Optimal Exchange Rate Policy

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January 30, 2022

UNDER CONSTRUCTION

Abstract

We develop a general policy analysis framework that features nominal rigidities and financial frictions with endogenous UIP deviations. The goal of the optimal policy is to balance output gap stabilization and international risk sharing using a mix of monetary policy and FX interventions. The nominal exchange rate plays a dual role. First, it allows for the real exchange rate adjustments when prices are sticky, which are necessary to close the output gap. Monetary policy can eliminate the output gap, but this generally requires a volatile nominal exchange rate. Volatility in the nominal exchange rate, in turn, limits the extent of international risk sharing in the financial market with risk averse intermediaries. Optimal monetary policy closes the output gap, while optimal FX interventions eliminate UIP deviations. When the first-best real exchange rate is stable, both goals can be achieved by a fixed exchange rate policy — an open-economy divine coincidence. Generally, this is not the case, and the optimal policy requires a managed peg by means of a combination of monetary policy and FX interventions, without requiring the use of capital controls. We explore various constrained optimal policies, when either monetary policy or FX interventions are restricted, and characterize the possibility of central bank's income gains and losses from FX interventions.

1 Introduction

What is the optimal exchange rate policy? Should exchange rates be optimally pegged, managed or allowed to freely float? What defines a freely floating exchange rate? Do open economies face a trilemma constraint in choosing between inflation and exchange rate stabilization, unlike the divine coincidence in a closed economy? These are generally difficult questions, as the exchange rate is neither a policy instrument, nor a direct objective of the policy, but rather an endogenous general equilibrium variable with directs close links in both product and financial markets. At the same time, equilibrium exchange rate behavior features a variety of puzzles from the point of view of conventional business cycle models typically used for policy analysis in open economy.

We address these questions by developing a general policy analysis framework with nominal rigidities and financial frictions that are both central for equilibrium exchange rate determination and result in an empirically realistic model of the exchange rate. We extend the framework in Itskhoki and Mukhin (2021b), where we studied a switch between floating and fixed exchange rate regimes, to allow for explicit policy analysis using both monetary policy and exchange rate interventions in the financial market. The goal of the policy is to balance output gap stabilization and international risk sharing. Financial market interventions are effective in segmented financial markets and affect the extent of UIP deviations, as well as the equilibrium exchange rate volatility, which in turn also endogenously feeds back into the equilibrium size of the UIP deviations.

The model features Balassa-Samuelson mechanism determining the value of the frictionless real exchange rate (departures from PPP) and segmented financial markets resulting in endogenous equilibrium UIP deviations. The presence of both endogenous PPP and UIP deviations is essential for the optimal exchange rate policy analysis, as exchange rates are crucial components of both deviations. We build on our earlier work that develops a tractable model of equilibrium exchange rate determination with realistic properties of PPP and UIP deviations. We show that this framework is easily amenable to normative analysis and characterize the optimal exchange rate policies implied by the model.

The nominal exchange rate plays a dual role. First, it allows for the real exchange rate adjustment when prices (or wages) are sticky, and in the absence of such nominal exchange rate movements, the economy features an output gap resulting in welfare losses. Monetary policy can eliminate the output gap, but this generally requires a volatile nominal exchange rate. Volatility in the nominal exchange rate limits the extent of international risk sharing in the financial market, as international financial transactions are intermediated by risk-averse market makers who need to hold the nominal exchange rate risk. This also leads to welfare losses. Financial market interventions can redistribute the risk away from arbitrageurs, stabilizing the resulting equilibrium UIP deviations and improving the extent of international risk sharing.

We prove a divine coincidence result in an open economy: if frictionless real exchange rate is stable, then fixed nominal exchange rate achieves both goals of output gap and UIP stabilization, and fixed nominal exchange rate is constrained optimal. Furthermore, direct nominal exchange rate targeting is favored over inflation stabilization, even though the goal of both policies is the same, as the latter policy may result in multiple equilibria in the international financial market, with and without nominal exchange rate volatility. Additionally, pegging the exchange rate with monetary policy may emerge as the second best policy, even when divine coincidence is not satisfied, yet there are tight constraint on the balance sheet of the central bank making effective FX interventions infeasible.

Second, we show that access to unconstrained monetary policy and FX interventions generally allows the implementation of the constrained optimal allocation, independently of whether the frictionless real exchange rate is stable or not. The resulting equilibrium generally features volatile nominal exchange rate and inflation targeting, with financial interventions stabilizing UIP deviations. We also show that economies with segmented financial markets do not feature a trilemma constraint, as market segmentation offers the financial regulator a powerful tool to stabilize the financial market, even when monetary policy focuses exclusively on domestic inflation and output gap stabilization.

Third, we explore various circumstances where either monetary policy is constrained (e.g., due to the zero lower bound) or the financial interventions are constrained (e.g., due to non-negative requirement on central bank foreign reserves or value-at-risk constraints for the central bank portfolio). In this case, there are two independent policy goals (the output gap and the risk sharing wedge) and only one unconstrained policy tool, thus making it generally impossible to replicate the constrained efficient allocation. Fixing the exchange rate using the monetary policy tool is generally feasible, but is also generally suboptimal. Similarly, targeting the output gap alone is also suboptimal, and monetary policy trades-off output gap and exchange rate stabilization (partial peg) in the absence of FX interventions. Using financial interventions to stabilize output gap is generally infeasible and does not achieve constrained optimality. We characterize the optimal policy mix under a variety of constrained environments.

Lastly, we explore the monopoly power of the government in the international financial market and the ability of the central bank to both earn monopoly rents and/or complete international risk sharing and reduce the volatility of national incomes (inclusive of international transfers). The government can generate expected rents only in the presence of noise traders by leaning against the wind of their liquidity currency demand. Arbitrageurs compete with the government for these rents, and greater equilibrium exchange rate volatility allows the government to capture a greater share of these rents by discouraging arbitrageurs from active intermediation. In general, the policymaker favors small departures from efficient risk sharing and expected UIP deviations which result in expected incomes of the central bank against the noise traders. Capital controls are only useful in the presence of international transfers from the financial sector.

Literature review

1. Financial frictions and exchange rates: Jeanne and Rose (2002), Gabaix and Maggiori (2015), Itskhoki and Mukhin (2021a,b), Gourinchas, Ray, and Vayanos (2019), Greenwood, Hanson, Stein, and Sunderam (2020), Gopinath and Stein

2. Costs of exchange rate interventions: Jeanne, Amador et al, Fanelli, Fanelli and Straub

3. Policy: Devereux and Engel (2003), Farhi and Werning (2012), Farhi, Gopinath, and Itskhoki (2014), Egorov and Mukhin (2020), Gopinath et al on integrated policy framework

4. Other: Brunnermeier and Sannikov, Fornaro, Kekre and Lenel
5. Objective approximation: Clarida-Gali-Gertler, Gali-Monacelli, Corsetti-Dedola-Leduc, Gopinath et al, Egorov-Mukhin
6. Depart from Ricardian (Modigliani-Miller) equivalence environment of Wallace (1981)
7. Marcet-Nicolini

2 The Model of Exchange Rate Determination

We consider a small open economy with a tradable and a non-tradable sector. In our baseline analysis, we assume a separable log-linear utility of the households:[1]

$$\mathbb{W}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t, \qquad U_t = U(C_t, L_t) = \log C_t - (1-\gamma)L_t, \qquad C_t = C_{Tt}^{\gamma} C_{Nt}^{1-\gamma},$$

who can borrow or lend using a one-period risk-free home-currency bond:

$$P_t C_t + \frac{B_t}{R_t} = B_{t-1} + W_t L_t + P_{Tt} Y_{Tt} + \Pi_t + T_t,$$

where R_t is the gross nominal interest rate, and thus $1/R_t$ is the price of a bond paying one unit of home currency next period.

The households own an exogenous stochastic endowment of the tradable good Y_{Tt}, which is homogenous and traded at a flexible international price that satisfies the law of one price:

$$P_{Tt} = \mathcal{E}_t P_{Tt}^*,$$

where P_{Tt}^* is the international price of the tradable good and $\mathcal{E}_t$ is the nominal exchange rate (units of home currency for one unit of foreign currency; thus, an increase in $\mathcal{E}_t$ corresponds to a home depreciation). We assume a stable price level in the foreign country, $P_{Tt}^* = 1$, and therefore the home-currency tradable price tracks the nominal exchange rate, $P_{Tt} = \mathcal{E}_t$. Therefore, home net exports equals $NX_t = P_{Tt}(Y_{Tt} - C_{Tt}) = \mathcal{E}_t(Y_{Tt} - C_{Tt})$.

The non-tradable good is produced using labor subject to productivity shocks, $Y_{Nt} = A_t L_t$, and the firm profits are $\Pi_t = P_{Nt} Y_{Nt} - W_t L_t$. The equilibrium labor supply (household FOC) satisfies $C_{Nt} = W_t / P_{Nt}$, and the market clearing requires $C_{Nt} = Y_{Nt}$. The competitive flexible price of non-tradables equals W_t / A_t, however, prices are permanently sticky at an exogenous level $P_{Nt} = 1$.[2]

[1] Many analytical results are exact and hold for a general utility function $U(C_{Tt}, C_{Nt}, L_t)$. The analysis also extends to large open economies and endogenous produced non-homogenous tradable goods with home bias. The set of our baseline assumptions, however, allows for the sharpest characterization with stark policy motives.

[2] We focus on the fully sticky price case as a limiting benchmark which simplifies the analysis by avoiding an additional dynamic equation, yet maintains all the qualitative tradeoffs of a more general environment. By having price stickiness only in the non-tradable sector we avoid the need to choose between PCP, LCP and DCP frameworks; alternatively, we could focus on sticky wages, which are equivalent in this case.

Therefore, the flexible (first-best) supply of the non-tradable good satisfies $L_t = 1$ and $Y_{Nt} = A_t$, while the output gap due to sticky prices is given by $X_t = Y_{Nt}/A_t = W_t/A_t$, and correspondingly the equilibrium labor supply is $L_t = W_t/A_t = X_t$.

The policymaker uses wage inflation as monetary instrument. For simplicity, we assume the policymaker has direct control over nominal wages and chooses W_t in conducting monetary policy.[3] When monetary policy sets wages to peg the non-tradable productivity, $W_t = A_t$, this results in the first best employment and output level, i.e. zero output gap, with a constant price $P_{Nt} = 1$.

The total consumption expenditure is split between tradables and non-tradables, $P_t C_t = P_{Tt} C_{Tt} + P_{Nt} C_{Nt}$, such that $\gamma P_{Nt} C_{Nt} = (1-\gamma) P_{Tt} C_{Tt}$. Using the law of one price $P_{Tt} = \mathcal{E}_t$ and the labor supply $C_{Nt} = W_t/P_{Nt}$, we obtain an equilibrium condition for the nominal exchange rate:

$$\mathcal{E}_t = \frac{\gamma}{1-\gamma} \frac{W_t}{C_{Tt}}. \tag{1}$$

Monetary policy W_t proportionally shifts nominal exchange rate, holding tradable consumption constant; in turn, holding constant monetary policy, greater tradable consumption appreciates the real exchange rate.[4]

The real exchange rate, defined as $\mathcal{Q}_t = \mathcal{E}_t/P_t = (\mathcal{E}_t/P_{Nt})^{1-\gamma}$, where the home price level $P_t = P_{Tt}^{\gamma} P_{Nt}^{1-\gamma}$ and $P_{Tt} = \mathcal{E}_t$. With sticky price, the real exchange rate tracks the nominal exchange rate, $\mathcal{Q}_t = \mathcal{E}_t^{1-\gamma}$, since $P_{Nt} = 1$. In the flexible price allocation, $\mathcal{Q}_t = \left(\frac{\gamma}{1-\gamma}\frac{A_t}{C_{Tt}}\right)^{1-\gamma}$, independently of the monetary policy W_t and hence the value of the nominal exchange rate $\mathcal{E}_t$. It is convenient to express the nominal exchange rate as a product of the real exchange rate and output gap, $\mathcal{E}_t = \mathcal{Q}_t^{1/(1-\gamma)} \cdot X_t$.

