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ECONOMY-WIDE PRODUCTION CAPABILITIES

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FOREWORD

Traditionally one thinks of the "productive capacity" of an economy as one number that expresses, in some appropriate unit, the maximum output of which that economy is capable in some stated period. The concept of gross national product provides a scale to which such a number can be referred. In geometrical language, it defines a line on which a "capacity point" locates the upper boundary of the collection of feasible rates of output.

This one-dimensional concept of capacity becomes inadequate, however, as soon as the product composition of output becomes as important as the aggregate output. For instance, in a Leontief input-output model with three industries, the rates of output of these industries are representable in a three-dimensional space. If a fourth commodity, say labor, is the only scarce primary input in the model, the productionwise feasible points fill a tetrahedron with one vertex in the origin and the other three vertices located on each of the three positive coordinate axes respectively.

Such geometrical language helps the imagination. It must yield to more practicable numerical procedures when the number of products, and the number of resource limitations bearing in specific ways on the various production processes, increase. In the programmatic first chapter the editors of this volume set forth their concept of process analysis as the procedures for representing the set of productionwise feasible points, and for making it accessible for such exploration as may be desired. The data are technological, the methods mathematical, and the results are estimates of alternative feasible compositions of output that are of possible interest to the policy maker. In more traditional terminology, one may say that process analysis aims at giving numerical access to the production function for a productive establishment, a multiplant firm, an industry, or an entire economy, in as much commodity detail as is both practicable and desirable.

Developments in input-output analysis, in activity analysis and in linear programming were necessary to make it possible to contemplate such an objective. At the same time, being defined in terms of a task rather than techniques, process analysis is not limited to the techniques from which it has developed.

On behalf of the Cowles Foundation, I wish to thank the editors and the authors of this volume for their innovating contributions, and for their consent to the inclusion of this collective work in our monograph series. To the readers, I express the hope that it may be found useful in a practical way in the making of economic decisions.

TJALLING C. KOOPMANS

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PART I

SCOPE AND METHOD OF PROCESS ANALYSIS

CHAPTER 1

INTRODUCTION

Alan S. Manne and Harry M. Markowitz

PROCESS ANALYSIS

The studies in this monograph analyze the production capabilities of industries and of industrial complexes. Models of technology are employed to answer questions concerning the product mixes achievable with various combinations of resources. The models are generaly based upon relationships well known to the industrial engineer or to his agricultural counterpart. The aim has been to cast these relationships into a form usable for the analysis of economy-wide capabilities.

In most cases, the models are of the activity analysis type, with the simplex technique of linear programming being used to compute numerical answers. Simulation techniques and integer programming also play a role (albeit a small one) in the present studies, and they will probably receive increasing emphasis in future work of this sort.

The phrase "process analysis" was chosen to identify studies, such as those presented in this monograph, which approach the analysis of industrial capability through models reflecting the structure of productive processes. Process analysis treats industrial capabilities in terms such as blast furnace capacity, petroleum product specifications, and metal machining operations—in contrast to approaches which treat capabilities in terms such as gross national product or interindustry sales and purchases. Process analysis is closely related to the conventional requirements calculation for critical resources. Unlike such requirements calculations, however, process analysis allows for alternate inputs per unit of output (reflecting alternate processes for making the same product), and uses mathematical programming techniques to determine the extent and circumstances under which one or another process should be employed.

Unfortunately, the phrase "process analysis" has at least two other meanings (one in economics and the other in industrial engineering) distinctly different from the meaning attached to it here. Over the years, we have attempted to find a better label for our studies, but none presented itself which was preferred by a majority of us.[1] An occasional resolution to do without a

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[1] The following are some alternate labels which were considered and the reasons they were rejected. *Capabilities analysis* or *feasibility analysis:* process analysis is but one

label broke down as soon as someone reported "Mr. A. is doing a process analysis of industry X."

As a practical matter the existence of two other, distantly related meanings of the phrase "process analysis" should rarely, if ever, cause confusion. Throughout this monograph in particular, "process analysis" always refers to the construction and use of industry-wide, multi-industry and economy-wide models which attempt to predict production relationships on the basis of technological structure.

A Division of Labor

We shall distinguish three areas of activity which are involved in process analysis. First is the model building activity itself. This starts with a study of technology and ends with a numerical mathematical model of an industry or industrial complex. Second is the development of algorithms. The object of this activity is to provide computing procedures to trace through the implications of models quickly and economically. The third area of activity is that of using models to throw light on practical problems of public policy. The prime interest here is not the model per se but the insight it can provide into the problem at hand.

We shall refer to these three areas of activity as model development, algorithm development, and policy application. Each is closely tied to the other two. In model development the size and form of the model is highly dependent on current computing capabilities, and the decisions as to aggregation and emphasis are based upon the needs of the potential policy maker. In the development of particular types of algorithms, the impetus and direction have frequently come from actual models and applications. Finally, to the policy maker, the usefulness of the process analysis approach depends on both cost and timeliness, which in turn depend on the state of the arts with respect both to models and to algorithms.

The construction of a process analysis model for a sector which has never been thus analyzed is time-consuming, and subject to substantial uncertainties. The same holds true for the development of a new type of computing algorithm. By contrast, the application of an existing model and/or algorithm—perhaps with minor modifications—requires relatively little time. The initial development activities represent an investment adding to our stock of multi-

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type of capabilities analysis; GNP analysis, for example, is another. *Activity analysis:* this phrase already has a well defined meaning; activity analysis is to process analysis as calculus is to physics; in both cases the latter uses the former as a technique of analysis to help deduce implications from initial postulates. *Economy-wide industrial engineering:* if industrial engineers cared to describe process analysis in this manner, they could make a strong case for it, particularly as they themselves increasingly make use of techniques such as activity analysis. But since most of the early process analysts were economists rather than industrial engineers, it would have been presumptuous for them to claim this label.

purpose models and computing procedures. When policy problems arise that demand decisions in weeks or months, only the models and algorithms in this available stock may be relied upon safely. Questions of industrial capability not covered by these models must be analyzed by cruder means or left to guesswork. As the stock of available models and algorithms increases, the more likely it becomes that a new problem will be related to an existing model, the cheaper it generally becomes to trace through implications, and the easier it becomes to put together broad-scope models involving industrial complexes or large portions of the economy.

This monograph is centered around the construction and application of models. The case studies deal at length with technological relationships—their measurement, aggregation, and representation—problems central to the building of process analysis models. Little emphasis is placed either upon algorithms or upon the broader issues of public policy. The policy maker may find one or more of the models presented here to be of practical value, but should take care to note the associated discussions as to the range of reasonable applications and the ways in which judgment may be needed to supplement formal analysis. The algorithm developer will find here several large linear and nonlinear programming problems with special (perhaps exploitable) structures.

Building a Process Analysis Model

Building models of industries and industrial complexes is closely related to the building of models of shops or plants for intra-enterprise analysis. Both require an understanding of technological relationships, both face problems of selection and aggregation, and both have a common body of analytical techniques upon which to draw. The differences are mostly a matter of degree. In models of broader scope, one faces a much more acute problem of selection and aggregation, does not have as immediate an access to primary data, and has a much more difficult time constructing meaningful tests of the model's validity. As a result, one should not expect the same degree of accuracy as is achievable with intrafirm studies. The task of constructing industry-wide models is simplified to the extent that detailed intrafirm models are available from which to borrow. With time, such intrafirm models should become increasingly available.

We will distinguish the following stages in the construction of a process analysis model: the study of technology, selection and aggregation, the choice of formal analysis, the collection of data, and the testing of the model. These stages are not isolated in time. While the analyst is studying the technology of the industry, he is mulling over alternatives with respect to selection; when he decides on aggregation he must worry about data as well as computation; at every stage he must return to find new information about technology. Thus the sequence in which these stages are listed reflects only the general order in which certain problem areas tend to command the principal attention of the analyst.

In brief outline, the nature of the stages may be characterized as follows:

THE STUDY OF TECHNOLOGY. An understanding of the principal products, processes, and resources of the industry is sought from its industrial engineering literature and trade journals. Contact with men who have production planning responsibilities provides valuable orientation. If the analyst is himself an engineer with experience in the industry, much of this is already part of his background.

SELECTION AND AGGREGATION. Some aspects of technology, although essential to the design of processes or the scheduling of equipment, can be ignored in making predictions of overall capabilities. Certain distinctions among products, processes, and equipment (although important for fine calculations of profit and loss) need not be made for the degree of accuracy required in a capabilities analysis. By selecting the important, aggregating the less essential, and ignoring certain distinctions altogether, the process analyst attempts to shape a manageable picture of technology.

THE CHOICE OF FORMAL ANALYSIS. The form in which the technological relationships should be cast depends on the nature of the questions to be asked and the availability of computing procedures with which to answer them. One or another type of analysis—such as linear programming, integer programming or simulation—will serve best depending on, among other things, the size of model, and the importance of features such as randomness and economies of scale. Algorithm availability frequently affects the size and nature of the model, not so much by laying down absolute limits as through differences in costs. The model builder should understand the costs involved with currently available algorithms, even if he does not completely master the details of their construction.

THE COLLECTION OF DATA. Frequently, much or all of the required data is to be found in engineering or trade sources, and in census sources. Care should be taken to understand the operational definitions—e.g., methods of measurement and aggregation—before using such numbers. In some cases, sample surveys may be undertaken to supplement the existing data.

TESTING THE MODEL. A common method of testing is to see if the model performs approximately as did the economy for some past period. The problem of validating a model is a difficult one, to which we will return in Chapter 3.

POTENTIAL APPLICATIONS

One area of application which particularly deserves attention is that of investment planning within the newly developing countries. Such planning, aimed at rapid economic growth, is subject to a variety of constraints in the form of bottleneck resources: foreign exchange, low-cost iron ore, skilled urban machinist labor, etc. Problems of substitution and of interindustry ramifications are of central importance in these investment programs. It is toward such problems that process analysis addresses itself.

Process analysis has other areas of potential use. For example, the early support of this work was motivated by the possibility of applications to military economics. At least before the advent of thermonuclear weapons, important aspects of military planning included the industrial ramifications of strategic bombing. Economic mobilization for nonnuclear warfare would still raise problems of industrial capability and program timing, as does any action which rapidly alters the composition of national output.

In general, a "potential application" of process analysis may be said to exist whenever part of a practical problem (albeit not the whole of the problem) concerns either the feasibility or the cost of achieving desired outputs in the presence of limitations on resources. Whether or not any particular potential application should be pursued as an actual application depends in part on the state of the art with respect to models and algorithms.

A PREVIEW

Parts II through IV of this monograph deal with individual sectors. Part II is concerned with petroleum and chemicals; Part III with agriculture; and Part IV with metals and metalworking.

Part V considers applications of process analysis to investment planning for newly developing countries. The Appendix (at the end of the book) discusses activity analysis and linear programming at an introductory level.

Although the studies in this monograph are interdependent with respect to objectives, approach, and areas covered, almost any part or chapter can be read before or after any other. There are a few exceptions: The four metalworking chapters form a whole, and the two petroleum chapters are best read in the sequence presented. If the reader is not already familiar with activity analysis and linear programming, he should first look at some introductory source such as the ones cited in the Appendix. Otherwise he can pursue the case studies and methodological discussions in whatever sequence his interests dictate.

CHAPTER 2

ALTERNATE METHODS OF ANALYSIS

Alan S. Manne and Harry M. Markowitz

INTRODUCTION

This chapter discusses three methods of analysis which—like process analysis—seek to predict the ability of the economy to meet desired objectives with limited resources. These three methods—namely, GNP analysis, requirements analysis and input-output analysis—together with process analysis may be grouped under the general heading of feasibility or capabilities analysis. There are some differences in the range of applicability of these methods, but each addresses itself to the following types of question: Is a particular national objective economically feasible? Are there resource limitations which will render the economy incapable of achieving its objective? What are the alternative economic targets that are feasible?

From the viewpoint of the analyst interested in practical policy development, the various methods should be judged in terms of applicability, accuracy, cost, and availability. The policy developer wants to know the types of question to which each method of analysis can be applied. Or, to state the "applicability" consideration conversely, he wants to know which of the available methods bears on his problems. "Accuracy" is concerned with whether or not the answers returned by the analysis are reasonably correct. When a method suggests conclusions counter to initial judgment, is it likely to be a true or a false oracle? If two methods of analysis were applicable to the questions at hand, and both provided equally acceptable levels of accuracy, then choice between them would properly depend on cost and availability. Here cost should include the total costs of preparing for, performing, and interpreting the results of the analysis; while availability may be thought of as the probability that any developmental work needed to make the method applicable to the specific problem at hand can be carried out without undue delay.

For each method, in turn, this chapter briefly discusses its nature and its salient characteristics with respect to applicability, accuracy, cost, and availability. We shall argue that for some problems—and for certain stages of other problems—the simplest method, GNP analysis, serves best. For other problems the more complex methods of requirements analysis, input-output analysis, and process analysis are desirable or essential. In areas where these methods are desirable but unavailable, the gap must be filled by that always available, sometimes accurate method called judgment.

GROSS NATIONAL PRODUCT ANALYSIS

The annual gross national product (GNP) is the money value of goods and services produced by the economy in a year. In contrast to *net* national product, GNP does not subtract depreciation allowances from the value of production. It does, however, exclude the value of intermediate products used within the year in the process of making other goods and services. Thus GNP represents the total money value of the economy's output available to satisfy "final demands" for household consumption, government expenditures, gross fixed capital formation, net inventory changes, and net exports.

Countless questions of detail must be resolved in order to produce an operational definition of GNP: How do we distinguish maintenance expenditures—which are excluded from GNP—from capital expenditures, which are included? Should GNP include the value of the food consumed by the farm families who grew it? Should it include the value of do-it-yourself activities in which people paint, repair, or build their own homes, boats, or furniture? With relative prices and product qualities changing from year to year, how should the value of goods and services be added together to permit reasonably meaningful comparisons of GNP over time? We shall not pursue these questions. Rather, we shall speak of GNP with no more qualifications than we speak of temperature or humidity—implicitly assuming that the various questions have been answered reasonably, and the resulting statistical procedure has been carried out consistently.

A GNP analysis estimates the total gross national product required to meet a proposed economy-wide program, and compares this with the GNP which the economy is likely to have at its disposal. If the proposed use of GNP exceeds the likely supply, something has to give way. Perhaps consumption or investment objectives should be reduced, or perhaps foreign funds should be sought. Or perhaps GNP can be raised through increased labor supply, e.g., increased female participation or longer working hours. In effect, a GNP analysis adds up the bill for proposed economy-wide objectives and compares this with the value of product available to cover this bill. The objectives will have to be reconsidered if the former exceeds the latter.

Although a GNP analysis is simpler to construct than the others, it is not without its difficulties. For example, projections of the supply of GNP usually involve estimates of labor force and productivity. Both of these factors are susceptible to errors of estimate. Nevertheless, in comparison with other methods, GNP analysis is by far the least expensive and most readily available.

Turning to the question of accuracy, we find that GNP analysis is subject to a definite weakness, but can nonetheless serve a useful function. GNP analysis, used by itself, tends to overestimate the capabilities of the economy. It fails to reject programs whose source of infeasibility is the shortage of specific, specialized resources as distinguished from resources in general. Suppose—to take an extreme example—that an agricultural nation attempted in the course of a single year to:

—substantially reduce its output of grain, and
—increase by an equal dollar value its output of steel.

The proposed program does not involve an increased requirement for GNP—yet it is clearly infeasible. Farm land and equipment cannot be converted quickly into steel mills, nor farmers into steel workers.

GNP analysis can serve as a coarse screen, catching proposals whose general demands on resources are out of line in total. Only those proposals which pass through this coarse screen are then subject to the finer screening of the more complex methods of analysis. The more complex methods look for specialized resources which will become bottlenecks if a proposed program is implemented. Not only do they attempt to answer with greater precision the question "Can the economy achieve the specified objectives with limited resources?" but they also address themselves to such questions as "Which resources will become bottlenecks and which will be plentiful?"

Thus, in the division of labor among methods of analysis, the inexpensive coarse screen approach of GNP analysis may have to be supplemented by some finer screen method. The rest of this chapter will discuss two such methods, and the balance of this monograph is concerned with a third.

REQUIREMENTS ANALYSIS

A requirements analysis for, say, steel might proceed as follows: X tractors and Y square feet of industrial construction are planned (or expected) for a forthcoming period of time. The average tractor requires A tons of steel; the average square foot of industrial construction requires B tons. Add A times X plus B times Y plus, similarly, any other uses of steel to obtain estimated total requirements. Compare this with the projected steel availability to see if a "steel problem" exists. The analysis may proceed through several levels, reflecting the fact that, e.g., automobile production requires steel, steel requires pig iron, pig iron requires coke, and coke requires coal. For computational convenience the requirements for several resources may be computed simultaneously, thus avoiding the duplicate recalculation of requirements for intermediate goods which contribute ultimately to the requirement for two or more resources.

The inputs to a requirements analysis consist of estimates of resource availabilities, desired levels of final demands, and requirements coefficients estimating the inputs needed per unit of output. From these, the analysis produces a shopping list of ingredients required to support the objectives. Comparisons between requirements and availabilities help to indicate possible trouble areas.

Requirements analysis is applied in practice to problems of various scope. Under different names it is used in manufacturing analysis to estimate expected needs for men, machines, standard materials and purchased parts. The military services use it to determine procurement quantities. At a national level it has been used, during war and peace, for tin, rubber, machine tools, blast furnace capacity, foreign exchange, and countless other potential bottlenecks. The problems of applying requirements analysis at a national level differ somewhat from those of applying it at an intrafirm level. Our concern will be with the former, broader-scope applications.

In an industry-wide steel study based, for example, on *U. S. Census of Manufactures* data, one could distinguish various mill shapes of "carbon steel," "alloy steel except stainless," and "stainless steel." But even "stainless steel" is an aggregate of many individual steels with differing applicabilities in manufacturing. Although the categories of a requirements analysis—e.g., stainless steel—are frequently discussed as if they were homogeneous commodities, in fact they are aggregates. The aggregation system of a requirements analysis in effect assigns specific goods and services to classes, and adds together the members of each class according to some criterion such as weight, volume, piece, or value. The characteristics and data requirements of each specific requirements analysis depend heavily on the aggregation system chosen. The choice of an aggregation system—whether done by design or by passive acceptance of existing categories—is the central, strategic decision in performing a requirements analysis.

Requirements analysis is not alone in its dependence upon aggregation. Aggregation is used with every practical technique of capabilities analysis at a national level. Aggregation problems constitute many, if not all, of the major problems of applying a technique. To a large extent, the differences between one technique and another may be viewed as differences in their approach to aggregation. GNP analysis, for example, represents an extreme form in which all goods and services are added together to form a single money value total. Formally, at least, GNP analysis may be viewed as a requirements analysis in which only one resource, national product, is distinguished. The cost of each good or service, then, is its requirement coefficient for GNP. Thus a GNP analysis is a requirements analysis with an extremely coarse aggregation of resources.

In a similar manner, requirements analysis may be viewed as a special case of process analysis. A process analysis can distinguish alternate ways of producing the same product. Requirements analysis aggregates these alternate productive processes into a representative activity with fixed inputs per unit of output. Thus, if a process analysis model did not distinguish alternate production activities, it would be a requirements analysis, and if it distinguished only one resource (gross national product) it would be a GNP analysis. Input-output analysis, finally, may be viewed as a form of requirements analysis using a somewhat different way of aggregating economic activity. The aggregation principles of input-output and their consequences will be discussed later in this chapter.

Thus each method of analysis has its own ground rules for aggregation. The specific aggregation system chosen within these ground rules determines the specific characteristics and data needs of the particular analysis.

The major cost involved in performing a requirements analysis is that of collecting and organizing data. Requirements analysis does not entail large calculation costs, as may be incurred for process analysis. With the available computing equipment, once the data for a requirements analysis are assembled in suitable form, calculations for even the largest analysis can be performed at a relatively small cost. With respect to the cost and availability of data, requirements analysis stands between GNP and process analysis. It needs a list of specific inputs per unit of output, as distinguished from the single money value figure which is sufficient for GNP analysis. Since requirements analysis does not develop coefficients for alternate productive processes, it can make extensive use of historical inputs and outputs to develop average requirements, avoiding (completely or in great part) the need for engineering data upon which process analysis frequently relies.

The major source of inaccuracy inherent in requirements analysis is its neglect of alternate modes of production. Often, substantially different inputs can be—and, in fact, are—used to produce the same product. Electricity can be produced by water power or, in steam electric plants, from either nuclear fuel, coal, oil, or natural gas; agricultural products can be produced using more or less fertilizer and irrigation; metals can be produced using varying ratios of scrap to ore; the same metalworking tasks can be performed on a variety of machines; and so on. A requirements analysis must attempt to estimate a typical process: e.g., the average use of nuclear fuel vs. coal vs. water power in the production of electricity. But the scarcity of one material relative to another will lead to the use of processes which conserve the one at the expense of the other. Thus, to an important extent, the use of one or another process will depend on the very shortages and surpluses that the analysis seeks to predict.

The manner in which a requirements analysis misestimates the capabilities of an economy, due to its failure to take account of substitution possibilities, depends on the aggregation system used. If two resources (e.g., lathes and milling machines) are aggregated together into the same category (e.g., machine tools) then they are assumed to be perfect substitutes for each other. If they are distinguished as different resource categories, then no substitution is assumed to exist. Thus an extremely coarse classification of resources will tend to overestimate the amount of substitution possible, and hence overestimate the capabilities of the economy. An extremely fine classification, on the other hand, will understate the amount of substitution possible between resources, and hence tend to underestimate the capabilities of the economy. In choosing an aggregation system for a requirements analysis the following dilemma must be faced: In order to anticipate bottlenecks among specific resources a fine classification is needed; but to avoid underestimating substitution possibilities a coarse classification of resources is needed.

Coal is sometimes a substitute for fuel oil, but not in all its applications; a lathe can sometimes substitute for a milling machine, but not always; aluminum and copper are competitors, but only in part of their range of applications. As a consequence, any aggregation system must be a compromise with, rather than a solution to, the dilemma. Any attempt to completely avoid one horn of the dilemma is bound to drive the analysis to the other horn.

In principle one could circumvent this dilemma by the following process of trial and error:

> Choose categories sufficiently fine to identify specific bottlenecks; estimate likely requirements; perform the analysis using these estimates; and inspect the results for bottlenecks.

Then, on the basis of this initial analysis:

> Modify coefficients to reflect processes which substitute plentiful for scarce factors of production; repeat the requirements calculation and again inspect the results; make further adjustments and repeat if necessary.

This is a tedious and time-consuming procedure which must, of necessity, be stopped short of its ultimate end. Process analysis in effect accelerates this procedure by distinguishing alternate processes at the outset, and by letting automatic techniques perform the substitution of one process for another according to overall resource availabilities.

Process analysis and requirements analysis are closely related, as is illustrated by the discussion of the metalworking industries in Part IV of this monograph. Data and procedures are presented in Part IV for a requirements analysis assuming fixed inputs of various kinds of men, machines, and materials per unit output of each metalworking industry. After this, data and procedures for analyzing a certain source of substitution possibilities are presented, plus suggestions concerning the analysis of another source of substitution. The requirements analysis serves as an immediately available technique to which information concerning alternate processes can be added, as appropriate and available.

In itself, without the addition of alternate process information, requirements analysis serves to identify potential trouble areas. In some cases the apparent bottlenecks are not bottlenecks at all. The economy would take care of the shortage naturally, by substituting plentiful for scarce resources. In other cases the bottlenecks are real; the timing and level of objectives should in fact be reconsidered in light of possible infeasibilities. By spotlighting possible trouble areas for further investigation, requirements analysis supplies a valuable service beyond that provided by GNP analysis.

INPUT-OUTPUT ANALYSIS

A difficult problem of requirements analysis is that of estimating total requirements as distinguished from direct requirements. For example, the production of electricity, say to light homes, requires coal; but the production of coal itself requires electricity, whose production in turn requires more coal; and so on ad infinitum. To make matters worse, the production of both coal and electricity have other requirements whose demands ramify through the economy, further augmenting the total requirements by electricity for coal. In the usual requirements analysis, indirect requirements are, after a point, accounted for by some rule-of-thumb procedure. An example of such a procedure would be to add together direct requirements, second order requirements, and third order requirements of each end item for a particular resource; see what fraction (X) of a particular year's use of this resource is thus explained; account for the rest by multiplying ($1/X$) times the sum of first, second, and third order requirements to form estimates of total requirements. Insofar as the fourth + fifth + sixth + · · · order requirement is not proportional to the first + second + third order requirement, the procedure is subject to error. The possible magnitude of this error depends on the extent to which first, second, and third order requirements account for the demands for the resource.

Input-output[1] approaches the problem of estimating total requirements through the use of a complete model of the economy. It classifies business establishments into an exhaustive set of industries and estimates the direct requirements by each industry for each other industry's output. These interindustry demands are arrayed in a square table with industries listed across the top and down the side. With this table (plus the assumption of fixed inputs per unit output) standard mathematical techniques can be used to answer questions such as "How much gross coal production is required to produce an extra one million dollars' worth of electricity, net of all intermediate interindustry requirements?"

As a theoretical matter, the notion of a complete input-output table dates back at least as far as the eighteenth century economist Quesnay. As a practical matter, the construction of an input-output table, in the sense used here, begins with the pioneering work of Wassily Leontief carried out during the 1930's and published in 1941. Subsequently—with electronic computers to trace through the consequences of ever larger systems, and with Leontief's work to demonstrate the feasibility of such an approach—various input-output tables have been built, including a 190-industry model of the United States economy for 1947.

Two forms of interindustry models are generally distinguished. The "closed model" (as first used by Leontief) includes households as an industry with inputs of consumption goods and outputs of labor. The "open model" (Cornfield, Evans, and Hoffenberg, 1947) does not include a household industry but treats demands by households as fixed requirements to be met by the economy. Also treated as fixed are the requirements for other components of "final demand" including government purchases, gross private capital formation, net inventory changes, and net exports. With respect to sectors other than households and other final demands, open and closed models can be identical. For problems of feasibility analysis, the open model is generally the more convenient.

In principle, an input-output analysis could use physical units of measure such as weight, volume, or count. In practice, however, dollar values have

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[1] Sometimes referred to as interindustry analysis.

been used almost exclusively. Thus the classic statement of procedure for estimating coefficients for interindustry requirements, expressed here for the closed model, is as follows:

> Each "industry" (including households) is treated as a single accounting entity—comparable to a "country" in official foreign trade statistics—with sales entered on one side of its trading account and purchases on the other. As in the trade between countries the sales of one industry are the purchases of another. Entering the sales and purchase accounts of all the separate industries in one large table we get a comprehensive view of the structure of the national economy as a whole.[2]

From this table of purchases and sales the direct requirement coefficients are calculated by dividing sales from industry i to industry j by the gross output of industry j.

The development of data for a large input-output analysis can require tens of thousands of man-hours. The 190-industry analysis of the U. S. economy, for example, required the estimation of thousands of coefficients. Most of these coefficients were not readily found in available statistics but had to be constructed from various sources, sometimes with the aid of rule-of-thumb estimation procedures. Computing costs for tracing out total requirements from the direct requirements, although not negligible, were small compared to the costs of constructing the basic table.

Because of the time required to collect and organize data for a complete interindustry table, "availability" is more of a problem with input-output than with GNP analysis or the usual requirements analysis. The development of a large input-output matrix should be viewed as a major construction project which is not to be rushed to answer some urgent policy question but is to be built carefully to serve many uses through the course of time.

INPUT-OUTPUT ANALYSIS (CONTINUED)

We shall note two general sources of inaccuracy to which input-output analysis is subject. The first concerns the existence of alternate methods of production. The second concerns the way in which "industry output" and "interindustry flows" are used as basic categories of analysis. Since the former problem area—the existence of alternate methods of production—was discussed previously, it can be dispensed with quickly in the present section. The nature and consequences of inaccuracies introduced through the other source will be discussed in some detail. Despite such inaccuracies in input-output analysis, the table itself—i.e., the basic tabulation of historical interindustry sales and purchases—is a valuable source of data concerning industrial activity. The basic table is tedious and expensive to develop, but so is much of the worthwhile economic data at our disposal.

In a preceding section we discussed difficulties of requirements analysis resulting from its failure to distinguish alternate methods of production. Radi-

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[2] Leontief (1951), p. 4. For an introduction to input-output, we also recommend Chenery and Clark (1959).

cally different methods of production exist for many goods and services. The choice of production method depends on relative scarcities of alternate resources, and hence average requirement coefficients depend on the very shortages and surpluses to be predicted. This consideration, already noted for requirements analysis, applies equally to input-output with its assumption of fixed interindustry flows per unit of output.

Input-output is also subject to inaccuracies due to the way in which it makes use of "industry output" and "interindustry flows." For concreteness, we shall illustrate the general nature of these inaccuracies by means of examples drawn from the 190-industry matrix of the United States in 1947. (See Evans and Hoffenberg, 1952.) Difficulties such as those illustrated below have been recognized by a number of analysts who have applied input-output to practical problems. To circumvent these difficulties, various special procedures have been introduced into particular input-output analyses. Our discussion cannot do justice to these various ways of not quite doing input-output. In the examples and generalizations below, we will be dealing essentially with the classical input-output formulation as characterized in the last section. Afterwards, we shall briefly argue our preference for a process analysis approach rather than supplementing input-output with ad hoc procedures.

Suppose that industry I sells to industries X, Y, and Z, and that it purchases from A, B, and C. In tracing out total requirements, the input-output procedure assumes that the proportions among the output of A, B, and C purchased by I to produce output destined for X is the same as those proportions purchased by I to produce output destined for Y or Z. This assumption can cause substantial distortion in estimates of total requirements.[3]

For example, the Non-Ferrous Foundries industry casts both aluminum parts (e.g., for aircraft) and brass parts (e.g., for plumbing fixtures). In tracing out total requirements, input-output analysis assumes that the proportions of aluminum, copper, and zinc in the castings purchased by the Aircraft industries are the same as the proportions purchased by the Plumbing Fixtures and Fittings industry. The Non-Ferrous Foundry industry is treated as if it receives materials destined for different end items, combines them into a homogeneous mixture, and sends this mixture to each purchaser of nonferrous castings.

The importance of this example depends on three points:

First: In a case such as the above the input-output procedure introduces substantial inaccuracies in the estimates of indirect requirements. In 1954,[4] for example, the Aircraft industries actually purchased $49 million worth of aluminum and aluminum-base castings vs. $4 million worth of copper and copper-base castings. The Plumbing Fixtures and Fittings industry, on the other hand, purchased $.5 million of aluminum and aluminum-base castings vs. $14 million worth of copper and copper-base castings. For these two, and

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[3] On this point, see also S. B. Noble (1960), especially p. 408.

[4] We use 1954 rather than 1947 figures here since statistics on castings purchased by the Aircraft Equipment n.e.c. industry are more complete for the later year.

for a number of other[5] large purchasers of nonferrous castings, the assumption of equal proportions is untenable.

Second: This difficulty cannot be avoided by a more detailed industrial classification. Manufacturing industries are collections of establishments. Interindustry flows are the sums of purchases by establishments in one industry from establishments in another. Since many establishments cast both aluminum and brass, no matter how finely we classify establishments into industries—even if we let each establishment be an industry in itself—brass for plumbing fixtures will appear to end up in aircraft, and aluminum for aircraft will end up in plumbing fixtures.[6]

Third: The Non-Ferrous Foundry industry is not alone in having this effect on the estimation of indirect requirements. Similar distortions are caused by any industry which supplies a service performed on a variety of materials on behalf of a variety of consuming industries. Examples include Iron and Steel Forging, Metal Stamping, Metal Coating and Engraving, Machine Shops, and Screw Machine Products.

Input-output analysis is frequently combined with the notion of industry capacity. The input-output analysis predicts gross production required from various industries. By comparing these gross required outputs with the available capacities, potential bottlenecks are identified. This procedure encounters difficulties when industries can, in effect, borrow capacity from each other. Such borrowing of capacity is particularly common among the metalworking industries, which fabricate and assemble metal parts for a great variety of military, household, and industrial durable goods. Skills and equipment needed to perform the tasks of one of these industries typically overlap with those required for other such industries. Many shops regularly or occasionally produce parts destined for commodities of other metalworking industries. It is for such reasons that we find, according to the U. S. input-output table for 1947, that 9 cents' worth of Motor Vehicles, 1.8 cents' worth of Aircraft and 1 cent's worth of Motorcycles and Bicycles were directly "required" to

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[5] The following examples present millions of dollars' worth of purchases of aluminum and aluminum-base castings vs. copper and copper-base castings (the data presented in that order) for some 4-digit census industries which consume large amounts of nonferrous castings and show a large discrepancy from the proportionality assumption. High aluminum consumers: Domestic Laundry Equipment (12.7, .1), Electric Appliances (10.1, .1), Metal Doors, Sash and Trim (4.7, .1), Internal Combustion Engines (21.8, 1.5); High copper consumers: Valves and Valve Fittings, except Plumbing (2.2, 22.8), Power Transmission Equipment (1.7, 5.7), Pumps and Compressors (4.4, 13.0).

Source: *U. S. Census of Manufactures*, 1954, Table 1B, pp. 210–238.

[6] This discussion is based upon (*a*) the definition of industry used by the *U. S. Census of Manufactures* and (*b*) the definition of interindustry flow used by Leontief (1951). One modification of these conventional definitions is to segregate the purchases and sales of establishments by product line, thereby departing from the establishment basis of classification. Since nonferrous foundry establishments have labor and equipment which may be used interchangeably for the casting of both brass and aluminum, the use of this product line classification would raise problems of the sort discussed immediately below for the metalworking industries.

make $1 worth of Locomotives; that 7 cents' worth of Motor Vehicles and at least 1 cent each of Aircraft, Ships, and Railroad Equipment were required to make Motorcycles and Bicycles.

Whether or not it is combined with the notion of industry capacity, the input-output procedure is inadequate here for at least two reasons. First, it fails to analyze the capabilities of one such industry to supplement another. Second, it assumes that since 1.8 cents' worth of Aircraft is required directly for Locomotives, a proportionate share of everything that went into Aircraft should also be incorporated into Locomotives. The effect is similar in nature, though less in amount, to the mixing of flows that occurred through intermediate industries such as forges, foundries, and machine shops.

The final difficulty to be discussed occurs when two or more joint products result from the same process. The way this can affect the analysis of economic capabilities may be illustrated by the case of coke production.

The Coke and Products industry produces coke (mostly for blast furnaces) as its main product, and basic organics (for the chemical industries) as a by-product. Suppose there were a fall in the demand for steel. This would reduce the demand by Steel for Blast Furnace output; reduce the demand by Blast Furnaces for Coke output; and thus, according to an input-output matrix based upon purchases and sales, release Coke industry capacity for use by the Organic Chemicals industry. But this implication is opposite in direction from what may be expected in fact. Since basic organics are a by-product of coke production, the reduced production of coke would *reduce* the by-products available for the chemical industries. The chemical industry would either have to use alternate sources of raw materials or reduce its production. Thus, according to a purchase-and-sales analysis, additional "Coke Oven Capacity" would be made available by the fall in steel production, whereas in fact the flow of organics from coke ovens to the chemical industry would be reduced.[7]

To a certain extent, difficulties such as the above can be circumvented without giving up the appearance of an input-output table. The problem of alternate methods of production, for example, can be handled by trial-and-error procedures similar to those described in connection with requirements analysis. The problem discussed in connection with the foundry, forge, stamping, and machine shop industries can be handled by treating the primary metal purchases of such industries as if they were direct purchases by the end item producer.

In some cases, it would be extremely difficult to characterize accurately an aspect of technology within an input-output framework. The sharing of capacity between metalworking industries, for example, could be handled by means of "conversion coefficients" which showed the extent to which the capacity of one industry could be converted to another. Such coefficients would still fail to characterize properly the possibilities for reducing output

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[7] An important instance of joint products will arise in multiperiod models of economic development. By investing in durable capital equipment, we obtain a sequence of joint products: capacity available for use during more than one time period.

in one set of industries to supply equipment and labor needed in another set. A more satisfactory approach is to explain the sharing of capacities in terms of the kinds of transferable resources used by these industries.

To many, the attractiveness of the input-output approach is that it permits the construction of a complete model of the economy wihout requiring an understanding of countless technological relationships. After N industries have been chosen, N^2 coefficients can be delegated to a data collection team. Data may not be immediately available, but at least the team has a well defined objective: "Find or estimate the amount sold from industry i to industry j during the specified year for each i and j."

We have argued above that various supplementary procedures must be used if the implications of such an analysis are not to be completely unreasonable. A serious difficulty with the input-output approach is that it provides no systematic way for seeking out those aspects of technology which require such special handling. Frequently input-output matrices are constructed and used without regard to such pitfalls. Sometimes these pitfalls are revealed through obviously absurd implications of the analysis. Other times pitfalls are found when someone looking at technology asks "What would happen if these technological relationships were forced into the input-output form?" There is no guarantee, however, that such ad hoc finding and patching of difficulties will not leave equally serious problems undetected.

SUMMARY

Gross national product analysis serves as a coarse screen to reject grossly infeasible programs. It does not detect programs whose infeasibility is due to excessive demands for particular specialized resources.

Requirements analysis compares the demands and supplies of specialized resources. Its chief drawback is its failure to account for alternate modes of production. Despite this difficulty, it can serve a valuable function in pointing out possible trouble areas.

Input-output is a form of requirements analysis which addresses itself particularly to the question of estimating total requirements—both direct and indirect. Input-output analysis fails to account for alternate methods of production. Additional difficulties in its use for capabilities analysis arise from the way in which it uses interindustry sales and purchases as the basic source of data.

Process analysis may be viewed as a generalization of requirements analysis which allows alternate modes of production to be distinguished wherever these are deemed important. Cost, availability, accuracy, and applicability characteristics of process analysis will be discussed in the next chapter.

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CHAPTER 3

PROBLEMS AND POTENTIALS OF PROCESS ANALYSIS

Alan S. Manne and Harry M. Markowitz

The experimental nature of much of process analysis, and the rapid rates at which certain contributing arts are advancing, make it hazardous to estimate the potentials and limitations of this approach. The methods discussed in Chapter 2 have been used extensively. Even the newest, input-output, has been applied more than once to the U. S. economy, and to an impressive list of other countries. In contrast, the studies in this volume generally represent first attempts of their type for the sectors covered. The most that can be claimed with certainty for the majority of these studies is that the models have been completed and have at least a familial resemblance to the industries represented. Further statements about cost and reliability are based largely on extrapolation and conjecture.

These extrapolations and conjectures, however, are central to our belief that the process analysis approach can make practical contributions to the understanding of an economy. This chapter presents these extrapolations and conjectures, at least to serve as a basis for further consideration.

APPLICABILITY: SCOPE OF ANALYSIS

In connection with process analysis applications, we shall consider two topics. In the present section, we discuss the scope of such models, and in the following section the type of decision to be influenced.

A process analysis application may cover a single industry or a group of industries. In principle, if models were available for all industries, a process analysis could cover the economy as a whole. Later, we discuss the feasibility of such all-embracing analyses. For the moment we will consider some problems of application which seem to apply most forcibly to smaller scope, industry-wide models, but in fact are equally serious as we widen the scope of the analysis.

Most of the studies in this monograph, being static in nature and covering a single industry or limited industrial complex, are closely related to the economist's "static, partial equilibrium analysis." The problems encountered in such uses of process analysis are those of any practical application of static, partial equilibrium analysis. They cluster in two groups around the phrases "static equilibrium" and "partial analysis," respectively.

The difficulty in using a static equilibrium analysis is that of interpreting its results for a world in constant change. Ideally we would prefer accurate dynamic models, thus avoiding the problems of static approximations. Thoughts concerning the construction and use of dynamic models are scattered throughout this monograph, but for the present, it is principally the static analysis which is available for application.

The details of a static model should depend upon the time horizon of the practical problem. For example, in setting up a linear programming computation, certain capacity restrictions or material availabilities, although fixed in the short run, should be regarded as expansible in the long run. There is no sharp boundary between the short and the long run. As we lengthen our view, an increasing number of resources are best treated as reproducible (or variable within limits) rather than rigidly fixed.

The major problems which arise from the "partial" nature of an analysis concern the handling of boundary conditions. Industries are connected with other industries via the products they sell or buy. In the long run, this includes equipment needed to expand capacity as well as inputs required on current account to manufacture the product. In a partial analysis (say, of a single industry) the products which the industry obtains from elsewhere can be treated either as fixed in supply, or as procurable (perhaps within limits), or as subject to a supply curve (perhaps as established in another analysis). The products which the industry supplies to other sectors can be treated as subject to fixed requirements, or salable at fixed prices or subject to a demand curve. Where there is uncertainty concerning availability or demand, several alternate levels can be explored.

Problems of boundary conditions do not disappear as we increase the scope of our analysis. When a model of industry A is combined with one of industry B to form a model of industrial complex AB, the interface between A and B is handled by formal computation, but the relations of these industries to other industries and to final demand must still be treated as boundary conditions. If all industries were combined into a single model, both the final demands for end items and the supplies of basic resources would still remain as boundary conditions to the analysis. The same options are available in treating these as are available in handling the boundary conditions of an individual industry in an analysis of smaller scope. For problems in which interrelations between industries play a crucial part, a multi-industry model may prove essential. One danger with such broader scope models is that the details concerning boundary conditions will perhaps not be considered with the same care as can more easily be given them in the smaller analyses.

APPLICABILITY: TYPE OF DECISION

Now we turn to the type of decision which can be influenced by a process analysis model, and comment on the apparent ability of such models to prescribe manufacturing procedures as well as to identify bottlenecks and to judge overall feasibility.

For its own use at least, a process analysis calculates the best method by which objectives may be achieved. In principle, one could use this solution to prescribe detailed production plans. In some cases, the analysis might contribute usefully to the choice of methods of production. In other cases, however, such an attempt to specify production details would be more likely to hurt than to help the economy.

In the case of complex industries in highly developed countries (e.g., U. S. metalworking, with thousands of heterogeneous establishments producing partially overlapping sets of products), any attempt to use the results of an industry-wide model to prescribe detailed operations would probably be detrimental. In an analysis of such industries, the categories of men, materials, equipment, and productive processes must be coarse aggregates. The industrial engineer with a knowledge of the particular circumstances of his establishment can better plan the allocation of his resources on the basis of analyses of narrower scope.

The desirability of a division of labor between broad-scope and narrower-scope analysis is not peculiar to either a market economy or a planned economy. In a market economy, broad-scope analysis is required to judge the feasibility and estimate the impact of major government actions, e.g., in the areas of military preparedness, regional unemployment, or agricultural policy. Problems of implementing these policies are passed on—in part through the government organization, in part through the market mechanism, in part through the business organizations of individual firms—to those responsible for specific production decisions. In a centralized economy, over-all programs must similarly be supplemented by detailed plans of smaller scope. The principal difference is in the channels of communication between the broad and narrow planning activities, and in the incentives at some of the stages of this process.

Thus, for complex sectors within highly developed countries, the chief use of industry-wide analyses consists of cost and feasibility estimates and general notions of shortage and surplus. Detailed planning decisions can and should be delegated.

The situation is very different in the case of a new industry in a less developed country. There an industry-wide analysis may encompass little more than would an establishment-wide or a firm-wide analysis within a highly developed country. In the chemical plant location study in Chapter 6, for example, the problem is to locate one, two, or at most five, plants of each of two types in the entire Latin American region. From an "industry-wide" analysis of such a nature, one could hope for reasonable prescriptions concerning detailed investment decisions.

ACCURACY AND TESTING

A process analysis model can be viewed as a complex hypothesis which, like hypotheses in the physical sciences, has implications concerning observable phenomena. The natural hope is that process analysis models can be tested by methods analogous to those of the physical sciences. Although the experimental methods of physics and chemistry are not available, the checking of theory against fact as in astronomy or meteorology seems appropriate. Towards this end are the tests reported in Chapters 4, 5, and 9.

It is important to realize some of the limitations inherent in such tests. Our remarks, however, are not intended to discourage testing. Checks of the model against recent history serve, if for no other purpose, to catch gross errors such as misplaced decimal points or reversed algebraic signs. Although they do not provide the level of confidence we would desire, their contribution is in an area where otherwise we have next to nothing.

The difficulty with historical testing of models is twofold: First, it is possible for an extremely inaccurate, untrustworthy model to do well at such tests. (We do not mean "do well" accidentally, one time, but do well consistently even though the model is of little use as a policy guide.) Second, it is possible for an extremely useful model to perform quite poorly.

In order to illustrate the possibility that a poor model can perform consistently well in tests against history, consider the following highly simplified example. Suppose that there are two products, *a* and *b*, and two limited resources, *A* and *B*. One unit of *a* can be produced only by using a unit of *A*; a unit of *b* can be produced only by using a unit of *B*. The model, however, incorrectly assumes that either a unit of *A* or one of *B* may be used to produce *a*, and that the same is true for *b*. In this case the production possibilities, in fact, are given by the region *OACB* in Figure 1, whereas the model would assert that the entire region *OA′B′* is attainable. If the economy is efficient it will produce *C*, and a historical test will vindicate the model. Suppose that the model was used to form policy decisions, and that plans were made requiring the product mix represented by *C′*. This mix could not be achieved by the economy since the only feasible points are those in *OACB*. If the economy remained efficient, *C* would be produced (perhaps disrupting plans based on *C′*). If, forgetting about the original plans to produce *C′* and the failure of the economy to accomplish this goal, we performed a test by history,

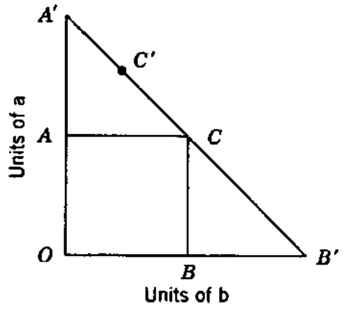

FIGURE 1

we would again find that the performance of the economy was consistent with the model.[1]

This example illustrates the possibility that a model can be quite useless for policy evaluation and yet score well according to the test of history. The converse may also be true. For example, in connection with requirements analysis, we have already noted that judgment may be used in an iterative fashion to modify the original model. Even though the initial model may fail the test of history, it nevertheless contributes to the general process of arriving at valid conclusions.

Cost and Availability of Broad-Scope Models

In principle, the process analysis approach permits a more accurate description of economic alternatives than do the alternate approaches discussed in Chapter 2. This superiority is merely academic, however, unless process analysis models can be developed and used economically. The studies in this monograph illustrate the feasibility of the process analysis approach in single industries. Now we turn to a more speculative issue concerning the feasibility of broader-scope (multi-industry and economy-wide) models.

We divide this question into two parts, the first concerned with computation, and the second with model building and data collection. In effect we ask: Is it possible, at reasonable cost and in a reasonable time, to formalize technological relations which characterize much or all of the economy? Can data be obtained to supply the parameters of such models? Can the consequences of such models be derived for problems with important economy-wide ramifications?

Computation

The art of linear programming has evolved rapidly, and gives ample evidence of a continued rapid evolution. Statements concerning computing costs become obsolete quickly. For example, in 1959 computing limitations restricted the size of linear programming models to 250 equations. By 1961—only two years later—a general-purpose linear programming code had run several 500-equation problems, and was capable of handling systems of up to 1000 equations. Without going beyond well-established principles (but by taking advantage of the special structure of large process analysis matrices), a code could be developed for handling systems with thousands of equations.

Linear programming is not applicable to all process analysis models. For example, in some instances involving economies of scale, it may be desirable to make use of integer programming optimization methods. In extremely complex cases (e.g., multisector spatial models involving sequential decisions over

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[1] We may, however, be led to somewhat greater confidence in a test by history if the relative implicit prices within the model (the slope of $A'C'CB'$) check reasonably well with the historical market prices. Such a test is described by Manne, pp. 40–42.

time in the face of uncertainty), there may exist no economical techniques for optimization. For such problems, simulation has proved useful.

A simulation analysis does not find an optimum policy. Rather it evaluates proposed policies under various contingencies. On the basis of initial simulation runs, new proposals may be devised and in turn tested by further runs. The evaluation of alternate proposals may be less desirable than would be the development of an optimum strategy; but for models beyond the range of optimization techniques such evaluation is preferable to implementing vast programs without prior testing, or to testing programs with models which seriously misrepresent the technological possibilities.

Despite the rapid progress in optimization techniques and simulation, computation capabilities remain a scarce resource. They constitute an important consideration in formulating economy-wide models. Depending upon the particular policy application, spatial, temporal, or technological details will be emphasized and others suppressed in order to fit within a computational budget. There remains great value in finding more economical computing methods to increase the analyst's ability to freely explore problems of economic capability.

Model Building and Data Collection

In the present section we discuss requirements for building economy-wide process analysis models. Of the three alternative approaches described in the preceding chapter, input-output is the most demanding of time and resources. We will be particularly concerned with comparing the demands of this approach, as a benchmark, with those of process analysis.

Process analysis models built for individual industries (or closely related industrial complexes) can have value in themselves. Examples of such sector studies are noted throughout this monograph. The sector studies do not individually make excessive demands on time and resources. They require an effort roughly comparable to that of a Ph.D. dissertation. The building of a one-sector model should lie well within the resources of any agency charged with responsibility for analyzing the capabilities of that sector.

Once the individual sector studies exist, multisector models can be obtained at relatively low incremental cost. If we include the time and resources needed to build the individual sector models, the requirements for an economy-wide process analysis model are substantially greater than those of a detailed input-output table. On the other hand, if we include just the incremental cost of combining sectors, the cost of an economy-wide process analysis model is only a small fraction of that of input-output.

A vast amount of information concerning productive processes is publicly available. The analyst's chief job is that of selection and aggregation. Insofar as needed data are not in the public domain, our own experience has been that managers are generally quite cooperative in supplying information for a scientific cause—provided that the nature of the information is clear, that it is readily available within their files, and that its confidential nature is carefully respected. Generally, the information needed for process analysis is more readily available than that needed for input-output. In running a business, it is not necessary to be able to classify sales and purchases by census categories, but it *is* necessary for someone to know what materials and resources are used and what products are produced by manufacturing operations.

Increasingly, techniques such as linear programming and simulation are being applied at an intrafirm level, e.g., to problems of equipment selection and plant operation. Such intrafirm models present both data and relationships in a form particularly suited to the needs of the process analyst.

Some sectors or aspects of the economy do not readily lend themselves to technological analysis. This is particularly characteristic of such service industries as retailing, advertising, and banking. The inputs to these sectors should probably be handled as boundary conditions of a process analysis.

Technological relationships are neither ageless nor universal. Productivity coefficients should be updated with time, and new processes and products added as they become important. In extrapolating from one country to another, coefficients must be modified to reflect, e.g., the average age of equipment in a given category, or the average skill and experience of labor in the performance of labor-paced tasks.

There is nevertheless a substantial degree of transferability of technological models in time and place. For an industry-wide model, the list of processes employed by the economy changes slowly. The updating of production coefficients (which should be done periodically for all models) is a much easier task, more subject to delegation and routine procedures, than is the original model building.

The transfer of models of technology from one country to another is most easily accomplished in the case of equipment-paced processes, although adjustments may have to be made to reflect differences in equipment age and efficiency. For labor-paced processes, further adjustments may have to be made to account for differences in work habits and experience between one region and another. Also, because of radical differences in the capital-labor availability ratio, productive processes may be relevant for one country but not for another. Nevertheless, as illustrated in Chapter 16 for Mexico, experience thus far suggests that the existence of models for one country is a great aid in the construction of models for similar industries elsewhere.

For previously unexplored sectors, process analysis models should not be mass-produced. They require an individual or a team familiar with detailed technological relationships and the types of models by which these relationships can be portrayed. Even for such an individual or team the construction of a model may be a time-consuming research activity subject to the unpredictability that research is generally expected to have.

The development of process analysis models can, nevertheless, be encouraged and assisted. For example, funds for Ph.D. candidates who wish to write dissertations in this area would help attract interested graduate students. A central clearinghouse for studies and data could help answer questions such as, "Now that I am done with my study, what should I do with the worksheets or punched cards of data which I no longer need but which may be valuable to someone in the future?" or "Has anyone already done a study in a particular area? If so, where is it published? And how can I get his final (and perhaps intermediate) data in a machine-sensible form?"

Considering the value of individual sector studies, and the nominal cost of multi-sector models once the sectors have been analyzed, the long-run prospects for broad-scope process analysis seem quite bright. But what about the short run? If no process analysis models existed at all, the policy planner would be well advised to use some form of requirements analysis on those proposals which passed the coarse screen GNP test. Insofar as process analysis models do exist, he can introduce greater technological detail in areas which he feels germane to the problem at hand. Each addition to our stock of reliable sector models makes it more likely that the policy planner will be able to represent satisfactorily those parts of the economy which play a critical role in his practical problem.

PART II

PETROLEUM AND CHEMICALS: PRODUCTION, TRANSPORTATION, AND PLANT LOCATION

A GLOSSARY OF TECHNICAL TERMS*

ALKYLATE. Product obtained in the alkylation process. Chemically, it is a complex molecule of the paraffinic series, formed by the introduction of an alkyl radical into an organic compound.

ALKYLATION. A synthetic process for the manufacture of components for aviation gasoline.

ANTIKNOCK AGENTS. Chemical compounds which, when added in small amounts to the fuel charge of an internal-combustion engine, have the property of suppressing or at least of strongly depressing knocking. The principal antiknock agent which has been developed for use in fuels is tetraethyl lead. Iron carbonyl and aniline (and other aromatic amines) have had limited use.

API GRAVITY. Arbitrary scale for measuring the density of oils, adopted by the American Petroleum Institute. Water is 10° API, gasoline approximately 55–60°.

ASTM DISTILLATION. A distillation test made on such products as gasoline and kerosene to determine the initial and final boiling points and the boiling range.

BARREL. Petroleum industry uses 42-gallon barrel as the standard barrel.

BOTTOMS. In a distilling operation, the portion of the charge remaining in the still or flask at the end of the run; in pipe stilling or distillation, the portion that does not vaporize.

Btu. Abbreviation for British thermal unit, a unit of heat commonly used in heat engineering. It is the amount of heat necessary to raise the temperature of one pound of water one degree Fahrenheit.

CATALYST. A substance which effects, provokes, or accelerates reactions without itself being altered.

CATALYTIC CRACKING. A method of cracking in which a catalyst is employed to bring about the desired chemical reaction.

CETANE NUMBER. Diesel fuel ignitability performance measured by the delay of combustion after injection of the fuel. It represents a comparison of a fuel with standards which are cetane in alpha-methyl-naphthalene.

COKING. The process of distilling a charge of oil to coke. In the last part of a coking run on a shell still, the bottom of the still is at a red heat and most of the volatile matter is driven out, leaving the coke hard and dry.

CRACKED GAS OIL. The gas oil formed as one of the products of a cracking reaction. It should not be confused with the term "gas oil cracking stock," one of the possible inputs *into* a cracking still; "cracked gas oil" is sometimes known as "catalytic gas oil" if the cracking process has involved the use of catalysts.

CRACKING. High temperature treatment of a given material (usually termed the "cracking stock" or "charging stock"). In this process, the long molecules of the cracking stock are broken up, with the attendant formation of gasoline. Other reaction products are gas oils, residual oils, and various gases.

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* Reprinted by permission from A. S. Manne, *Scheduling of Petroleum Refinery Operations,* Harvard University Press, Cambridge, Mass., 1956, pp. 5–9. Most of the definitions here are quoted directly from the glossary in *Fundamentals of Petroleum,* U. S. Bureau of Naval Personnel, NAVPERS 10883 (1953), pp. 161–172.

DISTILLATION. Distillation generally refers to vaporization processes in which the vapor evolved is recovered, usually by condensation, and a separation effected between those fractions which vaporize and those which remain in the bottoms. (See Fractional Distillation.)

END POINT (EP). The highest temperature indicated on the thermometer inserted in the flask during a standard laboratory distillation test. This is generally the temperature at which no more vapor can be driven over into the condensing apparatus.

FRACTIONAL DISTILLATION (SEE DISTILLATION). Fractional distillation implies the use of equipment for effecting a more complete separation between the low and high boiling components in a mixture being distilled than does the general term distillation. It is usually accomplished by the use of a bubble tower or its equivalent.

GAS OIL. Term originally used to mean oil suitable for the manufacture of illuminating gas. Now employed to designate an overhead distillate product with a boiling range intermediate between that of kerosene and residual fuel oil. The material is used as fuel for home furnaces and diesel engines and as a cracking stock. Also known as "distillate oil" or "middle distillate."

INITIAL BOILING POINT (IBP). The temperature at which the first drop of distillate falls from the condenser into the receiver in a standard laboratory distillation procedure.

KEROSENE. A petroleum overhead fraction with a boiling range intermediate between that of gasoline and gas oil. Used as an illuminant, stove oil, and tractor fuel.

MIDDLE DISTILLATES. A generic term for kerosenes and gas oils.

NAPHTHAS. Oils of low boiling range (80°F to 440°F), usually of good color and odor when finished. Sometimes refers to gasoline components and sometimes to special products, solvents, etc.

OCTANE NUMBER. Term used to indicate numerically the relative antiknock value of automotive gasolines, and of aviation gasolines having a rating below 100. It is based on a comparison with the reference fuels iso-octane (100 octane number) and normal heptane (O octane number). The octane number of an unknown fuel is the volume per cent of iso-octane with normal heptane which matches the unknown fuel in knocking tendencies under a specified set of conditions. Either the Motor method or the Research method may be used in determining octane rating of automotive gasolines; either the Aviation method or Supercharge method may be used in determining the octane rating of aviation gasolines. The test method employed *must* be reported with the octane rating.

POLYMERIZATION. A process for uniting light olefins to form hydrocarbons of higher molecular weight.

RECYCLING. The reuse of cracked distillate products as a charge stock in the same cracking process.

REDUCED CRUDE. The bottoms remaining from a distillation of crude oil.

REFORMING. A process for converting low octane number naphthas or gasolines into high octane number products.

REID VAPOR PRESSURE. The measure of pressure exerted on the interior of a special container (Reid Vapor Pressure apparatus), under specified test conditions.

RESIDUAL FUEL OILS. Fuel oils which include either reduced crudes or viscous cracked residuum. Used as fuel for industrial heat and power and also for marine and locomotive boilers.

RESIDUUM. The dark colored, highly viscous oil remaining from crude oil, after the more volatile portion of the charge has been distilled off.

STRAIGHT-RUN GASOLINE (RAW GASOLINE). A gasoline which is obtained directly from crude by fractional distillation.

TETRAETHYL LEAD. A volatile lead compound, $Pb(C_2H_5)_4$, which, when added in small proportions to gasoline, increases the octane rating.

THERMAL CRACKING. The process of cracking by heat or by heat and pressure.

VISBREAKING. A mild cracking process employed in order to reduce the viscosity of residual stocks.

CHAPTER 4

A LINEAR PROGRAMMING MODEL OF THE U. S. PETROLEUM REFINING INDUSTRY

Alan S. Manne

This chapter is addressed to the problems of estimating output capabilities for an entire economy. It represents a one-industry experiment in relying primarily upon engineering data for this purpose, rather than upon time series information alone. The use of linear programming, combined with the shift in emphasis upon sources of information, holds out the promise of greater forecasting reliability than is otherwise attainable.

This chapter reports upon the construction of one of a series of process analysis models that are being developed for the United States economy (see Markowitz, 1955). The general purpose of these studies is to give numerical answers to questions about production capability, and at the same time to take account of potential substitutions between alternative production processes. Among the sectors now under study are the following: metal machining, iron and steel, chemicals, and fuel and power.* Ultimately, it is believed that the technological capabilities of an entire economy could be covered in a similar fashion.

Typically, a process analysis model employs not only time series information on input-output relationships, but also engineering estimates of alternatives to these observed relationships. By using linear programming in place of the square Leontief interindustry flow matrix, process analysis models provide a more satisfactory allowance for both substitutability and complementarity effects. Success here depends upon progress in several distinct but allied fields—the formulation of meaningful problems, the collection of suitable data, the computation of large-scale systems, and the testing of results. Since a reader can best appreciate the interaction between these things after seeing something of the construction of a process analysis model, further discussion of the general problem will be deferred until the final section of this chapter.

In the case of the petroleum model, the basic question that has been asked is

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* *Editors' note:* The metal machining and iron and steel studies constitute Chapters 9–14 of this volume. Chemicals and fuel-and-power analyses were begun, but were not completed.

the following: Using the refining equipment and the raw materials available in the U. S. A. on January 1, 1953, what product-mix alternatives were possible as between the output of JP-4 jet fuel and the output of other refinery products, and how would these alternatives be affected by a reduction in the available capacity of refining equipment?

In making these estimates, the first step was to construct a linear programming model of the U. S. refining and crude oil production sectors. This contains details, not only upon the manufacture of JP-4 jet fuel, but also upon the following major end items: 115/145 avgas; 100/130 avgas; premium motor fuel; regular motor fuel; numbers 1, 2, and 6 fuel oil; diesel fuel; lube oils and asphalt; coke; liquefied petroleum gases; and the basic aromatic chemicals.

It must be emphasized that there were many simplifying assumptions involved in setting up the industry-wide model, and that there is a considerable margin for error in each of the detailed estimates. Fortunately, the model's predictions check with observable events in two important respects: (*a*) the model predicts that the industry could have produced only a slightly higher quantity of the 1952–53 product mix than was actually achieved; and (*b*) "shadow prices" on individual items in this particular product mix correlate fairly well with the actual market prices that prevailed during the 1952–53 period. The chief discrepancy between the market prices and the "shadow prices" (i.e., marginal rates of substitution within the model) occurred in the case of product categories that had not been of primary concern—liquefied petroleum gases and number 6 fuel oil. For this reason, the reader is cautioned that the model is not an all-purpose collection of technical data. Other industry-wide applications might well require a more detailed analysis of certain refining operations than is presented here, and would at the same time require less detail in other categories. There is no doubt that each of the estimates could be improved by careful scrutiny from members of the petroleum industry. Even in its present form, however, the model appears to give reasonable answers to a wide range of questions dealing with the substitutability of jet fuel and other refinery products.

MAJOR ASSUMPTIONS

Since this study is largely concerned with manufacturing technology, scant attention has been paid to either the geographical or the dynamic aspects of the problem. The analysis proceeds as though the crude oils, the refining equipment, and the products were concentrated at a single point in space and time. The model neglects the fact that transportation resources may be limitational, and it neglects the implications of new investment and of inventory accumulation.

This is not to say that the spatial and temporal aspects are of negligible importance—only that at the time these computations were performed (1954), an n-region, t-period model would have made the work inordinately expensive, except for the special case of $n = t = 1$. By suitable aggregation, of course, the amount of technological detail may be brought within more modest proportions, thereby making it feasible to attach time and location subscripts to individual items.[1]

It is apparent that a substantial change in the formal structure of the model would result from an attempt to relax the assumptions as to space and time. The other major assumptions could be altered with little more than a numerical change in certain constants or coefficients: (1) The reference date for all calculations is January 1, 1953. (2) Both the equipment capacities and the crude oil availabilities are taken to be a datum—at the overall maximum rates estimated for January 1, 1953.[2] (3) All end item specifications (octane number, performance number, boiling range, aromatics content, vapor pressure, and viscosity) are set at the average levels prevailing on this date. (4) Of the petroleum refinery inputs other than crude oil, only the following items are considered explicitly: isopentane, natural gasoline, tetraethyl lead, heat, and C2, C3, and C4 gases. These are taken to be available at the average rate prevailing between July 1, 1952, and June 30, 1953. All other inputs (e.g., labor, electric power, catalysts, and sulfuric acid) are considered to be nonlimitational, and are excluded from the analysis.

On the technical side of the process, two primary simplifications are made: (1) Just 25 categories of crude oil are considered here—despite the fact that there are well over 300 distinct oil fields within the United States. (2) Refining equipment types are distinguished by broad categories, but not by particular patents. For example, "catalytic cracking" and "catalytic reforming" are obviously taken as separate categories. No distinction is made, however, between the Fluid Catalytic Cracking of the Universal Oil Products Company and Thermofor Catalytic Cracking of the Houdry Process Corporation. Although the operating and investment costs do differ as between individual types of catalytic cracking processes, the product yields seemed sufficiently comparable to justify the aggregation.

Appendix A at the end of this chapter presents the list of products that are under study here. Although there are 14 distinct end items considered, this is not an exhaustive list of refinery products. For example, the catch-all phrase "vacuum distillation bottoms" refers to a whole variety of final products—numerous grades of lubricating oils and of asphalts. Without considerably more detail than is provided within the existing scheme of classification, it would be impossible to describe the interdependence within this group of

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[1] One study along these lines is already under way. T. A. Marschak has streamlined the technological details of the present model and used his small-scale version for constructing a four-region model (East, Midwest, Gulf Coast, and West), which includes explicit restraints upon transportation. It will be interesting to examine the effects of geography within this context. Marschak's study should provide a more satisfactory treatment of the interdependence between the three sectors of the industry: crude oil producing, refining, and transportation.

Editors' note: Marschak's study is presented in Chapter 5 of this monograph.

[2] The domestic crude oil availability on this date was 7.465 millions of barrels per calendar day—slightly in excess of the crude oil charging capacity of 7.285 millions of barrels. Even ignoring the possibility of imports, there would have been enough domestic crude oil to use up the crude charging facilities available within the U. S.

materials. All that the existing model pretends to do is to allow for the gross effects of these items upon the other refinery products.

THE MATHEMATICAL MODEL

The model used here was of the conventional linear programming type.[3] One way to describe this mathematical structure is to refer to the maximization of a linear form, subject to linear inequality restraints. An equivalent characterization is the following:

Subject to

$$x_j \geq 0 \qquad (j = 0, 101, \ldots, 1162)^4$$

and

$$\sum_j a_{ij}x_j \leqslant q_i \qquad (i = 101, 102, \ldots, 620),$$

choose values of x_j so as to maximize x_0.

Rather than reproduce the complete linear programming matrix (105 distinct rows and 205 columns) on one sheet of paper, the column vectors of the main a_{ij} matrix have been grouped into eleven families, and each such group is shown on one of the tables of Appendix B, B.1, B.2, . . . , B.11. A complete identification list of the individual equations is given in Appendix C. This appendix also lists the constants, q_i, the net initial availabilities of all items.

Figure 1 gives a simplified view of the connections between the different stages of processing. This figure may also be used as a general guide to the linear programming matrix of Appendix B. For example, the block labeled "1. Atmospheric crude distillation" corresponds to Table B.1, i.e., activities 101, 102, . . . , 125. Because of the intricacy of the refining operation, Figure 1 includes only the major types of stream flow. To cite just two of the omissions, this diagram does not show any of the fuel inputs or outputs, nor does it indicate any of the possibilities for shifting equipment between alternative uses. (See Tables B.6 and B.7.)

Within this model, there are numerous possibilities for varying the product mix. The key points at which choices are to be made correspond to the three main steps in the refining sequence: first atmospheric crude distillation, then conversion of the straight-run streams into blending stocks, and finally the blending of the end items. At the primary distillation stage, there is a choice of how much of each type of crude is to be used. Once this selection is per-

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[3] The standard reference on the theory of linear programming is T. C. Koopmans (1951). For applications to the oil refining industry, see A. Charnes, W. W. Cooper, and B. Mellon (1952); G. Symonds (1955); and A. S. Manne (1956). For a glossary of technical terms, see pp. 31, 32 of this book.

The entire numerical analysis of this model was performed upon IBM 701 equipment at The RAND Corporation. For a discussion of the actual method, see W. Orchard-Hays (1956).

[4] The 205 individual x_j variables (x_0, x_{101}, . . . , x_{1162}) are identified by the column headings in Appendices A and B.

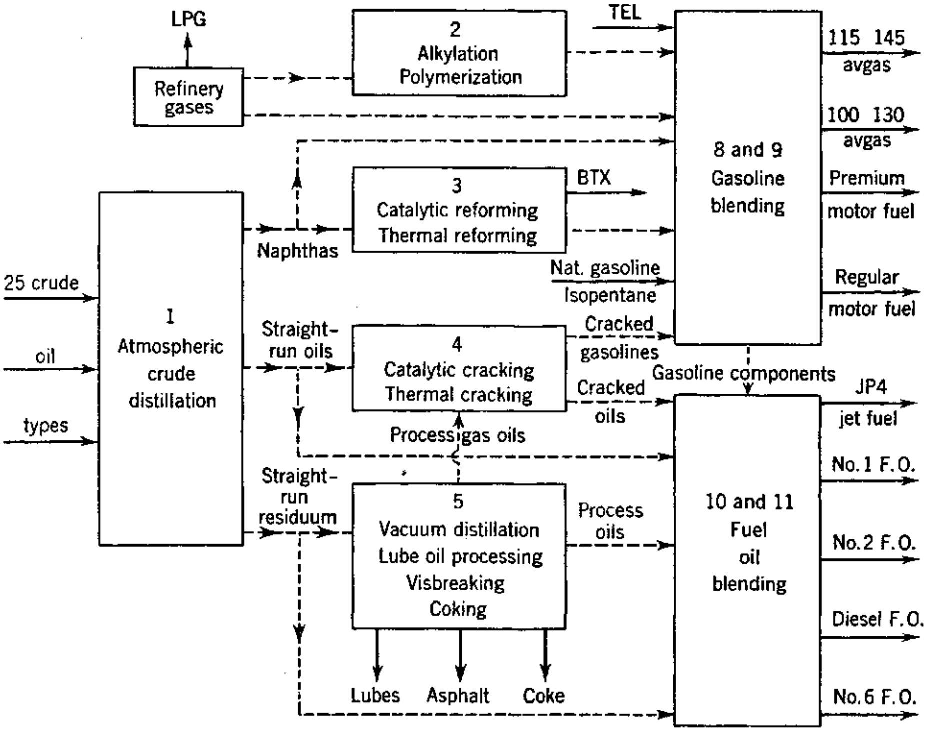

FIGURE 1. Schematic flow diagram.

formed, the quantities of each type of straight-run material (gases, naphthas, gas oils, and residuum) become determined. Next, at the conversion step (cracking, reforming, alkylation, etc.), there are choices as to how much of the straight-run materials will be sent directly to blending, how much to the individual types of conversion units, and how much of the converted materials sent in turn to other units. Finally, at the blending stage there is considerable freedom in the selection of components for any particular end item.

In addition to these options as to the routing of materials, there are choices as to the mode of operation of individual units—variations in the recycle ratio for cracking and in the severity for reforming operations. No allowance is made for the possibilities of manipulating the pressure or space velocity on individual units.

For presentation purposes, the restraint equations have been grouped into six classes: (1) equipment capacity, (2) raw material availability, (3) refinery gases and straight-run streams, (4) converted streams, (5) gasoline and jet fuel specifications, and (6) end item requirements. The equipment capacity equations, for example, ensure that no program will call for more refining equipment of any one type than is available within the U. S. economy. Similarly, the next ones (group 2) ensure that no program will exceed the net initial availabilities of crude oils and other raw materials. Groups 3, 4, and

6 constitute material balance equations on various intermediate and end items.[5] Group 5 constrains all gasoline and jet fuel blends to fall within acceptable specification limits.[6] Also included within this group are four equations (numbers 523, 524, 533, and 534) which permit the ethyl fluid (TEL) concentration in motor gasolines to fall below 3 cc/gal., but which ensure that an octane penalty will be exacted in return.[7]

Appendix B contains all non-zero coefficients of the a_{ij} matrix, with just one exception—the maximand (activity 0).[8] This maximand represents the level of output of a product mix which, except for JP-4 jet fuel, is proportional to the actual 1952–53 output of each item. (See Appendix A.) Per barrel of this standard mix, there are, for example, .13597 barrels of premium grade motor gasoline and .31350 barrels of regular. These proportions are identical with the 1952–53 ratios of premium and regular grade gasoline to the total volumetric output of all refinery products other than jet fuel. The objective, then, is regarded as the maximization of the standard product mix, subject to producing specified quantities of jet fuel. (The production requirement for jet fuel is varied by means of the parameter, q_{620}.) In this way, the linear programming model traces out a substitution curve between the one group of refinery products and the other. Two such loci—here referred to as "tradeoff curves"—are presented in the sections that follow. Should there be interest in any product mix other than the one studied here, the only revisions required would be in the coefficients shown in Appendix A.

RESULTS OF THE INITIAL LINEAR PROGRAMMING CALCULATIONS

Figure 2 contains the trade-off curve between JP-4 jet fuel and the standard product-mix—given all the assumptions that have been outlined previously.[9]

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[5] Some of the so-called "intermediates" are also end items. Item 375, "vacuum distillation bottoms," for example, may be used directly as the end item "lube oils and asphalts." Alternatively, this material may be utilized as an intermediate for visbreaking, coking, or for blending into number 6 fuel oil (activities 521, 532, and 1150).

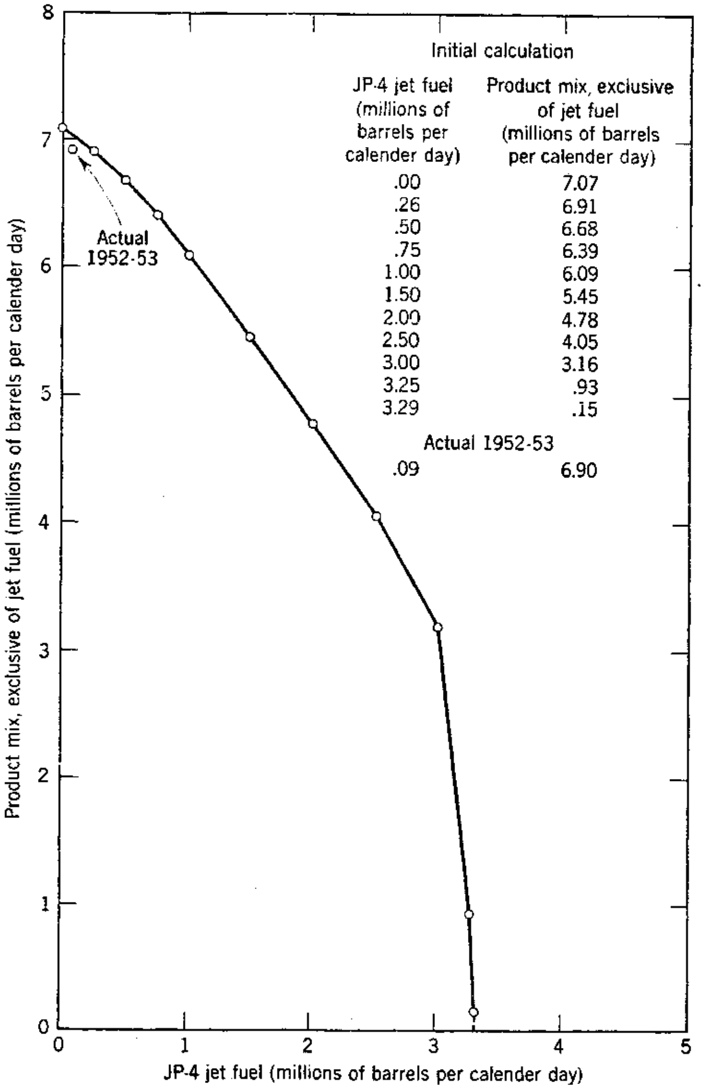
[6] In the case of diesel fuel and numbers 1, 2, and 6 fuel oil, no explicit specification equations have been written. Instead, the matrix contains a preselected *set* of acceptable blends for each of these products.

[7] The model is free to vary the concentration of ethyl fluid in preminum and regular grade motor fuel, up to a level of 3.0 centimeters per gallon (cc/gal.). The concentration of this item in aviation gasoline is fixed, however, at the maximum permissible level of 4.6 cc/gal.

The nonlinear relationship between ethyl fluid concentration levels and octane numbers is approximated by a two-segment linear curve. Reducing the TEL concentration in motor gasoline from 3.0 to 1.5 cc/gal. (63 cubic centimeters per 42-gallon barrel) is taken to be equivalent to a loss of 3 octane-barrels. (Vectors 949 and 999.) From 1.5 down to 0 cc/gal., the loss constitutes 8 octane-barrels. (Vectors 948 and 998.) This device implies that the effect of TEL upon the octane number of the blend can be approximated by taking an a priori average of the lead susceptibilities of the individual components.

[8] In order to preserve consistency with the linear inequality constraints written on p. 36, an input coefficient in Appendix B is represented with a positive sign and an output with a negative one.

[9] The curve shown here does not contain all facets of the Koopmans "efficiency fron-



Initial calculation

JP-4 jet fuel (millions of barrels per calender day)	Product mix, exclusive of jet fuel (millions of barrels per calender day)
.00	7.07
.26	6.91
.50	6.68
.75	6.39
1.00	6.09
1.50	5.45
2.00	4.78
2.50	4.05
3.00	3.16
3.25	.93
3.29	.15
Actual 1952-53	
.09	6.90

FIGURE 2. U. S. jet fuel tradeoff curve, initial calculation.

It is worth observing that, except over small segments, the relationship is not at all linear. According to this curve, the oil industry can neither substitute jet fuel for other products at a constant barrel-for-barrel rate, nor at a constant dollar-for-dollar rate. It is even more erroneous to suppose that jet

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tier" (Koopmans, 1951, p. 60), but rather connects up a number of arbitrary points along the periphery of that frontier.

The solid curve does not literally indicate the maximum attainable output of JP-4 jet fuel. Within the conditions of the model, it is possible to show that this maximum attainable output cannot be greater than 3.298 millions of barrels per calendar day, nor less than 3.292. The dashed line on Figure 2 extends to this upper bound.

fuel must be produced in fixed proportions with each of the other end items. Instead, the model indicates that the higher the level of jet fuel output, the greater will be the volume of other products that will have to be sacrificed in order to make an additional barrel of jet fuel. Over the initial segment of the trade-off curve, jet fuel can be produced at a marginal cost of only 0.6 barrels of the standard product-mix. This incremental cost rises steadily up to the point where 3.0 millions of barrels of jet fuel are obtained, and quite sharply beyond that point. At the extreme jet fuel output level of 3.29 millions of barrels, the indicated marginal sacrifice becomes 25.5 barrels of other products for one additional barrel of jet fuel.[10]

Figure 2 also contains the actual output point for 1952–53. It is reassuring to find such close agreement between actual and estimated results, but the reader should be reminded that this is a one-sided test. In view of the constraints imposed by the crude oil distillation capacity and the conservation of mass, it would have been impossible for the model to make a gross overestimate of the industry's production capabilities.[11]

The same objection does not apply to the possibility of an underestimate, for there was no built-in feature that guaranteed that the model would attain as high a point as was, in fact, reached by the industry. As things turned out, the tradeoff curve lies slightly above the actual historical point. The reader is free to choose between two possible interpretations of the results: (1) the industry was producing at less than 100% of capacity during this time period; or (2) the industry was really operating at capacity levels, but the model's imperfections resulted in an upward bias.

Perhaps a more satisfactory check upon the reliability of the model can be obtained by comparing the "shadow prices" of individual end items with the market prices prevailing at the midpoint of the 1952–53 period. "Shadow prices," it will be recalled, represent the change in payoff per unit change in the net requirement for a given item. Since payoff in this model is measured in terms of the quantity produced of a standard product mix, the shadow price of an item represents its incremental cost, as measured in terms of this standard product mix. In the case of premium motor fuel, for example, the shadow price of 1.525 means that if q_{604}, the net initial availability of this item, were increased by one barrel, it would be possible to produce an additional 1.525

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[10] The author has little confidence in the exactness of this incremental cost estimate for jet fuel at all-out production levels. In the face of such large-scale demand, there would undoubtedly be innovations for making this item available in a more economical fashion. One obvious expedient would be to expand coking facilities so as to convert a larger fraction of residual fuel oil into JP-4.

[11] For purposes of the model, the total available volumetric input amounted to 7.59 millions of B/CD. (This included 7.29 millions of crude distillation capacity, plus smaller amounts of natural gasoline, isopentane, and gases.) Since the model provided no opportunities for a liquid yield in excess of 100% of the initial material, the model could not have come up with an output rate in excess of this 7.59 millions. The solutions never indicated a volumetric production rate in excess of 7.18—this at a jet fuel level of .50 millions. The actual output during 1952–53 consisted of 6.99 millions, including jet fuel.

barrels of the standard product-mix, and at the same time hold jet fuel production constant. A similar meaning may be attached to the shadow prices associated with each of the other 104 rows of the matrix.

Under the following assumptions—neither of them obviously valid—the observed market prices ought to be proportional to the shadow prices within the model: (A) The technology of the model is identical to the real one. (B) The maximizing criterion produces results that are identical to those that would occur if refiners acted consistently as though they could sell unlimited quantities of each item at the stated market prices (see Koopmans, 1951, pp. 65–67).

To the extent that conditions A and B are invalid, one should expect to find nonproportionality—and even a lack of correlation—between the shadow prices and actual market prices. Figure 3 provides a comparison between the two price structures. On the horizontal axis is plotted the December 31, 1952, Gulf Coast bulk cargo price quotation (or range of quotations) for each item, and on the vertical axis the corresponding shadow prices. A 45° straight line has been drawn in to indicate the perfect-proportionality hypothesis. It is evident that there are deviations from this 45° straight line—particularly in the case of number 6 fuel oil and LPG—but that for all other items shown, the correspondence is reasonably good.

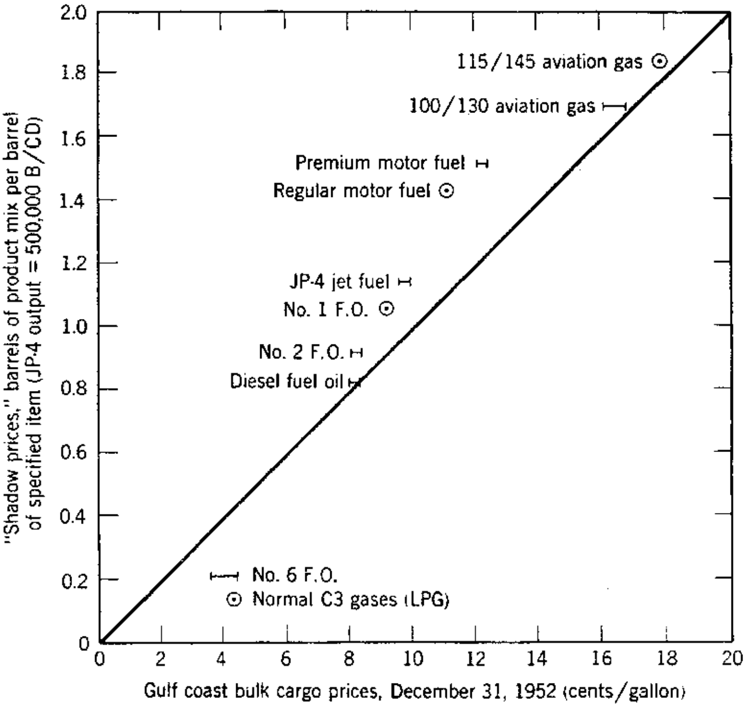

FIGURE 3. Comparison between market prices and shadow prices.

Details of the comparison are to be found in Table 1.[12] The shadow prices were taken from the linear programming solution at which the output of jet fuel was .50 millions of B/CD (barrels per calendar day), and that of other products was 6.68. Since shadow prices vary from facet to facet along the jet fuel tradeoff curve shown in Figure 1, this selection was somewhat arbitrary. No shadow price results had been computed for the point corresponding most closely to the 1952–53 actual product mix, i.e., .26 millions of JP-4 and 6.91 of other products. Shadow prices were available, however, from the solution for zero jet fuel and 7.07 millions of other products, and these are also given in Table 1. The reader will notice that at this output level, the correlation with market prices is not nearly as close as that for .50 millions of jet fuel.

TABLE 1

COMPARISON BETWEEN MARKET PRICES AND SHADOW PRICES

Equation number	Item	Shadow Prices (barrels of standard product-mix per barrel of specified item)		Gulf Coast bulk cargo prices, Dec. 31, 1952 (¢/gal.) Source: *Platt's Oil Price Handbook, 1953 and 1954. Ref.* [32]
		zero jet fuel output	.50 millions of B/CD of jet fuel output	
602	115/145 avgas	2.267	1.841	17.75
603	100/130 avgas	1.973	1.701	16.–16.75
604	premium motor fuel	1.740	1.525	12–12.25[b]
605	regular motor fuel	1.535	1.432	11.0[c]
620	JP-4 jet fuel	0[a]	1.123	9.5–9.75[d]
606	number 1 fuel oil	.984	1.055	9.0[e]
351	diesel fuel oil	.572	.839	8.125–8.5[f]
607	number 2 fuel oil	.697	.904	8.–8.25
608	number 6 fuel oil	.058	.204	3.57–4.40[g]
302	normal C3 gases (LPG)	.036	.128	4.375[h]

[a] i In this basis, the only vector entering into the jet fuel requirement equation was a "slack" vector. The basis is obvously infeasible for any jet fuel requirement greater than zero.

[b] *Platt's* quotation for 92 octane number gasoline. Within the model, premium motor fuel was considered to be 91.5 octane.

[c] *Platt's* quotation for 86 octane gasoline was 11–11.5, and for 83 octane, 10.75–11.25. Within the model, regular motor fuel was considered to be 85 octane.

[d] Price quotation as of December 31, 1953. None earlier available in *Platt's*. On December 31, 1952, price quotation for 41–43 gravity, water white kerosene was 9.0¢/gallon; and on December 31, 1953, was 8.875–9.75. Kerosene and jet fuel are sufficiently comparable to justify the use of the December 31, 1953 jet fuel price quotation.

[e] *Platt's* quotation for 41–43 gravity, water white kerosene.

[f] *Platt's* quotation for 48–52 Diesel Index Gas Oil. Within the model, diesel fuel was considered to be 52 cetane number.

[g] *Platt's* quotation for Bunker C fuel oil.

[h] *Platt's* quotation for industrial and commercial propane.

The most satisfactory check upon the goodness of the model will come from a careful scrutiny of the structure by refiners themselves. The fact that the

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[12] For these purposes, only one set of market quotations was employed, Gulf Coast bulk cargo prices. This one market was selected in preference to a national average—even though the model deals with the U. S. as a whole. Since the model does not include transportation as an explicit resource limitation, it was thought best to make the comparison in terms of the one geographical area which would be least affected by this omission.

model is set forth in detailed physical quantities rather than in dollar aggregates should facilitate any comparisons between these estimates and the data available within individual companies.

RESULTS OF THE SECOND CALCULATIONS

The only difference between the first and second set of linear programming calculations lies in the assumption that the refining equipment capacities are reduced by certain arbitrary amounts. The new level of crude oil charging capacity amounts to 47% of the January, 1953, level, while the capacities of other types of equipment range from 38% to 51% of the initial levels. Both sets of capacities are listed in Table 2.

TABLE 2

REFINING EQUIPMENT CAPACITIES

Equation number	Type of equipment	Charging capacity (millions of barrels per calendar day)	
		Total U. S. January 1, 1953[a]	Reduced capacity calculation
101	Crude oil distillation	7.285	3.411
102	Alkylation	.175[b, c]	.087[b, c]
103	Polymerization	.097[b]	.049[b]
104	Catalytic reforming	.282	.108
105	Thermal reforming	.341	.168
106	Catalytic cracking	2.471	1.227
107	Thermal cracking	1.457	.635
108	Vacuum distillation	.925	.475
110	Visbreaking	.278	.124
111	Coking	.230	.092

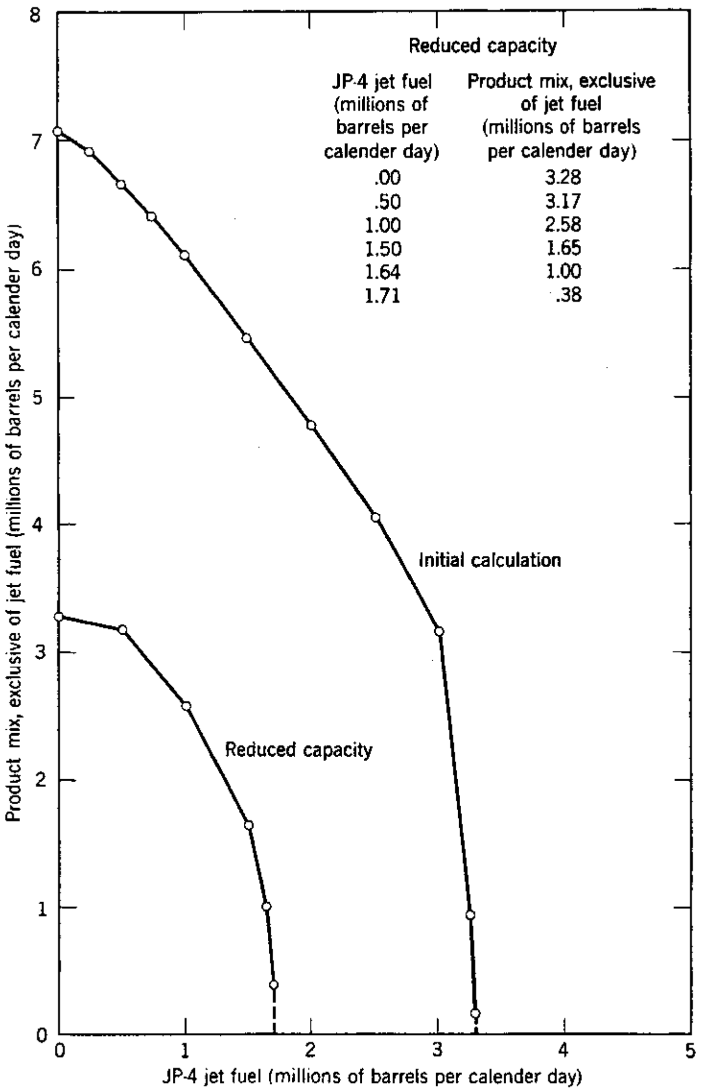
[a] Source: [2, pp. 312–330]. Capacities converted to daily rates by an on-stream efficiency factor of .90, except for catalytic reforming and polymerization. On these, the factor was taken to be .95. The totals used here are based upon the state-by-state information, and differ slightly from the U. S. totals appearing on p. 312 of the reference.

[b] Units stated in millions of barrels per day of output.

[c] The figure on U. S. alkylation capacity is not available in published sources, but is an educated guess, based upon the equipment surviving from World War II and the post-Korean expansion program.

Aside from equipment limitations, the matrix for the second set of calculations is identical with the previous one. The availabilities of crude oils and of other raw material inputs remain unchanged. Transportation facilities are still considered as nonlimitational. None of the end item specifications are altered. And finally, for the sake of comparability, the product mix, exclusive of jet fuel, is identical with that for 1952–53.

The linear programming calculation is again required to maximize the level of the non-jet-fuel product mix, subject to the production of stipulated amounts of jet fuel. The new substitution curve is shown on Figure 4, along with the curve derived previously. Again it turned out that there were markedly increasing costs in the production of jet fuel. Along this curve, the first increment of jet fuel is obtained with only a negligible sacrifice of other products.



Reduced capacity

JP-4 jet fuel (millions of barrels per calender day)	Product mix, exclusive of jet fuel (millions of barrels per calender day)
.00	3.28
.50	3.17
1.00	2.58
1.50	1.65
1.64	1.00
1.71	.38

FIGURE 4. U. S. jet fuel tradeoff curves, reduced capacity case.

The incremental cost of one barrel of jet fuel rises steadily up to 17.5 barrels of the product mix at an all-out production rate of 1.71 millions of barrels.

A priori, it had been expected that with the revised capacities it would be possible to convert a far larger proportion of the product mix into jet fuel than was possible under normal conditions. In the situation assumed here, it is only the refining capacity that becomes reduced; the quantity of each type of crude oil available remains unaffected. It then becomes possible to be far more selective in the choice of crude oils, and to distill only those that possess a high potential yield of jet fuel.

Despite this feature of selectivity contained within the model, there turned out to be only a slight difference in the maximum proportion of jet fuel produced under the two sets of conditions. For the initial capacity case, the maximum yield of jet fuel, 3.3 millions of barrels per calendar day, amounts to 45% by volume of the crude oil input. After the reduction in capacity, the maximum jet fuel yield comes to 1.7 millions, or 50% by volume of the crude.[13] True enough, the second jet fuel percentage yield is higher, but the difference is hardly a striking one. Such calculations suggest that for the majority of crude oils, the potential yield of jet fuel is not greatly different—provided that the available capacities of the refining equipment form a pattern similar to that shown in Table 2. This result was, of course, not anticipated prior to the actual numerical analysis. Had the model initially neglected the variability in crude oil composition, a perceptive critic would have immediately noted the omission.

SOURCES OF DATA

There are approximately 1,500 coefficients in the a_{ij} matrix of Appendix B. A detailed description of the parentage of each of these numbers might be of use to a number of readers of this paper, but would be of little interest to the majority. For the sake of this majority, it should suffice to name the original sources, to give a few examples of the problems that were encountered, and to indicate which of the coefficients are fairly reliable and which are particularly suspect.

All public sources used are listed among the references at the end of this paper. In addition, a number of estimates were made by individuals within the refining industry. For proprietary reasons, the names of these individuals and their company affiliations must remain anonymous.

Using sources that are widely available, it was possible to derive a number of the a_{ij} coefficients quite directly. For example, given the boiling-range definitions, there is a straightforward procedure for estimating the Reid vapor pressure of individual components—i.e., the entries in equations 503, 513, 522, 532, and 543.[14] Among other instances of easy-to-estimate coefficients were: the specification requirements for individual end items; the performance characteristics of such homogeneous materials as butane (C4 gases), isopentane, and alkylate; and the heat content of individual refinery fuels (i.e., the negative entries in equation 261).

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[13] It is worth observing that these estimates of jet fuel production capability check roughly with two off-hand statements found in independent sources. According to *Fundamentals of Petroleum* (1953), "its potential availability (i.e., JP-4) is considered as approximately 40 to 45% of the *average* barrel of crude petroleum" (p. 75). And according to *Aviation Fuels*, "low pressure JP-3 (roughly comparable to JP-4) can theoretically be produced to the extent of 48% of the crude" (Ethyl Corporation, 1951, p. 61).

[14] The entries in these equations actually represent the *differences* between the vapor pressure indices of the individual components and the specification level of the blend. See Charnes, Cooper, and Mellon (1952, pp. 141–146).

At the opposite end of the spectrum of difficulty were some coefficients whose values were based upon little more than informed guesses of people within the refining industry. Among those numbers that are particularly unreliable should be mentioned the following: the capacity degradation that results from shifting equipment into a secondary use (vectors 701 and 702); the octane number of straight-run naphthas (row 521 entries for vectors 905–909 and row 531 entries for vectors 955–959); and the average octane number effects of ethyl fluid upon motor gasoline (vectors 948, 949, 998, and 999).

In each of these cases, the difficulty stemmed from heterogeneity within the classification scheme, and was compounded by a lack of suitable countrywide data. A more thoroughgoing analysis, for example, might have included a detailed survey of equipment characteristics, and in this way obtained a better estimate of the possibilities for alternate use of this equipment. The fact of heterogeneity makes it quite hazardous to rely upon the usual practice that was followed—obtaining several observations of representative processes, and then extrapolating to the industry as a whole.

CATALYTIC CRACKING COEFFICIENTS. Our general procedure is perhaps best typified by the derivation of the catalytic cracking yield coefficients (activities 401–408). The starting point was the following statement by Sachanen (1948, p. 335):

> The average yield of motor gasoline by all catalytic (cracking) processes may be accepted as about 40 per cent by volume, with some excess of butane fraction. The gas formation is about 5 to 8 per cent by weight, and the total recovery of liquid products averages 90 per cent. . . . The yields given above relate to a once-through operation and to conventional straight-run gas oils. Lighter gas oils produce somewhat lesser yields of gasoline on the volume basis but the same ones on the weight or molar basis.

Sachanen's statement checked quite closely with data found in a 1946 pamphlet on catalytic cracking published by the M. W. Kellogg Company. According to this pamphlet (Kellogg, 1946, Table II), the yields of debutanized gasoline and of cracked oil vary as follows with the recycle ratio:

Charge stock: 28° API Wide-cut Midcontinent Gas Oil; 30 D + L Synthetic Catalyst. 900° F.

Recycle ratio (cycle stock ÷ fresh feed)	0	100.0%
Yield of debutanized motor gasoline – volumetric per cent of fresh feed	38.0%	53.2%
Cracked gas oil – volumetric per cent of fresh feed	50.0%	25.0%

The gasoline and gas oil yield coefficients that appear in Table B.4 were then estimated under the following assumptions: (1) the entire cracked gas oil belongs to the same boiling range as the initial charge stock; (2) the percentage yields of gasoline and of cracked gas oil are independent of the boiling range of the initial charge stock; and (3) the curve relating cracked gas oil to gasoline yield remains linear down to the point of zero gas oil.[15] Each of these is more or less unpalatable, but is no more objectionable than those frequently employed within the industry to extrapolate from pilot plant yields to full-scale refinery operations.

COMPOSITION OF CRUDE OILS. Because of the differences in composition between individual crude oils, it seemed undesirable to formulate the process analysis model in terms of a single "typical" material. The fact of variation means that at times when refining equipment is in short supply, it may pay to concentrate this equipment upon the more desirable crudes, and to leave the others in the ground. Hence, an effort was made to take some account of the field-to-field variations in the composition of this raw material.

In searching the literature dealing with crude oil characteristics, by far the best information at hand was that produced by H. M. Smith of the U. S. Bureau of Mines (1951, 1952). Smith tabulated the characteristics of 330 of the leading U. S. crude oils—65% by volume of the crudes produced within the U. S. For each of the following *individual* characteristics, he reports the frequency distribution of U. S. crude oil production: the content of naphtha, gas oil, asphalt, sulfur, aromatics, and naphthene ring. From his data, for example, one learns that 40% of the U. S. crudes have a naphtha content between 30 and 40%, and that 12% of the U. S. crudes have a gas oil content between 20 and 30%. From his work, however, one cannot determine what percentage of U. S. crudes have a 30–40% naphtha content and *also* a 20–30% gas oil content. The lack of information on the joint distribution of these material properties made it necessary to go back to a sample based upon Smith's initial data.

This sample was not a random one, but rather one that was selected on the hypothesis that the 25 leading U. S. crude oils represented a typical cross section of the U. S. domestic supply.[16] In this instance—as distinct from many statistical sampling problems—there is no special reason to suppose that the size of a field is correlated with any physical or chemical characteristics of the oil produced within that field.

To test this hypothesis, Smith's naphtha and gas oil frequency distribution was checked against that for the 25-field sample. Table 3 contains the results.

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[15] Recycling operations are reflected within the model by means of vectors 407 and 408. Per unit of these activities, one unit of catalytic cracking capacity is consumed; .25 units of cracked gas oil disappear ($.25 = .50 - .25$); and .152 units of gasoline are produced ($.152 = .532 - .380 = .0952 + .0314 + .0255$.) The allocation of gasoline yields to the individual boiling-range fractions of the process analysis model was calculated directly from the ASTM distillation data that also appeared in Table II of the Kellogg pamphlet.

[16] These 25 accounted for 23% of the U. S. 1950 output of 1.972 billions of barrels. The exact list of 25 crude oil fields may be found on p. 893 of the *Minerals Yearbook* for 1950 (U. S. Bureau of Mines, 1953). Each one was weighted on the basis of production during 1950. An individual analysis for 24 of these fields appears in McKinney and Blade (1948). The sole omission from this source was that of Levelland, the 19th ranking field. To take the place of Levelland in the sample of 25 fields, the 26th one was substituted—Long Beach.

From this table, it is obvious that there are discrepancies between the two sets of figures—especially at the extremes of each frequency distribution. Still these discrepancies did not appear sufficiently great to invalidate the use of the 25-field sample for purposes of approximating variations in the composition of all U. S. crude oils. Furthermore, the means of these distributions check quite well with one another.

TABLE 3

COMPARISON OF FREQUENCY DISTRIBUTIONS—25-FIELD AND 330-FIELD SAMPLES

Volume % naphtha		Per cent of crude oil produced having specified naphtha content	
Class interval	Assumed midpoint of interval	25 fields	330 fields
0–10.0%	5.0%	7.46%	7.25%
10–19.9%	15.0%	12.61%	15.04%
20–29.9%	25.0%	31.02%	26.84%
30–39.9%	35.0%	48.91%	40.06%
40–100.0%	50.0%	0%	10.81%
		totals......................100.00	100.00
		average naphtha content.....27.1%	28.8%
Volume % gas oil		**Per cent of crude oil produced having specified gas oil content**	
Class interval	Assumed midpoint of interval	25 fields	330 fields
0–20.0%	10.0%	2.33%	6.39%
20–29.9%	25.0%	76.98%	66.52%
30–39.9%	35.0%	10.71%	12.48%
40–49.9%	45.0%	9.99%	11.96%
50–100.0%	60.0%	0%	2.64%
		totals......................100.01	99.99
		average gas oil content......27.7%	28.6%

Having made the decision to use the 25-field sample as representative of the total U. S. crude oil producing sector, it was a simple matter to calculate the yield coefficients in vectors 101–125. McKinney and Blade (1948) give the Hempel cut percentage composition of each of the 25 crude oils.[17] For each of the 25 crudes, the Hempel cuts were aggregated into the boiling-range categories used here: iso and normal C4 gases, 100–250° naphtha, 250–325° naphtha, 325–400° naphtha, 400–550° straight-run gas oil, 550–725° straight-run gas oil, and straight-run residuum.

The boiling-range composition is not the only characteristic that dis-

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[17] Hempel cuts are fractions (a residuum plus 15 cuts—each 72°F. in width), obtained by a standard U. S. Bureau of Mines distillation procedure. See Holliman, Smith, McKinney, and Sponsler (1950), pp. 1–7.

tinguishes individual crude oils from one another. The octane numbers, cetane numbers, viscosity, and aromatics content of individual fractions are all relevant to this model. But since the study was focused largely upon jet fuel, and since the aromatics specification of this product was considered likely to be limitational, it seemed especially desirable to take some account of variations in the aromatics content of individual crude oils. Such characteristics as the octane number, the cetane number, and the viscosity were treated as secondary, and no attempt was made to go beyond an average value for each of these properties.

The variation in aromatics content is handled by subdividing each of three potential jet fuel boiling-range fractions into separate aromatics-content categories. The 250–325° cut, for example, is divided into just two categories: 5.0–9.9% and 10.0–14.9% aromatics. For each distinct crude, the 250–325° cut is then allocated entirely to the one or the other class.[18] In this way, the model contains a considerable degree of selectivity as between crude oil types. Not only is the model free to utilize those crudes that have a high content of straight-run materials within the jet fuel range, but also to concentrate upon those that have a low aromatics content.

A final word about the availability of each crude oil type, i.e., the constants in equations 201–225. For January 1, 1953, the total domestic U. S. crude oil availability was taken at 7.465 millions of B/CD.[19] The combined 1950 output amounted to 1.229 millions of B/CD for the 25 crudes used as a cross section of the entire industry (*Minerals Yearbook*, 1950, p. 893). For purposes of our model, the January 1, 1953, availability of each crude oil type was then estimated as follows:

$$\text{availability of crude oil type} = (\text{1950 output of type}) \times 7.465/1.229.$$

Considering the degree of arbitrariness in any estimate of an oil field's productive capacity, this approximation seemed as reasonable as any other. It is true that over long periods of time, the composition of the crude oils available in any one country is likely to change. An extrapolation of the 1950 to the 1952–53 composition, however, did not appear unduly hazardous.

THE FUTURE OF PROCESS ANALYSIS STUDIES

The preceding account should be enough to convince anyone that it is no easy task to construct a process analysis model of a single sector—let alone an entire economy, including spatial and temporal details. The formulation

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[18] McKinney and Blade (1948) provide the specific gravity, but not the aromatics content of each Hempel cut. In order to estimate the aromatics content from the specific gravity, a special correlation for each Hempel cut was devised on the basis of Holliman et al. (1950). Incidentally, the resulting frequency distribution of aromatics content checked reasonably well with that for Smith's 330-field sample.

[19] National Petroleum Committee (1953). This figure relates only to the availability of crude oil, and not to the combined availability of crude oil plus natural gas liquids. Isopentane and stabilized natural gasoline are shown as separate raw material inputs through the constants in equations 241 and 242.

of the problem, the gathering of data, and the numerical analysis are time-consuming and expensive.

Yet what are the alternatives? At any time that an analyst is concerned with predicting the production capabilities of an entire economy—whether during wartime, during a postwar period of repressed inflation, or during an economic development program—he finds that there is no altogether satisfactory procedure at hand. Predictions based upon the monetary aggregate of gross national product inevitably overstate the degree of production substitutability within an economy, and are congenitally over-optimistic on the short-run possibilities for a shift in the product-mix. At the other extreme, it is even less defensible to regard any single factor of production as *the* bottleneck in the performance of an entire economy—whether that bottleneck be a mineral resource or arable land or ball-bearing factories. A one-factor analysis understates the possibilities for substitution, and a model based upon gross national product overstates these possibilities.

The original Leontief interindustry flow model (Leontief, 1951) was an obvious improvement over both of the procedures sketched out above.[20] It allows for the possibility that a bottleneck may occur at one of many points—depending upon the vector of final demand. To this extent, the classical "open" Leontief system represents a happy compromise between the one-factor technique and the national income technique. But it too has a serious drawback. Extreme difficulties are encountered as soon as an attempt is made to take account of the pervasive phenomena of substitutability and complementarity.

It is true that a Leontief model can be aggregated so as to allow for perfect substitutability (i.e., a completely linear tradeoff curve between several products) and perfect complementarity (i.e., completely rigid proportions between several products). But these conditions are unduly strong. Everyday examples that violate *both* of these assumptions include the three following: (1) High-precision machine tools may be used in place of low-precision ones, but not vice versa. (2) Within limits, steel scrap may be substituted for pig iron in an open hearth furnace, but only at the cost of reducing the effective capacity of the furnace. (3) The basic aromatic chemicals—benzene, toluene, and xylene—are produced as by-products of destruction coal distillation (i.e., coking operations) within the iron and steel industry. But these same chemicals are also produced within the petroleum sector—beginning with an entirely different raw material (straight-run naphthas), and processing this raw material in an entirely different manner—catalytic reforming, followed by aromatics extraction. (See Appendix A, BTX production vector.)

It requires a confirmed optimist to contend that the examples just cited would produce mere second-order effects within an economy-wide model. Indeed, there is good reason to believe that much of the "variability" in input-output coefficients results precisely from a framework that ignores these ef-

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[20] This sketch was admittedly an unfair one. Even using crude tools of analysis, clever people may produce excellent results. But it is natural to wonder whether these same people might not do still better with improved tools.

fects. A considerable amount of the recent work done by Leontief's group has aimed at alleviating these rigidities through detailed engineering analysis (Leontief, 1953). Fortunately, linear programming provides a tool for dealing with such problems. Linear programming allows both for the possibility that a single item may be produced by more than one process, and for the possibility that a single process may produce more than one item. To the extent that a process analysis model uses linear programming to allow for these effects, it represents a logical extension of Leontief's original methods.[21]

The petroleum study described in this paper should help dispel the notion that a process analysis model of an entire economy represents a hopeless goal. (Nor should it generate the illusion that such models are inexpensive.) Neither the computational problems nor the difficulties in accumulating data constitute insuperable technical obstacles. Given time and effort, reasonably reliable results can be achieved.

This is not to say that the implementation of such models is a routine mechanical procedure. Without intimate collaboration between the model builder, the data collector, and the numerical analyst, there is little hope for success. From the very outset, both the model builder and the data gatherer must have some appreciation of the problems involved in the numerical analysis of their material. Failing this appreciation, it is quite likely that they wll either overestimate or underestimate the existing state-of-the-art of numerical solution. The linkage between the model builder and the numerical analyst is, of course, a two-way affair. As the model builder's needs become more clearly defined, it becomes increasingly possible to devise methods of solution that take advantage of special characteristics of his models.[22]

By far the more serious problem consists of the collaboration between those who are in a position to furnish numerical data and those who wish to utilize this material. Here again there is a two-way linkage. The gathering of data is not just a problem of filling out numerical coefficients within previously designated boxes. Meaningful data can only be collected with a view toward the overall purposes of a model. Conversely, the categories of a model cannot be defined without reference to the inevitable gaps in data coverage. (Cf. the discussion on page 49 about including variations in aromatics content, and excluding variations in octane number as between individual crude oils.)

This paper represents no more than a modest attempt to demonstrate the feasibility of constructing a process analysis model for a single sector. Before launching into a full-blown, economy-wide model, many similar intrasectoral studies will have to be performed. It is only upon a carefully prepared foundation that it will be possible to erect a reliable structure.

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[21] This is not to say that linear programming represents the best of all possible methods for computing numerical answers to optimization problems. There are numerous instances of nonconvex models for which no known computing methods are really satisfactory. Examples of such problems include: setup costs in machine shops, carload versus less-than-carload shipments, and economies of scale in the construction of chemical processing equipment. It is only by assuming that these phenomena produce second-order effects that linear programming can be applied to a given problem.

[22] Two such developments are described in Dantzig (1955) and Markowitz (1955).

APPENDIX A

DERIVATION OF PRODUCT-MIX VECTOR

Equation number	Item (Unit: barrels, unless specified otherwise)	U. S. 1952–53 output[a]		Inputs and outputs for .3375 barrels of BTX (benzene, toluene, and xylene)	Coefficients of x_0, product-mix vector (net of inputs and outputs for .00262 barrels of BTX/barrel of product-mix)
		(thousands of barrels, total for year)	(per cent by volume of product-mix, excluding jet fuel)		
104	catalytic reforming	—	—	1.0000	.007 763
261	British thermal units (thousands)	—	—	615.	4.774
301	refinery gases, C2 and lighter	—	—	−.1394	−.001 082
302	refinery gases, normal C3 (LPG)	26,775	1.062%	−.0202	.010 463
303	refinery gases, olefin C3	—	—	−.0122	−.000 095
304	refinery gases, iso and normal C4	—	—	−.0108	−.000 084
305	refinery gases, olefin C4 (LPG)	5,214	.207	−.0176	−.001 933
311	light naptha, 100–250°	—	—	.4835[e]	.003 753
351	diesel fuel oil	89,582	3.554		.035 540
375	vacuum distillation bottoms (lube oils and asphalt)	126,593	5.023		.050 230
602	115/145 avgas	36,555[b]	1.450		.014 500
603	100/130 avgas	36,555[b]	1.450		.014 500
604	premium motor fuel	342,673	13.597		.135 970
605	regular motor fuel	790,129	31.350		.313 500
606	number 1 fuel oil	129,382	5.134		.051 340
607	number 2 fuel oil	446,839	17.730		.177 300
608	number 6 fuel oil[c]	463,593	18.395		.183 950
610	coke	19,776[d]	.785[d]		3.14[f]
	BTX (benzene, toluene, xylene)	6,612	.262	−.3375	
	Total, excluding jet fuel	2,520,278	100.000		
620	jet fuel	32,357	1.284%		

[a] Primary sources: U. S. Bureau of Mines [44].

[b] Breakdown of avgas output is not available. The total was arbitrarily split 50–50 between the two main grades.

[c] Includes road oil.

[d] Coke converted to volumetric measure at rate of 400 pounds/barrel.

[e] This coefficient is based upon the assumption that the non-aromatic by-product from solvent extraction is equivalent in motor fuel performance to the initial 100–250° straight-run naphtha. The coefficient of .4835 measures the *net* consumption of this item.

[f] Pounds of coke/barrel of product-mix.

[a] Primary sources: U. S. Bureau of Mines, *Monthly Petroleum Statements* (1953).

