The Equilibrium Impact of Robots on Labor Markets*

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Abstract

We examine the impact of industrial robots on labor markets, focusing on changes in skill-premia and real wages by skill-type. We hypothesize the existence of robot-skill complementarity, appropriately embedding robots in our production function, and develop a static Ricardian trade model to derive the equilibrium effects. The model captures the displacement effect of robots as they compete with labor, as well as the productivity effect that comes through decreases of costs and consumption prices. We explicitly derive from our model a linearized equation which we can directly use to estimate the key elasticity parameters. The ordering of the obtained estimates supports our hypothesis of robot-skill complementarity, as well as -the established in the literature- assumption of capital-skill complementarity. Our estimates and derived equations suggest that automation benefits both skill-types. Nevertheless, the relative gains are at least 2.3 times larger for the high-skilled labor, widening the existing welfare gap. Our model allows us to use our estimates to also quantify the impact of trade on labor markets. As in the case of robot adoption, we find that while both skill-types gain from trade openness, the high-skilled are benefited 2.25 times more than the low-skilled; our results agree with the existing literature which claims that trade is skill-biased.

Keywords: automation, industrial robots, trade gains, skill-premium

JEL classification: E23; F11; F16; O33

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1 Introduction

With self-driving cars already roaming our streets and Artificial Intelligence (AI) getting seats in company boardrooms in Hong-Kong[1], the emergence of automation is more tangible than ever before. Frey and Osborne (2017) report that 46 percent of U.S. workers are at risk of losing jobs to automation over the course of the two next decades. Ominous science-fiction stories that warn of the dominance of robots over humankind seem to have perhaps influenced public opinion. According to a survey conducted by the Pew Research Center[2] twice as many Americans (72 percent) are expressing worry rather than enthusiasm (36 percent) about a future in which robots can do tasks currently performed by humans. What is more, the steady contraction of the labor income share during the last three decades (Karabarbounis and Neiman (2013)) seems to confirm the concerns about the future of automation (Brynjolfsson and McAfee (2014)). The same pessimism was also conveyed by John Maynard Keynes, who in 1930 stated that, "We are being addicted with a new disease of which some readers may not have heard the name, but of which they will hear a great deal in the years to come—namely, technological unemployment" (Keynes (1930)). Fortunately, his predictions did not come true, as technology, instead of substituting, has largely augmented the productivity of labor, bringing tremendous economic growth since the Great Depression.

Some scholars think that the effects of automation and AI will not prove to be any different. For example, consider that the introduction of automated teller machines (ATMs) coincided with the expansion in the employment of bank tellers. According to Bessen (2016) ATMs decreased substantially the cost of operations, inducing a scale

[1] Nikkei Asian Review
[2] Pew Research Center

effect that encouraged banks to open more branches and to hire more bank tellers to perform specialized tasks other than money withdrawals or deposits. Nevertheless, European experts who participated at the Initiative on Global Markets (IGM) forum at Chicago Booth are divided on whether "rising use of robots and artificial intelligence is likely to increase substantially the number of workers in advanced countries who are unemployed for long periods."[3] So, before we start planning a neo-Luddite movement, let us use economic reasoning to understand whether an alarmist reaction is even justified.

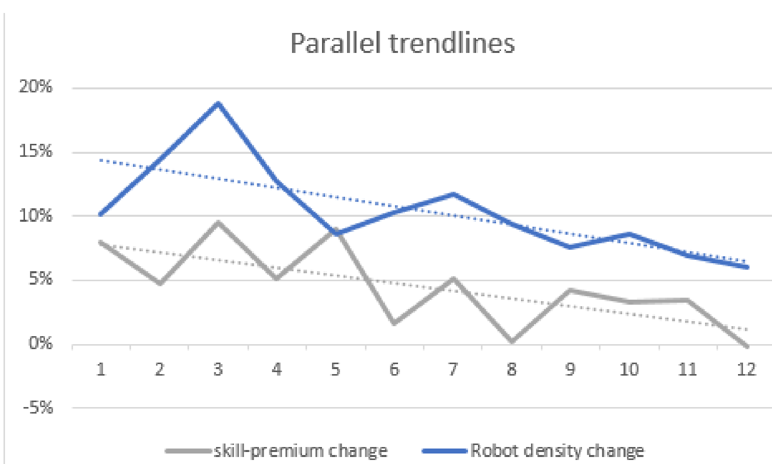
For our analysis we are going to develop a Ricardian trade model in a formulation firstly suggested in Eaton and Kortum (2002), which captures the equilibrium impact of robots on labor markets. As it has been clearly conceptualized in Acemoglu and Restrepo (2018), the automation effect manifests itself into two countervailing forces: a direct displacement effect and a productivity spillover effect. The displacement effect is a natural consequence of robots' substituting for human labor and therefore causes a downward pressure on wages. Opposing to it stands the productivity effect that results from decreases in the cost of production of intermediate goods and diffuses across sectors and across countries through trade, pushing wages up while lowering consumption prices. These countervailing forces exactly represent the dichotomy in the views about the impact of automation with each side of the debate believing that one of the effects will unambiguously dominate over the other. Our goal is to address this dichotomy by deriving equations that describe the combined effects.

We will focus our attention on the impact of industrial robots using data released by the International Federation of Robotics (2016). The IFR defines an industrial robot as an "automatically controlled, reprogrammable, multipurpose manipulator programmable in three or more axes, which may be either fixed in place or mobile for use in industrial

[3] IGM 2017

automation applications." Hence, we will not be concerned with examples of automation such as ATMs or AI applications, but we will limit ourselves to automation occurring through industrial robots. This limitation is well-justified since industrial robots have been the dominant form of automation technology for the last three decades (International Federation of Robotics (2016)). The number of installed industrial robots in Western Europe increased fourfold from 1993 to 2016. This increase was accompanied by a spectacular decrease in robot prices of almost 80 percent from 1993 to 2005, if we adjust the prices for quality improvements[4]. Current IFR reports put the number of operational robots globally close to 2 million, and this number is anticipated to double or triple by 2025 (BCG (2015)). To motivate our analysis, we plot the change of skill-premium on the change of robot density (defined as robots over efficiency hours of labor) in Germany from 1993 to 2005 as depicted in Figure 1. This positive correlation between robot densification and the change in the skill-premium suggests that there might be some sort of robot-skill complementarity, which we will now investigate in depth.

Figure 1: Percentage Change in Skill-Premium over Robot Densification(Germany 1993-2005)



4 IFR report(2006)

Our framework incorporates labor heterogeneity embedding a relative robot-skill complementarity, similarly to Krusell, Ohanian, RÌos-Rull, and Violante (2000), who analyze capital-skill complementarity. Labor comes in two flavors: high-skilled and low-skilled, and it is supplied inelastically. Although our approach does not separate between the two effects described above, it can completely characterize the aggregate impact of robots on the skill-premium (a proxy for inequality) and on the real wages for each skill-type (a proxy for welfare). We introduce this robot-skill complementarity through a Constant Elasticity of Substitution (CES) production function, where we hypothesize that the elasticity of substitution between robots and low-skilled labor is higher than the respective elasticity between robots and high-skilled workers; hence, robots and skill are relative complements. We maintain the capital-skill complementarity hypothesis, as it was proposed in Krusell, Ohanian, RÌos-Rull, and Violante (2000). We extend their specification by separating robots from the rest of intermediate capital inputs and embedding the production in a Ricardian trade model to derive the appropriate equilibrium equations. The parameters that dictate the magnitude and the sign of the effects on the skill-premia and the real wages are the elasticities of substitution between the factors, as well as the elasticity of substitution of trade that captures the productivity dispersion of the intermediate goods.

For our empirical analysis, we are going to estimate these parameters by using linearized forms of our equilibrium equations and data on skill-premia, skill-supply and volumes of intermediate capital inputs from the EUKLEMS data set (van Ark and Jäger (2017)), as well as data on the stock of operational robots, as reported by the IFR. To address potential endogeneity concerns, we use appropriate instruments for our regression analysis without sacrificing the statistical significance of our estimates. Obtaining these estimates will help us determine the impact of robots on skill-premia and real wages, in other words, whether there are gains from robotics and how they are distributed by skill-type. Moreover, they allow us to perform counterfactual exercises that test whether trade mitigates or intensifies the impact of automation. The importance of these considerations lies in their social and political implications. We have seen that recently there has been a populist backlash supported by those who consider themselves the losers of globalization. Similar reactions can be prompted if the gains from automation asymmetrically favor skilled-labor and capital-owners, sharpening the existing inequality.

Our results support the hypothesis of robot-skill complementarity, as well as the assumption of capital-skill complementarity, which is well-established in the literature (Krusell, Ohanian, RÌos-Rull, and Violante (2000),McAdam and Willman (2018)). Subsequently, the obtained estimates are used to predict the change in skill premia through equations that were not used for the regression. Our Ordinary Least Squares (OLS) estimates achieve good model fit, in contrast to the estimates obtained by the IV regression. As it is hinted by robot-skill complementarity, automation increases the skill-premium; nevertheless it benefits everyone in terms of real wages. The impact of trade openness is similar, as it is shown to be skill-biased staying consistent with the results of Burstein, Cravino, and Vogel (2013) and Parro (2013).

The starting point and source of inspiration for this paper has been the recently published series of papers by Acemoglu and Restrepo. Acemoglu and Restrepo (2017b) estimate the impact of robots on employment and wages on local U.S. labor markets (proxied by commuting zones) and find statistically significant negative effects that are slightly mitigated when the locations are allowed to trade. Since their level of analysis is local, they construct a measure of robot exposure from the IFR data, as the IFR does not provide information at the granular level of commuting zones. To obtain these results, they set up a task-based static model in which robots compete with labor for the performance of the tasks and increasing automation effectively improves the productivity of robots to a point that robots obtain the comparative advantage for performing the task; this is the displacement effect. Cleverly, they also incorporate the creation of new tasks in which labor has initially the comparative advantage; this is part of the productivity effect. Since our model is not task-based, we fail to capture this aspect. Moreover, Acemoglu and Restrepo (2018) complete their set-up by developing a dynamic model and characterizing the requirements for it to exhibit a balanced growth path. Lastly, in Acemoglu and Restrepo (2017a) they extend their model to incorporate labor heterogeneity by distinguishing between low-skill automation (through industrial and service robots) and high-skill automation (mainly through the introduction of AI).