Financial market Apart from households, three types of agents trade home and foreign currency bonds in the international financial market. Namely, these are the government, noise traders and arbitrageurs. The government holds a portfolio of (F_t, F_t^*) units of home- and foreign-currency bonds, respectively, with the value of the portfolio (government net foreign assets) given by $F_t/R_t + \mathcal{E}_t F_t^*/R_t^*$, where R_t^* is the gross nominal interest rate in foreign currency (dollar). Changes in F_t and F_t^* correspond to open market operations of the government.

Noise traders hold a zero capital portfolio (N_t, N_t^*) of the two bonds, such that $N_t/R_t + \mathcal{E}_t N_t^*/R_t^* = 0$, and $N_t^*/R_t^* = \psi_t$ is the liquidity demand for foreign currency of the noise traders, that is ψ_t is a random variable uncorrelated with macroeconomic fundamentals. A positive ψ_t means that noise traders short home-currency bonds to buy foreign-currency bonds, and vice versa.

In turn, B_t is the fundamental demand of home households for the home-currency bond, which is shaped by the macroeconomic forces resulting in the equilibrium of path net exports. The choice of B_t

[3]We show below that the first order condition for bond holdings implies $\beta R_t \mathbb{E}_t\{W_t/W_{t+1}\} = 1$, and thus an interest rate rule $R_t = \bar{R}_t \cdot (W_t/A_t)^{\phi}$ with a sufficiently large ϕ and $\beta \bar{R}_t \mathbb{E}_t\{A_t/A_{t+1}\} = 1$ implements $W_t = A_t$.

[4]Note that this does not violate exchange rate disconnect with aggregate consumption to the extent C_{Tt} is a small component of C_t, as is the case in the data due to the vast home bias in aggregate consumption.

is characterized by the household Euler equation:

$$\beta R_t \mathbb{E}_t \left\{ \frac{C_{Tt}}{C_{T,t+1}} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right\} = 1, \tag{2}$$

where we used $P_{Tt} = \mathcal{E}_t$. Using (1), this condition implies $\beta R_t \mathbb{E}_t\{W_t/W_{t+1}\} = 1$, a relationship between home currency interest rate and nominal wage inflation.

Finally, the arbitrageurs also hold a zero capital portfolio (D_t, D_t^*) such that $D_t/R_t + \mathcal{E}_t D_t^*/R_t^* = 0$, with a return on one foreign currency unit holding of such portfolio (i.e., $D_t^* = R_t^*$ and $D_t = -R_t D_t^* \mathcal{E}_t / R_t^*$ given by $\tilde{R}_{t+1}^* = R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$ in dollars. In other words, the income from this carry trade is given by $\pi_{t+1}^{D*} = D_t^* - D_t/\mathcal{E}_t = \tilde{R}_{t+1}^* \cdot \frac{D_t^*}{R_t^*}$ in foreign currency, where we used the zero-capital constraint linking D_t and D_t^*. Arbitrageurs choose their portfolio (D_t, D_t^*) to maximize min-variance preferences over profits, $V_t(\pi_{t+1}^{D*}) = \mathbb{E}_t\left\{\Theta_{t+1}\pi_{t+1}^{D*}\right\} - \frac{\omega}{2}\mathrm{var}_t\big(\pi_{t+1}^{D*}\big)$, where $\Theta_{t+1} = \beta \frac{C_{Tt}}{C_{T,t+1}}$ is the stochastic discount factor of home households, and the second term in $V_t(\cdot)$ reflects the additional risk penalty of the arbitrageurs with ω being the risk aversion parameter. The optimal portfolio choice satisfies:

$$\frac{D_t^*}{R_t^*} = \frac{\mathbb{E}_t\big\{\Theta_{t+1}\tilde{R}_{t+1}^*\big\}}{\omega\sigma_t^2},$$

where $\sigma_t^2 \equiv \mathrm{var}_t(\tilde{R}_{t+1}^*) = R_t^2 \cdot \mathrm{var}_t(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}})$ is a measure of the nominal exchange rate volatility.

The market clearing in the financial market requires that the home-currency bond positions of all four types of agents balance out:

$$B_t + N_t + D_t + F_t = 0.$$

The foreign-currency bond is in perfect elastic international supply at an exogenous interest rate R_t^*. The government budget constraint from operations in the financial market is given by:

$$\frac{F_t}{R_t} + \frac{\mathcal{E}_t F_t^*}{R_t^*} = F_{t-1} + \mathcal{E}_t F_{t-1}^* + \tau \mathcal{E}_t \pi_t^* - T_t, \qquad \pi_t^* = \tilde{R}_t^* \cdot \frac{N_{t-1}^* + D_{t-1}^*}{R_{t-1}^*},$$

where T_t is the lump-sum transfer to the home households and π_t^* is the combined income from the financial transactions of noise traders and arbitrageurs (in dollars). Note that parameter $\tau \in [0,1]$ can be viewed as either the home country's ownership share of the financial sector or a tax on financial transactions imposed by the home government.[5]

3 The Policy Problem

Define the net foreign asset (NFA) position of the home country, B_t^* in foreign currency, which has the home-currency value:

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} = \frac{B_t + F_t}{R_t} + \frac{\mathcal{E}_t F_t^*}{R_t^*},$$

[5]Note that the arbitrageur's problem omits τ without loss of generality, as a change in the tax rate τ is isomorphic to a change in risk aversion ω.

that is the value of the combined position of the home households and the government. Using B_t^*, we introduce a sequences of lemmas that characterize the equilibrium conditions for the open economy.

Lemma 1 *The NFA of the home country equals the combined foreign-currency bond position in the financial market:* $B_t^* = F_t^* + N_t^* + D_t^*$.

Proof: Using the market clearing for home-currency bond, $B_t + N_t + D_t + F_t = 0$, and the zero capital portfolios of noise traders and arbitrageurs, we have $\frac{B_t + F_t}{R_t} - \frac{\mathcal{E}_t(N_t^* + D_t^*)}{R_t^*} = 0$. Then using the definition of NFA and rearranging yields $B_t^* = F_t^* + N_t^* + D_t^*$. ■

The NFA position allows to characterize concisely the home country budget constraint:

Lemma 2 *The combined home country budget constraint in foreign currency terms is given by:*

$$\frac{B_t^*}{R_t^*} - B_t^* = (Y_{Tt} - C_{Tt}) - (1-\tau)\tilde{R}_t^* \frac{B_{t-1}^* - F_{t-1}^*}{R_{t-1}^*}. \tag{3}$$

Proof: See Appendix A. ■

Note that $NX_t/\mathcal{E}_t = Y_{Tt} - C_{Tt}$ is the real (or foreign-currency) value of net exports. The last term in the budget constraint reflects the international transfer of financial-sector income from the home country to the rest of the world. When $\tau = 1$, that is either all income is taxed away or the financial sector is owned by the domestic residents, there is no international transfer and the budget constraint is simply $B_t^*/R_t^* - B_t^* = Y_{Tt} - C_{Tt}$.

Finally, the equilibrium international risk sharing is characterized in:

Lemma 3 *The international risk sharing condition is given by:*

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} = 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \qquad \textit{where} \quad \sigma_t^2 = R_t^2 \cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big). \tag{4}$$

The international risk sharing wedge is $Z_t \equiv \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}$.

Proof: follows directly from the optimal portfolio of the arbitrageurs, which we rewrite expanding the expressions for Θ_{t+1} and $\tilde{R}_{t+1}^*$ as:

$$\omega\sigma_t^2 \frac{D_t^*}{R_t^*} = \mathbb{E}_t\big\{\Theta_{t+1}\tilde{R}_{t+1}^*\big\} = \mathbb{E}_t\left\{\beta \frac{C_{Tt}}{C_{T,t+1}} \cdot \left[R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right]\right\}.$$

Subtracting the household Euler equation (2) and substituting for D_t^* from Lemma 1 finishes the proof. ■

In the absence or risk-sharing wedge, $Z_t = 0$, the international risk sharing condition reduces to the conventional Euler equation for the foreign-currency bond, $\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} = 1$, a property of the constrained optimal risk sharing in this economy. Combining international risk sharing (4) with the home household Euler equation we obtain the modified UIP condition that holds in this economy:

$$\mathbb{E}_t\left\{\frac{C_{Tt}}{C_{T,t+1}}\left[R_t^* - R_t\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right]\right\} = \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{\beta R_t^*} = Z_t/\beta. \tag{5}$$

Note that Z_t/β is the UIP wedge. When $Z_t = 0$, whether due to $\omega\sigma_t^2 = 0$ or to $F_t^* = B_t^* - N_t^*$, the UIP holds from the perspective of the home household SDF. This is because we endowed the arbitrageurs who trade the home and foreign currency bonds with the SDF of the home households. Therefore, in the limit of risk neutral arbitrageurs ($\omega \to 0$), the international financial market converges to a frictionless two-bond market where UIP holds in expectation.

Equilibrium We can now define the equilibrium in this economy. Given the stochastic path of exogenous shocks $\{A_t, Y_{Tt}, R_t^*, N_t^*\}$, sticky non-tradable prices $P_{Nt} \equiv 1$, and the path of policies $\{W_t, F_t, F_t^*\}$, an equilibrium vector $\{C_{Tt}, B_t^*, \mathcal{E}_t, R_t\}$ and the implied $\{\sigma_t^2\}$ solve the dynamic system (1)–(4) with the initial condition B_{-1}^* and the transversality condition $\lim_{T\to\infty} B_T^* / \prod_{t=0}^T R_t^* = 0$. The other endogenous variables $\{Y_{Nt}, C_{Nt}, L_t, D_t^*, B_t\}$ are recovered from static side equations.[6] The policy vector contains monetary policy W_t and foreign exchange interventions F_t^*, and we can omit F_t since it merely crowds out B_t one-for-one without changing the equilibrium path of the economy due to Ricardian equivalence. Without loss of generality, the monetary policy tool could be changed to R_t, making W_t an endogenous variable instead, and one would need to ensure, as usual, the uniqueness of the implemented equilibrium path. Lastly, we note that exogenous shocks include non-tradable productivity A_t, tradable endowment Y_{Tt}, foreign interest rate R_t^* and noise trader liquidity shocks for foreign vs home currency N_t^*.

3.1 Exact policy problem

Given the equilibrium path, the resulting welfare of the country is given by:

$$\mathbb{W}_0 = \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_{Tt} + (1-\gamma) \left(\log W_t - \frac{W_t}{A_t} \right) \right], \tag{6}$$

where we used the fact that under sticky prices, $P_{Nt} = 1$, we have $C_{Nt} = W_t$ and $L_t = W_t/A_t$. The optimal policy problem is to pick the path of policies $\{W_t, F_t^*\}$, subject to possible constraints, that implement the equilibrium vector $\{C_{Tt}, B_t^*, \mathcal{E}_t, R_t\}$ that maximizes welfare. In other words, the planner maximizes (6) with respect to $\{C_{Tt}, B_t^*, \mathcal{E}_t, R_t, W_t, F_t^*\}$ and σ_t^2 subject to (1)–(4) and given the stochastic path of exogenous variables $\{A_t, Y_{Tt}, R_t^*, N_t^*\}$.

[6] For example, from market clearing and labor supply $Y_{Nt} = C_{Nt} = W_t$ and from production function function $L_t = W_t/A_t$, while from Lemma 1 $D_t^* = B_t^* - F_t^* - N_t^*$, and household assets satisfy $\frac{B_t+F_t}{R_t} = \frac{\mathcal{E}_t(B_t^*-F_t^*)}{R_t^*}$.

