SOME A POSTERIORI PROBABILITIES IN STOCK MARKET ACTION

By Alfred Cowles 3rd and Herbert E. Jones

In 1933, one of the authors[1] published an analysis of the results secured by 45 representative financial agencies in forecasting the prices of common stocks. This study, embracing the period of $4\frac{1}{2}$ years from January, 1928 to June, 1932, and including several thousand individual forecasts, showed that these were unsuccessful more often than successful and that, indeed, better results in the aggregate would probably have been secured by investors through following purely random investment programs. This result naturally suggested the question: Is stock price action random in nature or, if not, to what extent is it possible statistically to define the nature of its structure? Herewith is presented at least a partial answer to this question on the basis of evidence adduced from internal elements in the stock price series themselves.

Among the different viewpoints from which this problem may be approached are those which consider (1) the element of inertia, and (2) harmonic analysis for the purpose of disclosing evidence as to regular periodicity.

With respect to the latter, Professor Harold T. Davis[2] has presented evidence of periodicity in stock prices by means of Schuster's periodogram analysis, and has attempted tests of the significance of these periods by R. A. Fisher's technique. By this method, however, only *average* periods can be found and even then their significance can not be accurately determined because of uncertainty as to the independence of observations. Variable periodicity can be taken into account by a number of techniques, such as Frisch's changing harmonics,[3] or by using varying moving averages,[4] but there seems to be no way to determine the probabilities of significance in connection with any of these methods. For this reason in the following analysis a method more susceptible of interpretation in terms of probability has been used.

Carl Snyder,[5] in referring to measurements of the industrial growth

[1] Alfred Cowles 3rd, "Can Stock Market Forecasters Forecast?", ECONOMETRICA, Vol. 1, 1933, pp. 309–324.

[2] In a paper presented before the Econometric Society in St. Louis, January, 1936. See abstract in ECONOMETRICA, Vol. 4, 1936, pp. 189–190.

[3] Ragnar Frisch, "Changing Harmonics and Other General Types of Components in Empirical Series," *Skandinavisk Aktuarietidskrift*, 1928, pp. 220–236.

[4] Gerhard Tintner, *Prices In the Trade Cycle*, Vienna, 1935, p. 23.

[5] "Concepts of Momentum and Inertia," presented at the Atlantic City meeting of the Econometric Society, December, 1932, and published in *Stabilization of Employment*, Charles F. Roos, editor, 1933, p. 77.

of the United States since 1830, says, "The picture that these measures give is that of an amazingly even rate of growth not merely from generation to generation but actually of *each separate decennium* throughout the last century. As if there was at work a kind of momentum or inertia that sweeps on in spite of all obstacles."

The term "inertia," as employed by Mr. Snyder, may be said to be macroscopic, whereas its use in the present study is microscopic. He was concerned with the trend over a period of 100 years or more and concluded that all deviations from that trend were nothing more than inconsequential "jiggles." The present analysis, on the other hand, is concerned primarily with evidence as to inertia in movements of a few hours, days, weeks, months, or years, which Mr. Snyder, with his longer-range viewpoint, would designate as mere "jiggles."

Evidence of inertia may be disclosed in the following manner. In a penny-tossing series there is a probability of one-half that tails will follow heads and vice versa. If the stock market rises for 1 hour, day, week, month, or year, is there a probability of one-half that it will decline in the succeeding comparable unit of time? In an attempt to answer this question sequences and reversals were counted, a sequence occurring when a rise follows a rise, or a decline a decline, and a reversal occurring when a decline follows a rise, or a rise a decline.

A study of the ratio of sequences to reversals will disclose structure as defined above, if it exists within the series, and the significance of this structure can be defined by ordinary statistical methods. For instance, the probability can be determined that any ratio occurred by chance from a random population.[6] Also, the consistency of these ratios can be investigated and from their frequency distributions one can determine the probabilities of success in forecasting a rise or decline in stock prices. Samples, of adequate length where available, were ex-

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[6] L. Besson, "On the Comparison of Meteorological Data with Results of Chance," translated and abridged by E. W. Woolard, *Monthly Weather Review,* Vol. 48, 1920, pp. 89–94, pointed out that in a *random series*, the ratio of sequences to reversals will be 0.5, that is, there will be twice as many reversals as sequences. It should be noted that, in the present analysis, the data employed are not *random*, but rather *cumulated random series*, that is series in which the first differences, rather than the actual observations, are random. In a truly random series the auto-correlation drops to zero when the series is lagged against itself by so much as one observation. In a cumulated random series this is not the case. For example, in the stock price series under consideration, even when the first differences have been rearranged in a random manner, an auto-correlation with a lag of one observation will yield a very high coefficient. This correlation coefficient will be a function of the length of the series and is approximately equal to $1 - \log n/(n-1)$. In such a series, with the first differences random, the ratio of sequences to reversals will be, if n is large, 1.0 instead of 0.5 as observed in the case of a series in which the observations themselves are random.

amined, the intervals between observations being successively 20 minutes, 1 hour, 1 day, 1, 2, and 3 weeks, 1, 2, 3, $\cdots$, 11 months, and 1, 2, 3, $\cdots$, 10 years. The results of this investigation are presented in Table 1 and are shown graphically in Figure 1.

