Cleansing by Tight Credit: Rational Cycles and Endogenous Lending Standards*

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Abstract

Endogenous cycles emerge through the two-way interaction between lending standards and production fundamentals. When lenders choose credit quantity over quality, the resulting lax lending standards lead to low interest rates and high output growth but the deterioration of future loan quality. When the quality is sufficiently low, lenders switch to tight standards, causing high credit spreads and low growth but a gradual improvement in the quality of loans. This eventually triggers a shift back to a boom with lax lending, and the cycle continues. As such, credit standards play a dual role. If they help the economy through promoting loan quantity today, they hurt future loan quality, and vice-versa. Investors don't internalize either role, thus, the constraint efficient economy features both a static and a dynamic externality, and albeit often being cyclical, it differs from the decentralized equilibrium.

JEL codes: D82, E32, E44, G01, G10
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1 Introduction

A growing body of empirical evidence suggests that fluctuations in credit supply are strongly related to economic cycles. Booms often correspond to overheated credit markets with low interest rates and increased total quantity of credit, but deteriorating quality of newly issued credit. In the subsequent recessions, credit turns scarce and expensive even for ex-post high-quality investment. The recession finally turns into a boom, and the cycle continues. A major conundrum for policy makers and academics alike is how economic policy should respond to this phenomena. For this, it is essential to understand the mechanism which governs the transition between booms and recessions.

We provide a model where the interaction between the credit market and production fundamentals generates cycles. Booms are periods of positive growth, lax lending standards and abundant credit. Lax lending fosters good investment, but also leads to a gradual deterioration of loan quality during the boom, which in turn triggers lenders to tighten their lending standards, and causes a recession. In recessions, output collapses, growth is negative, and productive investment slows down: Economic activity is *stifled by tight credit.* On the other hand, tight lending induces high credit quality, prompts entry of healthier firms, and stimulates growth: Economy is *cleansed by tight credit.* That is, lending standards play a dual role.

We further show that although the constrained optimal economy tends to be cyclical, it generically differs from the equilibrium cycle. For instance in recessions lenders fail to internalize both the static stifling role, the negative externality, and the dynamic cleansing role, the positive externality, of tight lending standards. This often culminates in excessively long booms followed by exceedingly deep recessions. We evaluate the efficacy of various policy instruments in steering the economy towards higher welfare cycles.

In this economy firms borrow on the credit market to produce, but only some of them pay back. Most investors cannot distinguish between good, creditworthy and bad, not-creditworthy firms. However, investors have access to an assessment technology that can imperfectly reveal firm type. A *bold test* approves the credit application of all good firms along with some bad ones. A *cautious test* on the other hand rejects some good applications along all the bad ones. Thus, the bold test implies lax lending standards while the cautious test implies tight lending. As such, investors face a quantity-quality trade-off: tight lending standards improve the quality but decrease the quantity of the credit issued by an investor.

When there are few bad firms among borrowers, investors optimally choose lax lending standards and the credit market exhibits symptoms of overheating. A mixed quality of credit is issued at a low interest rate, which induces high credit growth and high output. At the

same time, availability of credit to bad firms enables them to expand and multiply, leading to the deterioration of borrower quality in future periods. When the average borrower quality sufficiently deteriorates, lenders rationally switch to tight standards. Tight lending coincides with high credit spreads, which dampens bad investment but also slows down some good firms. Put differently, tight standards not only suppress lending to non-creditworthy firms, but also harm some creditworthy ones, leading to a downturn where the quantity of good investment is stifled. However, the issued credit is of high quality. Thus the pool of credit applications improves, eventually triggering a shift back to lax lending standards. And the cycle continues.

As such, the information that investors choose to acquire and use as the basis for their lending decision implies that lending standards have a dual role: tight lending standards stifles good investment today, but cleanse the economy of bad projects in the future. On the other hand, lax lending standards enable good investment to thrive today, but sow the seeds of bad future investment. The two-way interaction between the choice of lending standards and the fundamentals of the economy sustains permanent endogenous cycles.

The model gives rise to a variety of cyclical behaviors depending on the underlying parameters. Often there are long booms interrupted by short recessions, akin to the usual US business cycle patterns. Alternatively, the cycle can feature a prolonged recovery period or a double-dip recession. The model can also rationalize extended periods of mediocre growth such as Japan's "Lost Decade" of 1991-2001.

We then use the model to study the welfare properties of the equilibrium cycle. Recessions exhibit the static welfare loss of low output, and slowdown of productive investment. However, they also have the dynamic welfare gain of cleansing by tight credit. Investors fail to internalize the effect of their individual choice of lending standards on the concurrent good credit quantity, a negative externality, as well as the effect on the future loan quality, a positive externality. In other words, investors ignore both the stifling and cleansing roles of tight lending standards in recessions. The opposite pattern characterizes booms. As such, the constrained optimal cycle often features shorter booms and milder recessions.

We further connect the constrained optimal economy to realistic monetary and macro-prudential policies. We show that to improve welfare, investors' choice of lending standards can be appropriately managed either by controlling the risk-free rate through monetary policy or by specifying capital requirements using a macro-prudential policy. However, the welfare gain comes at the expense of increasing the average cost of capital. This trade-off determines the ranking across policies. Under our representation, we show that macro-prudential and counter-cyclical monetary policy both strongly dominate a non-state contingent monetary policy. The counter-cyclical monetary policy can improve welfare slightly more than the risk-

weighted capital requirements, however, the former requires a more sophisticated regulator.

Finally, we show that the predictions of our model are consistent with a wide range of stylized facts regarding the role of credit in economic cycles. In particular, our mechanism is in line with the accumulating evidence that tight lending standards often have a crucial cleansing role in recessions by laying the groundwork for healthier subsequent booms (Caballero et al., 2008; Blattner et al., 2019; Gropp et al., 2020). Furthermore, our model generates the observed deterioration of credit quality in booms and the fact that exit rates in recessions tend to be higher among firms with poor credit (Leibovici et al., 2019; Demyanyk and Van Hemert, 2009; Palmer, 2015), as well as the strong negative correlation between the fraction of low-quality issued credit and subsequent returns (Greenwood and Hanson, 2013; López-Salido et al., 2017; Greenwood et al., 2020). At a broader level, our model emphasizes that credit cycles are driven by endogenous fluctuations of credit supply (Becker and Ivashina, 2014).

Literature To the best of our knowledge, our paper is the first to formalize the positive and negative externalities implied by the dual role of lending standards across booms and recessions. In particular, while both the equilibrium and constraint efficient outcomes are cyclical, they do not coincide. From a technical standpoint, we demonstrate that the rich two-sided heterogeneity in entrepreneur and investor types is crucial to capture this mechanism.

The paper contributes to a few strands of literature. First, it belongs to the growing body of literature on dynamic lending standards. In this literature, lenders' choice to acquire information about borrowers differs in booms and in recessions (Martin, 2005; Gorton and Ordonez, 2014; Asriyan and Vanasco, 2014; Gorton and Ordonez, 2016; Hu, 2017; Fishman et al., 2019; Asriyan et al., 2021).Gorton and Ordonez (2016) and the contemporaneous paper of Fishman et al. (2019) are the closest to our work. Similar to our model, the mechanism in Fishman et al. (2019) relies on the two-way interaction of lenders' information choice and borrowers' average quality. However, unlike our paper, their economy does not feature endogenous cycles. This is a common feature of most of the papers in this literature, with the exception is Gorton and Ordonez (2016). This paper has two long-run equilibria: a good steady state and an equilibrium that cycles between multiple periods in the good state and one in the bad one. Unlike our model, in this cyclical equilibrium, recessions and the corresponding tight lending standards have no welfare benefit. As such, the only constraint optimal outcome is a steady state with permanent lax lending standards. In our setup on the other hand, a planner often prefers a cyclical economy to a persistent boom, as tight lending standards during downturns have a dynamic cleansing role. Furthermore, a rich set of cycles with different properties emerge in our framework. Finally, in this literature good

firms predominantly benefit from information production in recessions. Alternatively, in our model a large set of creditworthy firms are stifled by tight credit due to investors being cautious in recessions, and making false negative mistakes.

The idea of the cleansing role of recessions goes back to Schumpeter (1939). A number of papers discuss this effect in the labor market (Caballero and Hammour, 1994, 1996; Haltiwanger et al., 2021). There is also a literature arguing that lax lending in the credit market harms growth during booms as it leads to misallocation. A number of theoretical explanations include lenders' unwillingness to terminate inefficient lending relationships due to existence of sunk costs (Dewatripont and Maskin, 1995), impairment of information production in booms (Asriyan et al., 2021), refinancing inefficient projects due to soft budget constraints close to regulatory minimum (Caballero et al., 2008), and gambling for resurrection. We provide an alternative explanation for how lax lending leads to deterioration of pool of borrowers in booms and prompts recessions with tight lending, which then improves the loan quality, leads back to a boom, and endogenously sustains the cycle. We believe this is a particularly powerful mechanism as it can simultaneously explain the transition between booms and recessions and vice-versa.

Our paper also contributes to the literature on endogenous credit cycles (Azariadis and Smith, 1998; Matsuyama, 2007; Myerson, 2012; Gu et al., 2013). These papers present different mechanism that leads to endogenous fluctuations in granted credit quantity. However, none of them capture the interdependence of investors choice of lending standards and economic activity.[1]

This paper is also connected to the literature on collateral based credit cycles (Kiyotaki and Moore, 1997; Lorenzoni, 2008; Mendoza, 2010; Gorton and Ordonez, 2014; Asriyan et al., 2021). As in these papers, we are also interested in how a change in credit availability induces boom and busts. However, these papers focus on how exogenous shocks are amplified by the effect through the price of the collateral. In our model the price of collateral or exogenous shocks play no role.

Finally, there is a literature connecting overheated credit markets to extrapolative expectations (Bordalo et al., 2018; Greenwood et al., 2019; Gennaioli and Shleifer, 2020). Our work is complementary to this literature as we demonstrate that certain features of overheated credit markets are consistent with the rational choice of lax lending standards.

From a methodological perspective, the structure of the credit market builds on Kurlat (2016) and Farboodi and Kondor (2018). Neither of these papers focus on endogenous

[1]In the search literature, a number of mechanisms have been suggested to explain the emergence of endogenous cycles. For instance, see Burdett and Coles (1998) for equilibrium and Shimer and Smith (2001) for optimal cycles.

economic cycles.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 and 4 characterize the static and dynamic equilibrium. Sections 5 and 6 discuss welfare and optimal policy, respectively. Section 7 provides empirical evidence supporting the implications of the model. Section 8 extends the model to accommodate stochastic cycles by introducing aggregate shocks. Finally, section 9 concludes.

2 Model

Time is discrete and infinite. Each day is divided into two parts: morning and evening. There is single perishable good. It can be consumed, invested, or stored at a rate of return $1 + r_f$ between morning and evening.[2] There are two types of agents, entrepreneurs and investors. Each agent is risk-neutral and endowed with one unit of the good in the morning.

Entrepreneurs. There is measure one of entrepreneurs and each one has a two-dimensional type. He is either good or bad, $\tau = g, b$, and either opaque or transparent, $\omega = 0, 1$. Let $\mu_{0,t}$, $\mu_{1,t}$, $\nu_{0,t}$ and $\nu_{1,t}$ denote the measure of opaque bad, transparent bad, opaque good and transparent good entrepreneurs at time t, respectively, with $\mu_{0,t} + \mu_{1,t} + \nu_{0,t} + \nu_{1,t} = 1 \; \forall \; t$. Entrepreneurs know their own type and maximizes their life-time utility. Each period, an entrepreneur (τ, ω) is endowed a unit of capital and a project. At time t, he obtains credit $\ell_t(\tau, \omega)$ at interest rate $r_t(\tau, \omega)$ and invests $i_t(\tau, \omega)$ in the morning and consumes in the evening. Each unit of investment in the morning produces $\rho > 1 + r_f$ the same evening.[3] The cost of investment has to be covered by the entrepreneur's initial endowment or credit, implying the following budget constraint

$$i_t(\tau, \omega) = 1 + \ell_t(\tau, \omega). \tag{1}$$

Furthermore, each entrepreneur has to pledge his investment as collateral to obtain credit. Seizing the collateral is the only threat to enforce repayment from borrowers, thus $(1 +$

[2] r_f can represent a physical return or a policy rate. In sections 3 and 4, we think of it as the rate of return on the storage technology, which can be normalized to zero. In section 5 we reintroduce r_f as the return on a risk-free asset provided by the policy maker.

[3] We have also solved the model under the alternative assumption that good (bad) investment returns $\rho_g > 1 + r_f$ ($\rho_b < 1$). The expressions are more complex without providing further intuition. Therefore, we have decided to use $\rho = \rho_b = \rho_g > 1 + r_f$. The more general solution is available in the previous circulated versions of the paper, as well as available upon request.

$r_t(\tau,\omega))\ell_t(\tau,\omega) \leq i_t(\tau,\omega)$. Using (1) this simplifies to

$$\ell_t(\tau,\omega) \leq \frac{1}{r_t(\tau,\omega)}. \tag{2}$$

The key friction of the model is that investors cannot seize the investment undertaken by bad entrepreneurs. As such, bad entrepreneurs do not pay back, i.e. they are not creditworthy. However, investors only have imperfect information about entrepreneur type. That is, if an investor can observe the type of an entrepreneur, she lends only to a good one, as repayment from bad entrepreneurs cannot be enforced.

At the end of each period, some entrepreneurs exit the market ('die'). An entrepreneur exits either because he is hit by an exogenous shock with probability δ, or because he has not been able to raise credit. Thus, we assume that credit is essential for survival. Entrepreneurs discount future to reflect the exit rate. When an entrepreneur exit, he is replaced with a newborn so as to keep the population fixed at 1. The type distribution of the new entrants is fixed. λ $(1-\lambda)$ of new entrants are bad (good), and $\frac{1}{2}$ $(\frac{1}{2})$ are transparent (opaque). The two dimensions of the type distribution of entrants are independent.

Investors. There are two types of investors. A small, w_1, measure of investors are skilled, while a large, w_0, measure are unskilled. Skill is privately observable. Each investor in endowed with one unit of capital. As such, measure of investors also represents their capital. We use h to index individual investors.

Each investor lives for one period and maximizes her period utility. She makes a portfolio decision in the morning and consumes and dies in the evening. A dead investor is replaced by the same type of investor the next day. A portfolio decision involves extending credit to entrepreneurs and/or storing part of their unit endowment until the evening.

Each investor chooses to participate in or stay out of the lending market. Skilled investors observe the type of each entrepreneur. Participating unskilled investors only observe imperfect signals of the type of entrepreneurs in the sample that they receive instead. These signals are generated by a test of the investor's choice. Each investor can opt for a *bold test* or a *cautious test*. We call the former a bold investor, and the latter a cautious investor. The fixed utility cost of any test is $c \in (0,1)$, and each unskilled investor runs exactly one type of test.

The tests differ in the signal they generate for opaque entrepreneurs. The bold test pools all opaque entrepreneurs, good or bad, with transparent good ones (a false positive error). The cautious test pools all opaque entrepreneurs with transparent bad ones (a false negative

error).[4] Intuitively, one can envision the bold test to reject projects of transparent bad entrepreneurs only and pass all other ones, while the cautious test passes only projects of transparent good entrepreneurs. When an investor is indifferent between the two tests, we break the tie by assuming that she chooses the bold test.

The size of the sample that an unskilled investor tests is limited by her unit endowment. She can test only as many applications as she could finance.

Credit Market. The credit market operates in the morning. After each unskilled investor chooses the type of her test, each participating skilled and unskilled investor advertises an interest rate, $\tilde{r}(h)$, at which she is willing to extend loans. Each entrepreneur chooses the measure of loan applications $\sigma(r;\tau,\omega) \in [0, \frac{1}{r}]$ he wishes to submit at each interest rate r. The credit market clears starting from the lowest interest rate. At each interest rate, the unskilled investors sample first.

We assume that there is no credit history for entrepreneurs. That is, investors cannot learn from the past.[5] Furthermore, in order to keep the problem analytically tractable we assume there is no saving technology available across periods. Therefore, entrepreneurs consume their wealth at the end of each period and if they survive, they start the new period with the unit endowment received in the morning. Moreover, we make the following assumption about skilled and unskilled investor wealth.

Assumption 1 *Skilled and unskilled investor capital w_1 and w_0 are such that*

(i) Skilled investor capital, w_1, is scarce. In particular, it is not sufficient to cover the credit demand of all opaque good entrepreneurs at any interest rate that any good entrepreneur is willing to borrow at.

(ii) Unskilled investor capital, w_0, is abundant. In particular, it covers the credit demand of all entrepreneurs that unskilled investors are willing to lend to at any equilibrium interest rate.

The formal optimization problem of investors and entrepreneurs, as well as further details on collateralization and market clearing protocol are stated in Appendix A. We next define the equilibrium within each period followed by the full dynamic equilibrium of the economy.

[4]For simplicity we restrict an investor's choice set to these two tests. In appendix D we enrich the model and allow the investors to choose among the continuum of tests lying between the bold and cautious tests. We prove that the dominant choice is always one of the extremes. Thus, this assumption is not restrictive.

[5]In Appendix E we relax this assumption by introducing a probabilistic signal of past defaults akin to real-world credit scores. We show that our main mechanism remains intact.

Equilibrium Definition. We focus on stationary equilibria, i.e. equilibria where the state variables are members of a finite ergodic set in the long run. We conjecture that in any stationary equilibrium, at each time t, $(\mu_{0,t}, \mu_{1,t})$ are sufficient statistics forthe entrepreneur type distribution, and in the stationary distribution, the corresponding measures of opaque and transparent good entrepreneurs are equal to $\nu_0 = \nu_1 = \frac{1-\mu_0-\mu_1}{2}$. We will prove this conjecture later.

Definition 1 (Stage Game Equilibrium) *For a fixed (μ_0, μ_1), the stage game equilibrium consists of entrepreneurs' investment schedule $i(\tau, \omega)$ and credit demand schedule $\sigma(r; \tau, \omega)$, investors' advertised interest rate schedule $\tilde{r}(h)$ and unskilled investors' choice of test, equilibrium interest rate schedule $r(\tau, \omega)$, equilibrium credit allocation schedule to entrepreneurs $\ell(\tau, \omega)$, and equilibrium allocation of applications to investors such that*

(i) each agent's choice maximizes the agent's stage game utility given the strategy profile of other agents, equilibrium interest rates, and allocations,

(ii) the implied interest rate schedule $r(\tau, \omega)$, credit allocation schedule for entrepreneurs $\ell(\tau, \omega)$, and allocation of applications to investors are consistent with agents' choices and the market clearing process.

Definition 2 (Dynamic Equilibrium) *The dynamic equilibrium consists of an infinite sequence of $\{(\mu_{0,t}, \mu_{1,t})\}_{t=0}^{\infty}$, individual entrepreneurs' $i_t(\tau, \omega)$ and $\sigma_t(\tau, \omega, r)$, individual investors' $\tilde{r}_t(h)$ and unskilled investors' choice of test, equilibrium $r_t(\tau, \omega)$, $\ell_t(\tau, \omega)$ and allocation of applications to investors, all within each period, such that*

(i) there exists a finite κ and a stable invariant set $\{(m_{0,i}, m_{1,i})\}_{i=1}^{\kappa}$ such that if $(\mu_{0,t}, \mu_{1,t}) = (m_{0,i}, m_{1,i})$ then

$$(\mu_{0,t+1}, \mu_{1,t+1}) = \begin{cases} (m_{0,i+1}, m_{1,i+1}) & if \quad i < \kappa \\ (m_{0,1}, m_{1,1}) & if \quad i = \kappa, \end{cases}$$

and $\nu_{0,t} = \nu_{1,t} = \frac{1-\mu_{0,t}-\mu_{1,t}}{2}$,

(ii) the dynamics of $(\mu_{0,t}, \mu_{1,t})$ are consistent with the birth-death process of entrepreneurs.

(iii) each agent's choice maximizes the agent's life-time utility given the strategy profile of other agents, equilibrium interest rates and allocations,

(iv) in each period t, the implied interest rate schedule $r_t(\tau, \omega)$, credit allocation schedule for entrepreneurs $\ell_t(\tau, \omega)$, and allocation of applications to investors are consistent with agents' choices and the market clearing process.

The dynamic equilibrium nests both a steady state and a cycle. If $\kappa = 1$, it is a standard steady-state equilibrium. When $\kappa > 1$, it is a *cyclical dynamic equilibrium* as it features a stable cycle of length κ.[6] In the next section, we start by describing the stage game equilibrium, and then show that each dynamic equilibrium is a sequence of stage game equilibria.

3 Stage Game Equilibrium

In order to analyze the stage game we fix the entrepreneur type distribution, summarized by state variables (μ_0, μ_1), and characterize the equilibrium in the credit market and in real outcomes within each period.

3.1 Credit Market

The following two lemmas describe the properties of entrepreneurs' and investors' optimal strategies in the credit market. The first lemma characterizes entrepreneurs' credit demand as a "threshold strategy," while the second lemma describe investors' financing decision.

Lemma 1 *Entrepreneurs' credit demand schedule $\sigma(r;\tau,\omega)$ is as follows.*

(i) Entrepreneur (τ,ω) chooses a reservation interest rate $r^{max}(\tau,\omega)$. He submits maximum demand, $\sigma(r;\tau,\omega) = \frac{1}{r}$ to all $r \leq r^{max}(\tau,\omega)$ and zero demand to all $r > r^{max}(\tau,\omega)$.

(ii) Good entrepreneurs never choose a reservation rate higher than $\bar{r} \equiv \rho - 1$, while bad entrepreneurs never choose a reservation rate lower than $\bar{r} \equiv \rho - 1$.

Lemma 2 *Each unskilled investor who participates in the lending market only extends loans to entrepreneurs who pass her test.*

A critical implication of Lemma 2 is that the choice of the test maps to different lending standards. A bold investor extends loans to all opaque entrepreneurs along with the transparent good ones. In other words, she applies *lax lending standards*. In contrast, a cautious investor applies *tight lending standards* as she issues credit to good (transparent) applicants only. The following lemma introduces interest rates that we will use in our first characterization result.

[6] As it is clear from the equilibrium definition, most of our formal analysis focuses on deterministic cycles for simpler intuition and analysis. In Section 8, we introduce an extension which leads to the replacement of deterministic cycles with more realistic stochastic ones.

Lemma 3 *There exist an interest rate* $r_B(\mu_0, \mu_1, c, r_f) > 0$ $\left(r_C(\mu_0, \mu_1, c, r_f) > 0\right)$ *at which an unskilled investor who runs a bold (cautious) test and receives a representative sample of all the entrepreneurs breaks even.* $r_B(\cdot)$ *and* $r_C(\cdot)$ *are both increasing in* μ_0*, and* $r_B(\cdot) < r_C(\cdot)$ *if and only if* $\mu_0 \in [0, \frac{c}{1+r_f})$.

Moreover, there exist an interest rate $r_M(\mu_0, \mu_1, c, r_f) > 0$ *at which an unskilled investor who runs a bold test and receives a representative sample of all bad and opaque good entrepreneurs breaks even.* $r_M(\cdot)$ *is increasing in* μ_0*,* $r_B(\cdot) < r_M(\cdot)$ *for all* μ_0 *and* $r_M(\cdot) < \bar{r}$ *if and only if* $\mu_0 < \tilde{\mu}_0(\mu_1, c, \rho, r_f)$ *implicitly defined by* $r_M(\tilde{\mu}_0(\mu_1, c, \rho, r_f), \mu_1, c, r_f) \equiv \bar{r}$.

