Panics and Prices*

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Abstract

Rumors of a shortage may create higher-order uncertainty and cause panic buying even when there is no real shortage and most consumers are aware of this fact. We study the role of prices in alleviating, or even preventing, panic buying caused by such rumors. Under some circumstances, flexible prices fail to do so and panic buying is the unique equilibrium outcome. In these circumstances, fixed prices prevent panic buying and lead to higher consumer surplus despite the possibility of rationing. Producer surplus may be higher as well.

1 Introduction

As news of the Covid-19 pandemic hit Japan in February 2020, rumors arose on social media—later proved to be unfounded—of disruptions in the supply of various household products. Sales of certain items jumped ten-fold even though most people received information that there was no real shortage (Iizuka et al., 2021). Similar occurrences were widespread all over the world.

What causes such "panic buying"? A conventional explanation goes as follows. Some people naively believe rumors that there is a shortage and try to stock up on the good by rushing to buy immediately. Sophisticated people themselves do not believe the rumors (or are persuaded by counter-rumors) but know that there are naive people who do and will buy in a panic. Because they know that this itself will cause a shortage, sophisticated people buy immediately as well. People who are even more sophisticated understand the behavior of naive people as well as that of other less sophisticated people and so they too buy immediately, etc. Panic becomes widespread. This "infection" argument is, of course, familiar from other settings (for

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example, Rubinstein 1989). It rests, however, on the assumption that consumers' decisions of when to buy are strategic *complements*—if more people buy in a panic, then it is more advantageous for me to do so as well.

But what about prices? If a lot of people rush to buy today, then this should cause today's price to rise, thereby discouraging consumers from doing so. A sharp rise in today's price may make it more advantageous to postpone buying. As a result, when prices respond to increased demand, consumers' choices of when to buy may not be complementary.

In this paper we study whether the price mechanism can alleviate, or even prevent, panic buying caused by rumors. We do this via a simple two-period model in which the total supply of a storable good is fixed. While the good will be consumed only tomorrow, consumers can either buy it today and store it until then or wait and buy the good tomorrow itself. Since the total supply is fixed, the amount available tomorrow is just what is left unsold today. Consumers are heterogeneous in the utility they derive from consumption tomorrow. Moreover, today consumers only have private, partial information about this utility. This means that there is an option value of waiting to see what the realized utility is. Because of this, panic buying results in a misallocation—those buying early may regret doing so.

To study panic buying, we study a situation in which consumers are unsure about the total supply of the good. Specifically, there are two states of nature. In one, the normal or "high" state, the supply is ample enough so that if there were no uncertainty, it is an equilibrium for all consumers to wait. In the other, perhaps rare, "low" state, there is a supply shortage. Rumors of a shortage are generated regardless of the state. But if the state is normal, then with some probability, a fraction, perhaps large, of the consumers hear counter-rumors—corrective messages saying that in fact there is no shortage. These messages can then be forwarded—retweeted—again and again via a process that mimics social media platforms like Twitter.[1]

We compare the case when prices are flexible, and so endogenously determined by market conditions, to one where prices are controlled and fixed at a "normal" level, that is, the price that would clear the market if there were no shortage. When prices are fixed, a shortage in supply will result in excess demand and so it will be necessary to ration what is available.

We find first, that flexible prices are unable to prevent panic buying caused by rumors. Second, in many circumstances in which flexible prices are unable to do so, fixed prices can prevent panic buying. Third, consumers are better off with fixed prices even though they lead to rationing. More precisely,

1. With flexible prices, there is always an equilibrium with panic buying. If the normal supply is not too large, then regardless of the shortage, the equilibrium is unique (Theorem 1).

[1] The spread of information in this manner is similar to Morris' (2001) multi-player extension of the Rubinstein (1989) E-Mail game.

2. With fixed prices, there is an equilibrium without panic if either (i) the normal supply is not too large; or (ii) the shortage is small. In case (i), the equilibrium is unique. (Theorem 2).

3. When the shortage is rare, consumer surplus in the waiting equilibrium under fixed prices is higher than that in the panic equilibrium under flexible prices.[2] Producer surplus may be higher as well. (Section 4.3).

The first result points to a market failure. In this environment it occurs because consumers are unable to insure against preference shocks—their consumption values tomorrow may be different from those today. Thus we are in a second-best situation and as the third result points out, fixed prices may increase both consumer and producer surplus. Fixed prices do not always prevent panic, however. We show below, by means of an example, that if the conditions for results 1. and 2. are not met, it is possible that the unique equilibrium with fixed prices involves panic whereas there is an equilibrium with flexible prices without panic.

To gain some intuition for the first result, note that there is always a positive fraction of the people who do not receive any corrective information and so believe that the shortage is very likely. Of these "naive" people, a substantial fraction—those who expect to have high values tomorrow—panic and rush to buy today. Their panic buying then "infects" other consumers—even those who are sure there is no shortage—in a way similar to that in Rubinstein's E-Mail game. Precisely, even consumers who know that there is no shortage suffer from higher-order uncertainty about other consumers and so panic as well. This "infection" argument leads to the conclusion that panic buying is the unique equilibrium outcome. Note that here panic buying emerges even in the normal state when there is no real shortage. This occurs even if the potential shortage is small and rare—a 1% chance of a 1% shortage may trigger panic buying!

Intuition suggests, however, that flexible prices should mitigate, or even prevent, the infection. When a lot of consumers rush to stock up today, this causes today's price to rise, thereby raising the cost of panicking. While this is true, panic buying also affects the price tomorrow. An increase in the number of consumers buying today, of course decreases demand for the good tomorrow. But since the total stock of the good is fixed, it also decreases the supply of the good tomorrow. A simple "price theory" argument shows that the effect on supply is stronger and so in fact, tomorrow's price rises as well. Thus panic buying not only raises the cost of buying today but also the cost of buying tomorrow. Although this leads to a lack of complementarity, when the total supply in the normal state is not too large, the latter effect dominates and so flexible prices are unable to stop panic buying.

The second result is simpler. Consider a situation in which prices are fixed at their "normal" level. Unlike in the case of flexible prices, now the price today is the

[2] Under weak conditions on demand function, this ranking holds no matter what the probability of the shortage. (Proposition 3).

same as it will be tomorrow. If the normal supply is not too large, then the fixed price—which is set at the normal level—is relatively high. But at this relatively high price, consumers are better-off waiting to see if their final values are high enough to justify the expense. Thus it is an equilibrium for everyone to wait.

The third result is rather straightforward. As a first step let us compare what happens in the normal state. In the fixed price regime, everyone waits and the price is set at a level that clears the market in the second period. In the flexible price regime, everyone buys today and from the price theory argument outlined above both the price today and the price tomorrow is higher than the normal level. Thus in the normal state, consumers are strictly better-off in the fixed price regime. In the shortage state, the fixed price is too low to clear the market and there is rationing. But if the shortage is rare, consumers still prefer the fixed price regime.

A word of caution is necessary here. The reader may wonder if consumers can learn from prices as in the rational expectations models of Radner (1979) and Grossman and Stiglitz (1980). In such models, it is never clear whether price formation precedes consumer behavior or vice versa. As detailed below, we use a specific price formation process—a uniform-price auction in each period—in which consumer choices precede, and determine, prices so that prices cannot convey decision-relevant information.

Rumors and Counter-Rumors With flexible prices, Theorem 1 shows that panic buying can emerge as the unique equilibrium outcome. This relies on the fact that even consumers who are sure that there is plentiful supply are unsure whether others are aware of this fact. In other words, consumers suffer from higher-order uncertainty. An interesting feature of our model is that the higher-order uncertainty is caused not by rumors of a shortage but rather by the counter-rumors that, in fact, there is no shortage. In other words, in our model it is "good news" that triggers panic buying, not "bad news."

The work of Iizuka et al. (2021) finds some support for this hypothesis. These authors studied how activity on Twitter affected daily sales of toilet paper in Japan in February 2020.[3] They report that starting on February 21, there were approximately 700 influential tweets and retweets about a shortage of toilet paper. By February 26, there was an attempt to counter this misinformation. Indeed, by March 10 there were over 300,000 "corrective" tweets and retweets and it is estimated that these were viewed over 100 million times! While there was no great jump in sales in response to the initial rumors (between February 21 and 26), sales increased ten-fold after the corrective tweets (between February 26 and 28). Interestingly, Iizuka et al. (2021) find that in total, corrective tweets increased sales by three times as much as misinformation tweets (see their Table 8).

[3]Out of population of 125 million, there are over 50 million Twitter accounts in Japan.

Related Literature In this paper we study how rumors can lead consumers to rush to buy in a panic when waiting is welfare superior. There are many other settings in which panic results in suboptimal outcomes. An important example is that of a bank run in which depositors rush to withdraw money from their accounts because they fear that if others withdraw, the bank will become illiquid and fail. In Diamond and Dybvig's (1983) model of bank runs there are multiple equilibria—in one, depositors panic, causing a bank run and in the other, they don't. Goldstein and Pauzner (2005) use a global game approach to isolate a single equilibrium in the Diamond and Dybvig (1983) setting with private signals. A key difference between a bank run and our model of consumer panic is that there is no analog of market clearing via prices in the former.

In our model, higher-order uncertainty leads to a unique equilibrium. This idea originates in the E-Mail game of Rubinstein (1989) and then has been studied at length in the related setting of global games due to Carlsson and van Damme (1993). In both settings, the arguments for uniqueness mostly rely on the assumption that players' actions are strategic complements (Frankel, Morris and Pauzner, 2003). Recently, Harrison and Jara-Moroni (2021) have established that the global game analysis goes through even in binary choice games with strategic substitutes. Hoffman and Sabarwal (2019) extend this to include games in which each player's marginal benefit is monotone in the actions of others; however, it may be increasing for some players and decreasing for others. In our model, the actions are neither complements nor substitutes. Indeed, because of the effect of flexible prices, the marginal benefits of buying today versus tomorrow are non-monotonic.

Shadmehr (2019) studies a model of regime change in the global games framework. Workers decide whether or not to revolt and the costs of revolting are forgone wages that are endogenously determined. As in our model, because of the endogenous adjustment of wages players' actions are neither complements nor substitutes.

While there is a vast literature applying the theory of global games to various economic settings (see the surveys by Morris and Shin, 2003 and Angeletos and Lian, 2016), the informational structure of the E-Mail game has rarely been used in economic contexts. The E-Mail game information structure is particularly well-suited to the modelling of the spread of information via rumors and social media. Awaya, Iwasaki and Watanabe (2021) use this kind of structure to model how rumors can result in a price bubble in markets for assets that can be sold and resold.

An altogether different mechanism that leads to panic buying has been explored by Noda and Teramoto (2020). Here it is not higher-order uncertainty that causes panic buying but rather an anticipated increase in search costs. Consumers then rush to buy today and because of complementarity, others follow. While the model displays a rich set of dynamics, prices are assumed to be fixed. Our main concern is with comparing fixed and flexible prices in the face of supply shocks.

Price and wage flexibility is, of course, a major concern in the macro literature in the so-called New Keynesian framework. In many environments an increase in price/wage flexibility may reduce welfare (for example, see Galí and Monacelli, 2016 or Bhattarai, Eggertsson and Schonle, 2018). The mechanisms by which this happens are quite different from that in our paper. For instance, in Bhattarai, Eggertsson and Schonle (2018), increased price flexibility reduces welfare because it leads to increased output volatility—a channel that is absent in our model.

In this paper, panic buying refers to a situation in which consumers purchase early with only partial information about the value of the good. The work on "unraveling" in matching markets studies similar phenomena in labor markets (see Roth and Xing, 1994 and Li and Rosen, 1998). The important difference from our model is that in this literature workers have differing qualities and prices, or rather wages, are assumed to be fixed.

The remainder of the paper is organized as follow. The basic model is introduced in the next section. As a first step, in Section 3 we study a situation when there is no uncertainty about the total supply and derive equilibria under both flexible and fixed prices. Section 4 then considers the full model with supply uncertainty and the spread of rumors. Again we compare equilibria under both price regimes. We identify circumstances in which equilibrium outcomes are unique—flexible prices lead to panic buying whereas fixed prices do not. In Section 5, we exhibit an example in which there is a no-panic equilibrium with flexible prices whereas the unique equilibrium with fixed prices involves panic.

