Online Appendix for "Trade Disputes and Settlement"

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1. Appendix

Proof of Remark 1: The claim follows from two observations: (1) Reducing c shrinks Region II, which is defined by $\gamma \in ((1-c)\gamma^*, (1+c)\gamma^*)$, thus a given γ can switch from Region II to Region I or III, but not vice-versa. And recall from Propositions 2 and 1 that only in Region II there can be a ruling; (2) If the initial value of γ is in Region II before and after the change, can we go from an early settlement to a ruling? The answer is no. Recall that, fixing the contract, the support of b^C is fixed. With reference to Figure 3, if initially there is no ruling, the possible disagreement points for the stage-4 bargain all lie on the P sub-frontier. Focus on the disagreement point that corresponds to the highest b^C: as c goes down, this point moves up vertically, while the central kink of the frontier moves up diagonally, so all possible disagreement points for the stage-4 bargaining will still lie on the P sub-frontier. QED

Proof of Lemma 1:

Consider an arbitrary contract $b^C(\gamma^{dsb})$. Recall that $\tilde{\omega}(b^C(\gamma^{dsb}), \gamma)$ and $\tilde{\omega}^*(b^C(\gamma^{dsb}), \gamma)$ denote the Home and Foreign payoffs in the stage-4 subgame, and the expected disagreement payoffs are $\int \tilde{\omega}(b^C(\gamma^{dsb}), \gamma) dG(\gamma^{dsb}|\gamma)$ and $\int \tilde{\omega}(b^C(\gamma^{dsb}), \gamma) dG(\gamma^{dsb}|\gamma)$, respectively. Recall also that, as we argued in the main text, in the stage-2 bargain governments obtain exactly their expected disagreement payoffs for all γ.

Now move back to stage 0, where $b^C(\gamma^{dsb})$ is chosen to maximize $E_\gamma(E[\tilde{\omega}(b^C(\gamma^{dsb}), \gamma)|\gamma] + E[\tilde{\omega}^*(b^C(\gamma^{dsb}), \gamma)|\gamma])$, which is given by:

$$\int \left[\int (\tilde{\omega}(b^C(\gamma^{dsb}), \gamma) + \tilde{\omega}^*(b^C(\gamma^{dsb}), \gamma)) h(\gamma^{dsb}|\gamma) d\gamma^{dsb} \right] h(\gamma) d\gamma$$
$$= \int \left[\int (\tilde{\omega}(b^C(\gamma^{dsb}), \gamma) + \tilde{\omega}^*(b^C(\gamma^{dsb}), \gamma)) h(\gamma|\gamma^{dsb}) d\gamma \right] z(\gamma^{dsb}) d\gamma^{dsb}$$

where $h(\gamma)$ is the marginal density of γ and $z(\gamma^{dsb})$ is the marginal density of γ^{dsb}. Clearly, maximizing the objective boils down to maximizing $\int [\tilde{\omega}(b^C(\gamma^{dsb}), \gamma) + \tilde{\omega}^*(b^C(\gamma^{dsb}), \gamma)] h(\gamma|\gamma^{dsb}) d\gamma$ for each given γ^{dsb}. QED

Proof of Proposition 5:

Given Lemma 1, the optimal contract is given by the schedule $b^C(\gamma^{dsb})$ that maximizes $\Omega^e(b^C, \gamma^{dsb}) \equiv E_{\gamma|\gamma^{dsb}}[\tilde{\Omega}(b^C, \gamma)|\gamma^{dsb}]$. We start by developing a figure that illustrates the outcome of the stage-4 negotiation given the DSB ruling b^C and the realization of γ.

