Imperfect Financial Markets and Investment Inefficiencies*

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Abstract

We analyze the consequences of noisy information aggregation for investment. Market imperfections create endogenous rents that cause overinvestment in upside risks and underinvestment in downside risks. In partial equilibrium, these inefficiencies are particularly severe if upside risks are coupled with easy scalability of investment. In general equilibrium, the shareholders' collective attempts to boost value of individual firms leads to a novel externality operating through price that amplifies investment distortions with downside risks but offsets distortions with upside risks.

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1 Introduction

We analyze the consequences of financial market imperfections for investment in partial and general equilibrium. We argue that noisy information aggregation in equity markets causes shareholders to distort investment decisions in an attempt to capture market rents and identify a new externality operating through share prices. We show that even small market imperfections can have severe consequence for investment. Our results suggest a new rationale for regulating financial risk-taking by publicly traded firms even when equity markets operate near efficiency.

We first develop a partial equilibrium model of a single firm whose incumbent shareholders make an investment decision prior to selling a fraction of their shares in a financial market populated by informed and noise traders. The share price then emerges as a noisy signal aggregating dispersed investor information about the firm's value.

In our model, the market-clearing share price must partially absorb shocks to demand and supply of securities, since informed traders are not willing or able to perfectly arbitrage perceived gaps between prices and expected fundamental values. This amplifies price fluctuations relative to the information about dividends that is aggregated through the market. The share price is therefore not just a noisy but also a biased estimate of the firm's dividends.

This bias has two important properties: (i) it inherits any asymmetries in underlying cash flow risks, and (ii) it scales with the firm's initial investment decision. Together these two properties result in an endogenous rent-seeking motive for shareholders that distorts corporate investment.

Property (i) implies that expected share prices are generally not an unbiased estimate of expected dividends: if cash flow risks are concentrated on the upside, the excess price fluctuations are primarily on the upside and lead to an upwards bias in average share prices relative to expected dividends. If instead the cash flow risk is concentrated on the downside, the downside price fluctuations dominate, resulting in a downwards bias of expected share prices. This wedge between the expected market value and the expected dividend value of a firm's equity is a transfer from final to initial shareholders (or vice versa), in other terms, a rent accruing to incumbent shareholders. Importantly, this wedge arises even when there is no risk premium.

Property (ii) then implies that incumbent shareholders can influence the magnitude of this rent through their investment decision. As our main result in this section, we show that rent-seeking incentives and investment distortions depend on two characteristics: risk asymmetries and scalability of investments. Firms with upside risks over-invest, while firms with downside risk under-invest. The scalability of investment then determines how flexibly a firm can adjust its investment to the gap between expected fundamentals and market returns. When investment is easy to scale, the surplus from investing is small but the scope for rent-seeking is particularly large. If easy scalability is coupled with upside risk, even small market frictions can induce incumbent shareholders to take excessively large risks purely to capture rents from selling their shares, while the firm in fact generates negative expected surplus. With downside risks, there can be severe under-investment, but surplus always remains positive. We then describe the taxes that implement the efficient investment level.

It is now useful to discuss the interpretation of upside vs. downside risks, and the scalability parameter. Regarding cash-flow risks, what really matters is that the price (being the expectation of the marginal investor) overweights the tails of the realizations of payoffs. In this sense any asymmetry of payoffs in the tails of the cash-flow distribution will result in either upside or downside risks. A natural way to think about this empirically is the comparison between mature value firms versus growth companies, or whether the tail risks for the firms are prevalently on the upside (say, an IPO) or on the downside (e.g., bankruptcy). The scalability parameter can be interpreted as the ability of scaling up investment to cater to the markets. Investment may not be easily scaleable for firms with technologies requiring large fixed capital expenditures, those facing tighter collateral constraint frictions, or firms with stronger corporate governance that limits the ease of catering investment to market prices.

We next embed the single-firm, partial equilibrium setup in an aggregate model with a continuum of heterogeneous firms, each subject to idiosyncratic investment risk. As our main result in this section, we show that equity market imperfections lead to a new externality which operates thorough the price and amplifies investment distortions in the case of downside risk but mitigates them in the case of upside risk. The externality arises because shareholders in any given firm do not internalize that by collectively distorting investment to boost their own share prices, they end up with lower aggregate dividends, and lower aggregate market values of equity shares. This introduces an intertemporal wedge that changes investment incentives. We formally derive a connection between the partial and general equilibrium level of investment and show how this wedge changes rent seeking incentives and equilibrium investment depending on the nature of the risks. With downside risk, this wedge reinforces the shareholders' desire to inflate share prices, which amplifies under-investment. With upside risk, it instead reduces the shareholders' desire to inflate market prices, which limits overinvestment and partially restores efficiency. We further show that for the highly scaleable investments even the small distortions may lead to large consequences in the general equilibrium setting. We then show that the tax that implements the efficient allocation in partial equilibrium has to be modified by a Pigouvian correction to account for the externality in general equilibrium.

Our model of the financial market builds on models of noisy information aggregation (Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981)), or more specifically the formulation in Albagli, Hellwig and Tsyvinski (2011) which characterizes returns for arbitrary securities in a nonlinear noisy rational expectations equilibrium model and argues that such a model can account for cross-sectional asset pricing puzzles. We depart from Albagli, Hellwig and Tsyvinski (2011) in two important aspects. First, we endogenize security cash flows as the outcome of firm's investment decisions. Second, we embed the firms in a general equilibrium environment. Endogenizing investment and cash flows is challenging even in partial equilibrium because of the interaction between how information aggregation affects investment incentives, and how investment in turn feeds into asset prices, payoffs and information aggregation. These challenges are compounded by the general equilibrium feedback from aggregate share prices to firm level incentives.

Our treatment of general equilibrium effects relates to the growing literature on externalities in financial markets. In our model, the externality results from market imperfections, when individual and aggregate share prices directly enter incumbent shareholder preferences. This is different from the pecuniary externalities commonly identified in the literature on financial constraints, where share prices indirectly affect investment incentives by relaxing or tightening collateral constraints (e.g., Lorenzoni 2008) or incentive constraints (e.g., Farhi, Golosov, Tsyvinski 2009).[1] With downside risk, our externality has the potential to generate significantly larger aggregate distortions because (i) it affects all firms, rather than a subset of financially constrained firms, and (ii) rather than being the primary source of inefficiency, it amplifies distortions caused by market imperfections. The interaction of trading frictions with externalities operating through price also appears in Asriyan (2016) but in a context where frictions in debt markets amplify balance sheet effects.

We then discuss the empirical relevance of our model. After briefly reviewing evidence of stock market pricing anomalies consistent with our main mechanism, we show that a straightforward extension of our partial equilibrium setting allows us to nest the empirical predictions of two types of models that study the sensitivity of investment to share prices. Specifically, information feedback from stock prices to investment decisions leads to excess price-investment sensitivity and to a negative co-movement of investment with future returns, as information feedback causes shareholders to cater to market expectations of returns. Depending on the realization of fundamentals and liquidity shocks, markets can be overly optimistic or pessimistic about the firm's return prospects, resulting in excessive investment when share prices are high, and foregone opportunities when share prices are low. Our setting thus delivers empirical predictions of the information feedback models (e.g.,

[1] See also Greenwald and Stiglitz (1986), Geanakoplos and Polemarchakis (1986), and Dávila and Korinek (forthcoming), among many other papers.

Chen, Goldstein and Jiang 2007) and the catering theory of investment (e.g., Polk and Sapienza 2009). Finally, we note that an important paper of David, Hopenhayn and Venkateswaran (2016), which augments the general equilibrium model of firm dynamics from Hopenhayn (1992) with the informational environment from our paper, documents large efficiency losses at the aggregate level arising from misallocation of capital.

2 Partial equilibrium

In this section, we describe a partial equilibrium environment with information frictions.

2.1 Baseline model

Our model has three stages. In the first stage, incumbent shareholders in a firm decide on an observable investment decision $k \geq 0$. In the second stage, they sell a fraction $\alpha \in (0,1]$ of the shares to outside investors. At the final stage, the firm's cash flow $\Pi(\theta, k) \equiv R(\theta) k - C(k)$ is a function of the investment k and a stochastic fundamental $\theta \in \mathbb{R}$, and paid to the final shareholders. The fundamental θ is distributed according to $\theta \sim \mathcal{N}(0, \lambda^{-1})$. The return $R(\cdot)$ on the investment is a positive, increasing function of the firm's fundamental, $C(k) = k^{1+\chi}/(1+\chi)$ denotes the cost of investment, and $\chi \geq 0$ is the scaling parameter that we refer as the firm's returns to scale or scalability. The expected dividends are given by $\mathbb{E}(\Pi(\theta, k))$. The ex-ante efficient investment k^* maximizes $\mathbb{E}(\Pi(\theta, k))$.

Stage 2: Description of the Market Environment. There are two types of outside investors: a unit measure of risk-neutral informed traders, and noise traders. Informed traders (indexed by i) observe a private signal $x_i \sim \mathcal{N}(\theta, \beta^{-1})$, which is i.i.d. across traders (conditional on θ). After observing x_i, an informed trader submits a price-contingent demand schedule $d_i(\cdot) : \mathbb{R} \to [0, \alpha]$, to maximize expected wealth $w_i = d_i \cdot (\Pi(\theta, k) - P)$. That is, informed traders cannot short-sell, and can buy at most α units of the shares.[2] An informed trader's strategy is then a function $d(x_i, P) \in [0, \alpha]$ of the private signal and the price. We assume that demand $d(x_i, P)$ is non-increasing in price P which is naturally satisfied if trading takes place through limit orders.