APPENDIX B

TABLE B.1

ATMOSPHERIC CRUDE DISTILLATION

Row Number	Item	Distillation of crude oils												
		101	102	103	104	105	106	107	108	109	110	111	112	113
101	atmospheric crude distillation capacity	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
201	availability—crude type 1	1.000												
202	availability—crude type 2		1.000											
203	availability—crude type 3			1.000										
204	availability—crude type 4				1.000									
205	availability—crude type 5					1.000								
206	availability—crude type 6						1.000							
207	availability—crude type 7							1.000						
208	availability—crude type 8								1.000					
209	availability—crude type 9									1.000				
210	availability—crude type 10										1.000			
211	availability—crude type 11											1.000		
212	availability—crude type 12												1.000	
213	availability—crude type 13													1.000
214	availability—crude type 14													
215	availability—crude type 15													
216	availability—crude type 16													
217	availability—crude type 17													
218	availability—crude type 18													
219	availability—crude type 19													
220	availability—crude type 20													
221	availability—crude type 21													
222	availability—crude type 22													
223	availability—crude type 23													
224	availability—crude type 24													
225	availability—crude type 25													
261	British thermal units (BTU), (billions of BTU/day)	199.	199.	199.	199.	199.	199.	199.	199.	199.	199.	199.	199.	199.
304	refinery gases, iso and normal C4	−.007	−.012	−.015	−.008	−.002	−.006	−.011	−.0005	−.012	−.002	−.003	−.010	−.011
311	light naphtha, 100-250°	−.184	−.090	−.004	−.105	−.089	−.118	−.163	−.143	−.095	−.067	−.173	−.060	−.096
321	medium naphtha, 250-325°, 5.0-9.9% aromatics								−.083	−.083				
322	medium naphtha, 250-325°, 10.0-14.9% aromatics	−.091	−.062	−.020	−.062	−.072	−.079	−.096			−.056	−.086	−.115	−.112
331	heavy naphtha, 325-400°, 10.0-14.9% aromatics	−.076			−.067		−.075	−.074	−.095	−.171	−.062	−.073		−.072
332	heavy naphtha, 325-400°, 15-19.9% aromatics		−.059	−.054		−.060							−.065	
341	light SR gas oil, 400-550°, 5.0-14.9% aromatics							−.165	−.315	−.171				
342	light SR gas oil, 400-550°, 15.0-24.9% aromatics	−.170			−.159		−.198					−.160		−.193
343	light SR gas oil, 400-550°, 25.0-29.9% aromatics		−.163			−.154					−.287		−.193	
344	light SR gas oil, 400-550°, 30.0-39.9% aromatics			−.214										
351	heavy SR gas oil, 550-725° (diesel fuel oil)	−.306	−.202	−.273	−.168	−.194	−.237	−.185	−.220	−.188	−.263	−.191	−.191	−.212
361	SR residuum, 725°+	−.260	−.402	−.405	−.402	−.426	−.280	−.294	−.343	−.264	−.242	−.311	−.317	−.393

Row Number	Item	Distillation of crude oils											
		114	115	116	117	118	119	120	121	122	123	124	125
101	atmospheric crude distillation capacity	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
201	availability—crude type 1												
202	availability—crude type 2												
203	availability—crude type 3												
204	availability—crude type 4												
205	availability—crude type 5												
206	availability—crude type 6												
207	availability—crude type 7												
208	availability—crude type 8												
209	availability—crude type 9												
210	availability—crude type 10												
211	availability—crude type 11												
212	availability—crude type 12												
213	availability—crude type 13												
214	availability—crude type 14	1.000											
215	availability—crude type 15		1.000										
216	availability—crude type 16			1.000									
217	availability—crude type 17				1.000								
218	availability—crude type 18					1.000							
219	availability—crude type 19						1.000						
220	availability—crude type 20							1.000					
221	availability—crude type 21								1.000				
222	availability—crude type 22									1.000			
223	availability—crude type 23										1.000		
224	availability—crude type 24											1.000	
225	availability—crude type 25												1.000
261	British thermal units (BTU), (billions of BTU/day)	109.	109.	199.	199.	1.99	199.	199.	190.	199.	199.	199.	199.
304	refinery gases, iso and normal C4	−.001	−.021	−.008	−.008	−.001	−.010	−.010	−.005	−.002	−.010	−.007	−.015
311	light naphtha, 100–250°	−.157	−.184	−.030	−.131	−.049	−.153	−.098	−.021	−.015	−.123	−.176	−.088
321	medium naphtha, 250–325°, 5.0–9.9% aromatics							−.054			−.112	−.094	−.063
322	medium naphtha, 250–325°, 10.0–14.9% aromatics	−.110	−.105	−.057	−.091	−.041	−.098		−.040	−.022			
331	heavy naphtha, 325–400°, 10.0–14.9% aromatics		−.081		−.090		−.076	−.043			−.119	−.081	
332	heavy naphtha, 325–400°, 15–19.9% aromatics	−.110		−.066		−.060			−.059	−.053			−.057
341	light SR gas oil, 400–550°, 5.0–14.9% aromatics												
342	light SR gas oil, 400–550°, 15.0–24.9% aromatics		−.182				−.174	−.136	−.187		−.339	−.177	−.160
343	light SR gas oil, 400–550°, 25.0–29.9% aromatics	−.298				−.254							
344	light SR gas oil, 400–550°, 30.0–39.9% aromatics			−.180	−.190					−.287			
351	heavy SR gas oil, 550–725° (diesel fuel oil)	−.227	−.187	−.214	−.188	−.297	−.190	−.170	−.259	−.303	−.208	−.193	−.198
361	SR residuum, 725° +	−.096	−.219	−.437	−.295	−.297	−.287	−.472	−.415	−.315	−.080	−.265	−.404

TABLE B.2
GAS CONVERSION

Row Number	Item	Alkylation		Polymerization		LPG Substitution		
		201	202	211	212	221	222	223
102	alkylation capacity	1.24	1.63					
103	polymerization capacity			.693	.755			
261	British thermal units (BTU), (billions of BTU /day)	2,418.	3,179.	662.	721.			
301	refinery gases, C2 and lighter	−.39	−.27	−.091	−.183			
302	refinery gases, normal C3 (LPG)					−1.	−1.	−1.
303	refinery gases, olefin C3	1.00		1.000		1.		
304	refinery gases, iso and normal C4	.90	1.17				1.	
305	refinery gases, olefin C4 (LPG)		1.00		1.000			1.
401	alkylate gasoline	−1.24	−1.63					
405	C3 polymer gasoline			−.693				
406	C4 polymer gasoline				−.755			

TABLE B.3

NAPHTHA REFORMING

Row Number	Item	Catalytic reforming								Thermal reforming							
		301	302	303	304	305	306	307	308	351	352	353	354	355	356	357	358
104	catalytic reforming capacity	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000								
105	thermal reforming capacity									1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
261	British thermal units (BTU), (billions of BTU/day)	615.	615.	615.	615.	615.	615.	615.	615.	767.	767.	767.	767.	767.	767.	767.	767.
301	refinery gases, C2 and lighter	−.1310	−.2022	−.1310	−.2022	−.1483	−.2850	−.1483	−.2850	−.0345	−.1725	−.0345	−.1725	−.069	−.1725	−.069	−.1725
302	refinery gases, normal C3 (LPG)	−.0189	−.0292	−.0189	−.0292	−.0214	−.0411	−.0214	−.0411	−.0050	−.0250	−.0050	−.0250	−.010	−.0250	−.010	−.0250
303	refinery gases, olefin C3	−.0114	−.0176	−.0114	−.0176	−.0129	−.0249	−.0129	−.0249	−.0030	−.0150	−.0030	−.0150	−.006	−.0150	−.006	−.0150
304	refinery gases, iso and normal C4	−.0220	−.0342	−.0220	−.0342	−.0220	−.0418	−.0220	−.0418	−.0050	−.0250	−.0050	−.0250	−.010	−.0250	−.010	−.0250
305	refinery gases, olefin C4 (LPG)	−.0360	−.0558	−.0360	−.0558	−.0360	−.0682	−.0360	−.0682	−.0025	−.0125	−.0025	−.0125	−.005	−.0125	−.005	−.0125
321	medium naphtha, 250-325°, 5.0-9.9% aromatics	1.000	1.000							1.000	1.000						
322	medium naphtha, 250-325°, 10.0-14.9% aromatics			1.000	1.000							1.000	1.000				
331	heavy naphtha, 325-400°, 10.0-14.9% aromatics					1.000	1.000							1.000	1.000		
332	heavy naphtha, 325-400°, 15-19.9% aromatics							1.000	1.000							1.000	1.000
411	catalytic reformed gasoline, 250-325°, low severity operation	−.865		−.865													
412	catalytic reformed gasoline, 250-325°, high severity operation		−.81		−.81												
415	catalytic reformed gasoline, 325-400°, low severity operation					−.852		−.852									
416	catalytic reformed gasoline, 325-400°, high severity operation						−.743		−.743								
421	thermal reformed gasoline, 250-325°, low severity operation									−.95		−.95					
422	thermal reformed gasoline, 250-325°, high severity operation										−.75		−.75				
425	thermal reformed gasoline, 325-400°, low severity operation													−.90		−.90	
426	thermal reformed gasoline, 325-400°, high severity operation														−.75		−.75

TABLE B.4

CRACKING

Row Number	Item	Catalytic cracking								Thermal cracking								
		401	402	403	404	405	406	407	408	451	452	453	454	455	456	457	458	459
106	catalytic cracking capacity	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000									
107	thermal cracking capacity									1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
261	Brit. ther. units (BTU), (billions of BTU/day)	461.	461.	461.	461.	461.	461.	414.	414.	537.	537.	537.	537.	537.	537.	537.	537.	537.
301	refinery gases, C2 and lighter	−.1117	−.1117	−.1117	−.1117	−.1117	−.1117	−.0751	−.0751	−.03961	−.03961	−.03961	−.03961	−.04190	−.01527	−.04898	−.04898	−.04737
302	refinery gases, normal C3 (LPG)	−.0161	−.0161	−.0161	−.0161	−.0161	−.0161	−.0108	−.0108	−.00935	−.00935	−.00935	−.00935	−.00990	−.00361	−.01157	−.01157	−.01119
303	refinery gases, olefin C3	−.0097	−.0097	−.0097	−.0097	−.0097	−.0097	−.0066	−.0066	−.01260	−.01260	−.01260	−.01260	−.01332	−.00486	−.01558	−.01558	−.01506
304	refinery gases, iso and normal C4	−.0448	−.0448	−.0448	−.0448	−.0448	−.0448	−.0255	−.0255	−.00855	−.00855	−.00855	−.00855	−.00905	−.00330	−.01058	−.01058	−.01023
305	refinery gases, olefin C4 (LPG)	−.0732	−.0732	−.0732	−.0732	−.0732	−.0732	−.0415	−.0415	−.01126	−.01126	−.01126	−.01126	−.01191	−.00434	−.01392	−.01392	−.01346
341	light SR gas oil, 400–550°, 5.0–14.9% aromatics	1.000								1.000								
342	light SR gas oil, 400–550°, 15.0–24.9% aromatics		1.000								1.000							
343	light SR gas oil, 400–550°, 25.0–29.9%, aromatics			1.000								1.000						
344	light SR gas oil, 400–550°, 30.0–39.9%, aromatics				1.000								1.000					
351	heavy SR gas oil, 550–725° (diesel fuel oil)					1.000								1.000				
361	SR residuum, 725°+														1.000			
371	vacuum distillation gas oil, 725°+						1.000											

431	catalytic cracked, light gasoline, 100–250°, SR charge	−.2379	−.2379	−.2379	−.2379	−.2379	−.2379											
432	catalytic cracked, light gasoline, 100–250°, cracked charge							−.0952	−.0952									
433	catalytic cracked, medium gasoline, 250–325°	−.0785	−.0785	−.0785	−.0785	−.0785	−.0785	−.0314	−.0314									
434	catalytic cracked, heavy gasoline, 325–400°	−.0637	−.0637	−.0637	−.0637	−.0637	−.0637	−.0255	−.0255									
435	catalytic cracked, light gas oil, 400–550°	−.500	−.500	−.500	−.500			.25								.300		
441	thermal cracked, light gasoline, 100–250°									−.114	−.114	−.114	−.114	−.101	−.051	−.077	−.077	−.072
442	thermal cracked, medium gasoline, 250–325°									−.057	−.057	−.057	−.057	−.050	−.025	−.038	−.038	−.036
443	thermal cracked, heavy gasoline, 325–400°									−.057	−.057	−.057	−.057	−.050	−.026	−.038	−.038	−.036
444	thermal cracked, light gas oil, 400–550°									−.700	−.700	−.700	−.700	−.042	−.057		.300	−.063
451	cracked heavy gas oil, 550–725°					−.500			.25	−.021	−.021	−.021	−.021	−.700	−.126	−.069	−.069	.300
455	cracked residuum, 725°+						−.500			−.006	−.006	−.006	−.006	−.012	−.700	−.024	−.024	−.045

TABLE B.5
HEAVY ENDS PROCESSING

Row Number	Item	Vacuum gas oil preparation 501	Visbreaking		Coking		
			521	522	531	532	533
108	vacuum distillation capacity	1.00					
110	visbreaking capacity		1.000	1.000			
111	coking capacity				1.000	1.000	1.000
261	British thermal units (BTU), (billions of BTU/day)	230.	268.	537.	307.	307.	307.
301	refinery gases, C2 and lighter			−.01527	−.030	−.030	−.030
302	refinery gases, normal C3 (LPG)			−.00361			
303	refinery gases, olefin C3			−.00486			
304	refinery gases, iso and normal C4		−.0058	−.00330			
305	refinery gases, olefin C4 (LPG)		−.0042	−.00434			
343	light SR gas oil, 400–550°, 25.0–29.9% aromatics				−.290	−.240	−.240
351	heavy SR gas oil, 550–725° (diesel fuel oil)				−.340	−.270	−.270
361	SR residuum, 725°+	1.00		1.000	1.000		
371	vacuum distillation gas oil, 725°+	−.40					
375	vacuum distillation bottoms (lube oils and asphalt)	−.60	1.000			1.000	
441	thermal cracked, light gasoline, 100–250°		−.030	−.051	−.100	−.100	−.100
442	thermal cracked, medium gasoline, 250–325°		−.015	−.025	−.050	−.050	−.050
443	thermal cracked, heavy gasoline, 325–400°		−.015	−.026	−.050	−.050	−.050
444	thermal cracked, light gas oil, 400–550°			−.057			
451	cracked heavy gas oil, 550–725°			−.126			
455	cracked residuum, 725°+			−.700			1.000
607	number 2 fuel oil—requirement		.31				
608	number 6 fuel oil—requirement		−1.24				
610	coke (millions of pounds/day)—requirement				−56.	−112.	−112.

TABLE B.6
HEAT PRODUCTION

Row Number	Item	Fuel inputs						
		601	602	603	604	605	606	607
261	British thermal units (BTU), billions of BTU/day)	−2,744.	−3,631.	−3,613.	−4,094.	−4,106.	−5,800.	−15.7
301	refinery gases, C2 and lighter	1.						
302	refinery gases, normal C3 (LPG)		1.					
303	refinery gases, olefin C3			1.				
304	refinery gases, iso and normal C4				1.			
305	refinery gases, olefin C4 (LPG)					1.		
608	number 6 fuel oil—requirement						1.	
610	coke—requirement							1.

TABLE B.7
CAPACITY SHIFTING

Row Number	Item	701	702
105	thermal reforming capacity	−.35	
107	thermal cracking capacity	1.00	
110	visbreaking capacity		−1.00
111	coking capacity		1.00

TABLE B.8

Aviation Gasoline Blending

Row Number	Item	115/145 avgas components							100/130 avgas components						
		801	802	803	804	805	806	807	851	852	853	854	855	856	857
241	isopentane	1.							1.						
251	tetraethyl lead (TEL), (millions of cubic centimeters/day)	193.2	193.2	193.2	193.2	193.2	193.2	193.2	193.2	193.2	193.2	193.2	193.2	193.2	193.2
401	alkylate gasoline		1.							1.					
412	catalytic reformed gasoline, 250–325°, high severity operation			1.							1.				
431	catalytic cracked, light gasoline, 100–250°, SR charge				1.							1.			
432	catalytic cracked, light gasoline, 100–250°, cracked charge					1.							1.		
433	catalytic cracked, medium gasoline, 250–325°						1.							1.	
441	thermal cracked, light gasoline, 100–250°							1.							1.
501	115/145 aviation gasoline—lean performance number	−11.6	−7.2	25.	22.	22.	25.	33.4							
502	115/145 aviation gasoline—rich performance number	8.4	−2.9	18.	15.	15.	18.	47.2							
503	115/145 aviation gasoline—Reid vapor pressure	15.0	−4.0	−5.	2.7	6.5	−5.	3.0							
511	100/130 aviation gasoline—lean performance number								−26.6	−22.2	10.	7.	7.	10.	18.4
512	100/130 aviation gasoline—rich performance number								−6.6	−17.9	3.	0.0	0.0	3.	32.2
513	100/130 aviation gasoline—Reid vapor pressure								15.0	−4.0	−5.	2.7	6.5	−5.	3.0
602	115/145 avgas—requirement	−1.	−1.	−1.	−1.	−1.	−1.	−1.							
603	100/130 avgas—requirement								−1.	−1.	−1.	−1.	−1.	−1.	−1.

TABLE B.9

MOTOR GASOLINE BLENDING

Row Number	Item	Premium motor fuel components																			
		901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920
241	isopentane	1.																			
242	natural gasoline		1.																		
251	tetraethyl lead (TEL), (millions of cubic centimeters/day)	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.
304	refinery gases, iso and normal C4			1.																	
305	refinery gases, olefin C4 (LPG)				1.																
311	light naphtha, 100–250°					1.															
321	medium naphtha, 250–325°, 5.0–9.9% aromatics						1.														
322	medium naphtha, 250–325°, 10.0–14.9% aromatics							1.													
331	heavy naphtha, 325–400°, 10.0–14.9% aromatics								1.												
332	heavy naphtha, 325–400°, 15-19.9% aromatics									1.											
401	alkylate gasoline										1.										
405	C3 polymer gasoline											1.									
406	C4 polymer gasoline												1.								
411	catalytic reformed gasoline, 250–325°, low severity operation													1.							
412	catalytic reformed gasoline, 250–325°, high severity operation														1.						
415	catalytic reformed gasoline, 325–400°, low severity operation															1.					
416	catalytic reformed gasoline, 325–400°, high severity operation																1.				
421	thermal reformed gasoline, 250–325°, low severity operation																	1.			

422	thermal reformed gasoline, 250-325°, high severity operation																		1.		
425	thermal reformed gasoline, 325-400°, low severity operation																			1.	
426	thermal reformed gasoline, 325-400°, high severity operation																				1.
431	catalytic cracked, light gasoline, 100-250° SR charge																				
432	catalytic cracked, light gasoline, 100-250°, cracked charge																				
433	catalytic cracked, medium gasoline, 250-325°																				
434	catalytic cracked, heavy gasoline, 325-400°																				
441	thermal cracked, light gasoline, 100-250°																				
442	thermal cracked, medium gasoline, 250-325°																				
443	thermal cracked, heavy gasoline, 325-400°																				
521	prem. motor fuel—Research octane no. 91.5 min.	−13.5	10.5	−13.5	−13.5	6.5	37.9	37.9	53.3	53.3	−13.5	−7.5	−13.5	−6.5	−14.5	−3.5	−16.4	22.1	5.8	22.6	10.5
522	prem. motor fuel—Reid vapor pressure, 10 lbs max.	12.0	4.0	55.0	55.0	0.0	−8.0	−8.0	−8.0	−8.0	−7.0	0.0	0.0	−8.0	−8.0	−8.0	−8.0	−8.0	−8.0	−8.0	−8.0
523	prem. motor fuel—reduction of TEL, 1.5-0 cc/gal.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.
524	prem. motor fuel—reduction of TEL, 3.0-1.5 cc/gal.,	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.
531	reg. motor fuel—Research octane no. 85.0 min.																				
532	reg. motor fuel—Reid vapor pressure, 10 lbs max.																				
533	reg. motor fuel—reduction of TEL, 1.5-0 cc/gal.																				
534	reg. motor fuel—reduction of TEL, 3.0-1.5 cc/gal.																				
604	premium motor fuel—requirement	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.
605	regular motor fuel—requirement																				

TABLE B.9—CONTINUED

Row Number	Item	Premium motor fuel components									Regular motor fuel components									
		921	922	923	924	925	926	927	948	949	951	952	953	954	955	956	957	958	959	960
241	isopentane										1.									
242	natural gasoline											1.								
251	tetraethyl lead (TEL), (millions of cubic centimeters/day)	126.	126.	126.	126.	126.	126.	126.	−63.	−63.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.
304	refinery gases, iso and normal C4												1.							
305	refinery gases, olefin C4 (LPG)													1.						
311	light naphtha, 100–250°														1.					
321	medium naphtha, 250–325°, 5.0–9.9% aromatics															1.				
322	medium naphtha, 250–325°, 10.0–14.9% aromatics																1.			
331	heavy naphtha, 325–400°, 10.0–14.9% aromatics																	1.		
332	heavy naphtha, 325–400°, 15–19.9% aromatics																		1.	
401	alkylate gasoline																			1.
405	C3 polymer gasoline																			
406	C4 polymer gasoline																			
411	catalytic reformed gasoline, 250–325°, low severity operation																			
412	catalytic reformed gasoline, 250–325°, high severity operation																			
415	catalytic reformed gasoline, 325–400°, low severity operation																			
416	catalytic reformed gasoline, 325–400°, high severity operation																			
421	thermal reformed gasoline, 250–325°, low severity operation																			
422	thermal reformed gasoline, 250–325°, high severity operation																			
425	thermal reformed gasoline, 325–400°, low severity operation																			

426	thermal reformed gasoline, 325-400°, high severity operation																			
431	catalytic cracked, light gasoline, 100-250°, SR charge	1.																		
432	catalytic cracked, light gasoline, 100-250°, cracked charge		1.																	
433	catalytic cracked, medium gasoline, 250-325°			1.																
434	catalytic cracked, heavy gasoline, 325-400°				1.															
441	thermal cracked, light gasoline, 100-250°					1.														
442	thermal cracked, medium gasoline, 250-325°						1.													
443	thermal cracked, heavy gasoline, 325-400°							1.												
521	prem. motor fuel—Research octane no. 91.5 min.	−4.5	−4.5	.5	8.5	−0.5	8.5	27.5	8.	3.										
522	prem. motor fuel—Reid vapor pressure, 10 lbs max.	−0.3	3.5	−8.0	−8.0	0.0	−8.0	−8.0												
523	prem. motor fuel—reduction of TEL, 1.5-0 cc/gal.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	1.											
524	prem. motor fuel—reduction of TEL, 3.0-1.5 cc/gal.	−1.	−1.	−1.	−1.	−1.	−1.	−1.		1.										
531	reg. motor fuel—Research octane no. 85.0 min.										−20.0	4.0	−20.0	−20.0	0.0	31.4	31.4	46.8	46.8	−20.0
532	reg. motor fuel—Reid vapor pressure, 10 lbs max.										12.0	4.0	55.0	55.0	0.0	−8.0	−8.0	−8.0	−8.0	−7.0
533	reg. motor fuel—reduction of TEL, 1.5-0 cc/gal.										−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.
534	reg. motor fuel—reduction of TEL, 3.0-1.5 cc/gal.										−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.
604	premium motor fuel—requirement	−1.	−1.	−1.	−1.	−1.	−1.	−1.												
605	regular motor fuel—requirement										−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.

TABLE B.9—CONTINUED

Row Number	Item	Regular motor fuel components																		
		961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	998	999
241	isopentane																			
242	natural gasoline																			
251	tetraethyl lead (TEL), (millions of cubic centimeters/day)	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	126.	−63.	−63.
304	refinery gases, iso and normal C4																			
305	refinery gases, olefin C4 (LPG)																			
311	light naphtha, 100–250°																			
321	medium naphtha, 250–325°, 5.0–9.9% aromatics																			
322	medium naphtha, 250–325°, 10.0–14.9% aromatics																			
331	heavy naphtha, 325–400°, 10.0–14.9% aromatics																			
332	heavy naphtha, 325–400°, 15–19.9% aromatics																			
401	alkylate gasoline																			
405	C3 polymer gasoline	1.																		
406	C4 polymer gasoline		1.																	
411	catalytic reformed gasoline, 250–325°, low severity operation			1.																
412	catalytic reformed gasoline, 250–325°, high severity operation				1.															
415	catalytic reformed gasoline, 325–400°, low severity operation					1.														
416	catalytic reformed gasoline, 325–400°, high severity operation						1.													
421	thermal reformed gasoline, 250–325°, low severity operation							1.												
422	thermal reformed gasoline, 250–325°, high severity operation								1.											
425	thermal reformed gasoline, 325–400°, low severity operation									1.										

426	thermal reformed gasoline, 325–400°, high severity operation										1.									
431	catalytic cracked, light gasoline, 100–250°, SR charge											1.								
432	catalytic cracked, light gasoline, 100–250°, cracked charge												1.							
433	catalytic cracked, medium gasoline, 250–325°													1.						
434	catalytic cracked, heavy gasoline, 325–400°														1.					
441	thermal cracked, light gasoline, 100–250°															1.				
442	thermal cracked, medium gasoline, 250–325°																1.			
443	thermal cracked, heavy gasoline, 325–400°																	1.		
521	prem. motor fuel—Research octane no. 91.5 min.																			
522	prem. motor fuel—Reid vapor pressure, 10 lbs max.																			
523	prem. motor fuel—reduction of TEL, 1.5–0 cc/gal.																			
524	prem. motor fuel—reduction of TEL, 3.0–1.5 cc/gal.																			
531	reg. motor fuel—Research octane no. 85.0 min.	−14.0	−20.0	−13.0	−21.0	−10.0	−22.9	15.6	−.7	16.1	4.0	−11.0	−11.0	−6.0	2.0	−7.0	2.0	21.0	8.	3.
532	reg. motor fuel—Reid vapor pressure, 10 lbs max.	0.0	0.0	−8.0	−8.0	−8.0	−8.0	−8.0	−8.0	−8.0	−8.0	−0.3	3.5	−8.0	−8.0	0.0	−8.0	−8.0		
533	reg. motor fuel—reduction of TEL, 1.5–0 cc/gal.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	1.	
534	reg. motor fuel—reduction of TEL, 3.0–1.5 cc/gal.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.		1.
604	premium motor fuel—requirement																			
605	regular motor fuel—requirent	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.		

TABLE B.10

JP-4 Jet Fuel Blending

Row Number	Item	Jet fuel components																
		1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015	1016	1017
311	light naphtha, 100–250°	1.																
321	medium naphtha, 250–325°, 5.0–9.9% aromatics		1.															
322	medium naphtha, 250–325°, 10.0–14.9% aromatics			1.														
331	heavy naphtha, 325–400°, 10.0–14.9% aromatics				1.													
332	heavy naphtha, 325–400°, 15–19.9% aromatics					1.												
341	light SR gas oil, 400–550°, 5.0–14.9% aromatics						1.											
342	light SR gas oil, 400–550°, 15.0–24.9% aromatics							1.										
343	light SR gas oil, 400–550°, 25.0–29.9% aromatics								1.									
344	light SR gas oil, 400–550°, 30.0–39.9% aromatics									1.								
431	catalytic cracked, light gasoline, 100–250°, SR charge										1.							
433	catalytic cracked, medium gasoline, 250–325°											1.						
434	catalytic cracked, heavy gasoline, 325–400°												1.					
435	catalytic cracked, light gas oil, 400–550°													1.				
441	thermal cracked, light gasoline, 100–250°														1.			
442	thermal cracked, medium gasoline, 250–325°															1.		
443	thermal cracked, heavy gasoline, 325–400°																1.	
444	thermal cracked, light gas oil, 400–550°																	1.
541	JP4 jet fuel—25% aromatics, max.	−21.	−17.5	−12.5	−12.5	−7.5	−14.3	−3.4	2.5	9.8	−18.5	11.	46.	46.0	−18.5	−10.	−5.	11.0
542	JP4 jet fuel—50% point at 370°, max.	−1.	−1.	−1.	0	0	1.	1.	1.	1.	−1.	−1.	0	1.	−1.	−1.	0	1.
543	JP4 jet fuel—Reid vapor pressure, 3 lbs, max.	7.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	7.	−1.	−1.	−1.	7.	−1.	−1.	−1.
620	JP4 jet fuel—requirement	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.

TABLE B.11
Fuel Oil Blending

Row Number	Item	Number 1 fuel oil blends				Number 2 fuel oil blends									Number 6 fuel oil blends			
		1101	1102	1103	1104	1121	1122	1123	1124	1125	1126	1127	1128	1129	1141	1142	1143	1144
331	heavy naphtha, 325–400°, 10.0–14.9% aromatics																	.0135
332	heavy naphtha, 325–400°, 15–19.9% aromatics																	
341	light SR gas oil, 400–550°, 5.0–14.9% aromatics	1.																
342	light SR gas oil, 400–550°, 15.0–24.9% aromatics		1.															
343	light SR gas oil, 400–550°, 25.0–29.9% aromatics			1.														
344	light SR gas oil, 400–550°, 30.0–39.9% aromatics				1.													
351	heavy SR gas oil, 550–725° (diesel fuel oil)					.4205	.4205	.4209							1.	.0293	.3285	
361	SR residuum, 725°+															.9707		.9865
375	vacuum distillation bottoms (lube oils and asphalt)																	
434	catalytic cracked, heavy gasoline, 325–400°																	
435	catalytic cracked, light gas oil, 400–550°					.5795			1.		.6514							
443	thermal cracked, heavy gasoline, 325–400°																	
444	thermal cracked, light gas oil, 400–550°						.5795			1.		.6514						
451	cracked heavy gas oil, 550–725°										.3486	.3486	.3490					
455	cracked residuum, 725°+																.6715	
606	number 1 fuel oil—requirement	−1.	−1.	−1.	−1.			.5791					.6510	1.				
607	number 2 fuel oil—requirement					−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.				
608	number of 6 fuel oil—requirement														−1.	−1.	−1.	−1.

TABLE B.11—CONTINUED

Row Number	Item	Number 6 fuel oil blends																	
		1145	1146	1147	1148	1149	1150	1151	1152	1153	1154	1155	1156	1157	1158	1159	1160	1161	1162
331	heavy naphtha, 325–400°, 10.0–14.9% aromatics								.1815							1.			
332	heavy naphtha, 325–400°, 15–19.9% aromatics	.0135								.1815							1.		
341	light SR gas oil, 400–550°, 5.0–14.9% aromatics																		
342	light SR gas oil, 400–550°, 15.0–24.9% aromatics																		
343	light SR gas oil, 400–550°, 25.0–29.9% aromatics																		
344	light SR gas oil, 400–550°, 30.0–39.9% aromatics																		
351	heavy SR gas oil, 550–725° (diesel fuel oil)																		
361	SR residuum, 725°+	.9865	.9865	.9865	.9664	.9785													
375	vacuum distillation bottoms (lube oils and asphalt)						.55												
434	catalytic cracked, heavy gasoline, 325–400										.1815							1.	
435	catalytic cracked, light gas oil, 400–550°		.0135																
443	thermal cracked, heavy gasoline, 325–400°			.0135								.1815							1.
444	thermal cracked, light gas oil, 400–550°																		
451	cracked heavy gas oil, 550–725°				.0336			1.					.3605						
455	cracked residuum, 725°+								.8185	.8185	.8185	.8185	.6395	.7371					
606	number 1 fuel oil—requirement																		
607	number 2 fuel oil—requirement					.0215	.45							.2629	1.				
608	number 6 fuel oil—requirement	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.	−1.

APPENDIX C

Row Assignments

(unit: millions of B/CD, unless specified otherwise)

Net initial availability, q_i		
		1. Equipment capacity.
7.285	101	atmospheric crude distillation
.175	102	alkylation
.097	103	polymerization
.282	104	catalytic reforming
.341	105	thermal reforming
2.471	106	catalytic cracking
1.457	107	thermal cracking
.925	108	vacuum distillation
.278	110	visbreaking
.230	111	coking
		2. Raw material availability:
1.624 365	201	availability—crude type 1
.771 386	202	availability—crude type 2
.387 082	203	availability—crude type 3
.348 541	204	availability—crude type 4
.342 117	205	availability—crude type 5
.312 961	206	availability—crude type 6
.308 634	207	availability—crude type 7
.225 193	208	availability—crude type 8
.223 380	209	availability—crude type 9
.220 384	210	availability—crude type 10
.214 110	211	availability—crude type 11
.200 248	212	availability—crude type 12
.199 665	213	availability—crude type 13
.199 549	214	availability—crude type 14
.194 823	215	availability—crude type 15
.189 947	216	availability—crude type 16
.187 151	217	availability—crude type 17
.186 968	218	availability—crude type 18
.174 403	219	availability—crude type 19
.173 738	220	availability—crude type 20
.170 193	221	availability—crude type 21
.169 711	222	availability—crude type 22
.155 965	223	availability—crude type 23
.143 866	224	availability—crude type 24
.140 621	225	availability—crude type 25
.050 000	241	isopentane
.196 824	242	natural gasoline
322.2	251	tetraethyl lead (TEL), (millions of cubic centimeters/day)
1,642.	261	British thermal units (BTU), (billions of BTU/day)
		3. Refinery gases and straight-run (SR) streams:
.000 608	301	refinery gases, C2 and lighter
.012 271	302	refinery gases, normal C3 (LPG)
0	303	refinery gases, olefin C3
.046 583	304	refinery gases, iso and normal C4
0	305	refinery gases, olefin C4 (LPG)
0	311	light naphtha, 100–250°
0	321	medium naphtha, 250–325°, 5.0–9.9% aromatics
0	322	medium naphtha, 250–325°, 10.0–14.9% aromatics
0	331	heavy naphtha, 325–400°, 10.0–14.9% aromatics
0	332	heavy naphtha, 325–400°, 15–19.9% aromatics
0	341	light SR gas oil, 400–550°, 5.0–14.9% aromatics
0	342	light SR gas oil, 400–550°, 15.0–24.9% aromatics
0	343	light SR gas oil, 400–550°, 25.0–29.9% aromatics

APPENDIX C—CONTINUED

Net initial availability, q_i		
	3. Refinery gases and straight-run (SR) streams:—*Continued*	
0	344	light SR gas oil, 400–550°, 30.0–39.9% aromatics
0	351	heavy SR gas oil, 550–725° (diesel fuel oil)
0	361	SR residuum, 725°+
0	371	vacuum distillation gas oil, 725°+
0	375	vacuum distillation bottoms (lube oils and asphalt)
	4. Converted streams:	
0	401	alkylate gasoline
0	405	C3 polymer gasoline
0	406	C4 polymer gasoline
0	411	catalytic reformed gasoline, 250–325°, low severity operation
0	412	catalytic reformed gasoline, 250–325°, high severity operation
0	415	catalytic reformed gasoline, 325–400°, low severity operation
0	416	catalytic reformed gasoline, 325–400°, high severity operation
0	421	thermal reformed gasoline, 250–325°, low severity operation
0	422	thermal reformed gasoline, 250–325°, high severity operation
0	425	thermal reformed gasoline, 325–400°, low severity operation
0	426	thermal reformed gasoline, 325–400°, high severity operation
0	431	catalytic cracked, light gasoline, 100–250°, SR charge
0	432	catalytic cracked, light gasoline, 100–250°, cracked charge
0	433	catalytic cracked, medium gasoline, 250–325°
0	434	catalytic cracked, heavy gasoline, 325–400°
0	435	catalytic cracked, light gas oil, 400–550°
0	441	thermal cracked, light gasoline, 100–250°
0	442	thermal cracked, medium gasoline, 250–325°
0	443	thermal cracked, heavy gasoline, 325–400°
0	444	thermal cracked, light gas oil, 400–550°
0	451	cracked heavy gas oil, 550–725°
0	455	cracked residuum, 725°+
	5. Gasoline and jet fuel specifications:	
0	501	115/145 aviation gasoline—lean performance number
0	502	115/145 aviation gasoline—rich performance number
0	503	115/145 aviation gasoline—Reid vapor pressure, 7 lbs max.
0	511	100/130 aviation gasoline—lean performance number
0	512	100/130 aviation gasoline—rich performance number
0	513	100/130 aviation gasoline—Reid vapor pressure, 7 lbs max.
0	521	premium motor fuel—Research octane number, 91.5 min.
0	522	premium motor fuel—Reid vapor pressure, 10 lbs. max.
0	523	premium motor fuel—reduction of TEL, 1.5–0 cc/gal.
0	524	premium motor fuel—reduction of TEL, 3.0–1.5 cc/gal.
0	531	regular motor fuel—Research octane number, 85.0 min.
0	532	regular motor fuel—Reid vapor pressure, 10 lbs max.
0	533	regular motor fuel—reduction of TEL, 1.5–0 cc/gal.
0	534	regular motor fuel—reduction of TEL, 3.0–1.5 cc/gal.
0	541	JP4 jet fuel—25% aromatics, max.
0	542	JP4 jet fuel—50% point at 370°, max.
0	543	JP4 jet fuel—Reid vapor pressure, 3 lbs, max.
	6. End item requirements:	
0	602	115/145 avgas
0	603	100/130 avgas
0	604	premium motor fuel
0	605	regular motor fuel
0	606	number 1 fuel oil
0	607	number 2 fuel oil
0	608	number 6 fuel oil
0	610	coke (millions of pounds/day)
0	620	JP4 jet fuel

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CHAPTER 5

A SPATIAL MODEL OF U. S. PETROLEUM REFINING

Thomas A. Marschak

I. INTRODUCTION

The model described in this chapter adds a spatial dimension to A. S. Manne's model of the U. S. petroleum producing and refining industry of 1952–53 (Chapter 4). It uses a condensed version of that model to describe petroleum production and refining in each of several regions and then adds the third major sector of the petroleum industry—petroleum transportation.

If the petroleum industry's capacity to transport its inputs and outputs effectively constrains its ability to deliver end items to consumers, then a model which omits transportation and regards all production and refining as concentrated at one point in space may seriously err in estimating maximum feasible amounts of delivered end items. Thus a nonspatial model could not have been used to estimate the maximum ability of the U. S. petroleum industry in World War II to deliver petroleum products to military consumers at required geographical areas in required proportions; such areas for example, were the East and Gulf coasts (for the European theater) on the one hand, and the West coast (for the Pacific theater) on the other. For in some periods it was the domestic petroleum transportation network that proved to be the effective limit to these capabilities; thus in 1942–43, before the Big and Little Inch pipelines were completed, utilization of the tanker and tank-car fleets was pushed to what was regarded as the limit of feasibility.[1]

Apart from differences in estimates of end-item availability, a spatial and a nonspatial model may well yield different answers with respect to the alternative levels of *production and refining* activities that achieve a given attainable end-item bundle. A given end-item bundle may be attainable in both models; but when transportation is an effective constraint there may be some set $x_1, \dots, x_n$ of activity levels which achieves the end-item bundle in the nonspatial model but not in the spatial model (when the refining and production activities are operated in each spatial region at levels that sum, over all regions, to $x_1, \dots, x_n$).

Finally, the relative importance—the shadow prices—of different items of refining equipment and different types of crude oil in achieving certain goals

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[1] See Chapter VII of *A History of the Petroleum Administration for War,* U. S. Government Printing Office, 1956. In the 1956–57 Suez crisis there was a similarly full utilization of the U. S. owned tanker fleet.

may be drastically altered if the spatial dimension is added and transportation is an effective constraint. Suppose the amount of a mix of end items is being maximized in the nonspatial model subject to refining capacities and crude oil availabilities. The nonspatial model may reveal refining equipment of a certain kind to be a crucial bottleneck, while the spatial model, maximizing the amount of the same mix[2] but subject in addition to transportation constraints, may reveal transportation equipment of a certain kind *or refining equipment of quite a different kind* to be a far more critical bottleneck.

There is no simple way to correct a nonspatial model's answers to allow for a spatial dimension and a transportation sector. The optimal choice of regional refining process levels and interregional commodity flows depends on refining technology and capacities, transportation capacities, and the weights given to regions in the overall maximand. The dependence is so complex that one has no choice but to embed the nonspatial model, or a model based on it, into a spatial model which repeats it as many times as there are regions. If, moreover, the model describing each region's refining and production were an extreme reduction of the original nonspatial model—a single point in the commodity space, giving *the* end-item bundle produced in each region—the dependence just described would degenerate into the trivial and totally unrealistic. Thus, for example, one of the major properties of a spatial model —that the transportation of intermediate commodities can relieve local equipment bottlenecks—would never come into play. Some choice as to refining processes and crude oils in each region—and hence a regional refining and production matrix of some nontrivial size—must be allowed to remain if the spatial model is to describe usefully the three sectors of the petroleum industry.

If the spatial model is constrained not to exceed a certain size—a certain number of rows in its matrix—then the use of a number of rows to portray a detailed refining and production technology for each of a few regions has to be balanced against the alternative of using fewer rows to describe each region's technology but including more regions in the model. If this balance has to be struck it is certainly helpful for the nonspatial matrix describing each region's technology to be "efficient," i.e., not further reducible in size without losing significant information. In the present study, moreover, the nonspatial matrix used *had* to be smaller than the nonspatial matrix constructed by Manne if more than two regions were to be used and if the constraint on model size imposed by available computing codes was to be met.[3]

An aggregation was performed which reduced the size of Manne's nonspatial model from the original 105 rows to 39 rows. This was a major part of the work and it would be a major part of any attempt to add useful spatial detail to a large existing nonspatial model of an industry. Accordingly, an appendix to the present chapter (Appendix I) briefly surveys the aggregation prob-

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[2] With or without a labeling of end items in the mixes according to the region in which they are delivered.

[3] Two-hundred-equation linear programming problems were the largest computing codes could handle at the time the computations described here were performed (1955–56).

lem in linear programming models and searches for an a priori justification for the main aggregation procedures which were used. Part II of the chapter describes the specific procedures used to obtain a small nonspatial model, the small model itself, and some comparisons between the small and large nonspatial models.

Part III describes the spatial model. The model has four regions (East, Midwest, Gulf, and West) and commodities flow between them by four means of transport (tanker, pipeline, barge, and tank car). The principal difficulties in constructing the spatial model, once a small nonspatial model of refining and production had been obtained, were (1) formulating *interregional* transportation activities given that "distances" between large regions do not exist, and (2) allowing for *intraregional* transportation.

Part IV reports some applications of the spatial model. Computation proved much more time-consuming than anticipated and the applications were accordingly limited. It was not possible to explore in detail the general question of what "difference" the incorporation of spatial detail makes—to make detailed comparisons of the answers given by the (small or large) nonspatial model and by the spatial model to a variety of questions.[4] The two main applications performed maximize the amount of a "spatial" product mix (in which each end item enters four times, once for each region), subject, in one application, to alternative tanker availabilities, and, in the other application, to the additional constraint that all movements originate only in the Gulf (currently the dominant exporter) and that all regions use only those refining activities which achieve maximization of the amount of the nonspatial product mix in the (small) nonspatial model. The results suggest some interesting properties of the spatial model, and so, probably, would many other applications not undertaken.

II. The Aggregation Performed to Obtain a Small Nonspatial Model

The flow diagram of Chapter 4 portrays schematically the refining technology of the large nonspatial model whose aggregation we now describe. The three main refining sectors—crude distillation, conversion, and blending—remain in the small model. The extent of the aggregation performed was different for the three sectors; by far the most extensive aggregation occurred with respect to distillation.

It was kept in mind throughout that the major questions to be put to the spatial model (after its construction using the small nonspatial model) would be of the following form: What is the maximum attainable amount of a spatial product mix in which each delivered end item is distinguished according to region of delivery? The product mix was to be roughly equivalent (after summing, for each end item, the regional amounts in a unit of the mix)

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[4] For example, to determine the maximum amounts of a mix of all end items but one attainable when alternative amounts of the omitted item are required (1) if all transportation equipment is free (here it suffices to use the nonspatial model), and (2) if the availability of some means of transport (say tankers) is varied.

to the small nonspatial model's product mix, so as to allow questions bearing on the difference that adding spatial detail makes. The small nonspatial model's mix was to correspond closely to the large nonspatial model's mix, so that a comparison of the large and small nonspatial models would be possible. Preservation of the end items in the product mix was thus an important constraint on the aggregation. The end item omitted from the product mix in the large model—jet fuel—was also omitted in the small model. The main guiding principle for the aggregation was that the small model should yield, for 1952–53 equipment capacities and for reduced capacities, a trade-off curve between maximum amounts of the product mix and given amounts of jet fuel close to the large model's trade-off curves.

The large model has 105 rows and 205 columns (not including slack columns); the small model has 39 rows and 81 columns. The reduction was just sufficient to permit the rows of the four-region spatial model (which include rows dealing with transportation) to be fewer than 200, the upper limit to the size of computable linear programming problems at the time. We now describe the main parts of the aggregation; the description will be best understood if reference is made to the discussion of the large model in Chapter 4.

1. CRUDE OILS. In the large model there are 25 types of crude oil; they differ with respect to the proportions in which the several distillation fractions occur. The 25 types are intended to approximate the choices actually open to the refining sector of the industry, which is in reality confronted with some three or four hundred crude types. The small model has three crude types which were chosen in the hope that they would help to yield tradeoff curves, for jet fuel versus product mix, close to those of the large model.

The possibility of choice between crude types comes most sharply into play for the case of a crude distillation capacity less than normal. The normal capacity is close to the normal total crude availability and virtually all crude will normally be used in maximizing the amount of product mix (given normal availabilities of other equipment). When distillation capacity is less than normal some available crude will go unused (left in the ground) and the possibility of selection of "good" crudes and rejection of "bad" ones becomes particularly important.

The reduction of 25 crude types to 3 is an aggregation task in which a problem of the type (iiib) discussed in Appendix I of this chapter is to be reduced in size. For each of the 25 crudes there is an activity which is its sole user—the distillation activity; in each such activity there is required an additional primary commodity (the use of distillation equipment) whose availability effectively constrains all of them (makes it infeasible to distill all of each crude). Intermediate commodities (distillation fractions) emerge from these activities. To reduce the 25 types to 3, a procedure was used which is described in general terms in Section 7 of Appendix I.

The crudes are divided into three groups or crude types; the availability of each group is the sum of the availabilities of the component crudes. The column characterizing the distillation of each of the three groups or crude types is a weighted average of the columns characterizing the distillation of that group's component crudes; the weights are the availabilities of the component crudes.[5] The three groups were chosen according to the gas oil content of the crudes they contain (gas oil is defined as the fraction boiling between 392° and 617°). The first group chosen—type 1 crude—aggregates those of the 25 crudes whose gas oil content (by volume) is less than 30%; type 2 crude aggregates those crudes with gas oil content between 30 and 40%; and type 3 those with gas oil content greater than 40%.[6, 7]

2. DISTILLATION. Refinery gases, the lightest of the distillation products, are omitted in the small model. They can be blended only in very small proportions into end items, and in the jet-fuel-versus-product-mix applications of the large model they proved to be fairly abundant by-products of distillation, with low shadow prices.

In the large model the 250–325°F and 325–400°F fractions emerging from a given crude are assigned to one of two aromatics classes and this requires three rows of the matrix for each of these fractions. The small model dispenses with aromatics classes (and thus saves four rows), letting a single number, an average, characterize the aromatics content of each fraction.[8]

[5] Note that normal distillation capacity is just barely an effective constraint—it is just slightly less than the total availability of crude (see Table 1). Hence the condition given in Section 7 of Appendix I, under which the aggregation just described would be error-free, is almost met for the case of normal distillation capacity.

[6] It will be noted that the gas oil fraction (392°–617°F) does not coincide with a sum of any of the large model's fractions. But the gas oil fraction nevertheless had to be used for grouping the crudes since the spatial model requires data on the availabilities of the three crude types in each region; and the only such data readily available (the data in the study by Harold M. Smith, cited in Table 4) relate to each region's availability of crudes falling in the indicated gas-oil-percentage intervals. In order to decide, for each of the large model's 25 crudes, into which gas-oil-percentage class it falls data were used (McKinney and Blade "Analyses of Crude Oils from 283 Important Oil Fields of the United States," *U. S. Bureau of Mines, Report #4289,* Washington, 1948) giving gas-oil percentages for these 25 crudes (which are, in fact, the crudes from the 25 largest U. S. fields). Thus a linear function which could be used for ranking a given one of the 25 crude distillation columns is, for example, a linear function of the amount of 400–725°F fraction obtained from the crude per barrel—a function which approximately yields the crude's gas oil content as given in McKinney and Blade. Simple regression would yield such a function.

[7] The choice of the three groups thus followed the principle of similarity suggested in Section 8 of Appendix I. There exists a linear function of column coefficients which, when evaluated for each of the 25 original distillation columns, yields a ranking from which the three groups actually chosen could be obtained. The linear function in question would approximate the amount of gas oil in the crude in question.

There appears to be no outstandingly "good" crude of high availability among the 25 that are aggregated (e.g., no crude yielding extraordinary amounts of the light fractions that can be blended directly into motor gas and avgas); hence there are no a priori grounds for believing that the aggregation procedure described "dilutes" such a crude, the possibility suggested at the end of Section 7 in Appendix I.

[8] This also permits dropping a row for the end item #1 Fuel Oil. For the 400–550° fraction is the sole material used for #1 Fuel Oil and a separate "#1 Fuel Oil" row is needed in the large model only because the 400–550° fractions of differing aromatics content can be used as #1 Fuel Oil. In the small model the 400–550° fraction enters the product mix directly.

3. CONVERSION. In the large model the main thermal operations—thermal cracking, thermal reforming, and visbreaking—each require equipment of a different kind. In the small model the three activities remain but the three types of equipment are pooled.

Coke (an end item), coking equipment, and coking activities are omitted in the small model.

In the large model, a distinction is made between the products of catalytic reforming and catalytic cracking, and between the products of thermal reforming and thermal cracking; furthermore, the reformed products are also distinguished by the "severity" of the operation. These distinctions are eliminated in the small model: A level of severity is chosen for the reforming process such as to yield a reformed material having the same octane rating as the cracked material. Thermally or catalytically reformed material of a given boiling range is then pooled in a single row with thermally or catalytically cracked material, respectively, of the same boiling range. In addition, no distinction is made in the small model between the products of catalytic cracking of straight-run and previously cracked (recycled) material, which differ only with respect to Reid vapor pressure.

Finally, the alkylate and polymer gasoline conversion processes are omitted in the small model. Alkylate is treated as an exogenous input, its availability equaling the large model's alkylation capacity. Polymer is made directly available to the model as motor gas.

4. BRITISH THERMAL UNITS (BTU'S). The production of Btu's (by burning of refinery gases and coke) and the requirements for Btu's are omitted in the small model. In applications of the large model more Btu's were available from burning otherwise unused by-products than were needed.

5. EXOGENOUS INPUTS. The small model drops TEL altogether (see Section 6 below) and drops the rows used for natural gasoline and butane. These are treated (together with polymer) directly as motor gas, i.e., their availability (assumed equal to the capacity of the equipment used to produce them) is entered in the motor gasoline row of the model's vector of capacities and availabilities. The excess octane which these three materials make available (the number of barrels of each times the amount by which it exceeds the octane requirement for motor gasoline, summed over the three materials) is similarly entered in the octane row of the vector. Isopentane is retained as an exogenous input.

6. END-ITEM SPECIFICATIONS. In the large model, there are four specifications by octane number for both premium and regular motor gasolines; in the small model, only one grade of motor gasoline is distinguished, and its octane number is 87, the weighted average of the octane numbers of the premium and regular grades.[9] In the large model, there are six aviation gasoline specifica-

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[9] The weights are the amounts of the two grades of gasoline in a unit of the large model's product mix.

tions. These are reduced to three in the small model (which retains two types of avgas) by eliminating the vapor pressure specification for each and by eliminating one lean performance number specification; these specifications have proved ineffective in all applications of the large model so far. In both large and small models, the aromatics content of jet fuel is restricted to not more than 25%; in the large model, but not in the small, there are two further jet fuel restrictions: (*a*) that at least 50% of the jet fuel be boiled off when its temperature is raised to 370°F, and (*b*) that its Reid vapor pressure not exceed 3 pounds. Finally, the motor gas Reid vapor pressure specification was dropped, since it is met by all the motor gas ingredients assigned separate rows in the small model.

If the small model retained the large model's apparatus for varying the use of TEL (and allowing it to influence motor gas octane number) three rows would be required.[10] These rows were initially retained (yielding a 42-equation model) but a computing run revealed that increasing TEL availability beyond its 1952–53 level gave a negligible further increase in product mix. It seemed reasonable, therefore, to save rows and drop TEL altogether.

7. END ITEMS. The small model omits coke, combines regular and premium motor gas, and lets the 400–550°F fraction serve directly as #1 Fuel Oil.[11] Otherwise the two end item lists are the same.

8. SUMMARY. The following table summarizes the reductions made to obtain the small nonspatial model; Table 1 presents the small nonspatial model (see pp. 105–110).

	Number of Equations (rows)	
	Large Model	Small Model
Number of crude oils	25	3
Equipment capacities	10	5
Refinery gases	5	0
Straight-run streams (and products of vacuum distillation)	8	8
Classification of SR streams by aromatics content	5	0
Btu's	1	0
Intermediate materials used or produced by conversion processes	22	10
Exogenous inputs	3	2
End items (not elsewhere listed)	9	6
End item specifications	17	5
Total	105	39

9. COMPARISONS OF THE LARGE AND SMALL MODELS. Two comparisons were made (Figures 1 and 2). Each comparison consists of a pair of trade-off

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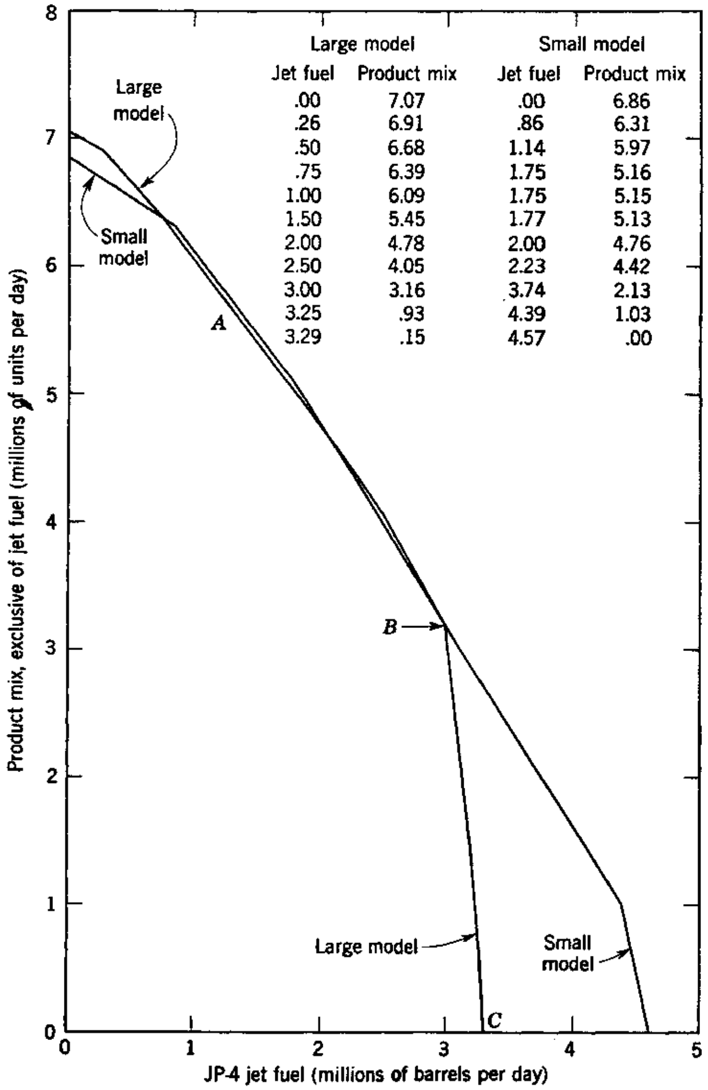
[10] See footnote 8, Chapter 4.

[11] This fraction is the sole material used for #1 Fuel Oil in the large model too, but there the end item "#1 Fuel Oil" is given a separate row.

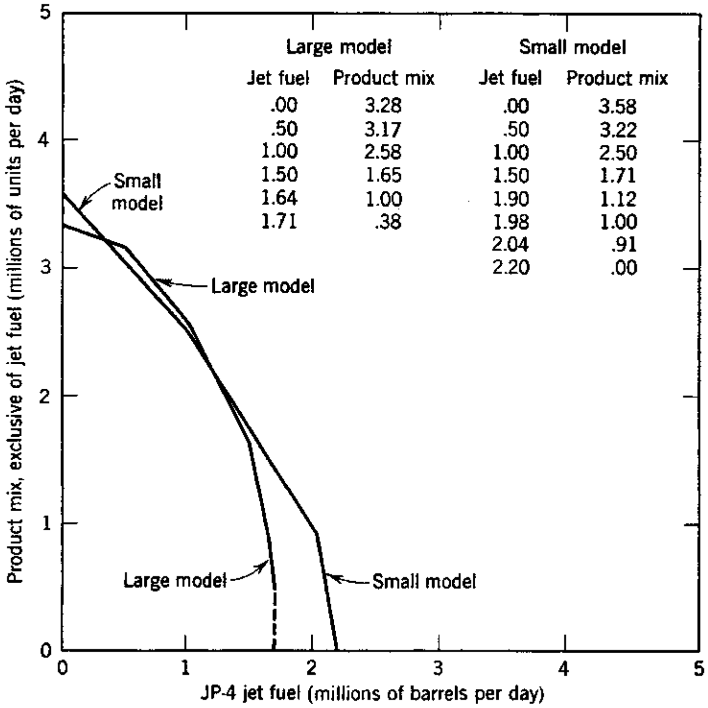
curves—jet fuel versus product mix—one computed using the small model and the other using the large model. For the first comparison 1952–53 equipment capacities and crude oil availabilities are imposed. For the second comparison the capacities and availabilities are drastically reduced below the 1952–53 levels. The exact reduced capacities are given in Chapter 4, Table 2.

a. *1952–53 Capacities.* The curves are close to the left of point *B*. The divergence to the right of point *B* (the small model permits increasingly more product mix than the large model) seems to be explained by the two extra constraints placed on the allowable jet fuel blends (50% point not higher than



Large model		Small model	
Jet fuel	Product mix	Jet fuel	Product mix
.00	7.07	.00	6.86
.26	6.91	.86	6.31
.50	6.68	1.14	5.97
.75	6.39	1.75	5.16
1.00	6.09	1.75	5.15
1.50	5.45	1.77	5.13
2.00	4.78	2.00	4.76
2.50	4.05	2.23	4.42
3.00	3.16	3.74	2.13
3.25	.93	4.39	1.03
3.29	.15	4.57	.00

FIGURE 1. Jet fuel tradeoff curves, 1952–53 capacities.



Large model		Small model	
Jet fuel	Product mix	Jet fuel	Product mix
.00	3.28	.00	3.58
.50	3.17	.50	3.22
1.00	2.58	1.00	2.50
1.50	1.65	1.50	1.71
1.64	1.00	1.90	1.12
1.71	.38	1.98	1.00
		2.04	.91
		2.20	.00

FIGURE 2. Jet fuel tradeoff curves, reduced capacities.

370°F and Reid vapor pressure not higher than 3 lbs.) in the large model. The aromatics content restriction, it is interesting to note, turns out to be ineffective in either model.[12]

This explanation is confirmed by the following "shadow prices" on jet fuel and "on" the three restrictions for three points of the large model's jet fuel–product mix curve:

	Point *A* (1.5 million bbls. per day of jet fuel)	Point *B* (3.0 million bbls. per day of jet fuel)	Point *C* (maximum amount of jet fuel: maximum amount of product mix = 0)
Shadow price of jet fuel	1.31	5.35	25.45
Shadow price "on" aromatics content restriction	0.00	0.00	0.00
Shadow price "on" 50% point restriction	0.042	0.00	27.61
Shadow price "on" Reid Vapor Pressure restriction	0.00	0.76	7.58

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[12] Except when the output of jet fuel is a maximum (with zero product mix), when the shadow price on the aromatics content restriction happens to be zero although the restriction is met exactly (is not overfulfilled).

At point A, for example, a small increment Δ in jet fuel requires an increment of $(.042/1.31)\Delta = .032\Delta$ in the sum of the products of the quantity of each jet fuel component blended into jet fuel with the amount by which the percentage of that component that is boiled off at 370°F exceeds 50; at point B, with more jet fuel produced, an increment of 0 is required; but at point C, with the most jet fuel possible being produced, a far bigger increment of $(27.61/25.45)\Delta = 1.08\Delta$ is required. Similarly at a point A a small increment Δ in jet fuel requires an increment of 0 in the sum of the products of the quantity of each jet fuel component blended into jet fuel with the amounts by which the Reid vapor pressure of that component exceeds 3 lbs.; at point B an increment of $(.76/5.35)\Delta = .14\Delta$ is required; but at point C an increment of $(7.58/25.45)\Delta = .30\Delta$ is required.

In the small model these two restrictions are omitted and between point B and point C the small model is able to meet the same jet fuel requirements as the large model while yielding more product mix, while to the right of point C (along the jet fuel axis of Figure 2), with the large model producing no more jet fuel at all, the small model continues to do so, to the extent of more than a million barrels per day. For the range to the left of point B, however, the two models yield "answers" which are very close.

b. *Reduced Capacities.* Not only does the choice of crudes (of which the large model provides far more) become important in the reduced-capacity case, but in addition one would expect the flexibility permitted (in the large model) by alternative levels of reforming severity to become more important as reforming capacity is diminished.

The two curves are close together over an interesting and fairly large range of jet fuel requirements (0.5 to 1.5 million bbls. per day). For requirements between 0 and 0.5 million bbls. per day and greater than 1.5 million bbls. per day, the small model allows somewhat more product mix. The divergence in the former interval is explained partially by the circumstance that here the large model "bottlenecks" on coking capacity, a capacity limit which is omitted altogether in the small model; the divergence in the latter interval suggests that the large model's two extra jet-fuel constraints were so effective that the best of the large model's 25 crude classes were not, in fact, much better than the best of the small model's three crude classes.

10. THE CLOSENESS OF THE TWO MODELS. It appears that the balance between the extra "stringencies" of the small model as compared with the large one (many less crude classes to choose from, reforming at one severity level only), and its extra "laxities" (coke omitted from product mix and coking capacity from equipment list, two jet fuel specifications omitted) is such that when the amount of product mix of the 1952–53 type is to be maximized subject to alternative capacity limits and alternative jet fuel requirements, the answers yielded by the two models over interesting capacity and jet fuel ranges will not be significantly different. For extreme cases (extremely high jet fuel requirements and extremely low capacities), there is divergence.

It seems likely that except for such extreme cases answers obtained from a spatial model based on the small nonspatial model would not differ much from the corresponding answers obtained from a spatial model based on the large nonspatial model.

III. Description of the Four-Region Model

1. 1952–53 SPATIAL DIFFERENCES IN U. S. CRUDE OIL PRODUCTION, REFINING, AND PETROLEUM PRODUCTS CONSUMPTION AND 1952–53 PRINCIPAL MOVEMENTS.

In broad outline the 1952–53 spatial distribution of the U. S. petroleum industry and its markets can be characterized as follows. The *Pacific Coast* is a virtually autonomous area. It produces its own crude, does its own refining, and consumes its own products with practically no importing or exporting. The *Gulf States* (Texas, Louisiana, Mississippi, Alabama, and Oklahoma) produce about half of the nation's crude, possess about two-fifths of its refining capacity (as measured by daily atmospheric distillation capacity), but consume less than one-ninth of its total petroleum products stream. The states, other than the Gulf States (augmented to include New Mexico and Arkansas) and the Pacific States (including those mountain states supplied by California), may be divided into two groups: East and Midwest. The *East* group (New England, the Atlantic Seaboard, and Ohio) consumes nearly half of the nation's product stream, possesses about a fifth of its refining capacity, and produces only about seven-hundredths of its crude. The *Midwest* group consumes about one-quarter of the nation's product stream, possesses about a third goes to Midwest refineries (where it constitutes about half of the crude

The Gulf group exports about two-fifths of its crude. Of this about one-third goes to Midwest refineries (where it constitutes about half of the crude received) and about two-thirds to East refineries (where it constitutes nearly all of the crude received). The Gulf group exports about seven-tenths of its product stream. Of this about one-fifth goes to Midwest markets (where it constitutes about one-fifth of the product stream consumed) and four-fifths to East markets (where it constitutes more than half of the product stream consumed). In addition, there is a small amount of products movement between East and Midwest, most of it from Midwest to East. Both the Midwest and East groups, then, are highly dependent on the Gulf group for both crude and products; but of the two the East is considerably more dependent.

Of the movement of crude from Gulf to East, about two-thirds is by tanker, a little less than a third by pipeline, and the remainder by barge, tank car, and truck. Of the movement of crude from Gulf to Midwest, about 93% is by pipeline, the bulk of the remainder by barge, and a very small amount by tank car. Of the movement of products from Gulf to Midwest, about six-tenths is by pipeline, three-tenths by barge, and one-tenth by tank car.

Within each region there are movements from oil fields to refineries and to ports or pipeline terminals (for further shipment), from ports or pipeline terminals to refineries or to consumers of products, and from refineries to consumers. Some of the principal intraregional movements are: the coastwise tanker shipment of crude and products along the Pacific Coast, pipeline and barge shipments of crude from Gulf fields to Gulf refineries, barge shipment of products on the Great Lakes within the East and the Midwest regions, coastwise tanker (and intraharbor barge) movements of products from East Coast refineries to East Coast markets. Tank trucks are everywhere used for short hauls between refineries and distribution centers or retail outlets.

2. CHOICE OF REGIONS AND TRANSPORTATION ACTIVITIES INCLUDED IN THE MODEL. The four regions, and the transportation activities allowed between them, were selected to permit the model:

1. To duplicate roughly the 1952–53 pattern of interregional flows (and to make allowance for intraregional transportation).
2. To reflect the principal 1952–53 constraints on pipeline flows (pipelines are unique among all the means of petroleum transportation for they are not movable and take a great amount of time and resources to build).
3. To yield answers to questions that involve interesting variations *as between the four regions* from the 1952–53 spatial patterns of consumption, crude production, and refining capacity (the consequences of variations from the 1952–53 spatial patterns *within* regions cannot be traced in the model, but the regions are so selected that such variations are generally less interesting than variations as *between* the regions).

The four regions selected—East, Midwest, Gulf, and West—are shown in Figure 3, together with the principal *interregional* crude oil and products pipelines and the principal barge routes. *Intraregional* pipelines are considerably smaller in capacity than the interregional ones on whose capacities the model explicitly imposes the present limits. They are chiefly gathering lines within the oil fields, or else short field-to-refinery lines whose capacity is not less than the normal producing rate of the originating fields; intraregional pipelines are ignored entirely in the model.

The means of transportation that the model distinguishes are four: (interregional) pipeline, tanker, barge, and railroad tank car. Tank trucks, which are largely used to serve local consumption areas surrounding refineries, appear to be of very little importance for interregional transport. We assume (*a*) that there is no means of transportation which can be substituted for tank trucks in providing such local service, and (*b*) that the nation's tank-truck fleet is adequate for any "interesting" distribution of the requirements for such service over the four regions. Accordingly, tank-truck transportation is entirely omitted in the model.

Every transportable commodity (crude oils, intermediate materials, end items, and exogenous inputs) distinguished in the small nonspatial model may, in the spatial model, be shipped from any one of the four regions to any other by any one of the four means of transportation, provided that it is physically possible to do so (i.e., there is no barge transportation to and from the West, nor tanker transportation to and from the Midwest, nor are "vacuum distillate bottoms" transported by pipeline).[13]

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[13] Or transportation of crude and "black" products in products pipelines and "white" products in crude lines (see Section 5 below).

3. THE ROWS AND COLUMNS OF THE MATRIX: A GENERAL SUMMARY. The general unit of measurement of crude and products flows is one million barrels per year (365 calendar days) in the spatial model; empty tankers, tank cars, and barges in the various regions are measured in numbers per year.

One hundred fifty-six of the rows and 324 of the columns are devoted to four successive repetitions of the 39-equation nonspatial petroleum refining matrix.

Thirty-nine rows of the spatial matrix, then, are devoted to the crude oils, intermediate materials, exogenous inputs, specifications, and end items of the nonspatial model *located in the East,* and 81 columns are devoted to the distillation, conversion, and blending activities of the nonspatial model *as carried on in the East* and involving the items of the 39 rows. Another 39 rows are devoted to the nonspatial model's items located in the Midwest and another 81 columns to its activities as carried on in the Midwest; and similarly for the Gulf and West regions.

Eight rows are devoted to intraregional transportation—for each region there is one "intraregional crude transportation" row and one "intraregional products transportation" row. These items will be discussed in Section 4 below. Seventeen rows are devoted to empty tankers, tank cars, and barges as they appear in the various regions, with a division of the empty tankers and tank cars into "clean" and "dirty" categories (Sections 6, 7). Eight rows are devoted to the capacities of crude and products pipelines between certain pairs of regions (Section 5.1). Three rows are used to allow for the effect of Venezuelan imports on U. S. capabilities (Section 8). Finally, three rows are devoted to the national fleet of movable transportation equipment: one row for tankers, one for tank cars, and one for barges.

Of the 1332 columns other than the 324 refining technology columns, 904 are devoted to transportation of each commodity from every region to every other by each feasible means of transportation (Section 5); 16 to the activities of "making available intraregional transportation" (Section 4.3); 13 to the activities of "cleaning or dirtying" tankers or tank cars; 36 to "interregional movement of empty tankers, tank cars, and barges" (Section 6); 19 to "shipment of Venezuelan crude to U. S. with transshipment within U. S. and atmospheric distillation at final destination" (Section 7); 19 to "shipment of Venezuelan residual fuel oil to U. S. with transshipment within U. S." (Section 7); and four to "movement of empty dirty tankers to and from Venezuela" (Section 7). One additional column is the "product mix" column (Section 9). Finally, since the system has 195 equations, there are 195 columns that are slack vectors, i.e., that define disposal activities.

4. INTRAREGIONAL TRANSPORTATION. We describe first the attempt that was made to deal with perhaps the most troublesome problem in constructing any multiregion model when the regions themselves are quite large. The difficulty is that a considerable proportion of the nation's movable transportation equipment is in fact used for transportation *within* the regions; the equipment used intraregionally must somehow be subtracted from the national pool if the nation's interregional transportation capabilities are not to be overstated.

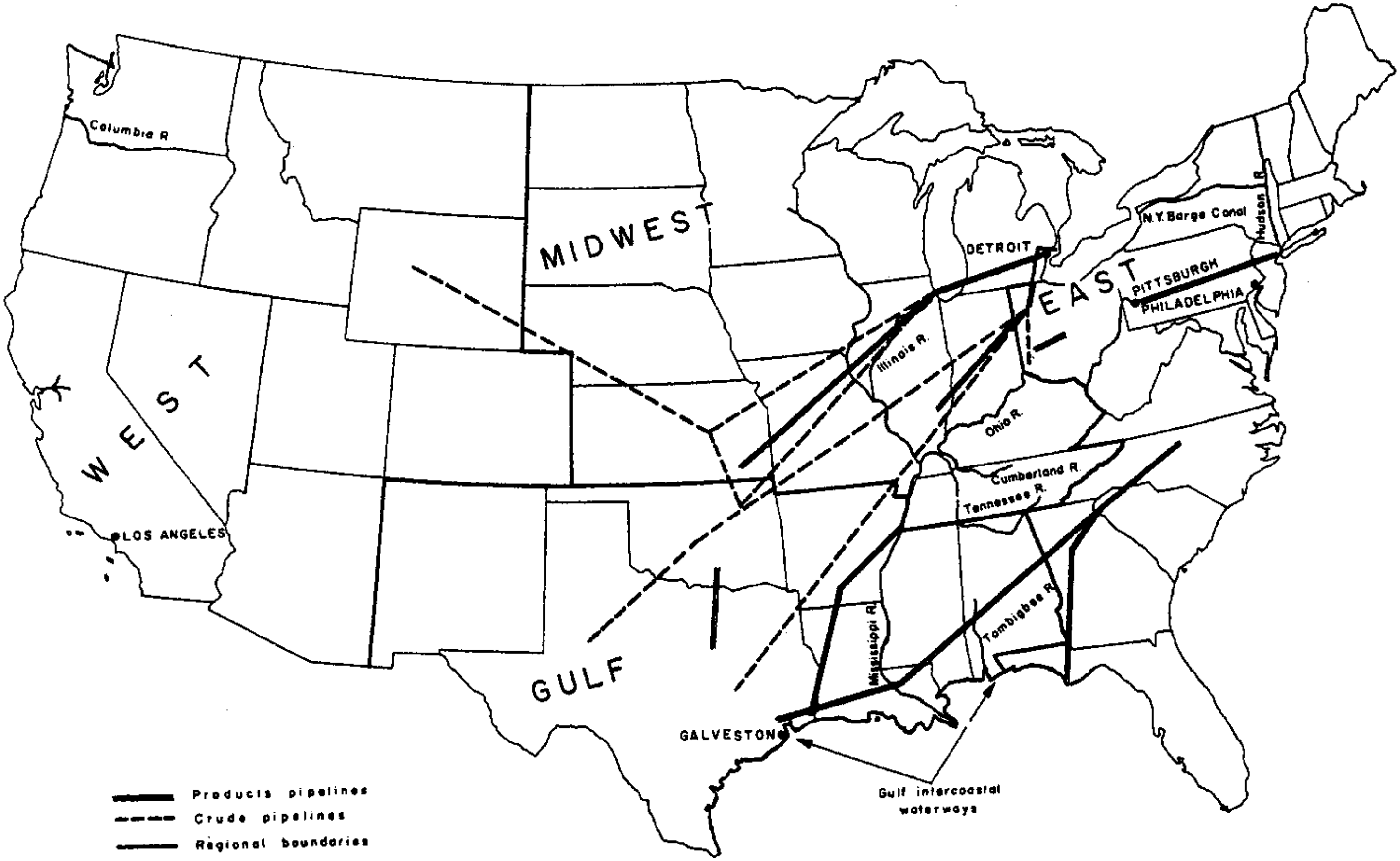

FIGURE 3. The four regions of the model. Principal barge routes and interregional pipelines are shown.

4.1. *Assumptions.* In the case of petroleum transportation there are several kinds of intraregional movement by movable transportation equipment; intraregional pipeline movements need not concern us, and the term "intraregional movements" henceforth excludes them. In the first place there are three conceivable kinds of intraregional crude oil movements: field-to-refinery movements, movements from fields to interregional transportation terminals (tanker ports, barge ports, pipeline terminals, and rail terminals), and movements from interregional transportation terminals to refineries. Next, the intraregional movements of products (except movements by tank truck) can be of four principal kinds: refinery to distribution center (from which deliveries to retail outlets are made), refinery to interregional transportation terminal, interregional transportation terminal to distribution center, and (for intermediate materials sent to a refinery in another region for further processing) interregional transportation terminal to refinery. Finally, there are small intraregional movements of intermediate materials that are shipped from one refinery to another in the same region for further processing; and small intraregional movements of exogenous inputs (alkylate, isopentane, butane, natural gasoline) from the plants where they are produced to refineries, movements which are largely in specialized tank cars (pressurized cars) that may be assumed always in adequate supply for this purpose. These two categories of small intraregional interrefinery movements are omitted altogether from the analysis.[14]

To estimate the extent of the three kinds of intraregional crude movements and the four kinds of intraregional products movement is very difficult. We make a number of simplifying assumptions. We assume

1. That no intraregional movement begins or ends at an interregional rail terminal. (This assumption simply recognizes that the rail network is generally sufficiently dense, and the cost of transferring crude or products between tank car and barge, tanker, or pipeline is sufficiently high, so that if any part of a movement from a point in one region to a point in another region can be made by rail then all of it can be; and if it is efficient to do any part of it by rail then it is efficient to do all of it by rail.)
2. That intraregional movements of crude are exclusively movements from fields to refineries and from interregional terminals to refineries. (No movements are from refineries to interregional transportation terminals—a fairly accurate assumption in 1952–53 for the Gulf, which was the dominant exporter.)

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[14] There is a slight asymmetry in our prohibiting in the model the increase of intraregional interrefinery movements of intermediate materials beyond their present negligible levels (assumed zero in the model) but permitting *interregional* interrefinery movements (currently also negligible) and allowing in addition for the intraregional transportation which such interregional movements require. But (1) it is a property of multiregion models that they shed little light on the effects of changes within regions; and (2) we have to allow explicitly for (to "charge" for) the intraregional transportation required by interregional interrefinery movements in order to avoid the computational pitfall mentioned in Section 4.3 below.

3. That when end items are shipped between regions (other than by tank car), intraregional movement is required in the receiving region (from interregional transportation terminals to consumption centers) but not in the originating region. (Again this was largely true in 1952–53, the significant receiving regions for end items being the East and Midwest, where a considerable quantity of Gulf-produced end items had to be moved from pipeline terminals and ports to distribution centers; Gulf ports and products-pipeline terminals, moreover, were nearly all refinery sites as well.)

4. That when exogenous inputs are shipped between regions no intraregional movement is required. (In fact, such shipments have always been very small in volume and are usually, and most efficiently, made in specialized tank cars; assumption 1 has excluded intraregional movements required as a result of interregional rail movements.)

5. That when intermediate materials are shipped between regions, intraregional transportation is required in the receiving region (for movements from terminals to refineries) but not in the originating region. (Actually interregional intermediate-material movements were negligible in 1952–53. But if the model *were* to make use of such movements we would want the proper charge for the resulting intraregional transportation to be made, and this assumption is a reasonable way of doing so.)

6. That intraregional transportation is required for field-to-refinery movements of crude and for refinery-to-distribution-center movements of end items.

If assumptions 1 to 6 were met in 1952–53—and they nearly were—then the amount of intraregional transportation of crude required in a region i depended only on Q_i, the amount of crude distilled in that region (and obtained from regional production as well as from imports). The amount of intraregional transportation of products (end items and intermediate materials) required in a given region i depended only on $\bar{Q}_i$, the amount of end items produced in that region less the amount exported plus the amount imported plus the amount of intermediate materials imported (the latter quantity was virtually zero in 1952–53).[15]

Within a given region, moreover, a certain minimal fleet of tank cars, barges, and (except for the Midwest) tankers was required in order to achieve the 1952–53 intraregional tank car, barge, and tanker movements of crude. This fleet can be regarded as made up of a certain number of identical bundles of tank cars, barges, and tankers, where the proportions in which the three kinds of vehicles occur in a bundle are the same as the proportions in which they occur in the fleet. Similarly, for a given region, another minimal fleet of tank cars, barges, and tankers made up also of a number of identical bundles (composed differently from the intraregional crude transportation bundles for the same region) was required in order to achieve the 1952–53 tank car, barge, and tanker movement of products. We now assume

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[15] Except for imports of Venezuelan residual fuel oil into the East.

7. That for a year other than 1952–53 the number of these bundles of intraregional crude transportation vehicles required in a region i bears the same ratio to the quantity Q_i as was the case in 1952–53. The number of bundles of intraregional products vehicles required in a given region i for the given years bears the same ratio to the quantity $\bar{Q}_i$ as was the case in 1952–53.

Assumption 7 justifies a procedure for estimating the amount of intraregional crude transportation (the number of minimal bundles) and the amount of intraregional products transportation required by each of the four regions when the region's crude and products production and imports are at given levels. The procedure and the data used are given in Appendix II. They yield the first eight columns of the intraregional transportation submatrix shown as Figure 4. The intermediate commodities "intraregional crude transportation" and "intraregional products transportation" (each distinguished according to region) are required in the model's refining and interregional transportation activities, in a manner to be described below (Section 4.3).

Assumption 7, and the intraregional transportation matrix based on it, will be the more inaccurate the more the spatial distribution of fields, refineries, consumption centers, and interregional terminals within each region differs from that of 1952–53.

4.2. *Varying the Intraregional Transportation Fleets.* If the first eight columns of Figure 4 were the model's entire treatment of intraregional transportation the model could bottleneck unrealistically in some interesting applications. If, for example, the effects of great reductions in the national barge fleet on the maximum attainable amount of a spatially differentiated product mix were being traced, then intraregional transportation in the Gulf, Midwest, and East would become increasingly "expensive" for increasing amounts of product mix and a given (abnormally small) barge fleet. For a barge fleet of size zero, very little product mix would be attainable, according to the model, since any production in Gulf, East, or Midwest requires intraregional transportation which in turn requires barges. Hence production in the West and export to other regions would be the only way of obtaining any product mix.

In order to avoid such unrealistic bottlenecking, we permit for each region some variations from the 1952–53 composition of the minimal intraregional transportation fleets; i.e., we add to the intraregional transportation submatrix eight "replacement" activities in addition to the eight "normal" activities based on the 1952–53 intraregional transportation fleets. The additional activities are shown in Figure 4.

It will be noted that it is always tank cars that replace the other kinds of (movable) transportation equipment; tank cars themselves are never replaced. The justification for this is that tank-car transportation is at present considerably more expensive than all other types.[16] Hence it seems reasonable to assume that the small amount of material now shipped by tank car (the exist-

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[16] See Chapter 2 of *Transportation of Oil,* Petroleum Administration for Defense, 1950.

Items	Activities of Making Available Intraregional Transportation															
	"NORMAL"								"REPLACEMENT"							
(Minus signs indicate inputs)	Crude, East	Products, East	Crude, Midwest	Products, Midwest	Crude, Gulf	Products, Gulf	Crude, West	Products, West	Crude, East	Products, East	Crude, Midwest	Products, Midwest	Crude, Gulf	Products, Gulf	Crude, West	Products, West
Diesel fuel oil, East (millions of bbls./yr.)		−.000554							−.000036	−.000681						
Diesel fuel oil, Midwest			−.000287	−.001139							−.000456	−.001681				
Diesel fuel oil, Gulf					−.000369	−.001438							−.000768	−.001638		
Diesel fuel oil, West							−.000100	−.000330							−.000195	−.003115
#6 fuel oil, East	−.000037	−.000930							−.000012	−.000310						
#6 fuel oil, West							−.000700	−.004160							−.000231	−.001373
Tankers (T-2)	−.003200	−.007900					−.014000	−.045000	−.001066	−.002600					−.004600	−.015000
Tank-cars (209 bbl.)		−1.600000	−.600000	−3.600000	−.028000	−6.300000	−.840000	−3.800000	−.120000	−3.300000	−3.600000	−8.900000	−10.228000	−16.900000	−2.44000	−23.500000
Barges (9570 bbl.)		−.460000	−.190000	−.450000	−.560000	−.940000				−.460000						
Intraregional crude transportation, East (units/yr.)	1.000000								1.000000							
Intraregional products transportation, East		1.000000								1.000000						
Intraregional crude transportation, Midwest			1.000000								1.000000					
Intraregional products transportation, Midwest				1.000000								1.000000				
Intraregional crude transportation, Gulf					1.000000								1.000000			
Intraregional products transportation, Gulf						1.000000								1.000000		
Intraregional crude transportation, West							1.000000								1.000000	
Intraregional products transportation, West								1.000000								1.000000

$i =$	East	Midwest	Gulf	West
1952–1953 value of Q_i (in millions of bbls.)	218.2	215.1	1008.3	443.2
1952–1953 value of $\bar{Q}_i$ (in millions of bbls.)	1107.3	617.6	335.9	449.6

If the coefficients of the "normal" intraregional crude (products) transportation activity in each region are multiplied by the 1952–1953 value of $Q_i(\bar{Q}_i)$, then an estimate of the minimal "bundle" of transportation equipment and fuel that could have achieved the 1952–1953 intraregional crude (products) movements is obtained.

FIGURE 4. Intraregional transportation submatrix.

ing tank-car fleet is currently being utilized far less intensively than it could be) cannot be shipped by other means of transportation.

Our alternative activities do not allow for the replacement of tankers by barges (which could travel, to some extent, on "parallel" inland waterway routes) or vice versa. The reason is that use of the vast and heavily underutilized tank-car fleet, which in most interesting applications would remain underutilized,[17] provides sufficient scope for replacement of parts of the "normal" intraregional transportation fleets.

The alternative activities do not allow for replacement of barges in the East, since, except for the small proportion used in intra-East Great Lakes movements, these are used largely for short movements within large harbor areas (New York, Philadelphia, Baltimore, Boston)[18] and tank cars do not seem a feasible replacement for this purpose.

Note finally that the replacement activities do not allow replacement of the *entire* tanker fleet in East and West. A two-thirds replacement compared with 1952–53, in the production of a unit of intra-East or intra-West crude or products transportation,[19] is the maximum permitted. This is meant to reflect, in an arbitrary way, the fact that coastwise rail routes, already in heavy use, would be seriously clogged if they had to carry much tanker traffic, and that complete replacement of tankers would be impractical.

The sixteen intraregional transportation activities provide a flexible mechanism for furnishing the intraregional transportation necessary to distribute end items and crude over each region, given that the 1952–53 spatial distribution of end-item consumers, refineries, fields, and interregional transportation terminals is approximately to prevail within each region.

4.3. *Intraregional Transportation Rows in Columns Relating to Refinery Operations, Interregional Shipments, and Disposal Activities.* We now describe the manner in which, under assumptions 1–6 of 4.1, the eight intraregional transportation commodities that are produced by the intraregional transportation activities we have just defined enter other activities (columns of the matrix).

In the 324 refining activities they enter as follows: At unit level, the atmospheric distillation activities in a given region (which, at unit level, require one million barrels of crude per year and produce .994 mllion barrels of distillates per year) require one unit of intraregional crude transportation (which is needed for field-to-refinery or terminal-to-refinery movement of the million barrels of crude within the region) and .994 units of intraregional products transportation (which would be needed for refinery-to-distribution-center movement within the region if all of the distillates obtained from the million barrels of crude became end items in the region's refineries and if all of these end items were consumed in the region). But any refining activity—other

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[17] The World War II situation is no longer likely ever to apply again since pipeline capacity has been so greatly expanded.

[18] Some barge movement does occur, however, along the Hudson River and the New York State Barge Canal.

[19] There are no intra-Gulf tanker movements. See Appendix II.

than the atmospheric distillation activities—in which there is "disappearance," i.e., in which the total volume of outputs at unit level is less than the total volume of inputs by an amount Δ, "produces" (has as an output), at unit level, of Δ units of intraregional products transportation. The "charge" made for intraregional products transportation in the atmospheric distillation activities, in other words, is corrected by the amount of intraregional products transportation required for refinery-to-distribution-center movement of a quantity of products equal in volume to the amount of material that disappears in the course of subsequent refinery operations. Thus, the activity "catalytic cracking of 400–550°F straight-run, East," which at unit level requires one million barrels a year of distillate but produces only 830,000 barrels a year of intermediate materials, has as a further output at unit level .17 units of intraregional products transportation in the East.

But the possibility of exporting products requires further correction of the "charges" made for intraregional products transportation in the atmospheric distillation activities. For when end items or intermediate materials are exported, then refinery-to-distribution-center transportation of the former, or of the end items which the latter become, is not required in the exporting region; neither, according to assumption 5 of Section 4.1, are intraregional movements of the refinery-to-interregional-transportation-terminal kind required in the exporting region. Hence a correction must be made: An activity of shipping an intermediate material or an end item from one region to another must have as an output at unit level an amount of intraregional products transportation in the exporting region equal to the ratio of the number of barrels shipped (at unit level) to one million.

In accordance with assumptions 2, 3, and 5 of 4.1, however, some "charges" must be made for intraregional transportation to support intraregional movements to and from interregional transportation terminals. In accordance with assumption 2, an activity of shipping crude oil by means other than tank car from one region to another—requires, at unit level, an amount of intraregional crude transportation in the importing region corresponding to the amount shipped. In accordance with assumption 3, an activity of shipping (by means other than tank car) intermediate materials or end items between regions requires, at unit level, an amount of intraregional products transportation in the importing region corresponding to the amount per year shipped. In accordance with assumption 4, the activities of shipping exogenous inputs interregionally do not require intraregional transportation.[20]

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[20] We note that the manner in which the eight intraregional transportation "commodities" enter the interregional shipment activities is such that the following computational pitfall is avoided: It cannot happen that, intraregional transportation being very scarce in some region (or rather the equipment and fuel required for it), the model is able to relieve the scarcity by calling for a back-and-forth interregional shuttling of some commodities in order to "produce" the required intraregional transportation. This *could* happen if, for example, in shipping an intermediate product by pipeline from one region to another, we "credited" the exporting region (i.e., included intraregional products transportation in the exporting region as an output)—as we in fact do—but did *not* "charge" the importing region (i.e., did not include intraregional transportation in the importing region as an input in the activity). It could also occur if we did the same with respect to

Finally, consider the 195 slack or disposal columns. Of these, 84 are concerned with "throwing away" the 21 intermediate materials and end items that may appear in each region. Whenever a quantity of an intermediate material or end item is "thrown away" in a region, however, its production does not create a need for intraregional products transportation (for refinery-to-distribution-center movement) in the region, although the region is "charged" with supplying such a need in the atmospheric distillation activity that originally brought the quantity of intermediate material or end item into being. Hence the following correction must be made: A disposal activity involving one of the end items or intermediate materials in a region has, at unit level, an input of one million barrels a year of the commodity in question in the region in question but it also has, at unit level, an output, namely one unit of intraregional products transportation (in the region in question).

5. INTERREGIONAL SHIPMENT OF COMMODITIES. The chief problems encountered in incorporating interregional transportation into the model arise because regions are not points and because there is no obvious method of assigning useful dimensions and numbers to the task of transporting a commodity unit from one region to another by a given means of transportation. We discuss the assumptions made and the numbers used for each of the four means of transportation. It will be noticed that the interregional shipment activities have a "feedback" property that is explicitly taken into account: There is consumption of certain petroleum products as fuel in the course of petroleum transportation.

5.1. *Interregional Pipeline Shipments.* In the case of pipelines, the daily (and hence the annual) 1952–53 capacities for delivery of products and crude lines crossing regional boundaries were readily obtained (and the totals are given in Table 3 at end of chapter).

Crude lines are assumed to be usable only for crude and for "black" products. These are #6 fuel oil, diesel fuel oil, 725+°F vacuum distillate, 550–725° cracked, vacuum distillate gas oil, the residual atmospheric distillation fraction—the 725+°F fraction—and 725+°F cracked. Vacuum distillate bottoms is a black product that cannot be piped. It is assumed that other products can be sent only by products pipeline. This is realistic, since conversion from crude to products use and vice versa, while feasible, takes considerable time (and resources); it would be easy, if desired, to add conversion activities to the model, in which a unit of one kind of pipeline capacity between two regions is converted during a year to less than one unit (allowing for conversion time) of the other kind of capacity.[21]

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the activity of shipping the same commodity by pipeline in the reverse direction. For then the model could artificially "produce" intraregional products transportation in one region by (fictionally) exporting this commodity by pipeline from one region to another and then importing it back again by pipeline, the only *net* input required being pipeline capacity. Our treatment manages to prohibit such a possible absurdity.

[21] Indivisibilities are, of course, (unrealistically) ignored if such an activity is allowed to be operated at any one of a continuous range of levels (for actually all of a pipeline's capacity must be converted if any part of its capacity is converted).

It is also assumed that, during the course of a year, commodities could be sent through a pipeline in both directions, the proportions between the two directions being unrestricted and the combined capacity for deliveries out of either end being equal to the 1952–53 capacity for flow in the "normal" 1952–53 direction. It is true that this assumption is somewhat unrealistic, for reversal of flow takes time (though less time than conversion between "black" and "white" use). But it seems very likely that in any interesting application, pipeline flow between each pair of regions will occur in one direction only for "white" products and one direction only for crude and "black" products in an optimal solution; if this direction is the opposite of the present "normal" one then a once-and-for-all reversal of flow is all that is required.

Further assumptions are: (1) products can follow each other through a pipeline without lag (which is not unrealistic if the sequence of products is from light to heavy, that is to say, from "clean" to "dirty"); (2) in any application the "capabilities" to be explored are stated in terms of annual flows, there being no preference for one part of the year over another with respect to the delivery of certain end items (perhaps the chief defect of this assumption is that seasonal peaks in heating oil demands are ignored).

An activity of sending a commodity by pipeline from one region to another, then, takes, at unit level, one million barrels a year of the commodity shipped in the region of origin, takes one million barrels a year of pipeline capacity between the two regions (crude pipeline capacity if the commodity is black, products pipeline capacity if the commodity is white), and produces one million barrels a year of the commodity in the region of termination. Intraregional transportation items also enter the activity in the manner described in Section 4.3.

Black and white pipeline shipment is permitted between Gulf and East, Gulf and Midwest, East and Midwest. In addition, black and white pipeline shipping activities between Gulf and West are included in the model in order eventually to explore the effects of constructing crude and products pipelines between Texas and California.

5.2. *Interregional Tanker Shipments.* In order to measure the time required to ship commodities from one region to another by tanker, each region (other than the Midwest) is represented in the model by a single port. The ports chosen are: Philadelphia in the East, Galveston in the Gulf, and Los Angeles in the West. It is assumed, in other words, that distance covered by the average tanker shipment between any two regions in 1952–53 equaled the distance between the representative ports.

The time required to ship a tanker load of a petroleum commodity from one region to another is taken as the sum of average loading time, average transit time between the representative ports, and average unloading time. This total is some fraction of a year, say x years, where $0 < x < 1$.

There is a further complication: In order to avoid contamination, white commodities cannot be shipped in tankers in which black commodities (crudes and vacuum distillate bottoms together with the other black products mentioned above) have just been shipped; black commodities, however, may be shipped in tankers that have just carried white commodities. If a transition from black to white is to be made, tanker cleaning is first required; cleaning ("butterworthing") of empty tankers may be done at sea or in port. In order to express the alternatives while using the smallest number of rows: (1) for each of the three tanker-accessible regions, a row of the matrix is assigned to empty "dirty" tankers (in which only black commodities may be shipped unless cleaning takes place), and a row is assigned to empty "clean" tankers (in which all commodities may be shipped); (2) dirty tankers traveling empty from any one region to another[22] are assumed always to be cleaned en route; (3) cleaning activities in each region[23] convert empty dirty tankers in the region into empty clean tankers in the region. In order to save some columns, a conversion (or "dirtying") activity is also defined for each region; this is a fictitious activity which instantaneously transforms a clean tanker in the region into a dirty one. Its inclusion means that we do not have to distinguish between the activity of sending a black commodity from one region to another using an originally clean tanker and the activity of sending the same commodity between the same two regions using an originally dirty tanker.

Sending a white (black) commodity by tanker from one region to another, then, requires, at unit level, one empty clean (dirty) tanker per year in the region of origin, an amount per year of #6 fuel oil in the region of origin sufficient for one trip between regions, one T-2 tanker load per year of the commodity shipped in the region of origin (the number of barrels in the tankerload depends on the density of the commodity and the length of the trip, a longer trip requiring more of the ship's tonnage capacity to be used for tanker fuel), *and* x *T-2 tankers, where* x *is defined* as above. The activity produces, at unit level, one tanker load per year of the commodity shipped in the region of termination and one empty clean (or dirty) tanker per year in the region of termination. It also involves intraregional transportation in the manner described in Section 4.3. Most of the numbers used in the interregional tanker shipment activities are included in Table 2.

5.3. *Interregional Tank Car Shipments.* The four regions are represented by four cities: Philadelphia, Houston, Chicago, and Los Angeles. Cleaning is again taken into account, chiefly in order to be able to examine the sensitivity of certain results to the inclusion of detail of this kind. Tank cars that have carried black commodities must be cleaned before they can carry white commodities; but "clean" tank cars can carry black commodities as well as white. Thus for each region there is an empty dirty tank car row and an empty clean tank car row. There is a cleaning activity in each region, which takes time (and is discussed in Section 13 below) and a conversion ("dirtying") activity which does not. (There is no cleaning of empties en route.)

Sending a white (black) commodity by tank car from one region to another requires, at unit level, one empty clean (dirty) tank car per year in the region of origin, an amount of diesel oil per year in the region of origin equal to the amount sufficient for transportation between the regions (or rather between

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[22] See Section 6 below.

[23] See Table 2.

their representative points) of one train of average length divided by the number of cars in such a train, one tank car load per year (the average tank car capacity is 209 barrels) of the commodity shipped in the region of origin, and y tank cars (where y is the fraction of a year required for loading of a tank car, shipment between the two representative cities on the average fast-freight schedule, and unloading of the tank car).[24] This activity produces, at unit level, one tank car load per year of the commodity in the region of termination and one empty clean (or dirty) tank car per year in the region of termination. Intraregional transportation also enters the activity, in the manner described in Section 4.3.

5.4. *Interregional Barge Shipments.* Three representative ports (all important interregional barge ports) were used for the three regions in which interregional barge movements may originate or terminate: New Orleans for the Gulf, Pittsburgh for the East, Chicago for the Midwest. Cleaning of barges is not allowed for in the model (the tanker and tank car cleaning activities are sufficient to test sensitivity to such detail; and, moreover, with the current distribution of barge flow with respect to black and white commodities barge cleaning seems seldom necessary for minimal barge use). Thus for each of the three regions there is only one "empty barge" row.

Shipping a commodity by barge from one of the three regions to another requires, at unit level, one empty barge a year in the region of origin, one barge load (the average barge capacity is 9,570 barrels) per year of the commodity in the region of origin, an amount of diesel fuel per year in the region of origin equal to the amount sufficient for one trip of the average barge tow from one representative port to the other divided by the number of barges in such a tow, and z barges (where z is the fraction of a year required for loading, unloading, and for the trip in question). This produces, at unit level, one barge load per year of the commodity in the region of termination and one empty barge per year in the region of termination. It also involves intraregional transportation in the manner described in Section 4.3.

5.5. *Interregional Movement of Empty Tankers, Tank Cars, and Barges.* For the model to select an optimal routing of empty vehicles in any application, it must permit the sending of any empty tanker, tank car, or barge from any region to any other region to which such movement is feasible.

Among the activities serving this purpose are the activities of sending empty clean tankers from one of the regions Gulf, East, or West to any other of these regions. This requires, at unit level, one empty clean tanker per year in the region of origin, sufficient #6 fuel oil per year in the region of origin to make one trip between the two relevant representative ports, and x^* tankers (where x^* is the fraction of a year required to make the trip). This produces, at unit level, one empty clean tanker per year in the region of termination. The remaining such activities are sending empty dirty tankers from one region (Gulf, East, West) to another and cleaning en route; sending empty clean (or dirty) tank cars from any one of the four regions to any other; and sending empty barges from one of the three regions East, Gulf, or Midwest to another.

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[24] See Table 2.

5.6. *Importing from Venezuela.* The importing of crude oil and of residual fuel oil (#6 fuel oil) from Venezuela is allowed for by the following two groups of activities:

1. An activity in the first group involves importing, at unit level, one million barrels a year of Venezuelan crude by tanker into the East, Gulf, or West, transshipping the crude by pipeline, barge, or tank car to another U. S. region (or else leaving it where the tanker brought it) and then atmospherically distilling it[25] in the final U. S. region of termination. This activity requires, at unit level, one million barrels a year of Venezuelan crude, appropriate numbers per year of empty dirty tankers in Venezuela, an appropriate number of tankers, and appropriate amounts of empty transportation equipment (other than tankers) per year in the U. S., of fuel for transportation in the U. S., and of transportation equipment other than tankers. The activity produces, at unit level, 994,000 barrels a year of distillation fractions in the U. S. region of final termination, as well as appropriate amounts of empty vehicles in the proper regions.
2. An activity in the second group involves importing, by tanker at unit level, one million barrels of Venezuelan residual fuel oil per year into East, Gulf, or West, and transshipping this amount to another U. S. region by some means of transportation (or else leaving it where the tanker brought it).

The possibility of Venezuelan imports requires that some additional empty-tanker-movement activities be added. Three activities are included in which empty dirty tankers are sent from Gulf, East, and West to Venezuela; and three in which they are sent from Venezuela to Gulf, East, or West.

5.7. *The Product Mix Activity.* The product mix activity requires, at unit level, an amount per year of each end item in each region and produces one unit a year of product mix. The proportions in which the regionally located end items occur in the product mix (jet fuel is excluded) are estimates of the proportions in which they were consumed in the period between July 1, 1952 and June 30, 1953. "Consumption" in a region is taken to equal refinery output plus imports (including imports from foreign countries) less exports *to other U. S. regions only.*[26]

To estimate these proportions required numerous sources and some adjustment to obtain end-item information for the regions in question from given information about somewhat different regions. The product mix obtained and the principal sources are given in Table 3.

5.8. *1952–53 Capacities.* Estimates were made of the regional equipment capacities, crude oil availabilities, and exogenous input availabilities, and the U. S. tanker, barge, and tank car fleets as of December 31, 1952. The size of the tanker fleet is perhaps the least meaningful of these magnitudes since (1)

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[25] The proportions in which the six fractions occur in Venezuelan crude were taken to be the proportions characterizing the crude of the principal Venezuelan field (Lagunillas). Source: A. E. Dunstan (ed.), *Science of Petroleum,* Vol. V, Oxford University Press, 1950, p. 21.

[26] U. S. exports of end items to foreign countries were roughly of the same volume as imports in 1952–53 (about 124,000,000 barrels, or 5% of total U. S. production).

tankers other than "U. S.-owned"[27] would presumably be at the disposal of the U. S. in certain situations; and (2) part of the U. S. fleet is currently engaged in hauling foreign petroleum between foreign ports and in some situations this task could not be abandoned.

Capacity estimates are given in Table 4 together with their sources.

5.9. *Transportation Times, Congestion, and the Model's Sensitivity to Transportation Coefficients.* The intraregional transportation analysis is a crude attempt, given the inadequate data and the small number of large regions in the model, to *allow* at least for the fact that intraregional transportation, as well as interregional transportation, requires transportation capacity. It contains many inaccuracies. The interregional transportation activities contains many inaccuracies as well.

These inaccuracies are not due simply to our use of points to represent regions in these activities. They arise also from our ignoring entirely the effects of *congestion* on transportation times. An "average time" required for the loading of a tanker, barge, or tank car at a given point, sending it to another point, and unloading it there is meaningful only if we are given the extent to which these loading and unloading facilities and the inland waterways and rails are used *by all other shippers.* Thus when we let a single transportation time characterize an interregional shipping activity, not only do we ignore the effects of congestion of such facilities in petroleum-shipping use when this activity is operated at a high level, but we assume, in effect, that all industries other than the petroleum industry will keep their demands on these facilities constant. The assumption is certainly a doubtful one in many national situations for which the model might be asked to estimate the industry's capabilities. On the other hand, there are virtually no data on which incorporation of a congestion effect into the model might be based.

We note finally that the inaccuracies inherent in using points to represent regions and in our analysis of intraregional transportation would be largely absent in a model containing not four regions but, say, thirty *points.* These points would be the principal U. S. refining centers, crude fields, and consumption centers (with the "tributary area" surrounding one of the latter regarded as concentrated at the center, petroleum transportation within the area being by tank truck and hence omitted from the model). *Distances* between the points would at least be accurately measurable, and the intraregional problem would not arise. The problem of rail congestion over these distances, and of congestion of other transportation facilities, would, however, remain.

5.10. *Remarks on Computation in a Typical Application of the Model.* A typical application of the model (and the first to be actually carried out) is maximization of the amount of 1952–53 product mix (i.e., of the level of the product mix activity) for 1952 capacities. The computer is asked, in other words, to maximize the level of the product mix activity subject to the nonnegativity of all activity levels and subject to the 195 equations in all the levels, corresponding to the 195 rows of the matrix. The 195 equations require

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[27] See Table 4.

that the following conditions be met. (1) The barrel-per-year refinery equipment capacities required for annual refinery operations in a given region must not exceed the capacities available. (2) The annual amount of each type of crude distilled and of each exogenous input used in each region must not exceed the regional (annual) availability of the crude or input plus the amount imported less the amount exported. (3) The annual amount of each end item entering the product mix in each region (and—for diesel and #6 fuel oil—the amount used to fuel interregionally bound and intraregionally moving vehicles) must not exceed the annual amount produced in the region plus the amount imported less the amount exported. (4) The annual amount of each intermediate material used up in each region must not exceed the amount produced plus the amount imported less the amount exported. (5) The annual amounts of intraregional crude and products transportation used in each region must not exceed the amounts made available. (6) The number of empty clean and dirty tankers, empty clean and dirty tank cars, and empty barges required per year in each region (and in Venezuela), for the purpose of sending empty tankers to other regions and for transport of commodities to other regions, must not exceed the number per year appearing in the region. (7) The annual imports of Venezuelan crude and residual fuel oil must not exceed the availabilities. (8) The number of tankers, tank cars, and barges required for all transportation (and cleaning) activities, interregional and intraregional, must not exceed the total nationally available. And (9) a crude or products movement from any region to any other by pipeline (in barrels per year) must not exceed the relevant barrels-per-year pipeline capacity.

Because of the large number of columns in the model, it is highly desirable to subdivide the columns into several smaller sets, as follows. The first set of columns is likely by itself to permit a level of the payoff to be maximized (e.g., the level of the product mix activity) quite close to the maximum required. Once maximization over the activity levels associated with the first set of columns is achieved, the second set is added and a further, but much smaller, increase in payoff may be expected. The addition of the third set permits a still smaller increase; and so on, until the possibility of increasing the payoff by changing the levels of all activities in the model has been examined.

Table 5 presents the coefficients of the matrix in a summary form (see pp. 116–122).

IV. Applications of the Spatial Model

1. INTRODUCTION. It proved possible to complete two applications of the spatial model: (1) maximization of the amount of spatial product mix (subject to the 1952–53 refining and transportation capacity restraints) in two stages—the first permitting only movements originating in the Gulf, and the second permitting all movements; (2) computing the maximum amount of spatial product mix attainable with two alternative tanker availabilities, both less than the 1952–53 availability.

2. MAXIMIZING THE AMOUNT OF SPATIAL PRODUCT MIX. With shipments from the Gulf only permitted—except for the critical commodity alkylate, whose shipment from all regions was allowed[28]—the maximum amount of spatial product mix attainable turned out to be 2311.3 units per year. In computing this result the subset of activities over which maximization was performed included initially those refining activities (in each region) which maximized product mix in the small nonspatial model, the commodity shipment activities originating in the Gulf, all shipment of empty vessels and tank cars, and all disposal activities.

Maximizing over this set of activities yielded 2295.4 as the maximum amount of product mix attainable. When all refining activities were permitted in each region, the maximum was 2311.3 and only one or two activities were altered in each region. The composition of each region's stock of refining equipment, and its commodity surpluses and deficits, thus appear to be such that operating the same set of processes in all regions obtains a near maximum of spatial product mix in the country as a whole. To obtain such a maximum, in other words, technology need not be "specialized" to particular regions.

Permitting shipment of commodities from all U. S. regions—but not from Venezuela—raised the attainable amount of spatial product mix to 2356.4. The principal shipments not originating in the Gulf which occur at this point are: shipment of 100–250° straight-run from East to Midwest, of Type 1 crude from West to Midwest, and of 550–725° cracked from East to Midwest. There are a number of small shipments not originating in the Gulf, including several which terminate in the Gulf. Otherwise the general pattern of movements is the one described in Section 1 of Part IV, and the well-known predominance of the Gulf as exporter of inputs and final products is given a measure: the proportion of the maximum attainable amount of spatial product mix which can be attained when the Gulf is the sole exporter. This proportion is 2311.3/2356.4 or about .98.

When the maximum is attained, there is still a great excess availability of tank cars—only ten and a half thousand out of 78,400 are needed—and of tankers as well—only 351 out of 756 are needed. There is excess crude and products pipeline capacity between East and Midwest. All other pipeline capacities, however, as well as the barge fleet, are completely utilized.

Making many more barges or more pipeline capacity available, however, would only negligibly increase the attainable amount of spatial product mix. The shadow price of a barge or of a unit of pipeline capacity is extremely small compared to the shadow price of a unit (in any region) of alkylate, the essential bottleneck, whose shadow price (in every region) is substantially higher than that of any other commodity. Transportation capacity, then, is

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[28] The availability of alkylate in a given region could easily become a bottleneck, since it is needed to produce avgas meeting the required specifications, and since there is some avgas in every region in a unit of the spatial product mix. The amount of alkylate required per unit of spatial product mix is small, however, relative to other inputs, and hence its transportation requirements per unit of product mix are small. It seems reasonable, therefore, to avoid at the start an alkylate bottleneck which could be broken by interregional shipment of alkylate.

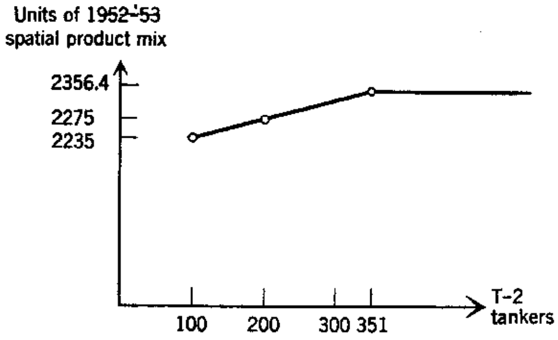

FIGURE 5

not, in the 1952–53 situation, the effective limit to the amount of spatial product mix attainable. The maximum amount—2365.4 units—is nearly the same (if we divide by 365 so that the time units are the same) as the maximum amount of product mix attainable in the nonspatial model for 1952–53 crude and equipment capacities. The difference can be accounted for by the spatial model's inclusion of the use of petroleum products in petroleum transportation.

Permitting shipment of crude and of residual fuel oil from Venezuela does not increase the amount of spatial product mix attainable. Evidently there is no chain of substitution by which such shipment can be made to relieve the alkylate bottleneck. This, one might have intuitively expected, but it requires computation to establish it.

3. REDUCING TANKER AVAILABILITY. The results of the sole parametric programming application which was performed—spatial product mix versus tanker availability—are as shown in Figure 5. The maximum amount of spatial product mix was computed for 200-tanker and 100-tanker availabilities. The previous computation, described above, yielded the maximum amount for tanker availabilities of 351 and greater. The three computed points are shown on the diagram, and linear interpolation yields the line segments connecting the first two points. Straightline interpolation between the 100-tanker point and the origin might clearly be grossly inaccurate and hence is omitted. Along the portion of the curve shown there is a substantial amount of substitution of tank-car and pipeline transportation for tanker transportation. At the 200-tanker point about 65,000 tank cars are used (as compared to 10,500 at the 351-tanker point), and there is no excess pipeline capacity; at the 100-tanker point all tank cars are used.[29] The World War II pattern of movements, in

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[29] At the 200-tanker level, the excess tank cars are not able to substitute completely for the tankers which have been taken away, because some of the activities which make available intraregional transportation (even the "replacement activities") require tankers. Intraregional transportation is, in effect, the new bottleneck—not present when the tanker fleet was left intact—which makes the maximum attainable amount of spatial product mix smaller; the excess tank cars cannot be used to break this bottleneck.

which the tanker availability for domestic transport was sharply curtailed, involved substitutions of this magnitude.

A point of some interest is that, at the 100-tanker level, the shadow prices of Gulf-Midwest crude and products pipeline capacity are nearly double, respectively, their values at the 200-tanker level, while the shadow prices of other pipeline capacities—including Gulf-West capacity (nonexistent in 1952–53)—remain about the same. The reason appears to be that additional Gulf-Midwest pipeline capacity can liberate tank cars, which are "free" at the 200-tanker level but not at the 100-tanker level. This illustrates that the relative "gain" to be obtained from an addition to one kind of transportation capacity may change quite sharply as the availability of other kinds of transportation alters by a small amount.

Finally, the relative shadow prices of the several types of refinery equipment in each region undergo only minor shifts as tanker availability is reduced. The normal bundle of equipment appears to be of no less appropriate composition when transportation severely constrains the model than when it is virtually "free."

4. CONCLUSION. The realism of the above quantitative results is, of course, open to serious question; sensitivity testing to determine the importance of errors in some of the transportation coefficients, which remains to be performed, might mitigate some of these doubts. The sole exception is the maximum attainable amount of 1952–53 spatial product mix for 1952–53 capacities, for here the historical figure (close, as we have seen, to the figure obtained for the nonspatial, and hence for the spatial, models) provides a check.

It is clear, on the other hand, that there are many interesting questions (including those which were answered in the computations just described) which can be put to the spatial model but not to the nonspatial model, whose answers, generally involving highly complex chains of substitution, cannot be estimated from "back-of-the-envelope" calculations. Some questions of this sort may be important enough to justify, in the future, the collecting and verifying of much additional data, and the expanding of the model, which unassailable realism would require. The present spatial model is only an exploratory beginning.