Our paper adopts a static approach measuring changes across equilibrium states with an international scope instead of a local one. To that end, we develop a Ricardian trade model à la Eaton and Kortum (2002), where production is given through a nested CES that intertwines the factors of production. Our model builds on the work of Burstein, Cravino, and Vogel (2013) and Parro (2013), who employ the same Ricardian model to characterize the skill-biased effect of trade. We extend their model to incorporate robots in our production function. Our results are comparable to theirs since we find similar effects of trade openness on the skill-premium and the real wages by skill-type.

More closely related to our work is the seminal on the topic paper of Graetz and Michaels (2015), who were the first to bring attention to and use the IFR data. They investigate the economic contributions of robots quantifying empirically the impact of robot densification on labor productivity, output prices and employment by using appropriate instruments. They provide a theoretical model of a task-based technological choice from the firms' side that predicts their results. In contrast, in our paper, the equations used for our empirical analysis are not simply implied, but explicitly derived. This enables us to recover the elasticities, whose cardinal order Graetz and Michaels (2015) simply assume. Therefore, our value-added is to quantify the impact of robots by estimating the key elasticity parameters. We do this by having developed a trade model rather than a task-based one which fully captures the equilibrium impact of robots while keeping our analysis at a cross-country level. Interestingly, our topic of interest is included in the World Bank research agenda with the work of Artuc, Bastos, and Rijkers (2018). They also use the Ricardian model of Eaton and Kortum (2002), but a task-based production function, to investigate the impact of robot adoption on trade, employment and welfare and how this effect percolates to economies that are not yet automatized. Having used panel data and IV strategies, they find that robotization increases volumes of trade and exhibits welfare gains for the automatized economy, gains that can also reach countries that have not used industrial robots yet.

The rest of the paper is organized as follows: Section 2 presents our static Ricardian trade model for automation, which incorporates robots in the production function and helps us develop further intuition about the equilibrium impact of automation on labor markets. In Section 3, we derive at a first-order approximation the equations that describe the changes in the skill-premium and real wages between two steady states and will be used for our empirical analysis that is described in Section 4. In Section 5, we get the results of our estimations, discuss their implications about our initial hypothesis of robot-skill complementarity and consider some counterfactuals. Finally, Section 6 concludes, while the Appendix 7 provides analytical derivations of our expressions.

2 A Static Ricardian Model of Automation

To guide our empirical analysis, we are going to develop a Ricardian trade model based on the model firstly described in Eaton and Kortum (2002). We follow the adaption of this model by Burstein, Cravino, and Vogel (2013) making the appropriate modifications to accommodate the scope of our question. Specifically, we incorporate robots in the production function. Then, we use this model to implement hat algebra for the change in variables between two static equilibria, as done in Dekle, Eaton, and Kortum (2007), and derive the equations that describe the change in skill-premium which will be used for the regression estimation of key parameters and our counterfactual analysis.

2.1 The Set-up

Our model describes a world economy with N countries that can freely engage in trade. Each country i is endowed with inelastically supplied efficiency units of high-skilled $(\bar{H}_i)$ and low-skilled $(\bar{L}_i)$ labor. We assume that there are three sectors $(s \in S)$: manufacturing $(s = M)$, capital equipment $(s = K)$ and non-tradable services $(s = NT)$. For each sector s there is a continuum of goods Ω_s that is produced with different productivity $A(\omega_s)$, where $\omega_s \in \Omega_s$. For simplicity, we take it to be have Lebesque measure 1, $\Omega_s = [0, 1]$. From each sector we can obtain the respective composite good given by $z_s = \left(\int_{\Omega_s} \omega_s^{\frac{\epsilon_s - 1}{\epsilon_s}} d\omega_s \right)^{\frac{\epsilon_s}{\epsilon_s - 1}}$, where ϵ_s is the elasticity of substitution between goods used for production in sector s.

Every producer in these continua combines the composites of intermediate goods from each sector with labor (high- and low-skilled) and robots, as described by the following production function, which is a hybrid of a Cobb-Douglass and a nested CES, without any spatial agglomeration forces. The output of good ω_s in location i is given by:

$$Y_i(\omega_s) = A_i(\omega_s)\, z_M(\omega_s)^{\gamma_{1,i}}\, z_{NT}(\omega_s)^{1-\gamma_{1,i}-\gamma_{2,i}} \times \left(\left(\left(z_K(\omega_s)^{\eta'} + H(\omega_s)^{\eta'} \right)^{\frac{\lambda'}{\eta'}} + R(\omega_s)^{\lambda'} \right)^{\frac{\sigma'}{\lambda'}} + L(\omega_s)^{\sigma'} \right)^{\frac{\gamma_{2,i}}{\sigma'}} \tag{1}$$

, where where z_s is the composite intermediate input from each sector, H(L) are the high(low)-skilled labor hours, R is the number of industrial robots used, $\gamma_{1,i}$ and $\gamma_{2,i}$ are the income shares of each factor, $A_i(\omega_s)$ is the productivity of location i in producing good ω, and $\eta' = \frac{\eta-1}{\eta}$ with η being the elasticity of substitution between capital and high-skilled labor (σ' and λ' are defined similarly).

A moment of reflection on the production function can be quite valuable. It is an extension of what was proposed in Krusell, Ohanian, RÌos-Rull, and Violante (2000) by incorporating composite intermediate goods and robots. Krusell, Ohanian, RÌos-Rull, and Violante (2000) by assuming a simpler functional form empirically confirmed the existence of capital-skill complementarity have found that the elasticity of substitution between capital and low-skilled labor is higher than the elasticity between capital and high-skilled workers (in our model that is manifested as $\eta < \sigma$ and $R = 0$). Similarly, we aim to investigate the relative complementarity between robots and skill. Intuitively, the set-up of the function foreshadows that our elasticities will be ordered as $\eta < \lambda < \sigma$, conforming with our robot-skill complementarity hypothesis. It remains to derive the appropriate equations and use them to estimate these elasticity parameters.

The countries are free to trade with each other subject to some "iceberg" costs $\left\{\tau_{ij,s}\right\}_{i,j\in N, s\in S'}$ where i is the source country and j is the destination $\forall i,j \in N$, $\tau_{ij} \geq 1$ with $\tau_{ii} = 1$. Of course, $\tau_{ij,NT} = \infty$ if $i \neq j$ for all non-tradable services. The composite manufacturing goods and services do not appear only as intermediate goods but also as final goods for consumption. A representative consumer at each location j gets utility from consumption according to:

$$C_j = C_M^{a_M} C_{NT}^{a_{NT}} = C_M^{a_M} C_{NT}^{1-a_M} \tag{2}$$

where a_M, a_{NT} are the shares of consumption of every sector with $a_M + a_{NT} = 1$. Households do not consume capital goods. For simplicity we can assume that the elasticity of substitution for the composite final goods is the same as for the composite intermediates (ϵ^s) . Moreover, assuming that there are not any amenities for consideration this consumption can be considered as welfare. We do not allow for migration in our model with efficiency units of each labor skill-type being exogenously provided. Hence, we should not expect welfare equalization across locations.

2.2 Prices

Assuming that markets are operating under perfect competition, the price of a good sent from a location i to a location j is given by:

$$p_{ij}\left(\omega_s\right) = \frac{c_{i,s}}{A_{i,\omega_s}} \tau_{ij} \tag{3}$$

where $\frac{c_{i,s}}{A_{i,\omega}}$ is the marginal cost of one unit of ω in location i. Notice that the marginal cost is country and sector specific. Since, each $\omega_s \in \Omega_s, \forall s \in S$ is homogeneous across locations, each location will choose the origin of each good by minimizing across prices, hence $p_j\left(\omega_s\right) \equiv \min_{i \in N} p_{ij}\left(\omega_s\right)$

Assume that for every location i and good ω_s , $A_i\left(\omega_s\right)$is an identically independently distributed random variable drawn by a Frechet distribution $F_i\left(A\right) = exp\left\{T_i A^{-\theta_s}\right\}$ where $T_i > 0$ expresses the the idiosyncratic production of location i and $\theta_s > 1$ governs the heterogeneity of the distribution. θ_s is a comparative advantage parameter that varies across sectors but not across locations (the higher it is the less heterogeneity there is).

Now as in Eaton and Kortum (2002), we can define $\Phi_{j,s} \equiv \sum_{i\in N}\left[T_i\left(c_{i,s}\tau_{ij,s}\right)^{-\theta_s}\right]$ and obtain the price index:

$$P_{j,s} = \Phi_{j,s}^{-\frac{1}{\theta_s}}\left[\Gamma\left(\frac{\theta_s + 1 - \epsilon^s}{\theta_s}\right)\right]^{\frac{1}{1-\epsilon^s}} \tag{4}$$

, where $\Gamma(y) = \int_o^\infty x^{y-1}e^{-x}dx$ is the corresponding gamma function. For convenience denote $C(\theta,\chi) \equiv \Gamma\left(\frac{\theta+1-\chi}{\theta}\right)^{\frac{1}{1-\chi}}$, where χ is the respective constant elasticity of substitution.

We take the price of robots $\left(\rho_j\right)$ to be exogenously determined for each location, following the approach of Graetz and Michaels (2015). Note that in our production function we did not account for quality improvements of robots. We simply take these improvements to be reflected through price discounts.