Noise traders place an order to purchase a random quantity $\alpha\Phi(u)$ of shares, where $u \sim \mathcal{N}(0, \delta^{-1})$ is independent of θ.

The aggregate demand for shares is $D(\theta, P) = \int d(x, P) d\Phi(\sqrt{\beta}(x - \theta)) + \alpha\Phi(u)$, where $\Phi(\sqrt{\beta}(x - \theta))$ represents the cross-sectional distribution of private signals x_i conditional on θ, and $\Phi(\cdot)$ denotes the cdf of a standard normal distribution. The orders submitted by informed and noise traders are executed at a market-clearing price P such that $D(\theta, P) = \alpha$.

Let $H(\cdot|x, P)$ denote the traders' posterior cdf of θ, conditional on observing a private signal x, and a market-clearing price P. A *noisy Rational Expectations Equilibrium* at stage 2 consists of a demand function $d(x, P)$, a price function $P(\theta, u; k)$, and posterior beliefs $H(\cdot|x, P)$, such that $d(x, P)$ is optimal given the shareholder's beliefs $H(\cdot|x, P)$; $P(\theta, u; k)$ clears the market for all (θ, u) and k; and $H(\cdot|x, P)$ satisfies Bayes' Rule whenever applicable.

Stage 2: Equilibrium Characterization. For a given level of investment k, it is straightforward to characterize the equilibrium share price in the unique noisy Rational Expectations Equilibrium (see Albagli, Hellwig, Tsyvinski 2011).

Lemma. ***Equilibrium Characterization and Uniqueness.*** *Define $z \equiv \theta + 1/\sqrt{\beta} \cdot u$. In the unique equilibrium, the market-clearing price function is*

$$P(z, k) = \mathbb{E}(\Pi(\theta, k) | x = z, z). \tag{1}$$

Each informed trader buys a share if the private signal is above a threshold

[2] We treat α as a parameter in the partial equilibrium setting and endogenize it in the general equilibrium setting. It is important to note that all our results carry through if we instead assume symmetric trading bounds, as long as the corresponding modifications to noise trader shocks are made to preserve tractability. In this sense, what matters is that investors face *some* limits to trading, and not whether such limits are more prominent in one direction of trading than the other.

$\hat{x}(P)$. The total demand of the informed traders is then

$$\alpha\left(1-\Phi(\sqrt{\beta}\,(\hat{x}(P)-\theta))\right).$$

Equating the sum of demand of the informed traders and of the uninformed traders ($\alpha\Phi(u)$) with the supply of shares (α), a price P clears the market in state (θ, u) if and only if

$$\alpha\left(1-\Phi(\sqrt{\beta}\,(\hat{x}(P)-\theta))\right)+\alpha\Phi(u)=\alpha,$$

which immediately gives the threshold characterization

$$\hat{x}(P)=\theta+1/\sqrt{\beta}\cdot u\equiv z.$$

That is, observing P is informationally equivalent to observing $z\sim\mathcal{N}(\theta,(\beta\delta)^{-1})$. Conditional on θ, z is distributed according to $z\sim\mathcal{N}(\theta,(\beta\delta)^{-1})$, while its unconditional distribution is $z\sim\mathcal{N}(0,\lambda_z^{-1})$, where $\lambda_z^{-1}=\lambda^{-1}+(\beta\delta)^{-1}$.

An intuitive way to understand this result is as follows. The sufficient statistic z represents the private signal of the trader who must be just indifferent between buying or not buying the stock if the market clears, which summarizes the demand for equity shares through noise traders (u) and informed traders (θ). The *identity* of this trader shifts in a systematic way with demand conditions: if informed traders become on average more optimistic (higher θ) or noise trader demand increases (higher u), the private signal defining the marginal trader must also increase to keep the market in equilibrium. To keep this marginal trader indifferent, the market price must increase with z and reveal z publicly to all market participants or outside observers. Thus, z acts as a sufficient statistic about the information contained in the price as a public signal, with a precision of $\beta\delta$.

The equilibrium share price differs systematically from the expected dividend value $V(z,k)\equiv\mathbb{E}(\Pi(\theta,k)\,|z)$, even though we assumed risk neutrality. Both are characterized as expected dividends conditional on the information contained in z. However, the share price $P(z,k)=\mathbb{E}(\Pi(\theta,k)\,|x=z,z)$ also incorporates the market clearing requirement of the equilibrium and thus additionally conditions on z. That is, it is the expectation of the payoff of an agent who infers z as the public signal contained in the price and also observes the private signal with the value $x = z$. Because the price is equal to the dividend expectations of this marginal trader, it places an additional weight on the signal z as if it had precision $\beta + \beta\delta$ (equal to the sum of the private and the price signal precision) compared to the weight of $\beta\delta$ that would be warranted from its precision as a public signal only when evaluating the expected dividends. Therefore, when z conveys sufficiently positive news about fundamentals, the price is upwards-biased, while if z conveys sufficiently negative news the price is biased downwards.[3]

We summarize the discussion of this section as follows. The price represents the expectation of the marginal trader who is indifferent between buying the asset or not. The identity of the marginal trader and hence the aggregate demand is determined by the signal he receives. Market clearing condition requires that the price is therefore a function of the signal of the marginal trader. Thus, the price is equal to the expectation of the dividends conditional on the private signal of the marginal trader and the information content of the public signal (the price) which is also given by the value of the marginal trader's signal. Because the identity of the marginal trader shifts in a systematic way with demand conditions due to market clearing forces (under limited arbitrage), the price reacts more to the realization of shocks than the expectation of dividends which uses only the informational content of prices. This is a general property of noisy REE models: when payoffs are nonlinear, expected prices and dividends will typically differ, giving way to systematic price premia or discounts and the corresponding distortions to investment decisions, which we explore next.

[3] We extensively discussed properties of this wedge between $P(z,k)$ and $V(z,k)$ in Albagli, Hellwig and Tsyvinski (2011). The closed form characterization extends to a general model with risk-neutral traders and arbitrary distributions and position limits. The functional form assumptions are convenient for comparative statics, but not otherwise crucial for our analysis.

Stage 1: Investment Decision We now describe how the investment decision and the information friction interact.

At the first stage, incumbent shareholders choose k to maximize the expected value of their equity:

$$\max_{k\geq 0} \mathbb{E}\left\{\alpha P\left(z;k\right)+\left(1-\alpha\right)\Pi\left(\theta,k\right)\right\} =$$

$$= \max_{k\geq 0}\left(\mathbb{E}\left\{\Pi\left(\theta,k\right)\right\}+\alpha\mathbb{E}\left\{P\left(z;k\right)-\Pi\left(\theta,k\right)\right\}\right), \tag{2}$$

where $P(z;k)$ is characterized by (1). The incumbent shareholder's objective differs from expected dividends by the term $\alpha\mathbb{E}\left(P\left(z;k\right)-\Pi\left(\theta,k\right)\right)$, which is a rent that accrues to incumbent shareholders.

Noisy information aggregation thus introduces a rent-seeking motive into incumbent shareholder preferences. When $\mathbb{E}\left(P\left(z;k\right)-\Pi\left(\theta,k\right)\right)\neq 0$, noisy information adds not just noise to stock prices, which would average out from an ex ante perspective, but also a bias. Importantly, the size of the rent is endogenous and its magnitude is influenced by the choice of investment k.

This rent-seeking motive arises because incumbent shareholders sell a fraction of their equity share at a price that differs in expectation from the shares' expected dividends. In the limit, where the incumbent shareholders keep all their shares (i.e. $\alpha\to 0$), or in an efficient market (i.e. if $P(z;k)=V(z;k)$), the rent-seeking motive disappears, and incumbent and final shareholder incentives are aligned on maximizing $\mathbb{E}\left(\Pi\left(\theta,k\right)\right)$.[4]

2.2 Investment distortions from market frictions

In this section, we characterize investment distortions due to noisy information aggregation in partial equilibrium. Investment and information frictions non-trivially interact. Information frictions engender endogenous rent and thus make payoffs endogenous. Investment decisions depend on and, in turn, also

[4] $P(z,k)=V(z,k)$ could result for example with free entry of uninformed arbitrageurs as in Kyle (1985), or when there is a public signal z, but no private information, and no heterogeneity among informed traders, so that they must all be indifferent about buying at equilibrium. This also corresponds to the limiting case of our model with $\beta\to 0$.

determine the size of the rent. We characterize how this mutual interrelation leads to inefficiencies in investment. We then determine a tax that implements the level of investment that maximizes ex-ante expected dividends.

