COWLES COMMISSION
FOR RESEARCH IN ECONOMICS

REPORT OF FIFTH ANNUAL
RESEARCH CONFERENCE ON
ECONOMICS AND STATISTICS
HELD AT COLORADO SPRINGS

July 3 to 28, 1939

THE UNIVERSITY OF CHICAGO

COWLES COMMISSION
FOR RESEARCH IN ECONOMICS

THE COWLES COMMISSION FOR RESEARCH IN ECONOMICS is a not-for-profit corporation, founded in 1932 for the purpose of conducting and encouraging investigations into economic problems. A function of the COMMISSION is to issue from time to time papers and monographs of an econometric or economic-statistical nature without, however, assuming responsibility for theories or opinions expressed therein. The COMMISSION is affiliated with the ECONOMETRIC SOCIETY, an international society for the advancement of economic theory in its relation to statistics and mathematics.

THE UNIVERSITY OF CHICAGO

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THE 1939 CONFERENCE

The Cowles Commission Fifth Annual Research Conference on Economics and Statistics was held at Colorado College, Colorado Springs, from Monday, July 3, to Friday, July 28, 1939. Two lectures, followed by discussion periods, were scheduled each morning, the sessions beginning at 9:30 and closing about 12:30. Saturdays, Sundays, and July 4 were left free for recreation. Several afternoon and evening discussion groups were organized by the participants to continue the consideration of points brought up in the morning lectures.

The scientific program was supplemented by recreation, which included a picnic supper for visitors in attendance on July 3. Teas were held at the residences of Dr. and Mrs. Charles H. Boissevain, Professor and Mrs. Harold T. Davis, and Mr. and Mrs. Dickson H. Leavens, and the members of the Cowles Commission staff were entertained at tea at the Broadmoor Hotel by the out-of-town guests. Swimming, horseback riding, golf, tennis, and other sports were enjoyed by many, and a number of hikes and picnics were arranged. Points of interest visited on afternoon or week-end trips included the summits of Pikes Peak and Cheyenne Mountain, the Royal Gorge of the Arkansas River, Rocky Mountain National Park (Estes Park), Cripple Creek gold camp, and the play festival at the old mining town of Central City.

The number of people participating in part or all of the sessions of the Conference was 234, including 112 from out of town.

PARTICIPANTS FROM OUT OF TOWN

BISHOP, MISS LOLA, Junior College, Bay City, Mich.
BORDSEN, MR. AND MRS. F. S., Watertown, Wis.

BOURNE, PROFESSOR AND MRS. RICHARD M., University of Wyoming, Laramie, Wyo.
BOWMAN, DR. MARY JEAN, Iowa State College, Ames, Iowa.
BRATT, PROFESSOR AND MRS. ELMER C., Lehigh University, Bethlehem, Penn.
BURGESS, DR. ROBERT W., Western Electric Company, 195 Broadway, New York, N. Y.
CAJORI, PROFESSOR AND MRS. FLORIAN, University of Pennsylvania, Philadelphia, Penn.
CANNING, PROFESSOR JOHN B., Stanford University, Calif.
COURT, MR. AND MRS. ANDREW T., Automobile Manufacturers Association, Detroit, Mich.
DAVIES, MISS, Fredonia, Kan.
DAVIES, MR. M. B., D.&R.G.W.R.R., Denver, Colo.
DAVIS, PROFESSOR AND MRS. HAROLD T., Northwestern University, Evanston, Ill.
DERKSEN, DR. J. B. D., Netherlands Central Bureau of Statistics, The Hague, The Netherlands.
DITTEL, MR. J. WALTER, 225 South Windomere, Dallas, Texas.
DODD, PROFESSOR AND MRS. EDWARD L., University of Texas, Austin, Texas.
DRESCH, DR. AND MRS. FRANCIS, University of California, Berkeley, Calif.
EZEKIEL, DR. AND MRS. MORDECAI, United States Department of Agriculture, Washington, D.C.
FERGER, DR. AND MRS. WIRTH F., Office of the United States Secretary of Agriculture, Washington, D.C.
FLEMING, MR. B. T., 820 South Classen, Norman, Okla.
FRIEDRICH, DR. WILLIAM G., Institute of Applied Econometrics, 405 Lexington Ave., New York, N.Y.
GARNSEY, PROFESSOR AND MRS. MORRIS E., University of Colorado, Boulder, Colo.
HAAVELMO, DR. TRYGVE, University Institute of Economics, Oslo, Norway.
HAMILTON, PROFESSOR THOMAS R., Agricultural and Mechanical College of Texas, College Station, Texas.
HARVILL, PROFESSOR RICHARD A., University of Arizona, Tucson, Ariz.
HAVAS, DR. EUGENE, 25 Broadway, New York, N.Y.
HAYES, MR. J. P., 2015 Merchandise Mart, Chicago, Ill.
HEDRICK, PROVOST E. R., University of California, Los Angeles, Calif.
HOAGLUND, MISS JEANNETTE, Northwestern University, Evanston, Ill.
INCE, MR. RICHARD W., Seasongood & Haas, 63 Wall St., New York, N.Y.
JACOBSON, MISS KATHARINE, 530 West 114th St., New York, N.Y.
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LAURSEN, MR. AND MRS. SVEND, L. E. Braunswej 19, Charlottenland, Denmark.
LEHMAN, MR. D. A., Goshen, Ind.

LERNER, MR. AND MRS. ABBA P., *London School of Economics*, London, England.
LEVIN, MR. JOSEPH H., 402 Eckhart Hall, University of Chicago, Chicago, Ill.
MCFADDEN, MR. J. R., Kansas Wesleyan University, Lawrence, Kan.
MCLAUGHLIN, PROFESSOR AND MRS. GLENN E., University of Pittsburgh, Pittsburgh, Penn.
MANLY, PROFESSOR L. F., Tufts College, Mass.
MARSCHAK, DR. AND MRS. JAKOB, University of Oxford, Oxford, England.
MOORE, DR. THOMAS W., United States Naval Academy, Annapolis, Md.
MORRISON, MR. PAUL L., 8 South Michigan Avenue, Chicago, Ill.
OPHEIM, MRS. C. O., Pierre, S. Dak.
PAULEY, MISS PAMELLA, Northwestern University, Evanston, Ill.
PAYDON, MR. JOSEPH FINDLAY, Northwestern University, Evanston, Ill.
PEITZSCH, MR. FREDERICK C., Northwestern University, Evanston, Ill.
PRIBRAM, DR. KARL, Social Security Board, Washington, D.C.
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ROOT, MRS. BURDETT, Wichita, Kan.
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ROSS, MR. EMERSON, Work Projects Administration, 1734 New York Avenue, Washington, D.C.
RULON, PROFESSOR P. J., Harvard University, Cambridge, Mass.
SARLE, DR. CHARLES F., United States Bureau of Agricultural Economics, Washington, D.C.
SCHWALLER, MR. HENRY, 1439 Tennessee, Lawrence, Kan.
SCOTT, PROFESSOR E. C., Sweet Briar College, Sweet Briar, Va.
SCOTT, MISS FRANCES V., Harvard Graduate School of Business Administration, Boston, Mass.
SCOVILLE, MR. AND MRS. JOHN W., Chrysler Corporation, Detroit, Mich.
SCOVILLE, MR. J., JR., Detroit, Mich.
SECRIST, PROFESSOR AND MRS. HORACE, Northwestern University, Chicago, Ill.
SHEPHERD, PROFESSOR AND MRS. GEOFFREY, Iowa State College, Ames, Iowa.
SIMPSON, MR. F. R., School of Business Administration, University of Minnesota, Minneapolis, Minn.
SMIT, DR. CAREL JAN, Ottawa Beach, Mich.
SMITH, MISS ELEANOR D., American Historical Review, 535 West 114th St., New York, N.Y.
STAFFORD, MR. AND MRS. JACK, The Victoria University of Manchester, Manchester, England.
STARK, MR. RICHARD HARLAN, Northwestern University, Evanston, Ill.
STURGES, MR. ALEXANDER, United States Bureau of Labor Statistics, Washington, D.C.

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TINTNER, PROFESSOR GERHARD, Iowa State College, Ames, Iowa.
TRIFFIN, DR. ROBERT, Harvard University, Cambridge, Mass.
TRUESDELL, DR. LEON E., United States Bureau of the Census, Washington, D.C.
WALD, DR. ABRAHAM, Columbia University, New York, N.Y.
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WISE, DR. OLIVER C., 2705 High St., Pueblo, Colo.
WOLFOWITZ, MR. AND MRS. J., 253 Cumberland Street, Brooklyn, N. Y.
WORK, MISS LAURA, National Institute of Public Affairs, 400 Investment Building, Washington, D.C.
WORKING, PROFESSOR ELMER J., University of Illinois, Urbana, Ill.
WORKING, PROFESSOR HOLBROOK, Stanford University, Calif.
YNTEMA, PROFESSOR THEODORE O., University of Chicago, Chicago, Ill.

HISTORY OF THE CONFERENCES

The Annual Research Conferences on Economics and Statistics, held at Colorado Springs under the auspices of the Cowles Commission for Research in Economics, originated in a series of informal meetings during the summer of 1935 following the sessions of the Econometric Society at Colorado Springs on June 22-24 of that year. At these gatherings various papers were presented and discussed by economists who remained in the vicinity. The meetings were so successful that it was decided to continue them in subsequent years. The following table gives a statistical record:

	1935	1936	1937	1938	1939
Number of conference lectures	8	32	40	38	38
Number of lecturers	7	20	27	27	32
Average attendance per lecture	15	27	36	47	51
Participants from out of town	5	55	67	93	112
" local	20	24	41	99	124
" total	25	79	108	192	234

AVAILABILITY OF COMPLETE TEXTS

In this report abstracts are given of all lectures presented at the Conference. In addition, as indicated by footnotes (pp. 23, 50, 70, and 90), complete versions of four lectures have been issued by the Cowles Commission through the plan of Auxiliary Publication, and are available as American Documentation Institute documents in the form of microfilm (images 1 inch high on standard 35 mm. safety photographic film comfortably usable in reading machines now widely available) or photoprints (readable without optical aid). For detailed information write American Documentation Institute, care Offices of Science Service, 2101 Constitution Avenue, Washington, D.C.

It is hoped that in future years it will be possible to make the complete texts of a large proportion of the Conference lectures available in this way to those interested.

PLANS FOR 1940 CONFERENCE

Although the offices of the Cowles Commission were moved to the University of Chicago in September, 1939, it is planned to hold the Sixth Annual Conference at Colorado Springs, from Monday, July 1, to Friday, July 26, 1940. Rooms and board for those attending the Conference will be available in dormitories of Colorado College at about $40 for the four weeks or $11 per week. Approximate summer round-trip railroad rates to Colorado Springs, first class including Pullman, are: from the Atlantic Coast, $110; from Chicago, $51; from the Pacific Coast, $72. Coach rates are about two-thirds of these.

There is no charge for attendance at the Conference. All serious students are welcome. Those who are interested should notify the Cowles Commission in order that they may receive in the Spring a preliminary announcement of the program.

ABSTRACTS OF CONFERENCE LECTURES

Monday, July 3—Problems in the Theory of Business Cycles, HAROLD T. DAVIS, Professor of Mathematics, Northwestern University, and Research Associate, Cowles Commission.

The problem of economic time series is concerned with the measurement and the explanation in terms of basic economic causes of four component variances into which the total variance, σ^2, of the time series can be resolved. This total variance may be written

$$\sigma^2 = \sigma_T^2 + \sigma_H^2 + \sigma_D^2 + \sigma_E^2 ,$$

where σ_T^2 is the variance of the trend, σ_H^2 is the variance of the harmonic element, σ_D^2 the variance of the disruptive, or inflationary, element, and σ_E^2 the variance of the erratic element. Thus, for the Cowles Commission index of industrial stock prices over the period from 1897 to 1913 a variance of 78.5 was divided into a trend variance of 42.8, a harmonic variance of 26.9, an erratic variance of 8.8, and a disruptive variance of zero. In the subsequent period, most of the variance was concentrated in the disruptive element. Evidence for the most part supports the conclusion that the correlations between the four characteristics are sufficiently small to be neglected; hence their variances may be added as in the above formula.

As a general observation, the trends of production series appear to be logistic, or semi-logistic, in character. This is partly due to the fact that production depends upon population growth, which is logistic. Moreover, new industries seem to grow in a biological manner, as is illustrated by the development of the automobile industry in the United States. The most disruptive economic phenomena appear to be associated with economies which have reached the upper asymptote of their general production curves, as is illustrated by the very

similar events in the Spanish economy of the seventeenth century and in the economy of the United States since 1929.

Price series, in contrast to production series, have large psychic elements. As a consequence, harmonic and disruptive features are their most interesting characteristics. One exception is to be noted in the great advance of prices in the sixteenth century, which resulted in the establishment of our present levels. This advance was logistic in character, since it was occasioned by the production of treasure in the New World and its importation into Europe.

Price series often supply excellent measures of events which reflect the psychic behavior of individuals. One of the best statistical estimates of the magnitude and influence of a war upon an economy is found in the behavior of the commodity price index, which is peculiarly sensitive to war psychology. The following ratio is suggested as a coefficient of war intensity:

$$\eta(t) = \frac{P(t) - P_0}{P_0},$$

where P_0 is the secular average of prices in the period prior to the beginning of hostilities, and $P(t)$ the price at any subsequent period t. During the American Civil War, the wholesale price index rose from approximately 62 at the beginning to 132 at the end. Hence the total intensity of the war was 1.12. For the World War the coefficient was 1.23 in so far as the economy of the United States was concerned.

The disruptive variance may be discussed from the point of view of the equation of exchange, $MV = PT$, where M is money (usually bank deposits), V is the velocity of money (usually bank debits divided by average circulating deposits), P is the general price index, and T the trade index. The trade index is highly, though not perfectly, correlated with total annual income. The ratio between MV and total annual income is around 10, although in inflationary periods it may be as much as twice this figure. The sensitivity of price and velocity to interest rates is indicated. Money tension, as measured in terms of interest rates, accompanied the sharp decline of prices in 1920; a moderate money tension, accompanied by a great drop in velocity, characterized the panic in stock prices

in 1929. Money tension and velocity have declined simultaneously since 1930 and the velocity of bank deposits is now about equal to the velocity of currency. Apparently all the advantages of our banking system have been swallowed up in this phenomenal behavior of the velocity factor. The fallacy of attempting to create a price or an industrial expansion, by means of a monetary inflation based on large bond issues spent for the most part to create nonproductive public works, is shown in this apparently causally related decline in interest rates and the velocity of bank deposits.

The variation in the concentration of income is related to various phases of the business cycle, since total annual income correlates very highly with the index of industrial production. One of the lowest observed concentrations in the United States, as measured by income-tax returns, occurred in 1920. A normal concentration, defined in terms of Pareto's distribution function, was restored the next year by the spectacular drop in prices. The highest observed concentration occurred in 1929, but this again became normal after the drop in stock prices of that year. Since 1932 the concentration has been steadily declining from its normal value.

A dynamic explanation of the disruptive phenomena in price series, such as that observed in the stock inflation of 1929, was attempted by means of the concept of resonance as defined for mechanical systems. If one maximizes the integral of profit,

$$\pi = \int_{t_0}^{t_1} [pu - Q(u)]\, dt\,,$$

where p is the price of some basic industrial good like steel, u is the demand, and $Q(u)$ is the cost of manufacturing and marketing u units, then the phenomenon of a harmonic variation in prices can be accounted for if u depends not only upon a linear function of price and its derivative, but also upon an impressed force, which correlates highly with the index of industrial production. If this impressed force is itself harmonic in character and if its period coincides nearly with the natural period of the system as defined by the coefficients of the price and cost functions, then resonance will be set up in prices, which can statistically account for the observed phe-

nomena. Supporting evidence for this explanation is found in the statistics of the production of steel.

Monday, July 3—The General Problem of Durable-Goods Demand, CHARLES F. ROOS, Director of Research, Institute of Applied Econometrics.

The variables that are useful in demand studies are of two types, those associated with the consumers of the commodity and those describing the commodity itself.

It often happens that several variables can be combined in a rational way into a single new variable. In this way it is possible to reduce the number of constants needed to secure a good fit. This is particularly important when the number of observations is limited.

The equations given for passenger-car and residential-building demand provide no clew as to how cyclical variations in demand originate. But they do show how cyclical variation or disequilibrium among the independent variables such as income or price is transmitted to automobile sales, how small-amplitude cycles in income, etc. and certain differences in phase between the independent factors produce large-amplitude swings in sales. The process described below by which these cycles become magnified is generally characteristic of durable goods. It is extracted from papers by the author and Victor S. von Szeliski.

The factors, consumer income, direct personal taxes, and necessary living costs, which affect automobile sales, go up and down with the business cycle. The last two lag behind consumer income. Thus, when consumer income flattens out after a cyclical rise, taxes and living costs continue to advance and supernumerary income, or the difference between income and living costs and taxes, may thus actually decline; it is pinched or squeezed between consumer income on the one hand and taxes and living costs on the other. The year 1937 furnished a very clear illustration of this squeezing process,

which was accentuated by a raising of tax rates, a sharp upward acceleration of the price level due mostly to labor-union activities, and monetary deflation by the Federal Reserve Board, leading first to a levelling out of income and then a decline.

An analogous sequence of events occurs at the bottom of the cycle: supernumerary income turns up more sharply than consumer income because taxes and living costs generally continue to fall after the turn. Variation in demand for consumers' durable goods may therefore be explained in part by the phase difference between incomes and prices.