2 Preliminaries

Demand There is a continuum of consumers in $I = [0, 1]$ each of whom wishes to consume a single unit of a *storable* good. There are two periods, 1 ("today") and 2 ("tomorrow"), and the good will be consumed only in period 2. The utility or value derived from consuming the good in period 2 varies across consumers and is determined as follows. Prior to period 1, each consumer $i \in I$ draws an *estimated value* v_i from a continuous distribution F on $[0, 1]$ with a positive density f on $(0, 1)$.[4] Prior to period 2, the consumer learns the *final value* w_i, which is determined as follows. With probability $\lambda \in (0, 1)$, the final value $w_i = v_i$, the estimate. With probability $1 - \lambda$, the final value w_i is the result of a new draw from F that is independent of the initial draw.[5] The initial and final values are drawn independently across consumers.[6] This means that in the first period, the expected final value conditional on the estimate $E[W \mid V = v_i] = \lambda v_i + (1 - \lambda) E[W]$, where $E[W]$ denotes the expectation of the values according to F.

[4] We will denote by μ the corresponding measure.

[5] This formulation has been used by Krasikov and Lamba (2020) in their work on dynamic pricing.

[6] We are glossing over the conceptual difficulties associated with postulating a continuum of independent random variables.

While all consumption takes place in period 2, each consumer decides whether or not to purchase the good in period 1 or, if available, in period 2. If a consumer purchases the good in period 1, he or she can store it at no cost. The fact that the final value could be different from the estimated value means that a consumer may wish to postpone purchasing until tomorrow. For instance, if price tomorrow is expected to be the same as that today, then a consumer will be better off postponing his or her purchase.

Supply The good is produced using raw material at a constant cost of c per unit. One unit of raw material produces one unit of the good and all production is to order. The total amount of raw material available is θ. In the first period up to θ units of the good can be produced and sold. If $d_1 \leq \theta$ units of the good are sold in the first period, then $\theta - d_1$ units can be produced and sold in the second period.

The total supply of the raw material is assumed to be fixed and insensitive to prices—consumer decisions are day-to-day and suppliers are unable to obtain more raw material that quickly. It is also assumed that in each period, suppliers are passive—they just try to sell all that they have on the market. In particular, they do not hold back some of what they have in the first period for strategic reasons.

Throughout, we will assume that the costs are such that for all consumers, there are gains from trade in the first period, that is,

Condition 1 (Gains from Trade)

$$(1-\lambda) E[W] > c$$

Prices We will compare two regimes. In one, prices are *flexible* and adjust to equate supply and demand in each period. Of course, if there is excess supply even when the price equals the cost c, then the market price is c. Depending on consumers' behavior, the market-clearing price p_1 today may be higher or lower than the market-clearing price p_2 tomorrow. Because there is a continuum of consumers, prices will be *competitive*. As a micro-foundation, we suppose that in each period market-clearing prices are determined via a *uniform-price auction* with a reserve price of c.[7]

In the other regime, prices in both periods are *fixed* at the market-clearing level

$$v_\theta \equiv F^{-1}(1-\theta)$$

In other words, they are fixed at a price which would clear the market if there were no panic buying. To see this, note that the mass of consumers with final values above

[7] Specifically, participating consumers submits bids which are ranked in descending order. If there is a market-clearing price $p > c$ such that the mass of bidders willing to pay p equals the supply, then all such bidders pay p for the good. If the mass of bidders willing to pay c is smaller than the supply, then all pay c. Each consumer who decides to buy today has a dominant strategy to bid his or her estimated value $\lambda v_i + (1-\lambda) E[W]$. Each consumer who decides to wait has a dominant strategy to bid his or her final value w_i.

v_θ is $1 - F(v_\theta) = \theta$, the available supply.[8] Of course, fixed prices may not clear the market if some consumers buy today. In that case, rationing may be needed and if so, we suppose that the good is uniformly *rationed* via a lottery.

2.1 Market-Clearing Prices

We begin the analysis by studying the prices that clear the market today and tomorrow. To do this, we first take consumers' decisions regarding whether they wish to purchase today or tomorrow as being exogenously specified. In this section, we derive three important properties of market-clearing prices. Later sections will endogenize these decisions as the result of consumers' optimal choices of when to buy.

Our first result is that if more people choose to buy today rather than to wait until tomorrow, then this (weakly) raises the price in *both* periods. Clearly today's price p_1 rises since today's demand has gone up while the supply is unchanged. The effect on tomorrow's price p_2 is more subtle since both tomorrow's demand and tomorrow's supply are affected. If more people buy today, then this decreases the amount available in period 2 one for one—tomorrow's supply curve is vertical and shifts to the left by an amount equal to the mass of additional customers today. Tomorrow's demand curve also shifts to the left, but by less than this amount because among those who buy today are some whose values tomorrow are so low that their absence does not decrease tomorrow's effective demand. As a result tomorrow's supply falls by more than tomorrow's demand and so the price tomorrow rises. This simple "price theory" result plays a key role in our analysis.

Lemma 1 (Price Theory) *Suppose that consumers in $C \subset [0,1]$ buy today and the resulting prices are (p_1, p_2). If consumers in $\overline{C} \supset C$ buy today, then the resulting prices $(\overline{p}_1, \overline{p}_2) \geq (p_1, p_2)$. If $\mu\left(\overline{C} \setminus C\right) > 0$ and $c < p_2 < 1$, then $\overline{p}_2 > p_2$.*

The formal proof of the lemma is in the Appendix but the basic idea is rather simple and can be seen in Figure 1. In the figure, the set C consists of all consumers with estimates above z while the set $\overline{C} \supset C$ consists all those with estimates above $\overline{z} < z$. If all those in C buy today, then the residual supply in period 2 is s_2 and the residual demand is d_2 and so the price tomorrow is p_2. If all those in $\overline{C}$ buy today, then tomorrow's price rises to $\overline{p}_2$. The reason is as follows. The residual supply tomorrow decreases by the amount $s_2 - \overline{s}_2 = z - \overline{z}$. Note also that although all those with estimates above z buy today, some customers with low estimates can have high values tomorrow and so the demand curve d_2 starts at $p_2 = 1$. The same is true for $\overline{d}_2$. Thus, $d_2(1) - \overline{d}_2(1) = 0$. Also, $d_2(0) - \overline{d}_2(0) = z - \overline{z}$ because if tomorrow's price is 0, then in both cases those who did not buy today are willing to buy tomorrow. Moreover, for any p, $d_2(p) - \overline{d}_2(p) < z - \overline{z}$. Thus the demand curve $\overline{d}_2$ shift to the left but by less than $z - \overline{z}$.

[8] Note that v_θ is just the *demand price* at quantity θ in a static setting where the demand function is $1 - F(p)$.

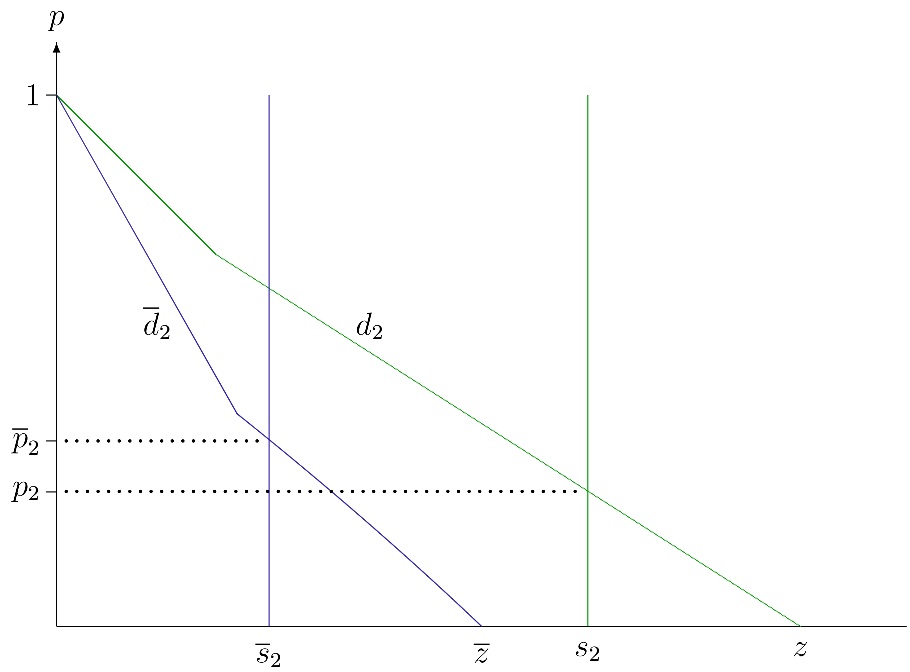

Figure 1: Price Theory Lemma

An immediate consequence of Lemma 1 is

Corollary 1 *All consumers are (weakly) better-off when those in C rush as opposed to when those in $\overline{C} \supset C$ rush. If $\mu\left(\overline{C} \setminus C\right) > 0$ and $p_2 < 1$, then consumer surplus is strictly greater when those in C buy today.*[9]

The next result uses Lemma 1 to put tight bounds on market-clearing prices. If the mass of consumers who buy today is less than θ, then there is excess supply today and so $p_1 = c$. On the other hand, if the mass of consumers who buy today is greater than or equal to θ, then the total supply of the good is exhausted today and so tomorrow's residual supply is zero. Thus, $p_2 = 1$. So we have that either $p_1 = c$ or $p_2 = 1$. In other words, either the price today is very low or the price tomorrow is very high. More precisely,

Lemma 2 (Price See-Saw) *Suppose (p_1, p_2) are market-clearing prices when the set of consumers who buy today is C. (i) If $\mu(C) < \theta$, then $p_1 = c$ and $p_2 \geq \max(v_\theta, c)$. (ii) If $\mu(C) \geq \theta$, then $c \leq p_1 \leq \lambda v_\theta + (1-\lambda) E[W]$ and $p_2 = 1$.*

Proof. Let C be the set of consumers who buy today. If $\mu(C) < \theta$, that is, there is excess supply in period 1, then the first-period price is $p_1 = c$. From Lemma 1 the price in the second period is at least as high as the price that results if everyone waits. But if everyone waits, the price in the second period is $\max(v_\theta, c)$.

[9]In the present context, consumer surplus is the same as the expected payoff of a consumer.

If $\mu(C) \geq \theta$, then the available supply is exhausted in period 1 and so $p_2 = 1$. Again, from Lemma 1, p_1 takes on its highest value when everyone buys today. But if everyone buys today, the supply is exhausted and so the marginal buyer today has value $v_\theta = F^{-1}(1-\theta)$. Thus,

$$p_1 \leq \lambda v_\theta + (1-\lambda) E[W]$$

the market-clearing price if everyone tried to buy today. ■

Lemmas 1 and 2 hint at why the price mechanism may fail to alleviate panic buying. If a lot of people panic and buy today, then this naturally raises today's price thereby raising the cost of panicking. But as Lemma 1 shows, this also raises tomorrow's price, thereby raising the cost of not panicking as well. Lemma 2 shows that if few people buy today, then today's price is c and thus lower than tomorrow's price. This gives more people the incentive to buy today. This effect is particularly strong when θ is small. On the other hand, if a lot of people buy today, then today's price cannot be higher than the market-clearing price and all those who are willing to pay this price have the incentive to buy today.

To study how the flexible prices affect the incentives to buy today versus tomorrow, suppose that the set of consumers who rush to buy today is C and let (p_1, p_2) be the resulting market-clearing prices. Given θ, define

$$\begin{aligned} \Delta_\theta(v, C) &= \underbrace{\max(\lambda v + (1-\lambda) E[W] - p_1, 0)}_{\text{Payoff from buying today}} \\ &\quad - \underbrace{(\lambda \max(v - p_2, 0) + (1-\lambda) E[\max(W - p_2, 0)])}_{\text{Payoff from buying tomorrow}} \end{aligned} \tag{1}$$

to be the gain from buying today versus tomorrow for a consumer with estimate v.

The gain from buying today $\Delta_\theta(v, C)$ is non-decreasing in v. To see this, note that from Lemma 2, we know that either $p_1 = c$ or $p_2 = 1$. If $p_1 = c$, then Condition 1 implies that

$$\begin{aligned} \Delta_\theta(v, C) &= \lambda v + (1-\lambda) E[W] - c \\ &\quad - \lambda \max(v - p_2, 0) - E[\max(W - p_2, 0)] \end{aligned}$$

On the other hand, if $p_2 = 1$, then

$$\Delta_\theta(v, C) = \max(\lambda v + (1-\lambda) E[W] - p_1, 0)$$

and in both cases, Δ, the gain from buying today versus tomorrow, is *non-decreasing* in v and may in fact be flat over certain regions.

