

# OPTIMAL TAXATION WITH ENDOGENOUS INSURANCE MARKETS\*

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## Abstract

We study optimal taxation in an economy where the skills of agents evolve stochastically over time and are private information and in which agents can trade unobservably in competitive markets. We show that competitive equilibria are constrained inefficient. The government can improve welfare by distorting capital accumulation with the sign of the distortion depending on the nature of the skill process. Finally, we show that private insurance provision responds endogenously to policy, that government insurance tends to crowd out private insurance, and, in a calibrated example, that this crowding out effect is large.

JEL Codes: E62, H21, H23, H53.

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# I. Introduction

The main question this paper addresses is whether there is a role for the government in designing social insurance programs. In dynamic optimal taxation environments with informational frictions it is often assumed that a government is the sole provider of insurance. However, in many circumstances, markets can provide insurance against shocks that agents experience. The presence of competitive insurance markets may significantly change optimal policy prescriptions regarding the desirability and extent of social insurance policies. In this paper we allow a rich set of competitive insurance markets, the structure of which is endogenously affected by informational constraints and by government policy. We show that while the markets can provide a significant amount of insurance, there is still a role for welfare improving distortionary taxes or subsidies imposed by the government. However, government interventions can be limited to correcting an externality that arises in dynamic provision of insurance rather than to the direct provision of public insurance.

We answer the question of optimal policy design in a dynamic economy in which workers receive unobservable skill shocks and can privately trade assets. In our benchmark case, as in the classical work of Mirrlees [1971], individual asset trades and, therefore, agents' consumption are publicly observable. In that environment, Prescott and Townsend [1984] and Atkeson and Lucas [1992] showed that allocations provided by competitive markets are constrained efficient. The only effect of government insurance provision is complete crowding out of private insurance leaving allocations and welfare unchanged. The case of observable consumption may have limited empirical relevance in modern economies. It is difficult to imagine that individual firms can preclude individual agents from engaging in credit market transactions or transactions with other firms. In a modern economy, it is very rare that a firm can condition its compensation on how much an agent saves in the bank, how much disability insurance it holds, etc.

Our main focus is on the environments in which asset trades are private information. In a competitive equilibrium, competition among different insurers implies that the interest rates at

which agents trade are equated to the marginal rate of transformation. We first consider two specific examples of skill processes – independently and identically distributed shocks to skills and absorbing disability shocks. For these two processes, we show that constrained efficiency requires that the interest rate at which agents trade assets is lower than the marginal rate of transformation. The intuition for this result is that a deviating agent chooses a higher amount of savings than an agent truthfully revealing his skills. A low interest rate affects deviating agents to a larger extent than truth-telling agents, thus improving incentives. We identify a specific tax instrument, a linear savings tax, that improves upon a competitive market allocation.

We then construct an example of a skill process for which it may be optimal to subsidize capital. In that example, the forces that call for taxation of capital, present in the case of independently and identically distributed shocks and in the disability case, still exist. With a more general skill process, there may be an additional effect of deviation that has a flavor of adverse selection: when an agent misreports his current skill, he may have better information about the probability distribution of his skills in the future than the planner. We show that this effect may lead to subsidization of savings. We numerically explore the tradeoff between these two effects and determine a range of parameters for which it is optimal to subsidize capital.

Privately provided insurance is inefficient because a competitive firm does not internalize the effect of hidden trades on the incentives to supply labor by agents insured by other firms. Because of this externality, we show that competitive equilibrium allocations can be improved by a government using distortionary taxes or subsidies to introduce a wedge between the interest rate and the marginal rate of transformation, an avenue not available to private insurers.

We then study how competitive markets for insurance respond to public provision of insurance. Even in the environment with unobservable trades, government insurance crowds out private insurance by changing the nature of private insurance contracts. We show that numerical estimates of the size of welfare gains from changes in public policy that do not take into account private market responses can give very misleading results. In particular, welfare gains to

government provision of insurance are smaller when private markets are endogenous. We apply our theory to a quantitative model of optimal disability insurance similar to that in Golosov and Tsyvinski [2006] to provide an illustration of the magnitude of the crowding out effect. Our benchmark is constrained efficient allocations with hidden trades. We consider the effects of complete elimination of optimally-provided public insurance in two environments. In the first environment, markets are exogenously restricted such that the only form of insurance available to agents is provided by trading risk-free bonds. In the second environment, we impose no restrictions on markets. We find that the welfare losses from elimination of public insurance are significantly smaller in the economy where private markets are endogenous. Private markets can provide most of the optimal level of insurance even in the absence of government interventions.

Our paper builds on the literature of government policy in private information economies stemming from the seminal paper of Mirrlees [1971]. Mirrlees showed that distorting taxes are optimal when the society wishes to redistribute income across agents with unobservable skills. More closely related to our work are papers by Green [1987], Atkeson and Lucas [1992], and Golosov, Kocherlakota, and Tsyvinski [2003] who studied efficient allocations in dynamic, private information economies.<sup>1</sup>

Hammond [1987] is one of the early contributors in the study of the economies with unobservable trades. Arnott and Stiglitz [1986, 1990] and Greenwald and Stiglitz [1986] argued that, in the presence of asymmetric information, competitive equilibria are generically constrained inefficient because of an externality similar to ours. Greenwald and Stiglitz [1986] also proposed linear taxation and uniform lump sum transfers as a Pareto-improving intervention. Guesnerie [1998] is an extensive study of models in which trades among agents are not observed. He investigates the structure of the tax equilibria and reforms in economies with a mix of a nonlinear income tax and a linear commodity tax. Several recent papers such as Geanakoplos and Polemarchakis [2004] and Bisin, et. al. [2001] argued in very general settings that economies with asymmetric information are inefficient and argued for Pareto-improving anonymous taxes. The

results in all of the above papers are derived mainly in static settings. Our contribution is to concentrate on the effect of asset trading on dynamic incentives and derive a precise characterization of inefficiency.<sup>2</sup> Another paper related to our work is Albanesi [2006] who considers optimal taxation of entrepreneurial capital in the model of private information under various market structures.

Our paper is also related to the literature on mechanism design with unobservable savings. An important early paper by Chiappori, Macho, Rey, and Salanie [1994] studies effects of unobservable saving and borrowing and commitment in the models of moral hazard. That paper forcefully argued that unobservable access to credit markets is an important constraint on the provisions of incentives. Also related are papers by Diamond and Mirrlees [1995], Cole and Kocherlakota [2001], Werning [2002], Abraham and Pavoni [2002], and Kocherlakota [2003]. In these papers, the authors assume that private insurance markets do not exist and the rate of return on savings is given. We show that, if private markets are unrestricted, competitive equilibria are efficient in these environments with hidden savings. In contrast, we study an economy in which market interest rates are endogenously determined by trading in markets.

Our results are also related to analysis of bank deposits as means of risk sharing in Diamond and Dybvig [1983] and Jacklin [1987]. Jacklin pointed out that in his model risk sharing breaks down if agents are able to trade among themselves unobservably. Farhi, Golosov, and Tsyvinski [2006] study a theory of liquidity and financial intermediation in those environments.

The rest of the paper is organized as follows. Section II describes the environment. Section III considers a benchmark case of observable trades. Section IV analyzes the economy with unobservable trades. Section V presents numerical results. Section VI discusses extensions and generalizations of our results. We conclude in Section VII.

## **II. Environment**

We consider an economy that lasts  $T$  ( $T \leq \infty$ ) periods, denoted by  $t = 1, \dots, T$ . In period 1, the economy is endowed with  $K_1$  units of capital. The economy has a continuum of agents with

a unit measure. Each agent's preferences are described by a time separable utility function over consumption of a private good  $c_t$  and labor  $l_t$ ,

$$\sum_{t=1}^T \beta^t U(c_t, l_t).$$

In the above specification,  $\beta$  is a discount factor, and  $\beta \in (0, 1)$ . To simplify the analysis we assume that the utility function is separable between consumption and labor:  $U(c, l) = u(c) + v(l)$ . The utility of consumption,  $u$ , is continuously differentiable, strictly increasing, strictly concave,  $u_c(0) = \infty$ , and  $u(0) = -\infty$ . The utility of labor,  $v$ , is strictly decreasing.

Agents are heterogeneous, and in each period they have idiosyncratic skills  $\theta$  that belong to a finite distribution  $\Theta = \{\theta(1), \dots, \theta(N)\}$  where  $\theta(1) < \theta(2) < \dots < \theta(N)$ . These skills evolve stochastically over time. Formally, in period 1 each agent gets an independently and identically distributed draw of a vector of skills for  $T$  periods from the distribution  $\Theta^T$  with a common probability  $\pi(\theta^T)$ . The  $t^{\text{th}}$  component of  $\theta^T$  is an agent's skill in period  $t$ . The probability  $\pi(\theta^T)$  is known but the specific realization of it is not. Each agent learns about his realization of  $\theta^T$  over time. In period  $t$ , he knows only his skill realization for the first  $t$  periods  $\theta^t = (\theta_1, \dots, \theta_t)$ . Skills are private information. We assume that the law of large numbers holds, and in each period there are exactly  $\pi(\theta^t)$  agents with the history of shocks  $\theta^t$ . At this stage, we do not restrict the process for skill, i.e., it can include persistent shocks, fixed effects, or any other evolution of skills. Our structure implies that there is no aggregate uncertainty.

An agent who supplies  $l$  units of labor and has a skill level  $\theta$  produces  $y = \theta l$  units of effective labor. The supply of labor is not observable. In the paper we use a common interpretation that, although it is possible to observe how many hours an agent spends at his workplace, it is impossible to determine if he works or consumes leisure there. This interpretation implies that in checking incentive constraints we only need to consider the possibility that agents underreport their skill level.

Effective labor is observable and is a factor of production. Production in this economy is described by a function  $F(K, Y)$ ,<sup>3</sup> where  $K$  is the stock of capital, and  $Y$  is the aggregate level of effective labor  $Y = \sum_{\theta} \pi(\theta)y(\theta)$ . We assume that  $F$  is continuous, increasing in  $K$  and  $Y$ , and has constant returns to scale. The output can be divided into consumption and investment.

An *allocation* is a vector  $\{c_t, y_t, K_t\}_{t=1}^T$  where  $c_t : \Theta^t \rightarrow \mathbb{R}_+$  and  $y_t : \Theta^t \rightarrow \mathbb{R}_+$ . Here,  $c(\theta^t)$  is private consumption of an agent with history  $\theta^t$ ;  $y(\theta^t)$  is the amount of effective labor units that such a person supplies, and  $K_t$  is the level of capital in period  $t$ . An allocation is *feasible* if, in every period  $t$ :

$$(1) \quad \sum_{\theta^t} \pi(\theta^t)c(\theta^t) + K_{t+1} \leq F(K_t, \sum_{\theta^t} \pi(\theta^t)y(\theta^t)).$$

We say that a history  $\theta^j$  *contains*  $\theta^i$  for  $j \geq i$  if the first  $i$  realizations of  $\theta^j$  are  $\theta^i$ , and we denote it  $\theta^i \in \theta^j$ . We also use notation  $c_t(\theta^T)$  which is equivalent to  $c(\theta^t)$  for  $\theta^t \in \theta^T$ . The probability of history  $\theta^{t+1}$  conditional of the realization of the history  $\theta^t$  is denoted by  $\pi(\theta^{t+1}|\theta^t)$ .

### III. A benchmark case: observable consumption

We first consider a benchmark model in which consumption of each agent is publicly observable. We define a constrained efficient allocation and a competitive equilibrium. We then prove that, as in Prescott and Townsend [1984] and Atkeson and Lucas [1992], the first welfare theorem holds, and competitive markets can provide optimal insurance.

#### III.A. Constrained efficient allocations

Consider a social planner who offers each agent a contract  $\{c(\theta^t), y(\theta^t)\}_{t=1}^T$ , where  $c(\theta^t)$  and  $y(\theta^t)$  are functions of the agent's reported type. Each agent chooses a reporting strategy  $\sigma$ , which is a mapping  $\sigma : \Theta^T \rightarrow \Theta^T$ . We denote the set of all such reporting strategies by  $\Sigma$ . An agent who chooses to report  $\sigma(\theta^t)$  after history  $\theta^t$  provides  $y(\sigma(\theta^t))$  units of effective labor and receives  $c(\sigma(\theta^t))$  units of consumption from the planner.<sup>4</sup>

The expected utility of an agent who is offered a contract  $\{c_t, y_t\}_{t=1}^T$  and chooses a strategy  $\sigma$  is denoted by  $W(c, y)(\sigma)$  and given by

$$W(c, y)(\sigma) = \sum_{t=1}^T \beta^t \sum_{\theta^t} \pi(\theta^t) U(c_t(\sigma(\theta^t)), y_t(\sigma(\theta^t))/\theta_t).$$

The strategy  $\sigma^*$  is *truth-telling* if an agent reveals his type truthfully after any history:  $\sigma^*(\theta^t) = \theta^t$  for all  $t$ . The allocation is *incentive compatible* if the truth telling strategy yields a higher utility than any other strategy

$$W(c, y)(\sigma^*) \geq W(c, y)(\sigma) \text{ for any } \sigma \in \Sigma.$$

An allocation  $\{c_t, y_t, K_t\}_{t=1}^T$  is *constrained efficient*<sup>5</sup> if it solves the planner's problem that follows:

$$\max_{c, y, K} \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t \{U(c(\theta^t), y(\theta^t)/\theta_t)\}$$

subject to

$$(2) \quad W(c, y)(\sigma^*) \geq W(c, y)(\sigma) \text{ for any } \sigma \in \Sigma,$$

$$(3) \quad \sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} \leq F(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)) \text{ for all } t.$$

The above program states that the planner maximizes the expected utility of an agent subject to the incentive compatibility constraint and to the feasibility constraint. We denote the solution to this social planner's problem as  $\{c_t^{sp}, y_t^{sp}, K_t^{sp}\}_{t=1}^T$ .

