The Pass-through of Productivity Shocks to Wages and the Cyclical Competition for Workers

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November 20, 2022

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Abstract

Using French matched employer-employee data, I document that after positive firm-level productivity shocks, the wages of stayers rise and job-to-job transitions fall. However, after positive sectoral productivity shocks, wages rise significantly more and job-to-job transitions rise. To explain this difference, I build a model with dynamic wage contracts subject to two-sided limited commitment and imperfect information and in which sectoral productivity shocks generate cyclical competition for workers. After a positive firm-level shock, a firm increases its wages to reduce the quit rate of its workers. This increase is limited because workers are risk-averse and value insurance against shocks and because there is no increase in the cyclical competition from other firms. In contrast, after positive sectoral shocks, the cyclical competition for workers heats up and workers become more likely to switch jobs. In response, all firms increase their wages more aggressively to retain them. I quantify how much insurance firms provide to workers over sectoral cycles and evaluate the role of policy. Firing costs play a new role when contracts are endogenous: lowering them reduces the commitment power of firms and makes income risk larger and counter-cyclical.

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I am sincerely grateful to my advisors Adrien Auclert, Luigi Bocola, Patrick Kehoe and Elena Pastorino for their advice and invaluable support. I am also grateful to Yuliy Sannikov for his help and encouragements. I would also like to thank Aniket Baksy, Daniele Caratelli, Nicolas Coeurdacier, Sebastian Di Tella, Robert Hall, Brian Higgins, Pete Klenow, Ellen Muir, Monika Piazzesi, Luigi Pistaferri, Xavier Ragot, Martin Schneider and Christopher Tonetti for their feedback as well as members of the Observatoire français des conjonctures économiques (OFCE) for help accessing data. This project was supported by the George P. Shultz Dissertation Support Fund and the Bradley Graduate Fellowship and B.F. Haley and E.S. Shaw Fellowship for Economics at SIEPR, Stanford. Access to some confidential data, on which this work is based, has been made possible within a secure environment offered by CASD - Centre d'accès sécurisé aux données (Ref. 10.34724/CASD). All errors are mine.

1 Introduction

How do firms and workers share risk when profits fluctuate? Do firms absorb productivity shocks into their profits or do they pass them on to workers through their wages? A prominent view among economists is that firms provide some form of insurance to workers through wage contracts (Knight, 1921, Baily, 1974, Azariadis, 1975). Estimates of the pass-through of productivity shocks to wages can be used to assess whether firms insure workers against these shocks or pass them through to wages. Recent evidence shows that the pass-through of firm-level productivity shocks is small (Guiso, Pistaferri and Schivardi, 2005) whereas the pass-through of sectoral productivity shocks is much higher (Carlsson, Messina and Skans, 2016). In this sense, the data suggests that firms provide relatively more insurance against firm-level shocks than against sectoral shocks.

There is essentially no existing work that simultaneously accounts for the patterns of pass-through of both firm-level and sectoral productivity shocks documented in the data. One strand of literature studies risk-sharing between firms and workers but focuses on firm-level and worker-level shocks (Balke and Lamadon, 2022). There is also a large macroeconomic literature that studies the response of wages to sectoral or aggregate productivity shocks (e.g. Moscarini and Postel-Vinay, 2013) but in models with risk-neutral workers, thus overlooking the risk-sharing problem between firms and workers.

This paper builds a model that generates these patterns of pass-through as a result of optimal contracting between firms and risk-averse workers. The key idea in this model, explained in more details below, is that firms face a trade-off between providing insurance to workers and competing against other firms to retain workers. On one hand, providing insurance makes contracts more attractive to workers, making it easier for firms to hire them. On the other hand, passing through productivity shocks to wages helps to retain workers when they generate the most profits. Firms face additional incentives to pass through sectoral productivity shocks relative to firm-level shocks because sectoral shocks also influence the intensity of the cyclical competition for workers. I derive analytical formulas for the pass-through, which describe how firms balance the worker preference for insurance with the cyclical competition for workers. I use the model to quantify how much insurance firms provide to workers over the cycle and revisit the role of firing costs when contracts are endogenous.

I start by using French matched employer-employee data between 2008 and 2019 to document the pass-through of firm-level and sectoral productivity shocks to the wages of workers employed at the same firm for two consecutive years, also called stayers. Consistent with existing literature, I find that wages respond significantly more to sectoral shocks than to firm-level shocks. After a positive productivity shock normalized to 100%, wages increase by 4% when the shock is firm-level and by 18% when it is sectoral. Since the competition for workers is central to my model, I also measure job-to-job transition rates and find that they respond very differently to firm-level and sectoral shocks. After a 100% increase in productivity, job-to-job transitions fall by about 2 percentage points when the shock is firm-level but rise by 4 percentage points when the shock is sectoral.

To understand these facts, this paper builds an equilibrium model of the labor market with risk-averse workers and dynamic wage contracts. Workers can switch jobs but face search frictions. They receive preference shocks for changing jobs, which effectively imply that only a fraction of workers choose to search for new jobs every period. Contracts are subject to limited commitment on the side of workers and firms as in Thomas and Worrall (1988), and there is imperfect information about the worker search decision and preference shocks. Firms are heterogeneous in their permanent productivity, and experience firm-level and sectoral productivity shocks. In a baseline version of the model, I assume that workers are hand-to-mouth and I later relax this assumption by allowing workers to trade in risk free bonds, which allows them to smooth their consumption in response to shocks. My model builds on Balke and Lamadon (2022) by considering market-level shocks, using results on directed search from Menzio and Shi (2011).

The model captures a trade-off between retaining workers when they are most productive, and insuring them against productivity shocks. Consider first how a firm responds to firm-level shocks. If a firm provided complete insurance against such shocks by paying constant wages, its workers would leave at a constant rate. Such a firm could increase its profits by raising wages when firm productivity is high to reduce the quit rate, and lowering wages when firm productivity is low to increase it. With this strategy firms would retain workers precisely when they generate the most profits. But this strategy of passing through productivity shocks to wages so much is not optimal because workers are risk averse and they value insurance against shocks. Indeed, if one firm adopted a strategy of close to complete pass-through of its shocks, it would have to offer much higher average wages to make its offer attractive relative to an offer that has a lower pass-through and, hence, better risk sharing. Hence, firms balance the benefits of varying wages with productivity to optimize worker retention against the benefits of providing insurance to workers against shocks so as to put together wages that are both attractive to the workers it meets and attractive for its own profit profile.

In sharp contrast to firm-level shocks, sectoral productivity shocks also affect the intensity of the competition for workers. After positive sectoral shocks, all firms are more productive and hence all of them are more eager to attract workers. This cyclical increase in competition means that if any one firm did not increase its wages, that firm would disproportionately lose its workers to poaching firms precisely when these workers would generate the most profits. Thus because of the cyclical upswing in competition from poachers, firms raise wages more aggressively when all firms become more productive. Nonetheless, in this scenario the larger increase in wages only partly offsets the upswing in competition so that firms lose workers at a faster rate than they would absent such sectoral shocks. In a symmetric fashion, negative sectoral shocks reduce the desire of competing firms to attract workers so firms can reduce wages significantly without causing an upswing in the quit rate of workers.

I derive a new analytical formula for the pass-through of firm-level productivity shocks to wages that yields further insights into the mechanism. Specifically, I compute the impulse response of wages to a mean-reverting productivity shock. I derive this result in continuous time using recent methods from Sannikov (2008), and using a novel approximation to the optimal contract that I introduce. The resulting pass-through formula shows that it is optimal to backload the wage increase in response to a positive firm-level shock, meaning that wages rise proportionately more in future periods than today relative to productivity shocks. Briefly, backloaded wages encourage workers to stay with the firm in order to benefit from these future higher wages.

The pass-through formula also shows that the tension between worker retention and insurance boils down to a ratio of a retention elasticity to the relative risk aversion of the worker, where the retention elasticity is the percentage point change in the worker job-to-job transition rate induced by a one percent increase in the present value of wages. When risk aversion is large, workers value insurance against shocks more and the optimal pass-through is low. In contrast, when the retention elasticity is large, increasing wages is an effective strategy to retain workers and the optimal pass-through is high. The retention elasticity is endogenous to the equilibrium, but is taken as an exogenous function of wages and shocks in the firm's problem. It turns out that this elasticity provides sufficient information for the determination of a firm's optimal policy.

A similar formula for the pass-through of sectoral productivity shocks shows when this pass-through is larger. First, it is larger when the firm value is larger. Intuitively, the larger is a firm's present value of profits, the stronger is the firm's desire to retain workers. Hence, this higher value induces firms to respond more aggressively to an increase in competition from outside firms following a sectoral shock. Second, the pass-through is larger when the retention elasticity increases in sectoral productivity. This second condition is especially strong for workers with currently high wages. The reason is that in normal times these workers are already paid more than nearly all poachers can offer so they are unlikely to leave. But in a boom wage offers from poachers become more attractive, these workers start searching for jobs and their retention elasticity increases greatly. Firm optimally respond by passing through a large fraction of sectoral shock to their wages.

In addition to these forces, the model features two important asymmetries in the response of wages to positive and negative shocks. First, positive productivity shocks at either the firm level or the sectoral level tend to generate wage increases. In contrast, most negative productivity shocks do not generate wage cuts but rather lead to slower wage growth. This asymmetry arises because, on average, workers start the match with a relatively low wage and experience positive wage growth, so the negative shock simply decreases this positive growth rather than turning it negative. The second asymmetry between positive and negative shocks is generated by the limited commitment of firms that implies a *walk-away constraint*. This constraint specifies that firms will terminate matches whenever their value turns negative. Hence, the contract will imply that after a sequence of negative productivity shocks that leads the value of the firm to be zero, there must be complete pass-through of negative shocks so that the firm value does not turn negative. Crucially, the competition for workers makes the walk-away constraint more likely to bind. The reason is that as part of their retention strategy, firms tend to increase wages over time in order to make workers less likely to leave but by doing so the firm value drifts back towards zero, where this constraint binds.

I bring the model to the data to quantify how much insurance firms provide to workers in response to various shocks. I calibrate the model using moments on firm and sectoral productivity shocks as well as labor market flows estimated on the matched employer-employee data. In the quantitative model I add a cost of terminating contracts, referred to as *firing costs*. This cost relaxes the walk-away constraint so that firms terminate matches only if their value is more negative than the firing cost. I calibrate these firing costs to be consistent with estimates from France. I find that the model accounts well for the differential response of wages and job-to-job transitions to firm-level and sectoral productivity shocks, which are not targeted in the calibration. Remarkably, the model generates about 40% of the cross-sectional dispersion in wage growth observed in the data, which is significantly more than what observable worker characteristics can explain.

To evaluate whether income risk is cyclical, I investigate when the pass-through of firm-level productivity shocks changes over sectoral cycles. In my baseline calibration there is little cyclicality: the pass-through of firm-level shocks is roughly constant over the cycle. This pattern is consistent with my estimates from the data. The reason for this lack of cyclicality is that the high firing costs I estimate imply that firms are willing to allow their profits to fall substantially following negative shocks rather than paying the firing costs, and hence protect the worker from experiencing sharp cuts in wages. To isolate the importance of this mechanism, I compute a counterfactual in which I reduce firing costs to a much lower level consistent with the U.S. With these lower firing costs, the pass-through of firm-level shocks increases significantly and becomes countercyclical: the pass-through is higher in downturns than in booms by 15%. Hence, workers benefit from less insurance and welfare falls by 1%. This result is in stark contrast with previous work on firing costs, which emphasized their ambiguous effects on firing and hiring (Bentolila and Bertola, 1990) and their perverse effect on the reallocation of workers towards more productive firms (Hopenhayn and Rogerson, 1993).

Finally, I find that income inequality increases in downturns because of the differential dynamics of wages for incumbent workers and new hires. In response to a negative sectoral shock, the wage of incumbent workers falls slowly over time whereas the wage of new hires falls sharply, which is consistent with existing work on dynamic contracts (Rudanko, 2009, Kudlyak, 2014, Basu and House, 2016). Since new hires are at the bottom of the income distribution, this implies that after a negative sectoral shock the bottom of the distribution expands whereas the top remains fairly stable. As a result, the cross-sectional dispersion in log wages is 10% larger in sectoral downturns than in booms.

An important assumption in my baseline model is that workers have no access to financial markets. In reality, workers have access to alternative forms of insurance, such as credit card debt, that can interact with the insurance provided by firms through wage contracts. To illustrate this point, I characterize a 2-period version of the model in which workers can trade risk-free bonds. Surprisingly, I find that risk-free bonds enhance the ability of firms to retain workers. The reason why workers are less likely to change jobs is that wages are extremely backloaded so workers forego a large part of their compensation when they switch jobs. Without risk-free bonds, backloading wages so much is not optimal because it implies a path for consumption that is also extremely backloaded, which is unattractive to workers with concave utility. With risk-free bonds, firms choose to backload wages more because they can make workers borrow and smooth consumption. In the model with risk-free bonds, a precautionary savings motive is the new force that limits the degree of backloading in wage contracts. When firms set the wage of workers, and effectively pin down borrowing, they take into account that borrowing is risky because workers might end up in the future with a lot of debt and very little income to pay for it, for example if they become unemployed. When trades in risk-free bonds are private information to workers, firms use the pass-through of productivity shocks to wages to manipulate this precautionary savings motive. In ongoing work, I extend this problem to a dynamic setting and evaluate whether the degree of wage backloading and the pass-through change significantly when workers have access to realistic asset markets.

Related literature My paper relates to the literature studying how firms compete for workers over the business cycle. These models have been used to explain the cyclicality of labor market flows (Menzio and Shi, 2011, Moscarini and Postel-Vinay, 2016b, Schaal, 2017, Fukui, 2020, Carrillo-Tudela, Clymo and Coles, 2021), to study the reallocation of workers towards more productive firms (Moscarini and Postel-Vinay, 2013, Coles and Mortensen, 2016, Lise and Robin, 2017, Acabbi, Alati and Mazzone, 2022) and to create a theory of the Phillips curve consistent with recent evidence on wage growth and worker mobility (Moscarini and Postel-Vinay, 2022). I study how firms insure workers against risk over cycles generated by sectoral productivity shocks. To do this I depart from most of the literature by introducing risk-averse workers[1] and considering a rich contracting environment. I use directed search because this leads to a property called Block Recursivity, which makes it tractable to study sectoral shocks as shown in Menzio and Shi (2011). The shocks that I call sectoral in my model are exactly identical to the aggregate shocks studied in this literature. In the context of dynamic wage contracts, this distinction is important because other forces than the cyclical competition for workers might prevent firms from insuring workers against aggregate shocks. In particular, it is often argued that aggregate risk is not insurable because it cannot be diversified by investors.

There are two branches of literature on job-to-job search. In the Burdett and Mortensen (1998) approach, firms do not make counteroffers. In a different tradition exemplified by Postel-Vinay and Robin (2002), firms do make counteroffers. From an applied perspective, I argue that the Burdett and Mortensen approach of *no counteroffers* seems more appropriate for my setting. In particular, while it is true that, at least anecdotally, counteroffers are often made for very skilled workers, they seem to be much less prevalent for the average worker in my dataset. In setting up my theoretical model, I impose restrictions on information and technology such that it is not incentive feasible to treat workers who have received offers differently from those who have not. As a result, it is not feasible for firms to make counteroffers in my model.

This paper also relates to the literature studying how firms and workers share risk when profits fluctuate. Balke and Lamadon (2022) use a similar model to evaluate whether firms insure workers against firm-level and worker-level productivity shocks. My paper differs in that it focuses on the risk that workers face over sectoral cycles. I also focus on different contracting frictions by assuming that workers experience preference shocks

[1] One recent exception is Acabbi et al. (2022) who use a model with risk-averse workers similar to mine. They focus on the persistent effects of recessions when workers have human capital.

for changing jobs that are private information and that firms have limited commitment (and in an extension that workers can trade risk-free bonds). My model is consistent with evidence that job-to-job transitions are an important driver of wage growth over the cycle (Moscarini and Postel-Vinay, 2016a, 2017, Karahan, Michaels, Pugsley, Sahin and Schuh, 2017) and can talk to the large empirical literature studying the pass-through of productivity shocks to wages (e.g. Guvenen, Schulhofer-Wohl, Song and Yogo, 2017, Guiso and Pistaferri, 2019, Chan, Salgado and Xu, 2020). In particular, my pass-through formulas show the critical role of the retention elasticity, a parameter that has been estimated in Kline, Petkova, Williams and Zidar (2019) and Dube, Giuliano and Leonard (2019).

My analytical results add to the literature studying the properties of dynamic wage contracts with job-to-job mobility. Building on the work of Burdett and Mortensen (1998), these papers sought to characterize the optimal hiring and retention policy of firms and the implications for equilibrium wage dispersion. In a model with random search and constant productivity within matches, Burdett and Coles (2003) show that firms face a trade-off between worker retention and smooth wages over time. Shi (2009) later extends this analysis to a model with directed search. I build on this work by characterizing the transmission of various productivity shocks to wages in a similar environment. I do it in continuous time, using recent methods introduced by Sannikov (2008) to write the contract recursively in a stochastic environment, and using a novel approximation to the optimal contract that I introduce. The pass-through formula that I derive is reminiscent of the Chetty-Baily statistic for optimal unemployment insurance (Baily, 1978, Chetty, 2006), highlighting a common structure behind their problem and mine. Finally, I study the implications of risk-free bonds with and without hidden trade and show that it alters the allocation and trade-off substantially. In doing so I extend work by Stevens (2004) who studied optimal wage contracts when financial markets are complete.

Layout The paper starts in section 2 with motivating evidence on the response of wages and job-to-job mobility to firm-level and sectoral productivity shocks. Section 3 presents the model, and I characterize the optimal contract in section 4. Section 5 brings the model to the data and quantifies the risk faced by workers over sectoral cycles. In section 6, I consider an extension of the baseline model that allows workers to trade in risk-free bonds. Proofs are in the appendix.

2 Motivating evidence

I start by I documenting using matched employer-employee data that wages and job-to-job transitions respond very differently to firm-level and setcoral productivity shocks. I will use these facts as testable implications of my model.

2.1 Matched employer-employee data from France

I use administrative data from France between 2008 and 2019 to discipline my analysis. I combine annual data on firm balance sheet with a panel of worker from social security data containing 1/12th of the French labor force. Using administrative data is critical for my analysis because I estimate the response of wages and job-to-job mobility decisions at the individual level to changes in firm and sectoral productivity.

I focus on a sample of workers with relatively strong attachment to labor markets and for which I can measure job-to-job mobility accurately. Specifically, I only keep in the sample workers with permanent full time contracts, and prime age workers (25-55 years old). I focus on private sector jobs in for-profit firms with at least 3 employees. Appendix A provides more details on the sample selection and data construction and summary statistics on the population of interest. I end up with about 530,000 workers and 130,000 firms per year.

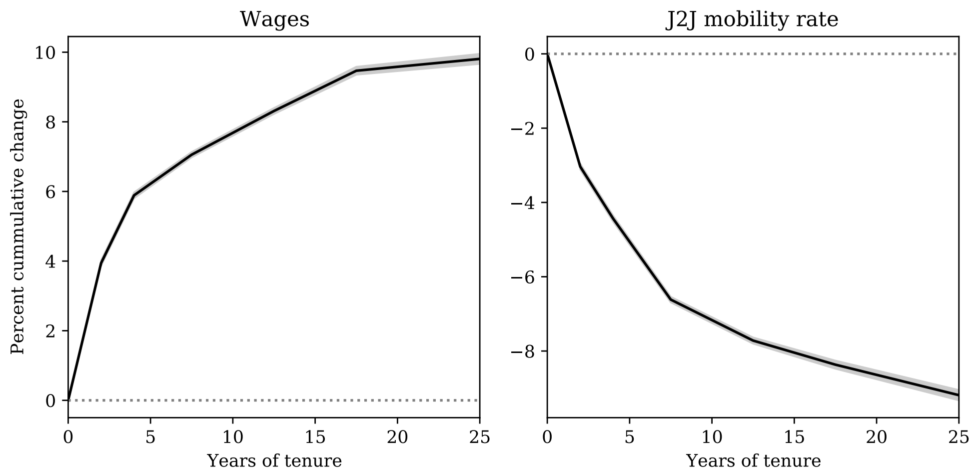
I measure labor productivity using value added per worker, controlling for the cost of capital. I measure the cost of capital as the product of tangible assets and interest rates plus depreciation rates, where interest rates are estimated from the balance sheet data and depreciation rates are estimated at the annual-sector level using national accounts data. I model labor productivity y_{jst} at firm j in sector s and at time t as

$$\log y_{jst} = \log a_t + \log z_{st} + \log x_{jst}$$

where a_t is an aggregate component, z_{st} a sectoral component and x_{jst} a firm-level component. I first residualize $\log y_{jst}$ on time dummies to extract the common component and on firm age dummies to control for the life cycle of firms, which is not in the model. I then measure the sectoral component $\log z_{st}$ as the average productivity across firms within a sector and compute the firm component $\log x_{jst}$ as the residual.