The dashed curves on the left panel of Figure 1 illustrate these interest rates, which we use in Proposition 1. This proposition is the first key result of the paper and characterizes the credit market equilibrium in the stage game.

Proposition 1 *When* $\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r}$,

(i) If $\mu_0 \in [0, \frac{c}{1+r_f}]$*, the economy is in a* **bold phase**. *Every unskilled investor chooses the bold test. The credit market is integrated. All good entrepreneurs and opaque bad ones obtain credit at common interest rate* $r_B(\mu_0, \mu_1, c, r_f)$.

(ii) If $\mu_0 \in (\max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}, 1]$*, the economy is in a* **cautious phase**. *Every unskilled investor chooses the cautious test. The credit market is fragmented. Transparent and opaque good entrepreneurs obtain credit at rates* $r_C(\mu_0, \mu_1, c, r_f)$ *and* $\bar{r} > r_C$*, respectively. Bad entrepreneurs do not obtain any credit.*

(iii) If $\mu_0 \in (\frac{c}{1+r_f}, \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}]$*, the economy is in a* **mix phase**. *Some unskilled investors choose the bold test while others choose the cautious test. The credit market is fragmented. Transparent good entrepreneurs obtain credit at rate* $r_C(\mu_0, \mu_1, c, r_f)$*, while all opaque entrepreneurs obtain credit at rate* $r_M(\mu_0, \mu_1, c, r_f) > r_C$.

Otherwise the economy is in autarky, where unskilled investors do not lend, bad entrepreneurs do not borrow, and good ones obtain credit at interest rate $\bar{r}$ *from skilled investors only.*

In a bold phase, the credit market is integrated with interest rate $r_B(\cdot)$ at which unskilled investors break even by lending to all good and opaque bad entrepreneurs. Since every good entrepreneur passes the bold test and borrows at this rate, skilled investors cannot attract any of them at a higher rate. No investor is willing to lend at a lower rate either. This confirms that there is a single prevailing market interest rate at which all good entrepreneurs and some bad ones raise funding from both skilled and unskilled investors. Skilled investors still make positive profits as they lend only to good entrepreneurs at a positive interest rate.

Alternatively, in a cautious phase the credit markets are fragmented. Unskilled cautious investors break even by lending at relatively low rate $r_C(\cdot)$ only to transparent good entrepreneurs. However, they reject opaque good entrepreneurs who can then be attracted by skilled investors at a higher interest rate. Since skilled capital is in short supply, the corresponding interest rate will be the highest rate that a good entrepreneur is willing to pay, $\bar{r}$.

In a mix equilibrium the credit market is fragmented as well. Unskilled investors are indifferent between being bold and cautious while breaking even. The cautious ones break even by only lending to transparent good entrepreneurs at low interest rate $r_C(\cdot)$. On the other hand, the bold ones break even by lending at a higher interest rate $r_M(\cdot)$ to both opaque good and bad entrepreneurs. Skilled investors lend at the same high rate $r_M(\cdot)$, but to good entrepreneurs only.

Since unskilled capital is abundant, unskilled investors break even in every phase. They lend at an interest rate that makes them indifferent between paying cost c, running the test of their choice, and lending to the entrepreneurs who pass the test, versus using the storage technology and earning the risk-free rate. Since entrepreneurs have a preference for cheaper borrowing, investors choose the test that leads to a lower interest rate which in turn determines the phase, as described in Proposition 1 and depicted in Figure 1.

Investors' choice of the test follows a quality-quantity trade-off. This is most intuitive to see when $\tilde{\mu}_0(\mu_1) < \frac{c}{1+r_f}$ and the only possible stage game equilibria are bold and cautious phases. The bold test is the high quantity, low quality option. Many applicants pass the bold test, however, the resulting loan portfolio involves some defaults since projects of opaque bad entrepreneurs pass the test as well. Therefore, μ_0 is the *quality cost* of a bold test and $r_B(\cdot)$ has to compensate investors for adverse selection. On the other hand, the cautious test is low quantity but high quality. Only projects of good entrepreneurs pass the cautious test. This leads to a high quality loan portfolio which always pays back. However, the rejection rate is high since even some good entrepreneurs fail the test. As the cost of testing a unit measure of applications is fixed at c regardless of the scale of lending and a dollar not lent out earns the risk free return $1 + r_f$, $\frac{c}{1+r_f}$ is the *quantity cost* of a cautious test and $r_C(\cdot)$ has to compensate the investor for the excess rejections at the same cost of the test.

As such, when the quality of the pool of loan applications is high, $\mu_0 \leq \frac{c}{1+r_f}$, the quality cost of a bold test is low, $r_B(\cdot) < r_C(\cdot)$, and a bold phase is realized. In this region investors are more concerned about losing out on good entrepreneurs. Thus lending standards are lax, and many entrepreneurs, including some bad ones, are able to raise financing at the same relatively low rate. On the other hand, if there are many bad entrepreneurs, investors are concerned about extending loans to the bad ones who will default, thus they require a

high $r_B(\cdot)$. It follows that in this region the quality cost of a bold test is higher than the quantity cost of a cautious test, $\mu_0 > \frac{c}{1+r_f}$, thus $r_C(\cdot) < r_B(\cdot)$ and a cautious phase realizes. Lending standards are tightened and credit market becomes segmented. Not only are bad entrepreneurs unable to raise financing, but even some good ones are able to do so only at extremely high rates.

Lastly, when $\tilde{\mu}_0(\mu_1) > \frac{c}{1+r_f}$ and the measure of opaque bad entrepreneurs is in an intermediate range, unskilled investors are indifferent between the two tests. Some apply lax and some tight lending standards such that they all break even. The credit market is still fragmented and a mix phase arises.

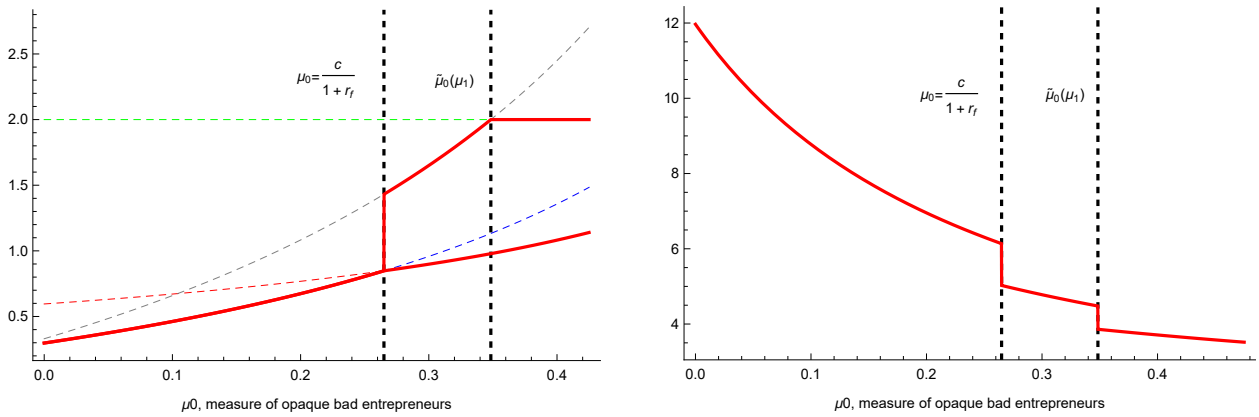
Figure 1a depicts the prevailing interest rates as a function of μ_0, the fraction of opaque bad entrepreneurs, for a particular level of μ_1. For this set of parameters all three phases arise for some μ_0. The left most region represents the bold phase, the intermediate region is the mix, and the rightmost region is the cautious phase.

It is worth mentioning that a bold phase exhibits several features of an overheated credit market. Interest rates are uniformly low and many entrepreneurs, including some bad ones, are financed. As such, the overall quality of initiated credit is low with a share of loans eventually defaulting. This is in contrast to the tight credit market in the cautious phase. Most importantly, the latter market is fragmented. Some good entrepreneurs (transparent ones) enjoy ample funding at relatively low interest rates while some other good entrepreneurs (opaque ones) can get only limited funding at very high rates. Furthermore, bad entrepreneurs are not funded at all. Therefore, although the total loan quantity is relatively low, its quality is high, which leads to high subsequent realized returns.

3.2 Investment and Output

In this section, we conclude the characterization of the stage game equilibrium by deriving the implied quantity of credit, investment and output for each phase. The critical observation is that the information friction influences the quantity of credit through two distinct channels: first, by increasing the prevailing rate and thus tightening the collateral constraint, and second by limiting supply of credit and rationing demand.

All good entrepreneurs in the bold phase and transparent good ones in the cautious phase are limited by the collateral constraint $\frac{1}{r(\tau,\omega)}$, while opaque bad entrepreneurs in the bold phase and opaque good ones in the cautious phase are restricted by the limited supply of capital and are rationed. The investment of entrepreneur (τ, ω) is given by $i(\tau, \omega) = 1+\ell(\tau, \omega)$ and his output is $y(\tau, \omega) \equiv \rho i(\tau, \omega)$. We will use the following two statistics to explore properties of output in our framework.



(a) Interest rates (b) Output

Figure 1: Interest rates and output as a function of μ_0, for a fixed μ_1. All three phases of stage game equilibrium occur for some μ_0. The left panel displays the break-even interest rates r_B (dashed blue), r_C (dashed red), r_M (dashed grey), the maximum feasible rate $\bar{r}$ (dashed green, horizontal), and the equilibrium interest rates (solid curves). The right panel displays the output. In the leftmost region the stage game equilibrium is bold, in the middle range it is mix, and in the rightmost region it is cautious. The parameters are: $\rho = 3, \lambda = 0.3, \delta = 0.55, c = 0.265, r_f = 0, w_0 = 5, w_1 = 0.01,\ \mu_1 = 0.11$.

Aggregate Output. Aggregate output in state (μ_0, μ_1) is given by

$$\begin{aligned} Y(\mu_0, \mu_1) &\equiv \frac{1-\mu_0-\mu_1}{2}\left(y(g,1)+y(g,0)\right) + \mu_1 y(b,1) + \mu_0 y(b,0) \\ &= \rho\left(1 + \frac{1-\mu_0-\mu_1}{2}\big(\ell(g,1)+\ell(g,0)\big) + \mu_0\ell(b,0)\right). \end{aligned} \tag{3}$$

In Equation (3), the terms in parenthesis correspond to different sources of investment. The first term is the endowment of all the entrepreneurs'. The second term is the outside financing raised by all the good entrepreneurs, and the last term is the outside financing raised by opaque bad entrepreneurs. Transparent bad entrepreneurs do not raise any outside financing.

Aggregate Output Growth. We define output growth in each period as the percentage difference between period output and initial capital of all agents,

$$g(\mu_0, \mu_1) \equiv \frac{Y(\mu_0, \mu_1)}{w_0 + w_1 + 1} - 1.$$

Given the OLG structure of the model and no inter-temporal transfer of resources, we believe this is the relevant measure of growth in our framework.

The main result of this section describes the equilibrium credit allocation within each

period, which in turn determines the investment and output.

Proposition 2

(i) In any equilibrium transparent bad entrepreneurs are not financed by any investors, $\ell(b,1)=0$.

(ii) In the bold phase, all entrepreneurs face interest rate $r_B(\cdot)$. *All good entrepreneurs borrow* $\ell(g,\omega)=\frac{1}{r_B}$. *Opaque bad entrepreneurs are limited by unskilled investors' false positives at interest rate* $r_B(\cdot)$, *implying* $\ell(b,0)=\frac{1}{r_B}-\frac{w_1}{1-\mu_0-\mu_1}$.

(iii) In the cautious phase, all transparent good entrepreneurs face interest rate $r_C(\cdot)$ *and borrow* $\ell(g,1)=\frac{1}{r_C}$. *Opaque good ones face* $\bar{r}$ *and are limited by the short supply of skilled capital, implying* $\ell(g,0)=\frac{2w_1}{1-\mu_0-\mu_1}$. *Opaque bad entrepreneurs are not financed,* $\ell(b,0)=0$.

(iv) In the mix phase, all transparent good entrepreneurs face $r_C(\cdot)$, *while opaque good ones face* $r_M(\cdot)$. *Neither are constrained by information frictions,* $\ell(g,1)=\frac{1}{r_C}$ *and* $\ell(g,0)=\frac{1}{r_M}$. *Opaque bad entrepreneurs are limited by unskilled investors' mistakes at interest rate* $r_M(\cdot)$, $\ell(b,0)=\frac{1}{2r_M}-\frac{w_1}{1-\mu_0-\mu_1}$.

(v) Aggregate output, $Y(\mu_0,\mu_1)$, *is decreasing in* μ_0 *and discontinuous at the threshold across any two phases.*

In a bold phase, all good entrepreneurs are fully financed at low interest rate $r_B(\cdot)$. Transparent bad entrepreneurs are excluded from the credit market. However, opaque bad ones obtain some credit since the bold test does not distinguish them from good entrepreneurs. Yet, their credit is limited by the mistakes of participating unskilled investors. Since all good entrepreneurs and even some bad ones raise credit at a low rate and invest, investment and output are high. Thus the bold phase tends to correspond to a "boom".

In a cautious phase transparent good entrepreneurs are financed by cautious unskilled investors at lower interest rate $r_C(\cdot)$. However, opaque good entrepreneurs can only obtain credit from skilled investors, limited by the restricted capital supply of these investors. We call this the *stifling role of tight credit*: good investment is dampened in a cautious phase. Furthermore, no bad entrepreneur can raise financing and bad investment contracts even more. As such, a cautious phase tends to correspond to a "downturn."

In a mix phase, transparent good entrepreneurs face the same credit conditions as a cautious phase, i.e. interest rate $r_C(\cdot)$. Opaque entrepreneurs, good and bad, borrow at a higher rate $r_M(\cdot)$, thus they are able to raise less credit. The credit raised by opaque bad entrepreneurs is further limited by the mistakes made by unskilled bold investors who lend

at rate $r_M(\cdot)$. Thus the output and investment is between that of the bold and cautious phases.

Figure 1b illustrates aggregate output as a function of μ_0, for a fixed μ_1. As we noted before, all three phases arise for this set of parameters. As part (v) of the proposition states, the aggregate output is continuous and monotonically decreasing in the measure of opaque bad entrepreneurs within each phase. The output decline is intuitive: within each phase, an increase in μ_0 (weakly) increases the equilibrium interest rates as the adverse selection problem worsens. This, in turn tightens the collateral constraint and suppresses investment and output. The output discontinuously drops when an increase in the fraction of opaque bad entrepreneurs leads to a change in phase.

4 Dynamic Endogenous Cycles

We next develop our main results on the cyclical dynamic behavior of the economy. We describe the defining features of the deterministic cycles that emerge under different conditions in both the credit market and the real economy.[7] Throughout, we use a *boom* or an *upturn* to refer to the times when output is high and output growth is positive. These real outcomes are accompanied by low yields in the credit market. Alternatively, a *bust*, *downturn*, or *recession* happens when output is low and output growth is negative. This is accompanied by a fragmented credit market.

We first establish that the dynamic equilibrium reduces to a sequence of stage games, established in the previous section.

Lemma 4 *In any dynamic equilibrium, the economy is in a stage game equilibrium in each period.*

This lemma demonstrates that maximizing life-time utility leads to the same outcome as maximizing stage game utility for each agent. We first argue that it is never optimal for an entrepreneur to take a loss in the stage game in a given period in order to increase his chances to obtain credit and, therefore, to survive to the next period.

The lemma further shows that at time t, the future dynamic path of the economy is independent of the past outcomes, conditional on all the variables that determine the stage equilibrium outcome at date t. We show in Propositions 1 and 2 that the credit and real outcomes in the stage equilibrium at date t are determined by $(\mu_{0,t}, \mu_{1,t})$. Thus a direct consequence of Lemma 4 is that $(\mu_{0,t}, \mu_{1,t})$ are sufficient state variables for the dynamic

[7] Section 8 presents an extension with aggregate shocks that leads to stochastic cycles.

economy. Furthermore, the lemma allows us to focus our discussion on the evolution of the state variables, determined by the prevalent lending standards in the previous period.

We first describe the law of motion for the state variables and then explain the emerging cycles. To ease the notation, we omit the time-subscript whenever it does not cause any confusion.

Law of Motion for State Variables. Let (μ_0, μ_1) and (μ_0', μ_1') denote the state variables today and tomorrow, respectively. When at least some investors are bold, only transparent bad entrepreneurs cannot raise financing. However, when all investors are cautious, opaque bad entrepreneurs are not financed either. Any entrepreneur who cannot raise financing exits and is replaced by a newborn. The next proposition summarizes the law of motion for measure of opaque and transparent bad entrepreneurs.

Proposition 3 *Assume* $\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r} = \rho - 1$ *so the economy is not in autarky.*

(i) If $\mu_0 \in \left[0, \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}\right]$*, then the law of motion for* μ_0 *and* μ_1 *follows*

$$\mu_{0B}'(\delta, \lambda, \mu_0, \mu_1) = (1-\delta)\mu_0 + \big(\delta + (1-\delta)\mu_1\big)\frac{\lambda}{2}, \tag{4}$$

$$\mu_{1B}'(\delta, \lambda, \mu_0, \mu_1) = \big(\delta + (1-\delta)\mu_1\big)\frac{\lambda}{2}. \tag{5}$$

(ii) If $\mu_0 \in \left(\max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}, 1\right]$*, then the law of motion for* μ_0 *and* μ_1 *follows*[8]

$$\mu_{0C}'(\delta, \lambda, \mu_0, \mu_1) = \big(\delta + (1-\delta)(\mu_0 + \mu_1)\big)\frac{\lambda}{2}, \tag{6}$$

$$\mu_{1C}'(\delta, \lambda, \mu_0, \mu_1) = \big(\delta + (1-\delta)(\mu_0 + \mu_1)\big)\frac{\lambda}{2}. \tag{7}$$

The laws of motion are intuitive. Consider the measure of opaque bad entrepreneurs, μ_0. When some investors are bold, function $\mu_{0B}'(\delta, \lambda, \mu_0, \mu_1)$ describes the evolution of μ_0. It consists of survivals from the current period, plus the newborns. From the existing opaque bad entrepreneurs, fraction $(1-\delta)$ survive. The newborns replace two groups of exiting entrepreneurs: δ measure of all entrepreneurs are exogenously replaced. Furthermore, the remaining transparent bad entrepreneurs cannot raise funding and are replaced as well. A

[8]Equations (6) and (7) govern the law of motion of the state variables if the economy is in autarky as well.

fraction $\frac{\lambda}{2}$ of newborns are opaque bad entrepreneurs. The law of motion for transparent bad entrepreneurs follows a similar intuition.

The law of motion for opaque and transparent good entrepreneurs are the same in both cases. The reason is two-fold: both groups always raise financing, and their measure among the newborns is the same. As such, in the long run both measures are equal to $\frac{1-\mu_0-\mu_1}{2}$. This validates that (μ_0, μ_1) are sufficient state variables for the economy despite four types of entrepreneurs.

If a single set of laws of motion were to govern the dynamic evolution of the state variables throughout, i.e. either equations (4)-(5) or (6)-(7), then (μ_0, μ_1) would converge to a steady state regardless of the initial conditions. This observation leads to the following Lemma, establishing conditions for the economy to converge to a long-run steady state.

Lemma 5 *Consider two pair of constants* $(\bar{\mu}_{0B}, \bar{\mu}_{1B})$ *and* $(\bar{\mu}_{0C}, \bar{\mu}_{1C})$ *such that*

$$\bar{\mu}_{0B} \equiv \frac{\lambda}{2-\lambda(1-\delta)}, \qquad \bar{\mu}_{1B} \equiv \frac{\lambda\delta}{2-\lambda(1-\delta)}$$
$$\bar{\mu}_{0C} \equiv \frac{\lambda\delta}{2-2\lambda(1-\delta)}, \qquad \bar{\mu}_{1C} \equiv \frac{\lambda\delta}{2-2\lambda(1-\delta)}.$$

For any λ *and* δ, $\bar{\mu}_{0B} > \bar{\mu}_{0C}$ *and* $\bar{\mu}_{1B} < \bar{\mu}_{1C}$. *Furthermore,*

(i) If $\bar{\mu}_{0B} \leq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\bar{\mu}_{1B})\}$, *then* $(\bar{\mu}_{0B}, \bar{\mu}_{1B})$ *is a bold steady state equilibrium.*

(ii) If $\bar{\mu}_{0C} \geq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\bar{\mu}_{1C})\}$, *then* $(\bar{\mu}_{0C}, \bar{\mu}_{1C})$ *is a cautious steady state equilibrium.*

$(\bar{\mu}_{0B}, \bar{\mu}_{1B})$ denotes the measure of opaque and transparent bad entrepreneurs in the steady states where investors are always bold. Similarly, $(\bar{\mu}_{0C}, \bar{\mu}_{1C})$ corresponds to the cautious steady state. Observe that $(\bar{\mu}_{0B}, \bar{\mu}_{1B})$ and $(\bar{\mu}_{0C}, \bar{\mu}_{1C})$ are the fixed points of equations (4)-(5) and (6)-(7), respectively. Lemma 5 simply states that if investors' optimal choice of the test is constant throughout time, then the economy converges to the corresponding steady state. Furthermore, the measure of opaque bad entrepreneurs in the bold steady state is higher than that of the cautious one, $\bar{\mu}_{0,B} > \bar{\mu}_{0,C}$, as the exit rate of opaque bad entrepreneurs is lower when investors are bold.

It is instructive to consider the quantity-quality trade-off to understand when the economy converges to a steady state. Recall that within each period, high μ_0 is the quality cost of a bold test, while $\frac{c}{1+r_f}$ is the quantity cost of a cautious test. The bold and cautious steady states correspond to the highest and lowest quality costs for a bold investor, respectively. If the quantity cost of being cautious is too high compared to even the highest quality cost, then investors choose to always be bold and a bold steady state emerges. Alternatively, if

this quantity cost is lower than the lowest quality cost, then investors choose to always be cautious and a cautious steady state emerges.

4.1 Credit and Output Cycles

Throughout the rest of the paper we focus on parameters where the conditions of Lemma 5 are violated and the dynamic equilibrium is cyclical, i.e. when the cost of testing is intermediate. Depending on the parameters, the economy admits a wide range of cyclical patterns where the two state variables cycle through a finite number of values in the long-run. We use the following two criteria to broadly classify the cycles. The first criterion is whether the cycle involves a mix phase or not. A *two-phase economy* is a cyclical economy which only consists of bold and cautious phases. Alternatively, a *three-phase economy* is one with all three phases, bold, mix, and cautious. The second criterion is whether the economy spends more time in the bold or cautious phase during the cycle.