It was found that, for every series with intervals between observations of from 20 minutes up to and including 3 years, the sequences outnumbered the reversals. For example, in the case of the monthly series

TABLE 1

RATIO OF SEQUENCES TO REVERSALS IN STOCK PRICE INDEXES

UNIT	INDEX	PERIOD	NUMBER OF OBSERVATIONS	RATIO OF SEQUENCES TO REVERSALS	PROBABILITY OF OCCURANCE BY CHANCE	UNIT	INDEX	PERIOD	NUMBER OF OBSERVATIONS	RATIO OF SEQUENCES TO REVERSALS	PROBABILITY OF OCCURANCE BY CHANCE
20 MINUTES	HARRIS UPHAM	1935-1936	2800	1.44	<.000001	8 MONTHS	INDEX OF RAILROAD STOCK PRICES	1835-1935	156	1.48	.01640
1 HOUR	DOW JONES HOURLY AVGS.	1933-1934	800	1.29	.00040	9 MONTHS	INDEX OF RAILROAD STOCK PRICES	1835-1935	138	1.57	.01016
1 DAY	DOW JONES HOURLY AVGS.	1931-1935	1200	1.18	.00094	10 MONTHS	INDEX OF RAILROAD STOCK PRICES	1835-1935	124	1.49	.03000
1 WEEK	STANDARD STATISTICS	1918-1935	938	1.24	.00386	11 MONTHS	INDEX OF RAILROAD STOCK PRICES	1835-1935	113	1.27	.21870
2 WEEKS	DOW JONES	1897-1935	976	1.02	.80258	1 YEAR	INDEX OF RAILROAD STOCK PRICES	1835-1935	100	1.17	.42952
3 WEEKS	DOW JONES	1897-1935	652	1.08	.30772	2 YEARS	INDEX OF RAILROAD STOCK PRICES	1835-1935	50	1.63	.08726
1 MONTH	INDEX OF RAILROAD STOCK PRICES	1835-1935	1200	1.66	<.000001	3 YEARS	INDEX OF RAILROAD STOCK PRICES	1835-1934	33	1.46	.28914
2 MONTHS	INDEX OF RAILROAD STOCK PRICES	1835-1935	600	1.50	<.000001	4 YEARS	INDEX OF RAILROAD STOCK PRICES	1835-1935	25	0.85	.68180
3 MONTHS	INDEX OF RAILROAD STOCK PRICES	1835-1935	400	1.29	.01242	5 YEARS	INDEX OF RAILROAD STOCK PRICES	1835-1935	20	1.00	1.00000
4 MONTHS	INDEX OF RAILROAD STOCK PRICES	1835-1935	300	1.18	.16452	6 YEARS	INDEX OF RAILROAD STOCK PRICES	1835-1931	16	0.67	.44130
5 MONTHS	INDEX OF RAILROAD STOCK PRICES	1835-1935	249	1.52	.00120	7 YEARS	INDEX OF RAILROAD STOCK PRICES	1835-1933	14	0.71	.56192
6 MONTHS	INDEX OF RAILROAD STOCK PRICES	1835-1935	208	1.40	.01778	8 YEARS	INDEX OF RAILROAD STOCK PRICES	1835-1931	12	0.22	.03486
7 MONTHS	INDEX OF RAILROAD STOCK PRICES	1835-1935	178	1.38	.03486	10 YEARS	INDEX OF RAILROAD STOCK PRICES	1835-1935	10	0.60	.74140

The Index of Railroad Stock Prices is composed of several series which were assembled and put together by the Cleveland Trust Company. Canal stock prices were used for the period 1831 through 1833. From 1834 through 1879 the index is based on three Harvard series. The one from 1834 through 1852 includes eight stocks, and that from 1853 through 1865 includes 18. The data are from *The Review of Economic Statistics* for August, 1928. The index from 1866 through 1879 includes 10 stocks, and the data are from *The Review of Economic Statistics* for 1919. The index from 1880 through 1896 includes 10 stocks, and from 1897 to date it includes 15. These two latter indexes were compiled by the Cleveland Trust Company. All the earlier indexes were adjusted to form a continuous series terminating with the final index of 15 stocks.

from 1835 to 1935, a total of 1200 observations, there were 748 sequences and 450 reversals. That is, the probability appeared to be .625 that, if the market had risen in any given month, it would rise in the succeeding month, or, if it had fallen, that it would continue to decline for another month. The standard deviation[7] for such a long series

[7] In a random penny-tossing series the probability of a sequence or reversal is

