COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut

COWLES FOUNDATION DISCUSSION PAPER NO. 204

GAME THEORY AND ECONOMIC BEHAVIOR

Martin Shubik

March 28, 1966

GAME THEORY AND ECONOMIC BEHAVIOR *

Martin Shubik

Game theory is a general term used in reference to some developments in a branch of mathematics and in applications to the social sciences. Several distinct branches of game theory exist and need to be identified before our attention is limited to economic behavior. Von Neumann and Morgenstern's book [1] presented three aspects of game theory which are so fundamentally independent from each other that with a small amount of editing their opus could have been published as three independent books.

The first topic was the description of a game or interdependent decision process in extensive form. This provided a language for the precise definition of terms, such as choice, decision tree, move, information, strategy and payoff, which has served as a basis for considering statistical decision-making [2], the study of artificial intelligence [3], and in the development of the behavioral theory of the firm. The definition of payoff has been closely associated with developments in utility theory.

The second topic was the description of the two-person zero-sum game and the development of the mathematical theory based upon the concept of the minimax solution [5]. This theory has formal mathematical connections

---

* Research undertaken by the Cowles Commission for Research in Economics under Contract Nonr-3055(00) with the Office of Naval Research.

to linear programming and has been applied successfully to the analysis of problems of pure conflict; however, its application to the social sciences has been limited as pure conflict of interests is the exception rather than the rule.

The third subject to which von Neumann and Morgenstern directed their attention was the development of a static theory for the n person ($n \geq 3$) constant sum game. They suggested a set of stability and domination conditions which should hold for a cooperative solution to an n-person game. It must be noted that the implications of this solution concept were developed on the assumption of the existence of a transferable, interpersonally comparable linear utility which provides a mechanism for side payments. Since the original work of von Neumann and Morgenstern twenty to thirty alternative solution concepts for the n-person non-constant sum game have been suggested, 6/, 7/. Some have been of purely mathematical interest but most have been based on considerations of bargaining, fair division, social stability and other aspects of human affairs. Many of the solution concepts do not use the assumption of transferable utility.

The major applications of game theory to economics have been to oligopoly theory, bargaining and general equilibrium. Perhaps the most pervasive concept underlying the writings on oligopoly is that of a non-cooperative equilibrium. This is basic in the works of Cournot 8/, Bertrand 9/, Edgeworth 10/, Chamberlain 11/, Stackelberg 12/, and many others.

Nash [13] has presented a general theory of non-cooperative games based on the equilibrium point solution. A group of individuals will be in a state of non-cooperative equilibrium if in the individual pursuit of his self-interest no one is motivated to change his strategy.

A distinction has been made between cooperative and non-cooperative solutions based upon the presence or absence of the requirement of Pareto optimality [14] for the final outcome. In much of the writing on oligopoly, quasi-dynamic models have been suggested and quasi-cooperative solutions have been advanced. Thus, while the Chamberlain large group equilibrium [15] can be interpreted as the outcome to a static non-cooperative game, the small group equilibrium and the market resolution suggested by Fellner [16] are cast in a quasi-dynamic, quasi-cooperative framework. A limited amount of development of games of survival [17] and games of economic survival [18] has provided a basis for the study of multiperiod situations and an extension of the concept of non-cooperative equilibrium to include the quasi-cooperative outcomes. The methodology of game theory has helped to clarify the different aspects of intent, behavior and market structure in oligopolistic markets [19] as well as to provide a unified way in which to investigage conjectural variations and lengthy "if-then" constructs in terms of strategies.

The theory of games has provided both a unifying basis for the mathematical and semi-mathematical works in oligopoly theory and has provided some new results. Some of these can be seen in the article of Mayberry, Nash and Shubik [20] and in Strategy and Market Structure [21]. Much of the use of game theory is derived from the methodology which requires explicit and detailed definition of the strategies available to the players and their payoffs.