2.2.1 Equilibrium investment distortions

We denote the efficient investment by k^* such that

$$C'(k^*) = \mathbb{E}(R(\theta)).$$

The initial shareholders instead choose $\hat{k}$ to equate the marginal cost of investment to a weighted average of expected market return $\mathbb{E}(\mathbb{E}(R(\theta)|x = z, z))$ and expected dividend return $\mathbb{E}(R(\theta))$:

$$C'\left(\hat{k}\right) = \alpha\left(\mathbb{E}\left(\mathbb{E}\left(R(\theta)|x = z, z\right)\right)\right) + (1-\alpha) E(R(\theta)),$$

or, alternatively,

$$C'\left(\hat{k}\right) = E(R(\theta)) + \alpha\left(\mathbb{E}\left(\mathbb{E}\left(R(\theta)|x = z, z\right)\right) - E(R(\theta))\right). \tag{3}$$

It then follows that $\hat{k} \gtreqless k^*$ if and only if

$$\mathbb{E}\left(\mathbb{E}\left(R(\theta)|x = z, z\right)\right) \gtreqless \mathbb{E}(R(\theta)).$$

Whenever the wedge between the expected price and expected dividends is positive, the initial shareholders find it optimal to overinvest to enhance the over-valuation of their shares. When instead the wedge is negative, the initial shareholders want to under-invest in order to limit the under-valuation of their shares.

We first relate the return ratio and hence the sign of the investment distortion to asymmetry between upside and downside risks. A return $R(\cdot)$ is *symmetric* if $R(\theta) - R(0) = R(0) - R(-\theta)$ for all $\theta > 0$. $R(\cdot)$ is *dominated by upside risk* if $R(\theta) - R(0) \geq R(0) - R(-\theta)$ for all $\theta > 0$, and *dominated by downside risk* if $R(\theta) - R(0) \leq R(0) - R(-\theta)$ for all $\theta > 0$. This classification compares gains and losses at fixed distances from the prior median to determine whether risks are concentrated on the upside or on the downside. The differences between upside and downside risks can be determined by comparing firms in different life-cycle stages, such as growth versus value firms, we return to this point in Section 4.

Standard arguments of compounding normal distributions imply that

$$\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)=\int_{-\infty}^{\infty}R\left(\theta\right)d\Phi(\sqrt{\lambda_P}\theta),$$

for some $\lambda_P^{-1}>\lambda^{-1}$. That is, from an ex ante perspective the market attributes too much weight to tail realizations of θ, which derives from the fact that the price places larger weight on the signal z than warranted from its precision as a public signal, as explained above. The parameter λ_P^{-1} depends on β, δ, and λ, and summarizes the severity of market frictions. If $R\left(\cdot\right)$ is symmetric, $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)=\mathbb{E}\left(R\left(\theta\right)\right)$ and investment is undistorted ($\hat{k}=k^*$). If $R\left(\cdot\right)$ is dominated by upside risk then $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)>\mathbb{E}\left(R\left(\theta\right)\right)$ and the firm over-invests ($\hat{k}>k^*$). If $R\left(\cdot\right)$ is dominated by downside risk then $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)<\mathbb{E}\left(R\left(\theta\right)\right)$ and the firm under-invests ($\hat{k}<k^*$).[5]

When returns to investment are asymmetrically distributed, the difference between the expectation of the share prices and the dividends that is due to the additional weighting of the market signal z results in the positive rent (in the case of upside risk) or negative rent (in the case of downside risk). With symmetric returns, posterior uncertainty is symmetric with respect to the realization of z and over- and under-pricing just offset each other. But in the case of upside (downside) risk, the overpricing of shares is higher (lower) on the upside, so that on average the rent is positive (negative).

The proposition that follows characterizes relates the magnitude of investment distortions and expected dividends to the return ratio $\frac{\mathbb{E}(\mathbb{E}(R(\theta)|x=z,z))}{\mathbb{E}(R(\theta))}$, the elasticity of marginal costs χ, and the fraction of shares sold α. Investment distortions are defined as $|\hat{k}/k^*-1|$. The ratio $\Delta=1-V/V^*$ is the loss in

[5] Furthermore, for upside risks, the return ratio $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)/\mathbb{E}\left(R\left(\theta\right)\right)$ is strictly increasing in λ_P^{-1}, while for downside risks the return ratio is strictly decreasing in λ_P^{-1}.

expected dividends $V = \mathbb{E}(R(\theta)) \cdot \hat{k} - C\left(\hat{k}\right)$, relative to the level without information frictions $V^* = \mathbb{E}(R(\theta)) \cdot k^* - C(k^*)$. If $\Delta > 1$, the losses are so large that expected dividends are negative.

Proposition 1. ***Investment distortions and information frictions in partial equilibrium.***

(i) ***Comparative Statics:*** *$|\hat{k}/k^* - 1| = \Delta = 0$ only if $\mathbb{E}(\mathbb{E}(R(\theta)|x = z, z)) = \mathbb{E}(R(\theta))$, $\chi \to \infty$, or $\alpha \to 0$. $|\hat{k}/k^* - 1|$ and Δ are increasing in α and $|\mathbb{E}(\mathbb{E}(R(\theta)|x = z, z))/\mathbb{E}(R(\theta)) - 1|$ and decreasing in χ.*

(ii) ***Bounded Distortions on the Downside:*** *If $\mathbb{E}(\mathbb{E}(R(\theta)|x = z, z)) < \mathbb{E}(R(\theta))$, then $\lim_{\chi\to 0} \hat{k}/k^* = 0$ and $\lim_{\chi\to 0} \Delta = 1$.*

(iii) ***Unbounded Distortions on the Upside:*** *$|\hat{k}/k^* - 1|$ and Δ become infinitely large, if either $\mathbb{E}(\mathbb{E}(R(\theta)|x = z, z))/\mathbb{E}(R(\theta)) \to \infty$, or $\chi \to 0$ and $\mathbb{E}(\mathbb{E}(R(\theta)|x = z, z)) > \mathbb{E}(R(\theta))$.*

(iv) ***Negative Expected Dividends:*** *Expected dividends are negative ($\Delta > 1$), whenever*

$$\alpha\left(\frac{\mathbb{E}(\mathbb{E}(R(\theta)|x = z, z))}{\mathbb{E}(R(\theta))} - 1\right) > \chi. \tag{4}$$

Proposition 1 shows that the magnitude of investment inefficiencies increases with return asymmetries, scalability of investment, and the proportion of shares traded α. Furthermore there is an asymmetry between over- and under-investment, since over-investment can result in arbitrarily large efficiency losses, to the point that the investment generates negative expected dividends.

The return ratio the proportion of shares traded α determine the initial shareholders' *incentive* to distort their investment due to the information friction, while the scalability parameter χ determines their *ability* to do so. Intuitively, scalability may be associated with either technological characteristics or certain corporate/institutional features. For example, firms with higher intangible capital can more easily expand operations, vis-a-vis businesses with large fixed capital expenses and rather long time-to-build investment which makes quick changes in the size of operations unfeasible. Alternatively, corporate/institutional features such as collateral constraint frictions or stronger corporate governance may limit the ease of catering investment to market prices.[6] With easy scalability (low χ), optimal investment is very sensitive to the size of the rent, and the scope for investment distortions and efficiency losses can become very large. At the other extreme, if marginal costs are very sensitive to k (high χ), investment is not easily scaleable, and investment distortions are small.

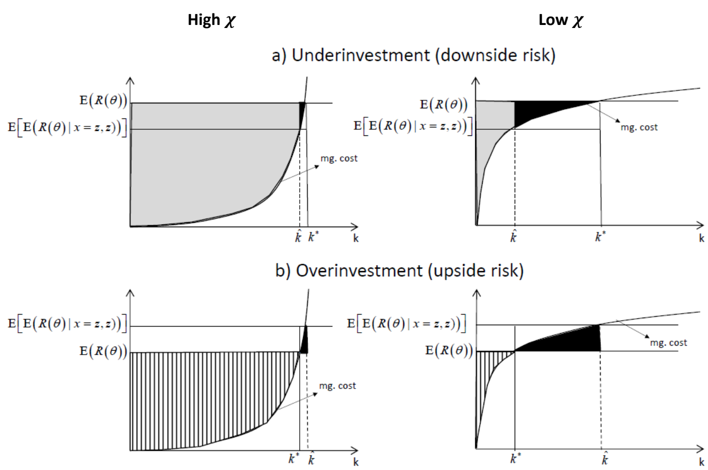
Figure 1 illustrates the comparative statics described by proposition 1. We plot marginal costs and expected market and fundamental returns, for high and low values of χ, and for the case with over- and under-investment, respectively. In all cases, the black triangular area corresponds to the loss in expected dividends, relative to those for the efficient investment level k^*. The upper panels consider the case with under-investment: the gray area corresponds to the expected dividends V, while the maximal expected dividends V^* corresponds to the combined gray and black areas. The lower two panels consider the case with over-investment: the striped area corresponds to V^*, the loss in the expected dividends corresponds to the black area, and the expected dividends V to the difference between the striped and the black areas. In both cases, a lower value of χ leads to a larger impact of frictions on investment and expected dividends.

In extreme cases, the firm generates negative expected cash flows. This occurs whenever the elasticity of marginal costs χ is less than the return distortion, which is given by the distance of the return ratio from 1, multiplied by the fraction of shares sold. Even a small rent (in terms of $\frac{\mathbb{E}(\mathbb{E}(R(\theta)|x=z,z))}{\mathbb{E}(R(\theta))}$) can have very large consequences for firms with easily scaleable investment, and with cash flows dominated by upside risk. On the other hand, with under-investment the firm's expected dividends always remain positive.