Our automobile study also makes use of the difference between current stocks, C, and the maximum stocks sustainable, M:

$$S = M - C .$$

It therefore has affinities with the "acceleration principle" associated with the names of Aftalion, J. M. Clark, Mitchell, Kuznets, Harrod, Pigou, and others. Both formulations describe the same mechanism, and, in broad outline, the view as to the causal relationships are the same.[1] The acceleration principle, however, is generally translated into mathematics as a function of the primary demand and its time derivative:[2]

$$S = f(I, \frac{dI}{dt}).$$

Our formulation is believed to have the advantage that it introduces explicitly the variable which, in the verbal statements of the acceleration theory, is assigned due importance, viz., consumer stocks. It makes use of a varying maximum-ownership level as a separate concept, and avoids the auxiliary explanations necessary for applying the acceleration principle in cases where there is unused capacity. Also it avoids the assumption of a constant ratio between primary demand and the stock of (consumers') capital goods.[3]

For capital-goods-demand calculation the income factor is

[1] See, for example, Haberler, *Prosperity and Depression*, pp. 80-88.
[2] For an example of the latter, see *Dynamics of Automobile Demand*, p. 12.
[3] Qualifications discussed by Haberler, *Prosperity and Depression*, p. 91.

again supernumerary income, but its operation is obscured by two more potent factors, i.e., the commitments of distributors of consumers' goods, both transient and durable, and the percentage of unused productive capacity. With these factors alone, that is, without the interest factor which has been made a central pivot of the theory of durable-goods demand, an excellent representation of capital-goods demand can be obtained. This indicates that economists must revise a considerable body of generally accepted theory, in which the interest factor plays the predominant role.

A rise in interest rates occurs at the top of the boom and a decline at or before the bottom of the depression. But, in each case, a condition leading to a change in the direction of business has already been set up. In fact, the interest rate can be anticipated from the business factors and certain monetary conditions *and not* the business from the interest rates.

The boom is characterized chiefly by a temporary squeeze in productive capacity, a resultant of a rapid increase in consumers'-goods orders induced by chance causes, and this seems to set various forces of adjustment into motion. While inventory in consumers'-goods lines is being worked down, production of consumers' goods declines, capacity is excessive, and capital-goods orders decline materially. To explain completely the business cycle one still needs to explain consumers'-goods orders. The durable-goods portion we have explained elsewhere.[4] At present the other seems to be simply an inventory cycle brought about by a succession of random events and influenced particularly by the rate of change of money supply.

[4] *Dynamics of Automobile Demand* and "Factors Influencing Residential Building," Chapter VI, *Dynamic Economics*.

Wednesday, July 5—A Simplified Economic System with Dynamic Elements, FRANCIS W. DRESCH, Instructor of Mathematics, University of California.

The economic model considered here is a simple generalization of a simplified system due to G. C. Evans. In it we are

concerned with an idealized economy involving two industries, one producing capital goods, the other producing consumption goods. Each of these two processes of production is assumed to employ three factors of production, namely, circulating capital, fixed capital, and labor. The amount of product turned out per year by each industry is assumed to be a function of the amount of factors used.

Thus, if the consumption industry uses up y units of capital and w units of labor annually and requires an average of u units of capital to be tied up in its production process, θ, the amount of consumption goods so produced, is assumed to be given as a function of y, w, and u. Similarly, the annual output, φ, of capital goods is assumed to be given as a function of x, z, and v, where these variables represent the amounts of circulating capital, of labor, and of fixed capital, respectively, involved in the process of producing capital goods.

If p_θ is the price of consumption goods, p_φ the price of capital goods (either fixed or circulating), and p_L the price (hourly wage rate) of labor, the net profits of the consumption-goods and the capital-goods industries respectively will be

$$\pi_\theta = p_\theta\,\theta(y,w,u) - p_\varphi\,y - p_L\,w - \lambda\,p_\varphi\,u$$

and

$$\pi_\varphi = p_\varphi\,\varphi(x,z,v) - p_\varphi\,x - p_L\,z - \lambda\,p_\varphi\,v\,,$$

where λ is the rate of interest.

Assuming strict competition, we have six additional equations obtained by setting

$$\frac{\partial\,\pi_\theta}{\partial\,y} = \frac{\partial\,\pi_\theta}{\partial\,w} = \frac{\partial\,\pi_\theta}{\partial\,u} = \frac{\partial\,\pi_\varphi}{\partial\,x} = \frac{\partial\,\pi_\varphi}{\partial\,z} = \frac{\partial\,\pi_\varphi}{\partial\,v} = 0\,,$$

which will permit us to solve for all the variables of the system in terms of λ and the price ratios $\mu = p_\varphi/p_\theta$ and $\nu = p_L/p_\theta$. By expressing the various quantity variables as functions of λ, μ, and ν in this fashion, we may construct offer functions for both capital and consumption goods as well as demand functions for both labor and capital, the last expressible in the form:

$$x(\lambda,\mu,\nu) + y(\lambda,\mu,\nu) + \frac{d\,u(\lambda,\mu,\nu)}{dt} + \frac{d\,v(\lambda,\mu,\nu)}{dt}\,.$$

If we append some assumed or empirically determined relations giving the demand function for consumption goods (the propensity to consume) and the offer function for labor, we may obtain three new equations by equating supply and demand functions for consumption goods, capital goods, and labor. In particular, if these new relations involve the prices only as ratios and not independently, this system of equations will determine the variables λ, μ, ν and the quantity variables, θ, φ, y, w, u, x, z, v. Whenever this solution can be obtained explicitly, each of the above variables will be given as a function of time. The dynamic character of the model comes, of course, from the fact that, upon equating supply and demand for capital, we obtain an equation involving the derivatives of u and v with respect to time. Omission of the variables, λ, u, v, throughout the system, reduces it to the static model of Evans.

In its generalized form, the model corresponds to a simple theory of economic dynamics based upon a slight extension of the usual notion of a general equilibrium. It suggests immediately further generalization by means of which it might be made into a systematic basis for comparative discussion of economic theories.

An even more interesting possibility lies in the fact that such a model might very well serve as an inductive device. If one assumes the a priori functions of the system, namely, the production functions and the offer and demand functions, to be of some simple form, general solutions are possible in a wide variety of cases. In such a solution each variable of the system will be expressed as a function of time in terms of the constants defining the assumed functions. By choosing the values of these constants in such a way that the theoretical time curves for these variables correspond as well as possible (in some sense or other) to the observed time series for these variables, one can "fit" such a model to the actual economy.

For example, if one describes the technical situation by taking the logarithm of the output for each industry as a linear combination of the logarithms of the amount of factors employed (i.e., approximates the technical functions by a logarithmic fit of first degree), takes the offer function for labor as a curve of constant elasticity, and finally assumes that a

certain fraction of the national income is spent on consumption goods, such a general solution is possible. In this case each variable has a solution of the form

$$\alpha(t+\beta)^{\eta}$$

where the constant β is the same for all variables. By fitting curves of this form to all the relevant time series simultaneously, one can estimate the constants entering the theoretical formulation of the system.

In this approach, intercorrelations, which cause so much trouble in any attempt to find static aggregate demand curves or production functions, are explicitly taken into account at the outset. Thus, in the empirical part of the work, no static relations need to be determined directly. One tries instead to find an appropriate model and to fit it as a unit to the actual economy employing only available time series as raw data for the process.

Wednesday, July 5—Some Economic Problems of the South, RICHARD A. HARVILL, Associate Professor of Economics, University of Arizona.

In the early days of the Republic, the South held leadership politically, possessed a prosperous economy, and was the leader in many social reforms. About 1830 the South began rapidly to lose its position of leadership.

The extent of the South's decline can be explained in terms of its large dependence upon a one-sided obsolete agrarian economy. With a much higher rate of human reproduction than the rest of the country, this region has an excess of people and suffers an exodus of many of its most able and best-trained persons that is not compensated by movements into the South. The illiteracy rate is higher and the educational facilities are more inadequate in relation to needs in the South than in the rest of the country although a larger percentage of its disproportionately small income is spent on education.

From its land devoted to agricultural production, the Southern people have long derived most of their income. The combination of soil-depleting crops and of the tenancy system in its worst forms has exhausted the soil and used up, without replenishment, other agricultural resources. The impact of political and economic forces operating on a national or world scale has destroyed much of the foreign market for Southern cotton. Of particular importance to the South are: the interaction of tariff and other foreign-trade policies of the United States and foreign countries; the rise of dictators motivated by self-sufficiency aims; the reduction in demand, due to the recent world-wide depression; and the increased production of raw cotton outside the United States. Trends in foreign countries toward increased cotton production are not new and will probably not be reversed even if freer international economic relations are re-established. Threatened by the introduction and wide use of substitutes for cotton and by a declining population, the annual domestic consumption of cotton will level off at an estimated figure of seven million bales. At the same time, changes in agricultural technology threaten to reduce the labor and land requirements necessary to the production of cotton and other agricultural products. In the face of this complexity of realized and potential changes, the South faces a reorientation of its agricultural economy.

Industrialization is increasing in the South. For forty years this region has more than held its own in the nation's industrial development. By 1937, the South had exceeded its 1929 level of manufacturing as measured by wage earners employed and by value of products, while the nation as a whole was considerably below the 1929 level. Lack of capital in the South is being overcome by rapid movement of capital from other regions to take advantage of the bountiful supply of low-wage labor and other resources.

Although the production of finer quality is increasing, Southern manufacturing is still confined largely to the initial processing of products of the farm, mine, and forests, and to the production of coarser finished materials. This requires a high proportion of unskilled labor so abundantly afforded by the high natural rate of population increase and the declining state of agriculture.

Labor unionization, of little consequence in the past, is becoming more important in the industrial South. Fear is being expressed that the rate of industrialization will be slowed down by higher wages due to unionization and national wages-and-hours legislation. High freight rates, allegedly discriminatory against the South, have probably retarded the rate of industrialization. Some of the Southern states are pursuing the objectionable policy of offering direct subsidies of considerable proportions to new industries. With investment opportunities in foreign countries virtually closed to American capital and with opportunities apparently reduced in other regions of the United States, the South should continue to experience an inflow of capital seeking profitable investment.

The economy of the South is being improved in some respects, and can be further improved by efforts of the Southern people. It is evident, however, that the South's problems are in large part the Nation's problems and await national policies that will permit more complete economic recovery.

Thursday, July 6—International Price Disparities and Their Measurement,* MORRIS E. GARNSEY, Assistant Professor of Economics, University of Colorado.

Attempts to describe international price relationships in quantitative terms are based, in most cases, on the assumption that under normal conditions the economic intercourse between national economies takes place under the assumptions of general-equilibrium theory. International price disparity thus may be defined as "the condition prevailing after a shift in (international) price relations to which there has not been complete adaptation among elements of the economic system at large."[1] In other words price disparity means price dis-

* For copy of complete lecture, order Document No. 1220 from American Documentation Institute, care Offices of Science Service, 2101 Constitution Avenue, Washington, D.C., remitting 36 cents per copy in microfilm or $1.80 for photoprints readable without optical aid.

[1] Mills, F. C., *Prices in Recession and Recovery*, p. 34.

equilibrium, and the term carries with it the implication of concomitant disequilibrium in other phases of the international trade relations between countries.

Probably the most significant effort to measure international price disparities has been made by the Institute for Economic Research at the University of Louvain. Their work was stimulated originally by the disturbances of the Belgian economy which followed the depreciation of sterling after 1931, and their findings served as the basis for the Belgian devaluation in 1935.

Three steps, each representing a progressive refinement in technique, may be distinguished in the attempt to measure international price disparities. The first step as developed by the Institute was to construct an index of purchasing-power disparity. This index is a ratio between an index of wholesale prices in country *A* and a similar index in country *B* adjusted for exchange fluctuations. The index indicates in a general way the extent to which prices and exchange rates in the two countries have diverged from a position of assumed equilibrium.

The second step was to calculate another ratio using cost-of-living indices instead of wholesale prices. This index focuses attention on the fact that fluctuations in the prices of international commodities create internal disparities between "international" and "domestic" prices. The ratio between the indices for two countries indicates the relative adaptation of internal, inflexible prices in each country to external changes in prices and exchange rates.

The third step involved the calculation of ratios of disparity for selected internal price groups, such as wholesale and retail prices or industrial and agricultural prices. Such indices, when interpreted in the light of the movements of the individual series from which they are calculated, can be used for the important purpose of distinguishing between the effects of external forces and internal forces (largely cyclical in type) in producing price disparities.

Indices of the three types described have yielded significant results when applied by the Institute to the Belgian experience. It is recognized, however, that further refinement of the indices is very desirable. A current proposal for such

refinement suggests the construction of a special-purpose index to take the place of a wholesale price index in the original index of purchasing-power disparity. The new index will be restricted to the prices of internationally traded commodities in order to indicate the presence of price disparity at the moment of its development. Such an index should provide a more sensitive and accurate measure of price disparity than is possible with the general index of wholesale prices.

Finally, it should be emphasized that each step in the measurement of international price disparities requires a careful evaluation of the numerous qualitative aspects of the given situation as well as the interpretation of the indices of disparity. Nevertheless, it is believed that the indices of disparity have proved to be invaluable aids in the quantitative description of international price disparities.

Thursday, July 6—The Fitting of Straight Lines if Both Variables Are Subject to Error, ABRAHAM WALD, Lecturer in Economics, Columbia University, and Research Fellow, Cowles Commission.

The problem of fitting straight lines if both variables x and y are subject to error has been treated by many authors.[1] It is a common feature of the methods given in recent years that the fitted straight line cannot be determined without a priori assumptions (independent of the observations) regarding the weights of the errors in the variables x and y. It is shown in this paper that under certain conditions (1) the fitted straight line can be determined without making a priori assumptions regarding the standard deviations of the errors, (2) the standard deviation of the errors can be well estimated by means of the observed values of x and y. The precision of the estimation increases with the number of observations. We could get the exact values if the number of observations were

[1] The most recent publication on this subject which came to my attention is R. G. D. Allen's interesting paper, "The Assumption of Linear Regression," *Economica*, May, 1939.

infinite, provided that a certain expression does not approach zero.

1. *Formulation of the problem.* Consider two sets of random variables:

$$x_1, \cdots, x_N ; y_1, \cdots, y_N .$$

Denote the expected value $E(x_i)$ of x_i by X_i and the expected value $E(y_i)$ of y_i by Y_i $(i=1, \cdots, N)$. We shall call X_i the true value of x_i, Y_i the true value of y_i, $x_i - X_i = \varepsilon_i$ the error in the ith term of the x-set, and $y_i - Y_i = \eta_i$ the error in the ith term of the y-set.

The following assumptions will be made:

I. The random variables $\varepsilon_1, \cdots, \varepsilon_N$ each have the same distribution and they are uncorrelated.

II. The random variables $\eta_1, \cdots, \eta_N$ each have the same distribution and they are uncorrelated.

III. ε_i and η_j are uncorrelated $(i, j=1, \cdots, N)$.

IV. A linear relation $Y_i = \alpha X_i + \beta$ $(i=1, \cdots, N)$ holds between the true values.

Denote by ε a random variable having the same distribution as ε_i and by η a random variable having the same distribution as η_i $(i=1, \cdots, N)$. We know only two sets of observations: $x'_1, \cdots, x'_N ; y'_1, \cdots, y'_N$; where x'_i denotes the observed value of x_i and y'_i denotes the observed value of y_i. We know neither the true values $X_1, \cdots, X_N ; Y_1, \cdots, Y_N$; nor the coefficients α and β of the linear relation. We have to estimate by means of the observations (1) the values of α and β, (2) the standard deviation σ_ε of ε, and (3) the standard deviation σ_η of η.

We shall use the notations x_i and y_i also for their observed values since no confusion can arise by it.

2. *Consistent estimates of* α, β, σ_ε, and σ_η. Let us assume that N is even $(N=2m)$, and introduce the notations:

$$\bar{x}_1 = \frac{x_1 + \cdots + x_m}{m}, \qquad \bar{x}_2 = \frac{x_{m+1} + \cdots + x_N}{m} ;$$

$$\bar{y}_1 = \frac{y_1 + \cdots + y_m}{m}, \qquad \bar{y}_2 = \frac{y_{m+1} + \cdots + y_N}{m},$$

$$\bar{x} = \frac{\bar{x}_1 + \bar{x}_2}{2}, \qquad \bar{y} = \frac{\bar{y}_1 + \bar{y}_2}{2},$$

$$s_x^2 = \frac{\sum_{i=1}^{m} (x_i - \bar{x}_1)^2 + \sum_{j=m+1}^{N} (x_j - \bar{x}_2)^2}{N},$$

$$s_y^2 = \frac{\sum_{i=1}^{m} (y_i - \bar{y}_1)^2 + \sum_{j=m+1}^{N} (y_j - \bar{y}_2)^2}{N},$$

$$s_{xy} = \frac{\sum_{i=1}^{m} (x_i - \bar{x}_1)(y_i - \bar{y}_1) + \sum_{j=m+1}^{N} (x_j - \bar{x}_2)(y_j - \bar{y}_2)}{N}.$$

If $\bar{x}_1 - \bar{x}_2$ does not approach zero with increasing N, the following expressions, a, b, s_ε^2, and s_η^2, are consistent estimates[2] of the unknown parameters, α, β, σ_ε^2, σ_η^2:

$$a = \frac{\bar{y}_1 - \bar{y}_2}{\bar{x}_1 - \bar{x}_2}, \qquad b = \bar{y} - a\bar{x},$$

$$s_\varepsilon^2 = \left[s_x^2 - \frac{s_{xy}}{a} \right] \frac{N}{N-2}, \qquad s_\eta^2 = \left[s_y^2 - a\, s_{xy} \right] \frac{N}{N-2}.$$

I observe that the expressions for s_ε^2 and s_η^2 are closely related to those which can be deduced from Allen's equations (4), *loc. cit.*, p. 194. I found these expressions independently of Allen's investigations.