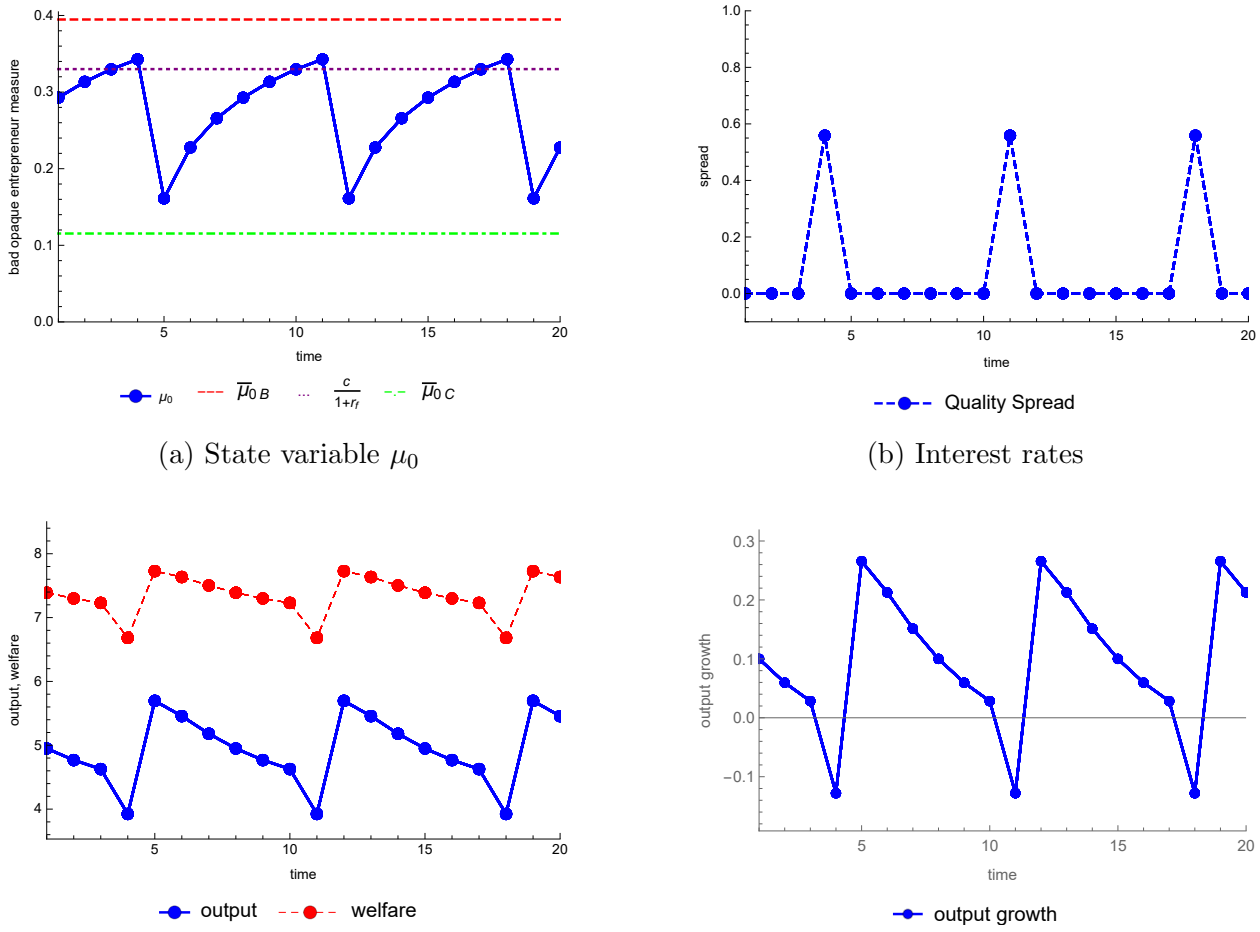
Two-phase Economy. From Lemma 5, a cyclical dynamic equilibrium arises when $\frac{c}{1+r_f} \in (\bar{\mu}_{0,C}, \bar{\mu}_{0,B})$. Moreover, Proposition 1 implies that an economy cycles through only bold and cautious phases if $\tilde{\mu}_0(\mu_1) \leq \frac{c}{1+r_f}$, for every realized μ_1. The next Proposition provides a more detailed description of the prevailing cycles as a function of $\frac{c}{1+r_f}$.

Proposition 4 *When $\frac{c}{1+r_f} \in (\bar{\mu}_{0C}, \bar{\mu}_{0B})$, for any λ and δ there exits constants $\mu^*_{0B} < \mu^*_{0C} \in (\bar{\mu}_{0C}, \bar{\mu}_{0B})$, such that if the prevailing cyclical dynamic equilibrium is a two-phase economy then*

(i) $\frac{c}{1+r_f} \in [\mu^*_{0B}, \mu^*_{0C})$ *implies a 2-period cycle with the two-point support (μ^*_{0B}, μ^*_{0C}). In the long-run, the economy oscillates between a one-period bold phase and a one-period cautious phase.*

(ii) $\frac{c}{1+r_f} \in [\mu^*_{0C}, \bar{\mu}_{0B})$ *implies a $\kappa > 2$ period long bold-short cautious cycle. The cycle consists of a long bold phase, $\kappa - 1$ consecutive periods where μ_0 increases, followed by a short cautious phase, a one period decline in μ_0. A larger $\frac{c}{1+r_f}$ implies a longer bold cycle.*

(iii) $\frac{c}{1+r_f} \in (\bar{\mu}_{0C}, \mu^*_{0B})$ *implies a $\kappa > 2$ period short bold-long cautious cycle. The cycle consists of a long cautious phase, $\kappa - 1$ consecutive periods where μ_0 decreases, followed by a short bold phase, a one period increase in μ_0. A smaller $\frac{c}{1+r_f}$ implies a longer cautious phase.*



(a) State variable μ_0

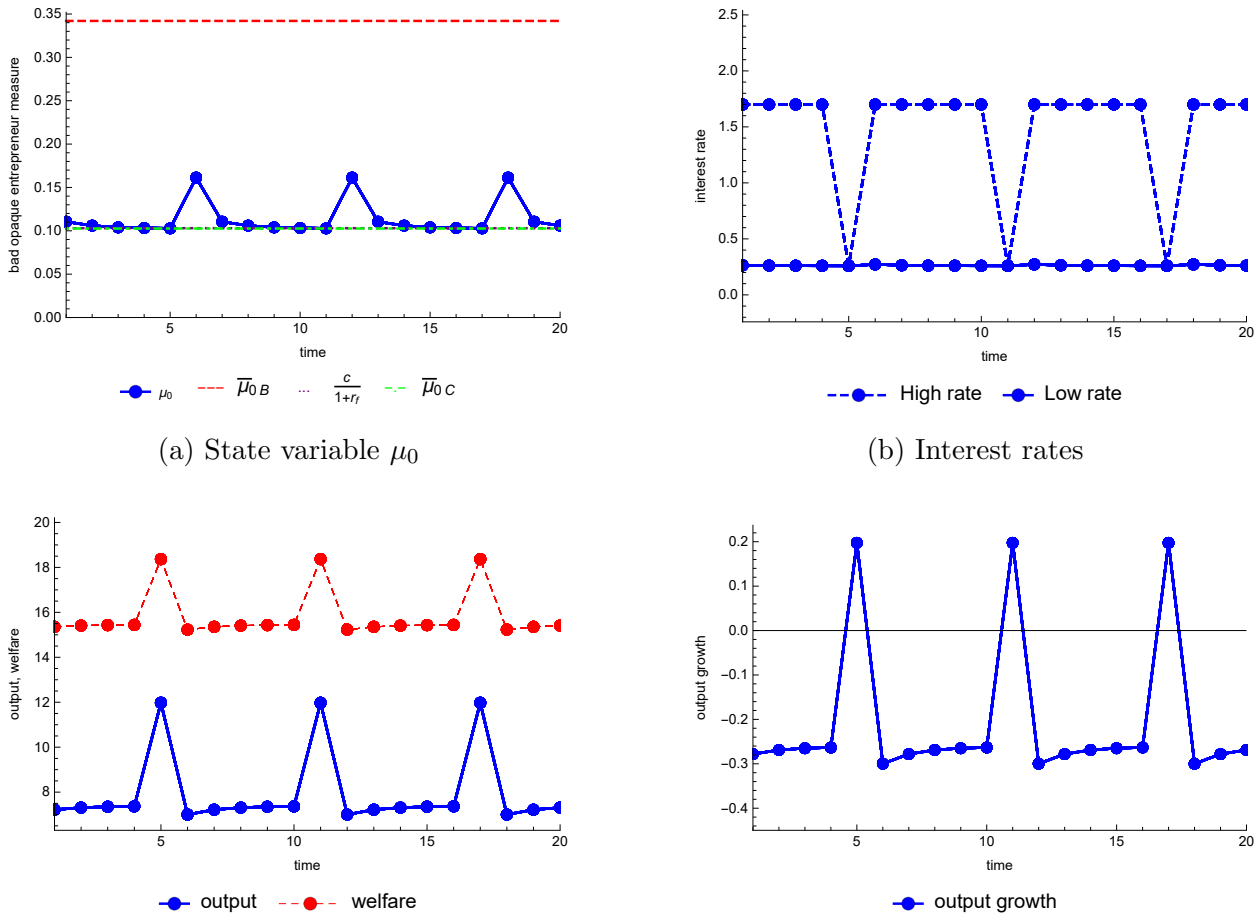
(b) Interest rates

(c) Output and Welfare

(d) Output growth

Figure 2: This figure plots a two-phase economy with a long bold-short cautious cycle. Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates. Panel (c) depicts the total gross output and welfare, and Panel (d) is the output growth. The parameters are: $\rho = 2.7, \lambda = 0.6, \delta = 0.2, c = 0.33, r_f = 0, w_0 = 3.3, w_1 = 0.2$.

For an intermediate range of cost, the economy features deterministic endogenous cycles. The cycles are an outcome of the two-way interaction between investors' choice of lending standards and the fundamentals of the economy. When the measure of opaque bad applicants is relatively low, there are few bad entrepreneurs who default in the portfolio of a bold investor, and thus the quality cost of the bold test is low and investors use lax lending standards: the interest rate is low, there is a lot of credit, and the economy is in a boom. However, as a result of lax lending standards, the quality of the credit pool deteriorates. Once the number of opaque bad applicants has grown sufficiently, the quality cost of the bold test becomes prohibitively high and investors prefer to turn cautious. Being cautious implies tight lending standards, high interest rates, a large credit spread, and little credit to



(a) State variable μ_0

(b) Interest rates

(c) Output and Welfare

(d) Output growth

Figure 3: This figure plots a short bold-long cautious two-phase cycle. Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates. Panel (c) depicts the total gross output and welfare, and Panel (d) is the output growth. The parameters are: $\rho = 2.7, \lambda = 0.54, \delta = 0.22, c = 0.103, r_f = 0, w_0 = 8.8, w_1 = 0.2$.

opaque entrepreneurs: a recession hits. However, this also stops opaque bad entrepreneurs from raising funding. We call this *cleansing by tight credit*: the quality of the pool of credit applicants improves in the cautious stage, which prompts the lenders to switch back to lax lending standards. The economy turns into a boom, and the cycle continues.

The output crashes when the investors tighten lending standards by switching from bold to cautious in line with Proposition 2(v). The tightening implies a discontinuous drop in credit as some or all unskilled investors stop lending to opaque entrepreneurs. Consequently, opaque good entrepreneurs can only borrow at a higher rate and opaque bad ones lose access to credit completely, which leads to discontinuously less investment and output.[9]

[9]Consistent with this interpretation, Leibovici et al. (2019) use Paydex credit score to document that plant shutdown increases during the financial crisis, and the increase is even more among firms with low

Proposition 4 illustrates the three different classes of two-phase economies that emerge in the long run. High quantity cost of tight standards implies longer bold and shorter cautious phases: a long boom is interrupted by a one period recessions. A short cautious phase is enough to improve the quality of loan applications sufficiently for investors to be bold again and not risk losing good investment at the cost of financing too many bad entrepreneurs.

Figure 2 depicts this case, a long bold-short cautious cycle. Panel 2a shows the evolution of the state variable, the measure of opaque bad entrepreneurs, μ_0.[10] Figure 2b plots the interest rates throughout the cycle. As shown in Proposition 1, there is no credit spread in the bold phase. However, the credit market is fragmented in the cautious phase, and the credit spread spikes. Panel 2c illustrates the cyclicality of output, and its crash when lending standards tighten. Comparison with panel 2a shows the negative co-movement of output with the measure of opaque bad entrepreneurs μ_0. Moreover, the output drop is amplified when there is a switch from lax to tight lending standards, which happens in periods 4, 11 and 18. While μ_0 increases only slightly in these periods, the drop in output is sizable. Finally, panel 2d shows that the growth rate is positive in the bold and negative in the cautious phase.

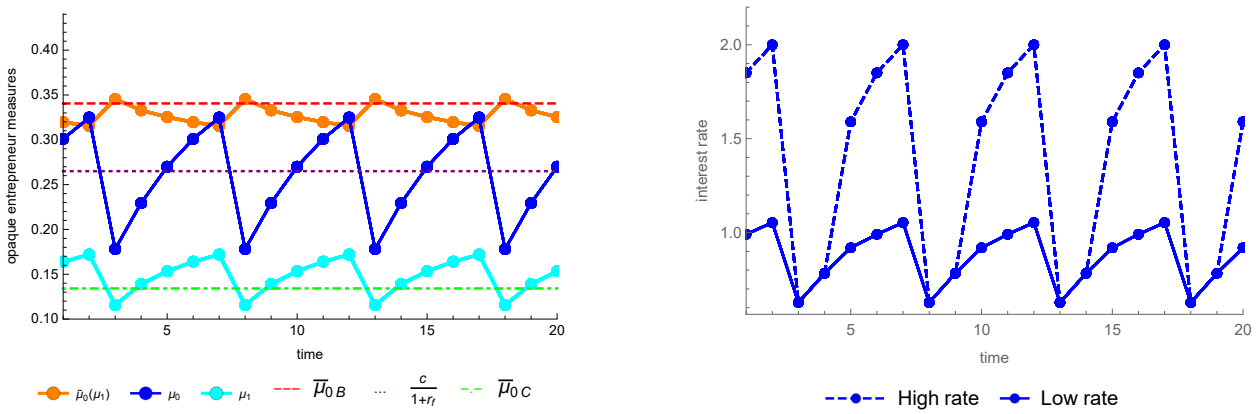
Alternatively, a low quantity cost of tight standards implies a short bold-long cautious cycle: longer downturns followed only by short booms. This corresponds to the economy in Figure 3. Lastly, an intermediate level of $\frac{c}{1+r_f}$ implies a short bold-short cautious cycle, alternating between short booms and short downturns. Proposition 4 also shows how the length of bold and cautious phases vary with $\frac{c}{1+r_f}$, fixing λ and δ.

Three-phase Economy. If $\tilde{\mu}_0(\mu_1) > \frac{c}{1+r_f}$, the economy does not directly transition from a bold phase into a cautious phase. Instead, it passes through an intermediate phase in which investors are indifferent between being bold and cautious, the *mix phase*. Here, a fraction of unskilled investors are bold and a fraction are cautious.

Proposition 5 *For any λ and δ, if the prevailing cyclical dynamic equilibrium is a three-phase economy, then the cycle has length $\kappa \geq 3$ and consists of a bold phase, followed by a mix phase, and a one period cautious phase. μ_0 increases during bold and mix phases and declines in the cautious phase.*

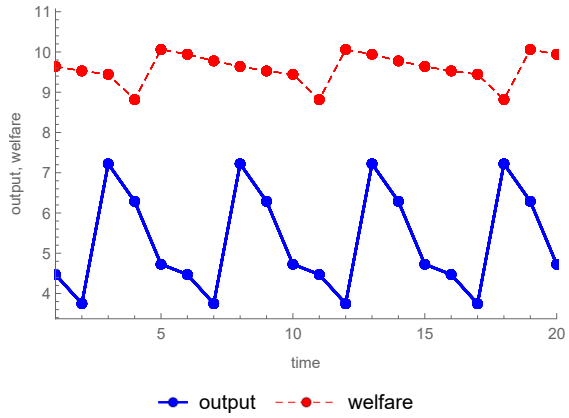
Paydex score. The Paydex score is a business credit score that is increasing in the likelihood of a business paying its debts on time.

[10] The indifference threshold $\frac{c}{1+r_f}$ is not a steady state equilibrium. With our tie-breaking assumption, Proposition 3 implies that the bold dynamics apply at the threshold and thus $\mu_0' > \mu_0$ if $\mu_0 = \frac{c}{1+r_f}$. Any other tie breaking assumption implies a change in μ_0 as well. In particular, if a positive measure of investors chooses to be bold, the bold dynamics apply. If all investors choose to be cautious, then the cautious dynamics apply.

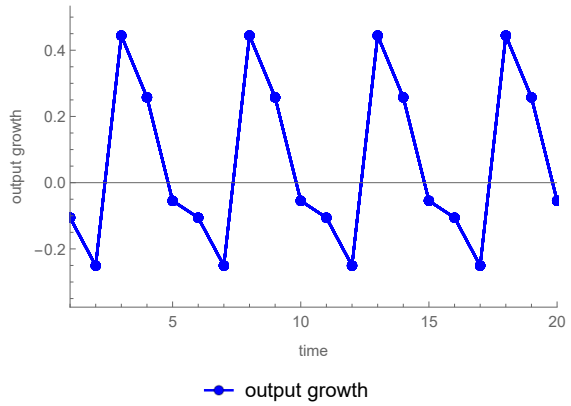


(a) State variables (μ_0, μ_1)

(b) Interest rates



(c) Output and Welfare



(d) Output growth

Figure 4: This figure plots a three-phase economy. Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates. Panel (c) depicts the total gross output and welfare, and Panel (d) is the output growth. The parameters are: $\rho = 3, \lambda = 0.3, \delta = 0.55, c = 0.265, r_f = 0, w_0 = 3.99, w_1 = 0.01$.

In the mix phase, the credit market is fragmented and interest rates rise relative to a bold phase. As such, the output experiences a first drop. However, the credit market fragmentation is not as extreme as a cautious phase and some investors stay bold and impose lax lending standards, which has two implications. First, the opaque bad entrepreneurs are still able to get some financing, which leads to a further decline of credit quality as the economy transitions through the mix phase. Moreover, although the output declines relative to the bold phase and decreases throughout the mix phase, it is still higher than the cautious phase.

The mix phase ends when the credit quality is sufficiently low that it is not optimal for any investor to be bold anymore. All investors switch to being cautious and impose tight standards. The economy enters a cautious phase and the output experiences a second drop.

However, this final output drop is accompanied by a dramatic improvement in quality of the credit applicants, to which the investors respond by switching to lax lending standards. The economy switches back to a bold phase, and the cycles continues. Figure 4 depicts a three-phase economy.

4.2 Diverse Cyclical Outcomes

The richness of the cyclical behavior generated by this framework allows us to consider a few different business cycle outcomes through the lens of the model.

Normal Expansion and Contraction. This is the common post-war business cyclice pattern in of the US according to the NBER US Business Cycle Expansions and Contractions categorization. It consists of a long boom and a short recession, followed by the same pattern. The credit market is integrated and the interest rate is low throughout the boom, while during the recession there is segmentation in the credit market and interest rates increase. This pattern is captured by a long bold-short cautious cycle, characterized in Proposition 4.(ii) and depicted in Figure 2.

Prolonged Recovery. If the fixed cost of testing is relatively low, the economy is trapped in a lengthy recovery period after each bust, before turning to a short boom. During the lengthy recovery period, the output and loan quality are only slowly improving, and the credit market is fragmented for a long time until credit quality improves sufficiently that investors choose to be bold and relax the lending standards. This corresponds to a short bold-long cautious cycle, as characterized in Proposition 4.(iii) and depicted in Figure 3.

Double-Dip Recession. The recession can be exacerbated if the initial decline in credit quality is not sufficiently bad to make all investors adopt a cautious strategy and impose tight lending standards. As such, the cleansing effect in the the credit market is delayed. Although the fragmentation of credit market leads to a drop in output, it does not entail an improvement in loan quality. For some time, the credit market is fragmented, but since some investors are still bold, bad entrepreneurs continue securing some financing and thus credit quality worsens. At some point however, the credit quality has deteriorated so much that every investor chooses to use tight lending standards. The output takes a second hit, but this time it is accompanied by an improvement in the loan quality and leads to a boom. This phenomena is captured in the three-phase economy, as explained in Proposition 5 and illustrated in Figure 4.

It is worth pointing out that the permanent bold steady state is associated with an extended period of low-quality lending, a high fraction of bad entrepreneurs, limited new entry, large share of non-performing loans, and, as a result, a low output growth.[11] This interpretation matches well the description of the Japanese economy in Caballero et al. (2008), which provide a credit market based explanation for the long Japanese slowdown after the asset price collapse in the early 1990s, as a market that involves widespread zombie lending and suffers from depressed restructuring.

5 Optimal Cycles

This section examines which credit allocations and real outcomes are socially optimal. We imagine a hypothetical social planner who can instruct investors to be bold or cautious, but who cannot directly alleviate the frictions in the credit market or influence entrepreneur investment.

The section first defines the constrained planner problem as well as an appropriate notion of welfare and establishes several formal result. We then explore the costs and benefits of realistic monetary and macro-prudential policies in this economy.

The natural measure of welfare in this economy is the average aggregate consumption of all entrepreneurs and investors. In order to accommodate the emergence of cycles, we will use the notion of *average welfare* defined below.

Definition 3 (Average Welfare) *For any collection of k states characterized by the pair of state variables $\{\mu_{0,j}, \mu_{1,j}\}_{j=1}^{k}$, the* average welfare *is*

$$EW\left(\{\mu_{0,j}, \mu_{1,j}\}_{j=1}^{k}\right) \equiv \frac{1}{k}\sum_{j=1}^{k} W\left(\mu_{0,j}, \mu_{1,j}\right),$$

where static welfare *is given by*

$$\begin{aligned} W(\mu_0, \mu_1) \equiv & \rho\big(1 + \mu_0 \ell(b, 0)\big) + \frac{1 - \mu_0 - \mu_1}{2} \sum_{\omega=0,1} \ell\left(g, \omega\right)\left[\rho - \left(1 + r\left(g, \omega\right)\right)\right] \\ & + w_0(1 + r_f) + w_1\left(1 + \max_{\omega} r\left(g, \omega\right)\right). \end{aligned} \tag{8}$$

$W(\mu_0, \mu_1)$ represents the static welfare of a given stage game. The first term is the total

[11] While a bold phase corresponds to higher output growth than a cautious phase for a fixed μ_0, if all unskilled investors are permanently bold, the economy is stuck with a high fraction of opaque bad entrepreneurs, $\mu_0 = \bar{\mu}_{0B}$. This leads to relatively low output and growth rate compared to an equilibrium cycle with the same parameters.

production of all bad entrepreneurs, which is fully consumed by them. The second term is the consumption of transparent and opaque good entrepreneurs, which is their production net of repayment. The third term is the consumption of unskilled investors, noting that they are indifferent between lending and storage at the risk-free rate. The last term is the consumption of the skilled investors.

The constrained planner seeks to maximize average welfare subject to a set of constraints. As we focus on the interaction of investors' choice of test with credit market and real outcomes, it is instructive to study the following constrained planner problem.