To summarize, our first main result of the paper is a characterization of how informational frictions can distort shareholder incentives to invest and take risks. Importantly, investment decisions endogenously determine the size

[6] Richardson (2006), for instance, documents how certain corporate governance structures can mitigate inefficient investment of free cash-flows.

Figure 1: Investment distortions and efficiency losses



of the rent that arises due to noisy information and thus amplify the effects on the difference of the expected dividends compared to those without information friction. The direction and magnitude of the resulting investment distortions and the effects on expected dividends depend on the firms' scalability of investment and on whether returns are characterized by upside or downside risks. If easy scalability is coupled with upside risks, even small frictions in financial markets can have very large consequences – so large, in fact, that the firm generates negative expected dividends.

2.2.2 Implementing efficient investment

In this section, we discuss a simple implementation of the efficient investment with taxes. We first note that trading in the markets can be thought of as a form of a friction in our environment. That is, prohibiting trades in the markets by, for example, by setting $\alpha = 0$ completely eliminates the distortions entailed by the information aggregation frictions. The idea that markets are a form of constraints can be traced to an important paper by Hammond (1987). This is, for example, a relevant restriction in the context of financial intermediation (see Jacklin (1987) and, more recently, Allen and Gale (2004) and Farhi, Golosov, and Tsyvinski (2009)). In what follows, we consider policies that indirectly affect the markets without completely shutting them down.

Consider now a tax, τ, that is imposed on the payoff $R(\theta)k$. Such a tax modifies the incumbent shareholder's objective function to

$$\alpha\mathbb{E}\left((1-\tau)\mathbb{E}\left(R(\theta)|x=z,z\right)k - C(k)\right) + (1-\alpha)\mathbb{E}\left((1-\tau)R(\theta)k - C(k)\right) =$$

$$= (1-\tau)\left(\alpha\mathbb{E}\left(\mathbb{E}\left(R(\theta)|x=z,z\right)\right) + (1-\alpha)\mathbb{E}\left(R(\theta)\right)\right)k - C(k).$$

The efficient level of capital k^* is implemented if the first order conditions for investment are:

$$(1-\tau)\left(\alpha\mathbb{E}\left(\mathbb{E}\left(R(\theta)|x=z,z\right)\right) + (1-\alpha)\mathbb{E}\left(R(\theta)\right)\right) - C'(k^*) = 0.$$

Noting that $C'(k^*) = \mathbb{E}(R(\theta))$, we find that a tax τ that implements the optimum satisfies

$$\tau = 1 - \frac{\mathbb{E}\left(R(\theta)\right)}{\alpha\mathbb{E}\left(\mathbb{E}\left(R(\theta)|x=z,z\right)\right) + (1-\alpha)\mathbb{E}\left(R(\theta)\right)}. \tag{5}$$

This tax realigns the investment incentives by correcting the effects of the informational friction. One can also consider a variety of other policies such as financial transaction taxes that we extensively discuss in the working paper version.

3 General equilibrium

In this section, we first show how individual firms' investment decisions interact in general equilibrium. We then identify and characterize a novel externality operating through prices. Finally, we characterize the optimal policy intervention in general equilibrium.

3.1 General equilibrium environment

There is a unit measure of firms, indexed by i and characterized by a firm-specific fundamental θ_i which is i.i.d. across firms and distributed according to $\theta_i \sim \mathcal{N}(0, \lambda^{-1})$. These firms are owned and controlled by incumbent shareholders who, in the first stage, control investment decisions k_i by each of these firms. Afterwards, incumbent shareholders sell a fraction α of shares to a new generation of final shareholders.

Our general equilibrium model combines three decision layers: i) the aggregate stock market determines aggregate market value of equity along with the fraction of shares sold, α; ii) the microstructure of equity markets determines individual firms' share prices; and iii) the incumbent shareholders' stage 1 decision determines investment in each firm. We describe each of these layers in turn.

Preferences and aggregate market values Let the aggregate market value of firms be denoted by

$$T = \int P_i di,$$

the aggregate dividends be denoted by

$$V = \int \Pi_i di,$$

and s be the fraction of shares sold by incumbent shareholders in period 1. The incumbent shareholders' preferences over stage 1 consumption $\left(C_1^I = sT\right)$ and stage 2 consumption $\left(C_2^I = (1-s)\,V\right)$ is $v_I\left(C_1^I\right) + u_I\left(C_2^I\right)$. The final shareholders' preferences over stage 1 consumption $\left(C_1^F = -sT\right)$ and stage 2 consumption $\left(C_2^F = sV\right)$ is $C_1^F + u_F\left(C_2^F\right)$, where the functions u_I, v_I and u_F satisfy standard Inada conditions. For a given value of V, T and s, the equilibrium values of s and T are uniquely determined from the incumbent and final shareholders' first-order conditions

$$\frac{T}{V} = \hat{Q}^{-1} = \frac{u_I'\left(\left(1-s\right)V\right)}{v_I'\left(sT\right)} = u_F'\left(sV\right). \tag{6}$$

Therefore, in the aggregate the financial market aligns the intertemporal marginal rates of substitution of incumbent and final shareholders. The aggregate market value of firms $T = Vu_F'(sV)$ is equal to aggregate dividends discounted at the shareholders' intertemporal MRS, $\hat{Q}^{-1}$.

The main purpose of this part of the microfoundations is to exposit the relationship

$$T = V\hat{Q}^{-1}$$

that relates the aggregate market value of the firms and the aggregate dividends. For the purpose of the description of the externality that arises in the general equilibrium setting, this is the only element that is needed. One can, otherwise, treat both the intertemporal marginal rate of substitutions of the shareholders, $\hat{Q}$, and fraction of the shares solved, s, as given. From now on, we assume that incumbent shareholder preferences are given by $v_I\left(C_1^I\right) + u_I\left(C_2^I\right) = \alpha lnC_1^I + (1-\alpha)\, lnC_2^I$, so that their supply of equity shares is inelastic at $s = \alpha$.

Microstructure of the equity markets We now turn to the central part of the description of the general equilibrium setting. We assume that neither final nor incumbent shareholders have inside information on the firms, and the incumbent shareholders sell a fixed fraction of each firm, which equals α given the assumption of log preferences. Final shareholders do not actively manage their investments but invest through two types of funds, mutual funds and hedge funds. As the owner of all hedge funds and mutual funds, the final shareholders indirectly purchase the aggregate market portfolio of firms. The funds $\alpha T = \alpha\mathbb{E}(P_i)$ invested by final shareholders are split such that in the aggregate, $\mathbb{E}(\alpha\Phi(u_i)\, P_i)$ are invested by mutual funds who purchase a random fraction $\alpha\Phi(u_i)$ of the shares in firm i. The remainder is allocated to hedge funds.

Mutual funds receive a stochastic inflow of funds and purchase $\alpha\Phi(u_i)$ fraction of shares in firm i where $u_i \sim \mathcal{N}(0, \delta^{-1})$ denotes a random, firm-specific liquidity.[7] Our modeling of the mutual funds is similar to that in Allen (1984) who models supply noise as that coming from the liquidity shocks. Hedge funds on the other hand acquire noisy private information about the different firms' fundamentals and then take positions in specific firms that are deemed sufficiently promising. There is a unit measure of such funds, who each obtain idiosyncratic private signals $x_i \sim \mathcal{N}(\theta_i, \beta^{-1})$ about each firms' fundamental, after which it decides in which firm to invest. To limit exposure to the risks associated with any individual firm, the hedge fund's positions are limited to no more than α shares per firm.[8]

Each hedge fund in turn either invests its funds directly in firms by buying up to α units of equity, by lending to other hedge funds at a market rate Q.[9] This market rate is a key object of interest in the general equilibrium analysis.

Incumbent shareholder's decision problem Incumbent shareholders of any given firm i maximize expected cash flow from equity sale in stage 1 and dividends in stage 2, weighted by their respective marginal utilities and taking the aggregate market values, α, V and Q as given:

$$\max_{k_i \geq 0} \mathbb{E}\left\{\alpha\hat{Q}P_i(z_i, k_i) + (1-\alpha)\Pi(\theta_i, k_i)\right\}. \tag{7}$$

3.2 Investment distortions in general equilibrium: an externality

We now describe the main result of the paper – an externality that arises in the general equilibrium model of information aggregation and endogenous investment.

[7] One can think of these funds' strategies as purchasing a fixed portfolio of firms' shares, whose overall expenditure varies exogenously with the random inflow/outflow of funds.

[8] Here we assume that the representative final shareholders' equity purchases through hedge funds and mutual funds scale with their aggregate demand for shares.

[9] This assumptions guarantees that all hedge funds have the same threshold return Q.