On the other hand, because of Lemma 1 consumers' actions are *not* strategic complements—that is, $\Delta_\theta(v, C)$ is not monotonic in C. This is depicted in Figure 2

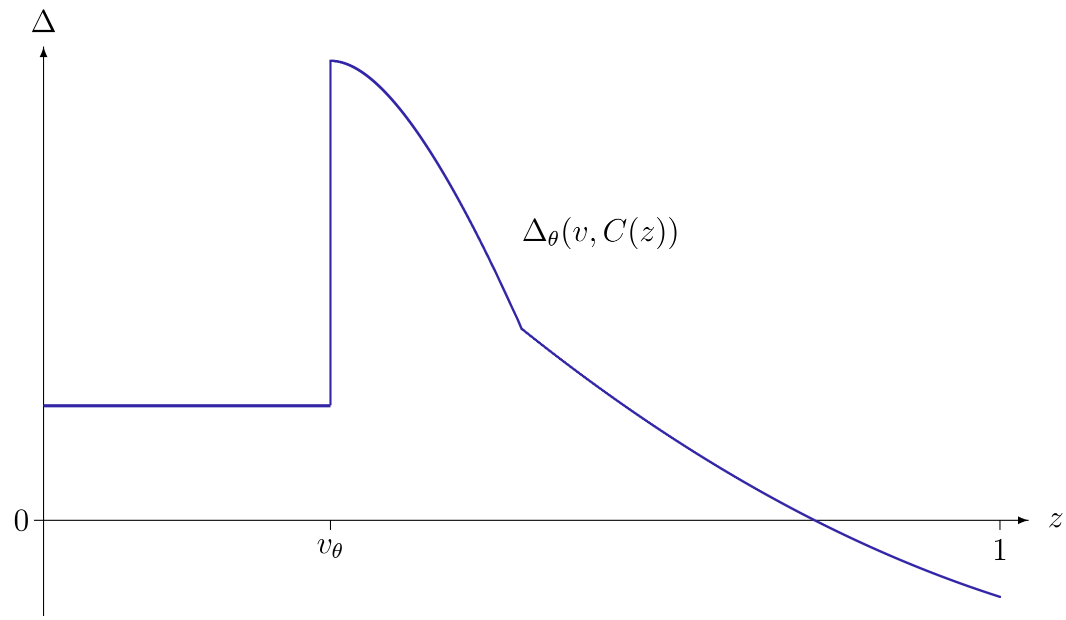

Figure 2: Lack of Complementarity

for an example in which consumers in $C(z) = \{i \mid v_i > z\}$ rush to buy today.[10] When $z < v_\theta$, the market-clearing price today $p_1 = \lambda v_\theta + (1-\lambda) E[W]$ and since the good is exhausted the payoff from waiting is zero. When $z > v_\theta$, the price today $p_1 = c$ since there is excess supply. But if $z = v_\theta$, then p_1 is indeterminate (see Lemma 2) and so $\Delta_\theta(v, C(v_\theta))$ is an interval.

The next result shows that $\Delta_\theta(v, \cdot)$ satisfies a limited version of quasi-concavity.[11]

Lemma 3 *For any* $C \subset \overline{C} \subset I$,

$$\Delta_\theta\left(v, \overline{C}\right) \geq \min\left(\Delta_\theta(v, C), \Delta_\theta(v, I)\right)$$

Proof. If the mass of consumers buying today $\mu\left(\overline{C}\right) < \theta$, then Lemma 2 implies that $p_1 = \overline{p}_1 = c$ and Lemma 1 implies that $\overline{p}_2 \geq p_2$. Since Δ is non-decreasing in tomorrow's price, we have $\Delta_\theta\left(v, \overline{C}\right) \geq \Delta_\theta(v, C)$.

If $\mu\left(\overline{C}\right) \geq \theta$, Lemma 2 implies that $\overline{p}_1 \leq \lambda v_\theta + (1-\lambda) E[W]$ and $\overline{p}_2 = 1$. But if everyone in I buys today, then the prices are $p_1(I) = \lambda v_\theta + (1-\lambda) E[W]$ and $p_2(I) = 1$. Thus, $\Delta_\theta\left(v, \overline{C}\right) \geq \Delta_\theta(v, I)$. ■

Observe that if $C = \varnothing$, then for any $\overline{C}$,

$$\Delta_\theta\left(v, \overline{C}\right) \geq \min\left(\Delta_\theta(v, \varnothing), \Delta_\theta(v, I)\right)$$

and so the incentive to buy today is minimized either when no one buys today or when everyone buys today.

[10] In the example of Figure 2, F is uniform, $\theta = 0.7$, $c = 0.3$, $\lambda = 0.1$ and $v = 0.9$.

[11] Recall that a function g is quasi-concave if $g(\beta x + (1-\beta) y) \geq \min(g(x), g(y))$.

3 Certain Supply

While our primary interest is in studying panic buying under supply uncertainty, as a first step, it is useful to see what happens when the total supply of the good, θ, is *commonly known* and so there is no role for rumors or corrective messages. All other elements of the model remain unchanged.

We will study equilibria under two regimes: (i) prices are fully flexible so that the market clears in both periods; and (ii) prices are fixed.

3.1 Flexible Prices

First, observe that if prices are flexible, then for all $\theta < 1$, there is an equilibrium in which *all sales occur today.* This is because if all consumers rush to buy today, by definition, the market-clearing price today p_1 equates supply and demand. Thus, the total supply is exhausted today and there is nothing left for tomorrow. All those willing to pay p_1 today are strictly better-off buying today and those who are not willing to pay p_1 today will not be able to buy the good tomorrow either.

When θ is relatively high, there is also an equilibrium in which *all sales occur tomorrow.* To see this, note that if everyone waits, then the market-clearing price today $p_1 = c$, since there are no customers today. The market-clearing price tomorrow $p_2 = \max(v_\theta, c)$. Using (1), the difference in payoffs from buying today versus waiting is

$$\begin{aligned}\Delta_\theta(v, \varnothing) &= \lambda v + (1-\lambda) E[W] - c \\ &\quad -\lambda \max(v - p_2, 0) - (1-\lambda) E[\max(W - p_2, 0)]\end{aligned}$$

If θ is so high that $p_2 = c$, then $\Delta_\theta(1, \varnothing) < 0$. This follows from the fact $E[W] - c < E[\max(W - c, 0)]$, that is, there is an option value to waiting. If θ is close to zero, then p_2 is close to 1 and so $\Delta_\theta(1, \varnothing) > 0$. Moreover, $\Delta_\theta(v, \varnothing)$ is strictly decreasing in θ unless $p_2 = c$. Thus, there exists a unique θ^* such that

$$\Delta_{\theta^*}(1, \varnothing) = 0 \tag{2}$$

Now if $\theta \geq \theta^*$, $\Delta_\theta(1, \varnothing) \leq 0$ and because $\Delta_\theta(\cdot, \varnothing)$ is non-decreasing, for all v, $\Delta_\theta(v, \varnothing) < 0$ as well. This implies that when $\theta \geq \theta^*$ it is an equilibrium for all consumers to wait. Thus, if $\theta \geq \theta^*$, there are *multiple* equilibria.

If $\theta < \theta^*$, then the equilibrium outcome is *unique*—everyone tries to buy today rather than wait. We will first argue that for all $v > v_\theta$ and for all $C \subset I$, $\Delta_\theta(v, C) > 0$. Lemma 3 implies that $\Delta_\theta(v, C) \geq \min(\Delta_\theta(v, \varnothing), \Delta_\theta(v, I))$ and we will argue that the right-hand side is positive.

First, if $C = \varnothing$ so that everyone waits, then $p_1 = c$ and $p_2 = v_\theta > c$.[12] Now

$$\begin{aligned}\Delta_\theta (v_\theta, \varnothing) &= \lambda v_\theta + (1-\lambda) E[W] - c - (1-\lambda) E[\max(W - v_\theta, 0)] \\ &= \Delta_\theta (1, \varnothing) \\ &> 0\end{aligned}$$

since $\theta < \theta^*$. Since $\Delta_\theta (\cdot, \varnothing)$ is non-decreasing, $\Delta_\theta (v, \varnothing) > 0$ for all $v > v_\theta$.

Second, if $C = I$ so that everyone buys today, then market-clearing price today $p_1 = \lambda v_\theta + (1-\lambda) E[W]$ and $p_2 = 1$. Now

$$\Delta_\theta (v, I) = \lambda (v - v_\theta) > 0$$

as well.

We have argued that it is dominant for all $v > v_\theta$ to buy today. But this means that the supply will be exhausted today and so there is no point in waiting. We thus obtain

Proposition 1 *Suppose the supply θ is known and prices are flexible. For all θ, there is an equilibrium in which all sales occur today. If $\theta < \theta^*$, then the equilibrium outcome is unique. If $\theta \geq \theta^*$, there is also an equilibrium is which all sales occur tomorrow.*

The proposition already points to a drawback of flexible prices—they clear markets! This means that they can never be high enough to choke-off demand in the first period and so there is always an equilibrium in which everyone rushes to buy today. Moreover, if everyone waits, there is excess supply today and so $p_1 = c$ while $p_2 = v_\theta$. If the total supply θ is small, then v_θ is high and so it is better to buy at a low price today than a high price tomorrow. In these circumstances, rushing to buy today is the unique equilibrium.

3.2 Fixed Prices

We now turn to consider a situation in which prices are *fixed* at a level that would clear the market if everyone waited—that is, $p_1 = p_2 = \max(v_\theta, c)$.

First, observe that with fixed prices, for all $\theta < 1$, there is an equilibrium in which *all sales occur tomorrow*. This follows from the fact that prices are fixed at a level that if everyone waits, then any consumer willing to pay $\max(v_\theta, c)$ for the good tomorrow can buy it for sure—there is no need for rationing. Since the price tomorrow will be the same as the price today, and there is an option value to waiting, it is optimal to wait.

Second, define

$$\underline{\theta} = 1 - F(\lambda + (1-\lambda) E[W])$$

[12] This follows from the fact, proved in Appendix A.2 that $v^* \equiv F^{-1}(1 - \theta^*) > c$.

If $\theta < \underline{\theta}$, or equivalently, if $\lambda + (1 - \lambda) E[W] < v_\theta$, then the *unique* equilibrium outcome is one in which all sales occur tomorrow. To see this, note that since $(1 - \lambda) E[W] > c$ the fixed price $\max(v_\theta, c) = v_\theta$. Now the fact that $\lambda + (1 - \lambda) E[W] < v_\theta$ implies that even a consumer with estimate $v = 1$ does not wish to purchase the good today. The same then holds for all consumers. The simple reason for the uniqueness result is that when the supply is small, the fixed price is high. In particular, today's price is high.

Finally, define

$$\overline{\theta} = 1 - F((1 - \lambda) E[W])$$

If $\theta > \overline{\theta}$, or equivalently, $(1 - \lambda) E[W] > v_\theta$, then there is an equilibrium in which all sales occur today. Again, since $(1 - \lambda) E[W] > c$, $(1 - \lambda) E[W] > \max(v_\theta, c)$, the fixed price. Now the payoff from buying the good today is positive for all consumers whether or not there is rationing. Thus, if all consumers buy today, the supply will be exhausted and there is no gain to waiting.

The arguments above lead us to conclude that in sharp contrast to Proposition 1, with fixed prices we have

Proposition 2 *Suppose the supply θ is known and prices are fixed. For all θ, there is an equilibrium in which all sales occur tomorrow. If $\theta < \underline{\theta}$, then the equilibrium outcome is unique. If $\theta \geq \overline{\theta}$, there is also an equilibrium in which all sales occur today.*

Note that Propositions 1 and 2 imply that for any θ, with flexible prices, there is an equilibrium in which all consumers rush to buy today while with fixed prices, there is an equilibrium in which all consumers wait. If $\theta < \min(\theta^*, \underline{\theta})$, there is a unique equilibrium under both regimes.[13]

3.3 Fixed versus Flexible Prices

Comparing flexible versus fixed prices, the latter are superior from the perspective of consumers in following sense. For any θ, consumer surplus in the waiting equilibrium under fixed prices is (strictly) higher than in the rush-to-buy-today equilibrium under flexible prices. This is because under fixed prices, all sales take place tomorrow at the price $\max(v_\theta, c)$ and all those with final value $w > \max(v_\theta, c)$ actually buy. But this is the same as the price and allocation under flexible prices if everyone waited. Now Corollary 1 implies that consumer surplus is strictly higher in the waiting equilibrium.