### III.B. Competitive equilibrium

In this subsection, we define a competitive equilibrium for the economy with observable consumption described above. Consider an economy populated by ex-ante identical agents each

of whom is endowed with the same initial capital  $k_1$ , so that the aggregate capital stock is  $K_1$ . There is a continuum of firms with the identical production technology  $F(K, Y)$ . We assume throughout the paper that all activities at the firm level are observable. All firms are owned equally by all agents. In the beginning of period 1, before any realization of uncertainty, each firm signs a contract  $\{c_t, y_t\}_{t=1}^T$  with a continuum of workers and purchases the initial capital stock  $k_1$  from them. We interpret  $c_t$  as the actual consumption of the agent. Such a contract is feasible since consumption and all transactions of agents are observable. The price paid for the initial capital is included in the contract.<sup>6</sup> The contracts are offered competitively, and workers sign a contract with the firm that promises the highest ex-ante expected utility. We denote the equilibrium utility by  $\underline{U}$ . After the contract is signed the worker chooses a reporting strategy  $\sigma$ , supplies  $y(\sigma(\theta^t))$  effective labor, and receives  $c(\sigma(\theta^t))$  units of consumption when his history is  $\theta^t$ . The agents do not participate in any markets.

Each firm accumulates capital  $k_t$  for  $t > 1$ , pays dividends  $d_t$  to its owners, and trades bonds with other firms. We denote by  $q_t$  the price of a bond  $b_t$  in period  $t$  that pays 1 unit of consumption good in period  $t + 1$ . All firms take these prices as given. We consider equilibria where all firms are identical, and we study a problem of a representative firm.

The maximization problem of the firm that faces intertemporal prices  $q_t$  and the reservation utility  $\underline{U}$  for workers is

$$\max_{c, y, d, k, b} d_1 + q_1 d_2 + \dots + \prod_{i=1}^{T-1} q_i d_T$$

subject to

$$(4) \quad \sum_{\theta^t} \pi(\theta^t) c(\theta^t) + k_{t+1} + d_t + q_t b_{t+1} \leq F(k_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)) + b_t,$$

$$(5) \quad W(c, y)(\sigma^*) \geq W(c, y)(\sigma) \text{ for any } \sigma,$$

$$(6) \quad W(c, y)(\sigma^*) \geq \underline{U}.$$

In equilibrium, competition among firms forces them to have zero profits. We now define a competitive equilibrium.

**Definition 1** *A competitive equilibrium is a set of allocations  $\{c_t, y_t, k_t\}$ , prices  $q_t$ , dividends  $d_t$ , bond trades  $b_t$  and utility  $\underline{U}$  such that*

- (i) *Firms choose  $\{c_t, y_t, d_t, k_t, b_t\}_{t=1}^T$  to solve firm's problem taking  $\{q_t\}$  and  $\underline{U}$  as given;*
- (ii) *Consumers choose the contract that offers them the highest ex-ante utility;*
- (iii) *The aggregate feasibility constraint (1) holds.*

It is easy to show in equilibrium the prices are  $1/q_t = F_k(k_{t+1}, Y_{t+1})$  and  $d_t = 0$  for all  $t$ .

We now show a version of the first welfare theorem. The result follows Prescott and Townsend [1984] and Atkeson and Lucas [1992] who show that, in environments with private information, competitive firms can provide the optimal allocation. The proof follows from an observation that the representative firm's problem is dual to the social planner's problem and, hence, gives the same allocations.

**Theorem 1 (Equilibrium without retrading is efficient)** *In an economy with no trades among agents (observable consumption), the competitive equilibrium is efficient.*

**Proof of Theorem 1.** Suppose the competitive equilibrium is not efficient. Consider an optimal allocation  $\{c_t^{sp}, y_t^{sp}, K_t^{sp}\}_{t=1}^T$  with utility level  $U^*$ . Such an allocation is feasible for the firm, satisfies incentive compatibility and delivers workers a utility  $U^*$  which is strictly higher than the equilibrium utility  $\underline{U}$ . This allocation also delivers zero profit for the firm, as in the candidate competitive equilibrium allocations. It is possible for the firm to offer another contract  $\tilde{U}$ ,  $U^* > \tilde{U} > \underline{U}$  with strictly less resources by reducing consumption of the agent with the lowest skill realization in the first period by  $\varepsilon$ . This deviation preserves the incentive compatibility, delivers the utility  $\tilde{U}$ , and firms have strictly positive profits  $\varepsilon$ . We arrive at a contradiction. ■

An immediate reinterpretation of the above result is that the only result of government provision of insurance is crowding out of private insurance. Suppose, for example, that the

government introduces a lump sum redistribution between agents  $T(\theta^t)$  where in each period  $\sum_{\theta^t} \pi(\theta^t)T(\theta^t) = 0$ . Such taxes leave the after tax allocations unchanged. Firms adjust their contracts optimally so that the new payments to the workers  $\{\hat{c}_t\}_{t=1}^T$  reflect the taxes:  $\hat{c}(\theta^t) = c(\theta^t) + T(\theta^t)$ . The higher level of insurance provided by the government is exactly offset by less insurance available through private markets.

Recent analyses by Golosov and Tsyvinski [2006], Albanesi and Sleet [2006], and Kocherlakota [2005] studied the implementation of dynamic Mirrlees problem via taxes. The assumption in these papers is that the government is the only provider of such insurance available to the agents. In our setup, in the absence of governmental policy, firms and agents write contracts that provide agents with insurance. We conclude that, even in the presence of private information, markets can provide optimal insurance if consumption is observable.

This analysis suggests that optimal allocations can be achieved without distortionary government interventions. It abstracts from many possible sources of inefficiencies. For example, by allowing agents to sign insurance contracts before any realization of uncertainty, “behind the veil of ignorance,” we abstract from issues arising because of adverse selection. We also assume that contracts are binding and neither the employer nor the agent can renege on them. The assumption of commitment may be important, especially in the context of the labor markets in which the law often requires that the employee can leave the contract at will.<sup>7</sup> Moreover, we assumed that government can commit to promises and is not time inconsistent. If a government or a planner cannot commit to the contracts then analysis becomes significantly more complicated as issues such as a ratchet effect [Freixas, Guesnerie, and Tirole 1985] have to be considered. Even though this analysis is outside of the scope of the paper, all of the above are important qualifications of results in this section and in the model with unobservable trades.

## IV. Unobservable trades

In this section, we relax the assumption of full observability of trades. We still maintain the assumption that an agent’s effective labor  $y$  is publicly observable. The agent can, however, trade

assets and consume unobservably.<sup>8</sup> We show that the competitive equilibrium is not constrained efficient. Finally, we show how distortionary taxes or subsidies can improve on the competitive market allocation.

#### IV.A. Retrading market

Consider an environment in which all agents have access to a market in which they can trade assets unobservably. We call this market a *retrading market*. In this market agents trade risk-free bonds. A purchase of the bond entitles the holder to one unit of consumption in the period that follows. In the appendix, we show that risk-free bonds are the only security traded in equilibrium.

All trades at period  $t$  occur at prices  $Q_t$ . The prices are such that the market for bonds clears each period. We assume that all trades are enforceable so that agents cannot default on their liabilities. This assumption precludes agents from borrowing more than they can ever repay in the future.

A social planner offers a contract  $\{c(\theta^t), y(\theta^t)\}_{t=1}^T$  to all agents, where  $y_t$  is the amount of effective labor that an agent provides, and  $c_t$  is the endowment of consumption goods that agent receives. Unlike the environment described in the previous section, the amount of consumption goods allocated by the planner is not necessarily equal to the actual consumption of an agent, since the planner has no possibility to preclude an agent from borrowing and lending on the retrading market.

An agent takes the contract offered by the planner and the equilibrium prices  $\{Q_t\}_{t=1}^T$  as given and chooses his optimal reporting strategy  $\sigma$  together with holdings (possibly negative) of a risk-free security  $s_t : \Theta^t \rightarrow \mathbb{R}_+$ . Total resources available to the agent are the endowment of consumption good  $c(\sigma(\theta^t))$  he receives from the planner and his asset holding from the previous period. The actual consumption after retrading is  $x_t : \Theta^t \rightarrow \mathbb{R}_+$ .

#### **Agent's Problem**

The agent maximizes his ex-ante utility:

$$\max_{\sigma, x, s} \sum_{t=1}^T \beta^t \sum_{\theta^t} \pi(\theta^t) U(x(\sigma(\theta^t)), y(\sigma(\theta^t)) / \theta_t)$$

such that for all  $\theta^t, t$

$$\begin{aligned} x(\sigma(\theta^t)) + Q_t s(\sigma(\theta^t)) &= c(\sigma(\theta^t)) + s(\sigma(\theta^{t-1})), \\ s(\theta^0) &= 0, \end{aligned}$$

where  $s(\theta^0)$  are the initial asset holdings of the agent before realizations of the shocks.

We denote the value of this problem at the optimum by  $V(\{c, y\}, \{Q\})$ . Sometimes we need to compute a value for an arbitrary reporting strategy  $\sigma$ , and we denote ex-ante utility from following this strategy by  $V(\{c, y\}, \{Q\})(\sigma)$ .

Equilibrium in the retrading market requires that in each period the total endowment of consumption goods should be equal to the total after trade consumption:

$$(7) \quad \sum_{\theta^t} \pi(\theta^t) x(\sigma(\theta^t)) = \sum_{\theta^t} \pi(\theta^t) c(\sigma(\theta^t)).$$

We define equilibrium in the retrading market.

**Definition 2** *An equilibrium in the retrading market given the contract*

$\{c(\theta^t), y(\theta^t)\}_{t=1}^T$  *consists of prices*  $Q_t$ , *strategies*  $\sigma$ , *and allocations*  $\{x(\theta^t), s(\theta^t)\}_{t=1}^T$  *such that*

- (i) *Consumers solve the Agent's Problem taking*  $\{c(\theta^t), y(\theta^t), Q_t\}_{t=1}^T$  *as given;*
- (ii) *The feasibility constraint on the retrading market (7) is satisfied.*

Although the equilibrium in hidden markets as defined above may not exist in pure strategies, it is straightforward to extend the model by allowing mixed strategies to prove existence. We use pure strategies to simplify the exposition. We assume that for any contract  $\{c_t, y_t\}_{t=1}^T$  offered

by the social planner there exists a unique equilibrium in the retrading market. The ex-ante utility of an agent in the equilibrium is denoted  $\hat{V}(\{c, y\})$ .

#### IV.B. Constrained efficiency with unobservable trades

The social planner chooses the allocations  $\{c_t, y_t, K_t\}_{t=1}^T$  that maximize the ex-ante utility of agents. Using the revelation principle it is easy to show that the social planner offers a contract  $\{c_t, y_t\}_{t=1}^T$  so that all agents choose to report their type truthfully to the planner and do not trade on the retrading market.<sup>9</sup>

**Definition 3** *A constrained efficient allocation  $\{c_t^r, y_t^r, K_t^r\}_{t=1}^T$  is the solution to the social planner's problem.<sup>10</sup>*

$$\max_{c, y, K} \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t \{U(c(\theta^t), y(\theta^t)/\theta_t)\}$$

such that for all  $\theta^t, t$

$$(8) \quad \sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} \leq F(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)),$$

$$(9) \quad \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t U(c(\theta^t), y(\theta^t)/\theta_t) \geq \hat{V}(\{c, y\}).$$

The market for hidden trades imposes a weakly stricter constraint (9) than the incentive constraint with observable asset trades (2). We show this by first showing that any allocation that satisfies (9) also satisfies (2). Consider an allocation  $\{c(\theta^t), y(\theta^t)\}_{t=1}^T$  that satisfies (9). Suppose there exists some reporting strategy  $\sigma$  for which the incentive constraint (2) is violated. Consider the same strategy  $\sigma$  on the market with hidden trades. The allocation  $\{c(\sigma(\theta^t)), y(\sigma(\theta^t))\}_{t=1}^T$  is feasible for the agent, and he can further improve upon it by trading bonds. Therefore, the strategy  $\sigma$  also violates the constraint imposed by the market for hidden trades (9).

The reverse relationship does not hold in general; it is typically not true that an allocation that satisfies (2) also satisfies (9). When consumption is observable by the planner, agents'

marginal rates of substitution defined by

$$\frac{u'(c(\theta^t))}{\beta \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u'(c(\theta^{t+1}))}$$

differ for different histories  $\theta^t$  under efficient allocations with observable trades and are smaller than the marginal rate of transformation. This wedge was first shown by Diamond and Mirrlees [1978] in the model with permanent disability shocks, by Rogerson [1985] in the context of a two-period model, and extended to a general skill process and optimal taxation setup by Golosov, Kocherlakota, and Tsyvinski [2003]. To the contrary, in the environment with hidden trades, agents' marginal rates of substitution are necessarily equated. For any reporting strategy  $\sigma$ , the allocations  $\{x(\sigma(\theta^t)), s(\sigma(\theta^t))\}_{t=1}^T$  that an agent chooses on the retrading market must satisfy the following conditions for all  $\theta^t$ :

$$(10) \quad x(\sigma(\theta^t)) + Q_t s(\sigma(\theta^t)) = c(\sigma(\theta^t)) + s(\sigma(\theta^{t-1})),$$

$$(11) \quad Q_t u'(x(\sigma(\theta^t))) = \beta \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u'(x(\sigma(\theta^{t+1}))),$$

$$(12) \quad s(\theta^T) = 0.$$

Condition (11) implies that agents equalize their marginal rates of substitution in each period for all histories  $\theta^t$ .

Another difference between the two environments is a possibility for agents to use a *double deviation* – agents choose not only a deviating reporting strategy but also hidden asset trades that maximize the utility of the deviation. The possibility of such deviations implies that even if agents' marginal rates of substitution were equalized for an allocation that satisfies (2), such an allocation would not necessarily satisfy (9).

To illustrate this point we rewrite social planner's problem.

**Lemma 1** *A constrained efficient allocation  $\{c_t^r, y_t^r, K_t^r\}_{t=1}^T$  together with the corresponding*

equilibrium prices on the retrading market  $\{Q_t\}_{t=1}^T$  is a solution to the problem

$$\max_{c,y,K,Q} \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t \{U(c(\theta^t), y(\theta^t)/\theta_t)\}$$

such that for all  $\theta^t, t$

$$(13) \quad \sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} \leq F(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)),$$

$$(14) \quad \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t U(c(\theta^t), y(\theta^t)/\theta_t) \geq V(\{c, y\}, \{Q\})(\sigma) \text{ for any } \sigma \neq \sigma^*,$$

$$(15) \quad Q_t u'(c(\theta^t)) = \beta \sum_{\theta^{t+1}} \pi(\theta^{t+1} | \theta^t) u'(c(\theta^{t+1})).$$

**Proof of Lemma 1.** In appendix. ■

In this problem the social planner chooses the prices  $Q$  on the retrading market directly. Although the planner does not control transactions on that market, he has power to determine these prices. By the revelation principle, a social planner chooses allocations such that each agent reveals his type truthfully and never retrades from the allocations he receives. The truth telling agent does not retrade if his marginal rate of substitution for consumption between periods  $t$  and  $t + 1$  is exactly equal to the interest rate. In other words, these intertemporal rates of substitution determine the prices of risk-free bonds. The incentive constraint should ensure that a deviating agent cannot achieve a higher utility by retrading at those prices.

