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Optimal Insurance: Dual Utility, Random Losses, and Adverse Selection†

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We study a generalization of the classical monopoly insurance problem under adverse selection (see Stiglitz 1977) where we allow for a random distribution of losses, possibly correlated with the agent's risk parameter that is private information. Our model explains patterns of observed customer behavior and predicts insurance contracts most often observed in practice: these consist of menus of several deductible-premium pairs or menus of insurance with coverage limits–premium pairs. A main departure from the classical insurance literature is obtained here by endowing the agents with risk-averse preferences that can be represented by a dual utility functional (Yaari 1987). (*JEL* D81, D82, D86, D91, G22)

A robust empirical finding in various insurance markets is that even moderate risks are often insured via contracts with low deductibles or with full coverage up to high limits. In a famous early study, Mossin (1968, p. 558) observed,"… the conclusion that full coverage is never optimal seems quite plausible, at least when considered as a normative guideline. Casual empirical evidence seems to contradict the conclusion, however; some of our best friends take full coverage."

Customers purchase insurance contracts with high coverage despite premium costs that are significantly above the value of the expected loss (see Barseghyan et al. 2013; Barseghyan, Molinari, and Teitelbaum 2016; Cohen and Einav 2007; and Sydnor 2010, among others). For example, Sydnor (2010) describes a large dataset of more than 50,000 households who choose among deductibles $\{\$100, \$250, \$500, \$1000\}$ for home insurance. About 48 percent of households chose the US$500 deductible paying an average premium of US$715, yet these all rejected a US$1,000 deductible whose average premium was just US$615. Interestingly enough, the average claim

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rate for this group is only 4.3 percent. Thus, a reduction in deductible worth less than $(\$1000 - \$500) \cdot 0.043 = \$21.5$ in expected terms is purchased by a large number of households who pay US$100 on average for it. Another 35 percent of households held the US$250 deductible. They have an average claim rate of about 5 percent, and on average paid an extra US$87 over the premium for the US$500 deductible. These households were willing to pay US$87 to protect against an expected loss of US$12.5. Barseghyan et al. (2013) and Barseghyan, Molinari, and Teitelbaum (2016) describe another large dataset of car and home insurance and find similar patterns of households' deductible choices.

Behavior as described above—for which very large degrees of risk aversion can be inferred—is hardly consistent with postulating that agents are expected utility maximizers: plausible calibrations of expected utility theory generally lead to risk-neutral behavior over small stakes (see, e.g., Rabin 2000).[1] In his survey about the econometric analysis of insurance data, Chiappori (2000, pp. 390-91) remarked,

> Finally, a better understanding of actual behavior is likely to require new theoretical tools. The perception of accident probabilities by the insurees, for instance, is a very difficult problem on which little is known presently. Existing results, however, strongly suggest that standard theoretical models relying on expected utility maximization using the 'true' probability distribution may fail to capture some key aspects of many real-life situations.

Several authors, e.g., Sydnor (2010); Barseghyan et al. (2013); Barseghyan, Molinari, and Teitelbaum (2016), have indeed shown that the insurance patterns in their datasets are consistent with alternative explanations based on theories of nonexpected utility that involve probability weighting.

Our main purpose in this paper is to provide a convenient analytic model that explains both the patterns of observed customer behavior as above and the pattern of insurance contracts most often observed in practice: these consist of simple menus of several deductible-premium pairs or menus of full insurance with coverage limits–premium pairs.

Our model departs from the classical insurance literature in two main directions: First, we assume that agents are endowed with risk-averse preferences represented by a dual utility functional (Yaari 1987) that incorporates a nonlinear function distorting probabilities rather than payoffs.[2] An alternative interpretation is that agents have a distorted belief that overweights more adverse events, leading to nonlinear probability weighting. Second, we allow for a much more general version of the classical monopoly insurance problem under adverse selection (see Stiglitz 1977):[3] our framework allows for a random distribution of losses that may be correlated with the agent's risk parameter, the latter being the agent's private information.

[1] Besides empirical findings, there is ample laboratory evidence that expected utility theory does not perform well in explaining agents' risk attitudes over small or modest stakes.

[2] Guriev (2001) offers a "micro-foundation" for dual utility: a risk-neutral agent who faces a bid-ask spread in the credit market will behave as if he were dual risk averse. This can be directly applied to insurance markets if credit is needed to cover accidental losses. The same happens if gains are taxed but losses are not.

[3] A large empirical evidence suggests that market power is prevalent in the insurance industry (see Dafny 2010; Robinson 2004; and Trish and Herring 2015, among others). For example, India's largest life insurer has a market share of 64 percent.

As we explain in Section IA, Kőszegi and Rabin's (2006) loss-averse preferences with linear utility over outcomes form a special case of Yaari's dual utility and are thus covered by our model. In addition, we note that, for the special, classical case with adverse selection and with a unique, fixed level of insured loss (see Rothschild and Stiglitz 1976 and Stiglitz 1977), our results apply more generally to the class of nonexpected utility displaying Constant Risk Aversion (CRA) (see Safra and Segal 1998). This class includes instances of many well-known utility formulations that have been proposed in the literature. Examples include Gul's (1991) and Loomes and Sugden's (1986) disappointment aversion theories with a linear utility over outcomes and mean-dispersion utility of the type used in the macro and finance literature (e.g., Rockafellar, Uryasev, and Zabarankin 2006); see online Appendix C for details.

One of the main features that distinguishes dual utility (and its many variants and generalizations) from expected utility is *first-order risk aversion* (see Segal and Spivak 1990 for definitions and a discussion of the various orders of risk aversion): in the limit where the stakes become small, the risk premium vanishes linearly in the size of the risk. This is in stark contrast to any EU preference represented by a twice-differentiable utility function that exhibits second-order risk aversion: in the small-stakes limit, EU agents become risk neutral and the risk premium they demand vanishes quadratically in the size of the risk. This difference can have far-reaching implications for behavior.[4] For example, under expected utility, full insurance is Pareto-optimal if and only if the premium is actuarially fair (Mossin 1968), i.e., if the cost of providing insurance equals the premium. In contrast, assuming complete information, Segal and Spivak (1990) showed that full insurance may be Pareto-optimal under first-order risk aversion even if the exogenous insurance premium is higher than fair. In our model, incomplete information and the associated possibility to screen agents endogenize the insurance premia and cause an inefficiency in the form of partial insurance being offered to some types.

Our main results characterize the incentive-compatible, individually rational, and profit-maximizing menus of insurance contracts. Each contract consists of an indemnity in case of loss (this depends on the loss and induces a retention share for the agent) and of a corresponding premium that must be paid up front. A main assumption underlying our analysis and often made in the finance and insurance literatures is that both indemnity and retention functions are increasing in the value of the loss (double-monotonicity). This assumption corresponds to so-called *ex post moral hazard* condition ensuring that the agent benefits neither from increasing the loss (arson) nor from hiding part of the loss.

Under a regularity condition, the optimal scheme is a layer contract: for each risk type, it consists of alternating intervals of losses where the agent's retention function has either slope zero or slope one. We also offer sufficient conditions under which the optimal contract consists either of a menu of deductibles or a menu of coverage limits with different premia, one for each risk type. The analysis allows us

[4] For example, Epstein and Zin (1990) argue that dual utility can resolve the *equity premium puzzle*: faced with small-stakes lotteries, a dual risk-averse (EU risk-averse) agent requires a risk premium proportional to the standard deviation (variance) of the lottery. Since for small risks the standard deviation is considerably larger than the variance, this generates a higher equity premium.

to distinguish structural differences between optimal contracts for the case where private information is about the probability of a loss (where we get deductibles) and the case where private information is about the loss magnitudes (where we get coverage limits). Other commonly used instruments such as coinsurance are not optimal in pure adverse selection frameworks.

Deductibles are common for medical and property insurance and also for product liability and professional liability policies for attorneys, accountants, corporate directors, and officers. It is well known that a deductible contract is welfare maximizing for any risk-averse agent in the class of contracts with a fixed expected cost for the insurer.[5] Hence, an insurance contract with a deductible is, in principle, consistent with the idea that the insurer needs to generate high welfare in order to extract a high revenue. As an illustration, consider the special case where the distribution of losses is independent of the probability of accident and where the losses can take a finite number of values. Then, in our framework, the optimal menu consists of a basic high deductible–low premium pair complemented by a ladder of additional fees that gradually reduce the deductible until, possibly, full insurance. As mentioned above, such a menu structure is ubiquitous in practice and taken almost for granted in most of the empirical literature.

On the other hand, deductibles are not common for medical malpractice, where rating based on the physician's individual claim record is relatively limited (see Harrington and Danzon 2000). Instead medical malpractice insurance uses menus of limits on coverage as a preferred screening instrument. As an example, US medical malpractice insurance typically offers doctors a choice between coverage with limits such as US$100,000, US$200,000, US$500,000, US$1,000,000.[6] Payments at these limits are often seen in practice.[7]

The optimality of contracts with coverage limits under some of our model's parameters is somewhat striking since we show that such a contract is worst for any risk-averse agent within the set of doubly monotonic contracts that yield the same expected cost to the insurer.[8] In other words, the benefits of screening via coverage limits can be higher than the welfare loss from choosing an extremely suboptimal allocation.

Technically, we study a principal-agent problem with interdependent valuations and with type-dependent outside options. For each type of the agent, the allocation is an entire retention function (i.e., for each possible loss, the part of the loss that remains to be covered by the agent) rather than a scalar. Dual utility yields here, for each risk type, a linear optimization problem. Hence, for each type, the optimum is achieved at an extreme point of the set of feasible retention functions that satisfy the ex post moral hazard constraints. We then offer sufficient conditions

[5] See, for example, Van Heerwarden, Kaas, and Goovaerts (1989). This result generalizes famous early results by Arrow (1963) and Borch (1960) about variances of such contracts.

[6] For example, Silver et al. (2015) survey a very rich dataset of Texas medical malpractice claims and find that 89 percent of policies have such limits; 6 percent of the policies have higher limits.

[7] For example, about 15 percent of all claims in the Texas malpractice dataset are paid at the limit, with the proportion rising to about 30 percent for those with limits less than US$500,000. Special disciplines such as perinatal medicine see about 40 percent of all claims paid at the limit (see Silver et al. 2015 and Zeiler et al. 2007).

[8] This should not be confused with the well-known "live-or-die" contract analyzed by Innes (1990). Such contracts are not doubly monotonic.

that render the obtained collection of retention functions, one for each risk type, incentive compatible.