constructed by random penny tossing would be 17.3; therefore the deviation of 149 from the expected value 599 is in excess of eight times the standard deviation. The probability of obtaining such a result in a penny-tossing series is infinitesimal. If the unit of time be increased to 6 months, we find that there are 120 sequences to 86 reversals or what appears to be a .583 probability that a sequence will occur in any successive pair of periods. The probability in this case is
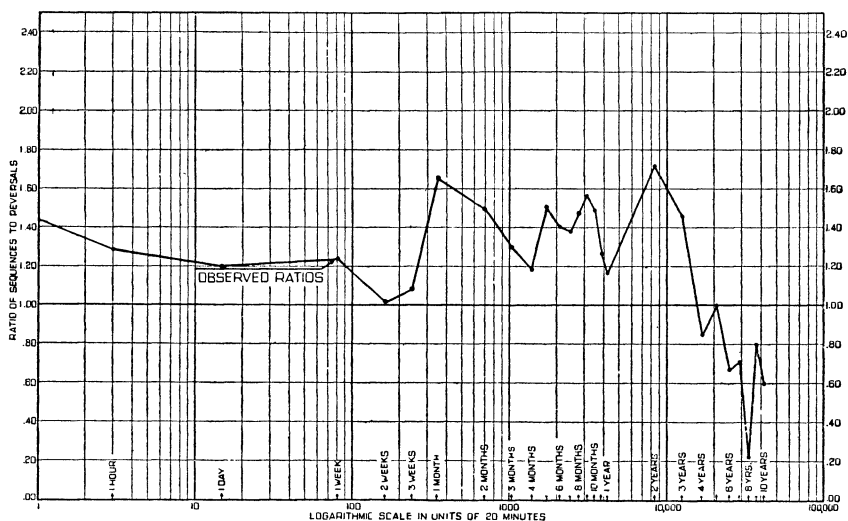

FIGURE 1.—Ratio of sequences to reversals in direction of stock price indexes for various time intervals.

.01778 that such a ratio of sequences to reversals might occur in a random series such as that of penny tossing previously referred to.

For annual series, the probability for chance occurrence, on the same basis as before, is .42952. In fact, the probabilities are inconclusive for all series using units of over 6 months, although this may be due to the limitations of the data. There seems to be a good chance that with more data it might be possible to demonstrate the existence of struc-

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$\frac{1}{2}$ and the standard deviation is $\sqrt{npq}$ or $\sqrt{n}/2$ where n is the number of observations and p and q are respectively the probabilities of success and failure. Sequences or reversals are determined from the first differences of the original price series. The total number of sequences and reversals is one less than the number of first differences. The standard deviation of the number of sequences, therefore, will be $\sqrt{n-2}/2$ where n is the number of observations of the original price series. The actual deviation will be $S-(n-2)/2$ where S is the observed number of sequences and the ratio $[S-(n-2)/2]/[(n-2)^{\frac{1}{2}}/2]$ is the observed deviation in terms of the standard deviation. The probability of as large a deviation occurring by chance can be found in the ordinary table of the normal probability function.

ture for all units of time up to and including 3 years. It is difficult, however, to explain the very small excess of sequences over reversals for series using intervals of 2 weeks and 3 weeks in view of the fact that for the slightly shorter unit of 1 week we have significant indication of structure and also for series using intervals of 1 month. In fact, as will be shown later, the series represented by units of 1 month proves to be the most significant from a practical point of view.

The above analysis was based upon a study of the stock market as a whole. It may now be of interest for us to examine the evidence with regard to industrial groups such as motors, oils, steels, and so forth. To this end the indexes of common stock prices of 61 industrial groups, prepared by the Standard Statistics Company, were analyzed. Their monthly deviations from the median were noted, the median being used because more groups are normally below, than above, the arithmetic means, in view of the fact that extremely large gains are larger in percentage than extremely large declines. Sequences and reversals were counted for the purpose of determining whether there was a tendency for such groups to persist in exceeding, or falling below, the median. In other words, if the oil stocks, as a group, were in the 30 of the 61 industrial groups which advanced more than the median group in January, what is the probability that the oils will also be found in the strongest 30 groups in February? For the 16 months from January, 1934 to April, 1935, when the general market movement was approximately horizontal, in 917 observations there were 570 sequences and 345 reversals. That is, if the oil stocks were among the strongest 30 groups in January the probabilities would appear to be .623 that they would also be found among the strongest 30 groups in February. The application of the theory of probability to interpret the significance of this result is hampered, as in many other analyses of economic time series, by uncertainty as to what is the number of independent observations in the sample. The action of the oils, for example, may be correlated with that of the motors or some other group, so that when one is stronger than the median, the other also tends to be stronger. Modern statistical technique appears to offer no ready solution for this problem of the independence of observations, and we must, therefore, content ourselves with obtaining unusually favorable probabilities.

In the period, May, 1935 to February, 1936, a rising market of 551 observations, there were 379 sequences and 170 reversals, indicating an apparent probability of .690 in favor of a sequence. Here, however, another factor has intruded itself. The stocks of certain industries can be shown to be more cyclical in nature than those of other industries. For instance, in severe depressions the building of new houses is almost completely stopped, and yet people under such conditions go on eating

almost as much food as in periods of prosperity. These tendencies are reflected by wider fluctuations in the earnings of producers of building materials than in the case of purveyors of food, and also by wider cyclical fluctuations in the stocks of the former corporations than of the latter. This tendency results, in the case of a rising market, in the building-stock prices being persistently stronger than the average, and the food stocks persistently weaker. A count of sequences and reversals under such conditions measures, therefore, to some extent, the differences in cyclical behavior among the various groups rather than the tendency toward inertia which is measured in the case of a period where the market as a whole is moving horizontally. Since there are not many periods of great length in which the market as a whole has moved horizontally, we are limited in the data available for this particular analysis. Considering only the data for the horizontal 16 months from January, 1934 to April, 1935, the excess of sequences over reversals is 7.5 times the standard error for a random series. Even assuming that half of the observations are not independent, there is still the exceedingly small probability of .00168 of occurrence in a random series.

