impact of the additional worker on the outcome of the vote. The new first-order condition is

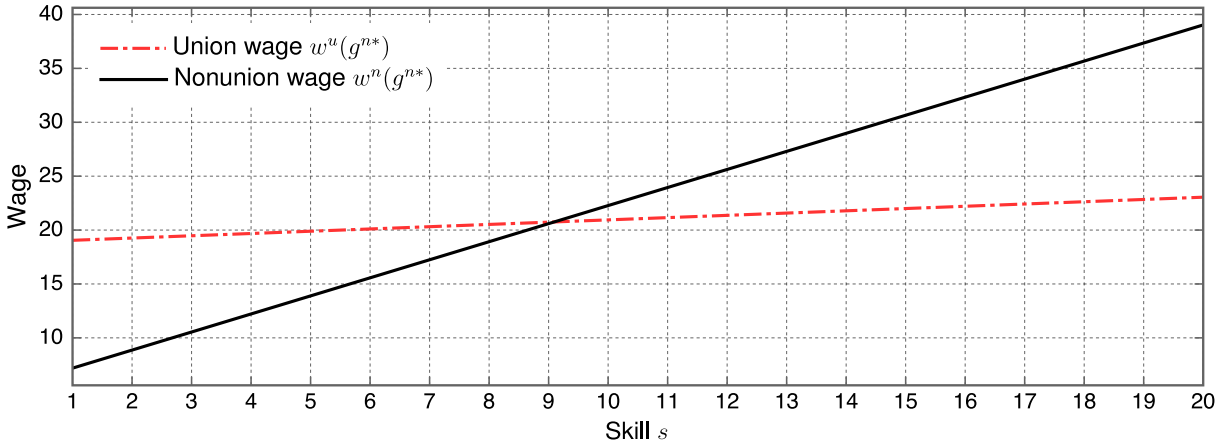
$$\mathrm{MC}_s^n = \Gamma^i \frac{\alpha A z_s}{(g_s^n)^{1/\sigma}} \left(\sum_{k=1}^{S} z_k (g_k^n)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1-\sigma(1-\alpha)}{\sigma-1}} - \lambda_{\mathrm{LM}} \frac{\partial V(g^n)}{\partial g_s^n} \tag{21}$$

[25] Appendix B provides the details of these simulations.

(a) Distributions of workers under exogenous union status



(b) Union and nonunion wages when the firms hires according to g^{n*}



(c) Union and nonunion wages when the firms hires according to g^{u*}

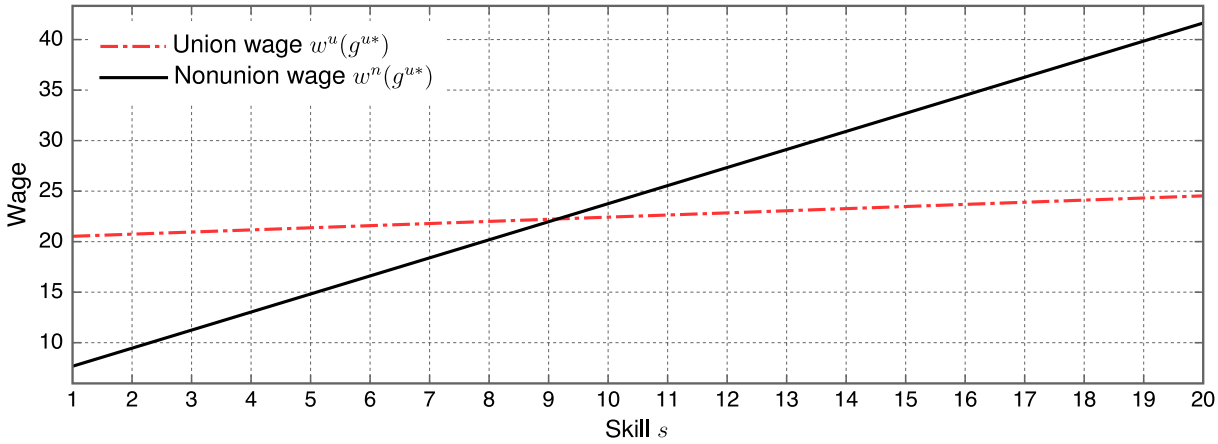


Figure 3: The hiring decision of an unconstrained firm. See Appendix B for the parameters of this simulations.

where $\lambda_{\text{LM}} \geq 0$ is the Lagrange multiplier associated with the voting constraint and where g^n denotes the optimal distribution for which workers actually reject the union. The term related to the unionization constraint can be expanded to highlight the mechanisms influencing the vote:

$$\frac{\partial V(g)}{\partial g_s} = \underbrace{\phi(\Delta_s(g)) - \frac{1}{2}}_{\text{A}} + \underbrace{g_s \frac{\partial \Delta_s(g)}{\partial g_s} \frac{\partial \phi(\Delta_s(g))}{\partial \Delta_s(g)}}_{\text{B}} + \underbrace{\sum_{s' \neq s} g_{s'} \frac{\partial \Delta_{s'}(g)}{\partial g_s} \frac{\partial \phi(\Delta_{s'}(g))}{\partial \Delta_{s'}(g)}}_{\text{C}} \tag{22}$$

where Δ_s is short notation for the union wage gap $w_s^u - w_s^n$ and where $\phi(\Delta_s)$ is, as before, the fraction of workers s who vote in favor of unionization when the wage gap equals Δ_s.

Each term of Equation 22 represents one mechanism that the firm takes into account to prevent unionization.

A. Fraction of voters for union Adding an extra worker of type s increases the unionization vote by $\phi(\Delta_s)$. With a decreasing union wage gap, increasing the number of workers with high-skills, or removing workers with low-skills, directly lowers the fraction of workers in favor of the union.

B. Wages of workers of the same skill group Adding an extra worker of type s changes the union wage gap for all workers of type s. In particular, it lowers their marginal product which, in turn, lowers their nonunion wage and makes a higher fraction of them vote in favor of unionization.

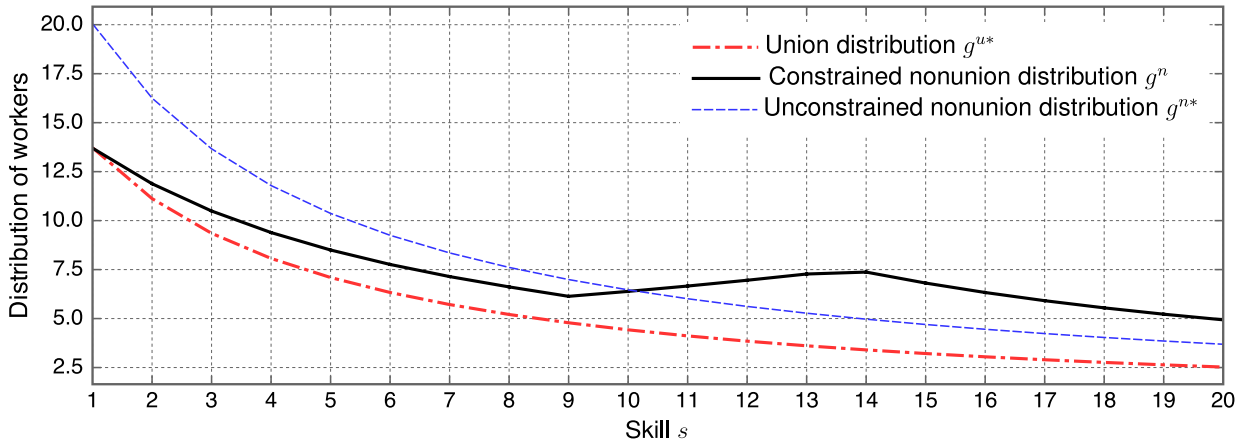
C. Wages of workers of a different skill group Adding an extra worker of type s changes the union wage gap for all workers of type $s' \neq s$. For instance, since union wages are determined by the average product of workers, increasing the number of high-skill employees increases the fraction of high marginal product workers which shifts the union wage schedule upwards. As a result, some workers will change their vote in favor of unionization. Similarly, if the firms hires a lot of low-skill workers, their relatively low marginal product pushes the union wage schedule downwards which tends to increase the number of workers against unionization.

Figure 4 shows the impact of the union threat on the same firm that was represented on Figure 3. Now the firm is not exogenously unionized or union free; the vote of the workers determines its union status. The firm therefore compares the profit it would make with two distributions of workers: the optimal one under which the workers unionize, g^{u*}, and the optimal one under which the workers reject the union, g^n.[26] These two distributions are featured on Panel (a) with, for comparison, the optimal nonunion distribution when the threat is absent, g^{n*}. Panel (b) shows the wages that workers will get if they unionized or not, conditional on the firm hiring g^n. The distance between these two wage schedules influences the vote. Panel (b) also depicts the nonunion wages $w^n(g^{n*})$ when there were no unionization constraint. Panel (c) shows the union and nonunion wages when the distribution of workers is g^{u*}. Notice that the firm does not have to distort g^{u*} for the workers to form a union.