I measure wages as annual labor earnings divided by the number of days worked. Given that I consider a sample of relatively stable workers, changes in hours within the day are unlikely to be large. Labor earnings are net of payroll taxes but before income taxes and they include all types of compensations, including bonuses and payment in

Figure 1: Tenure profiles of wages and job-to-job transitions



Note: the gray areas around the line denotes the confidence intervals estimated with block bootstrap in which firms are re-sampled. Number of observations is approximately 5530,000 workers per year for 12 years.

kinds, but excludes stock options. Unlike in the U.S., medical insurance is not an important part of pay in France. I residualize the log of labor earnings on observable worker and firm characteristics, such as occupation, industry or location dummies, a gender dummy, a polynomial in experience and dummies for firm age.

I then compute the growth rate of earnings and productivity and remove outliers at the bottom and top 0.5% of the distribution each year.

2.2 The tenure profile of wages and job-to-job transitions

I start by documenting in the data the profile of wages and job-to-job transitions with tenure within a match.

I regress residualized wages on dummies for tenures following

$$\log w_{ijst} = \alpha + \sum \delta^{\text{tenure}} + \epsilon_{ijst}$$

and use the estimates for dummy variables to measure the tenure profile of wages. Similarly, I compute the tenure profile of worker mobility by regression an indicator for job-to-job transitions on dummies for tenure.

The estimates for the tenure profiles of wages and job-to-job transitions are shown in 1. The results show that wages rise systematically over time for the duration of a match. At the same time, workers quit rate for another job fall. After 10 years in the match, workers are paid approximately 8% more and are 8 percentage point less likely to leave for another job than when they just matched with the firm. These results are consistent with existing literature and are often cited as evidence for dynamic wages contracts. In section 4, I show that my wage contracts are indeed consistent with this pattern.

2.3 The differential response of wages and job-to-job transitions to firm and sectoral productivity shocks

I measure the response of wages and job-to-job mobility as the percent change in wages, and the percentage point change in job-to-job transition rate after a 100% increase in firm-level and sectoral productivity. These responses are estimated using standard estimators from Guiso et al. (2005).

Define the growth rate of residualized wages for worker i in firm j sector s and between year $t-1$ and t as $\Delta \log w_{ijst}$ and define the growth rate of firm and sectoral productivity as $\Delta \log x_{jst}$ and $\Delta \log z_{st}$. The response of wages to firm-level and sectoral productivity shocks are defined as

$$\theta^{w,y} = \frac{\text{Cov}(\Delta \log w_{ijst}, \sum_{\tau=-1}^{1} \Delta \log y_{jst+\tau})}{\text{Cov}(\Delta \log y_{jst}, \sum_{\tau=-1}^{1} \Delta \log y_{jst+\tau})} \tag{1}$$

where $y \in \{x, z\}$ denotes firm-level or sectoral productivity and where $\sum_{\tau=-1}^{1} \Delta \log y_{jst+\tau}$ is the 3-year cumulative sum of productivity growth. In a model with permanent productivity shocks and static pass-through as in Guiso et al. (2005), this estimator recovers the true pass-through of productivity shocks to wages. It can be computed from a regression of wage growth on productivity growth, using the 3-year cumulative sum as an instrument for productivity growth. This instrument filters out the effect of transitory changes in productivity, which I interpret as measurement errors. In my model the pass-through is not static and shocks not permanent so these coefficients do not really measure the pass-through but I treat them as auxiliary statistics to compare my model with the data. I find it useful to report these statistics, as opposed to simple covariances, since they have been extensively documented in the literature. I measure the response of job-to-job transitions using an indicator variable J2J_{ijst} equal to 1 if worker i leaves firm j during a job-to-job transition in year t. I then recover $\theta^{\text{J2J},x}$ and $\theta^{\text{J2J},z}$ using similar estimators than (1).

Estimation results are shown in table 1 with standard errors in parenthesis. I find that wages and job-to-job transitions respond very differently to firm-level and sectoral productivity shocks. Wages respond almost 4 times more to sectoral productivity shocks than to firm-level shocks, while job-to-job transitions fall after a positive firm-level shock and increase after a positive sectoral shock.

	Wages	Job-to-job transition rate
Firm productivity shock	4.6% (0.61%)	- 1.7pp (0.89pp)
Sectoral productivity shock	18.5% (3.5%)	4.0pp (7.5pp)

Note: standard errors are shown in parenthesis and are estimated by block bootstrap in which firms are re-sampled. Number of observations is approximately 5530,000 workers per year for 12 years.

Table 1: Estimated response of wages and job-to-job transitions to productivity shocks in the data

3 Model

I now present a model of business cycles that features frictional labor markets with job-to-job mobility and dynamic wage contracts subject to rich contracting frictions. This model is meant to capture the differential response of wages and worker mobility to firm-level and sectoral productivity shocks documented in the data.

3.1 Environment

Time is discrete and runs forever at interval Δt. In the quantitative analysis in section 5 I set $\Delta t = 1$ quarter and when I characterize the contract in section 4 I take the continuous time limit $\Delta t \to 0$. The setup in continuous time can be found in appendix B.2.1.

Agents This is a small open economy model of a sector.

A continuum of ex-ante homogeneous workers can either be employed or unemployed. Workers have no access to financial markets so they consume their wage w when employed, and home production b when unemployed. I relax this assumption in section 6. They have period utility $u(w)$ and discount the future at rate β.

Firm are owned by outside investors with discount rate β. The justification for this assumption is that investors can diversify risk from firm-level and sectoral productivity shocks in financial markets since firms and sectors are atomistic. An active firm is one that is matched with a single worker. The output from that match is $\overline{x}\exp(x_t + z_t)$ with *firm specific-shocks* x_t and *aggregate shocks* z_t following mean reverting processes

$$x_t = (1-\alpha_x)x_{t-1} + \sigma_x \nu_{xt} \quad \text{and} \quad z_t = (1-\alpha_z)z_{t-1} + \sigma_z \nu_{zt}$$

where ν_{xt} and ν_{zt} are i.i.d. innovations with standard normal distribution, and $1-\alpha_x$ and $1-\alpha_z$ parameterize the persistence of these shocks. Firm fixed productivity $\overline{x}$ is drawn at the start of the match, independently across firms. This productivity stays constant over time and lasts for the length of the match. Thus, each firm is subject to one common aggregate shock z_t and has two firm-specific shocks $(x_t, \overline{x})$.

In each period workers receive preference shocks for moving ξ_t which increments utility only if the worker moves to any new job in that period. These shocks are i.i.d. over time and across workers with distribution $\mathcal{N}(0, \sigma_\xi^2)$. The motivation for these shocks is that they capture non-monetary reasons why workers change jobs. In practice, they will help the quantitative model match the large number of job transitions with negative wage changes. Briefly when workers receive large enough positive shocks for moving they can increase their utility by accepting jobs with lower wages. In contrast, when workers receive sufficiently large negative shocks they will not search because no firm is willing to offer a sufficiently high wage to compensate them for the cost of moving.

Timing Each period,

a) Firm-level shocks x_t and sectoral productivity shocks z_t are realized

b) Firms produce and pay current wages; workers consume

c) Job mobility phase: preference shocks ξ_t for moving are realized; employed and unemployed workers search for jobs; firms post vacancies; new matches are formed and new contracts are signed

d) Quits and exogenous separations into unemployment occur

Directed search There is a continuum of labor markets indexed by the promised value to a worker denoted v. Every period, workers choose in which labor market v to apply, and firms choose where to post vacancies. Both employed and unemployed workers search in the same labor markets. Firms post vacancies in these labor markets and, only after they match, learn about their productivity x and $\overline{x}$.

Denote $\phi_u(v, z)$ and $\phi_e(v, z)$ the mass of unemployed and employed workers searching for a job and denote $\phi_f(v, z)$ the mass of vacancies posted by firms. Let κ denote the search intensity of employed workers relative to unemployed workers. In the notation that I use, I rely on the result that only current sectoral productivity z is an aggregate state in this economy so that policies need not be indexed by time or distributions. This is a consequence of Block Recursivity (Menzio and Shi, 2010, 2011).

In each labor market, a constant returns to scale matching function $\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)$ turns workers searching for a job and vacancies into matches. Define the job finding rate

$\tilde{\lambda}_w(\phi_u + \kappa\phi_e, \phi_f)$ as the probability that an unemployed worker finds a job, and the vacancy filling rate $\tilde{\lambda}_f(\phi_u + \kappa\phi_e, \phi_f)$ as the probability that a vacancy finds a worker. These probabilities are defined as

$$\tilde{\lambda}_w(\phi_u + \kappa\phi_e, \phi_f) \equiv \frac{\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)}{\phi_u + \kappa\phi_e}, \quad \tilde{\lambda}_f(\phi_u + \kappa\phi_e, \phi_f) \equiv \frac{\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)}{\phi_f}$$

Since these matching probabilities will depend on v and z in equilibrium, we can write them in short-hand notation as

$$\lambda_w(v,z) \equiv \tilde{\lambda}_w(\phi_u(v,z) + \kappa\phi_e(v,z), \phi_f(v,z)), \quad \lambda_f(v,z) \equiv \tilde{\lambda}_f(\phi_u(v,z) + \kappa\phi_e(v,z), \phi_f(v,z))$$

I assume that the probability that a worker finds a job is at most 1, in that $\lambda_w(v,z) \leq 1$.

In equilibrium there will be an upper bound $\overline{v}$ on the set of active labor markets. For $v > \overline{v}$, the job finding rate is not defined because no firm post vacancies there. I extend this function by setting it to 0 for these values above $\overline{v}$. This convention will be useful when I describe the choice of employed workers with preference shocks. In particular, workers who receive extremely low preference shocks will choose not to search for a new job because there is virtually no firm offering a sufficiently high wage for them to want to switch jobs. Instead of writing explicitly that these workers do not search, I write that they search in labor markets with value $v > \overline{v}$ where the job finding rate is 0.

Unemployed workers Unemployed workers consume their endowment b and choose in which labor market v to search, for working in the next period. Given the job finding probability, $\lambda_w(v,z)$, the value of unemployed workers satisfies

$$U(z_t) = u(b) + \beta \max_{v} \left[\lambda_w(v,z_t)v + (1 - \lambda_w(v,z_t))\mathbb{E}_{z_{t+1}}\left[U(z_{t+1})|z_t\right]\right]$$

In choosing in which labor market v to search, workers face the following trade-off: searching in a high-v labor market brings a higher value v conditional on a match, but it will turn out that these matches occur with lower probability because $\lambda_w(v,z_t)$ will decrease with the value v in equilibrium. Here all unemployed workers search in the same labor market, denoted $v_u(z_t)$, that depends only on their common state z_t.

Employed workers Employer workers also search for new jobs. Their preference shocks for moving ξ_t are realized and then they decide which market v to search in. With probability $\kappa\lambda_w(v,z_t)$, they find a new match in market v. Existing matches break up and workers separate into unemployment for two reasons. First, with exogenous probability

δ a match is dissolved. Second, workers can quit voluntarily. Note that quits occur when a firm productivity is sufficiently low that it is efficient to dissolve the current match.

Contracts When firms and workers are first matched, they sign wage contracts. The contract is subject to limited commitment by workers and firms. In particular, workers cannot commit to turn down a job offer when they receive one, and cannot commit not to quit into unemployment. Contracts are also limited by private information in that a worker search decision and preference shocks for moving are both private information to that worker. This private information leads to a moral hazard problem with both a hidden action v and a hidden state ξ. Productivity shocks $(\bar{x}, x_t, z_t)$ are public information.

Firms also have limited commitment. Intuitively, this limited commitment captures the inability of a firm to commit to a contract that after some histories of shocks it would like to renege on. To model this limited commitment, I assume that if, after signing a contract, a firm chooses to walk away from it, by effectively firing the worker, it must pay a cost Φ to do so. Technically, this ability to walk away from a contract implies that in an incentive compatible contract, after any history the continuation value for the firm must be greater than $-\Phi$. Clearly, as this cost increases from 0 to some larger value, the set of contracts the firm can credibly commit to increases. I think of Φ as capturing a degree of commitment power. When $\Phi = 0$, firms have no commitment at all in that they can walk away from the match at any history for which the continuation value is negative. When $\Phi = \infty$, firms have full commitment in that they cannot walk away from a match no matter how negative the continuation value becomes. In the quantitative model, I will calibrate the commitment power of firms using estimates of firing costs. The idea behind this calibration is that if a firm decides to walk-away from a deal, it effectively has to fire the worker and must therefore pay a firing cost.

As mentioned in the introduction, I follow the Burdett and Mortensen (1998) approach in which firms do not make counteroffers. I now briefly describe two assumptions such that it is not incentive feasible for firms to make counteroffers to workers. First, job offers are private information to workers and poaching firms and workers cannot provide any (contractible) evidence to their current employer. Second, workers receive counteroffers at the end of a given period when their current employers are closed, and must accept this offer before the beginning of next period when their current employers open. This assumption on timing ensures that workers cannot renegotiate with their current employers using the new job offers as outside option since it would be too late to accept the new job. Taken together, these assumptions imply that if a firm wanted to make a counteroffer to one of its employee who has claimed to receive an offer, other employees who did not receive that offer would pretend that they did in order to benefit from the same deal. Therefore, firms cannot make counteroffers in any incentive compatible contract. In the appendix, I discuss a variant of the island model of Lucas and Prescott (1974) in which the physical transit times between islands justifies these assumptions.

Following previous work on dynamic contracts with hidden information, I write the contract recursively in terms of promised values and continuation values instead of histories of shocks. I also define incentive compatibility using *temporary incentive constraints* instead of constraints that depend on the entire history of shocks and reports[2] (Green 1987, Atkeson and Lucas 1992). A critical assumption that I use in doing so is that preference shocks ξ_t are i.i.d. over time. This implies that the continuation value of workers over contracts only depends on the reported preference shock, and not the realized shock[3].

Denote V_t the promised value of a worker at the start of the period and $s_t = (\overline{x}, x_t, z_t)$ the state of productivity, where $\overline{x}$ is the fixed firm productivity, x_t is the firm-level productivity shock and z_t is the sectoral productivity shock. Note that at time t, firms have different firm-specific productivity $(\overline{x}, x_t)$ but share the same sectoral productivity z_t. The state of a match at the beginning of the period are the worker promised value V_t as well as the current productivity s_t. After wages are paid and consumed, workers draw a preference shock ξ_t that becomes a part of the state at this point.

The components of the contract at time t are the wage paid today and a set of continuation values for each state tomorrow. The wage w_t is a function of the worker promised value V_t and productivity state s_t whereas the continuation values $V_{t+1}(s_{t+1}, \xi_t)$ are also functions of future productivity and of the realized preference shock ξ_t today. Formally, the wage and continuation values are represented by two functions

$$w_t\left(V_t, s_t\right) \quad \text{and} \quad V_{t+1}(s_{t+1}, \xi_t; V_t, s_t)$$

A *contract* is a collection of these functions for all t.

Given a contract, the worker chooses a search strategy and a quit strategy to maximize the present value of utility. Both the search strategy $v_t(\xi_t)$ and quit strategy $q_t(\xi_t)$ depend on preference shocks ξ_t because workers make these decisions after they observe ξ_t.

The value of a worker in state s_t given a contract and strategies $v_t(\xi_t), q_t(\xi_t)$ satisfies

$$V_t(s_t) = u(w_t) + \beta \mathbb{E}_{\xi_t}\left[\kappa\lambda_w(v_t(\xi_t), z_t)\left(v_t(\xi_t) + \xi_t\right) + (1 - \kappa\lambda_w(v_t(\xi_t), z_t))W_{t+1}(\xi_t)\right] \quad (2)$$

[2]In order to keep notations simple, I also abstract from randomized contracts.

[3]See Fernandes and Phelan (2000) for problems with hidden information and serially correlated shocks.

where $W_{t+1}(\xi_t)$ is the continuation value for non-matched workers

$$W_{t+1}(\xi_t) = (\delta + (1-\delta)q_t(\xi_t))\,\mathbb{E}_{z_{t+1}}[U(z_{t+1})|z_t] \\ +(1-\delta)(1-q_t(\xi_t))\mathbb{E}_{x_{t+1},z_{t+1}}\left[V_{t+1}(s_{t+1},\xi_t)|x_t,z_t\right] \tag{3}$$

In (2), the first term is the utility from consuming wages today. The second term depends on the probability that a worker finds another job $\kappa\lambda_w(v_t(\xi_t),z_t)$ and on the value that the worker receives if a job-to-job transition occurs $v_t(\xi_t)+\xi_t$, where we recall that a worker who prefers not to search selects an inactive labor market. In this expression, expectations over ξ_t are not conditioned on past shocks because preference shocks are i.i.d. A worker who does not find a new job gets a continuation value for non-matched workers of $W_{t+1}(\xi_t)$, which at this point can depend on the realization of the current preference shock ξ_t. This term is defined in (3). It is the sum of the value of unemployment if the worker decides to quit or if the match is disolved exogenously, and the continuation value at the same job for the following period $V_{t+1}(s_{t+1},\xi_t)$.

Denote the worker's report of preference shock at t by $\hat{\xi}_t$. A contract is temporary incentive compatible if, for any realization for the preference shock ξ_t and productivity s_t, and given continuation values $\{V_{t+1}(s_{t+1},\hat{\xi}_t)\}$, it is optimal for the worker to report the preference shock ξ truthfully and search v and quit q decisions are optimal

$$\xi_t \text{ solves } \max_{\hat{v},\hat{\xi},\hat{q}} \kappa\lambda_w(\hat{v},z_t)\,(\hat{v}+\xi_t) + (1-\kappa\lambda_w(\hat{v},z_t))W_{t+1}(\hat{\xi}) \tag{4}$$

where the continuation value $W_{t+1}(\hat{\xi})$ is defined in (3). Remember that that the preference shock ξ_t is private information to workers, so the state-dependent continuation values $\{V_{t+1}(s_{t+1},\hat{\xi})\}$, and hence $W_{t+1}(\hat{\xi})$, can only depend on the report of the worker $\hat{\xi}$ and not on the realized preference shock.

Fixing the continuation value for non-matched workers $W_{t+1}(\hat{\xi})$, a worker with high preference shock ξ_t will search in labor markets with a lower value $\hat{v}$ and a higher job finding rate $\lambda_w(\hat{v},z_t)$ because this worker really values getting a new job. Conversely, a worker with low preference shock ξ_t will search in markets with a high value $\hat{v}$ and a low job finding rate. A worker who draws a sufficiently low value for ξ_t will choose not to search since there does not exist an active market offering a sufficiently high value for the worker to be willing to move.

One might postulate that firms want to promise relatively high future wages to workers with high preference shock ξ_t in order to retain them, and relatively low future wages to workers with negative preference shocks ξ_t because they are unlikely to leave anyway. But, this strategy would not be incentive compatible. Indeed, from (4), it is immediate that the continuation value $W_{t+1}(\hat{\xi})$ cannot depend on the reported preference shock $\hat{\xi}$. Otherwise, any worker would benefit from reporting the preference shock with the highest value. I now argue that if $W_{t+1}(\hat{\xi})$ is independent of $\hat{\xi}$, then so are the state-dependent continuation values $\left\{V_{t+1}(s_{t+1}, \hat{\xi})\right\}$. Holding fixed $W_{t+1}(\hat{\xi})$, allowing the continuation values $V_{t+1}(s_{t+1}, \hat{\xi})$ to vary with the preference shock ξ_t will never be optimal for firms because workers are risk averse and this variation does not relax the incentive constraint for v, which by construction depends only on $W_{t+1}(\hat{\xi})$. Therefore, when I state the optimal contracting problem below I write the state-dependent continuation values $\{V_{t+1}(s_{t+1})\}$ as a function of future productivity states s_{t+1} only, and not of preference shocks ξ_t. Furthermore, since the quit decision only depends on these continuation values and on the value of unemployment, I will also write the worker quit policy q as independent of the current preference shock. Note, however, that the worker search decision $v(\xi_t)$ still depends on the realization of the preference shock because the value that a worker gets by changing job increases in it.

We are now ready to write the optimal contracting problem. Denote $\Pi(V, s_t)$ the present value of profits for a firm matched with a worker with promised value V and when productivity is currently $s_t = (\overline{x}, x_t, z_t)$. Taking as given the value of unemployment $U(z)$ and the job finding rate $\lambda_w(v, z)$, the value of a firm satisfies

$$\begin{aligned}\Pi(V, s_t) = \quad & \max_{w, V(s_{t+1})} \; \overline{x}\exp(x_t + z_t) - w \\ & + \beta\left(1 - \mathbb{E}_{\xi_t}\left[\kappa\lambda_w(v(\xi_t), z_t)\right]\right)(1-\delta)(1-q)\mathbb{E}_{x_{t+1}, z_{t+1}}\left[\Pi(V(s_{t+1}), s_{t+1}) | x_t, z_t\right]\end{aligned}$$

subject to

$$\begin{aligned}&\text{(PK)}: && V \leq u(w) + \beta\left(W_{t+1} + \mathbb{E}_{\xi_t}\left[\kappa\lambda_w(v(\xi_t), z_t)\left(v(\xi_t) + \xi_t - W_{t+1}\right)\right]\right) \\ &\text{(IC-v)}: && v(\xi_t) \in \arg\max\nolimits_{\hat{v}} \; \lambda_w(\hat{v}, z_t)\left(\hat{v} + \xi_t - W_{t+1}\right) \\ &\text{(IC-q)}: && q = 1 \quad \text{if} \quad \mathbb{E}_{z_{t+1}}[U(z_{t+1}) | z_t] \geq \mathbb{E}_{x_{t+1}, z_{t+1}}\left[V(s_{t+1}) | x_t, z_t\right] \\ &\text{(PC-F)}: && \Pi(V(s_{t+1}), s_{t+1}) \geq -\Phi\end{aligned}$$

where $W_{t+1} = (\delta + (1-\delta)q)\,\mathbb{E}_{z_{t+1}}[U(z_{t+1})|z_t] + (1-\delta)(1-q)\mathbb{E}_{x_{t+1}, z_{t+1}}\left[V(s_{t+1})|x_t, z_t\right]$.