3. *Test of significance and confidence region for* α *and* β *jointly.* It has been proved that the expression

$$(*) \qquad \frac{N-2}{2} \cdot \frac{(\bar{x}_1 - \bar{x}_2)^2 (a - \alpha)^2 + (\bar{y} - \alpha\bar{x} - \beta)^2}{s_y^2 + \alpha^2 s_x^2 - 2\alpha s_{xy}}$$

has the F (analysis of variance) distribution, the numerator having 2 and the denominator N—2 degrees of freedom. The F distribution is tabulated in Snedecor's book, *Calculation and Interpretation of Analysis of Variance,* Collegiate Press,

[2] An estimate is said to be consistent if it converges stochastically to the corresponding parameter value with $N \to \infty$.

Ames, Iowa, 1934. Denoting by F_0 the critical value of F corresponding to the chosen probability level, the deviations of a and b from the assumed population values α and β are significant if the value of the expression (*) is equal to or exceeds F_0. Representing a pair of values α, β by the point in the plane with the co-ordinates α, β, the confidence region R of α and β is the set of points α, β, for which the value (*) is less than F_0. It is shown that R is the interior of an ellipse.

4. *Remark.* The greater $|\bar{x}_1 - \bar{x}_2|$ the more efficient is the estimate $a = (\bar{y}_1 - \bar{y}_2)/(\bar{x}_1 - \bar{x}_2)$. The expression $|\bar{x}_1 - \bar{x}_2|$ becomes a maximum if we order the observations so that $x_1 \leq x_2 \leq \cdots \leq x_N$. The probability distribution of the expression (*) remains the same for any order of the observations which is defined independently of $\varepsilon_1, \cdots \varepsilon_N$; $\eta_1, \cdots, \eta_N$. Denote by x the median of $x_1, \cdots, x_N$ and by c a positive value for which the probability that $|\varepsilon| > c$ is negligibly small. It has been shown that if the number of observations contained in $[x-c, x+c]$ is very small, the distribution of (*) remains practically unchanged if we order the observations so that $x_1 \leq x_2 \leq \cdots \leq x_N$. Hence, in such cases, we get the best estimate for α if we order the observations so that $x_1 \leq x_2 \leq \cdots \leq x_N$ and then form the expression $a = (\bar{y}_1 - \bar{y}_2)/(\bar{x}_1 - \bar{x}_2)$.

Friday, July 7—Some Economic Aspects of Public Spending, CORRINGTON GILL, Assistant Administrator, Work Projects Administration.*

Until recently public spending was regarded as a form of recovery policy by both its advocates and opponents. At the present time there is much discussion of the need for a long-term public investment program, to fill the gaps left by stagnating private investment. Public spending is therefore seen

* Mr. Gill was prevented at a late date from attending the Conference and his paper was read by Mr. Emerson Ross, Director of the Division of Statistics of the Work Projects Administration, under whose general direction much of the research dealing with public spending was done.

by some as a means to overcome the depression phase of the cycle and by others as a desire to correct a structural maladjustment in the economy.

Looking back we can observe that public spending was resorted to because other measures of recovery policy failed. Laissez faire and deflation as efforts to halt the decline in income and employment proved disastrous both here and abroad. The early 1930's showed also the weakness of banking policy: low interest rates and easy money conditions were not enough to induce borrowing by business men. Devaluation of the dollar and attempts to raise wages likewise proved insufficient as the foundations of recovery. Hence the government turned to the policy of deficit financing, first with emphasis on public works and later with primary emphasis on work relief. In contrast with previous efforts to promote recovery, deficit financing, or spending, possesses a positive character. It permits a government to inject purchasing power into the stream of income. Unlike dollar devaluation or banking policy it does not merely establish the conditions for an increase of total spending.

The theory and policy of public spending have undergone not inconsiderable alteration since 1933. In this process of change the chief guide has been experience. Thus at first funds were provided in the National Industrial Recovery Act for a public-works program the purpose of which was to prime the pump of recovery. The theory behind this policy of a single large-scale emergency expenditure could be said to combine the multiplier and acceleration principle. Although such a program does stimulate certain parts of the economic system it became apparent that public works by themselves could not materially reduce unemployment in a short period of time and that public-works spending at the outset was very slow. Consequently emphasis was shifted to Federal work-relief programs, with their greater capacity for providing employment and with their greater flexibility. With this there developed the theory of compensatory fiscal policy, under which the Federal Government would spend in excess of its receipts during depressed years and receive in excess of its spending during prosperous years.

Since the recession of 1937 which took place before eco-

nomic activity had reached high levels, a further development in the theory of spending has come about, which sees in spending the vehicle for a long-run program of public investment. This school of thought rejects both emergency spending and compensatory fiscal policy as ill-conceived because they rest on the assumption that a recovery in private business can be generated. The advocates of the public-investment policy believe it probable that no important field for private investment will open up. Thus they believe that, if a high national income and low unemployment are to be achieved, large-scale public investment will prove necessary as a permanent policy.

We cannot be sure which of the major premises of the two schools of thought on spending possesses the greater element of truth. Only over a long period of time can the validity of these conflicting analyses be established. A practical program of public spending does not of necessity depend on the answer to a theoretical problem. What is of importance is the necessity for continued spending when the problem is viewed in the light of the existence of mass unemployment.

Friday, July 7—The Measurement of Tax Shifting: A Case Study in Economics and Law, WIRTH F. FERGER, Associate Economic Adviser, Office of the Secretary, United States Department of Agriculture.

Although the literature of public finance abounds with discussions of the theory of tax shifting and incidence, practically all of this discussion is focused on prediction rather than on explanations of the methods of measuring the shifting and incidence of a specific tax in its actual historical setting. The latter phase is important, however, not only for the scientific verification of theory, but to answer the practical needs of administrative law. An instance is found in the "windfall-tax" and processing-tax-refund provisions of the Revenue Act of 1936. The refund section provides that processing taxes illegally collected under the Agricultural Adjustment Act should

be refunded to a processor only on condition that he demonstrate that he himself bore their burden and did not shift it to others. The "windfall-tax" section likewise provides for an 80-per-cent income tax on the "unjust-enrichment" income resulting from the nonpayment of these taxes, under court injunctions, in case their burden was nevertheless shifted to others.

The procedure laid down by the refund provisions for determining, prima facie, the absorption or shifting of processing-tax burdens by the processor is the comparison of the processing margin in the tax-payment period with the margin during a comparative period just before the imposition and after the abolition of the taxes, the margin being defined as the difference between the cost of raw material (including the tax) and the current sales value of finished products. Provision is made for adjustments of the margin difference thus shown for demonstrated changes in factors other than the tax imposition, such as increased unit expenses of production, which are reflected in the margin.

Since the economic aspects of these refund provisions have not yet been tested in the courts on their merits, this paper examines the legal precedents which bear on their probable validity, and the economic precedents which bear on the reasonableness of the methods prescribed for measuring tax-burden shifting. On the legal side it is shown that, although the courts clearly recognize the economic fact of the indirect nature of certain types of taxes, there has never been presented for judicial determination the question of the proper methods of measuring the extent of this shifting, and no precedent exists either in support or in contradiction of the methods of measurement prescribed. Although the Jefferson Electric case upheld the legal principle that tax refunds could be limited to cases where the taxpayer could defend the equity of such refunds, the nature of the proof of shifting the sales tax there involved depended upon the technical methods used in computing the tax, and can not be applied generally in the measurement of the shifting of the economic burden of taxes. Other cases are examined to determine their status as precedent.

On the economic side it is shown that there is widespread precedent for employing the margin technique in the study of appropriate problems of price analysis, and that for the taxes under consideration, and for the limited problem of measuring the shifting rather than the final incidence of the taxes, the use of the margin technique is supported by the practice of both businessmen and economists. It is shown that margins defined exactly as in the statute are used as a common tool of business analysis and control in the industries here concerned as well as in others. It is reasonable, therefore, to expect judicial recognition of the validity of the methods of measuring tax-burden shifting employed by the statute.

Monday, July 10—Competitive Norms in Durable-Goods Industries, THEODORE O. YNTEMA, Professor of Economics, University of Chicago, and Director of Research, Cowles Commission.

The attempt to specify minimum standards of price competition for an industry raises perplexing questions to which economists have not given adequate attention. Too commonly the characteristics of perfect price competition have been offered as practical criteria for judgment, without recognition that these criteria are limits, not norms, and without due consideration of the complications created by business cycles.

These problems are most important in the durable-goods industries. In a major depression the production of durable goods declines much more than that of nondurable goods and, although the difference in price behavior is not so uniform, the prices of durable goods do, on the average, decline less in depression than the prices of nondurable goods. With the interest in administered and inflexible prices growing out of the observed inverse correlation between cyclical declines in production and price of commodities, it is natural to focus attention on the commodities with the least flexible prices, and thus on durable goods, and to inquire what degree of approach to perfect price competition is desirable and practicable in this field.

In dealing with these questions it is necessary to consider the underlying characteristics of demand and cost.

Almost without exception, the demand functions for new or additional durable goods are subject to great cyclical fluctuations, a phenomenon of major significance in determining competitive norms. In the second place, the demand for durable goods is dependent on expectations of future prices and on expectations of future incomes and costs; its timing, therefore, is largely speculative. Beyond these two characteristics it is difficult to generalize although there is some evidence that the demand is rather inelastic, especially for producers' durable goods.

On the cost side the production of durable goods usually involves considerable overhead. This tends to be high where much machinery is employed and where the operations are vertically integrated. On short-run incremental costs our information is fragmentary, but most of it indicates that for individual plants, or even for rather large aggregates of plants under a single management, incremental costs remain nearly constant over a very wide range of output and do not rise substantially until operations approach the physical limits of capacity. In most cases, therefore, the short-run average costs exceed the incremental costs except when plants are being operated practically at capacity.

The problem of selecting criteria for optimum conditions of competition in the durable-goods industries is complicated by these characteristics of demand and cost. If perfect price competition prevailed, producers would be operating at heavy losses during most of the cycle and would recoup these losses by extremely high prices and very great profits during the peak of prosperity. In major depressions the efficient as well as the marginal firms would fail to survive unless they had accumulated extraordinarily large cash balances. Under conditions of perfect price competition prices would conform to short-run incremental costs, remaining fairly stable in depression at levels which would fail to cover much more than out-of-pocket expenditures. In prosperity, when production pressed on capacity and incremental costs rose, price would rise correspondingly. At this stage there would be little pos-

sibility of increased production with existing facilities, and a further rise in the prosperity phase of the cycle would have to occur, if at all, in the nondurable goods field. Actually, of course, these characteristics of perfect price competition are not only unrealistic but they would not be tolerated. The cut-throat struggle in depression and the sharp increases in prices and the huge profits in prosperity would be the object of attack by politicians, labor-union leaders, and economists.

If perfect price competition is not a satisfactory standard, how far should it be modified? At what point are the recognized advantages of price competition in stimulating full use of capacity overbalanced by the depressing effects of declining prices, heavy losses, and high business mortality? Are full use of capacity in prosperity, low selling costs, and moderate profits over the cycle evidence of sufficient price competition? Is much damping of the business cycle possible by inducing more price competition in business, or must we look to deficit (and surplus) government financing or even to direct control of the desirability of money as a store of value?

Monday, July 10—The Possibilities and Limitations of Objective Sampling in Strengthening Agricultural Statistics, CHARLES F. SARLE, Principal Agricultural Economist, Division of Crop and Livestock Estimates, United States Bureau of Agricultural Economics.

The type of agricultural statistics demanded at any one time usually has reflected the nature of current agricultural problems in much the same way as current economic theories have reflected business and economic conditions of the times. The demand on the part of farmers for up-to-the-day crop reports issued by a neutral or nontrade agency became articulate at about the same time as the demand for agricultural colleges and agricultural experiment stations. In fact, the first official estimates of agricultural production were made at about the same time the state agricultural colleges were being organized.

The economic and social maladjustments in American agriculture growing out of the World War greatly increased the demand for more comprehensive and accurate agricultural statistics, especially those relating to the economic aspects of agriculture. The administration of the direct "action" programs of the Government in recent years has necessitated the use of available statistics, sometimes regardless of their obvious limitations for such uses.

The tremendous increase in the demand for more and better agricultural statistics has not been accompanied by adequate provision for improving methods and facilities for supplying them.

Agricultural statistics can be greatly strengthened by the development of objective methods for observing, sampling, and estimating. Considerable resourcefulness has been used in developing more objective methods for observing agricultural phenomena. The individual farm sample replaced (during the 1920's) the old judgment or opinion sample in estimating year-to-year change in crop acreages and kinds of livestock. The crop-meter has been developed for measuring the frontages of various crops along selected routes in obtaining a more objective sample for estimating changes in crop acreages. The assistance of the rural mail carriers since 1924 has been invaluable in securing a much larger and more representative sample of individual farms for both acreage and livestock estimates.

The method of sampling individual farms by mail, however, is basically unsound for data are obtained only from those farmers who are willing to reply to a mail questionnaire. Reports from many types and kinds of farms cannot be secured. An *enumerative sample* is needed each year, taken by paid enumerators and designed on the basis of sound statistical principles. An enumerative sample could be made representative of all kinds of farms in a given area.

Objective methods for estimating yield per acre at harvest time are being developed for several of the major crops—head samples for wheat immediately preceding harvest, boll counts and measurements for cotton, ear counts and measurements for corn. More objective methods of forecasting yield per acre that involve the use of both plant characteristics and

weather observations are being used experimentally along with the "condition" of crop as reported by crop correspondents. Research is under way in basic meteorology that eventually should make possible longer-range forecasts of weather.

The use of *split* schedules and estimation from a representative sample for items not of primary importance would permit a broadening of the scope of Census statistics relating to agriculture by from 50 to 200 per cent or more. A *rotating* partial census in intercensal years in commercial fruit and truck areas would greatly strengthen the statistics relating to such crops.

The greatest limitation in the case of more objective methods of sampling lies in higher cost as compared with sampling opinions by mail.

Tuesday, July 11—Forecasting Commodity Prices One to Ten Years Ahead, ROBERT W. BURGESS, Statistician, Western Electric Company.

As part of the routine of manufacturing-company operations, it is necessary to make forecasts of the prices which will be paid for commodities used as materials and supplies. For some purposes, these forecasts of commodity prices should aim to cover a relatively long period, say 5 to 10 years. Since it has proved to be impossible to predict the fluctuations of the business cycle with any accuracy more than a year in advance, the problem which it is practicable to try to solve has been recognized to be the estimation of the "normalized" prices for the period, that is to say, for each commodity the average price during the period which would be associated with normal levels of business activity.

In comment on the general nature of this problem, it should be noted that, as in the case of many other business problems, the essential point is to make an estimate which will usually be near enough to the outcome to serve as a helpful guide to business decisions. The solution must be based on

things as they are, all things considered, including facts, rumors, and general trends suspected or proved, in so far as they are known or can be known with an economic expenditure of time and effort. It is not essential, however, to find a solution in which a theoretician will find aesthetic satisfaction nor to contribute to the development of economic principles.

The general procedure which has been used for each commodity may be summarized as follows:

1. Determine the present normalized price by a review of prices in recent years and the factors which are believed to have tended to push prices up and down and of the market situation of the commodity. Scatter diagrams of average annual price compared with average industrial activity have been found useful.

2. Determine a general point of view as to the effect on all commodity prices or major groups of commodity prices of the chief economic, financial, and industrial trends which are expected to continue into the next ten years.

3. Estimate the probable effect of these general economic trends on the price of the particular commodity in view of its present and probable future market situation.

Under Point 2, the following conclusions underlay the estimates actually made:

1. General commodity-price inflation, because of monetary or financial factors, is not believed to be probable, although tight situations affecting particular commodities or particular types of skilled labor may very well lead to considerable, though perhaps temporary, increases in the corresponding prices and wage rates. The primary reason for this point of view is that excess stocks, excess productive facilities, and large numbers of unemployed workers are expected to have more influence on prices in this country than our fabulous stock of gold and possibilities of credit expansion.

2. A gradual increase in productivity per man-hour of factory labor may be expected to continue in the future as in the past thirty years.

3. The long-run effects on prices of government or cartel regulation of production, shipment, and marketing depend on the intelligent appreciation of economic factors by the regulatory agencies. Analysis of past and current decisions of

these agencies throws light on what may be expected in the future.

Tuesday, July 11—Budgetary Principles, ABBA P. LERNER, Assistant Lecturer in Statistics, London School of Economics.

It is necessary for a department of the government or of any other organization to limit its expenditures to its budgeted allowance because that is the mechanism whereby the activities of the department are co-ordinated into the general plan of the whole organization. It is better for an individual (or group of individuals within a society) to have a greater income and/or a smaller expenditure (other things being equal) because that makes him richer rather than poorer. It is advisable for an individual (or group) not to get too much into debt (by spending more than his income) because that increases the danger of bankruptcy.

These considerations do not apply to the sovereign government of a well-established modern state. All talk of the necessity, propriety, or usefulness of a government balancing its budget, whether weekly, annually, cyclically, or over any other period is nothing but an irrational transference to general public finance of principles that are appropriate only for departmental finance or private finance.

The government should spend money or subsidize expenditure by others wherever that is necessary to provide the effective demand for adequate employment or where a particular expenditure is in the public interest. It should tax away income wherever this is necessary to prevent too great an effective demand which would disorganize the economy through inflation or where a particular form of expenditure or income is considered to be socially undesirable. It should regulate its borrowing and lending of money entirely by the principle of keeping the rate of interest at a level that gives the ideal rate of profitable investment; hoarding or destroying the money

that it receives and dishoarding or printing any money it may need in carrying out the policy.

Although in this formulation of fiscal principles there is no room for any principle of attempting to make total revenue from taxation equal to total expenditure over any period of time, the idea of "balance" is not eliminated. Rather it is refined and preserved in the sense of a nice adjustment between expenditure and taxation so as to equalize their marginal significance for social welfare. The balance is only *marginal*; so that an equality between total revenue and total expenditure, if it should be reached (and there may be a tendency for it to be reached over the very long period) would be only an accidental result of policies framed in the light of other principles.