We reproduce the set of constraints (1)–(4) explicitly as:

$$
\begin{aligned}
\frac{B_t^*}{R_t^*} - B_{t-1}^* &= (Y_{Tt} - C_{Tt}) - (1-\tau)\left[R_{t-1}^* - R_{t-1}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\right]\frac{B_{t-1}^* - F_{t-1}^*}{R_{t-1}^*},\\
\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} &= 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*},\\
\beta R_t \mathbb{E}_t\left\{\frac{C_{Tt}}{C_{T,t+1}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right\} &= 1,\\
\mathcal{E}_t &= \frac{\gamma}{1-\gamma}\frac{W_t}{C_{Tt}},\\
\sigma_t^2 &= R_t^2 \cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big).
\end{aligned}
$$

The first is the budget constraint with the unconventional last term reflecting the ex post income transfer of the payoffs of noise traders and arbitrageurs (since $B_{t-1}^* - F_{t-1}^* = N_{t-1}^* + D_{t-1}^*$). The next two constraints are the Euler equations for foreign-currency and home-currency bonds, with the unconventional term in the former reflecting the risk premium charged by the arbitrageurs for intermediating the foreign currency risk (since $D_t^* = B_t^* - N_t^* - F_t^*$) in proportion with risk aversion ω and exchange rate volatility σ_t^2 defined in the last constraint. Finally, the next-to-last equation characterizes the equilibrium relationship between the nominal exchange rate, non-tradable wage W_t (monetary policy) and tradable consumption, reflecting the Balassa-Samuelson forces (since in equilibrium $C_{Nt} = W_t$). The complexity of this problem is exactly in that the equilibrium volatility of the nominal exchange rate σ_t^2 endogenously magnifies the wedge Z_t that distorts international risk sharing.

3.2 Linear-quadratic policy problem

While the exact policy problem (6) is sufficiently tractable for some sharp characterization, as we show below, consider further progress can be made with a linear-quadratic approximation. There are two challenges involved in the transition to a linear-quadratic environment. The first challenge relates to the quadratic approximation of the welfare function in an open economy, and in particular where the constrained optimal risk sharing is not full insurance, as the international financial market is incomplete and features risk free bonds only. Specifically, the optimal risk sharing corresponds to no UIP deviations in (5) rather than perfect consumption smoothing. The second challenge is associated with the risk sharing frictions that are proportional to second moments of the macro variables, namely the volatility of the nominal exchange rate σ_t^2 in (4). In our approximation, we must ensure that the risk sharing frictions remain in the linear-quadratic environment to maintain the key tradeoffs of the exact policy problem between output gap stabilization and risk sharing.

We denote with $x_t \equiv c_{Nt} - \tilde{c}_{Nt}$ and $z_t \equiv c_{Tt} - \tilde{c}_{Tt}$ the two wedges in our analysis, where by convention the small letters are the log deviations of the corresponding variables (e.g., $c_{Tt} = \log C_{Tt} - \log \bar{C}_T$) from a non-stochastic first-best equilibrium with $\bar{R} = \bar{R}^* = 1/\beta$ and $\bar{B}^* = \bar{N}^* = 0$. The variables with tildes denote the constrained optimum allocation. In particular, $\tilde{c}_{Nt} = a_t$ corresponds to the level of non-tradable consumption under flexible prices, or equivalently zero output gap, and thus

$x_t = \log X_t = \log \frac{W_t}{A_t}$ is the measure of the output gap (recall that $C_{Nt} = W_t$ in any equilibrium). In turn, z_t is the measure of the risk sharing wedge, equal to the proportional gap between C_{Tt} and the frictionless $\tilde{C}_{Tt}$ which is defined by $\beta R_t^* \mathbb{E}_t\{\tilde{C}_{Tt}/\tilde{C}_{T,t+1}\} = 1$ and the budget constraint.[7]

We focus here on the case without international transfers, $\tau = 1$ in (3), and consider the case with transfers separately in Section 5.

Lemma 4 *The equilibrium system (1)–(4) log-linearized around a non-stochastic equilibrium with $\bar{B}^* = \bar{N}^* = 0$ and a finite non-zero $\omega\tilde{\sigma}_t^2$ is given by:*

$$\beta b_t^* = b_{t-1}^* - z_t, \tag{7}$$

$$\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega}\sigma_t^2(\iota b_t^* - n_t^* - f_t^*), \tag{8}$$

$$\sigma_t^2 = \mathbb{E}_t e_{t+1}^2 - \big(\mathbb{E}_t e_{t+1}\big)^2, \tag{9}$$

$$e_t = \tilde{q}_t + x_t - z_t, \tag{10}$$

where $\tilde{q}_t \equiv a_t - \tilde{c}_{Tt}$ is (the log deviation of) the first-best real exchange rate, $b_t^* \equiv (B_t^* - \tilde{B}_t^*)/\bar{Y}_T$ with $\tilde{B}_t^*$ implied by the path of $\tilde{C}_{Tt}$, $n_t^* \equiv (N_t^* - \tilde{B}_t^*)/\bar{Y}_T$, $\bar{\omega} = \omega\bar{Y}_T/\beta$ and $\iota \in \{0, 1\}$.

The exogenous shocks in the linearized system are represented by two variables: (i) $\tilde{q}_t$, which reflects the evolution of non-tradable productivity A_t relative to tradable endowment Y_{Tt} shaping the path of $\tilde{C}_{Tt}$; and (ii) n_t^* which reflects the foreign currency demand by noise traders and households. The policy variables are the output gap x_t, chosen by the monetary policy, and the FX intervention $f_t^* \equiv F_t^*/\bar{Y}_T$.[8]

A distinctive feature of our approach and the key property of the linearized equilibrium system in Lemma 4 is that the second moment, namely the volatility of the nominal exchange rate σ_t^2, influences the first-order dynamics of the risk sharing wedge z_t, which in turn feeds back into the rest of the equilibrium system. The reason is that we take the approximation in a way that ensures that the risk premium approximated by $\bar{\omega}\sigma_t^2(\iota b_t^* - n_t^* - f_t^*)$ remains a first order object. Specifically, as shocks become small and $\tilde{\sigma}_t^2 = R_t^2 \cdot \mathrm{var}_t(\mathcal{E}_t/\mathcal{E}_{t+1}) \to 0$, we scale effective risk aversion of the financial sector ω to ensure that the sequence $\omega\tilde{\sigma}_t^2$ remains bounded away from zero by a constant (zero order term). We argue this provides a superior point of approximation for models that focus on the joint dynamics of macroeconomic variables and risk premia. Lastly, depending on the sequence of approximation, equation (8) features either $\iota = 1$ (baseline) or $\iota = 0$ (special case). The special case approximates the situation when macroeconomic demand for currency b_t^* is orders of magnitude smaller than financial (liquidity) demand for currency n_t^*, and disappears in relative terms in the limit.

[7]The relationship between z_t and Z_t from Lemma 3 is $\mathbb{E}_t e^{-\Delta z_{t+1}} = 1 + Z_t$, and $\mathbb{E}_t \Delta z_{t+1}$ is the linearized UIP wedge.

[8]Note that without international transfers, the Euler equation (2), approximated as $i_t = \log R_t - \log \bar{R} = \mathbb{E}_t\{\Delta c_{T,t+1} + \Delta e_{t+1}\} = \mathbb{E}_t\{\Delta x_{t+1} + \Delta a_{t+1}\} = \mathbb{E}_t w_{t+1}$, becomes a side equation defining the path of r_t, which follows the evolution of non-tradable productivity and output gap (or equivalently, the wage inflation, since $c_{Tt} + e_t = w_t = x_t + a_t$). The primitive exogenous shocks $\{A_t, Y_{Tt}, R_t^*, N_t^*\}$ affect the equilibrium system in Lemma 4 via $\{\tilde{q}_t, n_t^*\}$, with the effect of $\{Y_{Tt}, R_t^*, N_t^*\}$ contained in both n_t^* and $\tilde{q}_t$ via their effects on $\{\tilde{C}_{Tt}, \tilde{B}_t^*\}$, and the effect of A_t affecting $\tilde{q}_t$ only. Note that there is no one-to-one relationship between shocks to R_t^* and the equilibrium R_t, which only depends on the path of W_t chosen by the home monetary authority. However, equilibrium nominal exchange rate reflects both external and domestic shocks, and ensures that modified UIP (5) holds, or its log-linearized version $i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \mathbb{E}_t \Delta z_{t+1}$, where we used the fact that $i_t^* = \log R_t^* - \log \bar{R}^* = \mathbb{E}_t \Delta \tilde{c}_{T,t+1}$.

With Lemma 4, we can cast the policy problem as choosing the path of $\{x_t, z_t, e_t, \sigma_t^2, b_t^*, f_t^*\}$, where $\{x_t, f_t^*\}$ are policy instruments and $\{z_t, e_t, \sigma_t^2, b_t^*\}$ are endogenous variables solving the equilibrium system. The remaining element of the problem is the quadratic approximation to the welfare objective function around a constrained-optimal allocation with $x_t = z_t = 0$.

Lemma 5 *A second order approximation to the welfare maximization problem* (6) *around a constrained optimal allocation* $C_{Nt} = \tilde{C}_{Nt} = A_t$, $L_t = \tilde{L}_t = 1$ *and* $C_{Tt} = \tilde{C}_{Tt}$ *is given by:*

$$\min \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[\gamma z_t^2 + (1-\gamma)x_t^2\big]. \tag{11}$$

The difference of this approximation from a conventional approximation is that we do not use the first-best allocation with $C_{Tt} = const$, but rather a constrained optimal allocation with $C_{Tt} = \tilde{C}_{Tt}$ implied by $\beta R_t^* \mathbb{E}_t\{\tilde{C}_{Tt}/\tilde{C}_{T,t+1}\} = 1$. We prove Lemmas 4 and 5 in the appendix, and characterize the solution to the corresponding linear-quadratic minimization problem (11) subject to (7)–(10) in Section 5.

Equilibrium dynamics We now impose some structural assumptions on the dynamics of shocks to solve the equilibrium system (7)–(10) in certain special cases which prove useful in future analysis. In particular, this allows to characterize the equilibrium exchange rate volatility σ_t^2. First, we characterize the dynamics of $\{\tilde{c}_{Tt}, \tilde{q}_t\}$ when y_{Tt} follows an AR(1) with persistence ρ. Assuming $\beta R_t^* \equiv 1$, we have:

$$\mathbb{E}_t \Delta\tilde{c}_{T,t+1} = 0,$$
$$\beta\tilde{b}_t^* = \tilde{b}_{t-1}^* + y_{Tt} - \tilde{c}_{Tt},$$

which result in the following solution:[9]

$$\Delta\tilde{c}_{Tt} = \frac{1-\beta}{1-\beta\rho}(1-\rho L)y_{Tt} \sim \text{iid}, \tag{12}$$

which reduces to $\tilde{c}_{Tt} = y_{Tt}$ when y_{Tt} itself follows a random walk (i.e., $\rho = 1$). The first-best real exchange rate is given by $\tilde{q}_t = a_t - \tilde{c}_{Tt}$, and thus in general follows an ARIMA(1,1,1). When a_t and y_{Tt} both follow random walks, $\tilde{q}_t$ is also a random walk with innovations reflecting non-tradable productivity growth relative to tradable endowment growth.