The action of individual stocks was also investigated. Instead of the oils as a group, it was considered, for example, whether the Standard Oil Company of New Jersey, if it were stronger than the median of all stocks in January, would more likely than not be stronger in February. Taking 190 representative stocks for the years 1934 and 1935 and the first three months of 1936, and using the same technique as that employed in the case of the industrial groups, a total of 4659 observations, there was found, when the market moved horizontally, from January, 1934 to May, 1935, a ratio of sequences to reversals of 1.07 to 1 which is 9 times the standard error. In the last 12 months, a rising period, the ratio of sequences to reversals was 1.29 to 1. It follows from the previous discussion of cyclical behavior that the difference between the two periods is to be expected and that the evidence of the second period must be brought in question.

Taking 1 year as the unit of measurement for the period from 1920 to 1935, the tendency is very pronounced for stocks which have exceeded the median in one year to exceed it also in the year following. In 1837 observations were 1200 sequences and 635 reversals. The excess of sequences was about 13 times the standard error for a random series constructed on the basis of equal probabilities. During the period under consideration the market as a whole manifested 8 sequences and 7 reversals. The evidence therefore seems to indicate that, when 1 year is the unit of time, individual stocks will persist in doing better, or worse, than the median of all stocks.

This evidence of structure in stock prices suggests alluring possibilities in the way of forecasting. In fact, many professional speculators, including in particular exponents of the so-called "Dow Theory" widely publicized by popular financial journals, have adopted systems based in the main on the principle that it is advantageous to swim with the tide. The practicability of such forecasts, however, will depend,

TABLE 2

ABSOLUTE PERCENTAGE CHANGES IN STOCK PRICE INDEXES

UNIT	INDEX	PERIOD	NUMBER OF OBSERVATIONS	AVERAGE ABSOLUTE CHANGE IN PERCENT	STANDARD DEVIATION OF AVERAGE	UNIT	INDEX	PERIOD	NUMBER OF OBSERVATIONS	AVERAGE ABSOLUTE CHANGE IN PERCENT	STANDARD DEVIATION OF AVERAGE
20 MINUTES	HARRIS-UPHAM	JULY-9-1936 JULY-17-1936	111	0.12	.01	9 MONTHS	INDEX OF RAILROAD STOCK PRICES	JUNE 1859 JUNE 1934	100	12.73	1.29
1 HOUR	DOW JONES HOURLY AVGS.	SEPT-12-1935 OCT-8-1935	102	0.32	.03	10 MONTHS	INDEX OF RAILROAD STOCK PRICES	JAN 1851 APRIL 1934	100	13.00	1.31
2 HOURS	DOW JONES HOURLY AVGS.	AUG-1-1935 OCT-6-1935	103	0.47	.04	11 MONTHS	INDEX OF RAILROAD STOCK PRICES	DEC 1842 JULY 1934	100	13.99	1.25
1 DAY	DOW JONES	AUG-27-1934 DEC-31-1934	102	0.73	.07	1 YEAR	INDEX OF RAILROAD STOCK PRICES	JAN 1831 JAN 1934	103	14.70	1.43
1 WEEK	DOW JONES	JAN-6-1913 DEC-31-1934	1128	2.56	.21	2 YEARS	INDEX OF RAILROAD STOCK PRICES	JAN 1831 JAN 1933	51	22.58	2.78
1 MONTH	DOW JONES	JAN-1-1897 DEC-31-1934	451	3.70	.46	3 YEARS	INDEX OF RAILROAD STOCK PRICES	JAN 1831 JAN 1933	34	28.03	4.81
2 MONTHS	DOW JONES	APRIL-1-1916 DEC-1-1934	100	5.02	.91	4 YEARS	INDEX OF RAILROAD STOCK PRICES	JAN 1831 JAN 1932	25	30.59	4.77
3 MONTHS	INDEX OF RAILROAD STOCK PRICES	JAN 1835 DEC 1934	400	8.92	.79	5 YEARS	INDEX OF RAILROAD STOCK PRICES	JAN 1831 JAN 1931	20	33.95	5.71
4 MONTHS	DOW JONES	DEC 1900 DEC 1934	100	10.79	.99	6 YEARS	INDEX OF RAILROAD STOCK PRICES	JAN 1831 JAN 1933	17	38.59	8.90
5 MONTHS	INDEX OF RAILROAD STOCK PRICES	JAN 1893 SEPT 1934	100	8.62	.82	7 YEARS	INDEX OF RAILROAD STOCK PRICES	JAN 1831 JAN 1929	14	33.54	9.62
6 MONTHS	INDEX OF RAILROAD STOCK PRICES	DEC 1884 DEC 1934	100	10.04	1.20	8 YEARS	INDEX OF RAILROAD STOCK PRICES	JAN 1831 JAN 1927	12	32.38	9.15
7 MONTHS	INDEX OF RAILROAD STOCK PRICES	JUNE 1876 OCT 1934	100	11.81	1.30	9 YEARS	INDEX OF RAILROAD STOCK PRICES	JAN 1831 JAN 1930	11	45.98	13.64
8 MONTHS	INDEX OF RAILROAD STOCK PRICES	APRIL 1868 DEC 1934	100	11.30	1.13	10 YEARS	INDEX OF RAILROAD STOCK PRICES	JAN 1831 JAN 1931	10	51.64	10.90

not only on the ratio of sequences to reversals, but also on the brokerage costs and the average change in stock prices during the unit of time selected. The brokerage costs, of course, are known. To determine the average percentage change in the stock prices for various units of time an extensive study was made. The difference between the index at the beginning of one unit and the beginning of the next, given in percentage of the former value, was computed. The results are shown in Table 2. These averages are absolute values, that is, the directions of the moves were not considered. The values, plotted on a time scale, are shown in Figure 2. The increase is very regular, approximating a smooth exponential curve as shown in the diagram.