TABLE 1

THE SMALL NONSPATIAL MODEL

	Items (All capacities and product amounts are measured in units of one million bbls. per day)	Distillation				Conversion									
		Atmos. dist., type 1 crude	Atmos. dist., type 2 crude	Atmos. dist., type 3 crude	Vacuum dist.	Ther. cr. of 400–550 SR	Ther. cr. of 550–725 SR	Ther. cr. of 725+ SR	Visbreaking of vac. dist. bottoms	Ther. ref. of 250–325 SR (7 · % conv.)	Ther. ref. of 325–400 SR (95 % conv.)	Cat. cracking of 400–550 SR	Cat. cracking of 550–725 SR	Cat. cracking of vac. dist. 725+ gas oil	Cat. cracking of 400–550 cat. cracked (recycling)
	Activity Number}	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Crude and Equipment Availabilities	0. Product mix														
	1. Availability, type 1 crude	−1.00													
	2. Availability, type 2 crude		−1.00												
	3. Availability, type 3 crude			−1.00											
	4. Atmos. crude dist. capacity	−1.00	−1.00	−1.00											
	5. Vacuum distillation capacity				−1.00										
	6. Thermal operations capacity					−1.00	−1.00	−1.00	−1.	−1.00	−1.00				
	7. Catalytic cracking capacity											−1.00	−1.00	−1.00	−1.00
	8. Catalytic reforming capacity														
Intermediate Materials	9. 100–250 straight run	.138	.077	.049											
	10. 250–325 straight run	.090	.065	.041						−1.00					
	11. 325–400 straight run	.100	.067	.042							−1.00				
	12. 400–550 straight run (also #1 fuel oil)	.174	.218	.266		−1.00						−1.00			
	13. 550–725 straight run (also Diesel fuel oil)	.184	.256	.313			−1.00						−1.00		
	14. 725+ straight run	.308	.311	.283	−1.00			−1.00							
	15. Vac. distillate 725+ gas oil				.40									−1.00	
	16. Vac. distillate bottoms (also an end item)				.60				−1.00						
	17. 100–250 thermally cracked					.114	.101	.051	.030						
	18. 250–325 thermally cracked or reformed					.057	.050	.025	.015	.78					
	19. 325–400 thermally cracked or reformed					.057	.050	.026	.015		.95				
	20. 400–550 thermally cracked					.700	.042	.057							
	21. 550–725 cracked					.021	.700	.126					.50		
	22. 725+ cracked					.006	.012	.700						.50	
	23. 100–250 catalytically cracked											.24	.24	.24	.095
	24. 250–325 catalytically cracked or reformed											.079	.079	.079	.031
	25. 325–400 catalytically cracked or reformed											.064	.064	.064	.026
	26. 400–550 catalytically cracked											.50			−.25
Exogenous Inputs	27. Alkylate														
	28. Isopentane														
End Items Specs.	29. 115/145 avgas, PN rich														
	30. 100/130 avgas, PN rich														
	31. 100/130 avgas, PN lean														
	32. Motor gas, octane no.														
	33. Jet fuel, % aromatics														
End Item Requirements	34. 115/145 avgas														
	35. 100/130 avgas														
	36. Jet fuel														
	37. Motor gas														
	38. #2 fuel oil								−.31						
	39. #6 fuel oil								1.24						

TABLE 1 (*continued*)

THE SMALL NONSPATIAL MODEL

Conversion (*continued*) ← End Item Blending →

	Items (All capacities and product amounts are measured in units of one million bbls. per day)	Cat. cracking of 500–725 cat. cracked (recycling)	Cat. ref. of 250–325 SR	Cat. ref. of 325–400 SR	115/145 avgas "pre-blend" (1)	115/145 avgas "pre-blend" (2)	115/145 avgas "pre-blend" (3)	115/145 avgas "pre-blend" (4)	115/145 avgas "pre-blend" (5)	115/145 avgas "pre-blend" (6)	100/130 avgas "pre-blend" (1)	100/300 avgas "pre-blend" (2)	100/130 avgas "pre-blend" (3)	100/130 avgas "pre-blend" (4)	100/130 avgas "pre-blend" (5)	100/130 avgas "pre-blend" (6)
	Activity Number	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Crude and Equipment Availabilities	0. Product mix															
	1. Availability, type 1 crude															
	2. Availability, type 2 crude															
	3. Availability, type 3 crude															
	4. Atmos. crude dist. capacity															
	5. Vacuum distillation capacity															
	6. Thermal operations capacity															
	7. Catalytic cracking capacity	−1.00														
	8. Catalytic reforming capacity		−1.00	−1.00												
Intermediate Materials	9. 100–250 straight run															
	10. 250–325 straight run		−1.00													
	11. 325–400 straight run			−1.00												
	12. 400–550 straight run (also #1 fuel oil)															
	13. 550–725 straight run (also Diesel fuel oil)															
	14. 725+ straight run															
	15. Vac. distillate 725+ gas oil															
	16. Vac. distillate bottoms (also an end item)															
	17. 100–250 thermally cracked					−.625			−.57			−.625			−.57	
	18. 250–325 thermally cracked or reformed															
	19. 325–400 thermally cracked or reformed															
	20. 400–550 thermally cracked															
	21. 550–725 cracked	−.25														
	22. 725+ cracked															
	23. 100–250 catalytically cracked	.095					−.625			−.57			−.625		−.57	
	24. 250–325 catalytically cracked or reformed	.031	.91		−.75	−.375	−.375				−.75	−.375	−.375			
	25. 325–400 catalytically cracked or reformed	.026		.95												
Exogenous Inputs	26. 400–550 catalytically cracked															
	27. Alkylate							−.79	−.43	−.43				−.79	−.43	−.43
	28. Isopentane				−.25			−.21			−.25			−.21		
End Items Specs.	29. 115/145 avgas, PN rich				−15.6	−36.25	−16.1	.52	−25.7	7.3						
	30. 100/130 avgas, PN rich										−.6	−21.25	−1.1	15.5	−10.7	7.7
	31. 100/130 avgas, PN lean										−.85	−15.25	−8.13	23.1	−.94	5.56
	32. Motor gas, octane no.															
	33. Jet fuel, % aromatics															
End Item Requirements	34. 115/145 avgas				1.00	1.00	1.00	1.00	1.00	1.00						
	35. 100/130 avgas										1.00	1.00	1.00	1.00	1.00	1.00
	36. Jet fuel															
	37. Motor gas															
	38. #2 fuel oil															
	39. #6 fuel oil															

TABLE 1 (*continued*)

THE SMALL NONSPATIAL MODEL

	Items (All capacities and product amounts are measured in units of one million bbls. per day)	End Item Blending (*continued*)												
		Motor gas, component 9	Motor gas, component 10	Motor gas, component 11	Motor gas, component 17	Motor gas, component 18	Motor gas, component 19	Motor gas, component 23	Motor gas, component 24	Motor gas, component 25	Motor gas, component 27	Motor gas, component 29	"Removal" of TEL, 0–1.5 cc/gal.	"Removal" of TEL, 1.5–3.0 cc/gal.
	Activity Number	30	31	32	33	34	35	36	37	38	39	40	41	42
Crude and Equipment Availabilities	0. Product mix													
	1. Availability, type 1 crude													
	2. Availability, type 2 crude													
	3. Availability, type 3 crude													
	4. Atmos. crude dist. capacity													
	5. Vacuum distillation capacity													
	6. Thermal operations capacity													
	7. Catalytic cracking capacity													
	8. Catalytic reforming capacity	−1.00												
Intermediate Materials	9. 100–250 straight run													
	10. 250–325 straight run		−1.00											
	11. 325–400 straight run			−1.00										
	12. 400–550 straight run (also #1 fuel oil)													
	13. 550–725 straight run (also Diesel fuel oil)													
	14. 725+ straight run													
	15. Vac. distillate 725+ gas oil													
	16. Vac. distillate bottoms (also an end item)													
	17. 100–250 thermally cracked				−1.00									
	18. 250–325 thermally cracked or reformed					−1.00								
	19. 325–400 thermally cracked or reformed						−1.00							
	20. 400–550 thermally cracked													
	21. 550–725 cracked													
	22. 725+ cracked													
	23. 100–250 catalytically cracked							−1.00						
	24. 250–325 catalytically cracked or reformed								−1.00					
	25. 325–400 catalytically cracked or reformed									−1.00				
	26. 400–550 catalytically cracked													
Exogenous Inputs	27. Alkylate										−1.00			
	28. Isopentane											−1.00		
End Items Specs.	29. 115/145 avgas, PN rich													
	30. 100/130 avgas, PN rich	−2.00												
	31. 100/130 avgas, PN lean													
	32. Motor gas, octane no.		−33.40	−33.40	5.00	−4.00	−23.00	9.00	4.00	−4.00	18.00	18.00	−8.00	−3.00
	33. Jet fuel, % aromatics													
End Item Requirements	34. 115/145 avgas													
	35. 100/130 avgas													
	36. Jet fuel	1.00												
	37. Motor gas		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		
	38. #2 fuel oil													
	39. #6 fuel oil													

TABLE 1 (*continued*)

THE SMALL NONSPATIAL MODEL

	Items (All capacities and product amounts are measured in units of one million bbls. per day)	End Item Blending (*continued*) Jet fuel, component 9	Jet fuel, component 10	Jet fuel, component 11	Jet fuel, component 12	Jet fuel, component 17	Jet fuel, component 18	Jet fuel, component 19	Jet fuel, component 20	Jet fuel, component 23	Jet fuel, component 24	Jet fuel, component 25	Jet fuel, component 26	#2 fuel oil, "pre-blend"	#2 fuel oil, "pre-blend"	#2 fuel oil, "pre-blend"
	Activity Number	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57
Crude and Equipment Availabilities	0. Product mix															
	1. Availability, type 1 crude															
	2. Availability, type 2 crude															
	3. Availability, type 3 crude															
	4. Atmos. crude dist. capacity															
	5. Vacuum distillation capacity															
	6. Thermal operations capacity															
	7. Catalytic cracking capacity															
	8. Catalytic reforming capacity															
Intermediate Materials	9. 100–250 straight run	−1.00														
	10. 250–325 straight run		−1.00													
	11. 325–400 straight run			−1.00												
	12. 400–550 straight run (also #1 fuel oil)				−1.00									−1.00	−.579	
	13. 550–725 straight run (also Diesel fuel oil)														−.421	−.421
	14. 725+ straight run															
	15. Vac. distillate 725+ gas oil															
	16. Vac. distillate bottoms (also an end item)															
	17. 100–250 thermally cracked					−1.00										
	18. 250–325 thermally cracked or reformed						−1.00									
	19. 325–400 thermally cracked or reformed							−1.00								
	20. 400–550 thermally cracked								−1.00							−.579
	21. 550–725 cracked															
	22. 725+ cracked															
	23. 100–250 catalytically cracked									−1.00						
	24. 250–325 catalytically cracked or reformed										−1.00					
	25. 325–400 catalytically cracked or reformed											−1.00				
	26. 400–550 catalytically cracked												−1.00			
Exogenous Inputs	27. Alkylate															
	28. Isopentane															
End Items Specs.	29. 115/145 avgas, PN rich															
	30. 100/130 avgas, PN rich															
	31. 100/130 avgas, PN lean															
	32. Motor gas, octane no.															
	33. Jet fuel, % aromatics	21.00	7.60	13.00	18.30	18.50	10.00	5.00	−11.00	18.50	−11.00	−46.00	−46.00			
End Item Requirements	34. 115/145 avgas															
	35. 100/130 avgas															
	36. Jet fuel	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00			
	37. Motor gas															
	38. #2 fuel oil													1.00	1.00	1.00
	39. #6 fuel oil															

TABLE 1 (*continued*)

THE SMALL NONSPATIAL MODEL

	Items (All capacities and product amounts are measured in units of one million bbls. per day)	End Item Blending (*continued*)															
		#2 fuel oil, "pre-blend"	#2 fuel oil, "pre-blend"	#2 fuel oil, "pre-blend"	#2 fuel oil, "pre-blend"	#2 fuel oil, "pre-blend"	#2 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"
	Activity Number	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73
Crude and Equipment Availabilities	0. Product mix																
	1. Availability, type 1 crude																
	2. Availability, type 2 crude																
	3. Availability, type 3 crude																
	4. Atmos. crude dist. capacity																
	5. Vacuum distillation capacity																
	6. Thermal operations capacity																
	7. Catalytic cracking capacity																
	8. Catalytic reforming capacity																
Intermediate Materials	9. 100–250 straight run																
	10. 250–325 straight run																
	11. 325–400 straight run										−.014						−.182
	12. 400–550 straight run (also #1 fuel oil)		−.651														
	13. 550–725 straight run (also Diesel fuel oil)	−.421						−.029	−.966					−.329			
	14. 725+ straight run									−.979	−.986	−.986	−.986				
	15. Vac. distillate 725+ gas oil																
	16. Vac. distillate bottoms (also an end item)																
	17. 100–250 thermally cracked																
	18. 250–325 thermally cracked or reformed																
	19. 325–400 thermally cracked or reformed											−.014					
	20. 400–550 thermally cracked			−.651			−1.00										
	21. 550–725 cracked		−.349	−.349	−.349				−.034						−.361		
	22. 725+ cracked							−.971						−.671	−.639	−.737	−.818
	23. 100–250 catalytically cracked																
	24. 250–325 catalytically cracked or reformed																
	25. 325–400 catalytically cracked or reformed												−.014				
	26. 400–550 catalytically cracked	−.579			−.651	−1.00											
Exogenous Inputs	27. Alkylate																
	28. Isopentane																
End Items Specs.	29. 115/145 avgas, PN rich																
	30. 100/130 avgas, PN rich																
	31. 100/130 avgas, PN lean																
	32. Motor gas, octane no.																
	33. Jet fuel, % aromatics																
End Item Requirements	34. 115/145 avgas																
	35. 100/130 avgas																
	36. Jet fuel																
	37. Motor gas																
	38. #2 fuel oil	1.00	1.00	1.00	1.00	1.00	1.00			−.021						−.263	
	39. #6 fuel oil							1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 1 (*continued*)

THE SMALL NONSPATIAL MODEL

	Items (All capacities and product amounts are measured in units of one million bbls. per day)	← End Item Blending (*continued*) →									← Excess Availabilities →						
		#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	#6 fuel oil, "pre-blend"	Product mix (exclusive of jet fuel)	Excess availability of crude 1	Excess availability of crude 2	Excess availability of crude 3	Excess cap. of atmos. dist.	(Other excess vectors)	Excess #2 fuel oil	Excess #6 fuel oil
	Activity Number	74	75	76	77	78	79	80	81	100	101	102	103	104		142	143
Crude and Equipment Availabilities	0. Product mix									1.000							
	1. Availability, type 1 crude										−1.00						
	2. Availability, type 2 crude											−1.00					
	3. Availability, type 3 crude												−1.00				
	4. Atmos. crude dist. capacity													−1.00			
	5. Vacuum distillation capacity																
	6. Thermal operations capacity																
	7. Catalytic cracking capacity																
	8. Catalytic reforming capacity																
Intermediate Materials	9. 100–250 straight run																
	10. 250–325 straight run																
	11. 325–400 straight run						−1.00										
	12. 400–550 straight run (also #1 fuel oil)									−.0513							
	13. 550–725 straight run (also Diesel fuel oil)			−1.00						−.0355							
	14. 725+ straight run																
	15. Vac. distillate 725+ gas oil																
	16. Vac. distillate bottoms (also an end item)									−.0502							
	17. 100–250 thermally cracked																
	18. 250–325 thermally cracked or reformed																
	19. 325–400 thermally cracked or reformed	−.182						−1.00									
	20. 400–550 thermally cracked																
	21. 550–725 cracked				−1.00												
	22. 725+ cracked	−.818	−.818														
	23. 100–250 catalytically cracked																
	24. 250–325 catalytically cracked or reformed																
	25. 325–400 catalytically cracked or reformed		−.182						−1.00								
	26. 400–550 catalytically cracked																
Exogenous Inputs	27. Alkylate																
	28. Isopentane																
End Items Specs.	29. 115/145 avgas, PN rich																
	30. 100/130 avgas, PN rich																
	31. 100/130 avgas, PN lean																
	32. Motor gas, octane no.																
	33. Jet fuel, % aromatics																
End Item Requirements	34. 115/145 avgas									−.0145							
	35. 100/130 avgas									−.0145							
	36. Jet fuel																
	37. Motor gas									−.4495							
	38. #2 fuel oil					−1.00				−.1773						−1.00	
	39. #6 fuel oil	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	−.1840							−1.00

TABLE 2

Transportation Data Used in the Model

(Abbreviations: G for Gulf, E for East, M for Midwest, W for West, V for Venezuela)

Tanker In-Transit, Times between:

	E and G	E and W	G and W	V and E	V and G	V and W
(Years)	.01466	.03858	.03502	.01453	.01595	.02959

Philadelphia is used to represent E, Galveston (port for Houston) G, Los Angeles W, and Puerto Cabello V. The average speed of 14.2 knots is the average of "designed" tanker speeds for U. S.-owned fleet as of January 1, 1952, according to PAD study, *Transportation of Oil.* Distances obtained from *Table of Distances Between Ports,* prepared by Hydrographic Office, U. S. Navy Department.

Barge In-Transit, Times from:

	G to M	M to G (downstream)	G to E	E to G (downstream)	E to M and M to E
(Years)	.027	.014	.027	.016	.022

New Orleans is used to represent G, Pittsburgh E, and Chicago M. *Source:* PAD study, *Transportation of Oil.*

Tank Car In-Transit (i.e., departure-to-arrival), Times between:

	E and G	E and M	M and G	E and W	M and W	G and W
(Years)	.01918	.006849	.01370	.02603	.01918	.008219

Philadelphia is used to represent E, Houston G, Chicago M, and Los Angeles W. Times are "typical" fast-freight times as obtained from Southern Pacific, New York Central, and Pennsylvania Railroads.

Loading and Unloading Times ("typical" times)

Barge (9570-bbl.): 26 hours or .003 years for loading and same for unloading (*source:* PAD study)

Tanker (T-2, capacity 16,750 bbls.): 48 hours or .00548 years (*source:* Union Oil Company of California)

Tank car (209-bbl.), when in train of typical length: 30 hours or .0034 years (*source:* Union Oil Company of California)

Fuel Consumption

Bbls. of diesel fuel for barge trip between:

E and G	G and M	M and E
300 upstream 156 downstream	300 upstream 178 downstream	250

(*Source:* PAD Study)

Bbls. of diesel fuel per tank-car for trip between:

E and G	G and M	M and E	E and W	M and W	G and W
3	2	1	5	4	3

(*Source:* ICC, *Railway Statistics, 1950;* various tables were used to estimate fuel consumption per tank-car mile in diesel-powered trains.)

Bbls. of #6 fuel oil for tanker trip between:

E and G	G and W	E and W	V and E	V and G	V and W
3274	3887	4336	1491	1637	3240

(*Assumption:* 300 bbls. used per day, a figure obtained from Union Oil Company of California as typical for a T-2)

Cleaning times:		
	T-2 tankers in port	.0037 years
	Tank cars	.00137 years

(*Source:* Union Oil Company of California)

TABLE 3

THE 1952–53 SPATIAL PRODUCT MIX

One unit of product mix is composed of end items (jet fuel is excluded) in the following proportions:

	In East	In Midwest	In Gulf	In West
Motor gas	.17844	.13169	.06742	.07192
100/130 avgas	.00565	.00232	.00392	.00261
115/145 avgas	.00565	.00232	.00392	.00261
#1 fuel oil	.02516	.01130	.01438	.00500
Diesel fuel oil	.08750	.01009	.00640	.00842
#2 fuel oil	.09361	.04734	.01277	.02358
#6 fuel oil	.09790	.02264	.02153	.04196
Vacuum distillate bottoms	.01808	.01256	.00955	.01004

These proportions are obtained by taking the proportion in which each end item occurs in the national 1952–53 product mix (Table 1, column 100), adjusting the national diesel fuel oil and #6 fuel oil proportions downward *to take account of the use of these items in petroleum transportation* (so that an end item stream is a stream emerging from the *combined* petroleum production, refining, *and* transportation industry), and allocating each of these total national proportions over the regions. The allocation is made according to estimated 1952–53 consumption in each region. The principal data sources (and methods) used for estimating the consumption of each end item in each region were as follows:

Motor gas: Figures on consumption of "gasoline" by states in the 1952 and 1953 *Minerals Yearbooks* were used. "Gasoline," however, here includes avgas and certain blending stocks subsequently used in jet fuel. Estimated regional use of "gasoline" in jet fuel (total national proportion used was obtained from various Bureau of Mines *Monthly Petroleum Statements* and this was allocated over the regions) and estimated avgas consumption in each region (see below) were subtracted from the "gasoline" consumption figure to obtain estimated regional motor gas consumption in 1952 and 1953. For each region half of the 1952 total was added to half of the 1953 total to obtain estimated 1952–53 regional motor gas consumption and hence to obtain the proportions required for allocation of motor gas over regions in the product mix.

Avgas: Avgas consumption in each region was assumed split evenly (as in the nonspatial product mix) between 100/130 and 115/145 avgas. Total 1952–53 civilian consumption of avgas in each of the regions was estimated as follows: (1) Bureau of Public Roads publications *Highway Statistics*, 1952 and 1953, contain total sales of avgas to civilian users for states taxing such sales and reporting tax receipts; (2) the differences between the total 1952 and 1953 sales over these states and (larger) total 1952 and 1953 national sales-to-civilian-users figures obtained from the Ethyl Corporation were assumed attributable to the states not reported in the Bureau of Public Roads tables; (3) the 1952 and 1953 differences were each allocated between these nonreporting states in accordance with the annual number of aircraft departures of certificated carriers in these states, as given in a Civil Aeronautics Administration publication, *Enplaned Airline Traffic by Community, Calendar Year 1952*. Estimates of regional military avgas consumption for 1952–53 were then made and these were added to the 1952–53 regional civilian consumption estimates (obtained by adding half of the 1952 totals to half of the 1953 totals); and this yielded the required estimated regional allocation of total avgas consumption.

#1 fuel oil: This item is equivalent to the Bureau of Mines category "kerosine" less the kerosine blended into jet fuel. The required consumption allocation was obtained using 1952 and 1953 *Minerals Yearbook* tables of "sales of kerosine by states."

Diesel fuel oil: Railroads and inland vessels (barges) are the dominant consumers of diesel fuel oil. Diesel fuel oil is included in the Bureau of Mines category "distillate fuel oil"; the *Minerals Yearbook* gives national sales of distillate fuel oil (presumably all diesel fuel oil) to railroads and to inland vessels. The regional consumption by railroads was estimated using ICC diesel-locomotive-mile data for regions different than those of this study and making rough adjustments. From each estimated regional railroad-use figure an amount was subtracted which is roughly estimated (in accordance with ICC waybill data) to be the amount used in railroad transportation of petroleum products. One-third of the national consumption by inland vessels is assumed to be used in barge transportation of petroleum products and this third is allocated between the four regions in accordance with the data described in Appendix II. The remaining two-thirds is allocated between regions in accordance with analogous data dealing with cargoes other than petroleum. Summing for each region, an estimate of the proportion of national diesel fuel oil consumption (in use other than petroleum transportation) attributable to the region is obtained for 1952 and 1953, and hence for 1952–53.

#2 fuel oil: This item is principally heating oil and is equivalent to the Bureau of Mines category "distillate fuel oil" *less* diesel fuel oil. The allocation over regions is obtained from the *Minerals Yearbook* table "sales of distillate fuel oil by states" after subtraction of the estimates of regional diesel fuel oil consumption (in all uses, including petroleum transportation) described above.

#6 fuel oil: This item is the sum of the Bureau of Mines categories "residual fuel oil" and "road oil" (national use of the former is about sixty times as large as of the latter). Sales of residual fuel oil in each region are obtainable from *Minerals Yearbook* tables; consumption of road oil is assumed to have the same regional distribution as consumption of asphalt (see below). The amount of residual fuel oil in each region used to fuel intraregionally moving tankers and tankers bound for other regions in 1952–53 was estimated using the source of tanker movements data cited in Appendix II. After subtraction, the required estimates of the proportions of national #6 fuel oil consumption (other than that used in petroleum transportation) attributable to the four regions were obtained.

Vacuum distillate bottoms: This item is equivalent to the sum of the two Bureau of Mines categories "lubricants" and "asphalt." Regional consumption estimates were obtained using (1) production figures from the *Minerals Yearbook* and (2) interregional tanker and tank-car movements of these commodities from the ICC and Army Engineers sources cited in Appendix II.

TABLE 4

ESTIMATED REGIONAL CAPACITIES AND AVAILABILITIES, AND NATIONAL TRANSPORTATION EQUIPMENT AVAILABILITIES, AS OF END OF 1952

(Unit: millions of bbls. a year)

	East	Midwest	Gulf	West	Total U. S.
Availability of type 1 crude	19.616	211.224	1246.422	513.166	1990.428
Availability of type 2 crude	1.080	24.598	291.119	32.562	349.359
Availability of type 3 crude		3.217	380.155	1.603	384.975
Alkylate availability	17.7	14.1	23.5	8.6	63.9
Isopentane availability	1.095	1.278	14.235	1.642	18.250
Natural gasoline availability	2.241	4.070	55.411	10.148	71.870
C_4 polymer and butane availability	3.170	3.699	41.216	4.756	52.841
Motor gas "excess-octane-barrels" which are made available to model	43.614	32.640	409.422	24.720	510.396
Atmospheric distillation capacity	554.567	516.515	1098.091	489.852	2659.025
Vacuum distillation capacity	78.610	67.595	126.975	64.445	337.625
Thermal operations capacity	138.709	133.291	322.108	163.632	757.740
Catalytic cracking capacity	220.256	175.435	399.214	106.730	901.635
Catalytic reforming capacity (after subtracting 19% of each region's capacity to allow for BTX production)	23.733	41.207	11.816	6.610	83.366

Venezuelan crude (millions of bbls. a year): 119

Venezuelan residual fuel oil (millions of bbls. a year): 128

U. S.-owned tanker fleet, all flags (excluding Navy tankers): 756.4 T-2 tankers (capacity 16,750 bbls.)

U. S. tank-car fleet: 78,445 209-bbl. tank cars

U. S. barge fleet (except for barges used in West): 2,175 9,570-bbl. barges

Crude pipeline capacity, in millions of barrels per year, between
Gulf and East: 75.920
Gulf and Midwest: 485.085
East and Midwest: 262.070

Products pipeline capacity, in millions of barrels a year, between
Gulf and East: 98.550
Gulf and Midwest: 150.015
East and Midwest: 80.365

Sources of Estimates

Crude availabilities: To obtain total annual crude availability for each state, its 1952 crude production (as obtained from *Minerals Yearbook*) is multiplied by a number R; R is U. S. daily crude availability in January, 1953, according to a National Petroleum Council Statement, divided by U. S. daily crude production in January, 1953. R equals 1.170 for "Southwest" states, 1.048 for "Mid-continent" states, 1.165 for California, and 1.000 for "East Coast" states. Harold M. Smith, in "Characteristics of Crude Oils Currently Produced in the United States," *Oil and Gas Journal*, March 1951, gives the proportions in which crudes characterized by various gas oil intervals (each such interval corresponding to one of our three crude types) occur in each of these groups of states. For each state in each of our four regions (which do not coincide with Smith's regions), the availability of each of our three crude types can then be estimated using Smith's percentages. Summing over the states in each of our four regions for each of the three crude types then yields the required estimates of crude availabilities.

Alkylate availabilities: Each region's alkylation capacity and alkylate availability is assumed to be in the same proportion to its catalytic cracking capacity as is the case for the Gulf Coast. The proportion for the Gulf Coast is reported in *Oil and Gas Journal,* June 15, 1952. Total U. S. capacity is taken from Manne (Ch. 4, Table 2).

Isopentane and natural gasoline availabilities: Source: Bureau of Mines, *Monthly Natural Gasoline Staetments.*

C_4 *polymer and butane availabilities:* Assumed to have same distribution over regions as isopentane.

"Excess octane-barrels" made available to model: Procedure described Section 5 of Part II is used.

Refinery equipment capacities: Source: *Oil and Gas Journal,* March 23, 1953.

Venezuelan crude and residual fuel oil: Availabilities taken as equal to 1952 imports. Source: *Minerals Yearbook.*

Tanker fleet: Source: Petroleum Administration for Defense study, *Transportation of Oil,* which gives fleet as of end of 1951. Assumptions: (1) scrappage equaled new construction in 1952, and (2) the average tanker is idle 16 days a year for necessary maintenance and repairs (an estimate contained in PAD study).

Tank-car fleet: Source: PAD Study. 209 bbls. is average capacity.

Barge fleet: Source: PAD study. 9,570 bbls. is average capacity of non-Western barges.

Crude pipeline capacities: Source: Unpublished study prepared by Rice Institute.

Products pipeline capacities: Source: personal communication from Oil and Gas Division, Department of Interior.

TABLE 5

A SUMMARY OF THE SPATIAL PETROLEUM REFINING MATRIX

(Rows and columns are listed by code number. *Abbreviations:* E, East; G, Gulf; M, Midwest; W, West. Unit is one million bbls./yr. for all crudes, intermediate materials, end items, exogenous inputs. Units for transportation items are shown below.)

Rows

Code Number	Description
000	Spatial product mix
101–139	Rows 1-39 of Table 1 for the East
201–239	“ “ “ “ “ “ “ Midwest
301–339	“ “ “ “ “ “ “ Gulf
401–439	“ “ “ “ “ “ “ West
001	Empty clean tankers, E (numbers per year)
002	“ “ “ , G “ “ “
003	“ “ “ , W “ “ “
004	“ dirty “ , E “ “ “
005	“ “ “ , G “ “ “
006	“ “ “ , W “ “ “
007	Empty clean tank cars, E “ “ “
008	“ “ “ , M “ “ “
009	“ “ “ , G “ “ “
010	“ “ “ , W “ “ “
011	“ dirty “ , E “ “ “
012	“ “ “ , M “ “ “
013	“ “ “ , G “ “ “
014	“ “ “ , W “ “ “
015	Empty barges, E “ “ “
016	“ “ , M “ “ “
017	“ “ , G “ “ “
018	Intraregional crude transportation (ICT), E (units per yr.)
019	Intraregional products transportation (IPT), E “ “ “
020	ICT, M “ “ “
021	IPT, M “ “ “
022	ICT, G “ “ “
023	IPT, G “ “ “
024	ICT, W “ “ “
025	IPT, W “ “ “
026	Tankers (number)
027	Tank cars “
028	Barges “
029	Crude pipeline capacity between E and M (million bbls./yr.)
030	Same between E and G “ “ “
031	Same between G and M “ “ “
032	Same between G and W “ “ “
033	Products pipeline capacity between E and M “ “ “
034	Same between E and G “ “ “
035	Same between G and M “ “ “
036	Same between G and W “ “ “
037	Venezuelan crude “ “ “
038	Venezuelan residual fuel oil “ “ “
039	Empty dirty tankers in Venezuela (numbers per yr.)

TABLE 5 (*continued*)

A SUMMARY OF THE SPATIAL PETROLEUM REFINING MATRIX

COLUMNS

(When it is convenient to do so the digit 1 identifies E, 2 identifies M, 3 identifies G, 4 identifies W, 5 identifies tank cars, 6 identifies tankers, 7 identifies barges, 8 identifies pipeline.)

(1) Activity Description	(2) Column Code	(3) Rows with Nonzero Coefficients (The coefficient generally follows the row number and is in parentheses; omitted coefficients can be found in Tables 1 or 3)
Refining activities: columns 1–81 of Table 1 for region j (j = 1, 2, 3, 4)	00j01–00j81 (j = 1, 2, 3, 4)	j01–j39
	Within the group 00j01–00j81, the following columns have additional nonzero coefficients, not shown in Table 1:	
	00101	018(−1),019(−.994)
	00102	" , "
	00103	" , "
	00105	019(.046)
	00106	" (.045)
	00107	" (.015)
	00108	" (.001)
	00109	" (.220)
	00110	" (.005)
	00111	" (.117)
	00112	" (.117)
	00113	" (.117)
	00114	" (.098)
	00115	" (.098)
	00116	" (.09)
	00117	" (.05)
	00j01–j03, 00j05–j17 (j = 2, 3, 4)	same as for preceding group except 2(j − 1) is added to row number
Cleaning of tank cars		
In E	01500	007,011,027
In M	02500	008,012,027
In G	03500	009,013,027
In W	04500	010,014,027

TABLE 5 (*continued*)

A SUMMARY OF THE SPATIAL PETROLEUM REFINING MATRIX

(1)	(2)	(3)
Conversion ("dirtying") of tank cars		
In E	11500	007(−1),011(1)
In M	12500	008(−1),012(1)
In G	13500	009(−1),013(1)
In W	14500	010(−1),014(1)
Cleaning of tankers		
In E	01600	001(1),004(−1),026
In G	02600	002(1),005(−1),026
In W	04600	003(1),006(−1),026
Conversion ("dirtying") of tankers		
In E	11600	001(−1),004(1)
In G	12600	002(−1),005(1)
In W	14600	003(−1),006(1)
Shipping clean empty tank cars		
E to M	05120	007(−1),008(1),027,113
E to G	05130	007(−1),009(1),027,113
E to W	05140	007(−1),010(1),027,113
M to E	05210	007(1),008(−1),027,213
M to G	05230	008(−1),009(1),027,213
M to W	05240	008(−1),010(1),027,213
G to E	05310	007(1),009(−1),027,313
G to M	05320	008(1),009(−1),027,313
G to W	05340	009(−1),010(1),027,313
W to E	05410	007(1),010(−1),027,413
W to M	05420	008(1),010(−1),027,413
W to G	05430	009(1),010(−1),027,413
Shipping empty dirty tank cars (12 activities)	same as above twelve codes except last digit is 1	same as for preceding twelve columns except 4 is added to the first two row numbers
Shipping empty clean tankers		
E to G	06131	002(1),004(−1),026,139
E to W	06141	003(1),004(−1),026,139
G to E	06310	001(1),002(−1),026,339
G to W	06340	002(−1),003(1),026,339
W to E	06410	001(1),003(−1),026,439
W to G	06430	002(1),003(−1),026,439
Shipping empty dirty tankers and cleaning en route		
E to G	06131	002(1),004(−1),026,139
E to W	06141	003(1),004(−1),026,139
G to E	06311	001(1),005(−1),026,339

TABLE 5 (*continued*)

A SUMMARY OF THE SPATIAL PETROLEUM REFINING MATRIX

(1)	(2)	(3)
G to W	06341	003(1),005(−1),026,339
W to E	06411	001(1),006(−1),026,439
W to G	06431	002(1),006(−1),026,439
Shipping empty barges		
E to M	07120	015(−1),016(1),028,113
E to G	07130	015(−1),017(1),028,113
M to E	07210	015(1),016(−1),028,213
M to G	07230	016(−1),017(1),028,213
G to E	07310	015(1),017(−1),028,313
G to M	07320	016(1),017(−1),028,313
Interregional Commodity Shipment	2nd and 3rd digits of code identify row of Table 1 which corresponds to commodity shipped	
Shipping crude and black products by tank car	the pair of digits *ij* takes the values 01, 02, 03, 13, 14, 15, 16, 22, 39	
E to M	1*ij*25	007(−1),008(1),027,1*ij*(−.000209), 2*ij*(.000209),113
E to G	1*ij*35	007(−1),010(1),027,1*ij*(−.000209), 4*ij*(.000209),113
E to W	1*ij*45	007(−1),010(1),027,1*ij*(−.000209), 4*ij*(.000209),113
M to E	2*ij*15	007(1),008(−1),027,1*ij*(.000209), 2*ij*(−.000209),213
M to G	2*ij*35	008(−1),009(1),027,2*ij*(−.000209), 3*ij*(.000209),213
M to W	2*ij*45	008(−1),009(1),027,2*ij*(−.000209), 4*ij*(.000209),213
G to E	3*ij*15	007(1),009(−1),027,1*ij*(.000209), 3*ij*(−.000209),313
G to M	3*ij*25	008(1),009(−1),027,2*ij*(.000209), 3*ij*(−.000209),313
G to W	3*ij*45	009(−1),010(1),027,3*ij*(−.000209), 4*ij*(.000209),313
W to E	4*ij*15	007(1),010(−1),027,1*ij*(.000209), 4*ij*(−.000209),413
W to M	4*ij*25	008(1),010(−1),027,2*ij*(.000209), 4*ij*(−.000209),413
W to G	4*ij*35	009(1),010(−1),027,3*ij*(.000209), 4*ij*(−.000209),414
		In addition, Note 1 (end of table) applies for $C = .000209$.

TABLE 5 (*continued*)

A SUMMARY OF THE SPATIAL PETROLEUM REFINING MATRIX

(1)	(2)	(3)
Shipping white products by tank car	same eight column codes as before except that ij now takes the values 09, 10, 11, 12, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 34, 35, 36, 37, 38, 39	same as for preceding group except that 4 is subtracted from the first two row numbers. Note 1 now applies for all ij except $ij = 27, 28$ and for $C = .000209$.
Shipping crude and black products by tanker	the pair ij takes the first set of values given above	
E to G	$1ij36$	004(−1),005(1),026,$1ij$(−.016750), $3ij$(.016750),139
E to W	$1ij26$	004(−1),006(1),026,$1ij$(−.016750), $4ij$(.016750),139
G to E	$3ij16$	004(1),005(−1),026,$1ij$(.016750), $3ij$(−.016750),339
G to W	$3ij46$	005(−1),006(1),026,$3ij$(−.016750), $4ij$(.016750),339
W to E	$4ij16$	004(1),006(−1),026,$1ij$(.016750), $4ij$(−.016750),439
W to G	$4ij36$	005(1),006(−1),026,$3ij$(.016750), $4ij$(−.016750),439
		In addition, Note 1 applies for all ij occurring in this group, for $C = .016750$, and for $k = 1, 3, 4$. Note 2 applies for the same C and k and for $ij = 01, 02, 03$. Note 3 applies for the same C and k and for $ij = 14, 15, 22$.
Shipping white products by tanker	same column codes except that ij now takes the second set of values given above	same as for the preceding group except that now Note 1 applies for all ij occurring except 27, 28. Note 3 applies also. Note 2 does not apply.
Shipping by barge	ij takes both sets of values	
E to M	$1ij27$	015(−1),016(1),028,$1ij$(−.009570), $2ij$(.009570),113
E to G	$1ij37$	015(−1),017(1),028,$1ij$(−.009570), $3ij$(.009570),113
M to E	$2ij17$	015(1),016(−1),028,$1ij$(.009570), $2ij$(−.009570),213
M to G	$2ij37$	016(−1),017(1),028,$2ij$(−.009470), $3ij$(.009570),213
G to E	$3ij17$	015(1),017(−1),028,$1ij$(.009570), $3ij$(−.009570),313

TABLE 5 (*continued*)

A SUMMARY OF THE SPATIAL PETROLEUM REFINING MATRIX

(1)	(2)	(3)
G to M	3ij27	016(1),017(−1),028,2ij(.009570), 3ij(−.009570),313 *In addition*, Notes 1–3 apply for k(or m) = 1, 2, 3 and for C = .009570.
Shipping crude and black products by pipeline	ij takes the first set of values except ij = 16 is omitted	
E to M	1ij28	029(−1),1ij(−1),2ij(1)
E to G	1ij38	030(−1),1ij(−1),2ij(1)
M to E	2ij18	029(−1),1ij(1),2ij(−1)
M to G	2ij38	031(−1),2ij(−1),3ij(1)
G to E	3ij18	030(−1),1ij(1),3ij(−1)
G to M	3ij28	031(−1),2ij(1),3ij(−1)
G to W	3ij48	032(−1),3ij(−1),4ij(1)
W to G	4ij38	032(−1),3ij(1),4ij(−1) *In addition*, Notes 1–3 apply for C = 1.
Shipping white products by pipeline	same column-codes except that ij now takes the second set of values	same as for the preceding group except that 3 is now added to the first row number. Notes 1–3 again apply.
Importing Crude from Venezuela with Transshipment within U. S. and Distillation at Terminal Region		
Importing one tanker-load to E and leaving crude there	81100	004(1),018(.016750),026, 037(−.016750),038,039(−1), 104(−.016750),109,110,111,112, 113,114
Importing to E and transshipping to M by tank car	81205	004(−1),007(−.016750/.000209), 011(.016750/.000209),026, 037(−.016750),038,039(−1),113, 204(−.016750),209,210,211,212, 213,214
	The remaining seventeen activities in this group are analogous and are omitted here.	
Importing Venezuelan Residual Fuel Oil		
Importing one tanker-load to E and leaving it there	91100	004(1),026,038(−.016750 minus amount required to fuel tanker), 039(−1),139(.016750)
Importing one tanker-load to E and trans-	91205	004(1),011(−.017560/.00209), 012(.−17560/.000209),

TABLE 5 (*continued*)

A Summary of the Spatial Petroleum Refining Matrix

(1)	(2)	(3)
shipping to M by tank car		038(−.016750 minus amount required to fuel tanker), 039(−1),113,239(.016750)
The remaining seventeen activities in this group are analogous and are omitted here.		
Making Available Intra-regional Crude and Products Transportation (16 activities)		
"Normal"	900j0 and 900j1	See Figure 4
"Replacement"	901j0 and 901j1 ($j = 1, 2, 3, 4$)	
Disposal Activities		
Disposal activity for row *abc*	9*abc*9 (The triple *abc* goes over all row numbers.)	*abc*(−1) *In addition*, if bc = 9–22 or 34–39 and a = 1–4, then there is the coefficient 1 in row 19 if $a = 1$, 21 if $a = 2$, 23 if $a = 3$, 25 if $a = 4$.

Notes 1–3 (see Section 4.3 of Part III)

Note 1: For ij = 9–26 and 34–39, and for the first column-code digit k, there is the coefficient C in row 019 if $k = 1$, row 021 if $k = 2$, row 023 if $k = 3$, row 025 if $k = 4$.

Note 2: For ij = 01, 02, 03 and for the fourth column-code digit m, there is the coefficient $-C$ in row 018 if $m = 1$, row 020 if $m = 2$, row 022 if $m = 3$, row 024 if $m = 4$.

Note 3: For ij = 9–26 and 34–39 and for the fourth column-code digit m, there is the coefficient $-C$ in row 019 if $m = 1$,021 if $m = 2$,023 if $m = 3$,025 if $m = 4$.

Appendix I Aggregation in Linear Programming Models

Since a large part of the effort reported here consisted in aggregating a large matrix of petroleum refining technology in order to obtain a smaller one, it seems appropriate to survey briefly the general question of aggregation in linear programming models and to determine in particular whether any a priori argument can be made in favor of the aggregation procedures used in the present study.[1]

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[1] We must distinguish between the study of aggregation in linear programming and the study of computational methods for special classes of linear programming problems so as to take advantage of the problems' special structure. The first has to do with reducing the *size* of a problem (i.e., of its matrix). The second has to do with fast methods for solving a problem of *given* size and special structure. Doubtless the two topics will turn out to be closely related in many cases. A survey of results in the second topic has been given by Dantzig (1959)—see reference list at end of appendix.

1. *Aggregation in Input-Output Models.* An extensive literature deals with aggregation in input-output models.[2] Given a matrix of input-output coefficients its aggregation has an unambiguous meaning: the combining of several rows (columns), corresponding to several industries, into one row (column), corresponding to an industry group. The combined row (column) is a linear combination of the component rows (columns); in particular the linear combination may be a simple sum. "Good" aggregation also has a fairly unambiguous meaning. It is aggregation such that the aggregated model yields an answer close to the original model's answer to the question: What are the industry-group outputs needed to meet given final demands? With one major exception,[3] the literature deals essentially with two main problems:

a. *Deriving conditions for "acceptable" or "admissible" aggregation into industry groups.* The aggregated matrix is "acceptable" if for any list of unaggregated final demands the unaggregated matrix implies unaggregated industry outputs which, when aggregated into the given industry groups, equal the respective industry-group outputs implied by the aggregated matrix for the aggregated list of final demands. The conditions which the aggregated rows and columns of each industry group must satisfy in order that the aggregation be acceptable are extremely strong.

b. *Measuring and minimizing the error due to aggregation.* By the error due to aggregation, for a given aggregated matrix and given final demands, is generally meant some function of those discrepancies which would vanish under acceptable aggregation, namely the differences between the industry-group outputs implied by the original matrix and the industry-group outputs implied by the aggregated matrix. The difficulty is that to measure the error due to a given aggregation and to find that aggregation which minimizes the error out of all those that reduce the size of the original matrix by a given amount require, in general, no less work than using the unaggregated matrix in the first place.

One possibility would be to find a nontrivial, relatively easily calculable *bound* to the error. Little effort has gone in this direction. A second possibility is to make the aggregation error a random variable by making the final industry-group demands random variables.[4] It is then possible, under reasonable assumptions on the random variables, to find expressions for the expected aggregation error (or the expected error for a particular industry group) whose computation does not require inversion of the original matrix.

2. *Aggregation in Three Linear Programming Problems.* Good aggregation in linear programming models is a more ambiguous matter than good aggrega-

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[2] See, for example, Ara (1959), Hatanaka (1952), Fei (1956), Fisher (1958), Malinvaud (1955), Theil (1957).

[3] Fei (1956), who is concerned with methods of making inferences about the original matrix given only the aggregated matrix.

[4] Theil (1957) also adds random disturbances to the equations giving each industry's output as a function of its inputs.

tion in input-output models (which are, of course, linear programming models of a very special kind). We consider three linear programming problems:

(i) *The most general problem.* Find an $x^* \geqq 0$ for which $Ax^* = q$ and $c'x^* = \max c'x.$

$$x \geq 0,$$

$$Ax = q.$$

(A is a matrix of order m by $n + m$, c and x of order $m + n$ by 1, q of order m by 1; the m last components of x are the levels of m slack activities and the last m columns of A are slack vectors.)

(ii) *A problem in which the amount of a "product mix" is to be maximized.* Determine $\Pi\ (A,\ a_0,\ q,\ y) = \max x_{m+n+2}.$

$$x \geq 0,$$

$$\begin{pmatrix} a_0 \\ A \end{pmatrix} x = \begin{pmatrix} y \\ q \end{pmatrix}$$

where $y \geqq 0$ is a scalar such that there exists an $x \geq 0$ for which $\begin{pmatrix} a_0 \\ A \end{pmatrix} x = \begin{pmatrix} y \\ q \end{pmatrix}$; $a_0 \geq 0$ ($a_0 = a_{01}, \ldots, a_{0n+m+2}$) is a row vector and x a column vector, both of dimension $n + m + 2$; q is a column vector of dimension m; and $A = (a_{ij})$ is an m by $n + m + 2$ matrix.

The components of a_0 may be thought of as the amounts of one final commodity—commodity zero—emerging from the activities. The last column (column $n + m + 2$) of the matrix defines a mix of the remaining final commodities, and the activity level x_{m+n+2} is the amount of the mix produced by the other activities. Columns $n + 1$ to $n + m$ and column $n + m + 1$ define slack activities for commodities 1 to m and zero, respectively. In addition to commodity zero there are m further commodities, corresponding to rows $1, \ldots, m$; a commodity is either final, intermediate, or primary. For a *final commodity* i, $i \geqq 1$, $a_{ij} \geqq 0$, $j = 1, \ldots, n$; $a_{ij} = 0$, $n < j < n + m + 1$, and $j \neq n + i$; $a_{i,n+i} = -1$; $a_{i,n+m+2} < 0$; $q_j = 0$. For commodity zero, $a_{0j} \geqq 0, j = 1, \ldots, n$; $a_{0j} = 0, n < j \leqq n + m + 2$; $a_{0,n+m+1} = -1$. For an *intermediate commodity* i, a_{ij} may be $>$, $<$, or $= 0$, $j = 0, \ldots, n$, but if $a_{ij} \begin{Bmatrix} > \\ < \end{Bmatrix} 0$ then there must be a $j^* \neq j$, $1 \leqq j^* \leqq n$, for which $a_{ij} \begin{Bmatrix} < \\ > \end{Bmatrix} 0$; $a_{ij} = 0$, $n < j \leqq n + m + 2$, and $j \neq n + i$; $a_{i,n+i} = -1$; $q_i = 0$. For a *primary commodity* i,$a_{ij} \leqq 0$, $j = 1, \ldots, n$; $a_{ij} = 0$, $n < j \leqq n + m + 2$ and $j \neq n + i$; $a_{i,n+i} = -1$; $q_i < 0$.

(iii) *Special forms of (ii).* This problem is the same as (ii) with commodities 1 to k primary commodities (there may be other primary commodities in addition) *and:*

(iiia) for $i = 1, \ldots, k$, $a_{ij} = 0$, when $j = 1, \ldots, n + m + 2$ and $j \neq i$, $j \neq n + i$; $a_{ii} = -1$; for any primary commodity i, $i > k$, $a_{ij} = 0$, $j = 1, \ldots, k$;

(iiib) conditions (iiia), and in addition, there is a primary commodity $k+1$; $a_{k+1,j} < 0$, $j = 1, \ldots, k$; $a_{k+1,j} = 0$ for $j > k$ and $j \neq n+k+1$; $q_{k+1} > \sum_{i=1}^{k} q_i$.

Primary commodity i, $1 < i < k$, enters (with nonzero coefficient) into only one activity (other than a slack activity), and that activity—activity i uses no other primary commodity; at unit level activity i requires one unit of commodity i. In case (iiib) each of the first k activities also uses a second primary commodity, commodity $k+1$, which enters no other activities. Its availability, moreover, is an effective constraint: there is not enough of it to permit operation of the first k activities at levels which use up all of commodities 1 to k. The principal aggregation performed in the present study was a reduction of a problem of form (iiib).[5]

Aggregation in the above problems consists in replacing the matrix A by a smaller one A^* and performing corresponding reductions in q, a_0, and the variable x. The goodness of a given aggregation A^*, for any of the above problems, is the closeness of the answers obtained when using A^* to the answers obtained when using A.

3. *"Acceptable" Aggregation.* Can one define acceptability, in a manner analagous to the definition used in the input-output discussion, for problems (i) to (iii)? The analogy would have to involve aggregation of the answers obtained in the unaggregated problem (industry outputs in the input-output case) in order to compare them with the lower-dimensional answers obtained in the aggregated problems (industry-group outputs in the input-output case). Since the answers in the unaggregated problems (ii) and (iii) are already one-dimensional, an analogy seems to fail there. In problem (i) acceptability could be defined if A^* had fewer columns than A, each of them a linear combination of columns in A. In an acceptable aggregation a given activity level in the aggregated problem's solution equals a linear combination of the activity levels in the unaggregated problem's solution, where the corresponding column of A equals the same linear combination of the columns of A. This would be a strong condition on A^* and it is arguable whether it would be an interesting one to strive for.

4. *Computing Costs and the Reduction of Rows at the Expense of Columns.* With any general-purpose linear-programming computing code based on the simplex method, the number of rows (which determines the size of the inverse to be carried) weighs far more heavily in the computing time required for solution than does the number of columns. For this reason it is of some importance that for any matrix A the rows corresponding to intermediate commodities can be suppressed at the expense of additional columns. The following has been shown:[6]

Consider A and x of problem (i). There exists a matrix A^* with no intermediate commodity rows such that given any $x \geqq 0$ and any vector v for

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[5] See Part II of the main body of the paper.

[6] Koopmans (1951), pp. 57–59.

which $Ax = v$, there exists a vector $z \geqq 0$ such that $A^*z = v^*$, where v^* is obtained from v by deleting the components corresponding to intermediate commodity rows of A. A^* has generally more (perhaps enormously more) columns than A (and the activity-level vector z correspondingly more components than the vector x). It seems, moreover, that no general procedure has yet been worked out for finding A^* given A, although for many special forms of A it is clear how intermediate commodities can be suppressed.[7]

The suppression could, in principle, be used to save rows in obtaining answers to problems (ii) and (iii) if no commodity is both final and intermediate (for then the payoff x—the amount of product mix—is not a function of the outputs of intermediate commodities). To use suppression in problem (i) would require some way of getting back from the maximizing vector z ($A^*z = v^*$) to a vector $\bar{x}$ such that $A^*\bar{x} = v$ for some vector v to which v^* bears the relation indicated above. Again a general procedure appears unknown but in special cases it is clear how the required $\bar{x}$ can be found.

5. *Adding Rows Together: Ranking Some Alternative Aggregations in Problem* (ii). A simple method of shrinking the size of a linear programming problem is to add together some of the rows of the matrix A and the corresponding components of the vector q. This seems a natural method especially if the commodities corresponding to the rows added are measured in comparable units—in dollar amounts (for fixed prices) or in the same physical units (e.g., barrels per day). It is then clear that, for problem (i), a solution x to the original problem is always feasible in the aggregated problem (as is any vector x which is feasible in the original problem).

It follows, for problem (ii), that the maximum feasible payoff after aggregation is not less than the maximum feasible payoff before aggregation—that $\Pi^* (A^*, a_0, q^*, y) \geqq \Pi (A, a_0, q, y)$ where A^* and q^* are obtained from A and q by row addition (with row zero left intact). Hence of any two such aggregations, A^*, q^* and A^{**}, q^{**}, the first gives an answer closer to the unaggregated problem's answer if and only if

$$\Pi^*(A^*, a_0, q^*, y) < \Pi^{**}(A^{**}, a_0, q^{**}, y).$$

To compute either of the two sides of this inequality requires less effort than to solve the original (unaggregated) problem—how much less depends on how much the number of rows has shrunk. If the shrinkage is great enough it will be possible, with less effort than solving the original problem, to choose the best among a large number of alternative aggregations exhibiting the same shrinkage. Perhaps a computing routine could be developed which searches a given collection of proposed aggregations more efficiently than would a series of pairwise comparisons, each of which requires a fresh computation of some new maximum feasible payoff and sometimes duplicates work performed in another comparison.

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[7] Thus, suppose an intermediate commodity enters (with nonzero coefficient) into only two activities in A—one which produces it (and nothing else) and another which uses it (and nothing else) to produce several final commodities. The two columns are replaced in A^* by a single column which is their sum with the intermediate-commodity row deleted. More complex variations of this principle are easily written down.

If one wishes to vary one of the parameters of problem (ii)—say y or a coordinate of q—then parametric programming would permit a continuous comparison of two proposed row-addition aggregations, with respect to maximum feasible payoff, as the parameter is varied. This would yield the ranges of the parameter over which a given one of the two aggregations is preferred.

Note that for problem (ii) it is of no importance that the activity-level vector which achieves Π^* in the aggregated problem may be infeasible in the unaggregated problem. It is maximum feasible payoff alone which matters—we are concerned solely with finding a *scalar* Π^* close to the unaggregated problem's maximum feasible payoff Π.

6. *Using Knowledge about the Capacities Exhausted.* It may sometimes be known that a particular capacity (primary commodity availability) q_i is exhausted for a set of activity levels solving problem (i) or yielding the required maximum feasible payoff in problem (ii), i.e., that $x_{n+i} = 0$, where row $\bar{\imath}$ corresponds to some primary commodity. In problem (i) the number of columns can then be diminished by one but not the number of rows. For we can write $\sum_{j=1}^{n} a_{ij}x_j = q_i$ and hence, for example, $x_1 = (1-a_{i1})\,(q_i - \sum_{j=2}^{n} a_{ij}x_j)$ (provided $a_{i1} \neq 0$). This expression can then be substituted into the equations $Ax = q$ and the expression $c'x$. But at the same time the constraint $x_1 \geqq 0$ must be preserved and row $\bar{\imath}$ can now be put to this use. Thus the new form of problem (i) is:

Find an $\bar{x}^* \geqq 0$ for which $A^*\bar{x}^* = q^*$ and $c^{*\prime}\bar{x}^* = \max\ c^{*\prime}x^*$ ($x^* \geqq 0$, $A^*x^* = q$) where c^*, and x^* have $n + m - 1$ components, A^* has $n - 1$ columns, and the first $n - 1$ elements in row $\bar{\imath}$ of A^* are $(a_{\bar{\imath}2}/a_{\bar{\imath}1}, a_{\bar{\imath}3}/a_{\bar{\imath}1}, \ldots, a_{\bar{\imath}n}/a_{\bar{\imath}1})$ while the next m elements are all zero except for the $(n + \bar{\imath} - 1)$th, which is 1. The other rows of A^* are obtained from A by performing the indicated substitution, and so is c^* from c; q^* is obtained from q by substituting $q_{\bar{\imath}}/q_{\bar{\imath}}$ for $q_{\bar{\imath}}$. The same reduction is possible for problem (ii) if it is known that it is possible to exhaust a certain capacity while achieving the maximum feasible payoff Π.

Consider now the special case of problem (iiia). Each of a number of primary commodities (commodities i to k) enters only one activity which uses no other primary commodity. Suppose it is known that the maximum feasible payoff Π may be achieved by a set of activity levels for which the primary commodity $\bar{\imath}$, $1 \leqq \bar{\imath} \leqq k$, is exhausted. Then we can simply impose the constraint $x_{\bar{\imath}} = -q_{\bar{\imath}}$, so that row $\bar{\imath}$ can be deleted from A, q, and x. The value $x_{\bar{\imath}} = -q_{\bar{\imath}}$ is then substituted into any equations in which $x_{\bar{\imath}}$ occurs (with nonzero coefficient). The matrix A^*, with one less row and one less column than A, results. The maximum feasible payoff is the same for the reduced problem as for the original one.

Note that if the activity $\bar{\imath}$, $1 \leqq \bar{\imath} \leqq k$, involves no (nonzero) amounts of any intermediate commodity then no advance knowledge as to the exhaustion of primary commodity $\bar{\imath}$ is needed. We can always impose the constraint $x_{\bar{\imath}} = -q_{\bar{\imath}}$, for by disposing of any "unneeded" amounts of $\bar{\imath}$, or of commodities produced from $\bar{\imath}$, we can always attain the required maximum feasible payoff Π. More generally, consider any subset of the activities 1 to k. If, when each of them is operated at the level that uses up all of the primary commodity of which it is sole user, the subset of activities together have no net requirement for any intermediate commodity, then it does not affect the maximum feasible payoff if we impose the constraint that they be operated at these levels.

7. *Combining Columns as well as Rows in Problems* (iii). Consider shrinking the size of problem (iiia) by adding together some of the primary commodity rows 1 to k, say rows 1 to $k' \leqq k$. This "liberalizes" the problem and may make the maximum feasible payoff larger than that of the unaggregated problem. Can one make a further change to compensate for the liberalization, i.e., to pull the maximum feasible payoff down again? One such further change consists in reducing the number of columns. But a simple arbitrary deletion of columns, while a change that goes in the required direction, is certainly very risky. More satisfactory is the approach of combining the columns 1 to k', the activities that use the primary commodities being pooled by the summing of rows. A method of combining these columns which immediately suggests itself is to take their weighted average, the weights being the availabilities of the primary commodities 1 to k'. The new matrix $\begin{pmatrix} a_0{}^* \\ A^* \end{pmatrix}$ has then $k' - 1$ fewer rows and fewer columns than $\begin{pmatrix} a_0 \\ A \end{pmatrix}$, while the vectors q^*, x^*, corresponding to q and x, have $k' - 1$ fewer components. The first column of $\begin{pmatrix} a_0{}^* \\ A^* \end{pmatrix}$ is obtained from the weighted average just described and has the coefficient -1 in row 1 (the row which corresponds to the new combined commodity, whose availability is $-\sum_{i=1}^{k'} q_i\big)$, the coefficient $\sum_{j=1}^{k'} a_{r-1,j}\left(q_j / \sum_{j=1}^{k'} q_j\right)$ in row r, $r = 2, \ldots, m - k' + 1$, and the coefficient $\sum_{j=1}^{k'} a_{0j}\left(q_j / \sum_{j=1}^{k'} q_j\right)$ in row 0. Columns 2 to $n - k' + 1$ of $\begin{pmatrix} a_0{}^* \\ A^* \end{pmatrix}$ are the same as columns $k' + 1$ to n of $\begin{pmatrix} a_0 \\ A \end{pmatrix}$, except that rows 2 to k' have been deleted; columns $n - k' + 2$ to $n + m - k' + 3$ of $\begin{pmatrix} a_0{}^* \\ A^* \end{pmatrix}$ (the slack activities for all commodities including commodity zero, plus the product-mix activity) are the same as columns $n + k'$ to $n + m + 2$ of $\begin{pmatrix} a_0 \\ A \end{pmatrix}$ except that rows 2 to k' have been deleted.

The new aggregated problem is certainly more "stringent"—cannot have a higher maximum feasible payoff—than the old aggregated problem (obtained from the original one by summing of rows only). But does it *overcompensate* for the liberalizing effect of summing rows, i.e., can it be that the maximum feasible payoff is now *smaller* than the original problem's Π? There is one rather obvious sufficient condition for overcompensation to be impossible. This is that $-\sum_{j=1}^{k'} a_{ij}q_j \geqq 0$ if i is an intermediate commodity, the condition discussed at the end of Section 6 of this appendix. For then in the original problem the constraints $x_i = -q_i$, $i = 1, \ldots, k'$, can be imposed without affecting the maximum feasible payoff. The net commodity amounts which then emerge from the first k' activities in the original problem are exactly duplicated in the aggregated problem if the level of the first (the combined) activity is held at $-\sum_{i=1}^{k'} q_i$. To perform the aggregation described and to hold the combined activity at this level is simply equivalent to holding the activities 1 to k' at levels q_1 to $q_{k'}$, respectively, in the original problem, and eliminating $k' - 1$ of the k' variables x_1 to $x_{k'}$ as well as $k' - 1$ equations—all of which does not affect the maximum feasible payoff. Thus the aggregation described not only does not overcompensate with respect to the summing of rows only, it does not liberalize the original problem either; it is error-free aggregation.

In general, however, the condition on intermediate-commodity requirements is not met and overcompensation cannot be ruled out if the aggregation described is performed in problem (iiia).

In problem (iiib) the activities 1 to k share an additional primary commodity, commodity $k + 1$, whose availability is an effective constraint. Suppose the aggregation described is performed in problem (iiib), combining rows and columns 1 to $k' < k$. Even if the above condition on intermediate-commodity requirements is met, overcompensation is now possible. Consider, for the first k' activities of the original matrix, any levels $\bar{x}_1, \ldots, \bar{x}_{k'}$ which are feasible. Suppose $\bar{x}_1 = -q_1$, so that $-a_{k+1,1}q_1 + \sum_{j=2}^{k'} a_{k+1,j}\bar{x}_j \geqq q_{k+1}$ as well as $\bar{x}_j \leqq -q_j$, $j = 1, \ldots, k'$. Consider a single intermediate or final commodity $\bar{\imath}(\bar{\imath} > k,\ q_{\bar{\imath}} = 0)$ for which $a_{\bar{\imath}1} > 0$ and $\sum_{j=1}^{k'} a_{\bar{\imath}j}\bar{x}_j = 0$. At the levels $\bar{x}_1, \ldots, \bar{x}_{k'}$ the amount of commodity $\bar{\imath}$ emerging from the first k' activities is $h_{\bar{\imath}} = -q_1 a_{\bar{\imath}1} + \sum_{j=2}^{k'} a_{\bar{\imath}j}\bar{x}_j$. In the aggregated problem the new combined activity requires, at unit level, $-\left(\sum_{j=1}^{k'} a_{k+1,j}q_j\right) \Big/ \sum_{i=1}^{k'} q_i$ of commodity $k + 1$ and produces $\left(\sum_{j=1}^{k'} a_{\bar{\imath}j}q_j\right) \Big/ \left(\sum_{i=1}^{k'} q_i\right)$ of commodity $\bar{\imath}$. No more of commodity $\bar{\imath}$ can emerge from the combined activity than emerges when that activity is operated at the level $\left(-q_{k+1} \sum_{i=1}^{k'} q_i\right) \Big/ \left(\sum_{j=1}^{k'} a_{k+1,j}q_j\right)$, the level at which all of commodity $k + 1$ would be used up. Therefore an upper bound to the amount of commodity $\bar{\imath}$ emerging from the combined activity is

$\Theta = \left(-q_{k+1} \sum_{j=1}^{k'} a_{\bar{i}j} q_j\right) \Big/ \left(\sum_{j=1}^{k'} a_{k+1,j} q_j\right)$. Now the condition $h_{\bar{i}} > \Theta$ can be written

$$a_{\bar{i}1} \left[\left(q_{k+1} q_1\right) \Big/ \left(\sum_{j=1}^{k'} a_{k+1,j} q_j\right) - q_1\right] > -\left(q_{k+1} \sum_{i=2}^{k'} q_i\right) \Big/ \left(\sum_{j=1}^{k'} a_{k+1,j} q_j\right) - \sum_{j=2}^{k'} a_{\bar{i}j} \bar{x}_j$$

The inequality holds if $-q_{k+1} < -\sum_{j=1}^{k'} a_{k+1,j} q_j$ and if $a_{\bar{i}1}$ is large enough. If, in other words, (1) the availability of commodity $k + 1$ is an effective constraint on the k' activities which are being combined; (2) one of them, activity 1, yields at unit level a sufficiently great amount of an intermediate or final commodity $\bar{i}$; and (3) it is feasible to operate the activity at the level $\bar{x}_1 = -q_{\bar{1}}$, requiring all of the primary commodity it alone uses, than any feasible activity levels in the unaggregated problem, with $\bar{x}_1 = -q_1$, yield more of commodity $\bar{i}$ than the maximum attainable amount of that commodity in the aggregated problem. If the commodity $\bar{i}$ is sufficiently important in the payoff (the product mix)[8] then the maximum feasible payoff is larger in the unaggregated than in the aggregated problem. Moreover, the greater the availability of the primary commodity used solely in the activity which produces much of $\bar{i}$ (the greater $-q_1$), the greater is the reduction in maximum feasible payoff.

Thus in problem (iiib) we cannot state in general whether aggregating in the manner described liberalizes the problem or makes it more stringent. The counter-example suggests, however, that we would be particularly fearful of a major reduction in maximum feasible payoff if one of the activities being combined were far superior to the others with respect to its production of an important commodity (and could feasibly be operated at a high level), since the aggregation "dilutes" the superiority of such an activity.

8. *Selecting the Rows and Columns to Be Combined.* While the aggregation procedure just described seems no less reasonable a priori than any other for problem (iii), and seems preferable to the summing of rows only, the question as to *which* rows and columns to include in each combined group remains. A crude but intuitively appealing principle is that the columns combined be *similar*. This might mean, for example, that if one computes for each of the first k columns in problem (iii) a certain linear function of its coefficients (using the same linear function for each column), and ranks the columns according to the value of this linear function, then one group of columns (and corresponding rows) to be combined consists of the k_1 highest ranking columns, another group of the k_2 next ranking columns, and so on $\left(\sum_r k_r = k,\ r = 1,\right.$

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[8] Or if a commodity whose production requires commodity $\bar{i}$ as an input is sufficiently important.

2, . . .). (A simple such linear function is the sum of the column coefficients.) A complicated proposition may be conjectured: If for any linear function of column coefficients, columns 1 to k are divided into r groups of k/r columns each, according to the principle just described, and if for each group weighted-average column aggregation and summing of the corresponding rows is performed, then the aggregation error[9] is always less than the expected aggregation error if the r groups of the same size were filled with columns chosen at random. The intuitive grounds for this conjecture are essentially that the nonnegative activity levels achieving maximum feasible payoff in the unaggregated problem correspond to activities (columns of A) which are somehow similar—at any rate they all show zero profit at the shadow prices dual to these activity levels—while the remaining columns are also somehow similar. Grouping columns at random is likely to break up these two groups more than grouping according to the ranking described.

9. *Concluding Remarks.* A general form of the aggregation problem in linear programming is to find, given a computing cost C, a reduced version of problem (i) that costs no more than C to solve and has an aggregation error (the difference in payoffs or some other function of the differences in maximizing activity levels) no larger than any other reduced version of the problem costing no more than C to solve. It is clear that there is no general criterion that can be used to find such a best aggregation and is itself computationally less costly to apply than is solving the original problem.[10] The approaches adopted in studying aggregation in input-output models do not seem helpful for most other forms of the linear programming problem. One must be resigned to the individual study of many special linear programming problems and of the special aggregation procedures which are intuitively appealing in each. In the special problem (iiib) considered here, the suggested aggregation procedure seems defensible, subject to the cautionary note at the end of Section 7, for no alternative is clearly better. But fortunately the case for its use in obtaining a small petroleum refining matrix rests not on this defense, but on actual comparisons of maximum feasible payoffs in the unaggregated and aggregated problems.

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[9] I.e., the absolute value of the difference in maximum feasible payoff.

[10] Of course, if the reduced matrix is going to be used many times, or is going to be a repeated component of a much larger matrix (as in the present study) it may pay to make a very costly search for a good aggregation.

Koopmans, T. C., 1951, "Analysis of Production," in *Activity Analysis of Production and Allocation.*

Malinvaud, E., 1955, "Aggregation Problems in Input-output Models," in T. Barna, ed., *The Structural Interdependence of the Economy,* Chapter 8.

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APPENDIX II ESTIMATING THE COEFFICIENTS OF THE INTRAREGIONAL TRANSPORTATION SUBMATRIX

In accordance with assumption 7 of Section 4.1 (Part III of the body of this chapter) the procedure used begins by obtaining for each region estimates of the minimal fleets of tankers, barges, and tank cars that could have performed the intraregional movements of crude and of products, by tanker, barge, and tank car, *which actually occurred in the year 1952.* (The year 1952 is used to approximate the year 1952–53.)

1. *Intraregional Tanker Movements.* The primary source of data on these movements is the publication *Water-Borne and Lakewise Commerce of the U. S., Calendar Year 1952,* compiled by the Board of Engineers for Rivers and Harbors of the U. S. Army Corps of Engineers. This gives the number of barrels of crude and of petroleum products shipped by tanker from East ports to East ports, from Gulf ports to Gulf ports, and from West (Pacific) ports to West ports in 1952. Intra-Gulf tanker shipments of both crude and products are negligible (though barge shipments are not) and were ignored. To estimate the minimum number of tankers (T-2 tankers, which are the standard unit in terms of which the size of tanker fleets is expressed, and which have a capacity of 16,765 deadweight tons) required to achieve these movements in the East and West, the following assumptions were made:

(*a*) It was assumed (fairly realistically) that the East coastwise tanker movements of products (of which the 250-nautical-mile movement from Bayonne, New Jersey—a refinery port—to Boston—a distribution center port—is typical) could be approximated by 250-nautical-mile *northbound* movements from refinery ports to distribution center ports. Efficient routing of empty tankers can then be achieved by "shuttling," i.e., a tanker, having unloaded at a distribution center port, always returns empty to the refinery port from which it came.[1] Using the tanker speeds and turn-around times given in Table 2, we may then estimate the smallest tanker fleet that can perform such 250-nautical-mile movements (with shuttling), when the total number of barrels of products to be moved in a year is the 1952 intra-East total.

Intra-East tanker movements of crude were negligible in 1952 and were ignored.

(*b*) For the West coastwise movements (of both products and crude), similar assumptions were made; but here data are available on the total 1952

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[1] It is easy to show that for a "straight" (or "nonindented") coastline (in which the water distances from a given port always increase as one considers ports further and further along the coastline in a given direction) such shuttling always constitutes an optimal routing of empty vessels provided the required flows of cargo are all in the same direction.

movements along the California coast, along the Pacific coast north of California (these are negligible), and between the two parts of the coast. Crude movements by tanker were virtually all between California ports. It appears that about two-thirds of the crude moved by tanker was delivered in the San Francisco area and one-third in Los Angeles. Movements from Port San Luis (San Luis Obispo) to Richmond are assumed to approximate the movements ending in the San Francisco area. Movements from Ventura to Los Angeles are assumed to approximate the movements ending at Los Angeles. Efficient routing of empty tankers is then achieved if tankers unloading at Richmond always return empty to Port San Luis and tankers unloading at Los Angeles always return empty to Ventura.

Products movements between California ports are assumed to approximate Los Angeles-to-San Francisco movements; products movements from California to ports on the Pacific coast north of California are assumed to approximate San Francisco-to-Seattle movements. Again, "shuttling" achieves an efficient routing of empty tankers.

Using the assumed speeds and turn-around times, the minimal intra-West crude and products 1952 tanker fleets were estimated.

2. *Intraregional Tank Car Movements.* Here the primary sources are Interstate Commerce Commission publications: (a) the quarterly reports entitled *Distribution of Petroleum Products by Petroleum Administration Districts. One Percent Sample of Waybills for All Carload Traffic Terminated by Class I Steam Railways;* and (b) the annual reports entitled *Carload Waybill Statistics, State-to-State Distribution of Manufactures and Miscellaneous and Forwarder Traffic. One Percent Sample of Terminations* (for petroleum products), and *Carload Waybill Statistics, State-to-State Distribution of Products of Mines. One Percent Sample of Terminations* (for crude oil). The relevant data in (a) are, for each Petroleum Administration District, estimates[2] of the total quarterly amounts of crude oil and of petroleum products shipped by tank car, originating at some point in the District, and terminating at some other point in the District. (Districts 4 and 5 comprise our West region; District 3 plus Oklahoma, our Gulf region; District 2 less Ohio, Oklahoma, and Detroit, Michigan, our Midwest region; and District 1 plus Ohio and Detroit, our East region.) The relevant data in (b) are, for each of our regions (estimates of)[2] the annual amounts (in tons) and the annual number of ton-miles involved in tank car movements of crude and of petroleum products originating at some point in a state within the region and terminating at another point in a state (sometimes the same state) within the same region.

From these data one may estimate for each region the distance moved by the average ton (or barrel) of crude and of petroleum products shipped intraregionally by tank car and the total number of barrels of crude and of products so shipped in 1952. It was assumed that the 1952 tank-car movements in each region could be approximated by movements having the estimated average length and (a much more heroic assumption) that the optimal routing

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[2] After multiplying the given "one percent sample" figures by 100.

of the tank cars engaged in these movements, after they are emptied, can be approximated by "shuttling"—i.e., each empty tank car returns along a route of this same average length.

Using times required for "typical" fast-freight shipment of the relevant length in each region, and using the turn-around times given in Table 1 below, it was then possible to estimate roughly for each region the smallest tank-car fleet that could accomplish the 1952 intraregional crude movements and the smallest fleet that could accomplish the 1952 intraregional products movements.

3. *Intraregional Barge Movements.* Here the primary sources of data are: a publication of the Board of Engineers for Rivers and Harbors of the U. S. Army Corps of Engineers entitled, *Water-Borne Commerce Statistics. Domestic Inland Movements of Petroleum and Petroleum Products: Shipping Area by Receiving Area, Calendar Year 1952;* and the Great Lakes portion of the publication mentioned above, entitled *Water-Borne Commerce of the U. S., Domestic Deep Sea and Lakewise Traffic, Calendar Year 1952.* The relevant data in these sources for each region are the total amounts of crude and products shipped by barge from port to port within each region in 1952 (including Great Lakes barge movements within the East and within the Midwest).

It is assumed that the barge fleet in the West region (which travels on the Columbia River system and engages in some movements within Pacific Coast bays) is not transportable to other regions and that the fleet is adequate for the intra-West barge movements required in all interesting applications of the model. Intra-West barge movements, then, need not be considered.

In estimating the minimal fleets of barges that can achieve the 1952 intraregional movements in the Gulf, East, and Midwest, the following were the assumptions made and results (from another study) used:

(*a*) In the Gulf, barge movements of products were assumed to be such that (i) shuttling provided an efficient routing of empty barges, and (ii) the average round trip of a products-carrying barge took 11½ days. These assumptions are taken from the analysis of barge requirements after 1950 contained in the study *Transportation of Oil* prepared by the Petroleum Administration for Defense. The same study provided a direct estimate of the minimal barge capacity required for 1952 intra-Gulf barge movements of crude (this estimate was based in turn on an estimate of 1952 water-borne Gulf refinery receipts of crude, an estimate which turned out to be correct).

(*b*) Within the Midwest, there are barge movements along rivers and between Great Lakes ports. The PAD study provides an estimate of the minimal number of barges needed for all Great Lakes movements of products in 1952 (Great Lakes movements of crude are negligible). The total Lakes movements seem to be either between points in our Midwest region or between points in our East region, with only small movements between East and Midwest. The minimal Lakes barge fleet, as estimated in the PAD study, was accordingly assumed allocable between East and Midwest in the same proportion as lakeside refinery capacity in East and Midwest (about one-third East and two-thirds Midwest). The PAD study concludes that the intra-Midwest river barge movements (of both crude and products) are such that, as in the Gulf, average round-trip time is 11½ days and shuttling provides an efficient routing of empty barges. Thus it was possible to obtain an estimate of the total minimal barge fleet for crude and the total minimal barge fleet for products for the Midwest.

(*c*) The Great Lakes component of the minimal barge fleet required for the 1952 intra-East products movements is taken as half of the Midwest Great Lakes products fleet described above. Estimates of the minimal barge fleets required for 1952 movements of crude and of products inside Atlantic harbor areas and along the Hudson River and New York State Barge Canal (these comprise the intra-East movements not on the Great Lakes) are taken directly from the PAD study.

4. *The Intraregional Transportation Activities of the Model.* We now have estimates of the minimal fleet of tank cars, tankers, and barges required for the 1952 intraregional crude and products movements in each region. The next step is to divide each component (e.g., number of tankers) of the intraregional crude transportation fleet of each region i by the quantity Q_i = crude distilled in region i. Each component of the intraregional products transportation fleet of each region i is divided by $\bar{Q}_i$ = total volume of end items produced, less end items exported plus end items imported, plus intermediate materials imported. To estimate $\bar{Q}_i$ for 1952–53 requires the addition of many separate estimates (from the *Minerals Yearbook* and from the other sources cited above); Q_i is obtainable, after some adjustment, from the *Minerals Yearbook*.

We obtain:

i =	East	Midwest	Gulf	West
Estimated 1952–53 value of Q_i	218.2	215.1	1008.3	443.2
" " " " $\bar{Q}_i$	1007.3	617.6	335.9	449.6

Dividing each component of the eight intraregional transportation fleets by the appropriate Q_i or $\bar{Q}_i$ yields the first eight columns of the intraregional transportation submatrix of the model. The entry in the tanker row for the activity "Providing intraregional products transportation, East," for example, is an estimate of the number of tankers required for intra-East products movements per million barrels (a year) of Q_{East}. Each column also contains estimates of the fuel required for intraregional crude or products transportation (per million barrels a year of the appropriate quantity Q_i or $\bar{Q}_i$). The fuel inputs are based on estimates of the average per-mile fuel consumption of a tank car (i.e., the average diesel train-mile consumption divided by the number of cars in the average train, as obtained from ICC data), of a tanker, and of a barge (i.e., the average per-mile consumption of a tow of barges divided by the average number of barges in a tow, as obtained from the PAD study); and on estimates of the miles involved in the 1952 movements (with the assumed optimal routing of empty vehicles).

CHAPTER 6

CHEMICAL PROCESSES, PLANT LOCATION, AND ECONOMIES OF SCALE

Thomas Vietorisz and Alan S. Manne[1]

This chapter represents an example of process analysis applied to the chemicals industry. It deals with the choice of plant location in a system of interrelated chemical products, each of which is characterized by economies of scale. Our specific illustration has to do with the location of synthetic fertilizer plants within the Latin American reigonal market. As background to this specific problem, we will first review some of the general characteristics of process analysis in the chemicals industry, then some aspects of locational analysis, and then indicate some of the more crucial simplifications utilized in the synthetic fertilizer study.

This particular model can be formulated as one of mixed integer programming. In the absence of better numerical techniques, it was solved by brute-force numerical methods of complete enumeration. A similar approach is clearly inapplicable to problems of a more realistic and wider scope. However, because of the general shape of the cost distribution obtained in this example, we have become fairly optimistic on the possibility of attacking even the more complex cases through the use of statistical sampling.

PROCESS ANALYSIS OF CHEMICAL TRANSFORMATIONS

The roots of this model lie in three earlier locational studies of the chemicals industry: one by Isard and Schooler (1955), one by Isard, Schooler, and Vietorisz (1959), and one by Vietorisz and Szabo (1959). In this earlier work —after a considerable review of the chemical engineering literature—it was concluded that most of the inputs and outputs bear the simplest possible relation to scale, i.e., that of proportionality. The chief exceptions to strict proportionality consist of capital and labor inputs. These latter are generally believed to vary with the scale of the process as follows:

$$\text{capital or labor input} = \beta x^{\alpha} \qquad (1)$$

where β represents a constant of proportionality, x the scale of the process, and α an exponent that lies between zero and one.[2]

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[1] This is in every sense a joint paper. Vietorisz's name appears out of alphabetical sequence in order to emphasize that he performed most of the empirical work reported here, and that Manne concentrated on the computational aspects.

[2] In the case of direct operating labor inputs, the elasticity exponent α is typically reported within the range of 0 to .4. For capital investment costs, the range lies between .5 and .9.

The usual indicator of process scale consists of the rate of output of the principal product (e.g., pounds per year of ammonia production), or the rate of input of the principal raw material (e.g., barrels per day of naphtha charged to a reforming unit). Both "minimum" and "maximum" process scales are specified—primarily to indicate that no empirical evidence on plant costs is available outside this range. Even within this range of plant scales, there is a good deal of unpredictability of capital and labor inputs, e.g., because of differences in equipment design, because of inconsistencies in the definition of direct and indirect labor, and because of failure to define which auxiliary processes—if any—have been included within an *un*balanced addition to existing capacity.

Aside from the economies of scale implied by equation (1), it may be worth noting one other departure from strict proportionality to scale—the effect of seriality or length of production runs in batch-type processes. Just as in the case of the metalworking industries, there are significant setup and cleanup operations involved in the production of pharmaceuticals, dyes, and other fine organic chemicals. This means that a doubling of the run length leads to less than a doubling of the required equipment-hours. Fortunately from the analytical viewpoint, this setup time phenomenon is not as common in the chemical industries as in metalworking. It appears reasonable to ignore the run length problem in the case of most of the heavy chemicals, and that simplification will be adopted throughout the remainder of this study.

Except, then, for capital and labor inputs, the chemicals industry may be approximated tolerably well within an activity analysis framework. Activity analysis allows for two common possibilities within this sector: process alternatives and also joint products. Process alternatives result from the use of different initial raw materials (e.g., refinery gas versus natural gas for the production of ammonia), and also from the use of different operating conditions (e.g., variations in temperature, pressure, and catalysts). Because of the highly automated nature of the chemicals industry, there are only limited possibilities for process substitution as between direct labor and capital inputs. However, a certain amount of such substitution is apparently possible in the ancillary operations such as materials handling and equipment maintenance.

A typical process description is given in Table 1 for the case of ammonia and nitrogenous fertilizer production. The table is divided into two parts—one dealing with the inputs and outputs that are proportional to scale, and the other dealing with the principal nonproportional inputs, capital investment and direct operating labor. Inputs are denoted by negative signs, and outputs by positive quantities. Thus, activity (1) deals with the production of ammonia from refinery gas. For every 10 million pounds of annual ammonia output, Table 1 indicates that there are inputs of 31.5 billion kilocalories of refinery gas, 1.26 billion kilocalories of industrial fuel, etc. Plant scales are listed as ranging from 66 to 800, with a typical size or "reference scale" being 614 million pounds per year of the principal product, ammonia. The direct operating labor and initial investment inputs for this typical plant are estimated to be 134.3 thousand man-hours per year and 33.39 million dol-

TABLE 1

PROCESS DESCRIPTION FOR AMMONIA-FERTILIZER PRODUCTION[a]

	(1) Ammonia from Refinery Gas	(2) Ammonia from Natural Gas	(3) Ammonia from Coke Oven Gas	(4) Nitric Acid	(5) Ammonium Nitrate	(6) Sulfuric Acid	(7) Ammonium Sulfate
Proportional Inputs and Outputs[b]							
1. Ammonia, MM lbs./yr.[c]	+10.0	+10.0	+10.0	−2.9	−2.4		−2.6
2. Nitric acid, MM lbs./yr.				+10.0	−7.6		
3. Ammonium nitrate, MM lbs./yr.					+10.0		
4. Sulfuric acid, MM lbs./yr.						+10.0	−7.6
5. Ammonium sulfate, MM lbs./yr.							+10.0
6. Sulfur, MM lbs./yr.						−3.44	
7. Refinery gas, MMM kcal./yr.	−31.5						
8. Natural gas, MMM kcal./yr.		−47.1					
9. Coke oven gas, MMM kcal./yr.			−69.6				
10. Fuel, MMM kcal./yr.	−1.26		−0.05				
11. Steam, MM lbs./yr.	−45.0	−36.7			−6.5	+9.0	−2.0
12. Power, MM kwh./yr.	−5.5	−4.8	−0.5	−1.2	−0.20	−0.025	−0.14
Labor and Capital Investment Inputs							
1. Reference scale, MM lbs./yr. of principal product	614	614	614	100	400	490	400
2. Direct operating labor, M mhr./yr.	−134.3	−134.3	−97.9	−29.6	−54.6	−19.0	−91.6
3. Labor exponent, α	0.4	0.4	0.2	0.2	0.27	0.2	0.2
4. Initial capital investment MM$	−33.39	−33.39	−29.25	−3.05	−1.56	−2.52	−0.76
5. Capital exponent, α	0.81	0.81	0.78	0.63	0.68	0.63	0.65
6. "Minimum" scale	66	66	66	10	50	14	50
7. "Maximum" scale	800	800	700	100	400	490	400

[a] For illustrative purposes only. The numerical information may not be up to date.

Sources: Isard, Schooler, and Vietorisz (1959), Vietorisz and Szabo (1959), and Vietorisz (1961).

[b] Cooling and process water and catalyst inputs have been omitted.

[c] M, thousand
MM, millions
MMM, billions
lbs., pounds
kcal., kilocalories
kwh., kilowatt-hours
yr., year
mhr., man-hours

lars respectively. The labor and capital exponents α in relation (1) are indicated as .4 and .81 respectively.

PREVIOUS LOCATIONAL STUDIES INVOLVING ECONOMIES OF SCALE

The model to be developed below represents an example of the division of markets between several spatially separated points of production.[3] Whenever the cost function is convex with respect to the decision variables, the determination of market boundaries is a comparatively simple matter, and has been discussed by a number of students of classical location theory.[4] However, the case in which we are interested is one in which there *are* economies of scale in production—hence nonconvex cost functions. Under these circumstances, there is one principal factor which tends to discourage the concentration of all production in a single low-cost manufacturing point: the resulting increase in transport costs. Our model is intended to provide some guidance as to the optimal economic balance between transport and manufacturing costs.

There are three earlier locational studies that represent distinct stages in the evolution of nonconvex locational models for the chemical industry. The first study—that by Isard and Schooler (1955)—analyzes locational cost *differences* for some three dozen petrochemical processes or sequences of processes. The location of each process is analyzed by itself—independently of the influence of other locational decisions pertaining to the same region or to closely related processes. A model based upon locational differences represents the simplest possible formulation of the problem of geographical choice. Only two alternative locations are considered at one time, and the productive process is assumed to be identical with regard to scale and structure at both places. Under these assumptions, it is possible to obtain a comparison of the net benefits without reference to the absolute costs of individual inputs or outputs. Net benefits are calculated by multiplying the physical amount of each input or output by the difference between its local prices at the two locations. This method has two powerful advantages over more complex techniques of analysis. First, geographic price differentials tend to be easier to determine and remain more stable than absolute price levels. And second, no technological parameters need to be estimated for items associated with near-zero price differentials. The nonconvex labor and capital input functions do not create a difficulty in this model, since the scale of production is a constant.

The second study—Isard, Schooler, and Vietorisz (1959)—maintains the method of locational cost-benefit differentials as the foundation of the analysis, even though a set of secondary corrections are adopted to deal with major disparities between the structure or the scale of production at the competing locations. The model comprises about six dozen activities—not only petro-

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[3] The mirror image of this case would be one in which the sources of raw material (e.g., timber or sugar cane) were geographically dispersed, and in which there were economies of scale in processing the raw material at a small number of points (e.g., lumber mills or sugar refineries).

[4] For a detailed discussion of this literature, see Isard (1956), Chapter 2.

chemicals production, but also petroleum refining and synthetic fiber production. The important advance here consists of the simultaneous consideration of *complexes* of interrelated activities. This permits the analysis of economies of scale which result from the simultaneous demand of several processes for a given intermediate material. This also permits consideration of the transport cost savings obtained when successive links of a processing chain are operated jointly at a single location. Nonconvexity prevented the application of standard linear programming techniques in this case. However, the data are organized and presented as in Table 1 above—i.e., with the use of activity vectors, but distinguishing between proportional and nonproportional inputs. The quantitative investigation consisted of the detailed exploration of more than thirty programs, selected on the basis of process combinations which appeared attractive a priori, but without making an analytical attempt to arrive at an optimal program.

The third study—Vietorisz and Szabo (1959)—abandoned completely the locational cost difference method, and dealt with absolute costs and price levels at individual geographical points. This then permitted the analysis of five alternative production sites, twelve market areas, and over sixty production activities. Due to the nonconvexity of the labor and capital input functions, formal optimizing was again avoided, and a number of alternative programs which appeared attractive a priori were computed in detail.

In organizing the data for the purpose of preliminary identification of attractive programs, an important aid proved to be the breakdown of the technology matrix into several complexes of activities, e.g., a sodium-chlorine complex, an acetylene group, and a synthesis gas group. (Table 1, for example, represents a portion of the activities included within the synthesis gas complex.) In defining these complexes, the intention is to create a level of aggregation intermediate between the individual activity and the overall program. An attempt is made to group together activities in such a way that each complex may be analyzed as a more or less self-contained unit, thereby using these as building blocks for the construction of an overall industry program. Such an effort can never be entirely successful within the chemicals industry, since there will inevitably remain a few strong links between complexes. Nevertheless, this concept has proved to be a useful one in practice, and has made it possible to subdivide the larger problem in terms of a manageable number of individual analyses.

In selecting alternative programs for detailed computation, it was generally assumed that the production of intermediates was integrated with that of final products. This is a reasonable enough supposition whenever the intermediate is heavier or in general, costlier to ship than the final product into which it is fabricated. There are cases, however, in which the reverse is true. For example, Table 1 indicates that if activities (4) and (5) are combined into an integrated nitric acid plus nitration process, it takes only 4.6 million pounds of ammonia in order to produce 10.0 million pounds of ammonium nitrate. In this instance, it might well be economical to produce the intermediate, ammonia, at just a few points, and to convert it into nitric acid and then into nitrate fertilizer at a larger number of locations. The earlier work did not deal satisfactorily with this possibility of split locations, and appeared to deserve further study.

In the particular cases studied by Vietorisz and Szabo (1959), their work was greatly facilitated by a fortuitous relationship between the parameters characterizing the economies of scale, the transport costs, and the levels of demand. In most cases, it turned out that the economies of scale were dominant, and that it was preferable to serve all markets from a single large plant, rather than to split up the markets between two or more smaller units. Thus, the issue is reduced to selection of the best single location from among no more than five alternatives. The complex we have selected for study below—the synthetic fertilizers—was one of the few instances in which it was not immediately obvious that a single large integrated plant is best, and it was here that a more powerful method of analysis seemed needed.

In summary, these earlier studies may be viewed as attempts at programming of a nonconvex type. In the absence of formal optimizing methods, hand computations were employed to develop a series of alternative cases which appeared attractive on a priori grounds. The synthetic fertilizer complex was one of the few cases in which there were serious doubts that this procedure would yield a near-optimal result.

A SIMPLIFIED TWO-SITE EXAMPLE

In order to aid the reader's intuitive understanding of the plant location problem, we shall construct an elementary two-site example before proceeding to the more general case of several sites and nonintegrated production. Suppose that two sites are available, A and B. The demand for their production is uniformly distributed along a straight line between these two points. In this numerical example, we shall suppose that the two sites are 1,000 miles apart, and that there is a constant demand for one ton of product per day over each mile of the territory that lies between these two points. If production at site A is denoted by x, then production at site B will be $1{,}000 - x$. And if it costs \$.01 for each ton-mile's worth of transportation, the total daily transport costs will be as follows:

$$\begin{aligned}
&\$.01 \begin{bmatrix}\text{ton-miles to} \\ \text{deliver produc-} \\ \text{tion from site A}\end{bmatrix} &&+ \$.01 \begin{bmatrix}\text{ton-miles to} \\ \text{deliver produc-} \\ \text{tion from site B}\end{bmatrix} \\
&= .01 \int_0^x z\,dz &&+ .01 \int_0^{1{,}000-x} z\,dz \\
&= .01 \left[\frac{x^2}{2}\right] &&+ .01 \left[\frac{(1{,}000 - x)^2}{2}\right];
\end{aligned}$$

$$\therefore \qquad \text{transport costs (\$ per day)} = \frac{x^2}{100} - 10x + 5{,}000. \qquad (2)$$

Under these conditions, the value of x which minimizes transport costs will be 500 tons per day. That is, the market boundary lies equidistant between the two sources of production. How will this boundary be shifted if the two plants do not have equal production costs? Suppose that the variable costs of manufacture are \$4.00 per ton at location A, and \$6.00 at B. Variable manufacturing costs are then:

$$\begin{aligned} &\$4[\text{tons produced at A}] + \$6[\text{tons produced at B}] \qquad (3) \\ &= 4[x] \qquad\qquad + \; 6[1{,}000 - x]; \\ \therefore \quad &\text{variable manufacturing costs (\$ per day)} = 6{,}000 - 2x. \end{aligned}$$

Adding together the transport and variable manufacturing costs in equations (2) and (3), we find that the combined operating costs vary as follows with the decision variable x:

$$\begin{matrix}\text{combined transport plus variable} \\ \text{manufacturing costs (\$ per day)}\end{matrix} = \frac{x^2}{100} - 12x + 11{,}000. \qquad (4)$$

Again, it is a simple matter to apply calculus methods, equate the derivative of (4) to zero, and to observe that the optimal production level at plant A increases to 600 tons per day, and that the market boundary now lies 600 miles away from this plant. (See point C on Figure 1.)

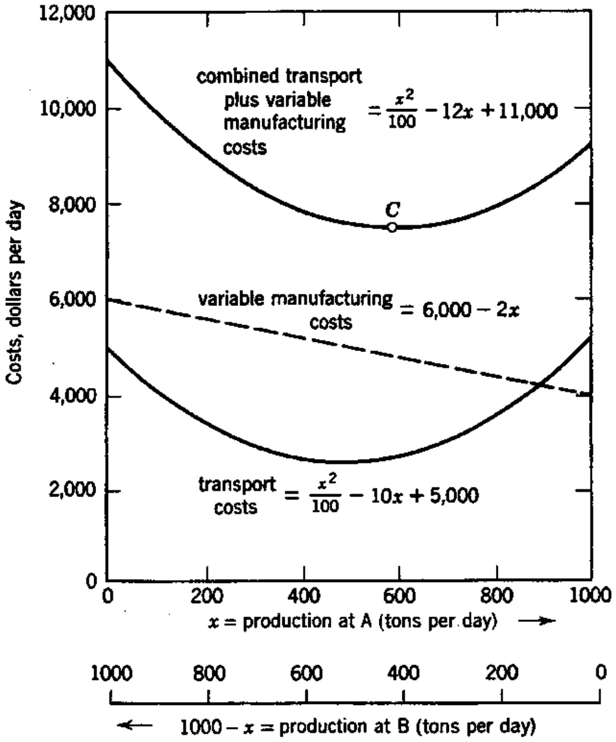

FIGURE 1. Transport and variable manufacturing costs in a two-site case, no fixed costs.

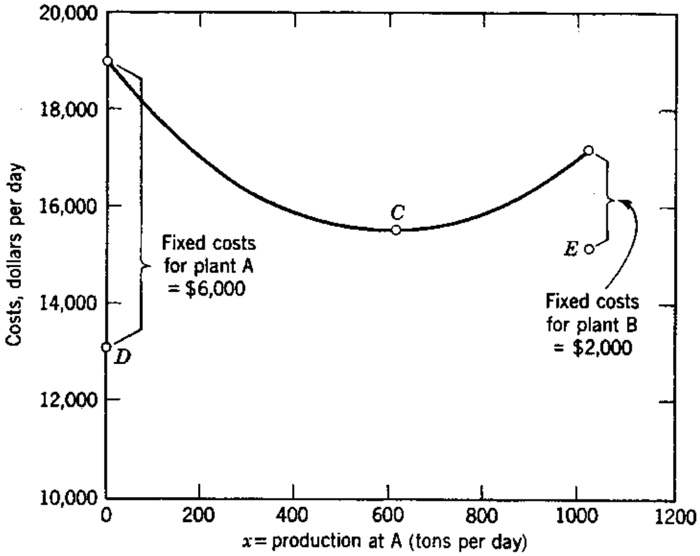

FIGURE 2. Transport, variable, and fixed manufacturing costs in a two-site case.

So far, so good. But what if there are economies of scale in manufacturing? Suppose, for example, that it takes a fixed cost of $6,000 per day to get plant A into operation and a fixed cost of only $2,000 for plant B.[5] These costs are taken to be independent of the size of A and B, and are avoidable only by installing zero capacity at these points. Now in order to find the minimum system costs, it is not sufficient to search for a *local* optimum along cost relation (4). Figure 2 indicates that there are three distinct local optima to be explored in this case: *C*, *D*, and *E*. As in Figure 1, point *C* refers to the cost minimum for the case in which both plants are constructed and fixed costs totaling $8,000 are incurred. Point *D* here refers to the case in which there is zero production at site A, and the entire output is concentrated at B. This means the elimination of $6,000 of fixed costs for plant A. Similarly, at point *E*, no plant is built at location B, and $2,000 worth of fixed costs are avoided there. In general—with n possible sites and with the requirement that at least one plant be constructed—there will be $2^n - 1$ local optima that need to be examined. This is no mean task, especially when one considers the possibility that a nonobvious solution, e.g., no plant at A as in Figure 2, may be the optimal one.

KEY ASSUMPTIONS OF THE PROGRAMMING MODEL

Our model is phrased in the following general way: Given a fixed demand for synthetic ammonia fertilizers within individual countries, what combina-

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[5] Even though marginal costs remain constant at positive levels of output, this is nevertheless a true case of economies of scale. Average costs exceed marginal costs over the entire output range.

tion of production plus imports would satisfy these requirements at a minimum cost to the Latin American region as a whole? This global cost criterion pays no attention to the distribution of gains among individual countries within the region, and should not be regarded as a *prediction* of what will occur. The purpose of this model is to trace an ideal without pretending that such an arrangement would necessarily be politically achievable. Nevertheless, the knowledge of such an ideal is believed to be an important piece of background information in reaching practical decisions related to the development of a regional common market.

This exploratory model is completely static. Demands are projected as of the year 1965, and no attempt is made at intertemporal optimization. Conclusions drawn from the present study are intended to aid in attacking the more ambitious problem: What is the optimal *sequence* of additions to capacity? For an intertemporal formulation that ignores locational aspects, see Manne (1961); but it is evidently more meaningful to combine the locational and the intertemporal considerations within a single model. This is an obvious next step in the analysis of optimal plant size.

In the original study—and in the present one also—five alternative production sites are considered. The combined markets of the twenty Latin American republics are represented by twelve distinct market points. The individual production sites and market combinations are not identified by name here. The empirical bases of the original study are under revision, and the results of these computations are accordingly to be construed as illustrative in nature.

The process data shown in Table 1 formed the basic information out of which the static cost minimization model was constructed. First, in order to use these data, we linearized the economies-of-scale relationship. Instead of a constant-elasticity labor or capital input relation of the form

$$\text{capital or labor input} = \beta x^{\alpha} \tag{1}$$

this was approximated with a piecewise linear function of the form:

$$\text{If } x \begin{Bmatrix} = 0 \\ > 0 \end{Bmatrix} \quad \text{capital or labor input} \begin{Bmatrix} = 0 \\ = a + cx \end{Bmatrix}. \tag{5}$$

In this linear approximation, the constants a and c were chosen so that function (5) would exactly coincide with (1) at the plant scale previously indicated as the "maximum" for which empirical evidence is available. The linear approximation of (5) is intended to be close to the nonlinear function (1) over the entire range between the "minimum scale" and the "maximum scale." The two functions also coincide at the origin, but diverge sharply at low scales near the origin (see Figure 3).

In order to keep the model in the simplest possible form, the linear approximation (5) has been adopted over the entire range of process scales, and has even been extrapolated into the region beyond the previously recorded maxi-

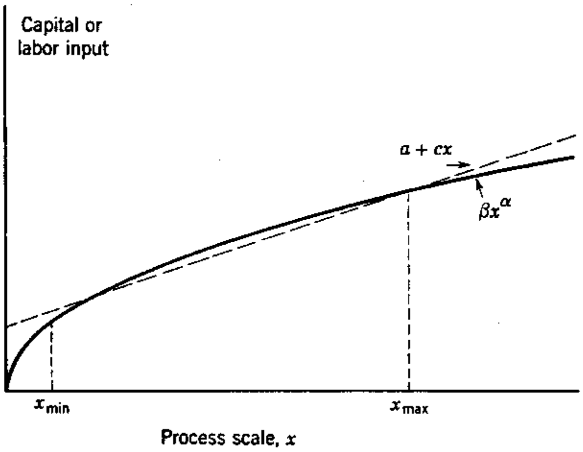

FIGURE 3. Linear approximation to capital and labor inputs.

mum-size plant. In view of the possibility that the marginal inputs of labor and of capital may either increase or decrease beyond this "maximum" plant size, the linear input function (5) is by no means a desirable one. It is employed here only in the absence of any better information. Total 1965 fertilizer demands for ammonia in the Latin American region are some 50% above the "maximum" plant size; and for sulfuric acid, ammonium nitrate, ammonium sulfate, and nitric acid, the fertilizer demand is respectively three, four, four, and twelve times their "maximum" size. Thus, if we have overstated the extent of economies of scale through the cost function (5), it is likely that we have *under*stated the number of plants that ought to be built so as to serve the region at minimum total cost.

After linearizing the capital and labor input relationships, the next step was to condense the seven activities of Table 1 into just two stages of production: first, ammonia production and then fertilizer production. In the case of ammonia production, Table 1 specifies three alternative processes based respectively on refinery gas, natural gas, and coke oven gas. Here only one of the three alternative activities is utilized—the one based on natural gas. This raw material is available at each of the five potential production sites, and generally represents a favorable alternative.

In the secondary stage—fertilizer production—the simplification consists of supposing that the two principal products—ammonium nitrate and ammonium sulfate—are produced in all plants and demanded in all markets in the somewhat arbitrary fixed proportions of 60:40 in terms of *nitrogen*. Once these proportions are set, the input mix of nitric acid and sulfuric acid becomes determined; and this in turn fixes the indirect requirements of ammonia for producing nitric acid. The product of the second stage is measured in terms of ammonia-equivalent units, i.e., as that quantity of a 60-40 nitrate-sulfate fertilizer product mix which is made from 1 million pounds of ammonia.[6] With this convenient normalization, we can translate each unit of demand for the final product directly into a unit of demand for the intermediate material, ammonia. This composite product mix excludes two forms of fertilizer that were believed to be of minor importance within the Latin American market: anhydrous ammonia and urea. Anhydrous ammonia demand is rapidly growing, but its handling and application are complex in comparison with the solid fertilizers.

COST ESTIMATING PROCEDURE

Having simplified the original production sequence, the next step was to convert into common units of process costs.[7] All costs have been calculated as annual rates over the life of the plant, not as discounted cash flows. The local prices assigned to capital and labor reflect not only the prevailing capital costs and wage rates, but also an allowance for accounting overhead cost items which have not been included separately. The estimating factors for the latter items have been taken to be approximately the same as in the United States. For example, indirect production cost is estimated at 55% of direct labor plus 2.3% per year of fixed investment. The estimating percentages are given in the literature as percentage ranges. A single percentage figure was obtained for each item by taking a reasonable average figure based on these ranges, and this average was then used for all processes.[8]

It should be noted that the above procedure is subject to criticisms on several counts: First, it disregards the variation of estimating percentages as between individual processes. Second, it transfers value ratios incorporating U. S. price relationships to Latin America. And third, it disregards the possibility of adapting the technology of the ancillary operations (materials handling, maintenance, etc.) to the factor-price relationship prevailing in the less industrialized countries. Despite these shortcomings, the above-mentioned procedure was adopted. The necessary information for a more refined estimate was unavailable.

For all activities, the percentages reflecting an allowance for the total of indirect accounting items are the following: 85% on direct operating labor and 23% on fixed capital, excluding interest but including 15% for depreciation. Thus, if the local price of labor at location i were L_i per man-hour, the annual direct man-hour requirements were multiplied by $1.85L_i$. And if the appro-

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[6] One million pounds of ammonia-equivalent product mix represents 1.365 million pounds of ammonium nitrate *plus* 1.430 million pounds of ammonium sulfate. The nitrogen content of this unit is .750 million pounds. Ammonium nitrate and ammonium sulfate contain 33% and 21% nitrogen, respectively.

[7] In addition to labor, capital, and raw materials, input costs had to be determined for steam and power. These latter were estimated at their opportunity costs. In particular, the price of power was estimated on the basis of initial investment and operating costs of a 50,000 kilowatt thermal plant, serving a complex of industries with a continuous pattern of operations and a resulting high (90%) power factor.

[8] See Isard, Schooler, and Vietorisz (1959), p. 59.

priate return on capital at location i was r_i, the initial investment was multiplied by $(r_i + .23)$. The capital return varied as follows: for plant sites 1, 4, and 5, 15%; for plant site 3, 12%; and for plant site 2, 10% per year.

The manufacturing cost estimates are summarized in Table 2. Annual costs are shown separately for the primary stage, ammonia production, and for the secondary stage, the production of both nitrate and sulfate fertilizers. In line with the piecewise linear approximation to labor and capital inputs discussed previously, these costs include a variable component proportional to the output capacity and a fixed component independent of the size of plant.[9] The reader should keep in mind that these cost estimates are inherently subject to a wide margin of error, and that no great accuracy is claimed for them.

TABLE 2

FIXED AND VARIABLE MANUFACTURING COSTS (UNIT: THOUSANDS OF DOLLARS PER YEAR)

	Location Number				
	1	2	3	4	5
Variable Costs per Unit of Annual Capacity Output					
Ammonia (unit: 1.000 million pounds of ammonia)	20.4	20.2	21.2	35.3	33.9
Ammonium nitrate (unit: 1.365 million pounds of ammonium nitrate)	11.7	10.8	11.2	13.9	13.7
Ammonium sulfate (unit: 1.430 million pounds of ammonium sulfate)	9.5	8.0	6.3	9.3	8.7
Fertilizer production, excluding costs of ammonia (unit: 1.000 million pounds of ammonia-equivalent product mix, i.e. 1.365 of nitrate plus 1.430 of sulfate)	21.2	18.8	17.5	23.2	22.4
Fixed Costs for Plant Installation					
Ammonia production	1,760	1,656	1,626	1,784	1,782
Ammonium nitrate fertilizer	458	500	425	477	476
Ammonium sulfate fertilizer	352	451	329	379	376
Fertilizer production	810	951	754	856	852

The final component in cost estimation consisted of the transport rates.

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[9] In calculating these costs, no allowance has been made for the value of working capital tied up within the system. Since working capital requirements are generally estimated as a percentage of final sales, and since these final sales are a fixed element of the model, total working capital requirements will be constant, regardless of the geographical pattern of production.

These were derived on the basis of the cost of two-year charters and operating expenses of cargo ships, allowing for partial utilization on return voyages. The resulting rates are considerably below the conference tariffs quoted for chemical products, but it should be remembered that the quoted rates on chemicals reflect very small-scale movements.

The transport rate on ammonia is taken as double the rate on ordinary solid chemicals; that of ammonium nitrate is taken as 20% over the latter, due to the existence of an explosion hazard. On these assumptions, it works out that the transport rate for the solid fertilizer (expressed in ammonia-equivalent units) is 1.534 times the transport rate per million pounds of the ammonia intermediate.

The potential manufacturing sites are not necessarily located at the same point as the center for fertilizer distribution within a market area. Thus, it would cost $11.0 thousands to ship an ammonia-equivalent fertilizer unit from plant site 1 to the market center in its own country; and only $10.3 thousands to ship it from that plant site to the market center in country 4.

Import prices of ammonia and of the fertilizer products are based upon world market quotations, and are taken to be independent of the Latin American demand.

ALGEBRAIC FORMULATION

There are four sets of decision variables here. Those variables known as x_{ij} and y_{jk} may take on any nonnegative values, but w_i and z_j are restricted to integer values of either zero or one:

x_{ij} = units of ammonia (i.e., millions of pounds per year) produced at location i and shipped to fertilizer production location j.

y_{jk} = number of ammonia-equivalent units of fertilizer (i.e., 1.365 million pounds of nitrate plus 1.430 of sulfate) produced at fertilizer production location j and shipped to market k.

w_i = fraction of fixed charge incurred for an ammonia plant at i ($w_i = 0$ or 1).

z_j = fraction of fixed charge incurred for a fertilizer plant at j ($z_j = 0$ or 1).

Associated with these decision variables, there are four sets of cost coefficients:

a_i = fixed annual charge for the construction and operation of an ammonia plant at i.

b_j = fixed annual charge for the construction and operation of a fertilizer plant at j.

c_{ij} = variable annual construction and operation cost per unit of annual ammonia production at i, plus the cost of transportation from i to j.[10]

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[10] In the case of imports from the world market, these are regarded as coming from production source 0. Hence, the coefficients c_{0j} and d_{0k} refer, respectively, to the cost per unit of ammonia imported into location j, and the cost per unit of fertilizer imported into market k.

d_{jk} = variable annual construction and operation cost per ammonia-equivalent unit of annual fertilizer production at j, plus the cost of transportation from j to market center k, excluding costs of ammonia to avoid double counting.

The annual rate of 1965 requirements for market k is denoted by R_k, and is measured in terms of ammonia-equivalent units. With these definitions, the programming problem may be phrased as follows:

$$\text{Minimize:}\quad \sum_i a_i w_i + \sum_j b_j z_j + \sum_i \sum_j c_{ij} x_{ij} + \sum_j \sum_k d_{jk} y_{jk} \tag{6}$$

subject to:

$$\sum_i x_{ij} = \sum_k y_{jk} \qquad (\text{all } j); \tag{7}$$

$$\sum_j y_{jk} = R_k \qquad (\text{all } k). \tag{8}$$

$$\text{If } w_i \begin{Bmatrix} = 1 \\ = 0 \end{Bmatrix}, \quad \text{then} \sum_j x_{ij} \begin{Bmatrix} \geq 0 \\ = 0 \end{Bmatrix} \qquad (\text{all } i). \tag{9}$$

$$\text{If } z_j \begin{Bmatrix} = 1 \\ = 0 \end{Bmatrix}, \quad \text{then} \sum_k y_{jk} \begin{Bmatrix} \geq 0 \\ = 0 \end{Bmatrix} \qquad (\text{all } j). \tag{10}$$

$$x_{ij},\ y_{jk} \geq 0; \tag{11}$$

and

$$w_i,\ z_j = 0 \text{ or } 1. \tag{12}$$

Expression (6) defines the costs that are to be minimized: the sum of fixed and variable costs of production, plus transportation charges, plus import costs. Note that the alternative of importation is recognized through the availability of source 0, and that there are zero fixed charges associated with the utilization of that source.

There is one material balance condition of type (7) for each of the possible fertilizer production sites j. The left-hand side measures the sum of ammonia inputs received from all possible primary locations and the right-hand side refers to the distribution of fertilizer product to all possible markets k. Since production is measured in terms of ammonia-equivalents, these two sides are necessarily equal.

Conditions (8) refer to the delivery requirements at each of the markets k. Demands must be satisfied out of fertilizer production at one or another of the sources j.

Conditions (9) and (10) are of an either-or type. If, for example, w_i is set at unity, the full fixed charge of a_i is incurred at location i, and the production of ammonia may be set at positive levels there. Otherwise, there are zero fixed costs and zero production.

Since the index 0 is employed to denote the import source, we shall adopt the convention that $w_0 = z_0 = 1$. This means that no fixed charges need to be incurred in order to utilize imports as a source of supply of either ammonia or fertilizer.

Table 3 contains the numerical values of the cost coefficients and market

TABLE 3

COST COEFFICIENTS AND MARKET REQUIREMENTS

Cost Coefficients (units: thousands of dollars)

Plant Location i or j	Ammonia Plant Fixed Costs, a_i	Fertilizer Plant, Fixed Costs, b_j
0	0	0
1	1,760	810
2	1,656	951
3	1,626	754
4	1,784	856
5	1,782	852

Variable Ammonia Costs, c_{ij}

		Fertilizer Plants, j					
		0	1	2	3	4	5
Ammonia Plants, i	0	0	56.0	47.4	47.6	54.4	51.0
	1		20.4	33.6	34.5	26.9	29.5
	2		33.4	20.2	26.7	31.3	28.1
	3		35.3	27.7	21.2	35.0	32.2
	4		41.8	46.4	49.1	35.3	42.1
	5		43.0	41.8	44.9	40.7	33.9

Variable Fertilizer Costs, d_{jk}

		Markets, k											
		1	2	3	4	5	6	7	8	9	10	11	12
Fertilizer Plants, j	0	86.1	76.4	76.8	87.0	83.6	90.3	75.4	76.4	81.2	96.4	82.4	86.4
	1	32.2	44.0	45.8	31.5	34.1	42.0	43.4	40.3	37.8	42.9	36.6	30.8
	2	36.6	24.4	29.2	37.6	34.2	41.0	26.1	24.2	29.9	48.9	31.9	37.6
	3	37.0	27.9	23.1	41.3	38.4	40.9	24.7	26.8	29.6	52.8	32.6	41.3
	4	38.5	41.9	48.0	23.2	32.2	48.5	44.6	44.1	43.1	34.5	42.4	28.9
	5	44.0	35.3	41.1	33.2	29.8	54.1	37.7	37.1	42.8	44.6	44.4	32.6

Market Requirements, R_k (unit: millions of pounds per year of ammonia-equivalent fertilizer product)

Markets, k											
1	2	3	4	5	6	7	8	9	10	11	12
9.2	59.9	95.8	185.3	344.1	66.1	168.6	50.0	34.0	11.1	109.5	47.0

requirements utilized in our calculations. These coefficients were put together on the basis of the process yields, cost estimates, etc. described in previous sections.