Finally, we note that Yaari's dual utility functionals also correspond to the so-called distortion or spectral risk measures, such as *expected shortfall*, often used in the finance and insurance literatures. These consist of weighted averages of the *values at risk* (*VaR*) for each quantile.[9] For the classical case with a unique insured loss, our results apply more broadly to the class of law-invariant, coherent risk measures that can be derived as minima over averages of values at risk with respect to a set of distortions (see Kusuoka 2001). Thus, our methodology applies to an insurer-reinsurer relation where the insurer (agent) uses such a risk measure to assess its portfolio.

The structure of the paper is as follows: We conclude this section with a literature review. In Section I we describe the risk environment, the agent's preferences, and the insurance contracts. Section II describes the set of feasible mechanisms that satisfy incentive compatibility and individual rationality constraints. In Section III we solve the optimal insurance problem within the general class of deterministic insurance contracts that respect two ex post moral hazard conditions. We also offer conditions under which simple contracts that consist either of menus of deductibles or menus of coverage limits are optimal. Section IV concludes. All proofs are in the Appendix.

Related Literature.—A large literature following Borch (1960) and Arrow (1963) focuses on models without adverse selection and studies the welfare-maximizing insurance policy under a pricing formula where the premium for each policy is proportional to its cost. If the insurer is willing to offer any insurance policy desired by the buyer at a premium that only depends on the policy's actuarial value, then the main finding is that the policy chosen by a risk-averse buyer will take the form of full coverage above a deductible minimum.

Raviv (1979) and Huberman, Mayers, and Smith (1983) obtain optimality of policies using additional instruments, such as upper limits on coverage only under exogenous constraints such as a bankruptcy constraint on the insurer, or under a limited liability constraint on the insuree created by legal provisions in personal bankruptcy law. Huberman, Mayers, and Smith (1983) also introduce restrictions on the retention and indemnity functions that avert several moral hazard problems arising after a loss has already occurred. In order to obtain the optimality of simple deductibles, Townsend (1979) assumes that the loss can only be verified at a cost.

The abovementioned strand of the literature does not incorporate adverse selection. Chade and Schlee (2012) offer a comprehensive and up-to-date study of monopolistic profit maximization in an insurance market subject to adverse selection where the agents are expected utility maximizers. Their model follows the pioneering work of Rotshchild and Stiglitz (1976) and Stiglitz (1977) by assuming that the private information pertains to the probability of an accident: all risk types in these models face the same fixed loss in case of an accident. This also holds for Szalay's (2008) alternative analytic approach to the same problem. Contrasting our framework, the allocation function for each risk type is then a scalar, i.e., the share

[9] For a good exposition, see, for example, Rüschendorf (2013, chap. 7).

of loss that is insured or retained. Thus, in such models, every deterministic feasible policy can be described in terms of a menu of deductibles. Hence, deductibles cannot be endogenously derived as being optimal. Chade and Schlee (2012) and Szalay (2008) exhibit some interesting properties of the profit-maximizing mechanism but are able to analytically solve for the optimum only for agents equipped with either exponential (CARA) or square root utility functions.

There is ample field and laboratory evidence showing that expected utility does not perform well in explaining agents' risk-taking behavior (e.g., Johnson et al. 1993 and Sydnor 2010 for insurance markets; Snowberg and Wolfers 2010 and Chiappori et al. 2019 for horse betting markets; and Bruhin, Fehr-Duda, and Epper 2010 in the lab). Many such papers argue that probability distortions play an important role in explaining behavior (see the survey of Starmer 2000). Some of them advocate Yaari's dual utility or, more generally, a rank-dependent utility a la Quiggin (1982) as an alternative to expected utility.

For instance, Barseghyan et al. (2013) analyze a large dataset obtained from a large US property and casualty insurance company and find that probability distortions play an important role in explaining households' deductible choices for auto and house insurance. Looking at a subset of 3,629 out of 4,170 households,[10] Barseghyan, Molinari, and Teitelbaum (2016) find that 80 percent of them make choices that are consistent with Yaari utility preferences, while 85 percent make choices that are consistent with rank-dependent utility preferences. Suhonen, Saastamoinen, and Linden (2018) find that dual utility well explains the behavior of pari-mutuel gamblers in Finnish horse races.[11]

Goeree, Holt, and Palfrey (2002) found that quadratic probability weighting, that is, a special case of dual utility, well explains bidding behavior in lab experiments. In another experiment that derived the demand for insurance, Papon (2008) showed that dual utility preferences describe the chosen insurance policies better than expected utility. In Papon's experiment a vast majority of the subjects chose either full insurance or no insurance, and only a small proportion selected partial insurance options. Such choices are expected under complete information and dual utility (Doherty and Eeckhoudt 1995) but not under expected utility.

Chiappori et al. (2019) also find that general models relying on rank-dependent utility and that allow for distortions of both payoffs and probabilities work relatively well in explaining betting data. But, in their data, and also in the laboratory experiment conducted by Hey and Orme (1994), dual utility is dominated by expected utility.[12]

Several other empirical papers, for example, Sydnor (2010) and Snowberg and Wolfers (2010), advocate loss aversion as an alternative explanation for behavior and show that models based on loss-averse preferences fit the data much better than expected utility. As we explain in Section IA, assuming linear utility over outcomes, Kőszegi and Rabin's (2006) loss-averse preferences constitute a special case of Yaari's dual utility.

[10] These households satisfy some consistency criteria.

[11] In pari-mutuel betting, odds are not fixed by a bookmaker but rather represent the gamblers' probability estimates of the winning horses.

[12] We note that the experiment had only 80 participants.

Finally, some papers do not support one formal model over others but argue instead that the probability weighting function takes an "inverse S" shape (e.g., Bruhin, Fehr-Duda, and Epper 2010; Dimmock et al. 2021). That is, agents tend to overweight small probability tail events. Our model can incorporate this possibility because in real-life insurance data, the probability of loss is relatively small.

Dual utility replaces the classical von Neumann-Morgenstern independence axiom behind expected utility (EU) with another axiom about mixtures of comonotonic random variables. It belongs to the family of *rank-dependent* utility functions. The idea of theoretically studying insurance markets while assuming that agents have a rank-dependent utility function is not new. Most of the relevant finance/insurance literature focuses on the related distorted or spectral risk measures.

In early work, Doherty and Eeckhoud (1995) study a model without adverse selection. Following Arrow (1963), they are interested in maximizing the agent's welfare under actuarial fair pricing plus a markup and focus solely on simple (not necessarily optimal) mechanisms, such as coinsurance or deductibles.

Bernard et al. (2015) and Xu, Zhou, and Zhuang (2019) focus on the optimization of the agent's welfare under a random loss but without adverse selection; i.e., there is a unique risk type. Thus, in their model premia are "exogenous," as there is no incentive constraint binding them to insurance level. In the same model without adverse selection, Xu, Zhou, and Zhuang (2019) impose conditions that constrain the agent's ability to manipulate the loss ex post—we impose analogous conditions here.

Assuming that the insuree has a dual utility function, Hindriks and De Donder (2003) add adverse selection a la Stiglitz (1977) to a model where the insuree has dual utility: the private information is about the probability of an accident, and the loss in case of accident has a fixed magnitude, independent of the probability of having an accident. They show that a profit-maximizing monopolistic insurer offers full insurance to relatively high-risk types, while leaving relatively low-risk types uninsured. Liang, Zou, and Jiang (2022) show that this result is not robust to the presence of random losses: in their model there are two risk types, and the lower-risk type is only partially insured.

Finally, Gershkov et al. (2022) analyze optimal auctions in a framework where bidders' preferences are represented by a nonexpected utility functional that exhibits constant risk aversion. Contrasting the present framework, their bidders face binary lotteries, and the optimal mechanism offers full insurance while distorting the allocation via endogenous randomization.

I. The Insurance Environment

An agent faces a random loss L distributed on the interval $[0,\bar{L})$, where the maximal loss $\bar{L}$ can be finite or infinite. The agent's private information, his type $\theta \in [\underline{\theta},\bar{\theta}] = \Theta$, parameterizes the distribution of losses $H_\theta : \mathbb{R}_+ \to [0,1]$ he faces. The distribution H_θ is increasing in first-order stochastic dominance, such that higher types face a stochastically larger loss. We assume that H_θ is uniformly Lipschitz-continuous in θ with constant $c < \infty$. We denote by $E[L(\theta)]$ the expected loss of type θ, and we assume that this is finite.[13] Finally, we denote by $F : \Theta \to [0,1]$ the distribution of types and by $f : \Theta \to (0,\infty)$ its density.

To illustrate the generality of the above setup, consider two important special cases.

Asymmetric Information about Loss Probabilities: Consider first the extreme case where the type θ represents the probability of an accident and where the distribution of losses conditional on an accident is given by a fixed distribution Q, independently of type. We obtain that

$$H_\theta(l) = (1 - \theta) + \theta Q(l). \tag{1}$$

This specification naturally captures health insurance where some agents face a greater risk of requiring certain medical procedures. Almost all of the insurance literature following Stiglitz's (1977) adverse selection model has focused on the special case where $Q(l) = \mathbf{1}\{l \geq l^*\}$ puts probability 1 on a single, deterministic loss $l^* > 0$.

Asymmetric Information about Loss Size: Consider another extreme case where the agent's type influences the size of the loss but not its probability. For example, consider $L = \theta K$, where K is a random variable with support $[0,\infty)$, so that the agent's type multiplies an exogenous damage K distributed according to Q, independent of the agent's type. For this example, we obtain that

$$H_\theta(l) = Q(l/\theta).$$

Here, all types face the same probability of accident, but some face higher losses in case of an accident. For example, the probability of an earthquake is the same for all agents, but an agent with a higher house value may face a higher damage should his house be destroyed.

Finally, we note here that our model is general enough to incorporate cases where the agent's private information concerns **both** the loss probabilities and the loss size.

A. *The Agent's Utility Function*

As it will become clear in the next subsection, the agent makes a payment for the insurance he purchases and then suffers a random loss. In this insurance context, it is sufficient to define risk preferences over random variables that capture the total loss suffered by the agent.