The direct connections between previous work and the new are the Nash theory of non-cooperative equilibrium [22] theories of monopolistic competition; the concept of the core (this and others will be defined below) with the Edgeworth contract curve [23]; various concepts of stable set [24] with the Pareto optimal surface; the Nash [25], Shapley [26], and Harsanyi [27] theories of fair division with Zeuthen's bargaining scheme [28] and cartel negotiations; and games of economic survival with quasi-cooperative theories.

Some of the new results concern the definition of optimum threat [29] in economic warfare, the reexamination and interpretation of the kinky oligopoly demand curve and the more general problem of oligopolistic demand; stability and the Edgeworth cycle in price variation oligopoly; duopoly with both price and quantity as independent variables and the development of diverse concepts applicable to cartel behavior such as blocking coalitions [30] discriminatory solutions and decomposable games.

Selten [31] has been concerned with the problem of calculating the non-cooperative equilibria for various classes of oligopolistic markets. His work has been directed both to the explicit calculations and the uniqueness of equilibrium points. Vickrey [32], Griesmer and Shubik [33] and others [34/] have studied a class of game models applicable to bidding and auction markets. Mills [35/], and others working from the viewpoint of marketing and operations research have constructed several non-cooperative game theoretic models of competition via advertising. Jacot [36/] has considered problems involving location and spatial competition.

Game theory can be given both a normative and a behavioristic interpretation. The meaning of rational behavior in situations involving elements of conflict and cooperation is not well defined. No single set of normative criteria has been generally accepted and no universal behavior has been validated. Closely related to and partially inspired by the developments in game theory, there has been a growth in experimental gaming [37/], some of which has been in the context of economic bargaining [38/] or in the simulated environment of an oligopolistic market [39/]. Where there is no verbal or face-to-face communication, under the appropriate circumstances, there appears to be some evidence in favor of the non-cooperative equilibrium.

The theory of bargaining has been of especial interest to economists in the context of bilateral monopoly. This can involve two firms, a labor union and a firm, or two individuals engaged in barter in the marketplace or trying to settle a joint estate. Formally, any two-person non-constant sum situation, be it haggling in the market or international negotiations, can be described in the same game-theoretic framework. However, there are several substantive problems which limit application and which have resulted in the development of different approaches. In non-constant sum games communication between the players is of considerable importance, yet its role is exceedingly hard to define. In games such as chess and even in many oligopolistic markets, a move is a well-defined physical act such as moving a pawn in a definite manner, or changing a price, or deciding upon a production rate; in bargaining it may be necessary to interpret a statement as a move. The problem of interpreting words as moves in negotiation is critical to the description and understanding of bargaining and negotiation processes. This "coding" problem has to be considered from the viewpoints of many other disciplines as well as game theory.

One approach to bilateral monopoly has been to regard it as a "fair division" problem and several solution concepts, each one embodying a formalization of concepts of symmetry, justice, and equity, have been suggested.

A desirable property of a solution should be that it predicts a unique outcome. In the context of economics, this would be a unique distribution of resources (and unique prices, if they exist at all). Unfortunately there are few concepts of solution pertaining to economic affairs which have this property. The price system and distribution resulting from a competitive market in general may not be unique; Edgeworth's solution to the bargaining problem was the contract curve which merely predicts that the outcome will be some point among an infinite set of possibilities.

The contract curve has the property that any point on it is jointly optimal (both cannot improve their position simultaneously from a point on this curve) and it is individually rational (no point gives an individual less than he could obtain without trading). The Pareto optimal surface is larger than the contract curve as it is restricted only by the joint optimality condition. If the assumption is made that a transferable comparable utility exists, then the Pareto optimal surface (described in the space of the traders' utilities) is flat; if not, it will in general be curved. Any point on the Pareto optimal surface that is individually rational is called an imputation. In the two-person bargain the Edgeworth contract curve coincides with two game theoretic solutions, the core [40/] and the stable set [41/]. The core consists of all undominated imputations (it may be empty). A stable set is a set of imputations which do not dominate each other, but which together dominate all other imputations. An imputation α is said to dominate another imputation β if there exists a coalition of players such that if they acted jointly but independently from the others they could guarantee for themselves at least the amounts they would receive if they accepted α; and if each player obtains more in α than in β [42/]. The core and stable set solutions can be defined with or without assuming transferable utilities.