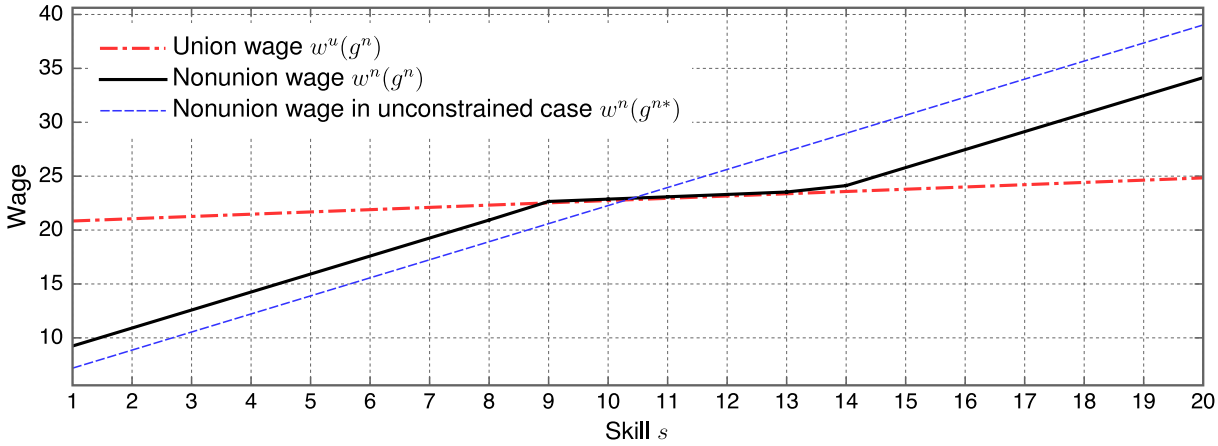
We can see on Panel (a) that, to prevent unionization, the firm distorts its hiring decision substantially. It reduces the number of low-skill workers and increases the number of high-skill workers it hires. Doing so increases the marginal product of low-skill workers which increases their

[26] In general, the workers vote for unionization when the firm hires according to g^{u*} so that $g^u = g^{u*}$. This is the case in the calibrated economy.

(a) Distributions of workers under endogenous union status



(b) Union and nonunion wages when the firms hires according to g^n



(c) Union and nonunion wages when the firms hires according to g^u

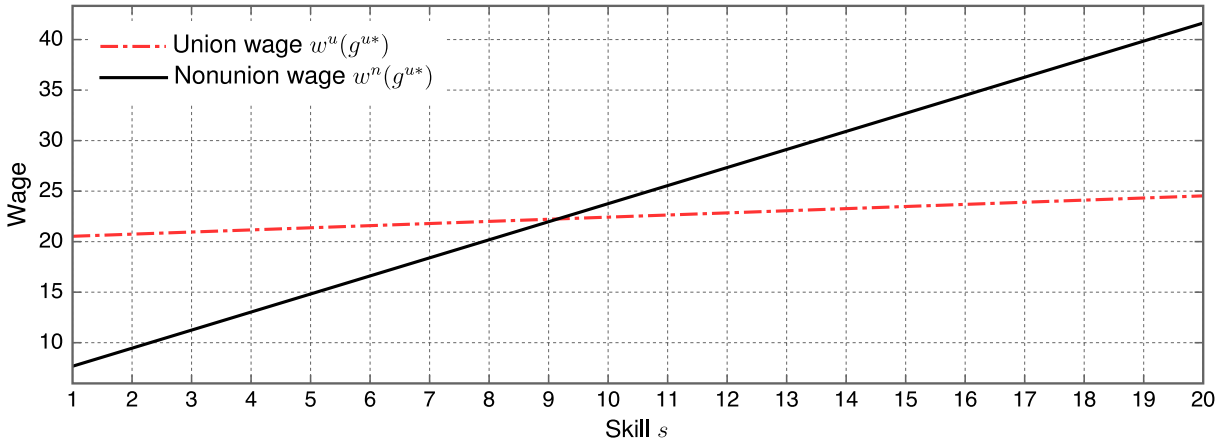


Figure 4: The hiring decision of a firm facing a threat of unionization. See Appendix B for the parameters of this simulations.

wages. The opposite effect take place at the top of the skill distribution and leads to lower wages for high-skill workers, as can be seen on Panel (b). By reacting to the fact that its workers *can* unionize, the firm pays a more compressed wage distribution. Notice that this distortion of the distribution of workers comes directly from the first order condition shown at Equation 21, where $\partial V/\partial g$ is positive for low-skill workers and negative for high-skill ones.

We also see on Panel (b) that, $w^u(g^n)$ and $w^n(g^n)$ are very close to each other for the workers with skills between $s = 9$ and $s = 13$. The firm would like to hire more of these workers: they vote against unionization and their relatively small marginal product has a smaller effect on the union wage schedule than the workers with higher skill. However, hiring an additional worker in this zone would lower his marginal product and push his nonunion wage under his union wage. If ρ is high, workers of this skill group would therefore massively change their votes to support unionization.

Table 1 shows some characteristics of a firm under the three following scenarios:

1. **Exogenously unionized** The firm hires according to g^{u*}.
2. **Endogenous union status** A vote on unionization takes place. The firm compares its profit under g^n and g^{u*}. In this example, it picks g^n.
3. **Exogenously union free** The firm hires according to g^{n*}.

These scenarios can be thought of as policy environments in which unions are mandatory, allowed or forbidden.

	1. Exogenously unionized	2. Endogenous union status	3. Exogenously union free
Union status of the firm	Union	Nonunion	Nonunion
Firm discounted profit ($\times 10^4$)	0.9	1.4	1.5
Number of workers	108	151	185
Fraction of voters for union	66%	50%	66%
Mean of wages	22	20	18
Standard deviation of wages	1.2	7.1	9.4

Table 1: The behavior of a firm the three scenarios.

We can see that the firm's profit is highest when it is exogenously union free and that, when unionization is endogenous, the firm still manages to find a nonunion distribution with higher profit then in the exogenous union case. The firm is therefore union free in scenario 2. Also, the fraction of voters in favor of a union is about the same in scenario 1 and 3. In scenario 2, the firm pushes down the fraction of workers who favor the union until it reaches 50%.

The number of workers employed by the firm is higher when it is exogenously union free than when it is exogenously unionized. As discussed when we introduced Equation 20, this is a consequence of the two bargaining frameworks. Because of the union threat, the size of the firm is smaller under scenario 2 than under scenario 3 even though the workers still bargain individually with the firm. The extra constraint on the firm's problem leads to an increase in the cost of producing an extra unit of goods. This inefficiency forces the firm to decrease its size to increase the marginal product of its workers. This, in turn, makes the constraint firm pay higher wages, on average, than the unconstrained nonunion firm. Because of the collective bargaining framework, the workers receive the highest average wage when the firm is exogenously unionized.

As for the variance of wages, it is the highest when the firm is exogenously union free. Allowing the workers to vote brings it down through the effect of the unionization threat. Finally, since collective bargaining mixes the characteristics of the workers, the variance of wages is lowest under scenario 1.

The unionization threat lowers the *range* of nonunion wages when compared to an unconstrained firm. However, the impact of the threat on the *variance* of wages needs to also take into account the change in the distribution of workers. In general, the threat also lowers the variance but this effect might be mitigated in firms that are extremely constrained by the vote. In these firms, the number of high-skill workers needs to be increased so much that w^n is almost equal to w^u for all the workers rejecting the union. The firm therefore tries to shift the w^u schedule downward by adding to its workforce a large number of workers with very low-skill. This leads to a U-shaped distribution of workers which tend to increase the variance of wages.

3.3 Impact of technology on unionization

Proposition 3 states that, when the bargaining powers are equal, a firm prefers to be union free for any distribution g. The intensity of this preference depends on the gap between the wage bill in the union and nonunion case and, therefore, on the curvature of the production function. As the production function becomes linear ($\alpha \to 1$) the firm becomes indifferent. Technology has also a strong influence on the vote of the workers and, through that channel, on the union status of the firm. The following lemma characterizes how returns to labor affect the workers preference for unionization.

Proposition 4. *In a given equilibrium, under the optimal distribution of workers g^{n*}, the union wage gap $w_s^u(g^{n*}) - w_s^n(g^{n*})$ decreases with α:*

$$\frac{d(w_s^u(g^{n*}) - w_s^n(g^{n*}))}{d\alpha} = -\frac{\beta_u}{\alpha^2 \sum_{s=1}^{S} \left(z_s/\mathrm{MC}_{j,s}^n\right)^\sigma} \sum_{s=1}^{S} z_s \left(\frac{z_s}{\mathrm{MC}_{j,s}^n}\right)^{\sigma-1} < 0. \tag{23}$$

Furthermore, the total fraction of workers in favor of unionization also decreases with α.

An increase in α lowers the union wage gap $w^u - w^n$ uniformly across skills which induces more workers to vote against unionization. It is therefore easier for the firm to prevent unionization. This result is broadly consistent with the empirical finding that industries with lower labor shares tend to be more unionized (Hirsch and Berger, 1984).

Finally, the following lemma shows that the productivity parameter A of the production function has no influence on the union status of a firm or on the wages it pays.

Lemma 4. *Consider two firms, identified by the subscripts 1 and 2, that have identical technologies except for $A_1 \neq A_2$. In equilibrium, if g_1 solves the problem of firm 1, then*

$$g_2 = \left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}} g_1$$

solves the problem of firm 2. Also, both firms have the same union status and pay the same wages.

This lemma will be useful to aggregate firms of the same type in the calibration.

4 Data and calibration

So far, I have emphasized the distorting effect that the union threat can have on the behavior of firms. To understand how this distortion influences the macroeconomy, we first need to calibrate the model so that we can look at the effect of different union policies.

I calibrate the model on the private sector of the United States in 2005. One period is one month and all monetary amounts are measured in thousands of dollars. I set the monthly discount rate to $\gamma = 0.996$ and the probability of job destruction to $\delta = 0.027$ (Shimer, 2012).[27] For the matching function, I use $q(\theta) = (1 + \theta^{\eta})^{-1/\eta}$ so that all probabilities are strictly between 0 and 1. To estimate η, I use data from the Job Opening and Labor Turnover Survey (JOLTS) for 2005 together with the probability of job finding from Shimer (2012). The estimate for η is 1.33.[28] For the elasticity of substitution between skills, I follow the literature and set $\sigma = 1.5$ (Johnson, 1997; Krusell et al., 2000). For the cost of posting a vacancy, I follow the analysis of Hagedorn and Manovskii (2008). They use a survey of employers to conclude that the cost of hiring one worker is 4.5% of quarterly wages. This translates to $\kappa = 0.36$ in the current model.[29] I also set the number of skill groups to $S = 6$, which is enough to observe the impact of union policies across skills while keeping the computational complexity at a reasonable level. Table 2 summarizes the parameters.