ENUMERATIVE SOLUTION

It is an elementary exercise to reformulate conditions (6) through (12) in terms of an integer programming model. However, in view of a number of disappointing results with the Gomory (1958) cutting plane technique, it was decided to try another approach, complete enumeration of the local optima. Only a small fraction of the number of variables in this particular case are of a zero-one variety: the ten unknowns w_i and z_j. This means that there are only $2^{10} = 1{,}024$ local optima to be examined here. Once a particular pattern of zeros and ones is assigned to the ten zero-one variables, the remaining unknowns x_{ij} and y_{jk} are related to each other via a "transshipment" problem—one of the easiest of all linear programming structures. See Orden (1956).

The computational approach may be summarized as follows: Enumerate all possible combinations of the zero-one variables, finding a *local* optimum to each of the resulting 1,024 transshipment problems.[10] Variable costs, $\left(\sum_i \sum_j c_{ij}x_{ij} + \sum_j \sum_k d_{jk}y_{jk}\right)$, are calculated for each local optimum and added to the fixed costs $\left(\sum_i a_i w_i + \sum_j b_j z_j\right)$ associated with that particular combination of zero-one variables. The combinations are then ranked in ascending order of total costs, thereby determining not only a minimum-cost solution, but also a cumulative distribution of all local optima. See Figure 4.

According to Figure 4, the optimal combination is one that will cost $64.9 millions per year. This solution is based upon utilizing a single integrated plant at location 2 to supply the entire Latin American market. But Figure 4 also indicates that there are many near-optimal solutions. For example, there are altogether 32 combinations (3.1% of the 1,024 combinations) with costs estimated to lie within the range of $64.9 to $66.9 millions. Considering the low precision of the initial data, it would be unwise to insist that the

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[10] A special-purpose computing algorithm was written by Donald Hester and Fred Brown for the IBM 650 computer. This algorithm took full advantage of the fact that there were no constraints in the problem that arose from *existing* plant capacity limitations; i.e., there were no upper bounds upon $\sum_j x_{ij}$ or upon $\sum_k y_{jk}$. Each local optimum could therefore be produced on the initial iteration of the transshipment problem. This meant that despite the comparatively slow speed of the IBM 650, it was possible to enumerate all 1,024 local optima within the comparatively short interval of four hours.

In retrospect, we realized that we had failed to take advantage of one special feature that would have virtually halved the computing time for this problem: It never pays to construct a greater number of ammonia plants than fertilizer plants. That is, $\sum_i w_i \leq \sum_j z_j$. In an enumerative solution, one ought to take full advantage of such a priori restrictions.

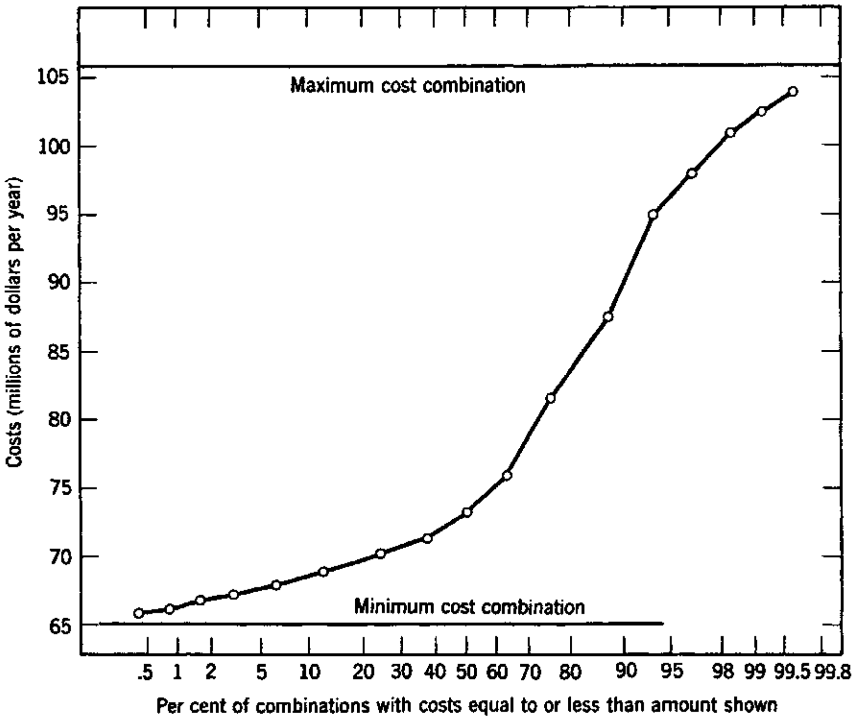

FIGURE 4. Cumulative cost distribution, 1,024 zero-one combinations.

optimum is really unique. Rather, there is a large number of alternative solutions that will do almost equally well.

Heller (1960) had reported some Monte Carlo experiments in which the sample distribution of schedule times in a job-shop sequencing problem appeared to be distributed in an approximately Gaussian fashion. These experiments led him to suggest that until such a time as an economical algorithm is found for obtaining an exact optimal solution to the scheduling problem, it would be wise to proceed by sampling. "Decision theory affords us a systematic procedure by which we can answer the question: How many samples should we take before the probable cost of taking another sample will be more than the probable gain in finding a better schedule?" [Heller (1960), p. 179.]

Our results lend further credibility to Heller's suggestion concerning statistical sampling. Unlike his experiments, this one does *not* result in a symmetrical frequency distribution of costs. Our median is $73.0 millions—much closer to the minimum-cost rather than to the maximum-cost combination. If this skewness is typical of other plant location problems, sampling techniques should indeed be even more efficient than in the Gaussian case studied by Heller. Further experiments are in progress—using random numbers for costs and requirements in the plant location model—and will be reported on some future occasion. The preliminary results of these experiments are generally consistent with the hypothesis of skewness.

How large a sample is needed in order to be reasonably confident that one will obtain a near-optimal result? Fortunately, there is a distribution-free statement that can be made about sample size. Let n represent the size of sample needed in order to ensure with probability α that there will be at least a proportion β of the population of all possible cost outcomes whose costs exceed the smallest observation in the sample. Then it can be shown that:[11]

$$n = \frac{\log (1 - \alpha)}{\log \beta}. \tag{13}$$

For example, in order to be 99% sure that the smallest member of a sample will have a cost within the lowest half of one percentile of the population, we would have $\alpha = .990$ and $\beta = .995$. Then the sample size n would have to be only 921 cases—regardless of the shape of the cost distribution, the total number of locations involved, etc. Needless to say, expression (13) represents a very crude criterion for choosing the best sample size. What is more relevant is an estimate of how many dollars could be saved by choosing a particular sample size—not an estimate of the *fraction* of the population with costs below those of the lowest member of the sample.

SOME ALTERNATIVE SOLUTIONS

It is an important advantage of an enumerative method that it permits us to examine many nonoptimal solutions, as well as the optimal one itself. From among these nonoptimal solutions, a selected group is presented in Figure 5. From this figure, what is clear is that the plant location problem does not just consist of picking the right *number* of plants. True, a single integrated plant in location 2 represents the optimal solution. However, this is closely followed by two two-plant combinations: locations 1 and 3 or, alternatively, locations 1 and 2. By contrast, a single integrated plant in either location 4 or 5 would come close to being the worst of all possible solutions.

It is sometimes suggested that problems of this type can be approximated by neglecting the fixed costs, minimizing the variable costs alone, and then setting the zero-one variables at levels of unity wherever needed in order to ensure satisfying conditions (9) and (10). This approximation would not have worked too badly in this particular instance. Total costs would come to $68.3 millions annually, around $3.4 millions away from the minimum. Note that this solution does not call for integrated plants at all points. The fertilizer plant at 4 would be supplied with ammonia raw material produced at location 1.

The bar labeled "Noncooperative" in Figure 5 is not one that corresponds to any of the 1,024 local optima that were enumerated. Instead, it corresponds to the results of pursuing a narrowly nationalistic policy on the part of the five potential major producing areas. In each of these five cases, it is supposed that the local market demand is the only one being considered. Imports from

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[11] T. N. Srinivasan called our attention to this criterion for sample size. A similar expression may be found in Bowker and Lieberman (1959), p. 232.

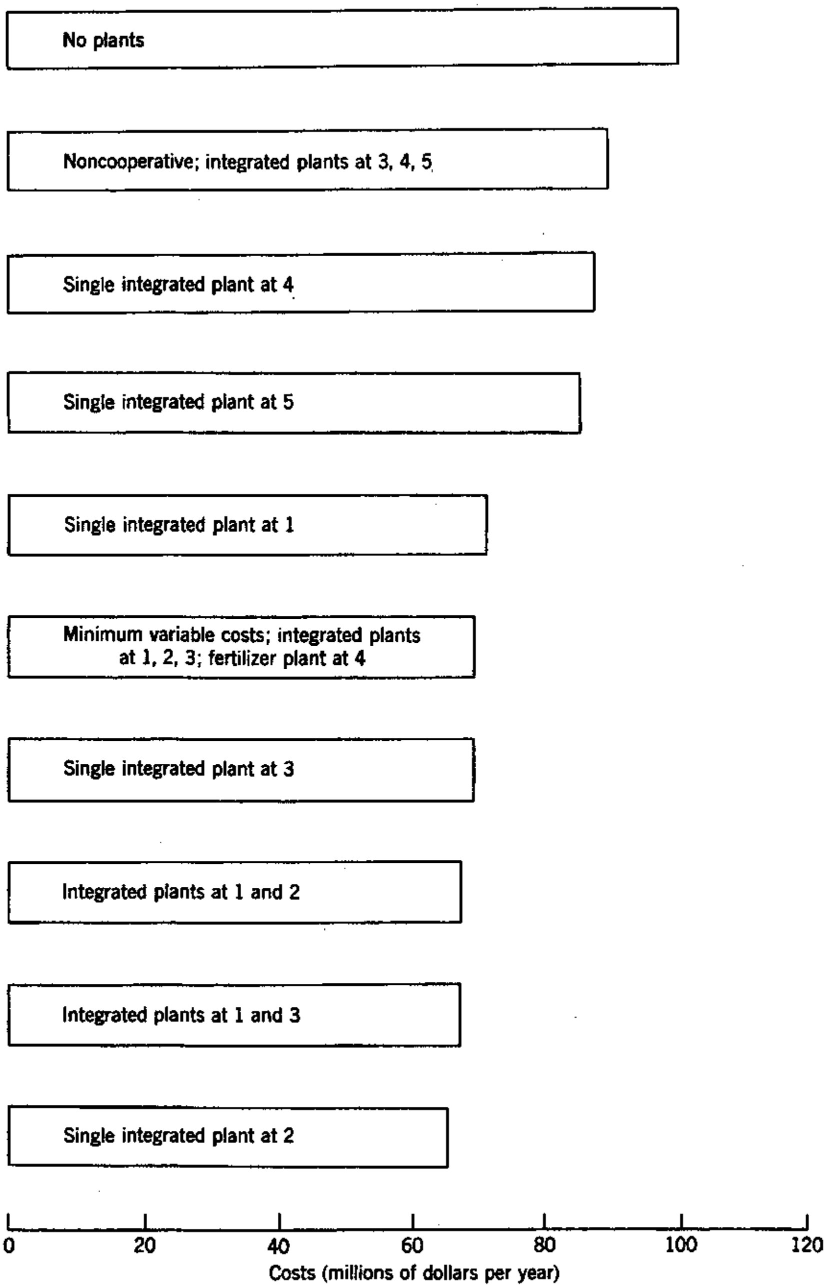

FIGURE 5. Some alternative solutions.

the outside world are taken to be the only alternative to domestic production. Then if the domestic demand is large enough so that the savings on imports pay for the fixed costs, a plant is constructed locally.[12] In all other cases, the area remains dependent on imports from the world market. This type of noncooperative solution evidently represents a cost improvement over complete reliance upon imports.[13] However, it is an extremely costly way to satisfy the market demand. Total costs here would come to $89.4 millions—38% higher than in the theoretically optimal solution. If this example is at all representative, there are substantial gains to be had from economic integration within Latin America.

EXPANSION OF MARKET SIZE

As the Latin American market grows in size, it is clear that the solution involving a single integrated plant will no longer remain optimal. Instead, the problem consists of determining an optimal *sequence* of locations and of plant capacities. Our computing resources did not permit us to attack this more ambitious problem. However, they did allow us to answer the following question: Suppose that all demands were multiplied by a uniform factor θ so that the requirements for market k amounted to θR_k, what then would be the optimal size and location of plants—neglecting the problem of the time path over which these units were constructed?

In analyzing this comparative statics version of the problem, we were aided by one feature of restrictions (7) and (8). Consider any particular plant combination, i.e., set of values assigned to the zero-one variables w_i and z_j. Then if the original local optimum is attained at activity levels of x_{ij}^* and y_{jk}^*, with the altered requirements the new local optimum is reached at θx_{ij}^* and θy_{jk}^*. This feature of homogeneity made it possible to trace out the solution ranges shown in Table 4. These ranges were generated from the same computing run that produced the 1,024 locally optimal solutions previously described, i.e., for the case of $\theta = 1.00$.

As θ is varied parametrically, six alternative plant combinations become optimal. For example, the single integrated plant in area 2 remains optimal for all values of θ in the interval between .08 and 1.31. At the latter point, it becomes desirable to construct a fertilizer plant in area 4. At $\theta = 1.83$, it becomes optimal to construct an ammonia plant in location 1 in order to supply the fertilizer plant in 4. After this, it pays to build a fertilizer plant in 1, and finally an integrated plant in area 3. For all levels of θ above 4.25, the

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[12] Area k builds an integrated plant of size R_k if, and only if:

$$a_k + b_k + (c_{kk} + d_{kk})R_k \leq d_{0k}R_k.$$

[13] The noncooperative solution calls for integrated plants to be built in locations 3, 4, and 5. Note that areas 4 and 5 have large internal markets, but that their production costs are high.

Through a side calculation, it can be shown that it would not pay for any country to supply its own requirements with a nonintegrated fertilizer plant, importing the intermediate material, ammonia, from the world market.

optimal combination is the one that minimizes variable costs. Minimization of variable costs must always be the asymptotic solution as θ increases indefinitely.

The sensitivity analysis of Table 4 suggests—but does not conclusively justify one rule of thumb that might be used to facilitate similar calculations. If a zero-one variable is not forced to operate at a unit level in the minimum variable cost solution, then nothing is lost by setting it at a zero level. Armed with a collection of similar rules of thumb—or rather, precepts for sample stratification—it does not seem unduly optimistic to expect that the intertemporal optimization problem, the optimal time path of plant construction, could be attacked by sampling methods within the near future.

TABLE 4

EFFECTS OF MARKET SIZE PARAMETER, θ

	Optimal Values of Zero-One Variables										Fixed Costs for Zero-One Variables	$\theta = 1$, Variable Costs
Limits of θ within Which the Optimal Zero-One Variables Remain Unchanged	w_1	w_2	w_3	w_4	w_5	z_1	z_2	z_3	z_4	z_5	$\sum_i a_i w_i + \sum_j b_j z_j$ (unit: millions of dollars per year)	$\sum_i \sum_j c_{ij} x_{ij} + \sum_j \sum_k d_{jk} y_{jk}$ (unit: millions of dollars per year)
0 – .08	0	0	0	0	0	0	0	0	0	0	0	97.03
.08 – 1.31	0	1	0	0	0	0	1	0	0	0	2.61	62.32
1.31 – 1.83	0	1	0	0	0	0	1	0	1	0	3.46	61.67
1.83 – 3.24	1	1	0	0	0	0	1	0	1	0	5.22	60.71
3.24 – 4.25	1	1	0	0	0	1	1	0	1	0	6.03	60.46
4.25 – ∞	1	1	1	0	0	1	1	1	1	0	8.41	59.90

STRATIFIED SAMPLING

One bit of evidence in favor of stratified random sampling will be recorded here. Prior to the date at which our computer solution was reached, six members of the Cowles Foundation at Yale University participated in the following experiment. They were all provided with the data shown in Table 3, and were each asked to deposit a small sum of money along with their guess as to the optimal plant combination. An anonymous donor contributed a further sum of money in order to bring this pool up to a grand total of $1.60.

None of the contestants guessed the optimal solution, but two of them submitted combinations that implied costs of $66.0 millions per year, only $1.1 millions above the minimum-cost solution. These two contestants each worked several hours, and had committed several arithmetic errors in their overall evaluation. By contrast, the most costly combination was submitted by someone who had flipped a coin ten times in order to construct his pattern of zeros and ones for the variables w_i and z_j. There are two morals to be drawn here: (1) Virtue—i.e., hard work—is sometimes rewarded. And (2), human beings have a knack for picking out plausible patterns of zeros and ones—even though they do not always do a good job at detailed arithmetical evaluation. Stratified sampling may provide the right kind of man-machine combination that is needed to attack integer programming problems of this class.

A Concluding Note

The reader is cautioned not to attach too much significance to the specific numerical outcome of this experiment. It is quite possible that the selection of a single integrated plant has resulted from one or more of the following kinds of bias on our part: an underestimate of demand; an underestimate of transport costs; an overestimate of natural gas costs in countries 4 and 5. Or perhaps the linear extrapolation rule (5) errs in failing to take account of diseconomies within extremely large plants. In order to guard against this latter possibility, it might well be preferable to decide in favor of one of the split-location solutions, e.g., two smaller plants in locations 1 and 3. But regardless of whether it is best to build one large plant or several smaller ones, our calculations point to one clear-cut conclusion: that there are significant gains to be had through regional cooperation, rather than through development of this branch of the chemicals industry on a country-by-country basis.

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PART III

FOOD AND AGRICULTURE

CHAPTER 7

SPATIAL PROGRAMMING MODELS TO SPECIFY SURPLUS GRAIN PRODUCING AREAS

Earl O. Heady and Alvin C. Egbert

This chapter reports the use of activity analysis to estimate optimal spatial patterns of grain production for U. S. agriculture. It is only the first phase of a larger and on-going analysis dealing with the interregional structure of the farm industry. The analysis reported covers programming of only wheat and feed grains. Subsequent analysis is being extended to include cotton, soybeans, and selected other crops and various classes of livestock and livestock commodities. Results become increasingly realistic and useful as more commodities are added, as coefficients are upgraded with time, and as models are devised to include other pertinent restraints. The overall study was initiated in 1955 as soon as data from the 1954 farm census became available. Hence, 1953 is used as the "point of time departure" for the analysis. Production is programmed against certain restraints defined by the year 1953 since it was the last cropping season in which government production controls were not in wide use and reflects some possible maximum restraints in management and physical resource supplies or possibilities. The on-going phases of the analysis will bring the programming framework up to date in terms of technical coefficients, production functions, and demand restraints. Furthermore, the analysis is being projected into the future as a means of specifying regional production patterns and suggested agricultural policies relating to prospects in food demand and agricultural production possibilities. The latter step is being followed because the technical coefficients of agriculture are not static but change with time as research investments in one time period give rise to new production functions used in subsequent time periods and a decline in input for a given output.[1] As an illustration of this trend, farm output in the United States has increased by about 110% in the last 40 years, while total inputs have increased only by 15%. Also the makeup of the input mix is changing greatly, with some specific categories of resources declining rapidly and others increasing at an equally rapid rate. Hence, we hope to be able to project new technology of one time period into coefficients of later periods, given the time path in adoption of new technology used by agriculture in

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[1] For further detail on data used and regions covered see Heady and Egbert (1959) and Egbert and Heady (1961).

aggregate. Our later analyses should become of increasing value for projections relating to national planning and policy. However, the production patterns explained in this summary for the initial phase of the study remain generally "stable" in our preliminary analysis of more complete models for agriculture. Addition of new commodity activities by regions and use of technical coefficients predicted for current and future time change the programmed spatial allocation of wheat and feed grains hardly at all in terms of major regions. We do, however, find some slight changes under technology including high fertilization rates; for example, some corn acreage is specified in the Southeast. Yet the change is small and our programmed results would generally specify the same aggregate modification of agriculture's spatial pattern; the changes specified by subsequent models and phases of our study being relatively minor intraregional modifications of the results summarized in this chapter.

Purpose of Analysis

This study was initiated for purposes which we hope to prove quite practical over the long run. The solutions generated are expected to have use in developing and implementing public policy and educational programs in agriculture. Our early data are, in fact, already finding use in these directions. The American public currently has been spending about $200 million per year in education to direct change in agriculture and around $2 billion to compensate agriculture for costs which are judged to fall on it because of economic change and to redirect structure of the industry to that more consistent with modern production possibilities and food demand under regime of high per capita income. The great difficulty in both of these programs has been, however, lack of sufficient knowledge of what the regional pattern of production should be, or would be under a use of resources more in conformity with current patterns in demand and production possibilities. In these attempts, surpluses of wheat have come to be over twice as great as annual food requirements and feed grain stocks have come to equal half of annual requirements. Educational programs within individual states have had an inadequate basis for guiding farm producers into or out of the industry and public price and production policy has lacked empirical foundation for guiding interregional adjustment to modern needs of factor and commodity markets. Aside from the war and period immediately following, U. S. field crop production has been under production policy restraint for 30 years. Even during the period of major free markets within that time, the wartime demand pattern was considerably different from that of the present and differential changes have taken place in technology and commodity supply functions among regions. Governmental production controls have held the spatial pattern of grain and cotton production quite closely to the mold of the past, even though changes in technology and factor prices exert pressure to bring realignment of total supply among regions. Public policy in recent years has expressed some increased flexibility, however; with some indication of allowing adjustment of the industry in differential regional pattern. Yet little has been, or still is, known about the extent of these adjustments if they were allowed to take place in a manner consistent with consumer demand and regional supply functions for commodities and resources. Which regions would shift out of production? Which would remain in, but simply lessen resource input to allow more extensive production of the same crop mix? How would population of various regions vary in extent of migration with decline of local farm industry? What treasury costs would be involved in bringing about these regional adjustments in supply of basic commodities? These are some of the more practical questions which we wished to answer from our programming solutions. The results reported here appear useful in this sense, but data forthcoming from subsequent phases will be even more so.

The general purpose of this study is to use linear programming models to specify how grain production should be allocated over the nation. The objective functions used as criteria for this allocation are those of minimizing costs or "maximizing profit" if production not only meets the land restraints of each region, but also meets two discrete national demand restraints for wheat and feed grains.

The results presented in this chapter refer only to grain production under the model restraints outlined later. They are simply preliminary results generated as a step in our more complete and detailed analysis. While the results presented refer to a particular point in time, subsequent models not only will refer to later time periods and include livestock production, but also will explore the impact of stepping up the rate of technical change in all producing regions. While these steps relate to the overall purpose of the study, we look upon the results presented in this chapter as also relating to these more specific objectives:

1. To formulate several programming models with special characteristics for analyzing particular facets of the grain surplus problem.
2. To obtain empirical solutions to the analytical models that will indicate comparative regional efficiencies of resource use in production of wheat and feed grains.
3. To use the empirical solutions to suggest optimum spatial production and land use patterns for wheat and feed grains.
4. To analyze weaknesses in the basic assumptions of the analyses and suggest ways of improving similar investigations.
5. To describe the problems encountered in collecting and processing data for the study, and to suggest means of acquiring improved data.
6. With the experience of this investigation as a basis, to suggest studies that would seem to be more adequate for analyzing regional resource-efficiency problems.

The results presented are tentative due to the complexity of the analysis problem and data deficiencies. They should be viewed as first approximations and the basis for further research. Extended analyses in progress are designed to erase many of the limitations evident in this phase. Due to the nature of the analysis problem, a particular methodology and its implications for solutions to surplus problems are emphasized in this report.

Alternative Empirical Techniques

Several major techniques are available for an analysis of the type reported here including linear programming, input-output, and time-series regression models. We selected process analysis over the other two because neither input-output nor regression models based on time-series data have any great meaning in an industry such as agriculture, where the coefficients have been changing rapidly and the mix of inputs and outputs are under obvious modification. While input-output and time-series regression or general equilibrium models allow generation of many coefficients, the coefficients are of more relevance as an academic exercise than in appraising structural change in agriculture. Agriculture has been passing through an era of extreme structural change and the coefficients and the factor and product mixes relating to a previous period or point in time have little value in determining how the interregional pattern will or should change.

We have made several input-output analyses of agriculture. The models used have included regional and industrial sectors important to agriculture.[2] But as indicated elsewhere, we look upon these as mainly of interest in providing a description of the structure of agriculture at a given historic point in time. Our studies show a great dependence of agriculture on nonfarm industry sectors, but trivial dependence of industry on agriculture. These types of facts are interesting, but tell us little about the internal structural changes of agriculture. In line with our previous comments, we believe that economic growth and extended food demand will not cause the product of agriculture to be drawn from regional sectors in the manner of the fixed-mix restraint imposed by input-output models.

Regressional models based on time series data also are mainly useful for projecting supply for a given structure of the industry and for extremely short-term forecasts. They have extreme limitations in predictions or analysis of potential interregional adaptations of agriculture in the framework of the purposes of our study. Predictions from time series regression models are obviously tied to past structure. Our purpose is to break from the constraints of the past when government policies obviated interregional changes which would have been forthcoming in response to new price and technical coefficients. The extreme limitations of regression models based on time series observations in prediction of supply and interregional or spatial adjustment of agriculture have been detailed elsewhere.[3]

Linear programming was selected as the method for analyzing potential interregional or spatial adjustments of agriculture in order to overcome the

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[2] *See:* Heady and Schnittker (1958), pp. 745–755; Heady and Carter (1960), pp. 978–991; Carter and Heady (1958); Schnittker and Heady (1958); and Peterson and Heady (1956).

[3] See the chapters by Heady, Learn, Staniforth, Diesslen, Jensen, and others in Heady et al. (1961).

specified limitations of input-output and regression models. Much work is still ahead before both data accumulation and relevant linear programming models allow us to attain the complete objectives of our study and to overcome empirical limitations of the type suggested above. Hence, until we have completed further steps in the study, our results are preliminary and might themselves be looked upon somewhat as academic exercise. Still we have found our preliminary data and model to have considerable utility in examining certain policy questions.

DATA ASSEMBLY

As in other mathematic programming of problems in "real world" context, the major professional input of the study was collecting and processing of data to forms required by the several models used. These data requirements are quite burdensome for an industry such as agriculture and for the detail of the models employed. They are much greater than for a single industrial firm or commodity, since they involve assembly of techniques and inputs used by different strata of farm firms in each region and weighting these to obtain regional coefficients.[4] The estimation of a single coefficient such as mean monetary or cash cost per unit of crop in a region requires first that estimates be obtained for various kinds and sizes of power units and machines, fertilizer types and amounts, tractor fuel, machine repair, insecticides and seeds used, and a large number of other individual inputs. Data on outputs are somewhat more readily available but also present difficulties in developing the data base for application of the programming models.

A great deal of effort went into accumulation of the relevant technical coefficients and yields. Most of three years was required to assemble basic coefficients. An attempt was made to provide as much accuracy and detail as possible in accumulating and measuring these coefficients. The task was large because coefficients and yields differ among regions and because the 104 regions used do not all correspond to the area basis on which certain crop and resource use statistics are reported. Statistics for some regions had to be assembled on the basis of county data. Research studies and the aid of agricultural economists in all states were used in building up the cost and technical coefficients for the various regions falling over the nation.

ALTERNATIVE PROGRAMMING MODELS

The overall objective of this study, then, is to determine the optimal allocation of wheat and feed grain production among regions, given the ends of

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[4] For some examples of firm applications of linear programming in agriculture and the detail of data used, see: Heady et al. (1961); Love and Heady (1961); Heady et al. (1958); Dean and Heady (1958); Heady and Gilson (1956); Mackie, Heady, and Howell (1958); Heady, Dean, and Egbert (1958): Smith and Heady (1960); and Heady and Candler (1958).

eliminating annual additions to surplus stock of grains and of adjusting agriculture in directions consistent with the competitive positions of various producing regions. While this phase of the study is partly methodological in nature, the results specify surplus grain producing regions in the United States, given the production restraints employed: An acreage of land "usable" for grain production in each region and quantities of wheat and feed grain to meet that required nationally for human and livestock consumption at a point in time. Given these restraints and the product prices and production costs of 1954, the optimal regional location of production is specified to include those areas that produce national grain requirements with objective function specifying either (*a*) minimum cost or (*b*) maximum profit, depending on the assumptions of the particular model. One hundred and four unique major grain producing regions were delineated for the analysis. These regions account for around 90% of United States' feed grain and wheat production. The small quantity of production in areas omitted is assumed to be independent of the system.

Five models were used and are designated as A, B, C, D, and E. The maximum regional acreage restraints are common to all five models. Food wheat, feed wheat, and a feed grain rotation represent the regional production activities in models A, B, C, and E. In model D the regional activities include food wheat, feed wheat, corn, oats, barley, and grain sorghums. Other differences between the models are: Models A, B, C, and D all have an objective function of minimum total production cost while model E has an objective function of maximum profit; annual land rents are included in the activity costs for model B, but not A, C, D, and E; and wheat and feed grain activities have separate regional production restraints for model C only. The structure and objectives of the five models are described below. Change is made from model to model in an attempt to add more realism to the analysis or to investigate a particular facet of the grain production and surplus problem. For each of the homogeneous grain producing regions, except where otherwise specified (model D), three types of grain producing activities are considered: food wheat, feed wheat, and a feed grain rotation. The quantity of grain produced by these three activities is limited by maximum acreage available in each region. Production costs of each activity include labor, power, machinery, seed, chemicals, and certain miscellaneous items. One central market is assumed for wheat and feed grain with cost of transporting grains from the producing regions being zero for three models.

MODEL A. The objective function for this model is

$$\min f(Y) = C_1Y_1 + \cdots + C_iY_i + \cdots C_mY_m \qquad (1)$$

where C_i is a subvector of per unit costs, containing r elements to represent costs of producing feed grains and wheat in the ith region; and Y_i is a subvector of crop outputs, with n elements representing production levels of the n crops. In this case, c_{ij}, the unit cost of producing the jth crop in the ith region, includes only the labor, power, machine, seed, fertilizer, and related inputs for each grain. In other words, land rent is not included as a cost. Neither are farm overhead or fixed costs included. We have $m = 104$ regions and $n = 3$ activities for model A and we now minimize equation (1) subject to restraints:

$$\begin{array}{llll} p_{11}y_{11} & + p_{12}y_{12} & + p_{13}y_{13} & \le s_1 \\ p_{21}y_{21} & + p_{22}y_{22} & + p_{23}y_{23} & \le s_2 \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ p_{i1}y_{i1} & + p_{i2}y_{i2} & + p_{i3}y_{i3} & \le s_i \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ p_{m1}y_{m1} & + p_{m2}y_{m2} & + p_{m3}y_{m3} & \le s_m \end{array} \tag{2}$$

where y_{i1}, y_{i2}, and y_{i3} refer respectively to outputs of food wheat, feed wheat, and feed grains (corn, barley, oats, and grain sorghums as one activity) in the ith region and p_{i1}, p_{i2}, and p_{i3} stand for the per unit land inputs for these activities in the ith region; while s_i is the acreage restriction in this same region. The total programming matrices for all models except C include 104 inequalities such as those in (2). The restrictions, or s_i, are set equal to the largest acreages devoted to feed grains and wheat in the previous year. In addition to these 104 inequalities to represent acreage restraints, there are two *discrete* demand restrictions:

$$y_{11} + y_{21} + \cdots y_{i1} + \cdots + y_{m1} = d_1 \tag{3}$$

$$y_{12} + y_{13} + y_{22} + y_{23} + \cdots y_{i2} + y_{i3} + \cdots y_{m2} + y_{m3} = d_2. \tag{4}$$

Coefficients in (3), a national demand restriction for food wheat, are 1, since no distinction is made between types and classes of wheat (a detail to be corrected in further analysis). In (4) a national "demand" restriction for feed grains, the coefficient of all y_{ij} are 1 because units of output are in terms of a feed equivalent expressed in corn. The feed grain demand restriction is measured in this same unit, with total units representing the 1954 level of feed grain disappearance adjusted for normal livestock production. For requirements restrictions in both (3) and (4), we use an equality to indicate that annual production must exactly equal annual requirements; with requirements at the 1954 level adjusted for normal livestock production, exports, population, and food uses.

In this model, feed grains other than wheat are combined into a single activity, with acreage in each region proportionate to the acreages in the period 1950–53. This procedure takes into account the fact that crops such as corn and small grains are grown in fixed rotational proportions in regions such as the Corn Belt.

MODEL B. This model is exactly the same as A, except that land rent is included in the c_{ij}, the per unit cost of producing the jth crop in the ith region. The modification represented by B was used because only grain crops are used as competitive alternatives in programming. Inclusion of land rent as a cost in model B gives some recognition to alternative crops. However, since grains are the major crops in the regions programmed, market rents are probably largely based on feed grains and wheat. For this reason, we believe the estimates arising under models A and E to be more appropriate than those of B. (We have computed the shadow prices or "rents" under each model, but do not present them here.) Neither model A nor model B takes into account transportation costs to regions of demand, or the magnitude of demand in each region.

MODEL C. The assumption stated earlier, that an acre of land could be used for either wheat or a feed grain rotation, is relaxed for model C where grain acreage in each region is divided into two components: a maximum for wheat and a maximum for feed grain. The number of land restraints in model C thus is 208, instead of the 104 in models A and B. All other variables (costs, demand requirements, etc.) are the same as for model A.

MODEL D. Agronomists pose the possibility of establishing meadow crops for rotation without use of a nurse crop such as oats. Feed produced from oats is much less than from corn. Hence, if oats could be eliminated from the customary rotation, a large increase in potential feed supply would be possible. Model D was designed to investigate impact of this innovation on optimum allocation of grain production among regions. For this model, there are $n = 6$ grain activities: food wheat, feed wheat, corn, oats, barley, and sorghum.[5] Costs and acreage and consumption restraints are the same as in model A with the objective being minimal national cost and land restrictions of $m = 104$.

MODEL E. This model is the same as A in terms of nature and number of activities and restrictions and the structure of production costs. However, it attempts to give some recognition to costs of transportation to regions of demand and also to give partial recognition to demand requirements in different regions. If transport costs between regions of production and regions of demand (as well as demand magnitudes in each region) were available, the pattern of production which minimizes costs, including transport costs, to meet the "fixed" demand of each region could be determined. We have such a model in the computational stage. However, for purposes here, we use a substitute. Instead of minimizing costs as in (1), we now maximize profit; Y_i

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[5] Implicit in the models described so far is that wheat land will be either continuously cropped, or grown in rotation with cultivated summer-fallow or crops such as peas, flax, and grasses (if other crops are normally grown in rotation with wheat in specific areas). Other crops in rotation with wheat are possible since their acreages are not part of the restraints. For the same reason, other crops can be part of the feed grain rotation acre.

is as before but C_i is now a vector of net prices for the ith region. Here we assume that net prices in each region account for transportation costs to consuming regions. In effect, we assume that prices in each region are equal to those in a central market (or a series of interrelated markets) less the cost of transportation from the region. Using historic price differentials between these regions to reflect these transport costs as they would be expressed in a purely competitive market, we have used an equation similar to (1) to indicate the pattern of feed grain and wheat production which maximizes profit. This is equivalent to a minimum-cost solution under the above assumptions and assuming that the geographic markets absorb programmed quantities at the implied prices. In an interregional competition sense, we assume that crops not included in Y_i are "lower" alternatives than those which are included. However, we do select a spatial pattern of feed grain and food wheat production which considers the comparative advantage among regions for the grain crops included. This is true for the particular objective function of all three models.

BASIC ASSUMPTIONS AND LIMITATIONS OF ALL MODELS. In order to reduce the analysis of the wheat and feed grain economy to a manageable size, certain simplifying assumptions were necessary. While these assumptions may not exactly describe within-region economic structure, they permit programming models of sufficient comprehension and detail for the general objectives of this study. The formal basic assumptions for the structure of the grain economy are: (*a*) There are m unique, spatially separated but interdependent production regions with many producers of wheat and feed grains. (*b*) All producers in each region have the choice of producing the same products or product mixes, and product is homogeneous among regions. (*c*) All producers in a region have identical input-output coefficients, and use the same production techniques. (*d*) Input-output coefficients are constant (i.e., constant returns to scale exist). (*e*) An acre of feed grain (or wheat) land substitutes for an acre of wheat (or feed grain) land at a constant rate within each region. (*f*) Total production in each region is limited only by maximum constraints on land in grain production. (*g*) Total grain consumption requirements are exogeneous, determined as discrete requirements per annum for human and livestock populations of the year.

The models used in this study provide refinement over calculations made and empirical approaches used in other studies for similar purposes. However, they do involve limitations of which we are aware. A complete model of general type projected into the future to avoid relevant historic and institutional attachments to production, logically appropriate for the problem, would imply prediction of all relevant production, cost, demand, and supply functions for all commodities under consideration and which compete and are jointly determined in production or consumption. However, data for an analysis within this framework are not available and alternative approaches must be used. Some of the major practical limitations of the general model used in this study are: (*a*) Specification of production regions is somewhat arbitrary. Distinct boundaries do not prevail and some difference in soil productivity exists in even the smallest regions specified. (*b*) Producers in regions do not have identical input-output and cost coefficients. Land also varies some between farms. (*c*) The quality of all the grain produced in the United States is not the same. Some quality differences are necessary to fulfill specialized demand (e.g., durum wheat and malting barley). (*d*) Total production within each region is not limited by land alone but can be increased by higher proportions of other inputs such as fertilizer (a consideration being included in later phases). (*e*) The consumption of grain is not independent of prices. (Due to degree of demand aggregation and the time period of one year, constant per capita consumption rates may give close approximation of demand restraints.) While these and other limitations exist in varying degree, the magnitude of computational burden, if all identifiable variables were considered, would exceed that possible with existing computer and research resources. But we look upon this as an aggregative analysis for purposes of "broad diagnostic designation" of regions to be withdrawn from production. Other more detailed intraregional analysis can be used, by those concerned, to overcome many of these limitations.

GRAIN-PROGRAMMING REGIONS

Delineation of meaningful grain-producing regions (in terms of the objectives of the study) was in itself a sizable job. At least one of the five grains under study is produced in all states and in most of the counties within these states. In many locations, however, grain production is only a small part of the total agricultural production and an insignificant part of the total grain economy. In many of these areas of sparse production, grain is either a complementary enterprise or has a special locational advantage. Thus, grain would be produced in certain areas with a wide range of prices. Also, for these sparse grain areas, data are very scarce. For these reasons, only major grain-producing areas of the United States were used for programming analysis.

Areas in which wheat and feed grain were harvested from 25% or more of the total cropland in 1954 were defined as major grain-producing areas for purposes of programming. To some extent, this demarcating percentage is arbitrary. But the major grain-producing areas thus defined represented 90% of the total wheat and feed-grain acreages in 1953. Furthermore, in 1954, the percentages of wheat, corn, oats, barley, and sorghum produced in these major grain areas were estimated to be 93.1, 93.4, 86.9, 72.7, and 91.0, respectively, of total production. Thus, the defined major grain-producing areas are the source of most of the wheat and feed grain produced in the United States and are also the areas that are most significant in the grain-surplus picture.

The programming regions are based primarily on state economic areas. To demarcate programming regions that were relatively homogeneous for grain production and to keep the computational work at a minimum, the following procedure was used: First, four classes of economic areas were defined:

1. Areas with grain production uniformly distributed, that is, the concentration of grain acreage within each county was approximately the same for all counties in the economic area.

a. Areas with total harvested acreage of wheat and feed grains combined equal to or greater than 25% of total cropland.

b. Areas with total harvested acreage of wheat and feed grains combined less than 25% of total cropland.

2. Areas with grain production not uniformly distributed.

a. Areas with total harvested acreage of wheat and feed grains combined equal to or greater than 25% of total cropland.

b. Areas with total harvested acreage of wheat and feed grains combined less than 25% of total cropland.

By using dot maps showing the geographic distributions and concentrations of the harvested acreages of wheat and feed grains in 1954, state economic areas were placed in either group 1 or group 2. Group 1 was divided into classes 1a and 1b by computing the required percentages, a and b above, from state economic area acreages. County acreages were used to divide group 2 into classes 2a and 2b. Thus classes 1a and 1b are state economic areas and classes 2a and 2b are counties.

Finally, classes 1a and 2a were aggregated to form the 104 programming regions. Criteria used to guide aggregation were as follows: state economic areas and counties within each region were required to be contiguous and to have similar grain yields, similar proportions of the five grains shown, and similar numbers of combines, cornpickers, and tractors per 1,000 acres of cropland. On the basis of these criteria, two or more state economic areas often could not be aggregated. Hence, some programming regions consist of only one state economic area. In other instances, it was possible only to aggregate one economic area and a group of counties. A few regions are made up of counties only.

The 104 programming regions shown in Figure 1 provided the basic units for making estimates of acreage, yield, and cost. But when the necessary data were not available for these regions for estimating input coefficients, state data were adjusted by other related data to compensate for within-state differences. In a few instances, state data were used without adjustment when a logical means of adjustment was not apparent.

The concept of "normal" is basic to the methods used in estimating the maximum regional grain acreages and regional yields. The word "normal" is used here to mean expected or average. The objective for yields was to obtain estimates that would reflect accurately the average quantity of inputs used per acre for production of wheat and feed grains in 1954. The general objective for all estimates was the obtaining of data that would reflect the relative competitive positions of the regions in production of wheat and feed grains.

REGIONAL ACREAGES. Grain acreages of 1953 were used as estimates of the maximum regional restraints. In this year, more grain was planted than in

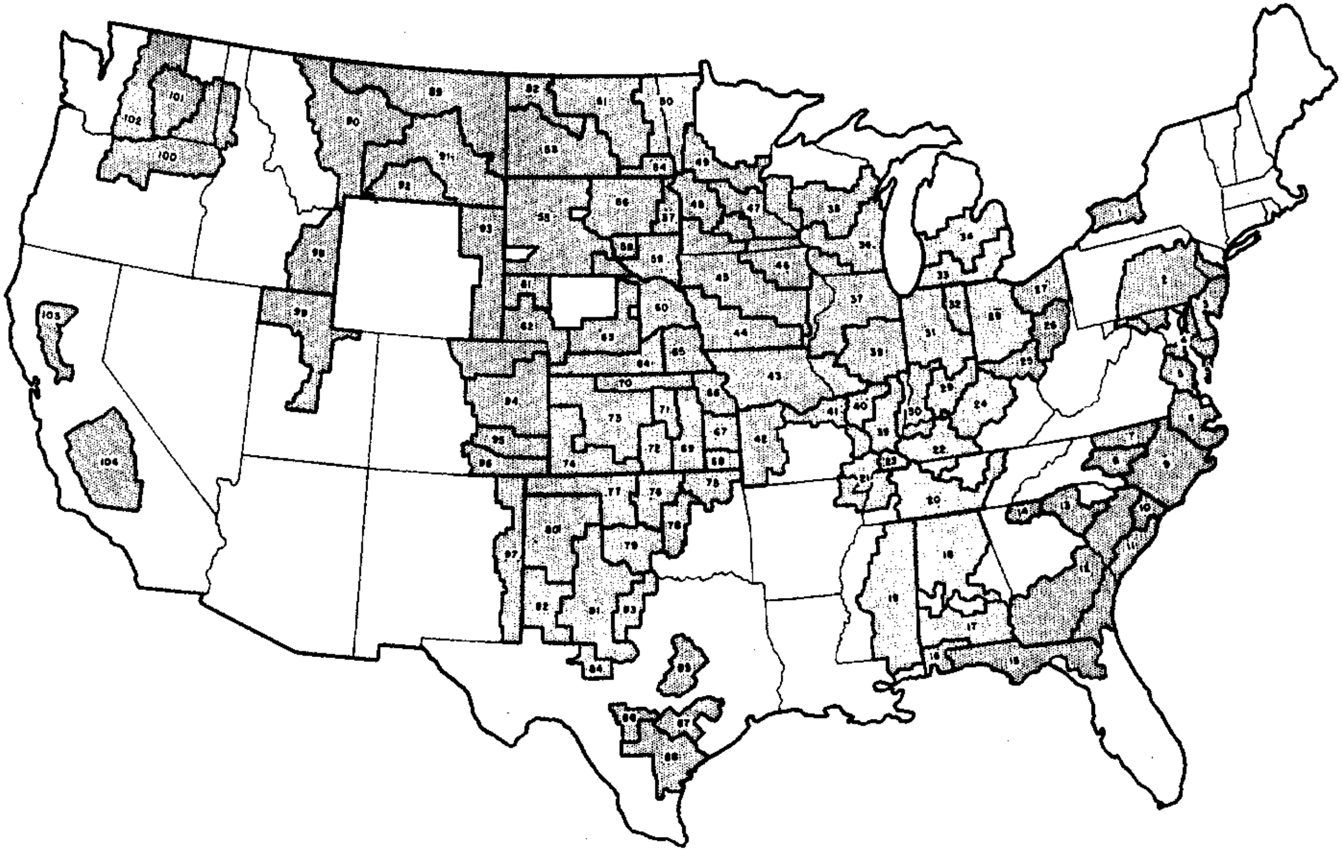

FIGURE 1. Identification of programming regions.

any year of history. Acreage control programs were not in effect and the large 1953 grain acreages perhaps represent maximum area adapted to these crops under peacetime economic conditions. Thus, later figures on production adjustment suggest the quantity of land that might need to be withdrawn, relative to the 1953 base acreage, if production of feed grains and wheat were balanced with annual use. Acreages planted to grain and summer-fallowed are the components of the regional acreage restraints [see discussion of equation (5)]. Acreages planted to grain were not easily estimated for many regions because: (*a*) estimates of planted acres were not available, or (*b*) when planted acre estimates are available, they include plants for hay, pasture, silage, cover crops, etc. These difficulties exist mainly for small grains. The total number of acres harvested for the various uses of corn are estimated by Federal-state agencies. Only for small grains are estimates of the acres harvested for grain only not made. A different method was used to estimate acreages of corn and small grains due to the nature of the data available. The acreages of corn planted for grain were estimated by the following formula:

$$\begin{bmatrix}\text{estimated acres}\\ \text{of corn planted}\\ \text{for grain in}\\ \text{the } i\text{th region}\end{bmatrix} = \begin{bmatrix}\text{acres of corn}\\ \text{planted for all}\\ \text{purposes in}\\ \text{the } i\text{th region}\end{bmatrix} - \begin{bmatrix}\text{estimated acres}\\ \text{of corn planted}\\ \text{for silage in}\\ \text{the } i\text{th region}\end{bmatrix}$$

$$(i = 1, 2, 3, \ . \ . \ . \ 104).$$

The acres of wheat, oats, barley, and grain sorghums planted for grain were estimated by the following relationship:

$$\begin{bmatrix}\text{estimated acres of the } g\text{th}\\ \text{grain planted for grain in}\\ \text{the } i\text{th region}\end{bmatrix} = \frac{\begin{bmatrix}\text{acres of the } g\text{th grain harvested}\\ \text{for grain in the } i\text{th region}\end{bmatrix}}{\begin{bmatrix}1 - \text{average abandonment rate of}\\ \text{the } g\text{th grain in the } i\text{th region}\end{bmatrix}}$$

$$(g = 1, 3, 4, 5).$$

The number of cultivated summer-fallow acres was included as a component of the regional acreage restraints because fallowed acreages are a necessary land input in semiarid wheat areas. Machinery and labor costs associated with fallowed land are a necessary part of the total per acre cost of production. Also, historic yields are based on production resulting from the use of cultivated summer fallow in rotation. Thus, the inclusion of cultivated summer fallow places estimates of acreage, yield, and cost in their proper relationship.

Estimates of cultivated summer-fallow acreages were obtained from the census and from unpublished data of the Crop Estimates Division, Agricultural Marketing Service. It was assumed that fallowed acreages did not change significantly from 1953 to 1954.

The 1953 crop acreages by regions, used to develop total acreage restraints, are presented in Table 1.

TABLE 1

Estimated Acreages of Land Used for Production of Wheat and Feed Grains, by Regions, 1953[a]

Region	Wheat	Corn	Oats	Barley	Sorghum
	1,000 acres	1,000 acres	1,000 acres	1,000 acres	1,000 acres
1	316.4	110.4	161.8	14.0	–
2	858.2	1,018.0	430.1	173.8	–
3	89.2	320.3	11.9	23.3	–
4	103.2	157.5	15.7	21.3	–
5	59.2	118.8	18.1	12.3	–
6	12.8	526.3	18.0	3.8	–
7	77.4	196.7	44.9	6.4	–
8	136.7	123.2	95.7	14.9	–
9	91.8	1,223.0	98.8	7.1	–
10	8.0	247.4	34.5	.2	–
11	1.0	241.6	18.0	.1	–
12	110.5	2,455.9	527.9	5.6	–
13	90.2	192.2	140.4	10.8	–
14	12.2	80.1	13.4	1.6	–
15	–	517.3	24.3	–	–
16	.4	83.8	6.9	–	.3
17	.9	698.2	21.9	–	6.2
18	18.1	1,120.3	72.5	–	19.1
19	2.7	1,132.9	89.7	–	2.7
20	107.9	741.4	66.7	52.5	–
21	161.7	647.9	38.7	3.9	.6
22	184.4	789.7	40.5	54.6	–
23	25.5	227.0	6.2	2.0	–
24	48.6	242.5	18.1	20.2	–
25	189.6	359.5	20.5	4.4	–
26	134.5	208.1	64.6	4.3	–
27	382.2	415.1	262.5	7.4	–
28	1,698.6	2,421.1	795.0	20.6	–
29	205.8	467.8	71.0	12.8	–
30	498.6	1,320.9	68.6	13.8	–
31	939.1	2,909.6	905.8	5.6	–
32	220.3	510.2	263.9	1.9	–
33	537.7	682.9	418.5	9.7	–
34	893.6	691.1	692.4	39.6	–
35	14.9	201.6	766.0	11.3	–
36	41.4	817.2	1,370.3	68.2	–
37	185.6	4,900.9	2,654.6	13.3	–
38	677.8	3,176.3	986.4	.8	–
39	305.9	769.4	52.5	5.2	–
40	461.1	450.6	92.9	8.3	–
41	252.3	333.7	99.2	7.7	–
42	442.6	500.1	492.5	77.0	22.3

TABLE 1 (*continued*)

Estimated Acreages of Land Used for Production of Wheat and Feed Grains, by Regions, 1953[a]

Region	Wheat	Corn	Oats	Barley	Sorghum
	1,000 acres	1,000 acres	1,000 acres	1,000 acres	1,000 acres
43	1,171.2	2,853.7	759.5	3.4	6.9
44	122.2	2,834.3	1,304.4	2.1	–
45	21.0	6,800.8	4,002.5	54.9	–
46	27.1	2,455.0	1,613.2	11.9	–
47	62.2	1,120.2	1,483.0	45.7	–
48	96.1	1,290.4	988.8	185.5	–
49	103.2	339.4	767.4	93.7	–
50	2,707.0	192.6	836.2	1,090.9	–
51	6,035.1	119.6	721.6	1,022.1	–
52	2,427.6	20.4	161.1	181.5	–
53	4,033.3	293.3	453.5	236.2	–
54	481.9	159.2	271.9	162.8	–
55	1,558.7	237.8	237.0	67.7	–
56	2,277.4	747.3	938.2	192.5	–
57	216.5	449.9	628.9	108.9	–
58	246.8	410.4	303.2	52.9	–
59	143.6	1,835.1	1,593.0	52.6	–
60	263.2	2,335.9	1,258.4	15.4	1.3
61	253.5	81.1	62.6	44.1	.1
62	3,647.4	233.8	109.9	266.1	24.9
63	491.1	1,157.2	271.0	52.5	18.2
64	1,358.2	939.4	80.7	13.9	117.0
65	1,596.1	2,426.8	647.0	9.8	38.1
66	398.1	584.6	208.4	1.9	26.9
67	292.9	208.4	175.4	10.3	101.2
68	484.9	96.0	127.4	7.4	42.9
69	615.5	173.1	178.7	8.2	143.5
70	1,076.3	532.2	79.3	4.6	100.0
71	1,063.1	97.4	112.8	1.8	96.2
72	2,347.6	36.4	161.7	7.5	182.8
73	6,565.5	99.7	106.7	78.0	814.3
74	4,473.6	8.0	14.1	26.1	592.4
75	155.0	76.2	154.1	4.0	31.5
76	2,568.2	12.2	94.8	17.4	46.4
77	2,485.9	6.7	27.1	8.4	383.8
78	277.9	78.4	102.3	5.8	39.1
79	1,763.5	23.7	109.2	11.9	116.8
80	1,820.1	5.0	9.4	28.7	1,017.3
81	1,342.4	12.4	182.7	18.6	398.7
82	75.4	4.4	1.7	3.7	1,090.5
83	277.5	14.9	65.9	5.4	5.7
84	48.4	2.8	11.5	.5	34.5

TABLE 1 (*continued*)

ESTIMATED ACREAGES OF LAND USED FOR PRODUCTION OF WHEAT AND FEED GRAINS, BY REGIONS, 1953[a]

Region	Wheat	Corn	Oats	Barley	Sorghum
	1,000 acres	1,000 acres	1,000 acres	1,000 acres	1,000 acres
85	36.6	296.2	-	-	106.1
86	22.7	43.3	26.4	2.2	13.8
87	1.2	289.2	-	-	35.6
88	5.2	291.9	-	-	312.3
89	6,095.1	103.6	147.5	146.8	-
90	3,485.9	1.2	75.4	270.3	-
91	505.2	22.9	44.8	38.9	-
92	629.1	1.2	25.2	36.5	-
93	685.8	30.2	71.8	42.2	-
94	3,884.4	155.6	40.0	95.7	118.0
95	492.0	32.9	10.8	21.5	52.1
96	505.2	1.8	2.7	9.3	42.1
97	412.6	4.5	.4	1.7	124.9
98	1,538.3	.5	40.0	171.1	-
99	489.1	1.2	4.7	23.8	-
100	4,334.2	.8	89.1	260.7	-
101	2,756.1	2.7	13.0	13.2	-
102	506.0	9.5	16.4	12.1	-
102	103.4	12.0	35.6	388.8	13.9
104	157.6	28.5	11.8	779.8	37.6
Total	94,716.0	67,084.5	34,163.6	7,272.0	6,378.7

[a] Acreages include cultivated summer fallow.

Acreage restraints. The acreage restraints—the maximum number of acres of land that can be used for all grain production in each region—are the sums of the individual grain acreages given in Table 1. The demand restraints for the programmed areas, that is, the quantities of food wheat and feed grain that must be produced within the system, are 677.5 million bushels of food wheat and 3,548.9 million bushels of feed grain. The acreage and production restraints used in programming are those in Table 2.

REGIONAL YIELDS

Normal regional yields, as defined previously, were estimated in two steps. First, the 1945–54 average yields were computed. These yields were then adjusted by a factor representing the average increase in yield between the midpoint of the period 1945–54 and the year 1954. Regression trends were computed from data for the period 1937–54 to accomplish this end.

TABLE 2

ACREAGE RESTRAINTS, BY REGIONS, AND TOTAL PRODUCTION RESTRAINTS FOR ALL MODELS BUT C

Region	Acreage	Region	Acreage	Region	Acreage	Region	Acreage
	1,000 acres		1,000 acres		1,000 acres		1,000 acres
1	603	27	1,067	53	5,016	79	2,025
2	2,480	28	4,935	54	1,076	80	2,881
3	445	29	757	55	2,101	81	1,995
4	298	30	1,902	56	4,155	82	1,176
5	208	31	4,760	57	1,404	83	369
6	561	32	996	58	1,013	84	98
7	325	33	1,649	59	3,624	85	439
8	370	34	2,317	60	3,874	86	108
9	1,421	35	994	61	441	87	326
10	290	36	2,297	62	4,282	88	610
11	261	37	7,754	63	1,990	89	6,493
12	3,100	38	4,841	64	2,509	90	3,833
13	434	39	1,133	65	4,718	91	611
14	107	40	1,013	66	1,220	92	692
15	542	41	693	67	788	93	830
16	91	42	1,535	68	758	94	4,293
17	727	43	4,795	69	1,119	95	609
18	1,230	44	4,263	70	1,792	96	561
19	1,228	45	10,879	71	1,371	97	544
20	969	46	4,107	72	2,736	98	1,750
21	853	47	2,711	73	7,664	99	519
22	1,069	48	2,561	74	5,114	100	4,685
23	261	49	1,304	75	421	101	2,785
24	329	50	4,827	76	2,739	102	544
25	574	51	7,898	77	2,912	103	554
26	412	52	2,790	78	504	104	1,015

Production Restraints	
Wheat	677,509 thousand bushels
Feed grain	3,548,911 " "

When annual data were available, 1954 average yields were computed by this method. The sources of the data are those listed for acreages. When annual data were not available for the period 1945–54, harvested yields per acre were estimated from state data and census economic area and county data. These yields per harvested acre were then adjusted by a factor representing the average percentage of the total acreage harvested, with total acreage equaling harvested acreage plus abandonment plus fallow.

The estimated yield for each grain by regions is shown in Table 3. These are net yields—per acre seed requirements were subtracted.

TABLE 3

ESTIMATED NET YIELDS PER ACRE FOR WHEAT AND FEED GRAINS, BY REGIONS, 1954[a]

Region	Wheat	Corn	Oats	Barley	Sorghum
	Bushels	Bushels	Bushels	Bushels	Bushels
1	26.9	45.6	39.0	30.0	–
2	21.3	50.0	36.4	37.3	–
3	18.1	45.9	29.2	26.2	–
4	18.4	49.8	32.2	27.2	–
5	21.2	39.6	35.2	30.7	–
6	16.2	36.6	27.4	21.9	–
7	19.3	29.4	32.0	29.9	–
8	18.3	31.2	32.2	30.6	–
9	17.7	29.2	29.9	24.1	–
10	17.8	21.3	28.2	21.6	–
11	16.5	18.6	23.1	17.0	–
12	16.6	16.2	27.2	23.6	–
13	16.5	18.9	27.2	23.6	–
14	16.1	18.6	24.0	21.0	–
15	–	15.0	21.2	–	–
16	22.8	20.9	21.7	–	19.0
17	20.4	15.4	22.1	–	14.8
18	19.6	21.5	29.5	–	17.1
19	15.7	19.8	20.9	–	15.0
20	14.9	27.6	25.3	14.9	–
21	18.0	25.6	24.4	18.6	19.2
22	17.1	36.4	27.0	19.0	–
23	16.6	32.6	27.7	19.6	–
24	15.8	36.3	28.6	24.4	–
25	17.4	50.8	27.7	24.5	–
26	23.0	51.3	37.1	29.8	–
27	26.0	50.4	40.9	32.8	–
28	24.1	56.6	39.5	29.1	–
29	19.0	44.4	30.0	24.6	–
30	19.1	39.8	28.5	26.5	–
31	24.3	55.6	38.0	25.5	–
32	27.0	56.0	39.8	26.2	–
33	26.6	43.4	37.1	28.7	–
34	27.6	43.3	37.2	32.0	–
35	20.6	44.6	37.9	33.0	–
36	27.3	58.6	53.6	38.4	–
37	25.2	59.9	41.2	30.6	–
38	27.1	57.0	36.6	26.6	–
39	18.8	36.1	23.7	25.0	–
40	19.4	35.2	25.2	25.5	–
41	21.3	36.0	23.9	25.9	17.0
42	19.7	28.2	24.5	23.3	16.1

TABLE 3 (*continued*)

ESTIMATED NET YIELDS PER ACRE FOR WHEAT AND FEED GRAINS, BY REGIONS, 1954[a]

Region	Wheat	Corn	Oats	Barley	Sorghum
	Bushels	Bushels	Bushels	Bushels	Bushels
43	22.7	42.8	27.1	27.8	22.3
44	15.5	46.1	28.1	22.4	–
45	14.7	50.1	31.2	17.8	–
46	17.6	51.4	37.1	27.5	–
47	17.0	47.6	38.2	26.9	–
48	13.6	39.5	32.6	23.6	–
49	14.6	40.3	33.2	27.8	–
50	9.2	26.4	30.0	25.0	–
51	8.0	20.2	24.6	18.7	–
52	7.0	17.8	24.4	17.6	–
53	7.5	17.8	25.5	18.5	–
54	7.9	22.1	25.8	18.9	–
55	8.1	19.0	22.4	16.7	–
56	9.0	22.2	25.5	17.0	–
57	8.6	29.7	30.0	20.1	–
58	8.5	21.6	24.2	16.4	–
59	9.6	36.5	29.5	18.8	–
60	16.2	38.9	22.8	16.8	21.4
61	12.8	24.4	23.7	19.3	16.7
62	10.0	26.4	24.4	21.8	15.3
63	10.6	32.2	17.8	12.7	21.5
64	11.2	25.2	19.2	14.3	21.4
65	17.5	37.0	22.7	16.3	30.5
66	17.8	31.5	17.4	17.6	25.5
67	17.9	25.5	18.7	19.2	19.8
68	17.1	22.1	20.9	18.1	17.5
69	17.4	24.0	19.9	18.5	18.4
70	10.8	22.1	13.7	11.7	19.8
71	13.3	22.2	18.8	14.3	19.9
72	13.8	21.0	19.7	13.3	18.6
73	9.4	20.4	16.0	12.8	18.6
74	7.3	16.1	15.6	10.3	17.0
75	12.0	18.4	13.5	12.7	12.5
76	13.0	16.5	17.3	11.6	14.8
77	6.6	11.2	10.1	7.3	12.9
78	10.4	19.5	15.7	9.8	13.1
79	10.3	18.0	15.4	10.1	14.7
80	6.1	27.2	16.6	12.2	27.5
81	7.5	13.7	17.9	12.1	10.0
82	5.0	14.5	15.9	13.2	15.0
83	8.3	13.7	16.0	12.4	9.1
84	4.5	11.3	14.2	9.1	12.7

TABLE 3 (*continued*)

ESTIMATED NET YIELDS PER ACRE FOR WHEAT AND FEED GRAINS, BY REGIONS, 1954[a]

Region	Wheat	Corn	Oats	Barley	Sorghum
	Bushels	Bushels	Bushels	Bushels	Bushels
85	5.8	17.7	–	–	19.0
86	4.2	14.9	16.6	9.9	16.1
87	4.5	17.6	–	–	15.9
88	4.5	17.1	–	–	23.6
89	8.0	14.6	28.0	29.6	–
90	8.9	16.4	29.4	27.0	–
91	6.5	13.0	23.9	16.4	–
92	10.6	25.4	40.6	30.2	–
93	8.7	24.2	22.8	22.5	–
94	7.0	16.3	15.8	12.7	8.8
95	5.2	42.8	17.9	14.8	16.5
96	2.5	16.7	11.9	10.1	8.6
97	1.6	10.0	19.7	10.6	10.8
98	12.9	45.2	39.4	30.6	–
99	9.9	38.1	49.4	47.2	–
100	16.9	64.5	40.0	31.0	–
101	12.6	52.5	37.6	30.7	–
102	11.6	71.7	51.5	33.1	–
103	12.5	36.1	18.2	23.2	33.5
104	9.8	25.4	17.0	27.1	36.4

[a] Estimated yield less seed.

PRODUCTION COSTS

The methods used in estimating per acre costs of grain production are described in this section. The basic items making up per acre cost are land, labor, machinery and power, seed, chemicals, and miscellaneous inputs. A charge for annual land services was considered for model B only. Indirect or overhead costs, such as management, purchasing, selling, housing, and so on, were not estimated because a satisfactory method and data for estimation were lacking. Some detailed unit cost studies have used 10% of the direct cost as an estimate of the indirect cost, but use of this method would not change the relative values of the activity costs. Hence, the inclusion of a proportional indirect cost would not affect the programming solutions in this study.

COMPOSITE ACRE. Uniform and complete data on average production costs for wheat and feed grains in each programming region are lacking. Hence, these costs had to be synthesized. To make realistic estimates of per acre cost, a composite acre was devised for each region. This composite acre was made up of 12 possible elements, each of which represents a unique production operation. These 12 acre-elements, or types of production situations, used for production-cost estimates are:

1. Mechanical, planted and harvested, not irrigated.
2. Mechanical, planted and harvested, irrigated.
3. Mechanical, planted but not harvested (abandoned).
4. Mechanical, cultivated summer fallow.
5. Semimechanical, planted and harvested, not irrigated.
6. Semimechanical, planted but not harvested, irrigated.
7. Semimechanical, planted but not harvested (abandoned).
8. Semimechanical, cultivated summer fallow.
9. Nonmechanical, planted and harvested, not irrigated.
10. Nonmechanical, planted and harvested, irrigated.
11. Nonmechanical, planted but not harvested (abandoned).
12. Nonmechanical, cultivated summer fallow.

Except for the mechanical items, these acre-elements are self-explanatory. They are defined as follows: Mechanical—tractor power is used for all tillage operations and harvesting is done by combine or cornpicker; semimechanical—tractor power is used for all tillage operations and harvesting is done by hand (for corn) or with binder and thresher (for small grain); and nonmechanical—a production technique in which animal power is used for all tillage operations and harvesting is done by hand (as for corn) or with binder and thresher (as for small grain). Also, acre-elements 2, 6, and 10 imply that no abandonment is assumed on irrigated acres.

The list of 12 acre-elements is not exhaustive. On the basis of regional data, however, they seemed to be complete enough to provide reasonable estimates of average production costs, and at the same time to facilitate computations for planned further investigations.

An example will help to explain the method used in deriving costs for each made up of 12 possible elements, each of which represents a unique production by mechanical techniques, (2) no irrigation, (3) no harvesting from land in cultivated summer fallow the preceding year, and (4) an average of 1% abandonment of the planted acres. Attached to each corn acre in region 1, therefore, were two types of acre-element costs—mechanical, planted and harvested but not irrigated; and mechanical, planted but not harvested. The weights, which are computed elsewhere on an acreage basis, are .99 for mechanical, planted and harvested but not irrigated; and .01 for mechanical, planted but not harvested. Furthermore, given per acre costs of $42.20 for the mechanical, planted and harvested acre and $34.50 for the mechanical, planted but not harvested acre, the estimated average per acre production cost for corn in region 1 is $42.12 ($42.20 \times .99 + 34.50 \times .01$).

Estimates of costs of labor, machinery, and power provided the greatest conceptual and empirical difficulties. Aggregate estimates of machinery and labor inputs exist for United States farms, but they are not broken down between individual farm enterprises. Hence, these costs were derived by estimating the average physical inputs per acre by type of operation (plowing, disking, harrowing, and so on) and then weighting physical inputs by the estimated per unit cost of the inputs involved. Because many of the published data on labor and machinery costs were either incomplete or out of date, supplementary data on these inputs were obtained from 25 different state agricultural experiment stations or colleges.

LAND. The annual value of land for grain production was used only in model B. The per acre value of land on cash-grain farms was assumed to be the best available basis for estimating the annual value of land services for grain production. The sum of the interest rate and tax rate was multiplied by the per acre value to obtain the annual input value of land. In region 1, for example, the interest and tax rates were .049 and .0184, respectively, per dollar of value, and the land value was $111 per acre.

LABOR. Inputs of physical labor were estimated for each production operation. The method is illustrated in the tabulation below for wheat production in region 1, which is based on the mechanical, planted and harvested, not irrigated acre-element.

Operation	Hours Required per Acre
Plowing	1.46
Disking	1.15
Harrowing	.69
Drilling	.82
Harvesting	1.54
Hauling	1.03
Total	6.69

The data on labor hours required for harvesting and hauling omit the portion of an "average" acre not harvested.

Data on the number of man-hours of labor required for each production operation were obtained from several publications and from the survey data. When possible, modal coefficients were used. When a modal production operation was not evident in the data, simple averages or single estimates were used. The per acre labor cost for each acre-element was obtained by multiplying the estimated number of man-hours required per acre by an estimate of the hourly wage rates on cash-grain farms. The per acre labor costs for each grain and each region were computed by weighting each acre-element labor cost by the proper coefficient.

POWER AND MACHINERY. The method used in estimating the power and machinery cost was similar to that used in estimating labor. The estimating problem was more complex, however, because of the multitude of items that compose machinery costs. Instead of one coefficient—hours per acre—and one price—wages—coefficients and prices for each implement required to produce each grain in each region were estimated. The tabulation that follows illustrates the procedure used in estimating this cost for an acre of corn in region 28 in Ohio. The example is for the mechanical, planted and harvested but not irrigated acre-element.

Implement	Size	Hours of Use Required per Acre	×	Cost per Hour	=	Cost for Implement per Acre
Tractor	19 hp	10.45		$0.81		$8.46
Plow	2–14″	1.30		.71		.92
Disk	7′T	1.00		.67		.67
Harrow	10′	.50		.22		.11
Drag	9′	.35		.26		.09
Cultipactor	10′	.40		.60		.24
Planter	2-R	.60		.65		.39
Cultivator	2-R	1.50		.80		1.20
Picker	1-R	1.80		1.71		3.08
Wagon	Std	1.00		.08		.08
Total						15.24

The machinery sizes and number of hours required per acre used in estimating machinery cost were modal values when these values could be determined. When a modal value was not apparent, simple averages or single observations were used. Machinery sizes and hours of use required per acre were obtained from U.S.D.A. data and from survey data. Extensive searching and many computations were necessary in order to estimate the per hour cost of each implement. Information was obtained or estimated for this purpose as to size, price, annual use; total life; interest, tax and insurance rates; grease and repair rates; and fuel and oil consumption rates. With these basic data, the items that make up the per hour cost of each implement—depreciation, insurance, interest, taxes, fuel, oil, grease, and repairs—could be computed.

SEED. The cost of seed was not included as a part of the total per acre production cost. Instead, the estimated quantity of seed required per acre was subtracted from the estimated yield. This method was used because total demand for seed is a function of the acreage grown in each region. But these acreages are variables to be determined within the system (that is, the model). Hence, the simplest way of allowing seed cost and demand for seed to be variables determined by the system is to deduct the seeding rate from the yield. To use this method, it is necessary that grain seed be planted in the region in which it is produced; and that planted acreages within each region be constants between years. Only state seeding rates were available. Therefore, adjustments were made in state rates to compensate for variations within the states.

CHEMICALS. Regional fertilizer costs for each of the five grains were calculated mainly from the U. S. Census of Agriculture. Specific data for only

TABLE 4
Estimated Production Costs per Acre, Excluding Land, for Specified Crops, by Regions[a]

Region	Wheat	Corn	Oats	Barley	Sorghum
	Dollars	Dollars	Dollars	Dollars	Dollars
1	29.23	34.76	28.34	28.64	–
2	28.08	33.08	26.40	25.14	–
3	29.86	29.29	28.28	30.61	–
4	28.14	32.36	27.58	27.81	–
5	24.36	30.59	24.31	24.31	–
6	25.25	32.12	26.87	24.92	–
7	32.35	39.01	32.31	32.38	–
8	30.17	32.57	29.04	30.16	–
9	28.35	35.48	28.22	28.56	–
10	22.79	30.85	23.62	24.08	–
11	27.24	31.93	24.53	25.43	–
12	23.37	25.49	23.39	23.49	–
13	22.79	27.56	21.90	22.30	–
14	26.46	30.08	24.57	25.91	–
15	–	26.53	25.49	–	–
16	23.36	35.09	28.85	–	27.42
17	23.73	29.61	28.66	–	29.13
18	23.42	29.17	28.21	–	28.10
19	22.87	28.65	22.92	–	26.29
20	25.84	28.61	24.55	24.40	–
21	22.60	24.12	19.89	19.53	–
22	24.25	30.64	23.62	23.53	–
23	26.68	28.35	25.84	26.29	–
24	28.04	30.33	27.07	27.57	–
25	25.93	33.07	27.16	21.65	–
26	29.64	34.72	24.03	24.16	–
27	30.28	34.78	26.02	26.49	–
28	25.72	32.85	21.15	21.75	–
29	26.25	30.96	22.81	20.95	–
30	20.68	20.82	18.82	17.81	–
31	23.70	26.99	20.75	19.12	–
32	20.45	26.45	18.67	17.24	–
33	28.11	29.57	23.45	26.40	–
34	30.45	30.40	29.62	28.68	–
35	21.85	30.93	24.03	22.33	–
36	21.57	30.34	21.28	22.06	–
37	20.37	23.82	18.85	19.78	–
38	18.52	18.52	14.17	15.77	–
39	20.65	20.14	15.09	16.10	–
40	18.74	22.70	14.40	15.57	–
41	20.06	23.34	17.12	19.85	22.21
42	20.79	23.34	16.91	18.32	21.60
43	19.86	21.78	16.76	23.29	26.01

TABLE 4 (*continued*)

Estimated Production Costs per Acre, Excluding Land, for Specified Crops, by Regions[a]

Region	Wheat	Corn	Oats	Barley	Sorghum
	Dollars	Dollars	Dollars	Dollars	Dollars
44	16.59	21.58	14.08	17.20	–
45	14.74	19.43	11.40	11.94	–
46	16.90	21.67	12.63	12.45	–
47	17.67	25.08	18.69	18.39	–
48	14.71	19.51	9.65	13.93	–
49	13.40	23.22	14.66	13.59	–
50	8.52	18.70	12.57	11.77	–
51	6.57	17.83	8.53	8.70	–
52	5.84	19.41	8.50	8.75	–
53	7.23	16.26	9.16	9.31	–
54	8.25	16.39	9.92	10.15	–
55	6.16	11.53	7.75	7.64	–
56	7.23	11.62	8.00	8.05	–
57	10.23	17.50	12.88	12.83	–
58	7.01	11.53	8.71	9.62	–
59	10.12	16.45	9.44	12.20	–
60	11.74	14.40	10.24	11.14	13.31
61	7.20	14.50	11.72	10.70	16.04
62	7.05	20.12	13.50	14.16	15.53
63	10.28	18.68	16.07	14.80	20.75
64	6.44	17.06	12.20	10.82	16.17
65	12.68	17.57	11.80	10.57	14.19
66	17.56	18.01	14.20	12.23	15.71
67	18.91	21.83	14.88	12.84	19.26
68	20.20	22.47	15.97	16.70	17.68
69	16.65	19.77	12.82	14.25	18.83
70	9.21	16.23	12.54	10.54	16.41
71	11.21	18.53	12.28	10.42	16.82
72	9.49	19.28	10.62	9.19	15.57
73	5.80	11.22	8.85	7.52	10.05
74	3.88	17.20	6.45	6.03	8.54
75	15.40	19.97	16.70	15.29	17.85
76	9.41	21.89	9.67	8.79	17.16
77	6.08	13.30	7.75	6.65	9.06
78	10.93	19.36	12.03	11.20	17.23
79	7.55	16.89	8.43	7.41	10.65
80	4.90	22.62	5.58	5.39	13.78
81	5.54	11.35	7.49	6.69	8.04
82	5.13	21.25	6.56	6.06	9.10
83	7.06	12.79	8.54	7.92	8.68
84	5.19	9.52	7.77	7.09	8.55
85	7.15	14.07	–	–	13.47
86	4.77	14.47	9.24	8.06	13.48

TABLE 4 (*continued*)
ESTIMATED PRODUCTION COSTS PER ACRE, EXCLUDING LAND, FOR SPECIFIED CROPS, BY REGIONS[a]

Region	Wheat	Corn	Oats	Barley	Sorghum
	Dollars	Dollars	Dollars	Dollars	Dollars
87	7.73	16.30	–	–	13.66
88	6.30	13.11	–	–	10.54
89	5.07	32.38	9.24	9.11	–
90	6.83	35.84	18.46	14.10	–
91	6.76	34.48	13.71	12.56	–
92	8.88	44.92	24.44	20.90	–
93	8.61	23.57	15.53	16.59	–
94	5.50	12.35	9.60	9.07	10.92
95	7.63	22.71	15.21	15.94	19.98
96	3.61	14.33	10.40	9.21	12.90
97	4.04	16.46	15.59	15.21	16.28
98	10.56	3.40	26.60	20.56	–
99	10.36	50.30	31.19	31.31	–
100	10.95	51.48	17.28	16.66	–
101	6.76	57.58	13.18	14.90	–
102	8.65	73.17	27.77	23.09	–
103	10.11	40.25	13.28	14.25	32.90
104	9.21	31.36	9.33	14.17	16.11

[a] These estimates are based on a composite acre; see previous section in text.

the "more important" crops are recorded in the census. When fertilizer applications were not tabulated for a grain crop in the census, this cost was estimated with the aid of unpublished data of the Farm Economics Research Division, A.R.S. The per acre cost of lime for each grain was estimated by dividing the total cost of lime applied in a region in 1954 by the total cropland.

Data were not available to show expenditures for insecticides, fungicides, and herbicides for wheat and feed grains by regions. Hence, these costs were first estimated for each state. The state estimates were then used to estimate chemical costs for regions within states. The basic data used for insect, pest, and chemical weed control expenditures were those compiled by U.S.D.A. workers.

MISCELLANEOUS. Miscellaneous costs include those involved in the spreading of manure, fertilizer, and lime, and those of water for acreages produced by irrigation. No attempt was made to estimate the value of manure applied to wheat and feed grains. The spreading cost alone was charged to crop enterprises. Costs of spreading manure were estimated only for the programming regions in the Northeast, Appalachian, Corn Belt, and Lake States regions and the corn-producing areas of the Northern Plains. For some of the fertilizer applied to grains, the cost of application was accounted for in the method used to compute machinery and labor cost. This accounting method was used for fertilizer applied by attachments on planters, drills, and cultivators. For fertilizer spread by other methods, an additional application cost, which included charges for labor, power, and machinery, was computed. Costs of lime spreading by custom operators were assumed to have been included in the lime expenditures reported by farmers. An additional spreading cost was computed for lime spread by farmers. In areas in which less than .5% of the grains were produced by irrigation methods, irrigation costs were not estimated.

Estimates of the production costs (except land) outlined above are summarized in Table 4. These costs are based on the composite acre described earlier. These costs are for each individual crop as used in model D. Per acre or unit costs for other models with an aggregate feed grain activity are included in Appendix Table A of this chapter.

DEMAND RESTRAINTS

Separate demand restraints were considered for food wheat and feed grain in aggregate for 1954. Hence, the calculations provided later show regional production patterns designed to meet aggregate demand at the 1954 level. Techniques of production also represent 1954 as a point in time. The year 1954 was used because more complete production data for it were available. These demand restraints, which are assumed to be fixed or constant, were based on the normal per unit requirements of the human or livestock populations, or both, and the actual net exports in the base year 1954.

Because it was believed that grain stocks "put an abnormal pressure" on grain disappearance in 1954, an attempt was made to estimate a normal domestic disappearance for each grain. No attempt was made, however, to estimate normal net exports, because of the many unmeasurable factors in the world market.

The total (domestic and foreign) estimated demand levels were approximately 757 million bushels of wheat and 3,887 million corn-equivalent bushels of feed grain. Although these estimates were derived by simple techniques, they seem quite reasonable and do not differ greatly from actual disappearances of wheat and feed grains in 1954. Seed requirements and grain for forage were not included in the estimates, as seed requirements were subtracted from yields and the study reported is concerned with grain production alone.

Since not all the land area in the United States was included in the programming regions, it was necessary to estimate the normal production of wheat and feed grains in these nonprogrammed areas in order to determine how much of the estimated total demand or requirements would need to be produced in the programming regions. Production from the nonprogrammed areas was subtracted from the total demand requirements mentioned above. This remainder formed the demand restraints that had to be met from production in the programmed regions.

The normal production in the nonprogrammed areas was estimated by a residual method. First, for each state and each grain, the total planted acreage in the programming regions within a state was subtracted from the 1953 acreage planted for grain in the state. When these residual acreages were multiplied by the estimated 1954 normal yields for the state, the total production in the nonprogrammed areas was obtained. With corn, oats, barley, and grain sorghums converted to corn-equivalents, these quantities were 80 and 338 million bushels of wheat and feed grain, respectively. Substracting these quantites from total requirements gave 677 million and 3,549 million bushels of wheat and feed grain, respectively, as the demand or requirement quantities to be provided from the programmed regions.

PRICES USED

Model E is based on the criterion of maximum profit. Hence, it was necessary, for this model, to estimate the regional grain prices. Estimating grain prices consistent with the fundamental concepts underlying model E was not simple. First, the differences in regional prices should be a measure of the relevant transportation cost between regions. Second, the regional prices should represent the relative values of each grain in a competitive market.

Briefly, regional grain prices were estimated as follows: The average wheat-corn price relative for the period 1932–41 provided the basis for estimating the price of wheat.[6] First, the 1945–54 United States average price of corn was multiplied by the 1932–41 United States wheat-corn price relative. This product was then subtracted from the actual United States average price of wheat for the period 1945–54. Next, this difference was subtracted from each average state wheat price for the period 1945–54. Finally, regional wheat prices were estimated by adjusting the calculated state average prices by the price gradients indicated on a wheat isoprice map. It was assumed that prices within each state were a linear function of distance. Regional corn prices were estimated with the aid of a corn isoprice map, by adjusting 1945–54 average state corn prices in a way similar to that used in adjusting state wheat prices. Individual prices for oats, barley, and sorghum were not estimated—these grains are converted to corn-equivalents for programming. Thus, in essence, the prices used for these three grains were the corn prices weighted by their respective feed values in terms of corn.

The regional wheat and corn prices used for programming of model E are presented in Appendix Table B.

STRUCTURE OF COEFFICIENT MATRICES AND WEIGHTING METHODS

To illustrate more clearly the nature of the programming models, the coefficient matrix for model A is outlined below, followed by a summary of the weighting methods used to form aggregate coefficients and restraints. In the

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[6] For more recent periods, the market wheat price has been maintained above the competitive level; for example, the price of wheat relative to corn increased from 122 for the period 1931–42 to 131 for the period 1945–54.

tableau below, P_{ij} is a vector of requirements for the jth crop activity for the ith restraint, while S is the vector of restraints. Using an acre as the unit of output, all activities have a land input requirement of 1, where they are relevant for the region. Considering a unit of output to be the product of an acre, the y_{ij} simply represent the per acre yield of the jth crop in the ith region. The first $m = 104$ equations are for the land restraints while the last two are demand restraints. The y_{ij} or per acre yields of grain crops are defined later in respect to method of computation. The number of P_{ij} is $mr = 312$ for model A (without vectors of identity matrix).

P_{11}	P_{12}	P_{13}	P_{21}	P_{22}	P_{23}	$\cdots P_{ij}$	$\cdots P_{m1}$	P_{m2}	P_{m3}		S
1	1	1	0	0	0	. . . 0	. . . 0	0	0	$\leq$	s_1
0	0	0	1	1	1	. . . 0	. . . 0	0	0	$\leq$	s_2
0	0	0	0	0	0	. . . 0	. . . 0	0	0	$\leq$	s_3
.	.	.	.	.	.	.	.	.	.		.
.	.	.	.	.	.	.		.	.		.
.	.	.	.	.	.	.	.	.	.		.
0	0	0	0	0	0	. . . 1	. . . 0	0	0	$\cdots$	s_i
.	.	.	.	.	.	.	.	.	.		.
.	.	.	.	.	.	.	.	.	.		.
.	.	.	.	.	.	.	.	.	.		.
0	0	0	0	0	0	. . . 0	. . . 1	1	1	$\leq$	s_m
y_{11}	0	0	y_{21}	0	0	. . . 0	. . . y_{m1}	0	0	$=$	d_1
0	y_{12}	y_{13}	0	y_{21}	y_{23}	. . . y_{mj}	. . . 0	y_{m2}	y_{m3}	$=$	d_2

RESTRAINTS AND OUTPUTS. The land restraints based on 1953 acreages are defined as follows:

$$s_i = \sum_{g=1}^{5} QA_{ig} + \sum_{g=1}^{5} F_{ig} \qquad (i = 1, 2, 3, \ldots 104) \tag{5}$$

where: s_i = acreage restraint in ith region; QA_{ig} = planted acreage in the ith region of the gth grain; F_{ig} = acreage of gth grain planted on summerfallow in the ith region ($g = 1$ = wheat, $g = 2$ = corn, $g = 3$ = oats, $g = 4$ = barley, and $g = 5$ = sorghum).

The y_{ij} or per acre yields have been computed on this basis.

$$y_{i1} = \frac{Q_{i1}T_{i1}}{H_{i1} + A_{i1} + F_{i1}}; \tag{6}$$

$$y_{i2} = \frac{Q_{i1}T_{i1}K_1}{H_{i1} + A_{i1} + F_{i1}}; \tag{7}$$

$$y_{i3} = \sum_{g=2}^{5} q_{ig} \frac{Q_{ig}T_{ig}K_g}{H_{ig} + A_{ig} + F_{ig}}; \tag{8}$$

$$\sum_{g=2}^{5} q_{ig} = 1.$$

where y_{i1} = estimated 1954 normal per acre yield of wheat in ith region; y_{i2} = estimated 1954 normal per acre yield of wheat in corn-equivalent bushels in ith region; y_{i3} = estimated 1954 weighted normal per acre yield of feed grains in ith region; Q_{ig} = 1945–54 trend adjustment factor for gth crop in ith region; H_{ig} = 1945–54 harvested acreage of gth crop in ith region; A_{ig} = estimated 1945–54 abandoned acreage (acreage seeded for grain minus harvested acreage) of gth crop in ith region; F_{ig} = estimated 1954 acreage of gth crop in ith region planted on fallowed land multiplied by the factor 10; K_g = corn-equivalent bushel conversion factor of gth grain; and q_{ig} = proportion gth crop acreage is of total corn, oats, barley, and grain sorghums acreage, 1953.

The demand restraints at the national level are the following for food wheat and feed grains:

$$d_1 = Nb_1 + E_w - R_w; \tag{9}$$

$$d_2 = Nb_2 + Lb_3 + E_f - R_f \tag{10}$$

where d_1 = estimated 1954 normal disappearance of wheat; N = U. S. population, January 1955; b_1 = estimated 1954 normal per capita consumption in bushels; E_w = net exports of wheat, 1954; R_w = residual wheat production in nonprogrammed areas; d_2 = estimated 1954 normal disappearance of feed grain; b_2 = estimated 1954 normal per capita use of feed grains in direct consumption; L = number of grain-consuming livestock units, 1954–55 and b_3 = equal normal feed production per livestock unit; E_f = net exports of feed grain, 1954; and R_f = residual feed grain in nonprogrammed areas.

ACTIVITY COSTS. The activity or per acre costs are defined as follows:

$$c_{ij} = \sum_{k=1}^{3} \sum_{l=1}^{4} q_{ig}'^{kl} e_{ig}^{kl} \qquad (g = 1); \tag{11}$$

$$c_{i3} = \sum_{g=2}^{5} \sum_{k=1}^{3} \sum_{l=1}^{4} q_{ig}^{kl} e_{ig}^{kl} \qquad (i = 1, 2, 3, \ldots n) \tag{12}$$

where $\sum_{k=1}^{3} \sum_{l=1}^{4} q_{ig}'^{kl} = 1, \quad \sum_{g=2}^{5} \sum_{k=1}^{3} \sum_{l=1}^{4} q_{ig}^{kl} = 1, \quad q_{ig} q_{ig}'^{kl} = q_{ig}^{kl};$ and

c_{ij} $(j = 1, 2)$ = estimated cost of producing a composite wheat acre for food or feed in the ith region, 1954 $(c_{i1} = c_{i2})$; c_{i3} = estimated cost of producing a composite feed grain acre in the ith region, 1954; e_{ig}^{kl} = estimated per acre cost of the lth crop acre component by the kth production technique for the gth crop in ith region; $q_{ig}'^{kl}$ = estimated proportion of per acre production cost of gth crop due to the kth technique on the lth crop acre component in the ith region; q_{ig}^{kl} = estimated proportion of per acre feed grain production cost due to the gth grain, kth technique on the lth crop acre component, in the ith region: where the g's are the same as above; where k = (1, 2, 3) = production techniques with 1 = mechanical production, 2 = semimechanical production, and 3 = nonmechanical production; where $l = (1, 2, 3, 4)$ = crop acre components with 1 = planted and harvested acre, not irrigated, 2 = planted, but not harvested acre (abandoned), 3 = planted and harvested acre, irrigated, and 4 = cultivated summer-fallow acre. Also,

$$e^{kl}_{ig} = w^{kl}_{ig} + v^{kl}_{ig} + h^{kl}_{ig} + m^{kl}_{ig} \tag{13}$$

where w = estimated per acre labor cost, v = estimated per acre power and machinery cost, h = estimated per acre chemical cost, and m = estimated per acre miscellaneous cost.

OTHER MODELS. The structure of the coefficients and model for B is the same as described above for A, except the e^{kl}_{ij} is redefined. Equation (13) becomes

$$e'^{kl}_{ij} = (a + w + v + h + m)^{kl}_{ij} \tag{14}$$

where a = estimated per acre land rent. All other symbols have the same meaning as before. The land restraints are partitioned for model C into a wheat land maximum and a feed grain land maximum. Thus

$$s_{iw} = QA_{i1} + F_{i1}; \tag{15}$$

$$s_{if} = \sum_{g=2}^{5} QA_{ig} + F_{ig}. \tag{16}$$

Consequently, there are two inequalities of the following type for each region:

$$s_{iw} \geq X_{ij} + X_{i,j+1}; \qquad (i = 1, 2, \ldots, 104) \tag{17}$$

$$s_{if} \geq X_{i,j+2} \qquad (j = 1, 2, 3) \tag{18}$$

where s_{iw} = wheat restraint in ith region, s_{if} = feed grain restraint in ith region, and the other symbols have the same meaning as above. For model C, therefore, the order of the A matrix is 210 × 312. The c_{ij}'s and y_{ij}'s are the same as those in model A.

The cost and yield coefficients for the wheat activities in model D are the same as in model A. Since corn, oats, barley, and sorghum are independent activities in model D, the following equations define the weighting method used for the y_{ij}'s and c_{ij}'s of these activities:

$$y_{ij} = \frac{Q_{ig}T_{ig}K_g}{H_{ig} + A_{ig} + F_{ig}}; \tag{19}$$

$$(g = 2, 3, 4, 5) \qquad (j = g + 1)$$

$$c_{ij} = \sum_{k=1}^{3} \sum_{l=1}^{4} q'^{kl}_{ig} e^{kl}_{ig} \tag{20}$$

where the notation has the same meaning as in equations (10) and (12) and

$$\sum_{k=1}^{3} \sum_{f=1}^{4} q'^{kl}_{ig} = 1.$$

The order of the coefficient matrix is 106 × 624 for model D. The only difference between model E and model A is a change in the objective function, as explained above.

SUMMARY OF OPTIMAL PROGRAMS

Adaptation of American agriculture to do away with surplus grain buildup would necessitate withdrawal of some land from production. Hence, one of our major interests is in specifying the amount of land needed for crops and available for withdrawal and shift to other crops. We summarize results in this direction in Table 5 where we assume that land not required for grain production would be shifted to "lower uses" such as grass and forestry. The acreage specified for food wheat and feed grains in Table 5 would allow attainment of the "discrete demand restraints" of 757 million bushels of food wheat and 3,887 million bushels of feed grain (in corn equivalent). Three models —A, B, and E—provide somewhat similar results. Model D, including a new crop technique not in wide use by farmers, would require a much smaller feed grain acreage and a higher wheat acreage because the latter crop would be grown more on land of lower yield and not so well adapted to the new rotational technique for feed grain. We do not believe model D is greatly applicable at the present time, but it might well be in another decade. Model C, which is somewhat unrealistic since it does not allow wheat and feed grains to compete for land, indicates the smallest surplus acreage and the largest amount of land required for feed grains. Model D has the lowest U. S. average per unit (bushel) costs for wheat and feed grains. Model C has the largest, as expected because crops are restrained from shift to greatest comparative advantage. Since we believe models A, B, and E to be most realistic at the present time, and since our space is limited, detailed discussion which follows will relate only to these three.

TABLE 5
UNITED STATES ACREAGE (000's) DEVOTED TO FOOD WHEAT AND FEED GRAINS UNDER SPECIFIED MODELS

Model	Food Wheat, Acres[a]	Feed Grain, Acres	Acres Unused and Available for Shift	Average Cost per Bushel	
				Food Wheat	Feed Grain
A	63,661	114,003	31,951	.73	.54
B	58,357	116,607	34,651	.75	.54
C	57,562	129,089[b]	22,964	.81	.57
D	65,712	81,511	62,392	.71	.45
E	67,121	113,639[b]	28,855	.80	.55

[a] Includes acreage necessary for summer fallow (as also true per footnote *b* for wheat).
[b] Includes acres of wheat designated for livestock feed.

The per unit costs indicated in Table 5 are simply as discussed in respect to

TABLE 6

Producing Regions, Acreages Utilized, and Production, Model A Solution

Region	Acreage	Wheat	Feed Grain[a]
	1,000 acres	1,000 bushels	1,000 bushels
2	92	–	3,600
3	445	–	19,189
4	298	–	13,075
25	574	–	27,833
26	412	–	17,770
28	4,935	–	233,287
29	757	–	30,303
30	1,902	–	72,903
31	4,700	–	222,916
32	996	–	43,444
36	2,297	30,121	45,623
37	7,754	–	356,616
38	4,841	–	231,170
39	1,133	–	39,025
40	1,013	–	31,522
41	693	–	20,970
43	4,795	–	175,338
44	4,263	–	153,258
45	10,879	–	403,933
46	4,107	–	157,062
47	2,711	–	84,314
48	2,561	–	73,085
49	1,304	–	30,795
50	4,827	–	89,054
51	7,898	–	111,446
52	2,790	–	37,225
53	5,016	37,722	–
54	1,076	–	17,072
55	2,101	–	31,183
56	4,155	–	68,643
57	1,404	–	28,884
58	1,013	–	17,430
59	3,624	–	94,739
60	3,874	–	113,009
61	441	5,667	–
62	4,282	42,692	–
63	1,990	–	53,852
64	2,509	28,104	–
65	4,718	–	148,795
66	1,220	–	31,068
69	1,119	19,469	–
70	1,792	19,394	–
71	1,371	18,213	–

TABLE 6 (*continued*)

PRODUCING REGIONS, ACREAGES UTILIZED, AND PRODUCTION, MODEL A SOLUTION

Region	Acreage	Wheat	Feed Grain[a]
	1,000 acres	1,000 bushels	1,000 bushels
72	2,736	37,617	–
73	7,664	72,121	–
74	5,114	–	82,640
76	2,739	35,469	–
77	2,912	19,301	–
79	2,025	20,898	–
80	2,881	–	76,304
81	1,955	14,562	–
82	1,176	–	16,450
83	369	3,063	–
84	98	–	1,088
88	610	–	12,359
89	6,493	52,009	–
90	3,833	34,035	–
92	692	7,086	–
94	4,293	29,964	–
98	1,750	22,569	–
100	4,685	79,077	–
101	2,785	35,147	–
102	544	6,316	–
103	554	6,895	–
104	1,015	–	30,643
Total	177,664	677,511	3,548,915

[a] Expressed in corn-equivalent bushels.

equations (11) through (13). They do not include (except for rent in C) imputed interested returns on capital employed in production. The per-unit (bushel) costs under the optimum programs (with cost minimization for A and B and profit maximization for E) are highly similar; greatest variance being for wheat.

REGIONAL PATTERNS OF WITHDRAWAL AND PRODUCTION. There is an important degree of similarity among some of the programming models in the production patterns specified. The total acreages specified to remain in grain production and the bushel production of wheat and feed grain for each region are specified in Tables 6 through 10. Where a region is not indicated in these tables, it is not specified to produce wheat or feed grains, given the objective function of the particular model.

To summarize more clearly the geographic production pattern specified by models A, B, and E, we include Figures 2, 3, and 4 to indicate the regions in which feed grains and wheat would be located if average annual production

TABLE 7

Producing Regions, Acreages Utilized, and Production, Model B Solution

Region	Acreage	Wheat	Feed Grain[a]
	1,000 acres	1,000 bushels	1,000 bushels
3	445	–	19,189
4	298	–	13,075
25	574	–	27,833
26	412	–	17,770
28	4,935	–	233,287
29	757	–	30,303
30	1,902	–	72,903
31	4,760	–	222,916
32	996	–	43,444
35	994	20,494	–
36	2,297	62,619	–
37	7,754	–	356,616
38	4,841	–	231,170
39	1,133	–	39,025
40	1,013	–	31,522
42	390	7,673	–
43	4,795	–	175,339
44	4,263	–	153,258
45	10,879	–	403,933
46	4,107	–	157,062
47	2,711	–	84,314
48	2,561	–	73,085
49	1,304	–	30,795
50	4,827	–	89,054
51	7,898	–	111,446
52	2,790	–	37,225
53	5,016	–	73,438
54	1,076	–	17,072
55	2,101	–	31,183
56	4,155	–	68,643
57	1,404	–	28,884
58	1,013	–	17,430
59	3,624	–	94,739
60	3,874	–	113,009
61	441	5,667	–
62	4,282	42,692	–
63	1,990	–	53,852
64	2,509	28,104	–
65	4,718	–	148,799
66	1,220	–	31,070
69	1,119	19,470	–
70	1,792	19,394	–

TABLE 7 (*continued*)

PRODUCING REGIONS, ACREAGES UTILIZED, AND PRODUCTION, MODEL B SOLUTION

Region	Acreage	Wheat	Feed Grain[a]
	1,000 acres	1,000 bushels	1,000 bushels
71	1,371	18,212	–
72	2,736	37,617	–
73	7,664	64,187	14,277
74	5,114	–	82,645
76	2,739	35,469	–
79	2,025	20,898	–
80	2,881	–	76,305
81	1,955	14,561	–
83	369	3,063	–
88	610	–	12,359
89	6,493	52,008	–
90	3,833	34,035	–
91	611	3,979	–
92	692	7,086	–
93	830	7,204	–
94	4,293	29,969	–
98	1,750	22,569	–
100	4,685	79,077	–
101	2,785	35,147	–
102	544	6,316	–
104	1,015	–	30,643
Total	174,965	677,510	3,548,912

[a] Expressed in corn-equivalent bushels.

were to equal requirements under the conditions assumed and if the geographic pattern of production were consistent with certain restricted comparative advantages of various regions. Figure 5 indicates the extent of agreement in number of times a particular region is specified for a particular use by the three models. The nonshaded areas include feed grain and wheat production at the same levels as in the base year. We assume that the small portion of grains produced in these nonshaded areas (8% of the total United States tonnage) is grown for complementary and supplementary reasons and would largely continue even under competitive markets and prices. These regions were not included in the programming model.

Under the assumptions of model A, regions would be withdrawn from production of all grains in southeastern Colorado, eastern New Mexico, northern Utah, and eastern Wyoming and Montana. Regions scattered among Texas, Nebraska, Wisconsin, Michigan, Oklahoma, Missouri, Kansas, and New York also would be withdrawn. In the Southeast, regions representing a large acreage would be withdrawn from production of grains (see Figure 2). It is interesting to note that the major wheat and feed grain areas would remain

TABLE 8

PRODUCING REGIONS, ACREAGES UTILIZED, AND REGIONAL WHEAT AND FEED GRAIN PRODUCTION, MODEL C SOLUTION

Region	Acreage	Food Wheat	Feed Wheat[a]	Feed Grain[a]
	1,000 acres	1,000 bushels	1,000 bushels	1,000 bushels
1	316	8,505	–	–
2	1,622	–	–	63,807
3	355	–	–	15,340
4	194	–	–	8,543
5	149	–	–	5,322
6	548	–	–	19,572
16	[b]	8	–	–
21	691	–	–	17,109
22	885	–	–	30,121
23	235	–	–	7,526
24	281	–	–	9,455
25	384	–	–	18,640
26	277	–	–	11,962
27	685	–	–	26,405
28	4,935	41,044	–	153,000
29	552	–	–	22,070
30	1,902	9,543	–	53,792
31	4,760	22,839	–	178,938
32	996	5,937	–	33,837
33	1,649	14,324	–	37,542
34	504	–	–	15,469
35	15	307	–	–
36	2,297	1,128	–	86,326
37	7,754	4,677	–	348,088
38	4,841	18,341	–	198,805
39	827	–	–	28,489
40	1,013	8,936	–	17,174
41	693	5,369	–	13,341
42	443	8,705	–	–
43	4,795	26,562	–	132,509
44	4,263	1,897	–	148,863
45	10,879	309	–	403,164
46	4,107	477	–	156,025
47	2,711	1,055	–	82,381
48	2,561	1,301	–	70,343
49	1,304	1,501	–	28,357
50	4,827	24,770	–	39,109
51	7,898	–	54,075	26,291
52	2,790	17,041	–	4,841
53	5,016	30,330	–	14,391
54	1,076	–	4,275	9,425
55	2,101	12,625	–	8,052
56	4,155	–	23,070	31,024
57	1,188	–	–	24,431

TABLE 8 (*continued*)

PRODUCING REGIONS, ACREAGES UTILIZED, AND REGIONAL WHEAT AND FEED GRAIN PRODUCTION, MODEL C SOLUTION

Region	Acreage	Food Wheat	Feed Wheat[a]	Feed Grain[a]
	1,000 acres	1,000 bushels	1,000 bushels	1,000 bushels
58	1,013	1,610	544	13,185
59	3,624	1,375	–	90,987
60	3,874	4,259	–	105,335
61	441	3,255	–	3,392
62	4,282	36,364	–	12,472
63	1,990	5,196	–	40,563
64	2,509	–	17,059	27,038
65	4,718	–	31,378	98,460
66	1,220	7,086	–	20,930
67	293	5,241	–	–
69	616	10,710	–	–
70	1,792	11,645	–	14,316
71	1,371	14,118	–	5,112
72	2,736	32,279	–	5,771
73	7,664	61,781	–	18,602
74	5,114	–	36,728	10,351
76	2,568	33,259	–	–
77	2,911	–	18,495	5,134
78	278	2,879	–	–
79	2,025	18,200	–	3,036
80	2,881	–	12,413	28,091
81	1,955	10,001	–	5,929
82	1,176	376	–	15,394
83	369	2,300	–	877
84	49	–	–	549
85	402	–	–	7,215
87	325	–	–	5,653
88	605	–	–	12,254
89	6,493	48,822	–	6,900
90	3,833	30,955	–	6,890
91	505	–	3,688	–
92	629	6,442	–	–
93	686	5,952	–	–
94	4,293	–	30,415	4,836
95	117	–	–	2,596
98	1,750	19,843	–	4,937
99	519	–	5,420	1,044
100	4,685	–	82,046	8,216
101	2,785	34,782	–	705
102	506	–	6,589	–
103	554	1,287	–	8,357
104	1,015	–	1,732	25,888
Total	186,645	677,508	327,927	3,220,984

[a] Expressed in corn-equivalent bushels.

[b] Less than 500 acres.

TABLE 9

Producing Regions, Acreages Utilized, and Production, Model D Solution

Region	Acreage	Wheat	Corn	Barley[a]	Sorghum[a]
	1,000 acres	1,000 bushels	1,000 bushels	1,000 bushels	1,000 bushels
30	1,902	–	75,600	–	–
31	4,760	–	264,562	–	–
32	996	–	55,826	–	–
36	2,297	21,472	88,528	–	–
37	7,754	–	464,324	–	–
38	4,841	–	275,705	–	–
39	1,133	–	40,905	–	–
43	4,795	–	205,113	–	–
44	4,263	–	196,652	–	–
45	10,879	–	545,036	–	–
46	4,107	–	210,991	–	–
47	2,711	–	129,180	–	–
48	2,561	–	101,049	–	–
51	7,898	63,108	–	–	–
52	2,790	19,589	–	–	–
55	2,101	17,020	–	–	–
56	4,155	–	92,416	–	–
58	1,013	–	21,897	–	–
59	3,624	–	132,320	–	–
60	3,874	–	150,826	–	–
61	441	5,667	–	–	–
62	4,282	42,692	–	–	–
64	2,509	28,104	–	–	–
65	4,718	–	174,700	–	–
66	923	–	29,115	–	–
70	1,792	19,394	–	–	–
71	1,371	18,212	–	–	–
72	2,735	37,617	–	–	–
73	7,664	72,120	–	–	–
74	5,114	37,436	–	–	–
76	2,739	35,469	–	–	–
79	2,025	20,898	–	–	–
80	2,881	–	–	–	78,143
81	1,955	14,561	–	–	–
83	369	3,063	–	–	–
88	610	–	–	–	14,176
89	6,493	–	–	151,806	–
90	3,833	34,035	–	–	–
92	692	7,086	–	–	–
94	4,293	29,964	–	–	–
98	1,750	22,569	–	–	–

TABLE 9 (*continued*)

PRODUCING REGIONS, ACREAGES UTILIZED, AND PRODUCTION, MODEL D SOLUTION

Region	Acreage	Wheat	Corn	Barley[a]	Sorghum[a]
	1,000 acres	1,000 bushels	1,000 bushels	1,000 bushels	1,000 bushels
100	4,685	79,077	-	-	-
101	2,785	35,147	-	-	-
102	544	6,316	-	-	-
103	554	6,896	-	-	-
104	1,015	-	-	-	49,954
Total	147,226	677,512	3,254,745	151,806	142,278

[a] Expressed in corn-equivalent bushels.

entirely in production under the construction and assumptions of the models. Southwestern Kansas and western Texas would shift to sorghums for feed.

Model B (Figure 3) provides a spatial production pattern differing somewhat from both A and E. The main differences under B are: All of Montana would be devoted to wheat for food, the Oklahoma panhandle and Pennsylvania would be shifted out of grains, and the region in southwest Missouri would be used for food wheat. Also, a large portion of Kansas would be used for both wheat and feed grain.

Under model C, as compared to model A, large parts of Montana, Washington, Oregon, Idaho, and Nebraska would be devoted to wheat for feed only. In parts of Nebraska and Colorado, wheat would be grown for both feed and food. In the upper plains, North Dakota and South Dakota, along with parts of Minnesota and Wisconsin, would be devoted to wheat for food. Also, slightly more feed grain would be produced along the Atlantic seaboard and the Gulf of Mexico. Under this profit-maximizing model, it is the relatively high wheat prices because of location near larger milling and consuming centers, and because of prices paid for hard red spring and durum wheats, that cause wheat for food to be specified in Minnesota and Wisconsin, as well as the Dakotas. While there is considerable difference in the food wheat and feed grain patterns specified by models A and E, they largely agree regarding regions specified to remain in grain production. Only five regions specified for production of some grain by model E are not specified by model A. Conversely, only one region specified to remain in grain production by model A is not specified by model E. Hence, only four more of the 104 regions would be needed to meet feed grain and food wheat requirements in model E than in A. The five additional regions for fulfilling feed or food requirements under E include regions in eastern Virginia, northeast Ohio, western Kansas, southern Alabama and northern Utah. The region specified by model A, but not by C, is in northeast South Dakota. Thirty-five entire regions and part of a small region in western Kentucky would not be required for grain production in model E. These 36 regions represent the 28.8 million acres which could be

TABLE 10

PRODUCING REGIONS, ACREAGES UTILIZED, AND PRODUCTION, MODEL E SOLUTION

Region	Acreage	Food Wheat	Feed Wheat[a]	Feed Grains[a]
	1,000 acres	1,000 bushels	1,000 bushels	1,000 bushels
2	2,480	–	–	97,567
3	445	–	–	19,189
4	298	–	–	13,075
5	208	–	–	7,434
16	91	2,087	–	–
23	231	–	–	7,402
25	574	–	–	27,833
26	411	–	–	17,770
27	1,067	–	–	41,137
28	4,935	–	–	233,287
29	757	–	–	30,303
30	1,902	–	–	72,903
31	4,760	–	–	222,916
32	996	–	–	43,444
36	2,297	62,620	–	–
37	7,754	–	–	356,616
38	4,841	–	–	231,170
39	1,133	–	–	39,026
40	1,013	–	–	31,522
41	693	–	–	20,970
43	4,795	–	–	175,339
44	4,263	–	–	153,258
45	10,879	–	–	403,933
46	4,107	–	–	157,062
47	2,711	46,006	–	–
48	2,561	–	–	73,085
49	1,304	18,955	–	–
50	4,827	44,165	–	–
51	7,898	63,108	–	–
52	2,790	19,589	–	–
53	5,016	37,722	–	–
54	1,076	8,509	–	–
55	2,101	17,018	–	–
56	4,155	37,519	–	–
58	1,013	–	–	17,430
59	3,624	–	–	94,739
60	3,874	–	–	113,009
61	441	–	6,351	–
62	4,282	21,340	23,922	–
63	1,990	–	–	53,854
64	2,509	28,104	–	–
65	4,718	–	–	148,802

TABLE 10 (*continued*)

PRODUCING REGIONS, ACREAGES UTILIZED, AND PRODUCTION, MODEL E SOLUTION

Region	Acreage	Food Wheat	Feed Wheat[a]	Feed Grains[a]
	1,000 acres	1,000 bushels	1,000 bushels	1,000 bushels
66	1,220	–	–	31,069
69	1,119	19,470	–	–
70	1,792	19,394	–	–
71	1,371	18,212	–	–
72	2,736	37,617	–	–
73	7,664	72,121	–	–
74	5,114	–	–	82,649
76	2,739	35,470	–	–
77	2,912	–	–	35,114
79	2,025	20,898	–	–
80	2,881	–	–	76,302
81	1,955	14,561	–	–
82	1,176	–	–	16,449
83	369	3,063	–	–
84	98	–	–	1,088
88	610	–	–	12,359
89	6,493	–	58,307	–
90	3,833	–	–	76,116
92	692	–	7,944	–
94	4,293	29,967	–	–
98	1,750	–	25,300	–
99	519	–	–	18,315
100	4,685	–	88,681	–
101	2,785	–	39,405	–
102	544	–	7,083	–
103	554	–	7,732	–
104	1,015	–	–	30,643
Total	180,764	677,515	264,725	3,284,179

[a] Expressed in corn-equivalent bushels.

shifted to nongrain uses. The pattern is the same, except for the six regions noted above, for model A. (See Figure 4.)

Consistency or lack of consistency in the three models is indicated by Figure 5. The major corn and winter and spring wheat areas are specified to remain in production of grain in all three models. In a similar manner, all three models specify withdrawal from grain production of eastern Colorado and New Mexico, parts of Kansas, Oklahoma, Texas, Michigan, and New York, and practically all of the Southeast—from Arkansas, Tennessee, and southeastern Virginia to the coasts. Only one model (B) specified grain production in eastern Wyoming, southeast Montana, western Missouri, and a few other scattered areas.

All three models are consistent for 88 of the 104 regions in the sense that

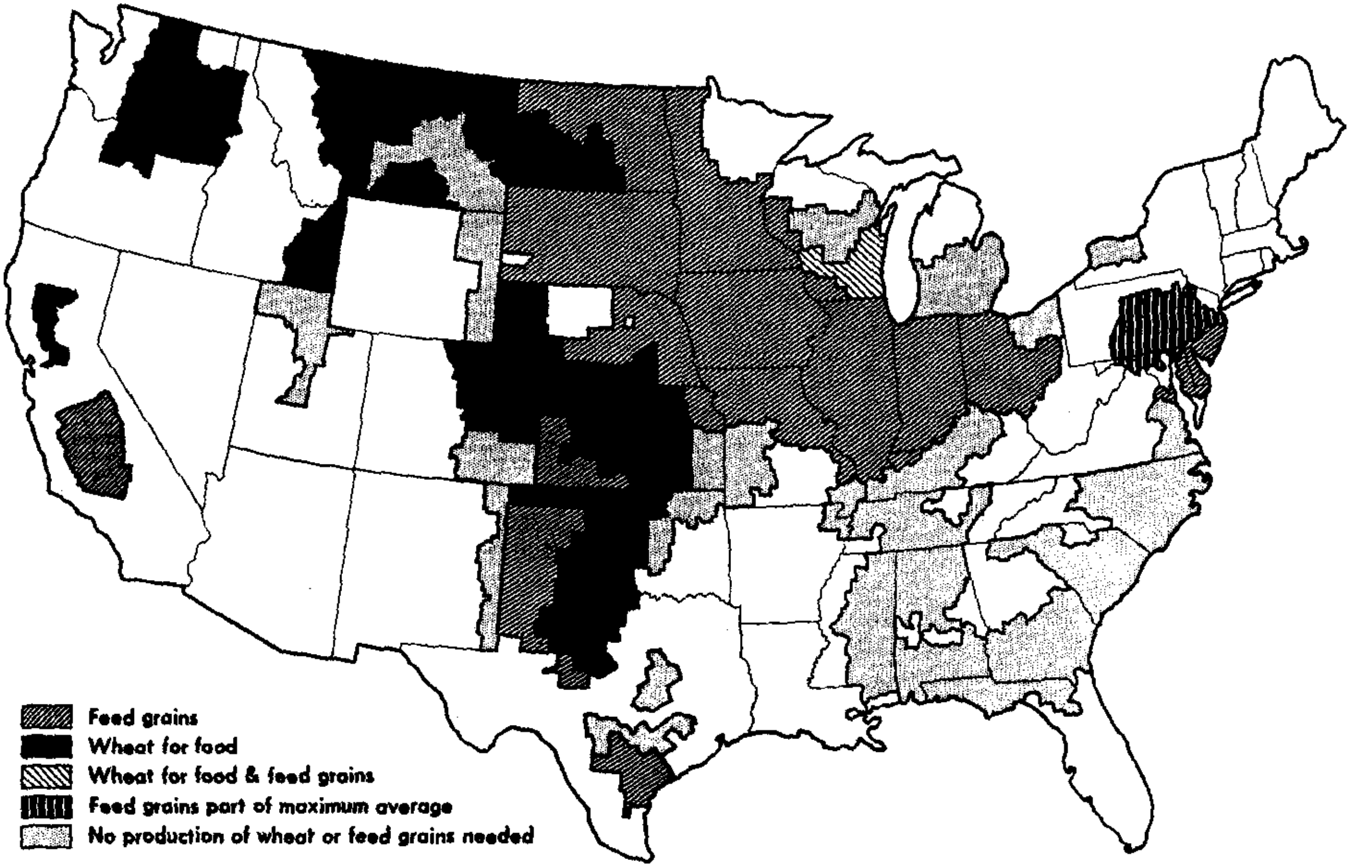

FIGURE 2. Production pattern specified by Model A.