What about producer surplus? In general, producer surplus could be higher or lower with fixed prices. But if $\theta < \underline{\theta}$, which is equivalent to $v_\theta > \lambda + (1 - \lambda) E[W]$, then producer surplus is higher in the waiting equilibrium as well. This is because the fixed price is now v_θ and so is greater than $\lambda v_\theta + (1 - \lambda) E[W]$, the market-clearing price in the first-period. So we have that good is sold at a higher price under the

[13] Depending on the parameters, θ^* may be higher or lower than $\underline{\theta}$.

fixed price regime than under the flexible price regime. Since the sales under the two regimes are the same—all those with values above v_θ get the good—the producer surplus under the fixed-price regime is now also higher than under the flexible-price regime. When $\theta < \underline{\theta}$, both consumer and producer surpluses are greater with fixed prices than with flexible prices.

Market Failure Why does the price mechanism fail? The reason, as usual, is missing Arrow-Debreu state-contingent markets. In the current context, a "state" is realization of estimated and final values for *each* consumer and so each contract would be commitment to deliver a unit of the good in period 2 depending on the complete profile of estimated and final values. Not only are such contracts too numerous but they are unrealistic as well—consumers' values are private information that cannot be verified by a contract enforcer such as a court.

The arguments above also show that the price mechanism is not constrained optimal either. Without creating any additional contingent markets, fixed prices result in a superior allocation.

Resale What if consumers could purchase in the first period and resell in the second? This, of course, opens the door to the possibility that someone could buy large quantities today for pure speculative purposes, gain substantial market power and sell at a high price tomorrow. Many countries have laws in place that forbid such "price gouging" in times of crisis.

One remedy is to impose quantity controls—a limit of "one per customer." Now with the possibility of resale, the equilibrium outcome is always first-best. This is easily seen in the case when $v_\theta > c$. Now regardless of what happens in the first period, the price in the second period $p_2 = v_\theta$. All those with final values less than v_θ who purchased a good in the first period (re-) sell it and all those with final values greater than v_θ who did not buy in the first period buy in the second. The payoff of a consumer with estimate v from buying today at price p_1 is

$$\lambda \max(v, p_2) + (1-\lambda) E[\max(W, p_2)] - p_1$$

With resale the effective value of the good is the maximum of the consumption value and the resale price. The first term is the value of today's purchase if the final value is the same as today's estimate. The second term is the payoff if the final value comes from a second draw from F. Similarly, the payoff from waiting is

$$\lambda \max(v - p_2, 0) + (1-\lambda) E[\max(W - p_2, 0)]$$

It may be verified that if $p_1 = p_2$, then a consumer is indifferent between (i) buying today and possibly reselling tomorrow and (ii) waiting. Since, as argued above, $p_2 = v_\theta$ this implies that $p_1 = v_\theta$. If each consumer buys today with probability θ and waits with probability $1-\theta$, then this constitutes an equilibrium with resale. The resulting outcome is first-best and so the possibility of resale, together with quantity controls, resurrects the first welfare theorem.

4 Uncertain Supply and Rumors

With the analysis of the complete information case in hand, we turn to a situation in which the supply is uncertain. There are two states of nature, H and L, with prior probabilities $1-\pi$ and π, respectively. In state H, the amount of raw material available is $\theta_H < 1$ while in state L, only $\theta_L < \theta_H$ is available. Although the analysis below does not require this, it will be useful to think of H as the *normal*, likely state and L as the unusual, rare state in which there is a raw material *shortage* of $\theta_H - \theta_L$.

Information Consumers receive two sorts of information—"rumors" of a shortage and "corrective messages" that try to dispel the rumors. We will suppose that rumors arise regardless of the state and are completely uninformative. Thus, they carry no information. In what follows, we will concentrate on the effects of "corrective messages"—saying that there is no shortage—spread via social media. These messages are sent *only* in state H—that is, when there is no real shortage. The message process, similar to that in Morris (2001), is as follows.

In the shortage state L, no corrective messages are sent.

In the normal state H, with probability $1-\varepsilon$, a corrective message m_1 ("tweet"), indicating that there is no shortage, is sent via social media to a randomly chosen set of consumers of mass $\alpha \in (0,1)$. Since the message is sent only in state H, it assures all who receive it that there is no shortage. With probability $1-\varepsilon$, the message is forwarded ("retweeted") to a new randomly chosen set of consumers, again of mass α. The set of consumers observing the second message (the "retweet") is independent of the set receiving the original message. But since the second message is just the first message that has been forwarded, any consumer who reads the second message also reads the first. Again, with probability $1-\varepsilon$, a third message (a "re-retweet") is sent to a new, randomly chosen, set of consumers, again of mass α. This process continues indefinitely and is non-strategic.

As a result, the *length* of the message sequence is a random variable N distributed according to a geometric distribution. The total number of messages read by a consumer is also a random variable and call this the *type* t_i consumer i. So if consumer i never received any messages, then $t_i = 0$. If i received the initial "tweet" and nothing after that, then $t_i = 1$. If i received the initial tweet and a retweet and no further messages, then $t_i = 2$. If i did not receive the original tweet but only the retweet, then again $t_i = 2$ because the second message includes the first. Thus $t_i = k$ means that consumer i received the first k retweets. Of course, $t_i \leq n$, the realized number of messages, since if i read the first t_i messages then at least t_i messages must have circulated.

Thus, if $t_i > 0$, the consumer knows that the state is H. But the consumer does not know the realized value of N and so is uncertain about how many other consumers have read at least one message. This creates higher-order uncertainty about the state, a feature that is central to the problem.

The message process is intended to capture the basic intuition about panic buying. Even those who believe that there is no shortage, buy in a panic because they think that others will panic.

As in the previous section, we will compare a flexible price regime with one with fixed prices.

4.1 Flexible Prices

In what follows, we will assume that

Condition 2

$$\theta_L < \theta^* < \theta_H$$

Recall that, as defined in (2), θ^* is the threshold supply so that if $\theta < \theta^*$ and θ were common knowledge, then with flexible prices, there would be unique equilibrium in which everyone buys today. Moreover, if it were commonly known that the state is $\theta \geq \theta^*$, then there are at least two equilibria, one in which rushes to buy today and another in which everyone waits (Proposition 1).

If corrective messages are spread in the manner outlined above, then panic buying can emerge as the unique equilibrium outcome.

Theorem 1 (Panic) *Suppose supply is uncertain and prices are flexible. There exists a $\delta > 0$ such that for all $\theta_H \in (\theta^*, \theta^* + \delta)$, panic buying is the unique equilibrium outcome.*

Note that the result places no restrictions on any of the other parameters of the model. Of course, the value of δ depends on these parameters. Also, if $\theta_H \leq \theta^*$ then the uniqueness is trivial.

A consequence of the theorem is that panic buying can emerge as the *unique* equilibrium outcome even when $\theta_H - \theta_L$ is small, that is, the amount of the shortage is seemingly inconsequential. Similarly, panic buying can occur even when the prior probability π of the shortage is small. It can occur even when both the amount and the probability of the shortage are small.

Also, panic buying can emerge as the unique equilibrium outcome even when almost everyone knows that the state is H, almost everyone knows that almost everyone knows that the state is H and so on.

Sketch of proof of Theorem 1 The proof of the theorem is somewhat involved and uses the iterated elimination of dominated strategies. Most of the complications arise from the fact that prices are *endogenously* determined. This in turn means that arguments relying on the complementarity among consumers' choices cannot be used. A detailed proof is in the Appendix and here we confine ourselves to a sketch of the essential arguments.

In what follows, it is convenient to define

$$v_H = F^{-1}(1 - \theta_H)$$

and similarly, $v_L = F^{-1}(1 - \theta_L)$ and $v^* = F^{-1}(1 - \theta^*)$. These are the demand prices at the respective quantities. Note that since $\theta_L < \theta^* < \theta_H$, it is the case that $v_H < v^* < v_L$.

Step 1 For any $\eta > 0$, when θ_H is close to θ^*, it is iteratively dominant for all consumers with estimates $v > v_L + \eta$ to buy today (Proposition 5 in the Appendix). This step itself has a few sub-steps:

Step 1.0 It is dominant for all consumers who do not receive any corrective messages—that is, those with $t_i = 0$—and with estimates $v > v_L + \eta$ to buy today (Lemma 4). In the ex post event that the state is L, it is strictly dominant for these consumers to buy today for the same reason that it is so when L is commonly known (Proposition 1). In the ex post event that the state is H, the supply θ_H is close to θ^* and it can be shown that these consumers lose very little by buying today.

Step 1.1 Given Step 1.0, it is now iteratively dominant for all those with $t_i = 1$ and estimates $v > v_L + \eta$ to buy today (Lemma 5). All those with $t_i = 1$ know that the state is H. But since they did not receive anything other than the initial tweet, they assign a significant probability that the message sequence ended after a small number of retweets, that is, they believe that n is small. But if n is small then the message sequence ended early and so there is a significant fraction of consumers who did not get a corrective message. Of these, we know from Step 0 that those with estimates above $v_L + \eta$ buy today. Lemma 3 guarantees that the gain from buying today is at least as large as that when either only those in Step 1.0 buy today or everyone buys today. First, suppose that the consumers identified in Step 0 are the *only* ones who buy today. This means that there will be excess supply in the first period and so the price $p_1 = c$. Lemma 1 guarantees that the price in the second period p_2 will be higher than v_H—the market-clearing price if everyone waits—and we will argue that it is so high that it is better for consumers with $t_i = 1$ to buy today. On the other hand, if everyone buys today then from Proposition 1 it follows that it is again better for consumer with $t_i = 1$ and estimates $v > v_L + \eta$ to buy today.

Note that despite a lack of complementarity, panic spreads from those with $t_i = 0$ and high values to those with $t_i = 1$. This is because if some people panic, then this increases not only the cost of panicking, but that of waiting as well (Lemma 1). Thus, prices are unable to prevent the spread of panic.

Step 1.k Now an induction argument can be applied. As above, while all those with $t_i = k > 1$ know that the state is H, they also believe that for a significant fraction of the consumers $t_j < k$ and from earlier steps these people buy today. But now the same logic as in Step 1.1 ensures that all those with $t_i = k$ and $v_i > v_L + \eta$ also buy today (Lemma 6).

Step 2 Now given Step 1, it is iteratively dominant for all consumers with estimates $v_i > v_H$ to buy today (Proposition 6 in the Appendix). Again Lemma 3 is key—the gain to a consumer from buying today is at least as large as when either only those in Step 1 buys today or when everyone buys today. If only those in Step 1—those with estimates above $v_L + \eta$—buy today then as argued in Step 1.1 above, it is better to buy today. On the other hand, if everyone buys today then the first period price will be $p_{1H} = \lambda v_H + (1 - \lambda) E[W]$ and the supply will be exhausted and so again it is better to buy today.

While the steps outlined above resemble existing arguments in Rubinstein (1989) and Morris (2001), they are carried out in an environment in players' actions are not strategic complements. Moreover, unlike those arguments, Step 0 above cannot be immediately strengthened to say that all consumers with $t_i = 0$—who assign high probability to θ_L—have a dominant strategy to buy today regardless of their estimates. This is because if all those with estimates $v_i > v_L$ buy today, then in state L, those with low estimates $v_i \leq v_L$ will not be able to buy in either period and so are indifferent as to when they buy. This means their decision of when to buy hinges solely on what happens in state H, a state those with $t_i = 0$ deem unlikely. Thus, we need to make the argument in a series of steps as outlined above.

Some Remarks on Theorem 1 Theorem 1 is deliberately stated so that the conclusion obtains regardless of the other parameters of the model. These include the distribution of values F, the cost of production c, the persistence of values λ, the proportion of consumers who receive messages α, the probability that the message process will terminate ε, the prior probability π and finally, the supply θ_L when there is a shortage. The result shows that once θ_H is close enough to θ^*, panic buying results no matter what these other parameters are. The workings of Theorem 1 can be seen in the following example.

Example 1 *Suppose F is uniform on $[0, 1]$, $c = 0.24$, $\lambda = 0.5$, $\alpha = 0.5$, $\varepsilon = 0.001$ and $\pi = 0.1$. In this case, $\theta^* \simeq 0.7436$. For panic buying to be the unique equilibrium, it is sufficient that (θ_L, θ_H) be in the set S depicted in Figure* 3.