Parameter	Definition	Value	Source/reason
γ	Discount factor	0.996	5% annual interest rate
δ	Job destruction probability	0.027	Shimer (2012)
η	Matching function parameter	1.33	JOLTS with Shimer (2012)
σ	Elasticity of substitution between skills	1.5	Literature - see text
κ	Cost of posting a vacancy	0.36	Hagedorn and Manovskii (2008)
S	Number of skills	6	See text

Table 2: Parameters taken directly from the data or the literature

I assume that each firm is endowed with one of two technologies, denoted by the subscript $j \in \{u, n\}$. In equilibrium, firms of type u are unionized while firms of type n are not. I denote the technologies of union and nonunion firms by (A_u, α_u, z_u) and (A_n, α_n, z_n) respectively. As Lemma 4 shows, it is equivalent to change the number of firms of type j or the parameter A_j of these firms' technology. We can therefore normalize the number of firms of each type and adjust A_j to fit their size.

In what follows, the data about individuals is coming from the Merged Outgoing Rotation Groups of the Current Population Survey (CPS) as it is made available by the National Bureau of Economic Research (NBER). Industry data comes from the Bureau of Economic Analysis (BEA). See Appendix C for details about how I cleaned the data.

[27] Robert Shimer constructed this data. For additional details, please see Shimer (2012) and his webpage.

[28] See Appendix C for the details.

[29] As a robustness check, I also calibrated the model with a substantially higher cost of vacancy posting, $\kappa = 0.85$. All the results from the policy simulations are similar. A higher value for κ leads to a bigger impact of the union threat on wage inequality but to smaller impacts on output and unemployment. The minimum reached by the loss function is also higher, indicating a worse fit of the model.

Skill distribution

The skill index is, well, only an index. Throughout this model it is used to characterize the heterogeneity of the agents and to identify variables that are related to them (θ_s, b_s, N_s, etc.). To calibrate the economy, I first define a skill index from the data and then calibrate the firms' technologies to match moments of the data. This way, the skill index and the technologies are consistently determined to make the model match the distributions of workers.

I use data from the CPS to build the skill index. To do so, I follow Card (1998) and regress log monthly *nonunion* wages on two types of variables. The first type includes variables related to each individual. The second type of variables depends specifically on the job in which the individual works. I then use the predicted variable given by the OLS estimator of the individual characteristics alone as the skill index. Explicitly, denote by w_i the monthly wage of an agent i, who is working in industry $j(i)$. The regression is

$$\log w_i = \Gamma X_i^1 + \Psi X_{i,j(i)}^2 + \epsilon_i.$$

and the skill index is therefore given by the predicted values $\hat{s}_i = \exp\left(\hat{\Gamma} X_i^1\right)$. This way of constructing the index isolates the impact of variables intrinsically related to the individual from match-related factors that could also influence the wage. The individual characteristics X^1 are *sex*, *age*, *race*, *education* and *occupation* (set of dummy variables). The job related characteristics X^2 are *industry* (dummy variables) and the current *U.S. state* in which agent i lives.[30] Notice that even though the regression is run only on nonunion workers, the predicted values $\hat{s}_i$ are computed for *all* members of the labor force. Figure 5 shows the distribution of $\hat{s}_i$. For the numerical simulations, I split the support of the distribution in six bins of equal size to generate the empirical skill distribution N_s.

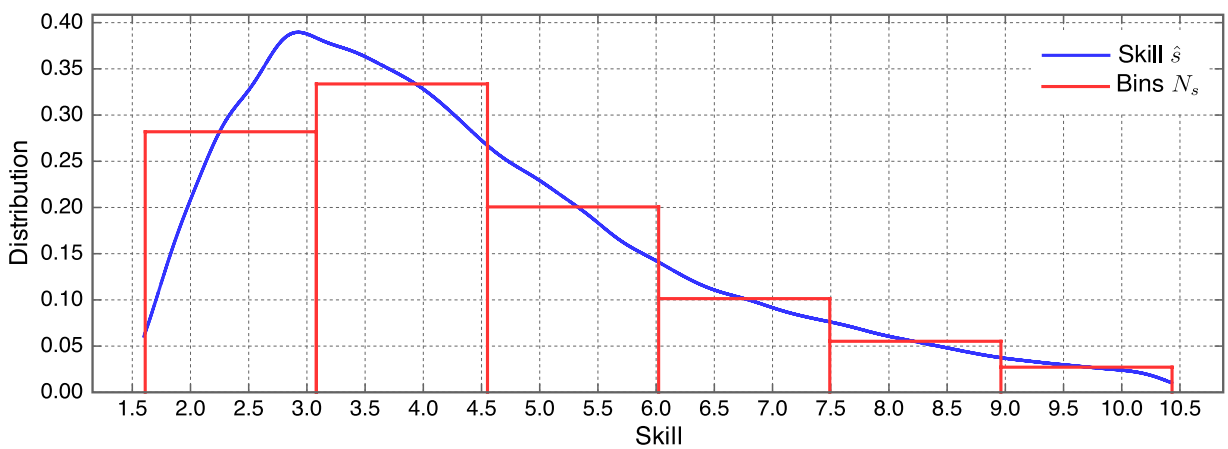


Figure 5: Normalized skill distribution and the bins.

This way of defining the skill distribution has the advantage of making the empirical wages and the empirical labor market tightness increasing with s, which makes the interpretation of the impact of unionization on different workers intuitive.

[30] Including *US state* as an individual characteristic instead has minimal impact on the distribution. For *industry* and *occupation*, I use the variables generated by the NBER. Both are at the 3-digit level. I drop from the sample individuals with skill index below the second percentile and above the 98th percentile.

Labor market tightness and the value of nonwork activities

In the United States, unemployment insurance programs are administered by the states. Krueger and Meyer (2002) provides the main characteristics of benefits for some U.S. states in 2000. The replacement ratio is about 50% in every state but the maximum weekly benefits vary considerably. In the model, b^0 also takes into account home production and the value of the extra leisure provided by unemployment, two elements that are harder to quantify. Different numbers have been used in the literature. For instance, Hall and Milgrom (2008) use a flow value of non-work that equals to 71% of productivity. Hagedorn and Manovskii (2008), on the other hand use, 95.5%. Because of the multi-worker production function used in this paper, setting b_s^0 to be a certain fraction of productivity is inconvenient. Instead, I rely on the analysis of Hall (2009) and set b_s^0 to be 85% of the average wage earned by workers of skill s.

I use Equation 17 together with the observed unemployment rates by bin to identify the labor market tightness θ for workers in each of these bins. Using the mean wages of union and nonunion workers together with the fact that, at the steady state, firms hire a fraction δ of their workforce every period, I compute the expected wage of a worker who just found a job. I then use Equation 2 to identify the outside option b for each of the skill bins.

Fraction of workers in favor of unionization

One of the moment that the calibration attempts to match is the fraction of workers of each skill bin voting in favor of unionization in each type of firm. Using data on workers who participated in union elections, Farber and Saks (1980) estimate a probit model to predict votes using worker and firm characteristics. I use their estimated coefficients to back out the fraction of workers of each skill voting for unionization.[31]

Loss function

To calibrate the remaining parameters, I pick the vector $\xi = (\rho, \beta_n, \beta_u, \alpha_n, \alpha_u)$ to minimize the following loss function:

$$\text{Loss} = \frac{\sum_{s=1}^{S}(N_s - \hat{N}_s)^2}{\sum_{s=1}^{S} N_s^2} + \frac{\sum_{s=1}^{S}(b_s^0 - \hat{b}_s^0)^2}{\sum_{s=1}^{S}(b_s^0)^2} + \frac{(\text{LS}_n - \widehat{\text{LS}}_n)^2}{\text{LS}_n^2} + \frac{(\text{LS}_u - \widehat{\text{LS}}_u)^2}{\text{LS}_u^2}$$
$$+ \frac{\sum_{s=1}^{S}(\phi_s^n - \hat{\phi}_s^n)^2}{\sum_{s=1}^{S}(\phi_s^n)^2} + \frac{\sum_{s=1}^{S}(\phi_s^u - \hat{\phi}_s^u)^2}{\sum_{s=1}^{S}(\phi_s^u)^2}$$

where LS_j denotes the labor share and where ϕ_s^j denotes the probability that a worker of type s vote for unionization while working in a firm of type j, for $j \in \{u, n\}$. In the previous equation, a hat indicates that the variable is from the simulated economy while the absence of a hat indicates that the variable is taken from the data.

The idea behind the calibration is straightforward. For any vector of parameters ξ, I set the technologies (A_u, z_u) and (A_n, z_n) to fit exactly the distribution of union and nonunion workers in the economy. I then compute the firms' decision assuming that the vectors b and θ observed in the data are equilibrium objects. The schedules $\hat{N}$ and $\hat{b}^0$ are those that make b and θ actual equilibrium objects. Appendix D explains how this can be done.

[31] See Appendix C for the details of the computations.

Calibrated economy

Table 3 shows the parameter values that minimize the loss function.

Parameter	Definition	Calibrated value
ρ	Preference for unionization parameter	1.06
β_n	Bargaining power of individual worker	0.44
β_u	Bargaining power of union	0.21
α_n	Returns to labor of nonunion firms	0.54
α_u	Returns to labor of union firms	0.49

Table 3: Calibrated parameters

Notice that β_n is bigger than β_u. This difference in bargaining powers is necessary to compensate for the fact that the decreasing returns provide the unionized workers with more leverage in the negotiations. In the calibrated model, workers always prefer to form a union and the firms need to fight to prevent unionization. Figure 6 shows the calibrated skill intensities z_u and z_n. We see that union firms are more intensive in workers with average skill. This comes from the fact that, in the data, the distribution of union workers is concentrated in the middle of the skill distribution.