2.3 Firm Cost Minimization

Now we turn our attention to the cost of production $c_{i,s}$ which is key in the determination of prices and trade shares. Every firm chooses inputs such that it minimizes the cost of production for a given amount of output.

Then we can put together the prices of inputs to get the following aggregate factor prices.

$$P_i^H \equiv \left(r_i^{1-\eta} + w_{S,i}^{1-\eta}\right)^{\frac{1}{1-\eta}}$$
$$P_i^R \equiv \left(\left(P_i^H\right)^{1-\lambda} + \rho_i^{1-\lambda}\right)^{\frac{1}{1-\lambda}} = \left(\left(r_i^{1-\eta} + w_{S,i}^{1-\eta}\right)^{\frac{1-\lambda}{1-\eta}} + \rho_i^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$$
$$P_i^F \equiv \left(\left(P_i^R\right)^{1-\sigma} + w_{L,i}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} = \left(\left(\left(r_i^{1-\eta} + w_{S,i}^{1-\eta}\right)^{\frac{1-\lambda}{1-\eta}} + \rho_i^{1-\lambda}\right)^{\frac{1-\sigma}{1-\lambda}} + w_{L,i}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

where for every location i, the price of capital is $r_i = \Phi_i^{-\frac{1}{\theta}} C\left(\epsilon_K\right)$, the price of robots is $\rho_i = \rho, \forall i \in S$. We assume that the price of robots is equal across locations and the price of skilled (unskilled)-labor $w_S\left(w_L\right)$ is equalized across sectors. We also get the price of the other composite intermediate goods $P_i^M = \Phi_i^{-\frac{1}{\theta}} C\left(\epsilon_M\right)$ for the manufacturing goods and similarly for the non-tradable goods $P_i^{NT} = \Phi_i^{-\frac{1}{\theta}} C\left(\epsilon_{NT}\right)$. Then the marginal cost for each unit of good ω_s is:

$$MC_{i,s,\omega}(q, P_i^M, P_i^F, P_i^{NT}) = \frac{B\left(P_i^M\right)^{\gamma_{1,s}}\left(P_i^F\right)^{\gamma_{2,s}}\left(P_i^{NT}\right)^{1-\gamma_{1,s}-\gamma_{2,s}}}{A_{i,\omega}} = \frac{c_{i,s}}{A_{i,\omega}} \tag{5}$$

, where $B \equiv \gamma_{1,s}^{-\gamma_{1,s}} \gamma_{2,s}^{-\gamma_{2,s}} \left(1-\gamma_{1,s}-\gamma_{2,s}\right)^{\gamma_{1,s}+\gamma_{2,s}-1}$. For a detailed derivation please consult the Appendix 7.1.

2.4 Gravity equations-Trade Shares

By the Law of Large Numbers the probability that $i \in S$ is the least cost provider of good ω is equal to the fraction of goods i sells to j, i.e. π_{ij}. By Eaton and Kortum (2002):

$$\pi_{ij}^s = \frac{T_i\left(c_{i,s}\tau_{ij,s}\right)^{-\theta_s}}{\sum_{k\in N}\left[T_k\left(c_{k,s}\tau_{kj,s}\right)^{-\theta_s}\right]} = \frac{T_i\left(c_{i,s}\tau_{ij,s}\right)^{-\theta_s}}{\Phi_{j,s}} \tag{6}$$

It turns out that π_{ij} is equal to the fraction of j's income spent on goods from i , $\lambda_{ij} \equiv \frac{X_{ij}}{Y_j} = \pi_{ij}$ where Y_j(total income)= E_j(total expenditure). Note that for sector $S = NT$, the trade costs $\tau_{ij} = +\infty, \forall i \neq j$. Hence, $\pi_{ij}\left(NT\right) = 1$.

By substituting, $\Phi_j = C^{\theta} P_j^{-\theta}$we get that

$$X_{ij} = C^{-\theta} \tau_{ij}^{-\theta} c_i^{-\theta} T_i E_j P_j^{\theta}$$

Most importantly, note that if we set $i = j$, then $\tau_{ii} = 1$. Hence we can obtain:

$$\pi_{ii} = \frac{T_i \left(c_{i,s}\right)^{-\theta}}{\Phi_i} \Rightarrow \Phi_i^{-\frac{1}{\theta}} = \left(\frac{\pi_{ii}}{T_i}\right)^{\frac{1}{\theta}} c_{i,s}$$

$$\Rightarrow P_i = C\left(\frac{\pi_{ii}}{T_i}\right)^{\frac{1}{\theta}} c_i \tag{7}$$

which is a pretty neat representation of the price index using only the domestic trade shares, as done in Arkolakis, Costinot, and Rodriguez-Clare (2012).

2.5 Welfare

For simplicity, assume that the welfare of a representative consumer/worker at location i is given by her real wage (i.e. consumption), -no amenities involved:

$$W_{S,i} = \frac{w_{S,i}}{P_i^C} \;\&\; W_{L,i} = \frac{w_{L,i}}{P_i^C}$$

where $P_i^C = \frac{\left(P_i^M\right)^{a_M}\left(P_i^{NT}\right)^{1-a_M}}{a_M^{a_M}\left(1-a_M\right)^{1-a_M}}$ is the consumption price index at location i, as it is implied by consumer welfare maximization. Since we have not allowed for labor mobility across countries, we do not expect welfare equalization in equalibrium.

2.6 Equilibrium conditions

To close our model, we derive the equalibrium conditions of market clearing. I follow Burstein, Cravino, and Vogel (2013) in their assumption of equalizing the parameters γ_1 and γ_2 across all sectors. They claim that this approach comes at the only cost of not modeling the Stolper-Samuelson effect, which has been estimated to be very close to zero. Under this assumption, we can integrate over all goods and sectors for each location.

Each equilibrium can be uniquely characterized by the following set of variables ($8N$ unknowns):

$$\left(\{w_{S,i}, w_{L,i}, c_i, R_i, z_{K,i}, z_{M,i}, z_{NT,i}, D_i\}_{i\in N}\right)$$

where $\bar{S}_i = \sum_{s\in S}\int_{\Omega_s} S_i(\omega)\,d\omega$, $\bar{L}_i = \sum_{s\in S}\int_{\Omega_s} L_i(\omega)\,d\omega$, $R_i = \sum_{s\in S}\int_{\Omega_s} R_i(\omega)\,d\omega$, $z_{K,i} = \sum_{s\in S}\int_{\Omega_s} z_{K,i}(\omega)\,d\omega$ and $z_{M,i} = \sum_{s\in S}\int_{\Omega_s} z_{M,i}(\omega)\,d\omega$.

For the price indices we have: $P_i^s = \Phi_i^{-\frac{1}{\theta}} C(\epsilon_s), \forall s \in S, i \in N$. Note that we do not have to solve the price of robots (ρ_j), which as mentioned is determined exogenously. You can imagine that robots are supplied by an exogenous agent that sets the prices for each country and is committed to deliver the demanded quantity.

We can define the income at location i as $Y_i \equiv w_{S,i}S_i + w_{L,i}L_i + \rho_i R_i$. The composite goods are not included because they are intermediate inputs. Then, from the feasibility/market clearing constraint, we obtain that:

$$Y_i = \left[\sum_{j\in N}\left(\pi_{ij}^M\left(\gamma_{1,j} + a_M\right) + \pi_{ij}^K \gamma_{K,j}\right) Y_J + \pi_{ij}^M a_M D_j\right] + \left(1 - \gamma_{1,j} - \gamma_{2,j}\right) Y_i + [1 - a_M]\left(Y_i + D_i\right) \tag{8}$$

where $\pi_{ij}^s = \frac{T_i\left(c_i\tau_{ij}\right)^{-\theta_s}}{\Phi_j^s}$ are the trade shares for each sector $\pi_{ii}^{NT} = 1$, where D_i is the deficit of i and γ_K is the income share of capital.

Moreover, total deficits clear:

$$\sum_{i \in N} D_i = 0$$

Having completely described the model, we will proceed by deriving the equations that characterize the change in skill-premium and real wages by skill-type.

3 Model Characterization & Further Derivations

3.1 The skill premium

By cost minimization we can derive the skill premium at a steady state. We drop the location subscripts to lighten up the notation. After some steps, which you can follow analytically in the Appendix 7.2, we get the expression:

$$\frac{w_S}{w_L} = \left(z_K^{\frac{\sigma-1}{\sigma}} + S^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\eta-\lambda}{\lambda(\eta-1)}} \left(\frac{1}{S} \right)^{\frac{1}{\eta}} \left(z_H^{\frac{\lambda-1}{\lambda}} + R^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\sigma-\lambda}{\sigma(\lambda-1)}} L^{\frac{1}{\sigma}} \tag{9}$$

We can compare this skill premium with what Burstein, Cravino, and Vogel (2013) got without robots in their production function:

$$\frac{w'_S}{w'_L} = \left(\left(\frac{z_K}{S} \right)^{\frac{\eta-1}{\eta}} + 1 \right)^{\frac{\sigma-\eta}{\sigma(1-\eta)}} \left(\frac{L}{S} \right)^{\frac{1}{\sigma}} = \left(z_K^{\frac{\eta-1}{\eta}} + S^{\frac{\eta-1}{\eta}} \right)^{\frac{\sigma-\eta}{\sigma(\eta-1)}} L^{\frac{1}{\sigma}} \left(\frac{1}{S} \right)^{\frac{1}{\eta}}$$

Notice that the introduction of robots just adds the extra factor $\left(z_H^{\frac{\lambda-1}{\lambda}} + R^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\sigma-\lambda}{\sigma(\lambda-1)}}$ in the expression of the skill premium.

3.2 Implementing Hat Algebra

We now follow the method of implementing hat algebra, which had been introduced by Dekle, Eaton, and Kortum (2007). This approach capitalizes on the findings of Arkolakis, Costinot, and Rodriguez-Clare (2012) who link gains from trade to domestic trade shares. We continue to operate under the assumption of equal γ's across sections.

