Learning between Buyers and Sellers Along the Global Value Chain

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Abstract

This paper analyses learning between buyers and sellers as a new channel through which international trade affects product introduction across different production stages within firms. Using detailed plant level data from the Indian manufacturing census, I find that (i) 45% of multi-product plants produce at least one product pair that is connected in the Input-Output matrix, (ii) 40% of new products added by plants every year are either upstream or downstream to products previously produced by them, and (iii) exogenous increases in upstream export market access cause firms to add new products that are downstream to their previous production sets. I attribute this effect to plants learning about new products from their downstream buyers. To analyze the effects of trade policy on firm scope I build a dynamic quantitative general equilibrium model of Global Value Chains with knowledge spillovers arising from buyer-seller linkages along the value chain. Potentially multi-product and multi-stage producers in the model invest in R&D to increase their product sets and benefit from knowledge spillovers from domestic and foreign markets. Trade policy counterfactuals show that cross-stage product innovation *decreases* as the economy liberalizes due to convergence in technology levels across countries in general equilibrium.

Keywords: Multi-Product Plants, Multi-Stage Plants, Vertical Relationship, Upstream-Downstream, Knowledge Spillovers, Growth

JEL Codes: F14, F62, F63, O33, O41

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1 Introduction

Global Value Chains (GVCs) bring firms from emerging economies in contact with firms at the global frontier of knowledge by creating firm-to-firm relationships across borders. The disparity between supplying firms in developing countries and buyers from the developed world generates previously unavailable opportunities for suppliers to change their production processes and outputs. These opportunities can either be demand driven or technology driven. For example, sophisticated demand from high-income countries can induce suppliers to provide higher quality goods.[1] On the other hand, firms can benefit from the sharing of know-how and technology transfers from other parties in the chain that improve overall efficiency of the chain.[2] In fact, the World Bank recognizes that "[the] relational nature of GVCs makes them a particularly powerful vehicle for technology transfer along the value chain" (World Bank, 2020).

A less explored aspect of knowledge transfers along GVCs is access to new product ideas through buyer-seller interactions. This is despite the considerable anecdotal and case study evidence of firms from developing countries that enter GVCs as input suppliers who then have expanded their production tasks over time.[3] These examples highlight that international trade can help in idea diffusion through firm-to-firm relationships. In this paper I study the expansion of production stages within a firm, and propose a channel of learning to explain firms' value chain expansion. Specifically I ask the question: do input suppliers learn how to make downstream products from their buyers? What is the role of international trade in providing access to (better) downstream foreign knowledge through buyer-seller interactions?

To answer this question, I use plant-level data from the Indian manufacturing sector to analyze the set of products produced by them over time. Using detailed product level input-output tables, I find that adding products that are either upstream or downstream to previous production baskets is an important dimension in which firms expand. I propose a channel of learning between buyers and sellers as a potential mechanism for the vertical expansion of the value chain within plants. Specifically, I explore the international dimension of learning where Indian upstream suppliers benefit from downstream knowledge flows from their foreign buyers. I find that Indian firms expand downstream after facing an exogenous shift to upstream foreign demand, and find that this phenomenon is consistent with learning.

In order to understand the causes and consequences of the interaction between trade and firm dynamics, I build a dynamic quantitative general equilibrium model of GVCs with innovating

[1] See Verhoogen (2008), Flach (2016), Auer et al. (2018) etc for evidence on firms in developing countries upgrading export quality in response to demand from high income consumers. For more specific examples of demand driven changes, see Barrientos et al. (2016) for process upgrading in the African horticultural industry, and Macchiavello and Miquel-Florensa (2019) for quality upgrading in the Colombian coffee industry.

[2] Smarzynska Javorcik (2004) and Alfaro-Ureña et al. (2019) study how FDI and multinational supply chains can result in productivity improvements for suppliers. Sampson (2020) takes a theoretical approach to study this problem. For a more specific example, see Costa and Delgado (2019) for the introduction of technology improvements in the Mozambican cashew production by international brands.

[3] Bangladesh, Srilanka, and Turkey in the textile and apparel industry now produce large fractions of the value chain after entering GVCs as cheap input suppliers (Gereffi, 1999; Fernandez-Stark et al., 2011). Korea (Samsung, LG), Taiwan (Acer), and China (Huawei) are lead firms in the electronics industry that started off as contract manufacturers (Sturgeon and Kawakami, 2010). China and India are playing an increasing role as independent auto-mobile producers from only being auto parts suppliers in the past (Sturgeon and Van Biesebroeck, 2011).

firms. Firms in the model expand their product portfolios either vertically, by adding new stages, or horizontally, by adding differentiated varieties within stages. Firms also benefit from knowledge spillovers arising from buyer-seller linkages along the value chain while innovating. The calibrated model shows that incumbent innovation is not a significant contributor to growth or the gains from trade. However, an important dimension of within-firm product expansion is explained by trade and international knowledge diffusion through buyer-seller linkages.

The first half of the paper establishes motivating facts about within-firm product expansion. I start by constructing a highly detailed product-level national input-output (IO) table by aggregating micro data at the plant level for close to 6000 products. This IO table enables me to define a vertical relationship between any two products produced in the economy. For example, leather is an input in the production of footballs, and hence leather is defined to be upstream to football. At the same time, football is downstream to leather. I then define plants as multi-stage if they produce at least one pair of products that are vertically linked in the IO table, implying that they have integrated at least two stages of production into their production baskets.

The first contribution of this paper is to establish two new stylized facts about manufacturing plants in India. First, multi-stage plants dominate multi-product production. Close to 45% of all multi-product plants are multi-stage, and 65% of all multi-product output is produced by multi-stage plants. Zooming out, while only 18% of *all* plants in the data are multi-stage, they account for more than 40% of all manufacturing output. This shows that multi-stage plants are different from the average plant and the average multi-product plant in the data. While the literature has established the dominace of multi-product plants in production activities, this is one of the first papers to establish the importance of multi-stage plants.[4] Second, while the incidence of potentially integrated plants is not surprising in itself,[5] a third of all new products added by plants are vertically related to their previous production sets. Of these, more than half are new downstream products. This establishes a quantitatively important dimension of firm-level product expansion that hasn't been studied before.[6]

The second main contribution of this paper is to provide causal evidence for knowledge spillovers between buyers and sellers that can potentially explain the vertical nature of within-plant product addition. In terms of a stylized example, the question I ask is this: Does an Indian leather producer that gets increased access to foreign football knowledge start to produce footballs in the future? Here, access to foreign football knowledge comes through access to foreign football producers that buy Indian leather through export market opportunities. Sellers of a

[4] Orr (2020) notes that a there is a prevalence of multi-stage producers in the Indian manufacturing sector, but excludes these producers from his analysis.
Note that a multi-stage plant has to be multi-product plant as well. Therefore, this fact does not invalidate the facts about multi-product plants in the literature (Bernard et al. (2010) and Boehm et al. (2020)), but brings in a new dimension of multi-product plants that can explain their importance.

[5] See the vast literature on vertical integration of suppliers into production processes based on Grossman and Hart (1986)'s incomplete-contracting and property-rights theory of firm boundaries like Antras (2003), Antràs and Chor (2013), Nunn and Trefler (2008), Nunn and Trefler (2013), Boehm and Oberfield (2020) among others.

[6] To the best of my knowledge, Chor et al. (2020) is the only study that documents the vertical scope of firms' production expansion for China, but they use measures of upstreamness and downstreamness at highly aggregated industry level to infer a firm's position on the value chain. My paper on the other hand used detailed product information and observes product addition over time.

product that experiences an exogenous increase in exposure to foreign downstream knowledge through exports are more likely to start producing a product downstream—to their original product—in the future. In particular, I find that a twofold increase in the measure of export access to foreign technology increases the probability of producing a downstream product by approximately 3.5 percentage points. My empirical specification corrects for the simultaneity bias in estimating the impact of a plant's export decisions and new product introduction using import demand shocks of the importing country to instrument for India's exports. This strategy guarantees that any India specific variation in the determination of export market activity is removed from the regression. This effect persists even after controlling for potentially correlated demand shocks across upstream and downstream products.

What are the general equilibrium effects of such knowledge spillovers on welfare and growth? How do trade policy changes affect firm level innovation decisions, and the resulting distribution of firm types? The reduced form results indicate that a trade liberalization episode may result in increased firm-level innovation into new stages of production. However, one cannot asses the effect of a policy change and the resulting firm innovations on aggregate outcomes in the economy, which in-turn affect the environment that firms operate in, purely from reduced form evidence. In order to answer these questions I build a dynamic general equilibrium model of GVCs with innovating firms, where international trade acts as a vehicle through which goods and ideas flow between countries.

Production in the economy is sequential in nature with two stages of production: upstream and downstream. As in Klette and Kortum (2004), firms are potential multi-product producers. In my model, however, they are also multi-stage producers. Firms add products in both production stages by investing in research and development and benefit from knowledge spillovers from both domestic and foreign producers while innovating. Specifically, they learn from their buyer or sellers to innovate in complementary or cross-stage production. While innovating in the same or own stage of production, firms learn from other products being sold in their domestic markets, similar to the knowledge diffusion in Buera and Oberfield (2020). Lastly, entrants in the model also learn from products sold in the domestic market for innovating new product. Knowledge spillovers in this model take the form of lower innovation costs for both incumbents and entrants. This model of innovating multi-product multi-stage firms is embedded into a dynamic selection model of growth a la Sampson (2016) and Perla and Tonetti (2014). New product ideas developed by incumbents and entrants are drawn from the existing distribution of ideas in the economy. With the endogenous exit of low-quality products at every instance, growth results from better incumbent technology diffusing to new products.

The third contribution of this paper is two-fold. First, I extend the models of trade and growth through dynamic selection, like in Sampson (2016) and Perla et al. (2019), to a setting with asymmetric economies. Introducing international knowledge spillovers that entrants benefit from enables me to solve for a balanced growth path equilibrium where all countries grow at the same rate.[7] Compared to these papers, knowledge spillovers across countries introduces a

[7]Without knowledge spillovers in this model of asymmetric economies, balanced growth path equilibrium exists only in knife edge cases.

new channel of dynamic gains from trade, wherein entrants in laggard economies benefit from decreased cost of innovation due to knowledge spillovers from frontier economies. The implies that the gains from trade liberalization also depends on the initial technology gaps between economies; gains are large when countries are further away from each other, and lower when countries are closer to each other in terms of technology levels. Further, similar to the results in Buera and Oberfield (2020), dynamic gains from liberalizing a relatively closed world are higher than liberalizing a relatively open world.

Second, a novel introduction in this model is the multiple types of innovation that allows a firm to span different production stages. Models of multi-product firms and innovation, like in Klette and Kortum (2004) and Akcigit and Kerr (2018), generally have product innovation that results in adding differentiated varieties in the same product space. This model remains tractable despite the extra dimension of incumbent innovation due to the symmetry of option values from innovations across products in different production stages.

The consequences of introducing incumbent innovation into a model of dynamic selection warrants discussion. Incumbent firms benefit from interacting with foreign buyers and sellers. As mentioned above, knowledge spillovers experienced by entrant firms results in higher growth and welfare. The effect of knowledge spillovers experienced by incumbent firms on welfare is, however, ambiguous. Improved access to knowledge makes incumbents more valuable resulting in more entry and an increase in static welfare. Nevertheless, the endogenous exit rate is lower because knowledge spillovers make firms more profitable in the future, leading to low productive firms to stay active, resulting in lower growth and lower dynamic welfare.[8] Further, increase in incumbent innovation results in in the reallocation of innovation activity from entrants to incumbents and vice versa. This results in minimal changes to aggregate outcomes in response to any policy change that leads to incumbent innovation changes. This adjustment of the entry margin in response to incumbent innovation is similar to the results outlined in Atkeson and Burstein (2010).[9]

While the contribution of knowledge spillovers experienced by incumbent innovation to aggregate outcomes is not large, changes to economic conditions affect firm-level dynamics through adjustments of knowledge spillovers. Calibrating the model to India and an aggregate for the rest of the world, I find that cross-stage spillovers experienced through buyer-seller interactions on the international market are important for explaining the variation in firm type distributions. Counterfactual analyses in this paper are comparisons of balanced growth path equilibria across policies. Trade policy counterfactuals show that cross-stage innovation rate by incumbents in

[8]In models with endogenous exit, wherein firms have to pay a fixed operating cost that leads to low productive/quality firms from exiting the market, introducing incumbent innovation increases the option value of staying active in the market. Therefore, products that would not have been profitable enough to enter, now have an incentive to stay active in the hopes of gaining a new product in the future through innovation. This interaction of incumbent innovation and endogenous exit resulting in lower average incumbent productivity is similar to the model in Acemoglu et al. (2018).

[9]Atkeson and Burstein (2010) studies a model of trade and its impact on firm level entry, exit and process innovation decisions. They find that any changes to welfare resulting from incumbent innovation in response to a trade policy change is offset by the entry margin with product innovation. In my model, any changes to welfare due to incumbent product innovation is mostly offset by entrant firms adjusting their rate of innovation so that in a balanced growth path the adjustment of the equilibrium mass of firms is minimal.

India increases in response to a protectionist policy, while they decrease in response to trade liberalization. This surprising outcome results from technologies between the two countries diverging in response to a tariff increase, which results in higher knowledge spillovers experienced by incumbents. On the other hand, trade liberalization results in a convergence of technology levels across the two countries, so there is less relative knowledge to learn for Indian firms.

Literature Review This paper relates to multiple strands of literature. Firstly, it contributes to the space of international trade and firm level innovation. My paper establishes a link between exporting and new product introduction within plants. In this regard, I contribute to the literature on exporting and innovation like Lileeva and Trefler (2010), Aw et al. (2011), Bustos (2011), and Atkin et al. (2017). Atkin et al. (2017) is the closest to my paper as it focuses on exporters learning from interactions with their buyers in the foreign market, however the the learning process is about improving products already being produced by the exporters. In terms of international trade and product introduction, my paper relates to papers like Goldberg et al. (2010) and Bas and Paunov (2018). However, these papers study access to cheaper or higher quality intermediate inputs as a mechanism for new product introduction. My paper on the other hand looks at how export market access leads to new product introduction. The other focus of the literature on international trade and firm-level innovation has been to study the changes in firm level incentives and costs to innovate in response to a trade shock. Papers that have analyzed the impact of import competition on firm-level innovation activity have found mixed evidence.[10] My paper contributes to this literature by identifying a new channel through which international trade can affect firm level product introduction through knowledge transfers along the value chain.

The second set of studies my paper is related is the multi-product firms literature. Papers such as Bernard et al. (2010), Eckel and Neary (2010), Bernard et al. (2019), Boehm et al. (2019), and Ding (2019) have looked at the incidence of multi-product firms from different lenses. A main argument in the multi-product firms literature is that firm produce multiple product due to input complementarities. I move away from this argument, and find evidence for a new dimension of heterogeneity. I find that plants produce multiple products that are vertically linked on the value chain. Reasons of contractual frictions for vertically integrating suppliers like in Antràs and Chor (2013) and Boehm and Oberfield (2020) is not sufficient to explain the incidence of plants that add products downstream. Alfaro et al. (2019) also use a property-right model of firm boundary decisions to explain expansion of production stages both upstream and downstream, while Chor et al. (2020) finds that Chinese firms span more stages of production in both directions as they grow more productive and experienced. To my knowledge, I am the first to document the systematic relationship between accessing export markets and increasing production states downstream.

My paper also contributes to the growing literature on knowledge diffusion through international trade, and the dynamic gains from trade. This literature has built on the seminal works

[10]See Pavcnik (2002), Trefler (2004), Amiti and Konings (2007), Topalova and Khandelwal (2011), Bloom et al. (2016), Medina (2020) among other. Shu and Steinwender (2019) summarizes the literature.

of Jovanovic and Rob (1989), Grossman and Helpman (1991), Coe and Helpman (1995), and Kortum (1997) who model innovation and diffusion as a stochastic process. My model is closest to the ones in Sampson (2016) and Perla et al. (2019). However there are two key distinctions from these papers. I extend the model of dynamic selection to incorporate asymmetric economies and introduce knowledge spillovers across the economies to solve for balanced growth path equilibria. Other papers that study knowledge diffusion through the lens of international trade are the models developed by Alvarez et al. (2008), Alvarez et al. (2013), Buera and Oberfield (2020), and Cai et al. (2018). The other key difference in my model is incumbent innovation, not just in one product space, but in multiple product spaces. This allows me to study firm-level dynamics in response to policy shocks.

2 Empirical Evidence

In this section, I document a set of new facts about the product mix of multi-product plants in India. I show that plants produce products that are vertically linked in the production process, and that a disproportionately large share of new products added by plants are either upstream or downstream to older product sets. A key element in doing this exercise is the construction a highly detailed product-level input-output table using micro firm-level production data. I then propose a channel of learning between buyers and sellers which can explain the vertical nature of new product addition. In the second half of the empirical analysis I provide causal evidence on international knowledge spillovers along the value chain. My identification strategy depends on variation in product scope of firms across time, and plausibly exogenous variation in export market opportunities. I show that sellers of a specific product that experience greater export exposure in a period are more likely to produce a product that is downstream to the said product in future periods. This effect persists even after controlling for correlated demand shocks across the two products. I attribute this effect to knowledge spillovers wherein Indian plants benefit from interacting with foreign producers of downstream products. I first start by describing my dataset, and some stylized facts about Indian manufacturing firms. I then describe my empirical strategy, explain the variable construction, and finally discuss the results.

2.1 Data

The primary dataset I use comes from the Indian Annual Survey of Industries (ASI henceforth) for the years 2001 to 2009. The ASI extends to the entire country, and covers all registered factories[11] employing 10 or more workers using power, and those employing 20 or more workers without using power. All plants employing 100 or more workers are sampled every year called census plants, while the rest are sampled randomly.[12] I construct an unbalanced panel spanning the years 2001-2009.

[11] Factories in this dataset correspond to plants or establishments, and not firms. See appendix A.1 for more details.

[12] The 100 employee threshold applies to most states with a few exceptions. For more details see: http://www.csoisw.gov.in/CMS/UploadedFiles/ASIWrite_Up_2017_2018.pdf

I chose to use this specific dataset to study the product mix of plants and the role of technology diffusion in determining the product mix for multiple reasons. *First*, a developing country such as India provides an ideal setting to study learning along the value chain. India plays an important role in international markets, specially as an intermediate goods supplier. The share of India's exports that comprise of "intermediate goods" was close to 35% in 2018, where as the world average was 20%.[13] While knowledge diffusion can flow either way, a developing country by being farther away from the technological frontier has more to gain from international knowledge spillovers. *Second*, a unique feature of the ASI is that it contains detailed information about all products produced by plants in a given year, which are classified according to a national product classification. There are more than 6000 five digit product codes defined by the classification, out of which 5755 products are listed as outputs produced by firms in my sample period. More importantly, the data set also lists the inputs used by firms in the same product classification. This classification is unique for the level of detail in product definitions, and allows one to construct an Input-Output (IO henceforth) table at a detailed product level, and thereby define the vertical relationship between any two products. *Third*, Indian firms exhibit a lot of product switching behavior, and the ASI allows one to keep track of products produced in the past, present, and future. This variation is especially useful for the empirical exercise I consider.

Before I can document features of the product sets produced by Indian plants, I need to define vertical relationships between product pairs. To that end, I construct a national input output database that will enable me to establish if two products are related vertically.

2.2 Vertical Relationship between Product Pairs

A key element in this empirical analysis is determining the relationship between two products: whether one is upstream to the other, or downstream to the other, or are unrelated to each other. I do this for the set of products produced by Indian firms by constructing a national IO table. I then use the elements in the matrix to determine the vertical relationship between any two products. The IO table is constructed using information on the inputs used and outputs produced by only the single product firms, which make up about 60% of all firms in my sample.[14] I use only single product firms as the dataset reports total inputs used by the plant, and not inputs used per product line. Using only single-product plants ensures that inputs are not wrongly attributed to an output. I then aggregate up all firms' input and output information to construct one national IO table that contains more than 33 million (5755^2) product pairs, out of which only 72,292 elements contain positive values.[15]

For every product pair from the IO table, (ω, ω'), I can now observe the amount of material

[13] China's intermediate good share in total exports is 22% and the United State's is 17%. Source: World Trade Integrated Solutions.

[14] I use data from the years 2003-2009 to construct the IO table. I drop the first two years of my sample as the maximum number of inputs required by the ASI to be declared changed from 5 to 10 in 2003. Hence, I use the years for which the most consistent input data is available.

[15] To avoid picking up unrelated products as vertically linked arising due to errors in data filing, I use only those links that make up at least 1% of the total input supply of an output product. Doing this cuts the number of positive linkages by close to 75%.

Table 1: Vertical Relationship: Examples

ω	ω'	VR(ω, ω')
Tanned Leather (43301)	Foot Ball (93134)	Upstream
Desk with Seat (51208)	Plywood Board (51275)	Downstream
Aromatic Chemicals (36102)	Perfume based Materials (36146)	Both
Iron/Steel Wire Nets (71575)	Bamboo Pulp (54101)	None

Table 2: Vertical Relationship: Classification of Product Pairs

Category	Number	Share (%)
Upstream	35,770	0.108
Downstream	35,770	0.108
Both	752	0.002
None	33,084,451	99.78
Total	33,120,025	100

flow between them. Let $m_{\omega\omega'}$ be the value of ω used as an input into the production of ω'. Then, the placement of ω on the value chain with respect to ω' is defined as:

$$\text{VR}(\omega, \omega') = \begin{cases} \text{Upstream}, U & \text{if} \quad m_{\omega\omega'} > 0 \quad \& \quad m_{\omega'\omega} = 0 \\ \text{Downstream}, D & \text{if} \quad m_{\omega\omega'} = 0 \quad \& \quad m_{\omega'\omega} > 0 \\ \text{Both}, B & \text{if} \quad m_{\omega\omega'} > 0 \quad \& \quad m_{\omega'\omega} > 0 \\ \text{None}, N & \text{if} \quad m_{\omega\omega'} = 0 \quad \& \quad m_{\omega'\omega} = 0 \end{cases} \tag{1}$$

where VR$(\cdot)$ is the vertical relationship function. Note that by this definition, if ω is upstream to ω', then ω' is downstream to ω. Table 1 gives some illustrative examples of the different types of relationships between product-pairs. Table 2 gives details on the number of product pairs in the IO table and the shares of the different vertical relationship categories they fall into. A very small fraction of all the potential product linkages are active in the IO matrix. For an average product in the IO table, the number of products that are vertically linked to it is a little over 12 out of a total potential of 5754 products. That is, the average product is either an input to, or an output of, about 0.2% of all products.

There are two aspects of this definition of vertical linkages worth discussing. The first one pertains to the fact that this construction picks up only *direct* inputs in the production of an output. This means that products that are upstream or downstream through multiple steps of the production value chain are not considered to be "vertically" linked. As Boehm and Oberfield (2020) document, single-product plants in India use a diverse set of recipes to produce a given product, and these recipes vary in terms of how many production stages are integrated within the plant. To the extent that recipes used by single-product plants captures a large part of the value chain for products, my definition of vertical relationship between product pairs will pick

up even products separated by multiple stages of production in between.

The second aspect relates to my application of these product relationships to products produced by multi-product plants. It is possible that multi-product plants use different production technologies and recipes over the ones used by single-product plants. If this were the case, then my definition picks up far fewer relationships than that exist in reality. This would bias the conclusions from my empirical analysis towards finding no impact of trade on value chain learning.

2.3 Stylized Facts about Indian Manufacturing Plants

Fact I: Multi-product plants are also multi-stage plants

Table 3 lists some summary statistics on the types of plants in the ASI. My unbalanced panel sample consists of 298,545 plant-year observations. The purpose of this table is to show variation across plants in India in terms of number of products and types of products produced. First, the average Indian manufacturing plant produces 1.8 products per year, with the number of products ranging from 1 to 22. Based on this observation, I first classify plants as either single-product or multi-product. While multi-product plants make up for only 40% of plant-year observations, they are responsible for more than 70% of total output in the economy.[16]

The second dimension of heterogeneity I am interested in is the type of products being produced by these plants. Specifically, I look for whether plants produce products that are linked via the IO table, or in other words, linked via the value chain. Based on this I classify plants as single-stage or multi-stage producers. A plant is multi-stage if it produces at least one pair of products that has a non-zero element in the IO table. This implies the plant has integrated multiple stages of a value chain within itself. Note that, by this definition single product plants are never classified as integrated. This definition of vertical integration is different from the one used in Boehm and Oberfield (2020), where integration is a continuous measure defined for only single product firms.[17] In contrast, my classification of vertical integration can only be applied to multi-product plants. This classification will be an underestimate of the true share of multi-stage producers in the sample.

Roughly 18% of all plants in my dataset and 45% of all multi-product plants are classified as multi-stage producers. More surprisingly, multi-stage producers dominate production activity even when compared to other multi-product plants. They account for more than 45% of all production activity, and 65% of all multi-product output specifically. This seems to imply that multi-stage producers are different from an average multi-product producer, and underlines the importance of studying these plants in more detail. This feature of multi-product plants is less explored in the literature, and contributes to a growing body of literature that studies

[16] The dominance of multi-product firms in production has been documented by other papers: Bernard et al. (2010) for the US, Goldberg et al. (2010) and Boehm et al. (2020) for India.

[17] Boehm and Oberfield (2020) constructs a measure of vertical distance between the inputs used and outputs produced by a plant. Vertical distance is intended to capture the typical number of steps between the use of a product, ω', and the production of another product, ω. Multi- product plants are excluded in their analysis as they cannot observe input use by each production line.

Table 3: Plant Types in Indian Manufacturing

Plant Type	Statistic	2001	2005	2009	2001-09
All	Observations	27710	36042	34787	298545
	Avg. Products	1.85	1.78	1.73	1.79

(a) Within All Plants

Single-Product	Plant Share	0.57	0.60	0.62	0.60
	Output Share	0.29	0.27	0.29	0.29
	Avg. Products	1	1	1	1
Multi-Product	Plant Share	0.43	0.40	0.38	0.40
	Output Share	0.71	0.73	0.71	0.71
	Avg. Products	3.00	2.96	2.95	2.98

(b) Within Multi-Product Plants

Single-Stage	Plant Share	0.56	0.54	0.57	0.55
	Output Share	0.32	0.39	0.45	0.38
	Avg. Products	2.63	2.58	2.59	2.60
Multi-Stage	Plant Share	0.44	0.46	0.43	0.45
	Output Share	0.68	0.61	0.55	0.62
	Avg. Products	3.45	3.39	3.41	3.44

Notes: This table summarizes the heterogeneity in the product mix of Indian manufacturing plants. Panel (a) classifies all plants into either-product or multi-product based on the number of products produced. Panel (b) further classifies all multi-product plants into either single-stage or multi-stage producers based on the type of products produced. Rows labeled "Plant Share" and "Output Share" report the share of plant observations and the share of output respectively of the corresponding plant type out of all observations in panel (a), and out of multi-product observations in panel (b). Rows labelled "Avg. Products" report the average number of products within each plant type.