The possibility of trading assets and using double deviations implies that constraint (14) is stricter than the incentive constraint (2). For any strategy  $\sigma$  the allocation  $\{c(\sigma(\theta^t)), y(\sigma(\theta^t))\}_{t=1}^T$  is feasible, but the agent can further improve upon it using hidden trades.

Although the economy with unobservable retrading typically has lower welfare than the economy with observable trades, we can identify one situation in which the allocations and welfare in both economies are the same. It is the economy analyzed extensively in Werning [2001] where all the uncertainty about skill shocks is realized after the first period. When all

uncertainty is realized in the first period, there is no longer any gain from hidden trades. Any asset trading occurs after agents have revealed their type to the planner. The possibility of hidden trade does not improve the value of any deviation, and the incentive constraints in the two economies become identical. In the rest of the paper we assume that there is need to provide incentives in each period, so that hidden trades play a non-trivial role. We summarize this intuition in the proposition below.

**Proposition 1** *Suppose that all uncertainty is realized after the first period, so that in each period  $t$  for each history  $\theta^t$  there exists some history  $\theta^{t+1}$  such that  $\pi(\theta^{t+1}|\theta^t) = 1$ . Then the efficient allocations in the economy with and without observable trades are the same.*

**Proof of Proposition 1.** In appendix. ■

Our economy differs from standard problems with unobservable savings such as Diamond and Mirrlees [1995], Werning [2002], Doepke and Townsend [2006], and Abraham and Pavoni [2003] where the rate of return on hidden trades is assumed to be exogenous. Moreover, there are no private markets in these papers, and the interest rate is fixed. In our environment, the social planner can choose the rate of return on private hidden trade markets by choosing allocations  $\{c_t, y_t\}_{t=1}^T$ . This additional instrument is important for the planner because it allows the planner to affect the return from deviations. We show below that since competitive environments typically lack this instrument, competitive equilibria are not efficient. This result is different from the environments with an exogenous rate of return in which competitive equilibria are efficient.

#### **IV.C. Competitive equilibrium**

In this subsection, we consider a decentralized version of this private information economy with unobservable trades. As in the section on the economy with observable trades, we assume that, before any uncertainty is realized, an agent signs a long-term contract with a firm which is binding for both parties. The environment is identical to the one described in Section III,

but now firms need to take into account that agents are able to retrade their allocations on the hidden trades market.

The retrading market is identical to the one in the social planner's problem. Every agent who has a contract  $\{c_t, y_t\}_{t=1}^T$  with a firm chooses his reporting strategy and asset trades optimally, taking prices  $Q_t$  for the risk-free bond on the retrading market as given.<sup>11</sup>

The contracts offered by firms take into account the possibility that agents may retrade. Firms may choose to provide such allocations that agents retrade from them along the truth-telling path. The incentive constraint for the firm has the form

$$V(\{c, y\}, \{Q\})(\sigma^*) \geq V(\{c, y\}, \{Q\})(\sigma)$$

for any  $\sigma \in \Sigma$ .

The problem of the representative firm is similar to the problem described in Section III.A. Each firm is a price taker, it chooses a contract offered to workers  $\{c_t, y_t\}_{t=1}^T$ , investments  $k_t$ , dividends  $d_t$ , and bond trades  $b_t$  to maximize profits.

### Firm's Problem 1

$$\max_{c, y, d, k, b} d_1 + q_1 d_2 + \dots + \prod_{i=1}^{T-1} q_i d_T$$

such that for all  $t$

$$(16) \quad \sum_{\theta^t} \pi(\theta^t) c(\theta^t) + k_{t+1} + d_t + q_t b_{t+1} \leq F(k_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)) + b_t,$$

$$(17) \quad V(\{c, y\}, \{Q\})(\sigma^*) \geq V(\{c, y\}, \{Q\})(\sigma) \text{ for any } \sigma,$$

$$(18) \quad V(\{c, y\}, \{Q\})(\sigma^*) \geq \underline{U}.$$

The first constraint in the firm's problem is feasibility. The second is the incentive compatibility. The last constraint states that the firm cannot offer a contract which delivers a lower expected utility than the equilibrium utility  $\underline{U}$  from contracts offered by other firms. In

equilibrium, all firms act identically and make zero profits.

The firm's problem in this economy is very similar to the firm's problem in the economy with observable trades. The only difference comes from the fact that the incentive constraint (17) now has to take into account side trades that are not observable. The definition of the competitive equilibrium is parallel to that in the economy with observable trades.

**Definition 4** *A competitive equilibrium is a set of allocations  $\{c_t, y_t, k_t\}$ , prices  $q_t$ , dividends  $d_t$ , bond trades  $b_t$ , utility  $\underline{U}$  and prices  $Q_t$  such that*

- (i) Firms choose  $\{c_t, y_t, d_t, k_t, b_t\}_{t=1}^T$  to solve the Firm's Problem 1 taking  $q_t, \underline{U}$  as given;*
- (ii) Consumers choose the contract that offers them the highest ex-ante utility;*
- (iii) For any  $\{c_t, y_t, Q_t\}_{t=1}^T$  agents choose their reporting strategy and asset trades optimally as described in the Agent's Problem;*
- (iv) The aggregate feasibility constraint (1) holds;*
- (v) The retrading market for the contract  $\{c_t, y_t\}_{t=1}^T$  is in equilibrium, and  $Q_t$  are the equilibrium prices.*

It is easy to see that the interest rates in the economy must be equal to the marginal product of capital, so that  $1/q_{t-1} = F_k(K_t, Y_t)$  for all  $t$ . The prices that firms and agents face are also equalized,  $q_t = Q_t$  for all  $t$ . Suppose it were not true, so that for example  $1/Q_1 < F_k(K_2, Y_2)$ . It is optimal for all firms to postpone any payments of the first period wages until the second period. Workers are able to borrow at the interest rates  $Q_1$  and repay from the wages they make in the second period. But since all the firms are identical, they all choose to pay no wages in the first period, and then  $Q_1$  cannot be the equilibrium interest rate. In other words, if  $q_t \neq Q_t$  firms can use agent's ability to borrow and lend at rate  $Q_t$  to create arbitrage opportunities. We summarize this result in the following proposition.

**Proposition 2** *In the competitive equilibrium  $1/Q_t = F_k(K_{t+1}, Y_{t+1})$  for all  $t$ .*

This result suggests that competitive equilibria typically are *not* efficient when asset trades are unobservable. From the maximization problem described in Lemma 1, the social planner has the power to choose the interest rates on the retrading market  $1/Q$  and usually these interest rates are different from  $F_k(K, Y)$ .

Although the competitive equilibrium may not be efficient it is generally not true that no insurance is provided by firms. In the numerical section that follows, we show that this privately provided insurance can be very significant. This finding stands in contrast with the environments where the agent's endowment is not observable, such as environments studied in Allen [1985]. There, no insurance is possible when agents can borrow and lend at the rate equal to  $F_k$ . The difference between our model and that of Allen is the structure of private information. In our model, the amount of resources is endogenously determined in each period by effective labor provided by agents. Firms in competitive equilibrium have to provide incentives for agents to work and, therefore, provide some insurance.

#### **IV.D. Constrained efficient allocations and tax policy with iid shocks**

In this section, we assume that the skill shocks follow an independently and identically distributed process:  $\pi(\theta^t) = \pi(\theta_t) = \pi(\theta)$  for  $\theta = \theta_t$  for all  $\theta^t$ . We consider only pure strategies and assume that  $T$  is finite.

We showed that any equilibrium allocation in the retrading market satisfies conditions (10), (11) and (12). When  $\theta$  is independently and identically distributed, the Euler equation (15) becomes

$$(19) \quad Q_t u'(c(\sigma(\theta^t))) = \beta \sum_{\theta} \pi(\theta) u'(c(\sigma(\theta^t), \theta)),$$

where  $c(\sigma(\theta^t), \theta)$  denotes the allocation to the agent who sent report  $\sigma(\theta^t)$  in period  $t$  and revealed his realization of the shock in period  $t + 1$  truthfully.

We also assume that consumption allocations are monotonic so that agents who report

higher types receive weakly higher consumption. This assumption holds in all the numerical experiments we conducted.

*Assumption* (monotonicity). For any  $\theta^t$ , and any  $\theta', \theta''$  such that  $\theta'' > \theta'$  it is optimal for the planner to choose consumption allocations such that  $c(\theta^t, \theta'') \geq c(\theta^t, \theta')$ .

We first show that an agent who deviates from the allocation prescribed by the planner chooses positive savings.

**Proposition 3** *The only binding incentive constraints in the social planner's problem are those where  $s(\sigma(\theta^t)) \geq 0$  for some  $\theta^t$ . Moreover, there are some  $\theta^t$  in every  $t$  for which this inequality is strict.*

**Proof of Proposition 3.** In appendix. ■

The intuition for this result is simple. The marginal rate of substitution of a truth telling agent is equal to the price of a risk-free bond. When an agent reports a lower type, he gets lower consumption allocations. When shocks are independently and identically distributed that implies that consuming these allocations without any additional asset trading increases agent's marginal rate of substitution above the bond price  $Q$ , since fewer resources are available in the next period. However, it is optimal for the agent to retrade his consumption allocations to equalize his marginal rate of substitution with bond prices. Since future deviations imply fewer resources, it is optimal for the agent to save in the anticipation of those deviations, and borrowing is always suboptimal.

We now can prove that in the efficient allocations the interest rates on the retrading market are lower than  $F_k$ , which is formally stated in the following proposition.

**Proposition 4** *Suppose that skill shocks are independently and identically distributed. In the constrained efficient allocations,  $F_k(K_t, Y_t) > 1/Q_{t-1}$  for at least one  $t$ .*

**Proof of Proposition 4.** In appendix. ■

Although the proof is lengthy, its intuition is quite straightforward. We show that a deviating agent chooses positive savings. Then we show that changing the interest rate on the retrading market negatively affects the return to deviations by a larger amount than the truth-telling agents are affected. This leads to a higher amount of insurance being provided.

**Theorem 2** *If trades (consumption) are not observable, the competitive equilibrium is not efficient.*

**Proof of Theorem 2.** Follows from Propositions 2 and 4. ■

Intuitively, the competitive equilibrium is not efficient because a contract offered by one firm to its workers affects the return on trades and thus incentives to reveal information truthfully for agents insured by other firms. Individual firms can not internalize this effect. Competition between different insurers implies that interest rates at which agents trade are equated with the marginal rates of transformation. The planner, however, is able to choose the interest rates optimally. Thus, privately provided insurance does not lead to efficient allocations in this setting. The technical reason for the failure of the first welfare theorem is that prices enter the production set of the firms as can be seen in the Firm's Problem 1. Here, an externality has real effects because of the asymmetric information.<sup>12</sup> In the next section, we explore how distortionary taxes can introduce the wedge between the equilibrium interest rates on the retrading market and the marginal product of capital.

We can also easily show that in the environments with hidden savings such as Werning [2001] and Abraham and Pavoni [2003] the competitive equilibrium is efficient. There, the planner does not have the ability to affect the rate of return on hidden technology as agents do not interact via markets but unobservably save using a backyard technology.

#### **IV.D.i. Tax policy with iid shocks**

We showed in the previous section that efficiency requires that the interest rates on the retrading market are lower than the marginal product of capital. In the competitive equilibrium

without government interventions, interest rates are equated to the marginal product of capital, and the equilibrium allocations are not efficient. We now identify what government interventions in a form of distortionary taxes on capital can reintroduce this wedge in competitive equilibrium. In this section we show that such policy improves welfare.

We proceed as follows. First, we rewrite the firm's problem in its dual form. The dual form is convenient to use since it maximizes total utility of agents similar to the social planner's problem. Second, we show that positive linear taxes on capital income improve welfare when agent's optimal deviations involve oversaving.

Consider a dual version of the firm's problem. Since all the firms are making zero profit in equilibrium, their problem can be rewritten in the following form.

**Firm's Problem 2**

$$\max_{c,y,k} \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t U(c(\theta^t), y(\theta^t)/\theta_t)$$

such that for all  $t$

$$\sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t u(c(\theta^t), y(\theta^t)/\theta_t) \geq V(\{c, y\}, \{Q\})(\sigma) \text{ for any } \sigma,$$

$$\sum_{\theta^t} \pi(\theta^t) c(\theta^t) + k_{t+1} \leq F(k_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)).$$

**Claim 1** *In a competitive equilibrium, the solution to Firm's Problem 1 coincides with the solution to Firm's Problem 2.*

**Proof of Claim 1.** In appendix. ■

This result allows us to directly compare the firm's problem and the social planner's problem. These two problems are very similar. The planner, however, has an additional choice variable – prices on the retrading market  $Q$ . The social planner choosing efficient allocations takes into account how these allocations affect the interest rates in the economy. The competition

among firms makes the interest rates on the retrading market equal to the marginal rate of transformation.

Unlike the economy with observable asset trades, distorting taxes are welfare improving in this environment. Consider a simple linear tax  $\tau$  imposed on capital income  $Rk$ , where  $R \equiv F_k(K, Y)$ . The revenues from this tax are distributed equally among all agents. As argued in Proposition 1, such a lump sum distribution has the same effect as returning lump sum rebates directly to firms. In the following proposition we show that such a tax system is welfare improving.<sup>13</sup>

**Proposition 5** *Suppose that skill shocks are independently and identically distributed. There exists a positive tax  $\tau$  on capital income and a lump sum rebate  $T$  that improves the welfare in the competitive equilibrium.*

**Proof of Proposition 5.** From Proposition 3, the only binding incentive constraints in the firm's problem must be those constraints that involve only savings. Let  $t$  be a time period for which there exists a binding strategy  $\sigma$  and a history  $\theta^t$  such that  $s(\sigma(\theta^t)) > 0$ . We know that for all other  $\hat{\sigma}, \hat{\theta}^t$  savings are non-negative:  $s(\hat{\sigma}(\hat{\theta}^t)) \geq 0$ .