Definition 4 (Constrained Planner Problem) *The constrained planner chooses a threshold $\hat{\mu}_0^P$ for the state variable μ_0, and one single test on each side of the threshold to maximize the average welfare of the ergodic state distribution. He cannot directly intervene in the credit market or choose the investment levels, and he has to ensure that investors' and entrepreneurs' participation constraints are satisfied.*

The constrained planner has a very restricted tool to influence the economic outcomes. He can only partition the state space into two parts, and in each part chooses the single test that is available to investors. For instance, the planner can implement a bold (cautious) steady state by choosing a threshold $\hat{\mu}_0^P > \bar{\mu}_{0,B}$ ($\hat{\mu}_0^P < \bar{\mu}_{0,C}$). Alternatively, the planner can implement various two-phase cyclical economies by choosing different levels of $\hat{\mu}_0^P \in (\bar{\mu}_{0,C}, \bar{\mu}_{0,C})$ to partition the state space.[12] In the next section, we show that the policies we consider cannot outperform this constrained planner, which makes it a reasonable benchmark.

We first show that static welfare is decreasing in the measure of opaque bad entrepreneurs.

Lemma 6 *Assume the stage game equilibrium is not autarky. Static welfare is decreasing in the measure of opaque bad entrepreneurs μ_0, and discontinuously drops in μ_0 when an increase in μ_0 implies a transition between two states.*

Keeping μ_1 constant, an increase in the measure of opaque bad entrepreneurs decreases static welfare as it exacerbates the borrower adverse selection problem. The cost of capital increases and thus investment and production falls. When some investors switch to be cautious the problem is intensified since not only some entrepreneurs lose some (or all) financing, but also some good ones can only borrow at the high rates that skilled investors are willing to lend at. Thus at the level of μ_0 that the equilibrium changes from bold to cautious, static welfare is higher in a bold phase than in a cautious one, re-enforcing our

[12]Note that the constrained planner cannot implement a three-phase economy and cannot partition the economy into more than two segments even with only bold and cautious phases.

interpretation of these phases as booms and busts. Figure 2c depicts the dynamics of static welfare and output under our baseline parametrization.

The next proposition provides a sufficient condition for the constrained optimum to be cyclical.

Proposition 6 *There are* $\lambda^{\min}, \lambda^{\max}$ *and* $\bar{\delta}(\lambda) \in (0,1)$ *such that for any* $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$ *and* $\delta < \bar{\delta}(\lambda)$ *the constrained optimum features a cycle.*

The proposition demonstrates that if the share of bad entrepreneurs is in an intermediate range and the measure of entrepreneurs who raise funding in the credit market is sufficiently persistent, then the constrained optimal economy features cycles. Intuitively, the choice of the test is the planner's instrument to influence the ergodic state distribution. In this cycle, tight lending standards have a cleansing effect: they keep the measure of bad entrepreneurs at bay. However, if the planner forces investors to always be cautious, opaque good entrepreneurs are always squeezed. Therefore, to maximize average welfare, the planner only periodically wants the investors to be cautious when the measure of entrepreneurs who do not repay their loans is high. It is optimal to have lax lending standards in the credit market otherwise as it leads to high output.

Externality. It is insightful to think about two types of externalities in the model: *static* and *dynamic* externalities, as characterized in the proposition below.

Proposition 7 *There are two types of externalities in this economy*

(i) Static externality: There exists μ_0^{ext} *such that for* $\mu_0 \in (\frac{c}{1+r_f}, \mu_0^{ext})$ *the equilibrium outcome in the stage game is strictly dominated by an alternative outcome in which strictly more investors are bold. In contrast, there is no* (μ_0, μ_1) *for which the equilibrium outcome in the stage game is weakly dominated by an alternative outcome in which strictly more investors are cautious.*

(ii) Dynamic externality: Consider two consecutive periods and let (μ_0, μ_1) *and* (μ_0', μ_1') *denote the corresponding state variables. There exist* $\bar{\rho}, \bar{w}_1$ *such that if* $\rho < \bar{\rho}$ *and* $w_1 < \bar{w}_1$*, for any* (μ_0, μ_1)

$$W\left(\mu_{0C}'(\mu_0,\mu_1), \mu_{1C}'(\mu_0,\mu_1)\right) > W\left(\mu_{0B}'(\mu_0,\mu_1), \mu_{1B}'(\mu_0,\mu_1)\right).$$

This proposition highlights a static and a dynamic externality going in opposite directions. The static externality is quite intuitive. Within a stage, markets are incomplete and

investors are atomistic, so they do not internalize that choosing a cautious test leads to tight lending standards and a credit crunch. In particular, some good creditworthy firms can borrow only at a very high cost in a cautious stage. Often, static welfare is higher if all investors are forced to be bold and lending standards are lax. In contrast, tightening the lending standards never improves the static welfare. This is the static welfare loss due to the stifling role of tight credit standards. Thus from a static perspective, investors choose to be cautious too often compared to what is constrained optimal.

The decentralized equilibrium features a dynamic externality as well. The law of motion for state variables under each type of equilibrium directly implies that the measure of opaque bad entrepreneurs is higher following lax lending standards compared to tight ones. For a wide range of parameters, this implies a lower welfare in the period that follows, and all entrepreneurs face a higher interest rate due to the more extreme adverse selection problem. However, lenders ignore that tight lending standards cleanse the economy from opaque bad types, leading to higher welfare in the subsequent period.[13] Thus from a dynamic perspective, investors choose to be bold too often compared to what is constrained optimal, exactly opposite the static externality.

Figure 5 compares constrained optimal with equilibrium, as well as policy outcomes that we will discuss in the next section. The solid green curve corresponds to the optimum. In figure 5a it represents the average welfare of the cycles realized for different values of planner threshold $\hat{\mu}_0^P$. The blue dot represents the average welfare in the decentralized equilibrium, which is achieved if the planner chooses $\hat{\mu}_0^P = c$. The vertical dashed lines partition the state space into three regions. The leftmost and rightmost regions are cautious and bold steady states respectively, while the middle region corresponds to two-phase cyclical economies of various bold/cautious lengths. Welfare changes discontinuously when a change in $\hat{\mu}_0^P$ changes the prevailing cycle, and it is flat otherwise.

This figure illustrates the case where the dynamic externality dominates. The constrained planner prefers to shorten the length of the boom and have more frequent albeit milder but less deep recessions. More frequent cautious phases in the optimal cycle keep the measure of bad entrepreneurs in the pool of applicants lower on average, which in turn makes the following bold phase more productive. Panel 5b contrasts the path of the state variable μ_0 chosen by the planner with the decentralized equilibrium, while panel 5c compares the path of static welfare across the equilibrium and the optimum. Because of the lower measure of bad types, both the booms and the recessions have a higher static welfare in the optimum.

[13]Since markets are incomplete and investors are on their outside option, the dynamic externality persists even if investors were infinitely lived or in a perpetual youth model.

6 Economic Policy

In this section we consider three different realistic monetary and macro-prudential policies and compare their efficiency in this economy. For clarity, we normalize the physical return of the storage technology to zero and model the monetary policy as the introduction of a risk-free asset by the government with positive return $r_{f,t}$, the *monetary policy rate*. The asset supply is perfectly elastic for entrepreneurs and investors alike. To ensure that the budget constraint of the policy maker is satisfied in every period, we assume a lump-sum tax is imposed on investors each period which exactly covers the government's aggregate expenditure on the risk-free asset. We further restrict attention to monetary policy rates that are constant within each phase, but can be different across phases. Let r_f^B, r_f^C and r_f^M denote the monetary policy rate in the bold, cautious, and mix phase, respectively.

As a macro-prudential tool, we consider risk-weighted capital requirements where the regulator imposes a risk weight $x \geq 1$ for each unit of risky investment. The macro-prudential policy is permanent and only depends on the risk characteristics of individual investor's portfolio. As such, it is non-state-contingent. As only bold unskilled investors lend to bad entrepreneurs, they are the only investors with a risky portfolio and subject to the macro-prudential policy. Let v_g and v_r be a bold investor's investment in the risky and risk-free asset per-unit-financing, respectively, thus $v_g x + v_r = 1$. If $x = 1$, this reduces to the budget constraint of the investor in our baseline economy. When $x > 1$, the capital requirement forces bold investors to forgo investing $v_g(x-1)$ units of their resources. We assume that the investor consumes this excess capital at the end of the period.

Let the tuple $\pi = (x, r_f^B, r_f^C, r_f^M)$ denote a policy profile. The following lemma expresses the equilibrium associated with each policy profile.

Lemma 7 *For policy profile π, the equilibrium is characterized by Propositions 1-2 where the modified interest rate functions $r_B^\pi(\mu_0, \mu_1, c, \pi)$, $r_C^\pi(\mu_0, \mu_1, c, \pi)$ and $r_M^\pi(\mu_0, \mu_1, c, \pi)$ replace* (A.7)-(A.9)*, respectively, and the modified thresholds $\hat{\mu}_0^\pi(\mu_1, c, \pi)$ and $\tilde{\mu}_0^\pi(\mu_1, c, \rho, \pi)$ replace $\frac{c}{1+r_f}$ and $\tilde{\mu}_0(\mu_1, c, \rho, \pi)$.*

To gain further insight about the relative efficiency of the commonly used policy instruments regularly, we compare three specific policy profiles. First, a *simple monetary policy* always pays the same interest rate r_f, $\pi_{r_f} = (1, r_f, r_f, r_f)$. Second, a *counter-cyclical monetary policy* allows for potentially different interest rate for different phases. It is straightforward to show that it is optimal for the policy maker to have a positive interest rate in bold and zero in the cautious phase. We further assume that in the mix phase, the planner uses

an interest rate above $\bar{r}$, which in turn implies that the mix phase never realizes.[14] Thus the counter-cyclical monetary policy profile is represented by $\pi_{r_f^B} = (1, r_f^B, 0, \bar{r})$. Third, a *macro-prudential policy* consists of risk-weighted capital requirements for risky investment without providing a safe asset, $\pi_x = (x, 0, 0, 0)$.

The next Corollary summarizes the effect of the policy tools on the cost of capital and the cycle in a two-phase economy.

Corollary 1 *For any state* (μ_0,μ_1)*, keeping the state constant, an increase in the monetary policy rate in each phase increases the cost of capital only in the corresponding phase. An increase in risk weight* x *increases the cost of capital in the bold and mix phases. Furthermore in a two phase economy, all three policies* π_{r_f}, $\pi_{r_f^B}$ *and* π_x *weakly shorten the bold and elongate the cautious phase.*

This corollary is a direct consequence of Lemma 7. Intuitively, all of these policies lead to a tightening compared to the laissez-fair equilibrium. Increasing the monetary policy rate increases the opportunity cost of lending to entrepreneurs directly, while increasing the capital requirement implies a higher opportunity cost for bold investors by decreasing the amount of capital that they can lend. Finally, the prevailing rate in the credit market is more sensitive to the monetary policy rate in bold versus cautious regime. Thus, an increase in the simple monetary policy rate increases the bold lending rate more than the cautious one. As such, an increase in x, r_f^B or r_f leads the economy to spend more time in the cautious phase.

In order to make the welfare effects of these policies comparable, we introduce the concept of *equivalence* bellow.

Definition 5 *Policy profile* π *is equivalent to another policy profile* π' *or to the planner threshold* $\hat{\mu}_0^P$ *if they imply the same ergodic set for the state variable in the dynamic equilibrium.*

Equivalence is useful to evaluate the relative effectiveness of policy tools in improving the efficiency of the equilibrium cycle. The critical observation is that policy equivalence does not imply the same welfare. Our main result below ranks the three policy instruments according to their relative efficiency in achieving the same constrained optimal cycle.

Proposition 8 *Consider a set of parameters where the cautious phase is more frequent or longer in the constrained optimum compared to the equilibrium. For any policy* $\pi_{r_f}, \pi_{r_f^B}$ *or*

[14] This is motivated by our simulations which indicate that the policy maker always finds it optimal to set the mix monetary policy rate sufficiently high such that the realized cycle is two-phase.

π_x, equivalent to a threshold $\hat{\mu}_0^P$ that leads to a two-phase economy, the following statements hold.[15]

(i) The policy achieves a lower average welfare compared to the constrained optimum.

(ii) Any pair of equivalent counter-cyclical monetary ($\pi_{r_f^B}$) and macro-prudential (π_x) policies imply the same equilibrium borrowing interest rate faced by every entrepreneur in every phase. However, the counter-cyclical monetary policy achieves a higher average welfare.

(iii) For $\lambda \leq \frac{8}{9}$,

(a) $\pi_{r_f^B}$ achieves a higher average welfare than π_{r_f},

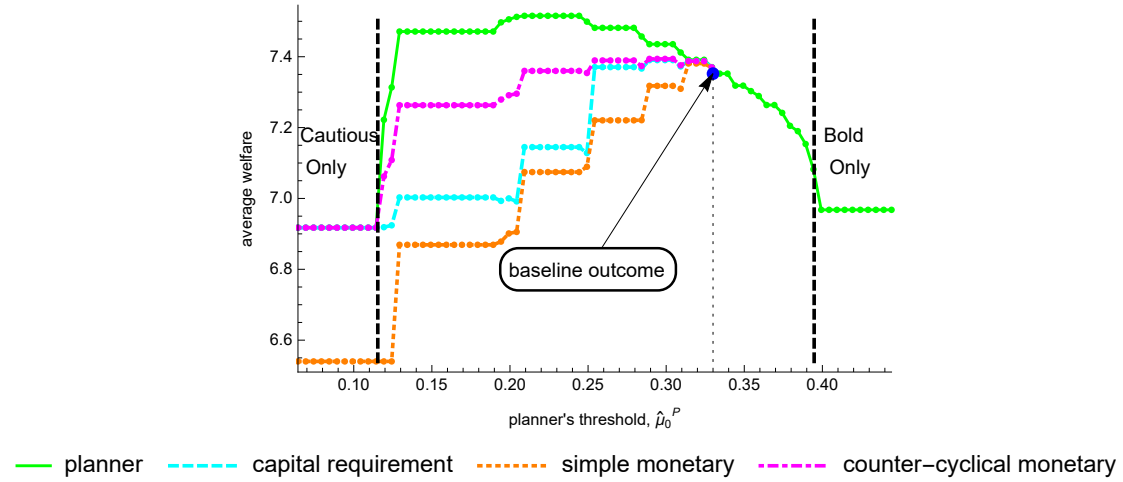
(b) There exits $\bar{c}$ such that if $c \leq \bar{c}$, π_x achieves a higher average welfare than π_{r_f}.

All three policies are socially costly compared to the constrained optimum. Unlike the constrained planner who directly chooses the lending standards, the policy maker has to influence investors' incentives to choose among the available tests in order to implement the desired lending standard. This leads to a higher interest rate in the credit market under all policies, which in turn dampens the output.

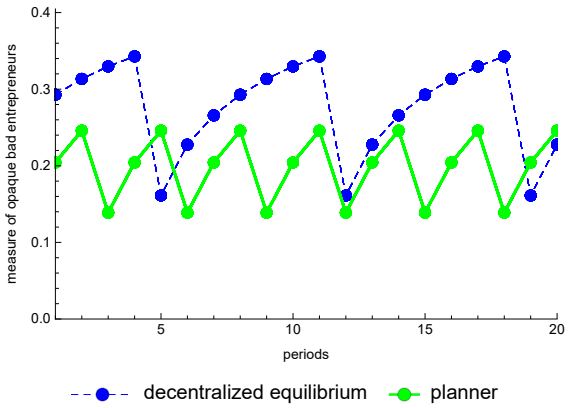
Each non-solid curves in Figure 5a corresponds to the average welfare of one of the three policies equivalent to the planner threshold $\hat{\mu}_0^P$ on the x-axis, with the minimum cost of capital. The pink dashed-doted, orange dotted , and blue dashed lines represent the counter-cyclical monetary, simple monetary, and macro-prudential policy, respectively. The welfare improvement associated with all three policies comes from shortening the boom compared to the laissez-fair equilibrium. The slightly more recurrent recessions have a cleansing effect: the amplified exit rate of bad entrepreneurs during recessions facilitates new firm entry which in turn warrants more future growth and leads to healthier subsequent booms.

Interestingly, the highest welfare counter-cyclical monetary policy and the capital requirements perform extremely closely, and outperform the simple monetary policy. The counter-cyclical monetary policy is aggregate state dependent while the capital requirements depend only on individual investor choices. As such, they require policy makers of different degrees of sophistication, while they reach almost identical maximum welfare.

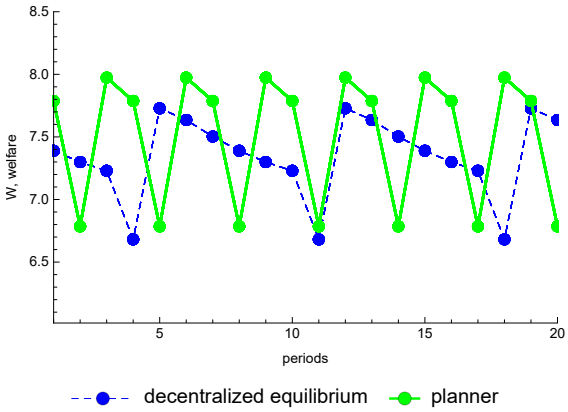
[15]Both the simple monetary policy and capital requirements can turn a two-phase economy to a three-phase economy by changing the incentives for different lending standards. In such cases a bold phase is replaced by a mix phase. While the policy remains equivalent to the corresponding $\hat{\mu}_0^P$, welfare is typically higher in a bold phase.



(a) Average welfare



(b) Measure of opaque bad entrepreneurs



(c) Welfare paths

Figure 5: Average welfare for different levels of planner choice of threshold $\hat{\mu}_0^P$, as well as the comparison between the implied paths for the measure of opaque bad entrepreneurs μ_0, and welfare, along the optimal versus the decentralized cycle. Baseline parameters are: $\rho = 2.7, \lambda = 0.6, \delta = 0.2, c = 0.33, r_f = 0, w_0 = 3.3, w_1 = 0.2$. In pannels (b) and (c), the planner's threshold is $\hat{\mu}_0^P = 0.21$.

7 Model and Facts

The model generates a rich set of empirical predictions despite its simple structure. In this section we explore a number of these predictions.

Tightness of credit, entry and exit, economic cycles An essential mechanism in our framework is the cleansing role of tight credit. That is, there is a casual link between tighter lending standards and a larger share of distressed firms exiting the economy, which in turn allows for more firm entry and implies higher growth in the subsequent years. Using

differences in supervisory forbearance across US regions as a proxy for differences in banks' lending standards, Gropp et al. (2020) show that such link exists.[16] Furthermore, Leibovici et al. (2019) use Paydex credit score to sort firms based on how reliable they are to pay back their suppliers, and document that plant shutdowns during the financial crisis are concentrated among firms with low Paydex score. This is in line with our mechanism in which bad firms which do not pay back their loans are forced to exit in recessions, but not in booms. At a broader level, we argue that aggregate credit cycles are driven by (endogenous) fluctations of credit supply. Becker and Ivashina (2014) present various measures to argue that this is the case, at least with respect to credit to small firms in the US.

The model also predicts that during a boom characterized by lax lending standards, the quality of issued credit keeps deteriorating. This is in line with the evidence that the quality of issued loans gradually deteriorated during the years boom years leading to the 2007 crisis in the subprime mortgage market (e.g. Demyanyk and Van Hemert, 2009; Palmer, 2015).

Furthermore, in our model longer booms tend to be followed by deeper recessions. This is the case because the average credit quality when the economy enters a recession is negatively correlated with both size of the crash and length of the boom. This is consistent with Jordà et al. (2011) and Müller and Verner (2021) who find that more credit-intensive expansions tend to be followed by deeper recessions and slowdowns.

Note that the above groups of predictions treat firms' opacity type as unobservable by the econometrician. An alternative approach is to assume that, at least ex-post, a proxy for opacity is available. This results in a more detailed set of empirical implications, explored below.

Credit composition, the quality spread and endogenous lending standards In order to map opacity to observables we consider that credit issued to firms who are rejected by a cautious test, i.e. credit to opaque firms, corresponds to junk bond issuance. Alternatively, loans to transparent good firms map to high-grade bond issuance.

With this interpretation, our model is consistent with the well-known fact that the quality spread, the spread between AAA and BAA corporate borrowers, is counter-cyclical. As such, our paper provides an information based alternative explanation for time-varying risk-premium.

We can also interpret our predictions in the context of the growing body of evidence

[16]The seminal work of Caballero et al. (2008) was perhaps the first to provide suggestive evidence for such link demonstrating that the continued presence of distressed firms due to lax lending standards in Japan was negatively associated with the entry of healthy firms, contributing to the prolonged recession in the 90s. Blattner et al. (2019) finds similar evidence for Portuguese banks using a regulatory event to establish causation.

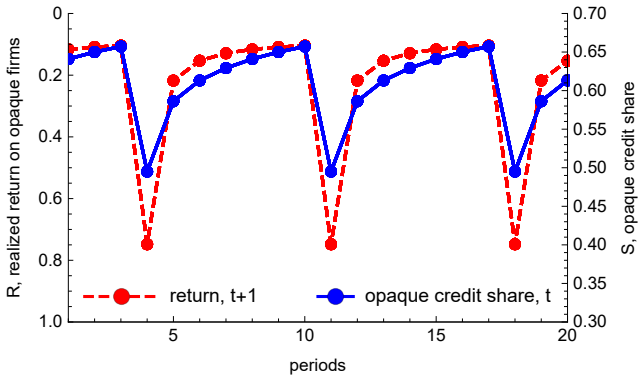


Figure 6: Model generated positive correlation between opaque credit share and its future realized excess return on the invested scale. The solid blue line plots the share of issued credit to opaque entrepreneurs relative to all credit in a given period on the right scale. The dashed red line depicts the realized excess return on opaque credit, one period later, on the left scale on an inverted scale.

that periods of overheating in credit markets forecast low excess bond returns. Importantly, Greenwood and Hanson (2013) show that the share of junk bond issuance out of total issuance inversely predicts the excess return on these bonds.[17]

Figure 6 illustrates the model equivalent of this empirical pattern documented in Exhibit 1 of Stein (2013), for the two-phase economy simulated on Figure 2. As in Exhibit 1 of Stein (2013), the model predicts a positive correlation between the share of junk bond issuance and its future realized excess return on the invested scale. Heightened values of curves correspond to overheated periods with low subsequent returns. Low values instead correspond to recessions and tight lending standards with high subsequent returns.[18] Note the strong comovement between share of opaque credit and the corresponding return on a

[17]The inverse relationship between credit expansion and subsequent returns is remarkably widespread across various financial markets. For instance, Baron and Xiong (2017) document the negative relationship between bank's credit expansion and banks' equity returns, Kaplan and Stromberg (2009) find a similar inverse relationship between venture capitalists aggregate flow to new investments and their subsequent returns. A related early work is Eisfeldt and Rampini (2006) who show that volume of transactions is pro-cyclical while return on transactions is counter-cyclical in the sales of property, plant and equipment.