Consider the problem of hedge funds. A hedge fund will invest in firm i if and only if its expectations about that firms' dividend satisfy $\mathbb{E}\left(\Pi\left(\theta_i, k_i\right)|x, P_i\right) \geq QP_i$, resulting in a characterization of an indifference threshold z that is a monotone function of the price P_i. As in Section 2, the equilibrium price is determined by a marginal hedge fund and is represented as a function of a sufficient statistic $z_i = \theta_i + 1/\sqrt{\beta}u_i$

$$P_i\left(z_i, k_i\right) = \frac{1}{Q} \cdot \mathbb{E}\left(\Pi\left(\theta_i, k_i\right)|x = z_i, z_i\right). \tag{8}$$

Taking expectations and combining with

$$\mathbb{E}\left(P_i\right) = T = V\hat{Q}^{-1},$$

the equilibrium value of Q is

$$Q = \hat{Q}\frac{\mathbb{E}\left\{\mathbb{E}\left(\Pi\left(\theta_i, k_i\right)|x = z, z\right)\right\}}{V}. \tag{9}$$

Relative to the incumbent and final shareholders' inter-temporal MRS, $\hat{Q}$, the market rate Q is distorted by a wedge $\mathbb{E}\left\{\mathbb{E}\left(\Pi\left(\theta_i, k_i\right)|x = z, z\right)\right\}/V$ that corresponds to the ratio between the expected market value and the dividend value of firms.

General equilibrium allocations are then fully characterized by values for Q, and k that solve the first-order condition for the firm's investment choice in (7) and the characterization of Q in (9).

Substituting (8) in (7), the firm's optimization problem becomes

$$\max_{k_i \geq 0}\left\{\alpha\frac{\hat{Q}}{Q}\mathbb{E}\left\{\mathbb{E}\left(\Pi\left(\theta_i, k_i\right)|x = z_i, z_i\right)\right\} + (1-\alpha)\,\mathbb{E}\left\{\Pi\left(\theta_i, k_i\right)\right\}\right\}. \tag{10}$$

Hence, as in our partial equilibrium model, the incumbent shareholders maximize a weighted average of share price and dividend value. However, the relative weight on these two objectives depends not just on the fraction of shares sold α, as in the partial equilibrium, but also on the ratio $\hat{Q}/Q$.

This ratio represents the relative valuation of period 1 consumption relative to period 2, $\hat{Q} = V/T$, according to shareholder's equilibrium intertemporal consumption allocations, and the interest rate Q faced by hedge funds in equilibrium – a ratio that shareholders take as given. This is because the market price $P_i(z_i, k_i)$ is determined in general equilibrium by how the decisions of shareholders in different firms interact with each other through the hedge funds' marginal return Q. If financial markets were efficient (in the sense that $P_i(z_i, k_i) = Q^{-1} \cdot \mathbb{E}(\Pi(\theta_i, k_i) | z_i))$, the same aggregation arguments as above imply that $Q = \hat{Q}$, i.e., hedge fund and incumbent shareholders' intertemporal marginal rates of substitution are aligned, and incumbent shareholders have an incentive to maximize expected dividends, i.e. $k = k^*$.

We now turn to characterizing how the informational heterogeneity interacts with the investment decisions in our general equilibrium setting. There are two main interrelated effects.

First, investment choices in individual firms exert an externality on each other through their effect on the equilibrium interest rate Q. For a given Q, incumbent shareholders in a specific firm gain from distorting investment to increase the market value of their own shares, but they do not internalize that if all firms engage in this behavior, then aggregate dividends will be lower, which lowers the aggregate market value of firms. The shareholders' rent-seeking incentives at the micro level turn out to be self-defeating in the aggregate. This feedback is similar to the collateral channel in Lorenzoni (2008) or the private trades channel of Farhi, Golosov and Tsyvinski (2009). However, the origin of the externality is different, as it emerges from market imperfections due to information heterogeneity rather than incentive problems. In addition, the share price directly enters the firms' objective thereby affecting incentives to invest, rather than affecting investment indirectly through incentive constraints or financial constraints.

Second, the equilibrium value of Q feeds back into the firm incentives, amplifying or dampening the rent-seeking incentives depending on whether the risk is upside or downside. The externality feeds back into the incumbent shareholders objective via the intertemporal investment wedge $Q/\hat{Q}$ that influences the relative weight associated with the share price.

The first-order condition for the optimal choice of capital in the general equilibrium setting, k_{GE}, is

$$\alpha\frac{\hat{Q}}{Q}\left(\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)-C'\left(k_{GE}\right)\right)+\left(1-\alpha\right)\left(\mathbb{E}\left(R\left(\theta\right)\right)-C'\left(k_{GE}\right)\right)=0.$$

The price Q satisfies:

$$Q=\frac{\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)k-C\left(k_{GE}\right)}{E\left(R\left(\theta\right)\right)k-C\left(k_{GE}\right)}\hat{Q}=$$
$$=\frac{\left(1+\chi\right)\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)-C'\left(k_{GE}\right)}{\left(1+\chi\right)E\left(R\left(\theta\right)\right)-C'\left(k_{GE}\right)}\hat{Q}.$$

where the first equality uses (9), and the second equality uses $C\left(k\right)=k^{1+\chi}/\left(1+\chi\right)$.

The equilibrium level of capital k_{GE} is then given by:

$$\alpha\underbrace{\frac{\left(1+\chi\right)E\left(R\left(\theta\right)\right)-C'\left(k_{GE}\right)}{\left(1+\chi\right)\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)-C'\left(k_{GE}\right)}}_{\text{GE Wedge: }\frac{\hat{Q}}{Q}}\left(\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)-C'\left(k_{GE}\right)\right)$$
$$+\left(1-\alpha\right)\left(\mathbb{E}\left(R\left(\theta\right)\right)-C'\left(k_{GE}\right)\right)=0. \tag{11}$$

The equation (11) can be easily solved in closed form. We, however, now show how the general equilibrium considerations modify the partial equilibrium results. Adding and subtracting $\alpha\left(\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)-C'\left(k_{GE}\right)\right)$ on both sides of this equation we get

$$\alpha\ \Big(\underbrace{\frac{\hat{Q}}{Q}}_{\text{GE Wedge}}-1\Big)\left(\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)-C'\left(k_{GE}\right)\right)+$$
$$+\underbrace{\alpha\left(\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)-C'\left(k_{GE}\right)\right)+\left(1-\alpha\right)\left(E\left(R\left(\theta\right)\right)-C'\left(k_{GE}\right)\right)}_{\text{Partial equilibrium}}=0. \tag{12}$$

With upside risks, we have $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)>E\left(R\left(\theta\right)\right)$ and, hence,

$$\frac{\hat{Q}}{Q} = \frac{(1+\chi) E (R(\theta)) - C'(k_{GE})}{(1+\chi) \mathbb{E} (\mathbb{E} (R(\theta) | x = z, z)) - C'(k_{GE})} < 1.$$

The general equilibrium wedge dampens the rent seeking present in partial equilibrium and there is less investment, $k_{GE} < k_{PE}$, compared to the partial equilibrium and, hence, less overinvestment compared to the efficient level.

With the downside risks, we have $\mathbb{E} (\mathbb{E} (R(\theta) | x = z, z)) < E (R(\theta))$ and

$$\frac{\hat{Q}}{Q} = \frac{(1+\chi) E (R(\theta)) - C'(k_{GE})}{(1+\chi) \mathbb{E} (\mathbb{E} (R(\theta) | x = z, z)) - C'(k_{GE})} > 1.$$

The general equilibrium wedge amplifies rent seeking present in partial equilibrium and there is less investment, $k_{GE} < k_{PE}$, compared to the partial equilibrium and, hence, more underinvestment compared to the efficient level. In both cases, the amount of investment is lower than in the partial equilibrium. We summarize these results in the proposition that follows, the proof of which is immediate from the previous arguments).

Proposition 2. ***Investment distortions in general equilibrium.*** *Equilibrium investment is lower in general equilibrium compared to partial equilibrium. In the case of the upside risks, overinvestment is offset compared to the efficient investment. In the case of the upside risks, underinvestment is amplified compared to the efficient investment.*

Further intuition can be gained by considering the limiting case when investment is highly scaleable, or $\chi \to 0$. This case is particularly interesting as it allows us to explore whether the small distortions may lead to large consequences in the general equilibrium setting. Let V^* denote the first-best dividends, let V_{PE} denote the level of dividends in partial equilibrium with $Q = \hat{Q}$ (characterized as V in section 3), and V_{GE} and k_{GE} denote the general equilibrium level of dividends and investment.

Proposition 3. ***Investment distortions for small*** χ***.***

(i) Bounded distortions with upside risk: If $\mathbb{E} (\mathbb{E} (R(\theta) | x = z, z)) > \mathbb{E} (R(\theta))$, *then for small* χ, $C'(k_{GE}) \approx \mathbb{E} (R(\theta)) (1 + \alpha\chi)$, $V_{GE}/V^* \approx (1-\alpha) e^{\alpha} < 1$, *and* $Q/\hat{Q} \to \infty$.

(ii) Unbounded distortions with downside risk: If $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)<\mathbb{E}\left(R\left(\theta\right)\right)$*, then for small* χ*,* $C'\left(k_{GE}\right)\approx\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)\left(1+\left(1-\alpha\right)\chi\right)$*,* $V_{GE}/V_{PE}\to 0$*, and* $Q/\hat{Q}\to 0$*.*

Without an intertemporal distortion introduced by general equilibrium, i.e. if $Q=\hat{Q}$, firms set marginal costs equal to $C'\left(k\right)=\alpha\mathbb{E}\left\{\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right\}+\left(1-\alpha\right)\mathbb{E}\left(R\left(\theta\right)\right)$.