The firm chooses the current wage w and state-dependent continuation values $V(s_{t+1})$ to maximize the present value of profits, where $(1 - \mathbb{E}_{\xi_t}[\kappa\lambda_w(v(\xi_t), z_t)])(1-\delta)(1-q)$ is the probability that the worker remains within the current match next period, often called the *retention probability*. When the firm computes this retention probability, it takes expectation over the different values that preference shocks ξ_t can take. By changing continuation values in the future, $V(s_{t+1})$, firms influence not only future profits but also the quit and search decisions of workers q and v today and therefore the retention probability. Section 4 charecterizes this trade-off between future profits and worker retention, and how it changes with firm-level and sectoral productivity shocks.

The first constraint (PK) is the *promise keeping constraint*, stating that the value the worker gets from the contract either through wage today or future values must deliver at least the promised value V. The second constraint (IC-v) is the *incentive compatibility constraint for the search* strategy v of the worker. It defines the search strategy that a worker chooses as a function of the preference shock ξ_t and the continuation value W_{t+1}. The third constraint (IC-q) is the *incentive compatibility constraint for quits* into unemployment. It states that a worker will quit if the expected value of unemployment exceeds the continutation value from the current match. The last constraint (PC-F) is the *participation constraint of the firm*, which states that the firm value cannot go below the cost Φ after any history. If this constraint was violated, firms would rather walk-away from the match than continue and deliver the value V to the worker.

Value of a match Consider the value of a firm when it is just matched with a worker in market v when sectoral productivity is z_t. At this point, the firm has not yet drawn its idiosyncratic shocks $(\overline{x}, x_t)$. Denote $\Pi_0(v, z_t)$ the value from this match. It solves

$$\begin{aligned} \Pi_0(v, z_t) = \quad & \max_{V(s_{t+1})} \mathbb{E}_{\overline{x}, x_{t+1}, z_{t+1}} \left[\Pi(V(s_{t+1}), s_{t+1}) | z_t \right] \\ \text{s.t.} \quad & \mathbb{E}_{\overline{x}, x_{t+1}, z_{t+1}} \left[V(s_{t+1}) | z_t \right] = v \end{aligned}$$

where the initial value of firm productivity x_{t+1} is drawn from $x_t \sim \mathcal{N}(0, \sigma_x^2)$ and the fixed firm productivity is drawn from $\log \overline{x} \sim \mathcal{N}(0, \sigma_{\overline{x}}^2)$.

Free entry Firms are subject to a free entry condition. They have to pay a unit cost for posting a vacancy k_1 and if they are successful at matching a worker, firms have to pay a training cost k_2. Training costs and vacancy posting costs are important quantitatively because they have different implications for the job finding rate. The vacancy posting cost k_1 influences how many vacancies firms post in a labor market, whereas the training costs k_2 also influences which labor markets are active.

The free entry condition is

$$-k_1 + \lambda_f(v, z_t)\beta \left(\Pi_0(v, z_t) - k_2 \right) \leq 0 \tag{5}$$

with equality for each active market v. Note that the firm value $\Pi(V, s_t)$ is decreasing in the worker value V, since raising V tightens the promise keeping constraint. Thus, the value of a match is lower in markets with a high worker value v than in markets with low worker value, that is $\Pi_0(v, z_t)$ decreases in v. Consequently, the free entry condition (5) implies that the vacancy filling rate $\lambda_f(v, z)$ is increasing in the worker value v in equilibrium. Intuitively, this condition requires that firms must be indifferent between posting vacancies in markets with a high worker value v, and therefore low match value and high vacancy filling rate, and in markets with a low worker value. Finally, this results implies that the job finding rate $\lambda_w(v, z)$ is decreasing in v from the matching function. Intuitively, in markets with low worker v, there are relatively more firms posting vacancies than workers search for a job so the job finding rate is high.

3.2 Definition of an equilibrium

A recursive equilibrium is a set of value functions, policies and matching rates for each labor market v such that i) the firm and worker strategies satisfy the optimal contract, ii) the free entry condition is satisfied and iii) the job finding and vacancy filling rates are consistent with the matching function.

Denote the probability density function of the distribution of unemployed and employed workers by $\{\psi_t^u, \psi_t(V, \overline{x}, x)\}$. The resulting equilibrium outcome path is a sequence of distributions such that, given the policies, the laws of motion of distributions, defined in appendix B.1, are satisfied. These distributions are functions of time, not just of sectoral productivity, because they depend on the entire history of shocks.

4 Characterizing the optimal contract

Before bringing this model to the data, I characterize optimal wage contracts. My main focus is on understanding the effects of job-to-job mobility on wage growth and on the pass-through of firm-level and sectoral productivity shocks to wages. For this reason I strip down the model to a simpler version in order to obtain clean analytical formulas. Specifically, I assume that firms have full commitment power ($\Phi \to \infty$) and have the same fixed heterogeneity $\overline{x} = 1$ and that workers cannot quit or fall into unemployment ($\delta = q = 0$) and have no preference shocks ($\sigma_\xi = 0$).

In order to derive analytical solutions, I use a continuous time formulation of the problem ($\Delta t \to 0$)[4]. Firm-level and sectoral productivity follow

$$dx_t = -\alpha_x x_t dt + \sigma_x dB_{xt} \quad \text{and} \quad dz_t = -\alpha_z z_t dt + \sigma_z dB_{zt}$$

[4]The optimal contract in continuous time is derived from primitives in appendix B.2.1.

which are the continuous time analogue of the AR-1 process of the quantitative model.

The optimal contracting problem becomes

$$\Pi(V_t, x_t, z_t) = \max_{w, \Delta_x, \Delta_z} \quad \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \int_0^t \kappa\lambda_w(v_s, z_s)ds\right)(\exp(x_t + z_t) - w_t)dt\right]$$

subject to

$$\begin{aligned}
&\text{(PK)}: \quad dV_t = (rV_t - u(w_t) - \kappa\lambda_w(v_t, z_t)(v_t - V_t))\,dt + \Delta_{xt}\sigma_x dB_{xt} + \Delta_{zt}\sigma_z dB_{zt} \\
&\text{(IC-v)}: \quad v_t = v(V_t, z_t)
\end{aligned}$$

where $r = 1/\beta - 1$ and the expectation is taken over the paths of productivity x, z. As before, the search decision $v(V_t, z_t)$ solves the static problem

$$v(V_t, z_t) \in \arg\max_v \; \lambda_w(v, z_t)(v - V_t) \tag{6}$$

In discrete time, firms were choosing the state-dependent continuation value of workers $V(x_{t+1}, z_{t+1})$ whereas in continuous time they choose the variables Δ_x, Δ_z, which measure how the worker value responds to firm-level and sectoral productivity shocks. These choice variables are critical for the contract because they characterize the pass-through of productivity shocks to the worker value. I characterize them in propositions 2 and 3. The law of motion of the worker's value dV_t can be understood as a first-order approximation of the policy function in discrete time, evaluated at current state, $V(V_t, x_t, z_t, x_{t+1}, z_{t+1})$. The HJB for this problem is written in appendix B.2.3.

Characterization method This optimal contract is difficult to characterize for two reasons: the problem is non-linear and dynamic. I now explain how I circumvent these difficulties to obtain an analytical characterization of the problem.

The first challenge with solving this contracting problem is that policies are non-linear. For example, from the HJB, the optimal pass-through Δ_x satisfies the condition $\Delta_x = -\Pi_{Vx}/\Pi_{VV}$ where $-\Pi_{Vx}$ measures the benefit of increasing the worker value when firm productivity x increases, and Π_{VV} captures the cost of varying the worker value because the worker is risk averse. The benefit $-\Pi_{Vx}$ does not admit a closed-form expression because it depends on the probability that the worker stays with the firm, together with the profits that the firm generates over time. The cost Π_{VV} also does not admit a closed-form expression because the cost of indexing the worker value on productivity depends on the risk aversion of the worker, but also on how much risk the worker is already exposed to.

To circumvent this difficulty, I introduce a novel approximation method. Specifically, I characterize the contract to first-order in the search efficiency κ of employed workers around $\kappa = 0$. This is an approximation in the value of a parameter κ that disciplines the ability of workers to change jobs. When $\kappa = 0$, there is no job-to-job mobility and the optimal contract features constant wage. To first order in κ, both Π_{Vx} and Π_{VV} are easy to compute as I show in proposition 2. In a sense, this is an approximation of the optimal contract in the contracting friction. As κ increases, the optimal contract provides more incentives and less insurance.

The second challenge with solving this contracting problem is that policies are dynamic. For example, the optimal pass-through of productivity shocks to wages depends on the persistence of these shocks. Furthermore, the pass-through itself is dynamic in that wages tend to respond to productivity shocks with delay as I show in sections 4.3 and 4.4. This makes it difficult to obtain closed-form expressions for the contract.

To circumvent this difficulty, I characterize the contract in terms of differential equations that can be solved in closed form for some assumptions about the process for productivity, and the job finding rate $\lambda_w(v_t, z_t)$. For example, I will write that the present value of output, denoted $g(x,z)$, satisfies $rg(x,z) = \exp(x+z) + \mathcal{D}g(x,z)$, where the differential operator $\mathcal{D}$ is defined as

$$\mathcal{D}g(x,z) \equiv -\alpha_x x g_x(x,z) - \alpha_z z g_z(x,z) + \frac{\sigma_x^2}{2} g_{xx}(x,z) + \frac{\sigma_z^2}{2} g_{zz}(x,z)$$

The term $\mathcal{D}g(x,z)$ should be interpreted as an adjustment due to the mean-reversion and the volatility of shocks. The expression for $g(x,z)$ is the equivalent of a Bellman equation in discrete time. If firm and sectoral productivity were following random walks ($\rho_x = \rho_z = 1$), this differential equation would admit a closed-form solution and the present value of output would satisfy $g(x,z) = \exp(x+z)/(r - (\sigma_x^2 + \sigma_z^2)/2)$. Even when it does not admit a closed-form solution, the expression for $g(x,z)$ shows that the present value of output mainly depends on output today $\exp(x+z)$, which is very intuitive. The operator $\mathcal{D}$ will allow me to describe the main determinants of the firm value $\Pi(V,x,z)$ in lemma 1, and of the pass-through Δ_x, Δ_z in propositions 2 and 3. I will also illustrate these results in specific cases where these expressions can be solved in closed-form.

4.1 The firm value $\Pi(V,x,z)$

I first derive an expression for the firm value $\Pi(V,x,z)$, which shows how costly worker mobility is to firms.

Lemma 1. *To first-order in κ, the firm value is given by*

$$\Pi(V,x,z) = g(x,z) - h(V) - \kappa\ell(V,x,z)$$

where $g(x,z) = (\exp(x+z) + \mathcal{D}g(x,z))/r$ is the present value of output and $h(V) = u^{-1}(rV)/r$ is the cost of providing value V to workers with constant wage. $\ell(V,x,z)$ represents the cost of job-to-job mobility for firms and satisfies

$$r\ell(V,x,z) = \underbrace{\lambda_w(v(V,z),z)}_{\text{Prob. of J2J transition}} \left[\underbrace{g(x,z) - h(V)}_{\text{Lost profits}} - \underbrace{[v(V,z) - V]\, h'(V)}_{\text{Gains from job mobility}} \right] + \mathcal{D}\ell(V,x,z)$$

Proof. See appendix B.2.3. □

Lemma 1 describes the firm value given productivity x, z and a promised value to workers V. It depends on the profits generated by the firm, captured by $g(x,z)$, and the cost of paying wages to the worker $h(V)$. This cost is measured at constant wages because when there is no worker mobility ($\kappa = 0$), it is optimal for firms to provide perfect consumption smoothing to agents.

The value of the firm also depends on the cost of job-to-job mobility $\ell(V,x,z)$ because firms lose their profits when workers change jobs. This is the main reason why firms want to retain workers when they generate positive profits. However, firms might also benefit from job-to-job mobility to some extent because it makes it easier to hire workers in the first place. In particular, the second term in $\ell(V,x,z)$ shows that firms benefit from job-to-job mobility because it allows them to provide worker with the same promised value V and pay them less. When workers switch jobs, they incur a utility gain of $v(V,z) - V$, which can be translated into units of output using $h'(V)$. The firm can then provide some of the promised value V through future job transitions, instead of future wages. For example, consulting jobs are often seen as a stepping stone for a career in management. As a result, consulting firms might find it easier to hire workers in the first place because they offer a high option value of future employment opportunities.

The moral hazard problem arises because there is an asymmetry in how much firms and workers value job-to-job mobility. To see why, it is useful to compare the optimal search decision of workers with hidden information from (6)

$$\lambda_w(v,z_t) + \frac{\partial \lambda_w(v,z_t)}{\partial v}(v - V_t) = 0$$

to the search decision with full information when firms control the search decision

$$\lambda_w(v^{\mathrm{FI}}, z_t) + \frac{\partial \lambda_w(v^{\mathrm{FI}}, z_t)}{\partial v^{\mathrm{FI}}} \left(v^{\mathrm{FI}} - \left[V_t^{\mathrm{FI}} + \Pi(V_t^{\mathrm{FI}}, x, z) u'(w_t^{\mathrm{FI}}) \right] \right) = 0$$

The first term in these expressions measure the benefit of applying to a market with higher value v, whereas the second term measures the cost. With full information, firms take into account both the worker and firm values when they compute the surplus from job-to-job transitions $v^{\mathrm{FI}} - \left[V_t^{\mathrm{FI}} + \Pi(V_t^{\mathrm{FI}}, x, z) u'(w_t^{\mathrm{FI}}) \right]$. By contrast, with hidden information workers only take into their own gains $v - V_t$. Intuitively, both the worker and the firm want the worker to find a better job, but the worker wants it more. This asymmetry is the reason why firms use wage contracts to influence the mobility choices of workers, and why wages vary with time and productivity.

4.2 The path of wages

I now derive an expression showing how wages change over time and in response to shocks. I will use this expression to derive the tenure profile of wages in this section, and the pass-through of productivity shocks in sections 4.3 and 4.4.

Before characterizing the path of wages, I introduce an important piece of notation. Define the *retention elasticity* $\epsilon(V, x, z)$ as the percentage point change in the retention probability induced by a 1% increase in the present value of wages for a match with state (V, x, z), that is

$$\epsilon(V, x, z) \equiv \underbrace{\frac{\partial \left(1 - \kappa \lambda_w(v(V, z), z)\right)}{\partial v(V, z)}}_{\text{Change in retention probability}} \times \underbrace{\frac{\partial v(V, z)}{\partial V}}_{\substack{\text{Change in} \\ \text{search decision}}} \times \underbrace{\frac{w(V, x, z) u'(w(V, x, z))}{r + \kappa \lambda(v(V, z), z)}}_{\substack{\text{Change in worker value after} \\ \text{1\% increase in PV of wages}}} \geq 0 \quad (7)$$

It turns out that this elasticity is a critical determinant of the retention strategy of firms because it measures the extent to which firms can influence the worker quit rate. It is positive in equilibrium: paying workers more makes them search in labor markets with a higher value v and a lower job finding rate $\lambda_w(v, z)$. This elasticity is an endogenous variable that depends on the assumption of directed search, the costs that firms pay to hire workers and the average productivity of new entrants among other things.

I now derive the path of wages.

Proposition 1. *The path of wages satisfies*

$$dw_t = (r + \kappa\lambda_w(v(V_t, z_t), z_t))\, \Pi(V_t, x_t, z_t)\, \frac{\epsilon(V_t, x_t, z_t)}{\gamma(w_t)} dt + 0 \times dB_{xt} + 0 \times dB_{zt}$$

where $\gamma(w) \equiv -wu''(w)/u'(w)$ is the coefficient of relative risk aversion.

Proof. See appendix B.2.3. This condition is derived by combining the optimality condition with respect to the wage w_t with the envelope condition. □

Proposition 1 is a generalization of theorem 1 in Burdett and Coles (2003) and lemma 3.2 in Shi (2009) to an environment with productivity shocks, and it is the continuous time analogue of proposition 2 in Balke and Lamadon (2022) with sectoral shocks. Note that this result does not require the approximation $\kappa \to 0$, and does not use the operator $\mathcal{D}$.

The intuition behind this equation are well established. If the firm value $\Pi(V_t, x_t, z_t)$ is positive, it is optimal for firms to increase wages to induce workers to stay. The optimal strategy to do so is to backload wages, namely to increase them with tenure. The wage continues to growth until the elasticity $\epsilon(V_t, x_t, z_t)$ reaches 0 or until the present value of profits $\Pi(V, x, z)$ becomes 0 and the firm is indifferent between retaining workers or letting them go. When the elasticity of inter-temporal substitution $1/\gamma$ is low, wages are less backloaded because workers dislike changes in consumption over time. When the retention elasticity $\epsilon(V_t, x_t, z_t)$ is large, wages grow faster because increasing wages makes workers much less likely to quit.

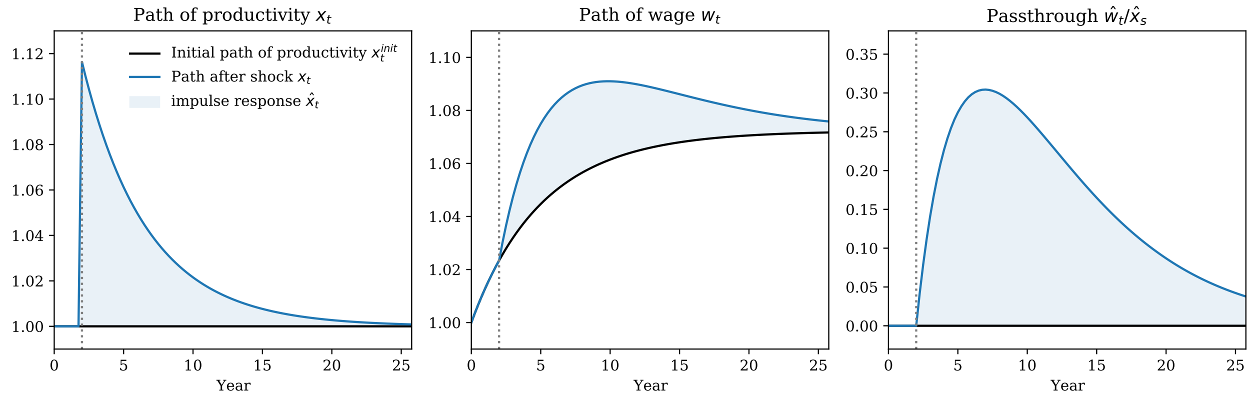
The tenure profile of wages and job mobility An implication of proposition 1 is that wages increase with tenure and job-to-job transitions fall with tenure, consistent with the empirical evidence documented in figure 1. To see this, note that the wage of new hires grows over time if the firm value $\Pi(V, x, z)$ is positive. From the free entry condition,

$$-k_1 + \lambda_f(v, z_t)\, [\Pi(v, x_0, z_t) - k_2] \leq 0$$

it is easy to see that this is true since $\Pi(v, x_0, z_t) = k_2 + k_1/\lambda_f(v, z_t) > 0$ for all active labor markets. Besides, the worker value also grows over time because the worker receives higher wages. Therefore, since the job-to-job transition rate $\kappa\lambda_w(v(V, z), z)$ decreases in the worker value, the job-to-job transition rate falls over time.

The observation that the wage of new hires grows over time will be important when I discuss the differential response of negative sectoral productivity shocks to wages in section 4.4. In particular, I will show that after most negative productivity shocks, workers do not experience wage cuts but lower wage growth.

Figure 2: Impulse responses from firm productivity shock



4.3 The pass-through of firm-level shocks

I now characterize the pass-through to firm-level productivity shocks to the wage and value of stayers. I first derive a formula for the impulse response of wages w_t following a 1% increase in firm productivity and then derive the response of the worker value V_t to the same shock.

The starting point to compute the pass-through to wages is proposition 1. Clearly, wages do not respond on impact to a change in productivity because the coefficient on the Brownian motion B_{xt} is zero. However, changes in productivity alter the path of wages through a change in the firm value $\Pi(V, x, z)$ in the drift (the retention elasticity in the drift is constant conditional on w from equation (7)). An increase in productivity x raises the firm value $\Pi(V, x, z)$ and thus increases the growth rate of wages going forward.

Solving for the path of wages in response to a productivity shock is difficult because this problem is non-linear. For this reason, I rely on my approximation as $\kappa \to 0$. In this case, the path of wages from proposition 1 becomes

$$dw_t = (rg(x_t, z_t) - w_t)\frac{\epsilon(V_t, x_t, z_t)}{\gamma(w_t)}dt$$

where the retention elasticity is defined in equation (7).