Consider a linear tax  $\tau$  on the return on capital  $Rk$  in period  $t + 1$ . The tax revenues are rebated in the lump sum amount  $T$  to the firms. Let  $k(\tau, T)$  denote the firm's investment in period  $t$  as a function of  $(\tau, T)$ . The feasibility constraint for the government is  $\tau Rk(\tau, T) = T$ . Using the implicit function theorem we obtain

$$T'(\tau) = \frac{Rk(\tau, T) + \tau Rk_\tau(\tau, T)}{1 - \tau Rk_T(\tau, T)}.$$

Let  $W(\tau, T)$  be the value of the objective function in the Firm's Problem 2 when the firm faces taxes  $T$  and  $\tau$ . It coincides with the ex-ante utility of agents and represents welfare in the

economy. Consider the derivative  $dW$  of this function at zero capital taxes

$$\frac{dW(\tau, T(\tau))}{d\tau} \Big|_0 = W_\tau(0, 0) + W_T(0, 0)T'(0) = W_\tau + W_T Rk.$$

All the variables on the right hand side are evaluated at zero taxes.

Let  $\gamma_{ic}(\sigma)$  be the Lagrange multiplier on the incentive constraint for a strategy  $\sigma$ , and  $\gamma_t$  be the multiplier on the feasibility constraint in period  $t$  in Firm's Problem 2. From the envelope theorem

$$\begin{aligned} W_T &= \gamma_{t+1}, \\ W_\tau &= -\gamma_{t+1} Rk - \sum_{\sigma} \gamma_{ic}(\sigma) V_Q(\sigma) \frac{\partial Q_t}{\partial \tau}. \end{aligned}$$

In equilibrium,  $1/Q_t = (1 - \tau) R_{t+1}$ , therefore,  $\partial Q_t / \partial \tau = 1 / (R_{t+1} (1 - \tau)^2) > 0$ . By Proposition 3, any deviation involves savings. Therefore, higher interest rates increase the return on savings, and  $V_Q(\sigma) \leq 0$  with at least one  $\sigma$  for which this inequality is strict. Combining these effects we see that capital taxes are welfare improving:

$$\frac{dW(\tau, T(\tau))}{d\tau} \Big|_0 = - \sum_{\sigma} \gamma_{ic}(\sigma) V_Q(\sigma) \frac{\partial Q_t}{\partial \tau} > 0. \quad \blacksquare$$

As in the economy with observable trades, lump sum taxes have no effect on the insurance that agents receive. Taxes on capital income have two effects. On one hand, they distort investment decisions of firms and create a deadweight loss. Note that a high tax would decrease the amount of savings accumulated by firms. On the other hand, a tax also lowers the return on savings in the retrading market. This improves the incentives of agents to reveal their private information truthfully, and firms are able to provide better insurance – private markets change endogenously in response to government policy. At least for small capital taxes the second effect dominates the first one, and improves welfare. The losses from distorting taxes are second-order while improvement in the insurance via worsening deviations is first-order.

The capital taxes alone are not sufficient to achieve the efficient outcome in the competitive settings. To see this, suppose taxes were set in such a way that the after-tax return on capital were equal to the interest rates on the retrading market under the efficient allocations,  $1/Q^{sp}$ . Then the firm would have the same incentive constraint (17) as the social planner. The feasibility constraint would be different, however. While the planner's decisions are undistorted, firms' savings are affected by distorting taxes. In general the government has to impose additional non-linear taxes on labor income to achieve efficient allocations.

#### **IV.E. Constrained efficient allocations with other shock processes**

In the previous section, we showed that when skill shocks follow an independently and identically distributed process, the optimal interest rate on the retrading market must be lower than the return on capital. The intuition for that result is that an agent who anticipates misrepresenting his type in the next period oversaves to smooth his consumption, and lower interest rates reduce the return to such deviations. In this section, we show that a deviating strategy of oversaving in anticipation of lying is present with other types of shocks that are not independently and identically distributed. We argue that, typically, incentive constraints can be relaxed by a lower interest rate, an effect that we highlighted with independently and identically distributed shocks. At the same time, when the skill shocks are not independently and identically distributed, there might exist other binding incentive constraints that involve borrowing, and lower interest rates would tighten such constraints. We identify one cause of such effects – a deviating agent may have information about the probability of the evolution of skills that the planner does not have. This adds an additional effect similar to adverse selection. In this section we present two examples of stochastic processes. In the first example, we extend our results from the independently and identically distributed case to the setup with absorbing disability shocks. Then we construct an example and identify a range of parameters in which the optimal interest rate is higher than the return on capital, implying that subsidization of savings improves upon the competitive equilibrium.

#### IV.E.i. Permanent disability shocks

Consider a stylized model of disability insurance. We assume that agent's skills can be one of only two types, productive or unproductive, with  $\theta(1) = 0$ . Assume that being unproductive is an absorbing state, so that if in period  $t$  any agent receives shock  $\theta(1)$ , he receives shock  $\theta(1)$  in all the subsequent periods. The assumption of absorbing shocks implies that there are only  $T$  possible incentive constraints, one for each period. In each period  $t$  an able agent with skill  $\theta(2)$  decides whether to reveal it truthfully or claim to be disabled. We can now generalize Propositions 4 and 5 to the case of absorbing disability shocks.

**Proposition 6** *Suppose that skill shocks are absorbing disability shocks. In the constrained efficient allocations,  $F_k(K_t, Y_t) > 1/Q_{t-1}$  for at least one  $t$ . A positive tax on capital income with a lump sum rebate improves welfare.*

The proof of this proposition closely follows proofs of Propositions 4 and 5 is provided in the appendix. The intuition is very similar to the case of independently and identically distributed shocks and relies on the necessity to deter deviations of joint lying and oversaving. In order to provide incentives for the able agent to work, the present value of consumption for the truth-telling agent should be higher than the present value of consumption for the agent who becomes disabled in period  $t$ . Therefore, an agent who deviates in period  $t$  chooses to save a positive amount in the previous periods to smooth his consumption. We can then see that lowering the interest rate relaxes the incentive constraint and improves upon the competitive market allocation.

#### IV.E.ii. Other shocks and a case for capital subsidies

In the previous examples of independently and identically distributed shocks and disability shocks *all* binding incentive constraints involved oversaving by agents. Before misreporting his type an agent oversaves in order to smooth his consumption. With a more general skill process,

there may be an additional effect of deviation that has a flavor of adverse selection: when an agent misreports his current skill, he may have better information about the probability distribution of his skills in the future than the planner. We show that this effect may lead to subsidization of savings.

In what follows, we construct an example that illustrates how with more general shock processes, the effect of asymmetric information may lead to an optimal interest rate above the marginal rate of transformation leading to subsidization of savings. We show that the effect calling for taxation of capital to deter deviating and oversaving is still present even in this example. We then show how the tradeoff between the two effects depends on the parameters of the model and explore conditions under which capital may be optimally subsidized.

Consider a two period economy, where types are drawn from a two point distribution  $\Theta = \{0, 1\}$ . In the first period, all agents face equal probability of becoming either of these types. If an agent is productive (has skill  $\theta = 1$ ) in the first period, he stays productive in the second period with probability one, i.e., being productive is an absorbing state. An agent who has a skill  $\theta = 0$  in the first period remains unproductive in the second period with probability  $\rho$ , and becomes productive with probability  $1 - \rho$ .

The planner maximizes the objective function:

$$(20) \quad \frac{1}{2} \{u(c_0) + (1 - \rho) [u(c_{01}) + v(y_{01})] + \rho u(c_{00})\} + \frac{1}{2} \{u(c_1) + u(c_{11}) + v(y_{11})\}.$$

In this example, there are two binding incentive constraints: (1) the type with the history of shocks (0, 01) should not have an incentive to claim to be unproductive in both periods:

$$(21) \quad u(c_0) + (1 - \rho) [u(c_{01}) + v(y_{01})] + \rho u(c_{00}) \geq u(c_0) + u(c_{00});$$

and (2) the productive type in the first period should not have an incentive to claim to be

unproductive in the first period:

$$(22) \quad u(c_1) + u(c_{11}) + v(y_{11}) \geq u(c_0) + u(c_{01}) + v(y_{01}).^{14}$$

The agents face the bond prices on the retrading market that are given by:

$$Q = \beta \frac{(1 - \rho)u'(c_{01}) + \rho u'(c_{00})}{u'(c_0)}.$$

In Figure I we provide results of a numerical computation in which we characterize the optimal price on the retrading market as we vary  $\rho$ . We assume the utility function is  $u(c, l) = c^{1-\sigma}/(1-\sigma) - \theta y^\gamma/\gamma$  and that there is no discounting. Let the production function be  $F(K, Y) = K + Y$ , and assume that agents have no initial endowment of capital. We plot the optimal price  $Q$  as a function of  $\rho$  when  $\sigma = 0.3$  and  $\gamma = 2$ . Note in the figure we observe for  $\rho \in (0, 0.8)$ , the interest rate  $1/Q < 1$ , implying implicit taxation of savings; for  $\rho \in (0.8, 1)$ , the interest rate  $1/Q > 1$ , implying an implicit subsidy to savings. We present the intuition for this result below.

As in the case of the independently and identically distributed shocks, an agent who follows the deviating strategy represented by the right hand side of the incentive compatibility constraint (21) saves a positive amount under such interest rates. The reason for that is he receives consumption  $c_{00}$  with probability one, which is less than  $c_{01}$  that he would receive with probability  $(1 - \rho)$  if he told the truth. On the other hand, the agent who follows the second strategy represented by the right hand side of the incentive compatibility constraint (22) knows with probability one that he is productive in the next period and receives consumption  $c_{01}$ . However, the planner assumes that an agent who was unproductive in the first period would be productive with probability  $(1 - \rho)$ . This is the effect of the asymmetry of information in which a deviator can exploit the informational advantage over the planner. The greater  $\rho$  is, the stronger incentives such a deviator has to borrow to smooth his consumption. Therefore, a lower interest rate relaxes the first incentive constraint (21) but tightens the second one (22).

Whether taxes or subsidies are optimal in equilibrium depends on the relative importance of the two incentive constraints.

Clearly, when  $\rho = 0$ , all the relevant information is revealed in the first period, and there is no need to distort intertemporal allocations. Therefore,  $Q = 1$ ; the interest rate on the retrading market is undistorted from the marginal rate of transformation. For small positive  $\rho$ , the high type in the first period has a relatively small informational advantage over the social planner, and lower  $Q$  tightens the incentive constraint (22) only by a small amount. At the same time, a lower value of  $Q$  significantly relaxes the incentive constraint (21). The optimal interest rates are below the technological rate of return which is equal to one. As  $\rho$  becomes larger, the relative importance of  $Q$  in the two incentive constraints changes. First, there are fewer agents who follow the first strategy, and the need to provide incentives for them diminishes. At the same time the agents who follow the second strategy gain more by borrowing. For  $\rho$  sufficiently far from zero these two effects imply that the optimal  $Q$  becomes eventually greater than one. In a decentralized economy that implies the optimal taxes on capital should be negative, i.e. capital should be subsidized. Finally, as  $\rho$  approaches 1, the need to provide incentives for the high types in the second period disappears, and the problem becomes again equivalent to a static problem with all information being revealed in the first period.

This effect of asymmetric information does not exist in the independently and identically distributed case as an agent who misreports does not have any additional information compared to the planner about the future skill. In the case of disability shocks, an agent who claims disability has better information than the planner – a deviator knows that he is going to be able with some probability in the future while the planner thinks that the deviator can only be disabled. However, the deviator cannot take advantage of the extra information. A planner would instantaneously know that an agent who previously claimed disability but now claims that he is able was a deviator, and the planner would punish such reports. Therefore, the second effect in the case of disability shocks does not influence the results that capital should be taxed.

This example illustrates several general points. First, there are typically incentive constraints that imply that agents choose to save when deviating, and lower  $Q$  relaxes these incentive constraints. At the same time, such  $Q$  might tighten the incentive constraints if the deviating agent has a sufficiently large informational advantage over the social planner. We conclude that the tradeoff of these two effects determines the exact prescription of the model, whether the capital should be taxed or subsidized. Theoretically, we showed two cases (independently and identically distributed shocks and absorbing disability shocks) in which the first effect dominates and capital should be implicitly taxed. This example presents an outline for the economic reasoning of under which conditions the informational advantage effect may dominate and call for implicit subsidies to capital.

## V. Numerical example

In this section, we compute optimal allocations and tax policy in economies with observable and unobservable asset trades. As a benchmark, we use a disability insurance environment analyzed in Golosov and Tsyvinski [2006]. We consider three types of experiments. First, we compute the efficient allocations in an economy where private trades are observable. In particular, we study the pattern and the size of intertemporal wedges. Second, we compute the optimal allocations for the economy in which agents are allowed to trade unobservably. We find that the intertemporal wedge in this economy is smaller than in the economy with observable trades. We then compare the welfare losses from the unobservability of trades. Third, we compute the competitive equilibrium in the economy with unobservable trades. We compare welfare in the competitive equilibrium to welfare of the optimal allocation with unobservable trades and with a version of Bewley's economy where the only form of insurance available to agents is trading of a risk-free bond. We find that, even in the environment with unobservable trades, private markets can achieve allocations that are nearly optimal. This result indicates that the large welfare gains from introducing government insurance found in the literature on

optimal dynamic contracting may be misleading as they treat private markets exogenously. To a large extent, public provision of insurance crowds out private insurance.<sup>15</sup>

We consider an economy with absorbing disability shocks that last ten periods. In the numerical exercises described below each period is assumed to be five years. The production function is  $F(K, Y) = rK + wY$ . We choose the following parameter values:  $\beta = 0.8$ ,  $r = \beta^{-1}$ ,  $w = 1.21$ . Each agent is endowed with  $k_1 = 0.69$  units of initial capital. The parameterization is described in Golosov and Tsyvinski [2003]. We adjust those parameters to represent a five year time period. The stochastic process for skills that we use matches disability shocks among the U.S. population for ages 20-65 years old. The utility function is  $u(c, l) = \ln(c) + 1.5 \ln(1 - l)$ .