We assume that the agent is endowed with a *Yaari* (*dual*) *utility* determined by a probability distortion function $g : [0,1] \to [0,1]$, where g is increasing, absolutely continuous, and satisfies $g(p) \leq p$ with $g(0) = 0$ and $g(1) = 1$. The certainty

[13] Note that, in view of the stochastic dominance assumption, it is enough to assume that the expected loss of the highest type $E[L(\overline{\theta})]$ is finite.

equivalent of a random total loss $x \geq 0$, distributed according to $H : \mathbb{R}_+ \rightarrow [0,1]$ where $H(s) = \Pr(x \leq s)$, is given by[14]

$$CE(x) = -\int_0^{\infty} 1 - g(H(s))ds.$$

Our agent is risk averse in a weak sense: the certainty equivalent of any lottery is less than the lottery's expected value. This is so because

$$CE(x) = -\int_0^{\infty} 1 - g(H(s))ds \leq -\int_0^{\infty} 1 - H(s)ds = -E[x].$$

While in the EU framework the above form of risk aversion is equivalent to aversion to mean-preserving spreads, here, the latter notion of risk aversion is stronger and is equivalent to g being convex. We refer below to the latter notion as *strong* risk aversion. The *weak* form of risk aversion that only requires $g(p) \leq p$ is sufficient for most of our present analysis.

For more intuition about dual utility, note that, for the case where H admits a density, integration by parts yields

$$CE(x) = -\int_0^{\infty} 1 - g(H(s))ds = -\int_0^{\infty} sg'(H(s))dH(s).$$

In other words, dual utility modifies the standard expectation operator $E[x] = \int_0^{\infty} sdH(s)$ by multiplying each loss level s with the weight $g'(H(s))$. Thus, $g(H(s))$ can be interpreted as the cumulative weight assigned to loss levels below s, while $1 - g(H(s))$ represents the cumulative weight assigned to loss levels above s. The assumption $g(p) \leq p$ says that the agent overweights the cumulative probability of large losses and underweights the cumulative probability of smaller losses, including no loss at all. The stronger notion of risk aversion where the agent is averse to mean-preserving spreads (i.e., where g is convex) requires the weights to be increasing in the loss level.

Finally, we note that Yaari's dual utility is additive with respect to constant random variables t (see Yaari 1987):

$$CE(x + t) = CE(x) + t.$$

This property implies that the agent has quasi-linear, separable preferences with respect to nonrandom transfers t, and it greatly facilitates the analysis of our screening problem.

We assume that the agent's willingness to pay for insurance is finite, independent of her type.[15] This is a weak assumption: for example, it is satisfied if the loss L is bounded or if g is bounded from below by a power function (see an illustration below).[16] Without such a restriction, the principal can obtain unbounded

[14] Yaari considered only bounded random variables. For extensions to integrable random variables in L_p see, for example, Baüerle and Müller (2006) and Rüschendorf (2013).

[15] This is equivalent to requiring that $\int_0^{\infty} \left[1 - g(H_{\bar{\theta}}(s))\right]ds < \infty$.

[16] To see this, note that if $g(p) \geq p^{\kappa}$, then $\int_0^{\infty} 1 - g(H_{\bar{\theta}}(s))ds \leq \int_0^{\infty} 1 - (H_{\bar{\theta}}(s))^{\kappa} ds = \int_0^{\infty} \kappa (H_{\bar{\theta}}(s))^{\kappa-1} xdH_{\bar{\theta}}(s) \leq \kappa \int_0^{\infty} xdH_{\bar{\theta}}(s) = \kappa E\left[L(\bar{\theta})\right] < \infty$.

profits (without any screening) by offering insurance that covers all losses above a threshold. Finally, as a technical condition, we also assume that for all θ, it holds that $\int_0^{\bar{L}} g'\big(H_\theta(l)\big)dl < \infty$. This suffices to render the uninsured agent's utility differentiable in type.

We conclude this section by noting that Kőszegi and Rabin's (2006) loss-averse preferences with linear utility over outcomes are a special case of Yaari's dual utility and are thus covered by our model. For a random loss x, Kőszegi and Rabin's (2006) formulation yields[17]

$$-E[x] + \int_0^\infty \int_0^\infty \mu(y - x)dH(x)dH(y),$$

where

$$\mu(z) = \begin{cases} z, & \text{if } z \geq 0; \\ \lambda z, & \text{if } z < 0. \end{cases}$$

As established by Masatioglu and Raymond (2016) in their Proposition 4, the above functional form is a special case of dual utility with distortion

$$g(p) = (2 - \lambda)p + (\lambda - 1)p^2.$$

Such preferences are consistent with monotonicity in the sense of First-Order Stochastic Dominance Order (FOSD) if and only if $\lambda \in [0,2]$. The agent is risk averse (either in the weak or strong sense) if and only if the agent is loss averse with $\lambda \in (1,2]$.[18]

B. *The Insurance Contracts*

There is a single, risk-neutral monopolistic insurance provider (she) who offers an insurance mechanism to the risk-averse agent (he). We restrict attention to (i) direct (static) and (ii) nonrandomized mechanisms. Two comments about these assumptions are in place:

In online Appendix A we illustrate through an example that the latter restriction is not without loss of generality: lotteries may help screen different types of the risk-averse agent. It is, however, suitable in order to address insurance applications: explicitly randomized insurance contracts offered to risk-averse agents are, to the best of our knowledge, never observed.

In his work about "Dutch books," Green (1987) showed that, if both designer and agent can observe the realization of the agent's random endowment, and if the designer can offer a series of lotteries, then she can extract money from a naïve agent who takes myopic decisions and whose utility does not satisfy the independence axiom (leading to nonlinearity in probabilities). Thus, for some notion of naïveté, the designer can use a random, dynamic mechanism in order to exploit the agent's dynamic inconsistency and extract more surplus than what would be possible in an

[17] See also the theory of disappointment without a prior, due to Delquié and Cillo (2006).
[18] When $\lambda = 1$, the model reduces to the standard EU risk-neutral preferences.

optimal static mechanism. But if one restricts attention to nonrandom mechanisms where also the agent uses a pure strategy, then the only remaining randomness in our environment is in the agent's endowment, i.e., the agent's stochastic loss. Then, as the agent's time-inconsistency stems from a gradual resolution of uncertainty, we conjecture that, under weak conditions, the restriction to static mechanisms is without loss of generality.

In a mechanism, the insurer offers a menu of contracts of the form $\left(I(\cdot,\theta),t(\theta)\right)_\theta$ where, for every type θ, $I(l,\theta) \in [0,l]$ is the amount covered if loss l occurs and where $t(\theta)$ is the associated premium. Equivalently, the insurer can be seen as offering a menu of *retention* functions $\left(R(\cdot,\theta),t(\theta)\right)_\theta$ where $R(l,\theta) = l - I(l,\theta)$ is the part of the loss l that remains to be covered by the agent of type θ.

ASSUMPTION 1: *We impose two natural monotonicity conditions* (*or ex post moral hazard conditions*) *on the retention function R for any* θ:

(*i*) $R(l,\theta)$ *is nondecreasing in l.*

(*ii*) $l - R(l,\theta) = I(l,\theta)$ *is nondecreasing in l.*

Part 1 of Assumption 1 ensures that the agent does not benefit from a smaller retention $R(l',\theta) < R(l,\theta)$ by artificially increasing an incurred loss from l to $l' > l$. Part 2 ensures that the agent does not benefit from a higher indemnity $l' - R(l',\theta) = I(l',\theta) > I(l,\theta) = l - R(l,\theta)$ by hiding part of the incurred loss with a report $l' < l$.[19] These *ex post moral hazard* assumptions were introduced by Huberman, Mayers, and Smith (1983) and are common in the insurance literature. Moreover, practically all contracts observed in practice satisfy these conditions.

Observe that any function that satisfies parts (i) and (ii) of Assumption 1 is Lipschitz-continuous with constant 1 and hence also absolutely continuous. Its derivative exists almost everywhere and satisfies $\partial R(l,\theta)/\partial l \in [0,1]$ for all θ, l.[20] For any θ, let $R^{-1}(\cdot,\theta)$ denote the *generalized inverse* of $R(\cdot,\theta)$.[21] Using the additivity of Yaari's utility with respect to constants, and noting that the distribution of the random variable $R(\cdot,\theta)$ is given by $H_\theta\left(R^{-1}(\cdot,\theta)\right)$, we obtain that the certainty equivalent of the contract $\left(R(\cdot,\theta),t(\theta)\right)$ offered to type θ equals

$$-t(\theta) - \int_0^{R(\bar{L},\theta)} \left[1 - g\left(H_\theta\left(R^{-1}(a,\theta)\right)\right)\right] da$$

$$= -t(\theta) - \int_0^{\bar{L}} \left[1 - g\left(H_\theta(l)\right)\right] \frac{\partial R(l,\theta)}{\partial l} dl,$$

where the last equality follows by the change of variables $l = R^{-1}(a,\theta)$.

[19] The monotonicity requirement on the insurance I can be technically dropped—see also the remark after Theorem 1 for the ensuing consequences for optimal insurance contracts.

[20] In particular, the function $R(\cdot,\theta)$ can be obtained as the integral of its derivative.

[21] $R^{-1}(l,\theta) = \sup\{z : R(z,\theta) \leq l\}$.

Finally, the cost of providing the insurance contract $\left(R(\cdot,\theta),t(\theta)\right)$ to a type θ agent is given by

$$E\left[L(\theta)\right] - \int_0^{R(\bar{L},\theta)} \left[1 - H_\theta\left(R^{-1}(a,\theta)\right)\right] da$$

$$= E\left[L(\theta)\right] - \int_0^{\bar{L}} \left[1 - H_\theta(l)\right] \frac{\partial R(l,\theta)}{\partial l} dl,$$

where the equality follows by the above change of variable.

II. Implementable Insurance Contracts

In this section we describe incentive-compatible and individually rational insurance contracts, i.e., we delineate the feasible set of contracts among which the insurer looks for the optimal one. The formulation of Yaari's dual utility allows us to develop a simple envelope condition that is necessary for incentive compatibility, and to formulate the design problem as a linear optimization problem. The key technical features behind this tractability are the separable additivity of constant random variables (here, these are the ex ante premia paid for insurance). This separability comes from the fact that risk aversion is induced here "dually" by distorting probabilities rather than payoffs. In other words, dual utility disentangles attitudes toward risk from the marginal utility of money, which is constant. Besides the technical convenience, this property makes the dual formulation appealing for settings where stakes are small or moderate, so that wealth effects are not significant: the linearity of the agents' utilities in monetary transfers can then coexist with any degree of risk aversion.