Recent tendencies—as in Sweden—to have longer-term and vaguer principles of budget balancing represent a compromise between the view here expressed and the traditional view that grew out of the necessity of limiting expenditures out of the king's (private) purse to the revenue that he was able to raise. While such compromise may be politically necessary, the economic scientist must free himself completely from the budget-balancing superstitions.

Wednesday, July 12—Vertical Horizons, ANDREW T. COURT, Automobile Manufacturers Association.

The new frontiers of economic expansion are "vertical." They lie not "extensively" West or South or North, but "intensively" upward into a fuller use of the national manufacturing and merchandising ingenuity.

In particular, extensive market saturation has radically influenced the social implications of durable-consumer-goods engineering, manufacturing, and marketing. Today practically every family has some sort of cooking stove, the majority of families that need a motor car have a vehicle of sorts, and most families have a roof over their heads. Therefore, any

material increase in our national standard of living must include substantial improvement in the average quality and condition of these and of other durable goods in use. Extensive saturation must evolve toward intensive satisfaction of consumer needs.

No conceivable reduction in cost of manufacture and distribution provides a satisfactory answer as far as supplying the major durable consumer goods for the least privileged third is concerned. This would still be true even at higher average levels of national real income. Subsidies and "sharing" can only benefit favored groups at the expense of the less articulate majority.

The democratic solution is to be found in an expansion of the flow of sound, second-hand, durable consumer goods. An adequate supply of such merchandise in good condition and at reasonable prices is the natural result of vigorous, well-dramatized competition which attracts those who already own substantial stoves, cars, and houses to buy new and more satisfying models. The resulting "trade-ins," still in good operating condition, go to low-income families at less than any possible cost of manufacture for articles of equivalent usefulness.

Strenuous quality competition among manufacturers of durable consumer goods offers many social benefits which do not accrue from reduction in unit price alone.

1. It makes possible the most rapid and widely shared expansion of the national standard of living.

2. It stimulates the economic energies of consumers, minimizing problems arising from "oversaving."

3. It provides the only fully effective vehicle for sustained and vigorous competition within a mass-production industry.

4. It minimizes the labor dislocation resulting from technological progress.

5. It provides the frontiers so essential to full functioning of the economy.

Wednesday, July 12—The Government Looks at Our Economic Past and Future: A Review of *Patterns of Resource Use* Submitted by the Industrial Committee to the National Resources Committee, JOHN W. SCOVILLE, Statistician, Chrysler Corporation.

Patterns of Resource Use[1] divides the American economy into 81 segments and shows for most segments formulas for consumption and man-power. Numerous curves show for period 1920 to 1935 a comparison of actual consumption (or man-power) with data calculated from the formulas.

In the formula for passenger-car sales, introduction of a factor determined by number of cars at least seven years old appears to be an error. However, the effect of this error was neutralized by introducing a rapidly declining time factor. Most formulas in the report are vitiated by failing to show the effect of price on consumption. The passenger-car formula in 1938 gives an error of 1,200,000 cars. Calculated figure in 1938 was too high by 63 per cent.

The curves on gasoline consumption on page 59 do not agree with the formula on page 123. The formula for tire and tube consumption fails to follow the usual and logical procedure of separating sales into two parts, original equipment and replacement. Both factors in the tire formula are illogical. In electric light and power industry, total man-hours were correlated with output. Man-hours should have been separated into two parts—operations and new construction. Operating hours should have been correlated with the capacity of the plants, rather than with output.

Patterns of Resource Use estimates man-hours and consumption as national income rises from 50 to 100 billion dollars. The report estimates that with national income of 100 billion dollars automobile output would be 12,992,000 vehicles annually, which would result in nearly four cars to a family, which is of course fantastic. The report estimates gasoline consumption about 50 per cent less than amount required to operate the vehicles, and allows for too little crude to produce the reduced amount of gasoline. At 100 billion dollars income, the

[1] Preliminary edition for technical criticism. Washington, 1939. 149 pages.

report calls for five times as many trucks in use as in 1935, but shows an increase of only 44 per cent in truck drivers.

The fundamental fallacy in *Patterns* is the assumption that the kind and number of articles bought when the national income temporarily reaches a high figure would continue if the national income remained at the high figure. The result is that the table in *Patterns* which shows what would be produced at the income level of 100 billions overestimates the production of durable goods and underestimates the output of perishable goods and services.

Another feature of *Patterns* which is unusual is that it proposes to help the rich more than the poor. According to *Patterns,* when the national income rises from 60 billions to 100 billions, the number of people employed will rise from 43.4 millions to 60.1 millions. If these persons receive on an average $1200 a year for their work, salaries and wages would rise from 52 billions to 72.1 billions. Hence, dividends, rent and interest would rise from 8 billions to 27.9 billions. Thus, while salaries and wages would rise by only 40 per cent, the returns on capital would increase by about 249 per cent. This program illustrates the Biblical saying, "To him that hath shall be given."

By assuming that a rise in national income will produce little or no increase in employment in agriculture, in the Government, and in the professions, the conclusion is reached that full employment would give a consumer income of about 88 billion dollars. We challenge this conclusion. It is more likely that full employment would result in a national income of around 80 billions and since we must allow for sickness, time lost in changing jobs, and other factors, perhaps the maximum income we can secure, with hours limited by the wages-and-hours law, is around 72 to 75 billions. If my analysis is correct, what becomes of the theory that we can boost the national income to 90 or 100 billion dollars or even to 80 billions and get enough revenue from existing tax laws to meet the expenditures of the Federal Government? But will even a 72-billion income be possible until business men and investors have confidence that the system of free enterprise in this country will not be destroyed? We have come to the end of the trail. We have studied some of the complicated formulas and the multi-

tudinous charts assembled by Dr. Means and his co-workers to determine the national income that would put all of our unemployed to work only to find that *Patterns of Resource Use* gives the wrong answer. The economic assumptions and correlations of the Industrial Committee end in statistical frustration.

We cannot project into the future economic trends based on the records of the past and say with any assurance what people will make and buy in some future year when a certain hypothetical value is reached for the national income. Nor under our free economy is it necessary to forecast the direction of economic activity in future years. Production automatically flows into those goods and services which people vote for when they spend their dollars in the marts of trade. It is only a planned economy, an economic autocracy, a totalitarian state, which has any need for a chart to show how many people shall work in each industry and how much of each product or service shall be produced.

Thursday, July 13—The Adaptation of Index-Number Construction to Punched-Card Equipment, FRANCIS MCINTYRE, Research Associate, Cowles Commission, and Associate Professor of Econometrics, Colorado College (now at Stanford University).

In a recent issue of the *Journal of the American Statistical Association*,[1] the argument is advanced that the "best" index-number formula for the measurement of the prices of common stocks, with a view to reflecting the experience of investors in such securities, is basically the simple value ratio $\frac{\sum p_1 q_1}{\sum p_0 q_0}$, where p and q denote price and number of shares outstanding, and the subscripts refer to different time periods. Modifying this formula to contain all corrections for changes in the capital structure of the corporations whose issues are included in

[1] *The Problem of the Stock Price Index Number*, Vol. 33, No. 203, September, 1938, pp. 557-563.

the index, and also all corrections for the addition of a new company or withdrawal of an old one, the final stock price index employed was, for the period i,

$$P_i = \prod \frac{\sum^{i} p_i q_i - \sum^{i} s_i (q_i - q_{i-1})}{\sum^{i} p_{i-1} q_{i-1}}, \qquad (i = 1, 2, \cdots i).$$

In this expression, Π is the customary notation for multiplication ($\Pi\, a_i = a_1\, a_2 \cdots a_i$), the superscript above the summation sign indicates that the summation is taken over the list of stocks permitted to affect the index in period i, p and q have the above meanings and s_i denotes the selling price at which new shares were issued or old shares withdrawn during period i from the number of shares outstanding. As companion indexes of dividends and earnings, there were employed $Y = \frac{\sum dq}{\sum pq}$ $R = \frac{\sum eq}{\sum pq}$, $D = PY$, and $E = PR$. In these expressions Y represents yield, d dividend per share, R earnings-price ratio, e earnings per share, and D and E the indexes of dividends and earnings. The multiplier P is, of course, the corresponding price index.

One tabulating card was punched for each corporation for each time it appeared in the index. Thus for period i, p_i, q_i, d_i, e_i, s_i, and $(q_i - q_{i-1})$ were entered on the cards directly from the original data sheets. The quantities p_iq_i, $p_iq_i - s_i(q_i - q_{i-1})$, d_iq_i, and e_iq_i were all obtained automatically and punched in other portions of the card by use of the multiplying punch. It was even found possible to handle the case of a corporation withdrawn from the index without separate treatment simply by punching $q_i - q_{i-1}$ as the negative of q_{i-1}, that is, by treating q_i as equal to 0. This did not preclude obtaining total dividends and earnings (dq and eq) because the multiplying punch can be wired so as to obtain the multiplier from the second of two fields of a card if it finds all zeros in the first field.

In the actual construction of the five indexes P, Y, R, D, and E no computation is made from the individual cards but only from the total of the quantities p_iq_i, $p_iq_i - s_i(q_i - q_{i-1})$, d_iq_i, and e_iq_i for all companies for a single period.

These totals can be obtained automatically, and once they are obtained the calculating machine work necessary to secure the final index numbers required less than an hour for the five indexes over 40 periods.

Thursday, July 13—Statistical Testing of Dynamic Systems if the Series Observed are Shock Cumulants, TRYGVE HAAVELMO, Research Associate, The University Institute of Economics, Oslo, Norway.

The main hypotheses of the shock theory[1] are: (1) that the *free* time expansion of the variables studied would be smooth and damped—being solutions of a theoretical dynamic system—and (2) that this type of expansion is *not* that one actually observed because of external forces—"shocks"—which from time to time suddenly change the initial conditions of the system. Because of these shocks the observed series will be more or less irregular and *not* systematically damped. The introduction of shocks, therefore, represents a simple way of explaining difference between theory and observation.

Accepting the hypotheses of shock theory, the *inversion problem* arises, viz., the problem of deriving the theoretical (free) time expansion from the observed shock-maintained series.

Let $\lambda_\kappa (\kappa = 0, 1, 2, \cdots)$ be the theoretical movement of a certain variable (e.g., stock prices), where $\sum_{\kappa=0}^{\infty} \lambda_\kappa^2 =$ constant, and let this movement be affected by a nonautocorrelated series of shocks e_t with variance σ_e and expectation $= 0$ ($t =$ time) taking for simplicity only integer values. Instead of λ_t we would then observe

$$(1) \qquad w_t = C \sum_{\kappa=0}^{\infty} \lambda_\kappa \, e_{t-\kappa}$$

$= a$ shock cumulant with weight system λ_κ,

where C is a constant which may be put equal to 1 by adjusting the λ's.

[1] References are made to works on this subject by Yule, Slutzky, Frisch, and Wold.

Then the automoment of w_t

$$(2) \qquad M_\tau(w) = \frac{1}{N} \sum_{t=t_0}^{t_0+N} w_t \, w_{t+\tau}, \qquad |\tau| = 0, 1, 2, \cdots$$

converges stochastically towards $\sigma_e^2 \, M_\tau(\lambda)$ as N increases, i.e., w and λ show for large N approximately the same automoment curve (apart from a constant). And $M_\tau(w)$ is approximately $= M_{-\tau}(w)$.

Suppose it has been found theoretically that λ should satisfy a linear difference equation

$$(3) \qquad \lambda_\kappa + a_1 \lambda_{\kappa-1} + a_2 \lambda_{\kappa-2} + \cdots + a_h \lambda_{\kappa-h} \equiv 0 \qquad \begin{cases} \kappa \geq h \\ \text{and} \\ \lambda_\kappa = 0 \text{ when } \kappa < 0 . \end{cases}$$

Then w_t satisfies identically the stochastical difference equation

$$(4) \qquad w_t + a_1 w_{t-1} + a_2 w_{t-2} + \cdots + a_h w_{t-h} \equiv \lambda_0 e_t + (\lambda_0 a_1 + \lambda_1) e_{t-1} + \cdots + (\lambda_0 a_{h-1} + \cdots + \lambda_{h-1}) e_{t-h+1} .$$

The problem is to find the a's. If the h initial values of the λ's are so chosen as to make the second member of (4) equal to e_t the a's may be estimated from the system of linear equations

$$(5) \qquad \sum_{i=0}^{h} M_{h-j-i}(w) \cdot a_i = 0 , \qquad j = 0, 1, 2, \cdots h-1 , \quad a_0 = 1$$

which for large N comes to the same as minimizing the sum of e_t^2. If the other terms in the second member of (4) do *not* vanish the a's cannot in general be estimated by classical regression methods. If the hypothesis of shock maintenance is accepted as true we are therefore in general *not* justified in fitting a theoretical relation to the observed series by means of the least-squares method.

The scheme of erratic shocks might be replaced by a scheme with stochastical variation of the *coefficients*, the *observations* themselves being considered as the "true" elements.[2]

[2] T. Haavelmo, "The Method of Supplementary Confluent Relations . . . ," *Econometrica*, Vol. 6, July, 1938, pp. 204-205.

Let $e_i(t)$, $i = 1, 2, \cdots h$, be h nonautocorrelated time series with finite variances and expectations equal to zero and such that $E(e_i e_j) = 0$ when $i \neq j$. We might then start from the hypothesis

$$(6) \qquad w_t + [a_1 + e_1(t)]w_{t-1} + [a_2 + e_2(t)]w_{t-2} + \cdots + [a_h + e_h(t)]w_{t-h} \equiv 0$$

where w_t is an observed series and the a's are certain unknown constants. Then the system of equations (5) would give a consistent estimate of the a's. More efficient estimates may be obtained if we have some a priori knowledge about the variances of the e's.

Friday, July 14—The Implications of a Stable or Declining Population, LEON E. TRUESDELL, Chief Statistician for Population, United States Bureau of the Census.

There are two outstanding problems in the field of population in the United States today: The problem of the approaching cessation of population growth, and the problem of persistent unemployment. These two problems are closely interrelated, especially in that if the second could be solved we should worry far less over the first.

The history of our population growth may be summarized as follows: From 1790 to 1930 the population of the United States increased from 3,929,000 to 122,775,000. The rate of increase, which at first was about 35 per cent per decade, had declined to 15 per cent in the final decade, and will be about 8 per cent between 1930 and 1940, with a forecast of only about 4 per cent for the next ten-year period, and no increase at all after 1970.

The first century of our national history was a period of settlement, with the growth of occupied territory almost keeping step with the growth of population. From 1820 to 1930 was a period of industrialization and urbanization, during which a simple, almost self-sufficing agricultural economy

gave place to a market-production economy, with increasingly complex problems of distribution, exchange, and employment.

Throughout the period of industrialization the production per worker has increased steadily and continuously, as the result of improved methods and the increased use of machinery. In many lines the increase in productivity has been spectacular (5-fold or 10-fold in a few years), rather than by small annual increments.

Partly as a result of this increase in production, the business world has in recent decades become more and more concerned over markets; one might even say that it had become market-mad. Advertising (largely brand-competitive) has become a major industry, and the sales force is now the most important element in many producing establishments.

This widespread market-mindedness is one source of the current pessimistic interpretation of the approach of a stable or declining population. An increasing population, say the marketeers, means more customers; a rapidly increasing population means many more customers; while a declining population means fewer customers—and that means ruin.

They seem to forget the possibility that even a smaller number of customers with more purchasing power per capita may afford larger sales; and that increases in per-capita purchasing power at least equal to the population increase of any recent decade are well within the realm of possibility. They forget, too, that a large fraction of the sales which have marked the best years of the proximate past have been sales of new types of goods—automobiles, radios, electric refrigerators, etc.; and that these very goods have represented market expansion resulting from improved living standards and increased purchasing power.

The needs and wants yet to be filled, however, bulk far larger than those which have so far been satisfied. Tens of millions of persons need better clothing, better shoes, better education, better medical care, better recreation, better houses, and dozens of other things.

But the pessimistic outlook with respect to the cessation of population growth is not limited to the market-minded business man. Economists and sociologists in the academic field are likewise pessimistic. Dr. O. E. Baker, for example, whose

interests are primarily identified with agriculture and the agricultural population, feels that a declining total population means not only a lessening demand for farm products, but fewer opportunities for the surplus sons and daughters from the farm areas to find urban employment. He visualizes increasing numbers of subsistence farms, with their necessarily low standard of living.

While relatively little attention has been devoted in this country to the intensive study of population problems, and even less to the formulation of plans whereby the threatened misfortunes could be forestalled, there is substantial agreement among present-day American economists on one point, that real prosperity will not return until much larger amounts are being spent for capital goods—for the expansion of our productive machinery—though for the moment we already have more productive machinery than we can sell the products of.

In some fashion our plan of economic activity is geared to the requirements of a growing country in which there was actual need for tremendous expenditures for new capital goods, and thus for the diversion of economic energies away from the production of goods for current consumption. Now that we have reached a point where relatively less capital expansion is required, and where large reservoirs of capital, so to speak, are in existence, it would be necessary, even if population continued to grow, to change the direction of some of these energies, and to think more of improving the standard of living. The decline or cessation of population growth simply means that this redirection of our economic energies must be made a bit more rapidly. If this can be done, it would seem quite obvious that the United States might be even more prosperous with a population of 100,000,000 in the year 2029 than it was in 1929 with 122,000,000—or than it bids fair to be in 1940 with 132,000,000. The task of bringing about this redirection of our energies offers a challenge, first, to the economists of the present generation, and after them to the business organizers and political leaders of the years just ahead.