### V.A. Observable trades

In this subsection, we compute optimal allocations and intertemporal wedges for an economy where trades are observable.

It is well known that in the economy with private information without hidden retrading, savings decisions of each agent are distorted. In particular, optimal allocations satisfy the following inequality for all  $\theta^t$ :

$$u'(c(\theta^t)) \leq \beta r \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u'(c(\theta^{t+1})).$$

This inequality is strict if  $\text{var}(c(\theta^{t+1})) > 0$ . We define the wedge  $\tau(\theta^t)$  that each agent faces as

$$(23) \quad \tau(\theta^t) = 1 - \frac{1}{r-1} \left[ \frac{u'(c(\theta^t))}{\beta \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u'(c(\theta^{t+1}))} - 1 \right].$$

The wedge is defined to be consistent with a wedge from a linear tax imposed on the net

capital income  $(r - 1)k$ . The standard Euler equation with linear taxes on capital income is

$$u'(c_t) = \beta [(1 - \tau)(r - 1) + 1] E_t u'(c_{t+1}),$$

and we use this expression to define the savings wedge  $\tau$ .

This wedge is history specific: agents who had a different history of shocks  $\theta^t$  face different wedges. The wedge is equal to zero for the agent whose current skill is zero (since it is an absorbing state) and is strictly positive for the other agents. In the computed example the wedge of the agent who has positive productivity increases over the lifetime and reaches eight percent (see Figure II).

### **V.B. Unobservable trades**

In this subsection, we compute the optimal allocation for the economy where trades are unobservable. We compare the welfare for this economy to that of the economy without private information and to the economy with private information but observable trades. When agents can trade assets unobservably, efficiency requires that equilibrium interest rates on the retrading markets are lower than  $r$ . Although the stochastic process for skills is not independently and identically distributed, it is straightforward to modify the proof of Proposition 3 to show that for any binding deviating strategy  $\sigma$ , savings are always non-negative,  $s(\sigma(\theta^t)) \geq 0$  with a strict inequality for some  $\theta^t$ . It implies that Proposition 4 holds in this economy.

We define the wedge in the same way as we defined it in (23) for the economy with observable trades. Figure III shows the computed wedge in this example. Note that it is strictly positive in each period but smaller than the wedge in the economy with observable trades. It never exceeds two percent.

The ex-ante utility of agents is lower in the economy with unobservable trades than in the economy with observable trades. When trades are not observable the set of incentive compatible allocations is smaller, and the provision of insurance to agents is more difficult.

We use the following measure to compare welfare in the two economies. Let  $\{c_t^{no}, y_t^{no}\}_{t=1}^T$  be the allocations that solve the social planner's problem with non-observable consumption. The ex-ante utility of such allocations is  $\sum_{t=1}^T \beta^t \sum_{\theta^t} \pi(\theta^t) u(c^{no}(\theta^t), y^{no}(\theta^t)/\theta)$ . If ex-ante utility in the economy with observable trades is  $U^o$ , we find such a number  $\kappa$ , that increasing consumption of each agent by  $\kappa$  percent would make the ex-ante utility of the agent equal to  $U^o$ , i.e.

$$\sum_{t=1}^T \beta^t \sum_{\theta^t} \pi(\theta^t) U((1 + \kappa)c^{no}(\theta^t), y^{no}(\theta^t)/\theta) = U^o.$$

We find the welfare losses from unobservable retrading, i.e., the difference between the utility of optimal allocations in which trades are observable and the utility of optimal allocation in which trades are not observable to be 0.2 percent. The welfare loss of the optimal allocation in which trades are unobservable compared to the first best outcomes - the economy with no private information - is 1.1 percent.

### **V.C. Crowding out**

In this subsection, we address the question as to what extent private markets are able to provide insurance in such an environment. We find that most optimal provision can be done privately with very small gains from public interventions. This contrasts with a large body of literature that studies social insurance when private markets are absent or exogenously restricted. For example, Hansen and Imrohoroglu [1992], Wang and Williamson [1996], Hopenhayn and Nicolini [1997], Alvarez and Veraciero [1998], and many others found large welfare effects of public policy when markets are exogenously incomplete. In this section, we show that this private provision of insurance, though not efficient, is a significant improvement over the autarkic allocations with self-insurance.

Consider an economy where there is no private provision of insurance. In the absence of taxes each agent is able to borrow and lend at the interest rate  $r$ , and, if he has a positive productivity, supplies labor at the wage rate  $w$ . This setup is equivalent to that in Aiyagari

[1994]. The agent's problem is

$$\max_{c,y,k} \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t \{u(c(\theta^t)) + v(y(\theta^t)/\theta_t)\}$$

such that for all  $\theta^t$

$$c(\theta^t) + s(\theta^t) = wy(\theta^t) + rs(\theta^{t-1}),$$

$$s(\theta^0) = k_1.$$

where we use a convention that if  $\theta = 0$  then  $v(y(\theta^t)/\theta_t) = v(0)$ .

Thus, similarly to Bewley [1986], Huggett [1993], and Aiyagari [1994], the only insurance available is self-insurance with a risk-free bond.

We find that competitive equilibrium allocations provide welfare which is 1.08 percent higher than welfare in the economy where a risk-free bond is the only form of insurance available to agents. Welfare under efficient allocations is 1.11 percent higher than in the economy with only risk-free bonds. These findings show that competitive equilibrium without government interventions provides about 97 percent of the optimal insurance in our numerical example.

This example suggests that it is important to consider responses of private markets to changes in the government policy. Consider the environment we described where the optimal insurance is provided by the government. Since there are no gains from additional insurance, all private insurance markets are absent. To an outside observer such an economy appears to be identical to Aiyagari's economy where the only private asset available is a risk-free bond. Taking exogenous such a structure of private markets would suggest that the removal of public insurance decreases welfare by 1.11 percent. This argument, however, does not take into account that private markets may emerge, and the actual welfare losses would be much smaller.

The analysis above assumes that private markets function perfectly. In such circumstances most of the optimal insurance can be provided with no government interventions. One may

argue that legal restrictions or market imperfections decrease the amount of insurance available privately, and public insurance is needed in such circumstances. The size of crowding out depends on the particular form of the assumed imperfections, and additional work would be needed to compute it. In general, unless such imperfections are assumed to be very severe, the welfare effects of the optimal public policy may be small.

## VI. Discussion and generalizations

One of the broad issues that this paper touches on is modeling the benefits of the markets in the models of optimal taxation. It can be argued that outcomes would be better if a) markets for trades among agents would be eliminated, or b) consumption was observable. We showed that the environment with observable consumption has higher welfare than the environment with private markets – an improvement can be achieved if markets for hidden trades are shut down. However, markets have multiple benefits including privacy or benefits from producing and disseminating information.

Our model can be generalized to the case where markets have benefits. An easy interpretation that would deliver the optimum in which the planner would choose not to shut down the markets is as follows. Suppose we do not model these benefits but assume they are large enough that the planner would choose not to shut down the markets. Alternatively, assume that monitoring transactions on markets is costly. In our paper, this reasoning manifests itself in assuming that it is *infeasible* to shut down markets, implicitly presuming that shutting down the markets would bring large negative welfare consequences.

The key difficulty in modeling the benefits of markets and showing that shutting markets down is suboptimal is that in *any model with a benevolent planner who can commit*, the centralized planner can always do at least as well as markets (or any other mechanism for that matter). The best we can hope for in that situation is for competitive markets to do just as well as the planner, but not better, as the planner can always replicate the market allocation. This is true in any standard mechanism design model or any optimal taxation model.

How can we specify a model in which a social planner would choose not to shut down markets? The only types of models that we are aware of in which allowing markets improves upon the allocation of the social planner are the models in which mechanisms are no longer run by a fictitious benevolent social planner. Acemoglu, Golosov, and Tsyvinski [2005] study a dynamic optimal taxation model in which the social planner is self-interested and lacks commitment. They show conditions under which markets are preferred to the governments. A similar comparison of markets versus governments would carry over to our model. Bisin and Rampini [2004] study a model in which markets are beneficial as they impose constraints on governments without commitment. We conjecture that similar arguments may be applicable to study the benefits of the markets in our setup.

We have shown in the paper that government interventions play an essential role in achieving optimal allocations when agents engage in hidden borrowing and lending. The intervention that we propose, namely a linear savings tax or subsidy, satisfies two appealing principles that preserve benefits of markets: anonymity and allowing functioning of the markets. Therefore, it is an appealing alternative to shutting down markets. First, to use a linear tax or subsidy a government does not need to know the identity of an agent; only the amount of the transaction needs to be known. Recall that the tax is levied at the side of the firm. The only thing the government needs to know is the aggregate amount of savings done by the firms. There is no need to know the identity of firm's consumers. In that sense, a linear tax respects agents' anonymity and privacy. Second, a linear tax or subsidy is minimally invasive to the functioning of the markets. The government armed with a linear optimal tax or subsidy has to balance the benefits (tax improves incentives) versus costs (deadweight loss of intervening in the markets).

An important assumption in our model is the enforceability of the contracts on the retrading markets, i.e., agents are not allowed to default on their obligations.<sup>16</sup> One straightforward justification of such an assumption is connected to our interpretation of the retrading markets as non-exclusive contracts. It is plausible to assume that a government chooses to enforce all debt

contracts yet would choose not to enforce exclusivity of contracts with each individual firm or insurer. The second, more technical justification of such assumption is as follows. Suppose that we restrict the contracts on the retrading market to be self-enforcing in the spirit of Kehoe and Levine [2001] and Alvarez and Jermann [2000]. Following Alvarez and Jermann [2000], one could model a retrading market as a market subject to type specific borrowing constraints. Limited enforcement of contracts would worsen the opportunities for retrading and relax incentive constraints. We believe that the general logic of Pareto improving government intervention would hold in such a model and leave this extension for future work.

We also would like to provide another interpretation of the retrading markets studied in this model. One can think about such markets as tax-arbitrage markets that preclude a government from imposing different capital income tax rates on various agents. This interpretation is appealing, especially in the context of the taxation of families in which tax arbitrage may happen between spouses or various generations of the families or in the context of established relationships between firms (when we interpret agents in our model as firms being affected by unobservable shocks). A careful analysis of such a model would have to model retrading possibilities not as competitive markets but as long-term interactions, perhaps, as in Kocherlakota [1996].

In the working version of the paper we also provided two straightforward extensions: an infinite horizon model and inclusion of public goods.

## **VII. Conclusion**

This paper studies dynamic optimal taxation in an economy with informational frictions and endogenous insurance markets. We relax the assumption of observable trades and study environments where trades are unobservable. We show that competitive equilibria are not constrained optimal. A government, even one that has the same information as private parties, can improve upon any allocations that can be achieved by markets. A linear tax or subsidy levied on

firms' capital income affects the rate of return in hidden asset markets and improves insurance provided to agents by insurance firms.

There are three substantive lessons that one learns from our framework. First, the structure of insurance markets and the extent of insurance that these markets provide respond endogenously to government policy. Taking these markets as given might lead to significant errors in designing the optimal policy. Second, competitive equilibria in the presence of hidden trades are inefficient, and there is a role for welfare improving taxes or subsidies. Third, the intervention we propose, a linear savings tax, is an appealing alternative to shutting down markets as it allows markets to provide most of the insurance while correcting an externality associated with such provision.

## VIII. Appendix

### VIII.A. Absence of shock-specific securities

The assumption that agents can trade only a risk-free bond is not restrictive. In many environments, risk-free bonds emerge as the only asset traded in equilibrium. Consider a market structure described in Section IV.A. Suppose each agent observes the identity of the agent with whom he transacts but not private characteristics of that agent. In these settings, no Arrow-type securities, for which the payment depends on the reports of the agents, are traded in equilibrium. The structure of securities markets is similar to the one studied in Bisin and Gottardi [1999]. Let  $a^i(\theta)$  be a security that pays one unit of consumption good if an agent  $i$  reports  $\theta$  to the planner in the next period, and zero otherwise. For simplicity we assume that the lowest skill,  $\theta(1)$ , is strictly positive, so that no agent incurs infinite disutility from reporting any other type.

**Claim 2** *There is no equilibrium where securities  $a^i(\theta)$  are traded. Only a risk-free bond is traded in equilibrium.*

**Proof of Claim 2.** We will show that, for any price  $q^i(\theta)$  of a security  $a^i(\theta)$ , either an agent  $i$  can make an infinite return or has a higher return on a risk-free security. Since, in bilateral

trades, agents can see each other's type, the price for each security may be different depending on whether the agent, who controls the outcome of it, buys or sells the security.

*Case 1.* An agent wants to buy a security that pays one unit of consumption good if he sends report  $\theta$  in the next period.

We show that a price for such a security will be  $q^i(\theta) \geq Q$ . Suppose, to the contrary, that  $q^i(\theta) < Q$ . Under such prices the agent could buy infinitely many securities that pay in state  $\theta$  and sell a risk-free debt for this amount. Then, in the next period, he claims the state  $\theta$ . Since an agent incurs only finite disutility from providing  $y(\theta)$  units of labor if his type is  $\hat{\theta}$ , this strategy yields an infinite utility for the agent. The seller of the security incurs losses, so it cannot be the equilibrium price.

If  $q^i(\theta) \geq Q$  an agent prefers not to sell such a security since it pays one unit of consumption in only one state  $\theta$ , while risk-free bond pays one unit of consumption in all states and is cheaper.

*Case 2.* An agent wants to sell a security that pays one unit of consumption good if he sends a report  $\theta$  in the next period.

The price of such a security is zero. Suppose not. Then the agent can sell infinitely many of such securities and in the next period claim any state other than  $\theta$ . The agent makes infinite profits and utility. Thus, this case is also not possible. ■

The intuition for the proof is simple. An agent can choose which skill to report in the next period. As long as there are gains from reporting any state  $\theta$ , he will report it with probability one. But that makes such a security  $a^i(\theta)$  equivalent to a risk-free bond, hence no type-specific securities are traded in equilibrium.

### **VIII.B. Proof of Lemma 1**

We show that any allocation satisfying (9) also satisfies (14) and (15), and vice versa.