Having obtained our price index in equation7, let us define $\hat{x} \equiv \frac{x'}{x}$; then we get:

$$\hat{c}_i = \left(\hat{P_i^M}\right)^{\gamma_1} \left(\hat{P_i^F}\right)^{\gamma_2} \left(\hat{P_i^{NT}}\right)^{1-\gamma_1-\gamma_2} \tag{10}$$

$$\hat{P}_i = \hat{c}_i \left(\frac{\hat{\pi}_{ii}}{\hat{T}_i}\right)^{\frac{1}{\theta}} \tag{11}$$

Now taking $P_i^{NT} = 1$ to be our numeraire at all steady states, we obtain $\hat{P_i^{NT}} = 1$. Moreover, we know that at every time, $\pi_{ii}^{NT} = 1 \Rightarrow \hat{\pi_{ii}^{NT}} = 1$

$$\Rightarrow \hat{c}_i = \hat{T}_i^{\frac{1}{\theta}} \equiv \hat{A}_i$$

$$\Rightarrow \hat{P_i^M} = \left(\hat{\pi_{ii}^M}\right)^{\frac{1}{\theta}}$$

$$\Rightarrow \hat{r}_i = \hat{P_i^K} = \left(\hat{\pi_{ii}^K}\right)^{\frac{1}{\theta}}$$

$$\Rightarrow \hat{P_i^F} = \left(\frac{\hat{A}_i}{\left(\hat{\pi_{ii}^M}\right)^{\frac{\gamma_1}{\theta}}}\right)^{\frac{1}{\gamma_2}} \tag{12}$$

Moreover, $\hat{P_i^F} = \left(\left(\hat{P_i^R}\right)^{1-\sigma} + (\hat{w_{L,i}})^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$, $\hat{P_i^R} = \left(\left(\hat{P_i^H}\right)^{1-\lambda} + (\hat{\rho})^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$, and $\hat{P_i^H} = \left((\hat{r}_i)^{1-\eta} + (\hat{w}_{H,i})^{1-\eta}\right)^{\frac{1}{1-\eta}}$.

Most importantly, for the change in skill-premium we get that:

$$\frac{\hat{w_S}^{\eta}}{\hat{w_L}^{\sigma}} = \left(\hat{P^H}\right)^{\eta-\lambda} \left(\hat{P^R}\right)^{\lambda-\sigma} \frac{\hat{L}}{\hat{S}} \tag{13}$$

Lastly, notice that the hat of the price of consumption is:

$$\hat{P_i^C} = \left(\hat{P_i^M}\right)^{a} \left(\hat{P_i^{NT}}\right)^{1-a} = \left(\hat{\pi_{ii}^M}\right)^{\frac{a}{\theta}} \tag{14}$$

, where $a = a_M$ from now on.

3.2.1 First-order approximation for the change in the skill-premium

Now, let us define $\tilde{x} \equiv log\,(\hat{x})$; then we have that:

$$\tilde{r_i} = \frac{\tilde{\pi}_{ii}^K}{\theta}$$

$$\eta\tilde{w_S} - \sigma\tilde{w_L} = (\eta-\lambda)\,\tilde{P^H} + (\lambda-\sigma)\,\tilde{P^R} + \left(\tilde{L} - \tilde{S}\right)$$

$$\tilde{P_i^F} = \frac{\tilde{A}_i - \frac{\gamma_1}{\theta}\tilde{\pi_{ii}^M}}{\gamma_2} = \frac{\tilde{T}_i - \gamma_1\tilde{\pi_{ii}^M}}{\theta\gamma_2}$$

Taking to -a first-order approximation, that $exp(\tilde{x}) = 1 + \tilde{x}$, we can obtain (steps omitted and presented thoroughly in the Appendix 7.2.1):

$$\tilde{w_S} - \tilde{w_L} = \underbrace{\left(\frac{\lambda}{\sigma} - 1\right)}_{<0}\tilde{\rho} + \underbrace{\left(\frac{\eta}{\sigma} - 1\right)}_{<0}\frac{\tilde{\pi_d^K}}{\theta} + \frac{\tilde{L} - \tilde{S}}{\sigma} + constant \tag{15}$$

where $\pi_d^K = \pi_{ii}^K$ is the domestic trade share. Hypothesizing robot-skill complementarity ($0 < \eta < \lambda < \sigma$), we notice that the signs of the coefficients in the equation confirm our intuition. A decline in the robot prices increases the skill-premium and so does globalization (a.k.a. $\tilde{\pi_d^K} < 0$), concurring with the results of Burstein, Cravino, and Vogel (2013) and Parro (2013). Moreover, the coefficient $\frac{1}{\sigma}$ agrees with the intuitive impact of changing the relative scarcity of the skill supplies (given $\sigma > 0$).

With a little bit of work (presented in the Appendix 7.2.1) we can arrive at an alternative expression for the skill-premium:

$$\tilde{w_S} - \tilde{w_L} = \underbrace{\left(\frac{1}{\eta} - \frac{1}{\sigma}\right)}_{>0} \tilde{z_K} - \frac{\tilde{S} - \tilde{L}}{\sigma} + \underbrace{\left(\frac{1}{\lambda} - \frac{1}{\sigma}\right)}_{>0} \tilde{R} + constant'$$

$$\Rightarrow \tilde{w_S} - \tilde{w_L} = \frac{\tilde{z_K}}{\eta} - \frac{\tilde{z_K} + \tilde{S} - \tilde{L} + \tilde{R}}{\sigma} + \frac{\tilde{R}}{\lambda} + constant' \qquad (16)$$

Again our intuition regarding the skill-premium is satisfied. The coefficient of $\tilde{z_K}$ is positive as dictated by capital-skill complementarity and the sign of the coefficient of $\tilde{R}$ relies on our robot-skill complementarity hypothesis ($\lambda < \sigma$).

Except from satisfying our theoretical priors and intuition, we have managed to derive an equation (explicitly from our model) where the elasticities are separated. This enables us to use this equation to estimate those key elasticities, assuming that there is some idiosyncratic or measurement error. We will provide more thorough explanations in the empirical sections of the paper. We expect the recovered estimates to be positive, satisfy the ordering $0 < \eta < \lambda < \sigma$ and be in the same order of magnitude as the elasticities of substitution obtained in the literature. For example, the values in the seminal paper of Krusell, Ohanian, RÌos-Rull, and Violante (2000) are $\eta = 0.67$ and $\sigma = 1.67$.

We can also substitute for $\tilde{z_K}$ using our first-order approximation and get the following expression for the skill premium (for details please refer to the Appendix 7.2.1):

$$\tilde{w_S} - \tilde{w_L} = \underbrace{\left(\frac{1}{\eta} - \frac{2}{\sigma} + \frac{1}{\lambda}\right)}_{>0} \tilde{R} + \underbrace{\left(\frac{\lambda}{\eta} - \frac{\lambda}{\sigma}\right)}_{>0} \tilde{\rho} + \left[\left(\frac{1}{\eta} - \frac{1}{\sigma}\right)\left(\frac{\lambda - \eta}{\eta}\right) - \frac{1}{\sigma}\right] \tilde{S} + \frac{\tilde{L}}{\sigma} + \underbrace{\left(\frac{\lambda}{\sigma} - \frac{\lambda}{\eta}\right)}_{<0} \frac{\tilde{\pi}_d^K}{\theta} + cons \tag{17}$$

Again the signs of the coefficients are consistent with our assumptions. Consistent with the results of Burstein, Cravino, and Vogel (2013) and Parro (2013) the skill-premium increases with trade openness. The impact of change in stock of robots $\tilde{R}$ and their price $\tilde{\rho}$ is positive. But, there is the caveat that these two changes are negatively related $\left(\tilde{R} \downarrow\uparrow \tilde{\rho}\right)$. Due to their simultaneous covariance, the counterfactuals of keeping the one constant while changing the other are not that helpful. However, these counterfactuals are not that meaningless if we revisit the idea presented in Arnaud (2018) that automation does not have to happen necessarily (i.e. change in $\tilde{R}$) but improvements in quality (manifested as decreases in $\tilde{\rho}$) can increase the threat to automate and pressure wages downward. This effect is manifested clearer in the following equations.

3.2.2 First-order approximation for the change in the real wages

We now use the same techniques to recover equations that describe changes in real wage of each skill-type. We use equation 28 from the Appendix 7.2.1 and obtain the following expressions. For the change in the real wage of the high-skilled labor we have:

$$\tilde{w_S} - \tilde{P^C} = \left(\frac{1}{\eta} - \frac{1}{\lambda}\right) \tilde{z_K} - \frac{\tilde{S}}{\lambda} + \frac{\tilde{R}}{\lambda} + \frac{1}{\eta} - \frac{1}{\lambda} + \tilde{\rho} - \frac{a}{\theta}\left(\tilde{\pi_d^M}\right)$$

and substituting for $\tilde{z_K}$ by equation 31 we get:

$$\tilde{w_S} - \tilde{P^C} = \frac{\tilde{R}}{\eta} + \frac{\lambda}{\eta}\tilde{\rho} + \left[\frac{1}{\eta}\left(\frac{\lambda}{\eta} - 2\right)\right] \tilde{S} + \underbrace{\left(1 - \frac{\lambda}{\eta}\right)}_{<0} \frac{\tilde{\pi_d^K}}{\theta} - \frac{a}{\theta}\left(\tilde{\pi_d^M}\right) + constant'' \tag{18}$$

The coefficient to the supply of high-skilled labor is negative if λ and η are close enough. While the real wage of skilled labor changes positively with trade openness since the coefficient of the domestic trade shares are negative under the assumption of capital-skill complementarity. The impact of change in stock of robots $\tilde{R}$ and their price $\tilde{\rho}$ is positive. As we argued above, the positive coefficient in front of $\tilde{\rho}$ suggests the downward pressure on wages due to the threat to automate, as in discussed in Arnaud (2018). Nevertheless, because of $(\tilde{R} \downarrow\uparrow \tilde{\rho})$, it more instructive for our counterfactuals to substitute for $\tilde{\rho}$ by using the equation 30. Then we obtain:

$$\tilde{w_S} - \tilde{P^C} = \frac{\tilde{z_K}}{\eta} + \left(\frac{1}{\lambda} - \frac{2}{\eta}\right)\tilde{S} - \frac{1}{\theta}\left(a\tilde{\pi_d^M} - \tilde{\pi_d^K}\right) + constant''' \tag{19}$$

Here, the impact of robots, which was manifested either through changes in prices or quantities, is absent. The coefficients of the skill-supply and domestic trade shares of manufacturing $\left(\tilde{\pi_d^M}\right)$ is unambiguously negative as expected.