Table 4: Plant Type Distribution

Plant Type	Single-Product	Multi-Product Single-Stage	Multi-Product Multi-Stage
Plant Share	0.60	0.22	0.18
Output Share	0.29	0.28	0.43

Notes: This table lists the distribution of plant types, and their corresponding share of total output, for all plant-year observations between 2001-2009 in my sample.

explanations for the incidence of multi-product plants.[18] A closely related paper is Boehm et al. (2019), which shows that multi-product plants in India produce products belonging to industries that share common intermediate inputs using the same dataset.[19] I provide a complementary explanation for the incidence of multi-product plants to the one provided in their paper.

Based on these dimensions of heterogeneity I classify all plants into three kinds: single-product, multi-product single-stage, and multi-product multi-stage. Only multi-product plants can be classified as a multi-stage producer as well. Table 4 summarizes these categories.

Fact II: New products are vertically linked to older product sets within plants

A key observation from the dataset is that a plant's type is not constant over time. Plants switch between being single-product and multi-product, and being single-stage and multi-stage. The statistics presented in this section are for the sub-sample of plants that appear in the dataset at least two years in a row, which is around 127,600 plants. Table 5 shows the transition matrix of firm types from one year to the next. Product adding makes plants transition into both multi-product plants and also multi-stage plants at comparable rates. Further, product dropping leads to plants losing their multi-product or multi-stage status.

A more important characteristic of product adding behaviour of plants is summarized in table 6. A disproportionate share of new products added by plants each year is vertically related to plants' previously produced product sets. For example, a plant producing tanned leather in one year may go on to produce footballs in the next, or a plant producing desks may start producing plywood in-house. Recall that an average product in the IO matrix is vertically related to only 0.2% of all products in the economy. However, close to 40% of new products added by plants at time $t+1$ are either upstream and/or downstream to product sets at time t. This implies that new products are not randomly chosen by these plants, but are chosen because they are linked via production processes. This is a key feature of the data that will shape the model in the next section.

2.4 Identifying Knowledge Spillovers Along the Value Chain

What could be driving this vertical nature of new product addition? I propose that a potential mechanism for adding new vertically related products is through learning about these products from their buyers and sellers. In the following empirical exercises, I attempt to tease out the effect of knowledge spillovers on Indian manufacturing plants' product decisions. I focus on a specific form of knowledge spillovers: input suppliers learning about downstream products from interactions with their buyers.[20] I hypothesize that buyer-seller interactions can lead to knowledge transfers. For example, the producer may share the design of his product with his input supplier in the process of providing the specifications for the input he requires. Or, the supplier learns how to organize his workers to incorporate a new stage of production. Knowledge

[18] Other papers have looked at multi product plants from different lenses: Eckel and Neary (2010), Bernard et al. (2019), De Loecker (2013), Nocke and Yeaple (2014).

[19] Industries in Boehm et al. (2019) are defined as the 3-digit aggregates of the 5-digit product codes.

[20] Note that buyers of an input are producers of products downstream to that input.

Table 5: Firm Type Transition

		year $(t+1)$		
		Single-Product	Multi-Product Single-Stage	Multi-Product Multi-Stage
year t	Single-Product	0.89	0.07	0.04
	Multi-Product Single-Stage	0.17	0.63	0.20
	Multi-Product Multi-Stage	0.11	0.22	0.67

Notes: This matrix represents the transition of firm types from one year to the next. Each row sums to 1, and each element in a given row represents a share.

Table 6: Types of New Products in year $(t+1)$

VR	2001	2005	2008	2001-08
Upstream	0.15	0.15	0.13	0.14
Downstream	0.16	0.17	0.16	0.17
Both	0.09	0.08	0.08	0.09
None	0.60	0.60	0.62	0.60
Total	1	1	1	1

Notes: This table summarizes the set of new products produced by plants in a year and the relationship to older product sets within the plant. New product added in the next year for 2009 cannot be calculated as it is the terminal panel year.

transfer could also happen when employees from the producer's firm moves to the supplier's firm. While I will not be taking a stance on the exact nature of knowledge spillovers, the regression specification lets me identify spillovers in an agnostic way.

My identification strategy is based on exogenous shocks to an Indian supplier's export market access. The export market access variation is exogenous to Indian specific trends, and captures changes in foreign countries' import demand. I show that increase in export market access for firms in India leads to an increase in the likelihood of firms producing products downstream to their present product set in the future. This can happen for two reasons: new exporters accessing new knowledge outside of the country, or exporters getting access to new technology in the downstream industry in these export markets.

2.4.1 Regression Specification

The following is the baseline specification to identify the effect of increased export exposure ($Exp^D_{\omega,t}$) of product ω on the likelihood that plant j producing ω introduces a downstream product, $D(\omega)$, in the future:

$$\mathbb{1}^D_{j,\omega,t+s} = \alpha_0 + \alpha_1 \log(Exp^D_{\omega,t}) + \mathbf{X}_{j,\omega,t}\boldsymbol{\alpha} + \delta_t + \delta_{j,\omega} + \epsilon_{j,\omega,t+s} \tag{2}$$

where $\mathbb{1}^D_{j,\omega,t+s}$ is the main variable of interest capturing new downstream production, and $\mathbf{X}_{j,\omega,t}$ are a set of firm-year and firm-product-year level controls, and δ_t and $\delta_{j,\omega}$ are year and firm-product fixed effects.

Downstream Upgrading I use the vertical relationship definition described in equation 1 to identify the set of downstream products to any product ω as:

$$D(\omega) = \{\omega' : \mathrm{VR}(\omega', \omega) = D\} \tag{3}$$

When constructing my dependent variables, I use only those products that are new in a firm's product set at time $t+s$ relative time t. Formally, let $\Omega_{j,t}$ be the set of products produced at time t by plant j. The set of new products at time $t+s$ would be: $\Omega^{new}_{j,t+s} = \Omega_{j,t+s} \setminus \{\Omega_{j,t+s} \cap \Omega_{j,t}\}$.

$$\mathbb{1}^D_{j,\omega,t+s} = \begin{cases} 1 & \text{if } \left|\Omega^{new}_{j,t+s} \cap D(\omega)\right| \geq 1 \\ 0 & \text{if } \left|\Omega^{new}_{j,t+s} \cap D(\omega)\right| = 0 \end{cases} \tag{4}$$

where $|A|$ is the cardinality of set A. The main variable of interest captures whether a firm j produces at least one new product at time $t+s$ that is downstream to a product ω from its time t production set.

Export Exposure The independent variable is constructed to capture exposure to foreign downstream technology that Indian firms have through exports. Export exposure of a product ω at t is the weighted average of India's product level exports to different countries. The weights

are constructed to capture how good importing countries are in producing products downstream to product ω. The weight for each country i, $\mathbb{w}^i_{D(\omega),t}$, is measured by the relative per-capita exports of downstream products to the world by country i. The assumption here is that if country i is a large supplier of downstream products, after controlling for the size of country i, then it is a good producer of downstream products. This is meant to capture the variation that I am interested in, namely knowledge of downstream production, and the potential for Indian firms to learn from this knowledge. The export exposure measure is constructed as follows:

$$Exp^D_{\omega,t} = \sum_i X^{Ind,i}_{\omega,t} \ \mathbb{w}^i_{D(\omega),t} \tag{5}$$

$$\mathbb{w}^i_{D(\omega),t} = \frac{X^{i,Wld}_{D(\omega),t}}{L^j_t} \Bigg/ \sum_j \frac{X^{j,Wld}_{D(\omega),t}}{L^j_t}$$

where $X^{Ind,i}_{\omega,t}$ is the value of Indian exports of product ω to country i in t, and $X^{Wld,i}_{D(\omega),t}$ is the value of exports from country i to the rest of the world of all the products that are downstream to ω. Note that this variable only has product and time level variation, and not firm level. Therefore, all firms in the regression receive the same shock. My coefficient of interest, α_1, is to be interpreted as the average country level effect.

There are two sources of variation in the above measure of export exposure: variation of Indian exports across different countries, and the variation in foreign countries downstream technology. Therefore, India's access to a foreign country's downstream knowledge increases for two reasons: Indian exports to said foreign country increases, or the foreign country's downstream knowledge increases.

2.4.2 Threats to Identification

I hypothesize that a positive value of α_1 in my estimating equation 2 would mean that there are knowledge spillovers across the stages of production. However, there are some concerns of endogeneity that have to be addressed. I discuss some threats to identification in my regression specification, and how I solve for them below.

Common Supply Shocks across Upstream and Downstream Products in India A concern may be that Indian firms are getting better at producing certain value chains, and exports increase due to this supply shock within India. This can lead to firms in India producing new downstream products, while simultaneously exporting upstream product from the same value chain. For example, Indian firms learn a new way to make both tanned leather and leather footwear more efficiently. This would make the OLS estimate upward biased. In order to correct for this, I use a import demand instrument, that would exclude India specific technology or policy shocks over time, and would capture export variation that is foreign demand driven. I describe this instrument in the next section.

Supply Shocks to Upstream Products only in India Another endogeneity concern is when supply shocks to products increases exports, which makes firms reluctant to switch into other products due to increased profits. For example, if Indian leather producers increase their efficiency and do not want to switch their production into footwear. This is especially a concern as the data shows that if plants start producing downstream products, it usually happens at the expense of their supply of upstream products. This would bias the OLS estimates downward. Table 7 shows this relationship where I run the following regression for different lead periods:

$$\mathbb{1}^{D}_{j,\omega,t+s} = b_0 + b_1 \mathbb{1}^{S}_{j,\omega,t+s} + \mathbf{X}_{j,\omega,t}\boldsymbol{\beta} + \delta_t + \delta_{j,\omega} + \epsilon_{j,\omega,t+s} \tag{6}$$

where $\mathbb{1}^{S}_{j,\omega,t+s}$ is an indicator for whether product ω that was produced in period t is still produced in period $t+s$. This regression captures the correlation between the probability of producing new products downstream to a given product in a firm's product set, and the probability of continued production of the said product in the future. This correlation is consistently negative, suggesting that production of new downstream products comes at the expense of incumbent products.

The import demand instrument described to correct for the first endogeneity issue is also used to correct for this bias. My identification assumption rests on the argument that foreign import demand shocks for a specific product should be exogenous to firm level production decisions.

World Demand shocks for Downstream Products Imagine a scenario where an increase in exports of a particular product ω is driven by increase in world demand for products downstream, $D(\omega)$. This would lead to countries demanding more of inputs used to produce the set of products in $D(\omega)$, one of which is product ω. So, any correlation we may find could be a result of Indian firms reacting to this world demand for downstream products. The import demand IV would not be able to correct for this bias, as the bias is in fact exacerbated by the use of import demand shocks. This would lead to a positive bias in OLS. To control for any such downstream demand shocks, I construct a variable that captures world demand shocks to downstream products that are relevant to India. It is as follows:

$$\text{World}^{D}_{\omega,t} = \sum_{\omega' \in D(\omega)} \gamma_{\omega',\omega} \sum_{i} s^{Ind,i}_{\omega',0} \;\; X^{-Ind,i}_{\omega',t} \tag{7}$$

where $X^{-Ind,i}_{\omega',t}$ is country i's imports of product ω' from all countries except India, $s^{Ind,i}_{\omega',0}$ is India's share in country i's imports of product ω' in a pre-sample period (2000), and $\gamma_{\omega',\omega}$ is the share of product ω' in total intermediate input demand faced by product ω. This variable captures world demand trends of downstream products that potentially shift India's demand curve for product ω.

2.4.3 Instrumental Variable Strategy

My instrumental variable strategy involves using shocks to a country's demand for a certain product to explain India's exports of said product to that country. Specifically I instrument

Table 7: Endogeneity Issue

	Produce Same Product, $\mathbb{1}^{S}_{j,\omega,t+s}$			
	$t+1$	$t+2$	$t+3$	$t+4$
	(1)	(2)	(3)	(4)
$\mathbb{1}^{D}_{j,\omega,t+1}$	-0.277***			
	(0.006)			
$\mathbb{1}^{D}_{j,\omega,t+2}$		-0.226***		
		(0.0006)		
$\mathbb{1}^{D}_{j,\omega,t+3}$			-0.209***	
			(0.007)	
$\mathbb{1}^{D}_{j,\omega,t+4}$				-0.202
				(0.008)
Observations	150,716	118,530	90,418	68,759
R-Squared	0.441	0.549	0.604	0.631

Notes: Standard errors are reported in parenthesis, and are clustered at the 3-digit product code. * Significant at 10%, ** Significant at 5%, *** Significant at 1%. All columns have the following controls: plant-product revenue in each year, total number of products produced by the plant in each year, year fixed effects, and plant-product fixed effects. All columns report ordinary least square (OLS) estimates. The independent variable is an indicator for a plant producing a new downstream product to its current product in a future year, $t+s$. These results show that the probability that a plant keeps producing its older product decreases with the probability of it producing a new product in the future. This can potentially bias the OLS results in the baseline specification downward.

country i's imports from India with its imports from the world except India. The variation captured here would potentially exclude Indian supply factors driving Indian exports. My instrument is then constructed as a weighted average of every country's import demand, weighted by the country's downstream market size at a pre-sample period:

$$\text{IV}_{\omega,t} = \sum_{i} X^{-Ind,i}_{\omega,t} \quad \mathbb{w}^{i}_{D(\omega),0} \tag{8}$$

2.5 Results

Table 8 reports the baseline results from OLS and IV strategies for lead year $s = 1$, i.e. I look for the impact of export exposure on downstream upgrading one year in the future. All regressions have year and plant-product fixed effects. The inclusion of plant-product fixed effects controls for any plant specific trends (that will affect all the products produced by the plant) and product level trends in the economy. The estimates should thus be interpreted as within plant, across product effects. All regressions are also clustered at the 3-digit industry level to capture correlated shocks across different industry markets.

Column (1) reports the OLS results which is close to zero and insignificant. Column (2)

reports estimates from the IV strategy. As described in the endogeneity section, ex-ante the OLS estimate could be upward or downward biased. The positive and significant IV estimate in column (2) suggests that the negative bias is stronger in OLS. This is potentially a result of firm inertia in switching products when hit with a supply shock as discussed in the edogeneity section. The results show that a doubling of exposure to foreign technology through exogeneous shocks to exports for a product increases a plant's likelihood of producing a new downstream product by close to 3.5 percentage points in the next year.

In column (3) I correct for the third simultaneity bias, world downstream demand trends, by including the world demand regressor $\text{World}^{D}_{\omega,t}$. This regressor captures any demand trends that Indian firms face in the downstream product market. As expected, the coefficient of interest is muted by the inclusion of this regressor, but not enough to cancel out the effect of export exposure. These results suggest that Indian firms when faced with an exogenous shock to their export market access learn from their buyers, which is then manifested as changes to their production decisions.

Table 9 reports the instrumental strategy results for different lead periods. Since, my sample is an unbalanced panel of plants, it is hard to compare results across the different columns. But, the main takeaway from these regressions is that the knowledge spillover results in the last table is not just spurious correlation from one year to the next, but is a consistent pattern over time. Firms learn about downstream products when given an opportunity, that they then may convert to production sometime in the future.

2.6 Placebo Tests

One may be concerned that the downstream upgrading results are not due to knowledge spillovers (specific to downstream products), but some other firm level effect of exporting that manifests as new products being produced. Improved export market access can lead to changes in firm outcomes, that can potentially make the firm more likely to produce new products. To exclude such concerns, I conduct placebo tests by re-running the baseline IV regression using other dependent variables. I replace new downstream products with new upstream products and new other unrelated products in my main regression specification 2. Table 10 reports the results for these regressions, where $\mathbb{1}^{U}_{\omega,j,t+s}$ and $\mathbb{1}^{O}_{\omega,j,t+s}$ are the number of new upstream products and new other products produced by the plant at time $t+s$.

The results of the placebo tests are reassuring, and provide more support to the argument that firms produce new downstream products due to knowledge spillovers. Columns (1)-(3) reports the placebo regression results for new upstream products for different lead years, and all estimates are zero. Similarly, Columns (4)-(6) reports the same for new unrelated products, with imprecise estimates. Export exposure for a given product does not affect the likelihood of a firm producing new products other than those flagged as downstream to the said product.

Taking Stock In the previous section, I have shown some new stylized fact about Indian manufacturing plants. Multi-product plants in India are also multi-stage producers, and multi-

Table 8: Knowledge Spillovers: Baseline

(a) OLS, Second Stage (IV), and Reduced Form Results

	New Downstream Production, $\mathbb{1}^D_{j,\omega,t+1}$				
	OLS	IV	IV	RF	RF
	(1)	(2)	(3)	(4)	(5)
$\log(\text{Exp}^D_{\omega,t})$	0.002	0.049**	0.048**	0.025***	0.026***
	(0.003)	(0.021)	(0.019)	(0.008)	(0.009)
$\log(\text{World}^D_{\omega,t})$			0.002		-0.007
			(0.008)		(0.007)
Observations	127,059	127,059	127,059	127,059	127,059
R-Squared	0.421	-	-	0.421	0.421
F-Stat	-	21.988	22.044	-	-

Notes: Standard errors are reported in parenthesis, and are clustered at the 3-digit product code. * Significant at 10%, ** Significant at 5%, *** Significant at 1%. All columns have the following controls: plant-product revenue in each year, total number of products produced by the plant in each year, year fixed effects, and plant-product fixed effects. Column (1) reports ordinary least squares (OLS) estimates, columns (2)-(3) reports second stage estimates from instrumental variable (IV) strategy, and columns (4)-(5) reports the reduced form (RF) estimates. Indian export exposure measure, $\text{Exp}^D_{\omega,t}$, is instrumented by the import demand IV defined in equation 8, $\text{IV}^D_{\omega,t}$.

(b) First Stage (IV) Results

	Export Exposure, $\log(\text{Exp}^D_{\omega,t})$	
	(1)	(2)
$\log(\text{IV}^D_{\omega,t})$	0.506***	0.545***
	(0.108)	(0.116)
$\log(\text{World}^D_{\omega,t})$		-0.193
		(0.189)
Observation	127,059	127,059
R-Squared	0.964	0.964

Table 9: Knowledge Spillovers: New Downstream Products over Time

	New Downstream Production, $\mathbb{1}^D_{j,\omega,t+s}$			
	$t+1$	$t+2$	$t+3$	$t+4$
	(1)	(2)	(3)	(4)
$\log(\text{Exp}^D_{\omega,t})$	0.048**	0.057***	0.078***	0.070*
	(0.019)	(0.016)	(0.025)	(0.041)
$\log(\text{World}^D_{\omega,t})$	0.002	0.019	0.049*	0.068**
	(0.008)	(0.013)	(0.027)	(0.029)
Observations	127,059	100,320	76,939	58,637
F-Stat	22.044	30.565	27.509	14.104

Notes: Standard errors are reported in parenthesis, and are clustered at the 3-digit product code. * Significant at 10%, ** Significant at 5%, *** Significant at 1%. All columns have the following controls: plant-product revenue in each year, total number of products produced by the plant in each year, year fixed effects, and plant-product fixed effects. All columns report second stage estimates from instrumental variable (IV) strategy. Indian export exposure measure, $\text{Exp}^D_{\omega,t}$, is instrumented by the import demand IV defined in equation 8, $\text{IV}^D_{\omega,t}$.

Table 10: Knowledge Spillovers: Placebo Tests

	New Upstream Products, $\mathbb{1}^U_{j,\omega,t+s}$			New Unrelated Products, $\mathbb{1}^O_{j,\omega,t+s}$		
	(t+1)	(t+2)	(t+3)	(t+1)	(t+2)	(t+3)
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(\text{Exp}^D_{\omega,t})$	0.004	-0.010	-0.023*	-0.024	-0.005	-0.026
	(0.017)	(0.017)	(0.013)	(0.024)	(0.020)	(0.019)
$\log(\text{World}^D_{\omega,t})$	0.008	0.010	-0.002	-0.007	-0.013	0.001
	(0.009)	(0.011)	(0.010)	(0.013)	(0.015)	(0.017)
Observations	127,059	100,320	76,939	127,059	100,320	76,939
F-Stat	22.044	30.565	27.509	22.044	30.565	27.509

Notes: Standard errors are reported in parenthesis, and are clustered at the 3-digit product code. * Significant at 10%, ** Significant at 5%, *** Significant at 1%. All columns have the following controls: plant-product revenue in each year, total number of products produced by the plant in each year, year fixed effects, and plant-product fixed effects. All columns report second stage estimates from instrumental variable (IV) strategy. The dependent variable in columns (1)-(3) is an indicator for the plant producing a product upstream to it's current product, and the dependent variable in columns (4)-(6) is an indicator for the plant producing unrelated products in the IO table to its current product in a future year $t+s$. Indian export exposure measure, $\text{Exp}^D_{\omega,t}$, is instrumented by the import demand IV defined in equation 8, $\text{IV}^D_{\omega,t}$.

stage production is a result of product adding over time. A disproportionate share of new products added by plants over time are vertically linked to previous production sets within plants. This results in previously single-stage producers to become multi-stage producers over time. I propose and explore a potential channel for why plants add vertically related products: learning between buyers and seller. I find evidence for such a mechanism whereby Indian plants are more likely to add downstream products when they are exposed to foreign downstream technology.

In order to asses the importance of product addition and international knowledge spillovers on aggregate outcomes, I build a dynamic general equilibrium model with innovating firms. A GE model is also particularly useful to study changes in firm dynamics in response to a trade policy change: Are there more of less firm innovating? Are there more or less multi-product and multi-stage producers? The model thus incorporates features of firms and firm dynamics documented in the empirical section. I first describe the structure of the economy, and then define firms and firm dynamics arising from innovation, and finally derive results for growth and welfare.

3 Model Setup

In this section, I develop a quantitative trade model with heterogeneous firms engaged in a sequential production process with dynamic idea diffusion. Consider a world made up of N asymmetric countries indexed by n, or l, and J industries indexed by j or k. The set of countries and industries will be denoted by $\mathcal{N}$ and $\mathcal{J}$ going forward. Each country is endowed with L_n amount of labor that is supplied inelastically by a representative household, and is the only factor of production. A non-traded final good, used for consumption and as material input (for roundabout production), is a composite of intermediate varieties sourced from all industries and all countries. Intermediate variety production involves two sequential stages: an upstream stage and a downstream stage, indexed by $s \in \{u, d\}$. Labor is mobile across all industry-stages of production but not across countries, and time is continuous.

3.1 Preferences

Each economy is populated by a representative household which seeks to maximize the following intertemporal utility function:

$$U_n = \int_0^\infty e^{-\rho t} log(C_{nt}) dt \tag{9}$$

where C_{it} represents consumption of final good at time t, and $\rho > 0$ is the discount rate. The budget constraint of the representative household is given by:

$$r_{nt} A_{nt} + w_{nt} L_n = P_{nt} C_{nt} + \dot{A}_{nt} \tag{10}$$

where r_{nt} is the return to asset holdings of the household, w_{nt} is the equilibrium wage received by workers, P_{nt} is the price of the final good, and A_{nt} is assets owned by the household at time t. Households also own all firms in the country, therefore the value of assets in each period should equal the totality of firm values at every time. Utility maximization implies the following Euler equation:

$$\frac{\dot{C}_{nt}}{C_{nt}} = r_{nt} - \rho - \frac{\dot{P}_{nt}}{P_{nt}} \tag{11}$$

The rest of the model is discussed in two main parts: production structure and innovation structure of the economy.

3.2 Production

Figure 1 illustrates the production structure of the economy. Each industry j has two stages of production indexed by s, upstream and downstream. Output from the upstream stage is used exclusively in the production of the downstream stage. Output from the downstream stage is then used in the production of the final good, which is used for consumption and as an input into both upstream and downstream stages. The details of the production technology at every point is described in detail below.

3.2.1 Final Good Production

A perfectly competitive sector produces a non-traded final good that is used for consumption and as a material input for production in each country. The final good is produced using a Cobb-Douglas aggregate of output from all industries as:

$$Y_{nt} = \prod_{j \in \mathcal{J}} \left(Q^j_{nt}\right)^{\alpha^j} \tag{12}$$

where $\sum_j \alpha^j = 1$, and Q^j_{nt} is the amount of industry j composite output. Final good price index is then given by:

$$P_{nt} = \xi \prod_{j \in \mathcal{J}} \left(P^j_{nt}\right)^{\alpha^j} \tag{13}$$

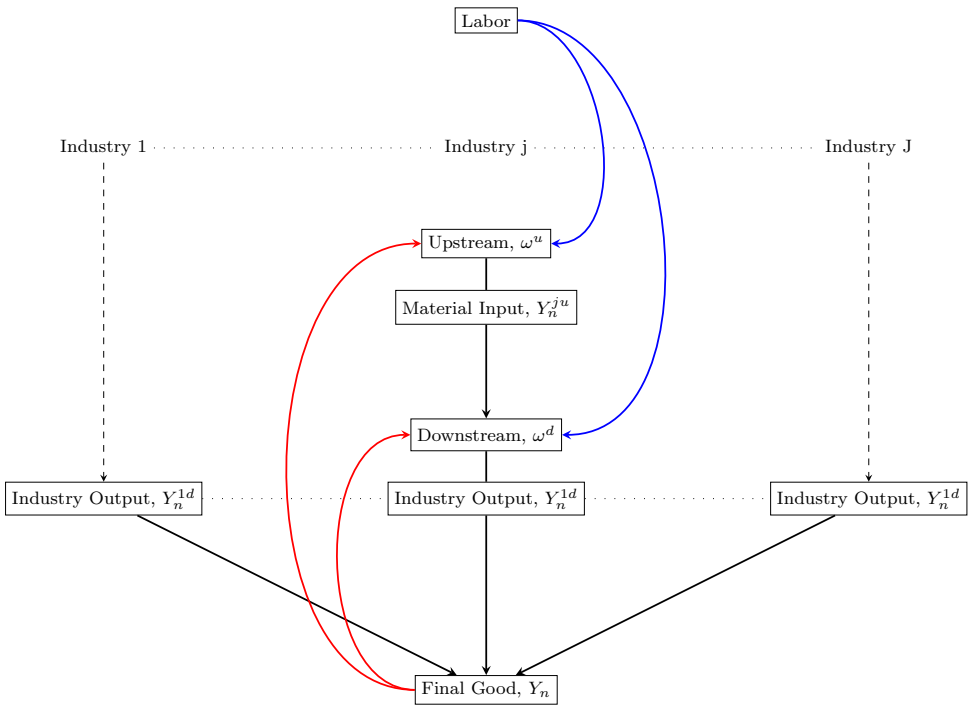
where P^j_{nt} is the price of industry j's composite output and $\xi = \prod_{j=1}^{J} \left(\alpha^j\right)^{-\alpha^j}$ is a constant.

3.2.2 Intermediate Good Production

Within each industry, a mass of differentiated intermediate varieties are produced in a sequential production process involving two stages: upstream and downstream. Differentiated varieties from the first stage of production, i.e upstream, are used in the production of a composite material input exclusively used in that industry's second stage of production, i.e. downstream. Differentiated downstream varieties are then used to produce the industry composite output which is exclusively used in the production of the final good.