A. *Incentive Compatibility*

Fix a mechanism $\left(R(\cdot,\theta),t(\theta)\right)_\theta$. Assuming that the agent has type θ, we denote by $U(\theta,\theta')$ the agent's certainty equivalent of the contract $\left(R(\cdot,\theta'),t(\theta')\right)$ designed for type θ'. We slightly abuse notation by using below $U(\theta)$ instead of $U(\theta,\theta)$ for the certainty equivalent the agent obtains when reporting truthfully. By the additivity of the dual utility with respect to constants, we obtain that an agent with type θ who reports to be of type θ' gets

$$U(\theta,\theta') = -t(\theta') - \int_0^{\bar{L}} \left[1 - g\left(H_\theta(l)\right)\right] \frac{\partial R(l,\theta')}{\partial l} dl.$$

A mechanism $\left(R(\cdot,\theta),t(\theta)\right)_\theta$ is *incentive compatible* if, for any pair of types θ and θ', it holds that

$$\text{(incentive compatibility)} \quad U(\theta) \equiv U(\theta,\theta) \geq U(\theta,\theta').$$

PROPOSITION 1 (Incentive-Compatible Mechanisms):

(*i*) *Fix any incentive-compatible mechanism* $\left(R(\cdot,\theta),t(\theta)\right)_\theta$. *Then, the agent's certainty equivalent is given by*

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta}\left[\int_0^{\bar{L}}\frac{\partial R(l,s)}{\partial l}g'\left(H_s(l)\right)\frac{\partial H_s(l)}{\partial s}dl\right]ds,$$

and the insurer's profit is given by

$$\pi(R) = \int_{\underline{\theta}}^{\bar{\theta}}\left[-E\left[L(\theta)\right] - \int_0^{\bar{L}}\frac{\partial R(l,s)}{\partial l}J(l,\theta)dl\right]f(\theta)d\theta - U(\underline{\theta}),$$

where

$$J(l,\theta) = H_\theta(l) - g\left(H_\theta(l)\right) + \frac{1-F(\theta)}{f(\theta)}g'\left(H_\theta(l)\right)\frac{\partial H_\theta(l)}{\partial\theta}.$$

(*ii*) *If R is submodular, then the above conditions are also sufficient for the menu of contracts* $\left(R(\cdot,\theta),t(\theta)\right)_\theta$ *to be incentive compatible.*

Submodularity is a very robust sufficient condition: it does not depend on the particular form of the distortion g that determines utility nor on the other features of the environment.

The "virtual value" J defined above captures the effect on the insurer's profit of marginally increasing the insurance coverage (or, equivalently, of marginally decreasing the agent's retention). We can split this effect into two parts. The first part measures the effect on the insurer's revenue:

$$\left[1 - g\left(H_\theta(l)\right)\right] + \frac{1-F(\theta)}{f(\theta)}g'\left(H_\theta(l)\right)\frac{\partial H_\theta(l)}{\partial\theta},$$

where $1 - g\left(H_\theta(l)\right)$ represents the agent's valuation for this marginal increase of insurance coverage and where $\left\{\left[1-F(\theta)\right]/f(\theta)\right\}g'\left(H_\theta(l)\right)\left[\partial H_\theta(l)/\partial\theta\right]$ is the agent's information rent (note that this term is negative, as $\partial H_\theta(l)/\partial\theta$ is negative). The second part

$$-\left[1 - H_\theta(l)\right]$$

measures the effect on the insurer's cost. The exact amount of insurance in an optimal mechanism is determined by balancing these two effects. We also note that the term $H_\theta(l) - g\left(H_\theta(l)\right)$ measures the size of efficiency gain resulting from providing insurance to a risk-averse agent. The more risk averse the agent is, the larger is the difference between the insurer's cost and the agent's gain.

B. *The Participation Constraint*

We furthermore restrict attention to mechanisms where each agent participates voluntarily. Since the distribution of losses is type dependent, the outside option from not purchasing insurance is also type dependent. Nevertheless, we show the participation constraint will be satisfied for all types if and only if it is satisfied for the *lowest* possible type $\theta = \underline{\theta}$ who has here the *highest* utility.

Define first

$$U_{NP}(\theta) = -\int_0^{\bar{L}} \left[1 - g\left(H_\theta(l)\right)\right] dl$$

to be type θ's certainty equivalent payoff from nonparticipation, i.e., not obtaining any insurance. The following *individual rationality*, or *participation*, constraint needs then to be satisfied:

(participation constraint) $\quad U(\theta) \geq U_{NP}(\theta).$

LEMMA 1: *In an incentive-compatible mechanism* $\left(R(\cdot,\theta), t(\theta)\right)_\theta$, *the participation constraint is satisfied for all types if and only if it is satisfied for the lowest type* $\underline{\theta}$.

The proof follows because both the utility from nonparticipation and the equilibrium utility are decreasing in risk type and because the latter function decreases more slowly due to the fact that $\partial R(l,\theta)/\partial l \in [0,1]$ for all θ, l.

C. *Strictly Positive Profit*

Our final result in this section shows that, under rather weak assumptions, a risk-neutral insurer necessarily makes a strictly positive profit by offering full insurance to at least some types. In particular, this shows that the insurer makes a strictly positive profit in the optimal mechanism.

LEMMA 2: *Assume that* $H_{\bar{\theta}}$ *is not degenerate*[22] *and that* $\partial H_\theta/\partial\theta$ *and* g' *are continuous.*[23] *Then, the insurer obtains a strictly positive expected profit in the optimal menu of contracts.*

The Proof of Lemma 2 explicitly computes the revenue from a mechanism that provides full insurance for sufficiently high types and no insurance for all lower types. It then shows that the full insurance cutoff can be chosen such that the mechanism generates strictly positive profits and such that all agents have an incentive to participate.

[22] A probability distribution is degenerate if it assigns probability 1 to a single value.

[23] Since H_θ is decreasing in θ for each z, and if g is convex, the derivative of these functions exists almost everywhere, and g is even twice differentiable almost everywhere.

Consider, for example, the case where

$$H_\theta(l) = (1 - \theta) + \theta Q(l)$$

and where θ represents the probability of an accident. We stress that the optimality of some trade holds even if $\bar{\theta} = 1$ (i.e., even if the highest type incurs some loss with probability 1), provided that $H_{\bar{\theta}}$ is not degenerate.

Assuming expected utility, Hendren (2013) derived conditions that are sufficient for insurance denial when there is a single loss magnitude. These conditions imply that, for any price set by the insurer, the distribution of the agent's types that accept this contract leads to negative profits for the insurer. This is never the case in our framework with nonexpected utility and with random losses: there always exists a contract such that the insurer gets strictly positive profits. This new phenomenon is caused by the combination of first-order risk aversion with the fact that the insurer has here finer instruments at her disposal: she can tailor the insurance to each specific loss.

Remark: If the insurer has additional information about observable characteristics and can divide agents into groups, then she could practice third-degree price discrimination by offering different insurance menus to different groups. By the same argument as above, the insurer would still find it profitable to provide some insurance to each group. In practice, there are other frictions that are not modeled here; e.g., the issuer cannot offer different menus to different groups or she faces reserve and liquidity costs, verification costs, and other administrative costs of issuing contracts and processing claims. These frictions may cause the insurer to refuse offering insurance to a high-risk group based on observable characteristics. If, however, additional information about risk groups is not available or cannot be used, selling insurance to low-risk types while excluding high-risk types cannot be incentive compatible: if a low-risk type wants to buy insurance, higher-risk types have incentives to pretend to be of a low-risk type in order to also be insured.

III. Optimal Insurance

We now provide a characterization of the optimal insurance menu under a regularity condition similar to the standard monotonicity condition on the virtual value.

THEOREM 1: *Suppose that the virtual value function,*

$$J(l,\theta) = H_\theta(l) - g\big(H_\theta(l)\big) + \frac{1 - F(\theta)}{f(\theta)} g'\big(H_\theta(l)\big) \frac{\partial H_\theta(l)}{\partial \theta},$$

is nondecreasing in θ for all l. Consider the maximization problem

$$\max_R \pi(R) = \int_{\underline{\theta}}^{\bar{\theta}} \left[-E\big[L(\theta)\big] - \int_0^{\bar{L}} \frac{\partial R(l,\theta)}{\partial l} J(l,\theta) dl \right] f(\theta) d\theta,$$

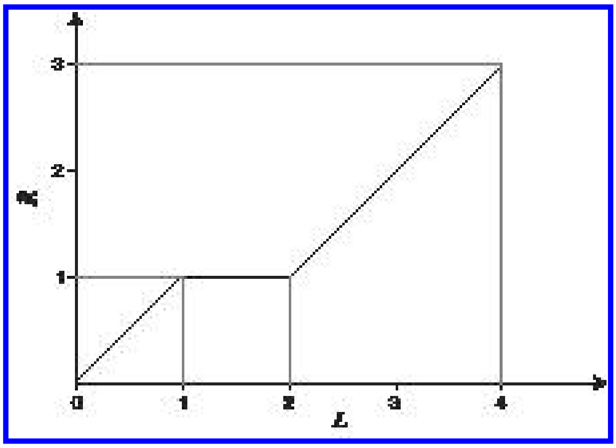

Figure 1. Contract with Deductible $D = 1$ and Coverage Limit $C = 2$

subject to

$$0 \leq \frac{\partial R(l,\theta)}{\partial l} \leq 1, \quad \text{for all} \quad \theta \in \Theta.$$

The above problem has a solution that is incentive compatible and thus optimal. In addition, at the optimum, it holds that $\partial R(l,\theta)/\partial l \in \{0,1\}$ *almost everywhere.*

The proof of Theorem 1 is in the Appendix. When J is monotonic in type, the solution to a relaxed version of the insurer's problem described above is submodular and thus incentive compatible. Therefore, it is also a solution to the original insurer's problem.

The monotonicity of J means that, in an incentive-compatible mechanism, it is more beneficial to increase the marginal indemnity (e.g., the extra indemnity due to a marginal increase in loss) if the type is more risky. It is important to note that this does not necessarily mean that a high-risk type is more valuable to the insurer—the cost of providing insurance to high-risk types is also higher, and this may lead to a lower profit level.