None of the solution concepts suggested so far predicts a unique outcome. For the two-person case, some of the various fair-division or arbitration schemes do. The Nash fair-division scheme assumes that utilities are measurable but does not need assumptions of either comparability or transferability [43] the Shapley value utilizes these assumptions. Other schemes have been suggested by Raiffa [44], Braithwaite [45], Kuhn [46] and others.

The other approach to bargaining is to treat it in the extensive form, describing each move explicitly and showing the time path taken to the settlement point. This involves attempting to parameterize qualities such as "toughness," "flexibility," etc. Most of the attempts to apply game theory in this manner belong to studies in social psychology, political science and experimental gaming. However, it has been shown that the dynamic process suggested by Zeuthen [47] is equivalent to the Nash fair-division scheme.

In association with problems in general equilibrium economics, game theory methods have provided several new insights. Under the appropriate conditions on preferences and production it has been proved that a price system that will clear the market will exist, provided that each individual acts as an independent maximizer. This result holds true independently of the number of participants in the market; hence, it cannot be interpreted as a limiting phenomenon as the number of participants increases. Yet in verbal discussions about the competitive market contrasting it with bilateral monopoly, the difference between markets with many participants, each with little if any control over price, and the market with few, where the interactions of each with all others are of prime importance, is stressed.

The competitive equilibrium best reflects the spirit of "the Invisible Hand" and of decentralization. The use of the word "competitive" is counter to both game theoretic and common language implications. It refers to the case in which, if each individual considers himself as an isolated maximizer operating in an environment over which he has no control, the results will be jointly optimal.

The power and the appeal of the concept of competitive equilibrium appears to be far greater than merely that of the property of decentralization. This is reflected in the results which show that under the appropriate conditions the competitive equilibrium may be regarded as the limit solution for several conceptually extremely different game theoretic solutions.

It has been noted that for bilateral monopoly the Edgeworth contract curve is the core. If the number of traders is increased on both sides of the market, Edgeworth [48/] suggested and presented an argument to show that the contract curve would shrink (interpreted appropriately, given the change in dimensions). Shubik [49/] observed the connection between the work of Edgeworth and the core; he proved the convergence of the core to the competitive equilibrium in the special case of the two-sided market with transferable utility and conjectured that the result would be generally true for any number of markets without transferable utility. This result was proved by Scarf [50] and improved by Debreu and Scarf [51]. Using the concept of continuum of players (rather than considering a limit by replicating the finite number of players in each category as was done by Shubik, Scarf and Debreu). Aumann [52] proved the convergence of the core under somewhat different conditions. When transferable utility is assumed the core converges to a single point as the competitive equilibrium is unique. Otherwise, it may split and converge to the set of competitive equilibria.

The convergence of the core establishes the existence of a price system as a result of a theory which makes no mention of prices. Its prime concern is with the power of coalitions. It may be looked upon as a formalization of countervailing power in as much as it rules out imputations which can be dominated by any group in the society.

Shapley and Shubik [53] have shown the convergence of the value in the two-sided market with transferable utility. A more general result for any number of markets has been proved by Shapley [54] and Shapley and Aumann [55] have worked on the convergence of a non-transferable utility value recently defined by Shapley [56]. Harsanyi [57] was able to define a value which generalized the Nash two-person fair-division scheme for situations involving many individuals without a transferable utility. This preceded and is related to the new value of Shapley. Convergence of this has not been proved.

There are several other value concepts 58/, all of which make use of symmetry axioms and are based upon some type of averaging over the contribution of an individual to all coalitions.