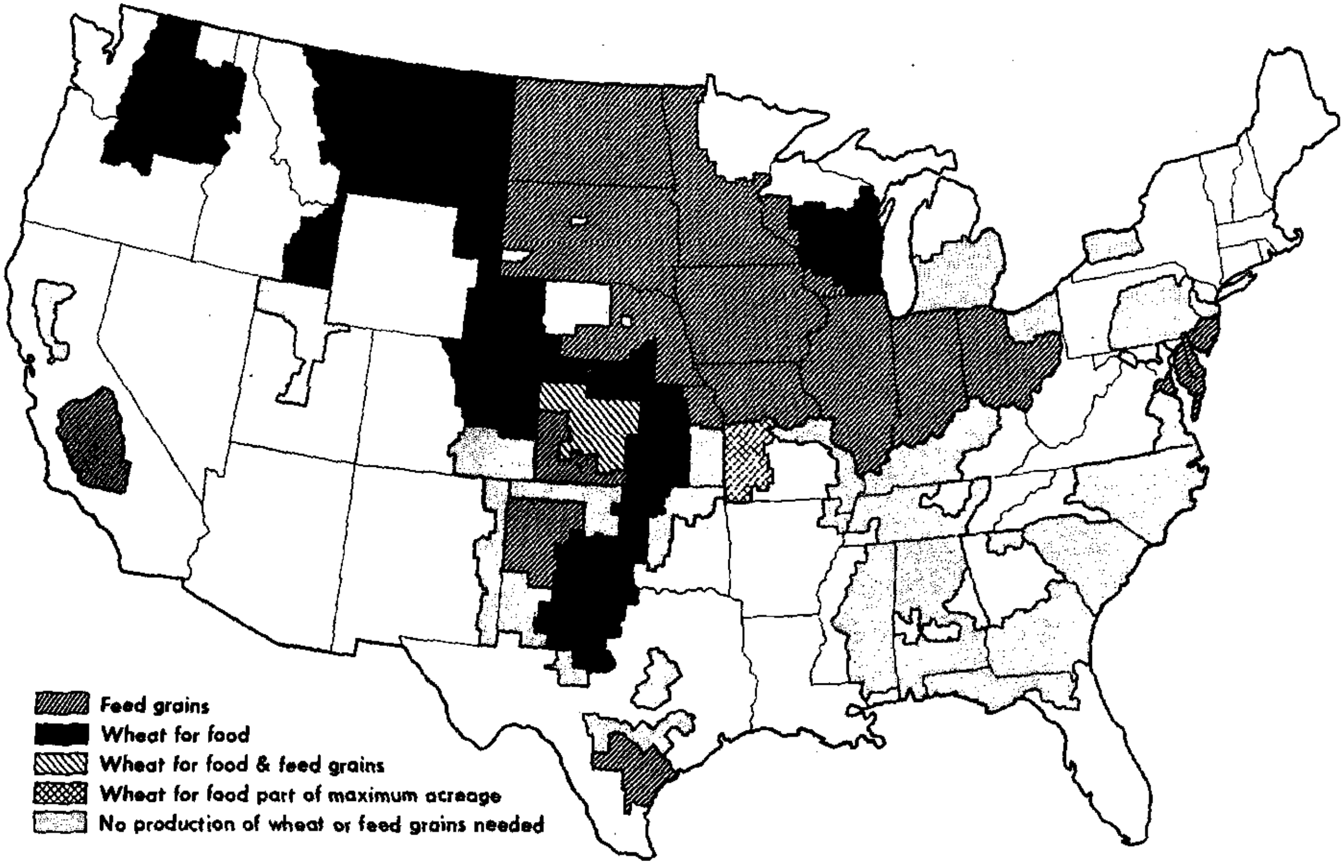

FIGURE 3. Production pattern specified by Model B.

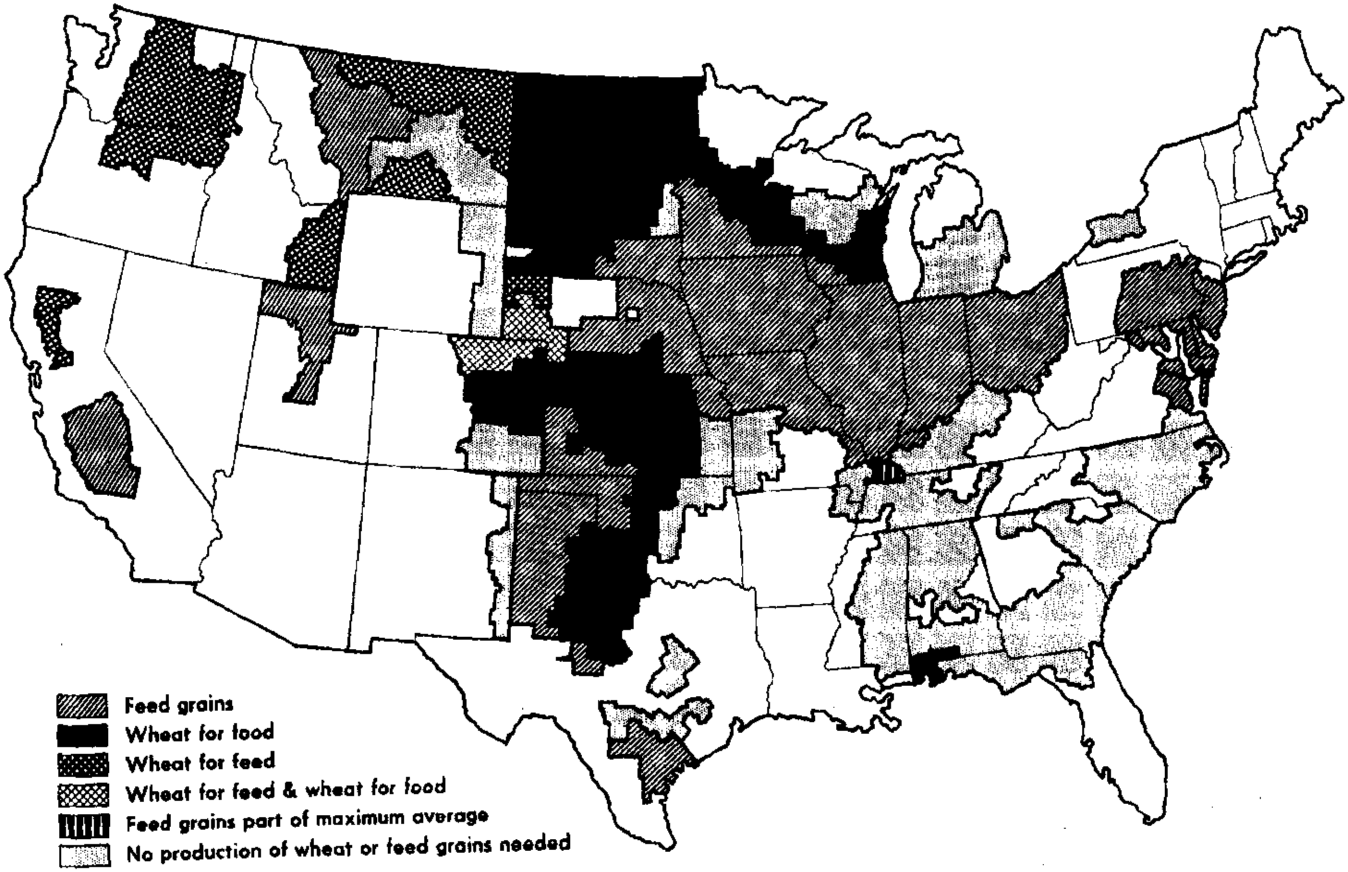

FIGURE 4. Production pattern specified by Model E.

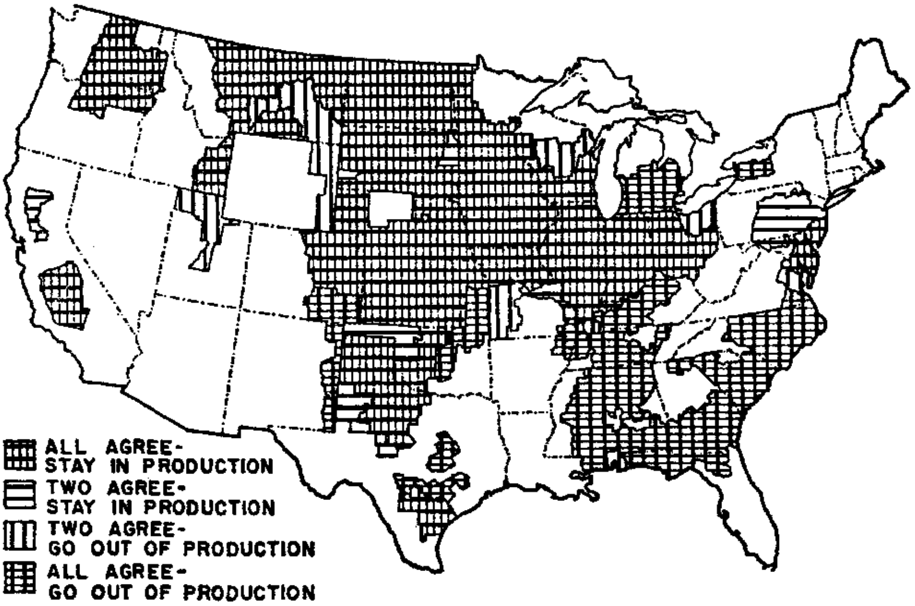

FIGURE 5. Consistency of three models.

they specify 88 regions (those indicated in Figure 5 as "all agree") that should remain in grain production or shift completely out of grains. Hence, disagreement among the three models existed for 16 regions. However, disagreement between models A and E, the two models deemed most appropriate by the writers, existed for only six regions.

Since this study was initiated, the American public has put into effect a national land withdrawal program. In contrast to the purpose of this study, however, the action program does not attempt to withdraw land where it has least comparative advantage in wheat and feed grain. It allows this pattern to develop somewhat, but restrains its extent so that the "population will not be thinned" too greatly in specific areas and tries to attain some withdrawal in all producing regions. This land is withdrawn under the 1956 Conservation Reserve Act, making payment for land withdrawn from production. We can say, however, to the extent allowed by policy restrictions, land has been withdrawn in a manner highly consistent with out optimal solutions. For example, largest amounts of land have been put under soil bank (conservation reserve) in the marginal areas of the Great Plains and the Southeast. We feel that our results, and especially those that we are now preparing, could provide firm bases for national policy formulation.

SIGNIFICANCE AND LIMITATIONS OF RESULTS

In interpreting these results for all models, it must be remembered that spatial production patterns were computed under the assumption of techniques (i.e., technical coefficients) equal to the average of each region and that the coefficients are constant within the delineated regions. Locational variations from the regional coefficients used would mean that some acreages in our "outgoing" regions should remain in grain production and some acreages in our "staying in" regions should be withdrawn. And only grain crops are used as competitive alternatives in programming, although inclusion of land rent as a cost in model B gives some recognition to alternative crops.

We are aware, perhaps more than anyone else, of the limitations of these assumption and of our data. We use them because of computational necessity. The empirical task involved is huge, involving about five man-years of professional work. Our efforts in gathering together data and in making computations were sizable indeed. Most of three years was spent in routine development of coefficients for this phase. Had we not lacked manpower and computational funds, and had we accepted "crude" coefficients rather than being concerned with as much accuracy as currently feasible, we could have constructed a somewhat more appropriate model in which we included both grains and nongrain crops and livestock activities and an objective function of return maximization.

Downward adjustments in production to meet demand might entail two types of input changes: (*a*) Withdrawal of land inputs and complementary inputs from grain production in extensive regions so that the geographic pattern of production would be consistent with restricted comparative advantages of various regions, and (*b*) maintenance of land in production but a lessening of other inputs—that is, a reduction of farming intensity—in regions remaining in grain production. However, we believe (and have some empirical indications) that this is the major adjustment and that consideration of the second would alter our results only slightly.

The study reported here was somewhat methodological in nature, to establish the steps necessary leading to data and models realistic for the uses mentioned above. The further steps in models and analysis, while still burdensome in data requirements, will be somewhat less difficult than the one reported here. In a sense, we have now made progress in establishing a "data bank" useful for further steps. Too, improvement in computer programming routines will increase capacity of computations available, both physically and in terms of research funds.

MODEL IMPROVEMENT. The new models which we have underway make several improvements over those summarized here. First, they bring technical coefficients up to date, allowing a somewhat larger acreage to be specified for withdrawal because the rate of advance in agricultural technology is more rapid than the rate of growth in food demand. Later models also incorporate cotton and soybeans. Initial work has been started on models which incorporate livestock activities, interregional transportation costs, and regional demand restraints. We have one model under construction which allows different technical coefficients within regions. Finally, using data available on technological trends and growth in population and food demand, projections are being made to 1975. We believe these models will eventually have great utility in national agricultural policy and educational programs designed to restore better economic balance to agriculture. Finally, we believe they can have use in foreign policy and international economic development goals—in specifying our food production possibilities and how these might be meshed with world growth in population and food demand. A hope for later analysis, too, is to incorporate factor demand into our models, so that we can specify regional requirements or migration needs in labor and capital.

A step now under way is the use of regional demand restraints and associated transportation costs in establishing the objective functions and restraints of the programming models. Freight tariffs are available for many origins and destinations, but other transportation costs, such as handling and commissions, cannot be ascertained easily. Also, total transportation costs apply to a product that takes many forms—wheat, flour, bread, corn, middlings, cornmeal, breakfast cereal, and so on—between producer and consumer. But the difficulties encountered in ascertaining transportation costs should not be more formidable than those of establishing production coefficients.

Further studies are needed in which known differences in input-output coefficients within grain regions can be considered. Additional activities and restraints for lands of different productivities might be used in future analyses. But these refinements are not feasible with current digital computers and research budgets.

Future programming needs to be based on models with variable-demand restraints or equations. Using such models, optimum solutions could be derived for an infinite number of demand levels. The variable-demand method has two advantages. It provides a "tailored" solution to fit most demand projections—as demand projections are changed from time to time, a production solution is available for each.

Quality is a variable that should be considered in later models. Soft wheat cannot be substituted for the hard varieties in the manufacture of some wheat products. It was assumed in the study reported that the regions in the model solutions would provide a variety mix of wheat that would meet the special demands for each variety. Apparently, this assumption was not contradicted by the results.

Many other aggregative problems need to be considered in linear-programming analyses of the grain economy. These include the determination of optimum-producing regions when crop failures are assumed in certain areas, the determination of the optimum level and location of grain stocks over time (dynamic programming), and a combination of the two. A model developed for the last two steps could easily exceed computational facilities if a large number of production regions and years were considered.

Models that use continuous supply and demand functions might be employed to describe the competitive position of various agricultural regions in the wheat and feed grain economy. Spatial equilibrium models using continuous supply and demand functions would seem, however, to be too complex for a detailed analysis of as many as 100 regions. Without the detail of many regions, analysis is general and is of little use in specifying needed adjustments. Too, these models must rest on regression coefficients from time-series observations and involve the limitations and "tie to the past" mentioned in an earlier section.

Analyses of the agricultural industry of the type and detail used in the study reported are desirable from the viewpoint of realism and complete analysis. Experience with the study, however, revealed the true magnitude of such an analysis. But if the regional interdependence of the agricultural industry is to be known or approximated, a programming type of analysis seems to be the most feasible of the several empirical methods presently available. Inclusion of the steps mentioned is necessary before realism and completeness can be achieved. For such analyses, however, sizable research funds and much time would be required.

AUXILIARY INFORMATION. Only summary presentation and interpretation has been made of the linear programming results in this chapter. The basic data and the solutions do lend themselves to by-product uses. One is an analysis of imputed land rents by regions as reported elsewhere.[7] Another is the examination of agricultural policy to meet certain political and institutional restraints. In computing for the latter problem, we examined the amount of land which would need to be withdrawn from production and the total public cost, with the treasury paying farmers to withdraw land from production, if (*a*) land could be withdrawn at lowest cost to the public, with any amount concentrated in individual regions, (*b*) land could be withdrawn in least-cost manner, but with the restraint that no more than 25% could be withdrawn in any one region, and (*c*) land could be withdrawn in least-cost pattern, but with a restraint allowing no more than 50% of land in any one region to be withdrawn. We made this analysis after the government had put into effect the soil bank program mentioned above, paying farmers to leave land idle in order that output might be reduced with a consequent increase in grain price. We examined acreage withdrawal under the above three conditions and with two price goals or restraints in respect to farm prices. Our results showed the total acres necessary to be withdrawn and the public cost to vary greatly depending on the "political" and price restraints used. Political restraints of the type mentioned are very real and have empirical reflection in government programs. For example, local and regional groups did specify that limits be applied to the amount of land withdrawn from production in localities under the soil bank program. These limits were imposed because concentration of land withdrawal caused large numbers of farmers to quit farming and migrate, thus reducing the sales of local merchants.

Political and institutional restraints were not included in the programming results presented on earlier pages. They cannot be ignored in public policy. We do believe, however, that to the extent they can be quantified, such restraints can be realistically examined in later models and analyses.

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[7] See Egbert and Heady (1961).

APPENDIX

The activity costs, per acre yields, and normal prices (for model E) are included below. Since feed grain activities were aggregated into a single activity for all models but D, the coefficients in Table A, rather than those in Table 3 for individual crops, were used in the programming computations for models A, B, C, and E. The price coefficients in Appendix Table B were used only for model E.

TABLE A

ACTIVITY COSTS AND YIELDS (OUTPUTS) PER ACRE, BY REGIONS, MODEL A

	Cost		Yield		
Region	Wheat	Feed Grain	Food Wheat	Feed Wheat	Feed Grain
	Dollars	Dollars	Bushels	Bushels	Bushels
1	29.23	30.83	26.9	30.1	29.7
2	28.08	30.45	21.3	23.9	39.3
3	29.86	29.34	18.1	20.3	43.2
4	28.14	31.47	18.4	20.6	43.9
5	24.36	29.31	21.2	23.8	35.7
6	25.25	31.90	16.2	18.2	35.7
7	32.35	37.67	19.3	21.6	26.8
8	30.17	30.97	18.3	20.5	24.5
9	28.35	34.90	17.7	19.9	28.1
10	22.79	29.96	17.8	19.9	20.4
11	27.24	31.42	16.5	18.5	18.1
12	23.37	25.12	16.6	18.6	15.7
13	22.79	25.01	16.5	18.5	16.7
14	26.46	29.23	16.1	18.1	17.6
15	–	26.49	–	–	14.8
16	23.36	34.59	22.8	25.6	20.2
17	23.73	29.57	20.4	22.9	15.2
18	25.42	29.09	19.6	21.9	21.0
19	22.87	28.22	15.7	17.6	19.0
20	25.84	28.04	14.9	16.7	25.4
21	22.60	23.84	18.0	20.2	24.8
22	24.25	29.88	17.1	19.2	34.0
23	26.68	28.27	16.6	18.6	32.0
24	28.04	29.93	15.8	17.7	33.7
25	25.93	32.62	17.4	19.5	48.5
26	29.64	32.07	23.0	25.8	43.2
27	30.28	31.32	26.0	29.1	38.6
28	25.72	29.90	24.1	27.1	47.3
29	26.25	29.66	19.0	21.3	40.0
30	20.68	20.69	19.1	21.5	38.3

TABLE A (*continued*)

ACTIVITY COSTS AND YIELDS (OUTPUTS) PER ACRE, BY REGIONS, MODEL A

Region	Cost		Yield		
	Wheat	Feed Grain	Food Wheat	Feed Wheat	Feed Grain
	Dollars	Dollars	Bushels	Bushels	Bushels
31	23.70	25.51	24.3	27.3	46.8
32	20.45	23.79	27.0	30.2	43.6
33	28.11	27.24	26.6	29.9	33.8
34	30.45	29.97	27.6	30.9	30.7
35	21.85	25.43	20.6	23.1	24.2
36	21.57	24.58	27.3	30.6	38.3
37	20.37	22.08	25.2	28.2	46.0
38	18.52	17.49	27.1	30.3	47.8
39	20.65	19.80	18.8	21.0	34.4
40	18.74	21.20	19.4	21.7	31.1
41	20.06	21.88	21.3	23.9	30.2
42	20.79	20.05	19.7	22.0	20.0
43	19.86	20.73	22.7	25.4	36.6
44	16.59	19.21	15.5	17.4	36.0
45	14.74	16.43	14.7	16.5	37.1
46	16.90	18.07	17.6	19.7	38.2
47	17.67	21.39	17.0	19.0	31.1
48	14.71	15.14	13.5	15.2	28.5
49	13.40	17.00	14.5	16.3	23.6
50	8.52	12.71	9.2	10.3	18.4
51	6.57	9.22	8.0	9.0	14.1
52	5.84	9.24	7.0	7.9	13.3
53	7.23	11.31	7.5	8.4	14.6
54	8.25	11.71	7.9	8.9	15.9
55	6.16	9.40	8.1	9.1	14.8
56	7.23	9.44	9.0	10.1	16.5
57	10.23	14.63	8.6	9.7	20.6
58	7.01	10.28	8.5	9.5	17.2
59	10.12	13.18	9.6	10.7	26.1
60	11.74	12.93	16.2	18.1	29.2
61	7.20	12.67	12.8	14.4	18.1
62	7.05	16.35	10.0	11.2	19.6
63	10.28	18.10	10.6	11.9	27.1
64	6.44	16.55	11.2	12.6	23.5
65	12.68	16.31	17.5	19.7	31.5
66	17.56	16.96	17.8	20.0	25.5
67	18.91	18.66	17.9	20.0	18.3
68	20.20	18.52	17.0	19.1	15.6
69	16.65	16.95	17.4	19.5	17.2
70	9.21	15.81	10.8	12.1	20.0

TABLE A (*continued*)

Activity Costs and Yields (Outputs) per Acre, by Regions, Model A

	Cost		Yield		
Region	Wheat	Feed Grain	Food Wheat	Feed Wheat	Feed Grain
	Dollars	Dollars	Bushels	Bushels	Bushels
71	11.21	15.66	13.3	14.9	16.6
72	9.49	13.73	13.8	15.4	14.9
73	5.80	9.86	9.4	10.5	16.9
74	3.88	8.50	7.3	8.2	16.2
75	15.40	17.75	12.0	13.5	10.8
76	9.41	12.49	13.0	14.5	10.8
77	6.08	9.00	6.6	7.4	12.1
78	10.93	15.45	10.4	11.6	12.7
79	7.55	10.13	10.3	11.6	11.6
80	4.90	13.53	6.1	6.8	26.5
81	5.54	7.90	7.4	8.4	9.7
82	5.13	9.13	5.0	5.6	14.0
83	7.06	9.20	8.3	9.3	9.5
84	5.19	8.41	4.5	5.0	11.1
85	7.15	13.92	5.8	6.5	17.9
86	4.77	12.54	4.2	4.7	12.8
87	7.73	16.00	4.4	5.0	17.4
88	6.30	11.78	4.4	5.0	20.3
89	5.07	15.27	8.0	9.0	17.6
90	6.83	15.14	8.9	10.0	19.9
91	6.76	17.75	6.5	7.3	12.5
92	8.88	22.77	10.2	11.5	22.4
93	8.61	17.52	8.7	9.7	15.9
94	5.50	10.90	7.0	7.8	11.8
95	7.63	19.57	5.2	5.8	22.2
96	3.61	12.22	2.5	2.8	8.6
97	4.04	16.28	1.6	1.8	10.5
98	10.56	21.67	12.9	14.5	23.4
99	10.36	32.03	9.9	11.1	35.3
100	10.95	16.88	16.9	18.9	23.4
101	6.76	18.14	12.6	14.2	24.4
102	8.65	37.68	11.6	13.0	37.3
103	10.11	15.43	12.4	14.0	18.6
104	9.21	19.83	9.8	11.0	30.2

TABLE B

Estimated Normal Prices per Bushel for Wheat and Corn, by Regions, 1954

Region	Wheat	Corn	Region	Wheat	Corn	Region	Wheat	Corn
	Dollars	Dollars		Dollars	Dollars		Dollars	Dollars
1	1.88	1.66	36	1.85	1.51	71	1.88	1.50
2	1.86	1.68	37	1.85	1.51	72	1.85	1.51
3	1.91	1.66	38	1.87	1.50	73	1.85	1.52
4	1.90	1.65	39	1.87	1.51	74	1.84	1.54
5	1.92	1.68	40	1.87	1.51	75	1.85	1.50
6	1.92	1.60	41	1.86	1.52	76	1.85	1.51
7	1.96	1.68	42	1.85	1.56	77	1.84	1.55
8	1.96	1.68	43	1.83	1.53	78	1.85	1.50
9	1.94	1.62	44	1.88	1.52	79	1.84	1.53
10	1.93	1.62	45	1.88	1.50	80	1.84	1.54
11	1.94	1.62	46	1.90	1.46	81	1.85	1.48
12	1.93	1.62	47	1.92	1.40	82	1.85	1.48
13	1.93	1.67	48	1.91	1.38	83	1.85	1.48
14	1.92	1.69	49	1.95	1.40	84	1.85	1.50
15	–	1.66	50	1.95	1.38	85	1.86	1.49
16	1.90	1.66	51	1.94	1.36	86	1.86	1.49
17	1.91	1.66	52	1.94	1.48	87	1.87	1.50
18	1.92	1.68	53	1.94	1.46	88	1.87	1.50
19	1.83	1.63	54	1.92	1.36	89	1.79	1.60
20	1.92	1.66	55	1.89	1.45	90	1.74	1.65
21	1.87	1.60	56	1.92	1.40	91	1.79	1.60
22	1.87	1.60	57	1.93	1.37	92	1.74	1.64
23	1.90	1.62	58	1.89	1.45	93	1.76	1.58
24	1.89	1.61	59	1.92	1.46	94	1.82	1.56
25	1.88	1.55	60	1.87	1.50	95	1.83	1.57
26	1.88	1.60	61	1.76	1.58	96	1.83	1.58
27	1.88	1.58	62	1.79	1.57	97	1.81	1.54
28	1.86	1.51	63	1.86	1.47	98	1.72	1.80
29	1.86	1.55	64	1.86	1.49	99	1.73	1.88
30	1.86	1.54	65	1.87	1.50	100	1.85	1.79
31	1.83	1.49	66	1.88	1.51	101	1.86	1.83
32	1.85	1.51	67	1.87	1.51	102	1.88	1.85
33	1.87	1.52	68	1.86	1.51	103	1.95	1.89
34	1.88	1.54	69	1.86	1.51	104	1.95	1.89
35	1.86	1.52	70	1.86	1.49			

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CHAPTER 8

SPATIAL PRICE EQUILIBRIUM AND PROCESS ANALYSIS IN THE FOOD AND AGRICULTURAL SECTOR

Karl A. Fox[1]

Agricultural economists have participated fully in the programming and computer revolutions of recent years. In 1951 Waugh published an application of linear programming to the selection of dairy feeds. Within a year or two after that economists at several of the land grant universities were conducting programming analyses of individual farms, feed mills, and marketing firms. Applications of programming and other formal models to major segments of the nation's agriculture began to appear in 1953.

This last type of research has mushroomed during the past three years. Since 1959, major advances have been made in the quantitative analysis of several aspects of the food and agricultural sector. Smith (1959, 1960) has greatly extended the least-cost diet model and emphasized its connection with the classical theory of consumer demand. Holdren (1960) has analyzed the nature of the demand functions confronting individual supermarkets and developed a framework which includes these demand functions, the corresponding internal cost curves, and the array of price and nonprice offer variations that are used by food supermarket operators to attract customers and extend their trading areas. Day (1960) has completed a highly detailed linear programming analysis of a small agricultural region. Henderson (1959) and Heady and Egbert (1959, 1961) have published multiregional programming studies of field crop production. Brandow (1961) has published a synthesized model of the demand for United States farm products in considerable commodity detail.

Thus, important new models have been developed at each end of the food production and marketing system. They pose for us an obvious question: What is the most fruitful way of connecting these new formulations of consumer demand and farmer supply to yield a complete model of the sector as a whole? In a country as large as the United States, such a model must also deal realistically with spatial factors.

At the national level, we might construct for each commodity a model including a consumer demand function, a farmer supply function, and an equa-

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[1] I am indebted to Jarvis M. Babcock and Alan S. Manne for comments on an earlier draft of this chapter.

tion connecting the farm price with the retail price—rationalized as a supply function for marketing services. As a second step, we might disaggregate the consumer demand and farmer supply functions by regions. This is easily done, and there is no conceptual difference between the regional and the national functions.

But the supply function for marketing services cannot be neatly subdivided. Food retailing services can be allocated in proportion to consumption in each region and certain assembling and processing activities can be allocated in proportion to regional production. However, the location of other processing and storage activities will depend upon the properties of the entire interregional system, including the levels and slopes of the demand and supply functions in all regions and the complete structure of transportation or transfer costs for farm products and processed foods. Furthermore, it is clear that retail and farm prices must differ regionally in accordance with the pattern of interregional shipments that exists at a given time. And the transportation costs are essentially interregional in character.

The structure implied in this second step is, of course, that of the spatial equilibrium model. In its present context, this chapter is intended (1) to extend the treatment given to transportation and locational problems in the monograph as a whole, (2) to complement the chapter by Heady and Egbert —Chapter 7—with a fuller treatment of the transportation and marketing aspects of food and agriculture, and (3) to suggest ways of integrating spatial equilibrium analysis with linear programming formulations of agricultural production.[2]

SIMPLE SPATIAL EQUILIBRIUM MODELS AND INTUITIVE SOLUTIONS

The first empirical application of the spatial equilibrium model (Fox, 1953) was not a direct outgrowth of the linear programming revolution. In connection with our official duties at the Bureau of Agricultural Economics, some colleagues and I had been thinking about the possibility of regionalizing the agricultural outlook work for farm products. The United States Department of Agriculture for many years had been publishing analyses of the short-run outlook for prices, production, and consumption of farm products at the national level. Behind such analyses lay a considerable body of statistical demand and supply functions in terms of national aggregates.

However, variations in crop yields sometimes caused prices in individual regions to change quite differently from the national average price. In the case of a homogeneous commodity, the reason evidently lay in the shifting of particular regions from surplus to deficit or self-sufficient status. In the case of a heterogeneous commodity, such as apples, the differential between prices in the states of Washington and New York would vary not only because of changes in regional surplus and deficit relationships for "all apples" but also

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[2] An article by Egbert and Heady (1961) indicates that they also have been moving toward a synthesis of the demand and supply aspects of the agricultural sector. See especially pp. 215–216, on "changes in demand."

because the varieties grown in the two states were not perfect substitutes. Hence, their relative prices at any given location could change sharply from year to year. Similarly, the differential between the prices of cottonseed meal at Memphis and soybean meal at Decatur would reflect substitutability as feeds in addition to changes in the regional demand and supply balances for "all oilseed meals."

The usual context of our outlook work involved forecasting price developments for a few months or a year ahead, taking advance estimates of consumer income and crop production as predetermined variables. Hence, regionalizing the outlook for a group of commodities interdependent in demand would logically require a demand function for each commodity in each region and a set of transfer costs for each commodity between all possible pairs of regions.[3]

The individual elements that belonged in a regional outlook model were readily apparent and could be measured or estimated. What was lacking until about 1951 was a convenient and exact method of obtaining solutions in a model with more than two or three regions. Conversations with Baumol in the summer of 1952 and the appearance of papers by Enke (1951), Baumol (1952), and Samuelson (1952) encouraged me to go ahead with a ten-region model of the livestock-feed economy. This was a one-commodity model with feed supplies in each region predetermined in addition to all variables (such as livestock numbers and prices) which influenced the levels of the regional demand curves for feed. These demand curves were assumed to be arithmetically linear.

The resulting model was so responsive to common sense or intuitive approaches that I did not formulate it in programming terms. Nor did I think of it as a problem in nonlinear programming. I did not use a simplex tableau, a transportation model algorithm, or other paraphernalia generally associated with programming studies.

The intuitive methods used in my earlier work (Fox, 1953; Fox and Taeuber, 1955) would be quite tedious for models involving more than two interdependent commodities and (say) 20 regions. And if spatial equilibrium analysis is to supplement models such as that of Heady and Egbert, it should have the capacity to deal with 20 or more interdependent commodities and perhaps 100 or more regions. Hence, a more formal statement of the computations required in large scale applications is indicated.

A ONE-COMMODITY MODEL WITH REGIONAL SUPPLIES PREDETERMINED

The structure of my 1953 model for feed represents the simplest case of spatial equilibrium analysis.

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[3] This formulation implies that production and consumption in a region takes place at a single point. However, we might argue that a model containing a demand function and a supply function for the United States as a whole implies that *national* production and consumption takes place at a single point. Obviously, there is a gain in information in proceeding from a national to even a five- or ten-region model, and in concept there is no limit to the number of regions into which the nation might be divided.

In the ith region the demand function for feed is

$$x_{1\cdot i} + b_{12\cdot i}x_{2\cdot i} + c_{11\cdot i}u^*_{1\cdot i} + c_{12\cdot i}u^*_{2\cdot i} = a_{1\cdot i}. \tag{1}$$

The supply "function" is simply

$$x^*_{3\cdot i} = a_{2\cdot i} = k_{2\cdot i}, \tag{2}$$

and a gross equilibrium requirement for the system as a whole is

$$\sum_{i=1}^{r} x_{2\cdot i} = \sum_{i=1}^{r} x^*_{3\cdot i} \tag{3}$$

where r is the number of regions. In these equations, $x_{1\cdot i}$, $x_{2\cdot i}$, and $x^*_{3\cdot i}$ are respectively price, consumption, and production of feed in the ith region. The asterisk indicates that $x^*_{3\cdot i}$ is a predetermined variable; $u^*_{1\cdot i}$ and $u^*_{2\cdot i}$ are predetermined variables (livestock numbers and livestock prices) affecting the level of the demand curve for feed and may be subsumed in a revised constant term, $[a_{1\cdot i} - c_{11\cdot i}u^*_{1\cdot i} - c_{12\cdot i}u^*_{2\cdot i}] = k_{1\cdot i}$.

If the whole nation is regarded as one self-contained region, neither importing nor exporting, the solution of the model is simply

$$x_1 = -b_{12}x^*_3 + k_1. \tag{4}$$

However, even a two-region case begins to suggest the nature of a solution of the spatial equilibrium model. If transportation costs were zero, then $x_{1\cdot 1} - x_{1\cdot 2} = 0$ and the model would be defined by the following matrix equation:

$$\begin{bmatrix} 1 & b_{12\cdot 1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & b_{12\cdot 2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1\cdot 1} \\ x_{2\cdot 1} \\ x_{3\cdot 1} \\ x_{1\cdot 2} \\ x_{2\cdot 2} \\ x_{3\cdot 2} \end{bmatrix} = \begin{bmatrix} k_{1\cdot 1} \\ k_{2\cdot 1} \\ k_{1\cdot 2} \\ k_{2\cdot 2} \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

or in compact form

$$Bx = k. \tag{6}$$

But if positive transportation costs are introduced, the sixth row of B and k is replaced by two linear inequalities,

$$x_{1\cdot 1} - x_{1\cdot 2} \geq -t_{12} \tag{7}$$

and

$$x_{1\cdot 1} - x_{1\cdot 2} \leq t_{21} \tag{8}$$

where t_{12} is the transport cost from Region 1 to Region 2 and t_{21} is the transport cost from Region 2 to Region 1. The two rates are not necessarily equal.

Obviously, no more than one of these inequalities can hold with equality in an equilibrium solution of the model. If Region 1 ships to Region 2, (7) applies with equality; if Region 2 ships to Region 1, (8) applies with equality. In either case, the zero in the sixth row of k is replaced by the applicable transport cost, $-t_{12}$ or t_{21}; the left-hand terms of equation (5) are unchanged. Prices in the two regions differ by whichever of the two transport costs prevails, and the system can be solved for the six variables—price, consumption, and production in each region.[4] In matrix form the solution is

$$x = B^{-1}k. \tag{9}$$

We can generate new solutions from equation (9) by modifying one or more elements of the vector k to reflect changes in predetermined variables affecting demand or supply or changes in the applicable transport cost from the surplus to the deficit region.

There is, of course, another possibility—each of the two regions may prove to be self-sufficient for certain combinations of values of the elements of k. When this happens, $x_{3\cdot1} - x_{2\cdot1} = 0$ and $x_{3\cdot2} - x_{2\cdot2} = 0$; also, neither of the two price-differential equations is satisfied. The system (5) dissolves into two sets of three equations, each representing an independent supply and demand equilibrium for one of the regions.

What happens to (5) as the number of regions is increased? For three regions, B will contain six demand and supply equations. It will contain a gross equilibrium row as before. And if transport costs are zero, it will contain two additional rows, one specifying that the price in Region 1 equals the price in Region 2 and the other specifying that the price in Region 3 equals the price in Region 2 (or in Region 1). Nine equations are available for determining the nine variables.

When positive transportation costs are introduced, however, the price differentials are restricted by six inequalities, running from $x_{1\cdot1} - x_{1\cdot2} \geqq -t_{12}$ through $x_{1\cdot2} - x_{1\cdot3} \leqq t_{32}$. No more than two of these relations can hold with equality in an equilibrium solution. If all three regions are linked by trade, two relations hold with equality and the matrix equation (9) applies to the resulting nine equation system.

To select the proper price differentials we must solve the familiar transportation model. Specifically, we minimize

$$T = \sum_{i} \sum_{j \neq i} t_{ij} s_{ij} \qquad (i, j = 1, 2, 3) \tag{10}$$

subject to

$$s_{ij} \geq 0, \tag{11}$$

$$S_1 = \sum_{j \neq 1} s_{1j} \quad \text{or} \quad -\sum_{j \neq 1} s_{j1}, \tag{12}$$

and

$$S_2 = \sum_{j \neq 2} s_{2j} \quad \text{or} \quad -\sum_{j \neq 2} s_{j2} \tag{13}$$

where T is the total transportation or transfer cost, s_{ij} is the quantity of feed shipped from Region i to Region j, and t_{ij} is the corresponding transportation cost per unit; S_i is the total quantity of feed shipped from (or to) Region i.

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[4] Regional production is a predetermined variable in equation (5) but would be an endogenous variable in a model which included price-dependent supply functions.

Any S_i can be either positive or negative, subject to the gross equilibrium condition, $\sum_{i=1}^{3} x_{2\cdot i} = \sum_{i=1}^{3} x_{3\cdot i}$. This condition also implies that $S_1 + S_2 + S_3 = 0$. Hence, once S_1 and S_2 are determined, S_3 is given immediately by $S_3 = -(S_1 + S_2)$.

It is clear, then, that the simplest spatial equilibrium model includes within it (*a*) a set of demand and supply functions and (*b*) a transportation model. In his 1952 article on spatial price equilibrium and linear programming, Samuelson showed that these two components could be handled in an iterative sequence.

Samuelson's procedure greatly simplifies some otherwise difficult calculations. But to deal with large numbers of regions and several interdependent commodities, some additional shortcuts are needed.

A Model for Two Interdependent Commodities: Livestock and Feed

The complications involved in handling two interdependent commodities may be illustrated with the Fox and Taeuber model (1955). For the *i*th region this model includes the following equations:

Demand for feed

$$x_{1\cdot i} + b_{12\cdot i}x_{2\cdot i} + b_{14\cdot i}x_{4\cdot i} + b_{16\cdot i}x_{6\cdot i} = a_{1\cdot i}; \tag{14}$$

Supply of feed

$$x^*_{3\cdot i} = a_{2\cdot i}; \tag{15}$$

Demand for livestock

$$x_{4\cdot i} + b_{45\cdot i}x_{5\cdot i} = k_{3\cdot i} = a_{3\cdot i} - c_{41\cdot i}u^*_{1\cdot i} - c_{42\cdot i}u^*_{2\cdot i}; \tag{16}$$

Supply of livestock

$$b_{41\cdot i}x_{1\cdot i} + b_{44\cdot i}x_{4\cdot i} + x_{6\cdot i} = a_{4\cdot i} \tag{17}$$

where $x_{1\cdot i}$, $x_{2\cdot i}$, and $x^*_{3\cdot i}$ are respectively the price, consumption, and production of feed and $x_{4\cdot i}$, $x_{5\cdot i}$, and $x_{6\cdot i}$ the price, consumption, and production of livestock. The predetermined variables $u^*_{1\cdot i}$ and $u^*_{2\cdot i}$ are respectively the number of consumers in the *i*th region and their average disposable personal income.

The ten-region model with which we worked required us to determine the values of 60 variables (including regional feed production, which was assumed predetermined in the 1955 article but could well be treated as endogenous in other applications). Equations (14) through (17) for each region provide us with 40 of the needed equations. Two more equations are given by the gross equilibrium conditions,

$$\sum_{i=1}^{10} x_{2\cdot i} = \sum_{i=1}^{10} x^*_{3\cdot i} \tag{18}$$

and

$$\sum_{i=1}^{10} x_{5\cdot i} = \sum_{i=1}^{10} x_{6\cdot i}. \tag{19}$$

These imply that the nation as a whole is a self-contained system with respect to both commodities.

Eighteen more equations appear in an equilibrium solution, provided by price-differential equations based on transport (or transfer) costs.[5] But these 18 must be selected optimally from a total of 180 possible restrictions by solving two independent transportation models, one for feed and the other for livestock. If prices of each commodity in all ten regions were solidly linked by trade, the matrix equation (9) could once more be used to predict the effects of changes in the predetermined variables and transport costs subsumed in the k vector.

However, B^{-1} would be a 60 by 60 matrix. If some change in data upset the original trading arrangements and price differentials, it would be necessary (after solving the new transportation models) to invert a new 60 by 60 matrix.

For concreteness, a solution of the Fox-Taeuber model is presented in Table 1. The values of the 60 endogenous variables are given in columns 2, 3, 4, 12, 13, and 14. The t_{ij}'s are implicit in columns 1 and 11, when taken in conjunction with the patterns of the s_{ij}'s in columns 6–9 and 16–18. All ten regions are linked by trade in feed, but one region is self-sufficient with respect to livestock. This means that one of the S_i's is zero and the corresponding price is not bound by any of the 18 possible equalities which might have linked it to the price of another region.[6]

It should perhaps be noted that the Fox-Taeuber model assumes a competitive market economy in which farmers and dealers are free to buy products from any source and to ship them to any destination. It implies also, that interregional price differentials are free to respond to changes in the geographical distributions of farm production and consumer demand.

This model would be quite compatible with certain types of subsidy programs under which market prices were permitted to vary as needed to clear the market but producers would be compensated for any adverse differences between markets' prices and some guaranteed unit return. Production in each region would be determined by the guaranteed unit return, while consumption would be determined by the market clearing prices, as in the spatial equilibrium model with regional supplies predetermined.

If a central authority undertakes to support market prices in each region at not less than some specified figure, a pattern of regional price differentials will emerge but the national market will in general not be cleared. The result will be a modified spatial equilibrium solution in which the market price in at least one region will be equal to the local support price.

In a country in which all agriculture was centrally planned and all distribution facilities governmentally owned, the planning authorities would presumably still be interested in the efficient use of resources. This efficient use would require the solution of a spatial equilibrium model; however, the "efficiency prices" would be bookkeeping items rather than determinants of

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[5] Assuming that prices in all ten regions are linked by trade in both commodities.

[6] However, the system is made determinate by the restriction that livestock consumption must equal livestock production in the self-sufficient region.

TABLE 1
JOINT SPATIAL EQUILIBRIUM FOR FEED AND LIVESTOCK UNDER APPROXIMATE 1949–50 CONDITIONS
A. FEED

Region	Price Differential from Corn Belt (1)	Equilibrium Price (2)	Feed Consumption (3)	Feed Production (4)	Net Trade (5)	Origins and Amounts of Net Imports				
						Corn Belt (6)	Lake (7)	Northern Plains (8)	Mountain (9)	Total (10)
	Dollars per bu.	Dollars per bu.	Million tons	Million tons	Million tons	Million tons	Million tons	Million tons	Million tons	Million tons
Northeast	.2156	1.4796	12.22	4.34	−7.88	7.73	0.15			7.88
Corn Belt	.0000	1.2640	45.18	54.70	9.52			(2.13)[a]		
Lake	−.0969	1.1671	15.24	15.39	0.15					
Northern Plains	−.1405	1.1235	13.23	17.28	4.05					
Appalachian	.1575	1.4215	9.96	8.92	−1.04	1.04				1.04
Southeast	.2182	1.4822	6.17	4.78	−1.39	1.39				1.39
Delta	.1454	1.4094	4.36	2.87	−1.49	1.49				1.49
Southern Plains	.1233	1.3873	6.85	5.85	−1.00			1.00		1.00
Mountain	−.0320	1.2320	2.84	3.09	0.25					
Pacific	.2656	1.5296	4.73	3.56	−1.17			0.92	0.25	1.17
Total			120.78	120.78	0.00	11.65[b]	0.15	4.05	0.25	13.97[c]

TABLE 1 (*continued*)
JOINT SPATIAL EQUILIBRIUM FOR FEED AND LIVESTOCK UNDER APPROXIMATE 1949–50 CONDITIONS
B. LIVESTOCK

Region	Price Differential from Corn Belt (11)	Equilibrium Price (12)	Livestock Consumption (13)	Livestock Production (14)	Net Trade (15)	Origins and Amounts of Net Imports: Corn Belt (16)	Origins and Amounts of Net Imports: Lake (17)	Origins and Amounts of Net Imports: Northern Plains (18)	Origins and Amounts of Net Imports: Total (19)
	Dollars per 100 pounds	Dollars per 100 pounds	Million units	Million units	Million units	Million units	Million units	Million units	Million units
Northeast	.9650	17.1045	50.56	17.41	−33.15	27.63	5.52		33.15
Corn Belt	.0000	16.1395	31.88	62.06	30.18				
Lake	−.2510	15.8885	14.73	20.45	5.72				
Northern Plains	−.0916	16.0479	4.96	17.37	12.41				
Appalachian	.7228	16.8623	14.80	14.03	−0.77	0.77			0.77
Southeast	.8792	17.0187	10.61	8.83	−1.78	1.78			1.78
Delta	.2900	16.4295	6.25	6.25	0.00				
Southern Plains	.8762	17.0157	11.61	9.46	−2.15			2.15	2.15
Mountain	.8384	16.9779	3.95	3.72	−0.23			0.23	0.23
Pacific	1.4588	17.5983	16.93	6.70	−10.23		0.20	10.03	10.23
Total			166.28	166.28	0.00	30.18	5.72	12.41	48.31

[a] Under the assumed structure of freight rates, this amount of feed is shipped to the Corn Belt and reshipped to deficit regions in addition to the 9.52 million tons classified as net exports from the Corn Belt.

[b] Includes 2.13 million tons received from the Northern Plains and reshipped to other regions.

[c] Excludes 2.13 million tons of imports into the Corn Belt offset by re-export.

personal incomes. The pervasiveness of spatial equilibrium considerations in the efficient long-run allocation of productive factors is well brought out by Lefeber (1958).

LARGER MODELS

The Fox-Taeuber model could be expanded to include more commodities, more regions, or both. Suppose we wished to include all domestically produced agricultural commodities and all parts of the continental United States. In aggregating commodities, we would try to recognize similarities in production and processing characteristics and interrelationships in demand. In aggregating regions we would try to recognize (1) homogeneities in soil, climate, types of farm products grown, and techniques of production and (2) the range of price variation among regions attributable to transfer costs.

CHOOSING APPROPRIATE NUMBERS OF COMMODITIES. To achieve a realistic description of agriculture and closely related sectors; it would seem desirable to subdivide feed into at least three categories—for example, feed grains, hay and forage, and by-product feeds. It would be important to separate meat animals and poultry into species; to treat eggs as a distinct commodity; and to distinguish between milk for fluid use, milk for manufacturing, and farm-separated cream. Furthermore, the location of meat and poultry processing facilities depends to some extent on the relative transportation costs of live animals and dressed meats. Thus, it seems that dressed meats, poultry, and the major manufactured dairy products should be included as additional commodities. At the farm level, other field crops in addition to feed should be recognized, as indicated in the Heady and Egbert models. Soybeans, cotton, cottonseed, tobacco, and wheat at the least deserve separate recognition.[7]

Ignoring orchard and vineyard crops, specialized production of vegetables, and various minor commodities, a comprehensive model might well include 15 or more commodity groups at the farm level and several more groups of processed foods—in round numbers, some 20 commodities.[8] Each commodity adds 3 equations per region to the size of the B^{-1} matrix in matrix equation (9).[9] Thus, 20 commodities would require 60 equations per region. Ten regions would require 600 simultaneous equations; 100 regions would require 6,000 equations.

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[7] Cotton and cottonseed are joint products at the farm level but must be treated separately in processing, transportation, and demand.

[8] Since this chapter was drafted, Brandow (1961) has published a demand model oriented toward the analysis of agricultural policies at the level of national aggregates. Brandow divides food consumption into 24 commodities and/or groups, and some additional nonfood commodities (primarily cotton and tobacco) are also included. Thus my reference to a 20-commodity model does not overstate the level of disaggregation required for a description of the food and agricultural sector that will be useful to policy makers.

[9] Including a price-differential relation if the region is linked to another by trade in the relevant solution or a requirement that consumption equal production if the region is self-sufficient.

In addition, 20 transportation models would have to be solved. Each of these would include 90 possible price-differential equations in a 10-region model or 9,900 in a 100-region model; the applicable price-differential equations in an equilibrium solution would number 9 and 99 respectively for each commodity in completely linked systems.

CHOOSING APPROPRIATE NUMBERS OF REGIONS. It would be desirable to analyze agricultural production in terms of quite a large number of regions. Many states contain two or more distinct type-of-farming areas. I believe Heady and Egbert have chosen a reasonable level of aggregation for the appraisal of production programs. Their 104 regions exclude portions of the country which grow little grain. For other purposes a moderate number of other regions would have to be added. If these are primarily mountain and desert range areas, perhaps the continental United States could be filled in completely with 120 regions.

If the United States were divided into 100 regions each accounting for 1% of national farm output, each region falling in Iowa would have an area of 5,000 or 6,000 square miles. Most of the produce in such an area would be within 40 miles of its center—about an hour's haul by truck. It seems doubtful that a national general-purpose model should concern itself with smaller regions than this.

Rules of reason for disaggregation in a spatial equilibrium model might also be based on the geographical ranges of prices for the various commodities included in it. In Table 1, feed prices range from $1.12 to $1.53 per bushel, with a midpoint of $1.32; livestock prices range from $15.89 to $17.60 per hundredweight, with a midpoint of $16.75. Thus, the range of feed prices is equal to about 30% of the midpoint, while the range for livestock is about 10%.

At first glance it might appear that transport costs have only a third as much impact on livestock as on crops. But livestock production depends upon the relative prices of livestock and feed, and these price ratios run from about 11.5 to 14.2—a range of 20%.

For all farm products flowing into domestic food use in 1960, long-haul transportation costs were equivalent to 20% of the net farm value—$4.1 billion as compared with $20.7 billion. There is no simple way to translate this figure into typical ranges of geographical price variation. First, a large proportion of the $4.1 billion cost is incurred in shipping products from one point to another within regions as large as those of the Fox-Taeuber model. Second, long-haul transportation costs are incurred on only a portion of total food production—though probably the major portion. Third, transportation costs are low relative to the equivalent farm value for some products and high for others.

Consider a farm product for which the extreme geographic range of prices is equal to 50% of its midpoint. In a 10-region model, the average price differential between successive regions arrayed from lowest to highest price would be 5 to 6%. In a 100-region model, the average price-step would be on the order of .5% of the national average price.

A 100-region model should permit us to recognize regional price differences arising from spatial equilibrium considerations with as much precision as is warranted, considering stochastic elements in crop yields, errors of measurement in official price and production data, and other errors, uncertainties, and aggregation problems in the real world. A 10-region or 20-region model might be perfectly satisfactory for a partial analysis of one or two commodities with low transportation costs. But different commodities might require different regional breakdowns if such small models were used.

EFFECTS OF THE TIME DIMENSION ON APPROPRIATE NUMBERS OF REGIONS AND THE FEASIBILITY OF PARTIAL ANALYSIS. In practice, it seems to me that two classes of spatial equilibrium models will be found useful.

In appraising long-run problems of interregional competition or the incidence of changes in transportation costs, state excise taxes, and other semipermanent elements, models of 100 regions or more seem appropriate. Long-run models might differ from short-run models in other respects as well, for the extent of supply response to a given change in prices depends upon the amount of time allowed for the adjustment. Certain variables which are predetermined for short-run purposes are endogenous in a long-run context. Long-run models would assume expected values of all stochastic variables, and empirical verification of predictions based upon them would involve (1) averages of actual realizations over a several-year period or (2) adjustments of observed values of the endogenous variables to reflect expected (rather than observed) values of the exogenous or predetermined variables.

Forecasts of regional price and shipment patterns for the year just ahead are subject to uncertainties about crop yields, consumer incomes, and other variables. They are also subject to standard errors of forecast from statistical demand and supply functions. These uncertainties and errors may be equivalent to errors of 2 to 5 or 10% in regional price forecasts. A 10- or 20-region model would probably give as much recognition to spatial equilibrium factors as is justified in a short-run forecasting context.[10]

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[10] The problem of estimating the parameters of regional demand and supply functions does not differ in principle from many other estimation problems in economics. For example, the Fox-Taeuber model was based in part upon the following two demand functions, fitted to national averages or aggregates:

$$p_f = a_f - \underset{(.21)}{1.93z_f} + \underset{(.20)}{.89p_l} + \underset{(.71)}{2.26n_l}; \qquad R^2 = .85$$

and

$$q_1 = a_1 - \underset{(.03)}{.52p_l} + \underset{(.10)}{.70p_0} + \underset{(.03)}{.40y}; \qquad R^2 = .95$$

where all variables are *first differences of logarithms* of annual observations for the period 1922–1941. Standard errors of the regression coefficients are shown in parentheses. The first equation expresses the price of feed (corn) as a function of the total supply of feed grains, an index of livestock prices, and an index of livestock numbers. The second equation expresses per capita consumption of food livestock products as a function of an index of retail prices of food livestock products, a price index for all other consumer goods and services, and real disposable income per capita. These equations were first published in Fox (1951) but are more generally accessible in Fox (1958) on pages 106 and

Also, for short-run forecasts some groups of commodities might be treated as substantially independent of one another in demand. This involves a judgment that the effect of neglecting such interdependencies would not materially increase the level of error and uncertainty already present in the situation.[11]

However, the smaller models would represent specialized offshoots of the general-purpose model with its 20 commodities and 100 or more regions. If we can solve the computational problems of the latter we can a fortiori deal with at least the nonstochastic aspects of the former.

SHORTCUT METHODS FOR HANDLING LARGE NUMBERS OF REGIONS

In practice, the size of the key matrix, B, in a spatial equilibrium model can be greatly reduced without loss of accuracy. We cannot, of course, aggregate over commodities once our 20 groups have been defined—these have different transportation cost patterns and even different units of measure. But we can aggregate over regions. The main point can be illustrated with a one-commodity model with regional supplies predetermined.

In such a model, the demand functions in each region can be aggregated to yield a single demand function at the national level. We assume each of the regional demand functions to be arithmetically linear. Given the predeter-

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116 respectively. The coefficients of these equations were rounded and in some cases adjusted to bring out the logical relations between them in an equilibrium context. (For example, if the feed supply is fixed, a 1% increase in the number of livestock reduces the supply per head by 1% and increases the price of feed in proportion to the coefficient of price flexibility. If the latter coefficient (the coefficient of z_f) is rounded to -2, the coefficient of n_1 should be rounded to 2. These changes are well within the standard error ranges of the coefficients.)

The regional demand functions were all derived from the national ones on the plausible assumption that elasticities of consumer demand for livestock products were similar in the different regions and that producers of a particular kind of livestock in different regions would respond similarly to changes in feed-livestock price relationships.

In principle, the parameters of the demand and supply functions in each region could be derived independently from family budget surveys, consumer panels, farm surveys, and the like. Or, as implied in Chapter 7 and in parts of this one, we could (given sufficient resources) develop a normative model for each region reflecting the resource mixes of its farmers and the food preferences of its consumers.

[11] The 24 by 24 matrix of elasticities and cross elasticities of demand for foods given on page 17 of Brandow (1961) would permit some interesting tests of the effects of different degrees of approximation in this regard. For example, Brandow's matrix, calculated on the basis of principles spelled out in Frisch (1959), contains 554 nonzero elements (there would be 576 except for a special handling of imported beverages—coffee, tea, and cocoa). However, 365 of the cross elasticities are smaller than .005 and another 95 would round to .01. Only 70 cross elasticities are greater than .01 and 44 of these are concentrated in two commodity groups, (*a*) meats and poultry and (*b*) fats and oils. Except for these two groups the matrix appears to be dominated by the own-price elasticities which form its diagonal elements. However, the cross elasticities of .01 or so may still be sufficiently numerous (and sufficiently concentrated) to have important effects upon the accuracy of consumption forecasts for particular foods.

mined national supply (say) of feed, we can immediately calculate the United States average price implied by the national demand function.

Suppose we wish to estimate the effects on feed consumption of a 10-cent increase in the U. S. average price. If a 10-cent change in the U. S. average price is associated with precisely a 10-cent change in *every* region, the change in national consumption calculated from the national demand function will be exactly equal to the sum of the changes calculated from the regional demand functions. Evidently, if the prices in all regions are linked together by trade into a definite structure, the price in each region is an exact linear function of the price in any region which might be chosen as a basing point. Similarly, the U. S. average price defined by our aggregative demand curve will be an exact linear function of the price in the basing point region. This obvious relationship was the key to the extreme ease with which my 1953 model could be manipulated.

It may turn out that as the entire price structure is raised or lowered, some region will pass from (say) a surplus to a self-sufficient status. In this case, the remaining $n - 1$ regions can still be manipulated as an aggregate to obtain a market clearing solution for the regions that are still interconnected.

With two interdependent commodities, a region might prove to be self-sufficient in one product but not in the other. In this case, the national model could be converted to a two-region system, requiring the inversion of only a 12 by 12 matrix.

These principles can be adapted to the handling of multicommodity models. However, space does not permit a more extended discussion, and I believe that most economists who decide to grapple with large scale empirical models will be resourceful enough to make such adaptations.[12]

It is true that shortcut solutions of the sort outlined must contend with a certain dilemma. If the trading pattern linking a group of regions is highly stable in the face of large changes in the level of the price structure, shortcut solutions which treat these regions as a single aggregate will also tend to be stable. This is a computational advantage. But on the other hand, one of the chief virtues of the spatial equilibrium model is its power to forecast and/or explain *changes* in the patterns of price differentials and interregional trade. For a spatial equilibrium model to be interesting, at least a few of its regions should be somewhat susceptible to changes in their net trade positions.

Actual experience with a large scale model (of perhaps 20 commodities and 100 regions) might also lead to the toleration of slight imperfections in the "final" solutions. For example, iterations might be suspended when estimated national consumption came within 1% of national production for every commodity. When we consider that there are significant errors of measurement in all our statistics on regional prices and production, and that regional consumption must often be inferred rather than measured directly, this tolerance level should be sufficient.

Also, the United States exports or imports significant quantities of a number

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[12] A few copies of an earlier draft containing additional suggestions on methods of solution are available on request.

of farm commodities. On the one hand, this suggests the desirability of linking the United States with other countries or regional groupings in a spatial equilibrium model of world trade and production. On the other hand, it suggests that errors in forecasting predetermined variables in other countries, let alone the possibilities of serious political disturbances, would limit the accuracy of our estimates of United States imports and exports to an extent greater than the 1% tolerance limit just suggested.[13]

INTEGRATION OF SPATIAL EQUILIBRIUM MODELS WITH LINEAR PROGRAMMING FORMULATIONS OF PRODUCTION

So far we have assumed continuous linear supply functions. But it would seem desirable to integrate the spatial equilibrium model also with a linear programming formulation of agricultural production. Heady, in collaboration with Egbert and with other colleagues and former students, has conducted many linear programming studies of individual farms as well as of agricultural regions. Day (1960), Henderson (1959), and others have also experimented with linear programming formulations of agricultural production at the regional level.

The principal problem in blending activity analysis formulations of production into a spatial equilibrium model lies in the discontinuous nature of (1) the regional supply functions for each commodity and (2) the derived demand functions for feeds as inputs into regional livestock enterprises. In the Fox-Taeuber model, for example, three of the four supply and demand equations in each region would become step functions. Exact market-clearing solutions might not always exist, and the pursuit of the optimal solution by standard programming methods might be extremely laborious.[14]

Something depends upon the sophistication with which regional production is to be portrayed. In actual farming practice we might expect to find a nearly continuous spectrum of production functions realized by different farmers as prices fluctuated within their usual ranges. Conceptually, it would seem

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[13] The suggestion of a world or international spatial equilibrium model implies that we operate at two different levels of aggregation, one for international trade and one for interregional trade. While this is inelegant, and would increase the difficulties of obtaining *exact* solutions for all regions which would be optimal in relation to the internal (regional) price and production patterns in every other country, it represents a practical compromise with the disturbances and measurement errors which characterize the real world.

We could also take the solution for one of our regions (say Region 3) from the national model and set up a highly detailed spatial equilibrium model for counties or townships within Region 3. This detailed model would accept regional production and consumption as predetermined, along with regional price, which would have to be identified with some particular point in the region.

If similar internal models were implemented for two contiguous regions, it is likely that some discontinuities would appear at the border. However, anyone operating with such a detailed locational model would probably be able to handle these problems satisfactorily on an ad hoc basis.

[14] Some further comments on iterative methods of solution are contained in the earlier draft mentioned in footnote 12.

desirable to take a sample of farms in each region, give each farm a linear programming formulation, calculate its profit-maximizing responses to predetermined sets of prices, and aggregate these to obtain regional supply functions and (in the case of feeds) derived demand functions. Depending upon the number of farms in the sample, the regional supply and derived demand functions might be very nearly continuous.

The next step would be to approximate these regional functions by straight lines over the range of price variation that seems likely for a few years ahead. Some experimentation would be needed to determine satisfactory routine methods for deriving these lines. A number of coefficients could be set at zero if the production processes on individual farms showed no direct connection between two commodities. Multiple regression techniques could then be used to estimate the nonzero coefficients in the supply functions in each region and in the derived demand functions for feed. Together with the maintained demand functions for livestock, the newly estimated supply and demand functions provide us with a standard spatial equilibrium model.

Alternatively, we might start out from the Heady and Egbert formulation which treats each region as though it were a single farm. The linear programming model for each region would be used to generate the discontinuous supply function for each product and the derived demand function for each feed. The wider the range of processes included in the regional model for producing each commodity, the nearer the corresponding step functions will approximate to continuous functions. Over the range of prices thought likely to prevail during the next few years, we might approximate these discontinuous functions with arithmetic straight lines. In the process of calculating the optimal regional production response to a change in the price of one commodity, the linear programming model would generate the step-function supply responses not only of the given commodity but of other interdependent commodities. Linear approximations could be made to these functions as well.[15]

In this fashion we could generate a complete set of linear supply functions for feed and livestock and derived demand functions for feed. The linear programming formulation of production would provide a clear-cut justification for the presence of some zeros in the coefficient matrix of the continuous functions. It would provide an explicit basis for the sizes of the coefficients in each linear equation. The demand functions for livestock products would continue to be based on statistical demand analyses of the traditional type.[16]

Either of the approaches just outlined could be extended to all farm products; they would not be limited to the livestock and feed complex.

[15] If the "trend" of a step function were nonlinear, the function might be approximated by two or three linear segments. In connection with any particular solution of the model, however, we must try to anticipate on which segment the equilibrium price-quantity combination will be found.

[16] Victor Smith's (1960) linear programming model of consumption could perhaps be added at the consumer end of the marketing system. Thus, we could treat each region as one big household with a linearized indifference surface based on (1) nutritional requirements and (2) food habits and preferences, including complementarities between foods

Thus, it seems possible to reinstate the computational convenience of continuous arithmetic functions in a manner closely related to the linear programming production base. If the restrictions included in the linear programming production model were based on short-run considerations, the coefficients of the parallel continuous model would be appropriate for the same short period. If the restrictions in the linear programming model reflected the degree of resource flexibility appropriate for a five or ten year period, the parallel continuous model would also apply to a five or ten year adjustment period.

CONCLUDING REMARKS

It is now possible to develop sophisticated and disaggregated models of the entire food and agricultural sector of an economy. There have been major breakthroughs recently in the analysis of several levels in the production and marketing sequence.

A model of regional price differentials, interregional trade, processing, distribution, and consumption at a level of aggregation comparable to that of the Heady and Egbert production model would be at the least a valuable construct for research workers. Students of particular industries and particular localities could ignore its existence and be no worse off than at present. But the large scale, nearly complete model of food and agriculture would be available if such students wished to check their *ceteris paribus* assumptions against the interdependencies indicated by it.

Interdependencies between food and agriculture and the rest of the economy can be approached from a number of viewpoints. In another connection, I have pointed out that a two-sector input-output matrix of the United States economy, one consisting of agriculture plus the food and fiber processing industries and the other of the remainder of the economy, is nearly triangular (Fox, 1962). In other words, there are very few processes or activities outside of the agriculture-food-fiber complex which require farm products or by-

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and upper and lower limits on the consumption of particular foods. These food habit restrictions could be varied in accordance with changes in the average per capita income of consumers in the region.

This approach would result in discontinuous demand functions and substitution relations directly analogous to the supply functions which result from the treatment of each region as one big farm. It would be conceptually possible to start out in each region from a sample of households, each represented by its own set of requirements and preferences. We could compute the normative responses of each such household to changes in retail food prices and expand them into regional aggregates. We could approximate these regional consumption surfaces by linear functions over limited ranges of expected price variation and compare them with the results of traditional demand analyses based on time series aggregates.

I am not recommending the early substitution of normative consumption surfaces for statistical demand functions, as the problems of concept and measurement need a thorough winnowing in the professional journals before this is done.

products as inputs. Based on 1947 technical coefficients, a 10% increase in deliveries to final demand from "all other industries" would increase requirements for farm output by only .5 or .6%.[17]

In percentage terms, agriculture is heavily dependent upon inputs from the rest of the economy. Nonagricultural input per unit of farm output has increased substantially during the past decade or two. But the absolute significance of this for the rest of the economy has been approximately offset by the declining share of agriculture in total economic activity. As of 1961, a 10% increase in farm output would probably call for less than a 1% increase in the output of "all other industries" as a group.

This suggests that some practical uses of a model of food and agriculture would require little attention to direct influences of this sector upon the rest of the economy and even less to possible indirect effects or feedbacks from it upon food and agriculture. However, some farm programs or endogenous developments could have considerable effects upon particular localities and nonfarm industries. It should be quite feasible to combine a detailed model of food and agriculture with a much more aggregative model of the rest of the economy, so that agricultural policies could be studied realistically without losing sight of the major interconnections between agriculture and other sectors.

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[17] An increase in nonfarm output would affect employment and disposable income and shift demand curves for food and fiber products. These effects would be in addition to the .5 or .6% which represents the use of farm products in industrial oils, soap, paint, and the like.

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PART IV

METALS AND METALWORKING

CHAPTER 9

PROCESS ANALYSIS OF THE U. S. IRON AND STEEL INDUSTRY

Tibor Fabian

INTRODUCTION

OBJECTIVES. This paper describes a general-purpose process analysis model of the iron and steel industry. Conception of the study in 1953 was strongly influenced by the U. S. Government's potential interest at that time to assess the industry's technological capability to operate under emergency conditions and to provide steel for using industries after a partial destruction of plants and equipment. Emergency conditions were understood to be (*a*) an unusual demand for steel by industries which supply the military, and (*b*) a decrease in the supply of certain materials, particularly imported materials.

Certain questions about each of the above problem areas no doubt could be answered without the use of an elaborate process analysis model. For example, independent studies had earlier been performed in connection with material stockpiling.[1] The models that were constructed appeared satisfactory. Why then, the need for this larger model? The previous studies considered only segments of the industry and its problems. For simultaneous consideration of several problems a means was needed both for efficient data reduction analysis and for answering questions likely to arise in military preparedness planning.

Once conceived, the model was seen as capable of serving peaceful needs as well. Suppliers to the iron and steel industry of its complex furnaces and mills, measuring and control equipment, and materials have need to assess the economics of different production processes in order to anticipate changes in the demand for their products and services. A process analysis model could become a prime tool in this planning process. Producers of materials which compete with steel can also use a process analysis model. And finally, a process analysis model could well be useful to members of the steel industry itself, all the way from the long range planning of a concern, a division, or a plant, down to the scheduling of the interlocking activities of a steel mill.[2]

The model permits prediction of the industry's responses to variations of a

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[1] For instance, Karreman (1957); Economics Research Project (1957).

[2] The use of mathematical programming in production planning and scheduling steel plants is expanding. See Blattner (1959).

range of exogenous factors such as the availability of certain input materials and the desired product mix. Within a more general view of economic equilibrium, such changes might well be regarded as endogenous. From the viewpoint of the single industry, however, these input and output limitations represent fixed parameters, rather than decision variables. Specifically, the model has been so designed that the following illustrative questions may be asked:

1. What is the consequence of a long run change in the ferrous content of some iron ores or concentrates on
   a. The demand for steel scrap in steel production?
   b. The marginal value of coke ovens, blast furnaces, and steel furnaces?
   c. The demand for coal by the steel industry?
2. What is the consequence of a short run change in the availability of steel scrap on
   a. The use of hot metal in steel production?
   b. The rate of output of steel?
   c. The marginal value of coke ovens, blast furnaces, and steel furnaces?
3. How does a shift in demand for the products of steel using industries affect the marginal value of the different units in steel plants under various operating circumstances? In other words, where do bottlenecks arise and do these bottlenecks vary depending on what input materials are available?

A number of problem areas are encompassed here. The first and second questions induce us to assess the relative values of domestic ore concentrates, such as beneficiated taconite, and the value of imported high grade ore in light of decreasingly available amounts of domestic ore and the future supply of steel scrap. The economic effect of new steel production technologies, such as the use of oxygen, can also be investigated within the context of the first question, sepecifically under 1*b*. With the use of a process analysis model earlier studies on the long range availability of retired steel scrap[3] can be extended into a quantitative analysis of the interaction between the rate of supply of retired scrap, scrap price, and the use of alternative production technologies. Such analysis is required to answer the second question. Finally, the last question is concerned with the effect of increased military or civilian demand for steel on the economy at large. The matrix in its present form can point out bottlenecks in the industry which would be of first concern in any expansion program. Similarly, one could analyze those critical points in steel production which, if inoperative for military reasons, would substantially reduce steel output. At the time this study was undertaken, the last question was of greater importance than it might be today.

The three questions can be answered by computing the solution of an appropriate process analysis model with a suitable optimizer. Reliability of the answers will depend on the accuracy of the formulation of the model and the coefficients. In this direction, the model presented here is but a first step; im-

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[3] For instance, "Iron and Steel Expansion and the Future Supply of Obsolete Ferrous Scrap," by B. E. Etcheverry, Kaiser Steel Corporation, 1951 (manuscript).

provements in both the formulation and figures should occur once that step has been taken.

SCOPE. In order to attain modest results within the broad objectives, several restrictions were imposed on the scope. The purpose of the study was to prove its feasibility rather than to prepare the ultimate model. Technological relationships are intended to be representative of those in a single country —the U. S.—during a single time period—the 1950's up to, but not including, institution of the oxygen converter process. The model is static. The industry's plant and equipment is taken as a datum. This convenient assumption makes it possible to bypass difficulties connected with the dynamics of plant expansion—specifically, the matter of economies of scale. The study has ignored the geographical aspects of the industry, and cannot be expected to provide insight into questions that involve plant location and/or transportation.

The model's constraints were based mostly on published material on the technology of production, and to a small extent on information gained in interviews with representatives of the industry. Such meager knowledge of the technology no doubt created an imbalance in the model by expanding the representation of some aspects of the production processes and at the same time limiting the consideration of others. Most of the data come from published material, but some were obtained from industry sources.

The analysis conceives the industry as one completely integrated steel plant. Such plant contains all phases of the production process, from the production of metallurgical coke through the production of iron and steel to the rolling and finishing of steel. Most iron and steel production takes place in such integrated plants, although some plants might be integrated back to steel or iron production only. Electric furnace plants do not require blast furnaces or coke ovens. The products of integrated plants usually do not completely overlap. Some plants may specialize in the production of structural steel or heavy plates, while others might produce sheets and pipes. The linearity of the input-output relations bypasses the problem that may arise from the economies or diseconomies of scale of one single plant producing *all* steel products.

DIFFERENT FORMULATION OF THE PROCESS ANALYSIS MODEL. The mathematical structure of the model used here is described in Fabian (1958). This study is based on manuscripts prepared on the formulation of a model on each stage of iron and steel production as discussion papers at the Management Sciences Research Project, U.C.L.A., between 1953 and 1955.

Other models of iron and steel production concentrated on one phase of the process. The blast furnace burdening problem drew the greatest interest and was exploited in Bailey (1956) and Weingartner and Reed (1957). More recently several steel firms have developed elaborate linear programming models of various stages of production. See Hilty, Taylor, and Gillespie (1959).

A Review of the Technology

THE INTEGRATED PLANT. The schematic diagram of Figure 1 presents the flow of materials, fuels, and products in an integrated plant. This is a highly simplified presentation of activities and units, and is designed to emphasize flows and processes which were incorporated into the model.

An integrated steel plant consists of the following major units or operating departments:

Coke ovens
Blast furnaces
Open hearth furnaces
Primary and secondary rolling mills

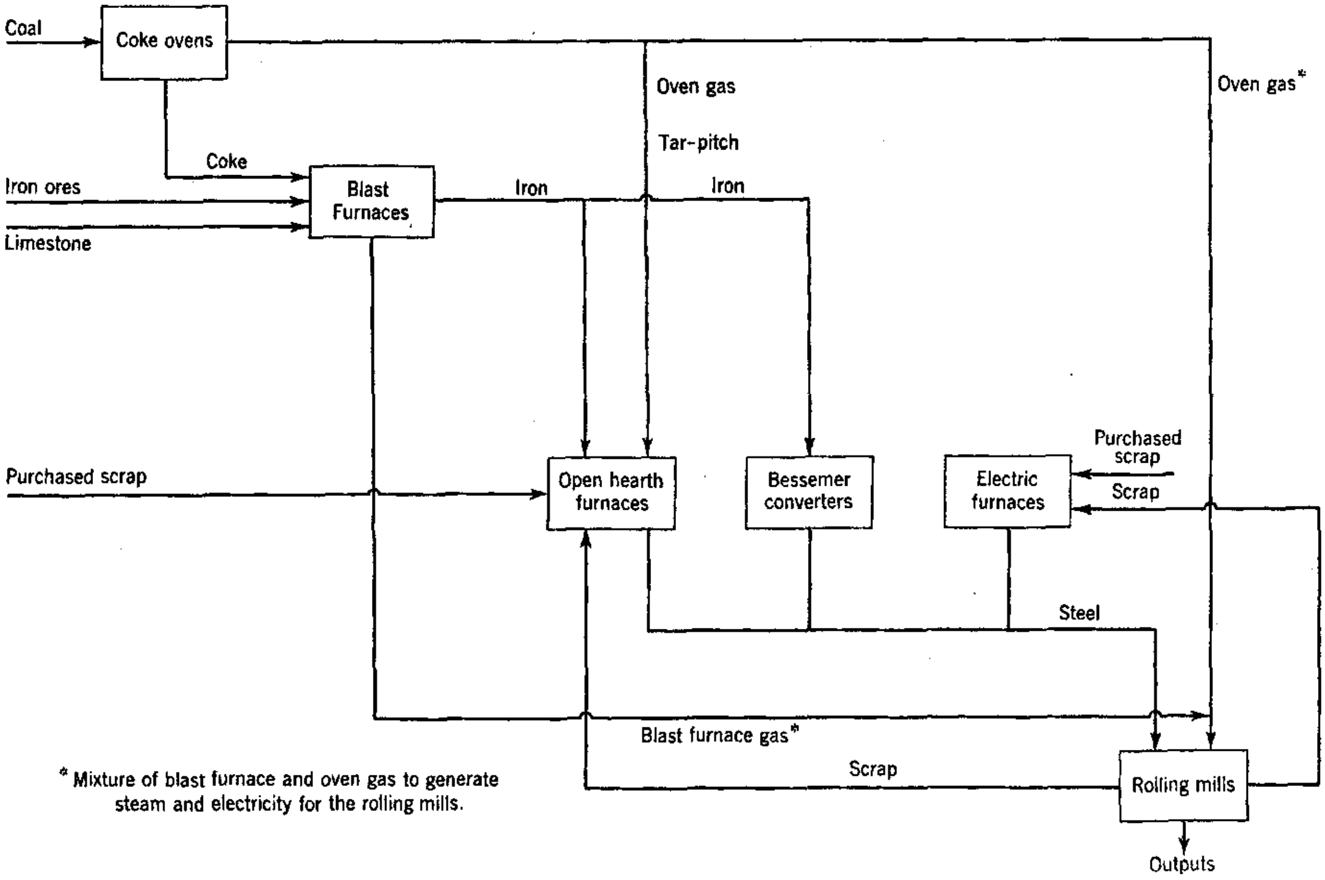
Each of these units may consist of several auxiliary components. Coke ovens may have a large plant for separating liquids and solids from the coke oven gas. Blast furnaces operate tandem with "stoves" which preheat the air that is blown into the furnace. Rolling mills include reheating furnaces. The auxiliary unit is not considered as a separate entity, but is treated as part of the technology of the major unit.

Bessemer converters are operated instead of open hearth furnaces or as a supplement to them. Electric furnace plants need not be operated in conjunction with other steel making furnaces or blast furnaces.

A transportation network exists in each plant to move the products. Specialized equipment is required to handle coke and hot metal (pig iron). This network will not be considered as a separate entity within the scope of this model, as if its capacity never limited production.

The flow of products is straightforward. Exogenous supply to the system is coal, iron ores of various types, and steel scrap (other than "home scrap"). The outputs of the integrated mill are the large variety of steel products, coke oven by-products and, to a small extent, oven gas and slag. Internally, coke from the ovens is used in the blast furnaces (some coke is used in the open hearth furnaces); iron is used in the open hearth and bessemer furnaces; rolling operations start with ingot steel input. Gas and tar-pitch mixture is used within the plant as fuel. Coke ovens constitute the main source of this fuel, although blast furnaces contribute some gas. Steel scrap, a by-product of rolling operations, is recycled into the steel furnaces.

At various points in the processes, choices exist as to which of several alternative technologies to use and, consequently, in what proportion to include certain materials. Decisions regarding (*a*) how much of each type of iron ore to use in the blast furnace, and (*b*) in what proportion the open hearth furnace should use iron and steel scrap, are a result of operating economy or necessity dictated by external factors, and affect the operations of the entire plant. If less iron is used in the open hearth furnaces, less coke and gas can be produced. Therefore, more fuel oil may perhaps be used for heating in the mills or furnaces. Similarly, a different ore mixture in the blast furnaces will affect the volume of coke produced and the fuel used.



* Mixture of blast furnace and oven gas to generate steam and electricity for the rolling mills.

FIGURE 1. Schematic flow chart of production in integrated iron and steel plants.

Each major step in the production process will now be examined briefly. Description of the technology will be limited to those elements which give a clue to the formulation of the process analysis model.

PRODUCTION OF METALLURGICAL COAL. The technologies were used in the reference year: beehive coke production and by-product coke production. In the beehive process the volatile components of coal are burned in limited air to obtain the required coke. The technology is simple: the input of the activity consists of coal; the output is coke. The weight ratio of input and output can be taken fixed for each type of coal used. A few plants use waste heat from the process to generate electricity.

The beehive process was widely used when high quality hard coal was abundant. By-product ovens replaced many beehive ovens, and to date the beehive process is primarily used to satisfy peak requirements.

A by-product oven consists of a battery of retort-type ovens. The oven charge, a mixture of high and low volatile content coals, is heated through the oven walls with coke oven gas, a by-product of the process. The volatile content of the coal is separated from the solid carbon and is led through groups of auxiliary equipment. Here coke oven gas is separated from the other by-products. The solid part of the coal becomes coke. Several by-products are identical to, or are substitutes for, by-products of crude oil distillation, and indeed the process of separating the coking by-products from each other resembles the distillation of crude oil.

The physical characteristics of coke and the quantities of various by-products depend to a large extent on the type of coal used in the coke ovens and on speed and temperature of the coking process. For our purposes, these factors indicate the possibility of using some mathematical programming technique to choose the best input mixture and process characteristics. This programming problem could in principle be incorporated into the process analysis model. To the extent that some by-product outputs are substitutes for outputs of the petroleum refining industries, certain activities of the iron and steel industry will become competitive with activities of the petroleum refining industry.[4]

An early attempt to formulate the programming of coke production and to determine the optimum mixture of coals failed. Apparently, while detailed knowledge of the coking characteristics of individual coals exists, much less is known about how to predict the attributes of a coal mixture from the characteristics of its components. Practical results are obtained by limited experiments with various mixtures.

As a consequence, by-product coke production is introduced into the steel

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[4] Up to 1946 all benzene, most toluene, and a substantial portion of xylenes were derived from coal. Increasing demand for the aromatics by the growing plastics, synthetic fiber, and other industries resulted in new technologies for the recovery of these chemicals from petroleum. By 1956 the petroleum refining industry was participating heavily in the substantial growth of the production of basic aromatics. (From a description of the UDEX extraction process.)

industry model as only one activity with coal as input, coke and coke oven gas as output. No opportunity is provided for a choice among different types of coal. Furnace gas output is used as a net figure after an allowance for the volume absorbed in the heating of the coke retorts. Chemical by-products other than oven gas are neglected, since in this formulation their production is proportional to that of coke and, in addition, they are not used in later stages of iron and steel production. In a general process analysis model, these by-products would appear together with their substitutes supplied by other industries. Such a general model may give unexpected results. For instance, one may find a price level for these by-products at which it might be economical to produce coke in a substantial quantity even though steel production were at a low ebb. For the time being, however, the observations are that the chemical synthetic fibers, cellophane, plastics, and perfume industries experience a shortage of some raw materials when steel production rates are down.[5]

PRODUCTION OF PIG IRON. The production of pig iron or hot metal is the first of the two metallurgical stages of steel production. The process is a sequence of reactions between the chemical components of the material inputs: iron ore, limestone, coke, and air. Each of these ingredients is a composite of free chemical elements and/or compounds, which, in simple terms, separate from each other and create new compounds at the high temperature in the furnace.

Blast furnace plants use several ores as a mixture to produce pig iron of a desired chemical analysis. A computational procedure, referred to sometimes as the "burdening problem," is applied to determine how much of each of the different types of ores and how much limestone and coke should be used in the mixture as blast furnace input. Subsequently the true chemistry of the output is determined by an analysis of the iron, and the input proportions are adjusted if needed. The chemical analysis of successive quantities of molten iron average out in a "hot metal mixer" prior to further use.

The outputs of the process are: metallurgical iron, low B.T.U. content blast furnace gas, slag, and waste heat. The first two will be incorporated into the model. Slag is a waste product without any positive economic value, and therefore, was not considered in the model. Waste heat was left out of the model as were all other components of power generation and usage.

STEEL PRODUCTION. Compared to the continuous process of iron production, steel production is a batch process. Individual "heats" of steel are produced in furnaces using molten pig iron ("hot metal"), steel scrap of different quality and source, lime and limestone, ferromanganese, and alloying materials.

The technology of the open hearth furnace is flexible. Hot metal can be used with steel scrap in widely varying proportions, from a low of about 30% scrap content in the charge to a high of up to 60–70%. The most usual furnace charge consists of half scrap and half hot metal. Chemical reactions in the open hearth furnace associated with steps in the operations apparently

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[5] See "Decline in production of steel results in a shortage of coke by-products," New York Times, September 4, 1960, Section 3.

require that at least about 30% of the metallic charge should be steel scrap. The upper limit is probably the consequence of the need to control the chemistry of the finished steel. Depending upon size, a furnace can produce from 100 to 500 tons of ingot steel at approximately 11 hour intervals. The actual time needed per heat to some extent depends on the percentage of hot metal used. Cold charge (steel scrap) takes longer to process than molten charge (hot metal) because of the time required for melting.

The charge of the oven is heated directly by coke oven gas or oil. The process analysis model incorporates the use of excess oven gas as furnace fuel.

Bessemer furnaces use almost all hot metal charge. Fuel is not required in Bessemer steel production; the process is endothermic. Electric furnaces can be operated with pure scrap charge; the charge is melted down and controlled by electric arc or induction heat.

Scrap is supplied (1) by the steel mills—about 30% of the output is recycled through the open hearth or electric furnaces; (2) by industry—as "prompt" scrap or scrap returned immediately to the mills as the production uses up steel; and (3) by the economy in the form of retired scrap. Scrap is graded by form, size, density of a baled bundle, and other characteristics. Grading appears to have some relationship to yield: the amount of steel produced per ton of scrap used; and to efficiency of use: production per hour. Heavy steel scrap gives a higher yield, and its use is more efficient, than light scrap. This difference is normally reflected in a price difference between the two types.

There is a similarity between the metallurgical aspects of iron and open hearth steel production. Both processes remove impurities from the iron. More correctly, both processes use lime based slag to absorb nonferrous materials, in particular, impurities of iron, in the furnace charge, and to yield a ferrous output with the desired characteristics. In the blast furnace process, lime binds the nonferrous rock of the iron ore, and the resulting slag absorbs some of the sulfur passed on by the coke. In the open hearth process, lime is available in excess amounts and is used to trap the remaining impurities of the iron (or steel scrap). The presence of manganese assists the chemical reactions.

ROLLING AND FINISHING OF STEEL. The last stage of steel production creates the shape and certain surface and physical characteristics of steel. We shall distinguish between rolled, forged, and cast steel.

Rolled steel is usually the product of a multistep process. Ingots are rolled to produce primary blooms, billets, and slabs; all other shapes result from rolling and forming the primary shapes. Thus, welded pipes are made of skelp, steel sheets or strips of sheet; wire is drawn from rods; nuts, bolts, rivets, nails are made of wire.

Tinning and galvanizing provide steel with a coat of nonferrous metal such as tin, zinc, or lead. The container and construction industries use such coated steel. Forging and casting create shapes which cannot be obtained with rolling and need not or cannot be obtained by machining. Figure 2 illustrates

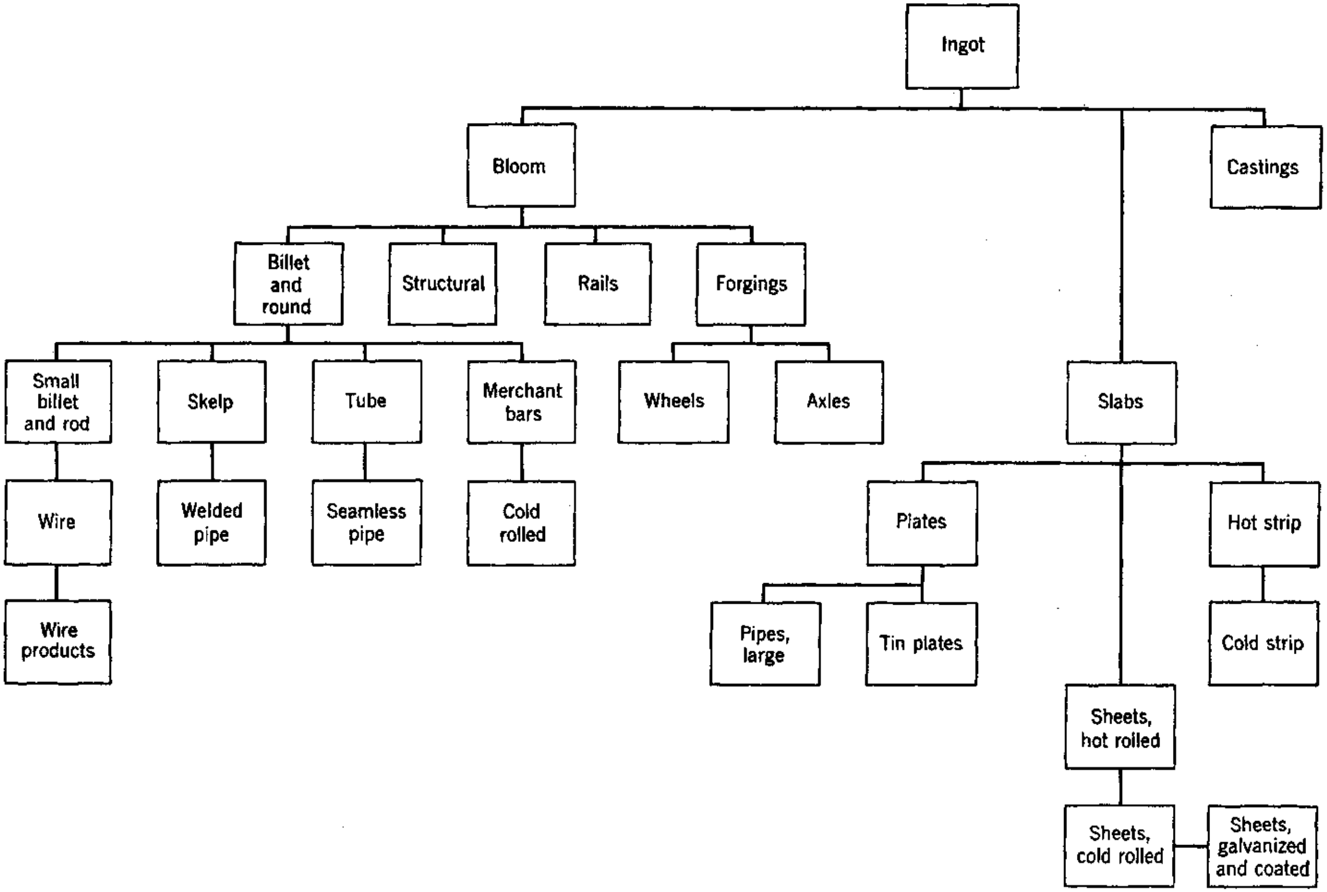

FIGURE 2. Flow of products among rolling mills.

the flow of finished steel through several stages of rolling. Each stage corresponds to a rolling mill. The diagram represents the interrelation between rolling mills but it is not a flow chart of any specific steel mill.

The model aggregates the production in all types of rolling mills into seven activities. An eighth activity represents forging of wheels and axles. (See Table 6.) The product flow originates at the rolling of primary shapes. The output of this activity (18) is allocated among other activities of the rolling mill segment of the model. Only two activities receive secondary rolled steel as input. Activity 22, representing the production of wire and wire products, uses bars as input (equation 38); activity 25, production of pipes, uses plates, sheet, and strip (equation 39). Normally, the reheating furnaces, manipulating equipment, and rolls of a mill are constructed to produce a certain range of items. While it is conceivable that other items could be produced, this would probably necessitate considerable modification of the entire mill set up. While the concept of substitution is similar to that of the machine working industry and in principle could be handled in the same way (see Chapter 12 of this book), in reality, since rolling mills constitute a custom made machinery complex, an accurate estimate of what the substitute items could be and at what rate they might be produced would entail intimate knowledge of each mill.

The Steel Industry Model

GENERAL. The model follows the outlines of the previously described technological processes. Consequently, the iron and steel industry is divided into the following subsections:

Production of coke and by-products	(477)
Production of pig iron	(336)
Steel production	(336)
Open hearth furnaces	
Bessemer furnaces	
Electric furnaces	
Rolling and finishing of steel	(336)
Primary rolling	
Rolling of bars	
Rolling of plate and sheet	
Rolling of structural steel	
Rolling of rails	
Rolling of pipes	
Wheels and forgings	(336)
Wire drawing	(336)

The numbers in parenthesis are the three-digit industry group classes of the corresponding subsection in the Standard Industrial classification.

Models for these subsections can be formulated separately. Equations which describe interactions between the subsections complete the industry-wide model.

Final demand for steel is appended to the model in the form of a "requirement matrix." This matrix indicates the rate of use of steel by type of rolling mill product and by type of using industry. The breakdown of steel usage by industry is exogenous to the model; it is incorporated here for illustrative purposes. Finished products are shapes expressed in terms of rolling mill output by type of mill. Surface finishes and hot or cold rolling are not distinguished.

In the following sections the model of each stage and the interacting relations are stated. Computations or sources of the numerical values of the parameters are explained. The matrix is presented in sections to the extent that is possible. Reference to the appropriate table is given with the title of the section.

COKE PRODUCTION (TABLE 1). The coke production sector is described with seven activities and five constraints. Of the seven, five are slack activities, and are not shown explicitly here. The two remaining activities represent the beehive and by-product oven processes respectively.

The constraints express available oven capacity, coal supply, and final disposition of the output. Oven capacity is measured in net ton output, and the industry figures of rated capacity were used as upper bounds. These were 2,092,000 NT and 69,416,000 NT respectively for beehive and by-product furnaces in 1956 (constraints 1 and 2).

Conversion of coal to coke is assumed to yield more coke per unit of coal in the beehive than in the by-product process. The beehive process may use coal with less volatile content, while the by-product oven normally uses a mixture of high and low volatile coals. The conversion coefficients were estimated at 1.20 NT of coal per net ton of beehive coke, and 1.35 NT of coal per ton of by-product coke. These figures are somewhat low. In an example, U. S. Steel (1957)[6] shows that the "typical American by-product oven practice" yields about 1,200 to 1,400 lbs. of coke per net ton of coal. This is more than the above figure. The lower conversion figure was accepted to account for losses in the form of coke breeze or other shrinkage. The upper bound on coal supply, 90,848,000 NT, is the 1956 coal consumption by the iron and steel industry.

The output of coke oven gas per ton of coke follows the figure given in the example of U. S. Steel (1957). As a result of deflating the coke output figure, oven gas output also is reduced.

Coke output is accounted for in net ton units (constraint 5).

PIG IRON PRODUCTION (TABLE 1). This process is formulated as a linear programming equivalent of the blast furnace burdening problem: What ferrous material inputs should be combined to give the required pig iron? The characteristics of pig iron depend on its use. The chemical composition of iron for casting differs from iron for steel making, and in the latter, distinction is made

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[6] Page 113.

TABLE 1

	Production of Coke		Production of Pig Iron						
			Iron Ore						
	Beehive Oven	By-product Oven	No. 1	No. 2	No. 3	Sintered Ore	Open Hearth Slag	Mill Scale	Ferro-manganese
{Row index Column index}	01	02	03	04	05	06	07	08	09
Constraint									
1. Beehive oven capacity	1.00								
2. By-product oven capacity		1.00							
3. Coal input	1.20	1.35							
4. Oven gas balance		1.02							
5. Coke balance	1.00	1.00	1.12	.93	.93	.93	.62	.61	1.10
6. Blast furnace capacity			.90	1.00	.88	.88	1.80	.40	
7. Ore No. 1 input			1.68						
8. Ore No. 2 input				1.95					
9. Ore No. 3 input					1.54				
10. Sintered ore input						1.68			
11. Open hearth slag balance							3.92		
12. Mill scale balance								1.34	
13. Manganese ore input									1.60
14. Limestone input			.50	.18	.18	.20	.11	.09	
15. Manganese balance			.92	.45	.17	.93	−7.22	.63	−48.00
16. Phosphorus balance			−.21	−.32	−.27	−.31	3.05	.15	
17. Sulfur balance			.07	−.11	−.06	−.09	.19	−.30	
18. Pig iron balance			−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
19. Furnace gas balance			−1.46	−1.22	−1.22	−1.22	−.80	−.80	−1.43

between open hearth and Bessemer grades. (See U. S. Steel (1957), page 221.) An average quality was chosen as the metal to be produced; its desired analysis is presented in Table 2. The iron is representative of basic open hearth grades. The model accounts for manganese, sulfur, and phosphorus, but not for silicon and carbon content. The presence of these latter impurities seems to depend on furnace operating condition, hearth temperature, and the proximity of other impurities, but the exact nature of the relationship is undetermined.

TABLE 2

ANALYSIS OF HOT METAL TO BE PRODUCED

Chemical Component	Content (per cent by weight)
Si	1.000
S	.030
P	.500
Mn	1.000
C	4.200

Analysis of the materials used as typical components of the furnace charge is contained in Table 3. Three types of ore were considered to represent the industry's main source of iron: two domestic ores, one with relatively high iron content and one with a lower iron content, and a very high iron bearing ore, typical of South America and some African deposits. One of the domestic ores is high in silicon and phosphorus, while the other has a relatively high manganese content. The sintered ore is about as good in ferrous content as the better of the two domestic ores; at the same time it is high in chemically basic compounds (CaO and MgO) but, together with the imported ore, it is relatively high in phosphorus. The analyses of open hearth slag (a candidate for charge material), mill scale, coke, and limestone were obtained from the operating data of one steel company. The relevant data in Table 3 are the Fe, CaO, MgO, S, Ph, Mn, and fixed carbon content. These become components of the parameters of the model.

The model of iron production is a linear programming mixing problem. The problem is to determine how much of each of the first six items in Table 3 to use for the production of pig iron of the required quantity. The production of ferromanganese and other ferro alloys was combined in one activity and incorporated into the iron production model.

The use of material supply per ton of iron output of the activity is computed on the basis of the actual Fe content of the source material and 95% Fe in the pig iron.

Blast furnace capacity usage of an activity per unit of output depends on the rate at which the iron content can be separated. This in turn depends on the volume of the material per unit of iron output. The figures in row 14 (Table 1) were computed on this basis; the two extremes, open hearth slag and mill scale, illustrate the point.

TABLE 3
Analysis of Materials Used in the Iron Production Sector

(Per cent by weight)

Chemical Components	Ore I[a]	Ore II[b]	Ore III[c]	Sinter[d]	Open Hearth Slag[d]	Mill Scale[d]	Coke[d]	Limestone[d]
Fe	56.82	47.47	65.00	56.00	24.00	70.00	.84	.77
SiO_2	15.47	6.09	5.00	9.54	21.00	.89	6.37	2.10
CaO	.67	.25		3.35	26.00	.45	.86	54.00
MgO	.33	.32	.006	2.37	5.00	.18	.14	.84
Al_2O_3	1.25	.83	2.00	2.22	4.53	.05	2.96	1.96
S	.007	.008	.400	.028	.150	.080	.890	.018
Ph	.135	.067	.120	.084	.900		.048	.016
Mn	.08	1.47	.90	.070	3.50	.46		
H_2O	.94	16.40						
Fixed carbon							87.89	

[a] Marquette range, natural basis. See U. S. Steel (1957), p. 144.

[b] Mesabi range, natural basis.

[c] Composite of various South American ores. Various components are not known; the value inserted for manganese content was chosen as the average of the manganese component of the other ingredients.

Source: U. S. Steel (1957), p. 140. Also Ted Metaxas, "Renaissance of Steel in South America," *Iron Age,* Sept. 25, 1949, pp. 92–93; "Chilean Ore for the United States," *Iron Age,* Dec. 20, 1951, p. 57; "Venezuelan Iron Ore," *Mining Congress Journal,* April, 1951, pp. 118–121 and 124.

[d] Data obtained by averaging a sample of data on blast furnace charge covering a year's operation of a major steel company.

Blast furnace capacity supply is expressed in net tons of iron output. In 1956 the rated capacity of furnaces was 85,450,000 NT; this figure was taken at the upper bound on the use of the capacity by the six ferrous inputs. Ferromanganese production does not use this capacity in the model, and no upper bound on capacity to produce ferromanganese is stated. The coke rate and the limestone requirement of each activity are determined here on the basis of Fabian (1958). A base to acid ratio of 1.1 is assumed. The resultant equations, for activities which use ore containing Fe_2O_3, are:

$$1092a_i - 3556c_i + 3102 = 0;$$

$$-503a_i + 92c_i + x_i = 0.$$

For ores containing FeO, the equations are:

$$1092a_i - 3556c_i + 2085 = 0;$$

$$-503a_i + 92c_i + x_i = 0$$

where the notation is:

a_i = amount of limestone used in activity i,
c_i = amount of coke used in activity i,
x_i = constant (values are given in Table 4).

TABLE 4
VALUES OF x_i[a]

i	x_i
1	305
2	14
3	11
4	12
5[b]	0
6[b]	−11

[a] For a base to acid ratio of 1.1. Values of x_i are higher for a higher ratio.

[b] Activity contains iron in the form of FeO.

Solution values of a_i and c_i are to be found in rows 14 and 5 of the matrix.[7]

The phosphorus content of each activity was computed as the sum of the phosphorus content of each component of the activity, such as the iron bearing material, coke, and limestone. The sulfur content of each activity was computed in Fabian (1958). The sulfur input of the activity was reduced by the amount which is expected to pass into the slag.

Maximum sulfur, maximum phosphorus, and minimum manganese content of the blast furnace charge are stated as

$$\frac{a_{ik}x_k}{\sum_j a_{ij}k_j} \leqq P$$

where x_j is the level of activity j ($j = 1, 2, \ldots, k, \ldots, J$), a_{ij} is the ith impurity content of the activity per unit of activity. The coefficients of inequalities $i = 15$, 16, and 17 were therefore computed as the coefficients of the derived inequality

$$(1 - P)a_{ik}x_k - \sum_{j \neq k} Pa_{ij}x_j \leqq 0$$

where P is the max (min) per cent impurity permitted (required) by the production technology. Furnace gas output was calculated on the basis of the coke content of the activity. Coefficients of equation 19 were computed as 130,000 cubic feet of gas per ton of coke in the activity. The supply and the allocation of coke oven and furnace gas were kept separate due to the different nature of the two gases.

STEEL PRODUCTION (TABLE 5). The steel production sector has three main elements: open hearth, Bessemer, and electric production. Of the eight ac-

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[7] The linear relations of the two equation system appear to be insensitive to extreme cases. The unit limestone requirements of activity 03 became .79, which is unexpectedly high. In order to meet normal operating experience, the figure was reduced in the matrix and the problem was relegated to further research.

TABLE 5

↓Row index	Open Hearth Steel Production					Electric Steel Production		
	Hot Metal	Home Scrap	Heavy Melting Scrap	Bundles and Light Scrap	Bessemer Steel Production	Home Scrap	Heavy Melting Scrap	Bundles and Light Scrap
Column index→	10	11	12	13	14	15	16	17
Constraint								
4. Oven gas balance		.68	.68	.68				
11. Open hearth slag balance	−.12	−.06	−.06	−.06				
14. Limestone input	.09	.07	.07	.07				
18. Pig iron balance	1.15				1.15			
20. Open hearth capacity	1.56	3.16	2.97	3.28				
21. Bessemer capacity					.48			
22. Electric furnace capacity						.12	.10	.13
23. Home scrap balance		1.11				1.08		
24. Heavy smelting scrap input			1.10				1.03	
25. Bundles and light scrap input				1.12				1.10
26. Upper limit on hot metal use	.41	−.72	−.71	−.72				
27. Lower limit on hot metal use	−.88	.33	.32	.34				
28. Ingot steel balance	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00

tivities representing the sector, four correspond to the use of open hearth, one to Bessemer, and three to electric furnace use.

The hot metal input of open hearth, per unit of steel output, was estimated on the basis of impurities in the iron relative to steel, and the yield of high hot metal production practice. Three to four per cent of the iron's impurities are removed in the slag in the open hearth process, and probably an additional ten to twelve per cent are lost through oxidation. An iron input of 1.15 tons per ton of steel for activity 10 seems a reasonable conversion factor. The same figure is used for the Bessemer process or activity 14.

The scrap usage rate was also computed on the basis of conversion and yield. Industry sources indicated that depending on the type of scrap used, and the type of steel produced, up to 95% of the weight of scrap is converted into ingot steel. Since public information on the effect of scrap use on open hearth yield by type of scrap (heavy, light) was not found, the coefficients of equations 23, 24, and 25 were obtained from private estimates. The estimates assumed that heavy scrap has a higher yield than light scrap. The yield factor of home scrap input was estimated on the basis of a "normal" scrap.

The heat requirement of a scrap using activity was estimated at approximately 3400 cubic feet per ton of finished steel.[8] The estimate refers to an efficient furnace using 50-50 practice (half steel scrap and half hot metal). One has to assume that all the heat requirement is the consequence of the cold charge—the steel scrap. For this reason, the activities using scrap were charged with twice the volume of gas used at 50-50 practice. To be exact, the different types of scrap may require different heat volume per unit: light scrap melts down faster than heavy scrap and therefore, less heat is lost in the process. This differential in heat requirement is balanced by the greater quantity of light scrap used per ton output (1.12 NT vs. 1.10 NT). An exact balancing of added and lessened heat requirement would be an unnecessarily fine point to make.

Limestone is used in open hearth production; 7% by weight of output usage was assumed on the basis of industry references.

All four open hearth activities "produce" slag. The volume of slag can be computed as the sum of limestone and impurities per ton of steel produced. Discrepancy between the figures is due to the loss in weight, through chemical reactions, of the limestone. In practice, slag should amount to more than the figures given in equation 11, since some iron ore also is used in conjunction with high hot metal or 50-50 practice.

The open hearth furnace capacity requirement of the activities had to be estimated through the use of operating time by activity. The hot metal using activity can reasonably be assumed to produce more steel per hour than the scrap using activity. Charging time of the hot metal using activity is short. Data from an open hearth shop which operates nine furnaces of equal size and about the same age indicates that the time required to produce a heat of steel is a monotonically increasing function of the cold part of the open hearth charge. These data and expert advice provided the parameters of equation

[8] U. S. Steel (1957), page 85.

20. The right-hand side represents the 1956 output of the average integrated plant consisting of nine 400 ton open hearth furnaces. The computations assumed that on the average, only eight furnaces would be in operation and one on repair. This amounts to approximately 5700 heats per year, provided that two heats are produced every 24 hours. The parameters fairly represent the furnace usage by the hot metal and by the scrap charge using activities. (See Table 10.)

The Bessemer and electric steel producing activities contain few parameters. Capacity requirements were computed along the lines of computation of the open hearth capacity figures. For electric steel activities, the time had to be apportioned among the three grades of scrap charge. The figures, .12, .10, and .13, were considered reasonable estimates by an expert reviewer.

Material inputs, hot metal in the case of Bessemer and scrap in the case of electric steel, were assumed to have a somewhat better yield (i.e., lower input per unit of output) than in open hearth furnaces, because of technological differences in the use of slag between the two processes. The figures used in the model were engineering estimates.

ROLLING AND FINISHING OF STEEL (TABLE 6). Because of the large number of different types of rolling mills and rolled steel products, this sector is treated in even broader aggregates than the previously discussed sectors of steel production. Each of activities 18 to 25 represents the output of a single class of rolling mill products, such as "plates and sheet." The eight activities contain the output of all rolling and forging, as represented in Figure 2. Since published figures on rolling mill production follow a different, somewhat more detailed grouping, Table 7 was prepared to show the relationship between the source data and the activities.

The structure of the model of steel rolling is simple. The activity levels are constrained by the supply of steel, oven gas, and rolling mill capacity. The activities supply rolled products to the steel using sectors of the economy and steel scrap and mill scale to the open hearth and electric furnaces. Interaction between the inputs and outputs of rolling mills follows the scheme of Figure 2.

The steel input of primary rolling is estimated as 1.20 tons per ton of blooms, billets, and slabs. Twenty per cent of this input becomes steel scrap; the remaining steel is used in secondary rolling. An additional five to ten per cent scrap loss occurs in secondary rolling.

Oven gas use by the rolling mills was computed from a published example of the heat allocation in steel mills.[9]

A detailed account of heat allocation by mills was unavailable. It was assumed, therefore, that at each stage, steel is heated to about the same temperature for rolling, and thus the per ton use of heat (or oven gas) will be the same for all rolling activities. This assumption neglects the heat used for heat treating processes and the effect of the volume of cold rolled steel on heat usage. A somewhat better figure could be obtained by a more detailed analysis of the heat usage.

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[9] U. S. Steel (1957), page 83.

TABLE 6
Rolling and Finishing of Steel

Row index	Constraint	Primary Shapes	Bars and Bar Shapes	Plates and Sheet	Structural Steel	Wire and Wire Products	Rails	Wheels and Forged Steel	Pipes
	Column index	18	19	20	21	22	23	24	25
4.	Oven gas balance	.09	.26	.26	.26		.26	.26	
12.	Mill scale balance		−.02	−.02	−.02		−.02	−.01	−.02
19.	Furnace gas balance	.65							
23.	Home scrap balance	−.20	−.06	−.06	−.06	−.10	−.06	−.06	−.05
28.	Ingot steel balance	−1.20							
29.	Primary rolling capacity	1.00							
30.	Bar mill capacity		1.00						
31.	Plate and sheet mill capacity			1.00					
32.	Structural mill capacity				1.00				
33.	Wire drawing capacity					1.00			
34.	Rail mill capacity						1.00		
35.	Wheel and forging capacity							1.00	
36.	Pipe mill capacity								1.00
37.	Primary rolled output balance	−1.00	1.06	1.06	1.06		1.06	1.06	
38.	Bar and rod output balance		−1.00			1.10			
39.	Plate and sheet output			−1.00					1.05
40.	Structural mill output				−1.00				
41.	Wire and wire products output					−1.00			
42.	Rail mill output						−1.00		
43.	Wheel and forging output							−1.00	
44.	Pipe mill output								−1.00

TABLE 7

GROUPING OF ROLLING MILL PRODUCTION BY PROCESS ANALYSIS ACTIVITIES

Activity Number	Activity	AISI Category[a]
18	Primary shapes	Ingots, blooms, billets, slabs, sheet, bars, and seamless tube rounds
19	Bars and bar shapes	Bars (all types), wire rods, tool steel
20	Plates and flat order products	Skelp, black plate, tin and terne plate, sheets (all types), electrical sheets and strip, strip (hot and cold rolled)
21	Structural steel	Structural shapes, plates, steel piling
22	Wire and wire products	Wire (drawn, nails and staples, barbed and twisted, woven wire fence, bale ties)
23	Rails	Rails, joint bars, tie plates, track spikes
24	Wheels and forgings	Wheels, axles
25	Pipes	Pipe and tubing (all types)

[a] *Annual Statistical Report,* 1956, American Iron and Steel Institute, p. 94 et seq.

Mill scale output of each rolling activity supplied to blast furnaces is a relatively small quantity. Strictly speaking, the sum of weight of steel, steel scrap, and mill scale should equal the steel input of each activity. A small discrepancy—one or two percentage points—due to the use of different sources of information for the coefficients, will be found here.

DEMAND FOR ROLLED STEEL PRODUCTS (TABLE 9). Final demand for steel products can best be illustrated with Figure 3 and Table 8. The former is an illustration of the steel components of a typical passenger automobile. These weigh 3,542 pounds, or about 1.75 short tons. Table 8 lists the components and the final product class of the steel mills used to produce each component. The total weight of steel by type built into all passenger cars in a model year can easily be determined by adding the numbers vertically and multiplying them by the total number of passenger cars produced. Inventory adjustments and the amount of scrap constitute the difference between the weight of steel actually used in the cars and the weight of steel purchased for automobile production.