[18]Formally, let $S(\mu_0, \mu_1)$ denote the share of credit to opaque firms, and $R(\mu_0, \mu_1)$ denote the net excess realized return on a portfolio of these loans. We have:

$$S(\mu_0,\mu_1) \equiv \frac{\mu_0 \ell(b,0) + \frac{1-\mu_0-\mu_1}{2}\ell(g,0)}{\mu_0 \ell(b,0) + \frac{1-\mu_0-\mu_1}{2}\left(\ell(g,0)+\ell(g,1)\right)} \quad R(\mu_0,\mu_1) \equiv \frac{\frac{1-\mu_0-\mu_1}{2}\ell(g,0)(1+r(g,0))}{\mu_0 \ell(b,0) + \frac{1-\mu_0-\mu_1}{2}\ell(g,0)} - (1+r_f).$$

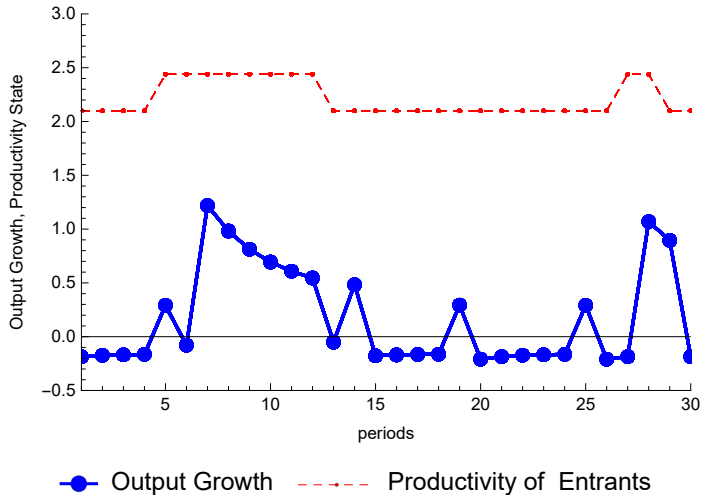


Figure 7: The figure plots a realization of the path for exogenous productivity of entrants (dashed red) along with the endogenous output growth path (solid blue). The parameters are: $\rho_g = 2.7, \rho_b = 1.7, \lambda_1 = 0.4, \lambda_2 = 0.6, \alpha = 0.8, \delta = 0.2, c = 0.116, r_f = 0, w_0 = 6.5, w_1 = 0.2$

reverse scale, both within the bold phase (boom) and across periods.[19]

8 Stochastic Cycles

In this section we introduce an aggregate shock to the model. This extension makes the framework more realistic while keeping the analysis tractable, and replaces deterministic cycles with more plausible stochastic ones.

Suppose that the fraction of bad entrepreneurs among the new entrants is stochastic, denoted by λ_t. Assume λ_t follows an S state Markov process with transition matrix $\Sigma \in [0,1]^S$. Furthermore, relax the assumption that the good and bad firms are equally productive and let $\rho_g > \rho_b > 1 + r_f$. As such, λ_t represents an exogenous productivity shock and the average productivity of entrants in a given period is $(1-\lambda_t)\rho_g + \lambda_t \rho_b$.

Observe that λ_t enters the equilibrium only through the law of motion for (μ_0, μ_1) described in Proposition 1. Therefore, the only change to the equilibrium characterization is replacement of (4)-(7) in the deterministic cycle with their state-dependent, stochastic counterparts. For instance, (4) is replaced by

$$\mu'_{0B}(\delta, \lambda_s, \mu_0, \mu_1) = (1-\delta)\mu_0 + \big(\delta + (1-\delta)\mu_1\big)\frac{\lambda_{s'}}{2} \qquad \text{w.p. } \Sigma_{s,s'}, \forall\ s'. \tag{9}$$

[19] Although our model generates a strong positive correlation between these variables, ie does not amounts to an exploitable anomaly based on the information set of unskilled investors. See Bordalo et al. (2018); Gennaioli and Shleifer (2020) for empirical facts pointing towards such anomalies, and bounded rational models designed to target those.

We study the properties of the extended model through a simple example.

Example 1 *Let λ_t follow a two state Markov process, where $\lambda_1 < \lambda_2$ and $\Sigma_{ss} = \alpha_s > \frac{1}{2}$ for $s = 1, 2$.*

As such, state $s = 1$ corresponds to high productivity of entrants. Figure 7 illustrates the implied stochastic cycle. The dashed red line is the average exogenous productivity of entrants, while the solid blue line is the endogenous aggregate growth of the economy. Aggregate growth depends on both the exogenous productivity of new entrants and the endogenous credit market outcomes, similar to the baseline economy where state variables (μ_0, μ_1) and lending standards determine which firms are financed and survive. This leads to an intricate relationship between productivity shocks and aggregate growth. In particular, output growth is not solely driven by exogenous productivity shocks. For instance, periods 14-26 correspond to an interval of continuously low exogenous productivity. Still, the economy switches between recessions and booms as lending standards endogenously fluctuate within this interval. Furthermore, this figure makes it clear that the introduction of an aggregate state leads to stochastic cycles in which booms and recessions of varying magnitude and length alternate.

It is also interesting to consider the highest aggregate growth episodes, periods 7 and 28 in this example. These episodes happen when the exogenous high productivity of entrants coincides with the endogenous cleansing effect of a long cautious phase leading to an unusually low fraction of surviving bad firms. As such, the aggregate growth is exacerbated.

9 Conclusion

The idea that economic fluctuations can be captured by models with endogenous cycles is not new. In fact, the earliest business cycle models by John Hicks and Nicolas Kaldor followed this approach. However, as Boldrin and Woodford (1990) explain, these models fell out of favor by the late 1950's because actual business cycles were found not to show regular cycling behavior.[20]

In this paper, we argue that despite real world cycles being complex and difficult to forecast, simple models with endogenous cycles are a useful apparatus for macroeconomic theory as indispensable analytical tools for policy analysis. To asses the effect of various policies on the length and depths of booms and busts, it is essential to understand the mechanism that turns booms into busts and vice-versa.

[20]See the recent work of Beaudry et al. (2020) for the argument that modern statistical techniques might refute this statement.

We propose a model where endogenous cycles are generated by the interaction between lenders' choice of lending standards in the credit market and the economic fundamentals. Tight credit standards screen out low quality entrepreneurs and thus the future quality of credit applications improves. Once the improvement is sufficiently significant, it triggers a switch to lax lending standards. This in turn leads to the deterioration of fundamentals, which prompts tight credit conditions again.

We show that simple policy tools allow the policy maker to control the cyclicality of the economy. By utilizing a macro-prudential policy to carefully choose capital requirements for risky investment, or through an appropriate counter-cyclical monetary policy, the policy maker can optimally use recessions to keep the stock of bad borrowers at bay. We further demonstrate that the predictions of the model match numerous stylized facts related to credit cycles.

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Appendix

A Agent Optimization Problem and Market Clearing Protocol

In this Appendix we formally define the problem of each agent, the market clearing protocol, and a robustness criterion. We also show how the agents' problem reduce to the ones set up in the main text. The structure of our credit market is a modified version of Kurlat (2016). The entrepreneur and investor problems are simplified versions of those in Farboodi and Kondor (2018).

A.1 Stage Game: Entrepreneur and Investor Problem

Let R denote the a set of trading posts, each of which identified by an interest rate r. The problem for an entrepreneur (τ, ω) is

$$\max_{\{\sigma(r;\tau,\omega)\}_{r\in R}} \rho i(\tau,\omega) - \mathbf{1}_{\tau=g}\ell(\tau,\omega)\left(1+r(\tau,\omega)\right) \tag{A.1}$$

$$s.t.$$

$$0 \leq \sigma(r;\tau,\omega) \leq \frac{1}{r} \quad \forall r \in R$$

$$\ell(\tau,\omega) = \int_R \sigma(r;\tau,\omega) d\eta(r;\tau,\omega) \tag{A.2}$$

$$r(\tau,\omega) = \frac{\int_R r\sigma(r;\tau,\omega) d\eta(r;\tau,\omega)}{\ell(\tau,\omega)} \tag{A.3}$$

$$\ell(\tau,\omega) \leq \frac{1}{r(\tau,\omega)} \tag{A.4}$$

$$i(\tau,\omega) = \ell(\tau,\omega) + 1.$$

$\sigma(r;\tau,\omega)$ denotes the number of credit units entrepreneur (τ,ω) demands at interest rate r. $\ell(\tau,\omega)$ and $i(\tau,\omega)$ denote the total amount of credit and the investment level for entrepreneur (τ,ω), respectively.

η is the *rationing function* that assigns $\eta\left(R_0;\tau,\omega\right)$ measure of credit, per unit of application, to entrepreneur (τ,ω) who has submitted applications to the subset of trading posts $R_0 \in R$. η is an equilibrium object, determined by the choices of the agents and the market clearing protocol as explained below. The entrepreneur takes η as given.

Let $\bar{\ell}$ denote the maximum available credit for a given entrepreneur,

$$\bar{\ell}(\tau,\omega) \equiv \int_R \frac{1}{r} d\eta\left(r;\tau,\omega\right).$$

We are interested in showing that an equilibrium exists. As such, we conjecture and then verify that there exist an equilibrium in which each entrepreneur only raises credit at one single interest rate. From equations (A.3) and (A.2), $r(\tau,\omega)$ denotes the *average* interest

rate that the entrepreneur raises credit at. Under the conjecture that he raises credit at a single interest rate, with some abuse of notation let $r(\tau,\omega)$ denote that unique interest rate. In particular, $r(\tau,\omega)$ does not depend on $\sigma(\cdot)$.

Under this conjecture, the entrepreneur's problem can be rewritten as

$$\max_{\ell(\tau,\omega),r(\tau,\omega)} \rho + \ell(\tau,\omega)\big(\rho - \mathbf{1}_{\tau=g}(1+r(\tau,\omega))\big) \tag{A.5}$$
$$s.t. \qquad \ell(\tau,\omega) \leq \min\left(\bar{\ell}(\tau,\omega), \frac{1}{r(\tau,\omega)}\right).$$

This form suppresses the choice over credit applications, $\sigma(\cdot)$, and focuses on the total obtained credit $\ell(\cdot)$. For any obtained credit $\ell(\tau,\omega)$ along with equilibrium $\eta(r;\tau,\omega)$ schedule, equation (A.2) determines $\sigma(r;\tau,\omega)$.

Each investor h advertises a single rate $r(h)$. Unskilled investor h solves

$$\max_{\chi(h),\tilde{r}(h)} (1+\tilde{r}(h))\left(S_u(r;g,1) + \mathbb{1}_{\chi(h)=B} S_u(r;g,0)\right)$$
$$+(1+r_f)\Big(S_u(r;b,1) + \mathbb{1}_{\chi(h)=C}\big(S_u(r;b,0) + S_u(r;g,0)\big)\Big),$$

while skilled investor h solves

$$\max_{\tilde{r}(h)} (1+\tilde{r}(h))\left(S_s(r;g,1) + S_s(r;g,0)\right).$$

$\chi(h)$ is the unskilled agent's choice of test. S_u and S_s are the sampling functions for unskilled and skilled investors.

An unskilled investors has one unit of wealth, thus she samples total one unit of applications at the interest rate she advertises. $S_u(r;\tau,\omega)$ denotes the measure of applications submitted by (τ,ω) entrepreneurs that the unskilled investor who has advertised interest rate r receives. Importantly, this measure is independent of unskilled investor's choice of test. $S_s(r;\tau,\omega)$ is the analogous object for skilled investors. The sampling functions are aggregate equilibrium objects determined by the market clearing protocol and the choices of agents, and are taken as given by investors.

We follow Kurlat (2016) to assume the following *robustness* criterion.

Assumption A.1 *Suppose that ε fraction of applications submitted at an advertised interest rate are granted unconditionally. We require that the equilibrium strategy of each entrepreneur is the limit of equilibrium strategies as ε goes to 0.*

This assumption has two implications. First, it prevents equilibrium multiplicity. Second, it implies that every type who chooses to submit loan applications at a given interest rate, submits the maximal amount. Thus $\sigma(r;\tau,\omega) > 0$ implies $\sigma(r;\tau,\omega) = \frac{1}{r}$. As such, the application pool at any given interest rate is independent of cross-sectional distribution of $i(\tau,\omega)$, and we can solve the credit market equilibrium independently of $i(\tau,\omega)$ choices. This simplifies the analysis considerably.

Market Clearing Protocol. Let r' denote the lowest interest rate which is both advertised by some investor and some entrepreneurs have submitted demand at this rate. If there is no such interest rate, then no applications are financed.

First, each entrepreneur who submits an application at that rate posts r' *down-payment* per unit of application from her endowment. Applications without a down-payment are automatically discarded. Then, each unskilled investor who has advertised rate r' obtains a sample of the (non-discarded) applications submitted at that rate with the underlying distribution. As such, $S_u(r';\tau,\omega)$ is equal to the fraction of non-discarded (τ,ω) application submitted at interest rate r'.

If there are not enough applications to fill up every unskilled present investor's capacity limit, then all applications have been sampled and the sampling process stops. Otherwise, all unskilled investors sample a measure (of value) one of applications and provide financing to all applications in their sample that passes their chosen test. Their remaining endowment is invested in the risk-free asset.

If all unskilled investors reach their sampling capacity and there are remaining applications from good entrepreneurs, then they are distributed pro rata across skilled investors up to their capacity given by their one unit of endowment. As such, $S_s(r';g,\omega)$ is the ratio of remaining non-discarded (g,ω) applications at interest rate r' relative to sum of remaining non-discarded $(g,\omega)+(g,\omega')$ applications after unskilled investors make their financing decision at rate r'. Skilled investors grant credit to these entrepreneurs.

Entrepreneurs who receive financing invest the credit they obtain along with the down-payment, and the invested units are posted as collateral for the loan. These invested units enter into a public registry, so they cannot serve as collateral to other loan applications. Applications that are submitted but do not receive financing are discarded, and the down-payment is returned to the entrepreneur who can only invest it in the risk-free asset.

Then, the process is repeated at the next lowest advertised interest rate at which there are applications. The process stops once there is no such rate anymore. $\eta(r;\tau,\omega)$ is computed by aggregating over all investors who grant credit to entrepreneur (τ,ω) at interest rate r.

A.2 Dynamic Economy: Entrepreneur and Investor Problem

Since each investor lives for a single period, she solves the identical utility maximization problem in the stage game and the full game.

For entrepreneurs the only change is that they maximize the expected sum of their future utility while alive. This consists of entrepreneur's period utility, as well as his expected continuation value. That is, instead of (A.5), the value function of the entrepreneur can be written as

$$
\begin{aligned}
&V(\tau,\omega;\mu_{0,t},\mu_{1,t}) = \\
&\max_{\ell_t(\tau,\omega),r_t(\tau,\omega)} \rho+\ell_t(\tau,\omega)\big(\rho-\mathbf{1}_{\tau=g}(1+r_t(\tau,\omega))\big)+(1-\delta)\mathbf{1}_{\ell_t(\tau,\omega)>0}V(\tau,\omega;\mu_{0,t+1},\mu_{1,t+1}) \qquad \text{(A.6)}\\
&s.t. \qquad \ell_t(\tau,\omega)\leq \min\left(\bar{\ell}_t(\tau,\omega),\frac{1}{r_t(\tau,\omega)}\right),
\end{aligned}
$$

where the entrepreneur takes the equilibrium dynamics of $(\mu_{0,t},\mu_{1,t})$ as given.

B Proofs

Proof of Lemma 1

The market clearing mechanism and Assumption A.1 implies that in the stage game if any agent would like to raise credit at an interest rate $r^{\max}$, she would want to submit a maximum measure of applications, $\sigma(r;\tau,\omega) = \frac{1}{r}$ at every interest rate smaller than $r^{\max}$ too. This makes it possible to receive a fraction of their credit at a lower rate (as markets clear from the lowest interest rate), and potentially even without the requirement to invest the received amount (Assumption A.1). This latter possibility is attractive for bad entrepreneurs. Because applications with no down-payment are discarded, there is no possibility of having more credit granted as intended. Agents also want to submit the maximum measure of applications at $r^{\max}$. Given the linear structure, if, at a given interest rate an agent would like borrow to invest, she also would like to borrow up to the limit $\frac{1}{r}$ and invest at that rate. This concludes the first part of the Lemma.

For the second part, observe that the objective function (A.1) implies that a good entrepreneur does not apply for credit at any interest rate $r(g,\omega) > \rho - 1$ as that would imply negative return on her investment. As we noted before, Assumption A.1 and objective (A.1) imply that bad entrepreneurs instead apply for maximum credit at any interest rate as they do not plan to pay back.

Proof of Lemma 2

As we explained in the text, in any equilibrium unskilled investors have to lend at the break-even interest rate which makes them indifferent whether to participate or not. Also, they never extend credit to entrepreneurs not passing their test. This is so, because tests are informative. Therefore extending credit to those entrepreneurs who do not pass an investors' test increases her break-even interest rate. Therefore, if there were such a group of investors in equilibrium, non-participating investors would deviate by entering at a slightly lower interest rate, extending credit only to those who pass their test, and stealing the business of the first group.

Proof of Lemma 3 and Proposition 1

Consider the interest rate functions

$$r_B(\mu_0,\mu_1,c,r_f) \equiv \frac{\mu_0 + (1-\mu_1)r_f + c}{1-\mu_1-\mu_0} \tag{A.7}$$

$$r_C(\mu_0,\mu_1,c,r_f) \equiv r_f + \frac{2c}{1-\mu_1-\mu_0} \tag{A.8}$$

$$r_M(\mu_0,\mu_1,c,r_f) \equiv \frac{2\mu_0 + (1+\mu_0-\mu_1)r_f + (1-\mu_1-\mu_0)c}{1-\mu_1-\mu_0}. \tag{A.9}$$

It is straightforward to check that the claimed properties of these functions in Lemma 3 hold. In what follows, we also show that they are indeed break-even interest rates under the

stated conditions and that they support the equilibrium in Proposition 1.

Bold phase We conjecture and verify that under the conditions of statement (i), all unskilled investors run a bold test, all investors advertise the rate r_B, all entrepreneurs submit maximum demand at that rate and all good entrepreneurs choose $r^{\max} = r_B$. Note that r_B must satisfy the indifference condition

$$(1 - \mu_1 - \mu_0)(1 + r_B) + \mu_1 (1 + r_f) - c = 1 + r_f \tag{A.10}$$

which is equivalent to (A.7). The entering measure of unskilled investors is determined by a market clearing condition spelled out in the proof of Proposition 2. This condition ensures that the fraction of bold unskilled investors' capital that finances good entrepreneurs, together with the capital of skilled investors (who only finance projects of good entrepreneurs), all good entrepreneurs, opaque or transparent, have all their credit demand satisfied.

We show that none of the agents have a profitable deviation. The left hand side of (A.10) is the expected utility of running the bold test on a proportional sample of applications, accepting all good and all opaque applications out of which only the good ones pay back (the first term), and investing the capital corresponding to rejected share of applications to risk-free assets (second term) minus the cost of the test (last term). The right hand side is the return of not entering the market and investing in risk-free assets only. That is, bold unskilled break-even at r_B. (Note that (A.10) is using the assumption that unskilled investors sample first.) Therefore, unskilled cannot offer a lower rate and participate. Skilled could offer a lower rate profitably but they lend out all their capital in equilibrium, so there is no advantage of offering a lower rate. Offering a higher rate for any investor would lead to a zero measure of applications from good entrepreneurs. Good entrepreneurs raise all the capital they need, hence they would not deviate to a higher r^{max}. None of the entrepreneurs would raise any capital at a lower r^{max} given strategies.

Finally, if the investor was to run a cautious test, she would break even at a rate r_C given by the condition

$$\frac{(1 - \mu_1 - \mu_0)}{2}(1 + r_C) + \left(\frac{(1 - \mu_1 - \mu_0)}{2} + (\mu_1 + \mu_0)\right)(1 + r_f) - c = 1 + r_f \tag{A.11}$$

which is equivalent to (A.8). As long as $r_B \leq r_C < \bar{r}$, switching to a cautious test leads to a loss. This condition is equivalent to $\mu_0 \in [0, \frac{c}{1+r_f}]$.

Cautious phase We conjecture and verify that under the conditions of statement (ii), all unskilled investors run a cautious test and advertise the rate r_C, all skilled advertise $\bar{r}$, transparent good entrepreneurs choose $r^{\max} = r_C$, while opaque good entrepreneurs choose $\bar{r}$. Bad entrepreneurs choose a higher $r^{\max}$ than $\bar{r}$. r_C must satisfies the indifference condition (A.11). Analogously to the bold phase, the entering measure of unskilled investors is determined by the market clearing condition ensuring that all transparent good entrepreneurs can obtain the capital they demand at r_C.

Cautious unskilled investors would make losses at a smaller rate, and would not pass any applications submitted at a higher rate. However, they could consider running a bold

test, advertising a higher rate, and finance the opaque good entrepreneurs (along with some bad ones). Under this strategy, they would break-even at the rate r_M determined by the indifference condition

$$\frac{\frac{(1-\mu_1-\mu_0)}{2}}{\frac{(1-\mu_1-\mu_0)}{2}+(\mu_1+\mu_0)}(1+r_M)+\frac{\mu_1}{\frac{(1-\mu_1-\mu_0)}{2}+(\mu_1+\mu_0)}(1+r_f)-c=(1+r_f)\,. \quad \text{(A.12)}$$

which is equivalent to (A.9). This deviation is not profitable if $r_C < r_M$, which is equivalent to the condition of statement (ii) with $\tilde{\mu}_0(\mu_1, c, \rho, r_f) = \frac{(\bar{r}-r_f-c)(1-\mu_1)}{2+c+r_f+\bar{r}}$. (Note that $r_M > r_B$ by Lemma 3.). Skilled investors do not deviate either as they are offering the highest rates any good investors is willing to pay and lend out all their capital. Transparent good entrepreneurs raise the capital they demand at the lowest rate available. Opaque good entrepreneurs are just indifferent to raise capital at $\bar{r}$ by definition. However, picking a lower r^{max} they could not raise any capital.

Mix phase In a mix phase, a group of unskilled investors enter as bold and advertise rate r_M and the reminder enter as cautious and advertise r_M. Skilled advertise r_M. Transparent good entrepreneurs choose $r^{\max} = r_C$, while opaque good entrepreneurs choose r_M. Bad entrepreneurs choose a higher $r^{\max}$ than $\bar{r}$. The entering measures of unskilled investors in each of these markets is determined by the market clearing conditions ensuring that all transparent good entrepreneurs can obtain the capital they demand at r_C, while all opaque good entrepreneurs can obtain the capital they demand at r_M taking into account that all skilled investors should be able to lend out all their capital. (This condition is spelled out in proof of Proposition 2.)

As long as $r_M < \bar{r}$ and $r_B > r_C$, by the definition of r_M and r_C, unskilled are indifferent whether to enter as part of the bold or cautious group or to stay inactive. These conditions are equivalent to those of statement (iii). If these conditions are met, skilled investors cannot offer a higher rate than r_M to opaque good entrepreneurs as they would be undercut by bold unskilled ones. Possible deviations of all other groups can be ruled out analogously to the other cases.