Let us consider whether or not a given level of damages b^C will be renegotiated at stage 4 given the realization of γ. Recall that the threat point in the stage-4 negotiation is defined by the DSB ruling b^C, so if the negotiation fails, Home may choose between $(T = FT, b = 0)$ and $(T = P, b = b^C)$. Letting $S(b) \equiv b + c \cdot |b|$ denote the total cost of the transfer b inclusive of deadweight loss, it is clear that for $\gamma < S(b^C)$ Home would choose $(T = FT, b = 0)$, while for $\gamma > S(b^C)$ it would choose $(T = P, b = b^C)$. The line $\gamma = S(b^C)$, where Home is indifferent between the two options, is depicted in Figure A1.[1] Consider first the case $\gamma < S(b^C)$. Here the threat point is $(T = FT, b = 0)$, and governments renegotiate to the policy P if and only if there exists a transfer b^e such that both governments gain by switching to $(T = P, b = b^e)$, which requires $\gamma > S(b^e)$ (for the importer) and $b^e > \gamma^*$ (for the exporter). Clearly, this is the case if and only if $\gamma > S(\gamma^*)$. Thus governments renegotiate toward policy P when $S(\gamma^*) < \gamma < S(b^C)$; the corresponding region is identified in Figure A1 by the label $\mathrm{P_R}$. Note that $b^e < b^C$ in this region, because $S(b^e) < \gamma < S(b^C)$ and $S(\cdot)$ is increasing. Consider next the case $\gamma > S(b^C)$. Here the threat point is $(T = P, b = b^C)$, and governments renegotiate toward policy FT if and only if there exists a (negative) transfer b^e such that both governments gain by switching to $(T = FT, b = b^e)$, which requires $S(b^C) - S(b^e) > \gamma$ (for the importer) and $\gamma^* > b^C - b^e$ (for the exporter). Clearly, such a transfer exists if and only if $\gamma < S(b^C) - S(b^C - \gamma^*) \equiv R(b^C)$. Hence, governments renegotiate toward policy FT when $S(b^C) < \gamma < R(b^C)$. The corresponding region is identified in Figure A1 by the label $\mathrm{FT_R}$. Note that $R(b^C)$ is a line with slope $2c$ satisfying $R(0) = (1 - c) \cdot \gamma^*$ and $R(\gamma^*) = S(\gamma^*) = (1 + c)\gamma^*$.

We now make an important observation: it can never be strictly optimal to have $b^C > \gamma^*$, because as Figure A1 makes clear, setting $b^C > \gamma^*$ induces the same policy outcome as setting $b^C = \gamma^*$ (namely FT for $\gamma < S(\gamma^*)$ and P for $\gamma > S(\gamma^*)$), but $b^C = \gamma^*$ implies a weakly lower expected transfer. This second claim can be seen as follows. Start with any $b^C = \tilde{b}^C > \gamma^*$. If this is replaced with $b^C = \gamma^*$, the expected equilibrium transfer falls (weakly), because: (1) if $\gamma > S(\tilde{b}^C)$, the importer would have chosen $(T = P, b = \tilde{b}^C)$ without renegotiating and now chooses $(T = P, b = \gamma^*)$, so the transfer obviously decreases, and (2) if $\gamma \in (S(\gamma^*), S(\tilde{b}^C))$, the contract would have been renegotiated under $b = \tilde{b}^C$ but will not be renegotiated under $b = \gamma^*$, and as we established above, when $\gamma < S(b^C)$ the equilibrium transfer b^e is higher than γ^*.[2]

[1] Figure 5 focuses on non-negative values of b^C. It is easy to show and intuitively clear that $b^C < 0$ can never be optimal for any γ^{dsb}.

[2] Note that we only claim that $b^C > \gamma^*$ is "weakly" dominated by $b^C = \gamma^*$, because if the support of γ around γ^* is sufficiently small, the expected equilibrium transfer is the same in the two cases, as all states $\gamma > S(\gamma^*)$

The next step is to write an explicit expression for $\Omega^e(b^C, \gamma^{dsb})$. First note that, since it can never be strictly optimal to have $b^C > \gamma^*$, we can focus on the range $b^C \in [0, \gamma^*]$. Second, note by inspection of Figure A1 that the equilibrium policy outcome is $T = FT$ for $\gamma < R(b^C)$ and $T = P$ for $\gamma > R(b^C)$. Third, recall that the range of γ for which a given level of damages b^C is renegotiated at stage 4 is the one depicted by the $\mathrm{FT_R}$ region in Figure A1. And finally, we need to derive the transfer b^e paid by the exporter when renegotiation occurs, that is in the $\mathrm{FT_R}$ region. Given the Nash bargaining assumption, it is direct to verify that $b^e = \frac{2b^C - (1-c)\gamma^* - \gamma}{2(1-c)} < 0$. Armed with these observations, and letting $V^{FT} \equiv v(FT) + v^*(FT)$, we can write the optimization problem as:

$$\begin{aligned}\max_{b^C \in [0,\gamma^*]} \Omega^e(b^C, \gamma^{dsb}) \;\; = & \left\{ V^{FT} + \int_{R(b^C)}^{\infty} (\gamma - \gamma^*) dH(\gamma|\gamma^{dsb}) \right\} \\ & - c \left\{ b^C[1 - H(R(b^C)|\gamma^{dsb})] + \int_{S(b^C)}^{R(b^C)} |b^e(b^C, \gamma)| dH(\gamma|\gamma^{dsb}) \right\}.\end{aligned}$$

The expression for $\Omega^e(b^C, \gamma^{dsb})$ is given by the difference between two terms. The term in the first set of curly brackets is the joint payoff associated with the FT policy, plus the gain in expected joint payoff associated with allowing the policy P for γ above $R(b^C)$. It is simple to see that this term by itself is maximized when b^C satisfies $R(b^C) = \gamma^*$. However, weighing against this first term is the deadweight loss associated with the transfers. The expected transfer is given by the second set of curly brackets, and is composed of the transfer b^C that accompanies the policy P (when $\gamma > R(b^C)$) and the transfer b^e that accompanies the renegotiated policy FT (when $\gamma \in (S(b^C), R(b^C))$).

We now prove part (i) of the proposition. We start by focusing on the case in which the optimal schedule $b^C(\gamma^{dsb})$ is continuous; we will later generalize the proof to allow for discontinuities. We will prove that the optimal level of b^C is (weakly) decreasing in γ^{dsb}, from which the statements in part (i) then follow. We can focus on non-prohibitive levels of b^C, because if the optimal b^C is prohibitive for a given level of γ^{dsb} it can only go down as γ^{dsb} increases.

have zero density.

Recalling that we can focus on $b^C < \gamma^*$, we can write:

$$\begin{aligned}\Omega^e_{b^C}(b^C, \gamma^{dsb}) &= c(1+c)|b^e(b^C, S(b^C))| \cdot h(S(b^C)|\gamma^{dsb}) \\ &\quad + \frac{c}{1-c} \cdot [H(R(b^C)|\gamma^{dsb}) - H(S(b^C)|\gamma^{dsb})] - c \cdot [1 - H(R(b^C)|\gamma^{dsb})].\end{aligned}$$

We now argue that $\Omega^e_{b^C}(b^C, \gamma^{dsb})$ is decreasing in γ^{dsb} when evaluated at a value of b^C satisfying $\Omega^e_{b^C}(b^C, \gamma^{dsb}) \leq 0$. Note that this encompasses both the case in which the optimal b^C is interior and the case of a corner solution at $b^C = 0$. To see this, note that the first and second terms in $\Omega^e_{b^C}(b^C, \gamma^{dsb})$ are positive, while the third term is negative. But then the log-supermodularity of $h(\gamma|\gamma^{dsb})$ ensures that, as γ^{dsb} increases, $h(\gamma|\gamma^{dsb})$ increases proportionally more for higher values of γ. This implies that as γ^{dsb} increases, the negative term increases proportionally more than the sum of the two positive terms. Coupled with the fact that, when evaluated at a value of b^C satisfying $\Omega^e_{b^C}(b^C, \gamma^{dsb}) \leq 0$, the negative term is equal or greater in magnitude than the sum of the positive terms, this implies that as γ^{dsb} increases, the negative term increases in magnitude by more than the sum of the positive terms, and hence $\Omega^e_{b^C}(b^C, \gamma^{dsb})$ decreases. And given that we were starting from an optimal level of b^C satisfying $\Omega^e_{b^C}(b^C, \gamma^{dsb}) \leq 0$, it follows that the optimal schedule $b^C(\gamma^{dsb})$ is non-increasing, at least if this schedule is continuous.