Information on steel content by type of a large group of manufactured items was collected by the Committee on Commercial Research of the American Iron and Steel Institute. The steel content of 173 items is presented in one publication.[10] Another source indicates the steel purchases of industry groups by type of industry.[11] A third source presents shipments to industry groups by the iron and steel industry.[12]

A matrix is appended to illustrate the relationship between the volume of

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[10] *Bills of Materials,* American Iron and Steel Institute (no date).

[11] *Census of Manufacturers,* 1954.

[12] *Annual Statistical Report,* American Iron and Steel Institute.

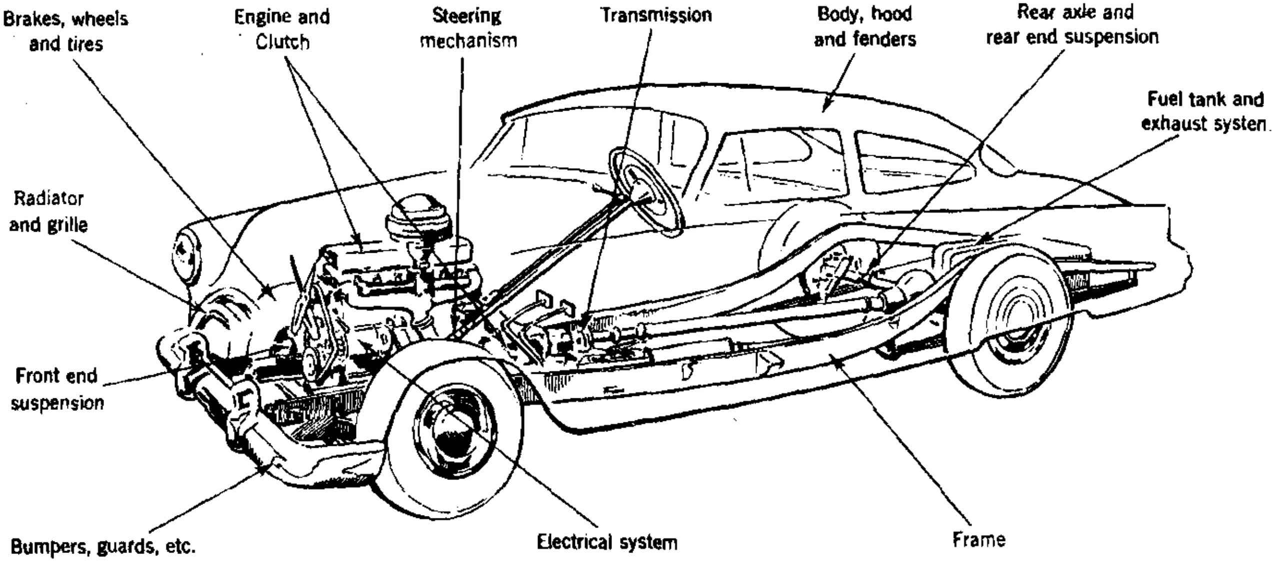

FIGURE 3. Iron and steel components of a passenger automobile. (Source: *Steel Facts,* October, 1957, page 7.)

TABLE 8

Steel Used in a Passenger Automobile by Rolling Mill Products

All weights are in pounds (weight of iron and steel castings not included)

	Steel Bars			Strip Steel		Sheet Steel							
	Hot Rolled	Cold Rolled	Forgings	Hot Rolled	Cold Rolled	Hot Rolled	Cold Rolled	Wire Products	Steel Plates	Terne Plate	Structural Shapes	Pipe and Tubes (Seamless)	Total
Steering mechanism	20.5	10.3	1.2	1.4	0.9	6.2	7.2	3.6	0.3				51.6
Engine and clutch	28.4	34.8	137.5	31.6	23.7	42.7	22.1	17.4	7.9	10.8			356.9
Front end suspension	47.2	7.7	57.4	9.5	2.2	18.2	0.2	2.6					145.0
Brakes, wheels, and tires	3.5	10.1	0.4	146.4	1.5	69.3	11.0	13.6					255.8
Bumpers, guards, etc.	5.4	1.4	25.1	26.7	78.2	7.4	51.7	3.4			5.8		205.1
Fuel tank and exhaust system	1.1	0.7		1.6	23.8	3.4	20.6	0.5		17.3		0.7	69.7
Frame						248.5	45.4	5.7					299.6
Electrical system	0.5	2.9		5.1	5.3	3.1	17.4	2.8		0.1			37.2
Transmission	4.6	22.4	42.9	1.0	2.1	0.4	4.1	2.1					79.6
Radiator and grille	0.5			4.2	1.8	14.7	51.1	0.9		5.3		0.4	78.9
Body, hood and fenders	7.5	8.3	0.9	53.9	115.7	160.2	1185.7	96.1		22.9		0.1	1651.3
Rear axle and rear end suspension	84.9	15.8	54.9	10.8	2.4	94.6	13.1	6.7	18.2			9.8	311.2
Total	204.1	114.4	320.3	292.2	257.6	668.7	1429.6	155.4	26.4	56.4	5.8	11.0	3541.9

Source: Steel Facts, October 1953, page 7.

TABLE 9

Demand for Finished Steel by Using Industries and Supply of Industrial Scrap

Row index / Constraint	Forgings, Other than Automotive	Bolts, Nuts, Rivets	Warehouses	Construction, Including Maintenance	Contractors' Products	Automotive	Rail Transportation	Shipbuilding and Equipment	Aircraft	Oil and Gas Drilling	Mining and Quarrying	Agriculture	Machinery and Industrial Equipment	Electrical Machinery	Appliances and Utensils	Other Domestic and Commercial Equipment	Containers	Ordnance and Military
Column index	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
24.	−.31	−.31	−.31	−.31	−.20		−.20	−.26		−.30	−.30							−.34
25.						−.26			.22			−.32	−.30	−.22	−.26	.30	−.30	
37.	.63	.03		.01		.04	.01	.01	.24	.08	.01	.02	.04					.34
38.	.35	.68	.19	.18	.09	.18	.09	.06	.37	.27	.38	.35	.31	.09	.03	.07		.19
39.		.06	.26	.09	.76	.72	.09	.04	.19	.03	.09	.46	.19	.68	.92	.74	.97	.19
40.	.02		.15	.48	.05	.03	.34	.85	.11	.02	.30	.12	.34	.08	.01	.01	.01	.22
41.		.22	.07	.01	.02	.02			.02	.11	.01	.01	.05	.03	.03	.17	.01	.01
42.				.01			.35				.17							
43.							.12				.01							
44.			.32	.21	.07	.01		.03	.06	.52	.03	.04	.07	.11	.01	.02		.04
Final Demand	14.73	14.85	167.52	104.41	40.75	141.42	42.27	7.60	1.35	7.78	3.50	10.82	50.32	24.38	21.29	22.64	68.18	5.24

total steel usage by industry groups and the demand for steel by rolling mill output. The matrix was constructed from data on steel shipments by industry and by type of steel in 1956.[13] Each column represents the proportion of steel by type used by one industry. It is sufficient to state total steel usage (see "Final Demand" line, Table 9) to obtain demand by type of rolling mill output. The demand for steel, say, by the automotive industry, will be multiplied by the coefficients in column 31, to apportion the demand for rolling mill type.

These technological coefficients are subject to change. Such changes can occur because (1) the product mix of the steel using industry changes; (2) the product is redesigned; (3) steel is substituted by other materials; and (4) other materials are replaced by steel. These changes require the periodical updating of the matrix from data on steel shipments or usage.

Steel using industries are normally the source of purchased scrap. *Prompt scrap* is the scrap sold or returned as a part of a steel purchase contract by steel fabricating industries or by industries such as construction, shipbuildings, mining, petroleum drilling. *Obsolete scrap* is retired steel, such as scrapped ships, machinery, automobiles.

Rows 24 and 25 indicate the estimated total scrap generated by the steel using industries. The estimates contain both prompt and obsolete scrap. The estimates are based on judgment and in many instances overstate the rate of scrap generation. The scrap output of activities 26 to 43 is classified into heavy and light scrap mostly on the basis of the predominant type of steel used by the activity.[14]

The present form of the inequalities 1–44 conforms to a "less than or equal" formulation of all constraints. Accordingly, the coefficients of the slack activities are all positive. These activities represent unused capacity, unused materials, impurity percentages below requirements, unused limits on material usage.

COMPOSITION OF THE MATRIX. The matrix is presented in Tables 1, 5, 6, 9, and 10. Activities which represent the same stage of the production and distributioin processes are grouped together in one table. Constraints which affect activities on different tables can be traced through their row numbers.

Two segments of the matrix were omitted in order to conserve space. These are: (1) a negative unit matrix of order 18 to be inserted under row 44 of Table 9—this matrix represents the steel supply for each consuming industry; (2) a matrix of 44 positive slack activities following column 43.

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[13] American Iron and Steel Institute (1956).

[14] More recently detailed information became available on the generation of prompt scrap. The U. S. Department of Commerce (1957) provides scrap generation ratios (scrap shipments per ton of steel consumed) by three and four digit industry groups of the Standard Industrial Classification for 1954. These ratios are in general not comparable to the data used in Table 9. They are based on primary source data on scrap generation derived from an industrial survey and data on steel consumption from the *Census of Manufacturers*. Furthermore, a breakdown is provided by scrap type (heavy melting, bundles, cast iron, all other).

The (nonzero) right-hand side of equations 1 through 36 is presented in Table 10. The numbers represent the quantity of each resource available to or used by the steel industry in 1956 based on American Iron and Steel Institute (1956). Furnace and oven capacities are expressed in net tons of production (scaled by 10^{-5}), while material usage is expressed in tons actually used during 1956 (also scaled by 10^{-5}).

TABLE 10

Resources for Iron and Steel Production, 1956

Constraint	Capital Equipment Capacity	Available Quantity (100,000 NT)
1	Beehive ovens	20.92
2	By-product ovens	694.16
6	Blast furnaces	855.00
20	Open hearth furnaces	2350.00*
21	Bessemer furnaces	127.10
22	Electric furnaces	170.00
29	Primary rolling mills	1152.16
30	Bar mills	195.86
31	Plate and sheet mills	546.20
32	Structural mills	156.86
33	Wire mills	54.11
34	Rail mills	17.50
35	Wheel production and forging	9.80
36	Pipe mills	104.97
Constraint	**Material for Consumption**	**Available Quantity (100,000 NT)**
3	Coal	908.48
7	Ore I	770.95
8	Ore II	208.23
9	Ore III	303.58
10	Sintered ore	302.17
13	Manganese ore	17.58
14	Limestone	336.08

* Open hearth furnace capacity is measured in units of "shop hours."

Experimental Solutions

The linear programming matrix presented in Tables 1, 5, 6, and 9 has a very simple structure and, in connection with certain optimizers, is capable of longhand solution.

Table 9 is a materials requirement submatrix. It can be used to evaluate the rolled steel requirement of the economy and to estimate the volume of industrial scrap generation. The last line of the table is not a part of the submatrix; it represents the (variable) demand for steel by industry. When the submatrix is multiplied by this (column) vector, the rolled steel equivalent of steel products use by type of rolled steel and the scrap supply by type of scrap is obtained. Table 11 contains the data for rolled steel. In our case, the data are equivalent to an appropriate summary of "Net Shipments" less exports, of "Shipments of Steel Products by Market Classification—year 1956" of the American Iron and Steel Institute (1956).

TABLE 11
ALLOCATION OF ROLLING MILL CAPACITY
(100,000 NT)

Mill	Shipped to Industry	Used by Steel Plants in Current Production	Total Production	Capacity	Unused
Primary rolling	21.91	774.61	796.52	1152.16	355.64
Bar and bar shapes	129.32	30.76	169.68	195.86	26.18
Plate and sheet	326.14	96.36	422.50	546.20	123.70
Structural	126.64		126.64	156.86	30.22
Wire drawing	27.96		27.96	54.11	26.15
Rail	16.43		16.43	17.50	1.07
Wheel production and forging	5.11		5.11	9.80	4.69
Pipe	91.77		91.77	104.97	13.20

This minimum requirement of rolled steel can be produced with only one rolling technology. Thus the equivalent level of the rolling mill activities will have to satisfy the rolling mill capacity constraints presented in Table 6. This is illustrated in Table 11. The table also presents the level of the appropriate slack activities (unused capacity).

Computation of the level of steel producing activities requires designation of an objective. If an answer is sought to question 2, parts *a* and *b* on page 238, then the objective is to find a solution to the remaining part of the matrix with a minimum level of the steel using activities. Clearly, the lowest level of steel scrap use is determined through equations 24, 25, and 26. Equation 23 can be excluded for two reasons. First, all available home scrap is normally used in current production within steel plants. Second, the problem behind question 2 is concerned with purchased scrap, since only by reducing the volume of purchased scrap can a change in the availability of steel scrap occur. This can arise for several reasons. For instance, if the world market price of scrap as compared with the domestic price, or the cost of domestic labor required to process retired scrap (ships for instance) is sufficiently high, the domestic supply of scrap will diminish to the minimum scrap usage level. As another example, because of military emergency or incapacitation due to hostile action, the transportation system may not be able to handle more than the minimum supply of scrap.

A solution can easily be traced by evaluating activity 11 of Table 5 at the level of available home scrap and then checking whether the appropriate technological constraint and the constraint on hot metal supply are satisfied. If either of the two constraints is not satisfied, more scrap will have to be added. The amount can easily be computed.

A similar approach can be used to answer parts *a* and *c* of question 1. Tracing the solution is perhaps more tedious. A change in the ferrous content of an iron ore can be attained by a change in the appropriate parameter of Table 1 or by deliberately excluding one iron ore from the mixture of ferrous inputs. The best candidate for exclusion is activity 05, which represents the use of high grade imported ores. (See Table 1.)

The remaining parts of the first two questions require the computation of shadow prices of the ovens and furnaces. An easy method to obtain these prices is to trace the solution of the dual problem along the lines described above.

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CHAPTER 10

THE METALWORKING INDUSTRIES

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This chapter is concerned with the capabilities of the metalworking industries. Metalworking industries are those whose primary manufacturing operations consist of fabricating or assembling metal parts. These industries produce a kaleidoscopic array of products, among them: electrical machinery (such as motors, generators, transformers, appliances, and communication equipment), other machinery (such as farm, construction, mining and metalworking equipment, office and store machines, engines, turbines, pumps, compressors, elevators, conveyors, furnaces and ovens), transportation equipment (such as automobiles, aircraft, ships and boats, railroad equipment, motorcycles and bicycles), and other fabricated metal products (such as hardware, heating and plumbing equipment, safes and vaults, nuts and bolts, nails, springs, lighting fixtures, and boilers). Despite the diversity of product there is (with certain exceptions) a high degree of overlap between the processes, skills, and equipment used by the various metalworking industries. It is this commonness of process, skills, and equipment—and the resultant ability of these industries to reallocate existing resources as new needs arise—that leads us to treat the metalworking industries as a whole. We will be principally concerned with the flexibility or bottlenecks involved when the metalworking industries attempt to alter the composition of their output.

The present chapter briefly characterizes the metalworking industries: their production processes; their use of men, machines and materials; the division of labor between different types of industries within metalworking. The next chapter discusses the use of requirements analysis in metalworking. Chapter 12 presents an analysis of direct substitution possibilities among machine tools. Chapter 13 presents thoughts on next steps in the analysis of metalworking capabilities.

In the present chapter our survey of metalworking will rely heavily on the 1954 Census of Manufactures and the 1953 Inventory of Metalworking Equipment by the *American Machinist*.[1] The survey will look at metalworking both at a macro and a micro level: from the point of view of the individual

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[1] At the time these chapters were written only fragments of the results from the 1958 *Census of Manufactures* were available. Both for this reason and for comparability with data presented in subsequent chapters, data circa 1953–54 is used in our introductory survey.

shop and in terms of aggregate statistics. Since the latter is the sum of the results of the actions of the former, important relationships in the one should be traceable in the other.

METALWORKING PROCESSES

MATERIALS. Metal parts are fabricated from castings and from shapes and forms such as sheet, plate, and bars. As shown in Table 1, the metalworking industries of 1954 consumed almost 46½ million tons of steel shapes and forms, over 5 million tons of iron and steel castings, plus over 2 million tons of copper and aluminum shapes, forms, and castings. In 1954 these 54 million tons of

TABLE 1

METALS CONSUMED BY METALWORKING INDUSTRIES

	Millions of Short Tons		Delivered Cost ($1,000,000,000)	
Steel shapes and forms[a]	46.4		7.04	
Carbon steel				
Bars and bar shapes		6.5		.88
Sheet and strip		18.8		2.42
Structural shapes		3.9		.45
Plates		4.8		.55
Wire and wire products		1.8		.34
Tin plate, black and terne plate		4.5		.73
Other shapes and forms		2.5		.44
Alloy steel (except stainless)				
Bars and bar shapes		2.0		.42
Other shapes and forms		1.2		.37
Stainless steel		.4		.44
Copper and aluminum shapes and forms	1.8		1.75	
Copper and copper base alloy				
Wire, bare and insulated[b]		.32		.36
Other		.69		.66
Aluminum and aluminum base alloy		.75		.73
Ferrous castings[c]	5.3		1.51	
Iron		4.5		1.10
Steel		.8		.41
Nonferrous castings[c]	.4		.46	
Aluminum and aluminum base alloy		.23		.31
Copper and copper base alloy		.13		.15
Total iron, steel, copper, aluminum	53.9		10.76	

[a] Includes small amounts of wrought iron in the appropriate categories.

[b] Excludes bare wire purchased by the Insulated Wire industry. In the case of insulated wire, only the metal content is included in the weight figures.

[c] Purchased castings only. Does not include castings produced and consumed in the same establishment.

Source: Census of Manufactures, 1954, Vol. I, Chapter X.

metal, worth \$10¾ billion, were transformed by metal fabricating men and equipment into parts for products.

FABRICATION EQUIPMENT. The metalworking equipment which fabricates metal parts is classified into two broad categories: machine tools and metal forming machines. *Machine tools* fabricate parts by cutting and grinding. They are defined as equipment which progressively removes metal in the form of chips (with grinding said to produce chips of microscopic size). Machines which fabricate parts by other means—such as by forging, pressing, punching, bending, and shearing—are referred to as *metal forming equipment.* As of about the middle of 1953 the metalworking industries had almost 2 million machine tools and over one-half million metal-forming machines.[2]

At least 92½% of these machines were "general purpose." They were of types which are specialized to functions rather than to products. The various types of lathes, for example, are particularly adept at generating surfaces of revolution; shapers and planers are good at making flat surfaces; the more precise types of grinding equipment can machine hardened materials to high tolerances. Such functions are needed for a variety of products; hence machines of this sort are generally used by some, many, or all of the metalworking industries. On the other hand, in performing operations on a limited number of mass produced parts (such as tin cans and engine blocks) special purpose equipment was used to reduce operating costs. In the *American Machinist's* figures on the existing stock of equipment (and in the *Census of Manufactures'* figures on new production) most types of special purpose metalworking machines are included in categories—such as "other boring machines," "other machine tools," "other hydraulic pressures (including hydropneumatic)"—which also include the rarer types of general purpose equipment. The combination of special and rarer general purpose equipment accounted for 7½% of metalworking's machine tools and metal forming equipment. The other 92½% were of the general purpose types specified in Table B of Chapter 14.

The durability of metalworking equipment, both physically and functionally, is suggested by statistics available concerning their age distribution. In 1953 about 19% of the machine tools and 25% of the metal-forming machines were over twenty years old. Many thousands of these old-timers were bought in the prosperity of the 1920's, worked intermittently through the depression of the 30's, served (along with more recently produced equipment) through World War II, the post-war boom, the Korean crisis, and were still fabricating metal parts when machines were counted in 1953.

PROCESSING EQUIPMENT. In addition to fabrication operations, the making of metal parts may require processes such as cleaning (to prepare the part for subsequent operations), plating (to deposit a thin coating of one material on a part made of another material), or heating (to harden the material or to relieve stresses caused by fabrication). Section *b* of Table 2 shows the amount

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[2] Source: *American Machinist* (1953).

of cleaning, plating, and heating equipment used in the metalworking industries.

ASSEMBLY, INSPECTION, MATERIALS HANDLING. Assembly, inspection, and materials handling rank with parts fabrication as major phases in the transformation of materials into products. The assembly of smaller products, or smaller subassemblies of larger products, may require only workers at benches, putting components together with the aid of simple hand tools. Larger assemblies may require special holding devices for supporting components, cranes for moving the assembly from one work station to the next, and riveting or welding equipment to bind parts together. Inspection and testing operations are inserted throughout production, wherever the cost of testing is deemed less than the potential gain from finding defects at this stage rather than later. Materials handling—frequently by conveyors where the routing of work can be standardized, by bulk means such as fork lift trucks otherwise—ties together the receipt of material, the fabrication, inspection, assembly, and storage of work, and the shipment of final product.

Sections *c* and *d* of Table 2 show, respectively, the amounts of certain assembly assisting and materials handling equipment. Equipment used in quality control (generally not identifiable in economy-wide data sources) range from simple gauges to complex, specially built product test facilities.

TABLE 2

SHOP EQUIPMENT IN THE METALWORKING INDUSTRIES

	1000's of Machines
a. Fabrication equipment	
Machine tools	1,933
Metal-forming machines	545
b. Cleaning, plating, and materials heating equipment	
Cleaning equipment (tumbling, blast, washing, and pickling)	93
Plating equipment	40
Materials heating equipment	106
c. Assembly assisting equipment	
Riveting machines (not portable)	41[a]
Electric and gas welding machines (not hand)	275
d. Materials handling equipment	
Cranes, overhead traveling	56
Hoists	191
Industrial trucks, power driven	95

[a] Riveting machines are included in the metal-forming machines figure.

Source: American Machinist (1953).

THE PRINCIPAL METALWORKING INDUSTRIES. For certain statistics quoted below—particularly those used in comparing numbers of men with numbers of machines, or comparing one class of metalworking industries with another

—we shall restrict our attention to the following major groups of Census industries:

19. Ordnance and Accessories
34. Fabricated Metal Products
35. Machinery Except Electrical
36. Electrical Machinery
37. Transportation Equipment

As a shorthand, we shall refer to these as the "Principal Metalworking Industries." These industries account for about 87½% of all machine tools and metalforming machines used in manufacturing. The major groups which account for most of the remaining 12½% are:

25. Furniture and Fixtures
33. Primary Metals
38. Instruments and Related Products
39. Miscellaneous Manufactures

Much of the employment and value added of the latter groups is not directly related to the production of fabricated metal products, hence the desirability of excluding them from certain comparisons.

EMPLOYMENT. The principal metalworking industries of 1954 employed about 5½ million people. Almost 4¼ million were classified by the Census as "production and related workers," i.e., they held jobs (from machinist to janitor) which fit the following description:

> workers (up through the working foreman level) engaged in fabricating, processing, assembling, inspection, receiving, storage, handling, packing, warehousing, shipping (but not delivering), maintenance, repair, janitorial, watchman services, product development, auxiliary production for plant's own use (e.g., power plant) record keeping and other services closely associated with these production operations. Excludes supervisory employees above working foreman level.[3]

The remaining 1¼ employees fell under one or the other of the following descriptions:

> Force account construction workers: Employees on the payroll of the manufacturing establishment who are engaged in construction of major additions or alterations to the plant. . . .
>
> All other employees: Nonproduction personnel of the manufacturing establishment, including those engaged in the following activities: factory supervision above working foreman level, sales (including driver salesmen), sales delivery . . . advertising, credit, collection, installation and servicing of own products, clerical and routine office functions, executive, purchasing, finance, legal, personnel (including cafeteria, medical, etc.), professional, and technical.[4]

Of the almost 4¼ million production workers, slightly over 1½ million were employed in shops related to parts making, namely: foundry, forge, electro-

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[3] *U. S. Census of Manufactures* (1954), Vol. I, p. XVII.

[4] *U. S. Census of Manufactures* (1954), Vol. I, p. XVII.

plating, galvanizing, heat treating, tool and die, pattern, plate or structural fabrication, stamping and machine (including automatic screw machine) shops. The Census does not tell us what the more than 2½ million remaining production workers were doing. Table 3, which presents data from World War II manning tables for four industries, gives us some hints. Despite the lapse of ten years, the change of conditions, and some differences in classification, there is a general similarity between these wartime manning tables and 1954 *Census of Manufactures* data, when and where comparable. The table provides at least rough orders of magnitudes, sufficient for the present brief survey, as to the requirements of metalworking industries for the performance of tasks such as bench assembly, structural assembly, inspection, maintenance and repair work, and simple physical labor.

TABLE 3
Summary of World War II Manning Tables for Four Metalworking Industries

Name	Tanks and Tank Components	Ball and Roller Bearings	Motor Vehicles	Motor Vehicle Parts and Accessories
Clerical	14.4	11.7	16.5	13.1
Accounting, legal, purchase, sales	1.8	1.3	2.3	1.3
Public, personal, and protective services	2.8	2.0	3.5	2.3
Engineering and drafting	2.8	1.5	2.0	1.9
Industrial supervision	3.3	3.9	4.6	5.4
Mechanical and electrical repair and maintenance	3.7	1.5	5.0	3.0
Welding, riveting, and other structural work	16.1	.6	16.5	5.4
Bench work (except inspect and test)	14.5	3.4	23.0	7.8
Inspecting and testing	3.0	19.6	2.6	8.5
Machine operating (including machining and forming)	28.0	42.1	14.1	37.0
Processing (including plating and heating)	1.0	1.9	1.4	4.1
Equipment serving and related work	4.4	9.6	6.4	8.9
Miscellaneous physical work	3.7	.2	2.2	.8
Laboratory, literature, and art work	.3	.6	.1	.4
Totals[a]	100.0	100.0	100.0	100.0

[a] Individual percentages do not sum to 100 because of rounding.

Source: Occupational Composition Pattern (1953).

THE RATIO OF MACHINES TO MEN. Returning to the 1.57 million production workers engaged directly in parts making in the principal metalworking industries: 1.40 million of these are engaged in shops using machine tools and metal forming equipment. This presents an apparent anomaly: almost 2.16

million machines tools and metal-forming machines (in the principal metalworking industries) are at the disposal of 1.40 million workers. Why?

Several causes tend to make the ratio of men to machines either less than or greater than 1:1. For example: For certain types of operations, one man can supervise several machines; while in the case of other operations, particularly with larger machines, more than one man may be required. The most common situation, however, is one man operating one machine. Employment in 1954 was slightly less than that of the peak year, 1953. This drop, however, is small compared to the greater than 3:2 ratio of equipment to men to be explained. There was some second shift operation in 1954, which makes the 3:2 ratio of machines to men an even larger puzzle.

The chief reason for the observed ratio, however, is none of the above, but the following: Even during the prime shift, many metalworking machines spend much of their time idle. This is not primarily due to bad planning or unemployment but as an economic consequence of the following conditions:

1. The parts for most metal products are so varied in their metalworking requirements that it is virtually impossible for most shops to schedule so as to prevent random variations in requirements for particular metalworking equipment.
2. A worker can typically operate several types or sizes of equipment.
3. For many types of the more common equipment the cost of idle machine hours is substantially less than the cost of idle man-hours. Thus the ratio of machines to men should be such that men rarely have to wait for machines, generally machines have to wait for men.

The converse applies to extremely expensive machines. Thus it is not rare to find a two or three shift operation in the large boring mills even though on the prime shift the smaller lathe and milling machines are seldom fully occupied and the simple drill presses are more frequently idle than not. In this manner the substitution of one factor for another, depending on their relative prices, is true in the aggregate (even ignoring alternate modes of production) despite the fact that for any operation one man uses one machine.

RESOURCES OF TRANSITION. Certain types of manufacturing operations—and the men and equipment that perform them—are particularly needed in times of change, when the metalworking industries attempt to alter either the relative amounts made of different types of products, the specific characteristics of products within product classes, or the methods by which existing products are made. The men and equipment needed for such transitions are used in large numbers even under "normal" conditions, since change (new types of cars, new types of aircraft, new models of old products, and new products completely) is the normal mode in U. S. metalworking. The resources of transition take on an added importance in times of exceptional change, however, as a major factor limiting the speed with which industry can convert from one program to another.

Some of the more tangible forms of transition resources are concerned with *special tooling*—jigs, fixtures, dies, and special cutting tools designed to be combined with general purpose equipment for the efficient production of specific parts. Dies for forge and press are essential to the process by which the image of parts is transferred to metal. Jigs and fixtures are part holding and machine guiding devices which permit less skilled labor to achieve higher rates of production than could skilled machinists without such tooling. The production of tooling requires highly skilled tool and die makers, plus precision equipment such as honing, lapping, and jig boring machines.

Other types of "transition resources" have to do with planning and organization. When new products are to be produced, decisions must be made and recorded as to:

Which parts and subassemblies will be produced in the establishment and which purchased from the outside.

What will be the nature and sequence of operations to be performed on components made inside the establishment?

What special tooling should be designed and produced?

Should equipment be rearranged for more efficient materials handling?

From which sources should materials and purchased components be obtained?

What procedures should be used for checking the quality of products and components?

If the product is still in the "idea" stage, design and drafting must precede the production, quality control, and procurement decisions. Like the production of special tooling, the planning and organizing activities tend to be a bottleneck in times of transition, as establishments adopt new product lines, modify the characteristics of products in old lines, or decide to make old products in new ways.

METALWORKING: PAST, PRESENT, AND FUTURE. Guided by various planning activities, assisted by special tooling, checked by quality control, and tied together by materials handling, millions of men and machines of the 1954 metalworking industries fabricated parts from tens of millions of tons of metal and assembled these parts into an endless variety of products. In broad outline the same is true today, and will remain true for the foreseeable future. Some of the newer or rarer types of equipment, now lost in the catch-all "other" categories, may emerge as important processors of metals. Certain metals whose consumption was small or negligible in 1954 are already playing an increasingly important role in the advanced technologies of flight and space vehicles.

The current development which will probably have the greatest general effect on metalworking is the numerically controlled machine. The setup or operation of such machines is controlled by a punched paper tape or other form of numerical memory. For such machines the punched tape takes over most or all the functions of special tooling and some of the functions of the operator. A present bottleneck to the widespread use of such equipment is the time required to form the minutely detailed instructions that are coded on the tape. This bottleneck will probably be overcome in time, partly through the use of electronic computers to develop such control instructions from a minimum of essential information concerning the part to be made. The introduction of numerically controlled machinery should:

Decrease the importance of special tooling and increase the importance of planning as "resources of transition."

Increase the importance of computers in the planning process.

Increase the number of operations for which one operator can supervise two or more machines.

Cause us to add some new equipment categories to our analysis of metalworking requirements and some new task categories to our analysis of equipment substitution possibilities.

It should leave unchanged, however, many salient features of metalworking such as:

The roles of fabrication, assembly, inspection, materials handling and planning, as sketched in the present section.

Many of the characteristics of industrial structure discussed in the next section.

The problems and techniques of economy-wide requirements analysis, as discussed in the next chapter.

The general nature of equipment substitution possibilities, discussed in Chapter 12.

Some of the numbers change, but most of the basic relationships remain.

INDUSTRIAL STRUCTURE

DEFINITIONS. The last section was mostly concerned with the use of men, machines, and materials by the metalworking industries as a whole. In this section we discuss differences between industries. To begin with, however, let us consider the nature of the industrial classification upon which Census data is based. The "establishment"—generally smaller than a company, bigger than a shop—is a crucial concept in the analysis of Census and related data.

> The Census of Manufactures is conducted on an establishment basis. That is, a company operating establishments at more than one location is required to submit a separate report for each location; also, companies engaged in distinctly different lines of activity at one location are required to submit separate reports if the plant records permit such a separation and if the activities are substantial in size.[5]

An establishment is classified into an industry according to the products it makes primarily—i.e., the product class which accounts for more of its production than any other product class. Thus to say that "the Radios and Related Products industry produced 15 million dollars worth of motors and generators" is but shorthand for the statement: The set of establishments

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[5] *U. S. Census of Manufactures* (1954), Vol. I, p. XIII.

which had more radio and related product shipments than shipments of any other product class, also shipped 15 million dollars worth of products classified as motors and generators. All statements concerning production, consumption, or resources of specific industries must be interpreted in this manner. Any further interpretation is hypothesis, not necessarily fact.

INTEGRATION, SPECIALIZATION, AND COVERAGE. Establishments classified in one industry frequently manufacture products which are primary to other industries. Table 4 shows the specialization and coverage ratios of the prin-

TABLE 4

Specialization and Coverage Ratios for the Principal Metalworking Industries

Industry		Specialization Ratio	Coverage Ratio
341	Tin cans and other tinware	98	99
342	Cutlery, hand tools, and hardware	87	90
343	Heating and plumbing equipment	85	86
344	Structural metal products	92	91
346	Metal stamping and coating	85	(NA)
347	Lighting fixtures	90	91
348	Fabricated wire products	89	57[a]
349	Metal products, n.e.c.	91	81
351	Engines and turbines	88	84
352	Tractors and farm machinery	87	92
353	Construction and mining machinery	87	85
354	Metalworking machinery	92	87
355	Special-industry machinery, n.e.c.	87	85
356	General industrial machinery	85	81
357	Office and store machines	80	96
358	Service and household machines	85	90
359	Miscellaneous machinery parts	90	88
361	Electrical industrial apparatus	88	91
362	Electrical appliances	76	74
363	Insulated wire and cable	96	27[a]
364	Engine electrical equipment	85	87
365	Electric lamps (bulbs)	94	99
366	Communication equipment	93	96
369	Electrical products, n.e.c.	95	83
371	Motor vehicles and equipment	96	98
372	Aircraft and parts	96	95
373	Ships and boats	94	98
374	Railroad equipment	83	90
375	Motorcycles and bicycles	86	92
379	Transportation equipment, n.e.c.	92	44

[a] The low coverage ratio reflects industry classifications which are based on manufacturing process rather than products. See footnote in source.

Source: U. S. Census of Manufactures (1954), Vol. I, Table 2A.

NA—Not Available.

cipal three-digit metalworking industries. The specialization ratio measures the extent to which the establishments of the particular industry produced products which are primary to other industries. Coverage measures the extent to which establishments of other industries manufactured the products primary to the particular industry. Thus in the case of the Electrical Appliance industry, its specialization ration of 76% indicates that 24% of the products produced by establishments of this industry are not electrical appliances. Its 74% coverage ratio indicates that 26% of all electrical appliances were produced by establishments in other industries. Among the products primary to other industries produced in significant amounts by the electrical appliance industry are:

> vacuum bottles and jugs; incandescent vehicular lighting equipment; lawn mowers; household mechanical washing machines; other household laundry machines; household refrigerators, electric and gas; misc. electrical equipment for industrial use; industrial blowers and fans; misc. service-industry and household machines; misc. aircraft parts and auxiliary equipment.[6]

While the 76% specialization ratio is unusually low, Table 4 shows that not infrequently industries have specialization ratios of 90% or less. Specific establishments within the industry will have both higher and lower specialization ratios. Almost every industry has at least a few establishments with a specialization ratio of less than 50%, i.e., establishments in which its primary products represent a plurality but not a majority of total output.

Establishments within an industry may perform different stages in the production of the same item. This introduces duplication into "costs of materials" and "value of shipments" figures:

> the total value of shipments figures for industry groups (2- and 3-digit) and a few individual industries (4-digit) have not been published by the Census due to the extensive duplication arising from shipments from one establishment to another in the same industry classification.[7]

Suppose that two establishments, across the street from each other, making two successive stages in the production of one item, were merged without changing any processes or payments to capital or labor. The effect on Census figures would be to substantially reduce the value of shipments and the value of materials for the industry. In this case, however, (where nothing "real" was changed by the merger) the integration would not change either the number of production workers employed, the value that was added by manufacture, the number of metalworking machines owned, or the values of direct purchases of primary metals. Statistics such as these—which are independent of integration within an industry—play an important role in the analysis of industrial capabilities.

Certain industries specialize in performing metalworking services such as casting, forging, electroplating, tool and die making, sheet metal work, and machine shop work. To various extents, establishments making end items

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[6] 1954 *Census of Manufactures*, Vol. II, p. 36B-8.

[7] *U. S. Census of Manufactures* (1954), Vol. I, p. 22, footnote 6.

either buy these services or perform them for themselves. Thus, in addition to a special Tool and Die Industry employing 65,500 workers (of which 34,800 are actually in tool and die shops), the other metalworking industries employ 146,600 production workers in their Tool and Die shops. Other comparisons of this sort are presented in Table 5.

TABLE 5

EMPLOYMENT OF WORKERS IN ESTABLISHMENTS SPECIALIZED TO A FUNCTION VERSUS EMPLOYMENT OF SUCH WORKERS IN OTHER INDUSTRIES[a]

Function	Production Workers (1,000's) Prime Industry (All Shops)	Prime Industry (Prime Shop)	Other Industries (Prime Shop)
Tool and die	65.5	34.8	146.6
Pattern shop (foundry only)	16.9	3.4	24.0
Foundry (including die casting)	254.7	(NA)	85.4
Forging	33.0[b]	(NA)	24.4
Electroplating	31.2	(NA)	26.6
Galvanizing	2.7	(NA)	12.3

[a] Covers major groups 19, and 33 through 37.

[b] Iron and Steel Forging industry only.

Source: U. S. Census of Manufactures (1954), Vol. I, Chapter XII.

NA—Not Available.

"MAKE OR BUY" DECISIONS, AND THE ESTIMATION OF ECONOMY-WIDE REQUIREMENTS. The purchase of metalworking services and metal fabricated components by establishments in end-product industries is the result of innumerable decisions concerning where it is best "to make"—i.e., manufacture the component within the establishment—and where it is best "to buy"—i.e., contract work or purchase components from the outside. The outcomes of these "make or buy" decisions depend on considerations such as whether or not the establishment has the capability of performing certain processes, whether or not it can purchase components from reliable suppliers more economically than it can produce them, and whether or not its current need for particular types of equipment exceeds the current availability of this equipment within the establishment. Such considerations are influenced, in turn, by previous decisions on capital expenditures—decisions which depended on predictability of need and availability of funds, as well as upon the costs of manufacture with and without particular equipment. The "buy decision," as it affects Census figures, includes drawing upon other establishments within the same company. The extent to which this is done is influenced by company policy as to the centralization or decentralization of processes and component production within or between the company's various establishments. Thus for a multitude of business reasons, available statistics showing the direct purchases of materials, employment of labor, and use of equipment by end-product industries provide us with only part of total end-product requirements. The rest have been fragmented and parceled out, reflecting themselves in the direct usage figures of supplying industries. One of the more difficult problems in the use of Census figures for estimating the capabilities of the metalworking industries concerns how total requirements can be pieced back together again. This problem is discussed at length in the next chapter.

Table 6 portrays, roughly, the extent to which production activity takes place in end-product industries versus the intermediate metalworking industries which make components or provide metalworking services. For this purpose we divided the principal metalworking industries into the following categories:[8]

I. *Fabricated and Simply Assembled Products.* These industries primarily manufacture end products of a simple nature in that they do not involve a large number of levels of assembly or incorporate complex components. Important items in this category are tin cans and structural metal products for buildings and bridges.

II. *Fabricated Parts and Simply Assembled Components.* These industries primarily provide metalworking services such as galvanizing, electroplating, machine shop and sheet metal work, or produce simple intermediate products such as valves and fittings.

III. *Complex Assembled Components.* These industries primarily manufacture complex, assembled intermediate products such as electric motors and internal combustion engines. The products of these industries are purchased directly for construction and other investment activities, as well as being incorporated into complex products of other metalworking industries.

IV. *Complex Assembled Products.* These industries primarily manufacture

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[8] Table 6 is based on a classification of three digit industries. The following are classified as Type I: 341 Tin Cans and Other Tinware; 342 Cutlery, Hand Tools and Hardware; 343 Heating and Plumbing Equipment; 344 Structural Metal Products; 347 Lighting Fixtures; 348 Fabricated Wire Products; and 365 Electric Lamps. The following are classified as Type II: 349 Metal Products, n.e.c.; 359 Miscellaneous Machinery Parts; 363 Insulated Wire and Cable; and 369 Electrical Products, n.e.c. Classified as Type III: 351 Engines and Turbines; 356 General Industrial Machinery; 361 Electrical Industrial Apparatus; and 364 Engine Electrical Equipment. Classified as Type IV: 19 Ordnance and Accessories; 352 Tractors and Farm Machinery; 353 Construction and Mining Machinery; 354 Metalworking Machinery; 355 Special Industry Machinery, n.e.c.; 357 Office and Store Machines; 358 Service and Household Machines; 362 Electrical Appliances; 366 Communication Equipment; 371 Motor Vehicles and Equipment; 372 Aircraft and Parts; 373 Ships and Boats; 374 Railroad Equipment; 375 Motorcycles and Bicycles; 379 Transportation Equipment, n.e.c.

In a few cases industries classified as one type produced significant amounts of products of another type. For example, 342 Cutlery, Hand Tools and Hardware was classified as Type I since most of its $1.6 billion of shipments were simple products not incorporated in the products of other metalworking industries. It produced, nevertheless, $.4 billion of transportation equipment hardware—a Type II product. No attempt was made, however, to separate measures of manufacturing activity by product line within three digit industries. For product detail see Table 6a for the particular industry group in Volume II of the 1954 *Census of Manufactures.*

complex end products including transportation equipment and various types of household and industrial machinery.

Table 6 shows the production workers, value added, number of machine tools, and amount of steel consumed by each type of industry as a percentage of total usage by the principal metalworking industries. We see that in the case of number of production workers, value added by manufacture, and number of machine tools, the end-product industries (I and IV) account for roughly 70 to 75% of the total. The remaining 25 to 30% is divided about equally between the simple and complex intermediate industries (II and III). In the case of steel, because of the high consumption by Type I industries, the end-item industries account for 82% of the total, while almost two-thirds of the remaining goes to industries of Type II. These figures suggest that, on the whole, direct usage represents the greater part of total usage, but that indirect usages, incorporated in services performed or components produced by II and III, cannot be dismissed as negligible.

TABLE 6

PERCENTAGE OF CERTAIN MEASURES OF PRODUCTION ACTIVITY ACCOUNTED FOR BY FOUR TYPES OF INDUSTRIES[a]

	Per Cent of			
Type of Industry	Production Workers (No. in 1954)	Value Added (1954)	Machine Tools (No. in 1953)	Steel Consumption (Tons, 1954)
I. Fabricated and simply assembled products	14.6	13.9	12.2	40.8
II. Fabricated parts and simply assembled components	12.2	10.6	14.3	11.6
III. Complex assembled components	11.8	13.1	14.9	6.4
IV. Complex assembled products	61.3	62.5	58.6	41.2
Total,[b] industry groups 19, 34, 35, 36, and 37	100.0	100.0	100.0	100.0

[a] See footnote 8, preceding page, for classification of industries.

[b] Detail may not add to 100% because of rounding.

PRINCIPAL USERS OF SELECTED MATERIALS, EQUIPMENT, AND LABOR. Tables 7 through 9 portray the extent to which various industries account for the economy's use of particular metals, metalworking labor, and metalworking equipment. Table 7 shows industries which purchase (directly) the greatest tonnage of certain metals; Table 8, those which employ the greatest number of certain types of labor; Table 9, those which have most of certain types of metalworking machinery. The source for each table includes the usage of the principal metalworking industries plus other significant users of metals, metal-

TABLE 7
MAJOR CONSUMERS OF SELECTED METALS

Material	Industry		% of Total Consumption	
Total steel	3717	Motor vehicles and parts	18.9	
	3441	Structural and ornamental work	10.9	
	3411	Tin cans and other tinware	8.2	
	3463	Metal stampings	4.5	
	3443	Boiler shop products	4.5	
	3991	Iron and steel forgings	3.0	
	3444	Sheet metal work	2.3	
	3489	Wire work, n.e.c.	2.1	
	3494	Bolts, nuts, washers, and rivets	2.1	
	3522	Farm machinery (except tractors)	1.9	
	3439	Heating and cooking equipment, n.e.c.	1.8	
				60.2
Carbon steel, sheet and strip	3717	Motor vehicles and parts	37.4	
	3463	Metal stampings	8.7	
	3444	Sheet metal work	5.2	
	3491	Metal barrels, drums, and pails	3.9	
	3441	Structural and ornamental work	3.5	
	3585	Refrigeration machinery	3.5	
	3429	Hardware, n.e.c.	2.7	
				64.9
Carbon steel, plate	3443	Boiler shop products	31.4	
	3441	Structural and ornamental work	15.6	
	3731	Ship building and repairing	6.1	
	3531	Construction and mining machinery	5.6	
	3742	Railroad and street cars	4.6	
	3717	Motor vehicles and parts	4.5	
				67.8
Carbon steel, bars and bar shapes	3717	Motor vehicles and parts	18.1	
	3441	Structural and ornamental work	17.7	
	3391	Iron and steel	9.7	
	3522	Farm machinery (except tractors)	6.3	
	3494	Bolts, nuts, washers, and rivets	4.7	
	3495	Screw machine products	2.8	
	3521	Tractors	2.6	
	3531	Construction and mining machinery	1.8	
				63.7
Carbon steel, structural shapes	3441	Structural and ornamental work	66.6	
	3443	Boiler shop products	4.1	
	3742	Railroad and street cars	3.3	
	3731	Ship building and repairing	3.1	
				77.1

TABLE 7 (*continued*)
MAJOR CONSUMERS OF SELECTED METALS

Material	Industry		% of Total Consumption	
Iron castings	3717	Motor vehicles and parts	51.1	
	3521	Tractors	7.3	
	3541	Machine tools	3.7	
	3519	Internal combustion engines	3.3	
	3522	Farm machinery (except tractors)	3.1	
	3561	Pumps and compressors	2.9	
	3585	Refrigeration machinery	2.5	
	3614	Motors and generators	2.2	
				76.1
Aluminum	3721	Aircraft	11.3	
	3442	Metal doors, sash, and trim	13.3	
	3497	Metal foil	10.5	
	3463	Metal stampings	10.3	
	3444	Sheet metal work	8.0	
	3585	Refrigeration machinery	5.5	
	3729	Aircraft equipment, n.e.c.	5.4	
				64.3
Insulated wire (Cu)	3641	Engine electrical equipment	20.4	
	3614	Motors and generators	19.7	
	3615	Transformers	16.1	
	3661	Radios and related products	10.3	
	3664	Telephone and telegraph equipment	2.6	
				69.1
Other copper	3717	Motor vehicles and parts	27.3	
	3614	Motors and generators	11.6	
	3431	Plumbing fixtures and fittings	11.1	
	3585	Refrigeration machinery	10.5	
	3463	Metal stampings	8.9	
				69.4

Source: U. S. Census of Manufactures (1954), Vol. I, Table X.

working labor, and metalworking equipment. Because of data availability, Tables 7 and 8 report percentages for four-digit industries, while Table 9 reports percentages for three-digit industry groups.

According to Table 7, almost 19% of the steel consumed by metal fabricating industries went to the Motor Vehicles industry, almost 11% went to the Structural Work industry, and over 8% went to the Tin Cans and Other Tinware industry. Thus, out of 91 industries whose consumption was reported, 38% of steel of all types was directly purchased by three industries whose principal products were automobiles, metal work for buildings and bridges, and tin cans. In the case of the various specific types of steel a few industries generally accounted for 50 to 60% of direct purchases. These largest

TABLE 8

MAJOR EMPLOYERS OF SELECTED TYPES OF LABOR

Type of Labor	Industry		% of Employment	
Plate or structural fabrication	3441	Structural and ornamental products	22.1	
	3443	Boiler shop products	13.0	
	3731	Ship building and repairing	9.8	
	3531	Construction and mining machinery	5.6	
	3721	Aircraft	5.6	
	3563	Conveyors	3.0	
	3729	Aircraft equipment, n.e.c.	2.8	
	3444	Sheet-metal work	2.4	
				64.3
Stamping, blanking, forming, or drawing	3717	Motor vehicles and parts	17.1	
	3463	Metal stampings	15.9	
	3429	Hardware, n.e.c.	5.6	
	3411	Tin cans and other tinware	5.4	
	3439	Heating and cooking equipment, n.e.c.	3.2	
	3585	Refrigeration machinery	3.2	
	3721	Aircraft	3.2	
	3444	Sheet-metal work	3.0	
	19	Ordnance and accessories	2.1	
	3489	Wirework, n.e.c.	2.1	
	3521	Tractors	.5	
				61.3
Automatic screw machine department workers	3495	Screw machine products	25.2	
	3717	Motor vehicles and parts	8.4	
	3593	Ball and roller bearings	4.2	
	3722	Aircraft engines	4.1	
	3494	Bolts, nuts, washers, and rivets	3.8	
	3591	Valves and fittings, except plumbing	3.5	
	3571	Computing and related machines	3.3	
	3599	Machine shops	2.6	
	3641	Engine electrical equipment	2.4	
	3614	Motors and generators	2.2	
	3729	Aircraft equipment, n.e.c.	2.0	
				61.7
Machine shop workers	3717	Motor vehicles and parts	12.1	
	3599	Machine shops	5.9	
	3541	Machine tools	4.8	
	3729	Aircraft equipment, n.e.c.	4.3	
	3722	Aircraft engines	3.7	
	19	Ordnance and accessories	3.6	
	3542	Metal-working machinery	3.0	
	3591	Valves and fittings, except plumbing	2.7	
	3519	Internal combustion engines	2.4	
	3721	Aircraft	2.4	
	3521	Tractors	2.4	
	3531	Construction and mining machinery	2.2	

TABLE 8 (*continued*)
MAJOR EMPLOYERS OF SELECTED TYPES OF LABOR

Type of Labor		Industry	% of Employment
Machine shop workers (*continued*)	3614	Motors and generators	2.1
	3559	Special-industry machinery, n.e.c.	2.0
	3545	Metalworking machinery attachments	2.0
	3544	Special dies and tools	1.8
	3552	Textile machinery	1.7
	3532	Oil-field machinery and tools	1.5
			60.6

Source: U. S. Census of Manufactures (1954), Vol. I, Table XII.

users were not the same for each type of steel, nor were they necessarily among the largest users of steel in total. For example, over 50% of carbon steel plate was accounted for by three industries—Boiler Shop Products, Structural and Ornamental Work, and Ship Building. In particular, Boiler Shop Products—which accounted for less than 5% of the total steel usage—purchased over 30% of the carbon steel plate. Other forms of steel which could be accounted for in large part by a very small number of using industries included Tin Plate (not shown in Table 7), most of which went to the Tin Can industry, and Structural Shapes, two-thirds of which went to Structural Work. Because of its size, the Motor Vehicles industry frequently appears among the larger users of various kinds and forms of metals. In 1954 it directly purchased 37% of the carbon steel sheet and strip, 18% of the bars and bar shapes, 51% of the commercially sold iron castings, and 27% of the copper other than copper wire. Among the large consumers of aluminum, the aircraft industry of 1954 had to share honors with two less-glamorous industries: Metal Doors, Sash and Trim; and Metal Foil. Over 65% of the insulated copper wire was accounted for by four industries: Engine Electrical Equipment, Motors and Generators, Transformers, and Radios and Related Products.

Turning to the four types of labor tabulated in Table 8, we find that three industries (Structural Work, Boiler Shop Products, and Ships and Boats) accounted for 45% of the Plate or Structural Fabrication Workers. The other three types of workers reported here were less concentrated. In the case of machine shop workers it required 10 industries to account for 45% of the employment, and 18 to account for 60%.

Table 9 shows the percentage breakdown, mostly by three-digit industry group, for a small assortment of equipment types. These equipment types vary noticeably in their degree of concentration. In the case of lathes (all types) 10 industry groups are required to account for 60% of the usage. In the case of gear grinders, three industry groups accounted for over 70% of the usage. Industries which appear prominently in Table 9 (as significant users of these particular kinds of metalworking and related equipment) include

TABLE 9

MAJOR USERS OF SELECTED TYPES OF METALWORKING AND RELATED EQUIPMENT

Equipment	Industry		Per Cent of Stock	
All boring	354	Metalworking machinery	11.6	
	371	Motor vehicles and equipment	10.3	
	355	Special-industry machinery, n.e.c.	7.5	
	356	General industrial machinery	7.5	
	19	Ordnance and accessories	6.9	
	372	Aircraft and parts	6.1	
	359	Miscellaneous machinery parts	5.2	
	351	Engines and turbines	5.0	
				60.1
Precision boring	371	Motor vehicles and equipment	26.2	
	351	Engines and turbines	15.0	
	372	Aircraft and parts	11.0	
	354	Metalworking machinery	7.0	
	358	Service and household machines	6.3	
				65.5
All gear making	371	Motor vehicles and equipment	21.2	
	356	General industrial machinery	19.2	
	352	Tractors and farm machinery	7.1	
	387	Watches and clocks	5.5	
	355	Special-industry machinery, n.e.c.	4.9	
	351	Engines and turbines	4.8	
	372	Aircraft and parts	4.4	
				67.1
Gear grinding	372	Aircraft and parts	35.3	
	354	Metalworking machinery	19.9	
	356	General industrial machinery	17.3	
	19	Ordnance and accessories	6.6	
				79.1
Honing and lapping (all types)	359	Miscellaneous machinery parts	17.1	
	371	Motor vehicles and equipment	12.8	
	354	Metalworking machinery	12.1	
	372	Aircraft and parts	6.5	
	19	Ordnance and accessories	5.6	
	356	General industrial machinery	5.1	
	355	Special-industry machinery, n.e.c.	3.4	
				62.6
Lathes (all types)	354	Metalworking machinery	8.2	
	349	Metal products, n.e.c.	7.4	
	355	Special-industry machinery, n.e.c.	7.2	
	359	Miscellaneous machinery parts	7.1	
	19	Ordnance and accessories	6.5	
	371	Motor vehicles and equipment	6.4	
	356	General industrial machinery	6.3	

TABLE 9 (*continued*)

MAJOR USERS OF SELECTED TYPES OF METALWORKING AND RELATED EQUIPMENT

Equipment		Industry	Per Cent of Stock	
Lathes (all types) (*continued*)	361	Electrical industrial apparatus	5.2	
	352	Tractors and farm machinery	3.1	
	366	Communication equipment	3.1	
				60.5
Milling (all types)	354	Metalworking machinery	11.8	
	19	Ordnance and accessories	11.7	
	355	Special-industry machinery, n.e.c.	8.0	
	371	Motor vehicles and equipment	6.4	
	372	Aircraft and parts	6.3	
	359	Miscellaneous machinery parts	4.1	
	349	Metal products, n.e.c.	4.0	
	356	General industrial machinery	4.0	
	366	Communication equipment	3.6	
	357	Office and store machines	3.6	
				63.5
Milling, die sinkers, and duplicators	372	Aircraft and parts	22.4	
	19	Ordnance and accessories	14.9	
	3391	Iron and steel forgings	8.3	
	371	Motor vehicles and equipment	6.7	
	336	Nonferrous foundries	5.2	
	354	Metalworking machinery	4.9	
				62.4
Planers	354	Metalworking machinery	20.9	
	355	Special-industry machinery, n.e.c.	15.7	
	373	Ship building and repairing	5.6	
	356	General industrial machinery	5.3	
	359	Miscellaneous machinery parts	4.9	
	19	Ordnance and accessories	4.8	
	344	Structural metal products	4.5	
				61.7
Hydraulic presses	371	Motor vehicles and equipment	10.8	
	361	Electrical industrial apparatus	10.2	
	352	Tractors and farm machinery	6.3	
	19	Ordnance and accessories	5.8	
	344	Structural metal products	5.0	
	356	General industrial machinery	4.7	
	355	Special-industry machinery, n.e.c.	4.3	
	359	Miscellaneous machinery parts	3.7	
	3393	Welded and heavy-riveted pipe	3.7	
	354	Metalworking machinery	3.7	
	372	Aircraft and parts	3.6	
				61.8

TABLE 9 (*continued*)

MAJOR USERS OF SELECTED TYPES OF METALWORKING AND RELATED EQUIPMENT

Equipment		Industry	Per Cent of Stock
Overhead cranes	344	Structural metal products	15.0
	354	Metalworking machinery	8.9
	372	Aircraft and parts	7.3
	355	Special-industry machinery, n.e.c.	6.9
	332	Iron and steel foundries	6.6
	373	Ships and boats	6.3
	356	General industrial machinery	6.0
	361	Electrical industrial apparatus	5.9
			62.9

Source: American Machinist (1953).

Motor Vehicles, Metalworking Machinery, Aircraft, Ordnance and Accessories, Special Industry Machinery, and General Industry Machinery.

REQUIREMENTS ANALYSIS. Tables 7 through 9 suggest industries or combinations of industries whose expansion might cause particular resources to become bottlenecks. The simultaneous expansion of the Boiler Shop Products and Structural Work industries would substantially increase the economy's requirements for carbon steel plate. The expansion of the Aircraft, Metalworking Machinery, and General Industrial Machinery industries would probably raise problems as to the availability of gear grinding equipment. But gear grinders are but one of many kinds of equipment used in various proportions by the various metalworking industries. To check the demand versus the supply of such potential bottlenecks we need a systematic way of determining which resources would be particularly taxed by a proposed program, taking into account the total requirements (direct and indirect) that end products generate. Such is the objective of the metalworking requirements analysis discussed in the next chapter.

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CHAPTER 11

METALWORKING REQUIREMENTS ANALYSIS

Harry M. Markowitz and Alan J. Rowe

THE ANALYSIS OF METALWORKING CAPABILITIES

The preceding chapter briefly described the resources, processes, and industrial structure of the metalworking industries. The remaining chapters of Part IV are concerned with techniques for analyzing the capabilities of these industries. The present section surveys the techniques presented, stating (in our view) what they accomplish and what remains to be incorporated in more refined analyses. The remainder of the part, beyond the present brief survey, is primarily devoted to technical discussions of concepts, sources of data, parameter estimation, and some problems of application.

Our first concern is with *requirements analysis*. In the following sections of the present chapter, and in Tables A through C of Chapter 14, usage figures are presented or cited for metalworking men, machines, and materials. Various topics are discussed concerning the application of such data.

As noted in Chapter 2 of Part I of this monograph, a requirements analysis assumes fixed inputs per unit output. It points out possible trouble areas where, if average current practice is followed, the demand for particular resources will exceed their supply. Since substitution possibilities exist in fact, some or all of these possible trouble areas may be "false alarms." Perhaps by substituting plentiful for scarce resources shortages can be alleviated and the desired program attained. Judgment and supplementary data can be used to modify the initial results of a requirements analysis, thus introducing substitution possibilities on a limited, ad hoc, basis.

The neglect of substitution possibilities by a requirements analysis tends to understate the capabilities of an economy. The opposite error—the overstatement of capabilities—can result when the classification system used in a requirements analysis is too coarse. If two different types of sizes of, e.g., machine tools, are aggregated into the same category they are assumed to be substitutes in all uses. This would cause difficulty if in fact they were not close substitutes and, for the program whose feasibility was in question, the use of one would decrease and the other increase substantially. The coarser the aggregation system the more the danger of overestimating capabilities. The finer the aggregation system, on the other hand, the greater the danger that the analysis will underestimate capabilities due to its neglect of substitution possibilities. This "dilemma of requirements analysis" is one of the principle reasons that, as emphasized earlier in this monograph, requirements analysis cannot be used "automatically" separated from judgment.

Despite limitations, a requirements analysis is substantially better than no analysis at all. With respect to industrial capabilities in general, it can serve as a stop-gap technique where more thorough analyses have not yet been developed. In metalworking in particular, where our analysis of substitution possibilities is still quite rudimentary, it must be relied upon heavily.

There are a variety of ways by which plentiful resources can be substituted for scarce ones in the metalworking industries. The type of substitution possibility discussed in Chapter 12 may be referred to as the "one-for-one" substitution of equipment. With such substitutions one of some type of machine is substituted for one of some other type of machine in the performance of a specified task. A different amount of time may be required by the substitute machine than was required by the original machine to perform the task. Thus the phrase "one-for-one" is not intended to imply that the two machines perform the task equally well. Rather it denotes the substitution of *one* machine of a particular type *for one* machine of a different type in the performance of a task which they both can accomplish, albeit at different rates.

Towards the analysis of one-for-one substitution possibilities among machine tools, Chapter 12 proposes a classification of machine tools and a classification of the tasks which machine tools perform, discusses problems of aggregation in terms of standard machines and standard tasks, presents a table of rates of productivity for standard machines performing standard tasks, describes two methods by which "task requirements" by industry or product line can be obtained at a reasonable cost, and discusses ways in which the productivity figures presented here plus tasks requirements can be incorporated into the analysis of the capabilities of the metalworking industries.

This type of analysis can be applied to some, if not all, of the remaining types of shop equipment. In particular a proposed classification of machines and tasks for forge and press equipment is to be found in Rowe (1955).

Since estimates of productivities and task requirements are subject to error, it is possible for the substitution analysis to overestimate the extent to which one machine can serve in place of another. However, if the task requirements are formed "conservatively"—in a sense spelled out in Chapter 12—we virtually assure that the analysis will generally understate, rather than overstate, substitution possibilities. It will not understate such possibilities as much as does a requirements analysis, however, since the latter ignores all such possibilities. Thus by estimating substitution possibilities conservatively—in the sense to be defined—one can make the substitution analysis a step in the right direction, adding substitution possibilities to requirements analysis with small danger of radically overstating such possibilities.

In addition to the one-for-one substitution of machine for machine there is a large (yet to be explored and structured) class of substitution possibilities involving combinations of men, machines, and/or materials. Examples of such and a discussion of the possibilities of incorporating these into process analysis models is presented by Thomas Vietorisz in Chapter 15.

Some types of such substitution possibilities seem to pose no inordinate problems of characterization and analysis. Others, such as the functional redesign of products to conserve scarce resources, seem to be beyond systematic characterization at present. Such unsystematizable (or just unsystemized) substitution possibilities would have to be handled on an ad hoc basis, as in a requirements analysis. Certain technical details concerning ad hoc procedure would be different when substitution possibilities were included. In the first place, the "requirements" might include demands for "tasks" or other abstract entities defined by the formalized substitution analysis. In the second place the extent to which any proposed alternative should be adopted would generally be determined by the formal analysis. The basic principle, however, is the same: Formal analysis shows consequences of such methods of production as are told to it (including alternatives where specified). On the basis of the results of initial runs, the analyst may introduce alternate methods. In the case of a substitution analysis such alternates could have been introduced from the start; but technology is much too rich, its potentials much too great, to permit the complete ennumeration of all potentially valuable alternatives.

The linear programming analysis of substitution possibilities, as presented in Chapter 12 and in the comments by T. Vietorisz, has difficulties in at least two areas. First, they have difficulty at present in characterizing indirect substitution via queueing phenomena. (Recall the discussion of the ratio of men to machines in the preceding chapter, and how this ratio can be varied because of queueing phenomena). Second they have difficulty characterizing the dynamics of transition (recall the discussion of resources of transition). The possibility of using simulation techniques to handle such areas is discussed in Chapter 13.

The problem of analyzing the capabilities of the metalworking industries, it seems to us, is neither hopeless nor solved. Requirements analysis is substantially better than no analysis at all. To a certain extent some of the weaknesses of requirements analysis can be alleviated by the data and techniques discussed in Chapter 12. To a much larger extent the problems of characterizing metalworking capabilities remain to be explored.

USAGE STATISTICS

Production requirements at a national level, for thousands of products classified into dozens of categories, are typically based on historical usage. These usage figures may be supplemented by engineering or other data, but they remain the backbone of the analysis of inputs per unit output. The present section discusses sources of such data for men, machines, and metals used by the metalworking industries.

A *Statistical Supplement on Metalworking* is available upon request from the Cowles Foundation for Research in Economics at Yale University, New Haven, Connecticut. The seventy pages of data contained in this *Supple-*

ment were omitted from the present volume because of space and cost considerations. For illustrative purposes, extracts from the tables of the *Supplement* are reproduced in Chapter 14. The letters, A through F, assigned to the tables in Chapter 14 are the same as those in the *Supplement*. Hence these discussions refer to the tables both as they are reproduced in the *Supplement* and as illustrated in Chapter 14.

Table C illustrated in Chapter 14 presents the number of various types of machine tools, metal forming machines, and other shop equipment in metalworking industries as of, roughly, the middle of 1953. This table is based on information collected by the *American Machinist* and summarized in various ways in *The Seventh American Machinist Inventory of Metalworking Production Equipment*.[1]

In Table C names of equipment types have been abbreviated, and names of industries have been replaced by their SIC numbers. Table B (extract in Chapter 14) presents the full titles of the equipment types, in the order of their appearance in Table C. Table A presents the industry titles corresponding to the SIC numbers. In addition, Table A shows the extent to which establishments responded to the *American Machinist's* questionnaire. Specifically, it shows the response percentage,

$$\frac{\text{employees in responding establishments}}{\text{employees in all establishments}} \times 100,$$

for each industry covered.

The *Census of Manufactures* periodically (e.g., 1947, 1954, 1958) collects comprehensive data on industrial activity and usage. In this essay we generally refer to the 1954 Census, partly because of its proximity to our mid-1953 data on equipment, and partly because only a small portion of the Census data for 1958 was available at the time this essay was written. The voluminous *Census of Manufactures'* information on men, materials, and industrial activity cited below has not been reproduced here, since these volumes are (or should be) readily available to any serious student of this subject.

The *Census of Manufactures* for 1954 shows the weight and value of metals consumed by four-digit industries.[2] The classification of materials distinguishes:

- –carbon steel: bars and bar shapes, sheet and strip, structural shapes, plate, wire and wire products, tin plate and other forms,
- –alloy steel (except stainless): bars, other shapes,
- –stainless steel: all shapes and forms,
- –castings: iron, steel, aluminum and aluminum base, cooper and copper base,
- –aluminum and aluminum-base alloy shapes and forms,
- –copper and copper-base alloy: bare wire, insulated wire, other shapes and forms.

The consumption of these materials is shown for the principal metalworking industry groups (19, 34–37), for furniture and fixtures (25), and for selected

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[1] *American Machinist*, November 1953, page A-2. We are indebted to the *American Machinist* for making this information available and for permission to publish it here.

[2] Vol. I, Table X.

industries in the following groups: 33 Primary Metals, 38 Instruments and Related Products, and 39 Miscellaneous Manufactures.

The *Census of Manufactures* shows the number of production workers in the following types of shops:[3]

Foundry
Die casting
Forging
Electroplating
Galvanizing and other hot dip coating
Heat treating and annealing
Tool and die shops
Foundry patterns
Plate or structural fabrication
Stamping, blanking, forming or drawing
Automatic screw machine departments
Machine shops

This information is provided for four-digit industries in major groups 19, and 33–37.

Among the vast amount of other data published by the *Census of Manufactures* is the number of units shipped and the value of shipments by various types of metalworking and other equipment.[4] From this, average replacement values may be computed.

PROBLEMS OF APPLICATION

Table 1 illustrates how usage information, such as that in Table C of Chapter 14, can be used with a minimum amount of analysis or supplementary data. Table 1 is extracted from a report by Markowitz (1954) based on the *American Machinist* survey for 1949 (five years earlier than the survey upon which Table C is based). The published statistics from the 1949 survey include detail on the geographical distribution of metalworking equipment, as well as information as to their use by industries. This information can answer certain questions of the following sort: Suppose that the metalworking equipment of a large number of counties in major metropolitan areas were no longer available (e.g., as if destroyed by nuclear attack), how would the remaining metalworking equipment compare with that used in strategic industries?

For a few types of equipment covered in the above references, Table 1 shows the total number of machines in the United States, the number outside of 93 metropolitan counties,[5] the number in each of four industries, the total for the four industries, and, finally, the ratio of the number outside the 93 counties to

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[3] Vol. I, Table XII.

[4] Vol. II, Section 35B, Table 6A.

[5] For a list of the 93 counties and a comparison with the 53 standard metropolitan areas see Markowitz (1954), p. 10.

TABLE 1

THE DISTRIBUTION OF CERTAIN METALWORKING MACHINES (1949)

				No. in U. S. in Selected Industries					
Selected Equipment Categories	(1) Total U. S.	(2) No. Outside 93 Counties	(3) % Outside 93 Counties	(4) Ordnance	(5) Ships and Boats	(6) Metal-working Equip. and Access.	(7) Aircraft and Parts	(8) Total Cols. 4–7	(9) Col. 2 Divided by Col. 8
Total All Equipment	3,118,342	1,237,581	40	110,958	78,651	201,373	80,106	470,984	2.64
Total Machine Tools	1,761,804	692,058	39	74,386	29,202	147,327	54,781	305,696	2.27
Total Metal Forming Machinery	471,257	164,838	36	7,532	8,778	7,193	8,013	31,516	5.34
Total Other Shop Equip.	885,281	380,685	43	29,037	40,671	46,853	17,312	133,772	2.85
Machine Tools									
Drilling Machines Total	361,935	143,516	40	11,445	4,870	20,706	11,135	48,156	2.98
Upright Single-Spindle	144,230	59,280	41	1,971	2,339	7,826	3,202	15,338	3.86
Deep-Hole Drills	2,772	1,142	41	840	–	168	143	1,151	.99
Gear Cutting and Finishing Total	44,216	18,094	41	927	242	4,822	1,786	7,777	2.35
Hobbing	15,867	6,387	40	333	100	1,516	312	2,261	2.82
Gear Grinders	1,729	481	28	45	7	399	455	906	.53
Lathes Total	396,464	153,198	39	16,236	5,910	31,282	8,783	62,211	2.46
Light Duty	71,360	27,081	38	3,388	592	5,216	959	10,155	2.67
Heavy Duty	131,198	54,041	41	6,812	4,014	14,013	3,140	27,979	1.93
Auto Screw	66,613	18,741	28	1,834	93	2,776	772	5,475	3.42
Gun and Shaft	1,286	842	65	499	14	58	4	575	1.46
Honing and Lapping Total	17,170	5,348	31	746	71	1,455	742	3,014	1.77
Honing	7,933	3,078	39	469	43	660	345	1,517	2.03
External Lapping	2,857	898	31	178	21	354	165	718	1.25
Rifle Working Machine	627	384	58	598	–	3	–	601	.64

TABLE 1 (*continued*)
THE DISTRIBUTION OF CERTAIN METALWORKING MACHINES (1949)

				No. in U. S. in Selected Industries					
Selected Equipment Categories	(1) Total U. S.	(2) No. Outside 93 Counties	(3) % Outside 93 Counties	(4) Ordnance	(5) Ships and Boats	(6) Metal-working Equip. and Access.	(7) Aircraft and Parts	(8) Total Cols. 4–7	(9) Col. 2 Divided by Col. 8
Machine Tools (*continued*)									
Threading (except pipe) Total	54,029	19,675	36	1,703	321	2,853	1,114	5,991	3.28
Tapping	22,689	7,813	34	632	50	547	428	1,657	4.72
Thread Grinders	3,008	1,049	35	196	21	781	320	1,318	.80
Metal Forming Equipment									
Hydraulic Presses Total	32,159	14,739	46	1,328	1,418	1,220	731	4,697	3.14
Mechanical Presses Total	230,392	73,757	57	2,348	578	2,844	1,404	7,174	10.28

the number used in the four industries. This ratio varies substantially by type of equipment. For example, there are 10 times as many mechanical presses outside the 93 counties than are used by the specified industries; whereas the number of gear grinders outside the counties is only one-half that used by the industries. Thus if the objective of the economy had been to support these industries. Thus if the objective of the economy had the help of the 93 counties, more than enough mechanical presses would have been available to accomplish simple sheet metal operations, but the grinding of hardened, high tolerance gears for the machine tool and aircraft industries would probably have faced a serious equipment bottleneck.

An analysis of usage data usually leaves many questions unanswered concerning any specified practical problem. In the case of equipment availability after nuclear attack, for example, there are problems of goals and organization which transcend the counting of machines. What would, or should, be the objectives of any economy after attack? Would there be sufficient social, political and economic organization to redirect the economy (e.g., in a manner similar to the complex of contracts and subcontracts which reorganized the economy for World War II)? With problems of resource availability at a national level generally (including problems of requirements for economic development as well as those of military support), social, political, and economic considerations affect the significance of answers to the purely technological question.

Even if we restrict ourselves to the technological aspects of problems, usage data does not provide a complete answer. For example, there is the possibility of substituting one machine for another: Perhaps some of the shortages indicated by Table 1 could be covered by machines which would be in more plentiful supply. In the next chapter we present data and analysis techniques which deal with this possibility, treating the substitution of one machine for another on a formal (rather than on an ad hoc) basis. Other technological considerations, however, such as some suggested by T. Vietorisz in Chapter 15, have not been formalized by the authors of this chapter.

Restricting ourselves still further, to the potentials of requirements analysis only, the analysis of Table 1 is still not all that could be desired. For one thing, in some places the classification of equipment is too coarse. Further information concerning the use and location of certain large machines would be particularly valuable. For another thing, Table 1 shows direct usage only. It tells us how many presses were used directly by the Aircraft industries, but does not tell us how many were used indirectly, via purchases from other industries. The problem of estimating total requirements from direct requirements will be discussed at length in the next section.

Despite all such drawbacks, the simple requirements tabulation can nevertheless contribute to our understanding and/or further investigation of resource availabilities under specified conditions. The 10-to-1 ratio in the case of mechanical presses versus the 1-to-2 ratio in the case of gear grinders gives us at least a sharper notion for orders of magnitude. The object of more refined analysis is to further sharpen such notions by dealing explicitly with considerations which can be handled formally, thus relieving some of the burden placed on judgment.

ESTIMATION OF TOTAL USAGES

Table C of Chapter 14, and the other sources of usage data cited above, show direct usages only. They show how much carbon steel sheet was directly purchased by the Motor Vehicles industry, for example, and how much was directly purchased by the Metal Stampings industry. They do not, however, show how much of the sheet steel purchased by Metal Stampings was fabricated into parts for motor vehicles. If all fabrication and assembly operations for end products were performed in establishments specialized to the particular product line, then direct usage would equal total usage. But in fact some requirements are parceled out to other industries, e.g., by the purchase of components such as motors and bearings, or by the contracting of machine shop, stamping, forging, foundry, or other work. The problem in estimating total usages is to piece back together again these parceled-out requirements.

A possible approach to the estimation of total requirements would be to use an input-output inverse to go from final demands to industry outputs, then use direct usage tables to obtain total requirements by type of man, machine, and material. Unfortunately, this approach is subject to the serious inaccuracies of input-output analysis discussed in Chapter 3. We recommend that the reader briefly review that discussion, since the proposal presented here would be needlessly complex if we could rely on input-output analysis for this purpose.

The present section proposes a method for reconstructing total usages. We cannot demonstrate the accuracy of this method (in the way one demonstrates the validity of a linear programming algortim or other deductive process). Perhaps further reflection will show that the proposed procedure, like the classic input-output approach, is subject to an error of first magnitude. Or perhaps the carrying out and spot checking of the procedure will show that it provides a reasonable reconstruction of total usages. This remains to be seen.

The proposed procedure requires three types of information for the period of time whose total usages are to be reconstructed. The first type of information is direct usage figures such as cited previously. In referring to such usage figures we shall let

V_{ij}^M = the value of the *i*th *material* directly purchased by industry *j*,
V_{ij}^L = the value of the *i*th type of *labor* directly employed by industry *j*, and
V_{ij}^E = the value of the *i*th type of *equipment* directly used.

A second type of information used by the procedure is the value of sales from each metalworking industry to each other metalworking industry (i.e., the metalworking portion of an input-output analysis, showing total "interindustry flow"). In our discussion we shall let

P_{jk} = the value of sales from metalworking industry j to metalworking industry k.

We shall also let

P_{jo} = the sales by j to either final demand or to industries outside of metalworking.

We shall refer to P_{j0} as the value of sales to final demand, although it differs from final demand in that it includes the value of purchases by nonmetalworking industrial sectors. A major component of the latter consists of the structural and other metal products purchased by the construction industries. A minor component are metal bits and pieces purchased by other industries to be incorporated into rubber, leather, and plastic products. Still another component is metal work purchased by various industries for maintenance purposes. All of these will be considered here as part of the final demand for metal products, along with the usual components of final demand such as equipment purchases on capital account, and the purchase of consumer durables by households.

In addition to direct usages and interindustry sales, the procedure can incorporate other data, such as engineering estimates and special survey data. The procedure does not require such data, but to achieve greater accuracy at critical points, it allows for the inclusion of these supplementary sources. The form in which this third type of information enters the analysis will be discussed later.

For expository purposes we will first describe the procedure under certain simplifying assumptions concerning the nature of interindustry sales. This will serve to illustrate the basic steps in an uncluttered case. After this we will turn to the general procedure as applicable in fact. Our initial assumptions are these:

1. All metalworking industries are either "end item industries" or "intermediate industries."
2. All interindustry sales between metalworking industries are from intermediate industries to end item industries.
3. End item industries make only their primary products.

We do not exclude the possibility of sales from intermediate industries to final demand, as in the case of a machine shop fabricating parts for maintenance.

Table 2 summarizes the procedure under our initial assumptions. We will first note briefly the objective of each step, to see the crucial steps in context, and then return to review these crucial steps in detail. The procedure makes three separate passes: one to estimate total material usages, another to estimate total labor usages, and a third to estimate total equipment usages. These passes are independent and may be made in any order. Table 2 shows

TABLE 2
OUTLINE OF PROCEDURE FOR ESTIMATING TOTAL USAGES (FOR SPECIAL CASE)

A. To impute indirect *materials* usages:
  1. For each intermediate industry (j_o) do the following:
    a. For each end item industry (k) estimate the *total value* T_{j_ok} of materials purchased by j_o to be processed for k.
    b. For each material (i) form *preliminary estimates* E_{ij_ok} of the amount of i purchased by j_o to be processed into parts for k.
    c. Form *final estimates* V_{ij_ok} (of the amount of i purchased by j_o to be processed into parts for k) so that

$$\sum_k V_{ij_ok} = V_{ij_o}, \qquad \text{for all } i;$$

$$\sum_i V_{ij_ok} = T_{j_ok}, \qquad \text{for all } k;$$

$$V_{ij_ok} \geq 0, \qquad \text{for all } i \text{ and } k;$$

and so that, subject to these constraints, the final estimates are "as close as possible" to the initial estimates.

  2. After 1 has been repeated for each intermediate industry, sum indirect usages, add these to direct usage to form total usage, and divide by final demand to obtain a_{ik} = estimates of total value of material i required per unit of final demand.

B. Repeat 1 and 2 for *labor* and for *equipment.*

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the procedure starting with the pass for materials. During this pass only the direct usages V_{ij}^M are needed, not V_{ij}^L or V_{ij}^E. Hence, in our discussion of this pass we may write V_{ij} for V_{ij}^M without ambiguity.

Within the pass for materials the procedure looks at each intermediate industry in turn. Let j_o represent the particular intermediate industry currently being processed. Step 1, parts a, b, and c, takes the amounts V_{ij_o} of the various materials purchased by j_o and breaks these into their components

$$V_{ij_o0}, \quad V_{ij_o1}, \quad V_{ij_o2}, \quad \ldots$$

where V_{ij_o0} is the value of the ith material used by j_o to fabricate parts or products for final demand;

V_{ji_o1} is the value of the ith material used by j_o to fabricate parts for industry 1;

V_{ij_o2} is the value of the ith material used by j_o to fabricate parts for industry 2; etc.

Once the V_{ijk} are estimated for all i, j, and k, the total usages follow readily.

Step a estimates the total value T_{j_ok} of materials used by j_o on behalf of k. Later, when we estimate the V_{ij_ok}, we will insist that

$$\sum_i V_{ij_ok} = T_{j_ok}.$$

In step a, however, we do not estimate the breakdown by type of industry, but only a total value (T_{j_ok}) of the material content of the sales from j_o to each end item industry k, and to final demand ($k = 0$).

Step b forms preliminary estimates (E_{ij_ok}) of the V_{ij_ok}. Three methods for forming these preliminary estimates are discussed below, the choice of method depending on the nature of industry j_o and the degree of accuracy desired.

Step c forms final estimates V_{ij_ok} which are as close as possible to the preliminary estimates E_{ij_ok} while satisfying the following three conditions:

For each material (i) purchased by j_o, the sum of the uses (V_{ij_ok}) of this material must equal the amount purchased (V_{ij_o}).

For each industry k that buys from j_o the value $\left(\sum_i V_{ij_ok}\right)$ of the material content of its purchases must equal the total value (T_{j_ok}) previously estimated.

The V_{ij_ok} are nonnegative.

The exact procedure in step c depends on the interpretation of "V_{ij_ok} as close as possible to E_{ijk_o}." Several criteria of "closeness" discussed below give rise to linear or quadratic programming problems at this point.

Having derived the V_{ij_ok} for industry j_o we move on to the next intermediate industry and repeat the process. When this is done for each intermediate industry, the indirect usages are added to the direct usages of each end item industry to form total usages. These, in turn, are divided by final demands for the period to form estimates of total requirements per unit of final demand. In a similar manner, the entire process is repeated to obtain estimates of total requirement coefficients for men and equipment.

This, in brief, is the procedure under our initial simplifying assumptions. Steps 1a through 1c are the critical ones. The other steps do simple bookkeeping with the estimates produced by 1a through 1c. Let us consider the critical steps once again, in greater detail, this time starting with c and working backwards to a.

Step c estimates the V_{ij_ok}'s so as to be close to the E_{ij_ok}'s, while satisfying the relationships

$$\sum_{k=0}^{N_I} V_{ij_ok} = V_{ij_o}, \quad \text{for all materials } i; \tag{1}$$

$$\sum_{i=1}^{N_M} V_{ij_ok} = T_{j_ok}, \quad \text{for all end item industries } k; \tag{2}$$

$$V_{ij_ok} \geq 0, \quad \text{for all } i \text{ and } k \tag{3}$$

(where N_M and N_I are, respectively, the number of kinds of materials and the number of industries). Constraints (1) insure that sources equal uses for each material purchased by j_o; constraints (2) insure that each purchasing industry receives the value of materials previously estimated (in step a); and constraints (3) tell the computer that negative V_{ij_ok} are not considered meaningful.

Several criteria can be suggested for measuring how close the V's are to the E's. One measure is the sum of the absolute deviations between the V_{ij_ok}'s and the corresponding E_{ij_ok}'s. In this case the problem of finding V's as close to their E's as possible requires us to

$$\text{minimize} \quad \sum_{i=1}^{N_M} \sum_{k=0}^{N_I} |V_{ij_ok} - E_{ij_ok}| \tag{4'}$$

subject to constraints 1, 2, and 3. Another measure of closeness is the sum of absolute values of the *percentage* deviations. This would require us to

$$\text{minimize} \quad \sum_i \sum_k \left| \frac{V_{ij_ok} - E_{ij_ok}}{E_{ij_ok}} \right| = \sum \sum \left| \frac{V_{ij_ok}}{E_{ij_ok}} - 1 \right| \tag{4''}$$

(where the summation includes all i, k with $E_{ij_ok} > 0$.) Another is the maximum percentage deviation:

$$\text{minimize maximum} \quad \left| \frac{V_{ij_ok} - E_{ij_ok}}{E_{ij_ok}} \right|, \qquad \text{for } E_{ij_ok} > 0, \tag{4'''}$$

or a quadratic criterion of closeness of fit, such as:

$$\text{minimize} \quad \sum_i \sum_k \left(\frac{V_{ij_ok} - E_{ij_ok}}{E_{ij_ok}} \right)^2, \qquad \text{summation as in (4'').} \tag{4$^{\text{iv}}$}$$

The finding of V's to minimize the (4′), (4″), or (4‴), can be expressed as a linear programming problem and solved with the standard simplex method. The use of (4^{iv}) as a criterion gives rise to a quadratic programming problem. Generally quadratic programming problems require several times as much computer time as linear programming problems with the same number of equations and variables. However, in the case of the quadratic in (4^{iv}) (which can be expressed as linear terms plus a weighted sum of squares) a special algorithm can minimize this function in little more time than its linear programming counterpart would require. The use of efficient computing procedures is important since the criteria minimization calculation must be performed once (under our present simplified assumptions) for each intermediate industry j_o, and may have to be performed more than once per industry in the general case. The cost of computation for any of these criteria—involving, in the case of large analyses, a number of hours even on the fastest available computers—is substantial, albeit still a small fraction of the labor and data processing costs of collecting reliable usage figures.

A criterion of closeness may be thought of as an instruction to the computer concerning when to let V_{ij_ok} be further from E_{ij_ok} so that $V_{i^*j_ok^*}$ can be brought closer to $E_{i^*j_ok^*}$. If we choose criterion (4″), for example, we are instructing the computer that an increase in

$$\left| \frac{V_{ij_ok}}{E_{ij_ok}} - 1 \right|$$

is worthwhile as long as it permits a greater amount of reduction in

$$\left|\frac{V_{i^*j_0k^*}}{E_{i^*j_0k^*}} - 1\right|.$$

Criterion $(4'')$ will make such a trade even if the latter difference is already substantially smaller than the former. Criterion $(4''')$, on the other hand, instructs the computer that trading an increase in

$$\left|\frac{V_{ij_0k}}{E_{ij_0k}} - 1\right|$$

for a decrease in

$$\left|\frac{V_{i^*j_0k^*}}{E_{i^*j_0k^*}} - 1\right|$$

is a matter of indifference unless one of these is the maximum deviation. Only in the latter case does the criterion take an interest in the trade, in favor of reducing the largest difference at any cost short of letting another difference become larger. The quadratic criteria (4^{iv}) lies between $(4'')$ and $(4''')$ in that it considers both the magnitudes of the two differences and the rate at which an increase in one can be traded for a decrease in the other. It acts on the rule that the worth of obtaining a decrease (or, conversely, the gain asked in compensation for an increase) is proportional to the difference being changed. Thus it is like $(4'')$ in the case of equal differences, and close to $(4''')$ in the case of extremely unequal differences.

A criterion of closeness may also be thought of as a hypothesis concerning how initial estimates are most likely to be in error. The choice of such a hypothesis—the selection of a closeness criteria—should depend on the manner in which the E_{ij_0k}'s are estimated. In the discussion of step b we suggest three different criteria for step c, depending on the derivation of the initial estimates. Each of these criteria, however, is a form of quadratic. The quadratic arises naturally since:

i. if we assume that certain components of our errors of estimate are normally distributed, then quadratic criteria provided maximum likelihood estimates of the V's; or

ii. even if we are not quite willing to assume normality, the quadratic criteria seems (to us at least) to provide more reasonable rules for fitting final estimates (in the sense discussed in the last paragraph) than do other easily optimizable measures of closeness.

This area could probably benefit much from statistical investigations related to optimum estimation procedures. In the meantime our recommendation is the quadratic.

Step b develops preliminary estimates E_{ij_0k}. These estimates may be formed:

i. by mechanical procedures based on direct usages and interindustry sales data only, or

ii. by procedures based on additional data from survey or engineering sources.

We shall first discuss two species of procedure of type i, and then discuss procedures based on supplementary information.

The two species of mechanical procedure we distinguish for estimating the E's using direct usages or interindustry sales data only are based, respectively, on the following two assumptions:

i′. For a particular intermediate industry j_o and end item industry k, the flows of various materials through j_o to k, i.e.,

$$V_{1j_ok},\quad V_{2j_ok},\quad V_{3j_ok},\quad \ldots$$

are roughly *proportional* to the *direct purchases* of these materials *by* j_o. Thus estimates may be formed according to the rule

$$E_{1j_ok} = \lambda_{j_ok}V_{1j_o};$$

$$E_{2j_ok} = \lambda_{j_ok}V_{2j_o};\text{ etc.}$$

where λ_{j_ok} is chosen so that $\sum_i E_{ij_ok} = T_{j_ok}$; i.e.,

$$\lambda_{j_ok} = \frac{T_{j_ok}}{\sum_i V_{ij_o}}.$$

i″. The preliminary estimates will result in better final estimates if they are taken to be *proportional to direct purchases by k*. In this case we let

$$E_{1j_ok} = \lambda_{i_ok}V_{1k};$$

$$E_{2j_ok} = \lambda_{j_ok}V_{2k};\text{ etc.}$$

where now

$$\lambda_{j_ok} = \frac{T_{j_ok}}{\sum_i V_{ik}}.$$

Assumption i′ is essentially the one used exclusively in forming an input-output inverse. It will tend to be correct insofar as j_o produces similar, standard components for all industries. Suppose that the same value of copper wire, iron castings, steel sheet, etc., were required per dollar of every kind of electric motor or generator. Then the inputs of materials to the Motor and Generator industry could be prorated to purchasing industries according to the formula in i′. For some intermediate industries it may be sufficient to develop special estimates for a few purchasing industries, using i′ for the rest. In the case of motors and generators, for example, perhaps if special estimates were developed for the aircraft industry (with its special need for lightness of airborne components) then assumption i′ would be sufficiently accurate for material requirements of the others. If, for a given j_o, assumption i′ is used for all k, the preliminary estimates are themselves consistent with the constraints 1, 2, and 3 of step c. Hence, the minimization of a criterion function is not needed. If special procedures are used to estimate the E_{ij_ok}'s for a few k's, and i′ for the rest, then the requirements for the special industries can be first subtracted from j_o's material purchases (V_{ij_o}) and the remainder of its material purchases distributed by i′, again without the minimization procedure in step c.

Assumption i″ is more appropriate for intermediate industries which provide metalworking services. These essentially do overflow work to the design of the purchasing industry. If industry k_1 directly purchases a greater ratio of aluminum to steel sheet than does k_2, we would expect this to be reflected in the material content of their purchases from, for example, the Metal Stampings industry. One apparent difficulty with using the direct purchases of the end item industries for forming initial estimates, as i″ would have us do, is that some intermediate industries (e.g., Metal Stampings) use substantially more of some materials (in the case of Metal Stampings, steel sheet) and less of others (iron castings) than do end item industries as a whole. Thus, for a particular intermediate industry j_o, the E_{ij_ok} of i″ will generally overestimate demands for some materials and underestimate them for others. Constraints (1) of step c will insist that, in the final estimates, sources equal uses for all materials. The manner in which step c adjusts the preliminary estimates to form final estimates depends on the criterion of closeness selected. Suppose we are willing to assume in the case of Metal Stampings that the requirements for sheet are to be scaled up, and those for castings scaled down, "more or less equally" for all purchasing industries. More specifically, suppose we are willing to assume that the relationship between the true flow $V^*_{ij_ok}$ and the initial estimate E_{ij_ok} as formed by i″ is given by

$$\frac{V^*_{ij_ok}}{E_{ij_ok}}, = \theta_{ij_o} + u_{ij_ok}$$

where θ_{ij_o} is the average correction factor for the uses of material i by industry j_o, and u_{ij_ok} is the deviation from average in the case of purchasing industry k, assumed to be normally distributed. The appropriate quadratic minimization for step c, under these assumptions, is:

$$\text{minimize} \quad \sum_{i=1}^{N_M} \sum_{k=0}^{N_I} \left(\frac{V_{ij_ok}}{E_{ij_ok}} - \theta_{ij_o} \right)^2 \tag{4^v}$$

where
$$\theta_{ij_o} = \frac{V_{ij_o}}{\sum_k E_{ij_ok}}.$$

The above is the closeness criterion which the writers recommend in the case of intermediate industries whose initial estimates are formed wholly on the basis of i″.

ii. For a given j_o, some or all the E_{ij_ok} may be based on data from engineering or survey sources. A 1956–58 U. S. study of the Soviet heavy machine building industry,[6] for example, constructed man, machine, and material requirements of typical products from drawings and data published in Soviet engineering literature. Requirements can be estimated more economically, however, when the establishments themselves are willing to devote a small amount of time of their regular production control apparatus for such a purpose. From a sample of orders over a period of time, material, labor, and equipment requirements can be generated by the same procedures as are regularly used to estimate the materials needed for orders, and shop load by station or labor class. In some cases further sampling to obtain factors concerning planned versus actual would be valuable, particularly in the areas of labor and equipment loading. The final requirement figures should be adjusted to allow for normal scrap and idle time.

If, for a given j_o, all the E's are formed from supplementary data and all are, percentage-wise, equally subject to uncertainty, then (4^{iv}) is the appropriate quadratic criterion. On the other hand, some estimates may be more trustworthy than others, e.g., because larger industries have been deliberately sampled more heavily than smaller ones, or because supplementary data have been used for the more important purchasing industries while the less important ones have been estimated by (i′). In this case the maximum likelihood estimate is provided by a weighted sum of squares

$$\sum \sum w_{ik} \left(\frac{V_{ij_ok} - E_{ij_ok}}{E_{ij_ok}} \right)^2$$

with weights inversely proportional to the variances of estimate. If, for a given j_o, supplementary data are used for some of the E's and procedure i″ for the rest, the appropriate criterion is a combination:

$$\lambda C^{i} + (1 - \lambda) C^{ii}$$

where C^{i} is the measure (4^{iv}) of closeness of fit of the terms based on i″, and C^{ii} the measure of closeness for those based on ii. The choice of λ determines the extent to which the optimization will "take the word" of the estimates based on ii versus those based on i″.

Estimates E_{ij_ok} based on supplementary data can also be used to test the adequacy of V_{ij_ok} generated by i′ and i″ alone. In testing i′ and i″ it should be remembered that the E_{ij_ok} can be systematically biased yet the V_{ij_ok} satisfactory. Furthermore, if (for a given i, k) the errors in V_{ijk} are moderate—but statistically unbiased—then errors in the final requirement coefficients will tend to be small. What we must watch for, then, is large errors in the V_{ijk} or systematic ones which will not tend to be canceled in the summation of total requirements.

Step a estimates (in the case of the materials pass) the total value of the

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[6] *Analysis of Production Processes in the Soviet Heavy Machine-building Industry. Interim Report, Phase 1* (Soviet Planning Study No. 5), Institute for Research in Social Science, University of North Carolina, 1956.

Input-Output Analysis of Soviet Heavy Machinery (Soviet Planning Study No. 6), Institute for Research in Social Science, University of North Carolina, 1958.

material content (T_{j_ok}) of the sales from j_o to k. These estimates are used in conditions 2 of step c. Step a may be performed mechanically by prorating materials to purchasing industries in proportion to their purchases from j_o:

$$T_{j_ok} = \lambda P_{j_ok}, \qquad k = 0, 1, \ldots, N_I,$$

where λ is determined so that $\sum_k T_{j_ok} = \sum_i V_{ij_o}$, i.e.,

$$\lambda = \frac{\sum V_{ij_o}}{\sum P_{j_ok}}.$$

Such proration assumes that all purchasing industries receive approximately the same value of material content per dollar spent in industry j_o. This assumption could be checked as a by-product of the supplementary investigations discussed under version ii of step b.

These, then, are the critical steps of the procedure applied under our initial, simplifying assumptions. Step a estimates the total value of materials (or, in subsequent passes, labor and equipment) incorporated in sales from j_o to k. Step b forms initial estimates of the breakdown by type of materials, type of equipment, or type of labor. Step c forms final estimates which are, in some sense, as close as possible to the preliminary estimates without violating the conditions that sources equal uses and sales are nonnegative.

Now let us reintroduce the complications previously ruled out. P_{jk} may now be positive for any pair of industries; P_{j0} may include shipments to final demand other than those for the primary product of the industry. Shipments to final demand by any industry j of products primary to industry k will be denoted by f_{jk}. In particular, f_{jj} equals final demand shipments of the primary product; hence

$$F_{j0} = \sum_{k=1}^{N_I} f_{jk}.$$

Steps a through c will now impute usages of men, machines, or materials to final demand categories as well as to interindustry sales. We will let

U_{ijk} = the value of the ith kind of material purchased by industry j to be processed into final demand primary to industry k.

As before we will let

V_{ijk} = the value of the ith kind of material purchased by industry j to be processed into components sold to industry k.

For a given pass (for materials, machines, or men) and for a given industry j_o the U_{ij_ok}'s and the V_{ij_ok}'s are generated, simultaneously, by steps a, b, and c, with P_{j_o0} replaced by $f_{j_o1}, f_{j_o2}, \ldots$, and with V_{ij_o0} replaced by U_{ij_o1}, $U_{ij_o2}, \ldots$.

The sequence in which industries are processed by steps a through c should depend on how much each purchases from other metalworking industries. Those which purchase least will be treated first. This ordering need not be exact, but the treatment of industries in approximately this order will speed the convergence of the process.

In the order described, the direct usage of men, machines and materials for each industry is imputed to final demand and interindustry sales by steps a through c. Each such imputation has two major effects:

1. Part of the industry's direct usages are imputed to final demand categories. These imputations are added to similar previously accumulated usage by product category. Our object is to eventually distribute all resource usage to these final demand categories.
2. The remainder of the direct usage is imputed to interindustry sales. These imputations are added to the direct usage of the purchasing industries. Thus by the time we come to distribute the direct usages of the Motor Vehicles industry, for example, they include imputations made to them as well as the original direct usage figures.

After passing through all industries the procedure next considers the extent to which the various factors of production have been distributed to final demand categories. If a sufficient amount (e.g., at least 99 per cent) of all types of materials has been distributed to final demand the remaining fractions (x_i) can be prorated mechanically by multiplying existing imputations to final demand by $1/(1-x_i)$. If the amount imputed to final demand is not sufficiently large—if imputations for later industries returned significant amounts to industries treated earlier—then the imputation process must be repeated for industries with nonnegligible usage still to be distributed. The process terminates when all types of materials (or men or equipment) have had enough usage imputed to final demand to justify the terminal proration to final demand categories.

The convergence of this process is accelerated by the structure of metalworking industries, which somewhat resembles the simple structure assumed in originally presenting the method. Thus after the second or third pass through, little usage should remain undistributed to final demand.

Because of the number of options it permits, the procedure described in this section is perhaps best thought of as a way of organizing the estimation of total requirements rather than as a specific proposal for their estimation. On the one hand, the procedure permits requirements to be estimated from direct usages, interindustry sales, and a classification of industries into two sets: (i′) those which tend to produce standard products for all industries, and (i″) those which do overflow work to the design of the purchasing industry. On the other hand, the procedure permits the use of engineering or survey data for some or all the requirements estimates, adjusting these estimates (as little as possible) to be consistent with aggregate usage figures. The classic input-output analysis may be thought of as a special case in which all industries are classified into i′, and the resource being distributed is "industry capacities" rather than types of men, machines, and materials.

SOME APPLICATION TECHNIQUES

The last section discussed the derivation of total usages per unit of final demand. We shall now consider the use of these figures, as requirements estimates, in the evaluation of proposed programs. Our initial discussion will be concerned with requirements for metalworking equipment. Thus, for the time being at least, it will be sufficient to write a_{ij}, rather than a_{ij}^E, as the value of the demand for the *i*th kind of equipment per unit of the *j*th final demand. We shall see how the simple structure of the requirements analysis allows us to answer broad classes of questions on the basis of a small amount of elementary calculation. Later we shall consider how the same, or similar, analysis apparatus can be used in conjunction with requirements estimates for labor and materials.

A requirements analysis for equipment addresses itself to questions such as the following: Suppose that the economy attempted to achieve a certain combination $(f_1, f_2, \ldots, f_n)$ of final demand per unit time: What would be the requirements $(R_1, R_2, \ldots, R_M)$ for equipment? The requirements analysis answers this question by the calculation:

$$R_i = \sum_{j=1}^{N} a_{ij} f_j.$$

The results R_i are expressed either in terms of value or physical units depending on the units of measurement of the a_{ij}. In the present discussion it will be convenient to assume that the a_{ij} are measured in value terms, and hence R_i is the value of the stock of required equipment. If $A_1, A_2, \ldots, A_M$ are the amounts of equipment available (measured in the same units as R_i), the analysis concludes that no additional equipment is needed as long as $A_i \geqq R_i$ for all i. If $R_i > A_i$ for some or all i, the investment required for new equipment is

$$I = \sum_{i=1}^{M} \max(0, R_i - A_i).$$

The above calculation does not address itself to the problem of how fast the economy can respond to sudden changes in the short run. If $I = 0$, for example, it would be unsafe to conclude that the economy could move instantly to the proposed new set of final demands. The calculation ignores the important bottlenecks of transition, such as planning and tooling; it fails to take into account the dynamics of what must be done first, before what must be done second, before what can be done third.

It considers, rather, the problem of equipment necessary to sustain a program, as distinguished from the speed and tactics by which this program is approached. The incremental (new investment) requirements $R_i \geqq A_i$ (where $R_i - A_i$) are relevant for the transition in that they constitute part, albeit not all, of the needs of going from the old to the new program.

It is generally desirable to compare investment requirements for a number of proposed sets of final demands. We shall show how a range of such proposals can be evaluated with the aid of a few basic computations. To begin let us consider the following problem: a nation, by choice or happenstance, equips so as to produce (among other things) amounts f_1 and f_2 of two products. Later, because of a change in internal needs or world markets it changes these amounts to f_1' and f_2', increasing one and decreasing the other. To what extent can equipment of the declining industry be transferred to the growing industry? We shall illustrate the nature and derivation of the solution to this problem in terms of the hypothetical five machine example whose requirement coefficients are presented in Table 2.

The heavy curve in Figure 1 shows the combinations of increases and decreases in f_1 and f_2 which can be obtained with $1 million of new investment. The axes represent changes (Δf_1, Δf_2) in f_1 and f_2 measured in million of dollars. In any particular case the graph applies as long as $f_i + \Delta f_i \geqq 0$. Within these limits the same graph applies to the costing of changes whatever the original f_1 and f_2.

TABLE 3

HYPOTHETICAL MACHINE REQUIREMENTS FOR TWO INDUSTRIES

Machine Number	Value of Stock Required per Dollar of Final Demand	
	Industry 1	Industry 2
1	$.20	$.50
2	.10	.05
3	.10	.10
4	.20	.00
5	.00	.10
Total	$.60	$.75

The line labeled (1) in the figure is the locus of changes (Δf_1, Δf_2) for which the amount of machine 1 released by the decreasing industry exactly equals the amount needed by the increasing industry. Above this line

$$a_{11}\,\Delta f_1 + a_{12}\,\Delta f_2 > 0;$$

the value of machine 1 needed exceeds the value of machine 1 released: the stock of machine 1 must be increased. Below the line

$$a_{11}\,\Delta f_1 + a_{12}\,\Delta f_2 < 0;$$

the value of machine 1 released exceeds the value needed: no increase in the stock of machine 1 is required. This "zero-requirement line" passes through the origin and has

$$\text{slope} = \frac{a_{11}}{a_{12}} = -.4.$$

In a like manner the lines labeled (2), (3), (4), and (5) represent "zero-requirement lines" along which the amount of machines 2 through 5, respectively,

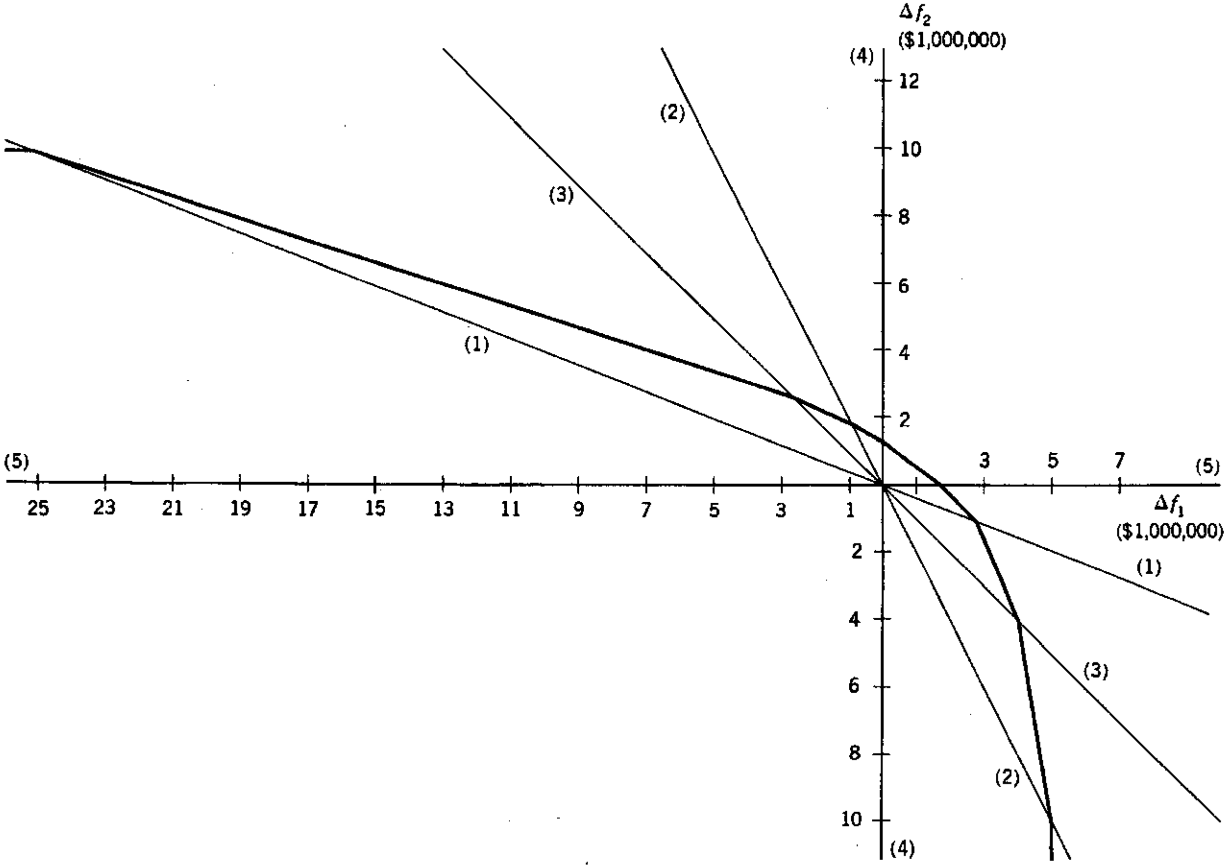

FIGURE 1. Combinations of $\Delta f_1, \Delta f_2$ obtainable with $1,000,000 new investment (hypothetical 5-machine example).

released by the decreasing industry equals that needed by the increasing industry. These lines pass through the origins and have slopes of $-a_{i2}/a_{i1}$. The line for machine i separates the Δf_1, Δf_2 plane into two half planes, in one of which the ith machine is a bottleneck, in the other the ith machine is in excess.

In the positive quadrant (when both f_1 and f_2 increase) the cost of additional equipment is

$$I = .60\,\Delta f_1 + .75\,\Delta f_2.$$

The coefficients of Δf_1 and Δf_2 in this equation are the total value of equipment required per dollar of each final demand, as shown in the total row at the bottom of Table 3 or in the middle of Table 4. Thus in the first quadrant of Figure 1 the isocost curve connecting all combinations costing $1 million is represented by the line

$$.60\,\Delta f_1 + .75\,\Delta f_2 = 1.0.$$

Let us, for a moment, consider the region of the second quadrant between lines (2) and (3). Every point in this region is below and/or to the left of lines (2) and (4); every point is above and/or to the right of lines (1), (3), and (5). Thus for every point within this region there will be a shortage of machines 1, 3, and 5, and a surplus of 2 and 4. The amounts bought of the former three machines will be

$$a_{i1}\,\Delta f_1 + a_{i2}\,\Delta f_2 \qquad (\text{for } i = 1, 3, 5)$$

(where $\Delta f_1 < 0$ in this region).

The amounts spent on the other two machines (2 and 4) will be zero. Hence, the total investment required by an increase in f_2 and decrease in f_1 represented by a point in this region is

$$I = A_1\,\Delta f_1 + A_2\,\Delta f_2$$

where $$A_1 = a_{11} + a_{31} + a_{51} = .30$$

and $$A_2 = a_{12} + a_{32} + a_{52} = .70.$$

The $1 million isocost curve in this region, therefore, is represented by the line

$$.30\,\Delta f_1 + .70\,\Delta f_2 = 1.0.$$

Similar remarks apply to the other cone-shaped regions bordered by successive zero-requirements lines. For example, each point in the region of the fourth quadrant between lines (1) and (3) has an investment requirement of

$$I = A_1\,\Delta f_1 + A_2\,\Delta f_2$$

where

$$A_1 = a_{21} + a_{31} + a_{41} = .40,$$

$$A_2 = a_{22} + a_{32} + a_{42} = .15.$$

The isocost curve in this region is represented by the line

$$.40\,\Delta f_1 + .15\,\Delta f_2 = 1.0.$$

Table 4 illustrates a procedure for obtaining the A_1, A_2 coefficients for all regions. The first three columns of the table present machine number, value of stock (a_{i1}) required for final demand 1, and value of stock (a_{i2}) required for final demand 2. The fourth column shows the ratio a_{i1}/a_{i2}. The first five rows of the table correspond to the five machines, arranged in order of increasing a_{i1}/a_{i2}. The sixth line is labeled Total; the seventh through eleventh lines again correspond to the five machines, arranged as before. The last column of the table represents the cost formulae for each region. For example, the fourth entry in the last column, which straddles the third and fourth rows (corresponding to machines 3 and 2), contains the cost formula for the region in the second quadrant which lies between zero-requirement lines (3) and (2). Similarly, the second, third, and fifth entries of the last column contain cost formulae for other regions of the second quadrant. The sixth entry, corresponding to the Total row, has the cost formula for the first quadrant. The seventh through tenth entries of the last column contain the cost formulae for successive regions of the fourth quadrant. The zeros in the first and eleventh positions of the last column are essentially the "cost formula" for the third quadrant. In short, the entries in the last column of Table 4 are the cost formulae for the various regions, arranged in the order they are reached by starting in the third quadrant, moving clockwise through the second, first, and fourth quadrants, returning finally to the third quadrant again. As we go from the first to the sixth entry in the last column, the A_1, A_2 coefficients of the cost formula are formed by successive addition:

TABLE 4

COST FORMULAE FOR Δf_1, Δf_2

Machine Number	Value of Stock Required		Ratio of Requirements (1)/(2)	Cost Formula $A_1\Delta f_1 + A_2\Delta f_2$
	Industry 1	Industry 2		
				0
5	\$.00	\$.10	.0	
				$.10\,\Delta f_2$
1	.20	.50	.4	
				$.20\,\Delta f_1 + .60\,\Delta f_2$
3	.10	.10	1.0	
				$.30\,\Delta f_1 + .70\,\Delta f_2$
2	.10	.05	2.0	
				$.40\,\Delta f_1 + .75\,\Delta f_2$
4	.20	.00	∞	
Total	\$.60	\$.75		$.60\,\Delta f_1 + .75\,\Delta f_2$
5	.00	.10	.0	
				$.60\,\Delta f_1 + .65\,\Delta f_2$
1	.20	.50	.4	
				$.40\,\Delta f_1 + .15\,\Delta f_2$
3	.10	.10	1.0	
				$.30\,\Delta f_1 + .05\,\Delta f_2$
2	.10	.05	2.0	
				$.20\,\Delta f_1$
4	.20	.00	∞	
				0

$$\text{next } A_1 = \text{previous } A_1 + a_{i1};$$

$$\text{next } A_2 = \text{previous } A_2 + a_{i2}.$$

As we go from the sixth to the eleventh entry in the last column the A_1, A_2 coefficients are formed by successive subtractions:

$$\text{next } A_1 = \text{previous } A_1 - a_{i1};$$
$$\text{next } A_2 = \text{previous } A_2 - a_{i2}.$$

To plot the isocost curve of Figure 1 it is sufficient to know points at which each zero-requirements line, of the form

$$a_{i1}\,\Delta f_1 + a_{i2}\,\Delta f_2 = 0$$

intersects the isocost line of one of its neighboring regions, the latter line being of the form

$$A_1\,\Delta f_1 + A_2\,\Delta f_2 = 1.0.$$

Solving the last two equations for the values of Δf_1, Δf_2, we get

$$\Delta f_1 = \frac{-a_{i2}}{a_{i1}A_2 - a_{i2}A_1};$$
$$\Delta f_2 = \frac{+a_{i1}}{a_{i1}A_2 - a_{i2}A_1}.$$

Table 5 uses these formulae for computing points on the $1 million isocost curve as it intersects various zero-requirements lines. Between pairs of successive points the isocost curve is a straight line. Beyond line (2) in the fourth quadrant the isocost curve is vertical; beyond line (1) in the second quadrant the isocost curve is horizontal.

TABLE 5

COMPUTATION OF Δf_1 AND Δf_2 AT KINKS IN THE ISOCOST CURVE

	a_1	a_2	A_1	A_2	$D =$ $[a_1A_2 - A_2A_1]$	$\Delta f_1 =$ $-a_2/D$	$\Delta f_2 =$ $+a_1/D$
(1)	.20	.50	.00	.10	.0200	−25.0	10.0
(3)	.10	.10	.20	.60	.0400	−2.5	2.5
(2)	.10	.05	.30	.70	.0550	−.9	1.8
(4)	.20	0	.40	.75	.1500	0	1.3
(5)	0	.10	.60	.65	−.0600	1.7	0
(1)	.20	.50	.40	.15	−.1700	2.9	−1.2
(3)	.10	.10	.30	.05	−.0250	4.0	−4.0
(2)	.10	.05	.20	0	−.0100	5.0	−10.0

In the same manner the $1 million isocost curve can be derived for an analysis involving M types of equipment with about M subtractions, $2M$ multiplications, $2M$ divisions, and $4M$ additions. Once the isocost curve for $1 million (or for any $I > 0$) is obtained, the isocost curve for any other level of I follows readily. The isocost curve for $I = \$10$ million, for example, can be obtained in one of two ways:

1. by drawing a curve parallel to the \$1 million isocost curve, but everywhere ten times as far from the origin; or
2. by changing the units of measurement in Figure 2 to tens of millions, so that, for example, the point (3.0, 0.0) on the horizontal axis represents a \$30 million increase in f_1. This done, the old curve in Figure 2, untouched, is now the \$10 million isocost curve.