Lastly, we turn our attention the change in the real wage of the low-skilled labor is obtained by:

$$\begin{aligned} \tilde{w_L} - \tilde{P^C} &= \left(\tilde{w_S} - \tilde{P^C}\right) - (\tilde{w_S} - \tilde{w_L}) \\ \tilde{w_L} - \tilde{P^C} &= \underbrace{\left(\frac{1}{\sigma} - \frac{1}{\lambda}\right)}_{<0}\tilde{z_K} + \underbrace{\left(\frac{1}{\sigma} - \frac{1}{\lambda}\right)}_{<0}\tilde{S} - \tfrac{\tilde{L}}{\sigma} + \tfrac{\tilde{R}}{\sigma} + \tilde{\rho} - \tfrac{a}{\theta}\left(\tilde{\pi_d^M}\right) + const \end{aligned}$$

$$\Rightarrow \tilde{w_L} - \tilde{P^C} = \frac{\tilde{z_K}}{\sigma} + \left(\frac{1}{\lambda} + \frac{1}{\sigma} - \frac{2}{\eta}\right)\tilde{S} - \frac{\tilde{L}}{\sigma} + \underbrace{\left(\frac{1}{\sigma} - \frac{1}{\lambda}\right)}_{<0}\tilde{R} + \frac{\tilde{\pi_d^K}}{\theta} - \frac{a}{\theta}\left(\tilde{\pi_d^M}\right) + const' \tag{20}$$

Here, the coefficient of $\tilde{R}$ is negative under our robot-skill complementarity assumption.

$$\tilde{w_L} - \tilde{P^C} = \underbrace{\left(\frac{2}{\sigma} - \frac{1}{\lambda}\right)}_{>0<} \tilde{R} + \frac{\lambda}{\sigma}\tilde{\rho} + \frac{1}{\eta}\left(\frac{\eta}{\lambda} + \frac{\lambda}{\sigma} - 2\right)\tilde{S} - \frac{\tilde{L}}{\sigma} + \underbrace{\left(1 - \frac{\lambda}{\sigma}\right)}_{>0} \frac{\tilde{\pi}_d^K}{\theta} - \frac{a}{\theta}\left(\tilde{\pi_d^M}\right) + cons \quad (21)$$

When we substitute for $\tilde{z_K}$ we get that the impact of $\tilde{R}$ is ambiguous depending on the relative magnitude of the elasticities. The coefficient of $\tilde{\rho}$ is positive but smaller in scale than the respective impact for high-skilled labor, while the coefficient of $\tilde{\pi}_d^K$ is positive. Assuming $\tilde{\pi}_d^K = \tilde{\pi}_d^M$ the overall impact of trade openness depends on the sign of $a + \frac{\lambda}{\sigma} - 1$.

All in all, with our assumptions of capital-skill and robot-skill complementarity, the increasing usage of robots $\tilde{R}$ is more beneficial to high-skilled labor. While improvements in their quality (these can be thought as declines in $\tilde{\rho}$) affect negatively high-skilled labor at a bigger extent. This is also true when it comes to trade openness: high-skilled labor is benefited from globalization; this might not be the case for and the low-skilled workers, as it depends on the exact magnitudes of the elasticity parameters.

4 Data description & Analysis

In this section we describe the data used and the exact steps for our empirical analysis that includes estimation of key elasticity parameters

4.1 Data sources & description

For our estimations we need data on the operational stock of robots by country and industry. The International Federation of Robotics (IFR) releases annual reports with analytical data on both the number of robots delivered and stock of operational robots for 50 countries at the industry level and by application, such as handling, dispensing, processing or welding. For our purposes, we use the data on the operational stock of robots. The data accounts for most countries starts since 1993. We also obtain data for changes in price with or without adjusting for quality improvements from the 2006 IFR World Robotics Report. At the industry level, a significant number of robots is denoted as unspecified. Following Acemoglu and Restrepo (2017b), we classify these unspecified robots to each industry according to the proportions observed in the specified data.

We combine these with data that come from EUKLEMS (March 2008 edition). EUKLEMS, due to the work of van Ark and Jäger (2017), is a comprehensive database that contains information on capital and labor inputs, outputs and growth accounting at an industry-level for most OECD countries. From this database, we extract the compensation share and hours-employed share by skill-type. Since, our model accounts for two skill-types while EUKLEMS includes three types we combine their middle- and low-skill type into one type. Hence, our skill characterization cut-off becomes high-school graduation. Knowing these share variables are enough to construct the relative skill-supply and the skill premium. We can also obtain data about the volumes of intermediates goods (denoted as II_QI) and capital services volumes (CAP_QI), which are readily available for every country at the industry level. Merging the data we have 24 observations at the country level and industry level observations for 8 countries (Germany, Italy, the United Kingdom, France, Spain, Japan, Finland , and Sweden) and 9 manufacturing industries. This brings our total sample size to 96.

We also extract data on product exports by country from the Trade Map of the International Trade Centre[5]. Specifically, we focus on exports of motor vehicles by Japan

[5] Trade Map

and Germany, counties that dominate the global exports of industrial robots; these data will be used for the construction of our instrumental variables. Lastly, we need data on the domestic trade shares, which we find in Adao, Costinot, and Donaldson (2017). Their data set includes all bilateral trade shares between a set of OECD countries, which largely coincides with the list of countries covered by EUKLEMS and IFR. By putting all data together we are left with 96 observations at the industry-sector–country level, which we will use for our subsequent analysis.

4.2 Analysis

Our main analysis is going to be the estimation of the elasticity parameters. Having recovered these parameters, their magnitude (or simply their relative order) can inform us about the impact of automation through industrial robots on the skill-premium and the real wages. Our work also allow us to conduct counterfactuals to determine the impact of trade openness as well.

The key equation we have derived is:

$$\tilde{w_S} - \tilde{w_L} = \frac{\tilde{z_K}}{\eta} - \frac{\tilde{z_K} + \tilde{S} - \tilde{L} + \tilde{R}}{\sigma} + \frac{\tilde{R}}{\lambda} + constant$$

Assuming that there is some idiosyncratic error ϵ_i that is not captured by our model (or information lost due to our approximation methods) we can uses this equation for a regression analysis. We can perform a simple OLS regression assuming that ϵ_i satisfies the moment condition. Since our equation is explicitly derived from a causal model, our regression does not simply recover the coefficients of the best liner predictor; these coefficients correspond to the key elasticity parameters of our model, which have direct implications for our hypotheses of capital-skill and robot-skill complementarity. We also perform an OLS regression by controlling for the level of the observation (country-sector-industry). We recognize that this variation within country variation might partially contradict our model since we have argued that within a country wages are equalized among sectors; hence, the skill-premium should vary only at the country level. However, such a claim is not supported by the data. We leverage on this fact and increase our sample size by constructing data at the industry-country level. If we limited our analysis to the cross-country level we would have been left with just 24 observations. The small sample size would have given less credibility to our estimates; making our subsequent analysis unreliable.

Even though our equation is derived from a causal model and there is not omitted variable bias, there can be other causes of endogeneity. In addition to the potential measurement error, a more serious consideration is reverse causality; robot adoption can be influenced by the changes in the skill-premium. To address the banes of endogeneity we need to come up with the appropriate instruments that safeguard us from biased estimators, hoping that this will not come at the cost of losing statistical significance. Since our observations come from the industry-country level, we need an instrument that varies by country and one that varies by industry; by interacting the two we get a valid instrument. We tried to follow Graetz and Michaels (2015) and instrument the robot adoption by industry with a variable that measures the fraction of hours in an industry that are replaceable given the replaceability of the occupations involved. The failure of this approach at the first-stage led us to an instrument that leverages on the fact that the idiosyncratic errors at the industry-country level are not correlated with the global adoption trend. Under this IV moment assumption, we used the robot adoption by industry at the global level, which yielded our instrument at the industry level (it varies across industries, not countries).

For our instrument at the country level (it varies across countries, not industries) we used the change in the exposure of countries to the top two exporters of industrial robots, Japan and Germany[6]. Very conveniently, these two countries also export a lot of vehicles. The changes in the imports of German and Japanese cars are used as instruments for the changes in robot adoption. And since we have both Japan and Germany, we effectively have two instruments (or even three if we take their average) to instrument for robot adoption at the country level. Last step is two interact these two instrument and obtain two (or even three) instrumental variables to use for a two-stage least squares (2SLS) regression. Since robots $\left(\tilde{R}\right)$ appear both alone and in $\tilde{z_K} + \tilde{S} - \tilde{L} + \tilde{R}$ we can use two of these instruments for our analysis. Finally, for our third variable, $\tilde{z_K}$, we can instrument II_QI (intermediate input volumes) with CAP_QI (capital services volumes). With three regressors and three instruments our 2SLS regression can be exactly identified, which allows to proceed to our analysis having accounted for potential endogeneity issues.