Suppose  $\{c_t, y_t\}_{t=1}^T$  satisfies (9) and the equilibrium prices on the retrading market are  $\{Q_t\}_{t=1}^T$ . Then the Euler equation (15) is satisfied. Otherwise, the truth telling agent can

improve his utility along some history, and (9) would not hold. Similarly (14) is also satisfied. Otherwise, if it did not hold for some strategy  $\sigma' \neq \sigma^*$ , this strategy  $\sigma'$  would also be optimal on the retrading market and the original allocation would not be incentive compatible.

Suppose  $\{c_t, y_t, Q_t\}_{t=1}^T$  satisfies (14) and (15). We need to show that on the retrading market in equilibrium agents choose to reveal their types truthfully, do not trade, consume their consumption allocations  $c(\sigma^*)$ , and the equilibrium interest rates are equal to  $Q$ . An agent who faces prices  $Q$  chooses the truthful revelation because of (14). The Euler equation (15) guarantees that the agent optimally chooses not to buy bonds along this truth telling path. That implies that the feasibility condition on the retrading market (7) is satisfied and  $Q_t$  are indeed the equilibrium prices.

### VIII.C. Proof of Proposition 1

Let  $\theta$  be the skill shock in the first period. Since all uncertainty is realized after the first period, it determines the future path of skills. It is a well known result from Werning [2001] and Golosov, Kocherlakota and Tsyvinski [2003] that when trades are observable, the optimal allocations satisfy  $u'(c_t(\theta)) = F_k(K_{t+1}, Y_{t+1})\beta u'(c_{t+1}(\theta))$  for all  $\theta$  and  $t$ .

We show now that these allocations are also feasible in the economy with unobservable retrading. Suppose prices on the retrading market are  $Q_t = 1/F_k(K_{t+1}, Y_{t+1})$ . Consider an agent who sends an arbitrary report  $\sigma(\theta)$  about his first period skill. Since all uncertainty is realized after the first period, in all the following periods the agent receives the allocations  $\{c_t(\sigma(\theta)), y_t(\sigma(\theta))\}_{t=1}^T$  that depend only in his report in the first period. Since allocations received from planner satisfy the Euler equation, it is optimal for the agent to consume these allocations without any additional trades:  $x_t(\sigma(\theta)) = c_t(\sigma(\theta))$  for all  $t$ . Therefore, efficient allocations in the economy with observable trades are still incentive compatible if there are hidden retrading markets. It remains to verify that the constructed  $Q_t$ 's are indeed the equilibrium prices. Since with such prices for all  $\theta$  and  $t$ , the following equality  $x_t(\theta) = c_t(\theta)$  holds, so the feasibility constraint (7) is satisfied.

### VIII.D. Proof of Proposition 4

To prove this result we first present a sequence of lemmas and propositions. We show that *any* deviating strategy  $\sigma \neq \sigma^*$  involves positive saving after some history and never borrowing. This result implies that the planner would want to decrease the return on deviations by lowering the interest rates on the retrading market.

Consider the optimal asset trades and consumption on the retrading market  $\{x(\sigma(\theta^t)), s(\sigma(\theta^t))\}_{t=1}^T$  for a given strategy  $\sigma$ . They must satisfy (10), (11) and (12).

**Lemma 2** *For any strategy  $\sigma$  consider the allocation  $\{x_t, s_t\}_{t=1}^T$  that satisfies (10), (11) and (12). This allocation must satisfy the following for all  $\theta^t$ :*

$$(24) \quad \frac{\sum_{\theta} \pi(\theta) u'(x(\sigma(\theta^t), \theta))}{u'(x(\sigma(\theta^t)))} \leq \frac{\sum_{\theta} \pi(\theta) u'(c(\sigma(\theta^t), \theta))}{u'(c(\sigma(\theta^t)))}.$$

**Proof of Lemma 2.** By the monotonicity assumption and the assumption that the only possible deviations are those in which an agent reports a lower type, it must be true that  $c(\sigma(\theta^t), \theta) \geq c(\sigma(\theta^t), \theta)$  for all  $\theta^t, \theta, \sigma$ . Here we use a notation  $\sigma(\theta^t, \theta)$  to denote a report of the agent who uses strategy  $\sigma$  after history  $(\theta^t, \theta)$ .

Equation (19) implies then that for any  $\theta^t$

$$Q_t u'(c(\sigma(\theta^t))) \leq \beta \sum_{\theta} \pi(\theta) u'(c(\sigma(\theta^t), \theta)).$$

Combining this inequality with (11) we obtain the lemma. ■

The intuition for the result is discussed in the text.

It is optimal for the agent to save in the anticipation of those deviations, and borrowing is always suboptimal. The following lemmas formalize this intuition.

**Lemma 3** *For any strategy  $\sigma$  consider the allocation  $\{x_t, s_t\}_{t=1}^T$  that satisfies (10), (11) and (12). Suppose  $s(\sigma(\theta^t)) < 0$  for some  $\theta^t$ . Then  $x(\sigma(\theta^t)) < c(\sigma(\theta^t))$  and  $s(\sigma(\theta^{t-1})) < 0$  for*

$\theta^{t-1} \in \theta^t$ .

**Proof of Lemma 3.** Suppose that  $x(\sigma(\theta^t)) \geq c(\sigma(\theta^t))$ . This implies that

$$\frac{\sum_{\theta} \pi(\theta) u'(x(\sigma(\theta^t, \theta)))}{u'(c(\sigma(\theta^t)))} \leq \frac{\sum_{\theta} \pi(\theta) u'(x(\sigma(\theta^t, \theta)))}{u'(x(\sigma(\theta^t)))}.$$

Combining this with (24) we obtain

$$\sum_{\theta} \pi(\theta) u'(x(\sigma(\theta^t, \theta))) \leq \sum_{\theta} \pi(\theta) u'(c(\sigma(\theta^t, \theta))).$$

This inequality implies that there must be at least one  $\theta$  such that  $x(\sigma(\theta^t, \theta)) \geq c(\sigma(\theta^t, \theta))$ . Then from (10) it follows that  $s(\sigma(\theta^t, \theta)) < 0$ . Using the previous argument since  $x(\sigma(\theta^t, \theta)) \geq c(\sigma(\theta^t, \theta))$  it must be true that there exists some node  $\theta'$  such that  $x(\sigma(\theta^t, \theta, \theta')) \geq c(\sigma(\theta^t, \theta, \theta'))$  and  $s(\sigma(\theta^t, \theta, \theta')) < 0$ . Continuing this induction there exists a node  $\theta^T$  such that  $x(\sigma(\theta^T)) \geq c(\sigma(\theta^T))$  and  $s(\sigma(\theta^T)) < 0$ . But this is impossible since in the last period it must be true that  $s(\sigma(\theta^T)) = 0$  for all  $\theta^T$ . A contradiction.

Negative assets in the previous period  $s(\sigma(\theta^{t-1})) < 0$  for  $\theta^{t-1} \in \theta^t$  follow from the budget constraint (10) and  $x(\sigma(\theta^t)) - c(\sigma(\theta^t)) < 0$ . ■

It is easiest to understand the intuition for this result in the case when consumption allocations that an agent receives along his deviation strategy  $\sigma$  satisfy the Euler equation, i.e. (24) holds with equality since inequality further strengthens this intuition. Agent's actual consumption  $x$  also satisfies the Euler equation. This implies that an agent chooses to have a higher consumption  $x(\theta^t)$  than his endowment  $c(\theta^t)$  only if his consumption is also higher in the future. This is possible only if an agent starts with a positive amount of assets and saves some resources for the next period.

The previous results imply that it is optimal for an agent to borrow only if he borrowed in the previous period. But then borrowing can never be optimal since each agent has a zero initial asset position. The next proposition formalizes this intuition.

**Proposition 7** Consider any strategy  $\sigma$  together with trades and after-trade consumption on the retrading market  $\{x_t, s_t\}_{t=1}^T$ . If  $s(\sigma(\theta^t)) < 0$  then there exists another pair  $\{\hat{x}_t, \hat{s}_t\}_{t=1}^T$  that is feasible and gives a higher utility.

**Proof of Proposition 7.** Consider any reporting strategy  $\sigma$ . The optimal consumption/saving pair  $\{x_t, s_t\}_{t=1}^T$  should satisfy (10), (11) and (12). The previous lemma showed that if  $s(\sigma(\theta^t)) < 0$  for some  $\theta^t$  than  $s(\sigma(\theta^{t-1})) < 0$ . Continuing this backward induction we obtain that it must be true that  $s(\theta^0) < 0$  which violates the initial condition  $s(\theta^0) = 0$ . Therefore there is no node in which it is optimal to borrow. ■

In the solution to the social planner's problem in Lemma 1, the incentive constraint (14) binds for some strategies  $\sigma$ . The next proposition shows that such strategies imply savings in some states and never borrowing.

**Proof of Proposition 3.** Consider any deviating strategy  $\sigma$  together with consumption/saving pair  $\{x_t, s_t\}_{t=1}^T$  that binds in the social planner's problem. We established before that for such allocations it must be true that  $s(\sigma(\theta^t)) \geq 0$  for all  $\theta^t$ . We show that the inequality is strict for some  $\theta^t$ . The allocations along the truth telling strategy  $\sigma^*$  are such that the optimal saving behavior is  $s(\sigma^*(\theta^t)) = 0$  for all  $\theta^t$ . For any other strategy there exists some  $\hat{\theta}^t$  so that  $c(\sigma(\hat{\theta}^t)) \neq c(\sigma^*(\hat{\theta}^t))$ . Since we assumed the incentive problem is non-trivial in each period, there must be at least one such  $\hat{\theta}^t$  in each  $t$ , and those constraints bind. But then (10) and (11) can not hold simultaneously with zero savings in each node, therefore there must be some  $\theta^t$  such that  $s(\sigma(\theta^t)) > 0$ . ■

The previous propositions showed that if an agent decides to deviate, he always optimally chooses to have positive savings. A decrease in the interest rates reduces returns on savings and lowers the utility from deviations. The next proposition shows that the social planner chooses interest rates to be lower than the return on capital.

We are finally ready to prove Proposition 4.

**Proof of Proposition 4.** The social planner's problem is as follows:

$$\max_{c,y,K,Q} \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t \{U(c(\theta^t), y(\theta^t)/\theta_t)\}$$

such that for all  $\theta^t, t$

$$(25) \quad \sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} \leq F(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)),$$

$$\sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t u(c(\theta^t), y(\theta^t)/\theta_t) \geq V(\{c, y\}, \{Q\})(\sigma) \text{ for any } \sigma \neq \sigma^*,$$

$$(26) \quad Q_t u'(c(\theta^t)) = \beta \sum_{\theta^{t+1}} \pi(\theta^{t+1} | \theta^t) u'(c(\theta^{t+1})).$$

Suppose  $Q_t \leq 1/F_k(t+1)$  for all  $t$ . The first order conditions with respect to  $Q_1$  imply that

$$-\sum_{\sigma} \mu(\sigma) \frac{\partial V(\sigma)}{\partial Q_1} + \sum_{\theta^1} \eta(\theta^1) u'(c(\theta^1)) = 0.$$

From Proposition 3,  $\frac{\partial V(\sigma)}{\partial Q_1} < 0$  which implies that  $\eta(\theta^1) < 0$  for some  $\theta^1$ .

Take the first order conditions for  $c(\theta^t)$ :

$$(27) \quad \begin{aligned} & \pi(\theta^t) \beta^t (1 + \sum_{\sigma} \mu(\sigma)) u'(c(\theta^t)) - \sum_{\sigma} \mu(\sigma) \frac{\partial V(\sigma)}{\partial c(\theta^t)} \\ & + \eta(\theta^t) Q_t u''(c(\theta^t)) - \eta(\theta^{t-1}) \beta \pi(\theta^t | \theta^{t-1}) u''(c(\theta^t)) \\ & = \lambda_t \pi(\theta^t). \end{aligned}$$

Take the first order conditions with respect to  $c(\theta^{t+1})$  for all  $\theta^{t+1}$  that follow  $\theta^t$  and sum

them:

$$\begin{aligned}
& \sum_{\theta^{t+1}} \pi(\theta^{t+1}) \beta^{t+1} (1 + \sum_{\sigma} \mu(\sigma)) u'(c(\theta^{t+1})) - \sum_{\theta^{t+1}} \sum_{\sigma} \mu(\sigma) \frac{\partial V(\sigma)}{\partial c(\theta^{t+1})} \\
& - \eta(\theta^t) \sum_{\theta^{t+1}} \pi(\theta^{t+1} | \theta^t) \beta u''(c(\theta^{t+1})) + \sum_{\theta^{t+1}} \eta(\theta^{t+1}) Q_{t+1} u''(c(\theta^{t+1})) \\
= & \lambda_{t+1} \sum_{\theta^{t+1}} \pi(\theta^{t+1}).
\end{aligned}$$

Consider an arbitrary deviating strategy  $\sigma$ . For such a strategy the first order condition on savings hold

$$Q_t \xi(\theta^t) = \sum_{\theta^{t+1} > \theta^t} \xi(\theta^{t+1}),$$

where  $\xi(\theta^t)$  is the Lagrange multiplier associated with constraint (10).