With upside risk, there is an upwards distortion in price, which implies that the equilibrium interest rate faced by hedge funds is larger than shareholder's marginal intertemporal substitution: $Q>\hat{Q}$. In the objective function (10), this is partially self-correcting, as it induces incumbent shareholders to place more weight on maximizing expected dividends, reduces the rent-seeking motive relative to the partial equilibrium analysis, and dampens investment distortions. Moreover, equation (6) implies that expected dividends and share prices must be positive in general equilibrium, and that therefore the extent of over-investment cannot become too large. In other words, marginal costs $C'\left(k\right)$ cannot stray too far from $\mathbb{E}\left(R\left(\theta\right)\right)$, and in the limit as $\chi\to 0$, $C'\left(k\right)$ must converge to $\mathbb{E}\left(R\left(\theta\right)\right)$. In this limit, the intertemporal wedge becomes large ($Q/\hat{Q}\to\infty$), dividends remain positive, yet strictly lower than at the first best, and investment remains distorted up by a factor e^{α} in the limit. Hence, the general equilibrium effects offset a large part of the partial equilibrium investment distortion but do not restore the expected dividends to the first-best case entirely. Depending on the fraction of shares traded in equilibrium, the loss relative to first-best can still be very substantial.[10]

With downside risk, the distortion in the price is downwards, $Q<\hat{Q}$. In the objective function (10), this pushes incumbent shareholders to shift even more weight towards expected share prices, which reinforces the rent-seeking motive, and the associated externality. In the limit as $\chi\to 0$, marginal costs $C'\left(k\right)$ must converge to $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)$. The amplification thus becomes so strong that it pushes shareholders to invest as if all their shares were sold in the market. Expected dividends vanish even relative to the partial equilibrium

[10] Recall that as $\chi\to 0$, $V_{PE}/V^{*}\to-\infty$ and $\hat{k}/k^{*}\to\infty$. Negative dividends are not possible in general equilibrium.

benchmark.[11] The general equilibrium effects thus vastly amplify the partial equilibrium distortion.

To summarize, in general equilibrium, shareholder rent-seeking generates an externality and an intertemporal investment distortion. The intertemporal distortion partly offsets the externality in the case of upside risk, but reinforces it in the case of downside risk. Interestingly, and in contrast to the partial equilibrium analysis, the expected dividend consequences of market frictions are now more severe with downside risk, but limited with upside risk. And once again, scalability of investment determines the severity of investment distortions and the externalities. The limiting results with $\chi \to 0$ illustrate that even small imperfections in equity markets can have very dramatic consequences for incentives and investment.

3.3 Implementing efficient investment in general equilibrium

We now turn to the analysis of the taxes that implement the efficient allocation. Consider, as in the partial equilibrium, a tax τ that is imposed on the payoff $R(\theta)\,k$. Such a tax modifies the incumbent shareholder's objective function to

$$\alpha \frac{\hat{Q}}{Q} \mathbb{E}\left((1-\tau)\,\mathbb{E}\left(R(\theta)\,|x=z,z\right)k - C(k)\right) + (1-\alpha)\,\mathbb{E}\left((1-\tau)\,R(\theta)\,k - C(k)\right).$$

The efficient level of capital k^* is implemented if the first order conditions for investment are:

$$\alpha \frac{\hat{Q}}{Q} \mathbb{E}\left((1-\tau)\,\mathbb{E}\left(R(\theta)\,|x=z,z\right) - C'(k^*)\right) + (1-\alpha)\,\mathbb{E}\left((1-\tau)\,R(\theta) - C'(k^*)\right) = 0.$$

Noting that $C'(k^*) = \mathbb{E}\left(R(\theta)\right)$, we find that a tax τ that implements the

[11] Recall that $V/V^* \to 0$ as $\chi \to 0$. It is straightforward to construct examples in which $V^* \to \infty$ as $\chi \to 0$, but $V_{GE} \to 0$, i.e. first-best dividends grow infinitely large, yet the realized surplus completely vanishes.

optimum satisfies

$$\tau = 1 - \frac{\left(\alpha \frac{\hat{Q}}{Q} + (1-\alpha)\right) \mathbb{E}\left(R\left(\theta\right)\right)}{\alpha \frac{\hat{Q}}{Q} \mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x = z, z\right)\right) + (1-\alpha)\,\mathbb{E}\left(R\left(\theta\right)\right)}, \tag{13}$$

where

$$\frac{\hat{Q}}{Q} = \frac{\chi E\left(R\left(\theta\right)\right)}{(1+\chi)\,\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x = z, z\right)\right) - E\left(R\left(\theta\right)\right)}$$

is the additional Pigouvian correction for the externality arising in general equilibrium.

The results so far have focused on implementing efficient investment. There still may be an intertemporal wedge in the first order conditions of the initial shareholders, and Q in general is not equal to $\hat{Q}$. Moreover, the discussion here assumes that there are no situations of excess demand or excess supply of equity shares and all sides find it optimal to trade equity. The working paper version of our work exposits the effects of financial transactions taxes on the shareholders' intertemporal distortions, as well as a variety of other policy interventions.

4 Empirical relevance

In this section, we begin with a brief review of the key asset pricing literature providing evidence consistent with the relationship between informational frictions and expected returns in stock markets. We then show that a straightforward extension of our partial equilibrium model nests the predictions of two important literatures: the link between real investment and share prices from information feedback theories, and models of catering to investor sentiments. We also discuss recent work which, using our asset market setup, quantifies the general equilibrium consequences of informational frictions for aggregate productivity.

4.1 Informational frictions and stock returns

A large empirical asset pricing literature studies the link between informational frictions and stock return anomalies. In an influential paper Diether, Malloy and Sherbina (2002) sort stocks by the dispersion of earnings forecasts across analysts covering each security. They find that stocks in the highest dispersion quintile have monthly returns which are about 0.62% lower than those in the lowest dispersion quintile, amounting to a yearly excess return over 7% for the strategy of going long/short on low/high dispersion stocks. Gebhardt, Lee and Swaminathan (2001) document that an alternative measure of stock risk premia, the cost of capital, is also negatively related to analyst forecast dispersion. Thus, using earnings forecast dispersion as a proxy of informational frictions, these results are consistent with the prediction of our model that larger frictions lead to larger overpricing in securities dominated by upside risks, such as stocks, and thus to lower ex-post returns.

Yu (2011) explores the issue further using bottom-up measures of disagreement for stock portfolios, and then studies the return dynamics associated with time variation in portfolio disagreement. Two results reported in the paper provide support to our mechanism. First, in line with Diether et al. (2002), an increase in bottom-up portfolio dispersion is associated with a large drop in one-year ahead market returns. Moreover, the paper documents that following an increase in portfolio forecast dispersion, stock prices rise contemporaneously, which is the main driver of ex-post lower portfolio returns, consistent with our mechanism. Second, the author sorts securities into growth and value portfolios, finding that price increases and subsequent low returns are stronger for growth stocks, as predicted by our model if we associate growth firms with cash flows more skewed towards upside risks.

Finally, Bassetto and Galli (2019) uses a partial equilibrium setting of our paper to argue for the importance of information in debt crises. There, rather than investment and, hence, rent-seeking endogenously determining the payoffs, it is the repeated interaction and information transmission over time that shapes the payoffs.

4.2 Investment feedback and catering theories

A large empirical literature explores the sensitivity of firm decisions, in particular corporate investment, to share prices.[12] One possible explanation for such investment sensitivity to stock prices is information feedback: the share price contains valuable information that helps shareholders and managers make more informed investment decisions.[13] A less positive view on the subject is taken by the catering theory models, stressing how investment managers aim to maximize market valuation by guiding investment towards the opinion of the market, whether such opinion is warranted by fundamentals or not. Indeed, in an influential paper, Polk and Sapienza (2009) show that investment is directly affected by the market deviations from fundamentals. Hoberg and Phillips (2010) also provide evidence for this mechanism, showing that high industry-level stock-market valuations coincide with higher investment and new financing, and are subsequently followed by sharply lower operating cash flows and abnormal stock returns in the US, a pattern particularly strong for highly competitive industries.

We now allow a direct informational feedback from the share price to investment, and consider how market frictions distort the use of information aggregated through share prices. We modify our benchmark model by assuming that the initial shareholders publicly commit to a price-contingent, or equivalently z-contingent investment function $k(z)$.[14] Market participants perfectly anticipate the investment level that will realize at a given price, and the incumbent shareholders internalize the impact of their decision rule on the share price. We also, for simplicity, set $\alpha = 1$, i.e., incumbent shareholders

[12] See Morck et al. (1990), Baker, Stein and Wurgler (2003), and Gilchrist, Himmelberg, and Huberman (2005).

[13] See Dow and Gorton (1997), Dow and Rahi (2003), Goldstein and Guembel (2008), Foucault and Fresard (2012), and Goldstein, Ozdenoren, and Yuan (2013).

[14] This requires implicitly that the price function is strictly monotone in z, a condition that is not automatically satisfied for all $k(z)$. Alternatively one may assume that shareholders have the means to infer z through other means than the price, or that there exists a "non-strategic" component of dividends $\pi(\theta)$ that is strictly increasing in θ and guarantees an upwards-sloping price function. Here we will ignore the invertibility issue, but note that monotonicity is satisfied via an envelope condition for the case of primary interest, where $\alpha = 1$ and incumbent shareholders maximize expected share price.

care only about the market value of their equity share.