Composite goods, both upstream material and downstream output, are non-traded and are

Figure 1: Production Structure of the Economy



Notes: This figure provides an illustration of the production structure of the economy. Labor and final good material are used in the production of all industry-stages of production. Upstream varieties are aggregated into a material composite input exclusively used in the production of downstream varieties. Downstream varieties are used in the production of the industry output.

sold by perfectly competitive producers. A stage $s \in \{u, d\}$ in industry j and country n has the following composite good production technology:

$$Y_{nt}^{js} = \left[\sum_{l \in \mathcal{N}} \int_{\omega^s \in \Omega_{lnt}^{js}} \left(q_{lt}^{js}(\omega^s)\right)^{\frac{1}{\sigma^{js}}} \left(y_{lnt}^{js}(\omega^s)\right)^{\frac{\sigma^{js}-1}{\sigma^{js}}} d\omega^s \right]^{\frac{\sigma^{js}}{\sigma^{js}-1}} \tag{14}$$

where Ω_{lnt}^{js} is the set of industry j stage s intermediate varieties sourced from country l, $q_{lt}^{js}(\omega^s)$ and $y_{lnt}^{js}(\omega^s)$ are the quality and quantity of the intermediate variety ω^s respectively, and $\sigma^{js} > 1$ is the elasticity of substitution across varieties. The corresponding price index for industry j stage s composite good is given by:

$$P_{nt}^{js} = \left[\sum_{l \in \mathcal{N}} \int_{\omega^s \in \Omega_{lnt}^{js}} q_{lt}^{js}(\omega^s) \left(p_{lnt}^{js}(\omega^s)\right)^{1-\sigma^{js}} d\omega^s \right]^{\frac{1}{1-\sigma^{js}}} \tag{15}$$

where $p_{lnt}^{js}(\omega^s)$ is the price of industry j stage s variety ω^s produced in country l paid by the composite good producer in country n. Demand faced by each variety is:

$$y_{lnt}^{js}(\omega^s) = q_{lt}^{js}(\omega^s) Y_{nt}^{js} \left(\frac{p_{lnt}^{js}(\omega^s)}{P_{nt}^{js}} \right)^{-\sigma^{js}} \tag{16}$$

Firms Firms in this model are potential multi-product producers, who operate in one industry. Within each industry, firms can produce multiple varieties from both stages of production. A firm f active in country n and industry j is defined by the portfolio of products it produces in equilibrium $\{\Omega^{ju}_{nt}(f), \Omega^{jd}_{nt}(f)\}$ at time t, where $\Omega^{js}_{nt}(f) = \{\omega^s : f \text{ produces } \omega^s\}$. Panel (a) of figure 2 illustrates some examples of firm portfolios: firms 1 and 2 are single-product firms, but in different stages of production, firm 4 is multi-product single-stage producer, and firm 3 is a multi-product multi-stage producer. Note that a firm's portfolio of products is time dependent. Firms can add products to their portfolio through innovation, and products get dropped from their portfolio as market conditions change their profitability over time. Total varieties produced in stage s of industry j is then the union of all stage s varieties produced by firms in that industry, $\Omega^{js}_{nt} = \bigcup_{f \in \mathcal{F}^j_{nt}} \Omega^{js}_{nt}(f)$, where $\mathcal{F}^j_{nt}$ is the set of active firms in country n and industry j.

Since varieties vary only along the quality dimension, I will represent each variety by its corresponding quality from here on to reduce notation. It is useful to define the following two objects that are used to represent the set of varieties in the economy. Let M^{js}_{nt} be the mass of stage s varieties active in country n and industry j, and $G^{js}_{nt}(\cdot)$ be quality distribution of these varieties.[21] I next discuss the sequential production of intermediate varieties produced within each industry. For ease exposition, I drop the industry indices. Production follows the outlined process in all industries, and across time.

Upstream Intermediate Varieties In each country and industry, monopolistically competitive firms in the upstream intermediate sector supply differentiated varieties to both domestic and foreign markets. Upstream varieties face demand from the producers of the composite material input used by the corresponding downstream stage producers. All upstream producers face the same production technology, but are differentiated in terms of quality of the products produced. Production is constant returns to scale and requires labor and material input from the final good sector. Producers also have to pay an overhead fixed cost of operation, f^u_{nn}, in terms of domestic labor. The production function for a variety ω is given by:

$$y^u_{nt}(q^u) = [l^u_{nt}(q^u)]^{\beta^{lu}} \left[Q^{fu}_{nt}(q^u)\right]^{\beta^{fu}} \tag{17}$$

where $l^u_{nt}(q^u)$ and $Q^{fu}_{nt}(q^u)$ is labor and material input (sourced from final good sector) used in the production of upstream variety q^u in country n, and $\beta^{lu} + \beta^{fu} = 1$. Cost minimization by firms implies the following constant marginal cost function (c^u_n):

$$c^u_{nt}(q^u) = c^u_n = \xi^u \left(w_{nt}\right)^{\beta^{lu}} \left(P_{nt}\right)^{\beta^{fu}} \tag{18}$$

where $\xi^u = \left(\beta^{lu}\right)^{-\beta^{lu}} \left(\beta^{fu}\right)^{-\beta^{fu}}$ is a constant, w_{nt} and P_{nt} are wage and final good price in country n at time t. Input demand function are then given by:

$$l^u_{nt}(q^u) = \beta^{lu} \frac{c^u_{nt}}{w_{nt}} y^u_{nt}(q^u), \quad Q^{fu}_{nt}(q^u) \ = \beta^{fu} \frac{c^u_{nt}}{P_{nt}} y^u_{nt}(q^u) \tag{19}$$

[21] Note that, mass of varieties is not equal to the mass of firms in this multi-product firms setting.

Downstream Intermediate Varieties Similar to the upstream stage of production, monopolistically competitive firms in the downstream intermediate sector supply differentiated varieties to both domestic and foreign markets. Downstream varieties face demand from the producers of the industry specific composite output, which is used in the production of the final good. All downstream producers face the same production technology, but are differentiated in terms of their qualities. Production is constant returns scale using labor and material inputs from the upstream stage and the final good sector. Production also entails an overhead cost of operation, f^d_{nn}, paid in terms of domestic labor. Each variety q^d is produced using the following technology:

$$y^d_{nt}(q^d) = \left[l^d_{nt}(q^d)\right]^{\beta^{ld}} \left[Q^{fd}_{nt}(q^d)\right]^{\beta^{fd}} \left[Q^{ud}_{nt}(q^d)\right]^{\beta^{ud}} \tag{20}$$

where $l^d_{nt}(q^d)$ is labor used, $Q^{fd}_{nt}(q^d)$ and $Q^{ud}_{nt}(q^d)$ is the material input from final good sector and the upstream stage respectively used in the production of downstream variety ω^d in country n, and $\beta^{ld} + \beta^{fd} + \beta^{ud} = 1$. The key feature in the production of downstream varieties is the exclusive use of upstream varieties, and this ends the sequential production process. Cost minimization implies the following constant marginal cost function:

$$c^d_{nt}(q^d) = c^d_n = \xi^d \left(w_{nt}\right)^{\beta^{ld}} \left(P_{nt}\right)^{\beta^{fd}} \left(P^u_{nt}\right)^{\beta^{ud}} \tag{21}$$

where $\xi^d = \left(\beta^{ld}\right)^{-\beta^{ld}} \left(\beta^{fd}\right)^{-\beta^{fd}} \left(\beta^{ud}\right)^{-\beta^{ud}}$ is a constant, and P^u_{nt} is the price index of the upstream material input composite. Input demand functions implied by the cost minimization problem are:

$$l^d_{nt}(q^d) = \beta^{ld}\frac{c^d_{nt}}{w_{nt}} y^d_{nt}(q^d), \quad Q^{fd}_{nt}(q^d) \;= \beta^{fd}\frac{c^d_{nt}}{P_{nt}} y^d_{nt}(q^d), \quad Q^{ud}_{nt}(q^d) = \beta^{ud}\frac{c^d_{nt}}{P^u_{nt}} y^d_{nt}(q^d) \tag{22}$$

3.2.3 International Trade

Upstream and downstream varieties can sell their products in foreign markets, but face standard iceberg trade costs. To deliver one unit of output to country l at time t, a stage s producer from country n has to produce and ship $\tau^s_{nlt} \geq 1$ units, with $\tau^s_{nnt} = 1\ \forall n, s$. Given this, total demand faced by each stage's varieties is:

$$y^s_{nt}(q^s) = \sum_{l \in \mathcal{N}} \mathbb{I}^s_{nlt}(q^s)\tau^s_{nlt} y^s_{nlt}(q^s) \tag{23}$$

where $\mathbb{I}^s_{nlt}(q^s)$ is an indicator function for whether q^s produced in country n is sold in country l at time t. In addition to ice berg trade costs stage s producers also have to pay a fixed exporting cost, f^s_{nl} in terms of domestic labor to export to country $l \neq n$. Given that there is no fixed cost of supplying to the domestic market, except the overhead operating cost f^s_{nn}, all active varieties supply the domestic market. Further, because firms have to pay a fixed cost of exporting, over the fixed production cost, exporting varieties are a weak subset of all domestic varieties.

3.2.4 Prices, Profits, and Trade Flows

Firms producing intermediate good varieties engage in monopolistic competition in both stages of production. As in Melitz (2003), I assume each variety is infinitesimally small relative to the size of each market that it supplies to. Firms choose to produce a variety only if their total returns from domestic and foreign markets are large enough to cover their fixed production costs and only export to a foreign market if the returns from that market are large enough to cover the fixed exporting cost. Returns to supplying to a specific market are the sum total of the variable profits made by the variety in that market and the option value generated by innovation activity associated with that market. The innovation process is discussed in subsection 3.3. Hence, production decisions are forward looking.

Prices Due to iceberg trade costs, the marginal cost of supplying one unit of a stage s variety differ by the market that the variety is sold to. Profit maximization leads to firms charging a constant markup over the marginal cost of supplying to a market:

$$p^s_{nlt}(q^s) = p^s_{nlt} = \frac{\sigma^s}{\sigma^s - 1}\tau^s_{nlt}c^s_{nt} \tag{24}$$

Note that prices charged in a given market are constant across varieties supplied by a country at time t, as the products are differentiated in terms of demand and not cost in this model.

Profits Given the above pricing strategy and the demand faced by heterogeneous varieties, variable profits generated from each market supplied to are strictly increasing in the quality of the variety. Market specific profits are given by:

$$\begin{aligned} \pi^s_{nlt}(q^s) &= \overline{\pi}^s_{nlt}q^s - f^s_{nl}w_{nt} \\ \text{where} \quad \overline{\pi}^s_{nlt} &= Y^s_{lt}\left(P^s_{lt}\right)^{\sigma^s}\left(p^s_{nlt}\right)^{1-\sigma^s}/\sigma^s \end{aligned} \tag{25}$$

Total profits generated by a stage s variety q^s is the sum of profits from all the markets that it is supplied to:

$$\pi^s_{nt}(q^s) = \sum_{l\in\mathcal{N}} \mathbb{I}^s_{nlt}(q^s)\pi^s_{nlt}(q^s) \tag{26}$$

Trade Flows The share of country l's expenditure on country n's stage s goods depends on the varieties that are exported, and is defined as follows:

$$\begin{aligned} \lambda^s_{nlt} &= \frac{X^s_{nlt}}{\sum_{n'\in\mathcal{N}} X^s_{n'lt}} \\ \text{where} \quad X^s_{nlt} &= \sigma^s\overline{\pi}^s_{nlt}M^s_{nt}\int \mathbb{I}^s_{nlt}(q^s)q^s dG^s_{nt}(q^s) \end{aligned} \tag{27}$$

3.3 Innovation and Knowledge Spillovers

Multi-product firms are engaged in innovation activities at every point in time in the form of new product creation. Following Klette and Kortum (2004), I model innovation as firms investing in research and development to discover new product varieties, with the innovation production function proportional to the number of products owned by the firm. Different from Klette and Kortum (2004) however is the type of innovation activities that a firm can invest in.

Research and Development For every product of a given stage q^s in a firm's portfolio, the firm can invest in two types of R&D: own-stage innovation (θ) and cross-stage innovation (η). Own-stage innovation leads to new product generation in the same stage as that of the product, s, and cross-stage innovation leads to new product generation in stage $s' \neq s$. Further, the firm can invest in cross-stage R&D for every market that it interacts with cross-stage producers from through product q^s. This means that an upstream product that is sold in i number of markets can invest in i different R&D activities to innovate on downstream products, and learn from its buyers in these markets. On the other hand downstream products in a country will invest in all markets from which upstream inputs are imported from. Since all downstream products use the same set of inputs, all downstream products will invest in the same number of upstream R&D activity.

Knowledge Spillovers For each R&D activity, firms choose the Poisson rate at which news varieties are generated. Each product benefits from having access to knowledge from other varieties it interacts with in own-stage markets and cross-stage markets. I model knowledge spillovers in a reduced form way in the R&D cost functions. Knowledge spillovers reduce the cost of innovation, i.e. products that have access to higher quality products on average learn more efficiently from them.

Table 11 lists all the different types of R&D activity per product that a firm can invest in. It also summarizes the type of knowledge spillovers the product benefits from for each of these innovation types. I now discuss these R&D cost functions in detail.

3.3.1 Own-Stage R&D

Each product, q^s, of stage s owned by a firm gives it the ability to discover more products in the same stage of production. This is akin to the external innovation processes described in Klette and Kortum (2004) and Akcigit and Kerr (2018). For own-stage innovation, firms learn from all the products being sold in their domestic market in the spirit of Buera and Oberfield (2020).[22] Knowledge spillovers across countries have two key aspects. First, innovation costs are low if the average quality of products sold in market n, $\tilde{q}_n^s$, is high. This captures that access to better quality ideas requires less resources to produce new ideas. Second, the ability of ideas *sold* in the market to reduce innovation costs depends of the relative quality of those ideas compared

[22] In Buera and Oberfield (2020) new innovations are drawn from the set of ideas sold in the domestic market, and are successfully produced only if they are better than existing domestically produced ideas.

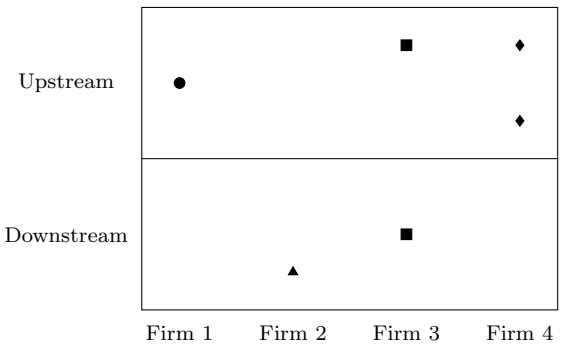
Table 11: Types of R&D per product

	Upstream product, q^u	Downstream product, q^d
Own-Stage R&D	1 R&D investment: θ^u_{nt} New u varieties invented Learns from other u varieties sold in the domestic market	1 R&D investment: θ^d_{nt} New d varieties invented Learns from other d varieties sold in the domestic market
Cross-Stage R&D	1 R&D investment per export market: $\left\{\mathbb{I}^u_{nlt}(q^u)\eta^d_{nlt}\right\}_l$ New d varieties invented Learns from d varieties in market l that buys q^u	1 R&D investment per input sourcing market: $\{\eta^u_{nlt}\}_l$ New u varieties invented Learns from u varieties from market l sold to q^d

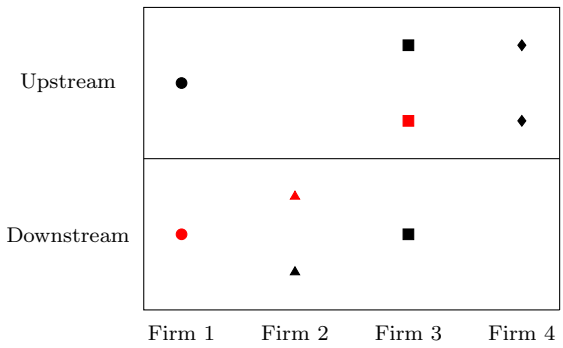
Notes: This table lists the types of R&D activity a firm producing a product can invest in, the outcomes of such investments, and the knowledge spillovers that the product has access to for each type of innovation.

Figure 2: Firms and Innovations

(a) Types of Firm Portfolios



(b) Types of Firm Innovations



Notes: Panel (a) illustrates different types of firm portfolios in the economy: firm 1 is a single-product upstream firm, firm 2 is a single-product downstream firm, firm 3 is a multi-product multi-stage firm, and firm 4 is a multi-product, but single-stage firm. Panel (b) shows how these firm's portfolio's change over time due to different types of innovations. New products are shown in red. Firm 1 becomes a multi-product, multi-stage firm through cross-stage innovation. Firm 2 becomes a multi-product firm, but stay single-stage through own-stage innovation. Firm 3 gains another product either through own-stage or cross-stage innovation, while firm 4 doesn't receive any successful innovations in either stage.

to domestically *produced* goods. When domestic ideas are already of higher quality, the relative importance of insights drawn from outside goods is less important.[23] The respective average qualities are defined as follows:

$$\text{Average quality produced}: \bar{q}_n^s = \int q^s dG_n^s(q^s) \tag{28}$$

$$\text{Average quality sold}: \tilde{q}_n^s = \frac{\sum_l M_{ln}^s \int q^s dG_{ln}^s(q^s)}{\sum_l M_l^s} \tag{29}$$

where M_{ln}^s is the mass of products from country l sold in country n, and $G_{ln}^s(q^s)$ is the distribution of country l product qualities sold in country n introduced in equation 27.

The firm chooses the own-stage innovation rate θ^s, by investing

$$R_{n,\theta}^s(\theta^s) = \chi_{n,\theta}^s (\theta^s)^\psi \left(\frac{\bar{q}_n^s}{\tilde{q}_n^s}\right)^\nu \tag{30}$$

units of domestic labor, where $\chi_{n,\theta}^s > 0$ is an innovation constant, and $\psi > 1$ is the curvature of the innovation function, the ratio $\tilde{q}_n^s/\bar{q}_n^s$ represents own-stage knowledge spillovers, and the spillover parameter $\nu > 0$ governs the degree to which knowledge spillovers affect the cost of innovation. Higher the quality of products that a firm can learn from, $\tilde{q}_n^s$, lower is the cost of innovation. At the same time, innovation costs are high if the average quality of products already produced in the domestic market, $\bar{q}_n^s$, is high.

A successful own-stage innovation results in a new product whose quality is drawn from the incumbent quality distribution of stage s varieties produced by country n firms, $G_{nt}^s(\cdot)$.

3.3.2 Cross-Stage R&D

Every product produced by the firm, q^s, also gives the firm an opportunity to learn from interactions with cross-stage producers. For example, an upstream producer can learn from interactions with his downstream buyers, and a downstream producer can learn from interactions with his upstream suppliers. In order to draw insights from buyer-seller relationships however, a firm has to invest in cross-stage R&D for each market that it wants to learn from. In order to innovate at rate $\eta^{s'}$ in the cross-stage s' by learning from ideas accessed in market l, a firm with product q^s in country n invests

$$R_{nl,\eta}^{s'}(\eta^{s'}) = \chi_{n,\eta}^{s'} \left(\eta^{s'}\right)^\psi \left(\frac{\bar{q}_n^{s'}}{\hat{q}_{nl}^{s'}}\right)^\nu \tag{31}$$

units of domestic labor, where $\chi_{n,\eta}^{s'} > 0$ is a country-stage specific innovation constant, $\hat{q}_{nl}^{s'}$ is the average quality of cross-stage s' products from market l that stage s firms may have access to, and $\bar{q}_{nl}^{s'}$ is the average quality of domestic stage s' products. Consistent with my empirical results, higher the value of cross-stage quality from market l relative to domestic quality, lower is the cost of cross-stage R&D for market l. Figure 3 illustrates the cross-stage knowledge spillovers

[23] Akcigit and Kerr (2018) also model R&D costs to increase with average productivity in the economy. This is done in part to capture that more advanced technologies are harder to innovate upon, and to make innovation effort independent of qualities in equilibrium.

that an upstream and downstream product may benefit from.

Cross-Stage Spillovers for Upstream Varieties Upstream varieties learn from downstream buyers of their product from each market. Therefore, only exporters to market l can learn from downstream varieties in market l by investing in market l specific R&D. The knowledge spillover term for market l is then given by:

$$\hat{q}^d_{nl} = \int q^d dG^d_l(q^d) \tag{32}$$

which is the average downstream incumbent quality in market l. Any upstream product that is sold to l will be sold to all downstream producers in l.

Cross-Stage Spillovers for Downstream Varieties Downstream varieties learn from suppliers of upstream input varieties from each source market. Since I do not model firm level import decisions, and all firms sourcing strategies are the same, every firm will access the same knowledge from each market l that exports upstream products to n. The knowledge spillover term from market l is then given by:

$$\hat{q}^u_{nl} = \int q^u dG^u_{ln}(q^u) \tag{33}$$

which is the average quality of upstream products sold in country n from country l. All downstream producers in country n buy these inputs.
Every cross-stage innovation results in a new s' variety being invented whose idea is drawn from the incumbent quality distribution in the domestic country, $G^{s'}_{nt}(\cdot)$.

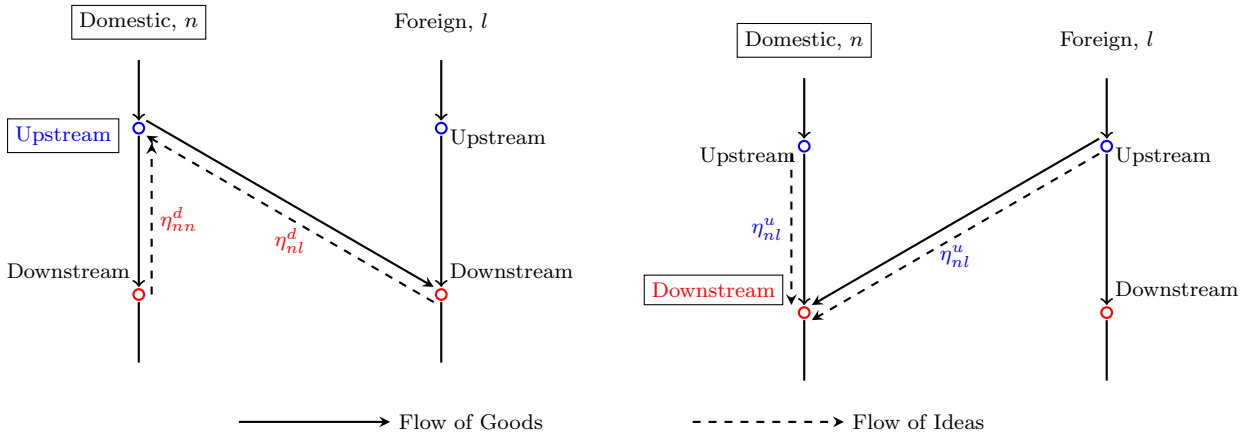
To conclude the section on incumbent innovation and to take stock, there are four different types of R&D a firm can potentially invest in: $R^u_{n,\theta}, R^d_{n,\theta}, R^u_{n,\eta}, and R^d_{n,\eta}$. A firm with products only from one stage of production, say s, will invest in own stage R&D in s and cross-stage R&D in s'. However, a firm with products in both stages of production will invest in all four types of R&D activities. Note that, there are two key implications of this innovation structure: (1) a single-product firm can potentially become a multi-product firm if it draws a successful innovation, and (2) a single-stage firm can potentially become a multi-stage firm if a successful cross-stage innovation is drawn. Panel (b) of figure 2 illustrates how firm portfolios can change in response to different innovation outcomes.

3.4 Incumbent Dynamic Problem

Having described the dynamic environment that firms function in, I now formalize the incumbent firm's problem, and it's value function. A firm is defined as a portfolio of products potentially from both stages of production. At every point in time, for every product in its portfolio, the firm makes decisions to supply to markets and makes profits, as well as invests in different types of R&D activity in order to expand its portfolio. Each product is also destroyed at an exogenous

Figure 3: Cross-Stage Knowledge Spillovers across countries

(a) Cross-Stage Innovations by Upstream Varieties (b) Cross-Stage Innovations by Downstream varieties



Notes: This figure provides a simple illustration of how international trade results in diffusion of ideas across countries through cross-stage innovations resulting from buyer-seller relationships. Upstream varieties gain insights from their domestic and foreign downstream buyers- panel (a), and downstream varieties gain upstream insights from their domestic and foreign suppliers- panel(b). Solid lines represents the flow of goods, and dashed lines represent the flow of ideas. Each market that a product from stage s interacts with leads to a potential innovation in it's cross-stage at the rate of $\eta^{s'}_{nl}$.

rate δ. I represent a firm by its portfolio of products, $\{Q^d, Q^u\}$, where Q^s is the set of stage-s products produced by the firm. The value function of a firm that owns $\{Q^d, Q^u\}$ is given by:

$$
\begin{aligned}
& r_{nt} V_{nt}\left(Q^u, Q^d\right) - \dot{V}_{nt}\left(Q^u, Q^d\right) \\
= & \max_{\substack{\{\mathbb{I}^u_{nlt}(q^u)\}_{Q^u,l} \\ \{\mathbb{I}^d_{nlt}(q^d)\}_{Q^d,l} \\ \theta^u_n \geq 0,\ \theta^d_n \geq 0 \\ \{\eta^d_{nl}\}_l \geq 0,\ \{\eta^u_{nl}\}_l \geq 0}} \sum_{s \in \{u,d\}} \sum_{q^s \in Q^s} \left\{ \begin{array}{c} \sum_l \mathbb{I}^s_{nlt}(q^s)\left[\overline{\pi}^s_{nlt} q^s - f^s_{nl} w_{nt}\right] \\ +\mathbb{I}^s_{nnt}(q^s)\delta\left[V_{nt}\left(\{Q^s \setminus q^s\}, Q^{s'}\right) - V_{nt}\left(Q^s, Q^{s'}\right)\right] \\ +\mathbb{I}^s_{nnt}(q^s)\theta^s_n\left[\int V_{nt}\left(\{Q^s, z^s\}, Q^{s'}\right) \mathrm{d}G^s_{nt}\left(z^s\right) - V_{nt}\left(Q^s, Q^{s'}\right)\right] \\ -\mathbb{I}^s_{nnt}(q^s) R^s_{n,\theta}\left(\theta^s_n\right) w_{nt} \end{array} \right\} \\
& + \sum_{q^u \in Q^u} \left\{ \begin{array}{c} \sum_l \mathbb{I}^u_{nlt}(q^u)\eta^d_{nl}\left[\int V_{nt}\left(Q^u, \{Q^d, z^d\}\right) \mathrm{d}G^d_{nt}\left(z^d\right) - V_{nt}\left(Q^u, Q^d\right)\right] \\ -\sum_l \mathbb{I}^u_{nlt}(q^u) R^d_{nl,\eta}\left(\eta^d_{nl}\right) w_{nt} \end{array} \right\} \\
& + \sum_{q^d \in Q^d} \left\{ \begin{array}{c} \mathbb{I}^d_{nnt}(q^s)\sum_l \eta^u_{nl}\left[\int V_{nt}\left(\{Q^u, z^u\}, Q^d\right) \mathrm{d}G^u_{nt}\left(z^u\right) - V_{nt}\left(Q^u, Q^d\right)\right] \\ -\mathbb{I}^d_{nnt}(q^d)\sum_l R^u_{nl,\eta}\left(\eta^u_{nl}\right) w_{nt} \end{array} \right\}
\end{aligned}
\tag{34}
$$

The firm's dynamic problem involves an optimal choice of production decisions, $\mathbb{I}^s_{nnt}(q^s)$, export decisions, $\mathbb{I}^s_{nlt}(q^s)\ \forall l \neq n$, and innovation effort in different types of R&D.[24] The first line on the right hand side of equation 34 represents the profits made from each market that the firm chooses to supply to net of the fixed costs of operating/exporting. The second line captures the change in value function due to an exogenous destruction of a product owned and produced by the firm, which occurs at Poisson rate δ. The third line shows the change in firm value when

[24]Note that $\mathbb{I}^s_{nnt}(q^s) = 1$ would imply that the product q^s is active, and is also supplied to the domestic market. Since there is no fixed cost of supplying to the domestic market, every product that pays the fixed cost of production will also supply to the domestic market.

the firm gets an own-stage innovation, and adds a new variety z^s to it's portfolio at Poisson rate θ_n^s. A product q^s has to be active ($\mathbb{I}_{nnt}^s(q^s) = 1$) in order to invest in own-stage R&D. When a new product is added through own-stage innovation, the quality is drawn from the incumbent quality distribution, $G_{nt}^s(\cdot)$. The fourth line is the cost of own-stage R&D.