Proof of Proposition 2

We described in the main text how entrepreneurs' decide on investment i and borrowing ℓ taking the interest rate $r(\tau, \omega)$ and the borrowing limit $\bar{\ell}(\tau, \omega)$ as given. Then, expressions in Proposition 2 follow from the determination of $r(\tau, \omega)$ in Proposition 1 and the borrowing limits $\bar{\ell}(\tau, \omega)$ which we derive here. We also derive here $k(\mu_0, \mu_1)$, the equilibrium fraction of unskilled investors who decide to not to enter the credit market in a given state. Consider the bold phase first. The market clearing condition for credit to transparent good and opaque entrepreneurs is

$$w_1 + (1 - k_B)\, w_0\, (1 - \mu_0 - \mu_1) = (1 - \mu_0 - \mu_1)\, \frac{1}{r_B}$$

where $k(\mu_0, \mu_1) = k_B$ in a bold phase. Then, $\bar{\ell}(b, 0)$ is determined by the endowment of unskilled investors which is allocated to bad, opaque credit by the false positives of the bold

test:

$$\mu_0 \bar{\ell}\,(b,0) = (1-k_B)\, w_0 \mu_0$$

implying

$$\bar{\ell}\,(b,0) = \frac{1}{r_B} - \frac{w_1}{(1-\mu_0-\mu_1)} \tag{A.13}$$

and

$$i\,(b,0) = \bar{\ell}\,(b,0)\,(1+r_B) = \frac{(1+r_B)}{r_B} - \frac{(1+r_B)\,w_1}{(1-\mu_0-\mu_1)}.$$

Assumption 1 requires $\frac{w_1}{(1-\mu_0-\mu_1)} < \frac{1}{r_B}$, thus the bad entrepreneurs are constrained in a bold phase.

In the cautious phase market clearing for opaque good firms gives

$$\frac{(1-\mu_0-\mu_1)}{2}\bar{\ell}(g,0) = w_1$$

implying

$$\bar{\ell}(g,0) = \frac{2w_1}{(1-\mu_0-\mu_1)} \tag{A.14}$$

and investment

$$i\,(g,0) = 1 + \frac{2w_1}{(1-\mu_0-\mu_1)}.$$

Assumption 1 requires $\frac{w_1}{(1-\mu_0-\mu_1)} < \frac{1}{2\bar{r}}$ implying that opaque good entrepreneurs are indeed constrained in this phase. The fraction of entering unskilled investors in a cautious phase, $(1-k_C)$, is determined by the market clearing condition for the low interest rate market,

$$\frac{(1-\mu_0-\mu_1)}{2}\frac{1}{r_C} = (1-k_C) w_0 \frac{(1-\mu_0-\mu_1)}{2}.$$

Turning to the mix phase recall from the proof of Proposition 1 that $\frac{1-\mu_0-\mu_1}{\mu_0+\mu_1+1}$ fraction of invested unskilled capital finances good, opaque entrepreneurs at the high interest rate market, $2\frac{\mu_0}{\mu_0+\mu_1+1}$ finances opaque bad entrepreneurs and $2\frac{\mu_1}{\mu_0+\mu_1+1}$ ends up at risk-free storage. Then market clearing for opaque good firms then is

$$\frac{(1-\mu_1-\mu_0)}{2}\ell(g,0) = (1-k_I)\, w_0 \frac{(1-\mu_1-\mu_0)}{1+(\mu_1+\mu_0)} + w_1$$

as opaque good entrepreneurs are not constrained, this implies

$$\frac{1}{2}\frac{1}{r_M} - \frac{w_1}{1-\mu_0-\mu_1} = (1-k_I)\, w_0 \frac{1}{1+(\mu_1+\mu_0)}$$

Then market clearing for bad, opaque entrepreneurs gives

$$\mu_0 \bar{\ell}(b,0) = (1-b_I)\, w_0 2 \frac{\mu_0}{\mu_0+\mu_1+1}.$$

Substituting back $(1-b_I)$ implies

$$\bar{\ell}(b,0) = \left(\frac{1}{2}\frac{1}{r_M} - \frac{w_1}{1-\mu_0-\mu_1}\right) \tag{A.15}$$

and

$$i\,(b,0) = (1+r_M)\left(\frac{1}{2}\frac{1}{r_M} - \frac{w_1}{1-\mu_0-\mu_1}\right).$$

Assumption 1 requires $\frac{w_1}{(1-\mu_0-\mu_1)} < \frac{1}{2r_M}$. Also, w_0 has to be sufficiently large that $k_I, k_B, k_C \in [0,1]$. We can summarize the requirements on w_1 for later use as:

$$\frac{w_1}{(1-\mu_0-\mu_1)} < \min\left(\frac{1}{2\bar{r}}, \frac{1}{2r_M}, \frac{1}{r_B}\right) = \frac{1}{2\bar{r}}. \tag{A.16}$$

For part (v), recall that $Y(\mu_0,\mu_1)$ is the population weighted sum of the outputs $\rho(1+\ell(\tau,\omega))$ for each group of entrepreneurs (τ,ω). The statement follows from the observation that (A.13)-(A.15) and that $\ell(\tau,\omega) = \frac{1}{r(\tau,\omega)}$ in the unconstrained cases and using (A.16). $\ell(g,0)$ discontinuously decreases in μ_0 as it crosses the threshold from below between a bold and a mix phase, or a bold and a cautious phase. Similarly, $\ell(b,0)$ discontinuously decreases in μ_0 as it crosses the threshold from below between a bold and a mix phase, a bold and a cautious phase, or a mix and cautious phase.

Proof of Lemma 4

Comparing (A.5) and (A.6) and using the equilibrium definitions, it is sufficient to show that maximizing life-time utility leads to the same outcome as maximizing stage game utility within each period. That is, introducing endogenous continuation does not change equilibrium strategy profiles.

First, consider a sequence of stage game equilibria consistent with the law of motion for state variables. We will show that in every period, there is no individual deviation from the optimal strategy in the stage game equilibrium which would increase the life-time utility of an entrepreneur who lives more than one period. That is, any sequence of stage game equilibria consistent with the equilibrium law of motion of the state variables (μ_0,μ_1) is a dynamic equilibrium. Proposition 1 implies that in any stage game equilibrium all good entrepreneurs obtain positive credit. That is, they hit the upper limit of their probability of survival, $1-\delta$. As such, they cannot increase the interest rate that they accept, compared to the stage game $\bar{r}$, in order to improve their survival probability. On the other hand, more credit always increases bad entrepreneurs' stage game utility. Furthermore, as long as they are able to raise credit they are indifferent about the corresponding interest rate. Hence, they have no incentive to reduce their reservation interest rate below $\bar{r}$. For them there is no trade-off between stage game utility and increasing the chance of survival by obtaining

more credit.

Second, we show that there is no dynamic equilibrium where the economy is not in a stage game equilibrium in each period. By contradiction, assume that such dynamic equilibrium exist. This implies that there is at least one period in which some good entrepreneur obtains credit at rate $r > \bar{r}$. First note that any good entrepreneur can obtain some credit if he demands a positive amount at an interest rate which a skilled investor advertises. Furthermore, by assumption, any amount of credit is sufficient for an entrepreneur to survive, i.e. maximizes the survival probability at $1 - \delta$. Thus, a necessary condition for such an equilibrium is that all skilled investors advertise an interest rate which is larger than $\bar{r}$.

In such an equilibrium, a good entrepreneur might be willing to borrow at interest rate above $\bar{r}$, lose in the short-term but in return survive with positive probability. Let $r' \equiv \bar{r} + \Delta$ denote the lowest advertised rate by any skilled investor. Note that since continuation value of an entrepreneur is finite, Δ cannot be arbitrarily large. Furthermore, all good entrepreneurs financed at $r > \bar{r}$ would submit only a diminishingly small demand at r' because that leads to minimal current loss and guarantees maximum survival probability. They submit 0 at every higher interest rate. Moreover, assumption A.1 implies that they demand maximum credit at all rates equal or lower than $\bar{r}$, where they make positive current profit and guarantees maximum survival probability. The first consequence is that all skilled investors must advertise the same rate r' as by advertising a higher rate would not lend anything. Second, each skilled investors can only lend out a diminishingly small fraction of her endowment and thus obtains a diminishingly small return on her capital. Hence, a skilled investor can deviate to $r \leq \bar{r}$ and lend a positive measure of her endowment, which is a contradiction. Thus, such an equilibrium does not exist.

Finally, since there is no profitable deviation from the strategies described by the stage game equilibrium, and investors' optimal strategies in the stage game depend only on the concurrent state variables (μ_0, μ_1) (determining the phases of Proposition A.6), there is no dynamic equilibrium where investors would condition on past values of (μ_0, μ_1). That verifies our conjecture that the contemporaneous values of (μ_0, μ_1) are the only state variable of the economy.

Proof of Lemma 5

See appendix C.1 for the proof.

Proof of Propositions 3

The proposition directly follows from birth-death process for entrepreneurs, the equilibrium information choice and lending choice of investors.

Proof of Proposition 4

See appendix C.4 for the proof.

Proof of Proposition 5

See appendix C.4 for the proof.

Proof of Lemma 6

The lemma follows from the following five Lemmas.

Lemma B.1 *Within the pooling region, welfare is decreasing in μ_0.*

Proof. Welfare in the bold phase is

$$\begin{aligned} W_B =& (1-\mu_0-\mu_1)(\rho-1)(1+\frac{1}{r_B})+\mu_0\rho(1+\frac{1}{r_B}-\frac{w_1}{(1-\mu_0-\mu_1)})+\mu_1\rho \\ &+w_0(1+r_f)+w_1(1+r_B) \end{aligned}$$

which we rewrite as

$$\begin{aligned} W_B =& \rho+w_0(1+r_f)+w_1\rho \\ &+(\rho(1-\mu_1)-(1+r_B)(1-\mu_0-\mu_1))\left(\frac{1}{r_B}-\frac{w_1}{(1-\mu_0-\mu_1)}\right) \end{aligned}$$

Note that

$$d(\frac{1}{r_B}-\frac{w_1}{1-\mu_0-\mu_1})/d\mu_0=(-\frac{1}{r_B^2}\frac{dr_B}{d\mu_0}-\frac{w_1}{(1-\mu_0-\mu_1)^2})<0$$

also

$$(1-\mu_1)(\rho-(1+r_f))-c=(1-\mu_1)(\bar{r}-r_B)+\mu_0(1+r_B)>0,$$

implying the result. ■

Lemma B.2 *Within the mix region, welfare is decreasing in μ_0.*

Proof. Welfare in the mix phase is

$$\begin{aligned} W_M =& \frac{1-\mu_0-\mu_1}{2}\left(\rho(1+\frac{1}{r_C})-\frac{1}{r_C}(1+r_C)+\rho(1+\frac{1}{r_M})-\frac{1}{r_M}(1+r_M)\right)+ \\ &\mu_0\rho(1+\left(\frac{1}{2}\frac{1}{r_M}-\frac{w_1}{1-\mu_0-\mu_1}\right))+\mu_1\rho \\ &+w_0(1+r_f)+w_1(1+r_I) \end{aligned}$$

which we rewrite as

$$\begin{aligned} W_M =& \rho+w_1\rho+w_0(1+r_f)+\frac{1-\mu_0-\mu_1}{2}\left((\rho-1)\frac{1}{r_C}-1\right)+ \\ &(\rho(1-\mu_1)-(1-\mu_1-\mu_0)(1+r_M))\left(\frac{1}{2r_M}-\frac{w_1}{1-\mu_0-\mu_1}\right) \end{aligned}$$

Then, the statement follows from the observations that

$$\frac{1}{r_C}, \frac{1-\mu_0-\mu_1}{2}, \left(\rho\left(1-\mu_1\right)-\left(1+\mu_0-\mu_1\right)\left(1+r_f\right)-\left(1+\mu_1+\mu_0\right)c\right)$$

are decreasing in μ_0,

$$\left(\rho-1\right)\frac{1}{r_C}-1>0$$
$$\left(\rho\left(1-\mu_1\right)-\left(1-\mu_1-\mu_0\right)\left(1+r_M\right)\right)=\left(1-\mu_1\right)\left(\rho-\left(1+r_M\right)\right)+\mu_0\left(1+r_M\right))>0$$

as $r_C \leq \bar{r}$, and

$$\frac{1}{2r_M} > \frac{w_1}{1-\mu_0-\mu_1}$$

by (A.16), finally

$$\frac{\partial\left(\frac{1}{2r_M}-\frac{w_1}{1-\mu_0-\mu_1}\right)}{\partial\mu_0}<0$$

as

$$\frac{\partial r_M}{\partial\mu_0}<0.$$

■

Lemma B.3 *Within the separating region, welfare is decreasing in μ_0.*

Proof. Welfare in the cautious phase is

$$\begin{aligned}W_C =&\frac{1-\mu_0-\mu_1}{2}\left(\rho(1+\frac{1}{r_C})-\frac{1}{r_C}\left(1+r_C\right)+\rho(1+\frac{2w_1}{1-\mu_0-\mu_1})-\frac{2w_1}{1-\mu_0-\mu_1}\rho\right)+\mu_0\rho+\mu_1\rho\\&+w_0(1+r_f)+w_1\left(1+\bar{r}\right)\end{aligned}$$

which we rewrite as

$$W_C=\rho+\frac{1-\mu_0-\mu_1}{2}\frac{\left(\rho-1-r_C\right)}{r_C}+w_0(1+r_f)+w_1\rho$$

Then

$$\frac{\partial\left(\frac{1-\mu_0-\mu_1}{2}\frac{\left(\rho-1-r_C\right)}{r_C}\right)}{\partial\mu_0}=\frac{1-\mu_0-\mu_1}{2}\left(-\frac{\rho-1}{r_C^2}\right)\frac{\partial r_C}{\partial\mu_0}-\frac{1}{2}\frac{\left(\rho-1-r_C\right)}{r_C}<0$$

where we used $\frac{\partial r_C}{\partial\mu_0}>0$. This implies the Lemma. ■

Lemma B.4 *Fix μ_1 and μ_0 at any level $\mu_0 \leq \frac{c}{1+r_f}$. Welfare is strictly larger in a pooling equilibrium than it would be in a – counterfactual – separating or mix equilibrium, $W_B\left(\mu_0,\mu_1\right) > W_C\left(\mu_0,\mu_1\right), W_M\left(\mu_0,\mu_1\right)$, as long as $\mu_0 \leq \frac{c}{1+r_f}$.*

Proof. As welfare is aggregate consumption, we can decompose $W_B(\mu_0,\mu_1) - W_C(\mu_0,\mu_1)$ as follows. The difference in transparent good entrepreneurs' consumption is

$$\frac{(1-\mu_0-\mu_1)}{2}(\rho-1)\left(\frac{1}{r_B}+1\right) - \frac{(1-\mu_0-\mu_1)}{2}(\rho-1)\left(\frac{1}{r_C}+1\right)$$

which is non-negative in any point when $r_B \leq r_C$, that is, in the pooling region. The difference in opaque good plus skilled consumption is

$$\left[\frac{(1-\mu_0-\mu_1)}{2}(\rho-1)\left(\frac{1}{r_B}+1\right) + w_1(1+r_B)\right] - \left[\frac{(1-\mu_0-\mu_1)}{2}\rho + w_1(1+\bar{r})\right] \quad \text{(A.17)}$$

note that the term in the first squared bracket is decreasing in r_B as

$$\frac{\partial\left(\frac{(1-\mu_0-\mu_1)}{2}(\rho-1)\left(\frac{1}{r_B}+1\right)+w_1(1+r_B)\right)}{\partial r_B} =$$

$$= -\frac{1}{r_B^2}\frac{(1-\mu_0-\mu_1)}{2}(\rho-1) + w_1 \leq -\frac{1}{r_B^2}\frac{(1-\mu_0-\mu_1)}{2}(\rho-1) + \frac{1-\mu_0-\mu_1}{r_B} =$$

$$= \frac{(1-\mu_0-\mu_1)}{r_B}\left(1-\frac{\rho-1}{r_B}\right) < 0$$

where we used (A.16), and equals to the term in the second left bracket when $r_B = \bar{r}$. That is, (A.17) is non-negative at any point as long as $r_B \leq \bar{r}$. Unskilled consumption is equal under the two regimes, while the difference in bad consumption is equal to

$$\mu_0\left(\frac{1}{r_B} - \frac{w_1}{1-\mu_0-\mu_1}\right) > 0.$$

The proof for $W_B(\mu_0,\mu_1) > W_M(\mu_0,\mu_1)$ is analogous, except that in the second step we show that use that opaque good plus skilled consumption has the form of

$$\left[\frac{(1-\mu_0-\mu_1)}{2}(\rho-1)\left(\frac{1}{r_x}+1\right) + w_1(1+r_x)\right] \quad \text{(A.18)}$$

with interest rates $r_x = r_B, r_M$ in the pooling and mix phases, respectively, which term is decreasing in r_x by (A.16). That is, (A.17) is non-negative at any point as $r_B \leq r_M \leq \bar{r} = \rho - 1$. Finally, the difference in bad consumption is

$$\mu_0\rho\left(\frac{1}{r_B} - \frac{w_1}{(1-\mu_0-\mu_1)}\right) - \mu_0\rho\left(\frac{1}{2}\frac{1}{r_M} - \frac{w_1}{1-\mu_0-\mu_1}\right) =$$

$$= \quad \mu_0\rho\left(\frac{1}{r_B} - \frac{1}{2}\frac{1}{r_M}\right) > 0.$$

■

Lemma B.5 *Welfare jumps downward in μ_0 at the mix-cautious threshold whenever that threshold exists. That is, $W_M(\mu_0,\mu_1) > W_C(\mu_0,\mu_1)$ at $\mu_0 = \tilde{\mu}_0(\mu_1)$.*

Proof. Consider the definition (8) where each element corresponds to the consumption of a group of agent of a given type. Recall that at $\mu_0 = \tilde{\mu}_0(\mu_1)$, $r_M = \bar{r}$ by definition. This the interest at which good investors are indifferent whether to borrow. Therefore, by Propositions 1 and 2, only the consumption of transparent bad types, $\rho(1 + \ell(b, 0))$ is discontinuous at $\mu_0 = \tilde{\mu}_0(\mu_1)$. $\ell(b, 0)$ jumps downward to 0 as opaque bad types cannot borrow when all investors turn to cautious which proves the Lemma. ■

Proof of Proposition 6

Let $\lambda^{\min} \equiv \frac{2c+2r_f}{3c+3r_f+1}$ and $\lambda^{\max} \equiv 2\frac{\rho-c-r_f-1}{2\rho-c-r_f-1}$. We show that under the conditions of the proposition, there is at least one cyclical economy (the one with short-booms and short recessions) which is preferred by the planner compared to both the always bold and always cautious economies. We will argue that for this conclusion, it is sufficient to show that $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$ implies

$$\max(\lim_{\delta\to 0} W_C\left(\bar{\mu}_{0C}, \bar{\mu}_{1C}\right), \lim_{\delta\to 0} W_B\left(\bar{\mu}_{0B}, \bar{\mu}_{1B}\right)) < \lim_{\delta\to 0} \frac{W_B\left(\mu'^*_{0B}, \mu'^*_{1B}\right) + W_C\left(\mu'^*_{0C}, \mu'^*_{1C}\right)}{2}.$$

Note that $\lim_{\delta\to 0} \bar{\mu}_{0B} = \frac{\lambda}{2-\lambda}$ and

$$\lim_{\delta\to 0} \bar{\mu}_{1B}, \bar{\mu}_{1C}, \mu'^*_{1C}, \mu'^*_{1B}, \bar{\mu}_{0C}, \mu'^*_{0C}, \mu'^*_{0B} = 0.$$

In an economy where investors are always bold or always cautious, welfare converges to $W_B\left(\mu'^*_{0B}, \mu'^*_{1B}\right)$ and $W_C\left(\bar{\mu}_{0C}, \bar{\mu}^*_{1C}\right)$ by definition. First, note that

$$\lim_{\delta\to 0} W_C\left(\bar{\mu}_{0C}, \bar{\mu}_{1C}\right) = W_C\left(0, 0\right) < \lim_{\delta\to 0} \frac{W_B\left(\mu'^*_{0B}, \mu'^*_{1B}\right) + W_C\left(\mu'^*_{0C}, \mu'^*_{1C}\right)}{2} = \frac{W_B\left(0, 0\right) + W_C\left(0, 0\right)}{2}.$$

This is implied by Lemma B.4. Then, we show that $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$ is a sufficient condition that

$$\lim_{\delta\to 0} W_C\left(\mu'^*_{0C}, \mu'^*_{1C}\right) > \lim_{\delta\to 0} W_B\left(\bar{\mu}_{0B}, \bar{\mu}_{1B}\right). \tag{A.19}$$

or

$$W_C\left(0, 0\right) > W_B\left(\frac{\lambda}{2-\lambda}, 0\right)$$

which we can rewrite as

$$\left(\rho - 1 - (r_f + c)\right)\frac{1}{2}\frac{1}{r_f + c} > \\ > \left(\rho - 1 - (r_f + c)\right)\left(\frac{1}{r_B(\frac{\lambda}{2-\lambda}, 0, c, r_f)} - \frac{w_1}{\left(1 - \frac{\lambda}{2-\lambda}\right)}\right).$$

This holds when $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$, because by (A.7) $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$ is the condition for

$$\frac{1}{2}\frac{1}{r_f + c} > \frac{1}{r_B(\frac{\lambda}{2-\lambda}, 0, c, r_f)}$$

and $r_B(\frac{\lambda}{2-\lambda}, 0, c, r_f) < \bar{r}$ to hold simultaneously. As all inequalities are strict and all relevant functions are continuous from the left in (μ_0, μ_1), for any $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$ we can pick a $\bar{\delta}(\lambda)$ that if $\delta < \bar{\delta}(\lambda)$ then our statement holds. Picking

$$\bar{\delta} = \max_{\lambda \in [\lambda_{\min}, \lambda_{\max}]} \bar{\delta}(\lambda)$$

defines the threshold for δ.

Proof of Proposition 7

(i) Recall that $W(\mu_0, \mu_1) = W_B(\mu_0, \mu_1)$ for $\mu_0 \leq \frac{c}{1+r_f}$ but $W(\mu_0, \mu_1)$ discontinuously drops at $\mu_0 = \frac{c}{1+r_f}$ in μ_0 by Proposition 6. However, $W_B(\mu_0, \mu_1)$ is differentiable in μ_0 around $\mu_0 = \frac{c}{1+r_f}$. This implies that $W_B(\mu_0, \mu_1) > W(\mu_0, \mu_1)$ if $\mu_0 > \frac{c}{1+r_f}$ as long as μ_0 is sufficiently close to $\frac{c}{1+r_f}$. That is, μ_0^{ext} with the given properties exists.

(ii) Consider the following trade-off:

Consider the case where $w_1 \to 0$, so by continuity, $\exists\ \bar{w}_1$ such that the result holds for $w_1 < \bar{w}_1$.

Let W_{xy} denote the welfare in the second period if the equilibrium in the first and second periods are x and y respectively (B or C). Also let $\mu'_{i,xy}$ denote μ_i in the second period with equilibrium y following the first period with equilibrium x. Finally, let (μ_0, μ_1) denote the state variables in the first period.

Then $\mu'_{0,By} > \mu'_{0,Cy}$ and $\mu'_{0,By} + \mu'_{1,By} > \mu'_{0,Cy} + \mu'_{1,Cy}$, but $\mu'_{1,By} < \mu'_{1,Cy}$. Now, the second period is either cautious or bold

(i) $y = C$: It is straightforward to show that $W_{BC} < W_{CC}$. in CC there are total fewer bad entrepreneurs in the second period, so more entrepreneurs get credit. Interest rate is also lower. Thus more entrepreneurs get cheaper credit too (more transparent good), and for sufficiently small w_1 the production of opaque good entrepreneurs is negligible.