Consider a worker with some initial path for productivity and wages $x^{\text{init}}, w^{\text{init}}$. I then simulate a small unanticipated shock to productivity $\hat{x}_0$ at time $t = 0$ and compute the impulse response relative to this initial path: $\hat{x}_t \equiv x_t - x_t^{\text{init}}$ and $\hat{w}_t \equiv w_t - w_t^{\text{init}}$, where t here denotes the time since the shock occurred.

Given the law of motion of productivity, the impulse response of x_t is

$$\hat{x}_t = \exp\left(-\alpha_x t\right)\hat{x}_0 \quad \text{for } t \geq 0$$

The paths of firm productivity before and after the shock are shown in the left panel of figure 2. In the middle panel is shown the paths of wages. I use the example of a new hire whose wage is increasing over time. After the shock, wage growth accelerates and eventually overshoots before falling back to its stationary level. On the right panel, I show the pass-through of productivity shocks to wages, defined as the impulse response of wages normalized by the initial shock to firm productivity.

For a small shock, we can write the change in the wage as

$$\hat{w}_t \approx \int_0^t \left(r g_x(x_s, z_s)\hat{x}_s - \hat{w}_t\right)\frac{\epsilon_0}{\gamma_0}dt \tag{8}$$

where $\epsilon_0 \equiv \epsilon(V_0, x_0, z_0)$ and $\gamma_0 \equiv \gamma(w_0)$ denote the retention elasticity and the risk aversion coefficient at the time of the shock. This equation is an approximation of the true wage response because I abstract from changes in the ratio ϵ/γ over time.

In appendix B.2.3, I show that equation (8) can be solved in closed-form for $\hat{w}_t$ given the path for $\hat{x}_t$. The pass-through is then given by

$$\frac{\hat{w}_t}{\hat{x}_0} \approx \frac{r}{\epsilon_0/\gamma_0 - \alpha_x} g_x(x_0, z_0)\frac{\epsilon_0}{\gamma_0}\left[\exp\left(-\alpha_x t\right) - \exp\left(-\frac{\epsilon_0}{\gamma_0}t\right)\right] \geq 0 \tag{9}$$

This expression shows that the pass-through is backloaded: wages do not respond on impact, and then changes for $t > 0$. When shocks are mean-reverting ($\alpha_x > 0$), the pass-through converges back to 0. When the shock is permanent ($\alpha_x = 0$), the pass-through converges towards $r g_x(x_0, z_0)$ and the worker eventually absorbs the entire shock. The speed and magnitude of the pass-through depends on the retention elasticity ϵ_0, the curvature aversion γ_0 and the persistence of shocks ρ_x. The term $g_x(x_0, z_0)$ captures how a change in productivity affects the present value of output.

This equation captures the trade-off that firms face between worker retention and insurance. A persistent increase in productivity makes firms eager to retain workers because profits go up. The more persistent this increase in productivity, the more firms want to retain workers. In order to retain workers, firms increase their wage to make their value go up and their job-to-job transition rate fall.

However, firms only increase the wage if it actually makes workers less likely to leave. In particular, a high retention elasticity $\epsilon(V, x, z)$ means that an increase in the wage leads to a sharp fall in the worker mobility rate. In this case, increasing the wage is a very effective strategy to retain workers and the pass-through is high.

Firms therefore want to pay workers more when productivity is high, and less when productivity is low. These ups and downs in wages generate risk for workers, who are risk averse. Furthermore, wage increases after positive productivity shocks are back-loaded and therefore wages are not constant over time. Because of risk aversion and preference for smooth consumption over time, workers must be compensated for the pass-through. The degree of worker aversion to time and state varying wages increases in γ, so the optimal pass-through falls in γ.

I now provide a formula for the pass-through into the worker value Δ_x.

Proposition 2. *To first-order in κ, the pass-through of a firm productivity shock x to the value of stayers satisfies*

$$\Delta_x(V,x,z) = (r+\alpha_x)^{-1}\left[g_x(x,z)\frac{\epsilon(V,x,z)}{\gamma(w(V))}u'(w(V)) + \mathcal{D}\Delta_x(V,x,z)\right]$$

where $w(V) = u^{-1}(rV)$ is the wage when $\kappa = 0$.

Proof. See appendix B.2.3. To first-order in κ, the optimality condition for the value pass-through is $\Delta_x = -\ell_{Vx}(V,x,z)/h''(V)$. Rearranging the terms gives the result. □

This proposition is easier to understand when $\mathcal{D}\Delta_x(V,x,z) = 0$. In appendix B.2.3, I show for example that this is true if sectoral productivity is constant ($\sigma_z = \rho_z = 0$) and output is given by xz instead of $\exp(x+z)$. In this case we have $g_x(x,z) = z/(r+\alpha_x)$.

To understand proposition 2, it is instructive to examine the relation between the value pass-through Δ_{xt} and the wage pass-through $\hat{w}_t/\hat{x}_0$ from equation (9). Define the pass-through to the present value of wages as $\widehat{PV} \equiv \int_s^\infty \exp(-rt)\hat{w}_t/\hat{x}_0 dt$. We can compute this pass-through from equation (9) and get $\widehat{PV} = (r+\alpha_x)^{-1} g_x(x_0,z_0)\epsilon_0/\gamma_0$ and therefore $\Delta_x \approx \widehat{PV}u'(w)$. This shows that for firm-level productivity shocks, the change in the worker value is only induced by a change in the present value of wages that the worker will receive at the current firm.

Relation to the Chetty-Baily formula for optimal unemployment insurance In appendix B.3, I show that the pass-through formula is reminiscent of the Chetty-Baily formula for optimal unemployment insurance (Baily, 1978, Chetty, 2006). In this literature, the planner wants to insure workers against unemployment risk but is wary that smoothing consumption too much will prevent workers from searching for a job. The optimal degree of insurance thus depends on the worker risk aversion coefficient, and the elasticity of the job finding rate with respect to unemployment benefits. In both this problem and mine, how much insurance workers receive influences the probability that the worker finds a new job. It is therefore not too surprising that the optimal policies are similar despite the problems being completely different.

4.4 The differential pass-through of sectoral shocks

I now turn to the pass-through of sectoral productivity shocks and show that it differs from that of firm-level shocks because of changes in the intensity of the competition for workers. I first derive the pass-through to wages, and then to the worker value.

I derive the pass-through of sectoral productivity shocks to wages using similar steps than in section 4.3 and detailed derivations are given in appendix B.2.3. The pass-through of wages is given by

$$\frac{\hat{w}_t}{\hat{z}_0} \approx \frac{r}{\epsilon_0/\gamma_0 - \alpha_z} \Bigg(\underbrace{g_z(x_0, z_0)\frac{\epsilon_0}{\gamma_0}}_{\substack{\text{Pass-through of} \\ \text{firm shocks}}} + \underbrace{\Pi(V_0, x_0, z_0)\frac{\epsilon_{z0}}{\gamma_0}}_{\substack{\text{Differential} \\ \text{pass-through}}} \Bigg) \left[\exp\left(-\alpha_z t\right) - \exp\left(-\frac{\epsilon_0}{\gamma_0} t\right) \right] \tag{10}$$

where $\epsilon_{z0} \equiv \partial\epsilon(V_0, x_0, z_0)/\partial z$ is the cyclicality of the retention elasticity evaluated at $t = 0$. The key difference between sectoral and firm-level productivity shocks is that sectoral productivity z enters directly as an argument of the job finding rate $\lambda(v, z)$.

Equation (10) shows that the pass-through of sectoral productivity shocks to wages exceeds that of firm-level shocks if the shock persistence is the same, $\alpha_x = \alpha_z$, and two conditions are met: the firm value is positive $\Pi(V_0, x_0, z_0) > 0$ and the retention elasticity is pro-cyclical $\epsilon_z(V_0, x_0, z_0) > 0$. I now explain the intuitions that these terms capture.

After a positive sectoral productivity shocks, workers receive more outside offers and incumbent firms thus have an additional incentive to increase the wage of workers to retain them. However, firms will only only the wage of workers if they are worth fighting for, that is if the firm value is positive $\Pi(V_s, x_s, z_s) > 0$. If the match did not generate any value to the firm, there is no reason to increase the wage when workers receive more offers from other firms.

After a negative productivity shock, firms reduce the wage because they become less eager to retain workers. The pass-through of productivity shocks is still amplified with sectoral shocks when profits are positive. In this case, wages were growing before the shock from proposition 1. After the shock, workers do not experience wages cuts but lower wage growth. When the competition for workers also cools down as a result of

Figure 3: Impulse responses from firm and sectoral productivity shocks

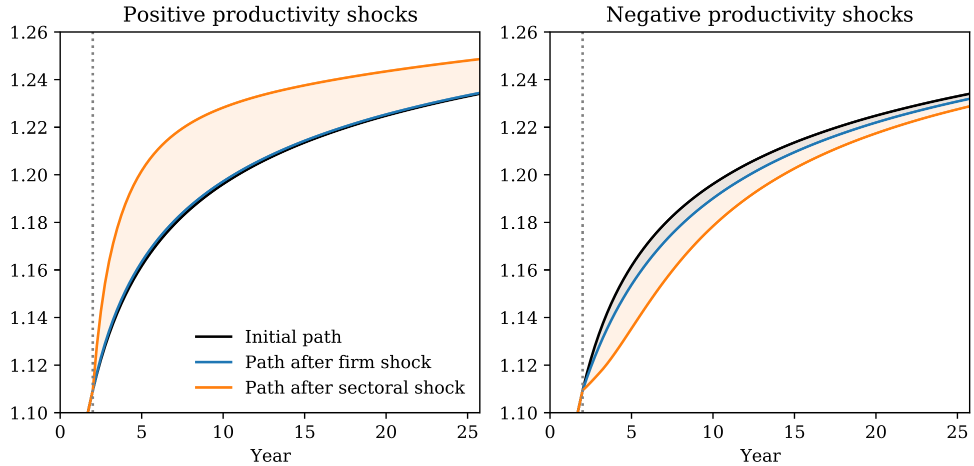
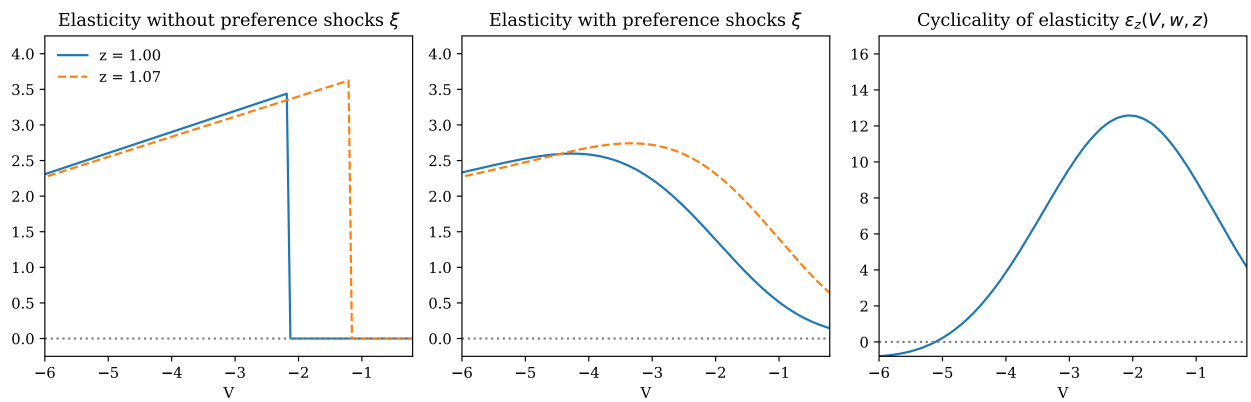


Figure 4: Level and cyclicality of the retention elasticity



the shock, firms can reduce wage growth even more because workers have become very unlikely to leave. To summarize, workers experience faster wage growth in a boom and in a recession they mostly suffer through missing wage growth. This asymmetric response is illustrated in figure 3, which shows the effect of positive and negative firm-level and sectoral productivity shocks on wages when the firm value is positive $\Pi(V_0, x_0, z_0) > 0$. In the figure I set $\alpha_x = \alpha_z$ and let the shocks be of the same size $\hat{x}_0 = \hat{z}_0 = 10\%$. The pass-through of firm-level shocks here is almost 0 because the retention elasticity is close to 0 in this specific example.

I now turn to the second condition that must be satisfied for sectoral shocks to have a higher pass-through: the pro-cyclicality of the retention elasticity $\epsilon_z(V_0, x_0, z_0) > 0$. The cyclicality of the retention elasticity and the cyclicality of job-to-job transitions are closely related but do not always move in the same direction. Consider for example a worker with a relatively low wage. This worker receives more job offers in boom than usual and therefore has a pro-cyclical job-to-job transition rate. However, in boom this worker can become so likely to leave that increasing the wage marginally does not affect her job-to-job transition rate as much as before and therefore her retention elasticity is counter-cyclical. Consider now a worker with a relatively high wage. This worker had almost no chance of changing job in normal time because she was too expensive to get poached. In boom, she suddenly receives more outside offers and both her job-to-job transition rate and her retention elasticity increase sharply. This worker has a pro-cyclical job-to-job transition rate and a pro-cyclical elasticity. The difference between low-wage workers and high-wage workers is illustrated in figure 4, which shows the retention elasticity without preference shocks in the left panel, the retention elasticity with them in the middle panel and the cyclicality in the right panel. It is easy to verify that workers with high wages have the most pro-cyclical elasticity while workers with low wages have a mildly pro-cyclical elasticity but a higher elasticity. This heterogeneity will turn out to be important when I evaluate the effects of sectoral productivity shocks on workers in section **??**.

I now derive the pass-through of sectoral productivity shocks to the worker value Δ_z.

Proposition 3. *To first-order in κ, the pass-through of a sectoral productivity shock z to the value of stayers satisfies*

$$(r+\alpha_z)\,\Delta_z(V,x,z) = \Big[\underbrace{g_z(x,z)\frac{\epsilon(V,x,z)}{\gamma(w(V))} + \Pi(V,x,z)\frac{\epsilon_z(V,x,z)}{\gamma(w(V))}}_{\text{Pass-through to present value of wages}}\Big]u'(w(V))$$

$$+ \underbrace{\kappa\lambda_{wz}(v(V,z),z)\,(v(V,z)-V)}_{\text{Change in expected gains from J2J transitions}} + \mathcal{D}\Delta_z(V,x,z)$$

where $\epsilon_z(V,x,z) \equiv \frac{\partial\epsilon(V,x,z)}{\partial z}$ and $\lambda_{wz}(v(V,z),z) \equiv \frac{\partial\lambda_w(v,z)}{\partial z}|_{v=v(V,z)}$.

Proof. See appendix B.2.3. □

Proposition 3 shows how changes in the intensity of the competition for workers impact workers after sectoral shocks. First, the worker value changes due to the larger response of wages. Second, sectoral productivity shocks impacts the worker value through changes in the probability of finding a job. In booms, the competition for workers heats up and workers are more likely to find a new job so that $\lambda_{wz}(v(V,z),z) > 0$. When they switch, they gain value $v(V,z) - V$. Workers therefore benefit in booms because it increases the probability of a job-to-job transition. In downturns, the opposite is true.

5 Quantitative analysis

The previous section established that the retention elasticity is a critical determinant of the pass-through of productivity shocks to wages. In this section, I calibrate the quantitative model using administrative data from France with a focus on moments that are informative about this elasticity. I then use it to evaluate how much insurance firms provide to workers over the cycle.

5.1 Quantification

I quantify the model by matching moments using the matched employer-employee data from France between 2008 and 2019.

Quantification strategy I set some parameters externally. Specifically, I use a CRRA utility function with coefficient γ to 1.5 following Balke and Lamadon (2022) and the discount factor to match an annual interest rate of 4%.

I use a Cobb-Douglas matching function

$$\mathcal{M}(\phi_u + \kappa\phi_e, \phi_v) = B\left(\phi_e + \kappa\phi_u\right)^{\nu}\phi_v^{1-\nu}$$

with $\nu = 0.5$, which is an intermediate estimate between Menzio and Shi (2011) and Shimer (2005). I calibrate B to get a market tightness $\phi_v/(\phi_e + \kappa\phi_u)$ of 0.6, following Hagedorn and Manovskii (2008), given the equilibrium job finding rate in my model.

I calibrate the degree of firm commitment Φ using estimates of firing costs. It is standard in the literature to justify firm commitment on the ground that firms have reputation concerns. This justification however only holds for relatively large and well-known firms. Instead, I argue that firing costs are a better proxy for firm commitment power. In my model firms only want to walk away from the contract if their value falls below the cost of firing the worker. Layoffs are tightly regulated in France and can lead to lawsuits and large compensations for workers. The parameter Φ captures the expected cost of a layoff, including severance payments and penalties that firms pay when layoffs are challenged in court. I calibrate the firing cost to account for approximately 6 months of labor earnings, $\Phi = 2$, following a methodology from Bentolila and Bertola (1990) that I update with recent data from the International Labor Organization (see appendix A.5 for details).

I normalize the search efficiency of employed workers κ to 1 and set the vacancy posting cost to be equal to 2 months of labor earnings, $k_1 = 0.66$[5].

[5] I am working on ways to discipline these two parameters.

Other model parameters are calibrated by matching moments in the data and in the model. Specifically, I simulate a panel of workers across different industries in the model and estimate the exact same set of moments in the model and in the data.

Section 4 showed that the critical determinants of the pass-through are the persistence of the productivity processes and the job mobility decision of workers. For this reason, I target several moments that inform these two aspects.

For productivity, I target the variance of productivity growth as well as the autocorrelogram to estimate its persistence. In the data I cannot identify the persistence of sectoral shocks with enough precision, and therefore I set $\alpha_x = \alpha_z$. In order to capture the amount of cross-sectional dispersion in productivity, I target the correlation between the average profit share of firms, defined as $1 - w/zx$, and average wages. The fact that firms paying higher wages do not necessarily have lower profits is indicative that they are also more productive. I add transitory measurement errors to firm and sectoral productivity, and recover their variances from the calibration.

To calibrate labor market transitions, I measure average flows of workers in and out of non-employment, as well as across jobs. Since the focus of this paper are wage contracts, and the separation rate is critical for the path of wages, I focus on non-employment rather than unemployment. I target the number of months spent out of employment to calibrate the job finding rate, and I measure the separation rate directly using flows into non-employment. I also target the average job-to-job mobility rate defined as the fraction of jobs ending up in a job-to-job transition each year.

Section 4 showed that the retention elasticity was a critical determinant of the pass-through. This elasticity measures the response of worker mobility to a change in the wage, so it is critical that my model captures well the reasons why workers decide to change jobs and how wage changes affect these decisions. For this reason, I target the share of job-to-job transitions with a positive wage change. It is well known that with heterogeneous productivity across firms some workers experience wage cuts when they change jobs because they expect higher wage growth in the future. In my quantitative exercise I find this to be insufficient to account for the large number of transitions with negative wage changes and therefore I calibrate the volatility of preference shocks v_ξ to match this moment.

The moments used in the paper are described in table 2. I find that about 6.5% of workers change job every year, and 5.5% of jobs end up in a separation into non-employment. Workers spend on average 14 months non-employed, which leads to a quarterly job finding rate of 20%. Remarkably, only 51% of workers changing jobs experience a positive change change when they do so, highlighting the critical importance of preference shocks

Moments	Data	Model
Average duration unemployed in months	14.3 (0.063)	13.4
Average annual separation rate into non-employment	5.5% (0.071%)	5.4%
Average annual job-to-job transition rate	6.6% (0.10%)	7.0%
Share of job-to-job transitions with a positive wage change	51% (0.39%)	59%
Correlation between average profit share and wages	0.051 (0.0022)	- 0.024
Variance of firm productivity growth	0.089 (0.00068)	0.089
Variance of sector productivity growth	0.0032 (0.00035)	0.0032
1st order auto-correlation of firm productivity growth	- 0.24 (.0023)	- 0.21
2nd order auto-correlation of firm productivity growth	- 0.046 (0.0020)	- 0.08
1st order auto-correlation of sector productivity growth	- 0.13 (0.053)	- 0.0044
2nd order auto-correlation of sector productivity growth	- 0.14 (0.058)	- 0.13

Table 2: Moments: data vs. model

for France.

Parameter values Parameters values are shown in table 3. I find that training costs k_2 are high, equivalent to about 1 year of wages. This is to be put in perspective with a long average match duration of about 8 years. The model requires a low value of home production b to rationalize the low job finding rate of 20% per quarter.

Productivity shocks are moderately persistent, with a quarterly AR-1 coefficient of 0.93, which is equivalent to about 0.75 annually. Firm-level shocks are four times more volatile than sectoral productivity shocks, but also more subject to measurement errors.

Does the model account for the differential pass-through? As validation of the model, I assess whether it matches the differential response of wages and job-to-job mobility to firm-level and sectoral shocks that I measure in the data.

Table 4 shows estimates of the response of wages and job-to-job mobility to firm-level and sectoral shocks. The first column repeats the data estimates from table 1 while the second column reports the same estimates from the model.