Friday, July 14—Demand Curves: Elasticity, Shifts, Rotation, Shape,* GEOFFREY SHEPHERD, Associate Professor of Economics, Iowa State College.

The pioneers in the field of price analysis measured changes in demand by removing trends (which were usually represented by straight lines or other simple mathematical curves fitted to the data by the method of least squares) or by expressing the data as percentages of the preceding year's data. After the World War inflation and deflation, price analysts attempted to eliminate changes in demand resulting from purely monetary causes by dividing price data by the corresponding index of the general price level.

The shortcomings of these early procedures are illustrated by the fact that even so painstaking a worker as Henry Schultz was led by their use to the erroneous conclusion that the trend of the per-capita demand for corn has declined precipitously during the past 20 or 25 years. What actually took place was a sudden downward shift during the World War, resulting from sharp increases in freight rates and other intervening charges; since the war the trend of the demand for corn has in fact remained practically horizontal. This is shown by more detailed analysis of longer series of data than were available to Schultz.

Deflating is inaccurate for well-known reasons—the relation between the deflator and the price series to be deflated is usually not 1 to 1, nor is it necessarily constant at different levels. These shortcomings can be circumvented by using the deflator as a separate variable. But this merely reveals the fundamental problem more clearly, without solving it; the problem remains—what variable or variables should be used to measure (in the earlier terminology, "remove") changes in demand?

In the past price analysts have often divided the factors affecting demand into two groups—monetary and real. A more useful division would seem to be: (1) general changes

* For copy of complete lecture, order Document No. 1219 from American Documentation Institute, care Offices of Science Service, 2101 Constitution Avenue, Washington, D.C. remitting 43 cents per copy in microfilm or $2.50 for photoprints readable without optical aid.

in demand affecting all commodities and (2) specific changes in demand for the particular commodity only.

(1) The first group (general changes in demand) may then be subdivided into: (1*a*) vertical shifts (monetary) as when prices leveled out about 50 per cent higher after the World War than before it; these changes can be measured by the general price level or the agricultural price level; and (1*b*) horizontal shifts (physical) as when general demand and supply both increased from 1922 to 1929 with practically no change in the price level; these changes can be measured by industrial production or pay rolls. But care must be exercised if the indexes measuring the two kinds of changes in demand are both used in the same analysis. For if they are intercorrelated, accidents of sampling may throw all the influence to the one or the other, or even convert a really positive influence to an apparently negative one. Thus Schultz concluded, puzzled as he was by the conclusion, that business activity had no effect on corn prices except for a slight negative effect in one of the three periods he studied. What he meant was that the index of business activity he used was correlated with the general price level (which he used as a deflator) so that the correlation between deflated corn prices and business activity was so reduced that it was determined more by accidents of sampling than by the actual correlation between undeflated corn prices and business activity, which was all too clearly evident in 1932 and 1933.

(2) Specific changes in demand then become an object for direct study, rather than a residual charged up simply to time. These changes are likely to lend themselves to measurement in concrete physical terms, such as (in the case of corn) the number and price of livestock eating up the corn, the displacement of horses and mules by tractors and trucks, etc. This procedure is less likely to lead the investigator into trouble than extrapolating trends in time.

Rotation refers to changes in elasticity. A demand curve may rotate because of changes in general price levels, in middlemen's margins, in consumers' tastes, etc. Changes in consumers' tastes may result from the introduction of new competing products, or simply from unusually long-continued scarcity or plenty of the original product itself. The reduction

of hog prices below their usual relationships to hog supply and demand since the drouths of 1934 and 1936 may be an illustration of this.

The shape or curvature of the demand curve is becoming important as programs for price stabilization and price discrimination develop. Study shows that the demand curves for most agricultural products are most elastic, not at the lower end as some economists have believed, but at the upper end. This means that price stabilization is likely to increase total returns as well as stabilize them. It means also that price discrimination is less likely to be profitable than product diversion.

Monday, July 17—Durable Goods and the Business Cycle, PROFESSOR MCINTYRE.

This study is a first attempt at the measurement of the relative prosperity in different phases of the business cycle of those industries which produce capital goods, consumers' goods, durable goods, and nondurable goods. The classification of more than 200 corporations into these four groups was extremely difficult. Space does not permit the display of this classification here. The separation of commodities into durable and nondurable goods is, of course, arbitrary. A customary durability in excess of three years entitled a good to the classification "durable." Since modern business-cycle theory is full of distinctions between durable and nondurable goods on the one hand, and between producers' goods and consumers' goods on the other, it would seem desirable to clarify the thinking on this point by subclassification of both producers' and consumers' goods into those which are durable and those which are not. With relevant data on corporations so subclassified, it should be possible to determine important lead and lag relationships among the earnings of these corporation groups and thus to answer the question whether it is the fact that goods are durable, or that they are capital goods, or that they fall

into some other classification, which causes the prosperity of the corporations producing them to be of such fundamental importance in business-cycle theory.

To make these studies of leads and lags more sensitive, it is essential to secure earnings data more frequently than once a year. Quarterly data involve the shortest practicable period, and these are available in comparable quantity only for the past decade (1929-1938). Subject to the limitations which this brief period presents—containing as it does, only 2 major upswings and 2 major downswings—it seems clear that the durable-consumers'-goods industries lead into and out of depression the other three basic groups. The typical lead is one quarter, but since this is the smallest time interval into which our data are divided, it is impossible to say more than that the lead apparently lies between two and four months. The lag correlations were tested for significance of difference by the standard probability techniques and these tests established the general superiority of the one-quarter lead over the zero and two-quarter leads, as well as over other leads and lags tested. Controlling in the list of durable consumers' goods are automobiles, houses, and household equipment.

While it does not necessarily follow from the consistent lead which this series has maintained, that stimulating industries of this type in some way which would provide them with greater profits would inevitably improve the profitability of industry as a whole; nevertheless, it seems fair to suppose, subject to the limitations of the data, that increased prosperity in our durable-goods industries at least helped to lead us out of the depression of 1929-33 and the down-swing of 1937-38. This should be viewed in the light of the discussions which have taken place with regard to the necessity of finding new inventions to do what the automobile did to restore prosperity in the early 1920's.

Again we may not conclude that the auto industry and other durable-consumers'-goods industries will necessarily be as effective now as they were 18 years ago in carrying us to higher ground. The argument might be advanced that the failure of the 1937 peak to survive was due in part to the fact that the automobile industry could not support it single-handed. Nevertheless there is encouragement in these data for the con-

clusion that existing industry can help a great deal. In tracing the manner by which prosperity in the durable-consumers'-goods industries benefits the remainder of the economy, it is important to consider two distinct ways in which orders for consumers' goods may be filled. If a larger plant or more plants are built, the result is to increase the demand for durable capital goods, that is to say, the durable instruments of production; but if increased output is secured merely by working a given plant more intensively, labor and raw materials (which are of course principally nondurable producers' goods) are the chief beneficiaries and even the so-called normal durable-capital-goods demand for replacement may be slighted. It has been characteristic of much of the government-created demand for consumers' goods that it has not inspired in entrepreneurs any assurance of a continually improved situation. As a consequence they have tended to fill these orders in the short-run way.

Monday, July 17—Keynes versus Chamberlin, MORDECAI EZEKIEL, Economic Adviser, Office of the Secretary, United States Department of Agriculture.

Classical and neo-classical economic theory assumed substantially full employment and full utilization of resources as the normal economic state. Now, after years of chronic unemployment here and in the other countries still depending on unregimented economies, theoreticians have developed modifications which explain why chronic underemployment and underproduction can take place.

The Keynesian theory relates to the balance between the funds set aside, for saving, from one income period, and the sums actually used for new capital-goods formation, in the subsequent income period. If the sums saved (and thus available for the purchase of new capital goods) exceed the sums invested in the subsequent period, a reduction in production and employment takes place. (In this statement, "savings"

and "investment" have not been defined with the close precision Keynes uses in his most recent writing, but the essential character of the balance remains.) Keynes further points out that "the propensity to consume" and the related "propensity to save" have no necessary relation to "the propensity to invest," and that hence there is no necessary reason for the two magnitudes to equal one another.

The Chamberlin theory relates to the existence of monopolistic competition, in that large part of our economy which lies between pure competition and pure monopoly. Where a small number of concerns dominate in an industry, they tend to arrive at production and price policies which approximate those of monopoly, and keep prices higher, and production lower, than would prevail with pure competition under the same conditions of production.

The cobweb theorem, which explains how industry cycles of over- and underproduction can occur even under pure competition, is an auxiliary explanation of failure to produce at capacity.

Statistical data for the verification of these theories is gradually becoming available. The work of Clark Warburton, Moulton, Kneeland, Kuznets, and Currie, is demonstrating that at levels of national income of $60 billion or better, and with the usual distribution of incomes and savings by income classes, savings would exceed investment if it were not for countervailing expenditures of government agencies. This tentative conclusion is supported by financial data on unused reserves; by observation of the stimulating effect of activity in debt-producing products such as automobiles and houses; and by the correlation between federal expenditures in excess of receipts, and subsequent industrial activity, for the period since 1932.

Statistical verification of Chamberlin is more scanty. Burns has shown descriptively that conditions conducive to monopolistic competition prevail in many industries; Means has demonstrated the importance of administered prices; and unpublished studies by Bean and Ezekiel have demonstrated the absence of competitive readjustments in steel, automobiles, cement, and other industries. Recent Scandinavian work has shown the cobweb reaction in shipbuilding and tankers,

parallel to the hog cycle here, and the housing cycle. In each case these contribute to underproduction.

Monopolistic competition produces excessive profits and an increased "propensity to save." Public policies may deal with the problem curatively, through controlling competition, or alleviatively, through compensating expenditures backed by loans, credit creation, or taxation on upper incomes.

Along one or both lines, the problem of chronic unemployment must be attacked.

From the data thus far available, one may draw the tentative conclusion that there is a pervasive tendency, in our economy as now organized, for the volume of savings at full production to outrun the volume of investment, and hence an inability to maintain the flow of payments necessary to maintain full employment. At the same time, there is monopolistic competition in many industries to an extent which tends to depress production, increase profits, and intensify the income maldistribution which contributes to excessive saving. Cyclical reactions of the cobweb type also are present in some of the competitive industries, further lowering the average volume of output.

The practical policies necessary to deal with these situations should attack all phases of the problem. The savings-investment balance may be attacked by government spending or investment from an unbalanced budget, either by borrowing or by continuous money issue; or the distribution of income may be altered by social expenditures backed by taxes which bear on those who oversave; or the propensity to save may be lowered, and the propensity to consume increased, by wage and price changes which reduce the inequality of income distribution. Simultaneously the problem of monopolistic competition must be dealt with, either by more powerful methods of anti-trust control, or by general agreements for increased production and reduced prices, like the Industrial Expansion proposal for concerted expansion under government guarantees.[1]

Unless economists can help make the proper diagnosis of the causes of chronic unemployment, and aid government to

[1] See *Jobs for All*, by Mordecai Ezekiel, Knopf and Co., 1939.

devise action powerful enough to correct it, our society cannot continue to endure along the lines which we have known heretofore.

Tuesday, July 18—The Use of Fourier Series in the Analysis of Seasonal Variation, ALEXANDER STURGES, Statistician, United States Bureau of Labor Statistics.

Methods devised for the study of seasonal variation may be divided into those which describe it simply as observed fact and those which measure it in terms of auxiliary series believed to be causally related. The method discussed here is definitely in the first class. It describes what has occurred in a way which may be a helpful preparation for the analysis in terms of causation.

For simplicity in the three definitions to follow suppose that the series under consideration is one of monthly prices.

(1) By "sequence of prices in a specific year" is meant the 12 observed prices.

(2) By "annual pattern of prices in a specific year" is meant any specified generalization representing these prices. Thus, if the 12 prices are represented as ratios to their average, then the table or graph of these ratios is an annual pattern for that year. In the method discussed here, however, the annual pattern consists of the Fourier series which fits the 12 prices. The important distinction between sequence and pattern is that the former is the observed fact while the latter, though still a statement of fact, is reorganized in an entirely arbitrary way depending upon the method of seasonal analysis being used. Inference is not involved, only convenience.

(3) By "seasonal variation of prices for a specific period" is meant the average of the annual patterns of the years included. No inference is made as to the validity of this same average annual pattern in any other period. The causal type of analysis would appear to be involved before such inferences could be made with confidence.

The use of Fourier series is a device for stating annual pattern in terms of several wavelike components. Often it is easy to interpret such components in terms of causal factors. Thus, the wave with one crest may be due to one set of influences, the wave with two crests in a year to an entirely different set, while waves of three and five crests usually have no good explanation and may be used as standards of comparison in a sense analogous to sampling error in a sampling series. The arithmetical routine of fitting Fourier series is simple, and with practice, rapid.[1]

The successive annual sets of coefficients obtained are useful in three ways:

(1) They suggest the presence of any systematic change in annual pattern.

(2) They provide a basis for study as to causes of fluctuation in annual pattern and possibly as to how it originates.

(3) They allow the ordinary adjustments for seasonal variation and the statement of a smooth seasonal variation.

If the graph for a series of years of a particular coefficient shows random fluctuations about a constant level, then that component of the annual pattern is stable and unchanging for the period under consideration. Within small limits the fluctuations may be ignored as being analogous to sampling fluctuations. Larger deviations suggest the need of a search for causes of difference of pattern. Likewise, if the level about which the coefficients fluctuate is not stable but is broken or sloping, then a causal analysis is in order, for there is a clear indication of a changing annual pattern. Corresponding pecularities must be sought in the same component of an appropriate causal series. Correlations between coefficients are more nearly valid than are ordinary time-series correlations because the coefficients appear to have little serial correlation and more nearly to follow theoretical sampling distributions.

The coefficients of the different components are not of equal size, which is a measure of their importance, nor of equal

[1] Running, T. R., *Empirical Formulas,* New York, John Wiley & Sons, Inc., 1917; Whittaker, E. T. and Robinson, G., *The Calculus of Observations,* London and Glasgow, Blackie & Sons, Ltd., 1932.

uniformity. In seeking causes of seasonal variation, components with large uniform coefficients would be of most interest, while in seeking causes of fluctuations in annual pattern, components with widely varying coefficients would be of most interest.

Tuesday, July 18—The Influence of Price on Exports, J. B. D. DERKSEN, Assistant Head of Division for Business-Cycle Research, Netherlands Central Bureau of Statistics.

For the Netherlands, and for many other countries too, the fluctuations of exports are an important factor in the fluctuations of general prosperity. A better quantitative knowledge of the factors causing these export fluctuations is therefore useful for the various problems of business-cycle policy. Such questions as, e.g., "What is the effect of a given reduction in wage rates on exports?" or "What is the effect of a given rate of depreciation of the currency on exports?" cannot be answered without a quantitative knowledge of the influence of prices on exports.

One way of solving this statistical problem would be the determination of the demand curve for Dutch exports in the usual sense.

In the present investigation a somewhat different method has been followed. Instead of "explaining" the fluctuations in the absolute level of exports, an endeavor has been made to "explain" the proportion between Netherland exports and competing foreign exports. In other words, an endeavor has been made to "explain" the fluctuations in the ratio between imports from the Netherlands into a certain market and imports from other countries into the same market. The advantage of this procedure is that it may be assumed that a number of factors influencing imports from the Netherlands have equally influenced those from other countries, and, therefore, have not influenced the ratio or at least influenced it to a small-

er degree. This will be true, e.g., for incomes and for some measures of commercial policy. It will not always or not always exactly be true. The procedure tends, however, to weaken the influence of the nonrelevant factors in our problem. As the chief explanatory variable the ratio of Netherland to foreign prices will remain. This ratio, and also the ratio between quantities, as a rule will be taken from the trade statistics of the importing countries, which yield the best available guarantee of comparability.

Two sorts of elasticity coefficients have been calculated, which may be given the names *elasticity of substitution*[1] and *elasticity of competition*, respectively. The elasticity of substitution indicates the percentage change in the proportion

$$H = \frac{\text{imports from the Netherlands}}{\text{imports from other countries}}$$

for a one-per-cent change in the proportion

$$P = \frac{\text{Netherlands price}}{\text{competing price}}.$$

The elasticity of competition indicates the percentage change in the proportion

$$Q = \frac{\text{imports from the Netherlands}}{\text{all imports}} \times 100$$

for a one-per-cent change in the same price proportion.

Therefore:

Elasticity of substitution $= \eta_H = -\frac{\partial H}{\partial P}\frac{P}{H}$, and elasticity of competition $= \eta_Q = -\frac{\partial Q}{\partial P}\frac{P}{Q}$.

It is easily found that

$$\eta_H = \frac{100}{100-Q}\eta_Q \quad \text{and} \quad \eta_Q = \frac{1}{1+H}\eta_H .$$

What seems to be remarkable in the results of the inves-

[1] Following Mrs. Robinson, and Messrs. Lerner, Allen, and Hicks. Cf. Allen and Hicks, "A Reconsideration of the Theory of Value," *Economica*, February and May, 1934.

tigation is the low elasticities found. There is a common belief among theorists that these elasticities of substitution are very high figures, since the proportion of the world market supplied by one country is small. From the figures found one would deduce that the imperfections in the market—of whatever nature they may be—are considerable. The low figures found for the elasticity of substitution mean—among other things—that the stimulating consequences of price reductions must not be exaggerated.

There is no full agreement between the figures found for the separate goods and those for the total of all exports. The weighted average for all goods considered is between 2 and 3, whereas the direct determination leads to figures from 1 to 2. One explanation of this discrepancy may be that the commodities with a higher elasticity of substitution will, as a rule, yield more successful calculations and the sample may for this reason be not quite fair. It may, however, be stated that both figures are rather low.

Finally it may be observed that the figures found for cases where a quota system was applied do not show the drastic decrease in elasticity one would expect.