By the above result, in the optimal mechanism, an increase in the loss l will be either completely passed on to the agent with type θ (in that case $\partial R(l,\theta)/\partial l = 1$) or not at all (in that case $\partial R(l,\theta)/\partial l = 0$). Here is one typical example where losses below D and above $C > D$ are passed on to the agent:

$$R(l,\theta) = \begin{cases} l, & \text{if } 0 \leq l \leq D; \\ D, & \text{if } D < l \leq C; \\ l - C + D, & \text{if } C < l \leq \bar{L}. \end{cases}$$

This kind of retention function R is induced by a contract with a deductible $D \geq 0$ and with a coverage limit $C > D$. Figure 1 illustrates this contract for $D = 1$ and $C = 2$.

Remark: It can be shown that overinsurance (i.e., having $I(l) > l$) is not optimal.[24] Moreover, the monotonicity requirement on the insurance I (the second part of Assumption 1) can be dropped under suitable technical conditions. We can then rewrite the objective function in terms of the retention R instead of R', under the constraint $0 \leq R(l,\theta) \leq l$. Any possible optimal retention R for this relaxed problem is piecewise continuous. In each continuity interval it first holds that $R(l) = l$ and $R(l)$ is a constant afterward. Thus, whenever it exists, $R'(l)$ is either zero or one, as in the current analysis. In other words, every additional marginal loss is either completely passed to the insurer or completely left with the agent. The optimal insurance contract for certain types may then be nonmonotonic: for example, it can consist of several loss-dependent deductibles.[25] Relaxing the monotonicity of R is technically more complex since it is then more difficult to compute the agent's utility and to formulate appropriate incentive compatibility conditions.

Some insurance contracts include *coinsurance*, where the agent retains a fraction of the loss and where the insurer covers the remaining fraction. By definition, coinsurance contracts have slopes strictly between zero and one. Therefore, these contracts cannot be maximizers of our linear functional. Our results thus suggest that the appearance of coinsurance contracts is not due to screening motives alone. It might be motivated by economic forces that we did not model here; e.g., linear or affine contracts are robustly optimal under moral hazard if the principal (insurer) is uncertain as to what actions the agent (insuree) can and cannot take (see Caroll 2015).

A natural question stemming from Theorem 1 is: when is the virtual value function J nondecreasing in θ? Observe that

$$\frac{\partial J(l,\theta)}{\partial \theta} = -\frac{\partial H_\theta(l)}{\partial \theta}\left\{ g'\big(H_\theta(l)\big)\left[\theta - \frac{1-F(\theta)}{f(\theta)}\right]' - 1\right\} + \frac{1-F(\theta)}{f(\theta)}\,\frac{\partial\left[g'\big(H_\theta(l)\big)\frac{\partial H_\theta(l)}{\partial \theta}\right]}{\partial \theta}.$$

By assumption, $\partial H_\theta(l)/\partial\theta < 0$ and g is increasing with $g(p) \leq p$. Assume here that g is convex; that is, the agent's preference exhibits aversion to mean-preserving spreads. A nondecreasing hazard rate $f(\theta)/\big[1 - F(\theta)\big]$ of the distribution of agent types together with $g'\big(H_\theta(l)\big) \geq 1$ are sufficient for the first term to be nonnegative. As g' is increasing and as H_θ is decreasing in θ, it is thus enough to require that $g'\big(H_{\bar\theta}(0)\big) \geq 1$. Because $\lim_{p\to 1} g'(p) > 1$, this last condition is satisfied whenever the probability of a loss faced by riskiest type, $1 - H_{\bar\theta}(0)$, is sufficiently small (see Example 1 for further illustration on this point). In addition, the increasing

[24] See also Gollier (2000), who shows that overinsurance is not optimal in a complete information framework where the insurer maximizes the agent's welfare.

[25] The indemnity could have the form of the "live-or-die" contracts that are optimal in the contracting framework of Innes (1990). In a live-or-die contract, low losses are fully insured (zero deductible), while high losses are not insured at all (deductible higher than any loss).

stochastic concavity[26] (in the usual stochastic order) of the family of random variables with distributions $(H_\theta)_\theta$ is sufficient for the second term to also be nonnegative, yielding the desired monotonicity. Roughly speaking, this last assumption says that the difference in loss distributions of risk types decreases as the type increases. Note that the increasing stochastic concavity condition always holds in the classical setting where the private information is about loss probability. In this case the family,

$$H_\theta(l) = 1 - \theta + \theta Q(l) = \theta\big[Q(l) - 1\big] + 1,$$

depends linearly on θ.

Lastly, we note that J being nondecreasing is a sufficient, but not necessary, condition for our current methodology to work. We present below an example that satisfies the above conditions, while in Section IIIA we provide another setting (see Example 3) where J is not monotonic but where the optimal menu is still a solution to the problem described in Theorem 1.

EXAMPLE 1 (Asymmetric Information about Loss Probabilities): *Consider an environment with asymmetric information about loss probabilities as defined in equation (1), i.e.,*

$$H_\theta(l) = \theta\big[Q(l) - 1\big] + 1.$$

In this case, the derivative $\partial J(l,\theta)/\partial\theta$ equals

$$-\big[Q(l) - 1\big]g'\big(H_\theta(l)\big)\left[\theta - \frac{1 - F(\theta)}{f(\theta)}\right]' + \big[Q(l) - 1\big] + \frac{1 - F(\theta)}{f(\theta)}\big[Q(l) - 1\big]^2 g''\big(H_\theta(l)\big).$$

For J to be nondecreasing in θ, we need

$$g'\big(H_\theta(l)\big)\left[\theta - \frac{1 - F(\theta)}{f(\theta)}\right]' - 1 + \frac{1 - F(\theta)}{f(\theta)}\big[1 - Q(l)\big]g''\big(H_\theta(l)\big) \geq 0.$$

Assuming that the hazard rate is increasing, the above inequality holds if $g'\big(H_{\bar\theta}(0)\big) = g'\big(1 - \bar\theta\big) > 1$. Intuitively, if the maximal possible probability of a loss $\bar\theta$ is sufficiently small, then the inequality will hold for all relevant types θ. For instance, if $g(p) = (1 - r)p + rp^2$, then $g'(p) > 1 \Leftrightarrow p > 1/2$, so that $g'\big(H_\theta(l)\big) > 1$ holds for any l as long as $\theta < 1/2$. Of course, real-life insurance data present accident probabilities that are much lower than 1/2.

We conclude the current section with the following comparative statics result showing that the insurance provider benefits from a higher risk aversion of the agent.

[26] See Shaked and Shanthikumar (2007, Section 8.C) for a definition. Beyond the stochastic monotonicity in θ already assumed above, this means that, for each loss l, $1 - H_\theta(l)$ is concave in θ.

PROPOSITION 2: *Assume that $g_2(p) < g_1(p)$ for any $p \in (0,1)$; i.e., g_2 represents an agent with a higher risk aversion. Assume also that the optimal retention function is submodular when the agent's preference is represented by g_1. Then, the insurer's profit in the case where the agent's preference is represented by g_2 is higher than the profit in the case where the preference is represented by g_1.*

A. *Optimality of Deductibles or Coverage Limits*

We now display conditions under which it is optimal to restrict attention to two special classes of mechanisms, most often used in practice: the first class consists of menus of contracts of the form $(D,t) = \big(D(\theta),t(\theta)\big)_\theta$, one for each type θ, where each contract specifies a *deductible* $D(\theta) \in [0,\bar{L}]$ and a *premium* $t(\theta) \in \mathbb{R}$. For a fixed risk type θ, the associated retention function is

$$R_D(l,\theta) = \begin{cases} D(\theta), & \text{if } l \geq D(\theta); \\ l, & \text{if } l < D(\theta). \end{cases}$$

The second class consists of menus of coverage limits $(C,t) = \big(C(\theta),t(\theta)\big)_\theta$ where all losses up to $C(\theta) \in [0,\bar{L}]$ are covered for type θ and where $t(\theta)$ is the corresponding premium. For a fixed risk type θ, the associated retention is

$$R_C(l,\theta) = \begin{cases} 0, & \text{if } l \leq C(\theta); \\ l - C(\theta), & \text{if } l > C(\theta). \end{cases}$$

For convenience, we sometimes refer to the first class as *deductible contracts* and the second class as *cap contracts*. Both contracts respect the ex post moral hazard conditions.

To see what is special about these two types of contracts, consider a setting without adverse selection, i.e., where there is a single publicly known risk type θ with corresponding loss distribution H_θ. Then, any strong risk-averse agent (in the sense of aversion to mean-preserving spreads) prefers the deductible contract to any other contract with the same expected cost to the insurer and prefers any contract to the cap contract with the same expected cost. The first argument is well known (see, for example, Van Heerwaarden, Kaas, and Goovaerts 1989).[27] We reproduce its short proof for completeness and also because we use it for proving the second, apparently new part about the contract with a coverage limit.[28]

Denote by $E[I]$ the expected cost of providing the insurance contract I to a type θ agent.[29]

[27] It generalizes famous results by Arrow (1963) and by Borch (1960), who showed that deductibles lead to the lowest variance among all contracts with the same cost.

[28] We recall here that the "live-or-die" contract studied by Innes (1990) is not doubly monotonic and is thus different from a contract with a cap.

[29] The classical literature following Arrow assumes that the premium is given by $P(I) = (1+\delta)E[I]$, where $\delta \geq 0$ is the load factor (or markup): thus, in that literature the premium is proportional to the expected cost of providing insurance.

THEOREM 2: *For a given contract* $0 \leq I \leq L$ *satisfying Assumption 1, let* $D \geq 0$ *be a solution to* $E\left[(L-D)_+\right] = E[I]$, *and let* C *be a solution to* $E\left[\min\{L, C\}\right] = E[I]$. *Then*

$$R_C(\cdot,\theta) \leq_{SOSD} R_I(\cdot,\theta) \leq_{SOSD} R_D(\cdot,\theta),$$

where SOSD denotes second-order stochastic dominance.

The above result implies that, if types are observable and fixing the insurance provision cost, **any** strong risk-averse agent is willing to pay most (least) for a deductible (coverage limit) contract. It follows that any deviation from deductible policies must be driven by the incentive constraints coming from types being unobservable. We show below that, with Yaari utility, these constraints induce the insurer to offer a coverage limit in some cases. This is the worst contract—with a given cost—for the agent.

We now present sufficient conditions under which the two simple forms of insurance discussed above are optimal within the general class of mechanisms. We then provide some examples to illustrate when the conditions hold.

THEOREM 3: *Assume that the virtual value* $J(l,\theta)$ *is nondecreasing in* θ *for all* l.