The remaining part of the incumbent value function describes the cross-stage innovation outcomes. Cross-stage innovation is different for upstream and downstream products. The fifth line represents the change in value function when an upstream product innovates a new downstream variety z^d from any of the markets it exports to. The sixth line is the associated R&D costs. The seventh line represents the change in value function when a downstream product innovates a new upstream variety z^u from any of the markets it imports inputs from. Note that in a trade equilibrium, all downstream varieties import from all countries, and hence the only requirement for them to invest in cross-stage innovation is producing in at least the domestic market. The last line represents the associated R&D costs. The quality of any new product added through cross stage innovation is also drawn from the domestic incumbent distribution , $G_{nt}^s(\cdot)$.

For every product that a firm owns, the decision to produce a product, decisions to export a product to different markets, and decisions to innovate are dependent on each other. The firm therefore decides on whether to produce a product based not only on the present profits it can make, but also takes into consideration the expected future returns to innovation (own-stage, and all cross-stage for downstream products). Similarly, the decision to export for upstream products depends on the profits from exporting the product plus the expected returns to cross-stage innovation from that export market. We can see from the value function that expected return to per-product per-market R&D investment is independent of the product and market in question, and is equal to a constant for all innovations. The only variable component to the total returns of producing/exporting a product are the variable profits which are linear in product quality. This means that a firm chooses to produce/export a product if it is above a certain quality threshold that makes it generate positive profits. We can then write:

$$\mathbb{I}_{nlt}^s(q^s) = \begin{cases} 1 & \text{if } q^s \geq q_{nlt}^{s*} \\ 0 & \text{if } q^s \leq q_{nlt}^{s*} \end{cases} \tag{35}$$

Note that this implies that the incumbent distribution G_{nt}^s has zero density below q_{nnt}^{s*} at time t.

3.5 Entry and Exit

Apart from incumbent innovation, a mass E_{nt}^s of stage s entrants also invest in R&D by employing $f_{ne,t}^s$ units of domestic labor. Entrants benefit from the same type of knowledge spillovers that incumbents investing in own-stage R&D benefit from. The cost of entrant R&D is cheaper if the average quality of products sold in the market is high (ideas they can learn from), but higher if the average quality of incumbent products in the domestic market are higher (ideas they are

innovating upon). Per entrant R&D cost is given by:

$$f^s_{ne,t} = \chi^s_{n,e} \left(\frac{\bar{q}^s_n}{\tilde{q}^s_n} \right)^{\nu} \tag{36}$$

Every entrant R&D investment results in the invention of a new product, whose quality is drawn from the incumbent distribution, $G^s_{nt}(\cdot)$. Free entry requires that the cost of entry equal the value of entering the market with a quality randomly drawn from the incumbent distribution.

$$f^s_{ne} w_{nt} = \chi^s_{n,e} \left(\frac{\bar{q}^s_n}{\tilde{q}^s_n} \right)^{\nu} w_{nt} = \int V_{nt}(\{q^s\}, \emptyset) dG^s_{nt}(q^s) \tag{37}$$

Exit in the economy occurs through two channels: (1) exogenous death at a Poisson rate δ as discussed above, (2) endogenous exit over time due to changing market conditions. Note that product quality is time-invariant, but, average product quality in the economy increases due to new product introduction by entrants and incumbents alike. This causes changes to the market profitability of a product over time. Once the product is no longer profitable to produce is dropped from the firm's portfolio.

3.6 Evolution of Product Mass and Quality Distribution

Innovations by incumbents and entrants alter the mass of varieties produced in the economy, and the quality distribution of the operated varieties. Before characterizing the law of motion for product mass and quality distributions, it is useful to define total cross-stage innovation intensity performed by an average product in the economy. Let $\eta^{s'}_n$ be the total cross-stage innovation intensity performed by the average product in stage s, then:

$$\begin{aligned} \text{Upstream Cross-Stage Innovation:} \quad & \eta^d_n = \sum_l \left(\int \mathbb{I}^u_{nlt}(q^u) \mathrm{d}G^u_{nt}(q^u) \right) \eta^d_{nl} \\ \text{Downstream Cross-Stage Innovation:} \quad & \eta^u_n = \sum_l \eta^u_{nl} \end{aligned} \tag{38}$$

Cross-stage innovation for upstream products depends on the export status of the product. Therefore, in line one of equation 38 the total cross-stage innovation performed by an average upstream product depends on the fraction of varieties that export to each market. Total cross-stage innovation for an average downstream product is simply the sum of all market specific innovation intensities.

Following Sampson (2016), I analyze the evolution of product masses and quality distributions by first discretizing time into periods of length Δ, and then taking the limit as $\Delta \to 0$. Also, as in Sampson (2016), I assume that the exit threshold q^{s*}_{nnt} is strictly increasing over time. The mass of active stage s products at time $t + \Delta$ that have quality less than q^s is:

$$\begin{aligned} M^s_{n(t+\Delta)} G^s_{n(t+\Delta)}(q^s) = & \left(M^s_{nt} + \Delta\theta^s_n M^s_{nt} + \Delta\eta^s_n M^{s'}_{nt} + \Delta E^s_{nt} \right) \left[G^s_{nt}(q^s) - G^s_{nt}(q^{s*}_{nn(t+\Delta)}) \right] \\ & - \Delta\delta M^s_{nt} \left[G^s_{nt}(q^s) - G^s_{nt}(q^{s*}_{nn(t+\Delta)}) \right] \end{aligned} \tag{39}$$

The first term on the right hand side of the equation represents the mass of products that were active in the market at time t with quality less than q^s that would also be active in the market at time $(t+\Delta)$. The set of products at time t include incumbents of mass M^s_{nt}, new innovations through own-stage R&D of mass $\Delta\theta^s_n M^s_{nt}$, new innovations through cross-stage R&D of mass $\Delta\eta^s_n M^{s'}_{nt}$, and finally new innovations through entrants of mass ΔE^s_{nt}. The second line represents the loss in the mass of products due to exogenous exit. Evaluating equation 39 as $q^s \to \infty$ will give us the total mass of products active at time $(t+\Delta)$,

$$M^s_{n(t+\Delta)} = \left(M^s_{nt} + \Delta\theta^s_n M^s_{nt} + \Delta\eta^s_n M^{s'}_{nt} + \Delta E^s_{nt} - \Delta\delta M^s_{nt}\right)\left[1 - G^s_{nt}(q^{s*}_{nn(t+\Delta)})\right] \tag{40}$$

Further taking the limit as $\Delta \to 0$ on equation 40, and rearranging the equation, gives us the law of motion for total mass of varieties in continuous time,

$$\frac{\dot{M}^s_{nt}}{M^s_{nt}} = \underbrace{\theta^s_n + \eta^s_n \frac{M^{s'}_{nt}}{M^s_{nt}} + \frac{E^s_{nt}}{M^s_{nt}}}_{\text{Rate of New Innovations}} - \underbrace{\delta^{s*}_{nt}}_{\text{Rate of Endogeneous Exit}} - \underbrace{\delta}_{\text{Rate of Exogeneous Exit}} \tag{41}$$

where $\dot{M}^s_{nt} = \frac{dM^s_{nt}}{dt}$, and $\delta^{s*}_{nt} = \frac{\partial G^s_{nt}(q^{s*}_{nnt})}{\partial q}\frac{\partial q^{s*}_{nnt}}{\partial t}$. The rate of change of mass of varieties depends positively on the rate of new innovations (own-stage, cross-stage, and entrant), and negatively on the rate of endogenous and exogenous exit. Endogenous exit occurs in the model due to the increased competition causing an increase in the exit threshold over time. This causes varieties at the edge of the quality distribution to exit over time.

Finally, substitution equation 40 into equation 39 and taking the limit as $\Delta \to 0$, gives us the law of motion for the quality distribution,

$$\frac{\partial G^s_{nt}(q^s)}{\partial t} = -\delta^{s*}_{nt}\left[1 - G^s_{nt}(q^s)\right] \tag{42}$$

Thus, the density of products less than quality q reduces over time with the growth of the exit cutoffs. See appendix C.1 for detailed derivations of the two laws of motion. I now solve for a stationary distribution of product qualities given an assumption on the initial quality distributions.

3.6.1 Stationary Distributions

In a stationary distribution, the distribution of relative quality $q^s/\bar{q}^s_{nt}$ (quality relative to average quality in the market) is constant over time. The following initial condition is sufficient to ensure that the relative quality distribution is stationary at every point in time.[25]

Assumption 1

The initial distribution of product quality is Pareto:

$$G^s_{n0}(q^s) = 1 - \left(\frac{q^s}{q^{s*}_{nn0}}\right)^{-\gamma^s} \tag{43}$$

[25] This is true for when the economy is in a balanced growth path, as well as the transition path

with $\gamma^s > 1$, and $q^{s*}_{nn0} > 0$ is the initial exit threshold in country n and stage s.

At every point in time, competitive pressures in the economy leads to the lowest quality products to exit the market. This results in a continual truncation of the quality distribution from below. As in Sampson (2016), the quality distribution has a stable shape, and resembles a traveling wave as the exit threshold grows. The quality distribution remains Pareto over time with the same shape parameter, γ^s, but a growing exit threshold $q^{s*}_{nnt} = \exp\left(\int_0^t g^s_{nt} dt\right) q^{s*}_{nn0}$, where g^s_{nt} is the growth rate of the exit threshold at time t.

Proposition 1

When assumption 1 holds, the equilibrium quality distribution is given by:

$$G^s_{nt}(q^s) = 1 - \left(\frac{q^s}{q^{s*}_{nnt}}\right)^{-\gamma^s} \tag{44}$$

Proof See Appendix C.2. ■

Given this result on the quality distributions, I can derive the rate of endogenous exit at any point in time as:

$$\delta^{s*}_{nt} = \gamma^s g^s_{nt} \tag{45}$$

The rest of the model is analyzed under the Pareto distribution assumption.

3.7 Evolution of Firm Portfolio Distribution

New innovations and exit in the economy alters not only the mass and distribution of varieties, but also the distribution of firm portfolios. Let $N_{nt}(n^u, n^d)$ be the mass of firms with n^u number of upstream varieties and n^d number of downstream varieties. Then, the change in the this portfolio mass of firms is given by:

$$\begin{aligned}
\dot{N}_{nt}(n^u, n^d) &= N_{nt}(n^u - 1, n^d)\left[(n^u - 1)\theta^u_{nt} + n^d \eta^u_{nt}\right] + N_{nt}(n^u + 1, n^d)\left[(n^u + 1)\delta^u_{nt}\right] \\
&+ N_{nt}(n^u, n^d - 1)\left[n^u \eta^d_{nt} + (n^d - 1)\theta^d_{nt}\right] + N_{nt}(n^u, n^d - 1)\left[(n^d + 1)\delta^d_{nt}\right] \\
&- N_{nt}(n^u, n^d)\left[n^u(\theta^u_{nt} + \eta^d_{nt} + \delta^u_{nt}) + n^d(\theta^d_{nt} + \eta^u_{nt} + \delta^d_{nt})\right] \\
\text{with} \quad \dot{N}_{nt}(1,0) &= E^u_{nt} + N_{nt}(2,0) \times 2\delta^u_{nt} + N_{nt}(1,1)\delta^d_{nt} - N_{nt}(1,0)\left[\theta^u_{nt} + \eta^d_{nt} + \delta^u_{nt}\right] \\
\dot{N}_{nt}(0,1) &= E^d_{nt} + N_{nt}(0,2) \times 2\delta^d_{nt} + N_{nt}(1,1)\delta^u_{nt} - N_{nt}(0,1)\left[\theta^d_{nt} + \eta^u_{nt} + \delta^d_{nt}\right]
\end{aligned} \tag{46}$$

where $\delta^s_{nt} = \delta + \delta^{s*}_{nt}$ is the aggregate exit rate. The change in mass of firms of a specific portfolio depends on the mass of firms that enter the portfolio from the cells surrounding it, and on the mass of firms that exit the portfolio (either by adding a product or dropping a product).

3.8 Equilibrium

In addition to producer and consumer optimization, equilibrium requires that labor market and goods markets also clear. Labor faces demand for production, fixed costs of production and

exporting, and finally R&D costs by incumbents and entrants. Upstream good varieties face demand from downstream producers from all countries in a trade equilibrium. Downstream varieties face demand from final good producers from all countries. Final good producers face consumption demand and input demand from upstream and downstream producers. See appendix C.3 for market clearing equations.

An equilibrium of the world economy is defined by the following time paths for $t \in [0, \infty)$, and for all countries $n \in \mathcal{N}$, industries $j \in \mathcal{J}$ and stages $s \in \{u, d\}$ (where applicable): prices $\{r_{nt}, w_{nt}, P^{js}_{nt}, P_{nt}\}$, final consumption and output $\{C_{nt}, Y_{nt}\}$, sectoral expenditures $\{X^{js}_{nt}\}$, expenditure shares $\{\lambda^{js}_{nlt}\}$, mass of products $\{M^{js}_{nt}\}$, quality distributions $\{G^{js}_{nt}\}$, market entry thresholds $\{q^{js*}_{nlt}\}$, incumbent innovation rates $\{\theta^{js}_{nt}, \eta^{js}_{nlt}\}$, flow of entrant innovations $\{E^{js}_{nt}\}$, value functions $\{V^{j}_{nt}()\}$ and growth rates of quality distributions $\{g^{js}_{nt}\}$, such that (i) consumers maximize utility subject to budget constraint 10 implying the Euler equation 11 is satisfied, (ii) producers maximize static profits implying the output rule 16 and pricing rule 24 are satisfied, (iii) incumbents solve their dynamic problem in 34 , thereby determining the market entry thresholds, and incumbent innovation rates, (iv) entrants solve entry decisions and free entry holds in 37, (v) market thresholds are strictly increasing, and the evolution of mass of products and quality distributions are governed by 41 and 42 respectively, (vi) labor markets clear, (vii) goods markets clear for upstream intermediate varieties, downstream intermediate varieties, and final good.

3.9 Balanced Growth Path

I will now analyze the balanced growth path (BGP) equilibrium of the world economy. In a BGP, consumption, output, wages, prices, sectoral expenditures, value functions, and market entry thresholds all grow at constant rates. Interest rates, expenditure shares, mass of products, flow of entrant innovations and incumbent innovations are all constant over time. Finally, the relative quality distribution is stationary. I will drop time subscripts when analysing the BGP henceforth, but as mentioned variables in BGP are either constant or grow at a constant rate.

3.9.1 Incumbent Decisions and Value Function

Proposition 2 solves the firm's dynamic problem in a BGP under an assumption, and lists the equilibrium firm decisions on producing, exporting, and innovating. See appendix for derivations.

Assumption 2

The fixed costs of exporting, f^{s}_{nl}, are large enough such that export thresholds to market l are strictly greater than the entry thresholds, i.e.

$$q^{s*}_{nl} > q^{s*}_{nn} \quad \forall l \neq n \tag{47}$$

Assumption 2 ensures that not all products that are active in the domestic market export to a foreign market. The set of exporting varieties is a strict subset of all active varieties in the economy.

Proposition 2

When assumptions 1-2 hold, the following firm level outcomes are true.

(i) Value function of an incumbent can be written as:

$$V_n\left(Q^u, Q^d\right) = \sum_{s\in\{u,d\}} \sum_{q^s\in Q^s} \sum_{l} \mathbb{I}^s_{nl}(q^s) v^s_{nl}(q^s) \tag{48}$$

where $v^s_{nl}(q^s)$ is the country-pair specific value of operating a stage s product of quality s.

(ii) Optimal innovation intensity of incumbents:

$$\begin{aligned} \textit{Own-Stage:} \quad \theta^s_n &= \left(\frac{f^s_{ne}}{\psi\chi^s_{n,\theta}}\left(\frac{\tilde{q}^s_n}{\bar{q}^s_n}\right)^{\nu}\right)^{\frac{1}{\psi-1}} = \left(\frac{\chi^s_{n,e}}{\psi\chi^s_{n,\theta}}\right)^{\frac{1}{\psi-1}} \\ \textit{Cross-Stage:} \quad \eta^s_{nl} &= \left(\frac{f^s_{ne}}{\psi\chi^s_{n,\eta}}\left(\frac{\hat{q}^s_{nl}}{\bar{q}^s_n}\right)^{\nu}\right)^{\frac{1}{\psi-1}} = \left(\frac{\chi^s_{n,e}}{\psi\chi^s_{n,\eta}}\left(\frac{\hat{q}^s_{nl}}{\tilde{q}^s_n}\right)^{\nu}\right)^{\frac{1}{\psi-1}} \end{aligned} \tag{49}$$

(iii) Market entry decisions take the form described in equation 35

(iv) Market entry threshold qualities are given by:

$$q^{s*}_{nl} = \frac{\tilde{f}^s_{nl} w_n}{\bar{\pi}_{nl^s}} \tag{50}$$

(v) $\tilde{f}^s_{nl}$ is the fixed cost of operating in market l net of expected return from market specific innovation:

$$\begin{aligned} \textit{Upstream:} \quad \tilde{f}^u_{nl} &= \begin{cases} f^u_{nn} - (\psi-1)R^u_{n,\theta}\left(\theta^u_n\right) - (\psi-1)R^d_{n,\eta}\left(\eta^d_{nn}\right) & \textit{if} \quad l = n \\ f^u_{nl} - (\psi-1)R^d_{n,\eta}\left(\eta^d_{nl}\right) & \textit{if} \quad l \neq n \end{cases} \\ \textit{Downstream:} \quad \tilde{f}^d_{nl} &= \begin{cases} f^d_{nn} - (\psi-1)R^d_{n,\theta}\left(\theta^d_n\right) - (\psi-1)\sum_l R^u_{n,\eta}\left(\eta^u_{nl}\right) & \textit{if} \quad l = n \\ f^d_{nl} \quad \textit{if} \quad l \neq n \end{cases} \end{aligned} \tag{51}$$

(vi) The components of the value function are given by:

$$v^s_{nl}(q^s) = \left(\frac{\bar{\pi}^s_{nl} q^s}{g^s_n(\kappa^s_n+1)} - \frac{\tilde{f}^s_{nl} w_n}{g^s_n \kappa^s_n}\right) - \left(\frac{\bar{\pi}^s_{nl} q^{s*}_{nl}}{g^s_n(\kappa^s_n+1)} - \frac{\tilde{f}^s_{nl} w_n}{g^s_n \kappa^s_n}\right)\left(\frac{q^s}{q^{s*}_{nl}}\right)^{-\kappa^s_n} \tag{52}$$

where $\kappa^s_n = (r_n + \delta - g_{w_n})/g^s_n$, g^s_n is the BGP growth rate of the domestic entry threshold q^{s*}_{nn}, and g_{w_s} is the BGP growth rate of wages in country n.

Proof See Appendix C.4. ■

This result extends the models of heterogeneous firms and growth as in Sampson (2016) and Perla et al. (2019) to a setting with asymmetrical countries and incumbent innovation. On a balanced growth path, firm values depend on the static profits made from selling to different

markets, as well as the dynamic returns from different types of R&D activities (which are incorporated into net fixed costs of market entry $\tilde{f}^s_{nl}$). Therefore, firms internalize that fact that entry into a market comes with the option value of developing new products in the future, which results in market entry thresholds to be different from a model with no incumbent innovation. More discussion on this is available in subsection 3.10.

When incumbents choose the optional innovation strategy, they weigh the costs and benefits for a given type of R&D activity. The benefit from a successful innovation is a new product added into the firm portfolio. Since this product is drawn from the existing distribution of qualities, the value of this added product is equal to the value of an entrant in the market. Since R&D costs for entrants and incumbent own-stage innovation benefit from the same knowledge spillovers,[26] free entry condition implies that own-stage innovation intensity is independent of the amount of knowledge spillovers experienced by incumbents. One can see this in the first part of equation 49. However, cross-stage innovation intensity increases with the amount of knowledge a firm can learn from accessing each market relative to the amount of knowledge an entrant learns from. This is evident in the second part of equation 49.

3.9.2 Growth

I now derive the BGP growth rates of quality thresholds and final output/consumption. Recall that for the while deriving the evolution of quality distributions in section 3.6 I assumed that domestic quality thresholds are strictly growing over time, implying $g^s_n > 0$. To ensure this condition is positive, I impose the following parameter restriction:

Assumption 3

For every country n and stage s, the parameters of the model satisfy:

$$\sum_l \left(\frac{q^{s*}_{nl}}{q^{s*}_{nn}}\right)^{-\gamma^s} \left(\frac{\tilde{f}^s_{nl}}{f^s_{ne}}\right) > (\delta+\rho)(\gamma^s - 1) \tag{53}$$

Proposition 3

When assumptions 1-3 hold, the growth rate of the domestic quality thresholds in country n and stage s is given by:

$$g^s_n = \frac{1}{\gamma^s(\gamma^s-1)} \sum_l \left(\frac{q^{s*}_{nl}}{q^{s*}_{nn}}\right)^{-\gamma^s} \left(\frac{\tilde{f}^s_{nl}}{f^s_{ne}}\right) - \frac{\delta+\rho}{\gamma^s} \tag{54}$$

Proof See Appendix C.6. ■

Proposition 3 highlights the relationship between international trade and growth. Growth is higher in a world with trade, as exposure to trade leads to higher competition in the economy causing low quality products to exit. The higher the competitive pressures, faster is the exit

[26] These are knowledge spillovers resulting from ideas being sold in the market.

of products on the lower end of the distribution. This feature of the model is identical to the mechanism in Sampson (2016), except that it has been applied to asymmetric countries.

A second key feature of the growth equation is the relationship between entrant R&D cost, f^s_{ne} and growth. Growth is higher when entry is cheaper. Also recall that $f^s_{ne} = \chi^s_{n,e}\left(\frac{\bar{q}^s_n}{\tilde{q}^s_n}\right)^{\nu}$. This implies that cross-country knowledge spillovers enjoyed by entrants also increases growth. Furthermore, an important implication of cross-country knowledge spillovers is that the growth rates in all countries in a given stage are equalized. Note that, if this were not true in a BGP, the knowledge spillover term, $\frac{\bar{q}^s_n}{\tilde{q}^s_n}$, would change over time causing the growth equation to not be constant over time. This is a contradiction to BGP. Therefore, in a world with stage specific knowledge spillovers stage specific growth rates are equalized across all countries, i.e. $g^s_n \equiv g^s$. The input-output structure, and round-about nature of production causes the growth rate of composite upstream and downstream output to be an amplification of the stage specific quality growth rates. In an economy with just one industry, i.e. $\mathcal{J} = 1$, the growth rates of the composite outputs can be written as:

$$\mathbf{g_{Y^s}} = \left(\mathbf{S} + (\mathbf{I} - \mathbf{B})^{-1}\mathbf{BS}\right)\mathbf{g^s} \tag{55}$$

where $\mathbf{g_{Y^s}} = (g_{Y^u}\ g_{Y^d})^T$ is a column vector of growth rates of stage specific composite good Y^s_n, $\mathbf{g^s} = (g^u\ g^d)^T$ is the column vector of growth rates of stage specific quality thresholds, q^{s*}_{nn}, and $\mathbf{B}$ is a matrix of production input-output linkages, and $\mathbf{S}$ is a diagonal matrix of elasticities of substitution:

$$\mathbf{B} = \begin{pmatrix} 0 & \beta^{fu} \\ \beta^{ud} & \beta^{fd} \end{pmatrix} \qquad \mathbf{S} = \begin{pmatrix} 1/(\sigma^u - 1) & 0 \\ 0 & 1/(\sigma^d - 1) \end{pmatrix}$$

Growth rate of final output and consumption under single industry assumption is simply equal to the growth rate of downstream output, $g_Y = g_{Y^d}$. Note that, final output and stage specific output growth rates are equalized across countries, which directly follows from growth rate of quality thresholds being equalized. Therefore, in a setting with asymmetric countries, knowledge spillovers will lead to growth rate of consumption being equal across all countries. The derivation for final output growth rates is in appendix C.7.

3.9.3 Welfare

Welfare along the BGP can be calculated to be a combination of a static component and a dynamic component of the gains from trade. Consider that the economy reaches a BGP at time t^*, then welfare (equivalent to utility) is given by:

$$U_{nt^*} = \int_{t^*}^{\infty} e^{\rho t}\log\left(e^{gt}C_{nt^*}\right)dt = \underbrace{\frac{log\left(C_{nt^*}\right)}{\rho}}_{\text{Static}} + \underbrace{\frac{g}{\rho^2}}_{\text{Dynamic}} \tag{56}$$

The static component of the welfare gains is similar to the gains from a static steady-state model of trade with no growth. The dynamic component of welfare gains comes from growth. As in Sampson (2016), trade affects both components of welfare.