The model accounts for the larger response of wages to sectoral productivity shocks than to firm level shocks, as well as the differential response of job-to-job transitions to firm and sectoral shocks. Wages respond 3.2 times more to sectoral shocks than firm level shocks in the model (14/4.4), compared to 3.8 times more in the data (18/4.7). The

Parameters	Value	Parameters	Value
Risk aversion coefficient γ	1.5	Discount factor β	0.99
Firing cost Φ	2	Elasticity of matching function ν	0.5
Search efficiency on the job κ	1	Vacancy posting cost k_1	0.66
Training cost k_2	4	Exogenous separation rate δ	0.015
Matching efficiency B	0.26	Value of home production b	0.4
Volatility of preference shocks σ_ξ	1	Dispersion in fixed productivity σ_{x_0}	0.3
Persistence of firm productivity $1-\alpha_x$	0.93	Persistence of sectoral productivity $1-\alpha_z$	0.93
Volatility of firm productivity σ_x	0.14	Volatility of sectoral productivity σ_z	0.035
Volatility of firm meas. errors σ_x^{meas}	0.15	Volatility of sectoral meas. errors σ_z^{meas}	0.005

Table 3: Model parameters

Moments	Data	Model
Response of wages to firm-level productivity shock $\theta^{w,x}$	4.7%	4.4%
Response of wages to sectoral productivity shock $\theta^{w,z}$	18.5%	14.0%
Response of job-to-job transitions to firm-level productivity shock $\theta^{\text{J2J},x}$	- 1.7 pp	- 2.5 pp
Response of job-to-job transitions to sectoral productivity shock $\theta^{\text{J2J},z}$	4.0 pp	8.1 pp

Table 4: Differential response of wages and job-to-job transitions to shocks

model overstates the response of job-to-job transitions to productivity shocks but get the difference in sign right.

From section 4, we already know that the pass-through of productivity shocks to wages is in general larger for sectoral shocks than firm-level shocks. In order to understand why the model also gets the differential response of job-to-job transition rates right, it is useful to consider the quit probability of an individual worker $\kappa\mathbb{E}_{\xi_t}\left[\lambda_w(v(W_{t+1},\xi_t,z_t),z_t)\right]$. Totally differentiating this probability with respect to firm productivity x gives

$$\left.\frac{d\text{J2J}}{dx}\right|_{\text{total}} = \underbrace{\frac{\partial \text{ J2J transition rate}}{\partial \text{ PV of wages}}}_{-\text{ret elasticity}<0} \times \underbrace{\frac{\partial \text{ PV of wages}}{\partial x}}_{\text{passthrough to x}>0} < 0$$

After a positive firm-level shocks, the job-to-job transition rate falls because firms increase wages precisely to reduce worker turnover.

	Data	Model
Cross-sectional dispersion in annual wage growth	0.178	0.0693

Table 5: Cross-sectional dispersion in wage growth - data vs. model

After a positive sectoral productivity shock, not only are firms retaining workers by increasing the wage but new entrants also try to hire workers by posting more vacancies. This new effects shows up in the job-to-job transition rate of an individual worker because sector productivity z_t enters directly as an argument. Totally differentiating this probability with respect to z gives

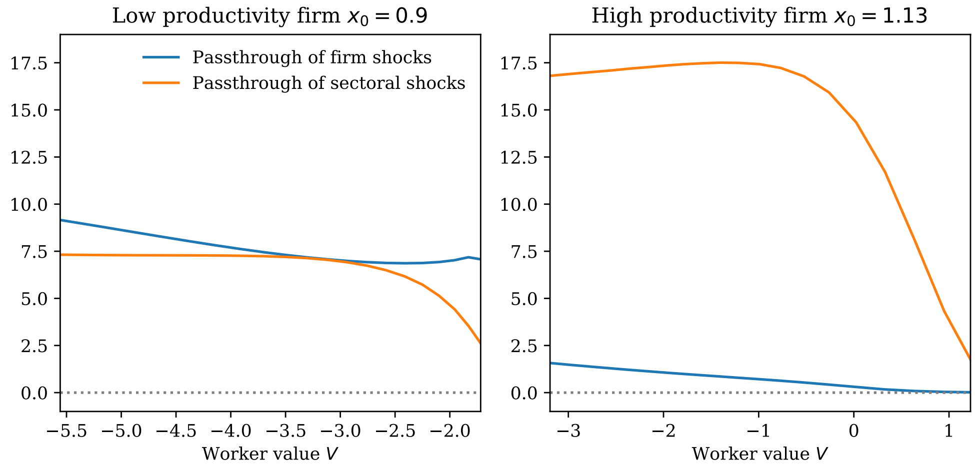
$$\left.\frac{d\text{J2J}}{dz}\right|_{\text{total}} = \underbrace{\frac{\partial \text{ J2J transition rate}}{\partial \text{ PV of wages}}}_{-\text{ret elasticity}<0} \times \underbrace{\frac{\partial \text{ PV of wages}}{\partial z}}_{\text{passthrough to z}>0} + \underbrace{\frac{\partial \text{ J2J transition rate}}{\partial z}}_{\text{change in worker outside option}>0} \lesseqgtr 0$$

In general, the response of job-to-job transitions to a sectoral shock is ambiguous. In the quantitative model, I find that the second term dominates and the total response of job-to-job transitions is positive, which is consistent with the data.

Cross-sectional dispersion in earnings growth The model generates a lot of dispersion in earnings growth from productivity shocks and from the tenure profile of wages. Table 5 shows the cross-sectional dispersion of annual earnings wage growth for job stayers in the data and in the model. The model accounts for almost 40% of the cross-sectional dispersion in the data, which is remarkable compared to what observable worker characteristics can account fro. For example, in appendix A.3 I show that the $R2$ from a regression of wage growth on detailed worker characteristics, such as occupation, industry, location or demographics, is only 1.1%. The vast majority of the dispersion in wage growth comes from the tenure profile of wages but some of it also comes from firm-level and sectoral productivity shocks as I show next.

Workers in high-productivity firms are disproportionately affected by the cycle The model predict that workers with different states (V, x, z) are differently exposed to firm-level and sectoral productivity shocks. In particular, workers in low-productivity matches experience a high pass-through of firm-level shocks but a relatively low pass-through of sectoral shocks, whereas workers in high productivity matches are more exposed to sectoral shocks than they are to firm-level shocks. Figure 5 shows the pass-through of firm and sectoral shocks for a match with relatively low fixed productivity $\overline{x} = 0.9$ and for

Figure 5: Pass-through from firm and sectoral productivity shocks or different $\overline{x}$



a match with relatively high fixed productivity $\overline{x} = 1.13$. To understand why there is no amplification of the pass-through for workers in low productivity matches ($\overline{x} = 0.9$), it is useful to remember from section 4.4 that the pass-through of sectoral shocks is amplified when firms have positive values and the retention elasticity is pro-cyclical. Workers in low-productivity matches generate little profits for firms, and their retention elasticity is only mildly pro-cyclical. By contrast, workers in high productivity matches ($\overline{x} = 1.13$) generate a lot of profits and have high wages. As a result, they are very unlikely to get poached in bad time, but quite likely in good time and their elasticity is strongly pro-cyclical. Intuitively, after a positive sectoral productivity shocks high-productivity firms realize that their workers are now going to get poached and that it must increase their wage in order to retain them. This result shows that workers in different match quality are subject to very different types of risk: workers in low productivity firms are more exposed to firm-level shocks whereas workers in high productivity firms are more impacted by the cycle.

5.2 Firm commitment and counter-cyclical income risk

The cyclicality of income risk depends on the degree of firm commitment.

As in Thomas and Worrall (1988), firm limited commitment implies that after large negative shocks wages sometimes have to fall sharply to prevent firms from walking away from the deal. Specifically, remember that contracts cannot be such that

$$\Pi(V, s_t) \geq -\Phi$$

	Firms at constraint	$\theta^{w,z}$	$\theta^{w,x}$ in booms	$\theta^{w,x}$ in downturns
Data	n.a.	18.5%	4.6%	4.8%
Baseline model ($\Phi = 2$)	0.64%	14.3%	4.5%	4.4%
Low firing costs ($\Phi = 0.33$)	4.73%	18.3%	6.9%	8.0%

Table 6: Firing costs and the cyclicality of income risk

where Φ captures the degree of commitment of firms, and is calibrated using estimates of firing costs. This constraint is more likely to bind when the worker value V is high and firm and sectoral productivity x, z are low.

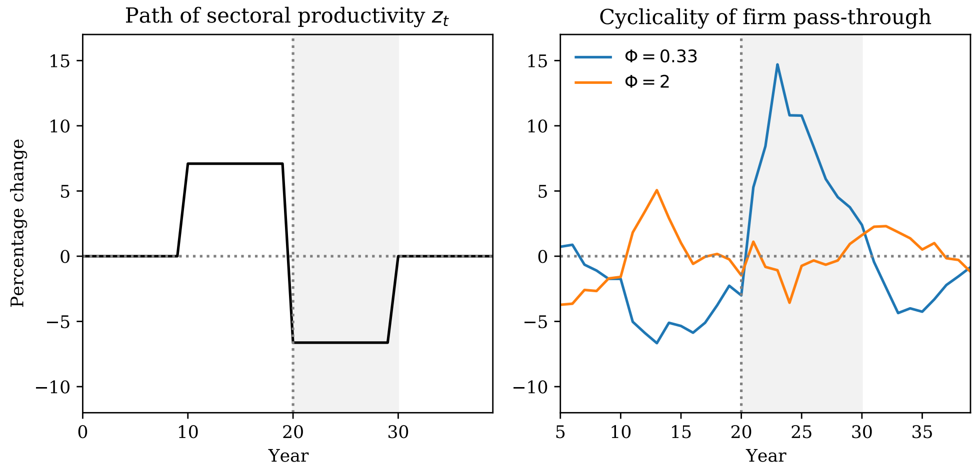
The competition for workers makes it more likely that this constraint binds because wages have a tendency to grow until the firm value $\Pi(V, s_t)$ becomes 0, from equation 1. This is especially true in boom when the competition for workers heats up. During a downturn following a boom, firms suddenly become much less productive and still pay relatively high wages. This is when the limited commitment constraint is most likely to be binding.

I first assess whether in my quantitative model calibrated for France the limited commitment constraint of firms is likely to be binding. This depends on the size of firing costs, the volatility of productivity shocks and any parameter governing how fasts profits fall towards 0. I report in table 6 the fraction of firms hitting this constraint each year. I find that only 0.64% of firms hit this constraint, which means that firing costs are sufficiently large in France that firms effectively have almost full commitment. Table 6 shows that as a result the pass-through of firm-level shocks is roughly constant in booms vs. downturns, which is consistent with my estimates from the data.

Counter-cyclical income risk with lower firm commitment Φ I now reduce the degree of firm commitment Φ using estimates of firing costs for the U.S. Effectively, I ask what would happen if a country like France with high firing costs were to implement labor market liberalization policies lowering firing costs to a level similar to the U.S.? In appendix A.5, I document that the U.S. has much lower firing costs and that a reasonable estimates is 1 month of monthly wage, equivalent in the model to $\Phi = 0.33$.

When firing costs are reduced, I find that firms are a lot more likely to hit their participation constraint. Specifically, this constraint is binding for almost 5% of firms each quarter. As a result, the pass-through of sectoral shocks $\theta^{w,z}$ and firm-level shocks $\theta^{w,x}$ rises. More importantly, the pass-through of firm-level shocks $\theta^{w,x}$ becomes counter-cyclical:

Figure 6: Cyclicality of the pass-through with high and low firing costs



it is 16% (8/6.9) larger in downturns than in booms. This makes income risk countercyclical. The reason for this is that in downturns sectoral productivity is low relative to wages. As a result, profit margins are depressed and firms are more likely to hit the constraint after a negative firm-level shock. Insurance contracts are critical for this result because they imply that wages are relatively more stable than productivity. They do fall in downturns but not quite as much as profits.

The state dependence of the pass-through of firm-level shocks is more complex than suggested in table 6 because it depends on the entire history of sectoral productivity shocks. In particular, the pass-through is at its maximum at the beginning of a downturn after a boom because productivity is low but wages remain high due to the intense competition for workers during the boom in past periods. This state dependence is illustrated in the simulations of figure 6. I simulate a panel of workers in a single industry with a path for sectoral productivity shown in the left panel: the economy is initially at the stationary level, then enters a boom (sectoral productivity increases by 2 standard deviations) for 10 years and finally a downturn for 10 years before returning to its stationary level. The right panel shows the pass-through of firm-level shocks to wages relative to the average pass-through with high firing costs ($\Phi = 2$) and low firing costs ($\Phi = 0.33$). When firing costs are low ($\Phi = 0.33$), the pass-through of firm-level shocks falls during the boom at $t = 10$ and then jumps at the beginning of the downturn at $t = 20$. The increase in downturn is larger than the fall in boom because the economy is initially in a state with relatively high wages due to the competition for workers in boom.

Novel implications for firing costs These results point to a novel role of firing costs when wage contracts are endogenous: they enhance firm commitment power and improve insurance for workers. As a result, I find that welfare falls by 1% when we introduce lower firing cost because workers are more exposed to shocks.

It is noteworthy that firing costs protect workers against large wage cuts more than they protect them against the risk of becoming unemployed. Indeed, the separation rate rises only marginally from 5.4% to 5.6% when I reduce firing costs. In my model, most workers keep their job after a large negative shock by experiencing large negative wage cuts. With firing costs, these workers enjoy a more stable wage. The small increase in the separation rate arises because the joint value of a match depends on the ability of firms to provide insurance to workers. If firms cannot insure workers through employment when firing costs are reduced, the joint value of matches falls and matches with low productivity are more likely to be efficiently terminated.

The results in this section stand in contrast to existing work on firing costs, which tend to find that they are detrimental to welfare. For example, Bentolila and Bertola (1990) study the effects of firing costs on the separation rate and the hiring rate. They find that firing costs could account for the poor employment dynamics among European countries after the oil shocks of the 1970s. Hopenhayn and Rogerson (1993) show that taxes on job destruction prevent the reallocation of workers towards more productive firms.

My results show that firing costs can improve welfare through their effect on insurance. The fact that firing costs are most useful at the start of recessions is useful for a concrete implementation of firing costs because other effects studied in the literature might also have a cyclical component. For example, the reallocation of workers towards more productive firms might be particularly important during recoveries when new businesses start growing. If this is true, an optimal firing cost strategy would be to implement firing costs permanently and relax them when the economy recovers from a recession because the benefits in terms of insurance are smaller whereas the costs in terms of reallocation are larger.

My model was not meant to capture all the effects of firing costs on welfare. In particular, I assume that firms only pay firing costs when they unilaterally walk away from a match, and not when workers quit or when separations are exogenous. In practice, firing costs do not apply when workers quit or under specific economic conditions, but the mapping between reality and the model is complex so I leave a more detailed analysis of firing costs when these competing forces are at play for future work.

5.3 Income inequality over the cycle

Sectoral fluctuations in productivity are not only a source of risk for employed workers, but also for workers transitioning through unemployment. In a downturn, these workers start with a much lower wage compared to workers who kept their job and experience a much more progressive fall in wages due to the insurance provided by contracts.

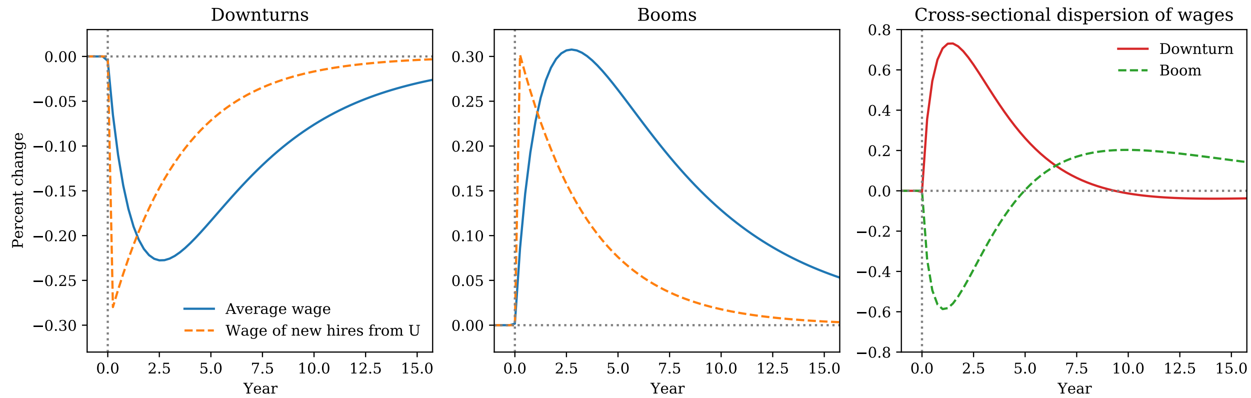
Figure 7 illustrates this differential response of wages for newly hired workers out of unemployment and for continuously employed workers. The left panel shows the percent change of wages for new workers out of unemployment and the average wage after a 1% negative and mean-reverting shock to sectoral productivity. The wage of workers out of unemployment falls on impact by about 0.3%, and then gradually recovers following the path of productivity. The wage of continuously employed on the other hand only falls gradually, which is reflected in the trajectory of the average wage. The average wage eventually becomes even lower than the wage of new hires out of unemployment after 2 years because the pass-through is backloaded. This differential dynamic of wages leads to an increase in income inequality during downturns, as illustrated by the right panel. The cross-sectional standard deviation of log wages among employed workers rises by about 0.75% in downturn because a large mass of new hires start with relatively low wages, while the wage of continuously employed worker adjusts slowly. After 9 years, wage inequality has reverted back to its pre-shock level and overshoots slightly as the wage of new hires from unemployment has recovered faster than the average wage.

The middle panel shows the response of wages to a 1% mean-reverting positive sectoral productivity shocks. The dynamics is similar except that average wages increase faster because the pass-through of sectoral shocks is larger for positive shocks, and because job switchers experience more job transitions with positive wage growth. As a result, the cross-sectional dispersion of log wages falls initially but by a smaller amount and for a much shorter period of time. After 5 years income inequality overshoots because the wage of current workers is significantly higher than the wage of new hires, and income inequality remains high for a significant period of time.

Figure 7 shows the surprising result that both downturns and booms amplify income inequality but for different reasons. Downturns amplify inequality because workers going through unemployment initially experience a much sharper fall in wages than continuously employed workers. Booms amplify inequality because continuously employed workers experience large wage increases due to the increased competition by poachers and these wages increases are very persistent.

In order to quantify this point, I compute the cross-sectional standard deviation of log wages in steady state when sectoral productivity shocks are turned off, and in the

Figure 7: Income inequality over the cycle



	No sectoral shocks	With sectoral shocks	in downturn	in boom
Cross-sectional sd. of log wages	0.178	0.187	0.198	0.179

Table 7: Sectoral shocks as a driver of income inequality

ergodic distribution when productivity shocks are present. The simulation results, in table 7, show that sectoral productivity shocks increase the cross-sectional dispersion in log wages from 0.178 to 0.187, a 5% increase. This shows that the cycle, not just negative sectoral shocks, amplifies income inequality. The last two columns report the dispersion of log wages in downturns (sectoral productivity below average), and in booms (sectoral productivity above average). Income inequality in downturns is 10% larger than in booms.

To summarize, changes in income inequality over the cycle reflect a differential exposure of continuously employed workers and new hires from unemployment to sectoral shocks. Unemployed workers suffer most from downturns because they do not benefit from the insurance provided by employers and start new jobs with a much lower wage. This result shows that changes in income inequality over the cycle are alarming because they signal that some workers are particularly affected by shocks.

6 Extension: introducing risk-free bonds

An important assumption I made so far in this paper is that workers do not have access to financial markets and consume their wage. In this section I show that when workers have access to risk-free bonds, the wage contracts offered by firms changes significantly: they feature much more wage backloading and face a new trade-off between worker retention and precautionary savings. When trades in risk-free bonds are private information, workers borrow more than what firms would like so firms increase the pass-through of productivity shocks to wages in order to make workers save for precautionary reasons.

6.1 Environment

I present a 2-period version of the model without preference shocks.

The timing works as follows: in period 1, workers are matched to firms and sign wage contracts given some exogenous promised value V_0. They receive a wage and produce. In period 2, firm and sectoral productivity shocks x and z are realized. Workers can lose their job with exogenous probability δ and if they do not lose their job, they search for a new job. Workers then produce and receive a wage from their employer. Workers can trade risk-free bonds when they receive their wage in period 1 and these bonds are due in period 2 when wages are paid.

Workers can trade risk-free bonds a at rate $R = 1/\beta$. They are not able to default on this asset. I assume first that firms observe and control this choice, and will relax this assumption in section 6.3. Firms can pay severance payments τ to worker in the event of a separation into unemployment. I assume that if the firm commits to a level of severance payment but does not fulfill its promise ex-post, it is subject to the firing cost Φ.

In the 2-period model, labor markets are indexed by the wage that workers receive from poachers $\tilde{w}_2$ and by the asset held by workers a. The expected profit from posting a vacancy in market $(\tilde{w}_2, a)$ is

$$\Pi_0(\tilde{w}_2, a, z_2) = -k + q(\tilde{w}_2, a, z_2)\left[x_0 z_2 - \tilde{w}_2\right]$$

where x_0 is the firm productivity of a new entrant. The free entry condition $\Pi(\tilde{w}_2, a, z_2) = 0$ implies that the vacancy filling rate $q(\tilde{w}_2, a, z_2) = q(\tilde{w}_2, z_2)$ is independent of assets, and from the matching function so is the job finding rate $\lambda(\tilde{w}_2, z_2)$.