Next, we consider the equilibrium path of $\{z_t, b_t^*\}$ when $n_t^* - f_t^*$ follows an AR(1) with persistence ρ. We conjecture and verify that $\sigma_t^2 = \sigma^2$. In this case, we can show that z_t satisfies:

$$z_t = (1-\beta\lambda_1)b_{t-1}^* - \frac{\beta\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1}(n_t^* - f_t^*), \tag{13}$$

where $\lambda_1 \le 1$ and $\lambda_2 \ge 1/\beta > 1$, such that $\lambda_1\lambda_2 = 1/\beta$, are the two roots of the equilibrium dynamic system, which in general depend on σ^2. When $\iota = 0$ in (8), we have $\lambda_1 = 1$ and $\lambda_2 = 1/\beta$ independently of the value of σ^2. Solving further for z_t, we can show that it follows an ARMA(2,1)

[9]The implied solution for $\tilde{b}_t$ is an ARIMA(1,1,0) given by $\Delta\tilde{b}_t = \frac{1-\rho}{1-\beta\rho}y_t$, which reduces to $\tilde{b}_t = 0$ when $\rho = 1$.

process with autoregressive roots ρ and λ_1 and moving average root $1/\beta$.[10]

Equations (12)–(13) allow us to evaluate the resulting conditional volatility of the exchange rate:

$$\begin{aligned}\sigma_t^2 = \text{var}_t(e_{t+1}) = \text{var}_t\big(\tilde{q}_{t+1} + x_{t+1} - z_{t+1}\big) &= \text{var}_t\big(w_{t+1} - \tilde{c}_{T,t+1} - z_{t+1}\big) \\ &= \text{var}_t\left(\varepsilon_{t+1}^w - \frac{1-\beta}{1-\beta\rho}\varepsilon_{t+1}^y + \frac{\beta\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1}(\varepsilon_{t+1}^n - \varepsilon_{t+1}^f)\right),\end{aligned}$$

where $(\varepsilon_{t+1}^y, \varepsilon_{t+1}^n)$ are exogenous innovations of tradable endowment and liquidity dollar demand shocks, respectively, and $(\varepsilon_{t+1}^w, \varepsilon_{t+1}^f)$ are innovations of monetary and FX policy, repsectively. Therefore, this equation characterizes a fixed point for σ^2, which is indeed constant as long as innovations $(\varepsilon_{t+1}^y, \varepsilon_{t+1}^n, \varepsilon_{t+1}^w, \varepsilon_{t+1}^f)$ have a constant covariance matrix Σ. In what follows, we consider various special cases in which the expression characterizing σ^2 simplifies and σ^2 can be solved explicitly. For example, when monetary policy stabilizes the output gap, $x_t = w_t - a_t = 0$, we have $\varepsilon_{t+1}^w = \varepsilon_{t+1}^a$, i.e. the innovation of non-tradable productivity.

4 Exact Optimal Policies

We start by exploring various special case with an exact optimal policy characterization.

4.1 Constrained optimum

First, we consider the case with $\tau = 1$, namely when all income in the financial sector remains in the home country and there is no international transfer associated with noise traders and/or arbitrageurs. The optimal policy problem in this case delivers the constrained optimum as there is no incentive to manipulate risk sharing or monetary policy to achieve a monetary transfer from the rest of the world.

The optimum problem in this case is still to pick $\{C_{Tt}, B_t^*, \mathcal{E}_t, R_t, W_t, F_t^*\}$ and associated $\{\sigma_t^2\}$ to maximize (6) subject to (1)–(4), but the country budget constraint (3) in this case simplifies to:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt}. \tag{14}$$

As a result, the policy instrument F_t^* (FOREX interventions) enters only the international risk sharing constraint (4), and thus it would be chosen to relax this constraint (that is, ensure a zero Lagrange multiplier). The optimal choice of B_t^* when (4) is relaxed requires:

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} = 1, \tag{15}$$

that is international risk sharing without a wedge (i.e., $Z_t = 0$ in Lemma 3). Combining this undistorted risk sharing condition with the budget constraint determines the unique optimal path of $\{C_{Tt}\}$.

[10] Specifically, $z_t = \lambda_1 z_{t-1} - \frac{\beta\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1}(1-\beta^{-1}L)(n_t^* - f_t^*)$ and $b_t^* = \lambda_1 b_{t-1}^* + \frac{\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1}(n_t^* - f_t^*)$, an AR(2). In the case with $\lambda_1 = 1$ (when $\iota = 0$), we have Δz_t follow an ARMA(1,1) and Δb_t^* an AR(1).

By consequence, this requires setting $F_t^* = B_t^* - N_t^*$ to ensure zero wedge $Z_t = 0$ independently of the equilibrium volatility of the nominal exchange rate σ_t^2. This characterizes the optimal foreign exchange interventions, which lean against the wind — in fact, fully eliminate the wind — by fully accommodating the NFA demand of the home country (households) B_t^* and the liquidity demand of the noise traders N_t^*. As a result, the arbitrageurs have no job left, and $D_t^* = 0$, the equilibrium risk premium is eliminated and UIP holds in expectation.

Next, consider the optimal monetary policy, namely the choice of $\{W_t\}$. Note that with simplified budget constraint and with eliminated risk premium, the nominal exchange rate $\mathcal{E}_t$ and the home interest rate R_t are no longer constraining the optimization, and are merely side variables determined from the respective constraints. The choice of W_t then becomes static:

$$W_t = \arg\max\{\log W_t - W_t/A_t\} = A_t.$$

Setting $W_t = A_t$ eliminate the output gap state-by-state (i.e., $X_t = W_t/A_t = 1$). The equilibrium nominal exchange rate obtains from (1) and equals $\mathcal{E}_t = \frac{\gamma}{1-\gamma}\frac{A_t}{C_{Tt}}$. Finally, the equilibrium interest rate satisfies $\beta R_t \mathbb{E}_t\{A_t/A_{t+1}\} = 1$, which follows from (1)–(2).

We summarize this discussion in:

Proposition 1 *The constrained optimum allocation denoted with $\{\tilde{C}_{Tt}, \tilde{W}_t, \tilde{B}_t^*, \tilde{F}_t^*, \tilde{\mathcal{E}}_t, \tilde{R}_t\}$ maximizes welfare subject to the budget constraint alone:*

$$\max_{\{C_{Tt}, W_t, B_t^*\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_{Tt} + (1-\gamma)\left(\log W_t - \frac{W_t}{A_t}\right)\right] \quad \textit{subject to} \quad \frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt},$$

and it is implemented with monetary policy $\tilde{W}_t = A_t$ which closes the output gap and foreign exchange rate interventions $\tilde{F}_t^* = B_t^* - N_t^*$ which eliminates risk premium, UIP deviations and the risk sharing wedge. The optimum consumption path $\{\tilde{C}_{Tt}\}$ is the unique path that satisfies the dynamic system (14)–(15). The nominal exchange rate is given by $\tilde{\mathcal{E}}_t = \frac{\gamma}{1-\gamma}\frac{A_t}{\tilde{C}_{Tt}}$. The optimal policy is time consistent.

Proof: See discussion above and Appendix A. ■

Intuitively, there are two distortions — output gap due to sticky prices and imperfect risk sharing due to limits to arbitrage — and two policy instruments (monetary policy and FOREX interventions), which allow to address both distortions and deliver the constrained optimum.[11] The property of the constrained optimum is zero wedges in production (output gap) and in international risk sharing, $X_t = 1$ and $Z_t = 0$. The maximal utility is given by:

$$\tilde{\mathbb{W}}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log \tilde{C}_{Tt} + (1-\gamma)(\log A_t - 1)\right].$$

[11]Note that the constrained optimum is not first best as international financial market is incomplete and only allows to share risk in expectation given the foreign interest rate R_t^*. This is equivalent to a single foreign-currency bond economy. Interestingly, the presence of the home currency bond is irrelevant for the optimum allocation, as R_t is merely a side variable and does not affect the equilibrium allocation in this case, and the planner has no incentive to use an additional instrument (e.g., capital controls) to relax this constraint (see below).

This is the benchmark for the remaining analysis, as we take:

$$\mathbb{W}_0 - \tilde{\mathbb{W}}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log \frac{C_{Tt}}{\tilde{C}_{Tt}} + (1-\gamma) \left(\log \frac{W_t}{A_t} - \frac{W_t - A_t}{A_t} \right) \right] \leq 0,$$

where the first term is the loss from risk sharing distortions and the second term is the loss from the output gap.

Importantly, the optimal policy is time consistent, as both instruments remove the respective distortions contemporaneously and require no intertemporal promises. As a result, the implementation of the constrained optimum allocation does not require commitment on the part of the monetary authority.

There is no closed form characterization of $\tilde{C}_{Tt}$ in the presence of uninsured country risk in Y_{Tt}, but $\tilde{C}_{Tt}$ follows a near martingale process with innovations approximately equal to the annuity value of the innovation to the NPV of tradable endowment Y_{Tt}. For example, when Y_{Tt} follows a random walk, $\tilde{C}_{Tt} \approx \frac{R_t^*-1}{R_t^*} B_t^* + Y_{Tt}$ with $B_t^* \approx B_{-1}^*$, i.e. $\Delta B_t^* \approx 0$. What are the implications of this for the nominal and real exchange rate? The nominal exchange rate $\tilde{\mathcal{E}}_t = \frac{\gamma}{1-\gamma} \frac{A_t}{\tilde{C}_{Tt}}$, as well as the real exchange rate $\tilde{\mathcal{Q}}_t = \tilde{\mathcal{E}}_t^{1-\gamma}$, appreciates with the relative productivity in the tradable sector, that is when tradable endowment Y_{Tt} increases faster than non-tradable productivity A_t. Indeed, this is the Balassa-Samuelson force which shapes the path of the real exchange rate as the interplay between tradable and non-tradable productivity. Under sticky prices, implementing this path for the real exchange rate requires the nominal exchange rate to follow the same relative productivities.

Implementing the constrained optimum in an economy with sticky prices and frictional financial market requires an active use of both monetary policy and foreign exchange interventions, but does not require the use of capital controls. The goal of foreign exchange interventions is not to eliminate exchange rate volatility, but rather to eliminate the risk sharing wedge (the UIP deviations). No UIP deviations are, in fact, consistent with a volatile nominal exchange rate, which itself is generally a consequence of the optimal monetary policy stabilizing output gap.[12] However, in certain circumstances, the constrained optimum may involve a stable exchange rate. With segmented financial market, foreign exchange interventions provide the government with an important additional tool, which allows to stabilize wedges in the international financial market — and, in some circumstances, this may involve stabilizing the nominal exchange rate. The use of foreign exchange interventions does not interfere with monetary policy, which should still be focused on output gap stabilization, as in the closed economy, and do not generally require the use of capital controls. In this sense, such economy does not feature the trilemma trade-off present in conventional monetary models with a frictionless financial market.

4.2 Divine coincidence: fixed exchange rate

In the constrained optimum allocation, foreign exchange rate interventions $\tilde{F}_t^* = B_t^* - N_t^*$ eliminates the risk-sharing wedge ($Z_t = 0$), but do not result in a stable exchange rate ($\mathcal{E}_t \neq const$ in general).

[12] As shown above, the nominal exchange rate implementing the first best follows the relative non-tradable productivity. Arguably, the volatility of relative productivities is not as large as the observed volatility of floating exchange rates, e.g. dollar-euro (10% annualized standard deviation). Thus, it is likely that optimal foreign exchange rate interventions partially stabilize the exchange rate relative to laissez-faire, as we further discuss below.

Indeed, the nominal exchange rate traces the frictionless real exchange rate, which in turn reflects the relative movements in non-tradable productivity (relative to tradable endowment). We now explore the special case when a fixed exchange rate implements the constrained optimum.

Note also that the constrained optimum implementation requires the use of both instruments — monetary policy W_t and foreign exchange rate interventions F_t^* — and, in general, it cannot be implemented with monetary policy alone. There exists, however, an important special, yet robust, case when monetary policy alone can simultaneously implement both goals — output gap stabilization and elimination of the international risk-sharing wedge — without any need to use foreign exchange rate interventions. This case relies on the full stabilization of the nominal exchange rate — the fixed exchange rate — which can be achieved by means of monetary policy and thus eliminates the need to use foreign exchange interventions. We refer to this special case as *divine coincidence* in an open economy.