(ii) $y = B$: This case is more complicated as there is a quantity-price trade-off.

  (a) Quantity. More entrepreneurs get credit in the second period in a BB sequence of equilibria compared to a CB sequence (μ'_1 lower).

  (b) Price. $r_{BB} > r_{CB}$, so investment per entrepreneur is lower in BB.

  In order to resolve the trade-off, note that $\exists\ \bar{\rho}$ such that if $\rho < \bar{\rho}$, in the second period the production of the opaque bad entrepreneurs who survive the first bold period and also get credit in the second bold period is sufficiently low, so that it does not compensate the lower production of everyone else who faces a higher interest

rate in the second period because of the presence of a higher measure of opaque bad entrepreneurs following a first-period B equilibrium, compared to a first-period C equilibrium. As such, the price effect in the quantity-price trade-off is stronger, which implies $W_{BB} < W_{CB}$.

Proof of Lemma 7

We give a draft here modifying the proofs of Proposition 1-2 when $x \geq 1$ and, possibly, $r_f^C \neq r_f^B \neq r_f^M \neq r_f$. Let the interest rate functions defined in (A.7)-(A.9) replaced by

$$\begin{aligned} r_B^\pi(\mu_0, \mu_1, c, \pi) &\equiv r_B(\mu_0, \mu_1, c, r_f^B) + \frac{(x-1)(c + r_f^B + 1)(1-\mu_1)}{1-\mu_1-\mu_0}, \\ r_C^\pi(\mu_0, \mu_1, c, \pi) &\equiv r_C(\mu_0, \mu_1, c, r_f^C), \\ r_M^\pi(\mu_0, \mu_1, c, \pi) &\equiv x r_M(\mu_0, \mu_1, c, r_f^M) + (x-1)(1 - \frac{2\mu_1}{1-\mu_0-\mu_1} c), \end{aligned} \tag{A.20}$$

and let $\frac{c}{1+r_f}$ and $\tilde{\mu}_0(\mu_1, c, \rho, \pi)$ replaced by

$$\begin{aligned} \hat{\mu}_0^\pi(\mu_1, c, \pi) &\equiv \frac{c}{1+r_f^C} - \frac{(1-\mu_1)}{1+r_f^C}\left((x-1)\left(1 + r_f^B + c\right) + \left(r_f^B - r_f^C\right)\right), \\ \tilde{\mu}_0^\pi(\mu_1, c, \rho, \pi) &\equiv \frac{(1-\mu_1)(\rho - (1+r_f^M) - (x-1)(c + r_f^M + 1)) - (1+\mu_1)c}{\rho + x(1 + c + r_f^M)}. \end{aligned} \tag{A.21}$$

Clearly, a risk weight of $x > 1$ does not influence the interest rate in a cautious phase as investors are lending to entrepreneurs which they all pay back.

In a bold phase, we require

$$v_g x + v_r = 1$$

but still assume that the technology of a bold test did not change implying

$$\frac{v_g}{v_g + v_r} = (1 - \mu_1).$$

Therefore,

$$v_g = \frac{1-\mu_1}{x(1-\mu_1) + \mu_1}, v_r = \frac{\mu_1}{x(1-\mu_1) + \mu_1}$$

which modifies the indifference condition determining the zero profit rate r_B^x as follows

$$\frac{1-\mu_1}{x(1-\mu_1) + \mu_1}(1 + r_B^x)\frac{(1-\mu_1-\mu_0)}{1-\mu_1} + \frac{\mu_1}{x(1-\mu_1) + \mu_1}(1 + r_f) - c = 1 + r_f$$

implying the expression for r_B^x in the lemma.

In the mix phase, the bold test on the high interest rate market (at which transparent

good entrepreneurs do not apply for credit) implies

$$\frac{v_g}{v_g + v_r} = \frac{\frac{(1-\mu_1-\mu_0)}{2} + \mu_0}{\frac{(1-\mu_1-\mu_0)}{2} + (\mu_1 + \mu_0)}.$$

Therefore

$$v_g = \frac{\mu_0 - \mu_1 + 1}{x + 2\mu_1 + x\mu_0 - x\mu_1}, v_r = 2\frac{\mu_1}{x + 2\mu_1 + x\mu_0 - x\mu_1}$$

in the mix phase. This implies that the indifference condition determining the zero profit rate r_M^x is modified as follows:

$$\frac{1 - \mu_0 - \mu_1}{x + 2\mu_1 + x\mu_0 - x\mu_1}(1 + r_M^x) + 2\frac{\mu_1}{x + 2\mu_1 + x\mu_0 - x\mu_1}(1 + r_f) - c = 1 + r_f$$

which gives the expression of r_M^x in the proposition. Finally, by analogous arguments to the baseline case, the threshold between the bold and cautious phases is given by identity

$$r_B^x\left(\bar{\mu}_0^x(\mu_1, c, r_f), \mu_1, c, r_f, x\right) \equiv r_C\left(\bar{\mu}_0^x(\mu_1, c, r_f), \mu_1, c, r_f\right)$$

while the threshold $\tilde{\mu}^x(\cdot)$ is given by identity

$$r_M^x\left(\tilde{\mu}_0^x(\mu_1, \rho_g, c, r_f), \mu_1, c, r_f, x\right) \equiv \rho - 1.$$

Otherwise, the proofs of Propositions 1 and 2 go through.

Proof of Proposition 8

First, we construct equivalent policies with the lowest implied cost of capital. Let $(m_{0,i}, m_{1,i})_{i=1}^{\kappa}$ the invariant set corresponding to the constrained planner's choice of $\hat{\mu}_0^P$. We define i' as the index of the smallest $m_{0,i} \in (m_{0,i})_{i=1}^{\kappa}$ such that $m_{0,i} > \hat{\mu}_0^P$. Note that Proposition 1 imply that $\max(\tilde{\mu}_0^{\pi}(m_{1,i'}, c, \rho, \pi), \hat{\mu}_0^{\pi}(m_{1,i'}, c, \pi)) = m_{0,i'}$ is sufficient to ensure that policy π is an equivalent policy to the planner's choice $\hat{\mu}_0^P$. Then, we can pick r_f, r_f^B and x for the equivalent policies $\pi_{r_f}, \pi_{r_f^B}, \pi_x$ as follows:

$$\begin{aligned}
r_f &= \max\left(\frac{(1 - m_{1,i'})(\rho - 1) - (1 + m_{1,i'})c - m_{0,i'}(\rho + 1 + c)}{m_{0,i'} + (1 - m_{1,i'})}, \frac{c - m_{0,i'}}{m_{0,i'}}\right), \\
r_f^B &= \frac{c - m_{0,i'}}{1 - m_{1,i'}}, \\
x &= \max\left(\frac{((1 - m_{1,i'})\rho - 2cm_{1,i'} - \rho m_{0,i'})}{((m_{0,i'} + (1 - m_{1,i'}))(c + 1)))}, \frac{c - m_{0,i'}}{((1 + c)(1 - m_{1,i'})) + 1)}\right).
\end{aligned}$$

In this proposition, we focus on those economies when the implied cycle does not feature a mix phase, that is, the relevant expression for r_f and x is the second term within the max

operator. Note that welfare in the bold and cautious phase is

$$\begin{aligned} W_B^{\pi}(\mu_0,\mu_1;\pi) = \rho + \\ (\rho-1)\left((1-\mu_0-\mu_1)\frac{1}{r_B^{\pi}(\mu_0,\mu_1,c,\pi)} + \mu_0\left(\frac{1}{r_B^{\pi}(\mu_0,\mu_1,c,\pi)} - \frac{w_1}{(1-\mu_0-\mu_1)}\right)\right) + \\ + (w_0+w_1) - c\left(\left(\frac{1}{r_B^{\pi}(\mu_0,\mu_1,c,\pi)} - \frac{w_1}{(1-\mu_0-\mu_1)}\right)\right) \\ - (1-\mu_1)(x-1)c\left(\frac{1}{r_B^{\pi}(\mu_0,\mu_1,c,\pi)} - \frac{w_1}{(1-\mu_1-\mu_0)}\right) \end{aligned} \quad \text{(A.22)}$$

and

$$\begin{aligned} W_C^{\pi} = \rho + \frac{1}{r_C^{\pi}(\mu_0,\mu_1,c,\pi)}\left(\frac{1-\mu_0-\mu_1}{2}(\rho-1) - c\right) \\ + w_0 + w_1\rho. \end{aligned}$$

These formulas follow the calculation in the baseline case with the additional adjustment in the last line of (A.22). For that last term, the market clearing condition in a bold phase is

$$w_1 + (1-k_P)\, w_0 v_g \frac{(1-\mu_1-\mu_0)}{((1-\mu_1-\mu_0)+\mu_0)} = (1-\mu_0-\mu_1)\frac{1}{r_B}$$

where v_g is the bold investor's credit to entrepreneurs, while $\frac{(1-\mu_1-\mu_0)}{((1-\mu_1-\mu_0)+\mu_0)}$ is the fraction of good firms passing her test. Then the fraction of entering unskilled investors $(1-k_P)$ has to satisfy

$$(1-k_P)\, w_0 = \frac{x(1-\mu_1)+\mu_1}{(1-\mu_1-\mu_0)}\left((1-\mu_0-\mu_1)\frac{1}{r_B} - w_1\right).$$

This implies that the total cost paid by these entrants is

$$-c(1-k_P)\, w_0 = -(x(1-\mu_1)+\mu_1)\, c\left(\frac{1}{r_B} - \frac{w_1}{(1-\mu_1-\mu_0)}\right).$$

As $(x(1-\mu_1)+\mu_1) > 1$, this implies an adjustment of

$$-((x(1-\mu_1)+\mu_1)-1)\, c\left(\frac{1}{r_B} - \frac{w_1}{(1-\mu_1-\mu_0)}\right)$$

which is the last term in (A.22).

For the first statement, note that monetary policy effects welfare only through the cost

of capital r_B^π and r_C^π. As

$$\begin{aligned}\frac{\partial W_B^\pi}{\partial r_B^\pi} &= -\frac{1}{\left(r_B^\pi\right)^2}\left(\left(\rho-1\right)\left(1-\mu_1\right)-c\left(\mu_1+\left(1-\mu_1\right)x\right)\right)= \\ &= -\frac{1}{\left(r_B^\pi\right)^2}\left(\left(\rho-1\right)\left(1-\mu_1\right)-\left(1-\mu_0-\mu_1\right)\left(1+r_B^\pi\right)+\left(r_f^B+1\right)x(1-\mu_1)\right)= \\ &= -\frac{1}{\left(r_B^\pi\right)^2}\left(\left(\bar{r}-r_B^\pi\right)\left(1-\mu_1\right)+\left(r_f^B x+\left(x-1\right)\right)\left(1-\mu_1\right)+\mu_0\left(1+r_B^\pi\right)\right)<0\end{aligned}$$

and

$$\frac{\partial W_C^\pi}{\partial r_C^\pi}<0$$

and $\frac{\partial r_B^\pi}{\partial r_f}, \frac{\partial r_C^\pi}{\partial r_f}>0$, any of our monetary policies lead to smaller welfare than the equivalent $\hat{\mu}_0^P$. The macro-prudential policy has a similar negative effect through cost of capital as $\frac{\partial r_B^\pi}{\partial x}>0$, along with an additional direct negative effect

$$\frac{\partial W_B^\pi}{\partial x}=-\left(1-\mu_1\right)c\left(\frac{1}{r_B^\pi(\mu_0,\mu_1,c,\pi)}-\frac{w_1}{\left(1-\mu_1-\mu_0\right)}\right)<0$$

The term in the bracket is the loan amount of an opaque bad borrower, hence it is positive.

The additional argument for the second statement is to show that equivalent π_x and $\pi_{r_f^B}$ implies the same r_B^π and r_C^π. None of them have an effect on r_C^π and

$$\begin{aligned}r_B^\pi(\mu_0,\mu_1,c,\pi_{r_f^B})-r_B^\pi(\mu_0,\mu_1,c,\pi_x)&= \\ &= \frac{1-\mu_1}{1-\mu_1-\mu_0}r_f^B-\frac{(x-1)(c+1)(1-\mu_1)}{1-\mu_1-\mu_0}= \\ &= \frac{1-\mu_1}{1-\mu_1-\mu_0}\left(x-1\right)\left(c+1\right)-\frac{(x-1)(c+1)(1-\mu_1)}{1-\mu_1-\mu_0}=0,\end{aligned}$$

where, in the second step, we used expression (A.21) and that $\mu_0^\pi(\mu_1,c,\pi_x)=\mu_0^\pi(\mu_1,c,\pi_{r_f^B})$ under the conditions of the statement.

For the first part of the third statement, consider first the simple monetary policy $r_f=\frac{c-m_{0,i'}}{m_{0,i'}}$, and the equivalent counter-cyclical monetary policy $r_f^B=\frac{c-m_{0,i'}}{1-m_{1,i'}}$. Note that, if $r_f\geq r_f^B$ in the two equivalent policies than welfare is weakly smaller in the bold phase and strictly smaller in the cautious phase under the simple monetary policy. Hence, it is sufficient to show that

$$\frac{c-\mu_0}{1-\mu_1}<\frac{c-\mu_0}{\mu_0}$$

whenever μ_0 implies a bold phase and μ_1 is within the support of the ergodic distribution of μ_1, $\mu_1\in[\bar{\mu}_{1B},\bar{\mu}_{1C}]$. As $\hat{\mu}_0^\pi(\mu_1,c,\pi)<\bar{\mu}_{0B}$ in any cyclical economy, it is sufficient that

$$1-\bar{\mu}_{1C}=1-\frac{\delta\lambda}{2((1-\delta)(1-\lambda)+\delta)}\geq\bar{\mu}_{0B}$$

or

$$\frac{\lambda}{2-(1-\delta)\lambda} \leq 1 - \frac{\delta\lambda}{2((1-\delta)(1-\lambda)+\delta)},$$

for which $\lambda \leq \frac{8}{9}$ is a sufficient condition.

Finally, the last result and the second statement implies the final statement: if c is sufficiently small than average welfare under π_x and $\pi_{r_f^B}$ is sufficiently close. As $\pi_{r_f^B}$ strictly dominates π_{r_f}, π_x is also more efficient than π_{r_f}.

C The Cyclical Dynamic Equilibrium: Characterization and Existence Conditions

In this appendix, we provide detailed characterization for a class of cyclical dynamic equilibria in our economy. This class is defined by the property that the finite invariant set $\{\mathbf{m}_i\}_{i=1}^{\kappa} \equiv (m_{0,i}, m_{1,i})_{i=1}^{\kappa}$, or a cyclical permutation of it, is monotonic in i. All the cases we highlight in the main text are within this class. Here, we present sufficient and necessary conditions for the existence of each member of this class. We also show uniqueness within this class, that is, at most one equilibrium within this class can exist for a given set of parameters. As we explain below, while for some parameter values cyclical equilibria exists outside of this class, they tend to have very similar properties to the ones exposed here.

C.1 Steady States; Proof of Lemma 5

Let

$$\boldsymbol{\mu}_t = \begin{bmatrix} \mu_{0t} \\ \mu_{1t} \end{bmatrix}$$
$$\mathbf{a} = \begin{bmatrix} \delta\frac{\lambda}{2} \\ \delta\frac{\lambda}{2} \end{bmatrix}$$
$$\mathbf{A}_C = \begin{bmatrix} (1-\delta)\frac{\lambda}{2} & (1-\delta)\frac{\lambda}{2} \\ (1-\delta)\frac{\lambda}{2} & (1-\delta)\frac{\lambda}{2} \end{bmatrix}$$

and

$$\mathbf{A}_B = \begin{bmatrix} (1-\delta) & (1-\delta)\frac{\lambda}{2} \\ 0 & (1-\delta)\frac{\lambda}{2} \end{bmatrix}$$

. By Proposition 3, if $\mu_0 \in \left[0, \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}\right]$ then

$$\mathbf{a} + \mathbf{A}_B\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t+1} \tag{A.23}$$

and $\bar{\boldsymbol{\mu}}_B$ solves

$$\mathbf{a} + \mathbf{A}_B\bar{\boldsymbol{\mu}}_B = \bar{\boldsymbol{\mu}}_B \tag{A.24}$$

or

$$\bar{\boldsymbol{\mu}}_B = -\left(\mathbf{A}_B - \mathbf{I}\right)^{-1}\mathbf{a} = \begin{pmatrix} \frac{\lambda}{-\lambda+\lambda\delta+2} \\ \lambda\frac{\delta}{-\lambda+\lambda\delta+2} \end{pmatrix},$$

a unique fixed point under the permanent bold regime. Clearly, the stationary steady state $\bar{\boldsymbol{\mu}}_B$ exists if $\bar{\mu}_{0B} \leq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\bar{\mu}_{1B})\}$.

If $\mu_0 \in (\max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}, 1]$ then

$$\mathbf{a} + \mathbf{A}_C \boldsymbol{\mu}_t = \boldsymbol{\mu}_{t+1} \tag{A.25}$$

and $\bar{\boldsymbol{\mu}}_C$ solves

$$\mathbf{a} + \mathbf{A}_C \bar{\boldsymbol{\mu}}_C = \bar{\boldsymbol{\mu}}_C \tag{A.26}$$

or

$$\bar{\boldsymbol{\mu}}_C = -\left(\mathbf{A}_C - \mathbf{I}\right)^{-1} \mathbf{a} = \begin{bmatrix} \frac{1}{2}\lambda \frac{\delta}{-\lambda+\lambda\delta+1} \\ \frac{1}{2}\lambda \frac{\delta}{-\lambda+\lambda\delta+1} \end{bmatrix} = \begin{bmatrix} \bar{\mu}_{0C} \\ \bar{\mu}_{1C} \end{bmatrix},$$

a unique fixed point under the permanent bold regime. The stationary steady state $\bar{\boldsymbol{\mu}}_C$ exists if $\bar{\mu}_{0C} \geq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\bar{\mu}_{1C})\}$. Note that $\bar{\mu}_{0C} \leq \bar{\mu}_{0B}$ but $\bar{\mu}_{1C} \geq \bar{\mu}_{1B}$. Furthermore,

$$0 < \tilde{\mu}_0(\bar{\mu}_{1B}) - \tilde{\mu}_0(\bar{\mu}_{1C}) = (\bar{\mu}_{1C} - \bar{\mu}_{1B}) \frac{\left(\frac{\rho}{(r_f+1)} + \frac{c}{(r_f+1)} - 1\right)}{1 + \frac{c}{1+r_f} + \frac{\rho_f}{1+r_f}} < (\bar{\mu}_{1C} - \bar{\mu}_{1B}) < (\mu'_{0B} - \mu'_{0C})$$

for any $\frac{\rho}{(r_f+1)} > 1$ and $\delta, \lambda \in (0,1)$. That is, at most one of the steady states can exist. Furthermore, both systems (A.23) and (A.25) are stable as the all eigenvalues of A_B and A_C are within the unit circle. This concludes Lemma 5.2.

C.2 Monotonicity Properties

Before, we proceed, it is useful to establish some monotonicity properties when $\mu_0 \in [\bar{\mu}_{0C}, \bar{\mu}_{0B}]$ and $\mu_1 \in [\bar{\mu}_{1B}, \bar{\mu}_{1C}]$. We will loosely refer to this range as $[\bar{\boldsymbol{\mu}}_C, \bar{\boldsymbol{\mu}}_B]$. Observe that under each dynamics, (A.23) and (A.25) both $\mu_{0,t}$ and $\mu_{1,t}$ monotonically converge to their respective steady states, but from opposite directions. For instance, under (A.25), $\mu_{0,t} > \mu_{0,t+1} > \bar{\mu}_{0C}$ and $\mu_{1,t} < \mu_{1,t+1} < \bar{\mu}_{1C}$ This can be seen by using (A.25)

$$\begin{aligned} \mu_{0,t} - \mu_{0,t+1} &= \mu_{0,t} - \delta\frac{\lambda}{2} - (1-\delta)\frac{\lambda}{2}\mu_{0,t} - (1-\delta)\frac{\lambda}{2}\mu_{1,t} > \\ &= \bar{\mu}_{0C}\left(1 - (1-\delta)\frac{\lambda}{2}\right) - \delta\frac{\lambda}{2} - (1-\delta)\frac{\lambda}{2}\bar{\mu}_{1C} = 0 \end{aligned}$$

and

$$\begin{aligned} \mu_{1,t} - \mu_{1,t+1} &= \mu_{1,t} - \delta\frac{\lambda}{2} - (1-\delta)\frac{\lambda}{2}\mu_{0,t} - (1-\delta)\frac{\lambda}{2}\mu_{1,t} < \\ &= \bar{\mu}_{1C}\left(1 - (1-\delta)\frac{\lambda}{2}\right) - \delta\frac{\lambda}{2} - (1-\delta)\frac{\lambda}{2}\bar{\mu}_{0C} = 0. \end{aligned}$$

Similarly, under (A.23)

$$\begin{aligned}\mu_{0,t}-\mu_{0,t+1} &= \mu_{0,t}-\delta\frac{\lambda}{2}-(1-\delta)\,\mu_{0,t}-(1-\delta)\,\frac{\lambda}{2}\mu_{1,t}< \\ &= \bar{\mu}_{0B}\left(1-(1-\delta)\right)-\delta\frac{\lambda}{2}-(1-\delta)\,\frac{\lambda}{2}\bar{\mu}_{1B}=0\end{aligned}$$

and

$$\begin{aligned}\mu_{1,t}-\mu_{1,t+1} &= \mu_{1,t}-\delta\frac{\lambda}{2}-(1-\delta)\,\frac{\lambda}{2}\mu_{1,t}> \\ &= \bar{\mu}_{1B}\left(1-(1-\delta)\,\frac{\lambda}{2}\right)-\delta\frac{\lambda}{2}=0.\end{aligned}$$

C.3 Monotonic Invariant Sets

Next, we construct all potential finite invariant sets, $\{\mathbf{m}_i\}_{i=1}^{\kappa}\equiv(m_{0,i},m_{1,i})_{i=1}^{\kappa}$ for our dynamic equilibria which are monotonic in i. For each $\kappa>2$, there exists two candidates.