Given the wage contract $\{w_1, w_{xz}\}$, severance payment τ and assets a, the value of the worker satisfies

$$\begin{aligned} V = {} & u(w_1 - a) + \delta\beta u(b + aR + \tau) \\ & +(1-\delta)\beta\mathbb{E}_{x_2,z_2}\left[\max_{\tilde{w}}(1-\kappa\lambda(\tilde{w}, z_2))u(w_{xz} + Ra) + \kappa\lambda(\tilde{w}, z_2)u(\tilde{w} + Ra)|x_1, z_1\right] \end{aligned} \tag{11}$$

The worker search policy $\tilde{w}_{xz} = \tilde{w}_2(w_{xz}, a, z_2)$ satisfies the optimality condition

$$\lambda_w(\tilde{w}, z_2)\left[u(\tilde{w} + Ra) - u(w_{xz} + Ra)\right] + \lambda(\tilde{w}, z_2)u'(\tilde{w} + Ra) = 0 \tag{12}$$

The optimal contract solves

$$\begin{aligned}
\max_{w_1, w_{xz}, \tau, a} \quad & x_1 z_1 - w_1 - \beta\delta\tau + (1-\delta)\beta\mathbb{E}_{x_2,z_2}\left[(1-\kappa\lambda(\tilde{w}_{xz}, z_2))\left(x_2 z_2 - w_{xz}\right)\right] \\
\text{s.t.} \quad & V_0 = u(w_1 - a) + \delta\beta u(b + Ra + \tau) \\
& \quad +(1-\delta)\beta\mathbb{E}_{x_2,z_2}\left[(1-\kappa\lambda(\tilde{w}_{xz}, z_2))u(w_{xz} + Ra) + \kappa\lambda(\tilde{w}_{xz}, z_2)u(\tilde{w}_{xz} + Ra)|x_1, z_1\right] \\
& \tilde{w}_{xz} = \tilde{w}_2(w_{xz}, a, z_2) \\
& x_2 z_2 - w_{xz} \geq -\Phi \\
& -\tau \geq -\Phi
\end{aligned} \tag{13}$$

I characterize the optimal contract in section 6.2 and discuss the implications of hidden trade in section 6.3.

6.2 Firms use debt to backload wages even more

I first show that introducing risk-free bonds enables firms to backload wages even more. When firms have limited commitment however, the extent to which firms can backload wages depends a precautionary savings motives.

To build some intuition for the results in this section, it is useful to remember the trade-off that firms faced when workers had no access to financial markets. Firms backload wages to prevent workers from leaving for another job, because it makes them apply for jobs with a lower job finding rate. However, when workers have no access to financial markets backloading wages implies backloading consumption too, and firms must compensate risk-averse workers for this. As a result, firms backload wages but only partially as illustrated by proposition 1. Trade in risk-free bonds allows firms to backload wages and smooth consumption over time by making the worker borrow. As a result, the firm is able to retain workers at minimal cost and the job-to-job mobility rate falls drastically. Said differently, risk-free bonds expand the set of instrument that firms can use to retain workers: a debt that workers must repay even if they change job.

I show in this section that the extend to which firms can use bonds is limited by a precautionary savings motive of workers. Workers face the risk of becoming unemployed before receiving their wage, or receiving a low wage tomorrow because of a negative productivity shocks. Because of this workers might not want to borrow too much today in anticipations of expected future income because this income is uncertain. If firms have full commitment power they can provide insurance to the worker against these risk but if commitment is limited, as in my baseline model, the worker can only borrow to some extend and the contract implies partial backloading.

In order to make these points as transparent as possible, I assume in this sub-section that utility is CARA $u(c) = -\exp(-\gamma c)/\gamma$. This assumption implies from condition (12) that the search policy $\tilde{w}(x_2, z_2, a) = \tilde{w}(x_2, z_2)$ is independent of assets a. I will relax this assumption in the next sub-section when I discuss the implications of hidden trades.

Combining the optimality conditions of the contracting problem gives

$$\frac{u'(b + Ra + \tau)}{u'(w_1 - a)} = 1 + \mu_U$$

$$\frac{u'(w_{xz} + Ra)}{u'(w_1 - a)} = 1 + \frac{\kappa \partial_{w(x,z)} [\lambda(\tilde{w}_{xz}, z_2)] (x_2 z_2 - w_{xz}) + \mu_{xz}}{1 - \kappa \lambda(\tilde{w}_{xz}, z_2)}$$

$$\begin{aligned} & u'(w_1 - a) = \delta u'(b + Ra + \tau) \\ & +(1-\delta)\mathbb{E}_{x_2, z_2} [(1 - \kappa\lambda(\tilde{w}_{xz}, z_2))u'((w_{xz} + Ra) + \kappa\lambda(\tilde{w}_{xz}, z_2)u'(\tilde{w}_{xz} + Ra)|x_1, z_1] \end{aligned} \tag{14}$$

where μ_{xz} and $\delta\beta\mu_U$ are the Lagrange multipliers of the firm participation constraints. The first equation is the optimality condition for severance payments τ. The second equation is the optimal wage growth condition. The third condition is the optimal saving condition, which here turns out to be the worker's Euler equation.

Full backloading with firm commitment Consider first the case with full commitment ($\Phi \to \infty$) as a benchmark. The two participation constraints of the firm drop out so $\mu_{xz} = \mu_U = 0$.

The optimality condition for severance payments shows that the firm insure the worker against unemployment risk since $c_1 \equiv w_1 - a = b + Ra + \tau = c_U$.

Combining the three optimality conditions, we find that

$$w_{xz} \geq x_2 z_2$$

so that workers get at least the value of output tomorrow. This is an extreme form of backloading in which firms pay more than the value of output to workers tomorrow irrespective of their promised value V_0 or wage at w_1. The wage at $t = 1$ is given by the promise keeping constraint and depends on V_0. It can even be negative, meaning that workers have to pay an upfront fee to firms. Because wages are so backloaded, the job-to-job transition rate falls drastically.

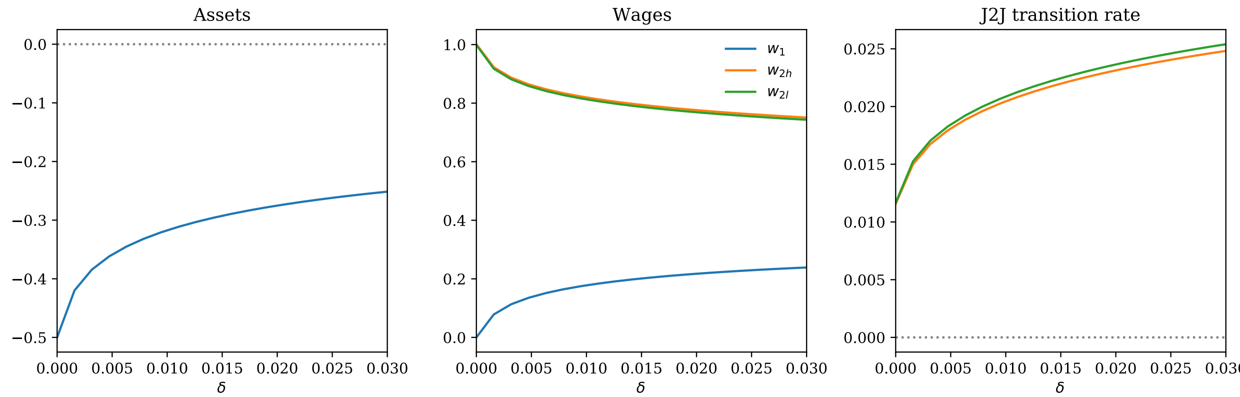
Consumption satisfies

$$c_{xz} \leq c_1 = c_U \leq \tilde{c}_{xz}$$

where $\tilde{c}_{xz}$ is the consumption of a worker in state (x_2, z_2) after a job-to-job transition.

This benchmark result is closely related to Stevens (2004) who studies optimal wage

Figure 8: Optimal contract with risk-free bonds: comparative statics with respect to δ



Note: illustrative example with two productivity states x_h and x_l.

contracts with complete financial markets. Remarkably, risk-free bonds are almost sufficient here to achieve the same results even though there are many sources of risk (e.g. unemployment, productivity shocks). This is because firms are able to insure workers against these risks. The only difference is that the firm is not able to insure workers against the upside risk of finding another job. Workers always get a higher wage when they changes jobs so their consumption must jump. The firm provides some form of limited insurance by making the worker borrow at $t = 1$ and backloading wages at $t = 2$. This is the reason why wages here are sometimes larger than output at $t = 2$.

Partial backloading with limited firm commitment Consider now the case with limited firm commitment ($\Phi < \infty$). In this case firms can only insure workers against negative events such as unemployment risk and negative productivity shocks if it is in their interests ex-post. In the limit case with $\Phi = 0$, firms cannot commit to make losses after adverse shocks so that $\tau = 0$ and $w_{xz} \leq x_2 z_2$.

As a result, workers dislike entering period 2 with too much debt because in the event of a negative shock or unemployment shock they will have very little income to repay it and will as a result consume very little. This prevents firms from backloading wages too much because it would make the worker either borrow or backload consumption too much at $t = 1$. Precautionary savings motives prevent workers from borrowing too much in anticipation of future income because this income is uncertain.

Figure 8 illustrates how the path of consumption changes as the exogenous separation rate δ rises from 0 to 3% when Φ is set to 0. When the risk of separation into unemployment increases, the optimal contract implies less borrowing since workers face the risk of having to repay the debt while unemployed. As a result, wages become less backloaded and the job-to-job transition rate increases.

6.3 Hidden trade

I have assumed so far that firms could observe and thus control the asset choice of agents. I now assess whether the allocation changes when agents can privately access financial markets, i.e. they are hidden trades.

I solve the problem with hidden trade using the first-order approach following Werning (2001) and Abraham and Pavoni (2008).

With hidden trade, the worker privately chooses in which labor market to apply $\tilde{w}_2$ and how much assets to hold a to maximize her present value (11). Taking the first-order condition with respect to asset a gives the Euler equation (14). The optimal contract now solves the problem (13) with the Euler equation (14) as an additional constraint.

With CARA utility the relaxed problem without hidden trade (13) solves the problem with hidden trade. To see this, note that the optimality condition with respect to assets a in the relaxed problem is the agent's Euler equation. Therefore the solution to the relaxed problem is also feasible in the problem with hidden trade, and since we can always do better in a relaxed problem it is also the solution with hidden trade. Intuitively, with CARA utility there are no wealth effects in that the level of assets does not influence the worker's search decision. As a result, given a choice for v, the worker and firm preferences towards savings are aligned.

For a general utility function there are profitable joint deviations for the worker. To understand this, it is useful to consider the optimal choice of assets of the firm in the relaxed problem (13) for a general $u(c)$ when firms control the level of assets directly. The optimality condition for a becomes, using the envelope theorem,

$$u'(w_1 - a)\left(1 + \kappa \partial_a \lambda(\tilde{w}_{xz}, z_2)(1-\delta)\beta \mathbb{E}_{x_2,z_2}\left[(x_2 z_2 - w_{xz})\,|x_1, z_1\right]\right) = \delta u'(b + Ra + \tau) \\ +(1-\delta)\mathbb{E}_{x_2,z_2}\left[(1 - \kappa\lambda(\tilde{w}_{xz}, z_2))u'((w_{xz} + Ra) + \kappa\lambda(\tilde{w}_{xz}, z_2)u'(\tilde{w}_{xz} + Ra)|x_1, z_1\right]$$

The firm takes into account that assets influence the search decision of workers, and therefore their job-to-job transition rate $\partial_a \lambda(\tilde{w}_{xz}, z_2) \neq 0$. When firms make positive profits, they alter the level of assets in a way that reduces the job-to-job transition rate so as to retain workers. From the optimal search condition (12), we can show by comparative static that workers with more assets search for jobs with a higher wage $\partial_a \tilde{w}_2(w_{xz}, a, z) > 0$. Since the job finding rate $\lambda(w, z_2)$ is decreasing in w, firms increase the worker's savings in order to reduce their job-to-job transition rate. Intuitively, workers enter period 2 with some assets a. The lower this level of asset, the higher her marginal utility of consumption. With low assets a, worker would rather apply for jobs that are easier to get and deliver a smaller increase in consumption, than jobs that are difficult to get. Conversely, if the worker enters with a high level of assets she is willing to apply for jobs that she is unlikely to get but delivers a high payoff if she gets it. Another way to see this result is that applying for a job is like buying a lottery ticket, and the worker is more willing to enter a risky lottery if her marginal utility is low (her asset level is high) than otherwise. Because of this, firms make workers borrow less and enter period 2 with a relatively low level of debt.

What would workers choose with hidden trade? In the allocation without private trade workers are borrowing constrained: relative to firms, they would prefer to borrow more at $t = 1$ to increase consumption. The joint deviation is therefore borrow more at $t = 1$, enter period $t = 2$ with more debt and apply to a labor market $\tilde{w}$ with a lower wage and higher job finding rate. In response, firms backload wages more to induce workers to apply in labor markets with a lower job finding rate. They also increase the pass-through of productivity shocks so as to increase the precautionary savings motive of workers, and make them save more at $t = 1$.

Future directions I am working on a dynamic version of this problem that I will use to evaluate whether wages are much more backloaded in optimal contracts when workers face a realistic amount of risks and have access to frictional financial markets, such as trades in risk-free bonds subject to a borrowing constraint. I will also use the dynamic model to study whether workers use bonds to smooth wages by borrowing when they receive a low income today. The 2-period model cannot be used to answer this question because trades in risk-free bonds are decided at $t = 1$ before shocks are realized.

7 Conclusion

I ask in this paper whether firms insure workers against risk over and from cycles generated by sectoral productivity shocks. I build a model with dynamic wage contracts consistent with the response of wages and job-to-job mobility to firm-level and sectoral productivity shocks estimated using administrative data. I find that workers in high-productivity firms are disproportionately affected by sectoral shocks and that the commitment power of firms is critical for the cyclicality of income risk. Because firing costs influence firm commitment, lowering them increases the risk faced by workers and makes it counter-cyclical. Finally, income inequality rises in downturns because newly hired workers from unemployment do not benefit from the same degree of insurance than continuously employed workers.

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Appendix

Contents

A Data appendix

A.1 Sample construction

I use administrative data provided by the CASD in France between 2008 and 2019. My analysis relies on two main files:

a) the panel version of the "DADS tous salariés" database, containing detailed information about employment history for 1/12th of the French population every year;

b) "FARE" database, with annual information about firm balance sheet and income statement for the entire private sector except firms in the agricultural sector

I complement my analysis with information about the structure of firms ("Contours des entreprises profilées") provided by the CASD and with national account information on depreciation rates and the price index provided by INSEE.

Sample selection From the FARE file on firms, I exclude firms with invalid information (e.g. missing ID), firms belonging to the public sector and household employers. I also drop firms from the financial sector because it is particularly challenging to estimate productivity for these firms as their income is mostly reported in their financial statement, unlike other firms. One challenge with this data is that it is reported at the legal unit level ("UL"), and several legal units can belong to the same firm. Since I want to measure job-to-job transitions across firms competing for the same workers, it is important that I aggregate firms within coherent economic units. To do so, I use information from the "Entreprise profilée" ("EP") files for available years, and extrapolate the information back in time when necessary.

From the DADS file, I exclude interns and apprenticeships as well as workers from the public sectors or working for non-profits. I keep prime-age workers (25 to 55 years old) and workers with full-time positions and permanent contracts (CDI). I focus on relatively stable jobs because I study the problem of worker retention, and it would not fit very well the case of temporary contracts (CDD) since they usually end after a short period of time. In my sample I find that full-time workers with permanent contracts account for about 60% of private sector jobs.

Definition of sectors I use detailed information about a firm industry to define a sector. Specifically, I use the NAF Rev 2. APE 2 classification, which contains about 81 sectors, including 24 sectors within manufacturing. Examples of sectors are pharmaceutical industry, retail trade or restaurants. My dataset features 6,761 workers and 1,751 per sector on average.

One advantage of using industry classifications instead of occupation or location to define a labor market is that industry is reported at the firm level, and it is therefore easy to aggregate and compute productivity at the sector level. By contrast, several employees within the same firm might belong to different occupations or live in different locations. The main drawback of using sectors is that workers might change sector when they change jobs more than they change occupation or location. I am working on a model extension with imperfect labor mobility across sectors to assess whether my results are robust to this modification.

The industry classification is available at different levels of aggregation. I choose an intermediate definition for two reasons. First, with more aggregated definition (e.g. manufacturing), changes in the sector productivity are likely affecting the entire economy. It is then more difficult to justify in my model that the interest rate is independent of sector productivity. With a more granular classification, idiosyncratic shocks to sector productivity are more likely to cancel out in the aggregate. At the other extreme, with the most granular definition most sectors are made of a few firms only and estimates of sector productivity (the average across firms) become contaminated by firm-level changes in productivity. In this case estimates of pass-through against sectoral productivity shocks are biased because they reflect the response of wages to firm-level productivity shocks. I run simulations to ensure that with the classification that I use (APE 2) and given the volatility of firm and sectoral productivity and the size of firms in my sample, this small-sample bias is negligible.

Definition of labor productivity I measure labor productivity as value added per worker, adjusted for the cost of capital

$$\text{LP} = \frac{\text{sales + variation in shocks - cost of materials - cost of capital}}{\text{number of employees}}$$

Sales includes products, services and merchandises sold while the number of employees is the average full-time equivalent number of workers in that year. The data contains information about depreciation costs reported by firms, but this information is known to be sensitive to accounting strategies followed by firms. Instead, I construct my own estimates for the cost of capital as follows. I first measure the depreciation rate at the year-industry level using national accounts data on consumption and stock of fixed capital (average of 6.5% annual). I then add the average interest rate paid by firms on their debt in my dataset for firms with positive debt (average of 10%) and multiply with firm tangible assets reported in the firm data.

I residualize the log productivity on dummies for firm-age to control for a life-cycle component. My measure of labor productivity is closely related to the accounting measure of operating profits, and therefore not surprisingly their correlation is very strong both across firms and over time within firms.

I decompose labor productivity into an aggregate, a sectoral and a firm component by assuming that they are log-additive

$$\log y_{jst} = \log a_t + \log z_{st} + \log x_{jst} \tag{A.1}$$

I measure aggregate productivity $\log a_t$ by average across firms each year. I then measure sectoral productivity $\log z_{st}$ by averaging the residual across firms within sector each year. Finally, firm-level productivity $\log x_{jst}$ is estimated as the residual. In ongoing work I investigate how my results change with alternative assumptions about productivity; for example one in which sectoral productivity is a function of aggregate productivity but with different loading coefficients. I confirm visually that there are no trends in sectoral productivity.

Definition of wages I define wages as daily labor earnings using the worker total worker earnings net of payroll taxes but gross of income taxes. This includes regular wages, overtime pay, bonuses and even payment in kind. It excludes however stock options, but these are less omnipresent in France than they are in the U.S. Note also that medical insurance is not a major component of pay in France, unlike in the U.S.

I divide total labor earnings in a year by the number of days worked at that firm. The data contains information about hours but for workers with full-time jobs and permanent contracts it usually refers to the legal number of hours and therefore does not represent the actual number of hours worked. For this reason I do not adjust for it.

Definition of labor market flows Identifying job-to-job transitions is challenging because workers sometimes hold multiple jobs at the same time. For this reason, I first identify the main job of a worker defined as the job with the earliest start date. I drop jobs that lasted for less than 35 hours during a year (a regular work week) and main jobs if they end up accounting for less than 50% of total earnings from simultaneous jobs. I also drop individuals with more than 5 jobs in a given year.

I use the exact start and end dates of jobs to identify a job transition. A job-to-job transition occurs if the new job starts 18 days or less after the previous job ends. This leaves a little bit of room for workers who take 2 weeks of holidays in between jobs. The risk is that it might also include workers who transit through unemployment for just 2 weeks and find a new job quickly. Note however that France is a country in which the job finding rate is fairly low (I estimate 20% per quarter) so most likely this risk is minimal. I also count as job-to-job transitions if the new and old jobs overlap for some time (i.e. the worker holds 2 jobs for some time), but my results are robust to remove them from the sample.

An important moment that I target in my quantitative exercise is the share of job-to-job transitions with positive wage growth. This moment is important because it is informative about why workers change jobs, and therefore has important implications for the retention elasticity. In France it is common for workers to change jobs to receive severance payments and compensations for vacations not taken when they switch job. As a result, average daily earnings at the current job is often larger than average daily earnings at the next job because it includes these extraordinary payments on top of the wage. Indeed, I compute that only 40% of workers experience a positive wage growth when daily earnings are computed in this naive way, and I find that workers who are about to make a job-to-job transition experience an average wage growth of 8%, compared to 1% for the entire population. To control for these exceptional payments, I compute the share of job transitions with a positive wage change by comparing daily labor earnings at the new job with daily labor earnings at the previous job the previous year. I use the same method in the model.