3.10 Discussion

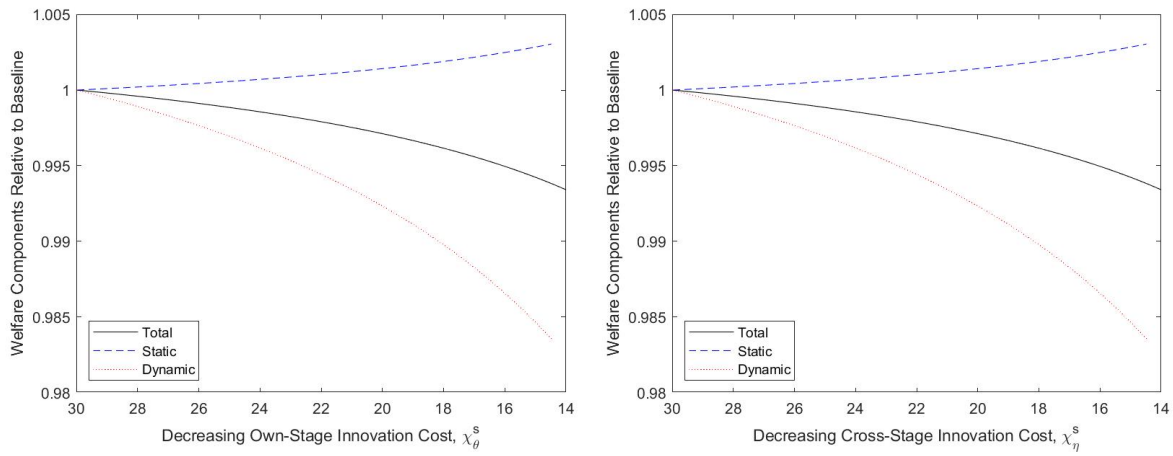
Before moving to the quantitative analysis of the model, there are some features of the model that are worth discussing. The above described model is an extension of Sampson (2016) and Perla et al. (2019) on two main fronts: (1) asymmetric countries and (2) incumbent innovation. Both Sampson (2016) and Perla et al. (2019) study symmetric country models for analytical tractability to analyze a channel of growth from dynamic selection, and domestic technology diffusion. As Perla et al. (2019) points out, balanced growth paths with asymmetric economies will exist only in knife-edge conditions without international technology diffusion. In this paper, I introduce knowledge spillovers between countries that enables me to study asymmetric countries. Specifically, entrant firms learning from all products sold in their domestic economy ensures that a fraction of knowledge developed in the foreign economy is internalized into the domestic economy. A consequence of this however is that in a world with knowledge spillovers that benefit entrants, all countries will grow at the same rate in the balanced growth path. This also results in the economy having two margins of growth: first, the dynamic selection channel highlighted in the above two papers, and a second new channel through international spillovers. As is evident from the growth rate equation in proposition 3, all else equal, lower costs of entry leads to higher growth. Recall that entry costs are a combination of R&D cost constant, and knowledge spillovers. The higher the quality of products sold in a country relative to domestically produced products, the lower is the cost of entry. This implies that growth is higher in a trade equilibrium.

Introducing incumbent innovation into a dynamic selection model results in two opposing effects on welfare. Incumbent firm values are higher in an economy where incumbents can add new products compared to an economy where incumbents are purely single product firms. Increases in firm values results in higher entry in equilibrium as entrants innovate more in order to capture the increased value of being an incumbent. This can be seen in equation 52 once we substitute for $\tilde{f}^s_{nl}$. Firm value increases with the expected value of receiving a future innovation. This results in a higher mass of firms and higher output and consumption levels in a steady state equilibrium. In terms of my model, this would result in an increase in the static component of welfare. However, at the same time, an increase in the option value of being an incumbent also makes lower quality firms stay in the market as they expect to become profitable in the future by adding new products into their portfolio. This results in a worse distribution of firms with lower average quality. One can see this in equations 50-51, where the domestic market entry cutoffs, q^{s*}_{nn} are lower, resulting in a lower average quality of products produced in the market. In a model where new innovation qualities are drawn from the incumbent distribution, this makes entrants draw worse products on average resulting in lower growth. While the static component of welfare increases from incumbent innovation, the dynamic component of welfare coming from growth is decreased due to worse products being active in the market. Which of these effects dominates in a BGP would depend on model parameters, and economy being calibrated.

Figure 4 plots the relationship between incumbent innovation costs and the two components of welfare for illustrative purposes, for a toy economy in Autarky. The graphs are intended to help one understand the mechanisms in play when incumbent innovation is introduced into a

Figure 4: Incumbent Innovation and Welfare

(a) (b)



Notes: This figure plots the relationship between incumbent innovation and the different components of BGP welfare for a toy economy in Autarky. The baseline economy's parameters are given by: $\chi_e^s = 1, \chi_\theta^s = 30, \chi_\eta^s = 30$. Panel (a) plots a decrease in own-stage innovation costs for both stages, χ_θ^s, and panel (b) does the same for cross-stage innovation costs.

growth model based on dynamic selection. Panel (a) plots welfare when own-stage innovation costs decrease for upstream and downstream, and panel (b) plots the same when cross-stage innovation costs decrease.[27] As discussed above, both types of incumbent innovation leads to opposing effects on the two components of welfare. A decrease in incumbent innovation increases static welfare as incumbent values increase. However, dynamic welfare decreases due to selection of worse firms over time.

This relationship between incumbent innovation and welfare is important to understand as it helps us understand the contribution of knowledge spillovers experienced by incumbents to growth and welfare. Knowledge spillovers due to trade experienced by a laggard economy reduces the cost of incumbent R&D, while the leading economy faces higher R&D costs.[28] Since both economies have to grow at the same growth rate, the impact of knowledge spillovers experienced by incumbents on dynamic welfare/growth is a combination of the effects experienced by both economies: laggard country faces as downward push to it's growth, while the leading economy faces the opposite. Individual consumption experiences due to knowledge spillovers, all else equal, should go in opposite direction. Decrease in R&D cost in the laggard economy increases static consumption, while the increase in R&D cost in the leading economy will decrease consumption. However, the actual contribution of knowledge spillovers on both static and dynamic components of welfare depends on other general equilibrium adjustments in the economies.

[27]This economy has no knowledge spillovers as it is in Autarky. The baseline economy is given by the following innovation cost parameters: $\chi_e^s = 1, \chi_\theta^s = 30, \chi_\eta^s = 30$. The simulations in figure 4 reduce the incumbent costs of both stages of production for each type of innovation keeping all other parameters constant.

[28]In a two country model, a leading economy is the one with higher avergae knowledge/quality, while the laggard economy is one with the lower average quality of produced varieties.

4 Calibration

This section outlines the calibration of my model. Since the goal is to study firm-level dynamics, both its impact on the economy and its response to economic changes, I calibrate the model to replicate firm-level innovation outcomes in the data. For computational tractability, I restrict the analysis to two "countries" and single industry with two stages of production.

4.1 Overview

I calibrate the model to a world with two countries, ($\mathcal{N} = 2$): India (domestic) and a foreign country, which is an aggregate of the ten major trading partners of India (in manufacturing): USA, China, Germany, France, Great Britain, South Korea, Italy, Japan, Australia, and Taiwan.[29] I use the World Input Output Database (WIOD) to gather trade flows at an aggregate level and choose the top trading partners. Henceforth, countries will be represented by "IND" for India and "ROW" for the foreign aggregate. Population size of "IND" is set to be half the size of "ROW".[30]

I also restrict analysis to have only one industry ($\mathcal{J} = 1$), and classify all goods produced in the world into upstream and downstream. I classify goods into the two stages of production based on the share of output that goes into final demand. I apply this rule to all the manufacturing sectors used in the WIOD, and use the median value of final demand share to divide sectors into upstream or downstream. This results in a total of 18 broadly defined manufacturing sectors in the WIOD divided equally.[31]

Finally, I bring this sector classification into the Indian dataset and classify products into upstream and downstream using a crosswalk.[32] Using this, I then categorize plants as either single-product, multi-product single-stage, and multi-product multi-stage plants.[33]

The model parameters to be calibrated include the preference parameters $\{\rho, \sigma^s\}$, production function parameters $\boldsymbol{\beta}$,[34], trade parameters $\{\gamma^s\}$ and $\{\{f^s_{nl}\}, \{\tau^s_{nl}\}\}$ for each origin-destination and stage pair, firm and R&D parameters $\{\delta, \psi\}$, the spillover elasticity parameter $\{\nu\}$, and $\{\{\chi^s_{n,\theta}\}, \{\chi^s_{n,\eta}\}\}$ for each country-stage pair, and entrant R&D parameters $\{\chi^s_{n,e}\}$ for each country-stage pair. I assume all primitives of the model except trade barriers and innovation

[29] I measure total trade as the sum of Indian imports from each country and Indian exports to each country, excluding Indian domestic expenditures. USA ranks first in total trade share at 12.8%, and Taiwan ranks tenth at 1.7%. See table D31 in appendix D for details on the set of countries in the foreign aggregate.

[30] I use data from the Penn World Tables for the year 2007 to calculate aggregate population for my "ROW" region, and get the relative population sizes.

[31] See table D32 in appendix D for industry details.

[32] The WIOD uses the ISIC Rev. 4 classification. I construct a mapping from ISIC Rev. 4 to the ASICC product classification used by the Indian manufacturing dataset. See appendix for details on these crosswalks.

[33] Note that this would result in a different categorization of plants compared to that used in the empirical analysis. The fine detail of product level relationships used in the empirical section, and the broad classification used here are different. In appendix D, table D33 lists the share of plants in each category of plant type resulting from this exercise, and table D34 shows the transition matrix of plants changing their types from one year to the next. A key objective of the calibration strategy is to mimic this plant type distribution and transitions.

[34] $\boldsymbol{\beta}$ is the matrix of input elasticities in the production technology for upstream and downstream varieties. $\boldsymbol{\beta} = \begin{pmatrix} \beta^{lu} & \beta^{uu} & \beta^{fu} \\ \beta^{ld} & \beta^{ud} & \beta^{fd} \end{pmatrix}$, where $\beta^{uu} = 0$.

costs to be common across countries.

Some of these parameters are calibrated externally and some are calibrated internally to the model. The shape parameters of the quality distributions, $\{\gamma^s\}$, are estimated using a gravity equation that is consistent with my model. Further, fixed costs of operating/exporting, $\{f^s_{nl}\}$, are then implied from the resulting gravity estimation procedure. More details are outlined in section 4.3.

4.2 External Calibration

I set the time discount parameter $\rho = 1\%$, which implies a 7% interest rate in the balanced growth path for India. The value for the constant elasticity of substitution for both upstream and downstream varieties is taken from Broda and Weinstein (2006) and set to $\sigma^s = 3$.[35] The production function parameters are implied from WIOD. Labor elasticities are set to be the share of value added in total expenditures, upstream input elasticity (for only downstream production) is set to be the share of upstream input expenditure, and final good elasticity is set such that all input elasticities sum to one. The exogeneous exit rate of varieties is set at $\delta = 0.1$ to reflect the exit rate of big plants in India.[36] The innovation cost curvature is set to $\psi = 2$ and is a value commonly used in the literature. Finally, the spillover elasticity parameter that governs the extent to which quality differences across countries contribute to R&D costs, ν, is set to 1 to reflect the evidence of foreign knowledge spillovers uncovered in the empirical analysis.[37]

Tariff data is taken from the Trade Analysis Information System (TRAINS) database for the year 2007. I use a weighted average of tariff rates between "IND" and "ROW" based on import and export share weights. The quality distribution shape parameter is implied from the trade elasticity parameter. I outline the gravity estimation and the resulting calibration in the next section. [38] Table 12 lists all the parameter of the model that are externally calibrated.

4.3 Gravity Estimation

I calibrate trade parameters, $\{\gamma^s, f^s_{nl}\}$, by leveraging the gravity equations implied by the model under assumptions 1-2. Under these assumptions, the quality distribution of the products imported by country l from country n is also Pareto with shape parameter γ^s, and minimum value being equal to export thresholds defined in equations 50-51. Using the definition of expenditure

[35] I take the average value of σ estimated in Broda and Weinstein (2006) across the three categories of goods: Commodity, Reference priced, and Differentiated, for the years 1990-2001.

[36] I use census plants that appear in the first year of my Indian manufacturing plant sample, and classify a plant as having "exited" the market if they do not appear again in any of the following years of the sample. This means a plant that is active in 2001, and does not produce anything for 8 years after. This is done in order to avoid over-estimating the death rate as plants that become small over time are sampled randomly.

[37] The results in the empirical section are suggestive of the fact that foreign technology diffuses in the domestic economy through trade. However, the regression specification and functional forms of the variables used are not model implied. Ideally, ν would be estimated from a model implied equation, which is work for future.

[38] See Akcigit and Kerr (2018)

Table 12: Externally Calibrated Parameters

Parameter		Value	Notes
Preferences & Production			
Discount Rate	ρ	0.01	BGP interest rate = 7%
CES Parameter	σ	3.00	Broda and Weinstein (2006)
IO linkages	$\boldsymbol{\beta}$	Table 13	WIOD, Expenditure shares
Trade			
Pareto Parameter	γ	5.00	Gravity Estimation, see Section 4.3
Variable Trade Costs	τ^s_{nl}	Table 14a	TRAINS database
Fixed Trade Costs	f^s_{nl}	Table 14b	Gravity Estimation, see Section 4.3
Firms & R&D			
Exogeneous Exit	δ	0.10	Exit rate of large firms
Innovation Cost Curvature	ψ	2.00	Literature
Knowledge Spillovers Elasticity	ν	1.00	Empirical Analysis

Table 13: Production Parameters

	β^l	β^u	β^f
Upstream	0.40	0.00	0.60
DownStream	0.40	0.25	0.35

shares from equation 27, one can derive the exact form in equilibrium to be:[39]

$$\begin{aligned}\lambda^s_{nl} &= \frac{M_n \left(q^{s*}_{nn}\right)^{\gamma^s} \left(\tau^s_{nl} c^s_n\right)^{-\gamma^s(\sigma-1)} \left(f^s_{nl} w_n\right)^{-(\gamma^s-1)}}{\sum_{n'} M_{n'} \left(q^{s*}_{n'n'}\right)^{\gamma^s} \left(\tau^s_{n'l} c^s_{n'}\right)^{-\gamma^s(\sigma-1)} \left(f^s_{n'l} w_{n'}\right)^{-(\gamma^s-1)}} \\ &= \Lambda^S_n \Lambda^D_l \left(\tau^s_{nl}\right)^{-\gamma^s(\sigma-1)} \left(f^s_{nl}\right)^{-(\gamma^s-1)}\end{aligned} \tag{57}$$

where Λ^S_n is the source-country n term, and Λ^D_l is the destination country l term. To address issues of incidence of zero trade flows between countries, and the inconsistent estimates resulting from a log-linearized regressiong of equation 57, I estimate the model using the pseudo Poisson maximum likelihood (PPML) estimator.[40] I use the following representation of the gravity equation to do so:

$$\lambda^s_{nl} = \frac{exp(S^s_n + \mathbf{V}^s_{nl}\boldsymbol{\phi}^s_V + \mathbf{F}^s_{nl}\boldsymbol{\phi}^s_F)}{\sum_{n'} exp(S^s_{n'} + \mathbf{V}^s_{n'l}\boldsymbol{\phi}^s_V + \boldsymbol{F}^s_{n'l}\boldsymbol{\phi}^s_F)} \tag{58}$$

where $S^s_n$is the source-country fixed effect, $\mathbf{V}^s_{nl}$ is the vector of variable trade barriers, and $\mathbf{F}^s_{nl}$ is the vector of fixed trade barriers. I classify the set of trade cost determinants in the data into those that would approximately affect variable costs and those that affect fixed costs of trade. Specifically, I consider variable trade costs to be affected by bilateral tariffs and distance between regions. Whereas, determinants of fixed costs are assumed to be the similarity between

[39]See appendix for derivation.

[40]See Silva and Tenreyro (2006) for more details on the benefits of estimating gravity using PPML over OLS.

Table 14: Trade Costs

(a) Variable Costs, τ_{nl^s}

		IND	ROW
Upstream	IND	1.00	1.02
	ROW	1.11	1.00
Downstream	IND	1.00	1.05
	ROW	1.12	1.00

(b) Fixed Costs, f^s_{nl}

		IND	ROW
Upstream	IND	1.00	1.44
	ROW	1.40	1.00
Downstream	IND	1.00	1.35
	ROW	1.33	1.00

the trading partners, like trading partners sharing a common border, a common language, have common legal origins, have common colonizers, and have common legal ties. This is a crude classification of trade costs, and as Head and Mayer (2014) point out many of the determinant of variable costs like distance can be reasonably expected to also affect the fixed exporting costs. However, this allows for a way to calibrate the fixed costs of exporting as a first pass, albeit a crude one. Note also that, tariffs are always considered to be only a variable trade cost, and hence my estimate on tariff will be correctly attributed to the elasticity of variable trade cost. All trade barriers data is from the CEPII Gravity Database.

I first get an estimate for γ^s by estimating the gravity equation using the PPML method for each manufacturing sector in the WIOD separately, and use all country-pairs to do so. Domestic trade barriers are normalized to one. I then take a weighted average of the estimates across the group of sectors that make up the two stage of production, using output weights of the sectors. Note that, the elasticity on tariff is equal to $-\gamma^s(\sigma^s - 1)$. Using the value of σ^s from Broda and Weinstein (2006), I then back out values for γ^s. Tables D35 and D36 in appendix D show the gravity estimation results for the set of sectors that make up upstream and downstream respectively. The implied Pareto shape parameters for both sectors are set to be equal to 5.[41]

Finally, using the calibrated values of γ^s and σ^s, the estimated coefficients for other trade barriers in Tables D35 and D36, I back out fixed exporting costs as:

$$\log(f^s_{nl}) = \frac{1}{1-\gamma^s}\mathbf{F}^s_{nl}\boldsymbol{\phi}^s_F \tag{59}$$

The implicit assumption here is that the set of variables used for fixed costs solely affect the fixed costs of exporting, and that these variables are translated into monetary units of fixed costs that is captured in the PPML estimates. Fixed costs implied through this procedure are listed in table 14b.

4.4 Internal Calibration and Identification

The remaining parameters to be calibrated are the innovation cost parameters for incumbents and entrants. I make the following assumptions about the cost parameters in order to reduce the number of parameters to be calibrated. The assumptions relate the cost of R&D by incumbents

[41] Upstream: $\gamma^u(\sigma^u - 1) = 10$ and Downstream: $\gamma^u(\sigma^u - 1) = 9.9$. With $\sigma^u = \sigma^d = 3$, $\gamma^u = 10.25, \gamma^d = 4.95$. I approximate both shape parameter to 5 to keep asymmetry in the computational analysis to a minimum.

to be linearly related to the cost of R&D by entrants upto a constant. This means that the variation across in R&D costs across countries is constant whether we compare incumbents or entrants.

$$\begin{aligned} \text{Entrant R\&D} : \chi^s_{ne} &= \bar{\chi}_e \chi_n \\ \text{Incumbent Own-Stage R\&D} : \chi^s_{n,\theta} &= \bar{\chi}^s_\theta \chi_n \\ \text{Incumbent Cross-Stage R\&D} : \chi^s_{n,\eta} &= \bar{\chi}^s_\eta \chi_n \end{aligned} \tag{60}$$

I use different moments to identify the R&D parameters. In order to pin down entrant R&D costs, I use the growth rate India's GDP percapita (taken from World Development Indicators (WDI)), and the average TFP difference India and the foreign aggregate (taken from the Penn World Tables (PWT)). The growth rate pins down the common entry cost parameter across both countries, $\bar{\chi}_e$. Higher the growth, lower is the cost of entry. The relative entry cost between the two countries on the other hand will determine relative productivity/quality differences. Therefore, TFP differences across "IND" and "ROW" pins down the ratio χ_{IND}/χ_{ROW}. The region with lower cost of entry will have higher equilibrium productivity.[42]

Since the goal of the quantitative exercises is to understand firm dynamics in response to policy shocks, I calibrate the rest of the parameters in the model to match incumbent product innovation activity in the Indian dataset. I use information on only single-product plant-year observations, and their product adding behavior in the following period. The moment used to calibrate the incumbent own-stage innovation constant, χ^s_θ is the share of single-product plants that transition into multi-product plants in the same stage of production. Similarly, to calibrate incumbent cross-stage innovation constant, χ^s_η is the share of single-product plants that add a product in the complementary stage of production. Note that I use moment by stages to calibrate incumbent innovation costs by stage. Again, higher the costs of innovation, lower is the rate of transitions of plants into different types over time. Table 15 lists the internally calibrated parameter of the model. Table 16 lists the targeted moments in the calibration exercise, with the model counterparts.

5 Counterfactuals

In this section, I study the quantitative implications of the theoretical framework. The goal of the quantitative model discussed so far is two fold: (1) to analyse the contribution of incumbent knowledge spillovers (in own-stage and cross-stage R&D) on aggregate outcomes of the economy, and (2) to analyse the implications of knowledge spillovers experienced by firms on firm-level dynamics in a general equilibrium setup. In a world with international exchange of ideas, the equilibrium level of technology differences, growth, and firm-level outcomes are interlinked.

[42] Note that, firm heterogeneity in my model is introduced as quality differences arising from relative demands. This is isomorphic to a model with cost heterogeneity, but upto an exponent. That is, average quality in my model will translate to average productivity to an exponent in a model with cost heterogeneity: $\bar{q} = \bar{z}^{\sigma-1}$, where q is productivity, z is productivity, and σ is the CES parameter. I translate the TFP differences across countries in the data into quality differences using this equivalence for the calibration exercise.

Table 15: Internally Calibrated Parameters

Parameter		Value	Moment
Country constant	$\frac{\chi_{IND}}{\chi_{ROW}}$	1.70	Relative TFP between IND and ROW
Entry cost Constant	$\bar{\chi}_e$	0.72	GDP per capita Growth rate IND
Own-Stage Constant	$\bar{\chi}_\theta^u$	7.87	Share of Single-Product firm's transitioning
	$\bar{\chi}_\theta^d$	6.34	into Multi-Product Single-Stage firms
Cross-Stage Constant	$\bar{\chi}_\eta^u$	90.00	Share of Single-Product firm's transitioning
	$\bar{\chi}_\eta^d$	28.00	into Multi-Product Multi-Stage firms

Table 16: Model Fit

Moment	Data	Model	Source
GDP per capita Growth rate IND	6.19%	6.06%	WDI, Average 2003-2007
Relative TFP IND and ROW	0.49	0.49: Up	PWT, 2007
		0.48: Down	
Share of Upstream single-product plants to Upstream multi-product plants	6.90%	6.92%	ASI, Average 2001-2007
Share of Downstream single-product plants to Downstream multi-product plants	8.56%	8.65%	ASI, Average 2001-2007
Share of Upstream single-product plants to multi-product multi-stage plants	2.38%	2.41%	ASI, Average 2001-2007
Share of Downstream single-product plants to multi-product multi-stage plants	2.51%	2.53%	ASI, Average 2001-2007

Hence, to understand the impact of a policy change on firm dynamics, an understanding of how the economy adjusts to these policy changes is required.

The first set of experiments I run are designed to understand the contribution of knowledge spillovers on incumbent innovation.[43] The second set of experiments pertain to different trade policy changes. In all counterfactuals, I analyze changes *across* BGPs. This means that dynamic changes in welfare do not reflect changes along the transition.

5.1 Contribution of Knowledge Spillovers

In this counterfactual exercise, I compare the calibrated baseline economy to an economy with no knowledge spillovers (KS) for incumbent innovation. To be specific, the R&D cost functions are independent of technology differences across the countries and are given by:

$$\begin{aligned} R^s_{n,\theta}(\theta^s) &= \chi^s_{n,\theta}\left(\theta^s\right)^{\psi} \\ R^s_{n,\eta}(\eta^s) &= \chi^s_{n,\eta}\left(\eta^s\right)^{\psi} \end{aligned} \tag{61}$$

Table 17 reports the results for the counterfactual economy with no KS for incumbents with respect to the Baseline economy. Both India and the rest of the world experience decreases in welfare when incumbents cannot benefit from KS. However, the mechanisms leading to this result are different for the two economies. Recall the discussion in section 3.10 regarding the relationship between incumbent innovation and welfare. Removing KS results in Indian incumbents facing higher R&D costs as they cannot learn from better foreign products.[44] This results in less incumbent innovation leading lower static welfare, but higher dynamic welfare. Over all, total welfare increases, albeit by a very small amount, when KS are switched off. Notice that with an increase in incumbent R&D costs, innovation is reallocated from incumbents to entrants in the counterfactual economy. In fact, the adjustment of the entry margin could be the reason for really small welfare changes when incumbent innovation changes.[45]

The effect of KS on ROW is slightly more complex than on India. Switching off KS for ROW results in incumbent R&D costs to decrease as firms now do not have to learn from worse off ideas from India. While this does result in increased incumbent innovation, static welfare does not increase as expected. The decrease in static welfare is a result of lower average quality of both upstream and downstream stages of production compared to IND. General equilibrium effects seem to act against ROW wherein average qualities go down relative to an economy with KS. This counterfactual exercise brings to light the complex mechanisms involved in the model.

A second set of results with respect to KS focuses on the gains from trade. Table 18 reports the gains from trade results for an economy with KS and an economy without KS, when the economy moves from the baseline level of trade to a free trade equilibrium. Not surprisingly,

[43] I cannot switch off knowledge spillovers for entrants in a model with asymmetric economies, as BGPs are not guaranteed without technology diffusion across economies.

[44] Recall that in the Baseline economy, India's average quality is half that of ROW.

[45] This result is similar what Atkeson and Burstein (2010) finds, where changes to incumbent innovation decisions, while important for firm-level dynamics, are largely offset by the adjustment of the entry margin resulting in almost no welfare changes.

Table 17: Contribution of Knowledge Spillovers

Outcomes	IND	ROW
(a) Welfare		
Static, $\log(C)$	0.9971	0.9857
Dynamic, g	1.0063	1.0063
Total	1.0028	0.9965
(b) Upstream Innovation		
Own-Stage Rate, θ^u	0.8463	1.0367
Cross-Stage Rate, η^u	0.4074	1.2181
Enrry Rate, e^u	1.3213	0.9543
Average Quality, q^{u*}	1	0.9232
(c) Downstream Innovation		
Own-Stage Rate, θ^d	0.7874	1.0304
Cross-Stage Rate, η^d	0.8285	1.0099
Enrry Rate, e^d	1.0511	1.0011
Average Quality, q^{d*}	1	0.9420

Notes: This table lists outcomes in the counterfactual economy with no knowledge spillovers on incumbent innovation, relative to the Baseline economy. Panel (a) reports the different components of welfare changes. Panel (b) reports outcomes in the upstream stage. Panel (c) reports outcomes in the downstream stage.

the contribution of KS to gains from trade are close to zero, in fact slightly negative. The total welfare gains from trade liberalization in an economy with KS is slightly lower than the welfare gains in an economy without KS. This may be a result of more convergence between IND and ROW in average qualities of the two stages of production in an economy with KS. A final observation in this counterfactual is to note that knowledge spillovers contribute to growth mostly through entrant innovation.[46] Gains from trade are mostly driven by entrants adjusting to changes in market environments.

Incumbent innovation therefore has little consequence on the overall growth of the economy, especially when compared to the contribution of entrant innovation to growth. However, it is important to study incumbent innovation within a general equilibrium model in order to understand the effect of a policy change on the equilibrium distribution of firm types, and the resulting firm-dynamics. The next set of counterfactuals study exactly this.

[46]In a model with asymmetric countries, KS on entrants is the only way to achieve a BGP equlibrium. Therefore, I cannot turn off KS on entrants to compare outcomes when entrants do not benefit from international spillovers.

Table 18: Knowledge Spillovers: Gains from Trade

Outcome	With Spillovers		Without Spillovers	
	Baseline Trade	Free Trade	Baseline Trade	Free Trade
(a) Welfare				
Static, C	1	0.9992	1	0.9997
Dynamic, g	1	1.1840	1	1.1880
Total	1	1.1133	1	1.1165
(b) Technology Gap				
Upstream, $q^{u*}_{ROW}/q^{u*}_{IND}$	1	0.52	1	0.55
Downstream, $q^{d*}_{ROW}/q^{d*}_{IND}$	1	0.55	1	0.57

Notes: This table reports results from a trade liberalization counterfactual for 2 types of economies: one with knowledge spillovers for incumbent innovation, and the other without knowledge spillovers for incumbent innovation. The trade liberalization exercise here is going from the baseline level of tariffs to zero tariffs (free trade). The outcomes for each type of economy are reported relative to the baseline tariffs case. Panel (a) reports the different components of welfare changes. Panel (b) reports the change in the equilibrium technology gaps between India and the rest of the world aggregate for the two states.