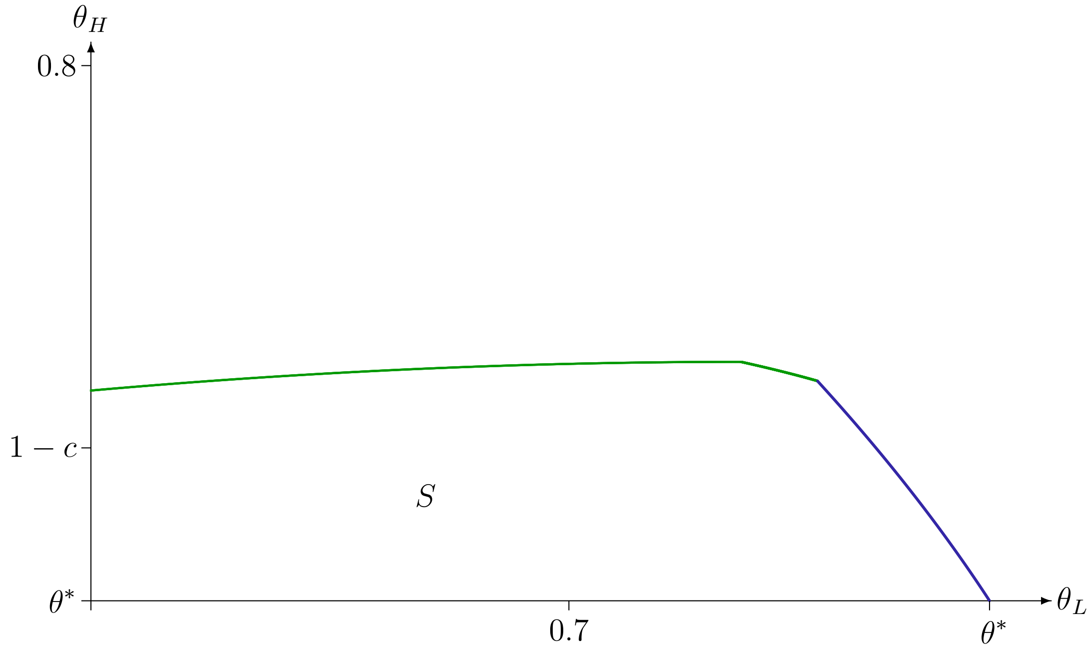

Figure 3: Unique Equilibrium in Example 1

The set S, partly depicted in Figure 3, is such that all the steps in the iterated argument in the proof of Theorem 1 go through (see Appendix A.2). The set is sufficient for uniqueness but it is not necessary—there may be configurations outside S for which uniqueness also obtains. The curves depicted guarantee that if consumers in the set $C_0 = \{(v_i, t_i) \mid v_i > v_L \text{ and } t_i = 0\}$ buy today, then it is iteratively dominant for consumers in the set $C_1 = \{(v_i, t_i) \mid v_i > v_L \text{ and } t_i \leq 1\}$ to buy today as well. This is Lemma 5 in Appendix A.2. For this example, this is sufficient to guarantee uniqueness.

Notice that when $\theta_L \simeq 0.7$, panic buying may occur even when $\theta_H > 1 - c$. Such a θ_H is so large that even if everyone waited and bought tomorrow, there would be excess supply even at $p_2 = c$.

To get some sense of the magnitudes, consider for example, $\theta_H = 0.75$. Now with corrective messages, even a 10% chance ($\pi = 0.1$) of a 2% shortage (so that $\theta_L = 0.735$) will lead to panic buying.

Finally, notice that if, as above, $\theta_H = 0.75$ and $\theta_L = 0.735$ then in the absence of any corrective messages, it is an equilibrium for everyone to wait because for any v,

$$(1-\pi)\,\Delta_H\,(v, \varnothing) + \pi \Delta_L\,(v, \varnothing) < 0$$

Thus, while there is a no-panic equilibrium when there are no corrective messages, such messages result in panic buying being the unique equilibrium outcome. This last feature echoes the findings in Iizuka et al. (2021) that corrective messages caused panic buying.

Learning from prices Recall that we have specified that market prices are determined via a uniform-price auction. This means that consumers cannot learn from each other via the prices as in the rational expectations models of Radner (1979) or Grossman and Stiglitz (1980). To us it is unclear how consumers can learn from prices and adjust their behavior, which in turn determines the prices. Rational expectations models are typically silent on the price formation process. In any case, the equilibrium with panic buying is immune to such learning. In the equilibrium, first-period price is different in the two states—in state L, $p_1 = \lambda v_L + (1-\lambda) E[W]$ while in state H, $p_1 = \lambda v_H + (1-\lambda) E[W]$—and so consumers can infer the state from the first-period prices. But in either state, the supply is exhausted in the first period and so there is no gain to waiting.

4.2 Fixed Prices

In the previous subsection we saw that with flexible prices corrective message can cause panic buying to emerge as the unique equilibrium outcome. As in the case without supply uncertainty, we now ask what happens if prices are fixed and do not adjust to changes in supply and demand. The process by which corrective messages spread remains unchanged.

In what follows, we suppose that prices are fixed in both periods at a level that would clear the market if supply were "normal" and all consumers waited, that is, $p_1 = p_2 = \max(v_H, c)$ where as defined above $v_H = F^{-1}(1-\theta_H)$. Note that the fixed price is the same in both states.

Proposition 2 implies that if there were no supply uncertainty and the state H were commonly known, then with fixed prices at level $\max(v_H, c)$, there would be an equilibrium in which everyone waits. A simple continuity argument now implies that even with supply uncertainty, if the shortage $\theta_H - \theta_L$ is small enough, then with fixed prices at level $\max(v_H, c)$, again there is an equilibrium in which everyone waits. Now in state L, there will be some rationing since the fixed price is too low to clear the market, but if θ_L is close to θ_H, the probability that a consumer with value $w > v_H$ is rationed in state L will be very small.

The condition that the shortage is small enough is not necessary for there to be a waiting equilibrium. If $\theta_H < \underline{\theta}$ or equivalently, $\lambda + (1-\lambda) E[W] < v_H$, then buying today is dominated no matter what the shortage is. Thus, not only is there an equilibrium with fixed prices in which everyone waits but it is *unique*.

Summarizing the findings above, we have

Theorem 2 (No panic) *Suppose supply is uncertain and prices are fixed. There exists an equilibrium in which everyone waits if either (i) $\theta_H < \underline{\theta}$; or (ii) $\theta_H - \theta_L$ is small enough. In case (i), the equilibrium is unique.*

4.3 Fixed versus Flexible Prices

What can we say about welfare under the two price regimes? We saw that without any supply uncertainty, there was an equilibrium with fixed prices that was (weakly) better for consumers than *any* equilibrium with flexible prices. From the perspective of consumers, fixed prices were superior to flexible prices. What can be said once there is supply uncertainty and corrective messages?

Consumer Surplus Without supply uncertainty, prices were fixed at a level such that when everyone waited, there was no rationing. This is no longer true once there is uncertainty. In state L, at the fixed price v_H the demand is θ_H while the supply is only θ_L and so rationing is necessary.

Suppose that there is an equilibrium with fixed prices in which everyone waits. Let us compare the waiting equilibrium under fixed prices to the panic equilibrium under flexible prices. First, if the probability of a shortage, π, is small, then consumer surplus under fixed prices is strictly higher than under flexible prices. This is because it is strictly higher when the state is H (Corollary 1) and since L is very unlikely, the same is true when there is supply uncertainty. Second, and perhaps more interesting, is the fact that under weak conditions on the distribution F, expected consumer surplus is strictly higher no matter what π is. Precisely,

Proposition 3 *Suppose F has an increasing hazard rate.*[14] *The expected consumer surplus in the waiting equilibrium under fixed prices is higher than in the panic equilibrium under flexible prices.*

Note that the proposition does not require that the shortages be rare—it holds even if shortages occur with high probability. It relies on the fact (proved in Appendix A.3) that when F has an increasing hazard rate, if everyone waits in state L, consumers prefer (ex ante) to be rationed at the fixed price $v_H < v_L$ than to be served for sure at the market-clearing price v_L. Since consumer surplus is higher when everyone waits as opposed to when everyone rushes to buy today, fixed prices are better in state L. Together with the fact that consumer surplus in state H is also higher under fixed prices then completes the proof.

From the perspective of consumers, the waiting equilibrium with fixed prices is better than the rushing equilibrium with flexible prices if (i) π is small enough; or (ii) F has an increasing hazard rate.

Producer Surplus Suppose that $v_H > \lambda + (1 - \lambda) E[W]$. Then since $v_H > (1 - \lambda) E[W]$ which is assumed to be greater than c, we have that the fixed price

[14] This is equivalent to the condition that the *semi-elasticity* of the demand function $D(p) = 1 - F(p)$ is non-decreasing.

$\max(v_H, c) = v_H$. In the waiting equilibrium under fixed prices, the expected producer surplus is

$$\Pi^{Fix} = (v_H - c) \times ((1-\pi)\,\theta_H + \pi\theta_L)$$

On the other hand, in the rushing equilibrium under flexible prices, the expected producer surplus is

$$\Pi^{Flex} = (1-\pi)\,(\lambda v_H + (1-\lambda)\,E\,[W] - c) \times \theta_H + \pi\,(\lambda v_L + (1-\lambda)\,E\,[W] - c) \times \theta_L$$

and now producer surplus is also higher in the waiting equilibrium with fixed prices.

An example illustrates that the surplus ranking derived above can occur even when there is a unique equilibrium under either price regime.

Example 2 *Suppose F is uniform, $c = 0.42$, $\lambda = 0.1$, $\alpha = 0.5$, $\varepsilon = 0.001$ and $\pi = 0.1$. Here $\theta^* \simeq 0.438$ and $\underline{\theta} = 0.55$. Suppose $\theta_H = 0.44$ and $\theta_L = 0.25$. With* fixed *prices, everyone waiting is the unique equilibrium outcome, while with* flexible *prices, panic buying is the unique equilibrium outcome.*

With fixed prices (at $p = v_H = 0.56$), the condition that $\theta_H < \underline{\theta}$ is satisfied so that $\lambda + (1-\lambda)\,E\,[W] < v_H$. Thus, the fixed price $p = v_H$ is so high that buying today is dominated for all. The fact that with flexible prices, panic buying is the unique equilibrium outcome can be verified by following the proof of Theorem 1. In this example, θ_H is close enough to θ^* so that the arguments underlying Theorem 1 go through.

In this example, the contrast between the flexible and fixed price regimes is stark. Both yield unique equilibrium outcomes. In one, panic ensues; in the other, there is no panic. The distribution F satisfies the conditions of Proposition 3 and so consumer surplus under fixed prices is greater than under flexible prices. The same is true for producer surplus.

5 Are Fixed Prices Always Better?

Fixed prices are not a panacea, however. There are circumstances in which panic buying is an equilibrium outcome even with fixed prices. To see this, suppose θ_H is large enough so that $(1-\lambda)\,E\,[W] > v_H$, the fixed price. If everyone rushes to buy today, then the payoff from waiting is zero since the supply will be exhausted in either state. The condition above guarantees that for all consumers the payoff from buying today at a price $p_1 = v_H$ is positive. Of course, since the demand in period 1 will exceed the supply in either state, there will be rationing but the expected payoff from buying today will still be positive.

On the other hand, there are circumstances in which (almost everyone) waiting is be an equilibrium outcome with flexible prices. Recall that Theorem 1 states that, once all other parameters have been fixed, for any fixed θ_L panic buying is the unique equilibrium outcome if θ_H is close enough to θ^*. If we reverse the order of limits—that is, for any fixed θ_H, if we take θ_L to zero— then there are other equilibria. Precisely,

Proposition 4 *For any $\theta_H > \theta^*$, if θ_L is small enough, with flexible prices there is an equilibrium such that in state H all consumers who receive corrective messages wait.*

The proof of the result is in Appendix A.4 and shows that there is an equilibrium with the property that all consumers who receive at least one message ($t_i > 0$) wait. Those consumers who do not receive any messages ($t_i = 0$) and have estimates $v_i < v_L = F^{-1}(1 - \theta_L)$ wait as well. The behavior of the remaining consumers depends on the parameters. Note that as θ_L goes to zero, the set of consumers who wait contains almost all consumers.

Some idea of why there is such an equilibrium may be gleaned by considering the case when ε is small. Now any consumer who does not receive any messages ($t_i = 0$) believes with very high probability that the state is L and so also believes with high probability that no one received any messages, that is, $t_j = 0$ for other consumers j. From the perspective, of a consumer with $t_i = 0$, the situation is very nearly the same as if θ_L were commonly known. Thus, their behavior is also very nearly the same as if θ_L were commonly known. Recall that if $\theta_L < \theta^*$ is commonly known, those with estimates $v_i > v_L$ have a strict incentive to buy today and so today's market-clearly price $p_1 = \lambda v_L + (1 - \lambda) E[W]$. But at this high price, consumers with $v_i \leq v_L$ do not want to buy today—flexible prices discourage those consumers from "panicking" (see Proposition 1). Finally, note that if θ_L is small, then $v_L = F^{-1}(1 - \theta_L)$ is large and so the fraction of consumers with $t_i = 0$ who actually buy today is small.