If one is willing to accept the value as reflecting certain concepts of symmetry and fairness, then in an economy with many individuals in all walks of life and with the conditions which are required for the existence of a competitive equilibrium, satisfied; the competitive equilibria will also satisfy these symmetry and fairness criteria.

One of the important open problems has been the reconciliation of the various non-cooperative theories of oligopolistic competition with general equilibrium theory. The major difficulty lies in the feature that the oligopoly models are open in the sense that the customers are usually not considered as players with strategic freedom, while the general equilibrium model has every individual considered in the same manner regardless of his position in the economy. As the firms are players in the oligopoly models, it is necessary to specify the domain of the strategies they control and their payoffs under all circumstances. In a general equilibrium model, no individual is considered as a player; all are regarded as individual maximizers; Walras' law is assumed to hold; supply is assumed to equal demand. When at attempt is made to consider a closed economic model as a noncooperative game, considerable difficulties are encountered in describing the strategies of the players. This can be seen immediately by considering the bilateral monopoly problem; each individual does not really know what he is in a position to buy until he finds out what he can sell. In order to model this type of situation as a game it may be necessary to consider strategies which do not clear the market and which may cause a player to become bankrupt, i.e., be unable to meet his commitments. Shapley and Shubik [59/] have successfully modeled the closed two-sided two-commodity market without side payments and have shown that the noncooperative equilibrium point converges from below the Pareto optimal surface to the competitive equilibrium point. They also have been able to consider some special cases with more goods and markets on the assumption of the existence of a transferable (but not necessarily comparable) utility [60/].

When there are more than two commodities and one market, the existence of a unique competitive equilibrium point appears to be indispensible in defining the strategies and payoffs of players in a noncooperative game. No one has succeeded in constructing a satisfactory general market model as a noncooperative game without using a side-payment mechanism. The important role played by the side-payment commodity is that of strategy decoupler. It means that a player with a supply of this type of "money" can decide what to buy even though he does not know what he will sell.

In summary, it appears that, in the limit, at least three considerably different game-theoretic solutions are coincidental with the competitive equilibrium solution. This means that by considering different solutions the competitive market may be interpreted in terms of decentralization, fair division, the power of groups and the attenuation of power of the individual.

The stable set solution of von Neumann and Morgenstern, the bargaining set of Aumann and Maschler [61], the "self-policing properties of certain imputation sets" of Vickrey [62] and several other related cooperative solutions appear to be more applicable to sociology and possible anthropology rather than to economics. There has been no indication of a limiting behavior for these solutions as numbers grow; on the contrary, it is conjectured that in general the solutions proliferate. When, however, numbers are few, such as in cartel arrangements and in international trade, these other solutions provide insights, as has been shown by Nyblem [63] dealing with stable sets.

When conditions other than those needed for the existence of a competitive equilibrium hold, such as when there are external economies or diseconomies present, when there is joint ownership, when there are increasing returns to scale, or when tastes are interlinked, then the different solutions in general do not converge. There may be no competitive equilibrium, the core may be empty and the definition of a non-cooperative game when joint property is at stake will call for a statement of the laws concerning damages and threats. (Similarly, even though the conditions for the existence of a competitive equilibrium are satisfied, if the numbers are few the various solutions will be different.) When the competitive equilibrium does not exist we must seek another criterion to solve the problem of distribution or change the laws (if possible) to reintroduce the competitive equilibrium. The other solutions provide different criteria. However, if, for example, a society desires to have its distribution system satisfy say both conditions of decentralization and fair division, or fair division and limits on power of groups, it may be logically impossible to do so.

Davis and Whinston [64/], Scarf [65/], and Shapley and Shubik [66/] have investigated game applications to external economies, increasing returns to scale and joint ownership. In the case of joint ownership the relation between economics and politics as a mechanism for the distribution of the proceeds from jointly owned resources is evident.

