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Relation Between Cowles and S & P Indexes

Cowles extended the Standard Stat. Service Indexes (which begin in 1918 or later) backward to 1871, using the same formula, with an adjustment only for the fact that monthly high and low data were used, instead of daily price data.

(See attached from Cowles Book Common Stock Indexes 1871-1937, 1938)

A second edition of the above, entitled Common Stock Indexes was published in 1939. (It corrected "such errors as have been discovered" p.viii)

In 1941 and 1942 Cowles published "Common Stock Indexes for 1939" and "Common Stock Indexes for 1940" in mimeographed form. Standard Stat. Service changed its base from 1926=100 to 1935-9=100. Cowles changed composite price index by conversion factor 1.056 (p. 13 of the 1942 publication).

Later, Standard Stat. Service changed base again, to 1941-3=10. There is no Cowles response to this, and Cowles never responded regarding his dividend or earning series.

There are slight differences between Cowles Series p-1 in 1938 edition and Cowles Series p-1 in 1939 edition going all the way back to 1871. These must be due to data errors, as the figures for recent years agree.

There are also slight discrepancies between DA-1 in the respective editions. The series I used in "Do Stock Prices..." and elsewhere is from the older edition.

The series S & P composite from S & P Statistical Service attached differs from the Cowles Series by a base year change, but there are other small discrepancies which I can't account for..

THE STANDARD STATISTICS COMPANY FORMULA

$$(18.3) \quad I_i = \prod_{t=1}^i \frac{\sum p_t q_t}{\sum p_{t-1} q_{t-1}}.$$

If also the original list of issues is retained throughout the construction of the index,  $\sum p_t q_t = \sum p_i q_i$ , and (18.3) reduces to

$$(4.2) \quad I_i = \frac{\sum p_i q_i}{\sum p_0 q_0}.$$

There are two minor differences between the Standard Statistics and Cowles Commission index formulas: (a) because of scarcity of price data for earlier years, the definition of  $p_i$  must be modified; and (b) because of the different definition of  $p_i$ , the correction for the sale of rights must be changed. Price data for the earlier years are confined in the case of each stock to tables of monthly highs and lows published in the *Commercial and Financial Chronicle*. No information is given regarding the day of the month to which any such quotation applies. It was therefore necessary, in the case of the Cowles Commission indexes, to let  $p_i$  represent the simple arithmetic average of monthly high and low, subject to adjustments as described on pages 18-20. These limitations in the data also necessitated resorting to approximations described on page 22 in the case of corrections for the issue of rights. The Standard Statistics indexes cover only the more recent period, for which daily quotations are available. It was not necessary, therefore, that they employ these approximations.

The general expression for the Cowles Commission indexes, Series P (and, if the corrections for payment of cash dividends are included in  $c_i$ , for Series C as well) is given by

$$(16.1) \quad I_i = \prod_{t=1}^i \frac{\sum c_t p_t q_t}{\sum p_{t-1} q_{t-1}}.$$

However, if daily closing prices had been available, a definition of  $p_i$  similar to that employed by Standard Statistics could have been used, and the correction for changes in capital structure could have been accomplished by subtracting from the new value of stock outstanding the new capital received, rather than by computing a reduction factor [formula (14)] by which the value of stock outstanding must be multiplied.

## COMMON-STOCK INDEXES

fective stocks is of considerable assistance. The Standard Statistics procedure is to adjust the numerator of (17) so as to include the value of all new issues to be added in period  $i$ , and exclude the value of all old issues to be withdrawn in period  $i$ . The numerator of (17) after adjustment represents simply the value of the effective stocks for the period  $i+1$ , and may be written  $\sum_{i=1}^{t+1} p_i q_i$ .