With these data it is possible to compute the net gain which would have resulted from an application of this type of forecasting to the stock market averages.

Let

$\quad I(t)$ = Expected annual net profit, in per cent,

$\quad R(t)$ = Ratio of sequences to reversals for time interval t,

$\quad C(t)$ = Average change per time interval t in per cent,

$\quad Y(t)$ = Number of time intervals, t, in one year,

$\quad B$ = Brokerage cost for one complete trade, in per cent.

In the long run there will be an average of $R(t)$ sequences for each reversal, that is, a speculator will, on the average, be on the right side of the market for $R(t)$ units of time for each unit that he is on the
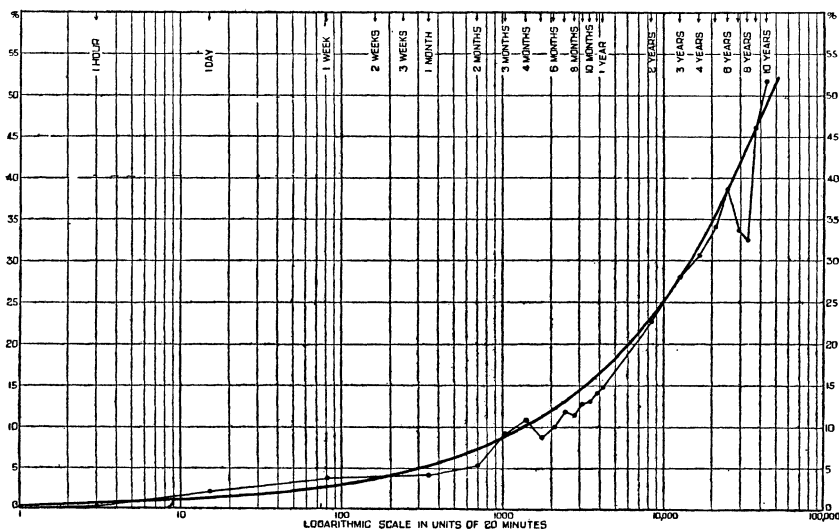
FIGURE 2.—Absolute percentage changes in stock price indexes.

wrong side, if his market position is changed only *after* the occurrence of each reversal. The average net time in the right direction between changes of position will then be $[R(t)-1]$ time units. The average move per unit of time is $C(t)$; therefore, the *gross* gain per position will then be $[R(t)-1]C(t)$. The *net* gain per position will be

$$[R(t) - 1] \cdot C(t) - B.$$

Since one will be in the market in the right direction $R(t)$ units of time and in the wrong direction 1 unit, the total time per position will be $[R(t)+1]$. The number of positions taken per year will be $Y(t)/[R(t)+1]$. The net annual gain, in per cent, will then be given by

$$I(t) = 100\left[\left\{ [R(t) - 1]C(t) - B \right\} \frac{1}{100} + 1 \right]^{Y(t)/[R(t)+1]} - 100.$$

When the values of $R(t)$ and $C(t)$ are substituted in this equation values of the expected annual net gain for various units of time and brokerage costs are obtained as shown in Table 3.

<div align="center">TABLE 3</div>

Time Unit	Ratio of Sequences to Reversals $R(t)$	Average Percent- age Changes $C(t)$	Expected Annual Net Profit		
			Brokerage Costs of:		
			1%	1½%	2%
1 day	1.18	0.73%	−67.4%	−83.0%	−91.1%
1 week	1.24	2.56	− 8.55	−18.6	−27.6
1 month	1.66	3.70	6.66	4.25	2.00
2 months	1.50	5.02	3.66	2.44	1.23
3 months	1.29	8.92	2.79	1.91	1.03

As might be expected, the daily and weekly units are too short. The probability of success is not sufficient to compensate for the fact that the changes per unit of time are small relative to brokerage costs. The average net gain *per trade* is largest for units of 2 months but, because with this unit so few trades are completed in a year, the annual net gain is less than when units of one month are used. It appears, indeed, that, for the period under consideration, one month is the optimum unit of time.

At this point a study was undertaken to determine whether it would be possible to select a group of stocks which would continue to be more volatile than the average and whether, if this could be done, the results of speculation, employing such a list, would be more successful than where average stocks were used. An investigation was made covering the period from 1900 to 1919. The number of stocks examined was 44 at the beginning of the period, increasing to 81 by 1918. The 10 per cent manifesting the greatest movement for 1 year, in the direction of the market, was chosen as the volatile group. It was found that such a group was about 1.7 times as volatile as the market for 6 months following its selection. The list of stocks making up the volatile group was adjusted at the end of each 6-month period.