What happens in state H? Any consumer with $t_j > 0$ knows that the state is H and expects that only a small set of consumers, close to that identified above, will buy today. Thus, any consumer with $t_j > 0$ expects today's price to be c and tomorrow's price to be close to v_H. With these prices, these consumers prefer to wait, as specified by the purported equilibrium.

An implication of the result is there are circumstances in which flexible prices are unable to prevent panic buying for some θ_L but are able to do so for smaller θ_L. In other words, the market mechanism fails for small shortages but may work for large shortages! Why is it that panic is infectious when θ_L is large but not when it is small? Again this is because of a lack of complementarity—this time between consumers' choices of when to buy and the state.[15] Just because a consumer is better-off buying today when θ_L is large does not mean she is better-off buying today when θ_L is small. As before, this lack of complementarity is a consequence of flexible prices

[15]Suppose all consumers buy today. Then in state θ the prices are $p_1 = \lambda v_\theta + (1 - \lambda) E[V]$ and $p_2 = 1$. In state $\theta' > \theta$, the prices are $p_1' = \lambda v_{\theta'} + (1 - \lambda) E[V]$ and $p_2' = 1$. For any v such that $v_{\theta'} < v < v_\theta$, $\Delta_\theta(v, I) = 0$ whereas $\Delta_{\theta'}(v, I) = \lambda(v - v_{\theta'}) > 0$.

Moreover, it may be that panic buying is the *unique* equilibrium with fixed prices while (almost everyone) waiting is an equilibrium with flexible prices. An example illustrates this possibility.

Example 3 *Suppose F is uniform, $c = 0.1$, $\lambda = 0.1$, $\alpha = 0.05$, $\varepsilon = 0.001$ and $\pi = 0.1$. Suppose $\theta_H = 0.9$ and $\theta_L = 0.05$. With fixed prices, panic buying is the unique equilibrium outcome, while with flexible prices, there is an equilibrium in which in state H, only a small fraction of consumers panic.*

In the example, $(1 - \lambda) E[W] > v_H$ so that the arguments above guarantee that with fixed prices, panic buying is the unique equilibrium. It is the only equilibrium because those consumers who do not receive any corrective messages (with $t_i = 0$) ascribe a high probability that the state is L. In this state, the supply θ_L is very small while the price v_H is low and since the supply will be exhausted, for all those with $t_i = 0$, it is best to buy today. With the specified parameter values, this is enough to guarantee that panic buying then infects even those who receive corrective messages ($t_i > 0$). With flexible prices, there are multiple equilibria and the construction of the equilibrium in which almost everyone waits mimics the arguments underlying Proposition 4.

A Appendix

A.1 Proof of Lemma 1

Proof. The claim that the price in the first period increases follows trivially from the fact that first-period demand does not decrease while the first-period supply is fixed. Thus, $\overline{p}_1 \geq p_1$.

To see that the second period price also increases, there are two cases to consider.

Case i. $\mu\left(\overline{C}\right) \geq \theta$.

In this case, the total supply is exhausted in the first period, so $\overline{p}_2 = 1$ and so the result is obvious.

Case ii. $\mu(C) \leq \mu\left(\overline{C}\right) < \theta$.

If $p_2 = c$, then certainly $\overline{p}_2 \geq p_2$ and again the result is obvious.

If $p_2 > c$, then it must be that in the second period demand equals supply. Let $B = I \setminus C$ denote the set of consumers who wait and similarly, let $\overline{B} = I \setminus \overline{C}$. Finally, let Z_p be the set of all consumers who are willing to pay p in the second period. With this notation, the demand in the second period at any price $p > c$ is just

$$D_2(p) = \mu(B \cap Z_p)$$

and $\overline{D}_2(p)$ is similarly defined. In the initial situation, the amount of the good remaining in the second period is $\theta - \mu(C)$ and so the market-clearing price p_2 is the solution to

$$\mu(B \cap Z_p) = \theta - \mu(C)$$

Such a price exists since the left-hand side is continuous and decreasing in p.

Now observe that at any price p,

$$\begin{aligned} D_2(p) &= \mu(B \cap Z_p) \\ &= \mu\left(\overline{B} \cap Z_p\right) + \mu\left(\left(B \setminus \overline{B}\right) \cap Z_p\right) \\ &\leq \overline{D}_2(p) + \mu\left(B \setminus \overline{B}\right) \end{aligned}$$

Note that if $p < 1$, then there is a subset of $B \setminus \overline{B} = \overline{C} \setminus C$ consisting of consumers whose willingness to pay is less than p. If $\mu\left(B \setminus \overline{B}\right) > 0$, then $\mu\left(\left(B \setminus \overline{B}\right) \cap Z_p\right) < \mu\left(B \setminus \overline{B}\right)$ because there will always be consumers whose willingness to pay in the second period comes from a redraw from the distribution F.

Thus we have that if p_2 is the second-period price, then

$$\begin{aligned} \overline{D}_2(p_2) &\geq D_2(p_2) - \mu\left(B \setminus \overline{B}\right) \\ &= \theta - \mu(C) - \mu\left(B \setminus \overline{B}\right) \\ &= \theta - \mu(C) - \mu\left(\overline{C} \setminus C\right) \\ &= \theta - \mu\left(\overline{C}\right) \end{aligned}$$

that is, the demand at p_2 exceeds the supply. Thus, $\overline{p}_2 \geq p_2$. Moreover, by the argument above, if $\mu\left(\overline{C} \setminus C\right) > 0$ and $p_2 < 1$, then the inequality is strict and so $\overline{p}_2 > p_2$. ■

A.2 Proof of Theorem 1

In what follows, it will be convenient to write the expected payoff from buying tomorrow at price p when the final value w is redrawn from F

$$\begin{aligned} \psi(p) &= E[\max(W - p, 0)] \\ &= \int_p^1 (v - p) f(v)\, dv \end{aligned}$$

and note that $\psi' < 0$. Note that since we can think of $1 - F(p)$ as a demand function, $\psi(p)$ is also the corresponding *consumer surplus* at price p.

It will also be useful to define

$$\Phi(p) = \lambda p + (1 - \lambda) E[W] - c - (1 - \lambda) \psi(p)$$

and note that $\Phi' > 0$. It is easy to verify that

$$\Phi(v_\theta) = \Delta_\theta(1, \varnothing)$$

and so using (2) we have that

$$\Phi(v^*) = 0$$

since v^* is defined as v_{θ^*}. Finally, since $\Phi(c) < 0$,

$$v^* > c$$

To prove Theorem 1 involves, we will first show

Proposition 5 *For any $\eta \in (0, 1 - v_L)$, there exists a $\delta > 0$ such that for all $\theta_H \in (\theta^*, \theta^* + \delta)$, it is iteratively dominant for all consumers with estimate $v > v_L + \eta$ to rush.*

It is convenient to use the following definition.

Definition 1 *For $k \geq 0$, let $C_k = \{(v_i, t_i) : v_i > v_L + \eta \text{ and } t_i \leq k\}$ denote the set of consumers i with $t_i \leq k$ and estimate $v_i > v_L + \eta$.*

The proposition will be established in steps. First, the conclusion will be shown to hold for C_0, then C_1 and then finally for all C_k $(k > 1)$.

Step 0: All consumers in C_0 rush.

Lemma 4 *For any $\eta \in (0, 1 - v_L)$, there exists a $\delta > 0$ such that for all $\theta_H \in (\theta^*, \theta^* + \delta)$, it is strictly dominant for all consumers in the set*

$$C_0 = \{(v_i, t_i) : v_i > v_L + \eta \text{ and } t_i = 0\}$$

to rush.

Proof. Lemma 3 implies that for any set of consumers C who rush to buy today

$$\Delta_H(v, C) \geq \min(\Delta_H(v, \varnothing), \Delta_H(v, I))$$

and similarly for state L.

First, notice that in state H, $\Delta_H(v, \varnothing) \leq 0$ because in this state it is an equilibrium for everyone to wait (Proposition 1). At the same time, $\Delta_H(v, I) \geq 0$ because it is also an equilibrium for everyone to rush to buy today (Proposition 1 again). Thus, $\min(\Delta_H(v, \varnothing), \Delta_H(v, I)) = \Delta_H(v, \varnothing)$.

Thus, the expected gain from buying today when those in C are rushing

$$\begin{aligned} & E_\theta[\Delta_\theta(v, C) \mid t = 0] \\ \geq\ & \Pr[H \mid t = 0] \times \Delta_H(v, \varnothing) + \Pr[L \mid t = 0] \times \min(\Delta_L(v, \varnothing), \Delta_L(v, I)) \end{aligned} \tag{3}$$

We now compute the three Δ's in the expression above.

(i) If everyone waits in state H, the market-clearing prices are $p_1 = c$ and $p_2 = v_H$. Thus, the gain from buying today for a consumer with estimate $v > v_L > v_H$ is

$$\begin{aligned} \Delta_H(v, \varnothing) &= \lambda v + (1 - \lambda) E[W] - c - \lambda(v - v_H) - (1 - \lambda)\psi(v_H) \\ &= \Phi(v_H) \end{aligned}$$

which is negative since $v_H < v^*$.

(ii) Similarly, if everyone waits in state L, the market-clearing prices are $p_1 = c$ and $p_2 = v_L$. The gain from buying today for any $v > v_L$ is

$$\begin{aligned} \Delta_L(v, \varnothing) &= \lambda v + (1-\lambda) E[W] - c - \lambda(v - v_L) - (1-\lambda)\psi(v_L) \\ &= \Phi(v_L) \end{aligned}$$

which is positive since $v_L > v^*$.

(iii) If everyone buys today in state L, the market-clearing prices are $p_1 = \lambda v_L + (1-\lambda) E[W]$ and $p_2 = 1$. Thus, the gain from buying today is

$$\Delta_L(v, I) = \lambda(v - v_L)$$

Using these facts we can rewrite (3) for $v = v_L + \eta$ as

$$\begin{aligned} & E_\theta [\Delta_\theta (v_L + \eta, C) \mid t = 0] \\ \geq \; & \Pr[H \mid t = 0] \times \Phi(v_H) + \Pr[L \mid t = 0] \times \min(\Phi(v_L), \lambda\eta) \end{aligned}$$

and as $\theta_H \downarrow \theta^*$, $\Phi(v_H) \uparrow \Phi(v^*) = 0$ while $\min(\Phi(v_L), \lambda\eta)$ remains bounded away from zero.

Finally note that $\min(\Phi(v_L), \lambda(v - v_L))$ is non-decreasing as a function of v. Thus if $E_\theta[\Delta_\theta(v, C) \mid t = 0]$ is positive when $v = v_L + \eta$ it is also positive for larger v (recall that Δ is non-decreasing in v). This completes the proof. ■

Step 1: All consumers in C_1 rush.

Lemma 5 *For any $\eta \in (0, 1 - v_L)$, there exists a $\delta > 0$ such that for all $\theta_H \in (\theta^*, \theta^* + \delta)$, it is iteratively dominant for all consumers in*

$$C_1 = \{(v_i, t_i) : v_i > v_L + \eta \text{ and } t_i \leq 1\}$$

to buy today.

Proof. Lemma 4 already argued that when θ_H is close to θ^*, it is dominant for all those in C_0 to buy today. It remains to argue that when θ_H is close to θ^*, it is then iteratively dominant for those with $t_i = 1$ and $v_i > v_L + \eta$ to buy today as well.

Since all those in C_0 buy today, consider any set $C \supseteq C_0$. A consumer with $t = 1$, knows that the state is H but does not know the length of the message sequence, n. The exact value of n determines the mass of consumers in C and so the prices that would result. Thus, for a consumer with $t = 1$, the expected gain from buying today is

$$
\begin{aligned}
& E_n\left[\Delta_H(v, C) \mid t=1\right] \\
= \; & \Pr[n=1 \mid t=1] \times \Delta_H^{n=1}(v, C)+\sum_{t=2}^{\infty} \Pr[n=t \mid t=1] \times \Delta_H^{n=t}(v, C) \\
\geq \; & \Pr[n=1 \mid t=1] \times \min \left(\Delta_H(v, I), \Delta_H^{n=1}\left(v, C_0\right)\right) \\
& +\sum_{t=2}^{\infty} \Pr[n=t \mid t=1] \times \min \left(\Delta_H(v, I), \Delta_H(v, \varnothing)\right) \\
= \; & \Pr[n=1 \mid t=1] \times \min \left(\Delta_H(v, I), \Delta_H^{n=1}\left(v, C_0\right)\right) \\
& +\Pr[n>1 \mid t=1] \times \min \left(\Delta_H(v, I), \Delta_H(v, \varnothing)\right)
\end{aligned}
$$

using Lemma 3. Here we are using the notation Δ_H^n to make the dependence of Δ on n explicit but note that $\Delta_H(v, I)$ and $\Delta_H(v, \varnothing)$ are not affected by n.