We next consider the possibility of a discontinuous optimal schedule $b^C(\gamma^{dsb})$. Above we have shown that $\Omega^e_{b^C\gamma^{dsb}}(b^C, \gamma^{dsb}) < 0$ when evaluated at a local (non-prohibitive) maximum, i.e. when $\Omega^e_{b^C}(b^C, \gamma^{dsb}) \leq 0$. This ensures that any local optimum decreases (weakly) with γ^{dsb}, but it is possible that, as γ^{dsb} changes, the global optimum jumps from one local optimum to another local optimum, and we need to ensure that the monotonicity continues to hold in this case.

The argument is in two steps. First, we argue that a sufficient condition for the monotonicity result is the following: if $b^{C'}, b^{C''}$ and γ^{dsb} are such that $\Omega^e(b^{C'}, \gamma^{dsb}) = \Omega^e(b^{C''}, \gamma^{dsb})$ (or equivalently, if $\int_{b^{C'}}^{b^{C''}} \Omega^e_{b^C}(b^C, \gamma^{dsb}) db^C = 0$), then $\int_{b^{C'}}^{b^{C''}} \Omega^e_{b^C\gamma^{dsb}}(b^C, \gamma^{dsb}) db^C < 0$. We will then argue that this condition is satisfied if c is sufficiently small.

Suppose that, for a given value of γ^{dsb}, there are two local maxima $b^{C'}$ and $b^{C''}$ (with $b^{C'} < b^{C''}$) that yield the same value of the objective function. Clearly in this case $\int_{b^{C'}}^{b^{C''}} \Omega^e_{b^C}(b^C, \gamma^{dsb}) db^C = 0$. We now argue that, under the sufficient condition mentioned above, if γ^{dsb} increases from this initial value then the value of the objective at $b^{C'}$ increases (weakly) more than at $b^{C''}$, or equivalently, $\Omega^e_{\gamma^{dsb}}(b^{C'}, \gamma^{dsb}) \geq \Omega^e_{\gamma^{dsb}}(b^{C''}, \gamma^{dsb})$. Noting that $\Omega^e_{\gamma^{dsb}}(b^{C''}, \gamma^{dsb}) = \Omega^e_{\gamma^{dsb}}(b^{C'}, \gamma^{dsb}) + \int_{b^{C'}}^{b^{C''}} \Omega^e_{b^C\gamma^{dsb}}(b^C, \gamma^{dsb}) db^C$, it is clear that our desired property holds if the sufficient condition mentioned above is satisfied.

We now argue that the above sufficient condition is satisfied if c is sufficiently small. We can write:

$$\begin{aligned}\Omega^e_{b^C\gamma^{dsb}}(b^C,\gamma^{dsb}) &= c(1+c)|b^e(b^C,S(b^C))|\cdot\frac{\partial h(S(b^C)|\gamma^{dsb})}{\partial\gamma^{dsb}} \\ &\quad+\frac{c}{1-c}\cdot\frac{\partial[H(R(b^C)|\gamma^{dsb})-H(S(b^C)|\gamma^{dsb})]}{\partial\gamma^{dsb}}-c\cdot\frac{\partial[1-H(R(b^C)|\gamma^{dsb})]}{\partial\gamma^{dsb}},\end{aligned}$$

In what follows we let $f(b^C)\equiv c(1+c)|b^e(b^C,S(b^C))|$. Note that, with c small, $R(b^C)\approx\gamma^*$ and $S(b^C)\approx b^C$, so we can approximate $\Omega^e_{b^C}$ as:

$$\Omega^e_{b^C}(b^C,\gamma^{dsb})\approx f(b^C)h(b^C|\gamma^{dsb})+\frac{c}{1-c}[H(\gamma^*|\gamma^{dsb})-H(b^C|\gamma^{dsb})]-c[1-H(\gamma^*|\gamma^{dsb})]$$

Consider $(b^{C'},b^{C''})$ s.t. $\int_{b^{C'}}^{b^{C''}}\Omega^e_{b^C}(b^C,\gamma^{dsb})db^C=0$. Then

$$\int_{b^{C'}}^{b^{C''}}f(b^C)h(b^C|\gamma^{dsb})db^C+\frac{c}{1-c}\int_{b^{C'}}^{b^{C''}}[H(\gamma^*|\gamma^{dsb})-H(b^C|\gamma^{dsb})]db^C-c[1-H(\gamma^*|\gamma^{dsb})](b^{C''}-b^{C'})=0 \tag{1.1}$$