It must be noted that the "many solution" approach to distribution is in contrast to the type of welfare economics which considers a community welfare function or social preferences which are not necessarily constructed from individual preferences.

Leaving aside questions of transferable utility, there is a considerable difference between an economy in which there is only barter or a passive shadow price system and one in which the government and possibly others have important monetary strategies. Faxen 67/ has considered financial policy from a game theoretic viewpoint.

There have been some diverse applications of game theory to budgeting and to management science as can be seen in the articles of Bennion 68/ and Shubik 69/. It remains to note the book of Nyblen 70/. The author has attempted to apply the von Neumann and Morgenstern concept of stable set to problems of macroeconomics. Nyblen notes that the Walrasian system bypasses the problem of individual power by assuming it away. He observes that in game theory certain simple aggregations procedures do not hold, thus the solutions to a four-person game obtained by aggregating two players in a five-person game may have little in common with the solutions to the original game. He outlines an institutional theory of interest based upon a standard of behavior and (at a primarily descriptive level) links the concepts of discriminatory solution and excess to inflation and international trade.

FOOTNOTES

1. Von Neumann, John, and Oskar Morgenstern, The Theory of Games in Economic Behavior, first edition, 1944, second edition, 1947; Princeton, New Jersey: Princeton University Press.

2. Blackwell, O., and M. A. Girschik, Theory of Games and Statistical Decisions, New York: John Wiley and Sons, 1954.

3. Minsky, M., "Steps Toward Artificial Intelligence," Proceedings of the Institute of Radio Engineers, 1961, volume 49, pages 8-30.

4. Cyert, R. M. and J. G. March, A Behavioral Theory of the Firm, Englewood Cliffs, New Jersey: Prentice-Hall, 1963.

5. See: McKinsey, J. C. C., Introduction to the Theory of Games, Chapters 2 and 10, New York: McGraw-Hill, 1952.

6. Many of these appear in: Luce, R. Duncan, and Howard Raiffa, Games and Decisions, Chapters 5, 6, and 9, New York: John Wiley and Sons, 1957.

7. See also: Dresher, M., H. W. Kuhn, R. D. Luce, A. W. Tucker, and P. Wolfe (eds.) Contributions to the Theory of Games - Volume I, 1950; Volume II, 1953; Volume III, 1957; Volume IV, 1959, Princeton: Princeton University Press.

8. Cournot, A. A., Researches Into the Mathematical Principles of the Theory of Wealth, pages 79-80, New York: MacMillan, 1897.

9. Bertrand, J., "Theorie Mathematique de la Richesse Societe" (Review), Journal des Savants: pages 499-508, Paris, September 1883.

10. Edgeworth, F. Y., Papers Relating to Political Economy, I, pages 111-142, London: MacMillan, 1925.

11. Chamberlain, E. H., The Theory of Monopolistic Competition, pages 89-94, Cambridge: Harvard University Press, sixth edition, 1950.

12. Stackelberg, Heinrich, Freiherr von, Marktform und Gleichgewicht, page 134, Berlin: Julius Springer, 1934.

FOOTNOTES (continued)

13. Nash, John, Jr., "Noncooperative Games," Annals of Mathematics, Volume 54, pages 286-295, Princeton: Princeton University Press, 1951.

14. For a definition of Pareto optimality see: Koopmans, T. C., The State of Economic Science, page 42, New York: McGraw-Hill, 1957.

15. Chamberlain, E. H., loc. cit.

16. Fellner, W., Competition Among the Few, Chapter 7, New York: Knopf, 1949.

17. Milnor, John W., and Lloyd S. Shapley, "On Games of Survival," 1957, Contributions to the Theory of Games, Volume III, No. 34, pages 15-45.

18. Shubik, Martin, and Gerald L. Thompson, "Games of Economic Survival," Naval Research Logistics Quarterly, Volume 6, Number 2; pages 111-123, June, 1959.