5 Results

We will be using the equation:

$$(\tilde{w_S} - \tilde{w_L})_i = constant + \frac{(\tilde{z_K})_i}{\eta} - \frac{\left(\tilde{z_K} + \tilde{S} - \tilde{L} + \tilde{R}\right)_i}{\sigma} + \frac{\left(\tilde{R}\right)_i}{\lambda} + \epsilon_i$$

Since our model describes static equilibria we need to consider the changes between two steady states. These states are the year 1996 and the year 2005 for all countries to avoid variations related to time between our observations. We chose these steady states since by 1995 all countries in our data set had already acquired some robots. Furthermore, the two time periods cover a range of 10 years; while by stopping at 2005,

[6] Top exporters of industrial robots

we avoid the distortions in the data because of the global financial crisis. Therefore, all values are expressed as log changes, $\tilde{x} = log\left(\frac{x_{2005}}{x_{1996}}\right)$. Having all needed data in hand, we proceed to our regression analysis.

Just for good measure, Table 1 reports the outcomes from the OLS regression using just the cross-country sample. The small size of the sample and the absence of statistical significance underscores the need expanding our data set by including country-industry level variation. Interestingly, the estimates, which are all positive, do not support robot-skill complementarity since $\lambda > \eta$.

Table 1
OLS cross-country data

sample size=24	coef.	std.error	p-value	95% Conf.	Interval	
$\tilde{z_K}$	0.2815	0.2029	0.18	-0.14	0.70	$\eta = 3.55$
$\tilde{z_K} + \tilde{S} - \tilde{L} + \tilde{R}$	-0.1888	0.1108	0.10	-0.42	0.042	$\lambda = 5.39$
$\tilde{R}$	0.1856	0.1098	0.11	-0.04	0.41	$\sigma = 5.3$
_cons	0.3745	0.3409	0.29	-0.37	0.19	

We now proceed with our augmented database and perform the OLS regression with the estimates being reported in Table 2. Table 3 reports the estimates for the same coefficient after controlling whether our observation is at the country or the industry level.

Table 2
Simple OLS

sample size=96	coef.	std.error	p-value	95% Conf.	Interval	
$\tilde{z_K}$	0.2802	0.084	0.001	0.1132	0.4470	$\eta = 3.57$
$\tilde{z_K} + \tilde{S} - \tilde{L} + \tilde{R}$	-0.2227	0.049	0.000	-0.3195	-0.1256	$\lambda = 4.22$
$\tilde{R}$	0.2367	0.048	0.000	0.1409	0.3325	$\sigma = 4.49$
_cons	0.0223	0.025	0.375	-0.0275	0.0725	

Table 3
OLS with level-control

sample size=96	coef.	std.error	p-value	95% Conf.	Interval	
$\tilde{z_K}$	0.2582	0.089	0.005	0.0797	0.4367	$\eta = 3.87$
$\tilde{z_K} + \tilde{S} - \tilde{L} + \tilde{R}$	-0.2193	0.048	0.000	-0.3160	-0.1225	$\lambda = 4.28$
$\tilde{R}$	0.2334	0.048	0.000	0.1379	0.3288	$\sigma = 4.55$
_cons	0.0506	0.028	0.375	-0.0275	0.0722	

We are pleased to see that the magnitudes of the elasticities are strongly positive and are relatively close to one. Just as a reference point, Krusell, Ohanian, RÌos-Rull, and Violante (2000) the corresponding elasticities, that they had recovered were $\eta = 0.67$ and $\sigma = 1.67$. Although the exact magnitudes will be also used for our subsequent analysis, let us most importantly appreciate their cardinal order $(\eta < \lambda < \sigma)$. This ordering strongly and statistically significantly supports our robot-skill complementarity hypothesis, while it also confirms the well-established capital-skill complementarity assumption.

Given our concerns about endogeneity, we perform a 2SLS regression using the instruments described above. The obtained estimates are moderately larger, hinting towards the existence of attenuation bias due to measurement error. The estimates and the implied elasticities are reported below in Table 4:

Table 4
IV regression

sample size=96	coef.	std.error	p-value	95% Conf.	Interval	
$\tilde{z_K}$	0.6493	0.258	0.012	0.143	1.155	$\eta = 1.54$
$\tilde{z_K} + \tilde{S} - \tilde{L} + \tilde{R}$	-0.5388	0.228	0.018	-0.986	-0.091	$\lambda = 1.81$
$\tilde{R}$	0.5514	0.24	0.022	0.081	1.021	$\sigma = 1.85$
_cons	0.136	0.09	0.134	-0.0275	0.0722	

The estimates obtained remain statistically significant with their ordering validating the robot-skill complementarity hypothesis. Nevertheless, we cannot reject the null hypothesis that $\lambda > \sigma$; hence the 2SLS estimates do not guarantee the existence of robot-skill complementarity. Interestingly, Wooldridge (1995)'s robust score test that gives a score of $chi^2(3) = 2.88831$ (p-value=0.48), does not allow us to reject the exogeneity hypothesis, leaving open the possibility that our regressors were exogenous in the first place and that the OLS estimates were sufficient. In any case our estimates confirm the existence of robot-skill and capital-skill complementarity.

5.1 Model accuracy

We can obtain the global average change in robot price from the IFR 2006 report to be $\tilde{\rho} = -0.372$. When we use the estimates from our OLS $\left[\eta = 3.57, \quad \lambda = 4.22, \quad \sigma = 4.49 \right]$ and IV $\left[\eta = 1.54, \quad \lambda = 1.81, \quad \sigma = 1.85 \right]$ regressions and the equation

$$\tilde{w_S} - \tilde{w_L} = \left(\frac{1}{\eta} - \frac{2}{\sigma} + \frac{1}{\lambda}\right)\tilde{R} + \left(\frac{\lambda}{\eta} - \frac{\lambda}{\sigma}\right)\tilde{\rho} + \left[\left(\frac{1}{\eta} - \frac{1}{\sigma}\right)\left(\frac{\lambda - \eta}{\eta}\right) - \frac{1}{\sigma}\right]\tilde{S} + \frac{\tilde{L}}{\sigma} + \left(\frac{\lambda}{\sigma} - \frac{\lambda}{\eta}\right)\frac{\tilde{\pi}_d^K}{\theta} + cons$$

, where $cons = \frac{\eta-\lambda}{\lambda(1-\eta)} + \frac{\lambda-\sigma}{\sigma(1-\lambda)} + \frac{(\sigma-\eta)(\lambda-\eta)}{\sigma\eta(\eta-1)}$. Then substituting for the respective values of elasticities we get $cons^{OLS} = 0.09$ and $cons^{IV} = 0.36$. For the trade elasticity, we take the value recovered by Simonovska and Waugh (2014) for the Eaton and Kortum (2002) model of $\theta = 4.17$. Then:

$$(\tilde{w_S} - \tilde{w_L})^{OLS}_{pred} \simeq 0.07\tilde{R} + 0.24\tilde{\rho} - 0.22\tilde{S} + 0.22\tilde{L} - 0.06\tilde{\pi}_d^K + 0.09 \tag{22}$$

$$(\tilde{w_S} - \tilde{w_L})^{IV}_{pred} \simeq 0.12\tilde{R} + 0.19\tilde{\rho} - 0.53\tilde{S} + 0.53\tilde{L} - 0.05\tilde{\pi}_d^K + 0.36 \tag{23}$$

According to our estimates and choice of parameters, we can see that the rising usage of robots increases the skill premium, while the decline in prices $\tilde{\rho} < 0$, tends to decrease the skill premium. Of course, as mentioned before these two forces function simultaneously and a "ceteris paribus" counterfactual is not that realistic, but it can have the interpretation discussed by Arnaud (2018).

Comparing the predicted values with the observed log-changes in the skill-premium in the cross-country-industry and cross-country level we obtain the respective model fits in Figure 2 and Figure 3:

Figure 2: OLS model fit

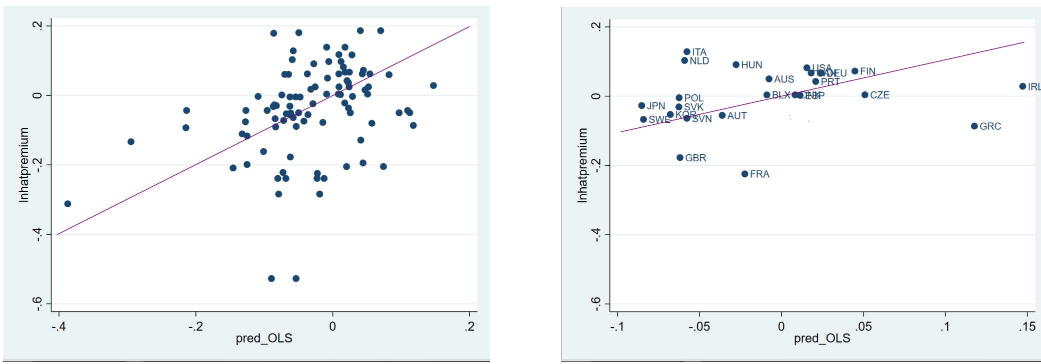
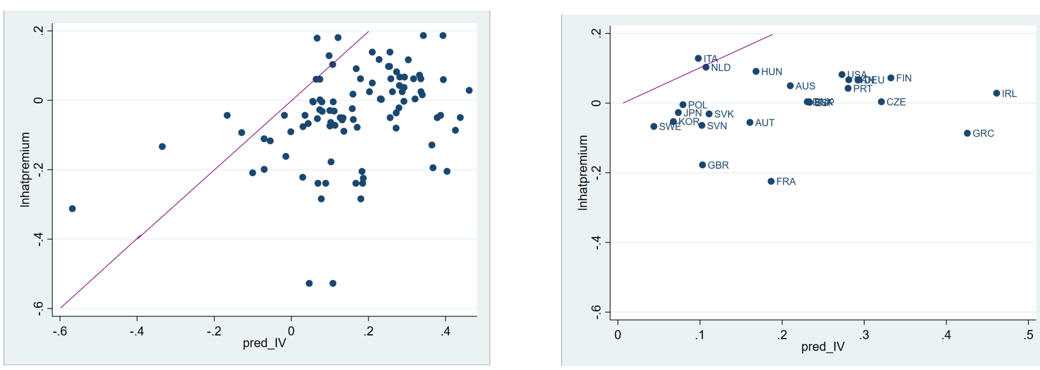


Figure 3: IV model fit



We can evidently see that the OLS estimates have larger predicting power, which supports the conjecture that our regressors are actually exogenous.