(*i*) *Suppose that, for each* θ, *there exists a unique* $l^*(\theta)$ *such that* $J(l,\theta) \leq 0$ *for* $l \leq l^*(\theta)$ *and* $J(l,\theta) \geq 0$ *for* $l \geq l^*(\theta)$. *Then the profit-maximizing mechanism consists of a menu of deductible-premium pairs* $(D,t) = \left(D(\theta),t(\theta)\right)_\theta$.

(*ii*) *Suppose that, for each* θ, *there exists a unique* $l^*(\theta)$ *such that* $J(l,\theta) \geq 0$ *for* $l \leq l^*(\theta)$ *and* $J(l,\theta) \leq 0$ *for* $l \geq l^*(\theta)$. *Then the profit-maximizing mechanism consists of a menu of cap-premium pairs* $(C,t) = \left(C(\theta),t(\theta)\right)_\theta$.

If the virtual value $J(l,\theta)$ is single crossing from above, then the insurer finds it profitable to cover small losses but not profitable to cover large ones. Therefore, a menu of coverage limits becomes optimal. If $J(l,\theta)$ is single crossing from below, then the insurer finds it profitable to cover large losses but not small ones. Therefore, a menu of deductibles becomes optimal. The formal proof (see Appendix) shows that the extreme contracts that maximize profit for each type have the above structure and that submodularity—and hence incentive compatibility—is also satisfied.

EXAMPLE 2: *Consider the case where the probability of a loss is the agent's private information* $H_\theta(l) = 1 - \theta + \theta Q(l)$. Let $r \in [0,1]$, and let [30]

$$g(p) = rp^2 + (1-r)p.$$

[30]This specification of g corresponds to Kőszegi and Rabin's (2006) loss-averse preferences with linear utility over outcomes (see Section IA), where we set $r = \lambda - 1$. A higher level of r indicates that the agent is more risk averse.

Suppose that $\bar{\theta} < 1/2$ *and that F has a monotonically increasing hazard rate. We obtain that*

$$J(l,\theta) = H_\theta(l) - g\big(H_\theta(l)\big) + \frac{1 - F(\theta)}{f(\theta)} g'\big(H_\theta(l)\big) \frac{\partial H_\theta(l)}{\partial \theta}$$

$$= r\big[1 - Q(l)\big]\left\{ \left[\theta - \frac{1+r}{r}\frac{1 - F(\theta)}{f(\theta)}\right] - \theta\big[1 - Q(l)\big]\left[\theta - 2\frac{1 - F(\theta)}{f(\theta)}\right]\right\}$$

is nondecreasing in θ. *Moreover, for any fixed* θ, $J(\cdot,\theta)$ *crosses zero at most once from below. Let* θ^* *denote the solution to*

$$\theta - \frac{1+r}{r}\frac{1 - F(\theta)}{f(\theta)} = 0,$$

and let θ^{**} *denote the solution to*

$$\theta - \frac{1+r}{r}\frac{1 - F(\theta)}{f(\theta)} = \theta\left[\theta - 2\frac{1 - F(\theta)}{f(\theta)}\right].$$

Then the profit-maximizing mechanism consists of a menu of deductible-premium pairs with a single deductible per risk type, denoted by $\big(D^*(\theta), t^*(\theta)\big)$. *This mechanism offers no insurance to agents with accident probabilities* $\theta < \theta^*$, *full insurance to agents with accident probabilities* $\theta > \theta^{**}$, *and a deductible*

$$D^*(\theta) = Q^{-1}\left\{1 - \frac{\theta - \frac{1+r}{r}\frac{1 - F(\theta)}{f(\theta)}}{\theta\left[\theta - 2\frac{1 - F(\theta)}{f(\theta)}\right]}\right\}$$

for $\theta \in [\theta^*, \theta^{**}]$. *Note that, for all* θ, $D^*(\theta)$ *is nonincreasing in r, the agent's degree of loss aversion.*

In the above example, both cutoff points θ^* and θ^{**} are independent of the loss distribution Q and are thus solely determined by the agent's distribution of loss-probabilities. In contrast, the optimal deductibles for types between the cutoff points are jointly determined by the loss distribution and by the type distribution.

For the special case $r = 1$, i.e., for $g(p) = p^2$, we obtain that $\theta^* = \theta^{**}$ and the optimal menu offers either full or no insurance. For all other cases, $\theta^* < \theta^{**}$. This suggests that, with a random loss, the full-or-no insurance policy can be optimal if the agents are sufficiently risk averse. This contrasts the finding of Chade and Schlee (2012) in a framework with expected utility: they show that full insurance is never optimal even if the loss is deterministic.

EXAMPLE 3: *Assume that* $g(p) = p^2$ *and that*

$$H_\theta(l) = 1 - e^{-\frac{l}{\theta}}.$$

Here, the agent's private information θ *is the mean of the exponential distribution of losses. We obtain that*

$$\begin{aligned} J(l,\theta) &= H_\theta(l) - g\big(H_\theta(l)\big) + \frac{1-F(\theta)}{f(\theta)} g'\big(H_\theta(l)\big)\frac{\partial H_\theta(l)}{\partial \theta} \\ &= H_\theta(l)\left\{1 - H_\theta(l) + \frac{2\big[1-F(\theta)\big]}{f(\theta)} \times \frac{\partial H_\theta(l)}{\partial \theta}\right\} \\ &= H_\theta(l)e^{-\frac{l}{\theta}}\left\{1 - \frac{2\big[1-F(\theta)\big]}{f(\theta)} \times \frac{l}{\theta^2}\right\}. \end{aligned}$$

As the function

$$1 - \frac{2\big[1-F(\theta)\big]}{f(\theta)} \times \frac{l}{\theta^2}$$

is decreasing in l, *we obtain that* $J(l,\theta)$ *always crosses zero from above. Moreover, for each* θ, *the solution to* $J(l,\theta) = 0$ *is given by*

$$C^*(\theta) = \frac{\theta^2 f(\theta)}{2\big[1-F(\theta)\big]}$$

that is increasing in θ. *Thus, a contract with coverage limits* $\big(C^*(\theta)\big)_\theta$ *is optimal.*[31]

Remark: If $\int l dQ(l) = 1$ and if F is supported on $\big[0,\bar{\theta}\big]$ with $\bar{\theta} < 0.5$, then $E\big[L(\theta)\big] = \theta$ both in Example 2, where the agent's private information is about the probability of an accident, and in Example 3, where the private information is about the mean size of a loss. Despite the fact that, for any θ, the cost of providing full insurance to an agent with type θ is exactly the same in both examples, the respective optimal incentive-compatible contracts look fundamentally different. In the former case (private information on accident probability), the profit-maximizing contract provides insurance in an optimal way for the agent: as we showed in Theorem 2, a deductible contract minimizes the expected cost to the principal of providing a given utility level to the agent. In contrast, in the case where the agent's private

[31] In this example $J(l,\theta)$ is not necessarily nonincreasing in θ for all l. But the ensuing solution is nevertheless incentive compatible.

information is about loss size, the realized loss is informative about the agent's risk type: a higher loss is indicative of a higher-risk type. By introducing a coverage limit—recall that, keeping the cost fixed, this is the worst insurance contract for the agent—the insurer most effectively discourages high-risk types from claiming to be low-risk types (as they would then suffer from the reduction in coverage limit). The revenue gain from this reduction in information rents dominates the efficiency loss due to the very inefficient provision of insurance.

Finally, we note that, in reality, the set of different risk types that can be plausibly discerned/screened without paying exorbitant costs is probably discrete. If the set of types is small (say low, middle, and high probability of accident), there will be an optimal menu of contracts that also consists of a small number of different contracts. Together with regulatory constraints on consumer obfuscation, this explains the prevalence of menus with a small finite number of options. In the next section we illustrate a related setting where there is a finite number of discernible losses—this also leads to an optimal menu that is finite.

B. *An Illustration: A Finite Number of Possible Losses*

In this section we specialize our model to the case where the type θ represents the probability of an accident and where the distribution of losses can only take a finite number of values, independent of type.[32] Thus,

$$H_\theta(l) \;=\; 1 - \theta + \theta Q(l),$$

where Q is a given distribution with discrete support. This finite-loss case is relevant in practice since the definition/verification of a loss cannot be too refined without incurring extra costs. We restrict attention here to contracts with deductibles. In Theorem 3 and in Example 2, we illustrated when this restriction is without loss of generality.

The optimal mechanism in the class of contracts with deductibles takes a commonly seen form: a basic deductible/premium contract, supplemented by a finite ladder of additional fees that, if added to the basic premium, gradually reduce the deductible until possibly reaching full insurance.[33]

PROPOSITION 3: *Assume that the probability of an accident is θ and that, conditional on an accident, there are n different levels of loss $l_1 < l_2 < \ldots < l_n$ with probabilities $p_1, \ldots, p_n$, respectively, where $p_i \geq 0, \forall i$, and $\sum p_i = 1$. Then, there exists an optimal contract in the class of contracts with deductibles that offers at most $n + 1$ of deductibles. For each offered deductible D, it holds either that $D = 0$ (full insurance) or there exists $1 \leq i \leq n$ such that $D_i = l_i$.*

A simple corollary can now be obtained for the focal case studied in almost the entire theoretical literature where the loss is deterministic, i.e., there is only one possible loss equal to l. By the above result, we obtain that there exists an optimal

[32] The proofs for all results in this section can be found in online Appendix B.

[33] See Hoppe, Moldovanu, and Ozdenoren (2011) for an alternative explanation of coarse menus.

insurance contract that offers either no insurance or full insurance (zero deductible).[34] This has been previously shown by DeFeo and Hindriks (2014).

COROLLARY 1 (Single Loss Case): *Consider the case of a single loss level* $H_\theta(z) = 1 - \theta + \theta\mathbf{1}\{z \geq l\}$. *Assume that the virtual value*

$$J(\theta) = (1-\theta) - g(1-\theta) - \frac{1-F(\theta)}{f(\theta)} g'(1-\theta)$$

crosses zero once from negative to positive at $\theta = \theta^*$.[35] *Then the optimal contract offers no insurance (i.e.,* $D(\theta) = l$*) to types* $\theta \leq \theta^*$ *and full insurance (i.e.,* $D(\theta) = 0$*) to types* $\theta \geq \theta^*$. *Moreover, the expected profit is given by*

$$l\left\{ \int_{\theta^*}^1 (1-\theta) f(\theta) d\theta - g(1-\theta^*)\left[1 - F(\theta^*)\right] \right\}.$$

The insurer makes higher profits from lower types that are buying this contract, while she makes lower profits (or even losses) from higher types. Yet the only possibility to attract lower types to acquire such a contract is to reduce its price.