From the envelope theorem  $\partial V(\sigma) / \partial c(\theta^t) = \sum_{\tilde{\theta}^t: \sigma(\tilde{\theta}^t) = \theta^t} \xi(\tilde{\theta}^t)$ . This implies that

$$Q_t \frac{\partial V(\sigma)}{\partial c(\theta^t)} = \sum_{\theta^{t+1}} \frac{\partial V(\sigma)}{\partial c(\theta^{t+1})}.$$

Premultiply (27) by  $Q_t$  and use the fact that  $Q_t \lambda_t \leq (1/F_k(t+1)) \lambda_t = \lambda_{t+1}$  to get

$$\begin{aligned}
& Q_t \pi(\theta^t) \beta^t (1 + \sum_{\sigma} \mu(\sigma)) u'(c(\theta^t)) - Q_t \sum_{\sigma} \mu(\sigma) \frac{\partial V(\sigma)}{\partial c(\theta^t)} + \eta(\theta^t) Q_t^2 u''(c(\theta^t)) \\
& - \eta(\theta^{t-1}) \beta Q_t \pi(\theta^t | \theta^{t-1}) u''(c(\theta^t)) \\
\leq & \sum_{\theta^{t+1}} \pi(\theta^{t+1}) \beta^{t+1} (1 + \sum_{\sigma} \mu(\sigma)) u'(c(\theta^{t+1})) - \sum_{\theta^{t+1}} \sum_{\sigma} \mu(\sigma) \frac{\partial V(\sigma)}{\partial c(\theta^{t+1})} \\
& - \eta(\theta^t) \sum_{\theta^{t+1}} \pi(\theta^{t+1} | \theta^t) \beta u''(c(\theta^{t+1})) + \sum_{\theta^{t+1}} \eta(\theta^{t+1}) Q_{t+1} u''(c(\theta^{t+1})).
\end{aligned}$$

Expressions containing  $\partial V(\sigma) / \partial c$  cancel so we get

$$\begin{aligned}
& \eta(\theta^t) Q_t^2 u''(c(\theta^t)) - \eta(\theta^{t-1}) \beta Q_t \pi(\theta^t | \theta^{t-1}) u''(c(\theta^t)) \\
\leq & -\eta(\theta^t) \sum_{\theta^{t+1}} \pi(\theta^{t+1} | \theta^t) \beta u''(c(\theta^{t+1})) + \sum_{\theta^{t+1}} \eta(\theta^{t+1}) Q_{t+1} u''(c(\theta^{t+1})).
\end{aligned}$$

After rearranging

$$(28) \quad \eta(\theta^t) \left[ Q_t^2 u''(c(\theta^t)) + \beta \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u''(c(\theta^{t+1})) \right] \\ - \eta(\theta^{t-1}) \beta Q_t \pi(\theta^t|\theta^{t-1}) u''(c(\theta^t)) - Q_{t+1} \sum_{\theta^{t+1}} \eta(\theta^{t+1}) u''(c(\theta^{t+1})) \leq 0,$$

with the boundary conditions  $\eta(\theta^T) = 0$  and  $\eta(\theta^0) = 0$ .

We know from optimality, there exists  $\theta^1$  such that:<sup>17</sup>

$$\eta(\theta^1) < 0 \text{ and } \eta(\theta^1) Q_1 - \eta(\theta_0) \beta \pi(\theta^1|\theta_0) < 0.$$

Assume inductively that there exists  $\theta^t$  such that:

$$\eta(\theta^t) < 0 \text{ and } \eta(\theta^t) Q_t - \eta(\theta^{t-1}) \beta \pi(\theta^t|\theta^{t-1}) < 0.$$

We want to prove that these inequalities also hold for  $(t+1)$ . Equation 28 implies that:

$$\sum_{\theta_{t+1} \geq \theta^t} [\eta(\theta^t, \theta_{t+1}) Q_{t+1} - \beta \eta(\theta^t) \pi(\theta^t, \theta_{t+1}|\theta^t)] u''(c_{t+1}(\theta^t, \theta_{t+1})) \\ \geq [\eta(\theta^t) Q_t - \eta(\theta^{t-1}) \beta \pi(\theta^t|\theta^{t-1})] Q_t u''(c_t(\theta^t)).$$

And from the inductive assumption:

$$\sum_{\theta_{t+1} \geq \theta^t} [\eta(\theta^t, \theta_{t+1}) Q_{t+1} - \beta \eta(\theta^t) \pi(\theta^t, \theta_{t+1}|\theta^t)] u''(c_{t+1}(\theta^t, \theta_{t+1})) > 0,$$

which implies that there exists  $\theta^{t+1}$  such that:

$$[\eta(\theta^t, \theta_{t+1}) Q_{t+1} - \beta \eta(\theta^t) \pi(\theta^t, \theta_{t+1}|\theta^t)] < 0 \text{ and } \eta(\theta^{t+1}) < 0.$$

The induction argument implies that there exists  $\theta^T$  such that:

$$[\eta(\theta^T) Q_{t+1} - \beta \eta(\theta^{T-1}) \pi(\theta^T|\theta^{T-1})] < 0 \text{ and } \eta(\theta^T) < 0,$$

which violates  $\eta(\theta^T) \geq 0$ . ■

### VIII.E. Proof of Claim 1

First we show that without loss of generality we can use utility of the consumer

$$\sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t U(c(\theta^t), y(\theta^t)/\theta_t)$$

instead of the indirect utility function  $V(\{c, y\}, R)(\sigma^*)$ . Consider any solution  $\{c_t, y_t\}_{t=1}^T$  to Firm's Problem 1 and the resulting equilibrium allocations of consumption  $\{x_t\}_{t=1}^T$ . For each history  $\theta^T$  the present value of firms' payment and agent's consumption must be the same

$$\begin{aligned} & x_1(\theta^T) + Q_1 x_2(\theta^T) + \dots + \prod_{i=1}^{T-1} Q_i x_T(\theta^T) \\ &= c_1(\theta^T) + Q_1 c_2(\theta^T) + \dots + \prod_{i=1}^{T-1} Q_i c_T(\theta^T). \end{aligned}$$

From Proposition 2,  $1/Q_{t-1} = F_k(t)$  for all  $t$ , which implies that the cost of providing  $\{x_t\}_{t=1}^T$  directly to agents must be exactly the same as the cost of providing  $\{c_t\}_{t=1}^T$ . Therefore without loss of generality we can assume that firms provide each agent with  $x$  directly so that the truth telling agent does not retrade.

Finally, since in equilibrium firm's profits are zero,  $d_t = 0$  for all  $t$ , and Firm's Problem 1 can be rewritten in its dual form as in Problem 2.

### VIII.F. Proof of Proposition 6

The proof of Proposition 6 closely mirrors the proofs of Propositions 3 and 4. First we prove the analogues of Lemma 3

**Lemma 4** *For any strategy  $\sigma$  consider the allocation  $\{x_t, s_t\}_{t=1}^T$  that satisfies (10), (11) and (12). Suppose  $s(\sigma(\theta^t)) < 0$  for some  $\theta^t$ . Then  $x(\sigma(\theta^t)) < c(\sigma(\theta^t))$  and  $s(\sigma(\theta^{t-1})) < 0$  for  $\theta^{t-1} \in \theta^t$ .*

**Proof of Lemma 4.** First, note that if an agent becomes disabled in some state  $\theta^{t+1}$  and has negative assets  $s(\theta^{t+1})$ , then (10), (11) and (12) imply that  $x(\sigma(\theta^{t+1})) < c(\theta^{t+1})$ .

Suppose that  $x(\sigma(\theta^t)) \geq c(\sigma(\theta^t))$ . This, together with the previous observation implies that there must be another state in period  $t+1$  where  $x(\sigma(\tilde{\theta}^{t+1})) \geq c(\sigma(\tilde{\theta}^{t+1}))$ . Otherwise, constraint (11) will not be satisfied. (10) then implies that  $s(\sigma(\tilde{\theta}^{t+1})) < 0$ . Continuing by induction we obtain that  $s(\theta^T) < 0$  for some  $t$ . ■

The remaining steps of the proof of Proposition 6 are identical to those for Propositions 3 and 4.

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## References

- Abraham, Arpad, and Nicola Pavoni, “Efficient Allocations with Moral Hazard and Hidden Borrowing and Lending,” mimeo, University College London, 2003.
- Acemoglu, Daron, Mikhail Golosov, and Aleh Tsyvinski, “Markets Versus Governments: Political Economy of Mechanisms,” NBER Working Paper No. 12224, 2006.
- Aiyagari, S. Rao, “Uninsured Idiosyncratic Risk and Aggregate Saving,” *Quarterly Journal of Economics*, CIX (1994), 659-684.
- \_\_\_\_\_, “Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting,” *Journal of Political Economy*, CIII (1995), 1158-1175.
- Albanesi, Stefania, “Optimal Taxation of Entrepreneurial Capital with Private Information,” CEPR DP 5647, 2006.
- Albanesi, Stefania, and Christopher Sleet, “Dynamic Optimal Taxation with Private Information,” *Review of Economic Studies*, LXXIII (2006), 1-30.

- Allen, Franklin, "Repeated Principal-Agent Relationships with Lending and Borrowing," *Economic Letters*, XVII (1985), 27-31.
- Alvarez, Fernando, and Urban Jermann, "Efficiency, Equilibrium, and Asset Pricing with Risk of Default," *Econometrica*, LXVIII (2000), 775-797.
- Alvarez, Fernando, and Marcelo Veracierto, "Search, Self-Insurance and Job-Security Provisions," Federal Reserve Bank of Chicago, Working Paper 98-2, 1998.
- Arnott, Richard, and Joseph E. Stiglitz, "Moral Hazard and Optimal Commodity Taxation," *Journal of Public Economics*, XXIX (1986), 1-24.
- \_\_\_\_\_, and \_\_\_\_\_, "The Welfare Economics of Moral Hazard," in *Information and Insurance: Essays in Memory of Karl H. Borch*, Henri Louberge ed., (Norwell, MA: Kluwer, 1990), pp. 91-121.
- Atkeson, Andrew, and Robert E. Lucas, Jr., "On Efficient Distribution with Private Information," *Review of Economic Studies*, LIX (1992), 427-53.
- \_\_\_\_\_, and \_\_\_\_\_, "Efficiency and Equality in a Simple Model of Efficient Unemployment Insurance," *Journal of Economic Theory*, LXVI (1995), 64-88.
- Attanasio, Orazio, and Jose-Victor Rios-Rull, "Consumption Smoothing in Island Economies: Can Public Insurance Reduce Welfare?" *European Economic Review*, XLIV (2000), 1225-1258.
- Bewley, Truman F., "Stationary Monetary Equilibria with a Continuum of Independently Fluctuating Consumers," in *Contributions to Mathematical Economics in Honor of Gerard Debreu*, Werner Hildenbrand and Andreu Mas-Colell, eds., (Amsterdam: North-Holland, 1986), pp. 79-102.
- Bisin, Alberto, John Geanakoplos, Piero Gottardi, Enrico Minelli, and Herakles Polemarchakis, "Markets and Contracts," *Journal of Mathematical Economics*, forthcoming.

- Bisin, Alberto, and Piero Gottardi, "Competitive Equilibria with Asymmetric Information," *Journal of Economic Theory*, LXXXVII (1999), 1-48.
- Bisin, Alberto, and Danilo Guaitoli, "Moral Hazard and Non-exclusive Contracts," *Rand Journal of Economics*, XXXV (2004), 306-328.
- Bisin, Alberto, and Adriano Rampini, "Exclusive Contracts and the Institution of Bankruptcy," *Economic Theory*, XXVII (2006), 277-304.
- Chamberlain, Gary, and Charles Wilson, "Optimal Intertemporal Consumption Under Uncertainty," mimeo, Social Systems Research Institute Working Paper 8422, University of Wisconsin, 1984.
- Chamley, Christophe, "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, LIV (1986), 607-622.
- Cole, Harold, and Narayana Kocherlakota, "Efficient Allocations with Hidden Income and Hidden Storage," *Review of Economic Studies*, LXVIII (2001), 523-542.
- da Costa, Carlos, "Yet Another Reason to Tax Goods," mimeo, Getulio Vargas Foundation, 2004.
- Diamond, Douglas W., and Philip H. Dybvig, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, XCI (1983), 401-419.
- Diamond, Peter, and James Mirrlees, "A Model of Social Insurance with Variable Retirement," *Journal of Public Economics*, X (1978), 295-336.
- \_\_\_\_\_, and \_\_\_\_\_, "Social Insurance with Variable Retirement and Private Saving," mimeo, MIT, 1995.
- Doepke, Matthias, and Robert M. Townsend, "Dynamic Mechanism Design with Hidden Income and Hidden Actions," *Journal of Economic Theory*, CXXVI (2006), 235-285.

- Farhi, Emmanuel, Mikhail Golosov, and Aleh Tsyvinski, "A Theory of Liquidity and Regulation of Financial Intermediation," mimeo, MIT, 2006.
- Freixas, Xavier, Roger Guesnerie, and Jean Tirole, "Planning under Incomplete Information and the Ratchet Effect," *Review of Economic Studies*, LII (1985), 173-91.
- Geanakoplos, John, and Herakles Polemarchakis, "Pareto Improving Taxes," mimeo, 2004.
- Golosov, Mikhail, Narayana Kocherlakota, and Aleh Tsyvinski, "Optimal Indirect and Capital Taxation," *Review of Economic Studies*, LXX (2003), 569-587.
- Golosov, Mikhail, and Aleh Tsyvinski, "Designing Optimal Disability Insurance: A Case for Asset Testing," *Journal of Political Economy*, CXIV (2006), 257-279.
- Green, Edward, "Lending and the Smoothing of Uninsurable Income," in *Contractual Arrangements for Intertemporal Trade*, Edward Prescott and Neil Wallace, eds., (Minneapolis, MN: University of Minnesota Press, 1987), pp. 3-25.
- Greenwald, Bruce C. and Joseph E. Stiglitz, "Externalities in Economies with Imperfect Information and Incomplete Markets," *Quarterly Journal of Economics*, CI (1986), 229-264.
- Guesnerie, Roger, "A Contribution to the Pure Theory of Taxation," (New York, NY: Cambridge University Press, 1998).
- Jacklin, Charles J., "Demand Deposits, Trading Restrictions, and Risk Sharing," in *Contractual Arrangements for Intertemporal Trade*, Edward Prescott and Neil Wallace, eds., (Minneapolis, MN: University of Minnesota Press, 1987), pp. 26-47.
- Hammond, Peter J., "Markets as Constraints: Multilateral Incentive Compatibility in Continuum Economies," *Review of Economic Studies*, LIV (1987), 399-412.
- Hansen, Gary D., and Ayse Imrohoroglu, "The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard," *Journal of Political Economy*, L (1992), 118-142.