	Number per year	Average duration in sample in years
Workers	532,005	7.89
Firms	129,576	8.16

Table A.1: Sample description

Avg. age	Shale male	Avg. firm size (firm obs)	Avg. firm size (worker obs)
38.4	66%	66.7	8999

Table A.2: Sample characteristics

When a worker separates from their previous jobs and does not make a job-to-job transition, I define it as a separation into non-employment. When a worker from my sample moves to another job that is not in my sample (e.g. transition from private sector to public sector), I do not count it either as a job-to-job transition nor as a separation into non-employment nor as a stayer.

I compute the duration of non-employment as the number of months until a worker reappears in my sample, conditional on the worker reappearing. By conditioning on whether a worker ever comes back in my sample I sort out workers who leave the labor force permanently (e.g. retirement, death). I only estimate this moment on the first half of my sample (2008-2015) so that workers have plenty of time to come back.

Summary statistics I merge the worker and firm data together and find that 95% of workers are successfully matched to a firm. I restrict my sample to workers and firms who at in the panel for at least 3 years, for firms with at least 3 employees (in the panel or not) and I keep sectors with at least 20 employees and 3 firms per year. I drop firms with negative or missing labor productivity and those with labor productivity growth below and above the 0.5 and 99.5 percentiles respectively. I also drop individuals with wage growth below or above the 0.5 and 99.5 percentiles.

A.2 Estimation of standard errors by Block bootstrap

The estimation of the moments used in the quantitative analysis is done in several steps, and for this reason I estimate standard errors by bootstrap. I sample firms with replacement and keep all the years and workers associated with a firm if it is sampled. I then create 1,000 samples and then apply my estimation procedure in each of them, including rezidualizing productivity, removing outliers, computing sectoral productivity or estimating the pass-through. The estimates that I report and their standard errors are the average estimates across bootstrap samples and the standard deviation across sample.

A.3 Additional moments

Pass-through estimates in boom vs. downturns Table A.3 reports the pass-through of firm-level and sectoral productivity shocks to wages estimated separately in booms, and in downturns. I define booms as periods in which sector-productivity in level is higher than the mean, and downturns as the complement. The estimates for the pass-through of firm shocks are meant to capture

	Booms	Downturns
Pass-through of firm-level shocks	4.63% (1.00%)	4.88% (0.75%)
Pass-through of sectoral shocks	16.81% (4.70%)	20.1% (5.41%)

Table A.3: Pass-through estimates in boom vs. downturns

	p1	p5	p10	p25	p50	p75	p90	p95	p99	mean	s.d.
Real wage growth	- 0.57	- 0.25	- 0.13	- 0.038	0.011	0.071	0.18	0.28	0.58	0.015	0.18

Table A.4: Distribution of real annual wage growth

how idiosyncratic risk faced by workers vary over the cycle. The results show that the pass-through in booms and in downturns are not significantly different from one another, which is consistent with my model calibrated for France.

Dispersion in earnings growth Table A.4 describes the distribution of real annual wage growth in the data for workers continuously employed at the same firm between year $t-1$ and year t. The mean annual wage growth is 1.5% and the distribution is remarkably symmetric. The standard deviation is 0.18, the vast majority of which cannot be accounted for by observable worker characterized. Specifically, I estimate

$$\Delta \log w_{ijst} = \alpha + X'\beta + \epsilon_{ijst}$$

where X is a vector of worker characteristics, including a polynomial in experience (age minus 20), dummies for gender and firm as well as dummies for occupation (4-digit), industry (4-digit) and commuting zones. The R^2 from this regression is only 0.011.

A.4 Aggregate shocks

In this paper I focus on the behavior of sectoral $\log z_{st}$ and firm productivity $\log x_{jst}$ because I want to isolate the effects of the cyclical competition from workers from that of time-varying price of risk. The assumption underlying this approach is that sectoral productivity shocks are diversifiable for firm owners because sectors are sufficiently small. Changes in aggregate productivity $\log a_t$ on the other hand cannot be diversified and will therefore influence both the cyclicality of the competition for workers and the ability of firms to provide insurance against these shocks. This distinction is especially important in the context of wage contracts because we know that there is perfect risk-sharing if workers and firms are both equally risk-averse and the contract is not subject to any friction.

Sectoral shocks also have the advantage that there is more data we can use to estimate these moments. For example in my data I have one time series of 12 years for aggregate shocks, but a panel of 81 sectors for sectoral shocks. I am working on extending my sample to the 1990s in order to get better estimates for aggregate and sectoral shocks. Doing so is difficult because there is an important break in the firm dataset in 2008 due to changes in the survey methodology.

	Response of wages (OLS)	Response of job-to-job transitions (OLS)	Variance of shocks
Firm shocks	0.011 (0.002)	- 0.0031 (0.0025)	0.0887 (0.0007)
Sectoral shocks	0.043 (0.015)	0.034 (0.028)	0.0032 (0.0004)
Aggregate shocks	0.138 (0.017)	0.321 (0.034)	0.00042 (0.00002)

Table A.5: The response of wages and job-to-job transitions to firm, sectoral and aggregate shocks

Table A.5 shows the response of wages and job-to-job transitions to firm, sectoral and aggregate productivity shocks estimated using equation A.1 as well as estimates of the variance of these different shocks. Aggregate shocks have an even higher pass-through than sectoral shocks, and an even more cyclical response of job-to-job transitions to shocks. This supports the idea that studying sectoral shocks is informative about the nature of aggregate cycles, since the coefficients move in the same direction relative to firm-level shocks, but that they are also different, since the response is much larger to aggregate shocks. The difference in the cyclicality of job-to-job transitions with respect to sectoral and aggregate shocks might also be informative for other line of research, such as the literature on unemployment volatility (Shimer, 2005).

Table A.5 also shows that the variance of sectoral shocks is about 10 times larger than the variance of aggregate shocks, which suggest that sectoral shocks might be a larger source of risk than aggregate shocks.

A.5 Estimates of firing costs

I update estimates on firing costs from Bentolila and Bertola (1990) using data from the International Labor Organization to discipline the degree of commitment of firms Φ. In appendix C, they define firing costs as

$$\Phi = N + (1 - p_a)SP + p_a\left[(1 - p_u)(SP + LC) + p_u(UP + LC)\right]$$

where N represents pay during the notice period, SP is the severance payment, LC are legal costs and UP are dismissal costs if the layoff is deemed unjustified in court. p_a is the probability that the layoff is brought to court, and p_u the probability that courts rule in favor of workers. This firing costs is evidently difficult to estimate since we do not have precise information about all of these elements, especially the probabilities p_a and p_u or legal fees.

Table A.6 reports information about layoff costs from the International Labor Organization, as well as the estimates that I use in this paper. I report estimates for a worker with an average of 8 years of tenure, which is the average tenure in my sample $(1/(\text{J2J} + \text{EU}))$.

I find that firing costs account for 4.2 months in France and 0.3 months in the U.S. on average. The average wage in the model is 1.08 per quarter so my estimates for firing costs is 1.5 and 0.11. These likely represent lower bounds since severance payments are sometimes increased at the industry level, and the probability I use most likely understate the chances that workers win in court nowadays. For this reason I use estimates of $\Phi = 2$ and $\Phi = 0.33$ for France and the U.S. Bentolila and Bertola (1990) report much higher estimates for France, of 8.2 months for the 1960s and 11 months for the 1970-80s because they use much higher severance payments for these periods.

The stringency of layoff restrictions in France relative to the U.S. is consistent with indicators

	France	United States
Notice period N (in months of pay)	2	0
Severance pay SP (in months of pay)	2	0
Redress costs UP	10	10 (max is \$50K to \$300K)
Probability of going to court p_a (from Bentolila and Bertola (1990))	0.05	0.05
Probability worker wins p_u (from Bentolila and Bertola (1990))	0.25	0.25
Legal costs LC (from Bentolila and Bertola (1990))	2	2
Estimates of firing costs	4.2	0.3

Table A.6: Estimates of firing costs in France and the U.S.

published by the Organization of Economic Co-operation and Development. The measure of firing costs published by the OECD is more comprehensive than the one I use, but it is more difficult to translate it into model parameters because it is an index.

B Model appendix

B.1 Quantitative model

In this appendix, I write the laws of motions of distributions. Define the retention probability as

$$p(V_t, s_t) = \left(1 - \mathbb{E}_{\xi_t}\left[\kappa\lambda_w(v(V_t, s_t, \xi_t), z_t)\right]\right)(1-\delta)(1 - q(V_t, s_t))$$

The distribution of employed workers satisfies

$$\begin{aligned}\psi_t(V_t, \overline{x}, x_t) = &\int_{V_{t-1}}\int_{x_{t-1}} \psi_{t-1}(V_{t-1}, \overline{x}, x_{t-1}) p(V_{t-1}, s_{t-1}) \pi_x(x_t|x_{t-1}) \mathbb{1}\left\{V_t = V(V_{t-1}, s_{t-1}, s_t)\right\} dx_{t-1} dV_{t-1} \\ &+ \psi^u_{t-1}\lambda(v_u(z_{t-1}), z_{t-1})\pi_{x_1}(x_t)\pi_{\overline{x}}(\overline{x})\mathbb{1}\left\{V_t = V_1(v_u(z_{t-1}), z_{t-1}, s_t)\right\} \\ &+ \int_{V_{t-1}}\int_{x_{t-1}}\int_{\overline{x}}\int_{\xi_{t-1}} \psi_{t-1}(V_{t-1}, \overline{x}, x_{t-1})\kappa\lambda(v(V_{t-1}, s_{t-1}, \xi_{t-1}), z_{t-1}) \times \\ &\qquad \times \pi_\xi(\xi_{t-1})\pi_{\overline{x}}(\overline{x})\pi_{x_1}(x_t)\mathbb{1}\left\{V_t = V_1(v(V_{t-1}, s_{t-1}, \xi_{t-1}), z_{t-1}, s_t)\right\} d\xi_{t-1} d\overline{x} dx_{t-1} dV_{t-1}\end{aligned}$$

and the distribution of unemployed workers satisfies

$$\begin{aligned}\psi^u_t = \psi^u_{t-1}\left(1 - \lambda(v_u(z_{t-1}), z_{t-1})\right) + \int_{V_{t-1}}\int_{x_{t-1}}\int_{\overline{x}} \left(1 - \mathbb{E}_{\xi_{t-1}}\left[\kappa\lambda_w(v(V_{t-1}, s_{t-1}, \xi_{t-1}), z_{t-1})\right]\right) \times \\ \times \left(\delta + (1-\delta)q(V_{t-1}, s_{t-1})\right)\psi_{t-1}(V_{t-1}, \overline{x}, x_t) d\overline{x} dx_{t-1} dV_{t-1}\end{aligned}$$

where $\pi_x(x_t|x_{t-1})$ is the probability of x_1 given x_{t-1}, $\pi_{x_1}(x_t)$ is the probability that firm productivity is x_t during the first period of production and $\pi_{\overline{x}}(\overline{x})$ is the probability that fixed productivity is $\overline{x}$. The first term contributing to $\psi_t(V_t, \overline{x}, x_t)$ represents stayers, the second represents new hires from unemployment and the third new hires from employment. The first term contributing to ψ^u_t represents unemployed workers who did not find a job, and the second term represents quits and exogenous separations.

B.2 Continuous time model

B.2.1 Environment

I describe the environment of the model studied in section 4.

Workers have utility $u(w)$ and have no accesss to financial markets. Their discount rate is r.

Firms are owned by risk-neutral investors with discount rate r. Output $\exp(x+z)$ is produced within matches with firm productivity x and sectoral productivity z following

$$dx_t = -\alpha_x x_t dt + \sigma_x dB_{xt} \quad \text{and} \quad dz_t = -\alpha_z z_t dt + \sigma_z dB_{zt}$$

where B_{xt} and B_{zt} are Brownian motions.

Search is directed so wokers apply for jobs in labor markets indexed by the present value that a worker would get v. The job finding probability follows a Poisson process with intensity $\kappa\lambda_w(v, z)$,

$$P(\tau > t) = \exp\left(-\kappa\int_0^t \lambda_w(v_s, z_s) ds\right)$$

where τ denotes the stopping time describing when the worker finds another job.

Contracts Contracts specify a wage for each history of firm and sectoral productivity shocks

$$w_t(\{x_s, z_s; 0 \le s \le t\})$$

After signing the contract, the worker chooses a job search strategy

$$v_t(\{x_s, z_s; 0 \leq s \leq t\})$$

to maximize expected utility.

Worker value function Given contract w and search strategy v, the value of a worker is

$$V^v(w) = \mathbb{E}\left[\int_0^\tau e^{-rt} u(w_t)dt + e^{-r\tau} v_\tau\right]$$

where the expectation is taken over the paths of (B_{xt}, B_{zt}, τ^v). We can rewrite this as

$$\begin{aligned} V_0^v(w) &= \mathbb{E}\left[\int_0^\infty e^{-rt}\left(\mathbb{1}(\tau^v > t)u(w_t) + \mathbb{1}(\tau^v = t)v_t\right)dt\right] \\ &= \mathbb{E}\left[\int_0^\infty e^{-rt}\left(P(\tau^v > t)u(w_t) + P(\tau^v = t)v_t\right)dt\right] \\ &= \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa\int_0^t \lambda_w(v_s, z_s)ds\right)\left(u(w_t) + \kappa\lambda_w(v_t, z_t)v_t\right)dt\right] \end{aligned}$$

where the second line uses the fact that B_{xt}, B_{zt}, τ^v are mutually independent and the last line uses the definition of τ. The expectation in the last line is taken over the paths of (B_{xt}, B_{zt}). In general, the value of a worker at time t is

$$V_t^v(w) = \mathbb{E}_t\left[\int_t^\infty \exp\left(-r(h-t) - \kappa\int_t^h \lambda_w(v_s, z_s)ds\right)\left(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h\right)dh\right]$$

Therefore, the value of a contract for a worker is defined as

$$V = \max_v \; V_0^v(w)$$

Optimal contract Given contract w and search strategy v, the value of a firm is

$$\begin{aligned} \Pi_0^v(w) &= \mathbb{E}\left[\int_0^\tau e^{-rt}\left(\exp(x_t + z_t) - w_t\right)dt\right] \\ &= \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa\int_0^t \lambda_w(v_s, z_s)ds\right)\left(\exp(x_t + z_t) - w_t\right)dt\right] \end{aligned}$$

Given the job finding rate $\lambda(v, z)$ and the processes for productivity x and z, the optimal contract solves

$$\begin{aligned} \Pi_0(V_0, x_0, z_0) = \max_w \quad & \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa\int_0^t \lambda_w(v_s, z_s)ds\right)\left(\exp(x_t + z_t) - w_t\right)dt\right] \\ \text{s.t.} \quad & V_0 = \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa\int_0^t \lambda_w(v_s, z_s)ds\right)\left(u(w_t) + \kappa\lambda_w(v_t, z_t)v_t\right)dt\right] \\ & v \in \arg\max_v \; V^v(w) \end{aligned}$$

B.2.2 Recursive formulation of the contract

In this section I use methods introduced in Sannikov (2008) to write this contract recursively. For simplicity of notations I write $V_t^v(w) = V_t^v$. I first derive the law of motion of the worker promised value V_t^v for any contract and strategy.

Lemma 2. *Given a contract w and a search strategy v, the worker value V_t^v satisfies*

$$dV_t^v = (rV_t^v - u(w_t) - \kappa\lambda_w(v_t, z_t)(v_t - V_t^v))\, dt + \Delta_{xt}\sigma_x dB_{xt} + \Delta_{zt}\sigma_z dB_{zt}$$

for some stochastic processes Δ_{xt}, Δ_{zt}.

Proof. Define the process H_t^v as

$$H_t^v = \int_0^t R_h^v \left(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h\right) dh + R_t^v V_t^v \tag{A.2}$$

where $R_t^v \equiv \exp(-rt - \kappa \int_0^t \lambda_w(v_s, z_s)ds)$ is the effective discount rate. Notice that

$$\begin{aligned} \mathbb{E}\left[H_t^v\right] = &\ \mathbb{E}\left[\int_0^t R_h^v \left(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h\right) dh\right] + \mathbb{E}\left[\mathbb{E}_t\left[\int_t^\infty R_h^v \left(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h\right) dh\right]\right] \\ = &\ \mathbb{E}\left[\int_0^\infty R_h^v \left(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h\right) dh\right] = H_0^v \end{aligned}$$

so H_t^v is a Martingale with respect to the filtration generated by x and z. By the Martingale representation theorem, there exist processes Δ_{xt}, Δ_{zt} such that

$$dH_t^v = \Delta_{xt} R_t^v \sigma_x dB_{xt} + \Delta_{zt} R_t^v \sigma_z dB_{zt}$$

Now using Ito's lemma on equation A.2 we find

$$dH_t^v = R_t^v \left(u(w_t) + \kappa\lambda_w(v_t, z_t)v_t\right) dt - R_t^v \left(r + \kappa\lambda(v_t, z_t)\right) V_t^v dt + R_t^v dV_t^v$$

Combining the two expressions for dH_t^v gives

$$dV_t^v = (rV_t^v - u(w_t) - \kappa\lambda_w(v_t, z_t)(v_t - V_t^v))\, dt + \Delta_{xt}\sigma_x dB_{xt} + \Delta_{zt}\sigma_z dB_{zt}$$

This concludes the proof. □

The next lemma characterizes incentive compatible strategies v in terms of the worker continuation value V_t^v.

Lemma 3. *A strategy v is incentive compatible if*

$$v_t \in \arg\max_v\ \lambda_w(v, z_t)\left(v - V_t^v\right)$$

Proof. Let v be an incentive compatible search strategy. We show that deviations are not profitable at any t. Assume that the worker deviates to an alternative strategy $\hat{v}$ until time t. Define $H_t^{\hat{v}}$ the process corresponding to this deviation,

$$H_t^{\hat{v}} = \int_0^t R_h^{\hat{v}} \left(u(w_h) + \kappa\lambda_w(\hat{v}_h, z_h)\hat{v}_h\right) dh + R_t^{\hat{v}} V_t^v \tag{A.3}$$

where the continuation value at time t is V_t^v because the worker follows the recommended strategy thereafter.

Note that $H_0^{\hat{v}} = H_0^v$. We want the process H_t^v to be a martingale under v and a super-martingale under any alternative strategy $\hat{v}$ so that $\mathbb{E}[H_t^v] = H_0^v = H_0^{\hat{v}} \geq \mathbb{E}[H_t^{\hat{v}}]$. This ensures that the worker will never choose to deviate from search strategy v since this would lower her expected utility $\mathbb{E}[H_t^{\hat{v}}]$. Using the law of motion for V from lemma 2 and equation A.3 we get

$$\begin{aligned} dH_t^{\hat{v}} &= R_t^{\hat{v}}\left(u(w_t) + \kappa\lambda_w(\hat{v}_t, z_t)\hat{v}_t\right)dt - R_t^{\hat{v}}\left(r + \kappa\lambda_w(\hat{v}_t, z_t)\right)V_t^v dt + R_t^{\hat{v}} dV_t^v \\ &= R_t^{\hat{v}}\left(\kappa\lambda_w(\hat{v}_t, z_t)(\hat{v}_t - V_t^v) - \kappa\lambda_w(v_t, z_t)(v_t - V_t^v)\right)dt + R_t^{\hat{v}}\Delta_{xt}\sigma_x dB_{xt} + R_t^{\hat{v}}\Delta_{zt}\sigma_z dB_{zt} \end{aligned}$$

$H_t^{\hat{v}}$ is a super-martingale if and only its drift is negative, i.e.

$$\lambda_w(v_t, z_t)\,(v_t - V_t) \geq \lambda_w(\hat{v}_t, z_t)\,(\hat{v}_t - V_t) \quad \text{for all } \hat{v}_t$$

This can be written as

$$v_t \in \arg\max_v \; \lambda_w(v, z_t)\,(v - V_t^v)$$

This concludes the proof. □

Using lemmas 2 and 3 we can rewrite the optimal contracting problem as

$$\Pi(V_t, x_t, z_t) = \max_{w, \Delta_x, \Delta_z} \quad \mathbb{E}\left[\int_0^\infty e^{-rt - \int_0^t \kappa\lambda_w(v_s, z_s)ds}(\exp(x_t + z_t) - w_t)dt\right]$$

subject to

$$\begin{aligned} &\text{(PK)}: \quad dV_t = \left(rV_t - u(w_t) - \kappa\lambda_w(v_t, z_t)(v_t - V_t)\right)dt + \Delta_{xt}\sigma_x dB_{xt} + \Delta_{zt}\sigma_z dB_{zt} \\ &\text{(IC-v)}: \quad v_t \in \arg\max_v \; \lambda_w(v, z_t)\,(v - V_t) \end{aligned}$$

B.2.3 Proofs for section 4

It will be useful to write the HJB corresponding to the optimal contract

$$\begin{aligned} (r + \kappa\lambda_w(v(V,z), z))\Pi(V,x,z) \quad &= \max_{w, \Delta_x, \Delta_z} \exp(x+z) - w \\ &+ \left(rV - u(w) - \kappa\lambda_w(v(V,z), z)(v(V,z) - V)\right)\Pi_V(V,x,z) \\ &- \alpha_x x \Pi_x(V,x,z) - \alpha_z z \Pi_z(V,x,z) \\ &+ \sigma_x^2\left[\tfrac{1}{2}\Delta_x^2 \Pi_{VV}(V,x,z) + \tfrac{1}{2}\Pi_{xx}(V,x,z) + \Delta_x \Pi_{Vx}(V,x,z)\right] \\ &+ \sigma_z^2\left[\tfrac{1}{2}\Delta_z^2 \Pi_{VV}(V,x,z) + \tfrac{1}{2}\Pi_{zz}(V,x,z) + \Delta_z \Pi_{Vz}(V,x,z)\right] \end{aligned}$$

Proposition 1 I first derive the optimal path of wages since it does not require my approximation in the degree of search efficiency $\kappa \to 0$.