Indeed, examining the general policy problem (6) (still assuming no international transfers, $\tau = 1$), the limiting case with commitment to $\mathcal{E}_t = const$ implies $\sigma_t^2 = 0$, and thus eliminates the risk sharing wedge, i.e. ensures $Z_t = 0$, irrespectively of the use of the other instrument F_t^*. Furthermore, since $\mathcal{E}_t = \frac{\gamma}{1-\gamma}\frac{W_t}{C_{Tt}}$, monetary policy can always ensure a fixed exchange rate by setting $W_t = C_{Tt}$. The only remaining question is when such monetary policy can also be optimal from the point of view of output gap stabilization, that is ensures that $X_t = W_t/A_t = 1$. While being a knife-edge case, it is an important one, and can be formulated as follows: if the first-best real exchange rate (i.e., the real exchange rate corresponding to the first-best allocation with zero output gap) is constant then fixed nominal exchange rate is the optimal policy stabilizing simultaneously output gap and international risk sharing. Indeed, recall that the real and nominal exchange rates perfectly comove under sticky prices, $\mathcal{E}_t = \mathcal{Q}_t^{1-\gamma}$, so that if the first-best real exchange rate $\tilde{\mathcal{Q}}_t = \left(\frac{\gamma}{1-\gamma}\frac{A_t}{C_{Tt}}\right)^{1/(1-\gamma)} = const$, then it can always be implemented with $\mathcal{E}_t = const$ independently of the degree of price stickiness. Furthermore, this is an "if and only if" statement, and the fixed exchange rate is necessarily suboptimal whenever $\tilde{\mathcal{Q}}_t \neq const$ and price are (at least partially) sticky.

Proposition 2 *The fixed nominal exchange rate implements the constrained optimum allocation if and only if the first-best real exchange rate is stable, $\tilde{\mathcal{Q}}_t = const$. In this case, monetary policy alone can achieve both goals of output gap stabilization, $X_t = 1$, and elimination of the international risk-sharing wedge, $Z_t = 0$, without the use of foreign exchange rate interventions or capital controls.*

Proof: See discussion above and Appendix A. ■

When can we expect the first-best real exchange rate to be stable? In our setup, this is the case when Balassa-Samuelson forces exactly offset each other, and in particular the non-tradable productivity and tradable endowment comove in lock-step. Formally, this would require a near random walk process in both Y_{Tt} and A_t, so that C_{Tt} tracks Y_{Tt} and thus $A_t/C_{Tt} = const$.[13] More generally, real exchange rate may also vary because of differential evolution of home and foreign tradable productivity under home bias in tradable consumption. The divine coincidence principle generalizes to those environments, and

[13]In a linearized environment, this is exactly the case, as $c_{Tt} = y_{Tt}$ under a random walk endowment, but in a full non-linear problem, the path of C_{Tt} differs from that of Y_{Tt} due to precautionary savings from uninsured idiosyncratic risk.

still suggests that if one can argue that the first-best real exchange rate is stable, then a fixed nominal exchange rate regime implements the constrained optimum and achieves both policy objectives without the need to use other instruments such as exchange rate interventions or capital controls. In other words, divine coincidence is exactly the case where inflation (output gap) stabilization does not come into conflict with a fixed exchange rate, and thus trilemma, if present, is not binding.

Implementation We focused above on the direct implementation of the peg using W_t. Two remarks are in order. First, the same allocation can be implemented using an interest rate R_t rule, as pointed out above. Second, and more importantly, either W_t or R_t implementation can either target output gap or nominal exchange rate directly. Indeed, divine coincidence implies that fixed exchange rate equilibrium corresponds to the zero output gap equilibrium. However, the implementation of the policy does matter, as targeting output gap may be consistent with multiple exchange rate equilibria, one with $\sigma_t^2 = 0$ and another with $\sigma_t^2 > 0$, and only the former one ensures undistorted international risk sharing.[14] Thus, in terms of implementation, a monetary policy that explicitly targets the nominal exchange rate can be superior to stabilizing the output gap, even when it achieves the same goal. In this sense, the model captures the idea of using a nominal peg to anchor expectations, although the focus is on the financial market expectations rather than inflation expectations of households and firms (*cf.* Marcet and Nicolini 2003).

4.3 Single instrument without divine coincidence

Proposition 1 characterized the optimal joint use of monetary policy and foreign exchange interventions, which allows to implement the first best allocation by eliminating both the output gap and the international risk sharing wedge state-by-state. Proposition 2 shows how monetary policy can fully stabilize the nominal exchange rate, which immediately eliminates the risk sharing wedge without the use of FX interventions, and further characterizes circumstances when it is also optimal from the point of output gap stabilization. As a corollary, when prices are flexible and thus the output gap is absent irrespective of monetary policy, the optimal risk sharing can be always achieved by monetary policy that stabilizes the nominal exchange rate, without the use of FX interventions. In other words, equilibrium nominal exchange rate volatility can be desirable only under sticky prices, when it needs to accommodate the real exchange rate variation that cannot be achieved via adjustment of prices.

We now consider the reverse case of whether the output gap can be stabilized by foreign exchange interventions alone, when monetary policy is constrained, e.g. by the zero lower bound $R_t \geq \underline{R}$ or fixed exchange rate $\mathcal{E}_t = \bar{\mathcal{E}}$.[15] In contrast to the previous case, it is not possible to implement the first-

[14] Formally, compare the case with $W_t = A_t$ and $W_t = \kappa C_{Tt}$ for some appropriately chosen $\kappa > 0$, which under divine coincidence are both consistent with the optimal allocation. While the latter implementation ensures $\mathcal{E}_t = const$ from (1) and thus $\sigma_t^2 = 0$, the former may be consistent with multiple equilibria that solve $\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} = 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^*}{R_t^*}$ where $\sigma_t^2 = R_t^2 \cdot \mathrm{var}_t\left(\frac{C_{T,t+1}/C_{Tt}}{A_{t+1}/A_t}\right)$, in addition to the budget constraint (14). The multiplicity of solutions for $(C_{T,t}, \sigma_t^2)$ translates into the multiplicity of solutions for $\mathcal{E}_t$, with the $\sigma_t^2 = 0$ solution favored over the others in terms of welfare.

[15] Recall that under sticky prices, $P_{Nt} = 1$, we have $C_{Nt} = W_t$, and the loss from the output gap can be written as $\log \frac{C_{Nt}}{A_t} - \frac{C_{Nt} - A_t}{A_t} \leq 0$. Furthermore, C_{Nt} must satisfy $\beta R_t \mathbb{E}_t \{C_{Nt}/C_{N,t+1}\} = 1$ and $\mathcal{E}_t = \frac{\gamma}{1-\gamma} \frac{C_{Nt}}{C_{Tt}}$, with the former possibly constrained by the ZLB and the latter by the fixed exchange rate.

best allocation with foreign exchange interventions. In particular, fixed exchange rate implies $\sigma_t^2 = 0$ in (4), and while it immediately eliminates the risk sharing wedge, it also makes FX interventions F_t^* irrelevant for the equilibrium allocation. F_t^* can still affect allocation $\{C_{Tt}, C_{Nt}\}$ under the zero lower bound constraint if $\sigma_t^2 > 0$. However, under separable utility, F_t^* is optimally used to only eliminate the risk sharing wedge in tradables without targeting the allocation of non-tradables and the output gap.[16]

This analysis in particular suggests that FX interventions cannot substitute for monetary policy. We next explore the optimal use of monetary policy in the presence of both frictions when FX interventions F_t^* are not available. In this case, optimal monetary policy closes the output gap on average and trades off the state-by-state variation in output gap with reducing the risk sharing wedge by partially stabilizing the nominal exchange rate. Formally, the optimal monetary policy ensures $\mathbb{E}_t X_{t+1} = 1$, where $X_{t+1} = \frac{W_{t+1}}{A_{t+1}} = \frac{C_{Nt}}{A_t}$ is the output gap, but varies $X_{t+1} \neq 1$ state-by-state to reduce σ_t^2, in particular in periods following large risk sharing wedges $Z_t = \omega\sigma_t^2 \frac{N_t^* - B_t^*}{R_t^*} \neq 0$.[17] The policy increases $C_{N,t+1} = W_{t+1}$ over and above A_{t+1} when $C_{T,t+1}$ is high, and vice versa, which reduces the volatility of $\mathcal{E}_{t+1} = \frac{\gamma}{1-\gamma}\frac{W_{t+1}}{C_{T,t+1}}$ by making tradable and non-tradable consumption more correlated. This is the optimal trade-off between the two frictions, namely giving up on fully stabilizing the output gap at $t+1$ to reduce the risk sharing wedge in borrowing from t to $t+1$.

We summarize these results in the following proposition and provide a formal proof in Appendix A):

Proposition 3 (i) *Monetary policy can eliminate the risk-sharing wedge and implement the optimal international risk sharing, while foreign exchange interventions cannot close the output gap when monetary policy is constrained, and can only ensure constrained-optimal international risk sharing.* (ii) *Optimal monetary policy in the absence of FX interventions eliminates the output gap on average and uses the state-by-state variation in output gap to reduce the volatility of nominal exchange rate and the risk sharing wedge.*

Discretionary policy An important property of the optimal policies in Proposition 1 was time consistency and no need for commitment to implement them. As described above, the optimal monetary policy in the absence of FX interventions trades-off output gap stabilization at $t+1$ for reducing the risk sharing wedge at t. This requires commitment on the part of the monetary authority, as the only time-consistent discretionary outcome is the state-by-state output gap stabilization, $X_{t+1} = 1$, which leaves a laissez-faire international risk sharing wedge Z_t. This is, of course, suboptimal, as shown in Proposition 3.

4.4 International transfers. Capital controls

We now consider the case with $\tilde{\tau} \equiv 1-\tau > 0$ which features an international income transfer $\tilde{\tau}\pi_t^*$ from the financial transactions in the country budget constraint (3). Under these circumstances, eliminating

[16] With non-separable utility in (C_{Tt}, C_{Nt}), foreign exchange interventions can depart from the optimal risk sharing $\beta R_t^* \mathbb{E}_t\{C_{Tt}/C_{T,t+1}\} = 1$ in order to relax the constraint imposed by $\beta R_t \mathbb{E}_t\{C_{Nt}/C_{N,t+1}\} = 1$ when R_t cannot adjust. As in the general theory of second best, the constrained optimal policy introduces a wedge into international risk sharing if it allows to reduce the domestic output gap. Unlike capital controls or other taxes, however, which can directly distort $\beta R_t \mathbb{E}_t\{\frac{C_{Tt}}{C_{T,t+1}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\} = 1$, foreign exchange interventions are less capable and operate exclusively via their indirect affect on C_{Tt} in (4). *Cf.* Farhi and Werning (2012), Correia, Farhi, Nicolini, and Teles (2013), Farhi, Gopinath, and Itskhoki (2014).

[17] In contrast, $X_{t+1} = 1$ state-by-state in periods following $Z_t = 0$.

the risk sharing wedge Z_t in (4) is still feasible, but no longer optimal. First, consider the optimal policies from Proposition 1, namely $W_t = A_t$ and $F_t^* = B_t^* - N_t^*$, which still eliminate both the output gap and the risk sharing wedge. In this case, the country budget constraint becomes:

$$\frac{B_t^*}{R_t^*} - B_t^* = (Y_{Tt} - C_{Tt}) - \tilde{\tau}\tilde{R}_t^*\psi_{t-1},$$

where $\psi_{t-1} = \frac{N_{t-1}^*}{R_{t-1}^*}$ is the exogenous noise trader liquidity demand (for dollar relative to the national currency), $\tilde{R}_t^* = R_{t-1}^* - R_{t-1}^*\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}$ is the realized carry trade return, and UIP (5) holds in expectation, $\mathbb{E}_{t-1}\Theta_t\tilde{R}_t^* = 0$, where $\Theta_t = \beta\frac{C_{T,t-1}}{C_{Tt}}$ is the stochastic discount factor. As a result, this allocation is associated with mean-zero idiosyncratic international transfers that contribute to national income volatility of the home country, and thus contribute negatively to welfare. Can the government improve upon this allocation? In particular, is it feasible to eliminate this risk or create systematic transfers from the rest of the world.