The value of new stocks added to the list of included issues does not represent a profit to investors, nor does the value of old stocks removed represent a loss. Therefore, the base value must be adjusted so that the additions and withdrawals will not result in a change in the level of the index for the period in which they occur. This adjustment is made by multiplying the previous base value by  $\frac{\sum_{i=1}^{t+1} p_i q_i}{\sum_{i=1}^t p_i q_i}$ . This factor simply restores the index to the figure which would have been obtained had no new issues been introduced, or old issues withdrawn, in period  $i$ . The most general form of the Standard Statistics formula is therefore

$$(18) \quad I_t = \frac{\sum_{i=1}^{t+1} p_i q_i}{\sum_{i=1}^t p_i q_i \prod \left[ \frac{\sum_{i=1}^t p_i q_i}{\sum_{i=1}^t p_i q_i - \sum_{i=1}^t p_{i+1} (q_i - q_{i-1})} \frac{\sum_{i=1}^{t+1} p_i q_i}{\sum_{i=1}^t p_i q_i} \right]},$$

or, more simply,

$$(18.1) \quad I_t = \frac{\sum_{i=1}^{t+1} p_i q_i}{\sum_{i=1}^t p_i q_i \prod \frac{\sum_{i=1}^t p_i q_i}{\sum_{i=1}^t p_i q_i - \sum_{i=1}^t p_{i+1} (q_i - q_{i-1})}}.$$

Since

$$\sum_{i=1}^t p_i q_i \prod \sum_{i=1}^t p_i q_i = \sum_{i=1}^{t+1} p_i q_i \prod \sum_{i=1}^t p_{i+1} q_{i-1},$$

(18.1) may be still further reduced to

$$(18.2) \quad I_t = \prod \frac{\sum_{i=1}^t p_i q_i - \sum_{i=1}^t p_{i+1} (q_i - q_{i-1})}{\sum_{i=1}^t p_i q_i}.$$

Actual  
Standard Stat  
Service  
formula

If no additional stock is sold in any period, the  $p_{i+1}$  are all zero, and (18.2) becomes

From S & P Statistical Service  
Security Price Index Record 1984 Edition

## Description of Method Used in Computation—Continued

especially important when long term comparisons are to be made. Certain modifications to the basic formula have been necessary to maintain the best possible representation over the years. The character of the stock market is subject to gradual, but continuous shifting, and it is only by periodic checks of coverage that true representation can be maintained.

**WEIGHTING**—Each component stock is weighted so that it will influence the index in proportion to its respective market importance. The most suitable weighting factor for this purpose is the number of shares outstanding. The price of any stock multiplied by number of shares outstanding gives the current market value for that particular issue. This market value determines the relative importance of the security.

**BASE VALUES AND GROUP INDEXES**—Market values for individual stocks are added together to obtain their particular group market value. These group values are expressed as a relative, or index number, to the base period (1941–1943) market value. As the base period market value is relatively constant, subject to change only as described in this text, the index number reflects only fluctuations in current market values.

**BASE PERIOD**—The base value for any group is the average of the weekly group values for the period 1941–1943. The current group value is expressed as a relative by dividing it by its base period value and multiplying the resulting quotient by 10. In this relative form an index number attains its maximum usefulness for statistical purposes.

**FORMULA**—The formula for the base-weighted aggregative index is  
As Cowles  
explains p. 284  
this is not  
their formula.

$$\text{Index} = \frac{\sum P_i Q_i}{\sum P_0 Q_i} \times 10$$

where  $P_i$  represents the current market price,  $P_0$  the market price in the base period,  $Q_i$  the number of shares currently outstanding, and  $Q_0$  the number of shares outstanding in the base period, subject to adjustment when necessary to offset changes in capitalization.  $\Sigma$  is the Greek letter Sigma, which always indicates addition, or the sum of, and in this instance indicates the addition of all the market values of the individual companies comprising the group.

### Variations in the Formula

**STOCK DIVIDENDS AND SPLIT-UPS**—It is possible to make absolute correction for a stock dividend or stock split-up, simply change the weighting factor to equal the number of stock shares outstanding after the dividend or split-up has become effective. This new weighting factor is introduced when computing the first index after the stock sells ex-dividend, or the new stock is traded.

**STOCK RIGHTS**—Many corporations, at the time of selling additional common stock, issue rights for its purchase to their stockholders. Two methods have been used at different times to offset the effect of the arbitrary price change caused by these rights.

During the 1918–1925 period the procedure was to increase the weighting factor just enough to offset the decline in the value of the issue caused by the decrease in price when the stock sells ex-rights. This method did not require any change in the base period values.

From the beginning of 1926 an improved method for adjusting for stock rights has been used. Now the weighting factor (shares outstanding) is increased by the number of shares actually sold, the market value reflecting this increase in capitalization based on the market price of the shares on the day they are traded on an ex-rights basis. An offsetting, or proportionate, adjustment is then made to the base period value. (See footnote on method of making base changes).