5.2 Trade Policy: Aggregate Outcomes

I now move on to analyze the impact of trade policies on aggregate outcomes for the Indian economy. Table 19 reports the main results. I conduct three policy experiments: (1) Autarky, (2) Trade Protection where tariffs are increased by 5 percentage points for all, and (3) Trade liberalization where tariffs are set at zero for all. Growth and welfare results go in expected directions for all trade counterfactuals relative to the baseline tariffs case: welfare decreases in Autarky, and when tariffs go up, and increase in a free trade equilibrium.

It is important to analyze the technology gaps between the two countries relative to the baseline case. These counterfactuals show the importance of trade as a vehicle for technology diffusion across countries. Increasing tariffs and reducing trade between countries reduces the extent to which knowledge moves between the two countries. This results in a higher gap between average qualities of ROW and India. In the baseline case, ROW was calibrated to have twice the value of India in average quality (for both upstream and downstream). In the trade protection counterfactual, this increase by more than 60%, resulting in ROW's average quality to be more than three times that of India. On the other hand, in the free trade equilibrium average qualities of the two countries are close to equal. Trade liberalization implies faster convergence between countries.

5.3 Trade Policy: Firm Dynamics

An interesting outcome of the adjustment of technology gaps across the different trade policy counterfactuals is the impact of the said gaps on firm level outcomes. Surprisingly, in the trade protection case the fraction of firms exporting from India increases in the BGP. This is a result

Table 19: Trade Policy: Aggregate Outcomes in India

Outcome	Baseline Trade	Autarky	Trade Protection	Trade Liberalization
Growth	1	0.39	0.95	1.18
Welfare	1	0.62	0.98	1.10
(a) Upstream Outcomes				
Technology Gap $q^{u*}_{ROW}/q^{u*}_{IND}$	1	-	1.63	0.52
Fraction Exporters	1	0.00	1.24	1.04
Cross-Stage Inn Rate, η^u	1	0.24	1.42	0.70
Own-Stage Inn Rate, θ^u	1	1	1	1
(b) Downstream Outcomes				
Technology Gap, $q^{d*}_{ROW}/q^{d*}_{IND}$	1	-	1.67	0.55
Fraction Exporters	1	0.00	1.06	0.98
Cross-Stage Inn Rate, η^d	1	0.81	1.20	0.95
Own-Stage Inn Rate, θ^d	1	1	1	1

Notes: This table reports results from trade policy counterfactuals. All outcomes are relative to the baseline level of tariffs. The trade policy experiments considered are: "Autarky" when no country trades, "Trade Protection" where tariffs have been increased by 5 percentage points across the board, and "Trade liberalization" where tariffs are zero across the board.

of an increase in the returns to cross-stage innovation for Indian firms from increased knowledge diffusion from ROW. Increased gaps in average qualities leads to incumbent firms being able to benefit from higher knowledge spillovers. Recall that the export cutoffs are a function of the fixed costs of exporting net of the expected returns from innovation.[47] In the free trade equilibrium however this channel is muted as firms do not benefit from knowledge diffusion with small technology gaps.[48] Table 19 shows that aggregate outcomes matter for firm-level outcomes.

Technology differences across countries also affect the equilibrium level of incumbent innovation intensity. Increases in technology gaps in the trade protection equlibrium leads to a drastic decrease in the cost of R&D.[49] This results in more incumbent innovation in a world with higher tariffs relative to the baseline economy. Conversely, because free trade leads to convergence in technology levels across countries, the equilibrium level of incumbent innovation decreases in BGP.

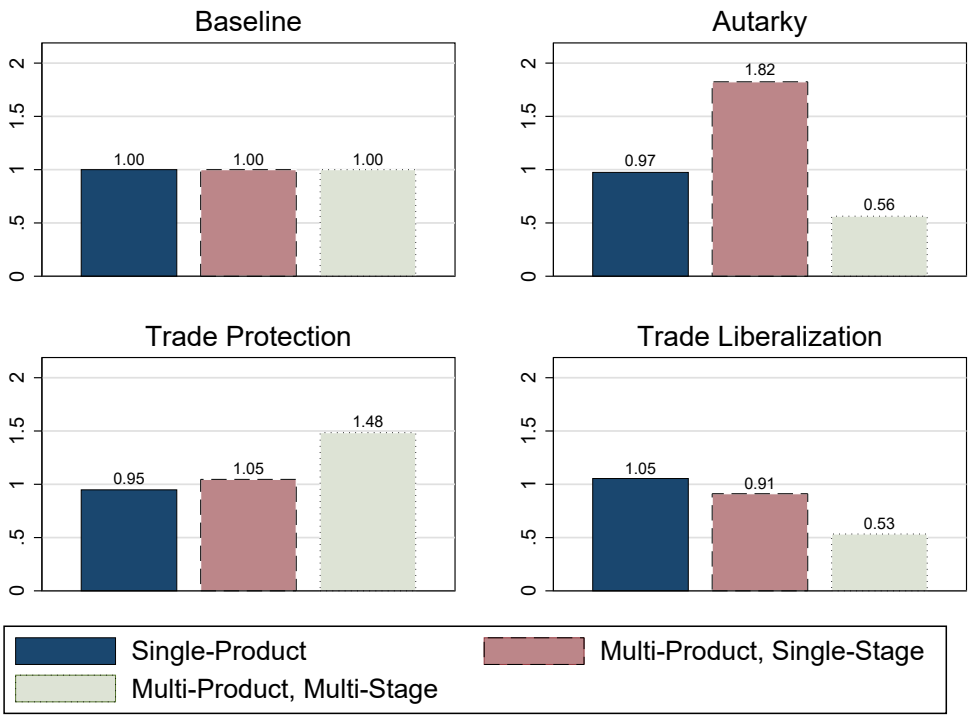
This leads me to come full circle and analyze the resulting firm type distributions from the trade policy experiments. Figure 5 plots the distribution of firms into three bins: (1) single-product, (2) multi-product but single-stage, and (3) multi-product and multi-stage. The figure

[47] Refer to equations 50-51 in Proposition 2.

[48] Recall that only upstream firms benefit from cross-stage knowledge diffusion from exporting. Downstream firms on the other hand experience knowledge spillovers from importing inputs. This is one of the reasons why upstream exporting margin seems to respond a lot more than downstream to changes in equilibrium technology gaps.

[49] Note that cost of all types of R&D decreases for India for both Upstream and downstream firms, unlike the export margin adjustment which was particularly more important for upstream firms.

Figure 5: Trade Policy: Firm Dynamics in India



Notes:This figure plots the firm type distributions under the different trade policy counterfactuals relative to the Baseline economy.

plots the share of each firm-type relative to the baseline economy. In Autarky, the share of multi-stage producers falls by half compared to the baseline economy. This is a result of decreased cross-stage innovation by incumbents due to two reasons. First, as firms cannot export in Autarky they cannot invest in market specific innovation activity on foreign markets.[50] Second, closing off trade results in zero knowledge diffusion across countries thereby affecting the ability of firms to take the opportunity to learn from foreign ideas.

The fraction of multi-stage producers in the trade liberalization counterfactual is close to the Autarky case, also reduced by 50% relative to the baseline. Note that the same fraction of firms end up being multi-stage even though firms now can invest in more cross-stage R&D activity for all the markets. The decrease is a result of decrease technology gaps between India and ROW resulting in lower incumbent innovation. Finally, in an economy that faces higher tariffs compared to the baseline, the fraction of multi-stage firms increases by 50%. This result is not surprising once we observe that the fraction of firms exporting, and the potential for knowledge spillovers is high in the trade protection economy.

These results bring to light the importance of macro-level adjustments in general equilibrium for quantifying and understanding micro-level firm dynamics in response to policy changes. While it is tempting to conclude from my empirical evidence that trade liberalization will lead to more incumbent cross-stage innovation, a general equilibrium analysis shows that it is not

[50]Recall that upstream firms in an open economy can invest in per-market cross-stage R&D for every market they export to, and downstream firms can invest in per-market R&D for every market they import from.

necessarily the case. Note that this is a comparision across balanced growth path of the different counterfactual economies, and does not speak to the effect of trade on firm dynamics along the transition paths, which I will leave for future research.

6 Conclusion

I study the impact of international trade on product introduction within Indian manufacturing firms. Diverging from most of the literature, I explore the possibility that plants learn about new products from their buyers or sellers. This line of inquiry is motivated by the finding that plants in India disproportionately produce products that are vertically linked to their previous production sets compared to other unrelated products. This makes multi-product plants also multi-stage plants, where they have integrated multiple production stages of a particular value chain. I find evidence for plants introducing new downstream products in response to an exogenous shock to downstream foreign technology access though exports.

I build a dynamic general equilibrium model with innovating firms in order to asses the importance of foreign knowledge spillovers on growth and welfare, and to assess how firm-level dynamics in response to a trade shock. Counterfactual analysis of the model calibrated to India and a rest of the World aggregate shows that while the contribution of knowledge spillovers on aggregate outcomes may be negligible, knowledge spillovers are important to explain the firm type distributions observed in the data. Trade policy counterfactuals show that incumbent innovation levels due to knowledge spillovers increase during a protectionist policy, and decrease during a liberalization policy. This is a result of technology gaps between countries, which determines the extent of spillovers, diverging when tariffs increase, and converging when tariffs decrease.

There are a number of paths worth exploring for future research. One major path forward is to understand the factors that determine the set of products that multi-product plants produce. This paper has shown that vertically related products are likely to be produced together, and that these products are being sold outside of the plant for revenue generation. More detailed analysis is required to understand what makes plants produce such vertically linked products. In the quantitative analysis, the first extension to consider is to solve for the transition path to analyze how firms adjust to changing economic conditions. Note that all my counterfactual comparisons are comparisons along the balanced growth path. Another promising avenue for future research is to extend the quantitative analysis to more countries. My current analysis is restricted to only two countries. An case to study would be a three country model to analyze the impact of a trade war between two countries on the third country. It would be most interesting to analyze the flow of ideas and equilibrium technology differences in such a scenario.

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A Data Appendix

A.1 Annual Survey of Industries

Plants vs Firms The primary data for my empirical analysis is the Indian Annual Survey of Industries (ASI). As mentioned in the main ext, the ASI mainly reports plant-level details, and not firm level. However, the ASI does allow consolidated returns to be filed for two or more establishments that operate in the same state, and within the same industry, belonging to the same firm. For the purpose of my analysis, I treat all filed returns equally, and do not distinguish between observations filed for more than one establishment. Hence, I use the terms firms and plants interchangeably. While ASI does allow consolidated returns to be filed, there is very little uptake. The ASI reports the total number of units within the firm in the country, in the state (for some years), and the number of units the return is filed for. Table A20 shows some summary statistics on the prevalence of such establishments in my final sample.

The incidence of observations that file consolidated returns is very small in the data, 6% in all of my sample. This is despite there being a large fraction of plants that belong to firms with multiple establishments within a state. The average number of establishments within a multi-establishment firm in the country is close to 5.8, and within a state is 5.5., indicating that multi-establishment plants mostly operate within a state.

Product Classification My sample includes data from 2001 to 2009, both years inclusive. The product classification used by the ASI to report outputs produced and inputs used by the plant remains fairly consistent over the years 2001-2009 (Annual Survey of Industries Commodity Classification- ASICC at the 5-digit level), with a minor revision in 2008. I bring all data to the first ASICC classification using product descriptions made available by the ASI. While data for 2010-2012 is also available, there was a major revamp of the product classification starting 2010 (National Product Classification for Manufacturing Sector-NPCMS at the 7-digit level). While the ASI does provide a concordance between the ASICC and the NPCMS, it is not a perfect one-to-one mapping. This results in the loss of product definition detail due to aggregation required to bring ASICC and NPCMS into one consistent classification. Table A21 provides details on the final number of product codes available after constructing crosswalks across all classifications. As a finely defined product classification is key to my empirical analysis, I make a sample choice that maximizes the number of clean observations, and hence I use only the data from 2001-2009. Table A22 lists the broad 1-digit ASICC product groups used by the ASI.

Data Cleaning The main variables that require cleaning for my empirical analysis are the input and output variables. The output products data is the main data used for the construction of plant types, and in my regression analysis. The input data on the other hand is used for the construction of the IO tables along with the output data. I start by collecting all year-plant observations between 2001 and 2009, and the corresponding outputs and inputs reported by them.

Table A20: Plants and Firms in the ASI

		Consolidated Returns		Part of a firm with > 1 plant in country		Part of a firm with > 1 plant in state	
Year	Obs.	Share	Avg. Plants	Share	Avg. Plants	Share	Avg. Plants
2001	22260	0.04	2.61	0.34	6.23	0.22	3.77
2002	23279	0.05	2.68	0.35	5.73	0.23	4.41
2003	28333	0.04	2.62	0.33	6.01	0.23	4.79
2004	20169	0.08	2.66	0.41	6.12	0.29	4.22
2005	22963	0.07	2.70	0.40	5.45	0.28	3.83
2006	24063	0.07	2.76	0.40	5.58	0.28	8.97
2007	18019	0.08	2.80	0.43	5.96	0.30	8.32
2008	15543	0.08	2.74	0.44	5.86		
Total	174629	0.06	2.70	0.38	5.86	0.26	5.52

Notes: This table shows the incidence of observations that file consolidated returns (more than one plant reporting), the incidence of observations that are part of a firm with more than one plant in the country, and the incidence of plants that are part of a firm with more than one plant in the state. the columns titled "Avg. Plants" is the average number of plants within the different types of observations identified. Data for 2009 is not reported as observations from 2009 are not part of my regression sample. Recall that my regression sample includes only those observations that have at least future year observation. Since my sample ends in 2009, no data from 2009 is used in the regressions. The ASI does not report information on multi-plant observations within a state from 2008 onwards.

Table A21: ASI Product Classifications

Years	Classification	# Codes
2001-2009	ASICC	6,176
2010-2012	NPCMS	6,182
Consistent Classification Over Years		
2001-2012	ASICC–NPCMS	4,199

Table A22: ASICC Product Groups

Code	Description
1	Animal, Vegetable, Horticulture, Forestry Products, Beverages, Tobacco & Pan Masala, and Non-Edible Water/Spirit & Alcohol Chiefly Used in Industry
2	Ores, Minerals, Mineral Fuels, Lubricants, and Gas & Electricity
3	Chemical and Allied Products
4	Rubber, Plastic, Leather, and Products thereof
5	Wood, COrk, Thermocol & Paper, and Articles thereof
6	Textile and Textile Articles
7	Base Metals, Products Thereof, Machinery Equipment and Parts thereof, Excluding Transport Equipment
8	Railways, Airways, Ships & Road Surface Transport, ans Related Equipment and Parts
9	Other Manufacturing Articles and Services n.e.c

For the output data, I drop all products that are coded as miscellaneous(ASICC 2-digit code 99) and service related items (ASICC 2-digit code 97). Similarly for input data I drop any input products that are coded as miscellaneous, fuels, non-basic items, and service related items. Further, I aggregate both domestic inputs and foreign inputs at the product code level to get the total value of a given product used as an input at the plant level. Finally, I drop all output and input observation within a plant that have zero or negative values reported. This results in roughly 1% of year-plant-output observations and less than 0.1% of year-plant-input observations from being dropped.

A.2 Trade Data

To construct my regressor and instrumental variables, I use imports and exports data from the Base for International Trade Analysis (BACI) dataset from Centre d'Études Prospectives et d'Informations Internationales (CEPII). BACI builds on trade data that is reported by countries to the United Nations COMTRADE division, and develops a procedure that reconciles the data reported by importers and exporters that may not coincide in the original data. I use the data at the Harmonized System 1996 (HS96) classification of products.

I construct a crosswalk in order to bring the trade data which is reported in the 6-digit HS96 product classification to the 5-digit ASICC product classification. I make use of the fact that the new Indian product classification NPCMS is built on the Central Product Classification version 2 (CPC2) deveped by the United Nations Statistics Division. The 7-digit NPCMS is made up of the 5-digit CPC code plus a 2-digit Indian requirement. I first bring the 5-digit ASICC codes to the 5-digit NPCMS using the crosswalk provided by the ASI. Note that the 5-digit nPCMS codes are the same as CPC2 codes. I then bring these CPC2 codes to the Harmonized System 2007 version (HS07) product classification. Finally, I bring the 6-digit HS07 codes to the 6-digit HS96 codes, which is the classification that the trade data is at.[51]

A.3 Construction of the National Input-Output Table

To be written.

[51] http://www.csoisw.gov.in/cms/En/1027-npcms-national-product-classification-fo-rmanufacturing-sector aspx for ASICC to NPCMS concordance
https://ec.europa.eu/eurostat/ramon/relations/index.cfm?TargetUrl=LST_REL&StrLanguageCode=EN&IntCurrentPage=7 for CPC2 to HS07 concordance
https://wits.worldbank.org/product_concordance.html for HS07 to HS96 concordance

B Empirical Appendix

B.1 Additional Regression Tables

Table B23: Knowledge Spillovers Over Time, Reduced Form Results

VARIABLES	(1) $\mathbb{1}^D_{t+1}$	(2) $\mathbb{1}^D_{t+2}$	(3) $\mathbb{1}^D_{t+3}$	(4) $\mathbb{1}^D_{t+4}$	(5) $\mathbb{1}^D_{t+5}$
$\log(IV^D_{\omega,t})$	0.026***	0.038***	0.054***	0.042**	0.026
	(0.009)	(0.011)	(0.013)	(0.020)	(0.028)
$\log(World^D_{\omega,t})$	-0.007	-0.001	0.013	0.018	0.005
	(0.007)	(0.008)	(0.015)	(0.019)	(0.020)
Observations	127,059	100,320	76,939	58,637	42,608
R-squared	0.421	0.484	0.522	0.552	0.586
Plant Controls	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y
Plt-Pdt FE	Y	Y	Y	Y	Y
Cluster	Ind	Ind	Ind	Ind	Ind

Table B24: Knowledge Spillovers Over Time, First Stage Results

VARIABLES	(1) $\log(Exp^D_\omega)$	(2) $\log(Exp^D_\omega)$	(3) $\log(Exp^D_\omega)$	(4) $\log(Exp^D_\omega)$	(5) $\log(Exp^D_\omega)$
$\log(IV^D_{\omega,t})$	0.545***	0.665***	0.693***	0.604***	0.684***
	(0.116)	(0.120)	(0.132)	(0.161)	(0.228)
$\log(World^D_{\omega,t})$	-0.193	-0.350*	-0.471*	-0.721*	-0.674*
	(0.189)	(0.192)	(0.246)	(0.424)	(0.393)
Observations	127,059	100,320	76,939	58,637	42,608
R-squared	0.964	0.965	0.966	0.966	0.971
Plant Controls	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y
Plt-Pdt FE	Y	Y	Y	Y	Y
Cluster	Ind	Ind	Ind	Ind	Ind

B.2 Additional Robustness Checks: Revenue Shares

Table B25: New Downstream Revenue Shares Over Time, IV Results

	(1)	(2)	(3)	(4)	(5)
	IV	IV	IV	IV	IV
VARIABLES	$RevSh^D_{t+1}$	$RevSh^D_{t+2}$	$RevSh^D_{t+3}$	$RevSh^D_{t+4}$	$RevSh^D_{t+5}$
$\log(Exp^D_{\omega,t})$	0.023*	0.034**	0.043**	0.025	0.027
	(0.014)	(0.014)	(0.017)	(0.025)	(0.033)
$\log(World^D_{\omega,t})$	-0.005	0.010	0.026	0.026*	0.019
	(0.005)	(0.010)	(0.017)	(0.013)	(0.020)
Observations	127,059	100,320	76,939	58,637	42,608
R-squared	-0.005	-0.014	-0.026	-0.007	-0.008
Plant Controls	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y
Plt-Pdt FE	Y	Y	Y	Y	Y
Cluster	Ind	Ind	Ind	Ind	Ind

Table B26: New Revenue Shares Over Time: Placebo Tests, IV Results

	(1)	(2)	(3)	(4)	(5)	(6)
	IV	IV	IV	IV	IV	IV
VARIABLES	$RevSh^U_{t+1}$	$RevSh^U_{t+2}$	$RevSh^U_{t+3}$	$RevSh^O_{t+1}$	$RevSh^O_{t+2}$	$RevSh^O_{t+3}$
$\log(Exp^D_{\omega,t})$	-0.000	-0.007	-0.022**	-0.037*	-0.027*	-0.037**
	(0.012)	(0.011)	(0.009)	(0.021)	(0.016)	(0.016)
$\log(World^D_{\omega,t})$	0.003	0.007	-0.002	-0.017	-0.039**	-0.036
	(0.008)	(0.008)	(0.008)	(0.014)	(0.018)	(0.023)
Observations	127,059	100,320	76,939	127,059	100,320	76,939
R-squared	0.001	0.002	-0.004	0.003	0.011	0.005
Plant Controls	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Plt-Pdt FE	Y	Y	Y	Y	Y	Y
Cluster	Ind	Ind	Ind	Ind	Ind	Ind

B.3 Additional Robustness Checks: Total number of New Products

Table B27: New Total Downstream Products Over Time, IV Results

VARIABLES	(1) N^D_{t+1}	(2) N^D_{t+2}	(3) N^D_{t+3}	(4) N^D_{t+4}	(5) N^D_{t+5}
$\log(Exp^D_{\omega,t})$	0.060**	0.063***	0.086***	0.076*	0.032
	(0.023)	(0.020)	(0.027)	(0.044)	(0.057)
$\log(World^D_{\omega,t})$	0.005	0.023	0.059*	0.074**	0.022
	(0.010)	(0.015)	(0.030)	(0.032)	(0.030)
Observations	127,059	100,320	76,939	58,637	42,608
R-squared	-0.007	-0.009	-0.024	-0.017	-0.001
Plant Controls	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y
Plt-Pdt FE	Y	Y	Y	Y	Y
Cluster	Ind	Ind	Ind	Ind	Ind
F-Stat	22.044	30.565	27.509	14.104	9.007

Table B28: New Total Products Over Time: Placebo Tests, IV Results

VARIABLES	(1) IV N^U_{t+1}	(2) IV N^U_{t+2}	(3) IV N^U_{t+3}	(4) IV N^O_{t+1}	(5) IV N^O_{t+2}	(6) IV N^O_{t+3}
$\log(Exp^D_{\omega,t})$	0.001	-0.013	-0.028*	-0.025	-0.019	-0.031
	(0.017)	(0.017)	(0.015)	(0.033)	(0.034)	(0.036)
$\log(World^D_{\omega,t})$	0.007	0.009	-0.003	-0.006	0.003	-0.002
	(0.009)	(0.011)	(0.011)	(0.020)	(0.025)	(0.032)
Observations	127,059	100,320	76,939	127,059	100,320	76,939
R-squared	0.005	0.006	0.004	0.036	0.067	0.070
Plant Controls	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Plt-Pdt FE	Y	Y	Y	Y	Y	Y
Cluster	Ind	Ind	Ind	Ind	Ind	Ind

B.4 Additional Robustness Checks: Effect of only Exports (without weights)

Table B29: Effect of only Exports: Baseline, IV Results

VARIABLES	(1) $\mathbb{1}^D_{t+1}$	(2) $\mathbb{1}^D_{t+2}$	(3) $\mathbb{1}^D_{t+3}$	(4) $\mathbb{1}^D_{t+4}$	(5) $\mathbb{1}^D_{t+5}$
$\log(Exp_{\omega,t})$	0.025*	0.031*	0.071**	0.117	0.077
	(0.014)	(0.017)	(0.032)	(0.087)	(0.071)
$\log(World^D_{\omega,t})$	0.002	0.013	0.047*	0.091*	0.054
	(0.006)	(0.008)	(0.027)	(0.055)	(0.035)
Observations	127,059	100,320	76,939	58,637	42,608
R-squared	0.001	-0.000	-0.022	-0.065	-0.026
Plant Controls	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y
Plt-Pdt FE	Y	Y	Y	Y	Y
Cluster	Ind	Ind	Ind	Ind	Ind
F-Stat	31.362	20.338	5.73	3.148	2.749

Table B30: Effect of only Exports: Placebo, IV Results

VARIABLES	(1) IV $\mathbb{1}^U_{t+1}$	(2) IV $\mathbb{1}^U_{t+2}$	(3) IV $\mathbb{1}^U_{t+3}$	(4) IV $\mathbb{1}^O_{t+1}$	(5) IV $\mathbb{1}^O_{t+2}$	(6) IV $\mathbb{1}^O_{t+3}$
$\log(Exp_{\omega,t})$	0.003	-0.006	-0.039**	0.008	0.043**	-0.014
	(0.016)	(0.018)	(0.020)	(0.020)	(0.021)	(0.024)
$\log(World^D_{\omega,t})$	0.008	0.010	-0.008	-0.005	-0.002	0.005
	(0.009)	(0.011)	(0.013)	(0.013)	(0.019)	(0.018)
Observations	127,059	100,320	76,939	127,059	100,320	76,939
R-squared	0.004	0.006	-0.001	0.015	0.020	0.026
Plant Controls	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Plt-Pdt FE	Y	Y	Y	Y	Y	Y
Cluster	Ind	Ind	Ind	Ind	Ind	Ind

C Model Appendix

C.1 Laws of Motion

This section outlines the derivations for the laws of motion for the mass of varieties and quality distributions in equations 41 and 42. I start by discretizing time into small intervals Δ. The mass of varieties that has quality less that q^s at time $t+\Delta$ is given by equation 39, which i start with:

$$M^s_{n(t+\Delta)}G^s_{n(t+\Delta)}(q^s) = \left\{ \begin{array}{c} M^s_{nt}\left[G^s_{nt}(q^s) - G^s_{nt}(q^{s*}_{nn(t+\Delta)})\right] \\ +\left(\Delta\theta^s_n M^s_{nt} + \Delta\eta^s_n M^{s'}_{nt}\right)\left[G^s_{nt}(q^s) - G^s_{nt}(q^{s*}_{nn(t+\Delta)})\right] \\ +\left(\Delta E^s_{nt} - \Delta\delta M^s_{nt}\right)\left[G^s_{nt}(q^s) - G^s_{nt}(q^{s*}_{nn(t+\Delta)})\right] \end{array} \right\} \tag{62}$$

Let's define the total rate of innovation/new varieties of stage s entering the market by: $\Delta Z^s_{nt} = \Delta\theta^s_n M^s_{nt} + \Delta\eta^s_n M^{s'}_{nt} + \Delta E^s_{nt}$. To get total mass of varieties, I first take limit as $q^s \to \infty$, and then taking the limit as $\Delta \to 0$:

$$\begin{aligned} \lim_{q^s\to\infty} M^s_{n(t+\Delta)}G^s_{n(t+\Delta)}(q^s) &= \lim_{q^s\to\infty}\left(M^s_{nt} + \Delta Z^s_{nt} - \Delta\delta\right)\left[G^s_{nt}(q^s) - G^s_{nt}(q^{s*}_{nn(t+\Delta)})\right] \\ M^s_{n(t+\Delta)} &= \left(M^s_{nt} + \Delta Z^s_{nt} - \Delta\delta M^s_{nt}\right)\left[1 - G^s_{nt}(q^{s*}_{nn(t+\Delta)})\right] \end{aligned} \tag{63}$$

where $G^s_{nt}(\infty) = 1$. Rearranging terms in the above equation and taking limit as $\Delta \to 0$, one gets:

$$\lim_{\Delta\to 0}\frac{M^s_{n(t+\Delta)} - M^s_{nt}}{\Delta} = \lim_{\Delta\to 0}\left(Z^s_{nt} - \delta M^s_{nt}\right)\left[1 - G^s_{nt}(q^{s*}_{nn(t+\Delta)})\right] - M^s_{nt}\frac{G^s_{nt}(q^{s*}_{nn(t+\Delta)})}{\Delta}$$