5.2 Measuring the impact of automation

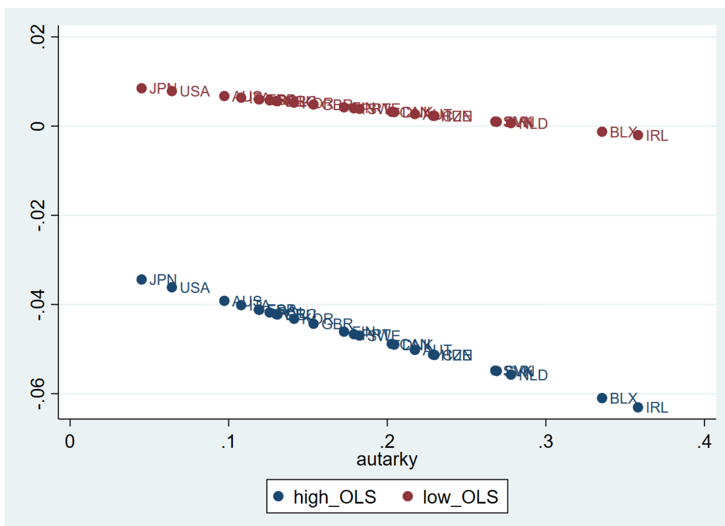
It is easy to extract from our estimates the counterfactual impact of the increasing robot adoption. "Ceteris paribus" (assuming prices do not change) a percentage increase in the stock of robots will decrease welfare for the high-skilled by 0.28% (0.65%) and for low-skilled by 0.21% (0.53%) according to our OLS (IV) estimates. We see that relative gains are 2.32 (2.54) larger for the high-skilled. Hence, if our estimates are close to reality the impact of robot is positive for the welfare of all consumers. Nevertheless, due to our hypothesized and empirically validated robot-skill complementarity, increasing automation widens the wage cap as they high-skilled claim a double share of the gains.

5.3 Is trade mitigating or amplifying the change in the skill-premium?

In our model, we did not have any direct interaction between the variables relevant to robots ($\tilde{\rho}$ and $\tilde{R}$) and domestic trade shares. However, there is an indirect link connected with the decreases in cost. We find that in agreement with the results of Burstein, Cravino, and Vogel (2013) and Parro (2013), trade openness amplifies the change in the skill-premium. Due to lack of separate data on bilateral trade shares by sector, we assume $\tilde{\pi_{ii}^{M}} = \tilde{\pi_{ii}^{K}}, \forall \in N$. Using equations 28 and 21, our elasticity estimates, a consumption share of $a_M = 0.2$ (as calibrated by Burstein, Cravino, and Vogel (2013) using the OECD Input-Output Database), and a trade elasticity of $\theta = 4.17$ (estimated for the Eaton and Kortum (2002) model in Simonovska and Waugh (2014)), we measure the impact of trade by skill-type. Using the OLS estimates, the impact of a percentage decrease in the domestic trade share of capital $\left(\pi_{ii}^{K}\right)$ benefits skilled-labor by 0.09% , compared to 0.03% for the low-skilled labor. These yield a ratio of 3; whereas, if we use the IV estimates we find that the gains are 0.09% for the skilled-labor and 0.04% for the unskilled labor, which give us a ratio of 2.25. Both are consistent with the 2.36 ratio found by Burstein, Cravino, and Vogel (2013).

To illustrate our results we construct the counterfactual countries moving to autarky. Since we do not have separate data on capital trade shares we assume due to greater dependence on imported capital equipment that the domestic shares are half of the respective manufacturing ones ($\pi_{ii}^M = 2\pi_{ii}^K$). This assumption is supported by the trade shares documented in Burstein, Cravino, and Vogel (2013); countries are more likely to import capital equipment. For the π_{ii}^M's we use the bilateral trade shares documented in Adao, Costinot, and Donaldson (2017). Then, we plot the log-change in real wages by skill-type over the log-change of trade shares from the 1996 levels to autarky. We can get the following counterfactual results using our OLS estimates depicted in Figure 4. As expected the return to autarky harms high-skilled labor more, but it is also harmful to the low-skilled labor.

Figure 4: Move to Autarky



6 Conclusion

With the recent rise of automation, there is an increasing concern about the future of jobs and wages. The clear displacement effect of robots is countervailed by their productivity effect that can increase the value-added of labor while decreasing overall costs of production and consumption. We enter this discussion by focusing our attention to a specific but dominant form of automation, industrial robots. We develop a trade-based model to capture the equilibrium impact of the increasing robot density in industries by hypothesizing robot-skill complementarity in addition to capital-skill complementarity. Our estimates which are obtained from a causal model regression support this conjecture. Our approach shows that there are gains from automation for both skill-types; these gains are favorably allocated to the high-skilled increasing the skill-premium. Our model also predicts the impact of trade openness showing that the gains are larger for the skilled-labor; this agrees with the existing literature. We hope that our analysis contributes interesting insights in the ongoing debate about automation and has soothed fears about the future to some extent. Although we should pay attention to the increasing inequality that can pose a serious challenge to social coherence, our findings about the benefits of automation suggest that a Luddite revolution should wait for the time being.

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7 Appendix

7.1 Cost-minimization

Given our production function: we can aggregate the factors as following:

$$z_H \equiv \left(z_K^{\eta'} + H^{\eta'}\right)^{\frac{1}{\eta'}}$$

$$z_R \equiv \left(z_H^{\lambda'} + R^{\lambda'}\right)^{\frac{1}{\lambda}}$$

$$z_F \equiv \left(z_R^{\sigma'} + L^{\sigma'}\right)^{\frac{1}{\sigma'}}$$

By cost minimization of a Cobb-Douglas production function for $\forall\omega$, we can use the Lagrangean to obtain:

$$\frac{P_i^F}{P_i^M} = \frac{\gamma_{2,s} z_M(\omega)}{\gamma_{1,s} z_F(\omega)}$$

$$\frac{P_i^{NT}}{P_i^M} = \frac{(1-\gamma_{1,s}-\gamma_{2,s})\, z_M(\omega)}{\gamma_{1,s} z_{NT}(\omega)}$$

Also, by using Lagrangeans cost minimization implies:

$$\frac{z_K(\omega)}{H(\omega)} = \left(\frac{r_i}{w_{S,i}}\right)^{-\eta}$$

$$\frac{z_H(\omega)}{R(\omega)} = \left(\frac{P_i^H}{\rho}\right)^{-\lambda}$$

$$\frac{z_R(\omega)}{L(\omega)} = \left(\frac{P_i^R}{w_{L,i}}\right)^{-\sigma}$$

By proceeding through the calculation it can be shown that the cost function is

$$C_{i,s,\omega}(q, P_i^M, P_i^F, P_i^{NT}) = \frac{qB\left(P_i^M\right)^{\gamma_{1,s}}\left(P_i^F\right)^{\gamma_{2,s}}\left(P_i^{NT}\right)^{1-\gamma_{1,s}-\gamma_{2,s}}}{A_{i,\omega}}$$

where $B \equiv \gamma_{1,s}^{-\gamma_{1,s}}\gamma_{2,s}^{-\gamma_{2,s}}\left(1-\gamma_{1,s}-\gamma_{2,s}\right)^{\gamma_{1,s}+\gamma_{2,s}-1}$

Then the marginal cost for each unit of good ω_s is

$$MC_{i,s,\omega}(q, P_i^M, P_i^F, P_i^{NT}) = \frac{B\left(P_i^M\right)^{\gamma_{1,s}}\left(P_i^F\right)^{\gamma_{2,s}}\left(P_i^{NT}\right)^{1-\gamma_{1,s}-\gamma_{2,s}}}{A_{i,\omega}} = \frac{c_{i,s}}{A_{i,\omega}} \tag{24}$$

7.2 The skill-premium

Given the total income at location i (i.e.$GDP_i - NX_i = X_i - D_i$), $Y_i = w_{S,i}S_i + w_{L,i}L_i + \rho R_i + r_i z_{K,i} + P_i^M z_{M,i} + P_i^{NT} z_{NT,i}$we can obtain the following (to lighten up the notation a little bit I have dropped the location subscripts i):

$$w_L L = \gamma_2 \left(\frac{w_L}{P^F}\right)^{1-\sigma} Y$$

$$\rho R = \gamma_2 \left(\frac{\rho}{P^R}\right)^{1-\lambda}\left(\frac{P^R}{P^F}\right)^{1-\sigma} Y$$

$$w_S S = \gamma_2 \left(\frac{w_s}{P^H}\right)^{1-\eta}\left(\frac{P^H}{P^R}\right)^{1-\lambda}\left(\frac{P^R}{P^F}\right)^{1-\sigma} Y$$

$$r z_K = \gamma_2 \left(\frac{r}{P^H}\right)^{1-\eta}\left(\frac{P^H}{P^R}\right)^{1-\lambda}\left(\frac{P^R}{P^F}\right)^{1-\sigma} Y$$

$$\Rightarrow \frac{w_S^\eta}{w_L^\sigma} = \left(P^H\right)^{\eta-\lambda}\left(P^R\right)^{\lambda-\sigma}\frac{L}{S} \tag{25}$$