- Hopenhayn, Hugo A., and Juan Pablo Nicolini, "Optimal Unemployment Insurance," *Journal of Political Economy*, LV (1997), 412-438.
- Hugget, Mark, "The Risk Free Rate in Heterogeneous-Agent, Incomplete Insurance Economies," *Journal of Economic Dynamics and Control*, XVII (1993), 953-969.
- Kehoe, Timothy, and David Levine, "Liquidity Constrained vs. Debt Constrained Markets," *Econometrica*, LXIX (2001), 575-598.
- Kocherlakota, Narayana, "Implications of Efficient Risk Sharing without Commitment," *Review of Economic Studies*, LXIII (1996), 595-610.
- \_\_\_\_\_, "Simplifying Optimal Unemployment Insurance: The Impact of Hidden Savings," mimeo, Stanford University, 2003.
- \_\_\_\_\_, "Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation," *Econometrica*, LXXIII (2005), 1587-1621.
- Krueger, Dirk, and Fabrizio Perri, "Risk Sharing: Private Insurance Markets or Redistributive Taxes?," mimeo, University of Pennsylvania, 2001.
- Ljungqvist, Lars, and Thomas J. Sargent, *Recursive Macroeconomic Theory*, (Cambridge, MA: MIT Press, 2000).
- Mirrlees, James, "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, XXXVIII (1971), 175-208.
- \_\_\_\_\_, "Optimal Tax Theory: A Synthesis," *Journal of Public Economics*, VI (1976), 327-358.
- Phelan, Christopher, "Incentives and Aggregate Shocks," *Review of Economic Studies*, LXI (1994), 681-700.
- \_\_\_\_\_, "Repeated Moral Hazard and One-Sided Commitment," *Journal of Economic Theory*, LXVI (1995), 488-506.

- Prescott, Edward C., and Robert Townsend, “Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard,” *Econometrica*, LII (1984), 21-45.
- Rey, Patrick, and Bernard Salanie, “On the Value of Commitment with Asymmetric Information,” *Econometrica*, LXIV (1996), 1395-1414.
- Thomas, Jonathan, and Tim Worrall, “Income Fluctuation and Asymmetric Information: An Example of a Repeated Principal-Agent Problem,” *Journal of Economic Theory*, LI (1990), 367-390.
- Wang, Cheng, and Stephen Williamson, “Unemployment Insurance with Moral Hazard and a Dynamic Economy,” *Carnegie-Rochester Conference Series on Public Policy*, XLIV (1996), 1-41.
- Werning, Ivan, “Optimal Dynamic Taxation,” mimeo, University of Chicago, 2001.
- \_\_\_\_\_, “Optimal Unemployment Insurance with Unobservable Savings,” mimeo, University of Chicago, 2002.

## Notes

1. See also Hopenhayn and Nicolini [1997], Werning [2002], Albanesi and Sleet [2006], Golosov and Tsyvinski [2006], Kocherlakota [2005].
2. See also Bisin and Guaitoli [2004] and Bisin and Rampini [2006].
3. One example of the production function is  $F(K, Y) = f(K, Y) + (1 - \delta)K$  where  $\delta$  is a depreciation rate.
4. This setup has two interpretations. One interpretation is that the planner controls the consumption of an agent directly, and an agent consumes goods that the planner allocates to him. Under the other interpretation, an agent is able to enter observable contractual agreements with other agents and trade various assets with them. The consumption allocations that the social planner allocates can be conditioned on these trades. Since we impose no restrictions on

the allocations, the social planner can make any additional contractual agreements unappealing to the agent such that he consumes  $c(\sigma(\theta^t))$  after each history  $\theta^t$ .

5. It is common to refer to this notion of constrained efficiency as “second best,” indicating that the planner faces constraints of unobservability of agents’ types.

6. Alternatively, we could assume that firms rent capital from workers. Then the contract would also specify the amount of savings of each agent. Our results are the same in this case.

7. For models with one-sided commitment see, for example, Phelan [1995]. Rey and Salanie [1996] study contracts that are renegotiable but cannot be broken before they expire.

8. Sometimes unobservability of trades is called “non-exclusivity” to stress the fact that the agents are not constrained to trade exclusively with one single partner - be it an insurance company in competitive equilibrium or the planner/mechanism designer in the definition of constrained efficient allocations.

9. The retrading market is a constraint on the social planner’s problem. The idea is similar to that in Hammond [1987] who studied a static environment with multiple goods and side markets where agents can trade unobservably. He showed that for any incentive compatible allocation, side markets must be in a Walrasian equilibrium. Guesnerie [1998] used that set up to study optimal taxation in static contexts.

10. This constrained efficient allocation may be called “third best” indicating that it has constraints that both agents’ types and trades are not observable to the planner. This constrained efficient allocation can be contrasted with the constrained efficient allocation with observable consumption, “second best,” in which the planner only faces constraints of unobservable types but not trades.

11. It is easy to extend the definition of the competitive equilibrium to the case in which consumers trade with intermediaries in addition to trades among themselves on the private markets. In that case, we can reinterpret our model as allowing access to credit markets.

12. See also Greenwald and Stiglitz [1986] for a discussion how economies with private

information are similar to the economies with externalities. Arnott and Stiglitz [1990] discuss how unobservable insurance purchases create externality-like effects in static moral hazard models.

13. Also see da Costa [2004] for a similar result in a two period model with two types of agents.

14. The third possible deviation, for high type to claim to be low in both periods, can be shown to be non-binding because of the other two incentive constraints.

15. Crowding out of private markets by government policies also occurs in Attanasio and Rios-Rull [2000] and Krueger and Perri [2001] who study economies with limited commitment.

16. See also Bisin and Rampini [2006] for a model of optimal contracting and hidden lending in which agents can default on some of their obligations.

17. We thank Narayana Kocherlakota for suggesting the following very elegant inductive argument that simplified our original proof.

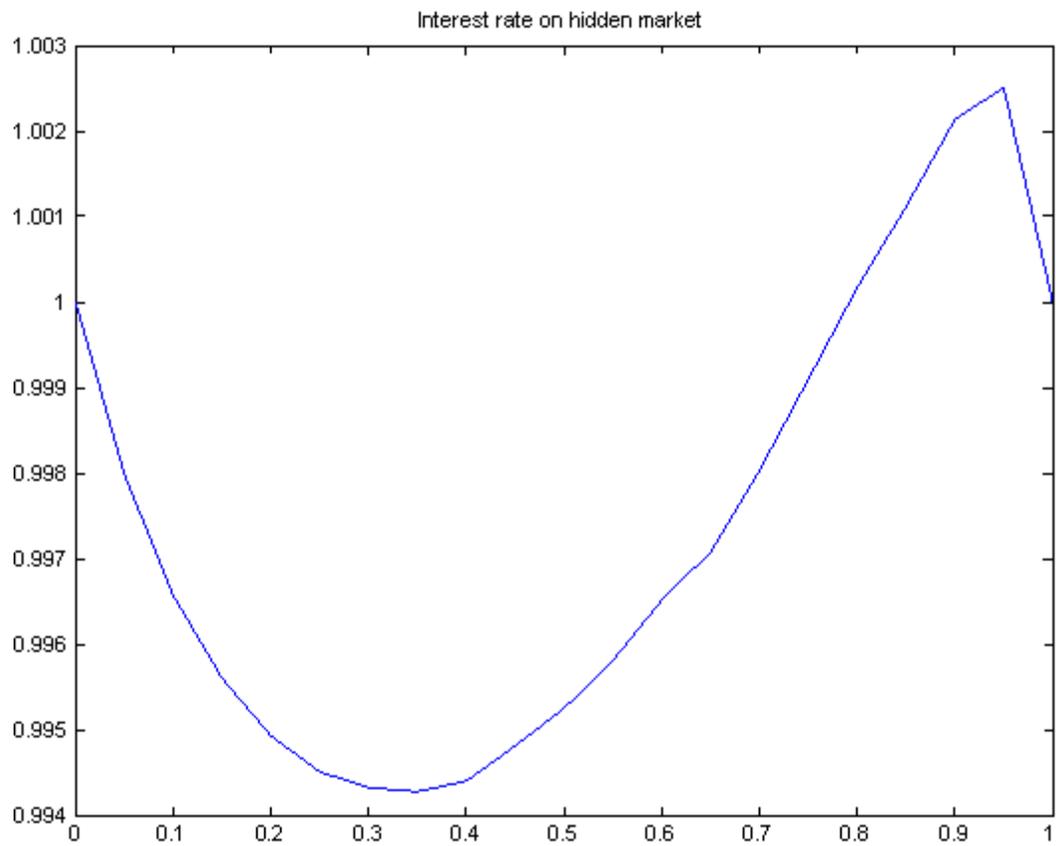


Figure I: Interest rate on hidden market

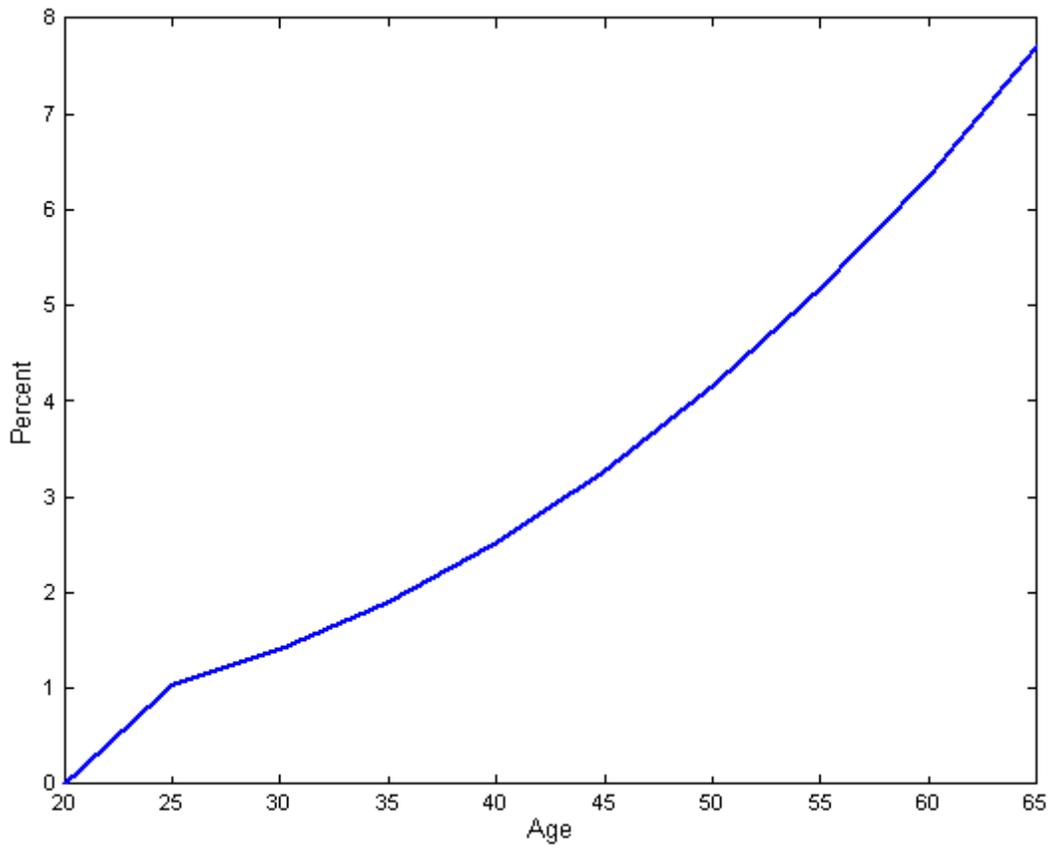


Figure II: Savings wedge when consumption is observable.

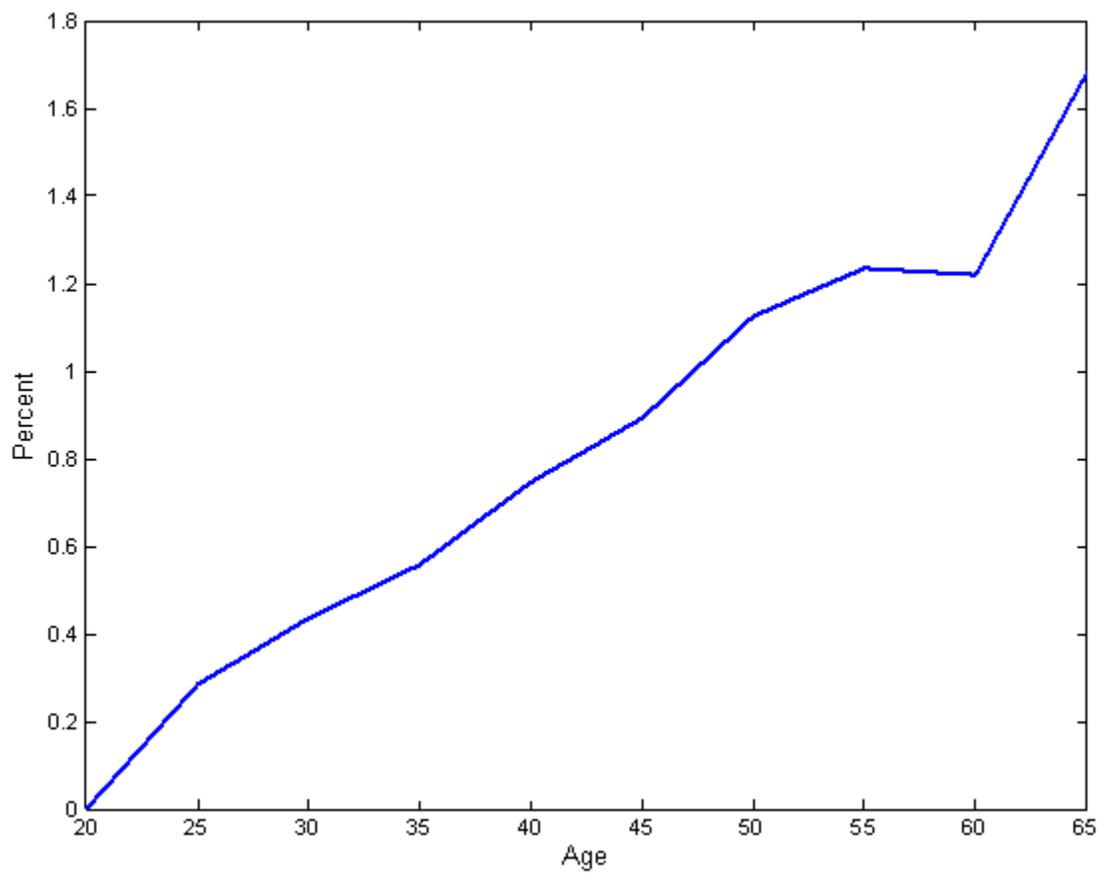


Figure III: Savings wedge when consumption is unobservable.