19. Shubik, Martin, "Market Form. Intent of the Firm and Market Behavior," in Zeitschrift Fur National oekonomie, volume XVII, number 2; pages 186-196, 1957.

20. Mayberry, J. P., J. F. Nash, and Martin Shubik, "A Comparison of Treatments of a Duopoly Situation," Econometrica, volume XXI; pages 141-145, January 1953.

21. Shubik, Martin, Strategy and Market Structure, New York: John Wiley and Sons, 1959.

22. Nash, J. F., Jr., op. cit.

23. Edgeworth, F. Y., Mathematical Psychics, London: Kegan Paul, 1881.

24. For the definition see: Luce, R. Duncan, and Howard Raiffa, op. cit., Chapter 9.

25. Nash, J. F., Jr., "Two Person Cooperative Games," Econometrica, Volume XXI; pages 128-140, January 1953.

FOOTNOTES (continued)

26. Shapley, Lloyd S., "A Value for n-Person Games," 1953, and pages 307-317 in Contributions to the Theory of Games, Volume II, number 28, Harold Kuhn, and Albert W. Tucker, (eds.), Princeton, New Jersey: Princeton University Press.

27. Harsanyi, John C., "Approaches to the Bargaining Problem Before and After the Theory of Games: A Critical Discussion of Zeuthen's, Hicks' and Nash's Theories," Econometrica, Volume 24, pages 144-157.

28. Zeuthen, F., Problems of Monopoly and Economic Welfare, London: G. Routledge and Sons, 1930.

29. See Nash, J. F., Jr., "Two-Person Cooperative Games," and also Martin Shubik, Strategy and Market Structure, Chapter 10.

30. See Scarf, H., "The Core of an n-Person Game," CFDP 182 (Mimeographed), Cowles Foundation for Research in Economics at Yale University: New Haven, September 1965.

31. Selten, Reinhard, "Value of the n-Person Game," paper presented at the First International Game Theory Workshop, Jerusalem: Hebrew University, October 1965.

32. Vickrey, William, "Counterspeculation, Auctions and Competitive Sealed Tenders," Journal of Finance, Volume 16, pages 8-37, 1961.

33. Griesmer, James H., and Martin Shubik, "Towards a Study of Bidding Processes," Naval Research Logistics Quarterly, Part I: "Some constant-sum games," Volume 10, Number 1; pages 11-21, Part II: "Games with Capacity Limitations," Volume 10, Number 2; pages 151-173, Part III: "Some special models," Volume 10, Number 3; pages 199-217.

34. For instance: Feeney, G. F., "Unilateral Markets, I: Price-Quantity Markets," CFDP 138 (Mimeographed), Cowles Foundation for Research and Economics at Yale University: New Haven, Conn., May, 1962.

35. Mills, H. D., "A Study in Promotional Competition," in Bass et al (eds.), Mathematical Models and Methods in Marketing, pages 245-301; Homewood, Illinois: Irwin, 1961.

FOOTNOTES (continued)

36. Jacot, Simon-Pierre, Strategie et Concurrence, Paris, 1963.

37. Rapaport, A., and C. Orvant, "Experimental Games: A Review," Behavioral Science, Volume 7, Number 1; pages 1-37, January 1962.

38. Siegel, S., L. E. Fouraker, Bargaining and Group Decision Making, New York: McGraw-Hill, 1960.

39. Hoggatt, A. C., "An Experimental Business Game," Behavioral Science, Volume 4, pages 192-203, 1959.

40. For a definition see: Luce, R. D. and H. Raiffa, op. cit., p. 194.

41. Ibid., Chapter 9.

42. Ibid., page 201.

43. Shubik, Martin, "Measureable, Transferable, Comparable Utility and Money," CFDP 200, (Mimeographed), New Haven, Conn.: Cowles Foundation for Research in Economics at Yale University, 1966.