One can show that departures from $W_t = A_t$, if UIP still holds in expectations, generate at most third order benefits, while creating second order losses from departures from output gap. Thus, we focus here for concreteness on monetary policy that stabilizes output gap, $W_t = A_t$, and explore the use of FX interventions F_t^* in the presence of international transfers. We rewrite the budget constraint (3):

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = (Y_{Tt} - C_{Tt}) - \tilde{\tau}\tilde{R}_t^*\left[\psi_{t-1} + \frac{\mathbb{E}_{t-1}\Theta_t\tilde{R}_t^*}{\omega\sigma_{t-1}^2}\right],$$

and the government has a direct control over the size of the UIP deviation, $\mathbb{E}_t\Theta_{t+1}\tilde{R}_{t+1}^* = Z_t$ by means of FX interventions, $Z_t = \omega\sigma_t^2\frac{B_t^* - N_t^* - F_t^*}{R_t^*}$, which simultaneously creates a risk-sharing wedge: $Z_t = \beta R_t^*\mathbb{E}_t\frac{C_{Tt}}{C_{T,t+1}} - 1$. Therefore, the tradeoff faced by the policymaker is whether to engineer ex ante UIP deviations, which distort risk sharing, yet can generate additional national income under certain circumstances.

The expected discounted income (using home SDF) from FX interventions that allow for UIP deviations ($Z_t \neq 0$) is given by:

$$-\tilde{\tau}\mathbb{E}_t\Theta_{t+1}\tilde{R}_{t+1}^*\left[\psi_t + \frac{\mathbb{E}_t\Theta_{t+1}\tilde{R}_{t+1}^*}{\omega\sigma_t^2}\right] = -\tilde{\tau}\left[\psi_t Z_t + \frac{Z_t^2}{\omega\sigma_t^2}\right].$$

Therefore, the expected income is (weakly) negative in the absence of noise trader demand (when $\tilde{\tau}\psi_t = 0$), and thus $Z_t = 0$ is optimal in this case as it guarantees both efficient risk sharing and no expected income losses. A corollary of this result is that, if noise traders are domestic and arbitrageurs are foreign, the government can also generate no expected income and should ensure $Z_t = 0$ by setting $F_t^* = B_t^* - N_t^*$ as in Proposition 1.

In the presence of international noise trader demand, the policymaker can generate expected incomes by partially "leaning against the wind" of their currency demand and choosing F_t such that:

$$\psi_t Z_t \propto N_t^* \cdot (B_t^* - N_t^* - F_t^*) < 0.$$

The income gains of the government are limited, however, by the arbitrageurs, who take positions in the same direction as the government and inversely proportionally to $\omega\sigma_t^2$. As a result, in the limit of $\omega\sigma_t^2 \to 0$, the government cannot sustain any expected income gains, even in the presence of noise traders, and should *not* attempt to choose $Z_t \neq 0$, which would be futile anyways. Finally, for any $\omega\sigma_t^2 > 0$, UIP deviations Z_t in response to $\psi_t \neq 0$ generate income gains that are first order in Z_t and welfare losses from the resulting risk sharing wedge that are second order in Z_t, around $Z_t = 0$. Therefore, non-zero UIP deviation Z_t are necessarily desirable in this case, if sufficiently small.[18]

We summarize these results in the following proposition:

Proposition 4 (i) *The expected discounted income from FX interventions is weakly negative in the absence of noise trader demand ($N_t^* = 0$), and thus it is optimal to fully offsetting the ex ante UIP deviations (with $F_t^* = B_t^* - N_t^*$) to ensure both no expected losses and efficient risk sharing.* (ii) *In the presence of noise trade demand ($N_t^* \neq 0$), and for $\omega\sigma_t^2 > 0$, there exist FX interventions F_t^* that partially lean against N_t^* and generate expected incomes that exceed welfare losses from the induced UIP and risk sharing wedges.*

The first part of this proposition generalizes the optimal policy and divine coincidence results of Propositions 1 and 2 by showing that they hold in frictional financial markets with arbitrageurs and international transfers, but without noise traders. Under these circumstances, the policymaker never wants to manipulate the UIP deviations, and thus two baseline policy tools are sufficient and are used exactly in the same way as before to close the output gap and the risk sharing wedge.

We leave full characteristic of the optimal FX interventions to the linearized environment, and note here two additional motivations of the policy. First, it may be possible to manipulate the ex post exchange rate realization (e.g., with the monetary instrument) to increase expected income or smooth national income fluctuations, that is increase the mean or reduce the variance of $Y_{Tt} - \tilde{\tau}\tilde{R}_t^*\big[\psi_{t-1} + \mathbb{E}_{t-1}\Theta_t\tilde{R}_t^*/(\omega\sigma_{t-1}^2)\big]$. Note that reducing the variance of national income first improves international risk sharing from the point of view of the home country. Second, the planner may want to increase the idiosyncratic volatility of the nominal exchange rate, σ_t^2, in order to reduce the arbitrageur activity and thus increase the maximum possible expected income transfers from the noise traders. This policy of "destabilization" makes intermediation by arbitrageurs more costly, and increases the space and effectiveness for the government FX interventions.

Volatility of central bank's balance sheet The policy of FX interventions, whether it results in UIP deviations or not, leads to ex post income or losses borne by the central bank, even when expected incomes and losses might be zero. In particular, the ex post income of the central bank is given by $\tilde{R}_{t+1}^*\frac{F_t^*}{R_t^*} = \Big[1 - \frac{R_t}{R_t^*}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big]F_t^*$ and its variance is given by $\sigma_t^2 \cdot (F_t^*/R_t^*)^2$. Thus, two of the possible constraints on the central bank's balance sheet may be non-negative foreign reserves $F_t^* \geq 0$ or a value at risk constraint $|F_t^*| \leq \alpha R_t^*/\sigma_t$. Both constraints may limit the ability of the central bank to implement the optimal policies, and in particular the policy $F_t^* = B_t^* - N_t^*$ from Proposition 1 may be infeasible.

[18]The maximum expected income equals $\frac{1}{4}\tilde{\tau}\omega\sigma_t^2\psi_t^2$, and it is achieved when $Z_t = -\frac{1}{2}\omega\sigma_t^2\psi_t$, or equivalently $F_t^* = B_t^* - \frac{1}{2}N_t^* = B_t^* - \frac{1}{2}R_t^*\psi_t$. The optimal intervention additionally takes into account the welfare loss from the risk-sharing wedge which is second order in Z_t.

Furthermore, the region of feasibility may not be connected, as there is a feedback between policy F_t^* and equilibrium exchange rate volatility σ_t^2. More specifically, limited interventions F_t^* may result in large equilibrium exchange rate volatility σ_t^2, while large interventions, vice versa, limit significantly the equilibrium σ_t^2, thus possibly making the intermediate levels of interventions infeasible — a vague analogy with an inverse Laffer curve in taxation.

Finally, in cases when sufficiently large interventions are infeasible, and the lowest achievable σ_t^2 with FX interventions is large, a fully fixed exchange rate by means of monetary policy may be superior relative to the output-gap stabilizing monetary policy and the best feasible FX interventions. This can be the case, in particular, even if the divine coincidence of Propositions 2 is not satisfied. Thus, this offers a justifications for some exchange rate pegs that are adopted despite the resulting output gaps and suboptimal real exchange rate under the peg.

Capital controls So far, we have left out capital controls from our considerations. Indeed, Propositions 1 and 2 show that optimal allocations can be attained without any use of capital controls, as long as there are no international transfers ($\tau = 1$ in (3)) and both monetary policy and FX interventions are available and unconstrained.[19] As soon as we consider the full policy problem (6) which features a general budget constraint (3) with a possibility of transfers, the capital controls become useful.

In particular, consider the full policy problem (6) with three constraints — the budget constraint (3) and two Euler equations, (4) with R_t^* and (2) with R_t, while treating equation (1) as the definition of $\mathcal{E}_t$ in the other equations (in the same way as the σ_t^2 definition in (4)). The only technological constraint is the budget constraint, and it cannot be relaxed, while the two Euler equations can be relaxed provided enough policy instruments. Indeed, FX interventions relax the risk sharing constraint (4), while capital controls on households (or other intertemporal taxes) relax the household Euler equation (2). This effectively makes R_t a free choice variable, allowing the government to manipulate UIP deviations with both F_t^* and capital controls, thus further maximizing the rents that can be extracted from noise traders. In general, these rents are limited by the intermediation of arbitrageurs, unless separate capital controls can be levied on the arbitrageurs as well.

[19]There remains a questions of whether FX interventions or capital controls are a more desirable policy instrument? Indeed, under certain circumstances, theoretically, capital controls are perfectly substitutable with FX interventions, with the only difference that FX interventions eliminate the UIP wedge, while capital controls compensate domestic households for the associated risk premium (provide a subsidy or a tax to ensure that their intertemporal decision is undistorted). From the informational point of view, both instruments require the observation of the expected UIP deviations, $\mathbb{E}_t \Theta_{t+1} \tilde{R}_{t+1}^*$. One can argue, however, that flexible state-specific FX interventions by the central bank are easier to implement than a comparable state-contingent capital taxes and subsidies on international financial transactions. On the other hand, to the extent there is a fixed permanent component of the UIP deviations, it may be easier to address it with a constant capital controls tax (or subsidy).

5 Linear-Quadratic Policy Problem

Lemmas 4 and 5 characterize the equilibrium system and the quadratic objective function respectively in the linearized environment, and we reproduce the general linear-quadratic policy problem here:

$$\min_{\{x_t, z_t, e_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2}\mathbb{E}_0 \sum\nolimits_{t=0}^{\infty} \beta^t \big[\gamma z_t^2 + (1-\gamma)x_t^2\big]$$

$$\text{subject to} \qquad \beta b_t^* = b_{t-1}^* - z_t,$$

$$\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega}\sigma_t^2(\iota b_t^* - n_t^* - f_t^*),$$

$$\sigma_t^2 = \mathbb{E}_t e_{t+1}^2 - \big(\mathbb{E}_t e_{t+1}\big)^2 \quad \text{where} \quad e_t = \tilde{q}_t + x_t - z_t,$$

given initial net foreign assets b_{-1}^* and the exogenous path of shocks $\{\tilde{q}_t, n_t^*\}$, where $\tilde{q}_t = a_t - \tilde{c}_{Tt}$ is the first best real exchange rate and n_t^* is the aggregate demand for currency (including both fundamental and noise sources), both described in Section 3.2. We can think of monetary policy as choosing directly the path of output gap x_t, while FX interventions f_t^* control the path of the risk sharing wedge z_t. The policies, thus, interact in determining the equilibrium volatility of the exchange rate, σ_t^2, which in turn feeds back into shaping the equilibrium risk sharing wedge without being directly a goal of the policy in itself. More specifically, the goal of the policy is to minimize the weighted average of the volatility (in the mean squared error sense) of the output gap and the risk sharing wedge.