The same two options apply for any $I > 0$. Finally, the changes in f_1 and f_2 which give rise to $I = 0$ lie in a region which always includes the third quadrant, and sometimes, as in the present example, is identical to it.

GENERALIZATIONS. The above analysis can be generalized in several ways, e.g., by including requirements for men and materials, or by defining in a different manner the alternate sets of final demands portrayed on the axes of the figure. We shall briefly characterize a few such generalizations. In some cases the analysis procedure described above applies with little or no modification. In other cases somewhat less simple procedures are required.

Δf_1 and Δf_2 can represent increases and decreases in "programs"—i.e., they can represent proportional changes in combinations of end products. The methods described above still apply, now letting a_{ij} equal the total requirements per unit of program.

In the long run requirements for materials are but requirements for the resources that produce these materials. The requirement coefficients for various kinds of steel, for example, can be translated into requirement coefficients for various kinds of steel rolling mills. The latter, then, can be incorporated into the requirements analysis in the same manner as metal fabricating equipment.

The simplicity of the analysis behind Figure 1 rested, in part, on the fact that there was a combination of f_1 and f_2, represented by $(\Delta f_1, \Delta f_2) = (0, 0)$, which exactly used up all available resources. If no such combination exists our zero-requirements lines no longer meet at a single point, but intersect in a more haphazard manner as in Figure 2. (*Note:* In Figure 2 we use f_1 and f_2, rather than Δf_1 and Δf_2, on the axes.) In this more general case the isocost curves (i.e., those for investment levels I_1 and I_2) are once again piecewise linear, breaking at zero-requirement lines. They are no longer necessarily parallel. Each region of Figure 2—i.e., each polygon bounded by zero-requirements lines—has associated with it a cost formula of the form

$$I = A_1 f_1 + A_2 f_2 - A_0$$

where A_1 is the sum of the requirements per unit of final demand 1 for equipment which is not in excess supply; where A_2 is the sum of such requirements for final demand 2; where A_0 is the original value (before new purchases) of equipment which is not in excess supply; and where "equipment not in excess

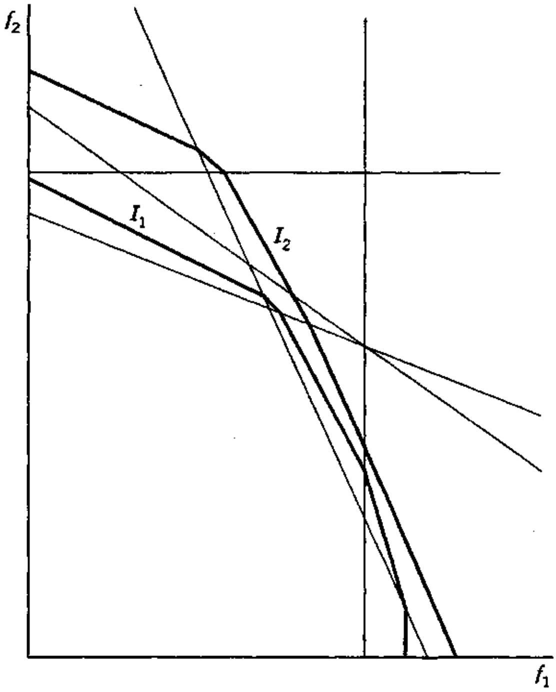

FIGURE 2. Isocost curves when no f_1, f_2 uses all resources.

supply" is that whose zero-requirements lines pass below or to the left of all points in the region. Any isocost curve I_1 which passes through the region is represented therein by the line

$$A_1f_1 + A_2f_2 - A_0 = I_1.$$

If we assume a single cost of training each type of labor, then labor development costs can be included in the analysis in the same manner as equipment purchase costs. If instead we let training cost depend on a worker's previous job as well as the job for which he is being trained, then the analysis of labor development costs becomes a linear programming problem, best done as a separate pass from the analysis of equipment procurement costs. The linear programming problem introduced by this handling of labor requirements, while more complex than the graphical procedures sufficient heretofore, is nevertheless of a simple structure requiring relatively little computer time for solution.

LOOKING BACK. Comparing this section with the preceding one, we see that the derivation of total usages from direct usages is a substantially more difficult problem than the use of these coefficients. Data collection must be left to agencies with substantial resources. Once derived, however, these figures can be used with relatively little difficulty in requirements analyses addressing themselves to a variety of objectives.

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CHAPTER 12

A MACHINE TOOL SUBSTITUTION ANALYSIS

Harry M. Markowitz and Alan J. Rowe

The analysis of the last chapter assumed fixed requirements for men, machines, and materials. Inputs per unit output are not fixed in fact, however, but can and do change depending on economic conditions. The present chapter presents an analysis of "one-for-one" substitution possibilities among machine tools. The same methodology is applicable to one-for-one substitution possibilities among metal forming and certain other shop equipment.[1] The possibility of analyzing certain other types of substitution is discussed in the next chapter.

Disaggregation is generally required to increase the ability of an analysis to detect bottlenecks under a variety of conditions. As we disaggregate, however, the importance of substitution possibilities increases. Suppose our analysis distinguishes two related machines, A and B, since under some conditions the special abilities of A will be most needed, while under other conditions B will be the bottleneck. Some tasks can be performed by A and B with equal efficiency; for others, A is more efficient; for still others, B is more efficient. The extent to which A and B can be substituted for each other, and the loss of efficiency which results when such substitutions are made, depends on the availability of tasks which the two machines can do equally (or almost) equally well.

The analysis presented here attempts to cast these notions into a usable, numerical form.[2] The sections of this chapter consider the classification of machines and tasks, the estimation of rates at which the various machines can perform different tasks, problems and potentials for obtaining task requirements by product class, and some problems and potentials with respect to applications. Table F, illustrated in Chapter 14, presents rates at which various machines can perform various tasks.

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[1] E.g., for a classification of machines and tasks involving some of the metal forming machines see Rowe (1955).

[2] The work reported in this chapter was originally presented in A. J. Rowe and H. Markowitz (1955).

An important precursor of this work is Mathilda Holzman (1953), pp. 326–359.

THE CLASSIFICATION OF TASKS

An object can be characterized by properties such as material, size, shape, surface finish, etc. A *task* is defined as the requirement to transform one or more objects with certain properties into one or more objects with certain (other) properties. The number of distinct objects which remain after a task is performed may be more or less than the number at the start, as when a bar is cut in two, or two parts are welded together. Complex tasks are equivalent to combinations of more elementary tasks. Thus the quite complex task of transforming raw materials of various sorts into automobiles is achieved by simpler tasks such as the production of lots (batches) of particular components. The production of a lot of some component typically involves operations at several machines; the processing of the lot through a particular machine requires the task of setting up the machine for the lot, then the task of processing each piece on the machine. These tasks, in turn, can be broken into more elementary tasks, as analyzed in time-and-motion studies. The tasks we shall consider in our analysis are such that, if accomplished in their usual manner, they require the setting up of a machine plus the processing of a lot with the machine thus set up. A combination of such tasks is typically required to fabricate a lot (batch) of parts.

Our analysis of machine tool substitution possibilities will require a classification of tasks into categories. For each category of tasks we will choose a specific *standard task*. Thus, if we had as a category "the set of all tasks which transform a piece of material into a surface of revolution," the standard task of this category might be "the task of transforming a piece of bar stock of specified dimensions and material, into a cylinder of specified dimensions, tolerance, and surface finish." Similarly, for each category of machines, a *standard machine* will be chosen. These standard machines and tasks will be used in estimating the times required to perform the jth kind of task with the ith kind of machine. All but one of the members of any task category will be nonstandard. We will speak of any such nonstandard task as being equivalent to a certain number of *units of the standard task*. This number of equivalent units will be defined by choosing for each task a machine which typically performs this task and letting the number of standard units of the task =

$$\frac{\text{time required to perform specific task}}{\text{time required to perform standard task}}.$$

Our analysis, therefore, does not assume that the time required to perform a given task in a category is the same as that required to perform every other task in the category. It does, however, assume that rate of substitution of one machine for another is the same for every task in the category. That is, our analysis assumes that

$$\frac{\text{time required to perform task A on machine 1}}{\text{time required to perform task B on machine 1}}$$

equals

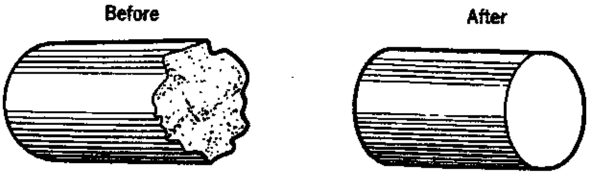
$$\frac{\text{the time required to perform task A on machine 2}}{\text{the time required to perform task B on machine 2}}$$

for every A and B in the given task category and for every machine whose ability to produce A or B is worth considering. It is no doubt impossible to classify all tasks into a small number of categories in such a manner as to make the above relation hold exactly. It is desirable, however, in constructing a classification of a given level of detail to choose the categories so that (in some sense) the relation holds as closely as possible. In particular it is important for the set of machines which can do the task well, the set of machines that can do the task less efficiently, and the set of machines which cannot do the task at all (for all practical purposes) to be essentially the same for all members of a task category. The choice of level of detail is itself a compromise between the needs of accuracy and the costs of data collection and computation.

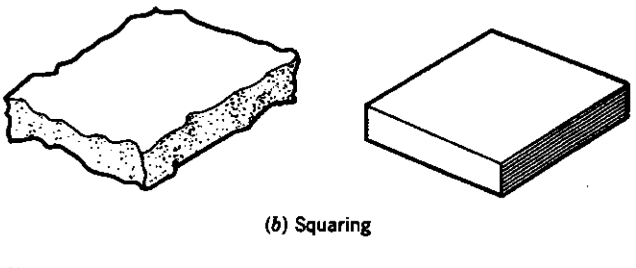
In an attempt to achieve reasonable homogeneity (in the sense defined above) and still have a manageable number of categories, the following ground rules were employed in defining categories of tasks:

(A) A task, by definition, is characterized by the set of properties of the material both before and after the task is performed. In practice the starting point of the task (the rough shape and size of the material) is "close" to that at the end of the task. We have therefore grouped together tasks which end

FIGURE 1
Configuration Categories, Sample Shapes

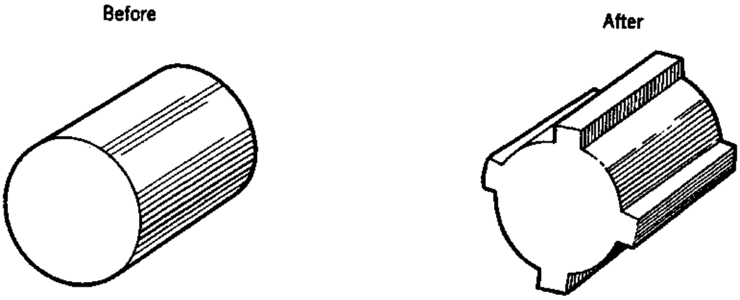


(a) Facing



(b) Squaring

FIGURE 1A. Flat surfaces—no contours or irregularities.



(*a*) Straight splines

(*b*) Serrations

(*c*) Slots

FIGURE 1*B*. Flat surfaces—external contour in one direction.

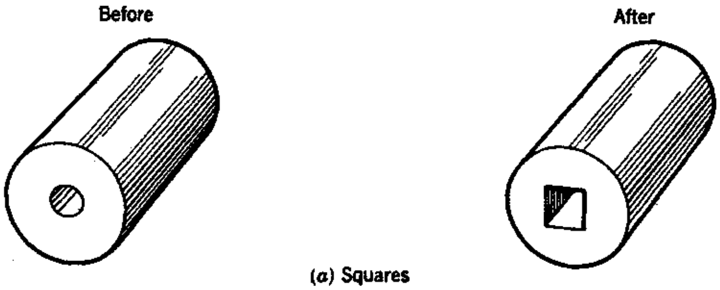
in the same final set of properties although their starting points are somewhat different. As explained above, the task of going from starting point *A* to final state *C* may represent a different number of "units of the standard" than does the task of starting at *B* and going to *C*, even though the two tasks are in the same category.

(B) Most of our task categories have five identifying "dimensions": (1) the geometric configuration which is to be obtained, (2) the pertinent dimensions of the work, (3) the precision required, (4) the number of pieces to be produced, and (5) the hardness of the material. Variations of these dimensions not only affect the amount of time required for a given machine to produce the task, but—more important in choosing a classification—they affect substantially the relative efficiences of different machines:

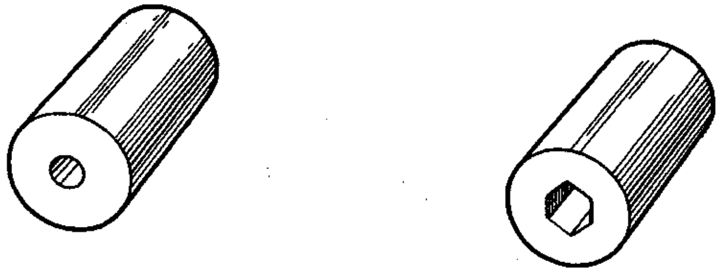
(1) The efficiency of the various major families of machines depends on the geometric configuration to be produced. The lathe family (e.g., engine lathes,

turret lathes, automatic screw machines) is especially suited to the production of surfaces of revolution. Planers and shapers are well suited for flat (or, more generally, ruled) surfaces. Gear hobbers, gear shapers, gear grinders, etc., are especially suited for making gears.

(2) The size of the part determines the sizes of machines which can perform



(*a*) Squares

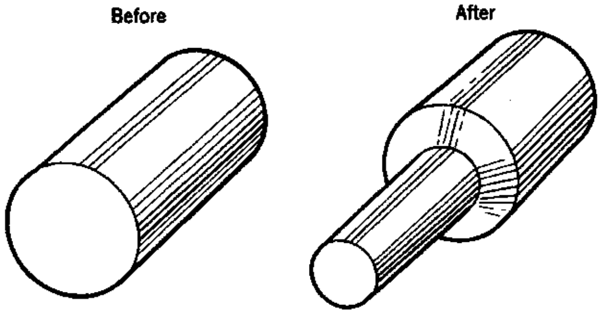


(*b*) Hexagons

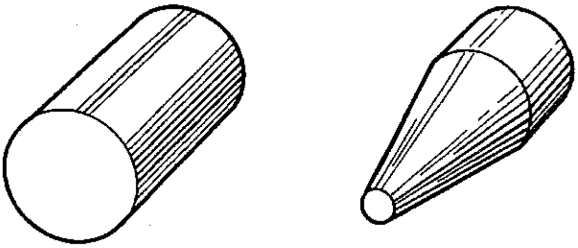
(*d*) Serrations

(*g*) Splines

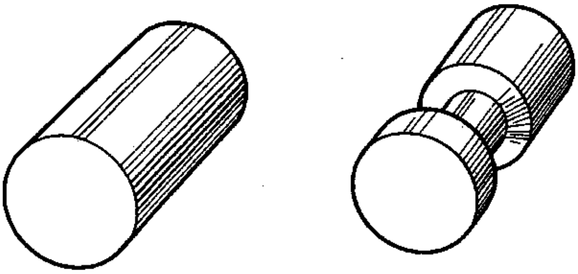
FIGURE 1*C*. Flat surfaces—internal contour in one direction.



(*a*) Stepped cylinders



(*b*) Cones



(*g*) Grooving

FIGURE 1*D*. Cylindrical surfaces—external—continuous.

a task. A large propeller shaft for a ship cannot be mounted on a small bench lathe. Conversely, while it is theoretically possible to turn a very small part on a lathe with a 10-foot swing and a 60-foot bed, such extreme machine-task combinations can be ignored for practical purposes. We should not ignore the possibility, however, that within certain limits, larger machines may be called on to perform smaller tasks.

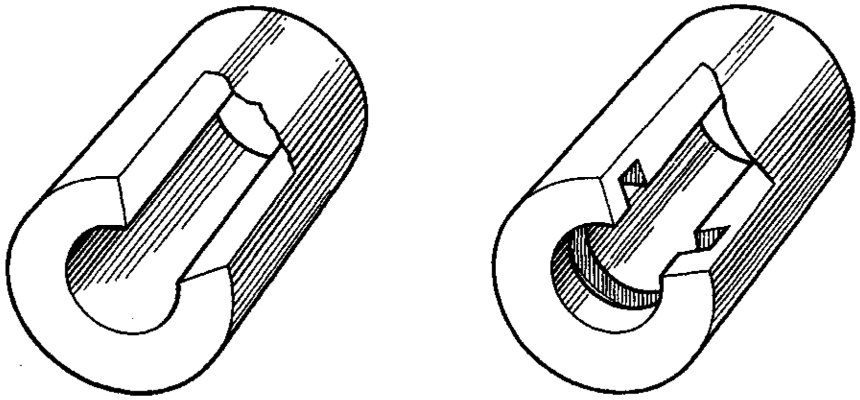
(3) When high tolerances are required, machines such as grinders, honers, and lappers become either absolutely essential or else of very high efficiency as compared to ordinary production machinery.

(4) As the number of pieces to be made increases, time required per piece falls. If this increased efficiency were approximately the same for all machines, we could treat a larger or smaller than standard lot as simply so many

units of the standard lot size, according to the time required. We do not have this proportional increase in efficiency, however, since some machines (e.g., automatic screw machines) are especially designed to handle large lots. We have therefore roughly distinguished the number of pieces to be made in classifying tasks. Within any lot size category, off-standard lot sizes are characterized by an equivalent number of standard lots in the usual manner. (We will return later to the problem of lot size.)

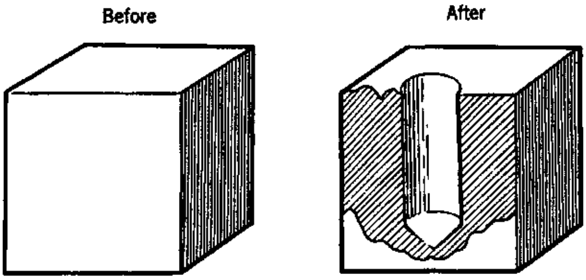
(5) Almost all machine tools are capable of machining a variety of metals such as steel, brass, and aluminum. There is, however, a degree of hardness, possessed by specially hardened steels, which sharply limits the machines which can be used. The general practice is to first machine the unhardened material until it is almost of the form desired; then to harden the material; and finally to grind the part to remove any slight distortions caused by the hardening process and to attain the precision and surface finish desired. Our analysis distinguishes tasks performed on hardened steel from those performed on softer material.

(*b*) Bored holes

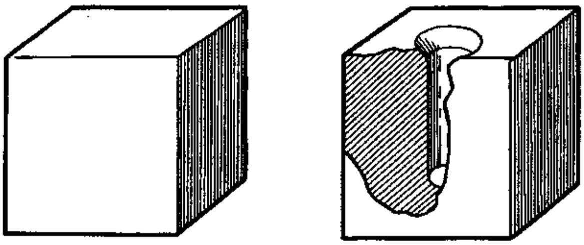


(*d*) Counterbore

FIGURE 1*E*. Cylindrical surfaces—internal—continuous.



(a) Drill hole



(c) Drill and countersink

FIGURE 1*F*. Drilled holes.

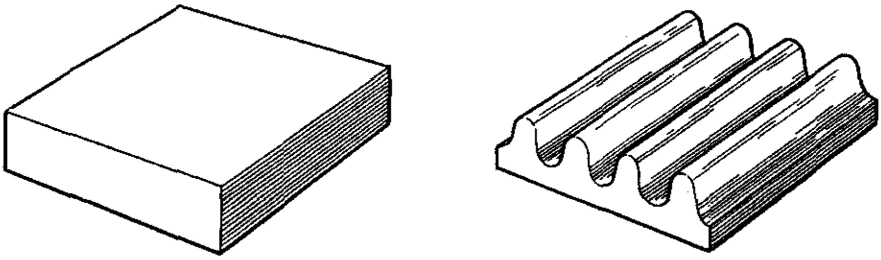
(a) Acme thread

FIGURE 1*G*. Cylindrical forms—external.

(a) Screw thread

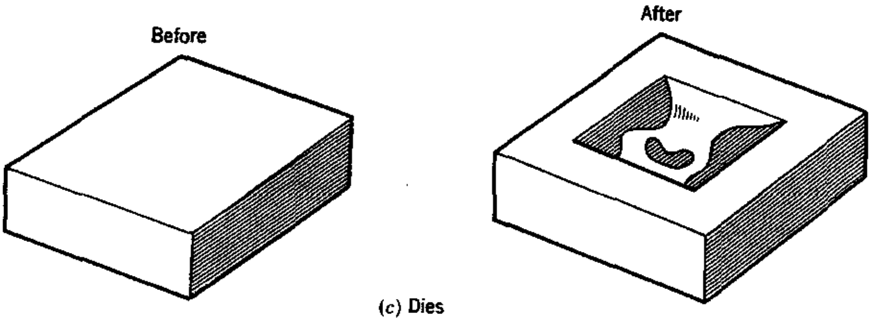
FIGURE 1*H*. Standard screw threads.

(*a*) Spur gears

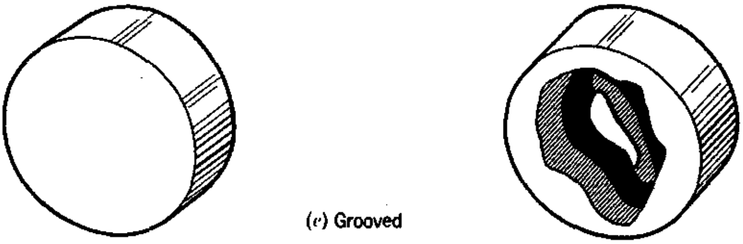


(*g*) Gear rack

FIGURE 1*I*. Standard gear shapes.



(*c*) Dies



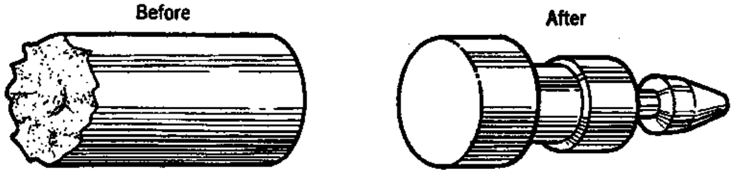
(*e*) Grooved

FIGURE 1*J*. Complex shapes—irregular.

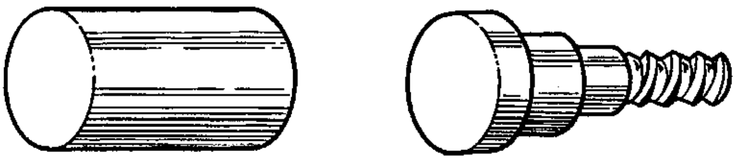
Thirteen kinds of geometric configurations are used in defining our task categories. These are listed in Table 1 and illustrated in Figures 1*A* through 1*M*. Five classes of size of part, three classes of tolerances, two classes of size of lot, and two classes of hardness are distinguished. The limits and typical members of these categories are also presented in Table 1. Not every one of the possible $13 \times 5 \times 3 \times 2 \times 2 = 780$ combinations of configurations, size, tolerance, lot size, and hardness is used to define task categories. We do not, for example, differentiate between large lots and small lots in cases of very large parts. In total we have 142 task categories which are listed in Table E of the *Statistical Supplement,* as illustrated in Chapter 14.

(*a*) Surface cams

FIGURE 1*K*. Irregular periphery—flat surface.



(*a*) Multiple external diameter and facing



(*b*) Multiple diameter and thread

FIGURE 1*L*. Multiple cylindrical surfaces.

TABLE 1

Task Specifications

I. Classification of Surface Shape

- 1. Flat surfaces—no contours or irregularities; e.g.,
  - (a) facing
  - (b) squaring
  - (c) planing
  - (d) surfacing
  - (e) spot face
  - (f) sawing
  - (g) parting
- 2. Flat surfaces—external—contour in one direction; e.g.,
  - (a) straight splines
  - (b) serrations
  - (c) slots
  - (d) dovetails
  - (e) vees
  - (f) keyways
  - (g) spur gears
  - (h) keyseats
  - (i) straight knurl—deep
- 3. Flat surfaces—internal—contour in one direction; e.g.,
  - (a) squares
  - (b) hexagons
  - (c) spur gears
  - (d) serrations
  - (e) slots
  - (f) keyways
  - (g) splines
- 4. Cylindrical surfaces—external—continuous; e.g.,
  - (a) cylinders, single and stepped
  - (b) cones and tapers
  - (c) radii
  - (d) spheres
  - (e) pointing
  - (f) chamfering
  - (g) grooving
  - (h) knurling
  - (i) spinning
- 5. Cylindrical surfaces—internal—continuous; e.g.,
  - (a) reamed holes
  - (b) bored holes
  - (c) turned holes
  - (d) counterbores
- 6. Drilled holes; e.g.,
  - (a) plain drill
  - (b) center drill
  - (c) countersink
  - (d) step drill
  - (e) deburr
- 7. Cylindrical forms—external; e.g.,
  - (a) threads—acme, square, etc.
  - (b) spirals
  - (c) lead screws
  - (d) regular contours, form tools
- 8. Standard screw threads; e.g.,
  - (a) tapped
  - (b) die
  - (c) chaser
- 9. Standard gear shapes; e.g.,
  - (a) spur
  - (b) spiral
  - (c) worm
  - (d) helical
  - (e) hourglass
  - (f) bevel
  - (g) rack
- 10. Complex shapes; e.g.,
  - (a) cams
  - (b) templates
  - (c) dies
  - (d) hourglass gears
  - (e) irregular contours
- 11. Irregular periphery—flat surface; e.g.,
  - (a) cams
  - (b) templates
  - (c) routings

TABLE 1 (*continued*)

TASK SPECIFICATIONS

12. Multiple surfaces; e.g.,
    - (a) multiple external diameter and facing
    - (b) multiple diameter and thread
    - (c) multiple diameter and drilled hole
    - (d) multiple diameter and bored hole
    - (e) multiple diameter and tapped hole
13. Multiple holes—drilled; e.g.,
    - (a) plain drill
    - (b) drill and cfm.
    - (c) drill and tap
    - (d) drill and counterbore

II. Size

(a) Machine requirement:

Size Category	Bed Length or Stroke	Swing
Very small	0–1″	0–1″
Small	1–12″	1–12″
Medium	12–36″	12–36″
Large	36–120″	36–120″
Very large	over 120″	over 120″

(b) Surface to be machined:

Size Category	Diameter	Length of Cut
Very small	1″	1″
Small	5″	5″
Medium	20″	20″
Large	50″	50″
Very large	100″	100″

III. Tolerance

		Typical Tol.	Depth of Cut
(a) Semi-precision	±.015″ to ±.001″	±.010″	.500″
(b) Precision	±.001″ to ±.0001″	±.001″	.030″
(c) High precision	±.0001″ to ±.00002″	±.0003″	.010″

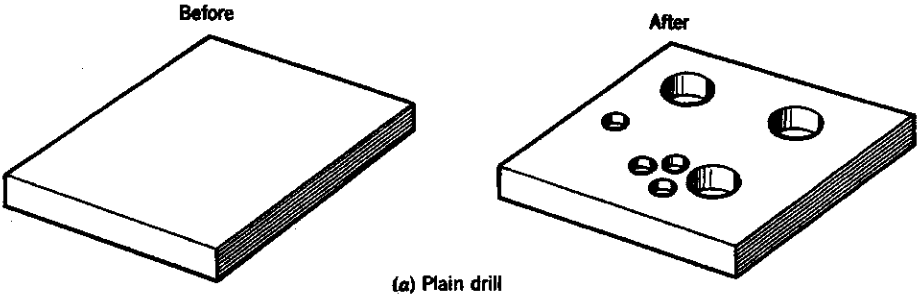
IV. Length of Run (Size of Lot)

	Very Small	Small	Medium	Large	Very Large
(a) Short	500	200	50	10	3
(b) Long	10,000	2000	500	100	3

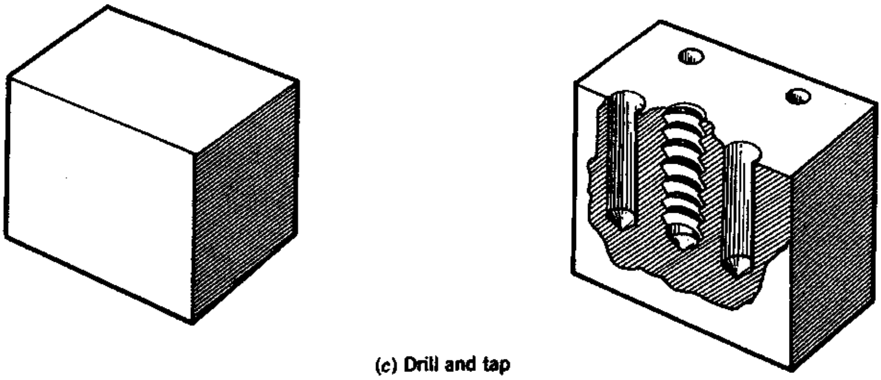
V. Hardness

(a) Ordinary material < 375 Brinell hardness

(b) Very hard material ≥ 375 Brinell hardness



(a) Plain drill



(c) Drill and tap

FIGURE 1*M*. Multiple drilled holes.

THE CLASSIFICATION OF MACHINES

A machine tool may be considered as a method of obtaining relative motion between a cutting edge or grinding surface and a piece of metal in order to generate a desired surface shape. Figures 2*A* through 2*H* show the principles of operation of some common machine tools by indicating the motion of the cutting tool, the motion of the work, and the flow of chips. Thus, the familiar drill press rotates a drill which it moves into the work to produce a hole. The common lathe rotates work past a cutting tool, the latter being moved left, right, in, or out to achieve the desired shape. The milling machine rotates the cutting tool (in contrast to the lathe which rotates the work) moving the work left, righ, in, out, up, or down. In the case of the planer (not illustrated) or the shaper (illustrated) cutting is accomplished by a linear motion, like a wood cutting plane in the hands of a carpenter. The vertical boring mill rotates the work, like a large lathe set on its side. And so on. With a little imagination even the reader unfamiliar with machine tool operations can visualize how these cutting actions can fabricate various of the geometric configuration which, in part, define the tasks of machine tools.

Table D, illustrated in Chapter 14, presents the classification of machine

TABLE 2

Typical Machining Operations for Various Machine Tools

1. Horizontal Boring Mill:
    - (a) without special attachments—
        - bore, drill, mill, ream, tap, face, chamfer, etc.
    - (b) with attachments—
        - turn, shape, form, duplicate, etc.
2. Precision Borer:
    - (a) without attachments—
        - bore, face, fly cutting, mill, chamfer, ream, groove, etc.
    - (b) with attachments—
        - turn, contour form, etc.
3. Jig Borer:
    - (a) without attachments—
        - bore, drill, counterbore, ream, grind, etc.
4. Broach:
    - (a) without attachments—
        - splines, serrations, keyways, keyseats, gear teeth, round holes, square holes, hexagonal holes, flat surfaces, irregular flat surfaces, sizing, burnishing, etc.
5. Drill Press:
    - (a) without attachments—
        - drill, ream, tap, counterbore, countersink, chamfer, spot face, etc.
    - (b) with attachments—
        - light hollow milling, light facing, etc.
6. Grinder:
    - (a) without attachments—
        - flat surface, stepped surface, contour surface, tapers, radii, cylinders, concave and convex surfaces, etc.
7. Gear Hobber:
    - (a) without attachments—
        - gears, splines, serrations, and generated shapes
8. Hone:
    - (a) without attachments—
        - size, hone, and bore
9. Engine Lathe:
    - (a) without attachments—
        - bore, face, turn, thread, tap, drill, ream, knurl, polish, groove, counterbore, centerdrill, etc.
    - (b) with attachments—
        - lap, spin, coil winding, mill, grind, gear cutting, etc.
10. Turret Lathe:
    - (a) without attachments—
        - recess, tap, bore, turn, face, drill, ream, thread, groove, centerdrill, countersink, knurl, etc.
11. Milling Machine:
    - (a) without attachments—
        - slot, face, surface, concave and convex forms, radii, chamfer, vee grooves, serrations, keyseats, keyways, etc.
    - (b) with attachments—
        - thread milling, gear cutting

TABLE 2 (*continued*)

TYPICAL MACHINING OPERATIONS FOR VARIOUS MACHINE TOOLS

12. Planer:
    (a) without attachments—
        flat surfaces, grooves, vees, etc.
    (b) with attachments—
        milling, boring, drilling, etc.
13. Screw Machine:
    (a) without attachments—
        turn, point, tapers, form, centerdrill, face, drill, ream, counterbore, recess, tap, thread, knurl, etc.
    (b) with attachments—
        light mill, hob, gear cutting, worm wheels, broaching, splines, screw slots, index drilling, etc.
14. Shaper:
    (a) without attachments—
        facing, plane surfaces, groove, slot, dovetails, splines, keyways, contours, cams, emergency gears

FIGURE 2

MACHINE TOOLS

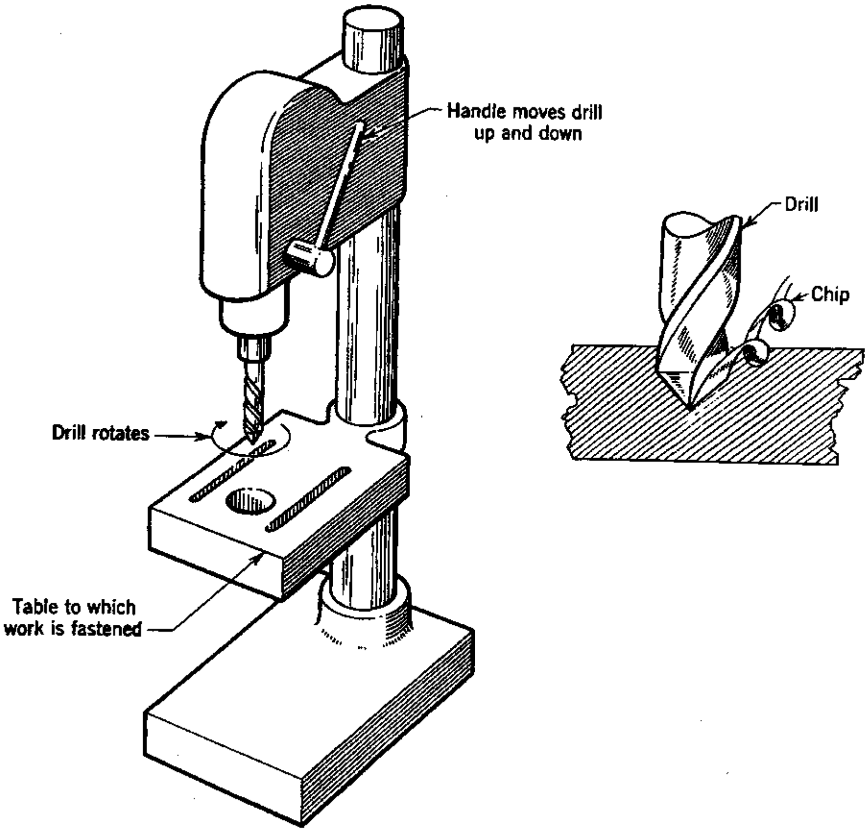


FIGURE 2A. Drill press.

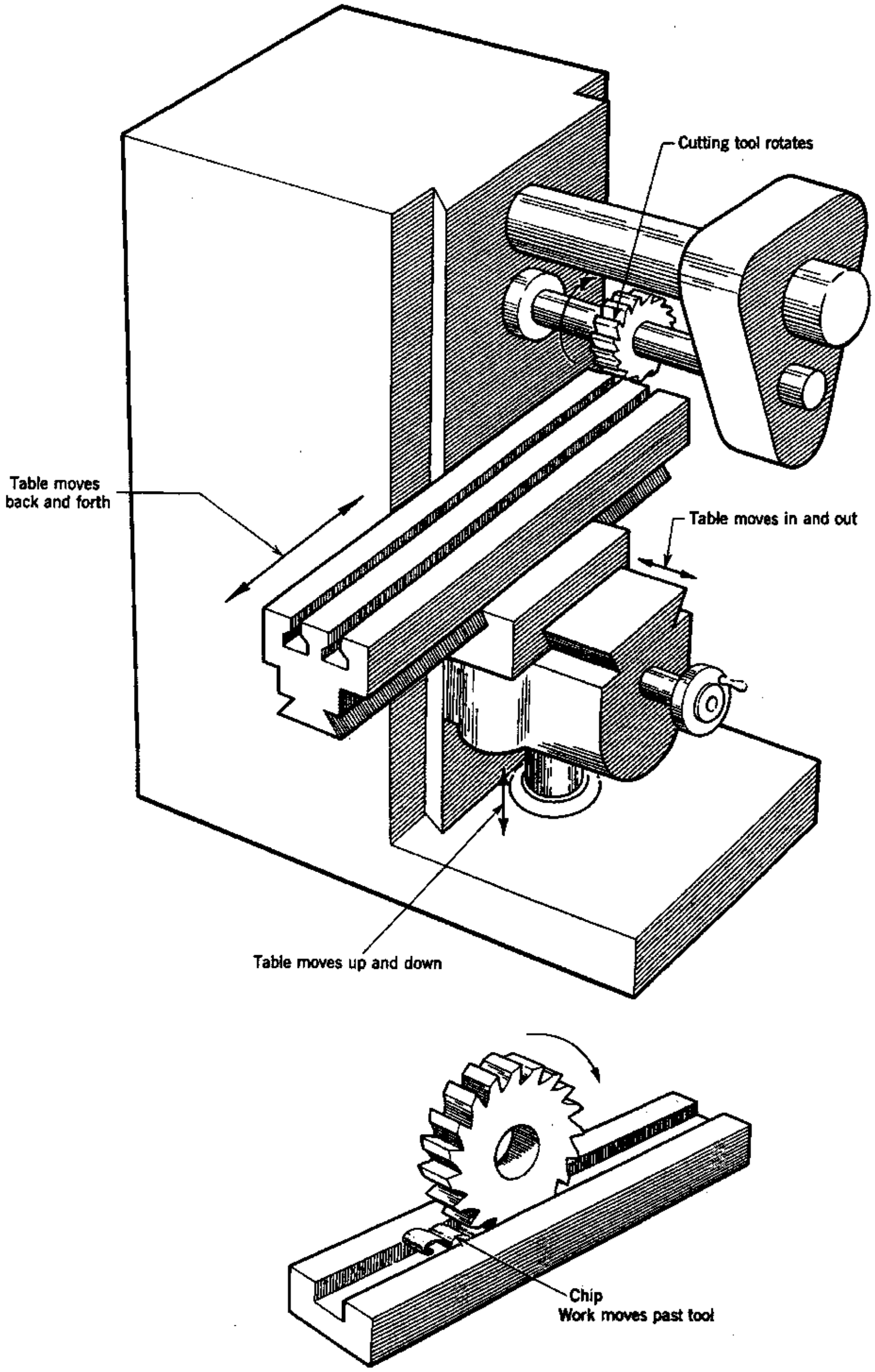


FIGURE 2*B*. Horizontal milling machine.

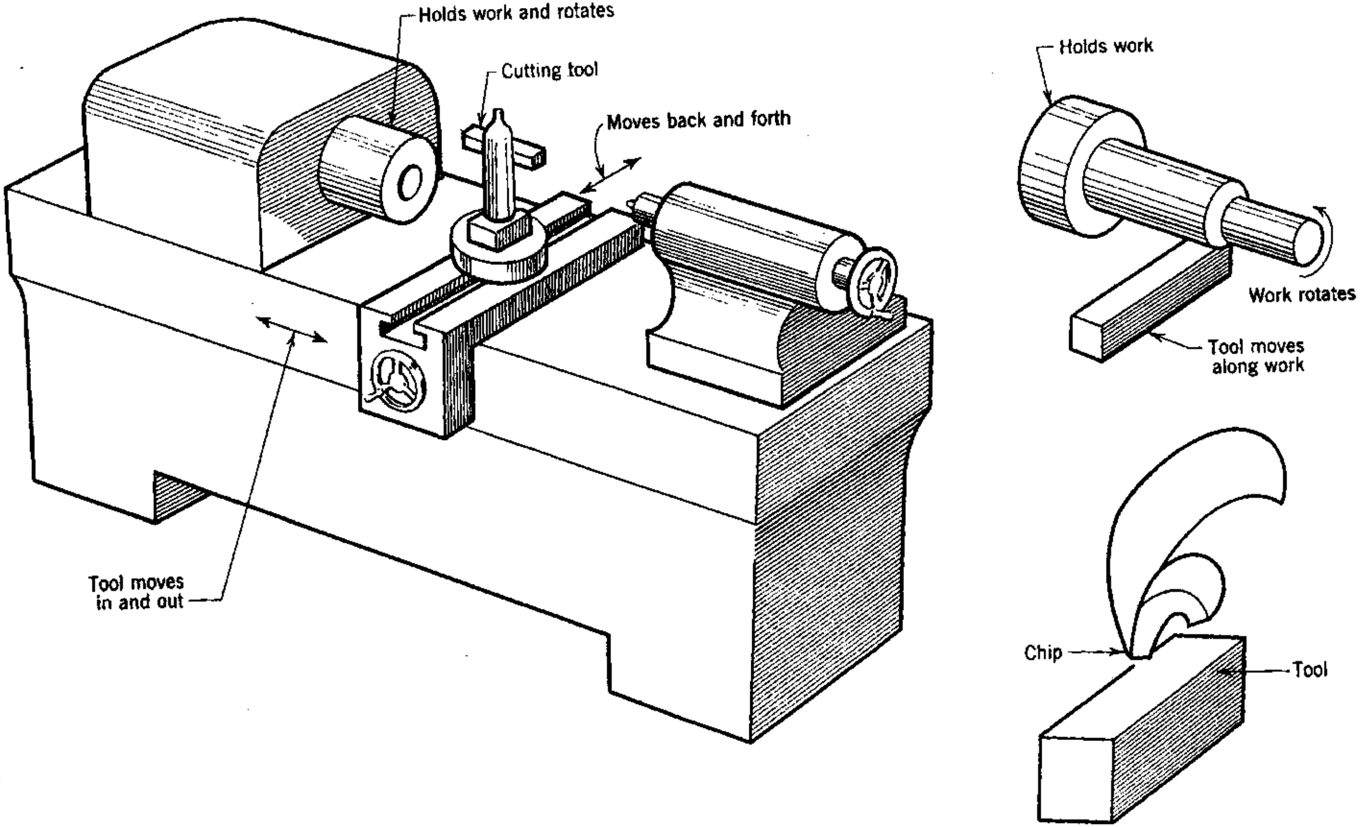


FIGURE 2C. Engine lathe.

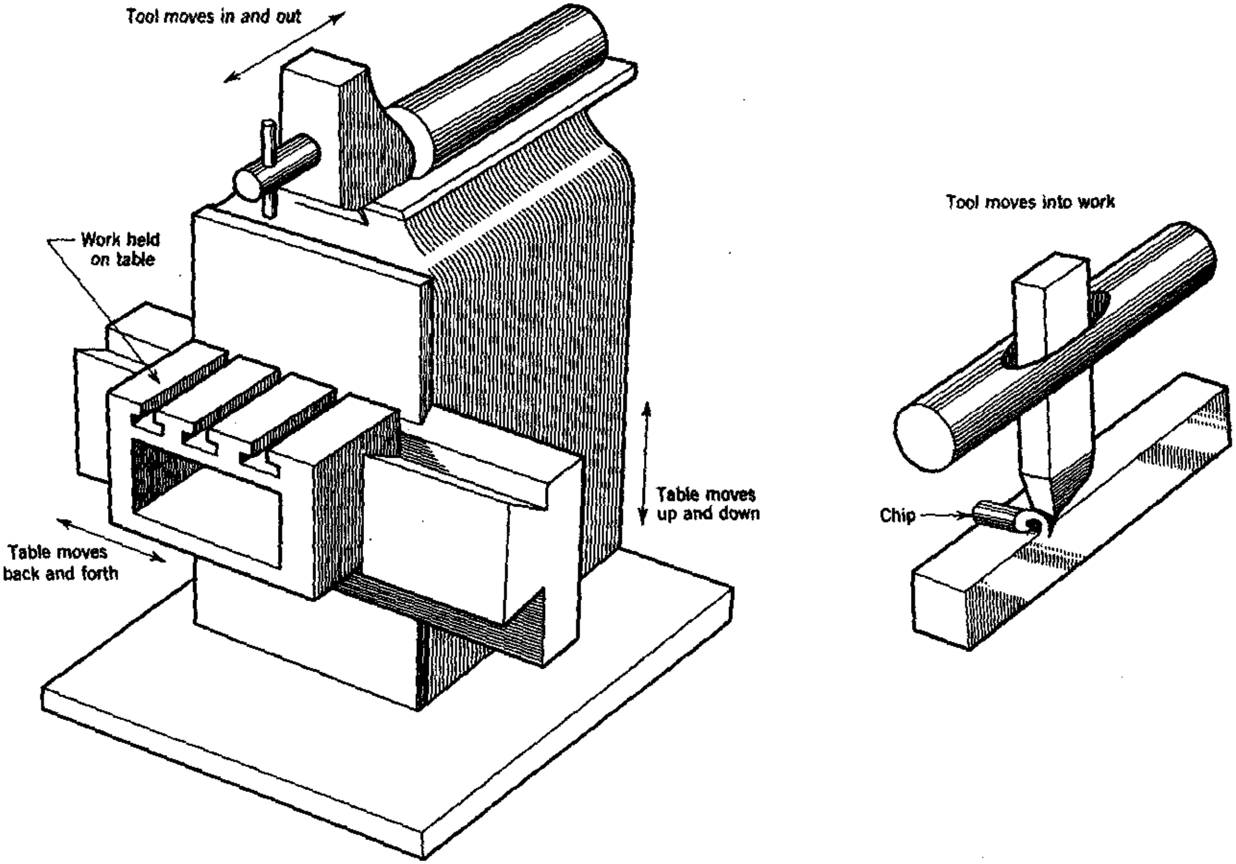


FIGURE 2*D*. Shaper.

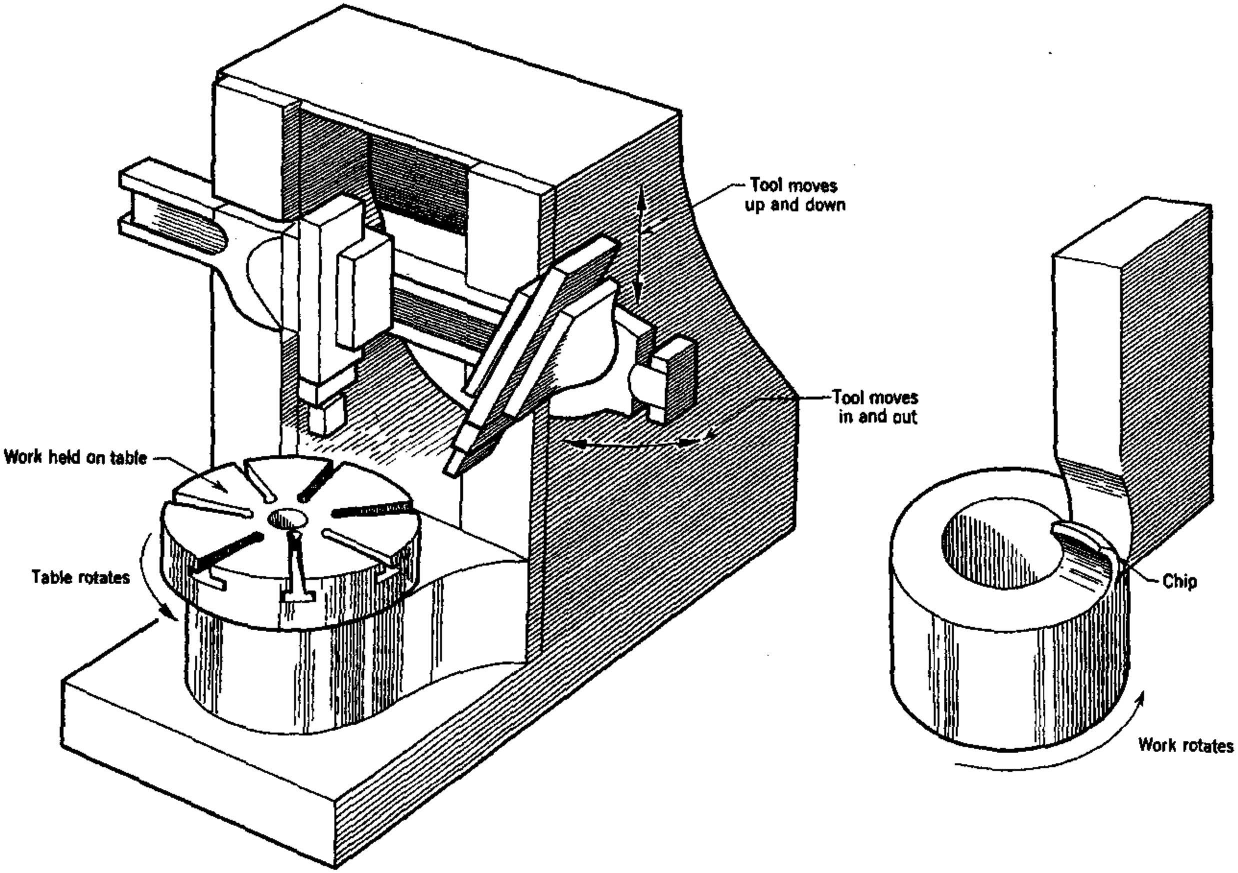


FIGURE 2*E*. Vertical boring mill.

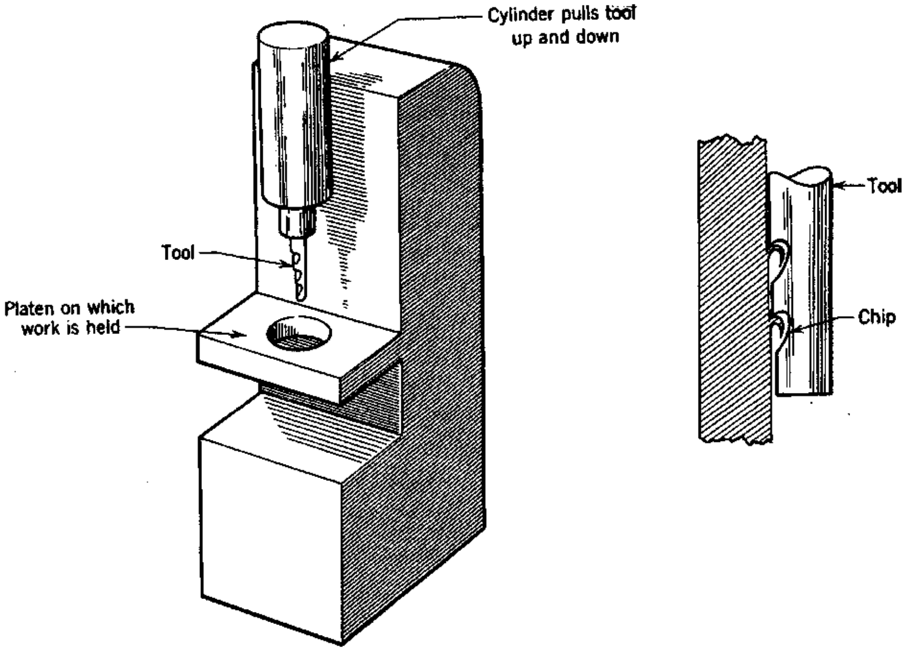


FIGURE 2*F*. Broach.

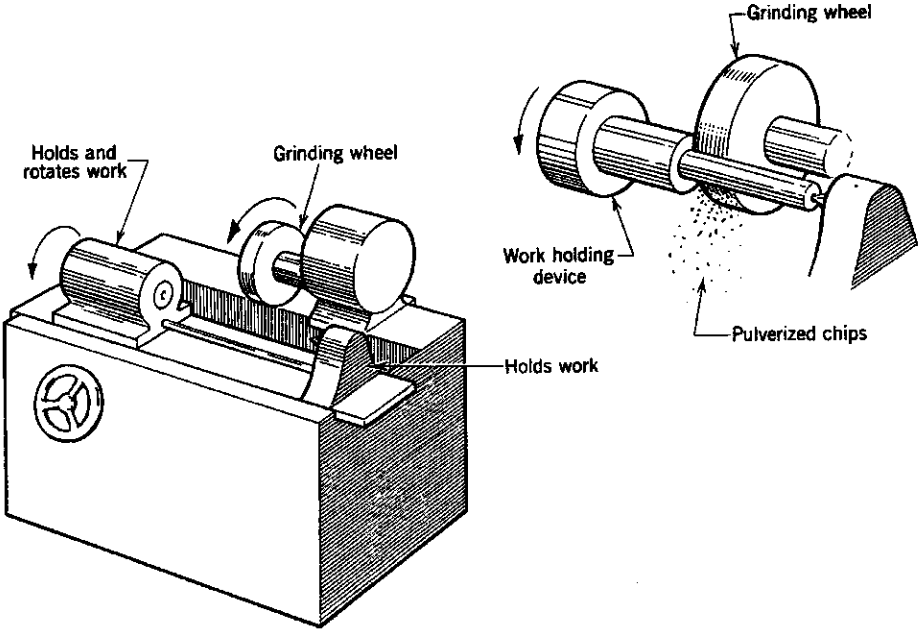


FIGURE 2*G*. External cylinder grinder.

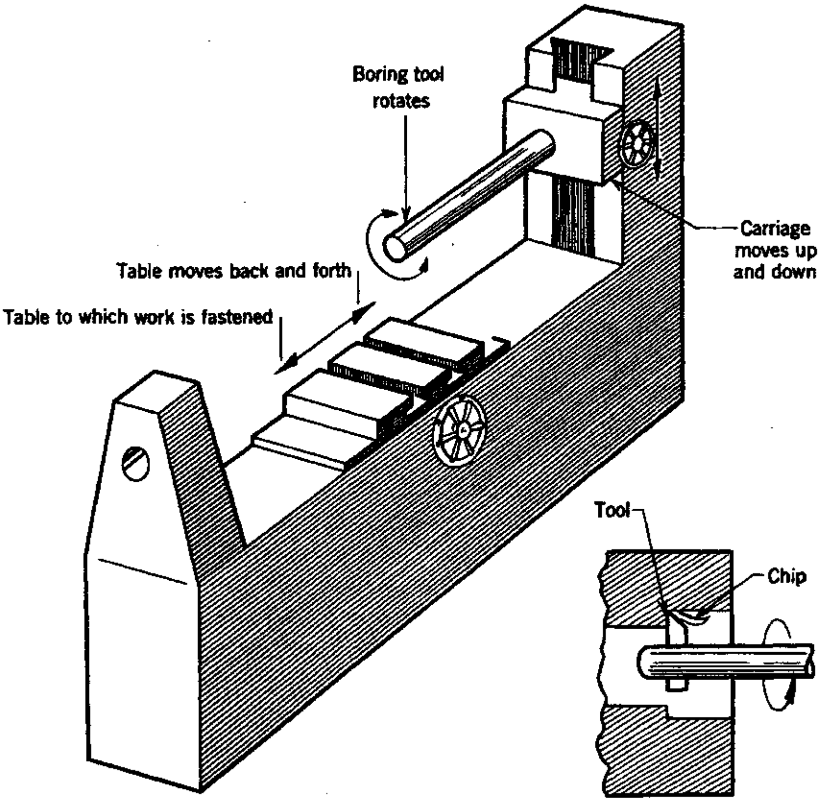


FIGURE 2*H*. Horizontal boring mill.

tools used in our analysis. This classification is slightly more aggregate by type than the *American Machinist* classification, but distinguishes size breakdowns within these categories. Thus, to use the present analysis for estimating the capabilities of the metalworking industries would require an "inventory" of metalworking equipment which included size detail. These, and more difficult data requirements, will be discussed in a subsequent section.

MACHINE-TASK PRODUCTIVITY ESTIMATES

Column 6 of Table F, illustrated in Chapter 14, presents estimates (circa 1954) of the rates at which various machines can perform various tasks. Column 1 of Table F identifies the machine, using the numbering system presented in Table D. Column 2, Table F, identifies the task, using the numbering system presented in Table E. Column 6 estimates the number of pieces upon which the specified task would be performed by a man assigned to the specified task for an 8-hour day.

These estimates of "pieces per day" for the given machine-task combination are derived from estimates of five components of productivity; namely:

1. *Machining time,* required for actual metal removal. Estimates of the number of minutes required to perform the task on one piece are presented in column 3, Table F.
2. *Handling time,* required for the insertion and removal of material and the adjustment of equipment between successive pieces. Estimates of handling time required per piece are presented in column 4 of Table F.
3. *Setup time,* required for the initial preparation of the equipment including any teardown of the previous setup. Estimates of the minutes required per lot are presented in column 5 of Table F. This time per lot must be prorated over lot size in obtaining the final productivity figures in terms of pieces per day.
4. *Inspection time,* during which the machine operator is not performing some other component of the task because he, or an inspector, is testing compliance with specifications. This does not include the inspection time spent (e.g., at a special inspection station) during which the machine operator continues with other components of this or other tasks. Estimates of minutes per piece required for inspection, as a function of the precision required by the task, are presented in Table 3, page 341 of the present chapter.
5. *Allowance time,* allowed for fatigue, personal requirements, and other work delays. Three components of allowance time are presented in Table 3, page 341. These figures are combined in Table 4, page 343, to form estimates of the total number of minutes, during an 8-hour day, available for machining, handling, setup, and inspection.

The relationship between the final productivity figures and the above five components of productivity is the following:

$$\text{output} = \frac{\text{productive minutes per day}}{\underset{\substack{\text{(minutes}\\ \text{per}\\ \text{piece)}}}{\text{machining}} + \underset{\substack{\text{(minutes}\\ \text{per}\\ \text{piece)}}}{\text{handling}} + \underset{\substack{\text{(minutes}\\ \text{per}\\ \text{piece)}}}{\text{inspection}} + \underset{\substack{\text{(minutes}\\ \text{per}\\ \text{piece)}}}{\text{setup}}/\underset{\substack{\text{(pieces}\\ \text{per}\\ \text{lot)}}}{\text{lot-size}}}$$

The separate tabulation of various components of productivity facilitates changing the final productivity figures to reflect changes in time, place, and classification. Procedures and problems involved in estimating these components are presented in a later section of this chapter.

Potential Applications

Subsequent sections discuss proposals for estimating task requirements (t_{jk}) for the jth task per unit of the kth final demand, as well as describing procedures used in estimating the rates (α_{ij}) at which the ith machine performs the jth task. In the present section we shall consider the use of these estimates (plus certain other information) for what may be described as a "static analysis with substitution." We begin with a discussion of units of measurement, a topic central to the understanding of both estimation procedures and applications.

The α_{ij} presented in Table F are measured in terms of pieces per day. We should note carefully: what kind of "pieces," what kind of "days," and what these units of measurement mean for the other magnitudes of the analysis. A "piece" is really shorthand for the requirement to perform, for one piece, one operation at one machine (or a series of operations usually requiring only one machine setup). The piece is standard with respect to its own size, shape, hardness, and precision characteristics, with respect to the size of lot of which it is a part, and with respect to the amount of work to be done on it in the particular operation. Nonstandard "pieces"—more precisely, tasks which are nonstandard because of the characteristics of the part, the size of lot, or the amount of work to be performed in the operation—are counted as a certain number of units of standard, depending on the relative time requirements for the standard and nonstandard tasks. Thus, "pieces per day" would be more accurately designated "standard task units (STU) per day." The units of measurement of final demand (f_k) are dollars per year. (For convenience, some multiple of this unit, e.g., $1 million per year, is used numerically.) The task requirements (t_{jk}) per unit of final demand are measured in terms of "pieces per dollar," where "pieces" again refers to the standard task units (STU). The total requirement for the jth kind of task is estimated by

$$t_j = \sum_k t_{jk} f_k.$$

Multiplying out the units of t_{jk} and f_k,

$$(\text{pieces/dollar}) \cdot (\text{dollars/year}),$$

we confirm that the task requirements (t_j) are measured in pieces (STU) per year.

The measurement of the α_{ij} depends on the definition of the standard "day," as much as it does on the standard piece. The "day" referred to in Table F is an 8-hour day of a man assigned to a machine. The number of productive minutes in the day, shown in Table 5 and used in the computation of Table F, deducts (from the 480 minutes of 8 hours) a rough estimate of time requirements for personal allowances for the man and maintenance for the machine. It does not take into account, however, the time that a machine is idle because labor has not been assigned to it. In our static analysis with substitution this loss of machine time is reflected in the machine availability figures. Machine availability, as used in conjunction with α_{ij}'s, will be measured in terms of "days per year." This is estimated by multiplying number of machines by days per year per machine. Thus, if 10,000 machines of a particular type provide, on the average, the equivalent of 200 full working days per year, then the time available on this machine, to be used for various tasks, equals

$$(10{,}000 \text{ machines}) \cdot (200 \text{ days per year per machine}) = 2.0 \text{ million days per year}.$$

We will let M_i represent the number of days per year available from existing machines and N_i from newly purchased machines. The costs of new equipment must be divided by "days per year per machine" to form (P_i) so that

$$I = \Sigma P_i N_i$$

will estimate total investment measured in dollars.

A substitution analysis can be used to trace out curves, like those in Figures 1 and 2 of the preceding chapter, showing the combinations of two end item programs that would require the same amount of new investment. To be most closely comparable with the curves of the last chapter, the one generated by the substitution analysis should be based on a normal or average days-per-year-per-machine. If expensive machine A is typically used two shifts per day for 350 days a year, while inexpensive machine B is used the equivalent of ½ shift a day for 250 days per year, then their respective machine availabilities would be calculated on the basis of 700 and 125 days per year per machine. If each task could be performed by one and only one machine then a "substitution analysis," with machine availabilities defined in this manner, would be the same as a requirements analysis. Insofar as tasks can be performed by several machines, the curve generated by the substitution analysis will tend to be higher than that of the requirements analysis as machines in excess come to the aid of bottleneck machines.

The tracing out of the curve for the substitution analysis can be expressed as a parametric linear programming problem. Let X_{ij} represent the number of days of the ith type of machine devoted per year to the jth task. Let T_{j1} and T_{j2} represent the requirements for task j per unit of the two programs F_1 and F_2. The iso-investment curve, for investment level I_0, is traced out by maximizing program level F_2 for all levels of F_1 subject to the constraints

$$\sum_j X_{ij} \leq M_i + N_i \qquad \text{[for each machine type } (i)];$$

$$\sum_i \alpha_{ij} X_{ij} = T_{j1}F_1 + T_{j2}F_2 \qquad \text{[for each task type } (j)];$$

$$\sum_i P_i N_i = I_0.$$

Labor and other operating costs increase as we substitute machines which are less efficient in performing a task for machines which are in short supply. A particular set of final demands, for example, might be achievable with no added investment but with high operating costs. The same final outputs per year, on the other hand, could be achieved with a minimum of operating costs by investing substantial amounts in machines best suited to the required tasks. Between these two extremes various other combinations of operating costs (C) and investment costs (I) could also achieve the specific final demands. If we are willing to assume that operating costs for machine i are a constant (per day of utilization per year) up to some limit of availability $(M_i + N_i)$ then the tracing out of possible combinations of operating costs versus investment costs becomes a parametric linear programming problem requiring the minimization of C for every value of I subject to the following constraints:

$$\sum X_{ij} \leq M_i + N_i \qquad \text{[for all machines } (i)\text{]};$$

$$\sum \alpha_{ij} X_{ij} = T_j \qquad \text{[for all tasks } (j)\text{]}$$

where

$$T_j = \sum_k t_{jk} f_k,$$

$$I = \sum P_i N_i,$$

$$C = \sum_i c_i \left(\sum_j X_{ij} \right).$$

In fact, however, operating costs are not linear up to the limit of machine availability. As the utilization of a machine increases, its unit operating costs increase—partly because of queueing phenomena, partly because of differences in wages and efficiencies between prime and nonprime shifts. If such increases in cost can be approximated by convex, piecewise linear cost curves, they can easily be incorporated into the linear programming analysis. At present we do not know the shape of such curves, or whether or not curves of this sort are sufficient to characterize variations of operating costs with changes in machine loads. In the next chapter we discuss the possibility of using simulation techniques for determining such relationships.

The linear programming problems posed above have a relatively large number of equations and variables. A machine tool substitution analysis based on Table F, showing about 1200 machine task combinations for 115 machines, and 142 tasks, would give rise to linear programming problems of about 257 $(= 115 + 142)$ equations with 1200 variables. These problems have a quite special structure, however, which can be exploited in building efficient computing procedures. A paper on "X_{ij} procedures,"[3] for example, describes an algorithm for a problem that was of interest to us when Table F was first built. With these procedures the IBM 701 (extremely slow by current standards) could trace out complete parametric solutions to the 257-equation-by-1200-variable problem in 1 to 3 hours. Similar procedures, adapted to the problems described above, should be many times faster on more recent computers.

The Decomposition Principle[4] of Dantzig and Wolfe can be used to connect sectors of a multisector linear programming analysis. The individual sectors can each use computing procedures appropriate to themselves, even though other sectors may not have the same structure. Thus, special X_{ij} procedures can be used to allocate machines to tasks within an analysis including industries which cannot use these procedures generally.

[3] Harry Markowitz (1955).

[4] George B. Dantzig and Philip Wolfe (1961).

The Rate of Generating Properties

The rates of performing the tasks described in Table E were based on information available in texts on machine shop estimating[5] and on standard data obtained from a number of manufacturing companies in the Los Angeles area. Differences in the data were due, in part, to the fact that in some sources the time for machining, handling, and setup included allowances for tool breakage, etc., while in other sources such allowances were kept separate. Another cause of variability was due to differences in the evaluation of fair and reasonable rates of output to be expected from man-machine combinations in performing given jobs. Insofar as possible the data were adjusted to provide consistent estimates.

MACHINING TIME. This time represents that portion of the total work cycle in which the equipment is engaged in the removal of metal. Although the estimation of machining time is often considered relatively accurate, it is not without difficulties. Metal removal rate depends on:

(a) the speed at which the machine rotates the tool or the commodity, i.e., the cutting speed, which is the relative motion of the tool and the commodity,
(b) the depth of the cut, and
(c) the feed or speed of tool travel.

These factors, in turn, depend on the horsepower, age, condition, and size of the machine, on the kind of metal being cut, on the type of lubricant being used, on the type of cutting tool being employed, etc. As a result, machining times differ from one firm to another, and are not constant for any given operation.

Some assumptions used in computing machining times included:

1. Material: aluminum or hardened steel where indicated.
2. Tooling: high speed steel.
3. Cuts: function of size and horsepower of the machine.
   a. typical cuts:
      (1) semiprecision, 2 rough cuts, 1 finish.
      (2) precision, 1 rough cut, 1 finish.
      (3) high precision, 1 finish cut.
   b. drilled holes—single cut.
   c. multiple tools—average of 4 tools.

To illustrate the method used to compute machining times consider, for example, task number one (small flat surface, no contours, semi precision tolerance, short run lot size). Referring to Table 1 the tolerance is ±.010 in. and

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[5] W. A. Nordhoff (1947).
A. A. Hadden and V. K. Genger (1954).
F. H. Colvin and F. A. Stanley (1940).
A.S.T.E. (1949).
D. W. Boston (1951).

depth of cut is .500 in. The size of part falls in the category of 1 in. x 1 in. to 11 in. x 12 in.

The cut depends on whether the part is flat or cylindrical. If flat, it is assumed to be a 5 in. x 5 in. cut; whereas, if it is a 5 in. diameter cylinder, the length of cut is assumed to be 5 in. Referring to Table 2, typical tasks performed on machines can be determined, and by referring to the size or swing of machine, the machines in each class which can do the task can then be determined.

The time to perform this task is determined either by reference to tables of standard data, reference to textbooks on machine shop practice, or by computation using a formula such as:

$$\text{machine time} = \frac{\text{length of cut in inches}}{\text{feed in inches} \times \text{revolutions per minute}}$$

Since tables of standard data are considered proprietary information by many companies, the example shown here will refer to tables in Nordhoff's *Machine Shop Estimating*.[6]

Three of the machines which can perform task number one are machine 77 (bench lathe), machine 85 (turret lathe), and machine 93 (shaper). Using the tables in Nordhoff, the following data were obtained:

Machine	Feed	Time per Inch	No. of Cuts	Length of Cut	Machining Time	Page Reference
77	.015 in.	.087 min.	2	5 in.	.87 min.	p. 212
	.125	.010	1	5	.05	
85	.012	.056	2	5	.56	p. 141
	.030	.022	1	5	.11	
93	.050	.330	1	5	1.65	p. 330

Summarizing these data, the times for machining task number one are:

Machine	Time
77	.92 min.
85	.67
93	1.65

The machining times shown are typical of the ones in Table F; however, the exact value used depended both on allowance for size of the machine and weighting by data from other available sources. For some of the machines, only a single source of data was available; although in most of the instances several sources were used. Thus, the times shown in Table F represent a composite of industrial standards and data from published books.

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[6] McGraw-Hill, 1947.

HANDLING TIME. The determination of the handling time component (Table F, column 4) of the total time is difficult since it is a manual portion of the operation and, as such, is less predictable. It depends on a number of factors, such as the arrangement of equipment, the placement of raw material, the means of disposing of completed parts, the type of jigs, fixtures, or special tooling, the availability of lifting devices such as hoists, the design of controls and levers on the equipment, and the pace or exertion of the operator.

In each task category, assumptions had to be made to each of these factors. For instance, it was assumed that there was an average time to perform each portion of the work, such as the time to start and stop the machine. Where the task involved working on large heavy parts, it was assumed that suitable lifting equipment was available. Special tooling, such as jigs and fixtures, was assumed for the more accurate tasks and for long-run tasks. For example, where a hole had to be drilled very accurately, it was assumed that a drill jig would be used. An average value was used for the time of handling a drill jig, since the time would vary according to the method of clamping, locating, etc.

SETUP TIME. The same difficulties arise in estimating the time necessary to prepare equipment for performing the various machining operations as in handling time. There is an additional problem, however, in that the setup is performed only once for each lot of material run and thus is not as well standardized as handling time. Setup may often include teardown time, which is the removal of tooling employed for a previous task. On the other hand, there are times when two successive tasks are so similar that relatively little setup time is required.

INSPECTION. The time to check whether the work met specifications was based solely on the precision or accuracy classification. Since the final dimensions of a piece of work depend both on machine adjustment and tool wear, it is necessary to periodically check the work piece. The frequency of checking is based on the accuracy required and was assumed inversely proportional to tolerance. Thus, the smaller the tolerance, the larger the inspection allowance.

There are instances where the machine operator does not require an inspection allowance since he can gauge the work while the machine is running. However, it was felt that this would be difficult to determine; therefore, it was assumed that all machining required a separate inspection by the operator and would be done when the machine was stopped.

DELAY ALLOWANCES. Allowance is made for that portion of the work day that is unavailable for productive purposes. For example, there are delays due to machine or tool breakage, material shortages, lack of work, personal requirements, and rest periods (in this analysis only scheduled or short delays are considered in contrast to long machine breakdowns). The average amount of time lost because of these delay factors depends on the particular firm and the kind of work done. In most firms, there is a lack of accurate information concerning this lost time, although techniques are available which provide efficient estimates of the average amount of delay (e.g., the work sampling technique).[7] In this analysis, however, arbitrary percentages were used to estimate the amount of time lost due to delays.

In computing the lost time, the kind of machine being used, the accuracy of the finished part, and the size of the finished part were considered. The larger the part, the more the physical effort was assumed, and hence the larger the allowance. Precision work is exacting, and hence a larger allowance was given for this class of work. Also, the time lost due to scrap or bad parts had to be accounted for by an additional allowance. The allowances for various tasks are shown in Table 3.

TABLE 3
ALLOWANCES AND INSPECTION TIME

I. Allowances

A. Fatigue Allowances

Task	Tolerance	Per Cent Allowance
1. Automatics, light work	.001″ tol.	5%
2. Power feed	.001″	10%
3. Power feed, short cycle	less than .001″	15%
4. Hand feed	close tol.	20%
5. Heavy work	hazards	25%

B. Personal Allowance

A constant personal allowance (time required for the personal needs of the worker) was assumed equal to 5% of the work day or .05 × 480 = 24 minutes per day.

C. Work Delay Allowance (including tool change time)

Semi-precision	4%
Precision	6%
High precision	8%

II. Inspection Time

	Single Cut	Multiple Cut
Semi-precision	.01 min./pc.	.02 min./pc.
Precision	.15	.60
High precision	.60	2.00

COMPUTATION OF TOTAL TASK TIME. The manner of computing the time for a given task can be illustrated by a specific example; e.g., drilling a very small, semiprecision hole (task 78) on a drill press (machine 26). In the references on machine shop practice,[8] the recommended cutting speed for

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[7] Barnes (1957).

[8] Barnes (1957).

aluminum is 300 feet per minute or 1146 revolutions per minute. Referring to the formula previously shown, the time per inch of cut is:

$$\text{machining time} = \frac{\text{length}}{\text{feed} \times \text{rpm}} = \frac{1 \text{ in.}}{.070 \times 1146} = .13 \text{ minutes}$$

This is based on a ½ in. depth of cut and ½ in. lead and breakthrough. The possible feed varied from .070 in. to .022 in., thus leading to many possible machining times. Since the purpose here is to be representative, the values shown in Table F were heavily weighted by the available industrial standard data, rather than by computation as shown above.

A typical set of elements for machine setup and handling are shown below, based on information from Nordhoff, page 250:

Setup Time	
1. Check job	1.00 min.
2. Study drawing	1.00
3. Obtain tools	5.00
4. Install jig	6.80
5. Adjust speed	.80
6. Install drill	1.60
7. First part, inspection	.80
	17.00 min.

Handling Time	
1. Pick up part	.120 min.
2. Position work	.060
3. Set aside part	.050
4. Clean jig	.050
	.280 min.

The times are summarized as follows:

1. Machining time on a bench drill, 0.13 minutes/piece.
2. Handling time on a bench drill, 0.28 minutes/piece.
3. Setup time on a bench drill, 17.00 minutes.
4. Inspection time, 0.01 minutes/piece.
5. Productive minutes available/day, 365 minutes.
6. Lot size, 10,000 pieces.

Computations required:

1. Prorated setup time $= \dfrac{17.00 \text{ minutes}}{10{,}000 \text{ pieces}} = .0017$ minutes/piece.
2. Total time per piece $= .13 + .28 + .0017 + .01 = .4217$ minutes/piece.
3. Estimated production per day $= \dfrac{365 \text{ minutes/day}}{.42 \text{ minutes/piece}} = 869$ pieces/day.

The final table of values (Table F, column 6) contains the rates at which the various machines can perform their respective tasks. These final values

TABLE 4

Productive Minutes Available per Day

(480—less fatigue, personal, and work delay allowances)

	Very Small and Small			Med., Large, and Very Large		
	Short and Long Run			Short and Long Run		
	Semi-Prec.	Prec.	High Prec.	Semi-Prec.	Prec.	High Prec.
Boring machines—horizontal	389	355	346	365	331	322
Boring machines—vertical	389	355	346	365	331	322
Precision boring	389	355	346	365	331	322
Jig boring—horizontal and vertical	389	355	346	365	331	322
Broaching—internal and external	389	355	346	365	331	322
Sawing, cut-off machines and bandsaws	365	331	322	341	307	298
Tapping and threading machines	365	331	322	341	307	298
Sensitive and upright drills—single spindle	365	331	322	341	307	298
Radial drill	365	331	322	341	307	298
Multiple spindle drills (9–60 spdls.)	413	379	370	389	355	346
Automatic drills (1–10 stations)	413	379	370	389	355	346
Gear hobbing	389	355	346	365	331	322
Gear shapers	389	355	346	365	331	322
Gear mills and planers and thread mills	389	355	346	365	331	322
Gear finishing (grinding, lapping, shaving)	389	355	346	365	331	322
Grinding, cylindrical—internal and external	389	355	346	365	331	322
Centerless grinders—internal and external	365	331	322	341	307	298
Surface grinding	389	355	346	365	331	322
Misc. grinders—disc, thread, tool, snag, bench, etc.	365	331	322	341	307	298
Honing—internal and external	389	355	346	365	331	322
Lapping—internal and external	389	355	346	365	331	322
Lathe—bench and light floor	389	355	346	365	331	322
Heavy duty lathes	389	355	346	365	331	322
Turret lathes	389	355	346	365	331	322
Automatic chucking	413	379	370	389	355	346
Automatic screw machines	413	379	370	389	355	346
Shapers and slotters	389	355	346	365	331	322
Milling machine—horizontal and vertical	389	355	346	365	331	322
Milling machine—planer and automatic	389	355	346	365	331	322
Profilers, duplicators and die sinkers	365	331	322	341	307	298
Planers—openside and double housing	365	331	322	341	307	298

indicate the expected productivity per 8-hour day by a qualified worker operating the machine tools at average speeds.

MACHINE AVAILABILITY AND TASK REQUIREMENT DATA

The present section considers possible sources for data to be used in conjunction with the productivity estimates discussed earlier. The need for additional data depends on the particular application. Chenery,[9] for example, has used the data of Table F to show how methods of production should vary from country to country, depending on economic lot size and the relative prices of labor and capital. Analyses such as this require little or no data in addition to that in Table F. The analyses discussed in the preceding section, on the other hand, require two types of information which are not readily available at present. These are

1. equipment availabilities broken down by size category, and
2. estimates of task requirements per unit of final demand.

The unavailability of such data at present does not mean that they could never, or will never, be collected. The questions we ask of our economy (regularly or occasionally) change with time as possible applications of such data emerge. What we consider here is the kinds of questions one might ask to obtain the above information, and the extent to which this would be a burden to the Asking and the Asked.

The estimation of equipment availabilities can be divided into two parts: the estimation of the number of machines, and the estimation of available days-per-year-per-machine. To begin with we shall discuss the former problem, the numbers of machines. The latter problem, the availability factor, will be discussed along with the t_{jk} later in this section, and will be considered again in connection with simulation techniques in the next chapter. For the potential applications discussed earlier the numbers of machines were not needed by industry, but only for the economy as a whole. Usage detail by industry would be valuable, however, for at least two reasons. First, it provides check totals for the estimation of task requirements (t_{jk}) per unit of final demand. Second, it would add important size breakdowns to the metalworking requirements analysis discussed in the last chapter. The latter use is probably sufficient to justify the collection of such data, independent of its use in the analysis of equipment substitution possibilities.

Several methods can be suggested for obtaining greater detail on metalworking equipment. First, the *American Machinist* questionnaire could be expanded. The 1953 questionnaire, covering two sides of a printed 8¼ x 14 in. form, provided places to record the number of units of equipment in each category. The inclusion of size detail for all classes of metalworking and related machinery might, roughly, triple the size of the questionnaire. In the case of a voluntary poll, as conducted by the *American Machinist*, this increase

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[9] H. B. Chenery (1957).

in bulk might seriously reduce the response percentage. Even with a mandatory poll, as the Census conducts, it is desirable to consider alternatives which might reduce the nuisance to the respondent. A second approach could involve a system of questionnaires such as the *Census of Manufactures* uses[10] to collect special data (e.g., material consumption and workers in metalworking operations) where the nature of the inquiry is tailored to the type of industry. In a like manner, equipment questions could vary by industry, e.g., by not asking for size breakdowns for kinds of equipment the industry has never owned, or for numbers of equipment in size categories which are clearly unreasonable in light of the product. Another approach would be to follow the lead of the survey of metalworking capabilities in York, Pennsylvania,[11] conducted during the Korean War. Among other things, this survey had establishments list individual machines, including pertinent size and precision characteristics. If a national survey of metalworking equipment had responding establishments list kind of equipment and (depending on kind) specific characteristics for each machine, this information could be punched onto cards and summarized by computer. This last approach would provide the greatest flexibility for later reaggregating the same information by different categories as new analysis needs arose.

Still another possibility would be to use a combination of the above, such as one form for tabulating the number of common types and sizes of equipment plus another form for listing individual detail on larger or less common equipment. The final choice of procedure should of course depend on experience with field tests using one or more possibilities.

REQUIREMENTS FOR TASKS. We shall present two possible approaches to the problem of estimating direct task usages. (The problem of estimating total task usages from such direct task usages is like the previously dealt with problem of estimating total usages of men, machines, and materials.) The first approach to direct task usage we propose for consideration is based on a questionnaire for supervisors of shop operations. This questionnaire would ask them—not for task requirements—but for information from which task requirements can be derived. The second approach involves information regularly generated for production control purposes. If a representative sample of establishments were willing to make such information available, then fairly accurate task requirement figures could be obtained with a moderately large research effort. In the case of task requirement information there are more than the usual reasons for field testing any theory on how to collect appropriate data. After we discuss the two suggested approaches, we will note possible important results of field testing one or both.

THE QUESTIONNAIRE APPROACH. The first approach is to question personnel —such as shop foremen—who are directly involved in the supervision of

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[10] Vol. I, Appendix C.

[11] *Survey of Production Machinery, Factory Space and Manpower, York, Pennsylvania,* (1951).

FIGURE 3. Questionnaire

This sheet applies to the
following type of equipment:
How many machines of the above type do you supervise............________
During the shift you supervise what
percentage of the time of these
machines is spent as follows:
Maintenance (scheduled or unscheduled)..........................________
Idle, because labor has not been assigned to the machine..........________
Setting up for or performing the following operations:
1.
.................________
2.
.................________
3.
.................________
Other operations (please note any
large uses of time on the
attached sheet) ....________
Other uses of time not listed above
(please note any large uses of
time on the attached sheet) ....________

machine operations. A questionnaire, such as roughly sketched in Figure 3, would inquire as to the approximate extent various kinds of machines spend time in various kinds of activities. The respondent would be provided one sheet of this type for each kind of machine in the area he supervises. Before being given to the respondent, the sheets would have descriptions of operations as well as equipment filled in. Ideally, small drawings illustrating configuration as well as verbal statements concerning configuration, dimension, tolerance, hardness, and lot size ranges for each type of operation would be preprinted on forms separately designed for each type of equipment. Rare machine-task combinations, distinguished in Table F for possible use in extreme cases, would be omitted from the questionnaire. The adequacy of the categories provided would be checked by questions concerning "other operations" and "other uses of time." The questionnaire should be filled out during nonworking hours, perhaps for a nominal compensation for the expected time (probably a fraction of an evening) usually required.

The questionnaire addresses itself to the estimation of available time per machine, and to the estimation of the proportions of the machine's time devoted to the performance of various tasks. Such information for a sample of establishments in each industry—together with the total number of machines of each type in each industry, perhaps based on a more complete sample—allows us to estimate:

X_{ijk} = the number of days machine i was used for task j in industry k during some period of time.

The generation of task j (measured in pieces, STU) by all machines in industry k is estimated by

$$T_{jk} = \sum_i \alpha_{ij} X_{ijk}$$

where α_{ij} is the entry of Table F for machine i, task j. The T_{jk} are, in effect, direct task usages comparable to the direct usages of men, machines, and materials discussed in the last chapter. The discussion there, concerning the derivation of total usages from direct usages, applies here as well.

FORMING "CONSERVATIVE" ESTIMATES. Since individuals can rarely give an exact estimate of the time they (or those they immediately supervise) spend on various types of work, the responses to the questionnaire are subject to error. If the results of such a survey are used in a "static analysis with substitution" they may overstate or understate the amount of substitution possible between machines. They cannot understate these possibilities more than a requirements analysis does, however, since a requirements analysis allows for no substitution. It is possible to modify the results of the survey so as to be virtually sure that the substitution analysis understates the amount of substitution, but of course by less than the requirements analysis understates it. In this case we can argue that the introduction of substitution possibilities is a conservative step in the right direction, providing results at least as good as the requirements analysis. For this purpose we invent a set of "fictitious tasks," one task corresponding to each machine. Each of these tasks can be performed by "its" machine only. To produce conservative estimates of substitution possibilities we form new estimates X^*_{ijk} (of the time machine i spent on various tasks j in industry k) by multiplying the original estimates by a fraction θ:

$$X^*_{ijk} = \theta X_{ijk} \qquad \text{(for all } j\text{)}.$$

The remaining usage of machine i, namely $(1 - \theta)$ times available time, is ascribed to the special task that only machine i can perform. For $\theta = 1$, we have a substitution analysis based on the original estimates. For $\theta = 0$, we have a requirements analysis. As θ decreases from 1 to 0 we have increasingly conservative estimates of substitution possibilities—estimates which tend to understate substitution possibilities, but not as much as a requirements analysis does.

THE "PRODUCTION CONTROL PAPER" APPROACH. Our second proposal for the estimation of task requirements relies on paper regularly generated as a result of the division of labor between production planning and actual shop operations. This division of labor is almost universal in the moderate to large size establishments which account for most metalworking production. Typically, the nature of every operation on every piece to be fabricated is determined by a production planner or process engineer, and is recorded on routing or operation sheets together with standard time data, material requirements, and any special tooling information.[12] Increasingly this basic information is being punched on cards; but, in any case, must be in a form permitting easy reproduction. Copies or summaries of this basic data are used for a variety of purposes: to indicate the material which must be supplied to the part's first shop; to indicate which shop the partially completed lot goes next; to tell the operator what operation he is to perform, which blueprint has the exact specifications, and what (if any) special tooling is to be used. The production planning information for the part may also be used to analyze the load waiting behind (or expected for) machines or work stations, and may be used as the basis for determining wages in the case of incentive plans.

Such operation information could be used for estimating task requirements in roughly the following manner:

> For each of a sample of establishments, draw a sample of products.[13]
>
> For each product generate all (or in the case of large products, a sample) of the operation or route sheets needed to characterize the nature of the operations and the standard times for each. These operation sheets are, of course, copies—generated by whatever mechanism the establishment would normally use to obtain copies for various uses.
>
> A team of analysts, familiar with both shop practices and the classification of tasks, codes task classification numbers opposite each operation.
>
> The coded task categories plus operation times and product codes are punched onto cards, so that the remaining steps of the estimation process (such as the weighting of the operation figures by the relative demands for the products) can rely on automatic data processing.

The whole procedure must be designed so as to completely protect information considered proprietary.

FIELD TESTING. A limited field test of one or both proposals might contribute substantially to our understanding of the feasibility and desirability of larger efforts along these lines. For the questionnaire of the first proposal, the field test might show ways of making the task descriptions more concrete and easily understood. The attempt to implement the second proposal in a few cases might develop efficient data collection procedures as a function of the type of production control paper used. If both proposals were tried, for some establishments, then differences in costs and results could be compared to determine the more desirable approach. There is also the possibility that

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[12] See the index entries on Route Sheets and related topics in the following or other books on production control:

L. P. Alford and John R. Bangs (1946).

Franklin G. Moore (1951).

Robert A. Pritzker and Robert A. Gring (1960).

[13] This description is applicable to establishments which make products rather than perform metalworking services. A modified procedure would be required in the latter case, perhaps based on a sample of work orders received.

these exploratory efforts might support one or another assumption which could substantially short-cut the estimation of task requirements. Suppose it turned out, for example, that for any particular machine the proportions among its utilization for different tasks did not vary considerably from industry to industry. True, machine A did different tasks than machine B; and industry I had more A and less B, in contrast to industry J. But the tasks which A did for I were about the same as A did for J, and similarly for B. If this turned out to be the case task requirements by industry could be inferred from the equipment used by the industry plus a set of typical machine-task usage proportions estimated from a much smaller sample than would otherwise be needed.

The individual establishment might find its task requirement estimates valuable for its own manufacturing analysis, particularly in the area of equipment selection. For a particular equipment expansion or modernization proposal, future operating costs versus future additional investment cost curves could be drawn for various possible shifts in product mix. This computation could be repeated for several alternate equipment expansion proposals to determine which would provide greatest flexibility for a given level of present investment outlays. For this purpose the task requirement data would have to be aggregated by products or product lines. (The coarser data needed for an economy-wide analysis could be obtained by further aggregation.) Rather than use an aggregate table of productivities, such as provided in our Table F, the analysis at the establishment level would use standard data based on the establishment's own operating practices and the characteristics of specific (present or proposed) machines. With electronic computers increasingly available within industry, and with the quite small computing times required for moderate size analyses once the data is available, the field tests discussed above might be justifiable as research into manufacturing analysis techniques, with insight into the capabilities of the metalworking industries produced as a joint product.

THE LOT SIZE PROBLEM

The formula used in estimating requirements for task j,

$$T_j = \sum_k t_{jk} f_k,$$

in effect assumes that changes in numbers of pieces required due to changes in final demands (f_k) are achieved by changes in the number of lots produced per year, lot sizes remaining unchanged. This assumption is in error insofar as changes in f_k produce changes in lot size. On the other hand, moderate variations in final demands—say the doubling of some at the expense of others—will probably lead to small errors due to lot size changes. In the first place, a change in final demand will generally result in a less than proportionate change in lot sizes:

The increase in final demand is likely to be accompanied by product differentiation. Not only is more of product k demanded, but also more kinds of product k.

The increase in final demand is likely to increase the number of establishments directly or indirectly producing k.

Even if the demand for the production of a specific component by a specific establishment were to double, lot size would usually increase less than proportionately. For example, if the traditional Economic Lot Size Formula is used, a doubling of annual production would multiply lot size by $\sqrt{2}$, an increase of about 41%, and would increase the average number of lots per year by the same factor.

In the second place a change in lot size does not lead to a proportionate change in "pieces per day."

Let M = productive minutes per day,
S = setup time in minutes,
U = unit time in minutes, including both handling time and machine time,
L = lot size (before the increase), and
$R = S/LU$ = the ratio of time spent setting up to time spent producing pieces after setup.

The ratio (P) of pieces per day after a doubling of lot size to pieces per day before the increase is given by

$$P = \frac{\dfrac{M}{S/2L + U}}{\dfrac{M}{S/L + U}} = \frac{2R + 2}{2R + 1}.$$

Thus if $R = 1$, i.e., if originally as much time was spent setting up as was spent processing, then $P = 1.33$. The doubling of lot size in this case leads to a 33% increase in pieces per day. On the other hand if $R = 10$, e.g., if one hour of setup was followed by ten hours of processing, then $P = 1.05$. The doubling of lot size leads to a 5% increase in the number of pieces per day.

The compounding of the above tendencies—product differentiation, changing numbers of establishments, changing numbers of lots per establishment, and less than proportionate effects of lot size on final pieces per day—should generally insure that moderate changes in final demands do not introduce serious inaccuracies in productivity estimates.

The lot size problem is nevertheless worth more consideration than we have been able to give it. It may be divided into

1. the problem of characterizing the distribution of lot sizes and how this distribution changes with changes in final demand, and
2. the problem of computing solutions when a large number (e.g., a continuum) of lot sizes are distinguished.

These two areas raise, in turn, interesting theoretical, empirical, and computational problems which seem substantial but not insurmountable.

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CHAPTER 13

FUTURE METALWORKING ANALYSIS

Harry M. Markowitz and Alan J. Rowe

It has been several years since the authors performed the analyses described in the preceding chapters, and (extrapolating from current activities) it will be at least as long before either of us actively engages in this area again. We would, however, like to devote a few pages to what seems to us to be a promising next step in the analysis of metalworking capabilities. Specifically, we wish to point out some potential applications of simulation techniques to the analysis of economy-wide capabilities.

We first discuss the nature of simulation, and its current use in manufacturing analysis. Afterwards we consider some of its potential contributions to the understanding of the metalworking industries as a whole.

SIMULATION TECHNIQUES

In the area of metalworking, simulation techniques are increasingly being applied to problems of manufacturing analysis. Simulation techniques are like optimization techniques (such as linear programming) in that both require a description of technological possibilities. Simulation differs from optimization, however, in that the former does not necessarily arrive at a best way of using production possibilities. A simulation analysis, rather, tries out different plans of action proposed to it—works through one or more "histories" using each proposed plan of action—and reports how well each plan scores with respect to a number of measures of performance. Since an analysis which can economically find a best strategy is generally preferable to one which simply evaluates specified proposals, optimization techniques are generally preferred to simulation techniques when the former are feasible. Simulation techniques, on the other hand, can be applied to problems which, at present, are hopelessly beyond our abilities to optimize.

We may visualize a simulation analysis in terms of what the human analyst gives the electronic computer, what the computer does with this information, and what the computer gives back to the analyst by way of results. In simulating a shop the computer must be given: a description of the shop; decision rules which prescribe shop action under all circumstances; and a description of orders to be processed by the shop. Let us consider each of these inputs in turn.

DESCRIPTION OF THE SHOP TO BE SIMULATED. The more advanced computer programs for shop simulation require an analyst to supply information such as the number of machine types, the number of labor classes, the number of machines of each type, the number of men in each labor class each shift, the possible assignments of types of men to types of machines, the basic hourly wage of each labor class, the premium for each shift, the length of each shift, etc. The simulation program allows these shop parameters to be varied from run to run.

DECISION RULES WHICH PRESCRIBE SHOP ACTION UNDER ALL CIRCUMSTANCES. Most shop simulation programs require a dispatch rule. Such a rule answers the question, "Suppose that a given man or machine can turn to one out of two or more different jobs; which of these alternatives should be selected?" Among the more commonly analyzed dispatch rules are:

First come first served.
Do the quickest job first.
Dispatch according to due date.
Do the most expensive job first.

In addition to dispatch rules the more complex manufacturing simulators may require decision rules in other areas, such as overtime rules to determine under what conditions labor should work extra hours, and alternate routine or alternate sequencing rules, to decide among the available options with respect to machine selection and operation sequencing. Such rules formalize actual or proposed operating procedures for running the shop.

DESCRIPTION OF ORDERS TO BE PROCESSED BY THE SIMULATED SHOP DURING A PERIOD OF TIME. In some simulation programs order information is read in, order by order, in the sequence these orders are released to the simulated shop. Information about each order includes routings and process times, and sometimes alternate routing, alternate sequencing, priority, and cost information. In other simulation programs the analyst supplies parameters of probability distributions from which the computer randomly generates order information as needed. For shops which make only a few standardized products, the routings and process times are supplied at the start of the simulation; then, during the course of simulation, the computer reads in orders by quantity and type, using the previously supplied information to determine routings and process times. Thus, in one manner or another, the analyst must characterize the jobs to be accomplished by the shop, including routings and process times, and any alternate routing or alternate sequencing possibilities which the shop may use.

The computer simulates by running the specified orders through the specified shop in accordance with the specified decision rules. At the beginning of a simulated shift it assigns men and machines to orders in accordance with the dispatch rule. As it makes these assignments it notes how long the order is to be processed and at what time processing is to be finished. When simulated time reaches this value the computer removes the order from its machine, sends it on to its next queue (or notes it as being completed) in accordance with its routing information, and allocates the man and machine (not necessarily to each other again) in accordance with the dispatch rule. This same working of the system through time could be done with pencil and paper. A human analyst could, in principle, keep track of the status of each item in the shop, applying the prescribed decision rules whenever a new action had to be decided upon, as he advanced the hands of the simulated clock to the point in time when the system next changed. Such simulation by hand is extremely tedious and time-consuming even for quite small shops. The computer, however, can trace the flow of large numbers of such orders through complex shops with fairly reasonable requirements for computer time.

As the computer (more precisely—as the simulation program directing the computer) moves the system through time, it keeps track of information concerning machine utilization, labor utilization, lengths and values of queues, the use of overtime or alternate routine, and the performance of the shop with respect to meeting its due dates. At the end of the run it labels and prints out this information, e.g., by simulated week and for the run as a whole.

To apply simulation successfully to a manufacturing problem, the analyst must conscientiously carry out a number of related analysis operations. Most, if not all, of these supplementary analysis operations were considered part of good practice even before the advent of simulation. In the analysis of the equipment selection problem for a new plant, for example, the analysis team must estimate typical operational requirements for various kinds of products within the product line, and one or more levels and mixes of possible demand. The analysis team must also design a good first guess as to a shop that could adequately handle such demands upon it. Before simulation, a static load analysis—which compared total demands for men and machines with total supplies of each—was used to judge the adequacy of the proposed shop. Since it was well-known that a shop cannot operate satisfactorily with a static load of 100% of capacity, judgment or "reasonable load" factors were applied. Even with the availability of simulation techniques the static load analysis provides useful information, particularly in forming the initial shop plan.

The static load analysis, however, frequently misses important dynamic interactions, and hence lets certain potential troublespots slip by undetected. A typical example is the following: Product A uses more of machine X than Y. Product B, on the other hand, uses more Y than X. Based on estimated total annual demand for A and B the static load analysis concludes that the shop is amply supplied with machines X and Y. But this may be an error, since at any one time the shop may be producing only A (with considerable idle time on machine Y), or only B (with considerable idle time on X). If both A and B are in the shop, their requirements for X and Y may dovetail—but to what extent? In actual shops the picture is further complicated by a multitude of types of work which route in various patterns through a variety of machines, perhaps to rendezvous with matching parts to route as subassemblies and assemblies. How, in such a maze of possible interactions, can we tell whether the available resources will keep work moving in a steady flow rather than in disrupted eddies and spurts? These are the questions to which simulation addresses itself.

A simulation analysis typically involves a number of computer runs. In equipment selection, for example, the first run would test the initial shop design against a likely level and mix of orders. This might show that the proposed shop had serious weaknesses even if the likely happened, and hence must be modified. The number of machines of various kinds would be altered in light of the queue and utilization experience of the first run. Perhaps, even after these corrections, the second run might reveal difficulties which suggest further modifications and further runs. It is not enough, however, to find a shop which works well for the likely level and mix of demands. It is important for the shop to remain manageable if possible alternate levels and mixes of demands hit it. Hence, further runs are required to test the flexibility of the shop. There is no assurance (in fact, no presumption) that this process reaches an optimal shop. It simply evaluates alternate proposals under a variety of conditions until—in the light of costs of analysis, need for a timely decision, and the apparent acceptability of the best solution thus far—it seems advisable to terminate the analysis.

When a simulation model is constructed for an existing facility the initial model can be run using approximately the actual shop configuration, decision rules, and orders for some past period of time. The results of this run can be compared with shop performance for the same period to see if the model reproduces the real world in important aspects with reasonable accuracy. For a complex shop the first runs of such a model frequently reveal some systematic error in data or some neglected aspect of shop performance that should be further investigated and incorporated into the model. Typically, after half a dozen or so runs, the model is deemed sufficiently accurate and is directed towards policy questions such as changes in shop configuration or operating procedures. In simulating a completely new facility, however, such testing of the model is not possible. Reliance on experience with similar plants, the use of somewhat conservative productivity estimates, and the use of the model itself to test its sensitivity to certain magnitudes are the principal means of treating this difficulty in modeling new facilities.

PROCESS ANALYSIS APPLICATIONS

The increasing application of simulation models to practical manufacturing problems should contribute to data and concepts needed for broader scope, industry-wide models. Not that the proper method for analyzing the metalworking industries is to build a detailed simulation involving 60,000 establishments. Rather the availability of simulation models permits us to test hypotheses concerning how such establishments can be characterized and thence aggregated.

One area in which simulation can contribute is that of static analysis with substitution. In the last chapter we presented a linear programming analysis describing direct substitution possibilities among machine tools. This analysis rested on the fact that the same task can be accomplished in more than one way. We can imagine more general linear programming analyses based on the fact that alternate sets of tasks can accomplish the same transformation of materials, or that alternate materials can be transformed into products serving the same ends. All such analyses have this in common: They portray substitution possibilities due to changes in actual, direct inputs per unit output. There is another major source of substitution among "requirements," however, which has to do with the extent to which various resources are fully utilized. Even if there were no substitution possibilities among direct inputs the actual use of resources by establishments would be effected by shift and overtime policy and by the ratios of load to availability for various resources. What we need is some way of characterizing substitution possibilities resulting from these sources. We need to know whether such substitution possibilities are substantial or negligible in magnitude. We need to know whether they can be characterized by relationships which, hopefully, could be incorporated into a linear programming analysis. Towards these ends simulation can serve as a laboratory in which to experiment with different shift and overtime policies, and with different ratios of availability to load. Just as the detailed simulation model can serve the manufacturing analyst in testing alternate shop configurations, it can also serve to test hypotheses of the analyst who seeks to characterize metalworking capabilities under a variety of circumstances.

Another area in which simulation can contribute is that of the *dynamics* of changing levels and compositions of output. It can do more than trace the effects of changing output levels in the production shop. Just as it is used to trace the flow of materials through various stages of production, it can also be used to trace the flow of pieces of paper through the preliminary steps of requisition engineering, drafting, tool design, and production and materials procurement.

CHAPTER 14

STATISTICAL APPENDIX ON METALWORKING

Harry M. Markowitz and Alan J. Rowe

The preceding chapters on metalworking refer to data contained in a *Statistical Supplement on Metalworking.* These data are not fully reproduced here because of space and cost considerations. The *Supplement* may be obtained from the Cowles Foundation for Research in Economics, Yale University, New Haven, Connecticut.

Tables A and B present the classification used in Table C. Tables D and E present the classification used in Table F. Chapter 10 (Table 9) and Chapter 11 refer to Table C; Chapter 12 to Table F.

TABLE A (*Complete*)

INDUSTRIAL CLASSIFICATION AND RESPONSE PERCENTAGES FOR TABLE C

	Industry	Response Percentage
19	Ordnance and accessories	20.4
25	Furniture and fixtures	11.8
332	Iron and steel foundries	21.1
336	Nonferrous foundries	18.8
3391	Iron and steel forgings	17.1
3393	Welded and heavy-riveted pipe	2.8
341	Tin cans and other tinware	3.8
342	Cutlery, hand tools, and hardware	21.1
343	Heating and plumbing equipment	10.4
344	Structural metal products	12.8
346	Metal stamping and coating	20.1
347	Lighting fixtures	21.8
348	Fabricated wire products	10.3
349	Metal products, n.e.c.	13.2
351	Engines and turbines	22.5
352	Tractors and farm machinery	34.5
353	Construction and mining machinery	26.5
354	Metalworking machinery	37.6
355	Special-industry machinery, n.e.c.	26.7
356	General industrial machinery	21.3
357	Office and store machines	23.4
358	Service and household machines	39.5
359	Miscellaneous machinery parts	18.1
361	Electrical industrial apparatus	26.0
362	Electrical appliances	36.4
363	Insulated wire and cable	8.5
364	Engine electrical equipment	43.1
365	Electric lamps (bulbs)	2.5
366	Communication equipment	23.0
369	Electrical products, n.e.c.	25.6
371	Motor vehicles and equipment	47.8
3721	Aircraft	12.7
3722–29	Aircraft engines, propellers, equipment, n.e.c.	38.3
373	Ships and boats	12.7
374	Railroad equipment	33.6
375	Motorcycles and bicycles	5.7
379	Transportation equipment, n.e.c.	15.3
381	Scientific instruments	11.1
382	Mechanical measuring instruments	40.0
383	Optical instruments and lenses	46.7
384	Medical instruments and supplies	29.5
385	Ophthalmic goods	39.4
386	Photographic equipment	7.7
387	Watches and clocks	10.0
39	Miscellaneous manufactures	8.7

TABLE B (*Extract*)

Equipment Classification Used in Table C

- Machine Tools
  - Boring Machines
    - Horizontal boring, drilling, and milling
    - Vertical boring mills including vertical turret lathes
    - Precision, horizontal and vertical
    - Jig boring, horizontal and vertical
    - Other
  - Broaching Machines
    - Horizontal internal
    - Horizontal surface
    - Vertical internal
    - Vertical surface
    - Rotary surface and other
  - Contour Saving and Filing
  - Cutoff Machines
    - Bandsaw
    - Abrasive disk
    - Hacksaws
    - Rotary, hot or cold
    - Other
  - Drilling Machines
    - Sensitive
    - Upright (single-spindle gang)
    - Radial
    - Multi-spindle cluster, adjustable and fixed center
    - Unit head and way type, multi-spindle
    - Deep-hole drills
    - Other
  - Gear Cutting and Finishing Machines
    - Hobbing machines
    - Shapers, gear
    - Cutters, form-milling type
    - Bevel gear cutters (not planer)
    - Cutters, planer type
    - Grinders, gear
    - Shavers, gear
    - Burnishing and lapping
    - Chamfering, tooth pointing, tooth rounding
    - Other
    - Gear checking machines
  - Grinding Machines
    - External cylindrical, plain and universal
    - Internal cylindrical
    - Centerless: External
      - Internal
    - Surface: Rotary table type
      - Reciprocating type
    - Disk, horizontal and vertical

TABLE C (*Extract*)
EQUIPMENT IN METALWORKING INDUSTRIES, 1953
(NUMBER OF MACHINES)

Industry	353	354	355	356	357	358	359	361
Equipment								
Boring	2454.	7343.	4756.	4738.	577.	1791.	3309.	2376.
Horizontal	840.	3178.	1930.	1351.	51.	633.	829.	660.
Vertical	1149.	1689.	2037.	2533.	30.	215.	1060.	806.
Precision	193.	837.	254.	483.	312.	746.	569.	357.
Jig	121.	1485.	423.	230.	171.	164.	299.	422.
Other	151.	154.	112.	141.	13.	33.	552.	131.
Broaching	292.	802.	579.	853.	172.	345.	817.	296.
Horiz. int.	212.	539.	370.	600.	73.	132.	375.	146.
Horiz. surf.	8.	69.	56.	42.	0.	46.	144.	0.
Vert. int.	49.	88.	112.	192.	56.	48.	182.	54.
Vert. surf.	19.	74.	30.	19.	43.	114.	88.	84.
Rot. othr.	4.	32.	11.	0.	0.	5.	28.	12.
Contour s. f.	238.	1464.	909.	572.	473.	407.	442.	672.
Cutoff	2608.	5053.	4810.	4705.	756.	2158.	3876.	5242.
Bandsaw	722.	1964.	1822.	1590.	320.	794.	1082.	2070.
Abr. disk	310.	651.	681.	525.	90.	291.	535.	445.
Hacksaws	1221.	1965.	1863.	1967.	260.	554.	1635.	1467.
Rotary	215.	308.	236.	356.	30.	324.	359.	749.
Other	140.	165.	209.	267.	56.	195.	265.	511.
Drilling	7464.	23758.	22464.	17759.	9905.	11213.	16048.	19631.
Sensitive	1009.	4416.	3815.	2734.	4522.	3294.	3571.	8744.
Uprt. singl.	2998.	9289.	8254.	7692.	1986.	4086.	7204.	6547.
gang	438.	2960.	3362.	1890.	1977.	1672.	1457.	1379.
Radial	2171.	4961.	3804.	3138.	214.	956.	1459.	1832.
MS. clstr.	280.	670.	1221.	1428.	777.	466.	850.	434.
Uh. wt. ms.	231.	364.	295.	244.	339.	180.	376.	142.
Deep hole	72.	165.	142.	75.	0.	78.	193.	61.
Other	265.	933.	1571.	558.	90.	481.	938.	492.
Gear	1443.	4543.	2598.	10224.	1029.	669.	422.	668.
Hobbing	480.	1483.	1062.	4104.	576.	417.	149.	449.
Shapers	476.	1042.	677.	1998.	184.	124.	144.	84.
Cutters	306.	547.	411.	882.	162.	18.	72.	31.
Bevel	83.	340.	161.	1534.	77.	46.	39.	19.
Planers	15.	98.	37.	295.	9.	0.	6.	15.
Grinders	8.	340.	64.	295.	0.	3.	6.	12.
Shavers	11.	194.	49.	464.	21.	38.	0.	8.
Brnsh, lp.	26.	162.	41.	324.	0.	23.	6.	31.
Chamfer	30.	162.	37.	169.	0.	10.	0.	0.
Other	8.	175.	59.	159.	0.	0.	0.	19.
Gear checking	113.	1028.	374.	1069.	26.	81.	11.	69.
Grinding	10239.	40664.	23828.	17930.	7434.	8291.	39738.	14749.
Ext. cyl.	949.	7004.	2918.	2471.	864.	1012.	3875.	1221.
Int. cyl.	688.	2707.	812.	1312.	222.	248.	3616.	549.
Cntrls. ext.	193.	670.	348.	361.	196.	486.	3650.	250.
int.	36.	68.	51.	45.	0.	81.	585.	88.
Surf. rot.	276.	1897.	808.	949.	90.	496.	1639.	503.
recip.	446.	7238.	2394.	1051.	1410.	1146.	1811.	2726.
Disk	163.	1010.	804.	638.	312.	326.	2208.	818.
Tool	2540.	8067.	4417.	3668.	766.	2176.	5360.	4881.
Bench	4150.	7571.	8347.	5981.	2660.	1784.	4033.	2346.
Abr. belt	363.	1222.	1182.	816.	517.	296.	800.	753.
Other	435.	3210.	1747.	638.	397.	240.	12161.	614.

TABLE D (*Extract*)

MACHINE TOOL CLASSIFICATION FOR TABLE F

Code Number	Boring Machines—Horizontal
1	(*a*) under 3″ spindle, under 36″ bed
2	(*b*) 3–5″ spindle, 36–120″ bed
3	(*c*) 6–10″ spindle, 10–23′ and over
	Boring Machines—Vertical
4	(*a*) under 36″ swing
5	(*b*) 36–120″ swing
6	(*c*) 10–25′ swing and over
	Precision Boring Machines
7	(*a*) under 8″ bore, 18″ table
8	(*b*) 8–14″ bore, 18″ table
	Jig Boring—Horizontal and Vertical
9	(*a*) under 15″ table
10	(*b*) 15–40″ table
11	(*c*) 40–120″ table and over
	Broaching—Horizontal and Vertical—Internal
12	(*a*) under 15″ stroke, under 5 tons
13	(*b*) 18–36″ stroke, 5–35 tons
14	(*c*) 36–120″ stroke, 35–60 tons
	Broaching—External
15	(*a*) under 15″ stroke, under 5 tons
16	(*b*) 18–36″ stroke, 5–25 tons
17	(*c*) 36–120″ stroke, over 25 tons
	Sawing and Cutoff Machines
18	(*a*) under 12″ work
19	(*b*) 12–30″ work
	Bandsaws
20	(*a*) under 12″ work
21	(*b*) 12–30″ work

TABLE E (*Extract*)

TASK CLASSIFICATION FOR TABLE F

Code Number	IV. Cylindrical—External (cont'd.)
52	Medium, Semiprecision, long run
53	Medium, Precision, short run
54	" " long run
55	Medium, High Precision
56	Large, Semiprecision, short run
57	" " long run
58	Large, Precision
59	Very Large, Semiprecision
60	Very Large, Precision
	V. Cylindrical—Internal
61	Small, Semiprecision, short run
62	" " long run
63	Small, Precision, short run
64	" " long run
65	Small, High Precision
66	Medium, Semiprecision, short run
67	" " long run
68	Medium, Precision, short run
69	" " long run
70	Medium, High Precision
71	Large, Semiprecision, short run
72	" " long run
73	Large, Precision, short run
74	" " long run
75	Very Large, Semiprecision
76	Very Large, Precision
	VI. Drilled Holes
77	Very Small, Semiprecision, short run
78	" " " long run

TABLE F (*Extract*)

Rate of Output of Machine-Task Combinations

Machine Number	Task Number	Machine Time	Handling Time	Set-up Time	Pieces per Day
1	61	.58	.90	40.00	230.17
1	62	.58	.90	40.00	257.61
1	63	.58	1.35	45.00	157.08
1	64	.58	1.35	45.00	169.05
1	65	.58	1.55	60.00	114.19
1	66	2.32	1.16	40.00	85.08
1	67	2.32	1.16	40.00	102.24
1	68	2.32	1.63	55.00	63.65
1	69	2.32	1.63	55.00	78.62
1	70	2.32	3.15	70.00	43.11
2	61	.60	.96	42.00	218.54
2	62	.60	.96	42.00	245.89
2	63	.60	1.40	49.00	151.71
2	64	.60	1.40	49.00	163.59
2	65	.60	1.60	64.00	110.90
2	66	2.50	1.16	43.00	80.57
2	67	2.50	1.16	43.00	97.07
2	68	2.50	1.70	59.00	59.86
2	69	2.50	1.70	59.00	74.05
2	70	2.50	3.25	74.00	41.12
3	66	2.70	1.20	45.00	75.88
3	67	2.70	1.20	45.00	91.25
3	68	2.70	1.75	63.00	56.48
3	69	2.70	1.75	63.00	69.98
3	70	2.70	3.35	79.00	39.13
4	1	.41	1.74	30.00	84.20
4	2	.41	1.74	30.00	89.22
4	3	.41	2.03	40.00	64.78
4	4	.41	2.03	40.00	68.01
4	5	.82	3.01	55.00	46.76
4	6	1.60	2.26	35.00	39.93
4	7	1.60	2.26	35.00	46.32
4	8	1.60	2.84	45.00	30.15
4	9	1.60	2.84	45.00	35.36
4	10	3.20	4.71	65.00	16.41
4	46	.41	1.74	30.00	84.20
4	47	.41	1.74	30.00	89.63
4	48	.41	2.03	40.00	64.78
4	49	.41	2.03	40.00	68.01

Note: Machine time and handling time are expressed in minutes per piece; set-up time is expressed in minutes per lot.