The list of rapidly moving stocks, selected as described above, was then subjected to a comparison with the results secured employing average stocks for the period under consideration. The average stocks would have yielded an average annual net gain of 8.9 per cent while the volatile stocks would have shown an average net gain of but 5.4 per cent. Though the volatile group manifested an average move considerably more than that of the market, this was more than offset by the

fact that its ratio of sequences to reversals was less favorable. The substitution of volatile for average stocks was therefore abandoned.

In the case of the stock market averages a study was undertaken to determine the degree of consistency in the data by considering distributions of the variables over finite periods of time. Figure 3 represents the frequency distributions of the ratios of sequences to reversals for units of 1 day, 1 week, 1 month, and 3 months. Periods of 12 and 24 units of time were considered in all cases except the last, where insufficient data made it necessary to consider periods of only 4 units. In the upper part of each diagram are the actual histograms, or frequency distributions. A better way to illustrate the probability of variations is by summation, or cumulative frequency distribution. The histograms, therefore, were smoothed, as shown by the solid continuous curve, and the cumulated curves computed from the smoothed distributions. The dotted curves represent the distributions that would be expected from series for which the probability of a sequence or reversal is $\frac{1}{2}$. They were computed from the formula

$$P(S) = \frac{{}_{n-2}C_S}{2^{n-2}},$$

where $P(S)$ is the probability of obtaining S sequences in a sample of n observations.[8]

For units of 1 day the periods of 12 units have a distribution very nearly random with a slight tendency towards a skewness to the right. This skewness is more pronounced in the diagram for periods of 24 units.

The cumulative frequency curves have a scale on the left giving the probabilities of a smaller ratio, and a scale on the right, the probabilities of a larger ratio. From the figure it will be seen that the probability of obtaining a ratio of sequences to reversals less than 1 is about .43. Or again, the probability of obtaining a ratio larger than 2 is about .20 in the case of periods of 12 days and about .09 for periods of 24 days. From such a diagram it is possible to determine the limiting ratio for any probability. For instance, the limits of a probability band of .50 can be determined by finding the ratios for which the probabilities are .75 and .25, respectively, of obtaining a larger ratio. In this case, for

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[8] In n observations there are $(n-1)$ first differences. In this set of $(n-1)$ first differences there can be S sequences and $(n-2)-S$ reversals. The number of different orders in which $(n-2)$ things can be arranged in two sets, S and $(n-2)-S$, is $(n-2)!/S!(n-2-S)! = {}_{n-2}C_S$. But a sequence can occur when either a rise follows a rise or a decline follows a decline. The total number of samples containing S sequences, therefore, will be $2 \cdot {}_{n-2}C_S$. Since there are 2^{n-1} possible samples, the probability of obtaining S sequences will be given by the above equation.
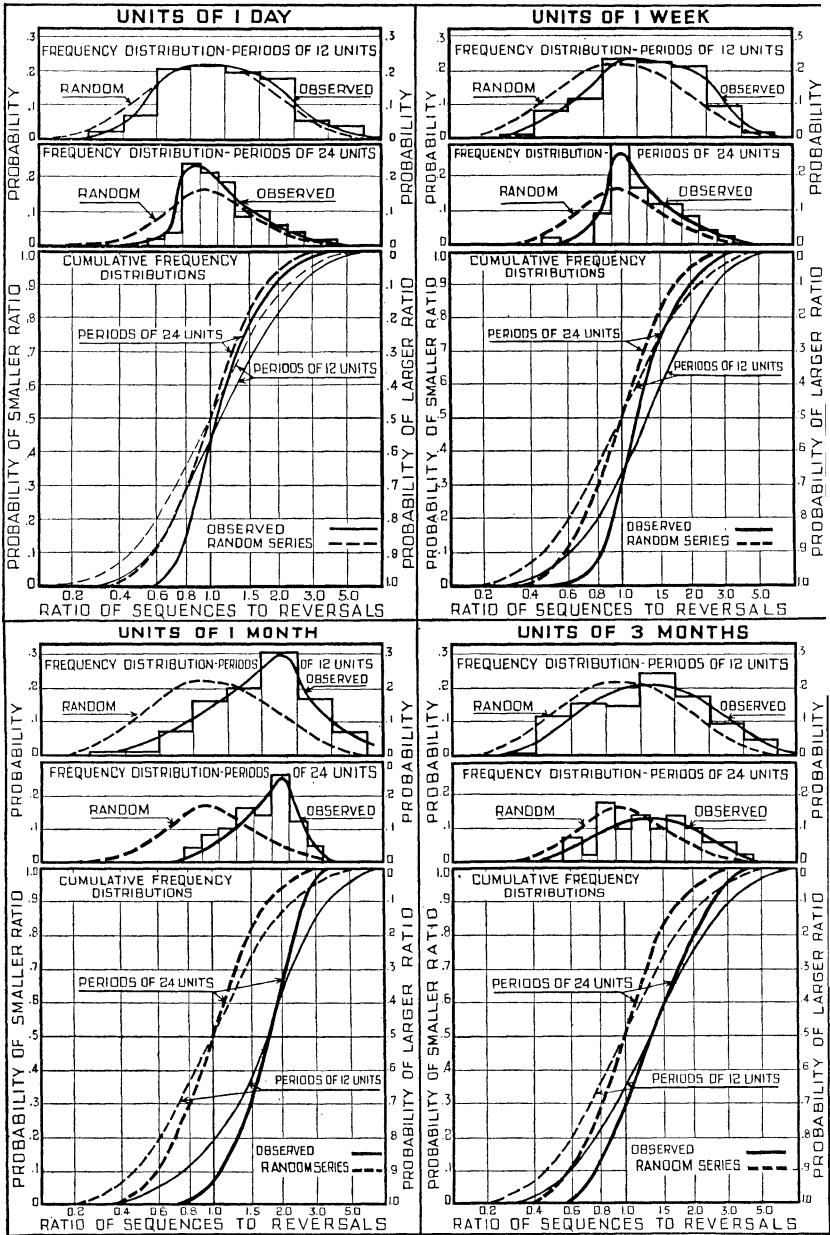
FIGURE 3.—Frequency distributions of ratio of sequences to reversals
in direction of stock price indexes.