5.1 First best

The first best allocation features $x_t = z_t = 0$ for all t, as it is the global minimum of the loss function. This allocation can be always delivered by a combination of monetary and FX interventions. Specifically, in addition to monetary policy that stabilized output gap, $x_t = 0$ (or $w_t = a_t$), the optimal FX interventions are $f_t^* = \iota b_t^* - n_t^* = -n_t^*$, since this policy ensures $z_t = 0$, and by consequence $b_t^* = 0$.[20] As a result, the risk sharing wedge is fully offset, and the optimal international risk sharing is restored independently of the currency demand shocks n_t^*. This solution is time consistent and its implementation requires no commitment.

Proposition 5 *If both policy instruments are available and unconstrained, the optimal policy fully stabilizes both wedges, the output gap $x_t = 0$ and the risk sharing wedge $z_t = 0$, by targeting home PPI inflation with monetary policy ($w_t = a_t$) and demand for currency with FX interventions ($f_t = \iota b_t^* - n_t^* = -n_t^*$). This solution is time consistent and its implementation requires no commitment.*

One notable feature of this result is that capital controls (see below) are not needed for implementation, as FX interventions are sufficient to achieve first best when combined with optimal monetary policy. The second implication of this result is that FX interventions do not target exchange rate or insure exchange rate stabilization. The optimal policy ensures $x_t = z_t = 0$, which in turn implies that

[20] Note that the policy rule $f_t^* = -n_t^*$ also necessarily implies $z_t = b_t^* = 0$ as the unique solution, thus resulting in the same outcome (see characterization in Section 3.2).

the nominal exchange rate equals the first-best real exchange rate:

$$e_t = \tilde{q}_t + x_t - z_t = \tilde{q}_t = a_t - \tilde{c}_{Tt}. \tag{16}$$

Instead, optimal FX interventions eliminate UIP deviations:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \mathbb{E}_t \Delta z_{t+1} = 0, \tag{17}$$

where $i_t = \log R_t - \log \bar{R} = \mathbb{E}_t\{\Delta c_{T,t+1} + \Delta e_{t+1}\}$ and $i_t^* = \log R_t^* - \log \bar{R}^* = \mathbb{E}_t \Delta \tilde{c}_{T,t+1}$, as we describe in footnote 8.

5.2 Divine coincidence

We now study the optimal monetary policy when FX interventions are not available, that is $f_t^* \equiv 0$. We start we the case in which the first best is attainable with a single monetary policy instrument, and by analogy with the closed economy literature we label this case *divine coincidence.*

Proposition 6 *If the first best real exchange rate is stable, $\tilde{q}_t = 0$, then monetary policy that fully stabilizes the nominal exchange rate, $e_t = 0$, ensures the first best allocation with $x_t = z_t = 0$, even in the absence of FX interventions $f_t^* = 0$. An exchange rate peg is superior to inflation targeting, as it rules out multiplicity of exchange rate equilibria.*

The first best solution (16) implies that the nominal exchange rate equals the first-best real exchange rate, $e_t = \tilde{q}_t$. Therefore, if $\tilde{q}_t = 0$, then $e_t = 0$ is part of the first best allocation, and this implies $\sigma_t^2 = 0$. With σ_t^2, however, $z_t = 0$ independently of (b_t^*, n_t^*, f_t^*). Hence, if $e_t = 0$ is consistent with $x_t = z_t = 0$, then such policy ensures this first-best outcome as the unique equilibrium. Indeed, this is a "divine coincidence", as targeting one margin — a zero risk-sharing wedge $z_t = 0$ — simultaneously ensures an efficient real exchange rate and an absence of the output gap $x_t = 0$.

This case provides a rationale for pegging the exchange rate. Moreover, in this case, a nominal exchange rate peg by means of monetary policy is not only efficient, but also effective, as it immediately eliminates the possibility of multiple equilibria. Consider the alternative policy of inflation targeting with $w_t = a_t$, which ensures $x_t = 0$ independently of $z_t = 0$. Under divine coincidence, such policy is consistent with an equilibrium $z_t = e_t = \sigma_t^2 = 0$, however, this is not a unique equilibrium. Indeed, consider our example in Section 3.2, where n_t^* follows an AR(1). In this case, in light of $\tilde{q}_t = x_t = 0$, the solution for the nominal exchange rate is $e_t = -z_t = -(1-\beta\lambda_1)b_{t-1}^* + \frac{\beta\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1} n_t^*$, which implies:

$$\sigma_t = \sigma = \frac{\beta\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1} \mathrm{std}_t\big(\varepsilon_{t+1}^n\big).$$

Thus, it generally adopts a solution with $\sigma = 0$ and with $\sigma > 0$, the latter being a suboptimal outcome with $z_t \neq 0$. Thus, under divine coincidence, exchange rate peg dominates inflation targeting, even though the outcome of exchange rate peg is stable inflation in this case.

How does divine coincidence work? The nominal exchange rate has a dual role. On one hand, its movements ensure expenditure switching in the goods market, changing the relative price of domestic (home or non-tradable) and international (foreign or tradable) goods. In the presence of sticky prices, without such exchange rate movements — and corresponding exchange rate volatility — the real exchange rate departs from its first-best level and, as a result, the goods market does not achieve the efficient allocation, as reflected in the output gap $x_t \neq 0$. At the same time, nominal exchange rate volatility, $\sigma_t^2 > 0$, results in UIP deviations for a given $(\iota b_t^* - n_t^* - f_t^*) \neq 0$, and thus departures from frictionless international risk sharing, $z_t \neq 0$. These deviations are increasing in the unpredictable exchange rate volatility, thus resulting in a conflict between the two policy objectives, or a policy tradeoff.

Divine coincidence is the situation when this policy tradeoff is absent, as it occurs when the first-best real exchange rate is stable $\tilde{q}_t = 0$, and thus $e_t = 0$ ensures both $x_t = 0$ and $z_t = 0$ — the latter due to $\sigma_t^2 = 0$, and the former as a coincidence due to $\tilde{q}_t = 0$. Note that in our baseline model, $\tilde{q}_t = a_t - \tilde{c}_{Tt}$ reflecting the Balassa-Samuelson forces in the model with non-tradables. In particular, if both non-tradable productivity a_t and tradable endowment y_t follow an identical random walk, the $\tilde{c}_{Tt} = y_t = a_t$, resulting in a divine coincidence $\tilde{q}_t = 0$. This is, of course, a knife-edge case which we do not expect to systematically hold in the data, yet it provides a key benchmark for our analysis and a stark illustration to the model's mechanism.

How special is the divine coincidence result? We explore its robustness below, where we show in particular that it does not rely on the specific model of the real exchange rate, namely the Balassa-Samuelson model with non-tradables and the law of one prices for tradables. What is crucial, however, is the model of the financial market in which ex post stable exchange rate, $e_{t+1} = 0$, implies ex ante certainty, namely $\sigma_t^2 = \mathrm{var}_t(e_{t+1}) = 0$, and it in turn guarantees that UIP holds and risk sharing is undistorted. This nests two assumptions. First, it requires that commitment to a peg is ex ante credible. Second, it relies on the structure of the model in which a fully stabilized exchange rate eliminates UIP deviations via the endogenous response of arbitrageurs, who are willing to take unbounded positions in the absence of exchange rate risk if UIP is violated. If either the peg is not credible, and there is a chance that $e_{t+1} \neq \mathbb{E}_t e_{t+1}$, or UIP deviations may coexist with $\sigma_t^2 = 0$, then divine coincidence result breaks down. To the extent a peg eliminates a large portion of UIP deviations, this result nonetheless may offer an accurate quantitative approximation, emphasizing robust economic forces at play.

Appendix

A Derivations and Proofs

Lemma 2 (country budget constraint) Substitute firm profits $\Pi_t = P_{Nt}Y_{Nt} - W_t L_t$ and household consumption expenditure $P_t C_t = P_{Nt}C_{Nt} + P_{Tt}C_{Tt}$ into the household budget constraint and use market clearing $C_{Nt} = Y_{Nt}$ to obtain:

$$\frac{B_t}{R_t} - B_{t-1} = NX_t + T_t,$$

where $NX_t = P_{Tt}Y_{Tt} - P_{Tt}C_{Tt} = \mathcal{E}_t(Y_{Tt} - C_{Tt})$. Next combine the household and government budget constraints to obtain:

$$\frac{B_t + F_t}{R_t} + \frac{\mathcal{E}_t F_t^*}{R_t^*} - B_{t-1} - F_{t-1} - \mathcal{E}_t F_{t-1}^* = NX_t + \tau\mathcal{E}_t\pi_t^*.$$

Define B_t^* such that $\frac{B_t^*}{R_t^*} = \frac{F_t^*}{R_t^*} + \frac{B_t+F_t}{\mathcal{E}_t R_t}$ and use the market clearing $B_t + D_t + N_t + F_t = 0$ and Lemma 1 that $B_t^* = D_t^* + N_t^* + F_t^*$ to rewrite:

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} - \mathcal{E}_t B_{t-1}^* + \mathcal{E}_t(D_{t-1}^* + N_{t-1}^*) + (D_{t-1} + N_{t-1}) = NX_t + \tau\mathcal{E}_t\pi_t^*.$$

Finally, recall that $\pi_t^* = \tilde{R}_t^* \frac{D_{t-1}^* + N_{t-1}^*}{R_{t-1}^*} = \left[1 - \frac{R_{t-1}}{R_{t-1}^*}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\right](D_{t-1}^* + N_{t-1}^*)$. Subtract $\mathcal{E}_t\pi_t^*$ on both sides of the budget of the budget constraint to obtain:

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} - \mathcal{E}_t B_{t-1}^* + \underbrace{(D_{t-1} + N_{t-1}) + \frac{R_{t-1}}{R_{t-1}^*}\mathcal{E}_{t-1}(D_{t-1}^* + N_{t-1}^*)}_{=0 \text{ as zero capital portfolio at } t-1} = NX_t - (1-\tau)\tilde{R}_t^* \frac{\mathcal{E}_t(D_{t-1}^* + N_{t-1}^*)}{R_{t-1}^*},$$

Divide through by $\mathcal{E}_t$, use the fact that $NX_t/\mathcal{E}_t = Y_{Tt} - C_{Tt}$, and Lemma 1 that $D_{t-1}^* + N_{t-1}^* = B_{t-1}^* - F_{t-1}^*$ to rewrite:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = (Y_{Tt} - C_{Tt}) - (1-\tau)\tilde{R}_t^* \frac{\mathcal{E}_t(B_{t-1}^* - F_{t-1}^*)}{R_{t-1}^*},$$

completing the proof of the lemma. ■

Proposition 1 (constrained optimum) The planner solves in this case:

$$\mathbb{W}_0 = \max_{\{C_{Tt}, B_t^*, \mathcal{E}_t, R_t, W_t, F_t^*, \sigma_t^2\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_{Tt} + (1-\gamma)\left(\log W_t - \frac{W_t}{A_t} \right) \right],$$

$$\text{subject to} \quad \frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt},$$

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} = 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \qquad \sigma_t^2 = R_t^2 \cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big),$$

$$\beta R_t \mathbb{E}_t \left\{ \frac{C_{Tt}}{C_{T,t+1}} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right\} = 1, \qquad \mathcal{E}_t = \frac{\gamma}{1-\gamma} \frac{W_t}{C_{Tt}}.$$

The Lagrange multipliers on all constraints, but the budget constraint must be zero, and thus the problem is equivalent to maximizing the objective with respect to $\{C_{Tt}, B_t^*, W_t\}$ subject to the budget constraint only. First, note that Lagrange multipliers on the two constraints in the third line must be zero: since F_t^* enters only one constraint, and σ_t^2 enters only one other constraint, and neither enter the objective, F_t^* can be chosen to relax both constraints (ensure zero multipliers). Second, dropping these constraints, optimization over R_t and $\mathcal{E}_t$, which are featured only in the two of the remaining three constraints and not in the objective, ensures zero Lagrange multiplier on those constraints as well.