Given that $G^s_{nt}(q^{s*}_{nnt}) = 0$, as q^{s*}_{nt} is the lowest quality of incumbent varieties, defined by market entry thresholds, I can rewrite:

$$\begin{aligned} \lim_{\Delta\to 0} -\frac{G^s_{nt}(q^{s*}_{nn(t+\Delta)})}{\Delta} &= \lim_{\Delta\to 0}\left(\frac{G^s_{nt}(q^{s*}_{nnt}) - G^s_{nt}(q^{s*}_{nn(t+\Delta)})}{q^{s*}_{nn(t+\Delta)} - q^{s*}_{nnt}}\right)\left(\frac{q^{s*}_{nn(t+\Delta)} - q^{s*}_{nnt}}{\Delta}\right) \\ &= \frac{\partial G^s_{nt}(q^{s*}_{nn})}{\partial q^s}\frac{dq^{s*}_{nnt}}{dt} = \delta^{s*}_{nt} \quad \text{(Endogeneous exit rate)} \end{aligned}$$

The endogeneous exit rate of varieties is given a combination of how quickly the threshold quality increases and the density of varieties at the threshold that drop out. The limit of the mass of functions is then given by:

$$\frac{dM^s_{nt}}{dt} = Z^s_{nt} - \delta M^s_{nt} - \delta^{s*}_{nt}M^s_{nt}$$

Defining $\dot{M}^s_{nt} = dM^s_{nt}/dt$, dividing by total mass M^s_{nt} and substituting for Z^s_{nt}, gives us the law of motion for mass of varieties:

$$\frac{\dot{M}^s_{nt}}{M^s_{nt}} = \theta^s_n + \eta^s_n \frac{M^{s'}_{nt}}{M^s_{nt}} + \frac{E^s_{nt}}{M^s_{nt}} - \delta - \delta^{s*}_{nt} \tag{64}$$

In a balanced growth path, the mass of varieties remains constant, i.e. $\dot{M}^s_{nt} = 1$. Then, we can solve for the the BGP mass entrants. Intuitively, in a BGP, the mass of new varieties discovered should be equal tot he mass of old varieties that die.

$$E^s_{nt} = (\delta + \delta^{s*}_{nt}) M^s_{nt} - \theta^s_n M^s_{nt} - \eta^s_n M^{s'}_{nt} \tag{65}$$

To derive the law of motion for the quality distributions, $G^s_{nt}(q^s)$, I start by substituting equation 63 into equation 62:

$$\begin{aligned}(M^s_{nt} + \Delta Z^s_{nt} - \Delta\delta M^s_{nt}) \left[1 - G^s_{nt}(q^{s*}_{nn(t+\Delta)})\right] G^s_{n(t+\Delta)}(q^s) &= (M^s_{nt} + \Delta Z^s_{nt} - \Delta\delta) \left[G^s_{nt}(q^s) - G^s_{nt}(q^{s*}_{nn(t+\Delta)})\right] \\ G^s_{nt}(q^s) - G^s_{n(t+\Delta)}(q^s) &= G^s_{nt}(q^{s*}_{nn(t+\Delta)}) \left(1 - G^s_{n(t+\Delta)}(q^s)\right)\end{aligned}$$

Dividing both sides by Δ and taking limit as $\Delta \to 0$:

$$\begin{aligned}\lim_{\Delta\to 0} \frac{G^s_{nt}(q^s) - G^s_{n(t+\Delta)}(q^s)}{\Delta} &= \lim_{\Delta\to 0} \frac{G^s_{nt}(q^{s*}_{nn(t+\Delta)})}{\Delta} \left(1 - G^s_{n(t+\Delta)}(q^s)\right) \\ \frac{\partial G^s_{nt}(q^s)}{\partial t} &= -\delta^{s*}_{nt} \left(1 - G^s_{nt}(q^s)\right)\end{aligned} \tag{66}$$

where I have substituted for the endogeneous exit rate derived earlier. This ends the derivation of the laws of motion for the mass of varieties and the quality distributions.

C.2 Proof of Proposition 1: Stationary Quality Distribution

The law of motion for quality distributions is given by equation 42:

$$\frac{\partial G^s_{nt}(q^s)}{\partial t} = -\delta^{s*}_{nt} \left[1 - G^s_{nt}(q^s)\right]$$

As shown in Perla et al. (2019), the solution to this partial differential equation is given by:

$$\begin{aligned}\partial G^s_{nt}(q)/\partial q^s &= \frac{\partial G^s_{n0}(q)/\partial q^s}{1 - G^s_{n0}\left(q^{s*}_{nnt}\right)} \quad \text{for } q^s \geq q^{s*}_{nnt} \\ \implies G^s_{nt}(q^s) &= \frac{G^s_{n0}(q^s) - G^s_{n0}\left(q^{s*}_{nnt}\right)}{1 - G^s_{n0}\left(q^{s*}_{nnt}\right)} \quad \text{for } q^s \geq q^{s*}_{nnt}\end{aligned} \tag{67}$$

Under assumption 1, which states that the initial distribution at time 0 is Pareto: $G^s_{n0}(q^s) = 1 - \left(\frac{q^s}{q^{s*}_{nn0}}\right)^{-\gamma^s}$, we can now show that the distribution at any instance in time is also Pareto.

Substituting for the initial distribution in equation 67, we get,

$$G^s_{nt}(q^s) = \frac{\left(1-\left(\frac{q^s}{q^{s*}_{nn0}}\right)^{-\gamma^s}\right) - \left(1-\left(\frac{q^{s*}_{nnt}}{q^{s*}_{nn0}}\right)^{-\gamma^s}\right)}{1-\left(1-\left(\frac{q^{s*}_{nnt}}{q^{s*}_{nn0}}\right)^{-\gamma^s}\right)} \quad \text{for } q^s \geq q^{s*}_{nnt}$$

$$G^s_{nt}(q^s) = 1-\left(\frac{q^s}{q^{s*}_{nnt}}\right)^{-\gamma^s} \quad \text{for } q^s \geq q^{s*}_{nnt} \tag{68}$$

which is a Pareto distribution with the minimum bound equal to the entry threshold. Note also that this law of motion preserves the shape of the initial distribution, γ^s.

C.3 Market Clearing Conditions

Before writing down the labor market and goods market clearing conditions, it is useful to derive labor demand from different economic activities in the economy. Labor is used for production, market access, and R&D activities by both entrants and incumbents. I will derive labor demand expressions under proposition 1, which results in quality distributions in all stages of production and in all countries to be Pareto. Cobb-Douglas production function technologies faced by upstream and downstream producers results in labor cost being a constant share of total production costs. Let X^s_n be the total expenditure on stage-s goods by country n. Then,

$$w_n L^P_n = \sum_s \underbrace{\beta^{ls}}_{\substack{\text{Labor share}\\ \text{in total expenditure}}} \underbrace{\epsilon^s}_{\substack{\text{Expenditure share}\\ \text{in total revenue}}} \sum_l \underbrace{\lambda^s_{nl} X^s_l}_{\substack{\text{Demand for stage-}s\\ \text{goods from country } l\\ = \text{Revenue}}}$$

where $\epsilon^s = (\sigma^s) - 1)/\sigma^s$. Labor used for market access is the sum of market access costs for all products that supply to different markets. Note that under Pareto distributions, and quality cutoffs for different markets, q^{s*}_{nl}, the share of products that are supplied from market n to market l is given by: $\left(\frac{q^{s*}_{nl}}{q^{s*}_{nn}}\right)^{-\gamma^s}$. Then total labor expenditure on market access is given by:

$$w_n L^{MA}_n = \sum_s M^s_n \sum_l \left(\frac{q^{s*}_{nl}}{q^{s*}_{nn}}\right)^{-\gamma^s} f^s_{nl} w_n$$

Finally labor expenditure used for R&D by entrants and incumbents is given by:

$$w_n L^{RD}_n = \underbrace{\sum_s E^s_n f^s_{ne} w_n}_{\text{Entrant R\&D}} + \underbrace{\sum_s M^s_n R^s_{n,\theta}(\theta^s_n)}_{\text{Owns-Stage R\&D}}$$
$$+ \underbrace{M^u_n \sum_l \left(\frac{q^{u*}_{nl}}{q^{u*}_{nn}}\right)^{-\gamma^u} R^d_{nl,\eta}(\eta^d_{nl})}_{\substack{\text{Cross-Stage R\&D}\\ \text{Upstream}}} + \underbrace{M^d_n \sum_l R^u_{nl,\eta}(\eta^u_{nl})}_{\substack{\text{Cross-Stage R\&D}\\ \text{Downstream}}}$$

Note that, only exporters in upstream stage can benefit from cross-stage R&D from other markets, while all firms in the downstrema market can invest in all market-specific cross-stage R&D. We are now equipped to write the labor market clearing condition:

$$w_n L_n = w_n L_n^P + w_n L_n^{MA} + w_n L_n^{RD} \tag{69}$$

Goods market clearing for upstream stage is given by the following:

$$X_n^u = \beta^{ud} \epsilon^d \sum_l \lambda_{nl}^d X_l^d \tag{70}$$

This follows similar logic to the labor demand for production activities. Total expenditure spent on upstream goods in country n depends on the share of upstream input used in the production of downstream goods. The amount of downstream goods produced in country n further depends on the demand for downstream varieties from all countries. In the case of single industry, $\mathcal{J} = 1$, final good is produced by only the single downstream output. Therefore, the demand faced by downstream varieties is equal to the demand faced by the final good produced in the country, which is consumption demand and material input demand. Therefore,

$$X_n^d = \underbrace{\sum_s \beta^{fs} \epsilon^s \sum_l \lambda_{nl}^s X_l^s}_{\text{Material input demand}} + \underbrace{I_n}_{\substack{\text{Consumption demand} \\ = \text{HH Income}}} \tag{71}$$

where I_n is household income that is completely spent on consumption. Since households in this model own firms, they receive profits from production activities, but also have to bear the cost of investment and operation of firms. Therefore household income is the sum of labor income and firm profits, and minus the firm costs:

$$I_n = \underbrace{w_n L_n}_{\text{Labor income}} + \underbrace{\sum_s (1 - \epsilon^s) \sum_l \lambda_{nl}^s X_l^s}_{\text{Firm profits}} - \underbrace{\left(w_n L_n^{MA} + w_n L_n^{RD}\right)}_{\text{Firm costs}}$$

C.4 Proof of Proposition 2: Balanced Growth Path Outcomes

In this subsection, I will solve for all BGP variables listed in proposition 2. Starting from the incumbent firm value function defined in 34, and suppressing time subscripts to reduce

notation:[52]

$$
\begin{aligned}
& r_n V_n\left(Q^u, Q^d\right) - \dot{V}_n\left(Q^u, Q^d\right) \\
= & \max_{\substack{\{\mathbb{I}^u_{nl}(q^u)\}_{Q^u,l} \\ \{\mathbb{I}^d_{nl}(q^d)\}_{Q^d,l} \\ \theta^u_n \geq 0,\ \theta^d_n \geq 0 \\ \{\eta^d_{nl}\}_l \geq 0,\ \{\eta^u_{nl}\}_l \geq 0}} \sum_{s\in\{u,d\}} \sum_{q^s \in Q^s} \left\{ \begin{array}{c} \sum_l \mathbb{I}^s_{nl}(q^s)\left[\overline{\pi}^s_{nl} q^s - f^s_{nl} w_n\right] \\ +\mathbb{I}^s_{nn}(q^s)\delta\left[V_n\left(\{Q^s \setminus q^s\}, Q^{s'}\right) - V_n\left(Q^s, Q^{s'}\right)\right] \\ +\mathbb{I}^s_{nn}(q^s)\theta^s_n\left[\int V_n\left(\{Q^s, z^s\}, Q^{s'}\right) \mathrm{d}G^s_n(z^s) - V_n\left(Q^s, Q^{s'}\right)\right] \\ -\mathbb{I}^s_{nn}(q^s) R^s_{n,\theta}\left(\theta^s_n\right) w_n \end{array} \right\} \\
& + \sum_{q^u \in Q^u} \left\{ \begin{array}{c} \sum_l \mathbb{I}^u_{nl}(q^u)\eta^d_{nl}\left[\int V_n\left(Q^u, \{Q^d, z^d\}\right) \mathrm{d}G^d_n\left(z^d\right) - V_n\left(Q^u, Q^d\right)\right] \\ -\sum_l \mathbb{I}^u_{nl}(q^u) R^d_{nl,\eta}\left(\eta^d_{nl}\right) w_n \end{array} \right\} \\
& + \sum_{q^d \in Q^d} \left\{ \begin{array}{c} \mathbb{I}^d_{nn}(q^s)\sum_l \eta^u_{nl}\left[\int V_n\left(\{Q^u, z^u\}, Q^d\right) \mathrm{d}G^u_n\left(z^u\right) - V_n\left(Q^u, Q^d\right)\right] \\ -\mathbb{I}^d_{nn}(q^d)\sum_l R^u_{nl,\eta}\left(\eta^u_{nl}\right) w_n \end{array} \right\}
\end{aligned} \tag{72}
$$

I start by conjecturing that the BGP value function of incumbents takes the following form:

$$
\begin{aligned}
V_n\left(Q^u, Q^d\right) &= \sum_{s\in\{u,d\}} \sum_{q^s \in Q^s} V^s_n(q^s) \\
&= \sum_{s\in\{u,d\}} \sum_{q^s \in Q^s} \sum_l \mathbb{I}^s_{nl}(q^s) v^s_{nl}(q^s)
\end{aligned} \tag{73}
$$

where $v^s_{nl}(q^s)$ is a origin-destination pair specific value of operating a stage s product of quality q^s. The above equation states that the total value of an incumbent firm in the BGP is simply the sum of values of all products that the incumbent owns, and further the value of each product is simply the sum of values received from all destinations that the product is sold to. Substituting equation 73 into the incumbent value function 72 on the right hand side, we can simplify the value of a new innovation that the firm receives. The change in the value function of the firm when it adds a new product of quality z^s is given by:

$$
\begin{aligned}
\int V_n\left(\{Q^s, z^s\}, Q^{s'}\right) \mathrm{d}G^s_n\left(z^s\right) - V_n\left(Q^s, Q^{s'}\right) &= \int \left(V_n\left(Q^s, Q^{s'}\right) + V^s_n(z^s)\right) \mathrm{d}G^s_n\left(z^s\right) - V_n\left(Q^s, Q^{s'}\right) \\
&= \int V^s_n(z^s) \mathrm{d}G^s_n\left(z^s\right) \\
&= f^s_{ne} w_n = \chi^s_{n,e} \frac{\bar{q}^s_n}{\tilde{q}^s_n} w_n
\end{aligned} \tag{74}
$$

where the last equality follows from the free entry condition under positive entry, equation 37. Therefore, the return to an innovation and adding a new product in stage s is simply the value of an entrant firm in stage s.

[52] Note that the following variables are still time dependent in the BGP: $V_{nt}(), \mathbb{I}^s_{nlt}(), \bar{\pi}^s_{nlt}, w_{nt}, \theta^s_{nt}, \eta^s_{nlt}, G^s_{nt}()$.

C.4.1 Optimal Innovation Intensity

In order to solve the incumbent firm's problem, I start by solving the inner most loops in the firm value function, and then solve for the outer-most decisions. In this case, I solve for the optimal innovation intensities conditional on market entry decisions being positive, i.e. $\mathbb{I}^s_{nl}(q^s) = 1$. Then, given the return from supplying to a market both from static profits and innovation returns, and the cost of supplying to that market, I solve for optimal entry-exit decisions. To solve for the optimal innovation intensity of an incumbent firm for both own-stage and cross-stage, I substitute the return to adding a product into the firm portfolio, equation 74 into the firm problem, equation 72. Let's start with the per-product R&D problem faced by the firm for own-stage innovation, conditional on the product being produced, i.e. $\mathbb{I}^s_{nn}(q^s) = 1$:

$$\begin{aligned}
&\max_{\theta^s_n \geq 0} \quad \theta^s_n \left[\int V_n \left(\{Q^s, z^s\}, Q^{s'} \right) \mathrm{d}G^s_n(z^s) - V_n \left(Q^s, Q^{s'} \right) \right] - \chi^s_{n,\theta} (\theta^s_n)^\psi \left(\frac{\bar{q}^s_n}{\tilde{q}^s_n} \right)^\nu w_n \\
&= \max_{\theta^s_n \geq 0} \quad \theta^s_n \chi^s_{n,e} \left(\frac{\bar{q}^s_n}{\tilde{q}^s_n} \right)^\nu w_n - \chi^s_{n,\theta} (\theta^s_n)^\psi \left(\frac{\bar{q}^s_n}{\tilde{q}^s_n} \right)^\nu w_n
\end{aligned}$$

The own-stage innovation intensity is given by:

$$\theta^s_n = \left(\frac{\chi^s_{n,e}}{\psi \chi^s_{n,\theta}} \right)^{\frac{1}{\psi - 1}} \tag{75}$$

Notice that the innovation intensity independent of the product that gives the firm the option to conduct own-stage R&D, q^s. Therefore, every incumbent that owns a product in stage s invests the same amount of resources, and receives new ideas at the same Poisson rate. It is useful to solve for the net return from own-stage innovation to solve for market entry-exit decisions later on. The net return is given by the expected return from innovation minus the cost of R&D at the optimal innovation intensity level. Notice that, from equation 75, one can write: $\chi^s_{n,e} = \psi \chi^s_{n,\theta} (\theta^s_n)^{\psi - 1}$. Substituting this in to the net return equation, one gets:

$$\begin{aligned}
&\theta^s_n \chi^s_{n,e} \left(\frac{\bar{q}^s_n}{\tilde{q}^s_n} \right)^\nu w_n - \chi^s_{n,\theta} (\theta^s_n)^\psi \left(\frac{\bar{q}^s_n}{\tilde{q}^s_n} \right)^\nu w_n \\
&= \psi \chi^s_{n,\theta} (\theta^s_n)^\psi \left(\frac{\bar{q}^s_n}{\tilde{q}^s_n} \right)^\nu w_n - \chi^s_{n,\theta} (\theta^s_n)^\psi \left(\frac{\bar{q}^s_n}{\tilde{q}^s_n} \right)^\nu w_n \\
&= (\psi - 1) R^s_{n,\theta}(\theta^s_n) w_n
\end{aligned} \tag{76}$$

where $R^s_{n,\theta}(\theta^s_n)$ is the cost of doing own-stage R&D. Therefore, the return to conducting R&D is proportional to the cost of R&D.

Similarly, one can solve the per-product per-market cross-stage R&D problem that an incumbent faces, conditional on the product being supplied to each market in case of an upstream product, $\mathbb{I}^u_{nl}(q^u) = 1$, and the product being produced in the domestic market in case of a

downstream product $\mathbb{I}^d_{nn}(q^d) = 1$,

$$\max_{\eta^s_{nl}\geq 0} \quad \eta^s_{nl}\left[\int V_n\left(\{Q^s, z^s\}, Q^{s'}\right) \mathrm{d}G^s_n(z^s) - V_n\left(Q^s, Q^{s'}\right)\right] - \chi^s_{n,\eta}\left(\eta^s_{nl}\right)^{\psi}\left(\frac{\bar{q}^s_n}{\hat{q}^s_{nl}}\right)^{\nu} w_n$$
$$= \max_{\eta^s_{nl}\geq 0} \quad \eta^s_{nl}\chi^s_{n,e}\left(\frac{\bar{q}^s_n}{\tilde{q}^s_n}\right)^{\nu} w_n - \chi^s_{n,\eta}\left(\eta^s_{nl}\right)^{\psi}\left(\frac{\bar{q}^s_n}{\hat{q}^s_{nl}}\right)^{\nu} w_n$$

The cross-stage innovation intensity per-product and per-market is given by:

$$\eta^s_{nl} = \left(\frac{\chi^s_{n,e}}{\psi\chi^s_{n,\eta}}\left(\frac{\hat{q}^s_{nl}}{\tilde{q}^s_n}\right)^{\nu}\right)^{\frac{1}{\psi-1}} \tag{77}$$

and the net return of this cross-stage R&D activity is:

$$\eta^s_{nl}\chi^s_{n,e}\left(\frac{\bar{q}^s_n}{\tilde{q}^s_n}\right)^{\nu} w_n - \chi^s_{n,\eta}\left(\eta^s_{nl}\right)^{\psi}\left(\frac{\bar{q}^s_n}{\hat{q}^s_{nl}}\right)^{\nu} w_n = (\psi-1)\, R^s_{n,\eta}(\eta^s_{nl})w_n \tag{78}$$

C.4.2 Optimal Market Entry Decision

Given the innovation intensity of own-stage and cross-stage R&D, we can now focus on the outer loop of the firm's problem, which markets to supply to. The firm's problem once innovation decisions are internalized can be simplified as follows:

$$\begin{aligned}
&r_n V_n\left(Q^u, Q^d\right) - \dot{V}_n\left(Q^u, Q^d\right) \\
&= \max_{\substack{\{\mathbb{I}^u_{nl}(q^u)\}_{Q^u,l} \\ \{\mathbb{I}^d_{nl}(q^d)\}_{Q^d,l}}} \quad \sum_{s\in\{u,d\}}\sum_{q^s\in Q^s}\left\{\begin{array}{l} \sum_l \mathbb{I}^s_{nl}(q^s)\left[\overline{\pi}^s_{nl}q^s - f^s_{nl}w_n\right] \\ -\mathbb{I}^s_{nn}(q^s)\delta V^s_n(q^s) \\ +\mathbb{I}^s_{nn}(q^s)\,(\psi-1)\,R^s_{n,\theta}(\theta^s_n)w_n \end{array}\right\} \\
&\qquad + \sum_{q^u\in Q^u}\left\{\ \sum_l \mathbb{I}^u_{nl}(q^u)\,(\psi-1)\,R^d_{n,\eta}(\eta^d_{nl})w_n\ \right\} \\
&\qquad + \sum_{q^d\in Q^d}\left\{\ \mathbb{I}^d_{nn}(q^d)\sum_l (\psi-1)\,R^u_{n,\eta}(\eta^u_{nl})w_n\ \right\}
\end{aligned}$$

Rearranging terms, we get:

$$\begin{aligned}
&(r_n+\delta)V_n\left(Q^u, Q^d\right) - \dot{V}_n\left(Q^u, Q^d\right) \\
= &\max_{\substack{\{\mathbb{I}^u_{nl}(q^u)\}_{Q^u,l} \\ \{\mathbb{I}^d_{nl}(q^d)\}_{Q^d,l}}} \quad \sum_{q^u\in Q^u}\left\{\begin{array}{l} \mathbb{I}^u_{nn}(q^u)\left[\overline{\pi}^u_{nn}q^u - f^u_{nn}w_n + (\psi-1)\,R^u_{n,\theta}(\theta^u_n)w_n + (\psi-1)\,R^d_{n,\eta}(\eta^d_{nn})w_n\right] \\ \sum_{l\neq n}\mathbb{I}^u_{nl}(q^u)\left[\overline{\pi}^u_{nl}q^u - f^u_{nl}w_n + (\psi-1)\,R^d_{n,\eta}(\eta^d_{nl})w_n\right] \end{array}\right\} \\
&\sum_{q^d\in Q^d}\left\{\begin{array}{l} \mathbb{I}^d_{nn}(q^d)\left[\overline{\pi}^d_{nn}q^d - f^d_{nn}w_n + (\psi-1)\,R^d_{n,\theta}(\theta^d_n)w_n + \sum_l (\psi-1)\,R^u_{n,\eta}(\eta^u_{nl})w_n\right] \\ \sum_{l\neq n}\mathbb{I}^d_{nl}(q^d)\left[\overline{\pi}^d_{nl}q^d - f^d_{nl}w_n\right] \end{array}\right\}
\end{aligned} \tag{79}$$

One can now easily solve for market entry decisions faced by the incumbent, $\mathbb{I}^s_{nl}(q^s)$. The variable profits made by each product in each market is proportional to its quality and the return to R&D is a constant in terms of labor. Therefore, every product that has a high enough quality to make up for the fixed cost of market entry is supplied to the market. Defining net fixed costs

as the value of fixed cost of entry to a market minus the fixed return to R&D from entering that market as:

$$\begin{aligned} \text{Upstream:} \quad \tilde{f}^u_{nl} &= \begin{cases} f^u_{nn} - (\psi-1)R^u_{n,\theta}\left(\theta^u_n\right) - (\psi-1)R^d_{n,\eta}\left(\eta^d_{nn}\right) & \text{if} \quad l = n \\ f^u_{nl} - (\psi-1)R^d_{n,\eta}\left(\eta^d_{nl}\right) \quad \text{if} \quad l \neq n \end{cases} \\ \text{Downstream:} \quad \tilde{f}^d_{nl} &= \begin{cases} f^d_{nn} - (\psi-1)R^d_{n,\theta}\left(\theta^d_n\right) - (\psi-1)\sum_l R^u_{n,\eta}\left(\eta^u_{nl}\right) & \text{if} \quad l = n \\ f^d_{nl} \quad \text{if} \quad l \neq n \end{cases} \end{aligned} \tag{80}$$

One can now solve for the optimal market entry decisions as:

$$\begin{aligned} \mathbb{I}^s_{nl}(q^s) &= \begin{cases} 1 & \text{if } q^s \geq q^{s*}_{nl} \\ 0 & \text{if } q^s < q^{s*}_{nl} \end{cases} \\ q^{s*}_{nl} &= \frac{\tilde{f}^s_{nl} w_n}{\bar{\pi}^s_{nl}} \end{aligned} \tag{81}$$

C.4.3 Incumbent Value Function

Given the structure of value function defined in equation 73 and optimal firm level decisions pertaining investment in R&D activities detailed in equations 75 and 77, and optimal market entry decisions for each product detailed in equation 81, the incumbent problem can further be simplified as:

$$\sum_{s\in\{u,d\}} \sum_{q^s \in Q^s} \sum_l \mathbb{I}^s_{nl}(q^s)\left[(r_n+\delta)v^s_{nl}(q^s) - \dot{v}^s_{nl}(q^s)\right] = \sum_{s\in\{u,d\}} \sum_{q^s \in Q^s} \sum_l \mathbb{I}^s_{nl}(q^s)\left[\bar{\pi}^s_{nl} q^s - \tilde{f}^s_{nl} w_n\right] \tag{82}$$