periods of 24 units, 0.9 is the lower limiting ratio and 1.4 the upper ratio. The median ratio is 1.1. Therefore, the ratio of sequences to reversals, for periods of 24 days, is 1.1; and 50 per cent of the time the ratios will lie between .9 and 1.4.

The distribution for units of 1 week shows a greater tendency towards structure than in the case of daily units. In this case the cumulative curve indicates that the probabilities are about 2 to 1 in favor of a ratio greater than unity. The tendency towards structure is even more
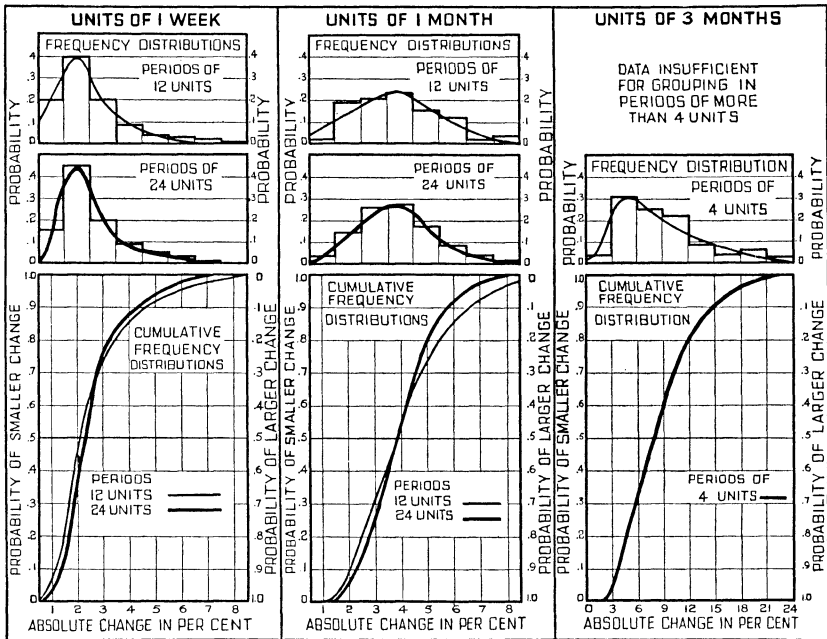

FIGURE 4.—Frequency distributions of absolute percentage changes in stock price indexes.

pronounced for units of 1 month where, for periods of 12 months, the chances are about 4 to 1 in favor of a ratio greater than unity. The cumulative curves are relatively steep, also, which indicates a high concentration around the average. This is especially true for the periods of 24 months. For units of 3 months the distributions are not nearly so favorable as in the previous case. They are flatter, indicating large variations, and the average ratio is much smaller.

Figure 4 shows the results of the same type of analysis applied to the data for the percentage changes. In this case only three units of time were investigated as the daily units already had proved to be impracti-

TABLE 4

COMPUTATION OF DISTRIBUTION OF $I(t) = 100 \left[\{[R(t) - 1]C(t) - B\} \dfrac{1}{100} + 1 \right]^{Y(t)/[R(t)+1]} - 100$

For Units of 1 Month, i.e., $Y(t) = 12$, and Brokerage $(B) = 1\%$
Roman type figures within table give values of $I(t)$
Italic type figures within table give probabilities of $I(t)$

AVERAGE PERCENTAGE CHANGES—C(t)

RATIO OF SEQUENCES TO REVERSALS—R(t)	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	PROBABILITY OF R(t)
.50	−13.17 *.0015*	−16.65 *.0042*	−20.00 *.0054*	−23.23 *.0043*	−26.34 *.0025*	−29.35 *.0010*	−32.25 *.0006*	−35.04 *.0004*	*.020*
.71	− 9.65 *.0060*	−11.50 *.0166*	−13.32 *.0216*	−15.10 *.0174*	−16.85 *.0100*	−18.57 *.0040*	−20.26 *.0024*	−21.92 *.0016*	*.080*
1.00	− 5.85 *.0109*	− 5.85 *.0302*	− 5.85 *.0392*	− 5.85 *.0315*	− 5.85 *.0181*	− 5.85 *.0072*	− 5.85 *.0044*	− 5.85 *.0029*	*.145*
1.40	− 1.98 *.0172*	0 *.0478*	2.02 *.0621*	4.07 *.0499*	6.15 *.0288*	8.26 *.0115*	10.41 *.0069*	12.59 *.0046*	*.230*
2.00	2.02 *.0029*	6.14 *.0634*	10.38 *.0824*	14.75 *.0662*	19.25 *.0381*	23.88 *.0152*	28.65 *.0092*	33.55 *.0061*	*.305*
3.00	6.12 *.0112*	12.49 *.0312*	19.10 *.0405*	25.97 *.0326*	33.10 *.0188*	40.49 *.0075*	48.15 *.0045*	56.09 *.0030*	*.150*
5.00	10.25 *.0038*	18.81 *.0104*	27.69 *.0135*	36.89 *.0108*	46.41 *.0062*	56.25 *.0025*	66.41 *.0015*	76.89 *.0010*	*.050*
PROBABILITY OF C(t)	*.075*	*.208*	*.270*	*.217*	*.125*	*.050*	*.030*	*.020*	