CHAPTER 15

ALTERNATIVE APPROACHES TO METALWORKING PROCESS ANALYSIS

Thomas Vietorisz

This chapter is based on a discussion of the Markowitz-Rowe study of metalworking as presented to the Cowles Foundation Seminar on Process Analysis, extended by a review of the Soviet Machinery Industry Study of the University of North Carolina (1959). The purpose of the paper is to compare these two alternative approaches to metalworking process analysis and to suggest ways of integrating them. Limitations of space have prevented including a review of several other relevant studies and the presentation of a technique for the formulation of development plans for the metalworking sector in underdeveloped countries. A more complete version of the paper is available as an IBM Research Report (Vietorisz, 1962).

The contribution of Markowitz and Rowe is an original, pioneering approach to the study of the metalworking sector. It offers a coherent set of principles for applying process analysis, at different levels of aggregation and with different degrees of complexity, to this sector which raises inherently more difficult problems of technological description than agriculture or the industries characterized primarily by chemical-type processes. The mechanical transformations dominant in metalworking involve the generation of three-dimensional geometric structures with prescribed properties of strength, hardness, and elasticity, which have to be evolved detail by precise detail, whereas the production processes of many other industries may rely on the simpler technique of controlling a limited number of key environmental or operating variables. Correspondingly, metalworking products show an almost endless variety, whereas the products of most other industries are relatively homogeneous. The technological description offered by Markowitz and Rowe cuts across this tremendous range of variation in a way which is not only applicable to the metalworking industries themselves, but also offers a guideline for work in other industries where mechanical transformations play an important role.

The central ideas of the Markowitz-Rowe approach may be summarized as:

1. The combined use of engineering and census data in requirements analysis, with a means of reconciling contradictions between them.

2. The description of technology in substitution analysis by means of the concepts of *standard tasks,* standard *machine classes,* the breakdown of a *time fund,* and machine *availability.*
3. The handling of demand in substitution analysis by a questionnaire survey of machine availabilities and task proportions or by the sampling of production paper.
4. The application of simulation techniques to the estimation of machine availabilities, to queueing, and to flexibility problems.

In the nature of a pioneering effort, the Markowitz-Rowe contribution is a mosaic of empirical work and methodological ideas. While the requirements analysis is formulated to cover the entire U. S. metalworking sector, substitution analysis is illustrated only by the case of machining and, at that, without an empirical demand estimate. The question arises: To what extent can the methodology of substitution between metalworking machine tools be extended to other metalworking operations? Furthermore, it is evident that the type of substitution between machines which is discussed by Markowitz and Rowe is only one of several kinds which may occur: others are substitution possibilities in product design or in materials; there are even substitution possibilities involving extrasectoral repercussions (e.g., numerous standard steel shapes combined with little machining, versus fewer standard steel shapes combined with more extensive machining). Finally, simulation is advanced only as a methodological suggestion, with no illustrative examples.

Another major independent investigation of the metalworking sector has been undertaken by the Institute for Research in Social Science of the University of North Carolina (1959). This is a study of Soviet machinery construction which contains an important methodological concept ("resource elements") and much useful empirical information. Both the above concept and the empirical data contained in this study are suitable for complementing the Markowitz-Rowe approach.

The Soviet Machinery Industry study was intended as a tool for estimating the economic capabilities of the Soviet Union. It was based on published Soviet engineering and industrial source material.

Technological coefficients were estimated in two stages. In the first stage, flow and capital coefficients were derived for basic metalworking operations including casting, forging, machining, stamping, upsetting, heat treatment, assembly, and welding. The coefficients were expressed as the physical quantities of inputs needed per unit of semifabricate output. In the second stage, typical end products (machines) were analyzed and estimates were made of the inputs of these semifabricates required per unit of machine output. The combination of these two types of coefficients subsequently yielded the estimates of flow and capital inputs required per unit of machine.

The level of aggregation chosen for the basic metalworking operations, in the final version of the study, corresponded approximately to the production shop or department. *Resource elements* or typical shops consist of coherent composites of capital equipment and floorspace, defined in such a way that a variety of end products might draw upon the same resource element. This approach served several purposes. First, it was a means of representing the large variety of equipment combinations used for machinery production by a number of typical combinations. Second, while information on the relationship of particular inputs to end products was often unavailable, the resource elements provided a means of estimating the corresponding ratios. Third, resource elements provided a simple way of taking into account the fact that given productive facilities can be converted from one end product to another, for example, that a given forge can produce semifabricates for many different kinds of machines. In order to guard against an overestimation of this convertibility, nevertheless, resource elements were divided into subclasses by the size of the product, the seriality of production, and other relevant factors. The total number of individually defined resource elements within the major classes enumerated above was fifty-three.

In order to permit a capability analysis of the economy, the production of various branches of the machine building sector was represented by statistically determined weighted averages of the typical individual machines which were analyzed in the course of the study.

The "resource element" concept summarized above can be integrated easily with the Markowitz-Rowe approach, both in requirements analysis and in substitution analysis.

In the Soviet Machinery Industry study of the University of North Carolina, the resource-element concept is used in connection with a simple requirements analysis, and no substitution between resource elements is permitted. Employing the resource element rather than the individual machine as the basic building block of a *requirements analysis* has the following advantages:

1. It simplifies the collection of information and permits the reconciliation of fragmentary data. Intermediate concepts such as this may perhaps appear superfluous from a purely analytical point of view because they are eliminated from the analysis at a subsequent stage. In the present instance, for example, they disappear in the course of a matrix multiplication. Nevertheless, they can be highly useful in practice, since they often correspond to the categories under which the original information is easiest to collect. In addition, they permit the focusing of attention on a limited number of variables at a time, and thus they facilitate the recognition of basic connections among the data. Finally, they create broad classes of phenomena within which statistical regularities can reveal themselves, whereas in working with the unaggregated elements (e.g., coefficients of machine-hour inputs directly into a particular product) the data are often so scarce that any potential relationships between them are masked by the accompanying random variations.

The convenience of the resource element concept from this point of view is illustrated by the fact that in the Soviet Machinery Industry study, earlier attempts (University of North Carolina, 1956, 1958) to evaluate directly the material, labor, and equipment inputs to classes of individual machines have met with serious difficulties, whereas the achievement of the same objectives in an indirect manner, through the intermediary of the resource element concept, has been successful and has permitted the compilation and organization of a large body of empirical information.

2. The resource element concept is a convenient means of bringing together statistical and engineering information. In the Soviet study, for example, the determination of resource element inputs into typical individual products within an industrial branch was undertaken principally by engineering techniques: the study of product blueprints, of shop layouts, equipment lists, and personnel classifications. All of this, however, was preliminary to the estimation of a weighted average of individual products for the purpose of representing the industrial branch as a whole, with the weights derived from statistical sources.

The methods used by Markowitz and Rowe[1] for deriving material, labor, and equipment input coefficients are based largely on Census data, and engineering estimates are suggested by the authors only for the purposes of secondary corrections. This method is useful and accurate in the case of structurally stable and well-studied economies, for example, the United States economy. Its applicability, however, becomes severely limited when *either* considerable structural change is taking place which renders statistical coefficients rapidly obsolete, *or* statistical data sources are scarce and unreliable, as is the case with all but a few countries (and more so with regions within countries). When working with underdeveloped economies, both structural instability and a lack of reliable data have to be contended with. In such situations, the combined engineering-statistical approach to requirements estimation, used in connection with resource elements, appears to be considerably superior to an attempt to transfer statistical information from one country for which it is available (e.g., the United States) to another country for which it is not.

It follows that for the purpose of most planning and locational studies in situations of development, where structural changes are of central interest, the combined engineering-statistical approach made possible by the resource element concept is likely to be superior to an approach based largely on statistical data.

3. The resource element concept, while maintaining the analytical techniques of ordinary requirements analysis, nevertheless introduces a considerable degree of flexibility into the investigation of capabilities, by allowing substitution between the *semifabricates* that can be obtained from a given resource element. Thus, the yearly capacity of a foundry or a forge, expressed in tons of semifabricate, is reasonably constant for pieces of equal general complexity and unit weight, produced under conditions of comparable average lot size. The great advantage of the resource element concept in this regard is that it takes care of the overwhelming majority of trivial substitutions between simple, standard metalworking machinery operations, by the elementary method of creating an aggregated concept. Thus, in the substitution analysis

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[1] See also *Process Analysis of the Metalworking Industries*, Markowitz et al. (1953).

(to be discussed below), attention can be centered on a relatively small number of critical substitutions, without burdening the model with a profusion of detail.

The advantage of the flexibility achieved by means of the definition of resource elements is, however, obtained at the expense of some product-mix problems which invariably accompany the use of aggregative concepts. Thus, the capacity of a resource element varies with the average composition of its output, and so does the ideal adjustment between the number and kind of machines, the amount of floorspace, the numbers and skills of workers and the kinds and amounts of flow inputs which characterize each resource element. When the average complexity or lot size of the semifabricates changes within reasonably narrow limits, an acceptable approximation is possible by means of defining simple capacity corrections, similar to the corrections for nonstandard tasks based on units of standard tasks in the Markowitz-Rowe substitution analysis. Nevertheless, this approximation does not allow for structural changes within the capacity and flow inputs of the resource elements. In order to improve the approximation substantially, it is necessary to increase the number of resource elements and to narrow the definition of each individual element. This brings the approach closer to ordinary requirements analysis as discussed by Markowitz and Rowe, but it increases the range of substitution that remains outside the confines of the individual resource elements.

A good compromise between the two objectives—flexibility on the one hand and homogeneity on the other—is afforded by the device of breaking out the specialized, outsize, or otherwise scarce items of machinery from the aggregated resource elements and handling them individually. This maintains the flexibility afforded by the substitution between common classes of machinery inside each resource element, but allows at the same time the identification of the major bottlenecks due to the limited capacities of individual machines during the course of the requirement analysis.

In regard to *substitution analysis*, the resource element concept appears at first sight in a somewhat ambiguous role. On the one hand, it can be regarded as a generalization of the "standard task" concept of Markowitz and Rowe. While geometry is not distinguished in defining resource elements, several of the other identifying dimensions (size class, precision, seriality) coincide between the Markowitz-Rowe *task* concept and the University of North Carolina *resource element* concept. On the other hand, a resource element can also be regarded as one of a set of differing "universal machines" each of which, in the University of North Carolina study, can perform but one single task. This ambiguity will be clarified in the discussion below.

While the Soviet Machinery Industry study contemplates no substitution possibilities between resource elements, this restriction is not necessary. In other words, the specification of one single input pattern of resource element units into a given product is not a built-in feature of the University of North Carolina method. On the contrary, it is logical to permit, for example, distinct resource elements which differ only in regard to seriality, to substitute for each other; likewise, resource elements which differ only in regard to the size of the heaviest fabricate they can turn out may substitute for each other in neighboring size classes, completely in one direction and at least partially in the other.[2] Moreover, substitution possibilities exist between unrelated resource elements; e.g., forge and foundry, since they can produce closely substitutable fabricates by their distinct operations: e.g., cast versus forged crankshafts.

Substitution possibilities between resource elements are reflected by alternative input patterns of resource element units into production activities. From these alternatives, process analysis models can be constructed in the usual way.

In order to clarify the relationship between the various concepts referred to above, three linear programming models are presented in Tables 1-A through 1-C. The first summarizes the key features of the Markowitz-Rowe approach; the second refers to the approach used by the University of North

TABLE 1-A

INTERPRETATION OF MARKOWITZ-ROWE MODEL

Code	Task generation activities based on machine classes				Production activities: one alternative				
	x 1010 0100	x 1090 0900	x 9010 0100	x 9090 0900	x 0001 0001	x 0008 0001	x 0009 0001	$\leqslant$	
Machine class									Machine availabilities by class*
y 1000 0000	a	a						$b = (n) \cdot (u)$	
y 9000 0000			a	a				$b = (n) \cdot (u)$	
Task									
y 0010 0000	-1		-1		a	a	a	0	Task balances
y 0090 0000		-1		-1	a	a	a	0	
Product									
y 0001 0000					-1			$-b$	Product demands
y 0008 0000						-1		$-b$	
$\geqslant$	0	0	0	0	0	0	1	→ MAX	
								↓ MIN	

* Total machine availability in each machine class equals number of machines in the class times unit availability.

Explanation of code and other notes are given following Table 1-C.

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[2] In other words, a heavier forging resource element can turn out all of the fabricates of the neighboring lighter element; while due to the fact that each forge handles a variety of fabricate sizes, many of which fall considerably short of the maximum, the lighter forge can also take over part of the work of the heavier forge.

Carolina; and the third is a general model from which both of the former approaches can be derived as special cases.

In *Table 1-A*, a coefficient a is given for the generation of each task by each machine:[3] this permits the substitution of one machine for another in generating given total task requirements. On the other hand, only one task combination exists for the production of each product: in other words, there are no alternative production activities embodying alternative task combinations for given products. Thus, flexibility in the manufacturing of products is obtained only by the substitution of machines in the generation of tasks, not by the substitution of tasks in the production activities. Evidently, an easy generalization of this model is possible by introducing alternative production activities.

TABLE 1-B

INTERPRETATION OF UNIVERSITY OF NORTH CAROLINA MODEL

	Resource element definition activities		Task generation by resource elements		Production activities				
	x 0100 1000	x 0900 1000	x 0100 0010	x 0900 0010	x 0001 0001	x 0008 0001	x 0009 0001		
CODE								≤	
Machine class									Total machine availabilities
y 1000 0000	a	a						b (not given)	
y 9000 0000	a	a						b (not given)	
Resource element									
y 0100 0000	−1		1					0	Resource element balances
y 0900 0000		−1		1				0	
Task									
y 0010 0000			−1		a	a	a	0	Task balances
y 0090 0000				−1	a	a	a	0	
Product									
y 0001 0000					−1			$-b$	Product demands
y 0008 0000						−1		$-b$	
≥	0	0	0	0	0	0	1	→ MAX ↓ MIN	

Explanation of code and other notes are given following Table 1-C.

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[3] When a given machine is not capable of generating a specific task, the corresponding coefficient a can be made sufficiently great to prohibit the use of this activity in an optimal or near-optimal solution.

TABLE 1-C

Generalized Metalworking Substitution Model

	Resource element definition activities				Task generation activities based on machine classes				Task generation activities based on resource elements				Production activities							
Code	x 0100 1000	x 0100 9000	x 0900 1000	x 0900 9000	x 1010 0100	x 1090 0900	x 9010 0100	x 9090 0900	x 0100 0010	x 0100 0090	x 0900 0010	x 0900 0090	x 0001 0001	x 0001 0009	x 0008 0001	x 0008 0009	x 0009 0001	x 0009 0009	≤	
Machine class																				Machine availabilities by class*
y 1000 0000	a	a	a	a	a	a													$b = (n) \cdot (u)$	
y 9000 0000	a	a	a	a			a	a											$b = (n) \cdot (u)$	
Resource element																				Resource element balances
y 0100 0000	−1	−1							1	1									0	
y 0900 0000			−1	−1							1	1							0	
Task																				Task balances
y 0010 0000					−1	−1			$-a$	$-a$	$-a$	$-a$	a	a	a	a	a	a	0	
y 0090 0000							−1	−1	$-a$	$-a$	$-a$	$-a$	a	a	a	a	a	a	0	
Product																				Product demands
y 0001 0000													−1	−1					$-b$	
y 0008 0000															−1	−1			$-b$	
≥	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	→ Max	
																			↓ Min	

* Total machine availability in each machine class equals number of machines in the class times unit availability. Explanation of code and other notes are given on following page.

NOTES TO TABLES 1-A, 1-B AND 1-C

The tables have been formulated in synoptic notation showing the direct and dual problems simultaneously. The direct variables (x) of the upper margin, when multiplied by the lower margin, give the direct criterion function, to be maximized; when multiplied by the coefficients of each row, they give the direct constraints. The dual variables (y) of the left margin, when multiplied by the right margin, give the dual criterion function (to be minimized); when multiplied by the coefficients of each column, they give the dual constraints. Among the technical coefficients in the tables, the positive ones represent inputs, the negative ones, outputs.

In each of the three tables, the complete parametric linear programming problem is summarized by the maximization of the last product with the other products held constant; evidently, any product can be maximized or any primary input (machine class) minimized, or any linear combination of these optimized, within the rules of parametric programming problems.

All variables and parameters are identified by an eight-digit code shown on the upper and the left margins. The code of the upper margin applies to the entire column; that of the left margin, to the entire row. Internal parameters at the intersection of a row and column receive the code obtained by the consolidation of the row and column codes.

In each digit of the code, the first element of a series is indexed as 1, the last element, as 9, and the next to the last element, as 8. A code position not relevant for indexing an element is filled by 0. In the tables, only the first and the last element of each one-dimensional series is shown.

The coding is explained in the following tabulation:

Code for Tables 1-A, 1-B, and 1-C

Digit	Commodities: Dual variables (y)	Activities: Direct variables (x)	Constraints	Other
1	Machine class	–	Total availability	n = total number of machines in a class u = unit availability of each machine in a class
2	Resource element	–	–	–
3	Task	–	–	–
4	Product	–	Fixed demand	–
5	–	Resource element definition	–	–
6	–	Task generation based on machines	–	–
7	–	Task generation based on resource elements	–	–
8	–	Production	–	–

Examples:

$x(0900\ 0010)$ denotes the first alternative way of generating tasks from the last resource element.

The coefficient a at the intersection of the row (0010 0000) and the column (0008 0009) represents the input of the first task in the production, by the last alternative method, of the next to the last product.

In *Table 1-B,* the University of North Carolina model is interpreted in linear programming terms. Resource elements are defined by specifying their contents of individual machines.[4] There is only one way of defining each resource element. The same rigid description is applied to the manufacture of products: there is only one resource-element combination for the manufacture of each product.

In order to facilitate the subsequent generalization of this model, the concept of "task" has been introduced as a formal intermediate stage between products and resource elements. As seen in Table 1-B, each resource element has been associated with the output of a unique task. This task is quantified in terms of the output of semifabricate from a given resource element: thus, for example, a forge of a given size and seriality class performs the unique task of producing a specified number of yearly tons of its own class of forgings, assumed to be homogeneous.

This concept of a task is entirely analogous to the Markowitz-Rowe concept. The fact that a resource element, as mentioned earlier, at times appears as a generalization of the task concept, and at other times as a "universal machine," becomes now understandable considering that in the University of North Carolina study there is a one-to-one correspondence between resource elements and tasks. Thus, speaking of units of task inputs into the production of given products comes to the same thing in this model as speaking of units of resource element inputs. In generalized versions of the model, however, this will no longer be necessarily so.

As there are no joint outputs in the original version of the model, the latter is equivalent to a simple Leontief input-output model as long as there is no more than one restriction on primary inputs (machine availabilities).[5] In fact, the original approach ignores maximization and starts with prescribed levels of *all* product demands (which at once determine the levels of production activities). This allows the derivation of resource element inputs, and, by a subsequent matrix multiplication, the computation of the levels of individual machine requirements. No limits are set on the latter.

In *Table 1-C,* a generalized model is given with four commodity classes: machines, resource elements, tasks, and products; and four classes of activities: the definition of resource elements, the generation of tasks by individual machines, the generation of tasks by resource elements, and the manufacture of products based on task inputs. The sets of activities are, however, generally more comprehensive than in the Markowitz-Rowe or the University of

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[4] Floorspace and flow inputs, not shown in Table 1-B, are likewise rigidly specified.

[5] See Dorfman, Samuelson, and Solow (1958), Chapter 9.

North Carolina models. Thus, alternative definitions of each resource element are permitted, in order to take into account the variations in the machine parks of typical shops, either in response to different proportions of semifabricates, or in response to differing local conditions. Likewise, each resource element is assigned several alternative combinations of task outputs, in order to allow for variations in the proportions of different kinds of fabricates falling into the same general size, seriality, or other classification. Finally, the manufacture of each product is represented not by one activity, but by several activities specifying alternative task combinations.

The Markowitz-Rowe model can be derived from this more general model by suppressing resource elements and eliminating alternative production activities. This leaves only the substitution of machines in task generation as an element of flexibility in the model. The University of North Carolina model can be derived by ruling out task generation by individual machines, eliminating alternative resource element definitions, and cutting down the generation of tasks from resource elements to one task per resource element.

The general model itself, representing a synthesis between the two approaches, can be used in the following fashion:

1. All simple, common and usual-sized machines are grouped into *resource elements*, leaving only the special, outsize, or otherwise scarce machines as individual entities. Even in the latter cases, it appears desirable to group all ancillary equipment used with such machines into a single resource unit.
2. Preferably, one *definition of each resource element* is adopted as a standard; alternative definitions are best handled by means of a sensitivity analysis in the form of joint changes in groups of parameters. Such joint changes might be used, for example, to represent a systematic shift of the definition of all resource elements in the direction of greater self-sufficiency.[6] If desired, nevertheless, alternative definitions may be left explicitly in the model.
3. In the case of individual machines, coefficients for the *generation of alternative tasks* are specified by the general model in the same way as by the Markowitz-Rowe model.
4. In the case of task generation by resource elements two alternative approaches are possible: *First,* a few typical alternative tasks may be associated with each resource element on the basis of common types of alternative fabricates falling into the same weight, size, or seriality class; subsequently, several alternative task generation activities may be specified for each resource element. These alternative activities differ in regard to the proportions of tasks generated by the resource element. *Second,* the one-to-one correspondence between tasks and resource elements may be retained. In this case, the task is regarded as a *standard task* in the sense of Markowitz-Rowe; it is taken to correspond to a given average composition of typical fabricates. Subsequently, requirements of fabricates in other proportions can be expressed in units of standard tasks, similarly to the Markowitz-Rowe formulation.

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[6] A comparison of typical shops in the U. S. A. and the U.S.S.R. shows that the latter typically comprise a higher proportion of large and specialized equipment and machines needed for internal repairs and maintenance (A. D. Little, 1961).

5. Whichever way the generation of tasks by resource elements is handled, the effect of the lack of homogeneity in semifabricate outputs from resource elements is only partially overcome. It has been mentioned earlier that the machine park of a resource element may vary in response to different proportions of typical semifabricates. When a process analysis model comprises an ample range of alternatives in regard to resource element definitions as well as in regard to variations in the proportions of typical semifabricates produced by individual resource elements, the model itself can be expected to determine the optimal definitions of resource elements and the optimal proportions in the outputs of semifabricates. In practice, however, it is hard enough to find the empirical information for specifying even a single alternative in regard to each one of these dimensions of variation: thus, the issues of resource element definition and the average composition of semifabricate outputs generally have to be decided before the model is constructed, on the basis of whatever information, often of a qualitative nature, happens to be available concerning the particular problem in hand.

6. It is a matter of detail, without effect on the structure of the model, whether there is an overlap between the tasks capable of being generated by individual machines and those capable of being generated by resource elements. Whenever a given machine or resource element does not lend itself to the generation of a specific task, the corresponding input coefficient can be set at a prohibitive level without a loss of generality.

7. For the manufacture of each product (or for several products jointly) alternative activities comprising alternative task combinations are given.

The generalized model, when used according to the above observations, will *concentrate the analysis* on the identification of bottlenecks that may exist in specialized, outsize, or otherwise scarce machinery; on the substitution of different kinds of resource elements (forge, foundry, machining) in the manufacture of products; on the substitution of shops with different size, weight, precision, or seriality characteristics; and on the distribution of typical semifabricates produced by each resource element. By the techniques of sensitivity analysis, the effects of systematic shifts in resource element composition on the optimal solution of the model can also be explored; and when the data permit doing so, the model can determine optimal machine parks and optimal proportions of semifabricate outputs for resource elements. At the same time, substitutions between simple, common and standard-sized metalworking machines; e.g., between different kinds of machine tools in a machine shop, are handled *implicitly* by the definition of aggregative concepts: the concept of a resource element and the concept of a task generated by a resource element. Since in most practical studies the overwhelming majority of substitutions are of the latter type, the resulting economy of effort is considerable.

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PART V

PROGRAMMING OF ECONOMIC DEVELOPMENT

CHAPTER 16

KEY SECTORS OF THE MEXICAN ECONOMY, 1960–1970

Alan S. Manne

During the summer of 1961, a small group within Nacional Financiera, S.A., constructed a model dealing with the interdependence between investment decisions in certain basic sectors of the Mexican economy.[1] It was the author's privilege to be associated with that group, and this chapter is intended as a summary of what was done—and what was left undone—in attempting to apply process analysis over a major segment of a rapidly changing economy.

The particular study was intended as an experiment to see what would be possible with a minimal expenditure of effort, less than a man-year altogether. In view of the time and manpower limitations, it would be rash to place much faith in the detailed numerical results with respect to investment requirements, foreign aid, imports, capacity expansions, etc. Far more checking and testing are needed before a cautious investigator would be able to conclude that the results of the model were fully dependable.

Perhaps the primary lesson to be learned from this work is that it is not only theoretically desirable but also quite practical to construct multisector aggregate investment planning models on the basis of a considerable amount of technological process detail. In addition to proving out the general feasibility of this approach, certain specific ideas have emerged—ways of reducing complexity without sacrificing too much information. (*a*) An interindustry model does not need to embrace all productive sectors of the economy, but only those that are most closely interdependent. (*b*) Without going through the effort involved in a multi-time-period model, there is a tolerably good shortcut method to allow for *en*dogenous generation of demand for capital equipment. And (*c*), much of the information needed for endogenous generation of demand for transport services can be included without an excessive amount of locational detail. Altogether, the linear programming model included 92 constraint equations and 156 nonslack activities.

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[1] Dr. Alfredo Navarrete R. initiated this study. The work was carried out within a group headed by Lic. Ignacio Navarro G. and Victor Navarrete R., and consisting of Lic. Pedro Galicia Estrada, Alfonso García Macías, Raul Garduño G., Fernando Torres V., and Tulio Espinosa Cabrera. The views presented here are those of the author, and do not necessarily represent the official policy of Nacional Financiera, S.A.

INSTITUTIONAL BACKGROUND

In order to understand some of the institutional factors that underlie the mathematical abstractions to be described below, the English-language reader would do well to turn to some such historical source as Simpson (1959). In the pages of Simpson—or better yet, in the murals of Diego Rivera—the non-Mexican will begin to understand how much of the post-Cortés history of the country is a history of exploitation. Many, but by no means all of the exploiters have been foreigners—Spaniards, Frenchmen, and Yankees. Against this background of social discontent and of genuine grievances against foreigners, it is not too difficult to understand why there is widespread popular support for the post-1911 Revolution policy of direct government ownership of a large percentage of the basic industries—electricity, railroads, oil, and steel—together with a considerable amount of public intervention in all other sectors of the economy. In 1960, investment in the public sector (excluding direct investment expenditures by the federal, state, and local governments) amounted to 5.6 billion pesos, in comparison with 12.3 for the private sector[2] (Banco de Mexico, 1960, pp. 15 and 69).

One of the several agencies concerned with long-term investment planning in Mexico is Nacional Financiera, S.A., hereafter abbreviated as Financiera. This official industrial development bank, founded in 1934, serves as the principal source from which public enterprises obtain long-term financing for amounts in excess of their own internal cash flow. In turn, Financiera obtains funds through the sale of its securities to banks, private individuals, and foreign institutions, and also through its own retained earnings.

One of Financiera's principal functions is to make recommendations with respect to investment project proposals submitted by individual enterprises. (Financiera recommends and persuades, but does not impose its decisions upon other governmental agencies.) In addition to specific projects, Financiera is often asked to submit its opinions with respect to establishing a new industry or expanding the aggregate capacity of an existing industry. These are the types of broad questions to which our model is addressed—not the more detailed short-run equipment scheduling problems which more properly lie within the domain of the individual decentralized enterprises.

There exist a variety of reasons for attempting to coordinate investment plans in different branches of the Mexican economy. Because of substantial economies of scale in plant construction, it is desirable for individual new installations to be large in relation to the rate of growth of the internal market. With long construction lead times for these plants, reliance upon decentralized trial and error becomes particularly dubious. It is worthwhile, for example, to attempt to coordinate the capacity expansion plans for aromatic chemicals which are produced in both the petroleum refining and the metallurgical coke industry.

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[2] During 1960, the exchange rate was 12.5 Mexican pesos per U. S. dollar. This will be the exchange rate utilized throughout the remainder of this paper.

Investment coordination is also regarded as a means of overcoming the irrationality of prices charged by individual enterprises and, in particular, the irrationality of the foreign exchange rate. For example, there is not great confidence that the price of residual fuel oil sold by the petroleum industry will lead to the socially optimal choice between hydroelectric and thermal plants by the electric power sector. There is even less confidence that the foreign exchange rate provides a sufficient stimulus to diversify Mexico's economy and to insulate it from outside disturbances. Many project proposals are premised upon the assumption that the official rate undervalues the advantages of economizing upon foreign exchange. Even those who advocate a minimum degree of coordination would agree that individual enterprises should evaluate their investment alternatives on the basis of similar "shadow prices."

In addition to these general reasons for being concerned with an interindustry model, there was one particularly urgent question in 1961: estimating how much foreign loans and aid it would be justifiable for Mexico to request for economic development under the "Alliance for Progress" program. More specifically, what would be the possible tradeoffs between foreign loans and aid, domestic savings, and the rate of growth of national product under conditions of full-capacity utilization in the industrial branches of the economy?

SCOPE OF MODEL

The model is concerned only with sectors that are of primary interest to Financiera: electricity, rail freight, heavy chemicals, paper, oil, steel, aluminum, copper, cement, and selected metal fabricated products, including capital equipment for these industries. For short, we shall hereafter refer to these as the "key sectors." An important amount of activity within these industries is carried out by private enterprise. However, we shall not be concerned with distinctions on the basis of ownership, and will suppose that similar criteria for investment planning are applied throughout the key sectors.

The reader would do well to note some of the more important sectors that are not included within the model: extractive industries, agriculture, food processing, textiles, wholesale and retail distribution, and housing. Demands from these excluded industries are regarded as exogenous—in much the same fashion as "final demand" by households and government. In retrospect, it was felt that some of the omissions were unfortunate—particularly those within the extractive industries. For example, there is no analysis of the production of raw sulfur used in the manufacture of sulfuric acid; nor of coal mining associated with the production of coke; nor of wood or bagasse for the manufacture of pulp. Several of the more important extractive sectors are, however, included within this matrix: iron and copper ore mining, limestone, crude petroleum, and natural gas. In subsequent work along these lines, it is hoped to include more branches of the extractive industry, but to continue to regard agriculture and the processing of agricultural raw materials as exogenous areas to be analyzed separately.

In all likelihood, the most important input omitted from the present analysis has been that of labor. A number of choices seemed open to us with respect to this item. It would have been possible to stipulate some minimum amount of new employment to be created within the key sectors. (This would have led to the possibility that the model would plan for the production of labor-intensive goods, and then throw away those goods.) Some upper limit might have been imposed upon the total work force available to the key sectors. (In view of Mexico's disguised unemployment—both urban and rural—this upper limit would almost surely have been redundant.) Perhaps the ideal procedure would have been to suppose that individual categories of skilled industrial workers could be created through the appropriate investment in education and in social overhead capital. Rather than devote the effort needed to handle this one aspect of the interindustry problem, labor inputs were entirely neglected. It is perfectly possible that the neglect of labor inputs has led to particular types of bias in the model's results, e.g., to an excessive emphasis upon the labor-intensive metal fabricating industries.

Model Formulation

The linear programming analysis has been focused upon a target year of 1970, with the implied changes in the structure of Mexico's economy taking place over the decade of 1960–1970. The analysis is intended to aid in the formulation of a rolling plan of investment projects, and is not intended as a once-for-all program. On the contrary, as time passes and new data become available, such a model should be brought up to date, and the target year should recede further into the future.

It can never be altogether satisfactory to construct a finite-horizon economic model. Here we followed the practice of Chenery and Kretschmer (1956) for southern Italy and of Sandee (1960) for India in choosing a decade as the time span. A decade seemed preferable to either a much closer or a more distant cutoff date. A five-year model would not have permitted us to ignore the lags between investment and output. There would be inadequate lead time to make major changes in the currently projected plant expansions. On the other hand, a 20-year or even more distant time horizon—although more representative of the service life of these investment projects—was rejected because of doubts that our group had the time to make reasonable estimates of technological change over such a long horizon. A decade was adopted as a pragmatic compromise, and not because the more distant future is of negligible interest.

The model is designed to predict on a simultaneous and consistent basis within the key sectors: (*a*) capacity expansions over the decade 1960–1970; (*b*) investment requirements; (*c*) individual imports; and (*d*) the requirements for foreign loans and aid. Largely because of the Alliance for Progress program, the model was intended to explore the likely effects of an increase in foreign aid upon the growth rate of gross national product (GNP). Foreign aid was regarded as a flexible item, and the calculations were phrased in terms of minimizing the total loans and aid necessary to enable the key sectors to expand so as to satisfy the demands associated with two alternative growth rates of GNP: 5.5% and 8.0% compounded annually over the entire decade.[3] (In order to allow for nonproportionality between GNP and the demands for individual goods and services, this formulation seemed slightly more convenient to compute than the alternative: maximize the rate of growth of GNP, given a fixed availability of foreign loans and aid.) Domestic savings available to the key sectors were regarded as an additional financial constraint that would have to be satisfied in order to avoid generating excessive domestic inflationary pressure.

With respect to foreign trade, imports and exports were treated in a distinctly nonsymmetrical fashion. Export demands for the key sectors' outputs were regarded as exogenous, and not dependent upon decisions taken within Mexico. (This seemed to be a particularly plausible assumption with respect to the country's major raw material exports: cotton, coffee, copper, sugar, and petroleum.) Since the key sectors produce industrial goods, there were numerous projections of zero for their growth in export demand over the 1960–1970 period.[4] As for imports, these were taken to be available in unlimited individual quantities at their prevailing price in U. S. dollars. The only restriction upon imports was of a global nature, dealing with the overall availability of foreign exchange.

ENDOGENOUS GENERATION OF DEMAND FOR CAPITAL EQUIPMENT

In describing the algebraic formulation of the model, it is convenient to begin with the shortcut method adopted to allow for endogenous generation of demand for capital equipment. Like the formulations of Frisch (1957, p. 20) and of Sandee (1960, p. 22) this one is intended to avoid the "edge effects" that are often to be found in finite-horizon models, where there is no rationale for investment activity toward the cutoff date. Unlike Sandee's formulation involving quadratic time paths, this one remains independent of the arbitrarily given initial growth rate at the beginning of the decade. And unlike Frisch's asymptotic growth model, this one avoids the supposition of proportional growth among *all* sectors during the years subsequent to the target date. In order to do away with the unpalatable assumption of proportional growth,

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[3] The rate of 5.5% was approximately equal to Mexico's actual performance in real terms over the decade 1950–1960, whereas 8.0% was regarded as a highly optimistic estimate of what could be achieved from 1960–1970. Since the country's population had been increasing at a 3% compound rate, the 8.0% growth rate for GNP would represent a *doubling* of the per capita growth rate from the previous decade.

[4] Mexico's brightest prospects for industrial exports seemed to be in the direction of other Latin American nations. It was doubted, however, that this trade would lead to *net* industrial exports. A more likely pattern would be for each of the Latin American countries to concentrate upon some particular range of industrial goods, and to give preferential import treatment to other members of the trading bloc.

this model makes a somewhat more modest claim than Frisch's. There is no longer any guarantee that the economy's material flows will be balanced during the post-cutoff years, but only during the cutoff year itself.

Each restraint equation within this model ensures the balancing of individual *flows* as of the year 1970.[5] The shortcut for generating investment demands by the key sectors has consisted of supposing that 15% of the ten-year expansion of each process takes place during the target year. Thus, the 1970 gross investment demand for good i is calculated as follows, neglecting equipment replacement:

$$\text{key sector gross investment demand for good } i\text{, 1970} = \sum_j (.15b_{ij})x_j \qquad (1)$$

where x_j denotes the ten-year increase in annual flow rate for process j from 1960 to 1970, and where b_{ij} represents the stock of capital good i required per unit of annual flow rate for process j. This investment equation is based upon the assumption that in the base year of 1960, the key sectors of the Mexican economy were working close to full capacity, and that any further increase in output over the 1960 levels would be dependent upon plant capacity expansions.

What is the rationale for using the 15% factor in equation (1) to convert stocks of investment goods into flows? Note that if process j grew by a constant absolute amount during each year between 1960 and 1970, the investment flow in 1970 would be only 10% of the cumulative total for the decade. However, instead of a constant absolute amount of growth, it is more reasonable to forecast the target year demand for capital good i as though the time path for installed capacity in process j were accumulating at a compound annual rate somewhere in the range of 5.0% to 10.0% per annum. Within this general range—but *without* any commitment to a specific growth rate for process j—the coefficient of 15% of the ten-year capacity increment yields a convenient linear approximation for estimating the demand for expansion during the target year. The closeness of this approximation may be judged from the following comparison, using three alternative growth rates within the range of 5.0% to 10.0% per annum, and letting the symbol X_j^0 denote the 1960 level of capacity for process j.

Annual percentage growth rate of capacity, g		5.0%	7.5%	10.0%
1970 capacity $= x_j + X_j^0 = (1+g)^{10}X_j^0$		$1.63X_j^0$	$2.06X_j^0$	$2.59X_j^0$
1970 capacity growth rate	linear approximation $= .15x_j = .15[(1+g)^{10} - 1]X_j^0$	$.094X_j^0$	$.159X_j^0$	$.239X_j^0$
	precise calculation $= g(1+g)^{10}X_j^0$	$.081X_j^0$	$.155X_j^0$	$.259X_j^0$

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[5] In a more refined version of this model, the 1960–1970 decade could be divided into several subperiods, each with its individual balancing restraints. However, with subperiods of five years or less, construction lags would become important, and would have to be estimated individually.

One side-effect is involved in using the 15% factor for stock-flow conversion, an effect that is reminiscent of von Newmann's theorem that under conditions of balanced growth, the rate of physical expansion will be identical to the rate of return on capital. (For an introduction to this theorem, see Kemeny, Snell, and Thompson, 1957, pp. 353–359.) The constraints of the *dual* of our linear programming calculation stipulate that the sum of the prices imputed to the outputs of each process must not exceed the prices imputed to the inputs. Since capital inputs are converted into annual flows at the 15% rate, these dual constraints stipulate, in effect, that the marginal productivity of capital goods be 15% per annum, neglecting depreciation. This rate has the virtue of being reasonably close to the estimates of capital productivity actually used by Financiera for purposes of project evaluation within the key sectors.

DEFINITIONS AND RESTRICTIONS OF THE LINEAR PROGRAMMING MODEL

definitions of unknowns

x_j = ten-year increase in annual flow rate for domestic process j, 1960–1970 (usually normalized in terms of principal output)

y_i = 1970 annual flow rate of imports of commodity i (units of commodity i)

z = 1970 annual flow rate of foreign loans and gifts required by the key sectors (U. S. dollars, 1960 prices)

definitions of parameters

a_{ij} = input ($-$) or output ($+$) of commodity i on current account per unit of process j

b_{ij} = stock of capital good i required per unit of annual capacity of process j

k_j = stock of miscellaneous imported capital equipment required per unit of annual capacity of process j (U. S. dollars, 1960 prices)

c_j = total value of stock of capital goods required per unit of annual capacity of process j (Mexican pesos, 1960 prices)

p_i = foreign exchange cost per unit imported of good i, (U. S. dollars, 1960 prices)

q_i = ten-year increase in annual rate of final demand for good i, 1960–1970

r_i = ten-year increase in annual rate of availability of resource i, 1960–1970

g = annual growth rate of GNP, 1960–1970

In order to make some rough allowance for anticipated technological changes over the decade, it appeared desirable to interpret the parameters a_{ij}, b_{ij}, k_j, and c_j as incremental or "best-practice" coefficients, rather than average-practice coefficients for the base year, 1960.[6] For this reason, most

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[6] In working exclusively with "best-practice" coefficients, we neglected the possibility that it would be desirable to decrease any domestic production process below its 1960 level. That is, the capacity increases over the decade, x_j, are constrained to be nonnegative.

of the constraint equations are stated in terms of balancing the *growth* in annual supply and demand flows, rather than in terms of the absolute value of these flows. Thus, in order to calculate the decade's growth in domestic availability of good i, we have:

$$\text{1960–1970 ten-year increase in domestic production rate of } i\text{, net of key sector current demands} = \sum_j a_{ij}x_j. \tag{2}$$

In accordance with the two alternative growth rates postulated for GNP, there were two separate projections made for the final demands. One of these projections is based upon the GNP growth rate $g = 5.5\%$ and the other upon $g = 8.0\%$ per annum.[7] The particular definition adopted for the parameter q_i stems from the fact that equation (1) defines the investment demand for good i in terms of the absolute flow rate for 1970, that the import variables y_i are also defined in terms of their 1970 absolute flow rate, but that the domestic availability of good i, net of key sector current demands, is defined through equation (2) in terms of the ten-year *increase* in flow rate:

$$\begin{aligned} q_i = {} & \begin{bmatrix} \text{ten-year increase} \\ \text{in household,} \\ \text{government, and} \\ \text{export demand for} \\ \text{good } i\text{, 1960–1970} \end{bmatrix} + \begin{bmatrix} \text{ten-year increase in} \\ \text{industrial demand} \\ \text{for good } i \text{ outside} \\ \text{key sectors, 1960–} \\ \text{1970} \end{bmatrix} \\ & + \begin{bmatrix} \text{imports of good } i\text{,} \\ \text{1960} \\ \end{bmatrix} - \begin{bmatrix} \text{key sector gross} \\ \text{investment demand} \\ \text{for good } i\text{, 1960} \end{bmatrix} \end{aligned} \tag{3}$$

With definitions (1), (2), and (3), the following general restriction covers the material balance for the 1970 flow of commodity i:

$$\begin{array}{cccccccc} & \begin{bmatrix} \text{ten-year increase} \\ \text{in domestic pro-} \\ \text{duction rate of} \\ i\text{, net of key} \\ \text{sector current} \\ \text{demands} \end{bmatrix} & - & \begin{bmatrix} \text{key sector} \\ \text{gross} \\ \text{investment} \\ \text{demand for} \\ i\text{, 1970} \\ \end{bmatrix} & + & \begin{bmatrix} \text{absolute} \\ \text{level of} \\ \text{imports} \\ \text{of } i\text{,} \\ \text{1970} \\ \end{bmatrix} & \geqslant & q_i \\ \therefore & \multicolumn{3}{c}{\sum_j (a_{ij} - .15b_{ij})x_j} & + & y_i & \geqslant & q_i \end{array} \tag{4}$$

Similarly, in order to make sure that the economy does not exceed the availability of three items utilized as current inputs (northern iron ore, ferrous scrap, and copper scrap):

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[7] Most of the final demand extrapolations were based upon the extremely crude assumption that the 1950–1960 change in final demand provided an estimate of the income elasticity for the particular item. This method resulted in income elasticity estimates between +.5 and +3.2, with typical values in the range of +1.0 to +2.0.

$$\begin{bmatrix}\text{ten-year increase}\\ \text{in key sector}\\ \text{current demand}\\ \text{for } i\text{, net of}\\ \text{domestic}\\ \text{production}\end{bmatrix} - \begin{bmatrix}\text{absolute}\\ \text{level of}\\ \text{imports}\\ \text{of } i\text{,}\\ \text{1970}\end{bmatrix} \leq \begin{bmatrix}\text{maximum in-}\\ \text{crease in}\\ \text{rate of ex-}\\ \text{ploitation}\\ \text{of resource}\\ i\text{, 1960–1970}\end{bmatrix} - \begin{bmatrix}\text{absolute}\\ \text{level of}\\ \text{imports}\\ \text{of } i\text{,}\\ \text{1960}\end{bmatrix} \qquad (5)$$

$$\sum_j - a_{ij}x_j \quad - \quad y_i \quad \leqslant \quad r_i$$

In addition to restrictions of types (4) and (5), there are two overall economic restraints: one dealing with foreign exchange and the other with the supply and demand for savings. These restrictions are both written in terms of the absolute flows for 1970, rather than in terms of the 1960–1970 increments.

According to restriction (6), there is no allowance for the foreign exchange available to the key sectors from any of Mexico's earnings on current account. Rather, it is supposed that the entire supply of foreign exchange to the key sectors will come from foreign loans and aid. (An alternative interpretation of restraint (6) is to say that the variable z measures the total foreign exchange required by the key sectors—regardless of whether these funds are provided by current account earnings or by capital transactions.)

The foreign exchange balance for 1970 is written as follows:

$$\begin{bmatrix}\text{foreign exchange}\\ \text{required by key}\\ \text{sectors for}\\ \text{miscellaneous}\\ \text{imported}\\ \text{capital}\\ \text{equipment}\end{bmatrix} + \begin{bmatrix}\text{foreign}\\ \text{exchange}\\ \text{required by}\\ \text{key sectors}\\ \text{for}\\ \text{merchandise}\\ \text{imports}\end{bmatrix} - \begin{bmatrix}\text{foreign}\\ \text{exchange}\\ \text{available}\\ \text{from aid}\\ \text{and loans}\\ \text{to key}\\ \text{sectors}\end{bmatrix} \leqslant 0 \qquad (6)$$

$$\sum_j (.15k_j)x_j + \sum p_i y_i \quad - \quad z \quad \leqslant 0$$

Note that in restriction (6), one component of the demand for imports consists of the broad category "miscellaneous imported capital equipment." The complementary capital input coefficients k_j are not technologically determined, but are intended to account for those types of capital equipment neglected as candidates for domestic production in 1970. The importance of possible errors in the coefficients k_j is underscored by the results of the linear programming calculations. According to these calculations, over 55% of the 1970 key sector requirements for foreign exchange would consist of miscellaneous capital equipment, and less than 45% would be required for raw material imports such as ferrous scrap.

The final restriction concerns the supply and demand for savings by the key sectors. It is expressed in terms of millions of Mexican pesos at 1960 prices, converting foreign gifts and loans into the supply of savings at the rate of 12.5 pesos per dollar. The gross domestic savings available to the key sectors is based upon the estimate that the 1960 domestic savings invested in these sectors amounted to 4,600 million pesos, and that the 1970 domestic availability would increase at the same annual rate, g, as the GNP from the base-year 1960 level:

$$\begin{bmatrix}\text{key sector}\\ \text{gross invest-}\\ \text{ment expendi-}\\ \text{tures, 1970}\\ \end{bmatrix} - \begin{bmatrix}\text{foreign}\\ \text{sources of}\\ \text{savings,}\\ \text{1970}\\ \end{bmatrix} \leqslant \begin{bmatrix}\text{gross domestic}\\ \text{savings available}\\ \text{for investment}\\ \text{in key sectors,}\\ \text{1970}\end{bmatrix} \qquad (7)$$

$$\sum_j (.15c_j)x_j - 12.5z \leqslant (1+g)^{10}(4{,}600)$$

Note that the foreign loans and aid variable z enters into two distinct restrictions, (6) and (7). Thus, each additional dollar of foreign aid is viewed as performing two distinct functions: it adds one dollar to the supply of foreign exchange, and it also adds 12.5 pesos to the supply of domestic savings. Alternatively, this means that in order to eliminate one dollar's worth of foreign loans and aid, it would be necessary for Mexico to earn an additional dollar on current account exports and *also* to increase domestic savings by 12.5 pesos.

The objective assumed for the linear programming calculation is that of finding the minimum value of z. This means finding the minimum inflow rate of foreign loans and aid required by the key sectors in order to satisfy the final demands confronting them, and also to remain within the resources available, i.e., to satisfy restrictions (4)–(7), together with nonnegativity of the unknowns x_j, y_i, and z.

Results of the Calculations

If we are to believe the linear programming results summarized in Table 1, a comparatively small change in foreign loans and aid could make a large difference in Mexico's growth between 1960 and 1970. In order to accelerate the country's annual growth rate from 5.5% to 8.0%, this table indicates that the 1970 rate of foreign loans and aid would have to be increased from \$172 to only \$245 millions.[8]

If foreign loans and aid are regarded simply as a substitute for domestic savings, it becomes quite difficult to explain why a comparatively small increase in foreign funds would help make it possible to achieve a difference in 1970 GNP of virtually \$5 billions. The two linear programming solutions yield a clue to this puzzle. The role of foreign aid appears as a complement, rather than as a substitute for domestic savings. At both growth rates, the

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[8] In order to make a rough estimate of both lower and upper bounds for the cumulative total of foreign loans and aid required over the decade, the 1970 figure could be multiplied by (1/.15) and by (1/.10) respectively.

program of key sector investments is estimated to absorb less than the gross savings available to these sectors, and accordingly the implicit price assigned to gross savings is zero.[9] Although a dollar's worth of foreign loans and aid performs two functions—first, providing a one dollar increase in the supply of foreign exchange, and second, substituting for 12.5 pesos of domestic savings—it appears that the first function is the crucial one. To the extent that Mexico's supply of foreign exchange could be augmented through increased export earnings on current account, it might be possible to achieve a substantial reduction in the volume of foreign aid requirements.

TABLE 1

Summary of Linear Programming Results

	Millions of U. S. Dollars, Mexican Currency Converted at 12.5 pesos per dollar			
	Parameters of Linear Programming Calculation			
	Gross National Product	Domestic Savings Available to Key Sectors	Gross Domestic plus Foreign Savings Available, but Unutilized by Key Sectors	Foreign Loans and Aid Required by Key Sectors
1960 actual	10,750[a]	368[b]	–	100[c]
1970 projections 5.5% annual growth	18,400	630	181[d]	172[e]
1970 projections 8.0% annual growth	23,200	795	71[d]	245[e]

[a] *Source:* Banco de Mexico (1961), p. 73.

[b] Equivalent to estimate of 4.6 billion pesos for 1960; see right-hand constant of restriction (7) in text.

[c] Author's estimate based upon figure of $156 millions for 1960 long-term capital inflow into all sectors of the Mexican economy. *Source:* Banco de Mexico (1961), p. 83.

[d] Result of linear programming calculation; value of slack activity for restriction (7) in text.

[e] Result of linear programming calculation; value of minimand variable z.

Admittedly, this particular matrix may have understated the capital coefficients c_j and thereby understated the domestic investment requirements of the capacity expansion program. Alternatively, these results may be attributable to *over*estimates of the coefficients k_j, the miscellaneous imported capital equipment required per unit of domestic production process j. However, be-

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[9] The model does not inevitably predict a zero implicit value for gross savings. In several preliminary calculations utilizing other final demand vectors, gross savings had a low implicit price but were nevertheless valued positively.

fore jumping to this latter conclusion, the reader should note that the model allows for explicit choice between imports and domestic production of many types of heavy capital goods, e.g., machine tools, steam engines and turbines, transformers, pumps and compressors, locomotives, etc. According to the optimal linear programming solutions, *all* of these items are to be produced domestically in 1970—a major structural change for the decade. It is difficult to escape from the conclusion that the country's low volume of domestic demand will lead Mexico to continue importing a considerable volume of specialized equipment and technical services, even after the major structural changes that are hoped for during the decade 1960–1970. Foreign loans and aid should be viewed as relieving a critical foreign exchange bottleneck, rather than as just a substitute for domestic savings.

STRUCTURE OF THE MATRIX

Within Financiera's files, detailed numerical values are available for the approximately 1,900 individual coefficients that entered into the linear programming calculation. In part because of the confidentiality of certain coefficients and in part because of the unreliability of others, the matrix is not being reproduced here. Instead, the appendix contains two tables—one to identify the rows and the second to identify the columns of the matrix. Although there are only 92 rows within this matrix, the data availability permitted considerably more detail for individual sectors than is shown within the 190-equation U. S. input-output table for 1947. (Even within the machinery sectors where the U. S. input-output classification was taken over directly, the machine tool products were disaggregated into five individual categories.) The row classification may be summarized as follows:

Row Identification	Sector	Number of Rows
5, 6	Electricity	2
8	Rail freight	1
9–11	Inorganic chemicals	3
12–18	Pulp and paper	7
7, 19–30	Oil and gas	13
31–42	Petrochemicals	12
43–50, 61–64	Iron and steel	12
51–53, 69	Aluminum	4
54–58, 70	Copper	6
59, 60	Limestone and cement	2
65–68	Metal castings	4
71–94	Metal fabricated products	24
95	Foreign exchange	1
96	Gross savings and investment	1
	Total	92

There was considerable variation in the number of activities specified as alternative sources of supply of each commodity. In most instances, only a single domestic production activity was specified. However, several alternative activities were included in such sectors as electricity, oil, and steel, where there exist major tradeoffs between domestic investment, foreign exchange, and current account inputs from other sectors. In the case of 60 commodities, importation was considered as an explicit alternative to domestic production.[10] The matrix also includes a number of instances of joint product relationships, e.g., electric energy as a co-product with peak electric power, natural gas as a co-product with crude oil, and ferrous scrap as a co-product with metal fabricated products.

No explicit allowance was made for handling economies of scale. Integer programming computer codes might have been employed to cope with this feature, but seemed too experimental to be applied as of 1961. Data were available for estimating economies of scale in investment in a number of the process-type industries, but not for metalworking. In several of these industries, the capital coefficients were predicated upon a typical Mexican plant size, e.g., 50,000 barrels per day for crude oil pipelines and for primary atmospheric distillation of petroleum. Whenever the investment coefficients were based upon some such specific plant size, this fact was noted in the group's worksheets as the "reference scale" for this process, and these scales were then available for subsequent checks of consistency with the aggregate capacity expansions projected over the decade.

Computing considerations did not preclude the case of *dis*economies of scale. One important possibility of this type occurs in connection with the country's iron ore deposits. Once the northern deposits are utilized to their full capacity, additional social overhead investment will be needed in order to develop the southern deposits. (This limitation on the availability of northern ore is specified in row 44 of the matrix.) Other instances of diseconomies of scale occur in connection with ferrous and copper scrap (rows 49 and 55).

It is also believed that the country will run into diminishing returns in the exploitation of its oil and natural gas deposits. Since no reliable numerical estimates of cost increases were readily available in this sector, a sensitivity analysis was conducted. Even after doubling the investment cost per additional barrel of crude oil available, the aggregate results were affected by a comparatively small amount. The 1970 requirements for foreign exchange were increased from $172 to $181 millions. However, the 1970 rate of unutilized domestic savings dropped from $181 to $44 millions. Interestingly enough, even with the increase in crude oil investment costs, the model still indicates that it would be optimal for Mexico to invest in thermal rather than in hydroelectric power plants. This same result would not necessarily hold

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[10] Import possibilities were neglected in the case of certain commodities with high transport costs, e.g., cement and electricity; in the case of items where Mexico has traditionally been an exporter, e.g., residual fuel oil and copper ore; and in the case of certain intermediate materials that are ordinarily integrated with other stages of production, e.g., catalytic cracked gasoline, styrene, and sponge iron.

true if the model were extended so as to include geographical details with respect to the location of oil and gas deposits and hydroelectric sites.

For the most part, locational details were introduced through assuming a "typical" locational pattern of production and sales. For example, electric power transmission investments were estimated separately on the basis of a thermal plant in the immediate vicinity of Mexico City, and a hydroelectric plant located at Temazcal. Similarly, the investment costs for crude oil and natural gas pipelines were based upon a typical distance: 800 kilometers between Ciudad Pemex and Mexico City.

Only in the case of the iron and steel industry was there an attempt made to relate rail transport inputs to the location of the processing and consuming plants. Ton-kilometer rail freight inputs for coke, limestone, iron ore, scrap, and mill products were derived individually, depending upon whether the iron ore originated in the north or in the south. For all other key sectors, rail freight inputs were regarded as exogenous, and were aggregated into the final demand.

SOURCES OF DATA

The work of gathering data for this matrix was considerably facilitated by the previous accumulation of experience with process analysis for individual sectors. This previous experience was used—not primarily as a direct source from which to transcribe coefficients—but rather to suggest an orderly framework within which to analyze the specific problems of the Mexican economy. For example, the work of Massé and Gibrat (1957) suggested the importance of analyzing the electric power sector in terms of joint product categories such as peak electric power and annual electrical energy. The work of Vietorisz (1961) provided the basic categories for the petrochemicals analysis. And similarly, the work of Fabian (Chapter 9 of this book) called attention to the possibility of alternative ratios of hot iron and ferrous scrap in the open hearth steel process. None of these technological features are novelties to engineers familiar with electric power, petrochemicals, or steel. The process analysis work consists of integrating these technological considerations to form a macro-economic investment decision model.

With the exception of the metalworking industries, Mexican sources were available for estimating the current account flow coefficients a_{ij}.[11] In the case of the metalworking industry, the current inputs of metals, fuel, and electricity were copied directly from the *U. S. Census of Manufactures* (1954).[12] Mexican industry has had only a limited range of experience with the metal fabricating industries, and it seemed preferable to extrapolate from U. S. material inputs rather than rely upon Mexican census data in this area.

Capital equipment investment coefficients for metalworking were also based

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[11] Certain of the current flow coefficients for petroleum and petrochemicals were also based upon U. S. sources: Manne (1961) and Vietorisz (1961) respectively.

[12] Also taken directly from the *U. S. Census* were the current inputs of certain complex assembled components produced by other metalworking industries: electric motors, generators, and internal combustion engines.

directly upon U. S. sources. The machine tool stocks per unit of output were taken from the 1953 *American Machinist* survey cited by Markowitz and Rowe (Chapters 10–14 of this book), and the miscellaneous equipment investment coefficients from H. Markowitz (1953). In turn, on the basis of a sample of just four Mexican metalworking plants, total construction costs were estimated at 33% of equipment costs. Finally, the coefficients for structural steel, reinforcing rods, and cement were extrapolated in relation to total construction costs on the basis of a detailed examination of the blueprints for a single Mexican plant. Whatever be the merits of such small-sample extrapolation procedures for an experimental effort, there is little doubt that further research could lead to a substantial improvement in the quality of the coefficients utilized in the metalworking industries.

Capital coefficients for other sectors of the Mexican economy—although difficult to estimate—did not present quite such a serious obstacle as those for metalworking. In certain sectors, capital coefficients were obtained directly from special analyses of expenditures upon recently constructed plants. In many cases, however, the plant construction cost data were available only in the form of total expenditures per unit of capacity installed. Here, the Mexican source was taken as a control total, and the U. S. capital coefficients published by Grosse (1953) were utilized to estimate the percentage breakdown of this total into detailed categories such as pumps and compressors, cranes and conveyors, etc.[13]

In the absence of a source such as Grosse (1953), it would have been virtually impossible to estimate detailed capital coefficients by branch of industrial origin, given the brief time available for this project. On the other hand, because Grosse's capital coefficients were stated in terms of money value units per unit of plant capacity, the output of most of the capital goods producing industries also had to be measured in money terms rather than in physical units. Altogether, 20 of the 92 rows are measured in terms of money units, and the remainder in physical units.

Incidentally, the general reliance upon diverse physical and technological units led to clerical difficulties. If all the engineering estimates of inputs and outputs had subsequently been converted into money values, this uniform yardstick would have served to detect a number of gross clerical errors such as misplaced decimal points, reversals of algebraic sign, and incorrect conversions between cubic feet of natural gas and kilocalories of industrial fuel. Only after painful experience was it realized that money value check totals could play a useful role in data processing, and in detecting major errors in the technological estimates.

CONCLUSIONS

The reader would do well to remain skeptical about the specific conclusions reported here, e.g., the highly complementary relationship between domestic

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[13] Grosse's capital coefficients were based upon engineering extrapolations as well as upon historical construction costs for balanced and unbalanced plant additions during World War II and during the Korean War.

savings and foreign loans and aid to Mexico. Results of this type are illustrative, but cannot be taken too literally. Any one of the 1,900 coefficients in the linear programming matrix could be in serious error, and each of these coefficients ought to be reexamined with care.

Despite many reservations, this work represents another step in the evolution of aggregate investment programming. Admittedly, from the viewpoint of overall economic development, a one-resource GNP model is more manageable than a multi-resource analysis of the type suggested here. It is also true that from the viewpoint of the individual enterprise, a specific investment project can be evaluated in far greater detail than is practical within an inter-industry framework. The role of a multisector model is to bridge the gap between the formulation of overall economic strategy and the implementation of that strategy in terms of specific investment projects. Process analysis—which relies heavily upon engineering estimates of alternative production methods—appears particularly well suited to aid in bridging this gap.

The importance of perfecting such analytical techniques represents more than the ability to generate a monolithic, consistent official program for economic development. Programming methods can be sufficiently flexible to permit planning experts to present not just one—but several *alternative* paths of development for consideration by other governmental officials and by the voters at large. If the assumptions and results are appropriately presented, mathematical models can serve to dispel some of the mystery that often surrounds central planning, and should make it possible for the electorate to be better informed with respect to the inherently difficult and subjective choices confronting their country.

APPENDIX

Key to Row Numbering

Row Identification	Item Identification	Exponent of Ten[a]	Unit of Measurement
5	Electric energy	6	kilowatt-hours
6	Peak electric power	0	kilowatts
7	Industrial fuel	9	kilocalories
8	Rail freight	6	ton-kilometers
9	Sulfuric acid	0	tons
10	Sodium hydroxide	0	tons
11	Chlorine	0	tons
12	Crude pulp	3	tons
13	Bleached pulp	3	tons
14	Mechanical pulp	3	tons
15	Sulfite pulp	3	tons
16	White paper	3	tons
17	Newsprint	3	tons
18	Other paper	3	tons
19	Crude oil and condensates	3	barrels

Key to Row Numbering

Row Identification	Item Identification	Exponent of Ten[a]	Unit of Measurement
20	Natural gas	3	cubic meters
21	Dry gases	3	tons
22	Propane, propylene (liquid gas)	3	barrels
23	Butanes, butylenes	3	barrels
24	Naphtha (solvents)	3	barrels
25	Middle distillates (jet fuel, kerosene, diesel oil)	3	barrels
26	Vacuum gas oil (lubricating oils)	3	barrels
27	Vacuum bottoms (asphalt)	3	barrels
28	Catalytic cracked gasoline	3	barrels
29	Motor gasoline, 87 octane	3	barrels
30	Residual fuel oil	3	barrels
31	Benzene, toluene, xylenes	3	barrels
32	Tetraethyl lead	0	liters
33	Ammonium nitrate	3	tons
34	Ammonium sulfate	3	tons
35	Nitric acid	3	tons
36	Ammonia, anhydrous	3	tons
37	Polyethylene	3	tons
38	Polystyrene	3	tons
39	95% ethylene	3	tons
40	Styrene	3	tons
41	Synthetic rubber	3	tons
42	Butadiene	3	tons
43	Coke	0	tons
44	Northern deposit limitation[b]	0	tons
45	Northern iron ore	0	tons
46	Southern iron ore	0	tons
47	Pig iron	0	tons
48	Sponge iron	0	tons
49	Ferrous scrap[b]	0	tons
50	Carbon steel ingots	0	tons
51	Bauxite	0	tons
52	Alumina	0	tons
53	Aluminum ingots	0	tons
54	Copper ore	0	tons
55	Copper scrap[b]	0	tons
56	Mata	0	tons
57	Blister copper	0	tons
58	Electrolytic copper	0	tons
59	Limestone	3	tons
60	Cement	3	tons
61	Carbon steel, reinforcing rods	0	tons
62	Carbon steel, flat products	0	tons
63	Carbon steel, other nonflat products	0	tons

Key to Row Numbering

Row Identification	Item Identification	Exponent of Ten[a]	Unit of Measurement
64	Alloy and stainless steel	0	tons
65	Iron castings	0	tons
66	Steel castings	0	tons
67	Aluminum castings	0	tons
68	Copper castings	0	tons
69	Aluminum mill shapes	0	tons
70	Copper mill shapes	0	tons
71	Fabricated structural steel	0	tons
72	Boiler shop products	3	U. S. 1960 dollars
73	Metal stampings	3	U. S. 1960 dollars
74	Steam engines, turbines	3	U. S. 1960 dollars
75	Internal combustion engines, excluding automotive	3	U. S. 1960 dollars
76	Farm tractors	3	U. S. 1960 dollars
77	Farm equipment, excluding tractors	3	U. S. 1960 dollars
78	Construction and mining machinery	3	U. S. 1960 dollars
79	Oil field machinery	3	U. S. 1960 dollars
80	Drilling machines	0	units
81	Grinding machines	0	units
82	Lathes	0	units
83	Milling machines	0	units
84	All other machine tools	0	units
85	Pumps and compressors	3	U. S. 1960 dollars
86	Cranes and conveyors	3	U. S. 1960 dollars
87	Blowers and fans	3	U. S. 1960 dollars
88	Motors and generators	3	U. S. 1960 dollars
89	Transformers	3	U. S. 1960 dollars
90	Electrical control apparatus	3	U. S. 1960 dollars
91	Motor vehicles and parts	3	U. S. 1960 dollars
92	Truck trailers	3	U. S. 1960 dollars
93	Locomotives and parts	3	U. S. 1960 dollars
94	Railroad cars	3	U. S. 1960 dollars
95	Foreign exchange[c]	3	U. S. 1960 dollars
96	Gross savings and investment[d]	6	Mexican 1960 pesos

N.B.: All rows—unless otherwise indicated—refer to restrictions of type (4) in text.

[a] Exponent of ten is to be read as follows: The unit of measurement in row 5 is 1,000,000 kilowatt-hours. The unit of measurement in row 6 is 1 kilowatt.

[b] Restriction of type (5) in text.

[c] Restriction (6) in text.

[d] Restriction (7) in text.

Key to Column Numbering

Column Identification	Activity[a]	Exponent of Ten	Unit of Measurement
UPxxx	Positive unit vector for row xxx; 5 activities of this type		
NUxxx	Negative unit vector for row xxx; 87 activities of this type		
YYxxx	Importation of one unit of commodity specified in row xxx; 60 activities of this type		
ZZZZZ	Foreign loans and aid	3	U. S. 1960 dollars
EL001	Thermal electricity, 90% power factor	0	kilowatts
EL002	Thermal electricity, 40% power factor	0	kilowatts
EL003	Hydroelectricity	0	kilowatts
RR001	Rail freight	6	ton-kilometers
INC01	Caustic soda	3	tons
INC02	Sulfuric acid	3	tons
PAP01	Crude pulp	3	tons
PAP02	Bleached pulp	3	tons
PAP03	Mechanical pulp	3	tons
PAP04	White paper	3	tons
PAP05	Newsprint	3	tons
PAP06	Other paper	3	tons
PET01	Production of crude petroleum and natural gas; pipeline transportation of crude petroleum	3	barrels
PET02	Pipeline transportation of natural gas	3	tons of dry gases
PET03	Crude petroleum to stills*	3	barrels
PET04	Vacuum bottoms to coking*	3	barrels
PET05	Middle distillates to catalytic cracking, 25% recycle*	3	barrels
PET06	Middle distillates to catalytic cracking, 100% recycle*	3	barrels
PET07	Vacuum gas oil to catalytic cracking, 25% recycle*	3	barrels
PET08	Vacuum gas oil to catalytic cracking, 100% recycle*	3	barrels
PET09	Tetraethyl lead	3	liters
PET10	Motor gasoline blend, reformate	3	barrels
PET11	Motor gasoline blend, catalytic cracked + naphtha, 1 cc TEL	3	barrels
PET12	motor gasoline blend, catalytic cracked + naphtha, 3 cc TEL	3	barrels
PET13	Residual fuel blend, vacuum bottoms + gas oil	3	barrels
PET14	Residual fuel blend, middle distillates	3	barrels
PET15	Input of dry gases into industrial fuel*	3	tons
PET16	Input of propane, propylene into industrial fuel*	3	barrels
PET17	Input of butanes, butylenes into industrial fuel*	3	barrels
PET18	Input of residual fuel oil into industrial fuel*	3	barrels

Key to Column Numbering

Column Identification	Activity[a]	Exponent of Ten	Unit of Measurement
PET19	Benzene, toluene, xylenes	0	330 barrels
PET20	Input of butanes into liquid gas*	3	barrels
PCM01	Ammonium nitrate	3	tons
PCM02	Ammonium sulfate	3	tons
PCM03	Nitric acid	3	tons
PCM04	Ammonia, anhydrous	3	tons
PCM05	Polyethylene	3	tons
PCM06	Polystyrene	3	tons
PCM07	95% ethylene	3	tons
PCM08	Styrene	3	tons
PCM09	Synthetic rubber	3	tons
PCM10	Butadiene	3	tons
STL01	Northern iron ore	3	tons
STL02	Southern iron ore	3	tons
STL03	Coke	3	tons
STL04	Pig iron from northern ore, blast furnace	3	tons
STL05	Pig iron from southern ore, electric process	3	tons
STL06	Sponge iron	3	tons
STL07	Carbon steel ingots, open hearth, minimum scrap	3	tons
STL08	Carbon steel ingots, open hearth, maximum scrap	3	tons
STL09	Carbon steel ingots, electric furnace	3	tons
STL10	Carbon steel ingots, sponge iron	3	tons
STL11	Carbon steel, flat products	3	tons
STL12	Carbon steel, reinforcing rods	3	tons
STL13	Carbon steel, other nonflat products	3	tons
STL14	Alloy and stainless steel, rolled products	3	tons
AL001	Alumina	3	tons
AL002	Aluminum ingots	3	tons
AL003	Aluminum mill shapes	3	tons
CPR01	Copper ore	3	tons
CPR02	Mata	3	tons
CPR03	Blister copper	3	tons
CPR04	Substitution of blister for scrap	3	tons
CPR05	Electrolytic copper	3	tons
CPR06	Copper mill shapes	3	tons
CEM01	Limestone	3	tons
CEM02	Cement	3	tons
MW101	Iron castings	3	tons
MW102	Steel castings	3	tons
MW103	Aluminum castings	3	tons
MW104	Copper castings	3	tons
MW201	Fabricated structural steel	3	tons

Key to Column Numbering

Column Identification	Activity[a]	Exponent of Ten	Unit of Measurement
MW202	Boiler shop products	6	U. S. 1960 dollars
MW203	Metal stampings	6	U. S. 1960 dollars
MW204	Steam engines, turbines	6	U. S. 1960 dollars
MW205	Internal combustion engines, excluding automotive	6	U. S. 1960 dollars
MW206	Farm tractors	6	U. S. 1960 dollars
MW207	Farm equipment, excluding tractors	6	U. S. 1960 dollars
MW301	Construction and mining machinery	6	U. S. 1960 dollars
MW302	Oil field machinery	6	U. S. 1960 dollars
MW303	Drilling machines	6	U. S. 1960 dollars
MW304	Grinding machines	6	U. S. 1960 dollars
MW305	Lathes	6	U. S. 1960 dollars
MW306	Milling machines	6	U. S. 1960 dollars
MW307	All other machine tools	6	U. S. 1960 dollars
MW401	Pumps and compressors	6	U. S. 1960 dollars
MW402	Cranes and conveyors	6	U. S. 1960 dollars
MW403	Blowers and fans	6	U. S. 1960 dollars
MW404	Motors and generators	6	U. S. 1960 dollars
MW405	Transformers	6	U. S. 1960 dollars
MW406	Electrical control apparatus	6	U. S. 1960 dollars
MW501	Motor vehicles and parts	6	U. S. 1960 dollars
MW502	Truck trailers	6	U. S. 1960 dollars
MW503	Locomotives and parts	6	U. S. 1960 dollars
MW504	Railroad cars	6	U. S. 1960 dollars

[a] All activities are normalized in terms of principal annual output except those indicated with an asterisk. These latter are normalized in terms of their principal input. Thus, activity EL001 refers to the production of one kilowatt of electric power with a thermal plant operated at a 90% power factor. In this case, there is also a co-product: .007,884 millions of kilowatt-hours per year. Activity PET03 refers to the input of 1,000 barrels per year of crude petroleum into stills.

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CHAPTER 17

SECTOR STUDIES IN ECONOMIC DEVELOPMENT PLANNING BY MEANS OF PROCESS ANALYSIS MODELS

Thomas Vietorisz

This volume contains a number of sector studies, both in manufacturing and in agriculture, which have been prepared by means of process analysis models. While most of these studies refer to the United States, they strongly suggest that the method of process analysis can also be applied to the formulation of sectoral development plans in underdeveloped countries. In what follows, the practical problems which arise in attempting such an extension will be discussed. It is convenient to begin by considering the manner in which sectoral planning problems present themselves to the economist in the field.

In a country where no significant amount of economic development planning has been undertaken previously, there is generally a lack of insight into the magnitude of the task of preparing a sector study. It is often hoped that an effort of a few weeks can show what the advantageous lines of new investment are in one or more sectors; at the same time, no adequate statistics exist, there is no overall development plan, and there are no sources of technical information. In such countries, it is indispensable to start out with a diagnosis of the position of the sector in the context of the entire economy, followed by a first-approximation trend projection. In other countries which have had more experience with planning, many elements of the diagnosis and the trend projection may be found ready at hand, and need perhaps only some complementation and systematization. In any event, the two preliminary phases of diagnosis and trend projection are a necessary foundation for the application of more sophisticated techniques, such as process analysis, to the preparation of sectoral development plans.

DIAGNOSIS

The process of diagnosis has to determine, first, what has been happening within the sector, how, and why; and second, what the relationship of the sector is to the rest of the economy.

In a country with little previous planning experience, the first question typically overloads the data system available. Censuses may not exist, and if they do, they are almost always out of date and of dubious reliability. Import and export statistics are generally better than other kinds of economic statistics, but they are seldom classified consistently with each other and with whatever production statistics may exist. The latter often cover only a few large products. Thus, the first task is to correct, to update, and to reclassify the statistical information relating to the sector. Sample surveys of domestic production, stratified by establishment size and by traditional versus commercial production methods, are among the most useful tools in these stages. Following the construction of an adequate statistical base, historic trends pertaining to the sector are identified and their causes, both economic and institutional, are analyzed.

The second question, concerning the relationship of the sector to the rest of the economy, may in turn overload the planning apparatus. Perhaps no overall plan has ever been formulated. In this case, the sector study must necessarily be expanded to include the preparation of at least a first-cut overall plan. The key questions to be answered by this phase of the diagnosis are the following:

How much import substitution must the sector contribute to the economy as a whole?

How much investment capital can be allocated to the sector?

How much must the sector contribute to the generation of employment opportunities? It should be noted that the economic context is typically one of rural-urban population shift, coupled with a rapid growth of the labor force.

What is the role of the sector in lending flexibility to the economy, and in reducing its vulnerability to outside shocks? The context here is mostly one of structural imbalance: a one-crop or monoexport economy, which is to be steered in the direction of a better structural equilibrium.

On the basis of the diagnosis, a first-approximation plan of the sector can be prepared by the simple method of trend projection. Both the overall analysis and the sectoral trend projection have to be revised successively as improved information becomes available in later stages of the planning process.

TREND PROJECTION

Sectoral trend projections are based on the assumption of a given overall growth rate for the economy. The sector is broken down into commodity classes, and the major components of supply and demand for each class are related to the overall growth rate, using to this end principally the historical trends identified in the course of the diagnosis. In certain regards, these trends may have to be modified: for example, a straight continuation of import and export trends generally leads to an intolerable foreign-exchange gap. The identification and resolution of such contradictions is one of the purposes of the projection.

The principal components of *demand* are: household consumption, projected by income (or total-consumption) elasticities; government expenditures, projected on the basis of historic trends and anticipated policy changes; exports, analyzed individually in their principal markets; investment requirements, derived from whatever is known about investments elsewhere in the economy, together with investments projected for the sector; and intermediate demands. The last may initially be projected by income elasticities, in order to arrive at a rough approximation. This can be improved greatly in a second step by allowing for key interactions with other sectors; for example, cement needs can be revised by reference to activity in the construction sector. The identification of such key interactions is a far simpler matter than a complete, formal interindustry analysis, yet, if judiciously applied, it may lead to close approximations. The identification of key interactions is especially important, but also especially convenient, in regard to interactions that arise between commodities within the sector itself.

The components of *supply* are national production and imports. The commodity-by-commodity breakdown of total supply into these components is the weakest point of the trend projection method, since it involves considerations of comparative efficiency in addition to the usual requirement of consistency. Concretely, the question is: where to substitute imports, and by how much? This question must be answered in every development plan; moreover, the plan as a whole is particularly sensitive to the kind of answer provided, since import substitution is the principal means for managing the ever-present foreign-exchange gap.

Export alternatives and technological choices in production can be expected to raise similar problems of comparative efficiency, but the practical significance of these is less in the formulation of sectoral development plans for underdeveloped countries. The bulk of the exports of these countries is of the traditional kind where no choice between alternatives arises (even though the instability of such exports may lead to difficult short-run policy problems of a different type); whereas the choice between new, nontraditional export opportunities, which raises a genuine efficiency problem, is reduced in significance by the relatively small volume of such potential exports when compared with traditional ones. In regard to technological alternatives, the ones which affect the plan most are precisely those which arise in connection with import substitution, where they appear as alternative new lines of national production. Elsewhere, the inertia of the economic system reduces substantially the degree of freedom of technological choice. As a country is progressively developing, however, both of these efficiency problems acquire increasing importance, and eventually they overshadow the problem of import substitution.

Process analysis models are particularly well suited to the exploration of efficiency problems. For this reason, they are a natural extension of the trend projection method in the formulation of sectoral plans. Before turning to a discussion of these models, it is nevertheless of interest to inquire how import substitution can be handled at all within the trend projection method.

Although this method offers no systematic approach to the problem, sectoral plans are, in fact, being formulated within its framework, and, in the hands of persons of intelligence and judgment, it appears to yield acceptable results.

There are two reasons why this should be so. First, plans which are defective under the criterion of efficiency may not appear unreasonable, since missed opportunities are inherently harder to perceive than obvious inconsistencies. Secondly, there are a number of practical considerations which are often used to restrict the range of variability, and which generally lead to an acceptable, if not necessarily to a highly efficient, import substitution plan. Thus, lines of production not customarily encountered in underdeveloped countries with a similar or slightly higher level of development, may be left out of consideration; import substitution ratios suggested by pre-existing isolated investment studies may be accepted without further appraisal; and it may be postulated that there will be no decline in the market share of any existing national production activity. For the ranges of variation of individual commodities remaining after the application of these limits, an average ratio of import substitution is derived on the basis of the total import substitution allotted to the sector in the overall plan. This average is then used as a benchmark in the final upward or downward adjustment of the substitution ratio for individual commodities—based, first, on interviews with knowledgeable persons and other local intelligence; secondly, on miscellaneous useful criteria including market size and growth in relation to usual production scales, differences between import prices and raw material costs, complexity of technology, and others; and finally, on un-useful but prevalent criteria, such as political and military factors.

A final note on trend projections: these can be made more sophisticated by applying formal input-output techniques to the determination of intermediate demands and investment needs. These techniques resolve consistency problems without the need for trial-and-error solutions, but they leave the entire problem of efficiency comparisons untouched. (The one-factor pricing scheme implied by input-output models is too grossly unrealistic even for a first approximation.) Thus, when using these techniques, import substitution and other problems of choice still have to be resolved by the same largely intuitive methods which have been discussed in the preceding paragraph.

PROCESS ANALYSIS MODELS: GENERAL PROBLEMS

Process analysis models are to be used for a *second approximation* in the formulation of sectoral development plans: for the extension and refinement, not the replacement, of trend projections. In order to ensure that process analysis models stay in close contact with reality, they should be checked constantly against the simpler models which, by virtue of being more intuitive, are less likely to lead inadvertently to gross distortions. Conversely, the initial version of a sector plan, defined by trend projection, is subject to major improvements of efficiency, because process analysis models can generally identify attractive alternatives which would otherwise almost surely remain hidden. *This task, the upgrading of a feasible plan, is held to be the central objective of process analysis as applied to the sectoral planning of economic development.*

The notion of efficiency implies two types of decision problems: the setting of goals and the selection of means. In the formulation of sectoral plans, the selection of means usually receives the major emphasis, since the definition of goals is more properly handled at the economy-wide planning level. Nevertheless, certain subsidiary goals which have escaped attention in the original formulation of the overall plan may be discovered as the initial versions of the sector plan are being inspected. For example, the optimal locational pattern of a major industry as determined on the basis of the originally stated development goals, may turn out to have unacceptable implications for regional development. Thus, a new constraint or a new weighting factor in the objective function may have to be added to the model, representing a social objective which has been discovered as a result of the formulation of the sectoral plan.

At times it is suggested that simulation models are preferable to optimization models as planning instruments. Such models are descriptive rather than normative, and they can incorporate a large amount of detail which is hard to analyze by formal methods (random elements, nonconvexities, time lags, etc.). In practical applications, however, the differences between the two methods tend to narrow down considerably. First, the more recent tools of optimization, including such techniques as integer programming, have greatly increased the complexity of particular cases that can be incorporated into formal models. Second, sensitivity analysis reduces the purely normative nature of optimization models, while conversely, the sequencing of simulation runs tends to be directed at the goal of improving the resulting trial solutions. Thus, the two approaches tend to converge.

When applied to the formulation of second-approximation sectoral development plans, simulation has the virtue of being able to stay close to the initial trend projections, while enriching them with large amounts of detail; on the other hand, optimization models are inherently more convenient for analyzing efficiency problems, since they have the ability of cutting across the enormous combinatorial range of alternatives. Perhaps the best way of integrating the two approaches would be to use optimization for identifying a narrow range of efficient solutions, and thereafter to explore the implications of these in more detail by means of a simulation model. Of particular interest in the process of exploration is the detailed tracing of the effect of *policy instruments* which can be used for the execution and control of sectoral development plans, including incentives and deterrents, direct quantitative controls, and various types of institutional regulation.

Turning next to a comparison of the sectoral process analysis models of this volume, most of them set in the context of the United States economy, with the kind of model required for sectoral planning in underdeveloped countries, an important difference in focus emerges. The two differing applications will be referred to as *capability analysis* on the one hand and *structural analysis* on the other. The contrasting features of these applications are summarized in the tabulation below.

	Capability Analysis	Structural Analysis
Aim:	Explore limitations on shifts in demand composition; explore effects of structural shocks.	Find "advantageous" new lines of investment; interrelate social objectives in defining advantage.
Typical structure of economy:	Approximate structural balance; given fixed capital or slow balanced growth.	Structural imbalance; rapid structural change with strong social impact; rapid discontinuous growth.
Principal alternatives explored:	Demand shifts; raw material changes; production scheduling; technological and locational alternatives.	Imports vs. national production; alternative locations for new plants; alternative markets for new exports; alternative scheduling of new investment; alternative social objectives.
Classification focused on:	Determination of characteristic averages based on broad product classes and broadly defined activity concepts.	Identification of individual products and production activities with desired characteristics; focus on narrowly defined products which can be taken as typical of a wide range of products with similar characteristics.

The change of focus required for the application of process analysis models to sectoral planning in underdeveloped countries introduces some new problems. First, nonlinearities and especially *nonconvexities* become considerably more troublesome, because in structural analysis, the scales of plants are variable, whereas in capability analysis they are generally fixed. Moreover, new investments in underdeveloped countries, especially the smaller ones, often introduce discontinuities, since in such countries the addition of, say, a single steel mill will often radically change the basic industrial structure, whereas in a more highly industrialized economy, the addition of the same plant represents merely a marginal change. For this reason, planning applications of process analysis stand to benefit greatly from advances in integer and nonconvex programming and related techniques.

Secondly, the description of the universe of technological choices introduces a major problem in *information processing*. Evidently, most of this information cannot be derived from statistical or other sources within the country itself, since the lines of production or export which are to be described do not yet exist there. This problem has sufficient ramifications to merit separate discussion in the next section.

INFORMATION PROCESSING

Technological and related information for the purposes of sectoral, especially industrial, planning in developing economies has never been compiled. The economist in the field would have an urgent need for a reference manual comparable to such manuals in their respective fields as the *Chemical Engineers' Handbook* (Perry, 1950) or other engineering and technical reference works. Such a volume would have to contain condensed outlines of the principal sectoral planning techniques, together with extensive tables describing the physical inputs and outputs and other important parameters of alternative production processes in all industries and industrial branches of interest.

While many of the conceptual problems of technological process description are discussed in the sectoral studies of this volume, some additional ones would require further analysis. The principal problem of constructing a reference manual, nevertheless, is a different one. It consists in the tremendous dispersion of the relevant information in thousands of technical journals, in a flood of company production paper, engineering reports, production norms, designers' manuals, cost estimating reference works, industrial statistics, and many other potential sources. The compilation poses a problem in information processing which is quite similar to the problem of the extraction of political and military intelligence from material collected by agencies for this purpose: the meticulous description of the exact information wanted, the development of leads on where to find it, the piecing together from various sources, the task of cross checking, organization, condensation, and presentation, are analogous in both cases.

Unfortunately, the most obvious sources are the most difficult of access. In the United States and Europe, many of the technical data which would be most vitally needed are treated as confidential business information and are not reflected in censuses and most other published material. To obtain them, personal contacts must be developed in individual companies and the process of data collection takes on the character of a confidential investigation. Soviet and related publications, on the other hand, reveal considerably more data of direct interest, in technical norms, designers' manuals, and industry reports; however, some of this material reflects desired rather than actual practice and is inherently difficult to double check.

From whatever source the information is compiled, it has to be updated constantly due to rapid technical progress in many industries.

The data collected in developed countries may not be fully transferable to underdeveloped ones. Productivity, for example, varies strongly between countries, and often in a way which is difficult to predict. Productivity measurements as customarily taken suffer from the deficiency of mixing the effect of factors characteristic of the labor force itself (educational level, health, etc.) with extraneous factors, such as capital intensity and the quality of management. If data are to be made more transferable between countries, these factors must be separated. A study of this problem exists for the textile sector (United Nations, 1951).

A related consideration is the variation of best technology between countries in response to climate, the availability and quality of local raw materials, and relative factor prices, primarily those of capital, labor, and skills. A description of current technology in highly industrialized countries will not comprise the alternatives which might be of the greatest interest to many underdeveloped areas. Thus, labor-intensive production processes are generally archaic in the advanced countries; a complete choice of alternatives, however, would have to extend to labor-intensive processes incorporating the most recent advances of modern technology. Many auxiliary operations such as materials handling, loading, packaging, etc., can be designed either with a high or a low degree of mechanization and automation, without affecting the core process which they serve. A compilation of technological alternatives for use in sectoral planning in underdeveloped countries would be severely deficient if it left this range of variability out of consideration. On the other hand, it is evident that covering such aspects goes beyond the mere compilation of information, into the area of technological research and development.

The above discussion indicates that the compilation of technological information in a planner's manual cannot hope to exhaust the universe of available choices. Such an aim would, in any case, not be a reasonable one, since there is a limit to the amount of information that is worth incorporating in a process analysis model. Even with unlimited computing abilty, it would make no sense to analyze in a single step hundreds of thousands of individually specified alternatives, since the formulation of such a monster model would be extravagantly wasteful of the limited resources available for planning.

The problem then arises: what to include in a process analysis model and what to leave out? Instead of attempting a direct answer, it is better to rephrase the question. How can the closest approximation to the ideal optimum solution of the "monster" model be obtained with a given effort?

Actual planning practice points the way to an answer. The selection of alternatives is usually carried through several *stages*, even though the sophistication with which each stage is handled varies widely. Three typical stages are: preselection, feasibility studies, and project engineering. From each stage to the next, the range of alternatives under active consideration is narrowed down, but the precision of evaluation is increased. Thus, the channeling of information into the planning process is not complete prior to the inception of analysis, but rather, *information feed-in and analysis are alternated.* Specifically, the analysis performed at each stage determines the information to be provided for the next stage.

This formulation opens up some questions of more general interest. What is the best number of stages in a given problem? What is the desirable degree of approximation at each stage? What are the criteria for resolving the dilemma between providing too much information at each stage, thus incurring excessive costs, and providing too little, thus increasing the danger of the premature exclusion of attractive alternatives?

While these issues cannot be further explored here, it is clear that process analysis models must be adapted to the stage-by-stage handling of information in the formulation of sectoral plans.

At the stage of preselection, the universe of alternatives should be as comprehensive as possible. At this stage, there is often a temptation to restrict the universe of selection to pre-existing project studies and to ideas current in the business and governmental circles of the country in question, or else, to apply mechanically the experience of industrialized countries in discarding certain alternatives from consideration. Both courses may result in overlooking attractive possibilities. For example, captive heavy forge capacity customarily found in certain lines of production in metropolitan countries, with high annual production rates, may lead to the mistaken assumption that these production rates are necessary for efficient operation; the possibility of a different institutional arrangement, i.e., the sharing of the forge capacity between different kinds of production, is being overlooked. Similarly, the scale of efficient operation of certain chemical processes may be overestimated if no allowance is made for the fact that a number of ancillary processes may be shared between different processes which are seldom found together in more developed countries.

The evaluation of alternative choices at the stage of preselection is customarily undertaken by reference to criteria such as market size, production scales required, skill and capital requirements, etc., which are applied to each alternative in isolation. It would be very useful to compile more technological information, of the semiquantitative nature needed for preselection, for a wide variety of processes, because the choices made at this stage affect critically the later stages. It does not appear attractive, however, to apply process analysis to the preselection stage, because it is too precise a tool for the degree of approximation desired here.

The technical alternatives which survive preselection are defined more closely during the subsequent stage of feasibility studies. Process analysis models come into their own during this stage. In addition to their other advantages, discussed elsewhere, such models have the virtue of permitting a further subdivision of the stages of information handling by means of the alternation of sensitivity analysis and progressively improved technological descriptions. More precisely, all technological alternatives received from preselection are initially described with a given (not too close) degree of approximation for inclusion in the process analysis model. An optimal solution is then derived and the sensitivity of this solution to the technical coefficients of the model is investigated. Those activities to the coefficients of which the optimal solution is most sensitive are described more closely and, if necessary, further disaggregated or subdivided; this is followed by another round of sensitivity analysis and a further improvement of the technological description, etc. Thus, the effort expended on the description of technology is concentrated on those activities to which the optimal solution proves to be most sensitive.

The same strategy can be carried over from the stage of feasibility studies also into the final stage of concrete project engineering. The bulk of effort in the exploration of alternatives, including whatever research and development may be undertaken in connection with this stage, should be concentrated on those activities or other technological decision units which have been singled out by earlier analysis as likely to have the greatest effect on the objectives of the sectoral plan. Process analysis may be one of the tools employed at this stage, but it is likely to be subordinate in importance to conventional techniques of engineering analysis and design.

The above discussion concerning the stages of information handling in the course of the sectoral planning process also suggests a convenient strategy for the problem of compiling reference manuals for sectoral planning. These manuals should be aimed primarily at the preselection stage and at the initial phases of the feasibility-study stage during which the early versions of process analysis models are being formulated. Instead of attempting to exhaust the finer detail needed during the later phases of feasibility analysis and a fortiori during project engineering, the manuals should be designed for more and more heavy reliance on complementation by technical specialists as the technological description is progressively narrowed and improved. The stage of final project engineering, in any event, must always be primarily the responsibility of such technical specialists.

SECTORAL MODELS IN A WIDER CONTEXT

In the present section, the relationship of a sectoral model to the overall plan and to other sectoral plans will be discussed.

The economy-wide context of sectoral models is taken into account to a first approximation while preparing sectoral trend projections. In the course of these, import substitution allotments, employment objectives, capital allocations and structural (flexibility) requirements for the sector are specified. With the exception of the flexibility requirement, the ties of the sector to the overall plan represent resource quantity constraints which can be assigned to a sectoral process analysis model in the form of quantity parameters. The sectoral model will determine shadow prices corresponding to each of these constraints, which can be interpreted as the opportunity costs of foreign exchange and capital and the subsidy for labor (assuming a prescribed minimum employment).

The procedure of setting resource quantity constraints for the sectors on the basis of economy-wide considerations has the drawback that the shadow prices of basic resources calculated within distinct sectoral models will generally not coincide. An alternative approach is to estimate common opportunity costs for the basic resources from economy-wide considerations, and to assign these to the sectoral models in the form of valuation parameters (coefficients of the objective function). Opportunity costs may be approximated by means of simple considerations, such as the marginal resource use in import substitution or export activities; or else they may be derived from an economy-wide, aggregated process analysis model. This approach will generally result in an inconsistency between the sums of sectoral resource balances and the economy-wide requirements or availabilities of the basic resources. Thus, either of the above two approaches will lead to a conflict between overall and sectoral plans which needs to be resolved.

In addition to the basic resources, the consistency requirement between sectoral and overall plans extends to all other resources which appear in more than one sector. In practice, overall plans and the plans for different sectors are prepared separately and are subjected to revisions, in order to eliminate the more troublesome inconsistencies between them. In many cases, however, plans are not prepared for all sectors; moreover, the commodity classification scheme and the planning procedure may be inconsistent between the different planning units. In such cases, the most that can be hoped for is the elimination of obvious conflicts between the partial plans, and this is often done in an arbitrary and inefficient manner: e.g., when a resource (capital, foreign exchange) is overdrawn, all sectoral planning units may be instructed to cut their requirements of the resource by a given percentage.

Evidently, a great improvement in planning can be expected when a planning organization is conceived as a unit and the interrelations between its parts are carefully adjusted. One way of analyzing an efficient scheme of interrelations between different planning units is to study the interrelations between process analysis models defined for different sectors and for the economy as a whole. While these interrelations cannot be developed in detail here, it is interesting to note that the decomposition method of solving linear programming problems (Dantzig and Wolfe, 1961) offers a convenient analogue of these interrelations for a simplified case. For one application of this method, the technology matrix of a problem may consist of blocks of coefficients which correspond exactly to independent individual sectors connected only by a number of basic resources. An optimal solution to the system as a whole is derived by the alternation of two steps: first, the derivation of shadow prices for the common resources in an "interconnecting subproblem" in which only the *total* resource requirements of each sector appear as variables; and second, the derivation of resource requirements in isolated sectoral subproblems in which the shadow prices of the common resources become valuation parameters inserted in the sectoral objective functions. The interconnecting solution is always pieced together from weighted averages of the successive sectoral suboptima. The process of weighting in the interconnecting problem contrives to eliminate conflicts between the independent sectoral resource requirements by means of allowing each sector to go only part of the way in the direction of its latest adjustment to current resource prices.

The similarity between the formulation of the problem of sectoral interrelations in the decomposition method and the form in which this problem is encountered in practice is striking. In fact, the practical recommendations of a United Nations working party (ECAFE, 1961) dealing with the problem of interrelations between overall and sectoral plans follow quite closely the iteration procedure of the decomposition method, except for the fact that they substitute a trial-and-error adjustment of common resource prices for the weighting of successive sectoral solutions.

The decomposition method serves not only as a model of the interrelations between sectoral and overall plans, but also raises the question of whether such plans need to be prepared separately at all, since it offers a workable approach to the solution of very large programming problems. In other words, if the decomposition method, or some modified version of it, is capable of interrelating and solving systems of sectoral models, then why not formulate the planning problem for the entire economy as a single process analysis model, incorporating in it all the detail which would otherwise be included in the individual sectoral models?

The answer to this question is twofold. First, the above-mentioned version of the decomposition method postulates that the sectors be independent, except for a limited number of common resources. This excludes inputs of the products of one sector into another as intermediate commodities, and also excludes secondary products cutting across sector classifications. More general versions of the decomposition method, for example, a version applicable to block triangular systems, require a much larger number of iterations. Thus, some form of aggregation becomes more attractive as an alternative to the analysis of a system by the decomposition method whenever the size and the complexity of the system increases beyond a certain point.

Secondly, it is an illusion to assume that a giant model for the economy can be formulated in one step and then solved in another subsequent step. As discussed in the foregoing section, the only reasonable way of handling the problem of information processing in connection with sectoral process analysis models is to alternate the channeling of information into the model with analysis performed on the model. The analysis at each stage determines the kind of information needed at the next stage. Moreover, especially in the later stages, much of the refinement of the technological information incorporated in the model has to be undertaken with the direct participation of technical specialists in the reformulation of the model. The model must also be updated continually as new information concerning technology or operating data within the sector becomes available. All of this clearly indicates the tremendous advantages inherent in the decentralization of the planning process sector by sector.

The problem, then, is to preserve the advantages of decentralization offered by the preparation of separate sector plans, while reducing the burden of making these plans consistent. A convenient approach is to allow the sacrifice of some consistency in order to reduce the effort and the cost involved in the revisions. This can be achieved by aggregative methods.

There is a considerable economy of effort in working with an aggregated overall planning model. In addition to its advantage in offering an easy-to-grasp, intuitive description of the workings of the economy, an aggregative overall model permits each sector to face a much smaller number of environmental variables than would appear to the sector if it were placed directly in touch with all other sectors. In order to formulate an overall model and to permit each sector to make its adjustment to this simplified environment, it is necessary to define a number of aggregation and disaggregation rules. First, the scales of aggregated activities and the shadow prices of aggregated resources have to be related to the detailed sectoral activity scales and commodity shadow prices by means of summing or averaging rules. Second, each sector must be given rules for deriving its quantity parameters (demands for sectoral products, supplies of sectoral inputs) from the aggregated activity levels of other sectors: for example, the chemical sector must be able to relate sulfuric acid demand by the steel sector directly to the aggregate level of steel production, without reference to the level of detailed individual activities, such as individual rolling operations, within this sector.[1] Third, each sector must be given rules for deriving its valuation parameters (prices of resource inputs from outside the sector) from the shadow prices of aggregated resources corresponding to other sectors and from the shadow prices of common basic resources: for example, the chemical sector must be able to relate its purchase price of a certain grade of reforming naphtha to the shadow price of the corresponding aggregated class of petroleum products, without reference to the variations of the relative shadow prices of detailed commodity classes within this sector. Fourth, a rule must be provided for revising the aggregated coefficients of the overall model on the basis of the commodity quantities and the value totals derived from the detailed sectoral models.

The convergence of the successive revisions of this system consisting of the sectoral models and the aggregated overall model, as well as the closeness of the consistency achieved, depend on the particular aggregation and disaggregation rules chosen. While the impression generally exists that under any reasonable set of rules, a fair degree of consistency may be achieved between the sectoral models and the overall model by means of a modest number of iterative revisions, this entire problem area requires a more definitive analysis. In practice, however, even a few rudimentary revisions would mean an important advance over sectoral planning as currently undertaken in many underdeveloped countries.

A discussion of the problem of flexibility has previously been deferred. It has been indicated that individual sectors may be allotted some portion of carrying the burden of structural flexibility within the overall plan. The term "flexibility," however, does not refer to a commodity like any other; it is, rather, a shorthand expression for summarizing the potential adaptation of a plan to certain disturbances in the underlying assumptions. A plan is flexible if it can be adjusted, with a relatively small loss in terms of its basic objectives, to a changed set of circumstances. Flexibility is especially important with regard to changes in export forecasts and with regard to changes in the supplies of strategic inputs, such as capital goods, replacement parts for machinery, or intermediate inputs needed by domestic production activities.

In the course of planning by means of process analysis models, a larger

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[1] If the steel sector is broken down into a few major aggregated activities in the overall model, coefficients of sulfuric acid consumption have to be defined separately for each of these.

degree of flexibility can generally be achieved at the cost of accepting a reduction in the value of the objective function below its optimal value for stable parameters; in other words, by accepting some combination of activities which yield a lower than optimal payoff if conditions turn out as forecast, but do not lead to a catastrophic fall in payoff if conditions should vary from those predicted. Thus, for example, the flexibility of a plan for an underdeveloped country can usually be increased by raising import substitution beyond the proportion found in the optimum solution which has been computed for stable exports. Some simple numerical examples of planning under uncertainty by means of an economy-wide process analysis model may be found in a paper by Sandee (1960).

A sector can be said to carry a part of the burden of flexibility if its optimum solution is modified in order to guard against an economy-wide risk, such as the export or supply risks-mentioned above. Evidently, it is efficient to allot the major share of the burden of flexibility to those sectors which have a favorable tradeoff between added flexibility and the sacrifice needed for achieving it. While this statement of the problem is intuitively clear, it is difficult to add a flexibility constrant (prescribed minimum flexibility) to a process analysis model, since such a constraint anticipates a knowledge of the sensitivity of the optimum solution to variations in certain parameters.

Notwithstanding the analytical difficulties, a revision of the optimal solution will usually be adequate for handling this problem in the concrete formulation of sectoral plans. Thus, the problem of flexibility is simply merged with the many other practical considerations which intervene to modify the conclusions derived from a process analysis model in the course of arriving at final planning decisions. Among the other practical considerations which have so far eluded adequate analytical treatment is the choice of certain policy instruments: thus, there is much doubt about the best way of bridging the gap between the shadow prices determined in a process analysis model and the market prices given by institutional conditions. The practical limit on the payment of public subsidies to socially attractive activities is an example of a constraint which is similar to the flexibility constraint in that it anticipates the optimal solution of a conventional process analysis model: in this case, it anticipates the knowledge of shadow prices. For the time being, these and similar problems must be handled by intuitive revisions of the results derived from formal process analysis models.

CONCLUSION

Process analysis models undoubtedly show great promise in their application to problems of sectoral development planning. In order to bring this promise to fruition, it will be necessary to advance along three distinct lines. First, a number of analytical problems which have been referred to during the course of this discussion will have to be explored in more depth. Secondly, it will be indispensable to devote considerable time and effort to the compilation of a satisfactory store of technological information for use in the rapid formulation of such models. Finally, more experience will have to be sought, by means of pilot studies, in regard to the practical problems which arise in concrete planning situations.

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APPENDIX

BASIC CONCEPTS OF ACTIVITY ANALYSIS

Alan S. Manne

For numerical analysis of the classical problem of production economics—allocation of limited resources among alternative uses—activity analysis is uniquely powerful. The activity analysis approach—and in particular, the simplex technique for linear programming calculations—can be applied to systems involving literally hundreds or even thousands of commodities. Activity analysis permits us to consider substitution and complementarity, as well as diminishing returns to scale in production processes. Optimization and economic choice are recognized explicity within this model. Moreover—unlike classical calculus methods—activity analysis will handle cases where "kinks," inequalities, and nonnegativity restrictions are important. Within an activity analysis framework, there is no need for production functions to be differentiable at all points.

It is needless to remind economists that this is a world in which every good thing has its price. The price of using the activity analysis and linear programming framework is this: that all production functions and all economic choices must be formulated in terms of *linear* relationship among the unknowns. At first glance this requirement of linearization appears highly restrictive. However, after examining the variety of empirical cases that can be handled within this framework, most readers will probably agree that linearization is not in itself an onerous requirement. The features that are likely to appear as more serious shortcomings in activity analysis are precisely those which are also troublesome in the more conventional models of production processes: the absence of economies of scale and the absence of stochastic elements. As of this date, the inclusion of stochastic elements within a *many*-commodity optimization model appears to be a formidable challenge to the mathematician.[1] Fortunately, economies of scale no longer appear as forbidding as they did prior to the discovery of integer programming.

AN ACTIVITY AS A "BLACK BOX"

Central to the models utilized throughout this volume is the concept of an "activity": a process for transforming inputs of goods and services into out-

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[1] Computer simulation often enables us to find good solutions—although not necessarily optimal ones—even in cases involving many commodities, stochastic elements, and economies of scale. See the bibliography on simulation compiled by Shubik (1960).

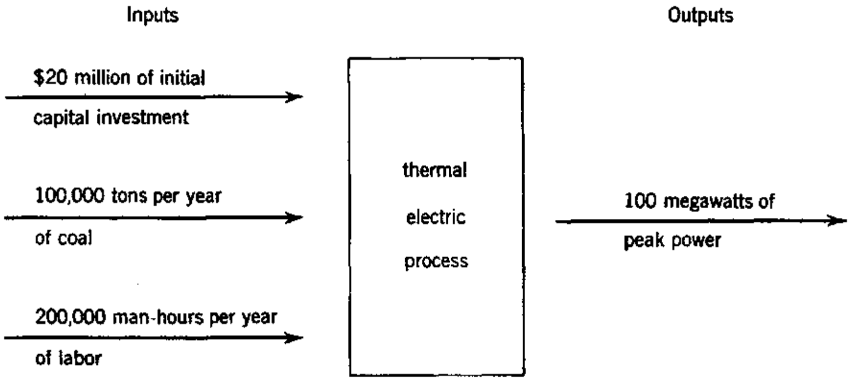

FIGURE 1

puts. If, for example, a coal-burning electric power plant were being analyzed from this viewpoint, we might be concerned with how much capital investment, how much coal, and how much labor would be required in order to produce the plant's rated output.[2] Suppose that in a plant capable of producing 100 megawatts of peak power, the required initial capital investment will be $20 millions; the annual rate of coal consumption will be 100,000 tons; and the annual labor inputs will amount to 200,000 man-hours. Then—assuming that these are the appropriate input and output categories for our purposes—the entire operation of the thermal electric plant can be summarized in terms of the black box shown in Figure 1.

According to this diagram, we note that the activity analysis viewpoint is one in which the producing sector of the economy is described in terms of: (*a*) "commodities" such as electric power, labor, coal, and capital equipment; and (*b*) "activities" such as the "thermal electric process" for transforming one group of economic commodities into another. Starting with these definitions, two axioms and one maxim are introduced: the axioms of proportionality and of additivity, and the maxim of economic efficiency.[3]

Beside these formal axioms, there is a further major assumption that is implicit here. Conventional activity analysis models are characterized by the absence of random elements and uncertainties. Randomness is regarded as a second-order effect, and is supposedly allowed for through converting all parameters into "certainty equivalents." Unfortunately, it is all too easy to cite cases where such deterministic shortcuts can be misleading. Both in the theory of inventory control and of waiting lines, the random nature of inputs and of outputs plays an essential role.

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[2] A pioneering application of process analysis in the electric power industry is to be found in Masse and Gibrat (1957).

[3] For an axiomatization of activity analysis that is both precise and readable, see T. C. Koopmans (1957).

THE TWO AXIOMS AND THE MAXIM OF ECONOMIC EFFICIENCY

AXIOM 1. PROPORTIONALITY. By "proportionality," we mean that if an activity can be operated at its base level, it can also be operated at any nonnegative fraction or multiple of that level, with all inputs and outputs varying proportionately. Figure 1 already provides a listing of the inputs and outputs corresponding to the installation of 100 megawatts of peak power output. The axiom of divisibility says that if we wish to depart from this base level and build a 50 megawatt plant, all the inputs will be halved; that if we build a 300 megawatt plant, all the inputs will be tripled; and that if we install x hundreds of megawatts, all of the inputs will be multiplied by x. (The quantity x is known as the "intensity" or "level" at which the thermal power process is operated.)

Activity analysis—in its conventional form—is incompatible with what the economist terms "increasing returns to scale." (By increasing returns we mean that if, say, the inputs into an activity are tripled, the output will exceed triple its base level.) The real world abounds with cases of increasing returns—including, in particular, some cases of investment in thermal electric power generating stations. Despite the real world, the usual activity analysis model is one in which the possibility of increasing returns is completely ignored. There is, however, an extension of activity analysis—an extension known as "integer programming"—through which it *is* possible to obtain numerical solutions in cases that involve increasing returns. For the numerical technique itself, see Gomory (1958); and for a discussion of increasing and of decreasing returns within activity analysis models, see Markowitz and Manne (1957).

AXIOM 2. ADDITIVITY. The axiom of additivity rules out most cases of what an economist would call "external economies." In terms of the power industry, this axiom implies that if there are two processes utilized together for producing electricity—the first one operated at an intensity of x_1 and the second at an intensity of x_2—the inputs required and the outputs produced will consist of the *sum* of the inputs and outputs corresponding to the operation of the two individual activities at levels of x_1 and x_2 respectively.

Additivity rules out certain possibilites for interactions between the individual processes. Suppose that we are constructing a model in which process 1 refers to the installation of a hydroelectric power plant at a downstream site, and process 2 to the installation of a hydroelectric plant and reservoir at an upstream site. It does not take a profound knowledge of hydrology to recognize the importance of a nonmarketable service produced by the upstream plant and consumed below, namely streamflow regularization. This by-product of the upstream reservoir will have a major influence upon the value of the downstream plant. The axiom of additivity implies that any physical interactions between processes have already been allowed for—e.g., through defining the activities in terms of integrated upstream and downstream plants. Additivity implies that the net output of the entire system be equal to the sum of what is independently produced (or consumed) by the individual activities—no more and no less.

THE MAXIM OF ECONOMIC EFFICIENCY. By "economic efficiency," we mean much the same thing that the economist ordinarily takes for granted about his "production function": Whatever activity levels are selected, there exists no other set of activity levels which generates a greater amount of net output (or a smaller net input) of one commodity from the system without reducing the net output (or increasing the net input) of some other commodity. This maxim of efficiency provides a partial ordering over all possible combinations of inputs and outputs.

Traditionally, the economist has taken it for granted that the responsibility for constructing such a partial ordering falls exclusively within the domain of the industrial engineer. For his own part, the industrial engineer has usually been kept busy providing optimal solutions geared to the particular needs of his employer, and has had no incentive to spell out the set of all possible efficient allocations of inputs and outputs. It is the aim of process analysis to explore some of the territory that lies between the domain of the economist and the industrial engineer—to exploit the latter's detailed knowledge of production processes in order to provide the economist with a better characterization of the set of efficient production possibilities.

LINEAR PROGRAMMING

"Linear programming" represents one particular form of activity analysis—a form which has proved particularly well suited for numerical calculations. In the case of linear programming, instead of attempting to construct a partial ordering over all possible combinations of inputs and outputs, we pose a much less ambitious question: Given the net input availabilities of certain commodities and the net output requirements of certain others, what is the maximum possible output of some item defined as the "maximand"? Or alternatively, what is the minimum possible input of the "minimand"?

It is typical for a linear programming model of an entire economy to be phrased in terms of maximizing some such physical quantity as the amount of a specific product mix or, alternatively, maximizing some such financial quantity as "national income" subject to possible side conditions on the product mix. An economy-wide model may also be phrased in terms of minimizing the input of investment or of foreign aid required to reach a predetermined national income and/or product mix target. If the system represents a single enterprise or an industry, the linear programming objective will often be phrased in terms of maximizing the output of a specific mix of commodities; or in terms of maximizing money profits; or minimizing the money costs of producing a certain product mix; or sometimes of minimizing the time elapsed before certain commodities have been produced.

For a concise introduction to linear programming computational methods, the reader should consult Gass (1958); or for a more comprehensive treatment, Dantzig (1963). A summary of the more important industrial applications is to be found in Vajda (1958). For economy-wide applications, see Chenery and Clark (1959); also Sandee (1960). For analogies between market mechanisms and linear programming computations, the economist will want to consult Koopmans (1951) and Dorfman, Samuelson, and Solow (1958).

The art of electronic computations is progressing altogether too rapidly for it to be safe to predict future developments in the numerical analysis of linear programs. In 1953, the IBM Card Programmed Calculator required eight hours to solve systems involving 27 equations. Inside the span of just four years—in 1957—the IBM 704 succeeded in solving a 195-equation system within a few hours (Orchard-Hays, 1958). By 1961, the IBM 704 system was already superseded by still more powerful machines and programs. By some date within the 1960's, it should be possible to handle systems involving upwards of 10,000 distinct equations. Note that success in computing large-scale models is not dependent solely upon improvements in computing machinery, but can also be achieved through improvements in the mathematical techniques that are employed. The "decomposition principle" represents one of the first practical attempts to take advantage of specialized matrix structures (Dantzig and Wolfe, 1961).

In planning linear programming computations, one further possibility should be borne in mind: Through the technique known as parametric programming, it is comparatively inexpensive to engage in sensitivity tests of a linear programming solution, to see what happens to the maximand or to the minimand as the availabilities of individual inputs or the requirements for individual outputs are varied. This makes it possible to end up with much the same result as activity analysis, a numerical description of the set of efficient combinations of inputs and outputs. Parametric programming is quite practical from a computational viewpoint, provided that the analysis is restricted to two or perhaps three dimensions, i.e., to the tradeoffs between just two or three groups of commodities.[4] Examples of parametric programming are to be found throughout the activity analysis chapters of this monograph.

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[4] For details on this technique, see Manne (1956, pp. 151–177) and Gass (1958, Chapter 8).

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