EXAMPLE 4: *Consider loss levels* $l_1 = 1$ *and* $l_2 = 2$ *with probabilities* $p_1 = p_2 = 1/2$, *respectively. Assume that* $g(p) = p^{3/2}$ *and that types* θ *distribute uniformly on* $[0,1]$. *The best full insurance contract sells to all types above* 0.803. *Now let*

$$\theta^* = \frac{1}{25p_2}\left(15p_2 - 2\sqrt{-15p_2 + 16} + 8\right) = 0.774,$$

$$\theta^{**} = 0.84.$$

The insurer can obtain a higher profit than that obtained by offering full insurance only: she can offer a basic insurance contract with deductible l_1 *and premium* $(l_2 - l_1)\left[1 - g\left(1 - (1/2)\theta^*\right)\right]$, *combined with an option to reduce the deductible to zero at the extra price of* $l_1\left[1 - g(1 - \theta^{**})\right]$. *Then types below* θ^* *obtain no insurance and pay zero, types in the interval* $[\theta^*, \theta^{**})$ *obtain partial insurance* $(D = l_1)$ *and pay* $(l_2 - l_1)\left[1 - g\left(1 - (1/2)\theta^*\right)\right]$, while types above θ^{**} *are fully insured* $(D = 0)$ *and pay* $(l_2 - l_1)\left[1 - g\left(1 - (1/2)\theta^*\right)\right] + l_1\left[1 - g(1 - \theta^{**})\right]$.

Remark: Whenever there is a unique loss level, our agent faces a binary lottery. Then, the above analysis holds for a wider class of utility functions that coincide with a Yaari utility for the class of binary lotteries, e.g., well-known (nonexpected) utilities displaying constant risk aversion (see Safra and Segal 1998). Some of these preferences are described in online Appendix C.

[34] In particular, offering full insurance for all insured types is optimal independent of the degree of risk aversion and independent of the distribution of accident probabilities. These last two model primitives only determine the set of insured types.

[35] The result is correct even if the virtual value crosses zero from negative to positive several times. Then θ^* must be one of the crossing values.

IV. Conclusion

We have analyzed an insurance model with adverse selection where the loss distribution depends on the risk type (that is private information) in a very general form. The insured agents have a dual utility function. In a reinsurance context this means that the primary insurer uses a coherent risk measure in order to assess its risk, while the reinsurer is risk neutral.

A main difference between our model and most of the literature without adverse selection is the pricing formula: instead of assuming formulas such as cost plus a markup, premia are here endogenously derived from the incentive compatibility and individual rationality constraints.

We have shown that layer contracts are optimal under some regularity conditions. We also focused on menus consisting of very simple contracts involving either deductibles or coverage limits, and we exhibited conditions under which such menus are optimal in the general class of insurance contracts where, for each risk type, higher losses lead both to a higher coverage and to a higher retention.

Appendix

PROOF OF PROPOSITION 1:

(i) We note that $U(\theta,\theta')$ is absolutely continuous in θ, with derivative

$$\frac{\partial U(\theta,\theta')}{\partial\theta} = \int_0^{\bar{L}} \frac{\partial R(l,\theta')}{\partial l} g'\big(H_\theta(l)\big)\frac{\partial H_\theta(l)}{\partial\theta}dl.$$

The above equality follows since

$$\left|\frac{\partial R(l,\theta')}{\partial l} g'\big(H_\theta(l)\big)\frac{\partial H_\theta(l)}{\partial\theta}\right| \le cg'\big(H_\theta(l)\big).$$

The bound holds because $|\partial R/\partial l| \le 1$ and because $|\partial H_\theta/\partial\theta| \le c < \infty$ by assumption. Note also that $\int_0^{\bar{L}} g'\big(H_\theta(l)\big)dl$ is finite by assumption. By the Envelope Theorem (see, e.g., Theorem 2 in Milgrom and Segal 2002), in any incentive-compatible mechanism, the agent's certainty equivalent is absolutely continuous and is given by

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta}\left[\int_0^{\bar{L}} \frac{\partial R(l,s)}{\partial l} g'\big(H_s(l)\big)\frac{\partial H_s(l)}{\partial s}dl\right]ds.$$

It follows that

$$\begin{aligned}
-t(\theta) &- \int_0^{\bar{L}}\big[1 - g\big(H_\theta(l)\big)\big]\frac{\partial R(l,\theta)}{\partial l}dl \\
&= U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta}\left[\int_0^{\bar{L}} \frac{\partial R(l,s)}{\partial l} g'\big(H_s(l)\big)\frac{\partial H_s(l)}{\partial s}dl\right]ds \\
\Leftrightarrow\ t(\theta) &= -U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta}\left[\int_0^{\bar{L}} \frac{\partial R(l,s)}{\partial l} g'\big(H_s(l)\big)\frac{\partial H_s(l)}{\partial s}dl\right]ds \\
&\quad - \int_0^{\bar{L}}\big[1 - g\big(H_\theta(l)\big)\big]\frac{\partial R(l,\theta)}{\partial l}dl.
\end{aligned}$$

The insurer's expected revenue equals

$$\int_{\underline{\theta}}^{\bar{\theta}} t(\theta)f(\theta)d\theta = -\int_{\underline{\theta}}^{\bar{\theta}}\int_0^{\bar{L}}\frac{\partial R(l,\theta)}{\partial l}\left[1 - g(H_\theta(l)) + \frac{1 - F(\theta)}{f(\theta)}g'(H_\theta(l))\frac{\partial H_\theta(l)}{\partial\theta}\right] \times f(\theta)dld\theta - U(\underline{\theta}),$$

where we used integration by parts to obtain the equality. Her profit equals

$$\begin{aligned}\pi(R) &= \int_{\underline{\theta}}^{\bar{\theta}}\left\{t(\theta) - E[L(\theta)] + \int_0^{\bar{L}}[1 - H_\theta(l)]\frac{\partial R(l,\theta)}{\partial l}dl\right\}f(\theta)d\theta \\ &= \int_{\underline{\theta}}^{\bar{\theta}}\left[-E[L(\theta)] - \int_0^{\bar{L}}\frac{\partial R(l,\theta)}{\partial l}J(l,\theta)dl\right]f(\theta)d\theta - U(\underline{\theta}),\end{aligned}$$

where

$$J(l,\theta) = H_\theta(l) - g(H_\theta(l)) + \frac{1 - F(\theta)}{f(\theta)}g'(H_\theta(l))\frac{\partial H_\theta(l)}{\partial\theta}.$$

(ii) Suppose that $R(l,\theta)$ is submodular. Taking any $\theta < \theta'$, we obtain

$$\begin{aligned}U(\theta,\theta') &= -t(\theta') - \int_0^{\bar{L}}[1 - g(H_\theta(l))]\frac{\partial R(l,\theta')}{\partial l}dl \\ &= U(\theta') - \int_0^{\bar{L}}\left[\int_\theta^{\theta'}g'(H_s(l))\frac{\partial H_s(l)}{\partial s}ds\right]\frac{\partial R(l,\theta')}{\partial l}dl \\ &= U(\theta) + \int_\theta^{\theta'}\left[\int_0^{\bar{L}}\frac{\partial R(l,s)}{\partial l}g'(H_s(l))\frac{\partial H_s(l)}{\partial s}dl\right]ds \\ &\quad - \int_\theta^{\theta'}\left[\int_0^{\bar{L}}g'(H_s(l))\frac{\partial H_s(l)}{\partial s}ds\right]\frac{\partial R(l,\theta')}{\partial l}dl,\end{aligned}$$

where the last equality follows from part (i) of this proposition. As $R(l,\theta)$ is submodular, we obtain that $\partial R(l,\theta')/\partial l < \partial R(l,s)/\partial l$ for any $s \in (\theta,\theta')$. Also, $g'(H_s(l))[\partial H_s(l)/\partial s] < 0$ because $\partial H_s(l)/\partial s < 0$. It follows that

$$\int_\theta^{\theta'}\left[\int_0^{\bar{L}}\frac{\partial R(l,s)}{\partial l}g'(H_s(l))\frac{\partial H_s(l)}{\partial s}dl\right]ds \leq \int_\theta^{\theta'}\left[\int_0^{\bar{L}}g'(H_s(l))\frac{\partial H_s(l)}{\partial s}ds\right]\frac{\partial R(l,\theta')}{\partial l}dl,$$

which implies that $U(\theta,\theta') \leq U(\theta)$. Similarly, we can show that $U(\theta',\theta) \leq U(\theta')$ also holds and conclude that the mechanism $(R(\cdot,\theta),t(\theta))_\theta$ is incentive compatible, as desired. ∎

PROOF OF LEMMA 1:

The condition is clearly necessary. For sufficiency, observe that both $U_{NP}(\theta)$ and $U(\theta)$ are decreasing in θ. Moreover, for all θ, it holds that

$$U'_{NP}(\theta) = \int_0^{\bar{L}} g'(H_\theta(z)) \frac{\partial H_\theta(z)}{\partial \theta} dz \leq \int_0^{\bar{L}} \frac{\partial R(z,\theta)}{\partial z} g'(H_\theta(z)) \frac{\partial H_\theta(z)}{\partial \theta} dz = U'(\theta)$$

because $0 \leq \partial R(z,\theta)/\partial z \leq 1$ and because $g'(H_\theta(z))[\partial H_\theta(z)/\partial \theta] \leq 0$ for all θ. Hence, we obtain that

$$U_{NP}(\theta) = U_{NP}(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} U'_{NP}(z)dz \leq U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} U'(z)dz = U(\theta),$$

as desired. ∎

PROOF OF LEMMA 2:

Consider the following simple mechanism: there exists a type θ^* such that

$$R(\theta, l) = \begin{cases} 0, & \text{if } \theta \geq \theta^*; \\ l, & \text{if } \theta < \theta^*. \end{cases}$$