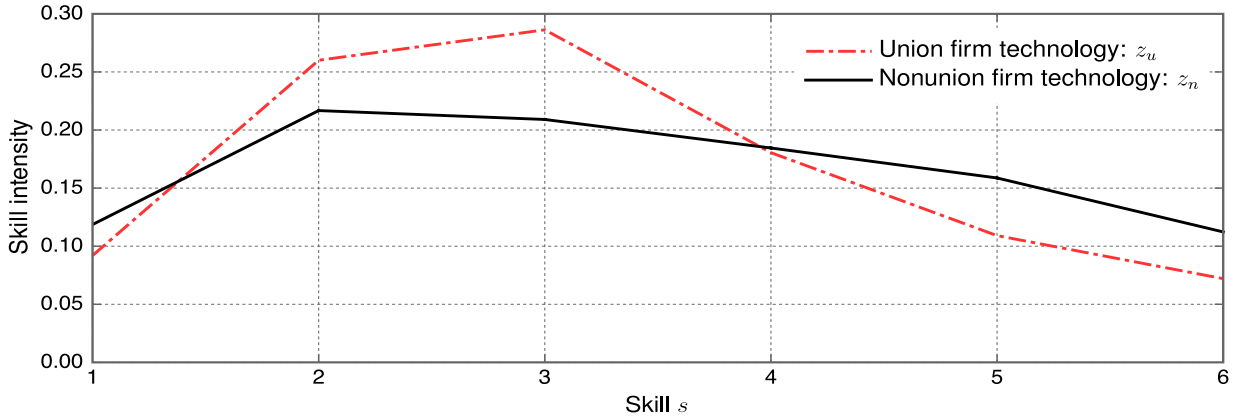


Figure 6: Calibrated skill intensities z_n and z_u.

Figure 7 shows how the model fits the wage schedules, the distribution of workers employed by the firms, the distribution of agents in the labor force N, the value of nonwork b_0 and the probability of voting for the union in the both types of firm: ϕ_u and ϕ_n. As a consequence of the calibration strategy and of the way the firms' technologies are identified in the data, the model fits perfectly the distribution of workers in each firm, the unemployment rate and the distribution of agents in the labor force. The labor shares are fitted very well.[32] The model also fits the nonunion wage schedule quite well. The fit of the union wage schedule is however less precise, a consequence of the rigid structure imposed on union wages by Equation 10. In fact, union wages vary in s only through the outside option b_s. Union wages in the calibrated economy are more unequal than in the data. Suggesting that the real equalizing effect of unions might be stronger than the one captured by the calibration. A better fit could be obtained by allowing β_u to vary by skill group.

[32]The labor share for the union firm is of 0.598 in the calibrated economy while its data value is 0.597. For the nonunion firm, the corresponding numbers are 0.611 and 0.613.

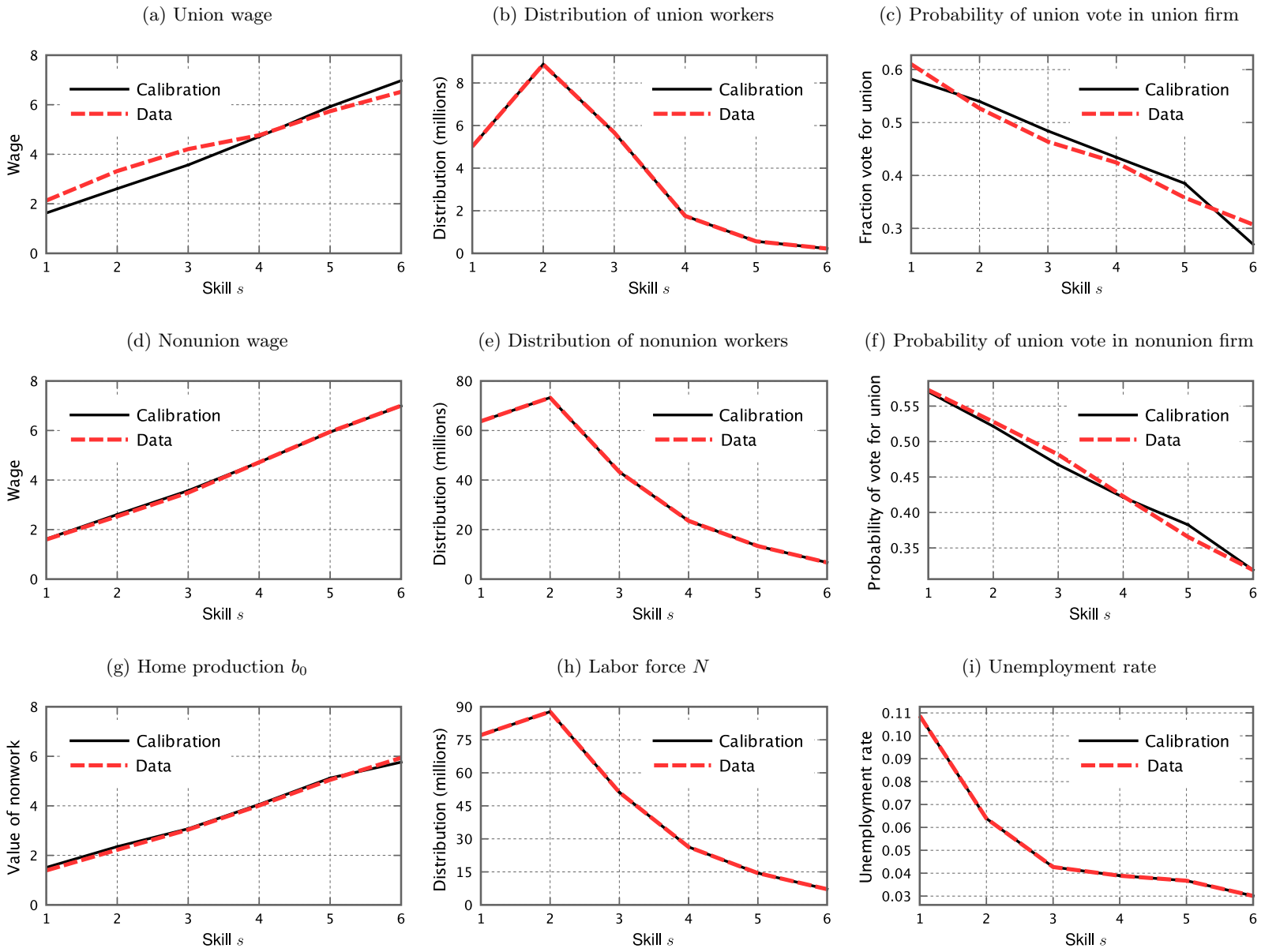


Figure 7: Fit of the calibrated model

5 Impact of union policy on the economy

Policy simulations

I do three policy simulations using the calibrated economy. Each of these simulations can be thought of as forcing firms in one of the branches of the tree depicted on Figure 2, such that the union vote does not matter anymore, or as a change in the legal environment. The policy simulations are:

1. **No union threat** All firms that are unionized in the calibrated economy stay unionized and all nonunion firms stay union free but they do not have to worry about the unionization vote anymore. They know that their workers will not unionized and they behave as such.

2. **Illegal unions** Unions are completely eliminated from the economy.

3. **Mandatory unions** All firms are unionized.

In all cases, I compute the new steady state general equilibrium using the algorithm described at Appendix E. Figure 8 shows the new equilibria. Consider first the nonunion wages shown on Panel (b). We see that removing the threat of unionization leads to higher wage inequality by increasing high wages more then low ones. Since nonunion firms do not have to distort their behavior to prevent unionization anymore, they hire more high-skill workers and fewer low-skill workers. Doing so changes the marginal product of each skill group which then influences wages. This has a direct effect on the value of unemployment b_s, which, in turn, modifies the wages paid by union firms, as seen on Panel (a). The impact of the threat removal on wages paid by union firms is purely through a general equilibrium mechanism.

Outlawing unions altogether amplifies the effect on wages further. In this case, the firms that were previously unionized now bargain wages individually with their workers. This leads to an increase in the slope of the wage schedule paid by these firms. This, in turn, increases the slope of b_s which leads to a higher variance for the wages paid by firms that were previously union free, further amplifying the effect on inequality.

The unemployment rates of all skill groups go down in all policy exercises. This comes as a direct consequence of the removal of the unionization constraint at the firm level. Once the threat is gone, the marginal cost associated with producing one unit of the good goes down and firms therefore increase their size substantially. Because of this higher demand for workers, the labor market tightnesses go up which then pushes wages upwards. This general equilibrium feedback helps to explain why the total changes in wages are positive for all workers.

As a consequence of lower unemployment rates and higher wages, removing the union threat and outlawing unions has a positive impact on the welfare of all skill groups, as can be seen on Panel (d).[33] Therefore, in this economy, even though some workers prefer their specific firm to be unionized, if an economy-wide referendum was organized to outlaw unions it would pass without a single objection.

Forcing all firms to be unionized has large macroeconomic implications simply because it affects the relatively large number of workers who were previously working in nonunion firms. As can be seen on Panel (a) of Figure 8, the wage of low-skill workers goes up substantially while high-skill wages suffer a steep decline. The change from individual to collective bargaining is responsible for this large reduction in wage inequality.

[33] The welfare schedule is computed by summing the welfare of all the agents, employed or unemployed, of a specific skill. In particular, the profits of the firms are not redistributed to the workers.