cable. For units of 3 months the data were insufficient for grouping in periods of more than 4 units. For units of 1 week and 1 month the difference between the periods of 12 units and 24 units is so slight as to be without statistical significance.

The next step was to combine these distributions in order to determine the distribution of the expected net gain. To do this mathematically gives rise to hopeless complications unless certain simplifying assumptions are made which themselves cast doubt on the value of the solutions. Therefore, it was necessary to compute the distributions by empirical means.
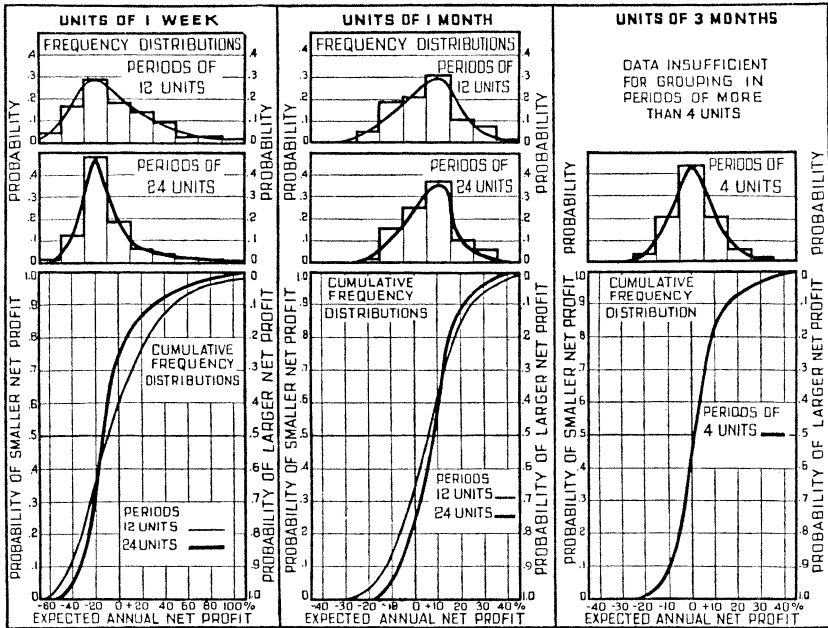

FIGURE 5.—Frequency distributions of expected annual net profits based on method of sequence probabilities. Brokerage computed at 1 per cent.

To determine these empirical distributions, values of $R(t)$ and $C(t)$ were taken which adequately covered their range of variation, and $I(t)$ was computed for all possible combinations of these values. The average brokerage charge was assumed to be 1 per cent per trade. To facilitate computation tables were employed of the type illustrated in Table 4.

The roman type figures in the body of the table give the values of $I(t)$ for the various values of $R(t)$ in the first column and $C(t)$ in the

first row. The probabilities for each value of $R(t)$ and $C(t)$ are shown in the last column and the bottom row respectively. The probabilities for the various values of $I(t)$ are the products of the probabilities of $R(t)$ and $C(t)$ and are shown by the figures in italics in the table.[9] The values of $I(t)$ have been classified and the sum of the probabilities for each value of $I(t)$ in any class taken as the probability of that class. The results of this analysis are presented in Figure 5.

Here again are shown the histograms and cumulative frequency curves representing units of 1 week, 1 month, and 3 months. For units of 1 week the cumulative curves indicate an average annual loss of about 10 per cent, with but 1 chance in 3 of obtaining a net profit over any period of 24 weeks. The same is true to a lesser extent when we use units of 3 months. In this case we see there is about an even chance of a loss or gain. For units of 1 month, however, an average net gain of about 7 per cent is indicated. But, even here, no great consistency is evident. In fact the cumulative curve indicates that the chance of loss for any one year is about 1 in 3.

Furthermore the results should be interpreted with caution in view of the fact that various units of time, other than 1 month, were considered and rejected. In all, 26 such units, ranging from 20 minutes up to 10 years, were examined. The series represented by units of one month, therefore, was selected by hindsight as the most favorable one in 26 trials.

This type of forecasting could not be employed by speculators with any assurance of consistent or large profits. On the other hand, the significant excess of sequences over reversals for all units from 20 minutes up to 6 months, with the exception of units of 2 weeks and 3 weeks mentioned previously, represents conclusive evidence of structure in stock prices.

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[9] The values of $R(t)$ and $C(t)$ for the periods studied are randomly distributed and a correlation analysis between $R(t)$ and $C(t)$ gave a coefficient of -0.2 with a standard error of 0.1. This coefficient is on the borderline of significance but is very small. Therefore, the probabilities of $I(t)$ may be computed in this manner without great error.