For a given $k(z)$, the equilibrium share price is

$$P(z, k(z)) = \mathbb{E}(R(\theta)|x = z, z) \cdot k(z) - C(k(z)).$$

The expected dividend is

$$V(z, k(z)) = \mathbb{E}(R(\theta)|z) \cdot k(z) - C(k(z)).$$

To illustrate the effect of information feedback, we compare expected dividends and shareholder rents with an increasing investment function $k(z)$, with a benchmark in which investment is constant at $\hat{k} = \mathbb{E}(k(z))$. The expected dividend takes the form

$$\mathbb{E}(V(z, k(z))) =$$
$$= \mathbb{E}\left(V\left(z, \hat{k}\right)\right) + cov\left(k(z), \mathbb{E}(R(\theta)|z)\right) - \left(\mathbb{E}(C(k(z))) - C\left(\hat{k}\right)\right).$$

The information feedback increases expected dividends by

$$cov\left(k(z), \mathbb{E}(R(\theta)|z)\right) > 0$$

relative to the constant investment case, and it reduces expected dividends by a term due to convexity of costs. The covariance term measures the value of conditioning investment on z, which strictly exceeds the second term if investment is not too volatile. Expected dividends increase because the information feedback aligns marginal costs and investment more closely with expected returns.

Likewise, we can characterize the effect of information feedback on expected shareholder rents:

$$\mathbb{E}(P(z, k(z))) - \mathbb{E}(V(z, k(z))) =$$

$$= \mathbb{E}\left(P\left(z, \hat{k}\right)\right) - \mathbb{E}\left(V\left(z, \hat{k}\right)\right) + cov\left(k(z), \mathbb{E}(R(\theta)|x = z, z) - \mathbb{E}(R(\theta)|z)\right).$$

If $R(\cdot)$ is symmetric and $\mathbb{E}(R(\theta)|x = z, z) \geq \mathbb{E}(R(\theta)|z)$ for $z \geq 0$, then this covariance is strictly positive. Information feedback thus generates endogenous upside risk: the firm invests more when z is high and expected market returns exceed fundamental returns.[15] This reinforces the incumbent shareholders' rent extraction incentive and increases shareholder rents. Moreover, shareholder rents are increasing in the sensitivity of $k(\cdot)$ to z. The efficient investment rule sets $k^*(z)$ such that $C'(k^*(z)) = \mathbb{E}(R(\theta)|z)$ and incorporates the information contained in the price according to Bayes' Rule. Also, scalability increase the potential value of information feedback, i.e. $lim_{\chi\to 0}\mathbb{E}(V(z,k^*(z)))/V^* = \infty$, but simultaneously increases shareholder rents, i.e. $\lim_{\chi\to 0}\mathbb{E}(P(z,k^*(z)) - V(z,k^*(z))) = \infty$. Thus even if the original returns are dominated by downside risk, incumbent shareholders in equilibrium capture arbitrarily large positive rents if investment is sufficiently easy to scale up.

Next, we discuss how rent-seeking by incumbent shareholders leads to excess sensitivity of investment to stock prices. Suppose that $R(\cdot)$ is such that $\mathbb{E}(R(\theta)|x=z,z)/\mathbb{E}(R(\theta)|z)$ is strictly increasing in z.

The initial shareholders choose $\hat{k}(z)$ to satisfy $C'\left(\hat{k}(z)\right) = \mathbb{E}(R(\theta)|x=z,z)$. Therefore, investment $\hat{k}(z)$ is dictated by market expectations of investment returns: investment responds more to z than would be justified by Bayes' Rule. In effect, information feedback with imperfect equity markets results in a theory of endogenous catering effects (see, e.g. Stein 1996). Capital market imperfections distort market valuations, and with information feedback, incumbent shareholders and managers have an incentive to cater investment decisions to these distorted market expectations of returns in an attempt to maximize shareholder rents.

We obtain a positive relation between investment and share prices:

$$\hat{k}(z) = ((1+1/\chi)P(z))^{1/(1+\chi)}.$$

[15] For general return distributions, $cov(k(z), \mathbb{E}(R(\theta)|x=z,z) - \mathbb{E}(R(\theta)|z))$ is nonnegative and can be arbitrarily large whenever (i) $k(z)$ is sufficiently responsive to z, and (ii) $\mathbb{E}(R(\theta)|x=z,z) > \mathbb{E}(R(\theta)|z)$ for sufficiently large realizations of z. With symmetric or upside risk, information feedback generates or strengthens the upside bias in market prices. With downside risk, information feedback mitigates or overturns downwards bias in prices.

Expected returns on equity are

$$\frac{V(z)}{P(z)} - 1 = \frac{1+\chi}{\chi}\left(\frac{\mathbb{E}(R(\theta)|z)}{\mathbb{E}(R(\theta)|x=z,z)} - 1\right),$$

and hence decreasing in investment and share price. The following proposition, which follows directly from the derivations above, summarizes the economic effects of information feedback for investment and equity returns.

Proposition 4. ***Information feedback causes excess investment volatility***

(*i*) ***Investment is increasing in share prices:*** $cov\left(\hat{k}(z), P(z)\right) > 0$.

(*ii*) ***Excess sensitivity of investment to stock prices:*** $\hat{k}(z)/k^*(z)$ *is increasing in* z.

(*iii*) ***Higher Investment leads to lower equity returns****:*

$$cov\left(\hat{k}(z), \frac{V(z)-P(z)}{P(z)}\right) < 0.$$

Our model thus merges the predictions of information feedback theories with models of catering to investor sentiments. Market signals convey valuable information to shareholders. But these signals are not unbiased and result in a catering of investment to market expectations of returns. Information feedback thus results in excess sensitivity of investment, higher expected share prices and shareholder rents, and lower subsequent returns. Proposition 4 summarizes these predictions.

Information feedback gives incumbent shareholders an additional margin along which to optimize their rents. Since shareholder rents are increasing in the sensitivity of investment to z, they take advantage through an investment rule that caters to market expectations. This causes excess volatility in investment: on the upside, shareholders over-invest to maximize the rents they extract from inflated share prices. On the downside, they under-invest to limit the losses they incur from the market price being below the fundamental value. Our model thus links investment volatility to stock market volatility by tying investment decisions to market expectations of returns.

Several papers confirm these empirical predictions. First, there is evidence in support of information feedback: Chen, Goldstein and Jiang (2007) find that real investment is more sensitive to share prices in firms whose shares are traded by more informed traders, as measured by PIN (probability of informed trading – Easley et al. (1996)). Roll, Schwartz and Subrahmanayam (2009) provide evidence that deeper options markets for a firm's share stimulate the entry of informed traders, and that such firms have a higher sensitivity of investment to share prices. These papers suggest that the equity market indeed conveys information about fundamentals that guide corporate investment decisions. Second, Polk and Sapienza (2009) offer direct support for catering effects in corporate investment by estimating the regression coefficients in proposition 4 (*i*) and (*iii*). Using discretionary accruals as a proxy for mispricing, they find a positive relation between share overvaluation and investment after controlling for Tobin's Q.[16] They also find that this relation is stronger for firms with higher share turnover, which can be interpreted as a reasonable proxy for the extent of short-termism in incumbent shareholders, or the fraction α of shares sold in period 1 in our model. Moreover, firms with high investment subsequently have low share returns, the more so the larger is their measure of mispricing. This suggests that such investment behavior is indeed inefficient.

Finally, additional support for our theory comes from recent work directly studying the link between proxyes of informational frictions and corporate investment efficiency. Chen, Xie and Zhang (2017) document that lower dispersion and/or higher accuracy of analysts' earnings forecasts increase investment efficiency –increasing (decreasing) investment in firms more likely to under (over) invest. They further show that such effects are stronger in firms with lower institutional stock ownership, another reasonable proxy for the degree of short-termism in stock trading (parameter α).

[16] Discretionary accruals measure the extent to which a firm has abnormal non-cash earnings. Firms with high discretionary accruals typically have relatively low share returns in the future, suggesting that discretionary accruals artificially drive up prices temporarily.

4.3 Information frictions and aggregate efficiency implications

We now address the empirical relevance of informational frictions in general equilibrium. An important paper of David, Hopenhayn and Venkateswaran (2016) augments a general equilibrium model of firm dynamics of Hopenhayn (1992) with the informational environment and the friction of our paper. They carefully calibrate the information friction and argue that it is responsible for 20-50% of the observed dispersion in the marginal (revenue) product of capital, and even a larger fraction if they control for firm-fixed effects.

While their model does not have a rent-seeking motive, such as the one we study, there are important similarities in terms of the general equilibrium mechanism having additional effects compared to those in partial equilibrium. Holding aggregate factors fixed, the informational friction affects aggregate productivity which in turn directly translates to the effects on output. There is, however, an important additional general equilibrium effect – misallocation reduces incentives to accumulate capital and thus amplifies the effects of informational friction. In our model, instead of the accumulation of capital, an externality operating through the price in general equilibrium has additional effects on capital and the value of the firms compared to the partial equilibrium. Furthermore, we qualify their results showing that depending on the nature of the risk, the general equilibrium effects may either amplify or dampen the partial equilibrium misallocation.

5 Conclusion

With unlimited arbitrage, equity markets can be trusted to accurately reflect firm fundamentals. This connection provides the intellectual basis for shareholder value as a measure of social surplus, and for the laissez-faire argument against interference with firm decisions. Its validity as a guiding principle for regulatory policy rests on the unstated assumption that departures from market efficiency cannot be too important, and have at worst minor effects on shareholder incentives.