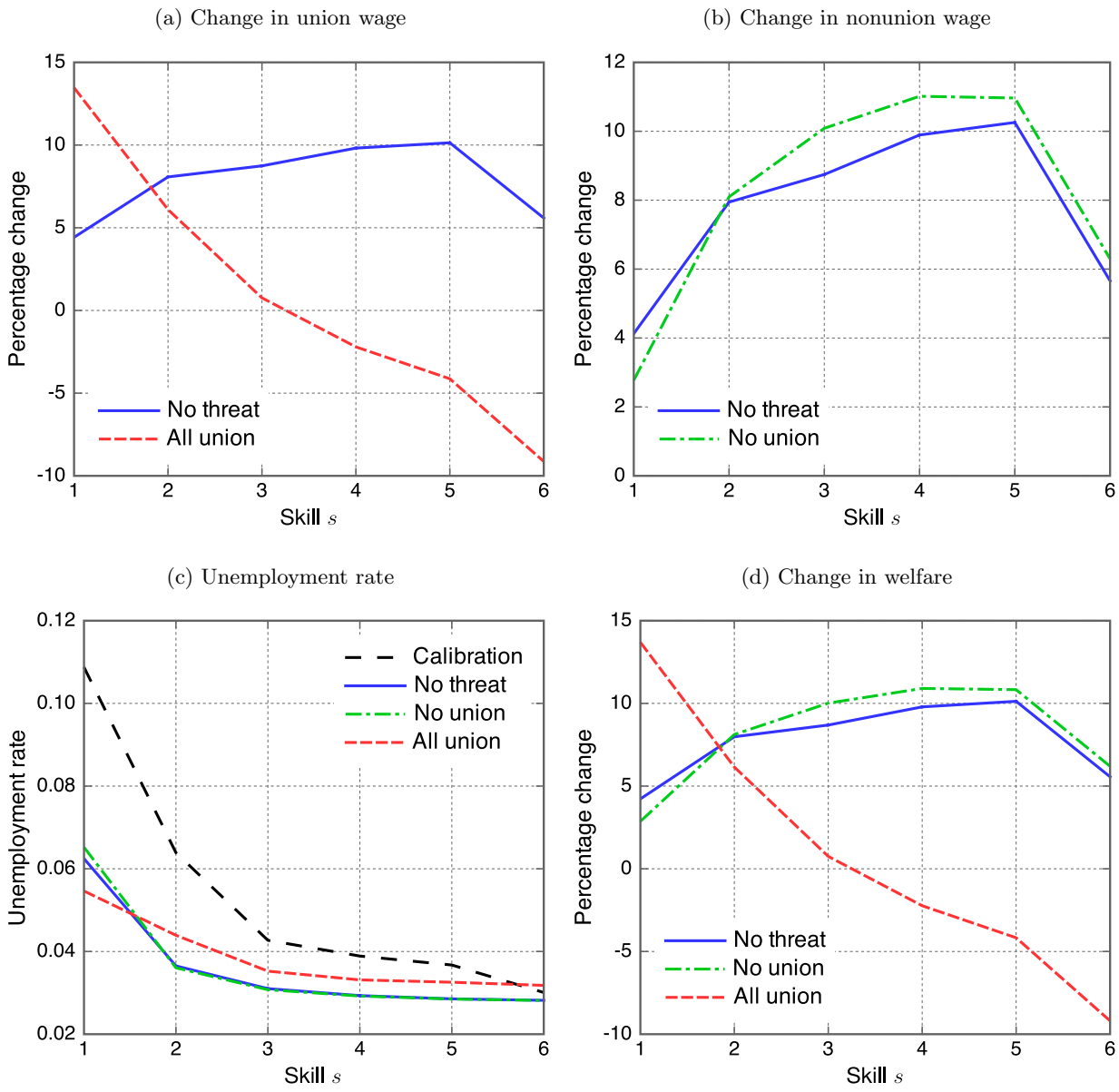


Figure 8: Impact of changes in union policies on wages, unemployment and welfare.

Table 4 presents the variance of wages, the unemployment rate, total output as well as welfare under the three changes in policy. We can see that removing the threat increases the variance of wages, lowers unemployment and increases output and welfare. These effects are further amplified when unions are outlawed. Overall, making unions illegal increases total production by 1.2%, welfare by 8.0% and lowers the unemployment rate by 2.6 percentage points. It also increases the variance of log wages by 12.3%. To give an idea of the magnitudes involved, in the U.S. the variance of the log of males annual labor earnings increased by about 25 % between 1980 and 2000 (Heathcote et al., 2010). Notice that the effects of the union threat are substantial even though the unionization rate, at 8.9%, is quite low. In fact, the threat could impact an economy even if no

firms are actually unionized.

The economy reacts differently when all firms are unionized (last column of Table 4). The change in bargaining regime pushes the variance of log wages down by 27 %, a substantial reduction in earnings inequality. Moreover, the removal of the threat inefficiency at the firm level is mostly responsible for an increase in output and welfare and for a lower unemployment rate. These movements are however not as strong as when unions are outlawed.

	Calibration (level)	No threat	Unions illegal	Unions mandatory
		(percentage change from calibration)		
Variance log union wages	0.128	+12.3%	-	+3.2%
Variance log nonunion wages	0.186	+7.3%	+9.1%	-
Variance log all wages	0.181	+8.1%	+12.3%	-27.0%
Total output ($\times 10^9$)	1.2	+1.1%	+1.2%	+1.0%
Welfare ($\times 10^{11}$)	2.0	+8.0%	+8.3%	+2.7%
Value of the firms ($\times 10^{11}$)	1.2	-13.8%	-14.4%	-4.7%

	Calibration	No threat	Unions illegal	Unions mandatory
		(all numbers are in level)		
Unemployment rate	6.8%	4.2%	4.2%	4.3%
Unionization rate	8.9%	7.6%	0%	100%

Table 4: Impact of policies on wages, unemployment, output and welfare.

Comparison with empirical estimation

These policy simulations suggest that labor union policies have a much bigger impact on the variance of log wages than standard econometric estimators would suggest. To illustrate this point, consider what a typical estimator would measure: the partial equilibrium reallocation of union workers on the nonunion wage distribution.[34] To do the comparison, we use the calibrated economy and build a counterfactual aggregate wage distribution by endowing each union worker with the wage that nonunion workers in the same skill group actually receive. In other words, each worker is now assigned the wage $w_s^{CF} = w_s^n$ regardless of the unions status of her firm. Comparing this new distribution with the wage distribution of the calibrated economy suggests that unions would be responsible for a reduction in the variance of log wages of 0.1%, a much smaller number than the 12.3% provided by the full simulation in general equilibrium.[35]

There are two main reasons for this. First, a reallocation estimator assumes that all general equilibrium forces are inactive such that nonunion wages and the hiring decision of each firm do not react to a change in labor union policies. In reality, we can think that, for instance, lower wages would induce additional hiring which would, in turn, influence wages. Second, a reallocation estimator cannot take into account the impact of the union threat on the behavior of nonunion firms, a channel that was shown to have a large impact in the simulations.

[34] See Card et al. (2004) for a summary of the empirical literature on unions and wage inequality.

[35] As explained in the introduction, I have also used the classical two-sector estimator of Freeman (1980) on the data set. It suggests that unions are responsible for a variance of log wages that is smaller by 0.4%. Note that this sample only contains private sector workers. Adding public workers would increase this number.

6 Concluding remarks

Empirical estimators of the effects of unions on inequality generally abstract from the decision process of the firms and from general equilibrium mechanisms. In particular, they neglect the possible consequences that the unionization threat exerts on firms.

This paper proposes a general equilibrium theory of firms' decisions and union formation to study the impact of unions on the economy. Workers and firms meet in a labor market characterized by frictions. Each period, the workers of a firm vote to create a union. If a union is created, wages are bargained collectively. Otherwise, each worker bargains his wage individually with the firm. This asymmetry of wage setting mechanisms causes unions to compress the wage distribution inside a firm. Furthermore, by fighting the threat of unionization, firms distort their hiring decisions in a way that also compresses wages.

I calibrate the model on the United States and show that outlawing unions increases the variance of wages substantially. This increase is much bigger than a standard empirical estimate would suggest. Furthermore, outlawing unions increases welfare and output while lowering unemployment. Forcing all firms to be unionized, on the other hand, reduces wage inequality while still improving output and unemployment.

This paper only deals with the private sector of the economy. Since the public sector is heavily unionized in the United States, it is likely that the counterfactual policy exercises underestimate the full impact of unions. One possible extension of the model would be to include a government in which the bargaining power of unions is different than in the rest of the economy. Another possible direction for future research would be to allow bargaining at the country level in order to compare the union legislations in some European countries with that of the United States. Also, the theory proposed in this paper could be used to study the interaction between the rise in inequality and the strong deunionization that has been observed in the United States during the last decades. In particular, it would be interesting to observe how a change in production technologies or in the skill distribution could impact the unionization rate and, through that channel, wage inequality. Finally, it would be interesting to investigate empirically the impact of the union threat on firms. Do to so, one could use changes in right-to-work legislations in the United States to identify potential variations in wage distributions and labor demand in *nonunion* firms. Such an approach could provide direct evidence of the impact of the threat on firm behavior.

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Appendices

A Proofs

This appendix contains the proofs from the previous sections.

Proof of Lemma 1. At a steady-state the aggregate variables are constant and the firm's problem is given by Equation 5, which we can rewrite

$$J\left(\sum_{s=1}^{S}\frac{g_{-1,s}}{q(\theta_s)}\right)=(1-\delta)\kappa\sum_{s=1}^{S}\frac{g_{-1,s}}{q(\theta_s)}+\max_{g}\left\{\pi(g)-\kappa\sum_{s=1}^{S}\frac{g_s}{q(\theta_s)}+\gamma J\left(\sum_{s=1}^{S}\frac{g_s}{q(\theta_s)}\right)\right\}.$$

The term that is maximized is constant with respect to g_{-1}. Denote that constant by B. Then, in particular

$$J\left(\sum_{s=1}^{S}\frac{g_s}{q(\theta_s)}\right)=(1-\delta)\kappa\sum_{s=1}^{S}\frac{g_s}{q(\theta_s)}+B.$$

The firm therefore solves

$$\max_{g}\pi(g)-\kappa\sum_{s=1}^{S}\frac{g_s}{q(\theta_s)}+\gamma\left((1-\delta)\kappa\sum_{s=1}^{S}\frac{g_s}{q(\theta_s)}+B\right)$$

and the result follows. ■

Proof of Lemma 2. In the first part of this lemma, I show how to derive the expression for the firm's surplus, given by Equation 8.