**CONSOLIDATION AND ACQUISITIONS**—Where a corporation absorbs another, wholly or in part through the issuance of common stock, or when a consolidation involves one or more corporations that are not already a part of the index, the weighting factor has to be changed to agree with the shares of stock outstanding and a new base value must be determined. The computation here is the same as that described in the footnote below. Here the adjustment is based on the value of additional shares outstanding, valued at the current market price.

**SPIN-OFFS**—Spin-offs to common stockholders of subsidiary companies produce an arbitrary downward price change at the time the stock sells ex-dividend. This situation has no effect upon the weighting factor for the stock involved. It does, however, necessitate a new base value for the group that will be smaller than the old base value. The method of adjusting the base is as described in the footnote below. This adjustment always reduces the base period value, the adjustment being dependent on the decline in price due to the spin-off.

**ADDING OR DROPPING STOCKS**—The addition of new stocks to any group involves only an increase in the old base value. (See footnote.) The reverse operation is employed when stocks are dropped from the group.

**SUBSTITUTIONS**—Substitutions are kept to an absolute minimum. However, to maintain current representation, and because of mergers, delistings, etc., substitutions are unavoidable. The adjustment to offset the difference between the two stocks involved is basically the same as for adding or dropping stocks, but based on the difference in market value between the two.

Weighting factors are revised to include small stock issues, such as those for minor absorptions, stock sold to employees, etc., as the multiplying factors are checked with the latest corporation report. An increase in stock outstanding involves an increase in the weighting factor for that stock, the stock added to the group is evaluated at current prices, and a new base value is computed (as shown in footnote) that offsets the value thus added.

### Dates of Indexes

The weekly data contained in this book are based on Wednesday's closing prices, or the last preceding sale price, this mid-week observation being considered most representative. The monthly averages are based on the indexes of the four or five weekly indexes of the month. Prior to 1954, the year's closing price index was the last Wednesday of the year; beginning 1954 it is on the last trading day of the year.

### Distribution of Indexes

Standard & Poor's Indexes for the main groups appear hourly on the tickers of the American Stock Exchange, the Commodity News Service, the Cotton Exchange Ticker, the Pacific Coast Stock Exchange, and the Montreal Stock Exchange. Also Bunker Ramo, Quotron, and ADP/F.I.S. (all electronic quotation services) carry the S & P 500 on a minute by minute basis, as well as the sub-indexes on an hourly basis. The Associated Press, United Press-International and Reuters (world wide) send the closing indexes over their financial news wires every day for publication by member papers. In addition, many of the national weekly news magazines report the indexes regularly.

The weekly indexes, by industrial groups, are published in the S & P weekly "Standard & Poor's Indexes of the Security Markets," and in the monthly "Current Statistics."

### FOOTNOTE

**BASE CHANGES**—Base changes are, in effect, proportional adjustments in its value to offset arbitrary changes in the market values upon which the index is based. The procedure is always as illustrated below.

$$\text{OLD BASE VALUE} \times \frac{\text{NEW MARKET VALUE (New Basis)}}{\text{OLD MARKET VALUE (Old Basis)}} = \text{NEW BASE VALUE}$$

or, assume a change which increases the market value by 12.0 million dollars, and that our aggregate market value after this increase is 1590.0 million dollars, and a base period value of 302.9. Then the new base period is

$$302.9 \times \frac{1590.0}{1578.0} = 305.2$$

It will be noted that the difference between the numerator and denominator in the above equation is 12.0, the amount for which the adjustment is being made.

Adjustments are always made in this manner. Reductions in base period values sometimes occur, if a stock is withdrawn or the substitution of a smaller company is made. In such cases the numerator is, of course, smaller than the denominator.

COMPOSITE

#### **From S&P Statistical Service**

MONTHLY STOCK PRICE INDEXES—Long Term

**Price Indexes** reflect the change in price of a basket of goods and services over time. The Consumer Price Index (CPI) measures the cost of a fixed basket of goods and services purchased by households. The GDP deflator measures the change in the average price of all goods and services produced in the economy.

1.1943 = 10

STERLINGSON

This section describes the construction of the GICS® Composite Stock Price Indexes. The GICS® Composite Stock Price Indexes are an extension of the Standard & Poor's Indexes. The same method of con-