(i) A $B-$cycle cycles through $\left(\mathbf{m}_i^{B,\kappa}\right)$ $i=1,..\kappa.$, a sequence of two-dimensional vectors with monotonically increasing first dimension and monotonically decreasing second dimension. In this cycle, the first $\kappa-1$ steps are implied by (A.23) and then a step implied by (A.25) pushes back the economy to its starting point $\mathbf{m}_1^B$. This implies that $\mathbf{m}_1^{B,\kappa}$ has to satisfy

$$\mathbf{m}_1^{B,\kappa}=\Sigma_{n=0}^{\kappa-1}\left(\mathbf{A}_B\right)^n\mathbf{a}+\left(\mathbf{A}_B\right)^{\kappa-1}\mathbf{A}_C\mathbf{m}_1^{B,\kappa}$$

implying

$$\mathbf{m}_1^{B,\kappa}=\left(\mathbf{I}-\left(\mathbf{A}_B\right)^{\kappa-1}\mathbf{A}_C\right)^{-1}\Sigma_{n=0}^{\kappa-1}\left(\mathbf{A}_B\right)^n\mathbf{a}.$$

Clearly, there is a unique such point. Then, for any $i\in[2,\kappa]$ we have

$$\mathbf{m}_i^{B,\kappa}=\Sigma_{n=0}^{i-2}\left(\mathbf{A}_B\right)^n\mathbf{a}+\left(\mathbf{A}_B\right)_1^{i-1}\mathbf{m}_1^{B,\kappa}.$$

(ii) A $C-$cycle has the support of $\left(\mathbf{m}_i^C\right)$ $i=1,..\kappa$, which is monotonically decreasing in its first element, and monotonically decreasing in its second one. That is, starting from $\mathbf{m}_{t_0+1}=\mathbf{m}_1^C$, for any non-negative integer k, if $t=t_0+k\kappa+i$ then $\boldsymbol{\mu}_t=\mathbf{m}_i^C$. In this cycle, the first $\kappa-1$ steps are implied by (A.25) and then a step implied by (A.23) pushes back the economy to its starting point $\mathbf{m}_1^C$. This implies that $\mathbf{m}_1^C$ has to satisfy

$$\mathbf{m}_1^{C,\kappa}=\Sigma_{n=0}^{\kappa-1}\left(\mathbf{A}_C\right)^n\mathbf{a}+\left(\mathbf{A}_C\right)^{\kappa-1}\mathbf{A}_B\mathbf{m}_1^{C,\kappa}$$

implying

$$\mathbf{m}_1^{C,\kappa}=\left(\mathbf{I}-\left(\mathbf{A}_C\right)^{\kappa-1}\mathbf{A}_B\right)^{-1}\Sigma_{n=0}^{\kappa-1}\left(\mathbf{A}_C\right)^n\mathbf{a}.$$

Clearly, there is a unique such point. Then, for any $i\in[2,\kappa]$ we have

$$\mathbf{m}_i^{C,\kappa}=\Sigma_{n=0}^{i-2}\left(\mathbf{A}_C\right)^n\mathbf{a}+\left(\mathbf{A}_C\right)^{i-1}\mathbf{m}_1^{C,\kappa}. \tag{A.27}$$

For $\kappa = 2$, two algorithms above imply the same $\{\mathbf{m}_i\}_{i=1}^2$ values

$$\mathbf{m}_1^2 = \mathbf{m}_1^{B,2} = \mathbf{m}_2^{C,2} = (\mathbf{I} - (\mathbf{A}_B)\,\mathbf{A}_C)^{-1}(\mathbf{a} + \mathbf{A_B a})$$
$$\mathbf{m}_2^2 = \mathbf{m}_2^{B,2} = \mathbf{m}_1^{C,2} = (\mathbf{I} - (\mathbf{A}_B)\,\mathbf{A}_C)^{-1}(\mathbf{a} + \mathbf{A_C a}).$$

In the main text, we denote the first element of $\mathbf{m}_1^2$ and $\mathbf{m}_2^2$ as $\mu_{0C}^{\prime *}$ and $\mu_{0B}^{\prime *}$ respectively.

C.4 Necessary Conditions; Proof of Propositions 4 and 5

Consider B-cycles first. For the invariant set $\left(\mathbf{m}_i^{B,\kappa}\right)_{i=1}^{\kappa}$ to be part of a cyclical dynamic equilibrium, we need that

$$\left[\mathbf{m}_\kappa^{B,\kappa}\right]_1 > \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\left[\mathbf{m}_\kappa^{B,\kappa}\right]_2)\} \tag{A.28}$$
$$\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1 \leq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_2)\}, \tag{A.29}$$

where $\left[\mathbf{m}_\kappa^{B,\kappa}\right]_1$ and $\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1$ denote the largest and second largest implied μ_0 value along this invariant set. Note, that under these conditions, this is a locally stable cycle because all the eigenvalues of $(\mathbf{A}_B)^{\kappa-1}\mathbf{A}_C$ are inside the unit cycle for any κ. (The largest eigenvalue is $\frac{1}{2^{\kappa-2}}\left(\lambda^{\kappa-1} + \Sigma_{i=0}^{\kappa-2}\lambda^i 2^{\kappa-2-i}\right)\lambda(1-\delta)^\kappa < 1$).

The corresponding equilibrium is a bold-cautious two-phase economy, if

$$\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1 \leq \frac{c}{1+r_f} < \left[\mathbf{m}_\kappa^{B,\kappa}\right]_1 \tag{A.30}$$

and

$$\tilde{\mu}_0(\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_2) < \left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1, \tag{A.31}$$

and a bold-mix-cautious three-phase economy[21] if

$$\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1 \leq \tilde{\mu}_0(\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_2) < \left[\mathbf{m}_\kappa^{B,\kappa}\right]_1 \tag{A.32}$$

and

$$\frac{c}{1+r_f} < \left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1. \tag{A.33}$$

and

$$\left[\mathbf{m}_1^{B,\kappa}\right]_1 < \frac{c}{1+r_f}. \tag{A.34}$$

Two important observations, which can be justified by tedious algebra, are that

(i) $\mathbf{m}_i^{B,\kappa} \in (\bar{\boldsymbol{\mu}}_C, \bar{\boldsymbol{\mu}}_B)$ and

[21] If (A.32)-(A.33) hold, but (A.34) is violated, we have a cautious-mix economy. This case is qualitatively similar to a bold-cautious two-phase economy, hence we do not discuss it in the main text.

(ii) $\left[\mathbf{m}_{\kappa}^{B,\kappa+1}\right]_1 - \left[\mathbf{m}_{\kappa}^{B,\kappa}\right]_1 > 0$, that is, the relevant intervals for the thresholds to imply a B-cycle of length κ are increasing and non-overlapping.

Given Proposition 1, the characterization in Proposition 5 and case (ii) in Proposition 4 follow.

Analogously, if $\left(\mathbf{m}_i^{B,\kappa}\right)_{i=1}^{\kappa}$ is part of a cyclical dynamic equilibrium then conditions

$$\left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1 < \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_2)\}$$
$$\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1 \geq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\left[\boldsymbol{\mu}_{\kappa-1}^{C,\kappa}\right]_2)\}$$

must hold, implying a locally stable cycle because all the eigenvalues of $(\mathbf{A}_C)^{\kappa-1}\mathbf{A}_B$ are inside the unit cycle for any κ. (The largest eigenvalue has the form of $\frac{1}{2}\lambda^{\kappa-1}(1-\delta)^{\kappa}(\lambda+1)$). Also, $\mathbf{m}_i^{C,\kappa} \in (\bar{\boldsymbol{\mu}}_C, \bar{\boldsymbol{\mu}}_B)$ for all i and $\left[\mathbf{m}_{\kappa}^{C,\kappa+1}\right]_1 - \left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1 < 0$. That is, the relevant intervals for the thresholds to imply a C-cycle of length κ are decreasing and non-overlapping. If the corresponding cyclical dynamic equilibrium is a bold-cautious two-phase economy[22], then

$$\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1 > \frac{c}{1+r_f} \geq \left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1 \tag{A.35}$$

and

$$\tilde{\mu}_0(\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_2) \leq \left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1 \tag{A.36}$$

must also hold. Case (iii) in Proposition 4 is implied by these conditions. Case (i) corresponds to a cyclical dynamic equilibrium of length $\kappa = 2$. A necessary condition for this case is

$$\left[\mathbf{m}_1^2\right]_1 \leq \frac{c}{1+r_f} < \left[\mathbf{m}_2^2\right]_1 \tag{A.37}$$

and

$$\tilde{\mu}_0(\left[\mathbf{m}_2^2\right]_2) < \left[\mathbf{m}_2^2\right]_1, \tag{A.38}$$

in line with the statement.

C.5 Sufficient Conditions

There is one additional condition to make sure that a given invariant set $\{\mathbf{m}_i\}_{i=1}^{\kappa}$ is part of a cyclical dynamic equilibrium. It is that the economy is not in autarky, or

$$\min\left(\frac{r_B(\mu_0,\mu_1,c,r_f)+1}{1+r_f}, \frac{r_C(\mu_0,\mu_1,c,r_f)+1}{1+r_f}\right) < \frac{\rho}{1+r_f}$$

[22] A mix-cautious 2-phase economy is also possible, if

$$\frac{c}{1+r_f} < \left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1 \leq \tilde{\mu}_0(\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_2) < \left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1.$$

.

for any $(\mu_0, \mu_1) \in \{\mathbf{m}_i\}_{i=1}^{\kappa}$.

The following Lemma is useful to establish sufficient conditions for a cyclical dynamic equilibrium.

Lemma C.6 *Suppose that* $1 - \bar{\mu}_{1C} - \bar{\mu}_{0B} > 0$, *and*

$$\frac{(1 - \bar{\mu}_{1C} - \bar{\mu}_{0B})}{2} \frac{\frac{-2\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}}}{\frac{(1-\bar{\mu}_{1C}-\bar{\mu}_{0B})}{2} - \frac{1-\bar{\mu}_{0B}-\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}}} > \bar{\mu}_{0C}. \tag{A.39}$$

Then, condition

$$\max\left(\frac{-2\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}} + \left(\frac{\rho}{1+r_f} - 1\right)\frac{1-\bar{\mu}_{0B}-\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}}, \bar{\mu}_{0C}\right) < \frac{c}{1+r_f} < \min\left(\frac{(1-\bar{\mu}_{1C}-\bar{\mu}_{0B})}{2}\left(\frac{\rho}{1+r_f} - 1\right), \bar{\mu}_{0B}\right). \tag{A.40}$$

defines a connected set of $\frac{\rho}{1+r_f} > 1$ *and* $\frac{c}{1+r_f}$ *values. When* $\frac{\rho}{1+r_f}$ *and* $\frac{c}{1+r_f}$ *are in this set, then for any* $\mu_0 \in [\bar{\mu}_{0C}, \bar{\mu}_{0B}]$ *and* $\mu_1 \in [\bar{\mu}_{1B}, \bar{\mu}_{1C}]$ *the economy is not in autarky, that is*

$$\min\left(\frac{r_B(\mu_0, \mu_1, c, r_f) + 1}{1+r_f}, \frac{r_C(\mu_0, \mu_1, c, r_f) + 1}{1+r_f}\right) < \frac{\rho}{1+r_f} \tag{A.41}$$

and $\max\left(\frac{c}{1+r_f}, \tilde{\mu}(\mu_1)\right) \in [\bar{\mu}_{0C}, \bar{\mu}_{0B}]$, *hence the economy is not stationary.*

Proof. For any $\mu_0 \in [\bar{\mu}_{0C}, \bar{\mu}_{0B}]$ and $\mu_1 \in [\bar{\mu}_{1B}, \bar{\mu}_{1C}]$, a sufficient condition for (A.41) is

$$\min\left(\left(\frac{1-\bar{\mu}_{1C}}{1-\bar{\mu}_{1C}-\bar{\mu}_{0B}} + \frac{1}{1-\bar{\mu}_{1C}-\bar{\mu}_{0B}}\frac{c}{1+r_f}\right), 1 + \frac{2}{1-\bar{\mu}_{1C}-\bar{\mu}_{0B}}\frac{c}{1+r_f}\right) < \frac{\rho}{1+r_f}$$

by the monotonicity of the functions $r_B(\cdot)$ and $r_C(\cdot)$ in μ_0 and μ_1. Note that if $1-\bar{\mu}_{1C}-\bar{\mu}_{0B} > 0$, this is equivalent to

$$\frac{c}{1+r_f} < \frac{(1-\bar{\mu}_{1C}-\bar{\mu}_{0B})}{2}\left(\frac{\rho}{1+r_f} - 1\right). \tag{A.42}$$

Now consider the condition $\tilde{\mu}(\mu_1) < \bar{\mu}_{0B}$. By the monotonicity of $\tilde{\mu}(\mu_1)$ in μ_1, it is sufficient that $\tilde{\mu}(\bar{\mu}_{1B}) < \bar{\mu}_{0B}$, which we rewrite as

$$\frac{-2\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}} + \left(\frac{\rho}{1+r_f} - 1\right)\frac{1-\bar{\mu}_{0B}-\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}} < \frac{c}{1+r_f} \tag{A.43}$$

The two conditions along with $\frac{c}{1+r_f} \in [\bar{\mu}_{0C}, \bar{\mu}_{0B}]$ aggregates to (A.40). Consider the space of $\frac{c}{1+r_f}$ values on the y-axis and $\left(\frac{\rho}{1+r_f} - 1\right)$ values on the x axis Then we need the set between two horizontal lines $(\bar{\mu}_{0C}, \bar{\mu}_{0B})$ and two increasing lines. The line corresponding to the left hand side of (A.43) starts at a negative value, while the one corresponding the right hand

side of (A.42) starts at 0. As long as their intercept is above $\bar{\mu}_{0C}$, the set exists. The intercept is at

$$\frac{\frac{-2\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}}}{\frac{(1-\bar{\mu}_{1C}-\bar{\mu}_{0B})}{2}-\frac{1-\bar{\mu}_{0B}-\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}}}=\left(\frac{\rho}{1+r_f}-1\right)$$

therefore we need (A.39). ■

It is simple to show that $(1-\bar{\mu}_{1C}-\bar{\mu}_{0B})>0$ if $\lambda<\frac{8}{9}$. With tedious algebra, one can also show that (A.39) holds if $\lambda<\bar{\lambda}$ where $\bar{\lambda}$ is a specific root of a six-order polynomial and $\bar{\lambda}>\frac{3}{4}$. (The numerically solution is $\bar{\lambda}=0.774388$). Therefore, $\lambda<\frac{3}{4}$ can be used to replace the conditions of the Lemma. One can also show that there is a real subset of $\frac{\rho}{1+r_f}$ and $\frac{c}{1+r_f}$ values satisfying (A.40) generating one of the cycles $B-$ or $C-$cycles we defined above. For this, note that sufficiently large κ, the interval $\left(\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1,\left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1\right)$ gets arbitrarily close to $\bar{\mu}_{0C}$ from above. Hence there must be a set of $\frac{c}{1+r_f}$ values, close to $\bar{\mu}_{0C}$, which simultaneously satisfy (A.40) and are within the interval $\left(\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1,\left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1\right)$, implying a C-cycle of length κ.

C.6 Other Classes of Cyclical Dynamic Equilibria

Suppose, that $\tilde{\mu}_0(\bar{\mu}_{1B})\leq\frac{c}{1+r_f}$, so we must have a two-phase economy. As we have established, intervals of the form $\left(\left[\mathbf{m}_{\kappa-1}^{x,\kappa}\right]_1,\left[\mathbf{m}_{\kappa}^{x,\kappa}\right]_1\right)$, $x=B,C$ are non-overlapping. That is, there must be a set of parameters that

$$\frac{c}{1+r_f}\in\left(\left[\mathbf{m}_{\kappa-1}^{C,\kappa-1}\right]_1,\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1\right).$$

This implies that the necessary conditions established in section C.4 for a cyclical dynamic equilibrium with monotonic $(\mathbf{m}_i^{\kappa})_{i=1}^{\kappa}$ are violated. Is there a cyclical dynamic equilibrium for such set of parameters? Our simulations show that in these sets, our economy still converge to a cyclical dynamic equilibrium where $(\mathbf{m}_i^{\kappa})_{i=1}^{\kappa}$ consists of a finite number of subsequent monotonic series. For instance, when $\frac{c}{1+r_f}$ is too high for a $\kappa=3$ B-cycle, but still too low for a $\kappa=4$ B-cycle, then the economy converges to a cycle which is in a bold phase for 4 periods, then cautious for a single period, then bold for 3 periods and only then, after an additional cautious period, returns to its starting point . By a trivial modification of our algorithm in section C.4, it is possible to establish necessary conditions for these slightly more complex cycles. However, given that the economic properties of these cycles are very similar to the ones with monotonic $(\mathbf{m}_i^{\kappa})_{i=1}^{\kappa}$, this would not add anything to the analysis, hence, we leave it for the interested reader.

D Continuum of Tests

Assume there is a continuum of tests, indexed by $s\in[0,1]$. Every test s passes all $\frac{1-\mu_0-\mu_1}{2}$ applications of transparent good entrepreneurs and rejects all μ_1 applications of transparent bad entrepreneurs. Furthermore, test s passes s fraction of the applications of opaque

entrepreneurs, i.e. $s\frac{1-\mu_0-\mu_1}{2}$ good projects and $s\mu_0$ bad opaque projects. Thus, $s=0$ corresponds to the cautious test, and $s=1$ corresponds to the bold test. Tests with $s\in(0,1)$ cover everything in between. We follow the logic as in proof of Proposition 1 to show that both the bold and the cautious equilibrium are robust to this modification. In particular, investors strictly prefer to choose the bold test when $\mu_0 < \frac{c}{1+r_f}$ and the cautious test when $\mu_0 > \frac{c}{1+r_f}$ even if the intermediate choices are also available.

Recall that the unskilled investors choose a test which allows them to advertise the lowest break-even interest rate under the conjecture that at that interest rate all types will submit an application. If that were not true, unskilled investors not entering in equilibrium could choose a test and advertise an interest rate which leads to higher profit than staying outside. (We rely here on Lemma 1 (i) ensuring that if an entrepreneurs applies for a given rate in equilibrium, he also applies for all lower rates, advertised or not.) The break-even interest rate for any test characterized by s is

$$\left(\frac{1-\mu_0-\mu_1}{2}+s\frac{1-\mu_0-\mu_1}{2}\right)(1+r(s))$$
$$+\left(\mu_1+(1-s)\,\mu_0+(1-s)\,\frac{1-\mu_0-\mu_1}{2}\right)(1+r_f)-c=1+r_f,$$

which in turn implies

$$\frac{(1+r_f)\left(1-\left(\mu_1+(1-s)\,\mu_0+(1-s)\,\frac{1-\mu_0-\mu_1}{2}\right)\right)+c}{\left(\frac{1-\mu_0-\mu_1}{2}+s\frac{1-\mu_0-\mu_1}{2}\right)}-1=r(s).$$

Note that

$$\frac{\partial r(s)}{\partial s}=-2\frac{c-\mu_0-\mu_0 r_f}{(s+1)^2(1-\mu_0-\mu_1)},$$

implying that whenever $\mu_0 < \frac{c}{\left(1+r_f\right)}$, the smallest interest rate is implied by the test $s=1$, while in the opposite case it is $s=0$. Thus, by the same argument as in the main text, if $\mu_0 < \frac{c}{\left(1+r_f\right)}$, the equilibrium advertised interest rate by unskilled investors corresponds to the test $s=1$ (bold test), and in the opposite case they choose $s=0$ (cautious test). In this sense, the continuum of intermediate tests are always dominated by either the bold or the cautious test, and restricting investor choice to these two tests is without loss of generality.

E Publicly Observable Credit Score

A simplifying assumption in the baseline model is that no credit score exists. This assumption has two important implications. First, in the period after a bold phase no information is revealed about the surviving bad firms. Thus if the stage game equilibrium remains bold, all of these bad opaque entrepreneurs keep getting financed. Second, in the period following a cautious phase, no opaque good surviving entrepreneur is known to unskilled investors as good. This is relevant when the stage game equilibrium remains cautious, as

these entrepreneurs are not identified by unskilled investors and thus they are rationed.

In this section we incorporate credit scores in the baseline framework in a simple fashion. Consider all the entrepreneurs who survive from period t to $t+1$. We model the credit score as a probability ζ that the true type τ of these surviving entrepreneurs is revealed to unskilled investors. ζ captures the precision of the credit score. When the true type of an entrepreneur is revealed to be τ, he is perceived as a $(\tau, 1)$ entrepreneur to investors, i.e. a transparent entrepreneur of type τ. We further assume $\zeta < \bar{\zeta}$.

Next we adjust the law of motion for the state variables of the benchmark economy, expressed in equations (4)-(7):

$$\mu_{0B}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = (1-\zeta)(1-\delta)\mu_0 + \big(\delta + (1-\delta)\mu_1\big)\frac{\lambda}{2}, \tag{A.44}$$

$$\mu_{1B}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = \zeta(1-\delta)\mu_0 + \big(\delta + (1-\delta)\mu_1\big)\frac{\lambda}{2}, \tag{A.45}$$

$$\mu_{0C}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = \big(\delta + (1-\delta)(\mu_0+\mu_1)\big)\frac{\lambda}{2}, \tag{A.46}$$

$$\mu_{1C}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = \big(\delta + (1-\delta)(\mu_0+\mu_1)\big)\frac{\lambda}{2}, \tag{A.47}$$

where superscript cs denotes the economy with credit scores. Consider equation (A.44). The first term is the measure of opaque bad entrepreneurs from last period who both survive and whose credit score is not revealed. The second term is the newborn additions and is identical to the baseline economy. Equation (A.45) is the complementary equation. It incorporates the measure of surviving opaque bad entrepreneurs whose type is revealed by the credit score to be bad, and are now transparent bad entrepreneurs from the perspective of unskilled investors. The last two equations do not change.

In this economy we need a third state variable, the measure of opaque good entrepreneurs, as the measure of the two groups of good entrepreneurs evolve differently. We have

$$\nu_{0B}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = (1-\zeta)(1-\delta)\nu_0 + \big(\delta + (1-\delta)\mu_1\big)\frac{\lambda}{2}, \tag{A.48}$$

$$\nu_{0C}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = (1-\zeta)(1-\delta)\nu_0 + \big(\delta + (1-\delta)(\mu_0+\mu_1)\big)\frac{\lambda}{2}. \tag{A.49}$$

Both equations incorporate the measure of opaque good entrepreneurs whose type is not revealed by credit scores. The complementary measure is added to the measure of transparent good entrepreneurs.

The most critical difference with the baseline economy is in deriving Lemma 4, i.e. proving that the dynamic equilibrium reduces to a sequence of stage game equilibria. In order to establish this result, the deviation to consider is that of a opaque bad entrepreneur in a market were investors are bold and all entrepreneurs behave as in a stage game. We need to show that a opaque bad entrepreneur does not have an incentive to deviate form his optimal stage game strategy and pay investors back, in order to be able to survive and earn future profits.

The key to the proof is to first observe that any opaque bad entrepreneur who does not pay back investors raises strictly positive profits in a bold equilibrium. The reason is that all

projects are positive NPV and unskilled lenders who determine the interest rate only break even. Thus even good entrepreneurs who do pay back make positive profits as they choose to participate. This implies that the opaque bad ones who do not pay back make higher, strictly positive profits. However, if an opaque bad entrepreneur does not pay back he will be excluded from the market with probability ζ. Alternatively, if he pays back, he makes strictly lower profits this period but his participation probability next period increases by $\zeta\%$, where he can make positive profits. Furthermore, depending on his repayment strategy he can make positive profits in the periods farther in the future as well.

As such, $\exists\ \bar{\zeta}$ such that if $\zeta < \bar{\zeta}$ no deviation exists for a opaque bad entrepreneur: the discounted probability of more likely survival by paying back at time t, $\frac{z}{1+r}$, is sufficiently small for a this entrepreneur that he prefers not to deviate.

In this case, the steady state levels of state variables, interest rates, the switching thresholds for choice of the test, as well as the stage game output adjust according to the new equations governing the evolution of the state variables, but the stage game and dynamic equilibrium logic remains the same. As such, similar to the baseline economy, the equilibrium will be cyclical for an intermediate range of c.