We will argue that as $\theta_H \downarrow \theta^*$, the right-hand side is positive.

(i) $\lim_{\theta_H \downarrow \theta^*} \Delta_H(v, I)>0$ **for** $v \geq v_L+\eta$.

$$
\begin{aligned}
\Delta_H\left(v_L+\eta, I\right) = \; & \max \left(\lambda\left(v_L+\eta\right)+(1-\lambda) E[W]-p_1, 0\right) \\
& -\lambda \max \left(v_L+\eta-p_2, 0\right)-(1-\lambda) \psi\left(p_2\right)
\end{aligned}
$$

and since everyone rushes to buys today, the price today is $p_1=\lambda v_H+(1-\lambda) E[W]$. This is because the mass of consumers willing to pay this amount is exactly θ_H. This means that the supply will be exhausted today and so the price tomorrow $p_2=1$. Thus,

$$
\Delta_H\left(v_L+\eta, I\right)=\lambda\left(v_L+\eta-v_H\right)>0
$$

since $v_L>v_H$ and so

$$
\lim_{\theta_H \downarrow \theta^*} \Delta_H\left(v_L+\eta, I\right)=\lambda\left(v_L+\eta-v^*\right)>0
$$

(ii) $\lim_{\theta_H \downarrow \theta^*} \Delta_H^{n=1}\left(v, C_0\right)>0$ **for** $v \geq v_L+\eta$.

In state H,

$$
\begin{aligned}
\Delta_H^{n=1}\left(v_L+\eta, C_0\right) = \; & \max \left(\lambda\left(v_L+\eta\right)+(1-\lambda) E[W]-p_1, 0\right) \\
& -\lambda \max \left(v_L+\eta-p_2, 0\right)-(1-\lambda) \psi\left(p_2\right)
\end{aligned}
$$

The state H is, of course, the total amount available and n determines the fraction of consumers in C_0. This fraction is at most $(1-\alpha) \theta_L$ and so $\mu\left(C_0 \mid n=1\right)<\theta^*$. Lemma 2 implies that $p_1^*\left(C_0\right)=c$ and from Lemma 1 $p_2^*\left(C_0\right)>v^*$. This is because $p_2^*(\varnothing)=v^* \in(c, 1)$ and since $\mu\left(C_0 \diagdown \varnothing \mid n=1\right)=\mu\left(C_0 \mid n=1\right)>0$, Lemma 1 implies that $p_2^*\left(C_0\right)>v^*$. Thus, as $\theta_H \downarrow \theta^*$, $\lim p_{1H}\left(C_0\right)=c$ and $\lim p_{2H}\left(C_0\right)>v^*$.

Thus,

$$\begin{aligned}
& \lim_{\theta_H \downarrow \theta^*} \Delta_H^{n=1} (v_L + \eta, C_0) \\
= \quad & \lambda (v_L + \eta) + (1 - \lambda) E [W] - c \\
& - \lambda \max (v_L + \eta - \lim p_{2H} (C_0), 0) - (1 - \lambda) \psi (\lim p_{2H} (C_0)) \\
> \quad & \lambda (v_L + \eta) + (1 - \lambda) E [W] - c - \lambda (v_L + \eta - v^*) - (1 - \lambda) \psi (v^*) \\
= \quad & \lambda v^* + (1 - \lambda) E [W] - c - (1 - \lambda) \psi (v^*) \\
= \quad & \Phi (v^*) \\
= \quad & 0
\end{aligned}$$

Since the incentive to buy today is non-decreasing in v, we have established that for all $v > v_L + \eta$,

$$\lim_{\theta_H \downarrow \theta^*} \min \left(\Delta_H (v, I), \Delta_H^{n=1} (v, C_0) \right) > 0$$

(iii) $\lim_{\theta_H \downarrow \theta^*} \min (\Delta_H (v, I), \Delta_H (v, \varnothing)) = 0$ **for** $v \geq v_L + \eta$.

Using the same argument as in Lemma 4, we have that the minimum is $\Delta_H (v, \varnothing)$ and this equals $\Phi (v_H) < 0$. Thus,

$$\begin{aligned}
\lim_{\theta_H \downarrow \theta^*} \Delta_H (v, \varnothing) &= \lim_{\theta_H \downarrow \theta^*} \Phi (v_H) \\
&= 0
\end{aligned}$$

This completes the proof. ■

Step k. All consumers in C_k rush. We now show that once δ has been chosen small enough so that for all $\theta_H \in (\theta^*, \theta^* + \delta)$ all consumers in C_0 rush to buy today (Lemma 4) and all those in C_1 also rush to buy today (Lemma 5), it is the case that without changing δ, all those in C_k ($k > 1$) will also rush to buy today. Clearly, a consumer who reads k messages is sure that the total number of messages $n \geq k$. We then have

Lemma 6

$$\Pr [n = k \mid t = k] \times \min \left(\Delta_H (v, I), \Delta_H^{n=k} (v, C_{k-1}) \right) + \Pr [n > k \mid t = k] \times \Delta_H (v, \varnothing)$$

is independent of k.

Proof. First, note that for any k,

$$\Pr [n = k \mid t = k] = \alpha (1 - \varepsilon) + \varepsilon$$

which is independent of k.

Second, note that for any $k > 0$,

$$\mu \left(C_{k-1} \mid n = k \right) = \mu \left(C_0 \mid n = 1 \right)$$

To see this note that for all k, the probability that a consumer last got a message at time k given that a message was generated at time k

$$\Pr \left[t = k \mid n = k \right] = \alpha$$

and so the mass of consumers with $t < k$

$$\Pr \left[t < k \mid n = k \right] = 1 - \alpha$$

And now since C_{k-1} consists of all consumers with estimates above $v_L + \eta$ and $t < k$, $\mu \left(C_{k-1} \mid n = k \right) = (1 - \alpha) \left(1 - F \left(v_L + \eta \right) \right)$ which is independent of k. ■

Lemma 7 *If*

$$\Pr \left[n = 1 \mid t = 1 \right] \times \min \left(\Delta_H \left(v, I \right), \Delta_H^{n=1} \left(v, C_0 \right) \right) + \Pr \left[n > 1 \mid t = 1 \right] \times \Delta_H \left(v, \varnothing \right)$$

is positive, then for all $k \geq 1$, it is iteratively dominant for all consumers in C_k to buy today.

Proof. The proof is by induction. The statement is true for $k = 1$ (Lemma 5). Suppose that it holds for all $t < k$. Then by the induction hypothesis, all those in C_{k-1} buy today and so

$$\begin{aligned} & E \left[\Delta \left(v \right) \mid t = k \right] \\ \geq \; & \Pr \left[n = k \mid t = k \right] \times \min \left(\Delta_H \left(v, I \right), \Delta_H^{n=k} \left(v, C_{k-1} \right) \right) \\ & + \Pr \left[n > k \mid t = k \right] \times \min \left(\Delta_H \left(v, I \right), \Delta_H \left(v, \varnothing \right) \right) \\ = \; & \Pr \left[n = k \mid t = k \right] \times \min \left(\Delta_H \left(v, I \right), \Delta_H^{n=k} \left(v, C_{k-1} \right) \right) + \Pr \left[n > k \mid t = k \right] \Delta_H \left(v, \varnothing \right) \\ = \; & \Pr \left[n = 1 \mid t = 1 \right] \times \min \left(\Delta_H \left(v, I \right), \Delta_H^{n=1} \left(v, C_0 \right) \right) + \Pr \left[n > 1 \mid t = 1 \right] \Delta_H \left(v, \varnothing \right) \end{aligned}$$

where the first equality is a consequence of the fact that $\Delta_H \left(v, \varnothing \right) < \Delta_H \left(v, I \right)$ and the second follows from the previous lemma. The last expression is positive by assumption. This implies that all those in C_k buy today. ■

We have shown that for all $k = 0, 1, ...$ any consumer with $t = k$ and $v > v_L + \eta$, buys today. Thus all consumers with $v > v_L + \eta$ buy today. This completes the proof of Proposition 5.

Final step We now show that given that all those with estimates $v > v_L + \eta$ rush to buy today (Proposition 5), it is iteratively dominant for all those with $v > v_H$ to buy today as well. Recall that $v_L > v_H$.

Proposition 6 *There exists a $\delta > 0$ such that for all $\theta_H \in (\theta^*, \theta^* + \delta)$, it is iteratively dominant for all consumers with $v > v_H$ to buy today.*

The proof of the proposition is in two steps. First, we show that the conclusion holds for all those with $t > 0$ and second, that it also holds for those with $t = 0$.

Lemma 8 *There exists a $\delta > 0$ such that for all $\theta_H \in (\theta^*, \theta^* + \delta)$, it is iteratively dominant for all consumers with $t > 0$ and $v > v_H$ to buy today.*

Proof. First, choose any $\eta \in (0, 1 - v_L)$. Proposition 5 implies that there is δ such that for all $\theta_H \in (\theta^*, \theta^* + \delta)$, it is iteratively dominant for all consumers with estimate $v > v_L + \eta$ to buy today. Suppose that $\theta_H \in (\theta^*, \theta^* + \delta)$ and so all consumers in the set $C_\infty = \{i \mid v_i > v_L + \eta\}$ buy today and so C_∞ is contained in the set of all consumers who rush to buy today. Then Lemma 3 implies that the gain from buying today is at least $\min(\Delta_H(v, I), \Delta_H(v, C_\infty))$.

(i) $\Delta_H(v, I) > 0$ **for** $v > v_H$

If everyone buys today, then the price today is $p_1 = \lambda v_H + (1 - \lambda) E[W]$ because the mass of consumers willing to pay this amount is exactly θ_H. Since the supply is exhausted in period 1, $p_2 = 1$. Thus, for $v > v_H$

$$\begin{aligned} \Delta_H(v, I) &= \max(\lambda v + (1 - \lambda) E[W] - p_1, 0) \\ &= \lambda (v - v_H) \\ &> 0 \end{aligned}$$

(ii) $\lim_{\theta_H \downarrow \theta^*} \Delta_H(v, C_\infty) > 0$ **for** $v > v_H$

Now the mass of consumers in C_∞ is at most θ_L and so $\mu(C_\infty \mid n \geq 1) < \theta_L < \theta^*$. Lemma 2 implies that in state θ^*, the prices are $p_1^*(C_\infty) = c$ and $p_2^*(\varnothing) = v^* \in (c, 1)$. Since $\mu(C_\infty \setminus \varnothing \mid n \geq 1) = \mu(C_\infty \mid n \geq 1) > 0$, Lemma 1 implies that $p_2^*(C_\infty) > v^*$. Thus, as $\theta_H \downarrow \theta^*$, the prices in state H satisfy $\lim p_{1H}(C_\infty) = c$ and $\lim p_{2H}(C_\infty) > v^*$.

Thus,

$$\begin{aligned} & \lim_{\theta_H \downarrow \theta^*} \Delta_H(v_H, C_\infty) \\ = \; & \lambda \lim v_H + (1 - \lambda) E[W] - c - \lambda \max(\lim v_H - \lim p_2, 0) \\ & - (1 - \lambda) \psi(\lim p_2) \\ > \; & \lambda v^* + (1 - \lambda) E[W] - c - (1 - \lambda) \psi(v^*) \\ = \; & \Phi(v^*) \\ = \; & 0 \end{aligned}$$

where the inequality in the second line stems from the fact that $\Delta_H(v_H, C_\infty)$ is increasing in p_2 and the last line is just a consequence of the definition of θ^*. Now recall that if the gain from waiting is positive for a consumer with estimate v_H, it is also positive for all consumers with estimates $v > v_H$.

Thus, we have argued that if $\theta_H > \theta^*$ is small enough, for all $v > v_H$,

$$\min\left(\Delta_H(v, I), \Delta_H(v, C_\infty)\right) > 0$$

■

Finally, we have

Lemma 9 *There exists a $\delta > 0$ such that for all $\theta_H \in (\theta^*, \theta^* + \delta)$, it is iteratively dominant for all consumers with $t = 0$ and $v > v_H$ to rush to buy today.*

Proof. Choose $\theta_H \in (\theta^*, \theta^* + \delta)$ from Lemma 8 which then implies that all consumers with $t > 0$ and $v > v_H$ buy today.