Wednesday, July 19—Some Remarks on the Dynamic Theory of Production, GERHARD TINTNER, Associate Professor of Economics and Mathematics, Iowa State College.

This is based upon the fundamental work of Hicks[1] and Allen[2] in this field and is also an adaptation of some of my own work in the dynamic theory of demand[3] to production theory. Suppose an entrepreneur is at the point in time 0 and plans for the period $1, 2, \cdots, n$. Denote by x_{ut} the amount of the

[1] J. R. Hicks, *Value and Capital*, Oxford, 1939, pp. 115 ff., 325 ff.

[2] R. G. D. Allen, *Mathematical Analysis for Economists*, London, 1938, pp. 502 ff.

[3] G. Tintner, "The Theoretical Derivation of Dynamic Demand Curves," *Econometrica*, Vol. 6, 1938, pp. 375 ff.; "Elasticities of Expenditure in the Dynamic Theory of Demand," *ibid.*, Vol. 7, July, 1939, pp. 266 ff.

product u he expects to produce at the point in time t or the negative amount of the factor u he expects to use at the same point in time. There are m factors and products. Let $G(x_{11}, \cdots, x_{mn})$ be the production or transformation function. p_{ut} is the expected price of the product or factor u at the point in time t. i_t is the expected rate of interest and $r_t = 1 + i_t$ the expected accumulation factor at the point in time t. $R_t = r_1 r_2 \cdots r_t$ is the expected total accumulation factor for the entire period $1, 2, \cdots, t$. $q_{ut} = p_{ut}/R_t$ is the expected discounted price of the product or factor u at the point in time t. All expectations are certain. The entrepreneur tries to maximize his total discounted profit from the production $J = \sum_{s=1}^{n} \sum_{v=1}^{m} q_{vs}\, x_{vs}$ under the condition of the given production function G. The equilibrium conditions are:

$$\lambda\, G_{vs} = q_{vs}\,, \qquad (v = 1, 2, \cdots, m;\ s = 1, 2, \cdots, n), \tag{1}$$

where G_{vs} is the partial derivative of G with respect to x_{vs}. λ is a Lagrange multiplier. Differentiating the system (1) and G we get the following set of linear equations in the demand differentials (for the factors) and supply differentials (for the products):

$$\sum_{s=1}^{n} \sum_{v=1}^{m} G_{vs}\, dx_{vs} = 0\,, \tag{2}$$

$$G_{ut}\, d\lambda + \sum_{s=1}^{n} \sum_{v=1}^{m} G_{vs,\, ut}\, dx_{vs} = dq_{ut} \qquad \begin{matrix}(u = 1, 2, \cdots, m,\\ t = 1, 2, \cdots, n),\end{matrix}$$

where $G_{vs,\, ut}$ is the second partial derivative of G with respect to x_{vs} and x_{ut}. Denote by D the following determinant:

$$D = \begin{vmatrix} O & G_{11} & \cdots & G_{mn} \\ G_{11} & G_{11,\, 11} & \cdots & G_{11,\, mn} \\ \cdot & \cdot & \cdots & \cdot \\ G_{mn} & G_{11,\, mn} & \cdots & G_{mn,\, mn} \end{vmatrix} \tag{3}$$

and by $D_{vs,\, ut}$ the cofactor of the element $G_{vs,\, ut}$. The general solution for the demand or supply differential is then:

$$dx_{ut} = \frac{\left(\sum_{s=1}^{n} \sum_{v=1}^{m} D_{vs,\, ut}\, dq_{vs}\right)}{D\,\lambda}\,. \tag{4}$$

The price and accumulation-rate derivatives and elasticities of demand, supply, expenditure, and revenue can easily be derived from this general formula.

Wednesday, July 19—On the Measurement of the Degree of Inequality of Income Distributions, HORST MENDERSHAUSEN, Research Fellow, Cowles Commission (now at Colorado College).

Distribution functions have been proposed by Pareto, Gini, Amoroso, Gibrat, Champernowne, and others. Pareto's function has become most famous. But none of these functions are very satisfactory. They either fail to fit empirical distributions, or cannot be interpreted in the light of a reasonable concept of inequality, or both.

In view of the technical and conceptual difficulties in deriving a satisfactory general income-distribution function, Lorenz, Gini, and others defined inequality and equality without presupposing a certain distribution function. To both Lorenz and Gini, perfect equality meant everybody receiving the same income. Gini defined as perfect inequality the situation where one out of a large number of persons holds all of the income of the community.

Gini's "concentration ratio," or the relative average difference between individual incomes, is a good measure of the degree of inequality. But it does not summarize all important aspects of the income distribution. It may be the same for two distributions, though in one the poor receive a much larger share of income than in the other. Some ambiguity is unavoidable in measures which are not based upon a—satisfactory—distribution function.

Another measure of the degree of inequality is proposed here. It is called β and defined as

$$\beta = \frac{E - M}{E},$$

where M stands for the median income and E for the "equa-

torial" income. If incomes, ordered by size, are cumulated, the equatorial income is the one which divides the aggregate amount of income into two equal parts. This measure is not based on a certain distribution function either. But it reflects the broad features of income distribution equally well as does Gini's R, seems to be more variable than the latter for the range into which the degree of inequality will fall in most practical cases, and is easier to calculate. β varies between 0 and 1. If it is 0, everybody receives the same amount of income (case of perfect equality). If $\beta = 1$, 50 per cent of the income receivers have no income at all, the other 50 per cent receiving the total income (case of perfect inequality). The definition of perfect inequality is less specific than that given by Gini. The covariation between R and β is very pronounced for 30 income distributions analyzed.[1]

Since the distribution data analyzed refer to samples, the problem of sampling errors of β has to be dealt with. If the attention nowadays given to changes in the income distribution continues, and if consequently periodic surveys of the entire income distribution are made, distribution data from sample studies will multiply, because it is cheaper to get a good sample than to question everybody about his income. Therefore, the sampling distribution of measures of income inequality can no longer be neglected, as it has been the custom in the past.

The formula for the standard error[2] of β is:

$$\sigma_\beta = \sigma_{M/E} = \pm \sqrt{\left(\frac{M}{E}\right)^2 (v_M^2 - 2\, r_{EM}\, v_M\, v_E + v_E^2)}$$

[1] These distributions are based upon samples taken in 51 cities of the United States under the Consumer Purchases Study of 1935-36, a WPA project conducted by various agencies of the Federal Government.

[2] For the general formula for the standard error of a ratio, see A. S. Merrill, "Frequency Distribution of an Index when both the Components Follow the Normal Law," *Biometrika*, Vol. XX_A, 1928, p. 56.

For the formula for the standard error of the median, see S. S. Wilks, *The Theory of Statistical Inference*, Lectures at Princeton University, 1936-37, p. 50.

The formula for σ_E, the standard error of the equatorial income, has been worked out by Dr. Abraham Wald on the suggestion of the author. Like several of the other formulae used it is an approximation formula holding true for a large number of observations. The samples studied contained several thousands of individuals each.

where $v_M = \sigma_M/M$, $v_E = \sigma_E/E$, $\sigma_M = 1/\psi(M)\sqrt{8(n-1)/2}$, and $\sigma_E = \sigma_X/2\,\psi(E)\,E\sqrt{N}$, σ_X = the square root of the second moment about zero income, N = the number of income receivers in the sample, $\psi(E)$ and $\psi(M)$ = the relative frequency of income receivers in the class including the sample equatorial income and median income respectively, divided by the income range of the class, r_{EM} = the coefficient of correlation between E and M.

If the number of observations is large, the standard error of β will generally be rather small. So, for instance, in the case of the income distribution in Columbus, Ohio, studied from a sample of about 12,000 families, $\beta = 0.42$ and $\sigma_\beta \infty 0.01$.

Applying the new measure of income inequality to distributions in various cities, the following general results were obtained: (1) Inequality increases with the size of the city. (2) Income inequality is larger among native-white than among native-negro urban families. (3) Among native-white urban families, income inequality tends to be largest in New England, increasingly smaller in the North Central, Southeastern, Mountain and Plains and Pacific regions, the last two having the same degree.

Thursday, July 20—Prolegomena to the Prewar International Gold Standard Reinterpreted in the Light of Its History, CAREL JAN SMIT, care of National Bureau of Economic Research.

The approach and the purpose of our group study[1] was defined in our original research project as follows: "The main

[1]This paper forms part of the fruits of a group study of the international gold standard undertaken on the basis of a research project entitled "The International Gold Standard Re-Interpreted" drawn up by the speaker in collaboration with his friend Professor William Adams Brown, Jr., of Brown University, in 1931. In the original research project a new, historical, consistent approach was presented to the vexing postwar international-gold-standard problems from the point of view of

scientific questions which we try to answer are not: What was the theory of the functioning of the gold standard before the war? Are the gold-standard facts of the postwar world in accordance with that theory? But, on the contrary, the main scientific questions we try to answer are: What was the practice of the functioning of the gold standard, in its historical setting, in the prewar world? What were the experiences with regard to its functioning in the postwar world? Can we learn from the past for the future, analyzing period by period?" The frame of reference for the various interconnected statistical investigations planned for the basic prewar period is contained in a brief paper entitled "The Prewar Gold Standard," read by me before the Academy of Political Science in New York City on March 21, 1934, and published in its *Proceedings*, Vol. XVI, No. 1.

The present lecture was announced at my own suggestion in February of this year in its subtitle as an institutional analysis of that important phase of the prewar international gold standard, the Weekly Accounts of the Bank of England from 1832 to 1911 inclusive, which have been compiled and systematically examined by me in relation to other statistical data in the field of the interdependence of money markets referred to in the last part of my frame of reference, the above-mentioned paper. There has been a change in the focussing point of my subject which is expressed in the changed title by the addition of the words "Prolegomena to" and the omission of that part that refers to the Weekly Bank of England Accounts. In handling and examining the various data I was at last faced by the definite question of my final attitude towards the classi-

institutional economics. The group study was carried out under the auspices of Brown University, thereto enabled by a generous gift from the Rockefeller Foundation, to both of which institutions I herewith express my heartfelt thanks. The relationship of this paper to the National Bureau of Economic Research is best explained by the following words of Dr. Wesley C. Mitchell, Director of Research of that institution, in his annual report for 1934-1935: "Other manuscripts which we expect to submit, but which we cannot schedule definitely as yet, include . . . a study tentatively entitled 'The International Gold Standard under Prewar and Postwar Conditions.' The last-named study is being conducted at Brown University by Professor William Adams Brown, Jr., Dr. Carel J. Smit, and Dr. A. G. Silverman. . . The Executive Committee has approved the acceptance of the report for publication in the National Bureau's series, provided it is satisfactory to the Directors."

cal and neo-classical theories of economics. As can be seen from the above quotations from the original research project, the starting point of our research analysis was definitely critical of the international-trade theory, which centers round the concept of a natural, general equilibrium that really goes back to Adam Smith. I felt it would be at least ungracious to a most imposing amount of learning, past and present, to be critical without explaining why.

My critique of the classical and neo-classical philosophy of economics behind the traditional interpretation of the gold standard has become not, as I first thought, a short introductory part, but has developed into the main topic of my one-hour lecture. My critique is focussed on the teleological, general-equilibrium concept of the classical theory of economics, the argument from design in Adam Smith's concept of a naturally equilibrating economic world, motivated as by one law of gravity by the individual self-interests of countless human atoms. This general-equilibrium concept cannot be understood without going into the preconceptions of a naturalistic, deistic religion which characterized the climate of opinion of the 18th century, as shown in Carl Becker's *Heavenly City of the 18th Century Philosophers*. The main thesis of my critique is that the concept of a general natural equilibrium of classical economics belongs to the faith category, to the category of that naturalistic, superficially optimistic and deistic faith which was typical for the 18th century. This faith, which was consciously present in Adam Smith's mind, has become subconscious in the minds of many of his successors. Nevertheless, this 18th century naturalistic, deistic faith is still there in the preconceptions of our traditional general-equilibrium economics. This faith goes down to the roots and spreads forth to the fruits of our economic science, one of the fruits being the traditional "faith" attitude towards an assumed "normal" automatic operation of the international gold standard as part of the international trade theory.

As a conscious and thinking Christian, who personally believes in original sin and in the atonement of Jesus Christ, the Son of God, on the Cross of Calvary, I realize that the naturalistic, deistic faith at the roots of our traditional economic science has nothing in common with the supernaturally revealed,

theistic religion of both the Old and New Testaments. But quite apart from my personal Christian faith, I tested Adam Smith's logical categories based on the universal human law of self-love and the assumed propensity to truck, barter, and exchange in the light of Karl Pearson's methodological standard work, *The Grammar of Science.* I found that Adam Smith's notion of a tendency towards a general equilibrium in the economic world is based with regard to the law of self-interest on a confusion of empirical and conceptual scientific laws and with regard to the propensity to truck, barter, and exchange on the logical *post hoc ergo propter hoc* fallacy which John Dewey illustrates by the quasi-explanation that opium puts men to sleep because of its dormitive potency. The validity of the conclusions built on the "faith" foundation of classical and neo-classical economics is not of the type exemplified by the formula which Newton propounded for the motion of the planetary system which will be accepted, so far as verified, by every rational mind which has once understood its terms and clearly analyzed the facts which it resumes. Economists treading nowadays in the footsteps of Adam Smith should become conscious of the fact that the validity of their logical categories really depends on the acceptance or rejection of certain philosophic or religious preconceptions regarding God, nature, and man, not just on the acceptance of certain a priori, axiomatic "faith" postulates like those in the science of mathematics.

Thursday, July 20—Individual and National Income and Consumption, JAKOB MARSCHAK, Director of the Institute of Statistics, University of Oxford.

The relationship $x = x(r)$ between a family's income r and its consumption x of a given commodity will be called "personal-budget (or Engel) function of the commodity x." Let $\int_0^h f(r)\,dr$ be the proportion of families having income smaller than h at the ("initial") time of budget studies. Then

$\int_0^\infty f(r)\,dr = 1$, $\int_0^\infty r\,f(r)\,dr \equiv R_1 \equiv$ initial average income. $\int_0^\infty x(r)\ f(r)\,dr \equiv X_1 \equiv$ initial average consumption. If the change of the average income from R_1 to R is accompanied by a change of the income of a given family from r to l, and $l = l(r, R/R_1)$, the "participation function," the new consumption amount,

$$(1) \qquad X = \int_0^\infty x[l(r, R/R_1)]\, f(r)\ dr\,,$$

is a function of R, say $X(R)$; this function will be called the "national-budget function for the commodity x." If, in particular, $l \equiv l(r, R/R_1) = r\,R/R_1$, i.e., if all incomes change proportionally,

$$(2) \qquad X = \int_0^\infty x(r, R/R_1)\, f(r)\, dr$$

is a "standard" national-budget function.

It is often tacitly assumed that $x(r)$ and $X(R)$ have the same properties, and "psychological laws" are extended from the individual to the community. The true connection between these functions depends, however, on the other functions involved: the initial income distribution $f(r)$ and the participation function $l(r, R/R_1)$. Define:

$x' \equiv dx/dr \equiv$ personal
$X' \equiv dX/dR \equiv$ national
} marginal propensity to consume the commodity x,

$\eta_x \equiv x'\,r/x \equiv$ personal
$\eta_X \equiv X'\,R/X \equiv$ national
} elasticity of demand for x with respect to income,

$$\eta_l(r) \equiv \frac{\partial \log l(r, R/R_1)}{\partial \log R} \equiv$$ elasticity of personal with respect to national income.

We find from (1):

$$X' = \int_0^\infty x'(l)\frac{\partial l}{\partial R} f(r)\, dr,$$

$$\eta_X = \int_0^\infty \eta_x\, \eta_l \frac{x}{X} f(r)\ dr,$$

$$X'' = \int_0^\infty x''(l)\left(\frac{\partial l}{\partial R}\right)^2 f(r)\ dr + \int_0^\infty x'(l)\,\frac{\partial^2 l}{\partial R^2} f(r)\, dr\,.$$

As an example the following propositions (given or implied in Keynes' *General Theory*, pp. 115 and 126, with regard to the total consumption expenditure) can be tested: (a) if $0 < x' < 1, 0 < X' < 1$; (b) if $\eta_x < 1, \eta_X < 1$; (c) if $x'' < 0, X'' < 0$. (a) is valid; (b) is valid in the "standard" case ($\eta_l = 1$), or if the poor have $\eta_l < 1$ ("inequality rising with prosperity") and have a major share in the nation's consumption of the commodity; and somewhat similarly for (c).

In the "standard" case, it is possible to construct $X(R)$ graphically from the budget and income-distribution data without any algebraic curve-fitting.

Friday, July 21—Growth and Cyclical Movements,* ELMER C. BRATT, Associate Professor of Economics, Lehigh University.

The distinction between long-run and short-run conditions is now more important than ever before. A reappraisal of the methods of distinguishing these forces is much needed, however. The approach of the classical economists has been dependent upon the distinction between market price and normal price. For the purpose of analyzing current-day conditions, this method has distinct limitations. Notable among these are: normal price is microcosmic, while balance must be viewed comprehensively; the demand curve locating market price shifts so rapidly, except for agricultural products, that short-run conditions are inadequately described.

The dividing of series into cyclical, irregular, long-time trend, and seasonal movements may be called the orthodox statistical procedure. The trend and cycle are an approach to the presentation of long- and short-run conditions. The meaning

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of the trend has often been particularly obscure, while arguments persist regarding the business cycle. The lack of definite periodicity is troublesome to many minds. Almost any trend will become absurd if extrapolated far enough.

Resort has been had more recently to still more empirical division of the movements taking place over time. Attempts are made to decompose time series into a number of periodicities of different lengths. First harmonic analysis, later successive sets of moving averages, and most recently bracket analysis have been employed. The assumption that the component movements are periodic presumes undue faith, even if such unlimited number of movements as shown by empirical analysis be allowed. Explanations are sometimes attempted, but they are too dependent upon wars, reinvestment cycles, and long-period swings in prices.