For the rest of this subsection, I will focus only on the market pair (nl) terms and derive the value function in a BGP. It is useful to normalize the value function so it is constant on the BGP. I normalize by the value function of a country by the wage of that country w_n, and define the following transformed functions and variables, that are constant on the BGP:

$$\begin{aligned} \hat{q}^s &= q^s / q^{s*}_{nn} \\ \hat{\mathbb{I}}^s_{nl}(\hat{q}^s) &= \mathbb{I}^s_{nl}(q^s) \\ \hat{\bar{\pi}}^s_{nl} &= \bar{\pi}^s_{nl} q^{s*}_{nn} / w_n \\ \hat{v}^s_{nl}(\hat{q}^s) &= v^s_{nl}(q^s)/w_n \end{aligned}$$

The normalized market pair level value function is given by dividing both sides of equation 82 by w_n, and separating the market pair terms:

$$(r_n+\delta)\hat{v}^s_{nl}(\hat{q}^s) - \frac{\dot{v}^s_{nl}(q^s)}{w_n} = \hat{\bar{\pi}}^s_{nl}\hat{q}^s - \tilde{f}^s_{nl} \tag{83}$$

Recall that terms in the above equation are all time dependent. Recognizing that the threshold quality grows at a constant rate in the BGP: $\frac{\mathrm{d}q^{s*}_{nn}/\mathrm{d}t}{q^{s*}_{nn}} = g^s_n$, and wage grows at a constant rate in

BGP: $\frac{\mathrm{d}w_n/\mathrm{d}t}{w_n} = g_{w_n}$, one can derive the time derivation of the value function as following:

$$\begin{aligned}\dot{v}^s_{nl}(q^s) &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\hat{v}^s_{nl}(\hat{q}^s)w_n\right)\\ &= w_n\left[\frac{\partial \hat{v}^s_{nl}(\hat{q}^s)}{\partial t} - \hat{q}^s g^s_n \frac{\partial \hat{v}^s_{nl}(\hat{q}^s)}{\partial \hat{q}^s} + g_{w_n}\hat{v}^s_{nl}(\hat{q}^s)\right]\end{aligned} \tag{84}$$

In the BGP, the normalized value function is time-invariant, and hence the first term in equation 84 is zero. Substituting equation 84 into equation 83, we get:

$$\begin{aligned}(r_n + \delta - g_{w_n})\hat{v}^s_{nl}(\hat{q}^s) + \hat{q}^s g^s_n \frac{\partial \hat{v}^s_{nl}(\hat{q}^s)}{\partial \hat{q}^s} &= \hat{\bar{\pi}}^s_{nl}\hat{q}^s - \tilde{f}^s_{nl}\\ \kappa^s_n\left(\hat{q}^s\right)^{\kappa^s_n - 1}\hat{v}^s_{nl}(\hat{q}^s) + \left(\hat{q}^s\right)^{\kappa^s_n}\frac{\partial \hat{v}^s_{nl}(\hat{q}^s)}{\partial \hat{q}^s} &= \hat{\bar{\pi}}^s_{nl}\left(\hat{q}^s\right)^{\kappa^s_n} - \tilde{f}^s_{nl}\left(\hat{q}^s\right)^{\kappa^s_n - 1}\\ \frac{\mathrm{d}}{\mathrm{d}\hat{q}^s}\left(\left(\hat{q}^s\right)^{\kappa^s_n}\hat{v}^s_{nl}(\hat{q}^s)\right) &= \hat{\bar{\pi}}^s_{nl}\left(\hat{q}^s\right)^{\kappa^s_n} - \tilde{f}^s_{nl}\left(\hat{q}^s\right)^{\kappa^s_n - 1}\end{aligned} \tag{85}$$

where $\kappa^s_n = \frac{r_n + \delta - g_{w_n}}{g^s_n}$, and the second line is a result of multiplying both sides of the equation by $\frac{(\hat{q}^s)^{\kappa^s_n}}{g^s_n}$. Under assumption 2, the solution to the the differential equation in equation 85 is given by (after re-normalizing):

$$v^s_{nl}(q^s) = \left(\frac{\bar{\pi}^s_{nl}q^s}{g^s_n(\kappa^s_n + 1)} - \frac{\tilde{f}^s_{nl}w_n}{g^s_n\kappa^s_n}\right) - \left(\frac{\bar{\pi}^s_{nl}q^{s*}_{nl}}{g^s_n(\kappa^s_n + 1)} - \frac{\tilde{f}^s_{nl}w_n}{g^s_n\kappa^s_n}\right)\left(\frac{q^s}{q^{s*}_{nl}}\right)^{-\kappa^s_n} \tag{86}$$

C.5 Prices and Trade Shares

In this subsection, I will derive closed form solutions for the price index and the trade shares when quality distributions follows Pareto. From the expression for the stage-specific price index in equation 15, the equilibrium prices charged in each market from equation 24, and the Pareto distribution of qualities specified in equation 44 we have:

$$\begin{aligned}P^s_n &= \left[\sum_{l\in\mathcal{N}}\int_{\omega^s\in\Omega^{js}_{ln}} q^s_l(\omega^s)\left(p^s_{ln}(\omega^s)\right)^{1-\sigma^s}\mathrm{d}\omega^s\right]^{\frac{1}{1-\sigma^s}}\\ &= \left[\sum_{l\in\mathcal{N}} M^s_l\left(p^s_{ln}\right)^{1-\sigma^s}\int_{q^{s*}_{ln}}^{\infty} q^s \mathrm{d}G^s_l(q^s)\right]^{\frac{1}{1-\sigma^s}}\\ &= \left[\sum_{l\in\mathcal{N}} M^s_l\left(p^s_{ln}\right)^{1-\sigma^s}\left(\frac{q^{s*}_{ln}}{q^{s*}_{nn}}\right)^{-\gamma^s}\frac{\gamma^s}{\gamma^s - 1}q^{s*}_{ln}\right]^{\frac{1}{1-\sigma^s}}\\ &= \left(\frac{\sigma^s}{\sigma^s - 1}\right)\left(\frac{\gamma^s}{\gamma^s - 1}\right)^{\frac{1}{1-\sigma^s}}\left[\sum_{l\in\mathcal{N}} M^s_l\left(q^{s*}_{ll}\right)^{\gamma^s}\left(q^{s*}_{ln}\right)^{1-\gamma^s}\left(\tau^s_{ln}c^s_l\right)^{1-\sigma^s}\right]^{\frac{1}{1-\sigma^s}}\end{aligned} \tag{87}$$

Import shares are defined as $\lambda^s_{ln} = \frac{X^s_{ln}}{\sum_l X^s_{ln}}$, where X^s_{ln} is the value of expenditure by country

n spent on stage s goods from country l, and is defined in equation 27. Using this expression for import expenditures, and substituting for market pair specific profits in equation 25, we get,

$$\begin{aligned} X_{ln}^s &= \sigma^s \overline{\pi}_{ln}^s M_l^s \int_{q_{ln}^{s*}}^{\infty} q^s dG_l^s(q^s) \\ &= Y_n^s \left(P_n^s\right)^{\sigma^s} \left(p_{ln}^s\right)^{1-\sigma^s} M_l^s \left(\frac{q_{ln}^{s*}}{q_{nn}^{s*}}\right)^{-\gamma^s} \frac{\gamma^s}{\gamma^s - 1} q_{ln}^{s*} \\ &\propto M_l^s \left(q_{ll}^{s*}\right)^{\gamma^s} \left(q_{ln}^{s*}\right)^{1-\gamma^s} \left(\tau_{ln}^s c_l^s\right)^{1-\sigma^s} \end{aligned}$$

It is useful to derive the export thresholds q_{ln}^{s*} in terms of only domestic entry thresholds. From the expression for market thresholds in equation 81 and market pair specific profits in equation 25, we have:

$$\frac{q_{ln}^{s*}}{q_{nn}^{s*}} = \frac{\tilde{f}_{ln}^s w_l}{\tilde{f}_{nn}^s w_n} \frac{\overline{\pi}_{nn}^s}{\overline{\pi}_{ln}^s} = \frac{\tilde{f}_{ln}^s w_l}{\tilde{f}_{nn}^s w_n} \left(\frac{\tau_{ln}^s c_l^s}{\tau_{nn}^s c_n^s}\right)^{\sigma^s - 1}$$

Using this expression for q_{ln}^{s*} in the expression for import values, we have,

$$X_{ln}^s \propto M_l^s \left(q_{ll}^{s*}\right)^{\gamma^s} \left(\tilde{f}_{ln}^s w_l\right)^{1-\gamma^s} \left(\tau_{ln}^s c_l^s\right)^{(1-\sigma^s)\gamma^s}$$

Trade shares can then be written as:

$$\lambda_{ln}^s = \frac{M_l^s \left(q_{ll}^{s*}\right)^{\gamma^s} \left(\tilde{f}_{ln}^s w_l\right)^{1-\gamma^s} \left(\tau_{ln}^s c_l^s\right)^{(1-\sigma^s)\gamma^s}}{\sum_{l'} M_{l'}^s \left(q_{l'l'}^{s*}\right)^{\gamma^s} \left(\tilde{f}_{l'n}^s w_{l'}\right)^{1-\gamma^s} \left(\tau_{l'n}^s c_{l'}^s\right)^{(1-\sigma^s)\gamma^s}} \tag{88}$$

C.6 Proof of Proposition 3: Growth

Given the BGP value functions from proposition 2, we can now derive the BGP growth rate of the minimum threshold of the stage-specific quality distributions from the free entry condition defined in equation 37. We have:

$$\begin{aligned} f_{ne}^s w_{nt} &= \int_{q_{nn}^{s*}}^{\infty} V_{nt}(\{q^s\}, \emptyset) dG_{nt}^s(q^s) = \int_{q_{nn}^{s*}}^{\infty} \sum_l \mathbb{I}_{nl}^s(q^s) v_{nl}^s(q^s) dG_{nt}^s(q^s) \\ &= \sum_l \int_{q_{nl}^{s*}}^{\infty} \left(\frac{\overline{\pi}_{nl}^s q^s}{g_n^s(\kappa_n^s + 1)} - \frac{\tilde{f}_{nl}^s w_n}{g_n^s \kappa_n^s}\right) - \left(\frac{\overline{\pi}_{nl}^s q_{nl}^{s*}}{g_n^s(\kappa_n^s + 1)} - \frac{\tilde{f}_{nl}^s w_n}{g_n^s \kappa_n^s}\right) \left(\frac{q^s}{q_{nl}^{s*}}\right)^{-\kappa_n^s} dG_{nt}^s(q^s) \end{aligned}$$

Using the stationary Pareto distribution from proposition 1, and the observation that $\overline{\pi}_{nl}^s q_{nl}^{s*} = \tilde{f}_{nl}^s w_n$, one can derive a closed form solution to the above integral.

$$f_{ne}^s = \frac{1}{g_n^s(\gamma^s - 1)(\gamma^s + \kappa_n^s)} \sum_l \left(\frac{q_{nl}^{s*}}{q_{nn}^{s*}}\right)^{-\gamma^s} \tilde{f}_{nl}^s \tag{89}$$

Note that terms inside κ_n^s can be simplified further. From the Euler equation, we have $g_{C_n}+g_{P_n} = r_n - \rho$ where $g_{C_n} = \dot{C}_n/C_n$ is the growth rate of consumption per-capita, and $g_{P_n} = \dot{P}_n/P_n$ is the growth rate of the final good price index. Further, aggregate income in the economy should equal final consumption, and in a BGP, it has to be the case that the share of labor income in aggregate income is constant. Therefore, $\frac{w_n L_n}{C_n P_n}$ is constant in BGP. Since labor does not grow in this model, the growth rate of wages should be equal to the growth rate of consumption plus price: $g_{w_n} = g_{C_n} + g_{P_n}$. Therefore, from the Euler equation and BGP labor income share condition we have, $r_n - g_{w_n} = \rho$. Substituting this into the expression for κ_n^s, we can write $\kappa_n^s = \frac{\delta+\rho}{g_n^s}$. Using this expression in equation 89 and rearranging terms, we have:

$$g_n^s = \frac{1}{\gamma^s(\gamma^s - 1)} \sum_l \left(\frac{q_{nl}^{s*}}{q_{nn}^{s*}}\right)^{-\gamma^s} \left(\frac{\tilde{f}_{nl}^s}{f_{ne}^s}\right) - \frac{\delta + \rho}{\gamma^s} \tag{90}$$

In a world with knowledge spillovers, we have $g_n^s \equiv g^s$. That is, for a given stage s all countries grow at the same rate since entrants from every country benefit from knowledge from every other country.

C.7 Output Growth

Now that we have the growth rate of average qualities of each stage,[53] it is useful to derive the growth rate of the stage-specific output. As mentioned in the previous subsection, on a balanced growth path, labor income is a constant share of total income in the economy, which gives us: $g_{w_n} = g_{C_n} + g_{P_n}$. Further, on the BGP, it should also be true that final consumption is a constant share of final good produced.[54] Given this, the growth rate of consumption should be equal to the growthrate of final good: $g_{C_n} = g_{Y_n}$. Combining these two observations regarding the BGP, we have:

$$g_{Y_n} = g_{w_n} - g_{P_n} \tag{91}$$

Note that the price of final good is the Cobb-Douglas aggregate of downstream price indices of all industries as defined in 13. From here on, for conciseness I solve for the economy wide growth rates in a world with single industry $\mathcal{J} = 1$. The derivation for the multiple industry case follows a similar path. In a single industry case, the price index of the final good is just the price of the downstream stage output, $g_{P_n^d}$:

$$g_{P_n} = g_{P_n^d} \tag{92}$$

I will now use the closed form expression for stage-specific price index derived in equation 87. Taking the time derivative of this equation gives us the relationship between the growth rate of price index $g_{P_n^s}$, growth rate of average quality g_n^s, and the growth rate of unit costs $g_{c_n^s}$ of a

[53]Note that average quality is proportional to the minimum threshold quality of the Pareto distribution. Therefore, the growth rate of average quality is equal to g^s

[54]Note that final good is used for both final consumption and as a material input into other production activities.

stage in an economy. Note that, on a BGP, both domestic entry cutoffs q_{nn}^{s*} and exports entry cutoffs q_{nl}^{s*} have to grow at the same rate g^s. Also on the BGP, the mass of varieties is constant. With these observations, we can see from the price index equation that:

$$g_{P_n^s} = g_{c_l^s} - g^s/(\sigma^s - 1) \quad \forall\, l \tag{93}$$

Since this equation has to hold for all origin countries l, it has to be true that the right hand side of equation 93 is equalized across all countries, leading to $g_{c_n^s} = g_{c^s}$. Given that the growth rate of unit cost, and average qualities are only stage-specific and not country-specific, the growth rates of stage-specific price index also equalizes across all countries. Therefore, equation 93 simplifies to:

$$g_{P^s} = g_{c^s} - g^s/(\sigma^s - 1) \tag{94}$$

Further, the price index of final good, from equation 92 also equalizes across countries: $g_{P_n} = g_{P^d}$.

From the expressions for stage-specific unit cost of production in equations 18 and 21, we can derive the growth rate of unit cost as a function of growth rate of other prices in the economy:

$$\begin{aligned} g_{c_n^s} &= \beta^{ls} g_{w_n} + \beta^{fs} g_{P_n} + \beta^{us} g_{P_n^u} \\ g_{c^s} &= \beta^{ls} g_{w_n} + \beta^{fs} g_{P^d} + \beta^{us} g_{P^u} \quad \forall\, n \end{aligned} \tag{95}$$

The second line follows from the discussion above.[55] Equation 95, one can see that the growth rate of wages have to equalize across countries for the equation to hold for unit costs in all countries. Therefore, we have the growth rate of stage-specific unit costs to be given by: $g_{c^s} = \beta^{ls} g_w + \beta^{fs} g_{P^d} + \beta^{us} g_{P^u}$.

Let $\bar{g}^s = g^s/(\sigma^s - 1)$. Substituting equation 93 for both stages into equation 95, and using the fact that $\beta^{ls} = 1 - \beta^{fs} - \beta^{us}$, we get:

$$g_{c^s} - g_w = \beta^{us}(g_{c^u} - g_w) + \beta^{fs}(g_{c^s} - g_w) - \beta^{us} g^u/(\sigma^u - 1) - \beta^{fs} g^d/(\sigma^d - 1)$$

In matrix form:

$$\begin{pmatrix} g_{c^u} - g_w \\ g_{c^d} - g_w \end{pmatrix} = \begin{pmatrix} \beta^{uu} & \beta^{fu} \\ \beta^{ud} & \beta^{fd} \end{pmatrix} \begin{pmatrix} g_{c^u} - g_w \\ g_{c^d} - g_w \end{pmatrix} - \begin{pmatrix} \beta^{uu} & \beta^{fu} \\ \beta^{ud} & \beta^{fd} \end{pmatrix} \begin{pmatrix} 1/(\sigma^u - 1) & 0 \\ 0 & 1/(\sigma^d - 1) \end{pmatrix} \begin{pmatrix} g^u \\ g^d \end{pmatrix}$$

$$\begin{aligned} (\mathbf{g_{c^s}} - \mathbf{g_w}) &= \mathbf{B}(\mathbf{g_{c^s}} - \mathbf{g_w}) - \mathbf{BSg^s} \\ (\mathbf{g_{c^s}} - \mathbf{g_w}) &= -\left(\mathbf{I} - \mathbf{B}\right)^{-1} \mathbf{BSg^s} \end{aligned} \tag{96}$$

From the goods market clearing condition, we have that expenditures on upstream output depend on expenditures on downstream output, which further depends on the expenditures on

[55] Note that $\beta^{uu} = 0$. The growth rate of final good price index is substituted by the growth rate of downstream price index as explained in equation 92.

final good.

$$X_n^u = \beta^{ud}\epsilon^d \sum_l \lambda_{nl}^d X_l^d \tag{97}$$

$$X_n^d = X_n \tag{98}$$

where $X_n^s = Y_n^s P_n^s$ and $X_n = Y_n P_n$. Differentiating equations 97 and 98, and using the result from equation 93 which shows growth equalization across countries for stage-specific price index, we get:

$$g_{Y_n^u} + g_{P^u} = g_{Y_l^d} + g_{P^d} = g_{Y_l} + g_P \quad \forall l \tag{99}$$

Since this equation has to hold for all l, it has to be true that the growth rates of downstream output and final output, across all countries are equalized, i.e. $g_{Y_l^d} = g_{Y^d}$ and $g_{Y_l} = g_Y$. With this the right hand side of equation 99 is constant for all countries n, which implies that the growth rates of upstream output has to be equalized across all countries, i.e. $g_{Y_n^u} = g_{Y^u}$. Combining these observations with equation 91, we have

$$g_{Y^u} + g_{P^u} = g_{Y^d} + g_{P^d} = g_Y + g_P = g_w \tag{100}$$

Substituting equations 100 and 94, we get:

$$g_{Y^s} = -(g_{c^s} - g_w) + g^s/(\sigma^s - 1)$$

Substituting for the first term on the right hand side from equation 96, and representing in matrix form, we get:

$$\mathbf{g_{Y^s}} = \left(\mathbf{S} + (\mathbf{I} - \mathbf{B})^{-1}\mathbf{BS}\right)\mathbf{g^s} \tag{101}$$

And since $g_P = g_{P^d}$, the growth of final good is simply equal to the growth of downstream output, $g_Y = g_{Y^d}$.

D Quantitative Appendix

Table D31: Calibration: Countries in the Foreign Aggregate

Country	Trade Share	Rank	Country	Trade Share	Rank
United States	0.128	1	South Korea	0.031	6
China	0.102	2	Italy	0.031	7
Germany	0.073	3	Japan	0.031	8
France	0.037	4	Australia	0.028	9
Great Britain	0.034	5	Taiwan	0.017	10

Notes: This table lists the countries included in the rest of the world aggregate for model calibration. Countries are chosen on the basis of their rank in trade shares with India. Trade share for a country with India is defined as the share of imports from India and exports to India of that country over total imports and exports.

Table D32: Calibration: Industries and Stages of Production

Code	Industry Description	FD Share	Stage
C24	Manufacture of basic metals	0.05	up
C17	Manufacture of paper and paper products	0.16	up
C16	Manufacture of wood and of products of wood and cork	0.17	up
C22	Manufacture of rubber and plastic products	0.19	up
C23	Manufacture of other non-metallic mineral products	0.21	up
C20	Manufacture of chemicals and chemical products	0.22	up
C25	Manufacture of fabricated metal products	0.26	up
C18	Printing and reproduction of recorded media	0.34	up
C26	Manufacture of computer, electronic and optical products	0.47	up
C27	Manufacture of electrical equipment	0 0.51	down
C19	Manufacture of coke and refined petroleum products	0.54	down
C13-C15	Manufacture of textiles, wearing apparel and leather products	0.57	down
C21	Manufacture of basic pharmaceutical products	0.58	down
C28	Manufacture of machinery and equipment n.e.c.	0.60	down
C29	Manufacture of motor vehicles, trailers and semi-trailers	0.61	down
C30	Manufacture of other transport equipment	0.69	down
C31-32	Manufacture of furniture; other manufacturing	0.76	down
C10-12	Manufacture of food products, beverages and tobacco products	0.77	down

Notes: This table lists the set of industries used in the calibration strategy. Industries are as defined in the World Input Output Database, and the column labeled "Code" represents the corresponding ISIC Rev.4 industry code. FD Share is the share of total industry output that is directly consumed by households and governments. Industries are categorized as upstream or downstream based on the share of output that goes into final demand. Higher share implies the industry is more downstream.

Table D33: Plant Types for Calibration

Plant Type	Share
Single-Product	0.69
Multi-Product Single-Stage	0.24
Multi-Product Multi-Stage	0.07

Table D34: Plant Type Transitions for Calibration

		year $(t+1)$		
		Single-Product	Multi-Product Single-Stage	Multi-Product Multi-Stage
year t	Single-Product	0.90	0.08	0.02
	Multi-Product Single-Stage	0.21	0.71	0.08
	Multi-Product Multi-Stage	0.18	0.24	0.58

This matrix represents the transition of firm types as categorized by the number of products and the number of production stages produced from one year to the next, classified for the purpose of calibration. Each row sums to 1, and each element in a given row represents a share.

Table D35: Gravity Estimation: Upstream

Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	Avg
log(Tariff)	-6.76***	-9.99***	-3.18	-5.02***	-12.04***	-6.04***	-16.75***	-4.79**	-13.61***	-10.58
	(2.38)	(2.21)	(5.79)	(1.84)	(2.45)	(2.16)	(3.72)	(1.93)	(3.45)	
log(Distance)	-0.30***	-0.37***	-0.55***	-0.35***	-0.32***	-0.45***	-0.34***	-0.41***	-0.29***	-0.35
	(0.05)	(0.05)	(0.08)	(0.03)	(0.07)	(0.05)	(0.07)	(0.05)	(0.05)	
Border	-1.71***	-1.87***	-0.70*	-1.33***	-1.26***	-1.04***	-0.98***	-0.96***	-0.61***	-1.07
	(0.20)	(0.17)	(0.36)	(0.16)	(0.20)	(0.21)	(0.31)	(0.19)	(0.23)	
Language	-0.38**	-0.16	-0.51	-0.03	-0.14	-0.19	-0.30	-0.00	0.05	-0.12
	(0.18)	(0.25)	(0.31)	(0.18)	(0.21)	(0.20)	(0.30)	(0.19)	(0.23)	
Colonizer	-1.11***	-0.31	-1.28***	0.05	-0.38	-0.68**	0.30	-0.76***	-0.60*	-0.27
	(0.24)	(0.29)	(0.49)	(0.21)	(0.33)	(0.28)	(0.37)	(0.23)	(0.33)	
Legal Origins	-0.05	-0.20*	-0.00	-0.21**	-0.26**	-0.21*	-0.18	-0.25**	-0.44***	-0.26
	(0.12)	(0.12)	(0.22)	(0.10)	(0.11)	(0.12)	(0.14)	(0.11)	(0.12)	
Colonial Ties	-0.67***	-0.16	-0.48*	-0.21	-0.41	-0.52*	-0.19	-0.29	0.21	-0.17
	(0.19)	(0.21)	(0.28)	(0.22)	(0.31)	(0.27)	(0.29)	(0.22)	(0.28)	
Sector	7	8	9	11	13	14	15	16	17	
Observations	1,060	1,069	1,046	1,098	1,098	1,088	1,067	1,055	1,103	

Notes: This table shows the gravity estimates from PPML for the sectors that are classified ad Upstream in the WIOD database. The last column gives the weighted average of the estimates across the sectors with sectoral output weights. Each of the following variables are coded as 0 if a country pair have a common feature, and 1 if they do not: Border, Language, Colonizer Legal Origins and Colonial Ties. The sector associated with the sector codes in the last but one row are listed in the appendix. Standard errors are in parenthesis. *** p<0.01, ** p<0.05, * p<0.1

Table D36: Gravity Estimation: Downstream

Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	Avg
log(Tariff)	-1.29**	-8.91***	-5.44	-30.37***	-15.14***	-10.48***	-15.02***	-6.79***	-18.57***	-9.89
	(0.64)	(1.42)	(4.30)	(3.18)	(2.87)	(2.66)	(2.73)	(2.02)	(2.40)	
log(Distance)	-0.48***	-0.40***	-0.62***	-0.38***	-0.19***	-0.23***	-0.43***	-0.37***	-0.37***	-0.41
	(0.04)	(0.05)	(0.06)	(0.05)	(0.06)	(0.04)	(0.06)	(0.06)	(0.05)	
Border	-0.72***	-1.23***	-0.94***	-0.76***	-1.09***	-1.11***	-1.43***	-0.73***	-0.97***	-1.04
	(0.16)	(0.26)	(0.27)	(0.27)	(0.26)	(0.16)	(0.19)	(0.23)	(0.21)	
Language	-0.58***	-0.51*	-0.29	-0.51*	0.32	-0.12	0.23	-0.73**	-0.21	-0.25
	(0.16)	(0.28)	(0.34)	(0.26)	(0.25)	(0.16)	(0.26)	(0.28)	(0.24)	
Colonizer	-0.64***	0.21	0.98**	0.60*	-0.86**	-0.40*	0.21	0.80**	0.07	-0.05
	(0.20)	(0.39)	(0.39)	(0.36)	(0.35)	(0.21)	(0.34)	(0.35)	(0.31)	
Legal Origin	-0.15	-0.08	-0.37**	-0.33***	-0.30**	-0.14	-0.09	-0.15	-0.17	-0.17
	(0.12)	(0.13)	(0.19)	(0.12)	(0.13)	(0.09)	(0.11)	(0.15)	(0.12)	
Colonial Ties	-0.28	0.60**	0.00	0.03	-0.64*	-0.56***	0.89***	-0.04	0.05	0.04
	(0.22)	(0.29)	(0.24)	(0.34)	(0.33)	(0.18)	(0.29)	(0.35)	(0.28)	
Sector	5	6	10	12	18	19	20	21	22	
Observations	1,079	1,097	889	973	1,061	1,104	1,079	1,009	1,098	

Notes: This table shows the gravity estimates from PPML for the sectors that are classified ad Downstream in the WIOD database. The last column gives the weighted average of the estimates across the sectors with sectoral output weights. Each of the following variables are coded as 0 if a country pair have a common feature, and 1 if they do not: Border, Language, Colonizer Legal Origins and Colonial Ties. The sector associated with the sector codes in the last but one row are listed in the appendix. Standard errors are in parenthesis. *** p<0.01, ** p<0.05, * p<0.1