Consider the firm's gain from agreeing on a wage schedule w with the union. At this point, the distribution g is fixed and the hiring cost is sunk. In a steady-state, the difference in discounted profits for the firm, denoted by $\Delta^u(w)$, is

$$\Delta^u(w)=\left[\pi(g,w)+\gamma J\left(\sum_{s=1}^{S}\frac{g_s}{q(\theta_s)}\right)\right]-[\pi(0)+\gamma J(0)] \tag{24}$$

where the first term between brackets is discounted profit if an agreement is reached and $\pi(0)+\gamma J(0)$ is the firm's discounted profit if negotiations break down. In such a case, the firm has no worker, it produces nothing and pays no wages. Therefore, $\pi(0)=0$. $J(0)$ is the value function of a firm that starts the period with no workers. Since the firm's employment decision is independent g_{-1}, it hires back to its steady-state optimal level g^* right away. Therefore,

$$J(0)=\pi(g^*,w^*)-\kappa\sum_{s=1}^{S}\frac{g_s^*}{q(\theta_s)}+\gamma J\left(g^*\right)$$

where w^* is the equilibrium wage schedule for this firm. We can therefore rewrite 24 as

$$\Delta^u(w)=\pi(g,w)+\gamma J\left(g\right)-\gamma\left(\pi(g^*,w^*)-\kappa\sum_{s=1}^{S}\frac{g_s^*}{q(\theta_s)}+\gamma J\left(g^*\right)\right).$$

But, the firm's value function is

$$J(g) = \pi(g^*, w^*) - \kappa\delta \sum_{s=1}^{S} \frac{g_s^* - (1-\delta)g_s}{q(\theta_s)} + \gamma J(g^*) \tag{25}$$

and therefore the firm's surplus from agreeing on a wage w is

$$\Delta^u(w) = \pi(g, w) + (1-\delta)\gamma\kappa \sum_{s=1}^{S} \frac{g_s}{q(\theta_s)}.$$

We now consider solutions to the bargaining problem of Equation 9. To keep a light notation, let us define

$$\Gamma \equiv F(g) + (1-\delta)\kappa\gamma \sum_{s=1}^{S} \frac{g_s}{q(\theta_s)} \geq 0.$$

By taking the log, we can define the objective function $P(w)$ as

$$P(w) \equiv \beta_u \sum_{s=1}^{S} \frac{g_s}{n} \log(w_s - c_s) + (1-\beta_u) \log\left(\Gamma - \sum_{s=1}^{S} w_s g_s\right)$$

and write the collective bargaining problem as

$$\max_{w \in M} P(w) \tag{26}$$

where

$$M \equiv \left\{ w \in \mathbb{R}_+^{\mathbb{S}} : w_s - c_s \geq 0 \; \forall \, s \in \mathcal{S}, \, \Gamma - \sum_{s=1}^{S} w_s g_s \geq 0 \right\}.$$

is the set of vectors w which might be agreed upon. For a w outside of M, some workers are better off unemployed or the firm will have a negative surplus.

The proof consists of four intermediary steps:

Step 1 The set of admissible functions M is convex.

If M is a singleton then it is convex. If not, take any $w_1, w_2 \in M$ and consider the convex combination $w_a = aw_1 + (1-a)w_2$ with $0 \leq a \leq 1$. Then $w_{a,s} \geq c_s$ for all $s \in \mathcal{S}$ and $\Gamma - \sum w_{a,s} g_s \geq 0$. Since w_1 and w_2 are in $\mathbb{R}_+^{\mathbb{S}}$, w_a is also in $\mathbb{R}_+^{\mathbb{S}}$ and therefore M is convex.

Step 2 The function P is strictly concave on M.

Take any $w_1, w_2 \in M$ such that $w_1 \neq w_2$ and consider the convex combination $w_a = aw_1 + (1-a)w_2$ with $0 < a < 1$. Since logarithm is a strictly concave function and $g > 0$, we can write

$$\begin{aligned}
P(w_a) &= \beta_u \sum \frac{g_s}{n} \log(w_{a,s} - c_s) + (1-\beta_u) \log\left(\Gamma - \sum w_{a,s} g_s\right) \\
&> \beta_u \sum a \frac{g_s}{n} \log(w_{1,s} - c_s) + (1-\beta_u) a \log\left(\Gamma - \sum w_{1,s} g_s\right) \\
&+ \beta_u \sum (1-a) \frac{g_s}{n} \log(w_{2,s} - c_s) + (1-\beta_u)(1-a) \log\left(\Gamma - \sum w_{2,s} g_s\right) \\
&= aP(w_1) + (1-a)P(w_2).
\end{aligned}$$

So P is strictly concave on M.

Step 3 The wage vector w^u is a critical point of P.

The vector w^u satisfies the first order conditions $\beta_u\left(\Gamma - \sum w_s g_s\right) = (1-\beta_u)n(w_s - c_s)$ for all $s \in \mathcal{S}$.

Step 4 If the joint surplus is strictly positive at w_u, then w_u is an interior point of M.

Explicitly, the assumption on the joint surplus is

$$F(g) - \sum_{s=1}^{S} c_s g_s + (1-\delta)\kappa\gamma \sum_{s=1}^{S} \frac{g_s}{q(\theta_s)} > 0.$$

This implies that $w_s^u > c_s$ for all $s \in \mathcal{S}$. Furthermore, a simple calculation shows that the firm's surplus is equal to a fraction $(1-\beta_u)$ of the joint surplus. Therefore, $\Gamma - \sum w_s^u g_s > 0$ and w^u is in the interior of M.

Putting the pieces together, P is a strictly concave function on the convex set M which has a critical point at w^u and w^u is in the interior of M. Therefore, w^u is the unique global maximum of the union bargaining problem. ■

Proof of Lemma 3. The Stole and Zwiebel (1996a,b) solution to the bargaining problem is the wage function that gives the worker a share β_n of the joint surplus. The bargaining takes place when all vacancies have been posted and the vacancy costs are therefore sunk. When bargaining with a single worker, the firm compares two scenarios. Either an agreement is reached, in which case production takes place as planned, or the negotiations break down and the firm produces without this individual worker. In this last case, that worker departs from the firm and additional vacancies will have to be posted in the next period for the firm to go back to its optimal distribution of workers. In equilibrium, an agreement is always reached.

To solve the problem, assume that each worker has as size h. We will later take the limit as $h \to 0$. The marginal discounted profit from hiring a worker of type s is proportional to

$$\begin{aligned}\Delta_s^n(w) =& F(g) - \sum_{k=1}^{S} w_k(g) g_k - \Bigg(F(\dots, g_s - h, \dots) - \sum_{k \neq s} w_k(\dots, g_s - h, \dots) g_k \\ & - w_s(\dots, g_s - h, \dots)(g_s - h) - h\gamma(1-\delta)\frac{\kappa}{q(\theta_s)} \Bigg)\end{aligned}$$

where the notation $(\dots, g_s - h, \dots)$ makes explicit the fact that we are considering the distribution g without a measure h of its sth member. Δ_s^n is simply the difference between value of the firm with and without an agreement. Notice that in the latter case, the firm loses value since it faces an additional hiring cost in the next period to get back to its equilibrium size.

A solution to the Stole and Zwiebel bargaining is a wage vector w that solves

$$\frac{\beta_n}{1-\beta_n}\Delta_s^n(w) = (W_s^e(w) - W_s^u)h$$

where the right hand side is the worker's surplus. By dividing Δ_s^n by h and taking the limit $h \to 0$, we get

$$\lim_{h \to 0} \frac{\Delta_s^n(w)}{h} = \frac{\partial F(g)}{\partial g_s} - \sum_{k=1}^{S} g_k \frac{\partial w_k(g)}{\partial g_s} - w_s(g) + \gamma(1-\delta)\frac{\kappa}{q(\theta_s)}.$$

Therefore, a solution must solve the following system of partial differential equations:

$$\frac{\partial F(g)}{\partial g_s} - \sum_{k=1}^{S} g_k \frac{\partial w_k(g)}{\partial g_s} - w_s(g) + \gamma(1-\delta)\frac{\kappa}{q(\theta_s)} = \frac{1-\beta_n}{\beta_n}\left(w_s(g) - c_s\right)$$

for all $s \in \mathcal{S}$. General solutions to this system are of the form

$$w_s^n(g) - c_s = \frac{\beta}{1-\beta(1-\alpha)} \frac{\alpha z_s}{g_s^{\frac{1}{\sigma}}} A \left(\sum_{k=1}^{S} z_k g_k^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1-\alpha(\sigma-1)}{\sigma-1}} - \beta_n c_s + \beta_n \gamma(1-\delta)\frac{\kappa}{q(\theta_s)} + C_s g_s^{-\frac{1}{\beta_n}}$$

where C_s is a constant term that could depend on $\{g_j\}_{j \neq s}$. To fix the constants, I use the convenient boundary conditions

$$\left\{ \lim_{g_s \to 0} w_s^n(g) g_s = 0 \right\}_{s=1}^{S}$$

which guarantees that $C_s = 0$ for all s.[36] ■

Proof of Proposition 1. We first start with the union wage. From Equation 10, we can write $w_s^u(g^{u*}) = c_s^u + D$ where D is a constant that does not depend on s. Also, from Equation 3, we have

$$c_s^u = \frac{b_s - \gamma(1-\delta) w_s^u(g^{u*})}{1-\gamma(1-\delta)}.$$

Combining these two equations, we get that $w_s^u(g^{u*}) = b_s + (1-\gamma(1-\delta))D$. Since, b_s is increasing in s so is $w_s^u(g^{u*})$.

For the nonunion wage, by combining Equations 13 and Equation 19, we find that

$$w_s^n(g^{n*}) = c_s^n + \frac{\beta_n}{1-\beta_n}\frac{\kappa}{q(\theta_s)}.$$

Using Equation 3 once again yields

$$w_s^n(g^{n*}) = b_s + (1-\gamma(1-\delta))\frac{\beta_n}{1-\beta_n}\frac{\kappa}{q(\theta_s)}.$$

Since b_s and θ_s are increasing in s so is $w_s^n(g^{n*})$.