The differentiation between long-period and short-period factors is logically significant. Complete avoidance of such classificatory convenience as abnormal, unbalanced, or maladjusted is impossible, for the assumption of regularity involves a recognition of the typical. Balance is an essential concept in economic analysis. The balanced situation is the long-run situation.

Many concepts of balance are possible. To be generally acceptable the concept must be dynamic and macrocosmic. The concept most adqeuately satisfying these conditions is a coordination of production and consumption.

Change in balance over time produces growth or decadence. The growth of integral industry is basic. An understanding of it is furthered by classifying the analytical determinants. The determining conditions of economic progress are capacity to produce and problems of group organization.

Lack of adjustment between production and consumption produces a short-period situation differing from that which exists at the hypothetical level of balance. It is not normal for conditions to lie at the balanced level, but it is for them to fluctuate about it. The recurrent upswing and downswing have both logical and empirical significance.

Lack of adjustment between production and consumption is most closely related to variation in the production of durable goods. This variation is more dependent than ever before

upon short-term market demand. Attainment of knowledge regarding the balanced situation and use of it to influence the production of durable goods is our greatest hope for modulating the severity of variation in the short-period situation.

The most pressing question today is concerned with the extent to which current levels of business activity are representative of short-run, unbalanced conditions. The answer to this may be as closely related to surprises of economists as some recent economic changes have been to surprises of business men. A major danger is acclimation to lower levels of activity.

Friday, July 21—Sources for Demand Analysis: Market, Budget, and Income Data, DR. MARSCHAK.

An econometrician's ideal: to estimate the parameters of the typical individual's indifference function

$$F(x, y, \cdots) \tag{1}$$

where $x, y, \cdots$ are the consumed amounts of various commodities. From this function, using the equilibrium condition

$$\frac{\partial F}{\partial x} / p = \frac{\partial F}{\partial y} / q = \cdots \equiv \mu \tag{2}$$

(where $p, q, \cdots$ are respective prices of $x, y, \cdots$, and μ defines the "marginal utility of money"), the individual's "consumption surface" for x,

$$x = x(r, p, q, \cdots) , \tag{3}$$

can be derived, where $r \equiv px + qy + \cdots \equiv$ income. If the proportion of families having income r is $f(r)$, the per-family consumption is

$$X = \int_0^\infty x(r, p, q, \cdots) \, f(r) \, dr , \tag{4}$$

the "national consumption surface." The observable data are:

Market (M) data: M_1: national quantities, X_h, Y_h, $\cdots$, for a large number of time points h;
M_2: prices, p_h, q_h, $\cdots$, for a large number of time points h;

Budget (B) data: B: r_{ik}, x_{ik}, y_{ik}, $\cdots$, for a large number of families i but a small number of time points k;

Income (I) data: I_1: income distribution, $f_l(r)$, for a small number of time points l;
I_2: per-family income,
$$R_m \equiv \int_0^\infty f_m(r)\, dr\ ,$$
for a large number of time points m.

Methods to determine function (3) or (4):

(α) *Frisch*: Fit (3) directly to B and M_2 data. Drawback: too small number of time points k.

(β) *Wald*:[1] Fit indifference function (1) of assumed (e.g., quadratic) form to B and M_2 data using (2). Same drawback as in (α) while same number of degrees of freedom required.

(γ) *Pigou* (1910): Using notation $u_v = (\partial u/\partial v)\,(v/u) \equiv$ "elasticity of u with respect to v," find x_r from B data; and use the fact

$$x_p : y_p = x_r : y_r\ , \tag{5}$$

valid under certain conditions, which can be obtained from (2) by assuming $\partial F/\partial x$ to be independent of $q \cdots$, and taking the ratio $x_p/x_r = r_\mu(1 + \mu_p)$; from this, (5) follows exactly if $\mu_p = \mu_q = 0$ ("small" commodities), but not (as Pigou thought) if $xp = yq$ (goods with equal expenditure shares). Drawback: Even with the independence requirement and the smallness requirement satisfied, we obtain ratios only (cf. discussion in *Quarterly Journal of Economics*, Vol. 50).

(δ) *Schultz*: To the M and I_2 data, fit the function

$$X = X(R, p, q \cdots)\ . \tag{6}$$

[1] Also suggested, for two commodities only, by the present writer in a paper read by title at the Econometric Conference, Stresa, 1934.

Drawback: (6) does not have the full economic meaning of (4), and leaves information from I_1 and B data unused. Even so, by using R as an independent variable Schultz probably succeeds in eliminating one major cause of demand shifts in time, better than in other attempts where M data only were used. (Pigou 1930).

(ε) *Staehle*: Using M and I data, fit a function $X = X(R, \alpha, p, q, \cdots)$, where α is Pareto's (or some other) measure of income inequality. Drawbacks and advantages analogous to (δ).

(ζ) *Pareto* (1895), *Roy* (1930), *Wisniewski,* (1935): In the economically significant equation (4), a priori functions $x = x(r, p)$ are inserted. Pareto assumes, for grain, $x = ms^b/(n + s^b)$, where $s = r/p$ and m, n, b, are unknown constants. Roy's assumptions are simpler and based on the concept of two saturation quantity indices: ξ_a for the aggregate of necessaries and ξ_b for the aggregate of luxuries. Denote by x_a, x_b, p_a, p_b the price indices and quantity indices for each aggregate; then $p_a\xi_a$ and $p_b\xi_b$ delimit the lower, middle, upper income range. The Engel functions are:

Income range	Necessaries (x_a)	Luxuries (x_b)
Lower	r/p_a	0
Middle	ξ_a	$(r - p_a)/p_b$
Upper		ξ_b

Integrating as in (4), national consumption surfaces for the two aggregates are obtained. To get them for single commodities, hypotheses are made as to the behavior of single goods within each aggregate. Drawback: validity of hypotheses doubtful although they usefully map out extreme cases; B information left unused.

To combine the M, I, and B data, methods (η) and (θ) are suggested.

(η) Example. Assume (3) to be quadratic in r and p (neglecting $q, \cdots$):

$$x = a + kp + br + lpr + mp^2 + nr^2 . \tag{7}$$

Putting $p = p'$ (the price prevailing at the time of the budget study) the Engel function is $x = u + vr + wr^2$, where

$$u = a + kp' + mp'^2 ,$$
$$v = b + lp' , \tag{8}$$
$$w = n .$$

From B data, u, v, w are found. Then (4) becomes (writing $\int_0^\infty f_i(r)\,dr = 1$, $\int_0^\infty rf_i(r)\,dr = R_i \equiv$ per-family income at time i, $\int_0^\infty r^2 f_i(r)\,dr = s^2 \equiv$ second moment about 0):

$$X_i = a + kp_i + mp_i^2 + bR_i + lp_iR_i + ns^2 .$$

Substituting for a, b, n from (8), a set of observation equations is obtained to yield estimates of k, l, m; substituting those in (8), a, b, n are found. The method can be generalized by introducing prices of other goods into (7) or choosing other forms for the function (3). Advantage: Changes in national income and its distribution can be eliminated more adequately than in (δ) and (ε).

(θ) Without fixing any a priori form for (3) calculate for each point of time i the hypothetical demand $X_i(p')$ which would have taken place if all incomes changed as they did but the prices were kept at the level (p') it had at the time of the budget studies:

$$X_i(p') = \int_0^\infty x(r, p')\, f_i(r)\,dr .$$

To the time series of (1) the actual $X_i(p)$, (2) the hypothetical $X_i(p')$, and (3) the prices p, fit a function

$$X = \phi[X(p'), p] ,$$

depending on the "standard" distribution of incomes at the time of the budget study (cf. the technique of "standardized death rates"). Drawback: does not enable forecasting response of demand to income changes. Advantage compared with (η): does not involve a priori formulae.

Monday, July 24—Relationships between the Distribution of Income and Total Real Income, PROFESSOR DAVIS.

In a paper on "The Significance of the Curve of Income" presented at the Fourth Annual Conference of the Cowles Commission last year the author suggested as a possible frequency curve for the distribution of incomes the equation

$$(1) \qquad \phi(x) = \frac{a}{z^n} \frac{1}{e^{b/z} - 1}, \qquad z = x - c,$$

where $\phi(x)$ represents the number of individuals who possess an income between x and $x + dx$, c is the income necessary to sustain life, and n is approximately 3.5.

This function is asymptotic, on the one hand, to the frequency function derived from Pareto's famous law for the distribution of incomes above the mode, and, on the other hand, it represents well the distribution of incomes around and below the mode.

Since the modal income as defined by this function may be shown to equal

$$x = c + b/(n-p),$$

where p is the nontrivial solution of the equation

$$p\, e^{-p} = n\, e^{-n},$$

it becomes a matter of interest to obtain some economic interpretation of the parameter b. The object of this paper is to determine the relationship between b and total real income, that is to say, total income corrected for the general level of prices.

If the Pareto distribution of incomes is plotted on double-log paper, on which it becomes a straight line for each of the years since and including 1914, when income-tax returns first became available in the United States, the significant observation may be made that the points on the lower end of the successive lines of the distributions show a large variation which tends to follow the business cycle, whereas those at the upper end exhibit the variation to a much less degree. That is to say, large-income receivers are more violently affected by variations in the business cycle than are small-income receivers.

From formula (1) we obtain the accumulated frequency

$$(2) \qquad y = \int_z^\infty \phi(z)\,dz = \frac{a}{b}\frac{z^{-n+2}}{n-2},$$

which is Pareto's law.

The assumptions are then made, first, that a/b varies as some power of total real income, I, and, second, that the total number, N, of income recipients varies as some other power of total real income. We thus may write

$$(3) \quad a/b = k_1 I^\gamma, \quad N = a\,b^{-n+1}\,\Gamma(n-1)\,\zeta(n-1) = k_2 I^\delta,$$

where γ and δ are to be determined.

By holding z fixed in equation (2), we may estimate statistically the dependence of large-income recipients upon I by means of the first equation in (3), and from the known values of N we may obtain similar information about the dependence of the total group upon total real income.

It is thus found that γ is approximately equal to 7 with a correlation of around 0.85 for each of the income groups examined. The data used in this determination ranged from those who received incomes of more than a million to those who received incomes in excess of $100,000. Similarly it is found that δ is approximately equal to 1, with a correlation of 0.95. This is equivalent, of course, to the reasonable proposition that total employment varies directly with total real income. From these two estimates, b is found to vary as the fourth power of total real income. Assuming that the subsistence income, c, varies approximately as the modal income, we reach the conclusion that the *modal income varies as the fourth power of total real income.*

A statistical test of this conclusion was afforded by a comparison of the per-capita modal incomes of 1918, 1929, and 1935-36. Using 1918, with a per-capita modal income of $347, the per-capita modal incomes of 1919 and 1935-1936 were estimated by the above law to equal $653 and $340 respectively. The actual per-capita incomes for these years, as estimated from available distributions, were respectively $630 and $340.

Monday, July 24—The Problem of Estimating the Length of the Cycles Created by the Moving Average and by Other Graduation Processes, EDWARD L. DODD, Professor of Actuarial Mathematics, University of Texas.

The convergence of *repeated* summations of chance elements to a sinusoidal form has been treated by E. Slutzky, V. Romanovsky, H. E. Jones, E. J. Moulton, and A. Wald. But here, I deal with the effects produced by a *single* graduation. Whether the linear operators used can be analyzed into summations is, moreover, immaterial.

Figures such as Slutzky exhibits in *Econometrica,* Vol. 5, pp. 105-146, illustrate what may be called cycles. One definition of a cycle-length is the average distance between successive maxima; another, the average distance between upward-crossings of the base line. In Figure 2, Slutzky exhibits maxima and minima *after* small "ripples" have been suppressed.

Now any "cycle length" may be regarded as the reciprocal of a "frequency" or probability. Thus, if the probability of an up-crossing is 0.05, we expect 5 up-crosses per 100 graduated values, making a cycle length of 20. Indeed, under conditions which I soon state, if twelvefold averaging by twos is applied to chance data, the probabilities for up-crossings, maxima, and inflections from concave to convex are 0.0628, 0.106, and 0.134, respectively. Among 100 such graduated values, then, 6.28 up-crossings are expected, 10.6 maxima, and 13.4 such inflections. This averaging was used by Slutzky for his Figure IVa, p. 111, which shows 6 or 7 up-crossings, 10 maxima, and 13 or 14 inflections. The inflections may be counted from up-crossings in Figure IVc, the second differences of IVa.

Now Slutzky's data were taken from lottery drawings, a "rectangular" distribution; and the several sets that I have used are likewise rectangular. But it is known that the average of a few such chance elements has a distribution not far from normal. Suppose the data, $x_1, x_2, \cdots$, independent, and normally distributed about zero, with constant variance. And let

$$y_r = a_{-m}\, x_{r-m} + \cdots + a_0\, x_r + a_1\, x_{r+1} + \cdots + a_m\, x_{r+m}\,.$$

Then $y_{r-1} = 0$ and $y_r = 0$ are "planes" in $2m + 2$ dimensions.

Let θ be the angle between these planes, that angle for which $y_{r-1} < 0$ and $y_r > 0$. This angle is readily found from its cosine. Then $\theta/360°$ is the probability that the curve through graduated values passes from below the base line where $y_{r-1} < 0$ to above where $y_r > 0$. Likewise, if $\Delta y_r = y_{r+1} - y_r$, and θ_1 is the angle determined from $\Delta y_{r-1} > 0$ and $\Delta y_r < 0$, then $\theta_1/360°$ is the probability of a maximum occuring at y_r. Again, if θ_2 is the angle for $\Delta^2 y_{r-1} < 0$ and $\Delta^2 y_r > 0$, then $\theta_2/360°$ is the probability of change of inflection from concave to convex.

If a third linear condition is imposed, we are led to a trihedral angle. For the Spencer 21-term graduation, the probability that a maximum will appear at y_r, with $y_r > (y_{r-7} + y_{r+7})/2$ is 0.0652, corresponding to a cycle length of 15.3. Moreover, under a simple assumption, 0.0626 is the probability that y_r be a maximum, with $y_r > y_{r+5}$ and $y_r > y_{r-5}$; cycle, 16.0. Now, in this Spencer formula, the coefficients are 1/350 of —1, —3, —5, —5, —2, 6, 18, 33, 47, 57, 60, 57, 47, etc.

Plotted as a curve, the central arch has a span of about 15 units. Moreover, for several similar formulas involving a central arch leading down to negative coefficients, I find that the *span of this central arch gives a number a little less than the cycle defined as the reciprocal of the probability of substantial maxima—ripples deleted.* Other methods, involving correlation and graduation of cosine curves, lead to about the same "cycle length." Moreover, computed variances for intervals between maxima indicate considerable regularity for waves produced by such graduation formulas.

Tuesday, July 25—Reflections on Certain Problems of Instability, JACK STAFFORD, Lecturer in Economics, The Victoria University of Manchester.

From being the variable of supreme importance in the economic system, the significance of the rate of interest has recently tended to be minimized. This is largely due to recent experience.

Economists have been impressed by the facts of recent years and advanced reasons for denying past beliefs. Thus, as regards inventories, it is price fluctuations and costs of storage which are the important determinants. For capital goods of short life, rates of depreciation, maintenance charges, and expected profits overshadow the effects of likely changes in interest rates. For capital goods of longer duration, where the interest factor is proportionately more important, obsolescence risks make necessary expected profit margins too broad for interest changes to be decisive.

These views are undoubtedly important. But the question may be asked whether the pendulum has not swung too far. For this view too there is certain empirical evidence. It is hard to doubt that in times of active trade there are unsatisfied borrowers, in the sense that they would like more funds than they acquire. If borrowers are not deterred by interest-rate charges, one might expect that such borrowers would offer terms which would tempt the market to supply them with funds. In which case, interest rates might be pushed up very high to induce the requisite supply, but there would then be no unsatisfied demand for capital.

Interest rates are not usually, however, pushed extravagantly upward in times of good trade, nor are all would-be borrowers fully satisfied. This suggests the view that there do exist regulatory influences which in times of active trade bring about an equation of supply and demand. An analysis of such regulatory influences constitutes a large field for inquiry, but it may be suggested that they fall into two categories. First, there exist various kinds of rationing which depend for their effects upon imperfections of the market. Secondly, there is the rate of interest, whose direct effects we have seen to be small, but whose indirect effects may be worth further consideration.

These indirect effects seem to be largely of a financial character, flowing from the reactions of changes in interest rates upon the financial health or prospective financial health of firms. Prospective profit is not the only thing to be measured against interest rates. There is also the effect of loss, either temporary or permanent, and the effects of the burden of loss upon the firm, which may be influenced by the cost of

capital. This consideration applies primarily to bond and prior-charge financing, but an extension of the argument makes it applicable to equity financing. In other words the effects of interest rates are likely to come not from nice calculations of receipts against cost, but from whether or not the potential borrower can afford to borrow in the broadest sense.

Tuesday, July 25—The Grouping of Data for Graphic Portrayal, TRUMAN L. KELLEY, Professor of Education, Harvard University.

This paper is concerned with the graphic portrayal of a frequency distribution by means of a histogram or frequency polygon. It is further limited to the presentation of samples drawn from a parent population involving a continuous variable which does not deviate greatly from the mesokurtic and is either unimodal or possessed of a major mode which is much more pronounced than any secondary mode which it may have.

If the original data are finely graduated, so that either fine or coarse grouping may be employed as the maker of the graph may decide, it is axiomatic that many modes will be shown in the sample if a very fine grouping is employed, while, with a very coarse grouping, there must of necessity be but one mode in the graph.