44. Raiffa, Howard, "Arbitration Schemes for Generalized Two-Person Games," pages 361-387 in Kuhn and Tucker (eds.) op. cit.

45. Braithwaite, R. B., Theory of Games as a Tool for the Moral Philosopher, Cambridge: Cambridge University Press, 1955.

46. Kuhn, Harold W., "On Games of Fair Division," Forthcoming 1966, in Martin Shubik (ed.) Essays in Mathematical Economics in Honor of Oskar Morgenstern, Princeton, New Jersey: Princeton University Press.

47. Zeuthen, F., op. cit.

48. Edgeworth, F. Y., op. cit., pages 20-25 et seq.

49. Shubik, Martin, "Edgeworth Market Games," Contributions to the Theory of Games, Volume IV, Albert W. Tucker, and Duncan R. Luce (eds.), pages 267-278, Princeton: Princeton University Press, 1959.

FOOTNOTES (continued)

50. Scarf, H., op. cit.

51. Debreu, G., and H. Scarf, "A Limit Theorem on the Care of an Economy," International Economic Review, Volume 4; pages 235-246, 1963.

52. Aumann, R. J., "The Core of a Cooperative Game Without Side Payments," (Trans.), American Math. Soc., Volume 98; pages 539-552, 1961.

53. Shapley, Lloyd and Martin Shubik, "Pure Competition, Coalition, Power and Fair Division," RM 4917 PR, Santa Monica, California: RAND Corporation, July, 1966, forthcoming.

54. Shapley, Lloyd, Unpublished Manuscript Notes, RAND Corporation, 1965-66.

55. Shapley, Lloyd, and R. J. Aumann, op. cit.

56. Shapley, Lloyd S., op. cit.

57. Harsanyi, John C., "A Bargaining Model For the Cooperative n-Person Game," pages 325-356 in Tucker and Luce (eds.), op. cit., 1959.

58. Selten, Reinhard, "Valuation of N-Person Games," pages 577-626 in M. Dresher, L. S. Shapley, and A. W. Tucker, (eds.), Advances in Game Theory, Princeton, New Jersey: Princeton University Press, 1964.

59. Shapley, Lloyd S., and Martin Shubik, "Concepts and Theories of Pure Competition," in Martin Shubik (ed.) op. cit., forthcoming 1966.

60. Shapley, Lloyd S., and Martin Shubik, Unpublished Manuscript Notes, RAND Corporation, 1965-66.

61. Aumann, R. J. and M. Maschler, "The bargaining set for cooperative games," pages 443-476 in Dresher, Shapley, and Tucker, (eds.), op. cit., 1964.

62. Vickrey, William, "Self-policing properties of certain imputation sets," pages 213-246 in Tucker and Luce, (eds.), op. cit., 1959.

FOOTNOTES (continued)

63. Nyblen, Goren, The Problem of Summation in Economic Science, Lund., 1951.

64. Davis, O., and A. Whinston, "Welfare Economics and the Theory of Second Best," CFDP (ONR) No. 146, (Mimeographed), New Haven, Conn: Cowles Foundation for Research in Economics at Yale University, 1965.

65. Scarf, H., "Notes on the Core of a Productive Economy," unpublished manuscript, New Haven, Conn.: Cowles Foundation for Research in Economics at Yale University, 1964.

66. Shapley, Lloyd S., and Martin Shubik, "Ownership and the Production Function," CFDP No. 167, (Mimeographed), New Haven, Conn: Cowles Foundation for Research in Economics at Yale University, March 1964.

67. Faxen, Karl O., Monetary and Fiscal Policy Under Uncertainty, Stockholm, 1957.

68. Bennion, E. G., "Capital Budgeting and Game Theory, Harvard Business Review, Volume 34; pages 115-123, 1956.

69. Shubik, Martin, "The Uses of Game Theory in Management Science," Management Science, Volume 2, number 1, pages 40-54, October, 1955.

70. Nyblen, Goren, op. cit., 1951.