Solving the remaining problem, as stated in the proposition, results in the solution $\{\tilde{C}_{Tt}, \tilde{B}_t^*, \tilde{W}_t\}$ with $\tilde{W}_t = A_t$ and $\{\tilde{C}_{Tt}, \tilde{B}_t^*\}$ the unique solution of:

$$\beta R_t^* \mathbb{E}_t \{ C_{Tt}/C_{T,t+1} \} = 1 \qquad \text{and} \qquad \frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt}.$$

Using the remaining constraints of the problem, we back out $\{\tilde{\mathcal{E}}_t, \tilde{R}_t, \tilde{F}_t^*, \tilde{\sigma}_t^2\}$, and in particular we have $\tilde{F}_t^* = \tilde{B}_t^* - N_t^*$ and $\tilde{\mathcal{E}}_t = \frac{\gamma}{1-\gamma}\frac{A_t}{\tilde{C}_{Tt}}$. ■

Proposition 2 (divine coincidence) ... ■

Proposition 3 (single instrument)

$$\max_{\{C_{Tt},C_{Nt},B_t^*,\mathcal{E}_t,R_t,F_t^*,\sigma_t^2\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_{Tt} + (1-\gamma)\left(\log C_{Nt} - \frac{C_{Nt}}{A_t}\right)\right],$$

$$\begin{aligned}
\frac{B_t^*}{R_t^*} - B_{t-1}^* &= Y_{Tt} - C_{Tt}, \\
\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} &= 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \\
\beta R_t \mathbb{E}_t \frac{C_{Nt}}{C_{N,t+1}} &= 1, \\
\mathcal{E}_t &= \frac{\gamma}{1-\gamma}\frac{C_{Nt}}{C_{Tt}}, \\
\sigma_t^2 &= R_t^2 \cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big).
\end{aligned}$$

$$\max_{\{C_{Tt},C_{Nt},B_t^*,\mathcal{E}_t,R_t,\sigma_t^2\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_{Tt} + (1-\gamma)\left(\log C_{Nt} - \frac{C_{Nt}}{A_t}\right)\right],$$

$$\begin{aligned}
\text{subject to} \qquad \frac{B_t^*}{R_t^*} - B_{t-1}^* &= Y_{Tt} - C_{Tt}, \\
\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} &= 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^*}{R_t^*}, \\
\beta R_t \mathbb{E}_t \frac{C_{Nt}}{C_{N,t+1}} &= 1, \\
\mathcal{E}_t &= \frac{\gamma}{1-\gamma}\frac{C_{Nt}}{C_{Tt}}, \\
\sigma_t^2 &= R_t^2 \cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big).
\end{aligned}$$

which reduces to by solving out R_t and $\mathcal{E}_t$ using the 3rd and 4th constraints:

$$\max_{\{C_{Tt},\Gamma_t,B_t^*,\sigma_t^2\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_{Tt} - (1-\gamma)\left(\log \Gamma_t + \frac{1}{A_t\Gamma_t}\right)\right],$$

$$\begin{aligned}
\text{subject to} \qquad \frac{B_t^*}{R_t^*} - B_{t-1}^* &= Y_{Tt} - C_{Tt}, \\
\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} &= 1 - \omega\sigma_t^2 \frac{N_t^* - B_t^*}{R_t^*}, \\
\beta^2 C_{Tt}^2 (\mathbb{E}_t\Gamma_{t+1})^2\sigma_t^2 &= \mathbb{E}_t\big(\Gamma_{t+1}C_{T,t+1}\big)^2 - (\mathbb{E}_t C_{T,t+1})^2\,(\mathbb{E}_t\Gamma_{t+1})^2,
\end{aligned}$$

where $\Gamma_t \equiv 1/C_{Nt}$. Use Lagrange multipliers $(\lambda_t, \mu_t, \delta_t)$ for the three constraints:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \left[\gamma \log C_{Tt} - (1-\gamma)\left(\log \Gamma_t + \frac{1}{A_t \Gamma_t}\right)\right] \\ & + \lambda_t \Big[B^*_{t-1} + Y_{Tt} - C_{Tt} - \frac{B^*_t}{R^*_t}\Big] + \mu_t \Big[1 - \omega \sigma_t^2 \frac{N^*_t - B^*_t}{R^*_t} - \beta R^*_t \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}}\Big] \\ & + \delta_t \Big[\beta^2 C_{Tt}^2 (\mathbb{E}_t \Gamma_{t+1})^2 \sigma_t^2 + (\mathbb{E}_t C_{T,t+1})^2 (\mathbb{E}_t \Gamma_{t+1})^2 - \mathbb{E}_t \big(\Gamma_{t+1} C_{T,t+1}\big)^2\Big] \Bigg\}.\end{aligned}$$

Note that μ_t has the same sign as $\sigma_t^2(N^*_t - B^*_t)$ so that $\mu_t \sigma_t^2 (N^*_t - B^*_t) \geq 0$ and $\delta_t \geq 0$, with equalities only if $\sigma_t^2(N^*_t - B^*_t) = 0$. Also note that $\mathbb{E}_t$ in the Lagrangian stands for $\sum_{s_{t+1}} \pi_t(s_{t+1})$, where $\pi_{t+1} = \pi_t(s_{t+1})$ is the probability of state s_{t+1} at $t+1$ conditional on state s_t at t. We need to take FOCs with respect to σ_t^2 and Γ_{t+1} in state s_{t+1}:

$$\begin{aligned}& -\mu_t \omega \frac{N^*_t - B^*_t}{R^*_t} + \delta_t \beta^2 C_{Tt}^2 (\mathbb{E}_t \Gamma_{t+1})^2 = 0, \\ & \beta \pi_{t+1} (1-\gamma) \frac{1}{\Gamma_{t+1}} \left(\frac{1}{A_{t+1}\Gamma_{t+1}} - 1\right) + 2\delta_t \pi_{t+1} \Big[\left(\beta^2 C_{Tt}^2 \sigma_t^2 + (\mathbb{E}_t C_{T,t+1})^2\right)(\mathbb{E}_t \Gamma_{t+1}) - C_{T,t+1}^2 \Gamma_{t+1}\Big] = 0.\end{aligned}$$

We simplify and rewrite:

$$\begin{aligned}\delta_t \beta^2 C_{Tt}^2 (\mathbb{E}_t \Gamma_{t+1})^2 &= \mu_t \omega \frac{N^*_t - B^*_t}{R^*_t}, \\ \beta(1-\gamma)\left(\frac{1}{A_{t+1}\Gamma_{t+1}} - 1\right) &= 2\delta_t \Big[(\Gamma_{t+1} C_{T,t+1})^2 - \left(\beta^2 C_{Tt}^2 \sigma_t^2 + (\mathbb{E}_t C_{T,t+1})^2\right)(\mathbb{E}_t \Gamma_{t+1})\Gamma_{t+1}\Big].\end{aligned}$$

Next take the expectation $\mathbb{E}_t$ of the second condition and use the definition of σ_t^2 to simplify:

$$\beta(1-\gamma)\mathbb{E}_t \left(\frac{1}{A_{t+1}\Gamma_{t+1}} - 1\right) = 2\delta_t \Big[\mathbb{E}_t (\Gamma_{t+1} C_{T,t+1})^2 - \left(\beta^2 C_{Tt}^2 \sigma_t^2 + (\mathbb{E}_t C_{T,t+1})^2\right)(\mathbb{E}_t \Gamma_{t+1})^2\Big] = 0$$

as the RHS corresponds to the definition of σ_t^2. Thus, average output gap is zero. Now substitute out δ_t:

$$\beta(1-\gamma)\left(\frac{1}{A_{t+1}\Gamma_{t+1}} - 1\right) = \frac{2\omega\mu_t}{\beta^2} \frac{N^*_t - B^*_t}{R^*_t} \left[\frac{(\Gamma_{t+1} C_{T,t+1})^2}{C_{Tt}^2 (\mathbb{E}_t \Gamma_{t+1})^2} - \frac{\mathbb{E}_t \big(\Gamma_{t+1} C_{T,t+1}\big)^2}{C_{Tt}^2 (\mathbb{E}_t \Gamma_{t+1})^2} \frac{\Gamma_{t+1}}{\mathbb{E}_t \Gamma_{t+1}}\right],$$

where we used:

$$\beta^2 \sigma_t^2 + (\mathbb{E}_t C_{T,t+1}/C_{Tt})^2 = \frac{\mathbb{E}_t \big(\Gamma_{t+1} C_{T,t+1}\big)^2}{C_{Tt}^2 (\mathbb{E}_t \Gamma_{t+1})^2}.$$

Rewrite in terms of C_{Nt} and $\mathcal{E}_t$:

$$\beta(1-\gamma)\left(\frac{C_{N,t+1}}{A_{t+1}} - 1\right) = \frac{2\omega\mu_t}{\beta^2} \frac{\mathcal{E}_t (N^*_t - B^*_t)}{R^*_t} \left[\frac{(1/\mathcal{E}_{t+1})^2}{(\mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}})^2} - \frac{\mathbb{E}_t \big(1/\mathcal{E}_{t+1}\big)^2}{(\mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}})^2} \frac{1/(\mathcal{E}_{t+1} C_{T,t+1})}{\mathbb{E}_t 1/(\mathcal{E}_{t+1} C_{T,t+1})}\right],$$

and further simplify by noting that $\mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} = \mathbb{E}_t \frac{W_t}{W_{t+1}} = 1/(\beta R_t)$:

$$\beta(1-\gamma)\left(\frac{C_{N,t+1}}{A_{t+1}} - 1\right) = 2\omega R_t^2 \mu_t \frac{\mathcal{E}_t(N_t^* - B_t^*)}{R_t^*}\left[\frac{1}{\mathcal{E}_{t+1}^2} - \mathbb{E}_t \frac{1}{\mathcal{E}_{t+1}^2} \frac{1/(\mathcal{E}_{t+1}C_{T,t+1})}{\mathbb{E}_t 1/(\mathcal{E}_{t+1}C_{T,t+1})}\right],$$

or

$$\beta(1-\gamma)\left(\frac{C_{N,t+1}}{A_{t+1}} - 1\right) = 2\omega R_t^2 \mu_t \frac{\mathcal{E}_t(N_t^* - B_t^*)}{R_t^* \mathcal{E}_{t+1} C_{T,t+1}}\left[\frac{C_{T,t+1}}{\mathcal{E}_{t+1}} - \frac{\mathbb{E}_t\big(1/\mathcal{E}_{t+1}\big)^2}{\mathbb{E}_t 1/(\mathcal{E}_{t+1}C_{T,t+1})}\right],$$

or

$$\beta\gamma C_{N,t+1}\left(\frac{C_{N,t+1}}{A_{t+1}} - 1\right) = 2\omega R_t^2 \mu_t \frac{\mathcal{E}_t(N_t^* - B_t^*)}{R_t^*}\left[\frac{C_{T,t+1}}{\mathcal{E}_{t+1}} - \frac{\mathbb{E}_t\big(1/\mathcal{E}_{t+1}\big)^2}{\mathbb{E}_t 1/(\mathcal{E}_{t+1}C_{T,t+1})}\right],$$

or

$$\beta\gamma C_{N,t+1}\left(X_{t+1} - 1\right) = \frac{2\mathcal{E}_t R_t^2 \mu_t Z_t}{\sigma_t^2}\left[\frac{C_{T,t+1}}{\mathcal{E}_{t+1}} - \frac{\mathbb{E}_t\big(1/\mathcal{E}_{t+1}\big)^2}{\mathbb{E}_t 1/(\mathcal{E}_{t+1}C_{T,t+1})}\right],$$

Increase $C_{N,t+1}$ above A_{t+1} when $C_{T,t+1}$ is particularly high, and vice versa, to reduce σ_t^2 and thus the period t risk sharing wedge Z_t.

■

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