If $t = 0$, the gain from buying today for $v > v_H$ is at most

$$\Pr[L \mid t=0] \times \min\left(\Delta_L(v, I), \Delta_L(v, C_0)\right) + \Pr[H \mid t=0] \times \min\left(\Delta_H(v, I), \Delta_H(v, C_\infty)\right)$$

This is because if the state is L, then no consumer gets a message and Lemma 4 implies that the set of consumers rush contains C_0. Lemma 3 then implies the gain from rushing is at least $\min\left(\Delta_L(v, I), \Delta_L(v, C_0)\right)$. If the state is H, then from Proposition 5, the set of consumers who rush contains C_∞. Lemma 3 now implies the gain from buying today is at least $\min\left(\Delta_L(v, I), \Delta_L(v, C_\infty)\right)$.

First, suppose that the state is L. Then, if everyone buys today, $p_1 = \lambda v_L + (1-\lambda) E[W]$ and $p_2 = 1$. Thus,

$$\begin{aligned} \Delta_L(v_H, I) &= \lambda \max(v_H - v_L, 0) \\ &= 0 \end{aligned}$$

since $v_H < v_L$. On the other hand, suppose consumers in the set C_0 buy today. The mass of consumers in C_0 when the state is L is just $1 - F(v_L + \eta)$. Since $1 - F(v_L + \eta) < \theta_L$, we have that $p_1 = c$ and Lemma 1 implies that $p_2(C_0) > p_2(\varnothing) = v_L$ and since $v_L > v_H$, we have

$$\begin{aligned} \Delta_L(v_H, C_0) &= \lambda v_H + (1-\lambda) E[W] - c - \lambda \max(v_H - p_2(C_0), 0) - (1-\lambda)\psi(p_2(C_0)) \\ &> \lambda v_H + (1-\lambda) E[W] - c - (1-\lambda)\psi(v_H) \\ &= \Phi(v_H) \end{aligned}$$

and so we have

$$\lim_{\theta_H \downarrow \theta^*} \Delta_L(v_H, C_0) > \lim_{\theta_H \downarrow \theta^*} \Phi(v_H) = 0$$

This implies that for $v > v_H$,

$$\lim_{\theta_H \downarrow \theta^*} \min \left(\Delta_L (v, I), \Delta_L (v, C_0) \right) \geq 0$$

Second, suppose that the state is H. Then repeating the argument from Lemma 8, we have that for all $v > v_H$

$$\lim_{\theta_H \downarrow \theta^*} \min \left(\Delta_H (v, I), \Delta_H (v, C) \right) > 0$$

Thus, we have shown that for all $v > v_H$, the gain from buying today is strictly positive. ■

This completes the proof of Proposition 6.

A.3 Proof of Proposition 3

To prove the proposition, we will argue that consumer surplus is higher with fixed prices in both state H and state L. The fact that this is true in state H is obvious since in that state, the equilibrium outcome with fixed prices is the same as the outcome when everyone waits and prices are flexible.

So it remains to consider state L. We will argue that if everyone waits under either price regime, then consumer surplus under fixed prices is higher. This is enough to prove the result since in the flexible price regime, consumer surplus if everyone waits is higher than if everyone rushes to buy today. So suppose that under either regime everyone waits.

If everyone waits, then the distribution of final values in the second period is just F and the resulting demand function is $1 - F(p)$. Thus, the consumer surplus at price p is

$$\psi(p) = \int_p^1 (v - p) f(v) \, dv$$

When prices are fixed at v_H, the area under the demand curve is $\psi(v_H)$ but since there is rationing in state L not every consumer willing to pay v_H is served. Thus, under fixed prices the consumer surplus

$$CS^{Fix} = \left(\frac{\theta_L}{\theta_H} \right) \psi(v_H)$$

because the amount demanded at price v_H is θ_H while the supply is only θ_L. When prices are flexible,

$$CS^{Flex} = \psi(v_L)$$

and so

$$
\begin{aligned}
CS^{Fix} - CS^{Flex} &= \left(\frac{\theta_L}{\theta_H}\right) \psi(v_H) - \psi(v_L) \\
&= \theta_L \left[\frac{\psi(v_H)}{1 - F(v_H)} - \frac{\psi(v_L)}{1 - F(v_L)}\right]
\end{aligned}
$$

Lemma 10 *Suppose F has an increasing hazard rate. Then $\psi(p)/(1 - F(p))$ is decreasing over $[0, 1]$.*

Proof. Write

$$
\begin{aligned}
G(p) &= \frac{\psi(p)}{1 - F(p)} \\
&= E[W \mid W > p] - p
\end{aligned}
$$

and note that $G(0) = E[W]$, $G(1) = 0$ and for all $p < 1$, $G(p) > 0$.

We want to show that G is a decreasing function. Differentiating G with respect to p and noting that $\psi'(p) = -(1 - F(p))$

$$
\begin{aligned}
G'(p) &= \frac{(1 - F(p))\psi'(p) + \psi(p) f(p)}{(1 - F(p))^2} \\
&= \frac{\psi(p)}{1 - F(p)} \times \frac{f(p)}{1 - F(p)} - 1 \\
&= G(p) \times h(p) - 1
\end{aligned}
$$

where $h(p) = f(p)/(1 - F(p))$ is the hazard rate function of F.

Differentiating again, we obtain

$$G''(p) = G'(p) h(p) + G(p) h'(p)$$

Now at any $p \in (0, 1)$ such that $G'(p) \geq 0$, we have $G''(p) > 0$. This means that if for some $p_0 \in (0, 1)$, $G'(p_0) > 0$, then for all $p \in (p_0, 1)$, it is positive as well. But this means that $G(1) = 0$ is impossible and this is a contradiction.

Thus, for all p, $G'(p) < 0$. ■

To complete the proof of Proposition 3, note that $v_H < v_L$ and so

$$CS^{Fix} > CS^{Flex}$$

A.4 Proof of Proposition 4

Proof. We will show that when θ_L is small enough, there is an equilibrium with the following strategies: For some $z_0 \in [v_L, 1]$, (i) all consumers with $t = 0$ and $v \geq z_0 \geq v_L$ buy today; (ii) everyone else waits.

For an arbitrary $z \geq v_L$ define

$$C_0(z) = \{i \in I \mid v_i \geq z \text{ and } t_i = 0\}$$

Suppose that all consumers in $C_0(z)$ buy today and everyone else waits.

The expected gain from buying today for a consumer with estimate v and $t_i = 0$ is

$$\begin{aligned} E_\theta[\Delta(v, C_0(z)) \mid t_i = 0] &= \Pr[H \mid t_i = 0] E_n[\Delta_H^n(v, C_0(z)) \mid t_i = 0] \\ &\quad + \Pr[L \mid t_i = 0] \Delta_L(v, C_0(z)) \end{aligned}$$

where n is the number of messages transmitted and Δ_H^n is the gain from buying today in state H when n messages are transmitted. The gain depends on n via the price in the second period p_{2H}, which depends on z as well as n. This is because the fraction of consumers who do not get a message when n messages are transmitted is exactly $(1-\alpha)^n$.

The expected gain from buying today for a consumer with estimate v and $t_i > 0$ is

$$E_n[\Delta_H^n(v, C_0(z)) \mid t_i]$$

since everyone that gets a message knows that the state is H.

Claim: When θ_L is small enough, $\Delta_H^n(v, C_0(z))$ is negative for all n and v.

The gain is monotonic in v, and so it is enough to show that $\Delta_H^n(1, C_0(z))$ is negative for all n.

Now, since $z \geq v_L$ the fraction of consumers in $C_0(z)$ is less than $\theta_L < \theta_H$. Thus, the first-period price in state H, $p_{1H} = c$. Denote by p_{2H}^n the second-period price when n is the number of messages transmitted. For all n,

$$\begin{aligned} \Delta_H^n(1, C_0(z)) &= \lambda p_{2H}^n + (1-\lambda) E[W] - c - (1-\lambda)\psi(p_{2H}^n) \\ &= \Phi(p_{2H}^n) \end{aligned}$$

Note that since $\mu(C_0(z) \mid n)$ is a decreasing function of n, from Lemma 1, we have that p_{2H}^n is a decreasing function of n as well. Thus, for all n, $\Phi(p_{2H}^n) \leq \Phi(p_{2H}^0)$.

Moreover, as $\theta_L \downarrow 0$, $v_L \uparrow 1$ and so $\mu(C_0(z) \mid n = 0) \downarrow 0$. Thus, as $\theta_L \downarrow 0$, $p_{2H}^0 \downarrow v_H$. This implies that when θ_L is small enough, $p_{2H}^0 < v^*$ and so $\Delta_H^n(1, C_0(z)) = \Phi(p_{2H}^n) < \Phi(v^*) = 0$. This establishes the claim.

We now choose z_0 and then show that for this choice of z_0, the strategies prescribed above constitute an equilibrium. To do this, consider the mapping $\Gamma : [v_L, 1] \to \mathbb{R}$ defined by

$$\Gamma(z) = E_\theta[\Delta(z, C_0(z)) \mid t_i = 0]$$

Notice that for any $z > v_L$, $\Gamma(z)$ is single-valued since at any such z, the prices $p_{1L}(z)$ and $p_{2L}(z)$ are uniquely determined. But $\Gamma(v_L)$ is an interval since when $z = v_L$ the market-clearing price is not unique and we only know that $p_{1L}(v_L) \in$

$[c, \lambda v_L + (1-\lambda) E[W]]$. The smallest element of the set $\Gamma(v_L)$, denoted by $\Gamma^-(v_L)$, results when the first-period price is as high as possible, that is, when $p_{1L}(v_L) = \lambda v_L + (1-\lambda) E[W]$. In this case, since $p_{2L}(v_L) = 1$, $\Delta_L(v_L, C_0(v_L)) = 0$ and so using the claim above, $\Gamma^-(v_L) = E_\theta[\Delta(v_L, C_0(v_L)) \mid t_i = 0] < 0$. The largest element of $\Gamma(v_L)$, denoted by $\Gamma^+(v_L)$ results when the first-period price is as low as possible, that is, when $p_{1L}(v_L) = c$.

Note that Γ is an upper-hemicontinuous correspondence and $\lim_{z \downarrow v_L} \Gamma(z) = \Gamma^+(v_L)$ since $p_{1L}(z) = c$ for all $z > v_L$.

We now choose $z_0 \in [v_L, 1]$.

(a) If $\Gamma^+(v_L) < 0$ and for all $z > v_L$, $\Gamma(z) < 0$ as well, then choose $z_0 = 1$.

(b) If $\Gamma^+(v_L) < 0$ and for some $z > v_L$, $\Gamma(z) \geq 0$, then choose $z_0 > v_L$ so that $\Gamma(z_0) = 0$.

(c) If $\Gamma^+(v_L) \geq 0$, then choose $z_0 = v_L$ and $p_{1L}(v_L) \in [c, \lambda v_L + (1-\lambda) E[W]]$ so that $E_\theta[\Delta(z, C_0(z)) \mid t_i = 0] = 0$.

It is now easy to see that the following strategies constitute an equilibrium: (i) all consumers with $t_i = 0$ and $v_i \geq z_0 \geq v_L$ buy today; (ii) everyone else waits.

To see this note, first that every consumer with $t_i = 0$ is acting optimally. In case (a), since $z_0 = 1$, if no one buys today, then it is optimal for all consumers with $t_i = 0$ to buy tomorrow as well. In case (b), all consumers with $t = 0$ and $v = z_0$ are indifferent between buying today or buying tomorrow. Since $E_\theta[\Delta(v, C_0(z)) \mid t_i = 0]$ in non-decreasing in v, all consumers with $t_i = 0$, and $v_i > z_0$ should buy today and all with $v_i < z_0$ should wait. This is what the equilibrium strategy prescribes. In case (c), $z_0 = v_L$ and all consumers with $v_i > v_L$ should buy today while those with $v_i < v_L$ should wait.

Finally, note that it is optimal for any consumer with $t_i > 0$ to wait. Any consumer with $t_i > 0$ knows that the state is H but is uncertain about n, the number of messages sent. We have already argued that when θ_L is small enough given any $n > 0$, for all v,

$$\Delta_H^n(v, C_0(z_0)) < 0$$

and so certainly it is optimal to wait. ■

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