We will be solving from bottom-up:

$$\left(\frac{r}{w_S}\right)^{1-\eta} = \left(\frac{z_K}{S}\right)^{\frac{\eta-1}{\eta}}$$

$$\frac{P^H}{w_S} = \left(\left(\frac{r}{w_S}\right)^{1-\eta} + 1\right)^{\frac{1}{1-\eta}} = \left(\left(\frac{z_K}{S}\right)^{\frac{\eta-1}{\eta}} + 1\right)^{\frac{1}{1-\eta}}$$

$$\frac{w_S}{\rho} = \left(\frac{P^H}{w_S}\right)^{\frac{\eta-\lambda}{\lambda}} \left(\frac{R}{S}\right)^{\frac{1}{\lambda}} = \left(\left(\frac{z_K}{S}\right)^{\frac{\eta-1}{\eta}} + 1\right)^{\frac{\eta-\lambda}{\lambda(1-\eta)}} \left(\frac{R}{S}\right)^{\frac{1}{\lambda}} \tag{26}$$

$$\frac{r}{\rho} = \left(\left(\frac{S}{z_K}\right)^{\frac{\eta-1}{\eta}} + 1\right)^{\frac{\eta-\lambda}{\lambda(1-\eta)}} \left(\frac{R}{z_K}\right)^{\frac{1}{\lambda}}$$

$$\left(\frac{P^H}{\rho}\right)^{1-\lambda} = \left(\frac{z_H}{R}\right)^{\frac{\lambda-1}{\lambda}}$$

$$\frac{\rho}{w_L} = \left(\left(\frac{z_H}{R}\right)^{\frac{\lambda-1}{\lambda}} + 1\right)^{\frac{\lambda-\sigma}{\sigma(1-\lambda)}} \left(\frac{L}{R}\right)^{\frac{1}{\sigma}} \tag{27}$$

Hence the skill premium is given by:

$$\frac{w_S}{w_L} = \left(\left(\frac{z_K}{S}\right)^{\frac{\eta-1}{\eta}} + 1\right)^{\frac{\eta-\lambda}{\lambda(1-\eta)}} \left(\frac{R}{S}\right)^{\frac{1}{\lambda}} \left(\left(\frac{z_H}{R}\right)^{\frac{\lambda-1}{\lambda}} + 1\right)^{\frac{\lambda-\sigma}{\sigma(1-\lambda)}} \left(\frac{L}{R}\right)^{\frac{1}{\sigma}}$$

$$\Rightarrow \frac{w_S}{w_L} = \left(z_K^{\frac{\sigma-1}{\sigma}} + S^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\eta-\lambda}{\lambda(\eta-1)}} R^{\frac{1}{\lambda}} \left(\frac{1}{S}\right)^{\frac{1}{\eta}} \left(z_H^{\frac{\lambda-1}{\lambda}} + R^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\sigma-\lambda}{\sigma(\lambda-1)}} L^{\frac{1}{\sigma}} \left(\frac{1}{R}\right)^{\frac{1}{\lambda}}$$

$$\Rightarrow \frac{w_S}{w_L} = \left(z_K^{\frac{\sigma-1}{\sigma}} + S^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\eta-\lambda}{\lambda(\eta-1)}} \left(\frac{1}{S}\right)^{\frac{1}{\eta}} \left(z_H^{\frac{\lambda-1}{\lambda}} + R^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\sigma-\lambda}{\sigma(\lambda-1)}} L^{\frac{1}{\sigma}}$$

7.2.1 First-order approximation for the skill-premium

Given $\eta\tilde{w_S} - \sigma\tilde{w_L} = (\eta-\lambda)\tilde{P^H} + (\lambda-\sigma)\tilde{P^R} + (\tilde{L}-\tilde{S})$, we can take the first-order approximation that $exp(\tilde{x}) = 1+\tilde{x}$ and obtain that:

$$\tilde{P_i^R} = \tilde{P_i^F} - \tilde{w}_{L,i} + \frac{1}{1-\sigma}$$

$$\tilde{P_i^H} = \tilde{r}_i + \tilde{w}_{S,i} + \frac{1}{1-\eta}$$

$$\tilde{P_i^R} = \tilde{P_i^H} + \tilde{\rho} + \frac{1}{1-\lambda}$$

$$\Rightarrow \eta\tilde{w_S} - \sigma\tilde{w_L} = (\lambda-\sigma)\left(\tilde{\rho} + \frac{1}{1-\lambda}\right) + (\eta-\sigma)\tilde{P^H} + (\tilde{L}-\tilde{S})$$

$$\Rightarrow \eta\tilde{w_S} - \sigma\tilde{w_L} = (\lambda-\sigma)\left(\tilde{\rho} + \frac{1}{1-\lambda}\right) + (\eta-\sigma)\left(\tilde{r} + \tilde{w_S} + \frac{1}{1-\eta}\right) + (\tilde{L}-\tilde{S})$$

$$\Rightarrow \tilde{w_S} - \tilde{w_L} = \left(\frac{\lambda-\sigma}{\sigma}\right)\left(\tilde{\rho} + \frac{1}{1-\lambda}\right) + \left(\frac{\eta-\sigma}{\sigma}\right)\left(\tilde{r} + \frac{1}{1-\eta}\right) + \frac{\tilde{L}-\tilde{S}}{\sigma}$$

$$\Rightarrow \tilde{w_S} - \tilde{w_L} = \left(\frac{\lambda}{\sigma} - 1\right)\tilde{\rho} + \left(\frac{\eta}{\sigma} - 1\right)\frac{\tilde{\pi}_d^K}{\theta} + \frac{\tilde{L}-\tilde{S}}{\sigma} + constant$$

We can also obtain a different expression. Given the relationship $\frac{w_S}{\rho} = \left(z_K^{\frac{\eta-1}{\eta}} + S^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta-\lambda}{\lambda(1-\eta)}} R^{\frac{1}{\lambda}} S^{-\frac{1}{\eta}}$, we apply our first-order approximation techniques to get:

$$\tilde{w_S} - \tilde{\rho} = \left(\frac{1}{\eta} - \frac{1}{\lambda}\right)\tilde{z_K} + \frac{\tilde{R}-\tilde{S}}{\lambda} + \frac{\eta-\lambda}{\lambda(1-\eta)} \tag{28}$$

And from $\frac{\rho}{w_L} = \left(z_H^{\frac{\lambda-1}{\lambda}} + R^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\lambda-\sigma}{\sigma(1-\lambda)}} L^{\frac{1}{\sigma}} R^{-\frac{1}{\lambda}}$ we obtain:

$$\tilde{\rho} - \tilde{w_L} = \left(\frac{1}{\lambda} - \frac{1}{\sigma}\right)\tilde{z}_H + \frac{\tilde{L}-\tilde{R}}{\sigma} + \frac{\lambda-\sigma}{\sigma(1-\lambda)}$$

where $\tilde{z_H} = \tilde{z}_K + \tilde{S} + \frac{\sigma}{\sigma-1}$

$$\Rightarrow \tilde{\rho} - \tilde{w_L} = \left(\frac{1}{\lambda} - \frac{1}{\sigma}\right)\left(\tilde{z}_K + \tilde{S} + \frac{\sigma}{\sigma-1}\right) + \frac{\tilde{L} - \tilde{R}}{\sigma} + \frac{\lambda - \sigma}{\sigma(1-\lambda)}$$

Hence, a different expression for the change in the skill-premium is:

$$\tilde{w_S} - \tilde{w_L} = \left(\frac{1}{\eta} - \frac{1}{\sigma}\right)\tilde{z_K} - \frac{\tilde{S} - \tilde{L}}{\sigma} + \left(\frac{1}{\lambda} - \frac{1}{\sigma}\right)\tilde{R} + constant$$

$$\Rightarrow \tilde{w_S} - \tilde{w_L} = \frac{\tilde{z_K}}{\eta} - \frac{\tilde{z_K} + \tilde{S} - \tilde{L} + \tilde{R}}{\sigma} + \frac{\tilde{R}}{\lambda} + constant \quad (29)$$

Moreover, by the relationship $\frac{r}{\rho} = \left(\left(\frac{S}{z_K}\right)^{\frac{\eta-1}{\eta}} + 1\right)^{\frac{\eta-\lambda}{\lambda(1-\eta)}}\left(\frac{R}{z_K}\right)^{\frac{1}{\lambda}}$ we obtain:

$$\tilde{r} - \tilde{\rho} = \left(\frac{1}{\eta} - \frac{1}{\lambda}\right)\tilde{S} + \frac{\eta - \lambda}{\lambda(1-\eta)} + \frac{\tilde{R} - \tilde{z_K}}{\lambda} \quad (30)$$

$$\Rightarrow \tilde{z_K} = \tilde{R} + \lambda\left(\tilde{\rho} - \tilde{r}\right) + \left(\frac{\lambda - \eta}{\eta}\right)\tilde{S} + \frac{\lambda - \eta}{\eta - 1} \quad (31)$$

Hence, the above expression for the skill-premium becomes:

$$\tilde{w_S} - \tilde{w_L} = \left(\frac{1}{\eta} - \frac{2}{\sigma} + \frac{1}{\lambda}\right)\tilde{R} + \left[\left(\frac{1}{\eta} - \frac{1}{\sigma}\right)\left(\frac{\lambda - \eta}{\eta}\right) - \frac{1}{\sigma}\right]\tilde{S} + \frac{\tilde{L}}{\sigma} + \left(\frac{1}{\eta} - \frac{1}{\sigma}\right)\lambda\left(\tilde{\rho} - \tilde{r}\right) + constant$$

$$\Rightarrow \tilde{w_S} - \tilde{w_L} = \left(\frac{1}{\eta} - \frac{2}{\sigma} + \frac{1}{\lambda}\right)\tilde{R} + \frac{1}{\eta}\left(\frac{\lambda}{\eta} - \frac{\lambda}{\sigma} - 1\right)\tilde{S} + \frac{\tilde{L}}{\sigma} + \left(\frac{1}{\sigma} - \frac{1}{\eta}\right)\lambda\left(\frac{\tilde{\pi}_d^K}{\theta} - \tilde{\rho}\right) + constant$$