Proof. The optimality conditions of the HJB with respect to w is

$$w(V,x,z) = (u')^{-1}\left(-\frac{1}{\Pi_V(V,x,z)}\right) \tag{A.4}$$

and with respect to , Δ_x and , Δ_z are

$$\Delta_x(V,x,z) = -\frac{\Pi_{Vx}(V,x,z)}{\Pi_{VV}(V,x,z)}, \quad \Delta_z(V,x,z) = -\frac{\Pi_{Vz}(V,x,z)}{\Pi_{VV}(V,x,z)} \tag{A.5}$$

Applying Ito's lemma on the optimality condition for w gives

$$dw_t = -\frac{w_t u'(w_t)}{\gamma(w_t)} d\Pi_V(V_t, X_t) \tag{A.6}$$

and applying Ito's lemma on $F_V(V, x, z)$ gives

$$\begin{aligned} d\Pi_V = & \left(\mu_V \Pi_{VV} + \mu_x \Pi_{Vx} + \mu_z \Pi_{Vz}\right) dt \\ & + \left(\tfrac{1}{2}\left(\Delta_x^2 \sigma_x^2 + \Delta_z^2 \sigma_z^2\right) \Pi_{VVV} + \sigma_x^2 \left(\tfrac{1}{2}\Pi_{Vxx} + \Delta_x \Pi_{VVx}\right) + \sigma_z^2 \left(\tfrac{1}{2}\Pi_{Vzz} + \Delta_z \Pi_{VVz}\right)\right) dt \end{aligned} \tag{A.7}$$

where we used optimality condition for $\Delta_x(V, X)$ to get ride of the diffusion terms. The terms μ_V, μ_x and μ_z denote the drift of V, x and z.

Differentiate the HJB equation with respect to V gives

$$\begin{aligned} \kappa \frac{\partial \lambda_w(v(V,z),z)}{\partial V} \Pi & = \mu_V \Pi_{VV} + \mu_x \Pi_{Vx} + \mu_z \Pi_{Vz} \\ & + \tfrac{1}{2}\left(\Delta_x^2 \sigma_x^2 + \Delta_z^2 \sigma_z^2\right) \Pi_{VVV} + \sigma_x^2 \left(\tfrac{1}{2}\Pi_{Vxx} + \Delta_x \Pi_{VVx}\right) + \sigma_z^2 \left(\tfrac{1}{2}\Pi_{Vzz} + \Delta_z \Pi_{VVz}\right) \end{aligned}$$

where we used the envelope theorem to get $\partial_V \lambda_w(v(V,z),z)(v(V,z) - V) = -\lambda_w(v(V,z),z)$. Combining this expression with A.6 and A.7 gives

$$dw_t = -\frac{w_t u'(w_t)}{\gamma(w_t)} \Pi(V_t, x_t, z_t) \, \kappa \frac{\partial \lambda_w(v(V,z),z)}{\partial V} dt + 0 \times dB_{xt} + 0 \times dB_{zt}$$

Rewriting this expression using the retention elasticity (7) gives the desired result. □

Lemma 1 Before solving the for firm value when $\kappa \to 0$, I solve it when $\kappa = 0$.

Lemma 4. *If $\kappa = 0$, wages are constant and the firm value is*

$$\Pi(V, x, z) = g(x, z) - h(V)$$

where $g(x,z) = r^{-1}\left(\exp(x+z) + \mathcal{D}g(x,z)\right)$ is the present value of output and $h(V) = u^{-1}(rV)/r$ is the cost of providing a value V to workers. The policy functions are

$$w(V) = u^{-1}(rV), \quad \Delta_x = 0, \quad \Delta_z = 0$$

Proof. We prove this result by guess and verify. Conjecture that the firm value takes the form

$$F(V, x, z) = g(x, z) - h(V)$$

for two functions $g(x,z)$ and $h(V)$ to be determined. This implies that $\Delta_x = \Delta_z = 0$ and that $w(V, x, z) = w(V)$ from equations (A.4) and (A.5). Plugging this conjecture in the HJB together with $\kappa = 0$ gives two conditions that $g(x,z)$ and $h(V)$ must satisfy

$$\begin{aligned} rg(x,z) &= \exp(x+z) + \mathcal{D}g(x,z) \\ rh(V) &= w(V) + (rV - u(w(V)))\, h'(V) \end{aligned}$$

Since these equations are independent, this confirms our guess. It is straightforward to verify that $h(V) = u^{-1}(rV)/r$ and $w(V) = u^{-1}(rV)$ satisfy the second ODE. Finally, plug in the value of $w(V)$ in dV_t together with $\Delta_x = \Delta_z = 0$ to show that the worker value and therefore the wage are constant over time. □

I now prove lemma 1.

Proof. Consider a first-order Taylor expansion of $\Pi(V,x,z)$ around $\kappa = 0$

$$\Pi(V,x,z) = g(x,z) - h(V) + \kappa\partial_\kappa \Pi(V,x,z;\kappa=0) + O(\kappa^2) \tag{A.8}$$

using lemma 4. Introduce the function

$$\ell(V,x,z) \equiv -\kappa^{-1}\left[\Pi(V,x,z) - (g(x,z) - h(V))\right] \tag{A.9}$$

and therefore $\partial_\kappa \Pi(V,x,z;\kappa=0) = \lim_{\kappa\to 0}\ell(V,x,z)$. The function $\ell(V,x,z)$ can be interpreted as the cost of retaining workers to the firm, scaled by the degree of worker mobility κ.

Plugging (A.9) in the HJB and using the optimality conditions (A.4) and (A.5) gives

$$\begin{aligned}
0 &= \exp(x+z) - w(V,x,z) \\
&+ (rV - u(w(V,x,z)) - \kappa\lambda_w(v(V,z),z)(v(V,z)-V))\,(-h'(V) - \kappa\ell_V(V,x,z)) \\
&-\alpha_x x g_x(x,z) - \alpha_z z g_z(x,z) \\
&+\frac{\sigma_x^2}{2}\frac{(g_{xx}(x,z)-\kappa\ell_{xx}(V,x,z))(h''(V)-\kappa\ell_{VV}(V,x,z))+\kappa^2\ell_{Vx}^2(V,x,z)}{h''(V)+\kappa\ell_{VV}(V,x,z)} \\
&+\frac{\sigma_z^2}{2}\frac{(g_{zz}(x,z)-\kappa\ell_{zz}(V,x,z))(h''(V)-\kappa\ell_{VV}(V,x,z))+\kappa^2\ell_{Vz}^2(V,x,z)}{h''(V)-\kappa\ell_{VV}(V,x,z)} \\
&-(r+\kappa\lambda_w(v(V,z),z))\,(g(x,z)-h(V)-\kappa\ell(V,x,z))
\end{aligned}$$

We now subtract $rg(X)$ and $rh(V)$ on both sides and use their definition from lemma 4 to get

$$\begin{aligned}
-(r+\kappa\lambda_w(v(V,z),z))\kappa\ell(V,x,z) &= \kappa\lambda_w(v(V,z),z)h(V) - \kappa\lambda_w(v(V,z),z)g(x,z) - [w(V,x,z)-w(V)] \\
&- (rV - u(w(V,x,z)) - \kappa\lambda_w(v(V,z),z)(v(V,z)-V))\,\kappa\ell_V(V,x,z) \\
&+\kappa\lambda_w(v(V,z),z)[v(V,z)-V]h'(V) + [u(w(V,x,z)) - u(w(V))]\,h'(V) \\
&-\kappa\alpha_x x\ell_x(V,x,z) - \kappa\alpha_z z\ell_z(V,x,z) \\
&-\frac{\sigma_x^2}{2}\frac{\kappa\ell_{xx}(V,x,z)(h''(V)-\kappa\ell_{VV}(V,x,z))-\kappa^2\ell_{Vx}^2(V,x,z)}{h''(V)-\kappa\ell_{VV}(V,x,z)} \\
&-\frac{\sigma_z^2}{2}\frac{\kappa\ell_z(V,x,z)(h''(V)-\kappa\ell_{VV}(V,x,z))-\kappa^2\ell_{Vz}^2(V,x,z)}{h''(V)-\kappa\ell_{VV}(V,x,z)}
\end{aligned} \tag{A.10}$$

where $w(V)$ is the wage policy when $\kappa = 0$.

Consider the term

$$\lim_{\kappa\to 0}\frac{w(V,x,z)-w(V)}{\kappa} = \lim_{\kappa\to 0}\frac{1}{\kappa}\left((u')^{-1}\left(\frac{1}{h'(V)-\kappa\ell_V(V,x,z)}\right) - (u')^{-1}\left(\frac{1}{h'(V)}\right)\right)$$

Take a Taylor expansion of the first term around $\kappa = 0$

$$(u')^{-1}\left(\frac{1}{h'(V)-\kappa\ell_V(V,x,z)}\right) = (u')^{-1}\left(\frac{1}{h'(V)}\right) + \kappa\frac{\ell_V(V,x,z)}{h'(V)}\frac{u'(w(V))}{u''(w(V))} + O(\kappa^2)$$

where we used $h'(V) = 1/u'(w(V))$. Therefore,

$$\lim_{\kappa\to 0}\frac{w(V,x,z)-w(V)}{\kappa} = \frac{\ell_V(V,x,z)}{h'(V)}\frac{u'(w(V))}{u''(w(V))}$$

Similarly, we get

$$\lim_{\kappa\to 0}\frac{u(w(V,x,z))-u(w(V))}{\kappa} = -\frac{\ell_V(V,x,z)}{h'(V)^2}\frac{u'(w(V))}{u''(w(V))}$$

Now divide equation A.10 by κ and take limit as $\kappa \to 0$

$$\begin{aligned} r\ell(V,x,z) \quad &= -\lambda_w(v(V,z),z)h(V) + \lambda_w(v(V,z),z)g(x,z) \\ &-\lambda_w(v(V,z),z)[v(V,z)-V]h'(V) \\ &-\alpha_x x\ell_x(V,x,z) - \alpha_z z\ell_z(V,x,z) \\ &+\tfrac{\sigma_x^2}{2}\ell_{xx}(V,x,z) + \tfrac{\sigma_z^2}{2}\ell_{zz}(V,x,z) \end{aligned}$$

where we used $rV - u(w(V)) = 0$ from lemma 4. We can reformulate this equation using the differential operator introduced at the start of section 4 as

$$r\ell(V,x,z) = \lambda_w(v(V,z),z)\left[g(x,z) - h(V) - [v(V,z)-V]h'(V)\right] + \mathcal{D}\ell(V,x,z)$$

This concludes the proof. □

Proposition 2

Proof. To first order in κ, the optimality condition with respect to Δ_x (A.5) in the HJB becomes

$$\Delta_x(V,x,z) = -\kappa\frac{\ell_{Vx}(V,x,z)}{h''(V)}$$

Now consider $\ell_{Vx}(V,x,z)$ from lemma 1. We have

$$(r+\alpha_x)\,\ell_x(V,x,z) = \lambda_w(v(V,z),z)g_x(x,z) + \mathcal{D}\ell_x(V,x,z)$$

Now differentiate with respect to V to find

$$(r+\alpha_x)\,\ell_{Vx}(V,x,z) = \frac{\partial\lambda_w(v(V,z),z)}{\partial V}g_x(x,z) + \mathcal{D}\ell_{Vx}(V,x,z)$$

Multiply by $-\kappa/h''(V)$ to get

$$(r+\alpha_x)\,\Delta_x(V,x,z) = -\kappa\frac{\partial\lambda_w(v(V,z),z)}{\partial V}\frac{g_x(x,z)}{h''(V)} + \mathcal{D}\Delta_x(V,x,z)$$

Now use $h''(V) = r\gamma(w(V))/\left(w(V)u'(w(V))^2\right)$ and the definition of $\epsilon(V,x,z)$ from equation (7) when $\kappa \to 0$ to get

$$(r+\alpha_x)\,\Delta_x(V,x,z) = g_x(x,z)\frac{\epsilon(V,x,z)}{\gamma(w(V))}u'(w(V)) + \mathcal{D}\Delta_x(V,x,z)$$

This concludes the proof. □

Proposition 3

Proof. To first order in κ, the optimality condition with respect to Δ_z (A.5) in the HJB becomes

$$\Delta_z(V,x,z) = -\kappa\frac{\ell_{Vz}(V,x,z)}{h''(V)}$$

Now consider $\ell_{Vz}(V,x,z)$ from lemma 1. We have

$$(r+\alpha_z)\,\ell_z(V,x,z) = \lambda_w(v(V,z),z)g_z(x,z) + \tfrac{\partial\lambda_w(v(V,z),z)}{\partial z}\,(g(x,z)-h(V)) \\ -\lambda_{wz}(v(V,z),z)\,(v(V,z)-V)\,h'(V) + \mathcal{D}\ell_z(V,x,z)$$

where I used the envelope condition of the worker search problem (6) for the third term. $\frac{\partial\lambda_w(v(V,z),z)}{\partial z}$ denotes the total derivative with respect to z whereas $\lambda_{wz}(v(V,z),z)$ denotes the partial derivative with respect to the second argument.

Differentiate with respect to V to find

$$(r+\alpha_z)\,\ell_{Vz}(V,x,z) = \tfrac{\partial\lambda_w(v(V,z),z)}{\partial V}g_z(x,z) + \tfrac{\partial^2\lambda_w(v(V,z),z)}{\partial V\partial z}\,(g(x,z)-h(V)) \\ -\lambda_{wz}(v(V,z),z)\,(v(V,z)-V)\,h''(V) + \mathcal{D}\ell_{Vz}(V,x,z)$$

Multiply by $-\kappa/h''(V)$ to get

$$(r+\alpha_z)\,\Delta_z(V,x,z) = -\kappa\tfrac{\partial\lambda_w(v(V,z),z)}{\partial V}\tfrac{g_z(x,z)}{h''(V)} - \kappa\tfrac{\partial^2\lambda_w(v(V,z),z)}{\partial V\partial z}\tfrac{(g(x,z)-h(V))}{h''(V)} \\ +\kappa\lambda_{wz}(v(V,z),z)\,(v(V,z)-V) + \mathcal{D}\Delta_z(V,x,z)$$

Now use $h''(V) = r\gamma(w(V))/\left(w(V)u'(w(V))^2\right)$ and the definition of $\epsilon(V,x,z)$ from equation (7) when $\kappa \to 0$ to get

$$(r+\alpha_z)\,\Delta_z(V,x,z) = \left[g_z(x,z)\tfrac{\epsilon(V,x,z)}{\gamma(w(V))} + (g(x,z)-h(V))\,\tfrac{\epsilon_z(V,x,z)}{\gamma(w(V))}\right]u'(w(V)) \\ +\kappa\lambda_{wz}(v(V,z),z)\,(v(V,z)-V) + \mathcal{D}\Delta_z(V,x,z)$$

where $\epsilon_z(V,x,z) \equiv \partial\epsilon(V,x,z)/\partial z$. Finally, use $(g(x,z)-h(V))\,\epsilon_z(V,x,z) = \Pi(V,x,z)\epsilon_z(V,x,z)$ to first order in κ to rewrite this expression as in the proposition. □

The pass-through of firm-level shocks to wages in section 4.3 Start from the path of wages in proposition 1 and take a first-order approximation in κ. This equation becomes

$$dw_t = (rg(x_t,z_t) - w_t)\;\frac{\epsilon(V_t,w_t,z_t)}{\gamma(w_t)}dt$$

where I wrote the retention elasticity as a function of the wage directly, and to first order in κ,

$$\epsilon(V,w,z) \equiv -\kappa\frac{\partial\lambda_w(v(V,z),z)}{\partial v(V,z)} \times \frac{\partial v(V,z)}{\partial V} \times \frac{wu'(w)}{r}$$

To first order in x, we can rewrite the wage growth equation as

$$\hat{w}_t \approx \int_0^t \left(rg_x(x_0,z_0)\exp\left(-\alpha_z s\right)\hat{x}_0 - \hat{w}_s\right)\;\frac{\epsilon(V_0,w_0,z_0)}{\gamma(w_0)}ds$$

where we used the definition of $\hat{x}_s$ and made the additional approximation that the ratio $\epsilon(V_0,w_0,z_0)/\gamma(w_0)$ was constant over time. We can solve this ODE in $\hat{w}_t$ and find

$$\hat{w}_t \approx \frac{r}{\epsilon_0/\gamma_0 - \alpha_x}\frac{\epsilon_0}{\gamma_0}g_x(x_0,z_0)\left[\exp\left(-\alpha_x t\right) - \exp\left(-\frac{\epsilon_0}{\gamma_0}t\right)\right]\hat{x}_0$$

where $\epsilon_0 \equiv \epsilon(V_0, w_0, z_0)$ and $\gamma_0 \equiv \gamma(w_0)$.

The pass-through of sectoral shocks to wages in section 4.4 Again start from the path of wages with $\kappa \to 0$,

$$dw_t = (rg(x_t, z_t) - w_t) \frac{\epsilon(V_t, w_t, z_t)}{\gamma(w_t)} dt$$

To first order in z, we can rewrite the wage growth equation as

$$\begin{aligned} \hat{w}_t \approx & \int_0^t \left(rg_z(x_0, z_0) \exp\left(-\alpha_z s\right) \hat{z}_0 - \hat{w}_s\right) \frac{\epsilon(V_0, w_0, z_0)}{\gamma(w_0)} ds \\ & + \int_0^t \left(rg(x_0, z_0) - w_0\right) \frac{\epsilon_z(V_0, w_0, z_0)}{\gamma(w_0)} \exp\left(-\alpha_z s\right) \hat{z}_0 ds \end{aligned}$$

where $\epsilon_z(V_0, w_0, z_0) \equiv \partial\epsilon(V_0, w_0, z_0)/\partial z$. This equation approximates the true wage response because we keep the ratio $\epsilon(V_0, w_0, z_0)/\gamma(w_0)$ and the firm value $rg(x_0, z_0) - w_0$ constant over time. The key difference with the pass-through from-firm level shocks derived in appendix B.2.3 is that sectoral productivity z enters directly as an input in the retention elasticity $\epsilon(V_0, w_0, z_0)$.

We can solve this ODE in $\hat{w}_t$ and find

$$\hat{w}_t \approx \frac{r}{\epsilon_0/\gamma_0 - \alpha_z} \left(g_z(x_0, z_0) \frac{\epsilon_0}{\gamma_0} + \Pi(V_0, x_0, z_0) \frac{\epsilon_{z0}}{\gamma_0} \right) \left[\exp\left(-\alpha_z t\right) - \exp\left(-\frac{\epsilon_0}{\gamma_0} t \right) \right] \hat{z}_0$$

where $\epsilon_{z0} \equiv \partial\epsilon(V_0, w_0, z_0)/\partial z$.

Special case: no aggregate shocks and mean-reverting productivity To illustrate the formulas derived in section 4, I focus on a special case in which these formulas can be derived in closed form.

Assume that sectoral productivity is constant ($\sigma_z = \rho_z = 0$), and assume that the production function is simply xz. This means that productivity follows a normal distribution, instead of a log-normal distribution as in the quantitative model. In this case, we can solve for $g(x, z)$ by guess and verify as

$$g(x, z) = \frac{zx}{r + \alpha_x}$$

so that $g_x(x, z) = z/(r + \alpha_x)$. To solve for $\Delta_x(V, x, z)$ notice that the elasticity $\epsilon(V, x, z)$ is independent of x to first order in κ. Then, by guess and verify we get

$$\Delta_x(Vx, z) = (r + \alpha_x)^{-1} g_x(x, z) \frac{\epsilon(V, x, z)}{\gamma(w(V))} u'(w(V))$$

B.3 Relation to Chetty-Baily statistic

Equation 1 in Chetty (2006) is

$$\gamma \frac{\Delta c}{c}(b^*) \approx \epsilon_{D,b}$$

where $\Delta c/c(b^*)$ is the optimal change in consumption following an unemployment shock, $\epsilon_{D,b}$ is the elasticity of the duration of unemployment D with respect to unemployment benefits b and γ is the coefficient of relative risk aversion.

Note that the duration is defined as $D = 1/(1-p)$ where p is the probability that the worker stays unemployed (the “retention probability"). Therefore,

$$\epsilon_{D,b} = \frac{dD}{db}\frac{b}{D} = \frac{dp}{db}(1-p)^{-2}b(1-p) = \frac{dp}{db}\frac{b}{1-p} = \frac{\epsilon_{p,b}}{1-p}$$

where $\epsilon_{p,b}$ is the semi-elasticity of the retention probability (the probability that the worker remains unemployment) with respect to the unemployment benefit. Thus, the Chetty-Baily formula can be written as

$$\frac{\Delta c}{c} \approx \frac{\epsilon_{p,b}}{\gamma}(1-p)^{-1}$$