For the union wage gap, notice that

$$w_s^u(g^{u*}) - w_s^n(g^{n*}) = (1-\gamma(1-\delta))\left(D - \frac{\beta_n}{1-\beta_n}\frac{\kappa}{q(\theta_s)}\right).$$

Since D does not vary with s and θ_s is increasing in s, the union wage gap is decreasing in s. ■

Proof of Proposition 2. We first start with the unionized firm. This firm hires according to g_s^{u*}. From Proposition 1, we know that $w_s^u(g^{u*}) = c_s^u + D$ and that $c_s^u = b_s - \gamma(1-\delta)D$. Consider

[36] Cahuc et al. (2008) study a similar bargaining problem in a more general class of production functions.

now the off-equilibrium nonunion wage that the union workers *would* get if they voted against the union. From Equation 13 we have

$$w_s^n(g^{u*}) = (1-\beta_n)c_s^u + \frac{\beta_n}{1-(1-\alpha)\beta_n}\frac{\mathrm{MC}_s^u}{1-\beta_u} + \beta_n\gamma(1-\delta)\frac{\kappa}{q(\theta_s)}.$$

Using the definition of MC^u, it is straightforward to show that

$$w_s^n(g^{u*}) = c_s^u + c_s^u\frac{\beta_n^2(1-\alpha)}{1-(1-\alpha)\beta_n} + \frac{\beta_n}{1-(1-\alpha)\beta_n}\frac{\kappa}{q(\theta_s)}\underbrace{\left(\frac{1}{1-\beta_u} - (1-\alpha)\beta_n\gamma(1-\delta)\right)}_{>0}$$

Since b_s and θ_s are increasing, we have that $w_s^u(g^{u*})$ and $w_s^n(g^{u*})$ are both increasing in s and that

$$w_s^u(g^{u*}) - w_s^n(g^{u*}) = D - \left(c_s^u\frac{\beta_n^2(1-\alpha)}{1-(1-\alpha)\beta_n} + \frac{\beta_n}{1-(1-\alpha)\beta_n}\frac{\kappa}{q(\theta_s)}\left(\frac{1}{1-\beta_u} - (1-\alpha)\beta_n\gamma(1-\delta)\right)\right).$$

so that the union wage gap $w_s^u(g^{u*}) - w_s^n(g^{u*})$ is decreasing in s.

We now turn to the nonunion firm. This firm hires according to g_s^{n*}. From Proposition 1, we know that

$$w_s^n(g^{n*}) = c_s^n + \frac{\beta_n}{1-\beta_n}\frac{\kappa}{q(\theta_s)}.$$

Furthermore, from Equation 10, we have that $w_s^u(g^{n*}) = c_s^n + D'$ where D' is a constant that does not depend on s. Therefore, the union wage gap is given by

$$w_s^u(g^{n*}) - w_s^n(g^{n*}) = D' - \frac{\beta_n}{1-\beta_n}\frac{\kappa}{q(\theta_s)}.$$

Since θ_s increases with s, the union wage gap decreases with s, which completes the proof. ■

Proof of Proposition 3. We need to compare the value of a firm under the two optimal distributions of workers g^{u*} and g^{n*}, for a given vector c_s. From Lemma 1, we know that we can compare

$$\pi(g^{i*}) - \kappa(1-(1-\delta)\gamma)\sum_{s=1}^{S}\frac{g_s^{i*}}{q(\theta_s)}$$

Using Equations 11 and 14 together with Equation 20 and after simplification we can now compare

$$(A\Gamma_i)^{\frac{1}{1-\alpha}}\,\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left(\sum_{s=1}^{S} z_s\left(\frac{z_s}{\mathrm{MC}_s^i}\right)^{\sigma-1}\right)^{\frac{\alpha}{(1-\alpha)(\sigma-1)}}$$

for $i \in \{u, n\}$. A few simplifications yield the result. ■

Proof of Proposition 4. With simple algebra, we find that

$$\begin{aligned} w_s^u(g^{n*}) - w_s^n(g^{n*}) = &\frac{\beta_u}{\sum_{k=1}^{S} g_k^{n*}}\left(F(g^{n*}) - \sum_{k=1}^{S} g_k^{n*}\left(c_k - \gamma(1-\delta)\frac{\kappa}{q(\theta_k)}\right)\right) \\ &- \left(\frac{\beta_n}{1-\beta_n}\mathrm{MC}_s^n + \beta_n\gamma(1-\delta)\frac{\kappa}{q(\theta_s)} - \beta_n c_s\right) \end{aligned}$$

The second term does not depend on α. We can expand the first term by using the value of g^{n*} given by Equation 20:

$$\frac{\beta_u}{\sum_k \left(\frac{z_k}{\mathrm{MC}_k^n}\right)^\sigma}\left(\frac{1-\beta_n(1-\alpha)}{(1-\beta_n)\alpha}\left(\sum_{k=1}^S z_k\left(\frac{z_k}{\mathrm{MC}_k^n}\right)^{\sigma-1}\right)-\sum_{k=1}^S\left(\frac{z_k}{\mathrm{MC}_k^n}\right)^\sigma\left(c_k-\gamma(1-\delta)\frac{\kappa}{q(\theta_k)}\right)\right)$$

The first result of the lemma follows directly by taking the derivative of this last equation with respect to α.

For the second result, we consider

$$\frac{d}{d\alpha}\left(\frac{1}{\sum_{s=1}^S g_s^{n*}}\sum_s\frac{g_s^{n*}}{1+\exp(-\rho(w_s^u(g^{n*})-w_s^n(g^{n*})))}\right).$$

Using Equation 20 we find that this last quantity is equal to

$$\frac{d}{d\alpha}\left(\frac{1}{\sum_{s=1}^S\left(\frac{z_s}{\mathrm{MC}_s}\right)^\sigma}\sum_{s=1}^S\left(\frac{z_s}{\mathrm{MC}_s^n}\right)^\sigma\frac{1}{1+\exp(-\rho(w_s^u(g^{n*})-w_s^n(g^{n*})))}\right)$$
$$=\frac{1}{\sum_{s=1}^S\left(\frac{z_s}{\mathrm{MC}_s^n}\right)^\sigma}\sum_{s=1}^S\left(\frac{z_s}{\mathrm{MC}_s^n}\right)^\sigma\frac{\rho\exp(-\rho(w_s^u(g^{n*})-w_s^n(g^{n*})))}{(1+\exp(-\rho(w_s^u(g^{n*})-w_s^n(g^{n*}))))^2}\frac{d\left(w_s^u(g^{n*})-w_s^n(g^{n*})\right)}{d\alpha}<0$$

which completes the proof. ■

Proof of Lemma 4. Assume first that the equilibrium schedules c_1 and c_2 are identical and denote that schedule by c. This result will be shown later in the lemma. We can write the problem of firm $j\in\{1,2\}$ as

$$\max_g \mathcal{J}(A_j, w(A_j,g), g)$$

such that

$$w(A_j,g)=\begin{cases}w^u(A_j,g) & \text{if } V(A_j,g)>0\\ w^n(A_j,g) & \text{if } V(A_j,g)\le 0\end{cases}$$

where w^u is the union wage function, w^n is the nonunion wage function and V is the excess number of workers for unionization.

The proof proceeds by showing that if g_1 solves the FOC of firm 1 than

$$g_2=\left(\frac{A_2}{A_1}\right)^{\frac{1}{1-\alpha}}g_1$$

solves the FOC of firm 2.

We therefore start with the FOC of firm 1 given by Equation 21 and Equation 22. First, notice that

$$\frac{A_1}{(g_{1,s})^{1/\sigma}}\left(\sum_{k=1}^S z_k g_{1,k}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\sigma(1-\alpha)}{\sigma-1}}=\frac{A_1\left(\frac{A_1}{A_2}\right)^{\frac{1-\sigma(1-\alpha)}{\sigma(1-\alpha)}}}{\left(\left(\frac{A_1}{A_2}\right)^{\frac{1}{1-\alpha}}g_2\right)^{1/\sigma}}\left(\sum_{k=1}^S z_k g_{2,k}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\sigma(1-\alpha)}{\sigma-1}}=\frac{A_2}{(g_{2,s})^{1/\sigma}}\left(\sum_{k=1}^S z_k g_{2,k}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\sigma(1-\alpha)}{\sigma-1}}.$$

This also implies that $w^n(A_1, g_1) = w^n(A_2, g_2)$. It is also straightforward to show that $F(A_1, g_1)/n_1 = F(A_2, g_2)/n_2$ such that $w^u(A_1, g_1) = w^u(A_2, g_2)$. We have so far shown that the left-hand side and the first term on the right-hand side of Equation 21 are the same, which completes the proof if firm 1 is unconstrained ($\lambda^1_{\text{LM}} = 0$). In which case, firm 2 is also unconstrained.