We need to evaluate the sign of $\int_{b^{C'}}^{b^{C''}}\Omega^e_{b^C\gamma^{dsb}}db^C$, that is the sign of

$$\begin{aligned}&\int_{b^{C'}}^{b^{C''}}f(b^C)h(b^C|\gamma^{dsb})\cdot\frac{\frac{\partial}{\partial\gamma^{dsb}}h(b^C|\gamma^{dsb})}{h(b^C|\gamma^{dsb})}db^C \\ &+\frac{c}{1-c}\cdot\int_{b^{C'}}^{b^{C''}}[H(\gamma^*|\gamma^{dsb})-H(b^C|\gamma^{dsb})]\cdot\frac{\frac{\partial}{\partial\gamma^{dsb}}(H(\gamma^*|\gamma^{dsb})-H(b^C|\gamma^{dsb}))}{H(\gamma^*|\gamma^{dsb})-H(b^C|\gamma^{dsb})}db^C \\ &-c[1-H(\gamma^*|\gamma^{dsb})](b^{C''}-b^{C'})\cdot\frac{\frac{\partial}{\partial\gamma^{dsb}}(1-H(\gamma^*|\gamma^{dsb}))}{1-H(\gamma^*|\gamma^{dsb})}\end{aligned}$$

The expression above is a weighted sum of (infinite) terms, where the *unweighted* sum of the same terms is zero, by (1.1). All of these terms are positive except the last one, namely $-c[1-H(\gamma^*|\gamma^{dsb})](b^{C''}-b^{C'})$, which is negative. The key is to note that the weight on the last term, namely $\frac{\frac{\partial}{\partial\gamma^{dsb}}(1-H(\gamma^*|\gamma^{dsb}))}{1-H(\gamma^*|\gamma^{dsb})}$, is higher than the weight on each of the other terms, namely $\frac{\frac{\partial}{\partial\gamma^{dsb}}h(b^C|\gamma^{dsb})}{h(b^C|\gamma^{dsb})}$ and $\frac{\frac{\partial}{\partial\gamma^{dsb}}(H(\gamma^*|\gamma^{dsb})-H(b^C|\gamma^{dsb}))}{H(\gamma^*|\gamma^{dsb})-H(b^C|\gamma^{dsb})}$, for any b^C in $(b^{C'},b^{C''})$, because (i) $h(\gamma|\gamma^{dsb})$ is log-supermodular and (ii) we are focusing on values of b^C that are lower than γ^*. It then follows directly that $\int_{b^{C'}}^{b^{C''}}\Omega^e_{b^C\gamma^{dsb}}db^C\leq 0$.

We finally turn to part (ii). Note that: (a) $R(b^C)$ is increasing in γ^*; (b) $|b^e(b^C, \gamma)|$ is increasing in γ^*; (c) $S(b^C)$ is independent of γ^*; and (d) $H(\cdot|\gamma^{dsb})$ is increasing. These observations imply that $\Omega^e_{b^C}$ is increasing in γ^*, and the claim follows. QED

Proof of Proposition 6:

Consider Figure A1. Note first that, if the support of $\gamma|\gamma^{dsb}$ does not include γ^*, clearly the optimal b^C is prohibitive (zero) if the support lies below (above) γ^*. Next suppose the support of $\gamma|\gamma^{dsb}$ includes γ^*. If this support is sufficiently small, and in particular if it is contained within the interval $[(1-c)\gamma^*, (1+c)\gamma^*)]$, then it can be seen by inspection of Figure A1 that when b^C is prohibitive or zero there is no renegotiation for any γ, and hence no transfers in equilibrium. Setting b^C at a positive but non-prohibitive level may achieve a state-contingent policy, but the associated benefit is small because the support of γ around γ^* is small. On the other hand, the cost of achieving this state-contingency is not small, because it requires a non-negligible level of transfer payments in equilibrium. To see this, recall that, given b^C, the policy outcome is FT for $\gamma < R(b^C)$ and P for $\gamma > R(b^C)$. Thus, when the support of $\gamma|\gamma^{dsb}$ is small around γ^*, if we want to induce a state-contingent policy outcome the transfer b^C needs to be close to $R^{-1}(\gamma^*) = \frac{\gamma^*}{2}$. Clearly, this transfer level does not become negligible as the support shrinks. Note that for $\gamma > R(b^C)$ the equilibrium transfer will be exactly b^C, while for $\gamma < R(b^C)$ the contract will be renegotiated, and the equilibrium transfer will be $b^e = \frac{2(b^C - \frac{\gamma^*}{2}) + c(\gamma^* - \gamma)}{2(1-c)} - \frac{\gamma}{2}$. This renegotiated transfer b^e may be smaller in magnitude than $\frac{\gamma^*}{2}$, but is unrelated to the size of the support of γ and hence does not become small as the support shrinks. The claim then follows. QED