In this paper we question this assumption by taking a different view of price formation in asset markets that is based on limits to arbitrage and noisy information aggregation. We argue that informational frictions introduce a rent-seeking motive to shareholder value: markets no longer fully align shareholder value with social surplus, and initial shareholders can no longer be trusted to act in the interest of future shareholders or society. What's more, even small departures from market efficiency can have large aggregate consequences either through firm-level scalability of investment, or through the externalities in general equilibrium.

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6 Appendix: Proofs

Proof of Proposition 1:

To simplify notation, let $\Upsilon = \alpha\left(\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)/\mathbb{E}\left(R\left(\theta\right)\right)-1\right)$. We begin with the results concerning $\hat{k}/k^{*}$. Part (i). Efficient investment: Since $\hat{k}/k^{*} = \left(1+\Upsilon\right)^{1/\chi}$, it is immediate that $\hat{k}/k^{*} = 1$ if and only if $\Upsilon = 0$, which occurs whenever $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right) = \mathbb{E}\left(R\left(\theta\right)\right)$; $\chi \to \infty$, or $\alpha \to 0$. Comparative statics: $\hat{k}/k^{*}$ is increasing in Υ. If $\Upsilon > 0$, then $\hat{k}/k^{*} > 1$ is increasing in α, increasing in $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)/\mathbb{E}\left(R\left(\theta\right)\right)$,and decreasing in χ. If $\Upsilon < 0$, then $\hat{k}/k^{*} < 1$ is decreasing in α, decreasing in $\mathbb{E}\left(R\left(\theta\right)\right)/\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)$ and increasng in χ, and so $|\hat{k}/k^{*}-1|$ inherits the opposite comparative statics w.r.t. these objects, as stated in the proposition. Part (ii). If $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right) < \mathbb{E}\left(R\left(\theta\right)\right)$, then $\Upsilon < 0$ and $\lim_{\chi\to 0}\hat{k}/k^{*} = 0$ follows immediately. Part (iii). If $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right) > \mathbb{E}\left(R\left(\theta\right)\right)$, then $\Upsilon > 0$ and both limit results follow immediately.

We now turn to the results concerning Δ. Note that we can write $\Delta = 1+\left(1+\Upsilon\right)^{1/\chi}\left(\Upsilon/\chi-1\right)$. Part (i). Efficient investment: it follows that $\Delta = 0$ iff $\Upsilon = 0$. Comparative statics. Since

$$\frac{\partial\Delta}{\partial\Upsilon} = \frac{1}{\chi}\frac{1+\chi}{\chi}\left(1+\Upsilon\right)^{1/\chi}\frac{\Upsilon}{1+\Upsilon} \text{ and } \frac{\partial\Delta}{\partial\chi^{-1}} = \left(1+\Upsilon\right)^{1/\chi}\left(\Upsilon-\left(1-\Upsilon/\chi\right)\log\left(1+\Upsilon\right)\right),$$

Δ is increasing in Υ (and therefore positive) if $\Upsilon > 0$, and Δ is decreasing in Υ (and therefore again positive) if $\Upsilon < 0$. Furthermore, $\frac{\partial\Delta}{\partial\chi^{-1}} > 0$ if $\Upsilon/\chi \geq 1$, and $\frac{\partial\Delta}{\partial\chi^{-1}} = \Upsilon - \log\left(1+\Upsilon\right)\left(1-\Upsilon/\chi\right) > \Upsilon - \Upsilon\left(1-\Upsilon/\chi\right) > \Upsilon^{2}/\chi > 0$ if $\Upsilon < \chi$. Parts (ii), (iii) and (iv) then also follow immediately. ■

Proof of Proposition 3:
The first-order condition for k_{GE} is

$$\alpha_0 \frac{\hat{Q}}{Q} \left(\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right) - C'\left(k_{GE}\right)\right) + \left(1-\alpha_0\right)\left(\mathbb{E}\left(R\left(\theta\right)\right) - C'\left(k_{GE}\right)\right) = 0,$$

where $\hat{Q} = 1/u_F'\left(\alpha_0 V_{GE}\right)$ satisfies

$$\hat{Q} = \frac{v_I'\left(\alpha_0 T_{GE}\right)}{u_I'\left(\left(1-\alpha_0\right)V_{GE}\right)} = \frac{V_{GE}}{T_{GE}} = \frac{\left(1+\chi\right)\mathbb{E}\left(R\left(\theta\right)\right) - C'\left(k_{GE}\right)}{\left(1+\chi\right)\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right) - C'\left(k_{GE}\right)} Q.$$

(i) Suppose that $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right) > \mathbb{E}\left(R\left(\theta\right)\right)$. We write the FOC for $C'\left(k_{GE}\right)$ as

$$\frac{\alpha_0}{1-\alpha_0} = \frac{C'\left(k_{GE}\right) - \mathbb{E}\left(R\left(\theta\right)\right)}{\left(1+\chi\right)\mathbb{E}\left(R\left(\theta\right)\right) - C'\left(k_{GE}\right)} \frac{\left(1+\chi\right)\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right) - C'\left(k_{GE}\right)}{\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right) - C'\left(k_{GE}\right)}$$

As $\chi \to 0$, the second ratio converges to 1, and therefore $C'\left(k_{GE}\right) \approx \left(1+\alpha_0\chi\right)\mathbb{E}\left(R\left(\theta\right)\right)$, and $Q/\hat{Q} \to \infty$. Expected dividends are $V_{GE} \approx \frac{\chi}{1+\chi}\mathbb{E}\left(R\left(\theta\right)\right)^{1+1/\chi}\left(1-\alpha_0\right)\left(1+\alpha_0\chi\right)^{1/\chi}$ and therefore $V_{GE}/V^* \approx \left(1-\alpha_0\right)\left(1+\alpha_0\chi\right)^{1/\chi} \to \left(1-\alpha_0\right)e^{\alpha_0}$.

(ii) Suppose that $\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right) < \mathbb{E}\left(R\left(\theta\right)\right)$. We write the FOC for $C'\left(k_{GE}\right)$ as

$$\frac{1-\alpha_0}{\alpha_0} = \frac{C'\left(k_{GE}\right) - \mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)}{\left(1+\chi\right)\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right) - C'\left(k_{GE}\right)} \frac{\left(1+\chi\right)\mathbb{E}\left(R\left(\theta\right)\right) - C'\left(k_{GE}\right)}{\mathbb{E}\left(R\left(\theta\right)\right) - C'\left(k_{GE}\right)}$$

The second ratio converges to 1 as $\chi \to 0$, and therefore it follows that as $\chi \to 0$, $C'\left(k_{GE}\right) \approx \left(1+\left(1-\alpha_0\right)\chi\right)\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)$ and $Q/\hat{Q} \to 0$. Expected dividends are

$$\begin{aligned}\frac{V_{GE}}{\left(\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)\right)^{1+1/\chi}} &\approx \left(\frac{\mathbb{E}\left(R\left(\theta\right)\right)}{\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)} - \frac{1+\left(1-\alpha_0\right)\chi}{1+\chi}\right)\left(1+\left(1-\alpha_0\right)\chi\right)^{1/\chi} \\ &\to \left(\frac{\mathbb{E}\left(R\left(\theta\right)\right)}{\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)} - 1\right)e^{1-\alpha_0}\end{aligned}$$

The expected dividends in the partial equilibrium benchmark are

$$\frac{V_{PE}}{\left(\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)\right)^{1+1/\chi}}$$
$$=\left(\frac{\mathbb{E}\left(R\left(\theta\right)\right)}{\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)}\left(1-\frac{1-\alpha_0}{1+\chi}\right)-\frac{\alpha_0}{1+\chi}\right)\left(1+\left(1-\alpha_0\right)\left(\frac{\mathbb{E}\left(R\left(\theta\right)\right)}{\mathbb{E}\left(\mathbb{E}\left(R\left(\theta\right)|x=z,z\right)\right)}-1\right)\right)^{1/\chi}$$

and therefore

$$\frac{V_{GE}}{V_{PE}}=\frac{\left(\frac{\mathbb{E}(R(\theta))}{\mathbb{E}(\mathbb{E}(R(\theta)|x=z,z))}-\frac{1+(1-\alpha_0)\chi}{1+\chi}\right)\left(1+\left(1-\alpha_0\right)\chi\right)^{1/\chi}}{\left(\frac{\mathbb{E}(R(\theta))}{\mathbb{E}(\mathbb{E}(R(\theta)|x=z,z))}\left(1-\frac{1-\alpha_0}{1+\chi}\right)-\frac{\alpha_0}{1+\chi}\right)\left(1+\left(1-\alpha_0\right)\left(\frac{\mathbb{E}(R(\theta))}{\mathbb{E}(\mathbb{E}(R(\theta)|x=z,z))}-1\right)\right)^{1/\chi}}\rightarrow$$
$$\frac{1}{\alpha_0}\frac{1}{\left(1+\left(1-\alpha_0\right)\left(\frac{\mathbb{E}(R(\theta))}{\mathbb{E}(\mathbb{E}(R(\theta)|x=z,z))}-1\right)\right)^{1/\chi}}\rightarrow 0.\blacksquare$$