We now consider the derivatives in Equation 22. Notice that, for any $s' \neq s$, we have

$$g_{1,s'} \frac{\partial w^n_{s'}(A_1, g_1)}{\partial g_{1,s}} = \frac{\alpha\beta}{1-\beta(1-\alpha)} \frac{1-\sigma(1-\alpha)}{\sigma} A_1 \left(\sum_{k=1}^{S} z_k g_{1,k}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\alpha\sigma-2\sigma+2}{\sigma-1}} z_s g_{1,s}^{-1/\sigma} z_{s'} g_{1,s'}^{\frac{\sigma-1}{\sigma}}$$

$$= \frac{\alpha\beta}{1-\beta(1-\alpha)} \frac{1-\sigma(1-\alpha)}{\sigma} A_2 \left(\sum_{k=1}^{S} z_k g_{2,k}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\alpha\sigma-2\sigma+2}{\sigma-1}} z_s g_{2,s}^{-1/\sigma} z_{s'} g_{2,s'}^{\frac{\sigma-1}{\sigma}} = g_{2,s'} \frac{\partial w^n_{s'}(A_2, g_2)}{\partial g_{2,s}}.$$

Similarly,

$$g_{1,s} \frac{\partial w^n_{s}(A_1, g_1)}{\partial g_{1,s}} = \frac{\alpha\beta z_s A_1}{1-\beta(1-\alpha)} \left(-\frac{1}{\sigma} \left(\sum_{k=1}^{S} z_k g_{1,k}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\alpha\sigma-\sigma+1}{\sigma-1}} g_{1,s}^{-\frac{1}{\sigma}} + g_{1,s}^{\frac{\sigma-2}{\sigma}} \frac{1-\sigma(1-\alpha)}{\sigma} \left(\sum_{k=1}^{S} z_k g_{1,k}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\alpha\sigma-2\sigma+2}{\sigma-1}} z_s \right)$$

$$= g_{2,s} \frac{\partial w^n_{s}(A_2, g_2)}{\partial g_{2,s}}.$$

Similar computations yield that, for any $s' \in \mathcal{S}$

$$g_{1,s'} \frac{\partial w^u_{s'}(A_1, g_1)}{\partial g_{1,s}} = g_{2,s'} \frac{\partial w^u_{s'}(A_2, g_2)}{\partial g_{2,s}}.$$

Combining these results, it follows that $V(A_1, g_1) = V(A_2, g_2)$ and that

$$\frac{\partial V(A_1, g_1)}{\partial g_{1,s}} = \frac{\partial V(A_2, g_2)}{\partial g_{2,s}}$$

for all s. This completes the proof since, in the case in which firm 1 is constrained, there exist a $\lambda^2_{\text{LM}} = \lambda^1_{\text{LM}} \geq 0$ such that g_2 solves the problem of firm 2 and $V(A_2, g_2) = 0$. Notice that firm 2 is also constrained. Notice also that since the two firms have the same union status and are paying the same wages, we find $c_1 = c_2$ for every s which justifies our initial assumption. ■

B Parameters for the simulations in partial equilibrium

This appendix contains the parameters used for the partial equilibrium simulations of Figure 3 and 4 as well as for Table 1.

C Data appendix

Job destruction probability δ

I use the data available on Robert Shimer's webiste (Shimer, 2012). Specifically, I rounded the 2005.5 entry of the Employment Exit Probability dataset to $\delta = 0.027$. Note that the series provided by Shimer are quarterly averages of monthly transition probabilities.

Parameter	Interpretation	Value
S	Number of skills	20
ρ	Shape of voting preference	50
δ	Probability of job destruction	0.05
γ	Discount rate	0.95
κ	Cost of posting a vacancy	3
β_n	Bargaining power of individual workers	1/2
β_u	Bargaining power of union workers	1/2
η	Parameter of matching function	1
c_s	Outside option of workers	Linear from 1 to 5
θ_s	Labor market tightness	Linear from 1 to 10
A	Firm's total factor productivity	1000
α	Firm's return to scale parameter	0.7
z_s	Skill intensity	$1/S$
σ	Elasticity of substitution	1

Table 5: Parameters for the simulations

Elasticity of the matching function η

To estimate η, I use data from the Job Opening and Labor Turnover Survey (JOLTS) for 2005 together with the probability of job finding from Shimer (2012). I take the yearly average of the job finding probability for 2005 (its value is 0.4136) and find the aggregate labor market tightness by using the vacancies and job searchers numbers provided by the JOLTS. I take the average over all months in 2005 of the seasonally adjusted non-farm job opening series together with the seasonally adjusted number of job searchers for part-time and full-time jobs. I find η by solving the equation $0.4136^{\eta} = \theta^{\eta}(1+\theta^{\eta})^{-1}$. The solution is $\eta = 1.33$.

CPS and BEA data

The data about individuals is coming from the Merged Outgoing Rotation Groups of the 2005 Current Population Survey (CPS) as it is made available by the National Bureau of Economic Research (NBER). I clean the sample by removing agricultural workers and individuals with hourly wage higher than \$100 or lower than \$5. I also remove individuals younger than 16 or older than 65 years old or those who are out of the labor force. Finally, I remove public sector workers. Industry data comes from the Bureau of Economic Analysis (BEA) Annual Industry Accounts as made available on the BEA's website.

Labor shares $\widehat{\mathbf{LS}}_n$ and $\widehat{\mathbf{LS}}_u$

I merge the data from the BEA and the CPS to compute a measure of the labor shares for union and nonunion firms. For each industry in the BEA dataset, I divide total workers compensation by value added to get an estimate of the labor share in that industry. I then associate each worker in the CPS sample with the labor share of the industry in which he is currently working. I then average this variable separately over all union and nonunion workers and find a labor share of 0.597 for union firms and of 0.613 for nonunion firms.

Fraction of workers voting in favor of unionization by skill bin

I use the statistical analysis provided by Farber and Saks (1980). They use a dataset covering 29 union votes that took place between 1972 and September 1973 in 29 establishments. The establishments were located in Illinois, Indiana, Iowa, Missouri and Kentucky, and in manufacturing, transportation, wholesale trade, retail trade and services. The elections involved a total of 2788 workers. I used the regression results presented in their Table 2 and then use their Equation 9 to find the probability of voting for the union per skill bin. I set all coefficients to their estimated value and set all variables for which there is no equivalent in the model at their mean. I set all coefficient that are not statistically significant at the 95% level to zero.[37] Explicitly,

$$\text{Prob(Worker } s \text{ votes for union)} = \Phi((1+\alpha_1 T_{1s} + \alpha_2 T_{2s})(\gamma_0 + \gamma_1 \text{DEV}_s + \gamma_2 \text{RDET}_s \tag{27}$$

$$+\gamma_3 \text{FIMP}_s + \gamma_4 \text{PRO}_s + \delta X_s) + \gamma_5 \text{DIFF}_s + \gamma_6 (\text{DIFF}_s \times \text{DS}_s)) \tag{28}$$

where Φ is the CDF of a standard normal random variable.

The variable of interest to us is DEV_s which is defined as[38]

$$\text{DEV}_s = \frac{W_s - \bar{W}}{\sigma}$$

where W_s is the wage of workers of skill s, $\bar{W}$ is the average wage in the firm they work at and σ is the standard deviation of wages in the firm.

Plugging in the values of their Table 1 and Table 2 into Equation 27, we get

$$\text{Prob(Worker } s \text{ votes for union)} = \Phi\left(-0.0865 - 0.161 \times \text{DEV}_s - 0.042\right).$$

This last equation gives us an estimate of how agents would vote if they were in the real economy. I use this equation together with the CPS data to construct Figures 6c and 6f.

D Algorithm to calibrate the model

This appendix contains the detailed algorithm used for the calibration. The values of some parameters taken directly from the data or the literature are shown in Table 2.

The labor market tightness vector θ is identified directly by using the steady-state Equation 17 and the unemployment rate for each skill group provided by the CPS. The outside option vector b is also fixed to its data value, as the main text explains. The calibration is constructed in such a way that the vectors θ and b provided by the data are equilibrium object in the calibrated economy.

The calibration algorithm for a given vector of parameters ξ is:

1. Find the the technologies:
   (a) Guess two technology schedules z^u and z^n for the firms
   (b) Find the schedules c^u and c^n:
      i. Guess the net outside option schedules c^u and c^n for the firms

[37] I also tried setting the coefficients that are not statistically significant to their estimated value. This has minimal impact on the final equation.

[38] The equation in Farber and Saks (1980) also includes a shift coefficient denoted λ. Since this coefficient is not statistically significant in their analysis I set it to zero.

ii. Given these schedules compute the hiring decision of the union and nonunion firms and compute the wage schedules.
iii. Update c^u and c^n using Equation 3.
iv. Measure the distance between the new c^u and c^n and the old ones.
v. If there is convergence stop. Else use the new schedules and go back to step 1(b)ii.

(c) Use lemma 4 to set A_n and A_u to match the total size of the nonunion and union sectors. Verify if the hiring decisions of the firms coincide with the distributions of workers in the data. If not, use Equation 21 to back out new guesses z^u and z^n and go back to step 1b.

2. See if any firm wants to deviate from the tentative equilibrium (for instance, the union firm has a higher profit if it fights the union). If so, discard the current parameter vector ξ.[39]

3. Use Equation 17 to reverse engineer the skill distribution $\hat{N}$ that makes θ an equilibrium outcome.

4. Use Equation 2 to reverse engineer the outside option schedule $\hat{b^0}$ that makes b an equilibrium outcome.

5. Compute the loss function at point ξ.

Notice that this algorithm exploits the fact that the decisions of the firms do not depend on N and b^0 directly but only through θ and b.

E Algorithm to solve the general equilibrium

This appendix contains the algorithm to find a general equilibrium given the vector of parameters $(\beta_n, \beta_u, \delta, \gamma, \kappa, \rho, \sigma, \eta, b^0, N)$ as well as the firms' technologies (z_j, A_j, α_j) for $j \in \{u, n\}$ and the union status of firms (given by the policy experiment).

The algorithm to find the aggregate variables θ and b that sustain this equilibrium is:

1. Make an initial guess on the aggregate variables: θ, b.

2. Find the schedules c^u and c^n:

   (a) Guess the net outside option schedules c^u and c^n.
   (b) Given these schedules compute the hiring decision of the firms and compute the wage schedules.
   (c) Update c^u and c^n using Equation 3.
   (d) Measure the distance between the new c^u and c^n and the old ones.
   (e) If there is convergence stop. Else use the new schedules and go back to step 2b.

3. From the wages and the distribution of hired workers, compute the new (θ, b).

4. Measure the distance between the old and the new (θ, b). If there is convergence, an equilibrium has been found. If not, go back to step 2 using the new (θ, b) as the current guess.

For all simulations, I tried different starting points for (θ, b) and the equilibrium seems to be unique.

[39] When considering deviations, firms take the equilibrium net outside option schedule c_j as given. Firms are also forced to pay wages higher than b, otherwise the workers would quit.