The problem is to determine what grouping interval to employ so as just to eliminate the minor modes which are spurious. A coarser grouping than this lacks a permissible meaningful detail, while a finer grouping throws into relief fluctuations which are merely incidents of the sample and do not reflect the population trend.

The solution reached is embodied in a table giving the size of interval (expressed both in terms of the population standard deviation and in terms of the sample range) to employ for samples of different sizes.

These intervals are recommended for graphic portrayal only and are not considered to be of optimum size for com-

putational work leading to means, standard deviations, etc. They do, however, have a certain merit in connection with the arithmetical computation of a smoothed mode.

Wednesday, July 26—Some Dynamic Aspects of the Urban Ground Rent: I. Europe, KARL PRIBRAM, Expert, Bureau of Research and Statistics, United States Social Security Board.

The urban ground rent is a residual rent; any surplus return on the land and building above that necessary to pay operating expenses, taxes, interest, and depreciation on the building is properly assigned to the nonreproducible element in the combination, i.e., land (H. Hoyt).

According to the factors which are instrumental in producing the urban ground rent, two main types of rent can be distinguished which may be termed differential rent and absolute rent.

The differential rent is due to monopolistic elements: buildings erected on privileged sites are likely to yield higher returns than buildings erected on less privileged sites, even though the costs per unit of construction have been the same in both cases. The tendency to exploit to the fullest possible degree such sites as are privileged by location results in intensifying the utilization of land of this type both horizontally and vertically. Hence the "intensity" rent is a variety of the differential rent.

The theory of the differential urban ground rent which has first been developed by the Austrian economist von Wieser is derived, on principle, from a rather static concept of economic system. It is inadequate for explaining certain striking features of real-estate valuation, particularly the fact that, according to common belief, there exists a general tendency for all urban sites to increase in value, regardless of whether or not they are privileged by location. Hence, in addition to the differential rent, an "absolute" urban ground rent can be assumed to exist.

The concept of "absolute rent" implies that any land within the suburban area, when used for housing purposes, is likely to yield a rent higher than that which had been produced by its agricultural use; moreover that, in the long run, the ground rent of urban land of every type is increasing.

In order to explain the existence of the absolute urban ground rent it appears to be necessary to take account of the dynamic features of the economic system.

The present approach to this problem may be made in the first place in terms of European housing and building conditions which are, in many respects, far simpler and easier to analyze than the more complex conditions obtaining in this country.

It is particularly important to note that the European building cycles, when building activity was left free from governmental interference, were clearly determined by the same forces which were instrumental in generating business fluctuations; but European building cycles were in so far anticyclical as they moved, as a rule, contrary to the business cycle.

The reasons for this striking phenomenon are likely to be found in the fact that the movements of European building activity were largely influenced by the level of the interest rate and costs of construction:

Building activity took its start when the deadlock of general business activity as experienced during the depth of the depression had given way to beginning recovery, when costs of construction and interest rates were low while rentals had fallen, if at all, at a much lesser rate than would have corresponded to the reduction of the cost factors. Consequently the urban ground rent was likely to be high during the course of recovery, and this situation was primarily instrumental in stimulating building activity.

On the other hand building activity fell off when costs of construction and interest rates reached a level which rendered construction of additional dwellings unprofitable since, the more prosperity developed into the boom, the less increases in rentals kept pace with increasing costs of construction.

General increases in rentals, however, as a rule went far enough to cause the returns of all buildings to be adjusted to

the level of yield required by the costs of construction of those houses which were erected when the rapid decline in building activity began.

Since the level of these rentals was more or less maintained even during the subsequent depression, a general increase of the ground rent of all urban real-estate properties was bound to occur when costs of construction were reduced in the course of the depression and building activity recovered.

This increase in the ground rent showed all characteristics of an absolute rent; it was, to a large extent, fixed in the sales prices of vacant lots and improved sites and thus entered as a cost element into any subsequent transactions.

This analysis of the European building conditions also demonstrates the preponderant influence exerted by the development of the absolute urban ground rent upon the course of building activity. Although perhaps less conspicuous than the differential rent the absolute rent appears to have been even more important economically.

Wednesday, July 26—The Mode in Graphic and Computational Statistics, PROFESSOR KELLEY.

This paper is concerned with the computation of the mode, in the case of seemingly unimodal distributions, by means which are less involved than the fitting of a mathematically smoothed curve to the entire data, and with the reliability of such computations.

Not only is the mathematically fitted curve laborious to determine, but also in that it is an endeavor to fit all the data by means of an equation having but a few parameters it may actually give a less excellent fit, i.e., represent the population less accurately, in the neighborhood of the mode, than does a method that aims to give a good fit in this region only.

If a parabola, cubic, or biquadratic is fitted to a frequency distribution in the neighborhood of the mode, we should expect a close approximation to the population curve at this

point. However, the fitting of such a curve involves the initial step of deciding upon the grouping interval. This should have some demonstrable merit and not be consequent to any arbitrary decision of the computer.

It is then a first purpose to decide upon a grouping interval, second to show by illustration how well the method works upon data for which the mode has been determined by alternative methods, and finally it is aimed to provide simple rules for procedure and for the determination of the standard error of the mode thus determined.

In this process it has been necessary to solve a related problem—determination of the probable range covered by a sample of N cases. This has been done utilizing and extending the work of L.H.C. Tippett (*Biometrika*, Vol. 17, 1925).

Thursday, July 27—Some Dynamic Aspects of the Urban Ground Rent: II. United States, DR. PRIBRAM.

As far as the differential urban ground rent is concerned no fundamental differences appear to exist between the American conditions and their European counterparts, although rent differentials resulting from location and intensity of land utilization have attained far greater proportions in this country.

This is largely due to the violence of the dynamic forces which is characteristic of American city developments. These forces appear to be only partly responsible for a particular phenomenon connected with the differentiated uses to which residential districts are put: the real-estate cycles of residential districts which are not directly produced by fluctuations in building activity but rather by secular trends (changes in living habits and standards of housing, population movements, and the like).

Far more striking differences between the features of the European ground rent and the corresponding American features are revealed when the cyclical fluctuations of ground

rents and land values are made the object of comparative analysis. The view prevails that, as distinct from European building fluctuations, cycles in American building activity are not primarily, if at all, ruled by the same forces which are instrumental in determining the course of general business fluctuations.

The hypothesis that building costs are the controlling factor in constructional activity is not borne out by statistical evidence, nor does it appear probable that building booms are initiated by changes in land values; land values rather tend to follow a major rise in building activity. Building cycles do not only extend over much longer periods than business cycles; the amplitude of their oscillations also exceeds any similar amplitude in changes of general business activity. Varying trends in population movements have mainly been made responsible for the existence and the characteristics of the building cycles.

When the American building cycles are examined in the light of European experience, the question is to be raised to what an extent also in this country changes in the development of the absolute urban ground rent do exert a strong influence upon the course of building activity.

The connections which exist between the behavior of the urban ground rent and the course of building activity have, at least indirectly, been statistically analyzed by Charles F. Roos for the city of St. Louis.

From this analysis and other studies dealing with the fluctuations in building activity and land values the conclusion can be drawn that, as distinct from corresponding European developments, the absolute ground rent does not appear to control the course of building activity to the same extent as in Europe.

This is shown by the fact that even considerable declines of the absolute ground rent do not result in checking building activity when it is once under way in a prosperity period. Such situations could hardly develop but for the structure of the American mortgage market which is marked by a far greater capacity of expansion than the corresponding European markets.

Again, in the course of the ensuing depression, the entire general increase in rentals which had been instrumental in

stimulating the preceding building boom may be lost, with the result that an entire business cycle may take its course without producing such a rental-cost relationship that revival of building activity is stimulated.

But even re-establishment of a favorable rental-cost relationship may not result in initiating building activity during such prosperity periods during which the real-estate market is depressed by a large overhang of real-estate properties which are repossessed by financial institutions owing to foreclosures.

Since such situations cannot be cleared unless the demand for buildings has been brisk for a considerable period, increased building activity can hardly develop during the first stages of beginning recovery, as in Europe; its incidence may be postponed to a later stage when costs of construction have already reached a relatively high level.

If this analysis of the American conditions is fairly correct the assumption can be made that the American building cycle, like its European counterpart, is originated by the same forces that are instrumental in creating business fluctuations; but that certain structural characteristics of the American mortgage market prevent the absolute ground rent from exerting full influence upon the course of building activity.

Hence, one of the major objectives of official housing policy appears to consist in adopting such measures as might result in adjusting the fluctuations of building activity more closely to the European pattern, and in making the development of the absolute urban ground rent a force of paramount influence upon these fluctuations.

Thursday, July 27—Monopoly in Particular-Equilibrium and in General-Equilibrium Economics, ROBERT TRIFFIN, Tutor in Economics, Harvard University.

From the discussions that centered, a few years ago, around the theories of monopolistic and imperfect competition a definite advance has resulted: the institutional assumptions

of our theory of value have been made more realistic and its social or political significance more carefully weighted than in the past. The *pure economics* of monopolistic competition, however, is less widely understood and is obscured by many misunderstandings and confusions. In England especially, little distinction, if any, is made between monopolistic competition on the one hand, and, on the other, monopoly and oligopoly. Mr. Kahn, for instance, gives as a definition for imperfect competition the very same criterion which Mr. Lerner uses to define the case of monopoly. And oligopoly is subsumed by Mrs. Robinson under the general case of imperfect competition, through a peculiar definition of her main tool of theoretical investigation: the demand curve for the product of the individual seller. This definition, however, can be shown to imply a circular reasoning that makes it unfit for the purpose to which it is used.

Bundling together monopoly, monopolistic competition, and oligopoly because some characteristics are common to the three cases is an oversimplification that ignores essential differences between them. The distinction, however, cannot be made on the basis of the criterion that prevails today in the literature of the subject: the elasticity of the demand curve for the firm's product. Another criterion is proposed that directs our attention not to any characteristic pertaining to a single firm, taken in isolation, but to interdependence relationships between the firms. This criterion is simply the partial derivative of the revenue of the firm, with respect to the price charged by any other seller: *pure monopoly* (which is merely a limiting, not a descriptive concept) is defined by the value 0 of this criterion; the product-differentiation case (or *heterogeneous competition*) is characterized by the finite values; *homogeneous competition* (covering both pure oligopoly and pure competition) by the value ∞ .

This tripartite classification makes no reference as yet to the problem of oligopoly. Etymologically, the term oligopoly applies to a situation where the sellers are few. In fact, however, the fewness of the sellers is merely an indication, or a *presumption,* that difficulties of a particular kind will appear that will imperil the determinateness of the equilibrium solution, or, at least, call for additional assumptions or informa-

tion to render the case determinate. A cross-classification of the cases previously distinguished is introduced that corresponds to the Chamberlinian subdivision of monopolistic competition between large and small numbers, and to the traditional difference made between pure oligopoly and pure competition (within the general case of homogeneous competition).

The whole classification is independent of any concept of groups (or industries) and of numbers. It is argued that the concept of group, or industry, uncritically inherited from the language of everyday life, was used arbitrarily by the classicists to assume gratuitously the functioning of very definite and precise economic mechanisms (the mechanisms of pure competition). To give it a precise content, the notion of industry must be derived a posteriori from an empirical investigation of market reactions. Even so, the concept remains somewhat vague and flexible when product differentiation has to be taken into account. It is in any case superfluous to the pure theory of value; this can be built up from the individual firms in competition with one another throughout the entire economy.[1] Such a treatment simplifies considerably the exposition, while nothing of value is lost through the abandonment of the group concept. On the contrary, this approach forces us to think more clearly and logically, and helps us to remove some inexactitudes or ambiguities from the traditional statements of general-equilibrium theory.

Due to the implications of his concept of industry coupled with particular-equilibrium methodology, Marshall could know only of two types of market relationships: pure monopoly and homogeneous competition. When the products of the firms are not identical, so that the mechanisms of pure competition do not apply, each firm becomes an industry, each seller a monopolist. The real significance of the Marshallian monopoly case—the singleness of the seller—is to identify a firm with an industry, to substitute for the shifting demand curve of the former the stable demand curve which particular-equilibrium methodology bequests to the latter. The analytical expression of this setup is provided by the criterion suggested above. In opposi-

[1] Cf. forthcoming dissertation in the Harvard Economic Studies, entitled *Monopolistic Competition and General Equilibrium Theory.*

tion to Marshall, Pareto supplements his analysis of the individual equilibrium of the monopolist with a study of the equilibrium conditions external to the firm. As distinct from monopolistic-competition procedure (evolved within particular-equilibrium traditions), this study of the conditions of equilibrium external to the firm takes in the competitive interrelationships of the sellers throughout the entire economic collectivity.

Friday, July 28—Some Neglected Variables in the Theory of Commodity Speculation,* JOHN B. CANNING, Professor of Economics, Stanford University.

The casual and inchoate comments on contracts markets found in the writings of theorists seem to be almost exclusively concerned with certain elementary relations between two particular points on an unspecified function. The operator, say in a wheat future, is thought of as focussing his study on the cumulating information on the supply and demand for wheat with intent to forecast the most probable price of such wheat in the spot market at the closing date of the future. Having forecast such a price and having taken carrying charges into account, he can make either a bid or an offer such that, if his forecast is realized in the event, he can make an acceptable gain. With the day-to-day changes in the aggregate and composition of market information this operator's bid and offer limits change.

The theory of even this oversimplified market leaves out of account several important independent variables such as:

(1) Differences in mode of evaluating market news;

(2) Changes in operations due to effects of antecedent operations upon limited funds;

(3) Changes in the number and identity of operators;

(4) The systems of premiums and discounts on tenderable grades;

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(5) The conditions of substitution;

(6) The conflict between minimizing errors of estimate and "beating the market" upon the "breaks" in the news; and

(7) The degrees of nonindependence of estimates of operators.

A full development of these—even within the limits imposed by an oversimplified market and a two-point residual function—may very well lead to important changes in the end products of our consideration. This is especially true of the sixth and seventh items.

But operations are not restricted to considerations relevant to the values of the limiting ordinates of the residual price function and the interval between them. Any other two or more residual ordinate values will serve. In fact the course of quotations and its rates of change—not the mere destination price—afford opportunities for acting on expectations.

Moreover, some operators may specialize in estimating changes in the expectation of other operators. Indeed, it is quite possible (and on plausible a priori grounds) to suppose that a preponderance of operators do so. In fact, the phenomena of movement trading bears all the marks of market instability to be expected if operators try to guess each other's expectation about market expectations—and do so to the more or less complete neglect of commodity news.

When scalping and movement trading become highly developed there are grounds for supposing that they make the commonly considered types of trading next to impossible.

In addition to consideration of immediate or proximate consequences of adverse events to entrepreneurs on unhedged risks as against the corresponding consequences to speculators on the hedged risks it is desirable to consider balances of consequential damages—as against consequential gains—to the two market groups. This takes on special importance if means should be found—say, via the Federal Crop Insurance Corporation—of hedging farmers' risks between seedtime and harvest. Even superficial study of the chains of consequential damage due to adverse price change in the seedtime-to-harvest interval discloses very powerful incentives to extend hedging to this stage of production.

Friday, July 28—The Money Factor in Trends of Trade, PROFESSOR DAVIS.

The equation of exchange, $MV = PT$, is a tautology, which says that what we buy is equal to what we spend. It loses its tautological character, however, when we seek the relationships between the four variables. Carl Snyder has investigated these relationships statistically and his conclusions have been formulated in a system of postulates by E. V. Huntington.[1] The present paper re-examines Snyder's system from the point of view of recent events.

Snyder's system states (1) that the trend of the velocity of circulating deposits is constant; (2) that the percentage variation in trade equals the percentage variation in velocity. He concludes that prices will remain constant provided money is kept proportional to the trend of trade.

A statistical examination of this system shows that the theory is satisfactory as an explanation of most of the changes in price from 1896 to 1930. Major discrepancies are found in the computed price as compared with the real price for the years 1920 and 1929. The latter may be partially explained by the fact that the general price index contained a fixed system of weights and that the weight of 4 per cent for common stock prices was unrealistic in 1929. The former variation apparently created a strain in the economic system, which was measured by an increase in money tension rather than by a change in the velocity of money.

Since 1930, however, there has been a continuous disagreement between the computed price and the actual price. This disagreement finds its root in the curious behavior of the velocity factor, which has remained abnormally low during the current period. The average velocity from 1896 to 1930 was approximately 45.6; from 1930 to date the velocity has been 26.0. Hence, we must assume either that velocity has established a new trend, or that current velocities are depressed below the normal trend by factors other than those recognized by Snyder's theory.

The most outstanding coincidental variation that one ob-

[1] *Econometrica*, Vol. 6, April, 1938, pp. 177-179.

serves in present series is the index of money tension, measured by the change in interest rates on 4- to 6-months commercial paper. In 1920 money tension was abnormally high, but velocity had attained an abnormal value. Since 1930 both money and velocity have declined simultaneously to the lowest levels ever recorded in this country. It seems possible to assume, therefore, that differences between Snyder's price and real price are accompanied by an abnormal variation in money tension. Thus, in 1920 money tension was reduced by a spectacular fall in prices without any disruption of velocity. In 1929 the disparity was corrected by a diminishing of money tension and a continuous fall in velocity. Since 1933 the disparity may be corrected by assuming that the trend of velocity is functionally related to money tension. Hence, while this tension is low, the trend of velocity will be low and prices again may be represented accurately by Snyder's theorem.

The creation of low money tension appears to be related to government deficit spending, which is financed by the issuing of large quantities of government securities. These securities lower the interest rate, but do not affect price. As deficit spending continues, higher and higher taxes are levied and this apparently is the mechanism which attacks the velocity factor.

The paper also gave statistical definitions of the various quantities in the equation of exchange and illustrated their behavior both for American data and for data associated with the economy of Spain from 1500 to 1650.