Proof of Proposition 7:

Recall from the proof of Proposition 5 that it can never be strictly optimal to have $b^C > \gamma^*$. If $b^C(\gamma^{dsb})$ is a liability rule (with or without escape), then $b^C(\gamma^{dsb})$ must be below the prohibitive level for all γ^{dsb}. Next recall that we have defined b^C as "prohibitive" given γ^{dsb} if it is such that Home would choose $T = FT$ for all γ in its conditional support $(\underline{\gamma}(\gamma^{dsb}), \bar{\gamma}(\gamma^{dsb}))$, which implies $S(b^C) > \bar{\gamma}(\gamma^{dsb})$. Hence, it follows that if $\bar{\gamma}(\gamma^{dsb}) > S(\gamma^*)$ for all γ^{dsb}, then the optimal $b^C(\gamma^{dsb})$ must be below the prohibitive level for all γ^{dsb}. And with $\underline{\gamma}(\gamma^{dsb})$ below γ^* for at least some γ^{dsb} by our assumption that the empty contract is not optimal, it follows that if $\bar{\gamma}(\gamma^{dsb})$ and $\underline{\gamma}(\gamma^{dsb})$ are sufficiently far apart for all γ^{dsb}, the optimum must be a liability rule (with or without escape). QED

Proof of Proposition 8:

We focus on showing the first part of the proposition, namely that if the optimal contract is a property rule then early settlement can never occur; the other parts of the proposition are easy to show. Notice first from the top right panel of Figure 3 that, when the optimum is a property rule with escape, early settlement can never occur for γ in Region II: this is because either $b^C(\gamma^{dsb})$ defines a property rule with escape over the support $[\underline{\gamma}^{dsb}(\gamma), \bar{\gamma}^{dsb}(\gamma)]$, in which case as we have observed above a ruling will occur; or $b^C(\gamma^{dsb})$ defines a noncontingent property rule, i.e. either a prohibitive or zero level of b^C, over the support $[\underline{\gamma}^{dsb}(\gamma), \bar{\gamma}^{dsb}(\gamma)]$, in which case as we have observed above there will be no dispute. Consider next a realization of γ in Region III, where $\gamma > (1+c)\gamma^*$. Here it is not possible for the DSB to receive a signal γ^{dsb} such that $b^C(\gamma^{dsb})$ is prohibitive, because otherwise for that signal we would have $\bar{\gamma}(\gamma^{dsb}) > (1+c)\gamma^*$ and hence $b^C(\gamma^{dsb})$ prohibitive would require $b^C(\gamma^{dsb}) > \gamma^*$ which cannot be optimal; and so when the optimum is a property rule with escape as we have supposed, it must then be that for realizations of γ in Region III we can only have DSB signals such that $b^C(\gamma^{dsb}) = 0$, and hence again no dispute. Finally, consider a realization of γ in Region I, where $\gamma < (1-c)\gamma^*$. We have argued earlier that $b^C(\gamma^{dsp}) = 0$ is inconsistent with $\underline{\gamma}(\gamma^{dsp}) < (1-c)\gamma^*$, and so if the optimum is a property rule with escape it must then be that for realizations of γ in Region I we can only have DSB signals such that $b^C(\gamma^{dsb})$ is prohibitive, hence a noncontingent property rule and once again no dispute. We conclude that early settlement cannot occur when the optimum is a property rule with escape. QED

Figure A1