Thus, all types $\theta \geq \theta^*$ are being offered full insurance (i.e., a zero deductible), while types $\theta < \theta^*$ are being offered no insurance at all (i.e., a deductible $\bar{L}$). The expected profit from this mechanism is given by

$$\begin{aligned}
\pi(R) &= -\int_{\underline{\theta}}^{\bar{\theta}} E[L(\theta)] f(\theta) d\theta - \int_{\underline{\theta}}^{\theta^*} \int_0^{\bar{L}} J(\theta, z) dz f(\theta) d\theta - U(\underline{\theta}) \\
&= \int_{\underline{\theta}}^{\theta^*} \int_0^{\bar{L}} \left[g(H_\theta(z)) - H_\theta(z) - \frac{1 - F(\theta)}{f(\theta)} g'(H_\theta(z)) \frac{\partial H_\theta(z)}{\partial \theta} \right] dz f(\theta) d\theta \\
&\quad - \int_{\underline{\theta}}^{\bar{\theta}} E[L(\theta)] f(\theta) d\theta - U(\underline{\theta}).
\end{aligned}$$

Since $E[L(\theta)] = \int_0^{\bar{L}} [1 - H_\theta(z)] dz$, we can rewrite

$$\begin{aligned}
\pi(R) &= -\bar{L} + \int_{\theta^*}^{\bar{\theta}} \int_0^{\bar{L}} H_\theta(z) dz f(\theta) d\theta - U(\underline{\theta}) - \int_0^{\bar{L}} \left\{ [1 - F(\theta)] g(H_\theta(z)) \right\}_{\underline{\theta}}^{\theta^*} dz \\
&= -\bar{L} - U(\underline{\theta}) + \int_{\theta^*}^{\bar{\theta}} \int_0^{\bar{L}} H_\theta(z) dz f(\theta) d\theta + \int_0^{\bar{L}} g(H_{\underline{\theta}}(z)) dz \\
&\quad - \int_0^{\bar{L}} [1 - F(\theta^*)] g(H_{\theta^*}(z)) dz.
\end{aligned}$$

At $\theta^* = \bar{\theta}$,

$$\begin{aligned}
\pi(R) &= -\bar{L} - U(\underline{\theta}) + \int_0^{\bar{L}} g(H_{\underline{\theta}}(z)) dz \\
&= -\bar{L} + \int_0^{\bar{L}} [1 - g(H_\theta(z))] dz + \int_0^{\bar{L}} g(H_{\underline{\theta}}(z)) dz = 0.
\end{aligned}$$

Moreover, we have that

$$\begin{aligned}\frac{\partial\pi}{\partial\theta^*} &= -\int_0^{\bar{L}} f(\theta^*)H_{\theta^*}(z)dz + \int_0^{\bar{L}} f(\theta^*)g\big(H_{\theta^*}(z)\big)dz \\ &\quad - \int_0^{\bar{L}} \big[1 - F(\theta^*)\big]g'\big(H_{\theta^*}(z)\big)\frac{\partial H_{\theta^*}(z)}{\partial\theta}dz \\ &= -\int_0^{\bar{L}} f(\theta^*)\big[H_{\theta^*}(z) - g\big(H_{\theta^*}(z)\big)\big]dz \\ &\quad - \int_0^{\bar{L}} \big[1 - F(\theta^*)\big]g'\big(H_{\theta^*}(z)\big)\frac{\partial H_{\theta^*}(z)}{\partial\theta}dz.\end{aligned}$$

At $\theta^* = \bar{\theta}$,

$$\frac{\partial\pi}{\partial\theta^*} < 0$$

whenever $H_{\bar{\theta}}$ is not degenerate. Continuity guarantees that the derivative remains negative in some interval to the left of $\theta^* = \bar{\theta}$. Hence, this simple mechanism where sufficiently high types are fully insured while all other types remain uninsured generates a strictly positive expected profit for the insurer. The optimal contract generates expected profits not lower than this simple contract. ∎

PROOF OF THEOREM 1:

A retention function $R: [0,\bar{L}) \times \Theta \to [0,\bar{L})$ is feasible and satisfies Assumption 1 if and only if $R(0,\theta) = 0$ and $\partial R(l,\theta)/\partial l \in [0,1]$ almost surely. Fixing θ, any subsequence of functions $R(\cdot,\theta)$ with these properties is uniformly bounded, Lipschitz-continuous, and hence uniformly continuous. The Arzela-Ascoli Theorem yields that, for every θ, any such sequence has a uniformly convergent subsequence.[36] Thus, absent incentive constraints, a collection $\big(R(\cdot,\theta)\big)_\theta$ that maximizes profit must exist by the continuity of the profit functional, by the compactness of the set of retention functions that satisfy Assumption 1 for each θ, and by Tychonoff's Product Theorem. Fixing again θ, note that the set of feasible policies is convex and that the objective is linear in $R(\cdot,\theta)$. Bauer's maximum principle yields that, for each θ, a maximum is attained at an extreme point. The extreme points of the set of feasible retention functions $R(\cdot,\theta)$ that satisfy Assumption 1 are continuous in l and are such that $\partial R(l,\theta)/\partial l \in \{0,1\}$ almost everywhere.[37] Thus, at the maximum, an increase in the loss l is either

[36] The classical Arzela-Ascoli Theorem assumes a compact support. In order to obtain compactness and the existence of extreme points for the case where the support of losses is unbounded, we use instead the extension to a σ–compact and locally compact Hausdorff space (see, for example, Theorem 4.44, p. 137 in Folland 1999).

[37] We wish to thank Martin Pollrich and Andreas Kleiner for insightful discussions about this set of functions. See also the related characterizations of extreme points of the unit ball of Lipschitz functions (without any monotonicity assumptions), e.g., Smarzewski (1997) and the papers cited there.

completely passed on to the agent with type θ (i.e., $\partial R(l,\theta)/\partial l = 1$) or not at all (i.e., $\partial R(l,\theta)/\partial l = 0$). In order to show that the obtained pointwise solution to

$$\max \pi(R) = \int_{\underline{\theta}}^{\bar{\theta}} \left[-E[L(\theta)] - \int_0^{\bar{L}} \frac{\partial R(l,\theta)}{\partial l} J(l,\theta) dl \right] f(\theta) d\theta,$$

subject to

$$0 \leq \frac{\partial R(l,\theta)}{\partial l} \leq 1, \quad \text{for all} \quad \theta \in \Theta$$

is indeed optimal, we need to show that the resulting retention $R: [0,\bar{L}) \times \Theta \to [0,\bar{L})$ is incentive compatible. To establish this property, it is enough to show that R is submodular. Take any $\theta < \theta'$. The assumption that $J(l,\theta)$ is nondecreasing in θ for all l ensures that the pointwise maximization solution to the above problem satisfies

$$\frac{\partial R(l,\theta')}{\partial l} \leq \frac{\partial R(l,\theta)}{\partial l}$$

for all l. That is, R is submodular, as desired. ■

PROOF OF PROPOSITION 2:

Let R_{g_1} be the optimal retention function if preferences are represented by g_1. Since $R_{g_1}(l,\theta)$ is submodular, it is also implementable if the preferences are represented by g_2. For every θ, we show that if we use the retention function $R_{g_1}(l,\theta)$ also for the agent with preferences represented by g_2, then

$$t_{g_2}(\theta) > t_{g_1}(\theta),$$

where t_{g_i} is the premium in case the preferences are represented by g_i and the retention function is $R_{g_1}(l,\theta)$. Recall that the premium is given by

$$\begin{aligned} t(\theta) = {} & -U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \left[\int_0^{\bar{L}} \frac{\partial R(l,s)}{\partial l} g'(H_s(l)) \frac{\partial H_s(l)}{\partial s} dl \right] ds \\ & - \int_0^{\bar{L}} [1 - g(H_\theta(l))] \frac{\partial R(l,\theta)}{\partial l} dl. \end{aligned}$$

As $R(l,s)$ is submodular, it follows that $\partial R(l,\theta)/\partial l$ is decreasing in θ. Furthermore, as $\partial R(l,\theta)/\partial l \in [0,1]$ for every fixed value of l, the function $\phi_l(\theta) = -\partial R(l,\theta)/\partial l$ defines a measure over Θ. We can rewrite the transfer of type θ as follows:

$$\begin{aligned} t(\theta) = {} & -U(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \left[\int_0^{\bar{L}} \frac{\partial R(l,s)}{\partial l} g'(H_s(l)) \frac{\partial H_s(l)}{\partial s} dl \right] ds \\ & - \int_0^{\bar{L}} [1 - g(H_\theta(l))] \frac{\partial R(l,\theta)}{\partial l} dl \end{aligned}$$

$$= -U(\underline{\theta}) + \int_0^{\bar{L}} \int_{\underline{\theta}}^{\theta} \phi_l(s) g'(H_s(l)) \frac{\partial H_s(l)}{\partial s} ds dl$$

$$- \int_0^{\bar{L}} [1 - g(H_\theta(l))] \frac{\partial R(l,\theta)}{\partial l} dl$$

$$= \int_0^{\bar{L}} [1 - g(H_{\underline{\theta}}(l))] dl - \int_0^{\bar{L}} \{\phi_l(s)[1 - g(H_s(l))]\}_{s=\underline{\theta}}^{\theta} dl$$

$$+ \int_0^{\bar{L}} \int_{\underline{\theta}}^{\theta} [1 - g(H_s(l))] d\phi_l(s) dl - \int_0^{\bar{L}} [1 - g(H_\theta(l))] \frac{\partial R(l,\theta)}{\partial l} dl$$

$$= \int_0^{\bar{L}} \left[1 - \frac{\partial R(l,\underline{\theta})}{\partial l}\right] [1 - g(H_{\underline{\theta}}(l))] dl + \int_0^{\bar{L}} \int_{\underline{\theta}}^{\theta} [1 - g(H_s(l))] d\phi_l(s) dl,$$

where the third line follows from integration by parts and from the property that in the optimal mechanism type $\underline{\theta}$ is indifferent whether to participate or not. As $\partial R(l,\theta)/\partial l \in [0,1]$ and as ϕ is a measure, submodularity of $R(l,s)$ implies that $d\phi_l(s) \geq 0$. Hence, the above term is decreasing in g. Therefore, for every fixed retention function, the premium is higher if the agent becomes more risk averse, while the expected cost (given the same retention function) is the same. Adjusting further to the optimal retention for g_2 yields the desired result. ∎

PROOF OF THEOREM 2:

(i) Since $0 \leq I \leq L$, it follows that $R_I \leq L$ and hence that $F_{R_I}(l) \geq F_L(l)$ for all $l \geq 0$, where F denotes here the distribution of the respective random variable. Moreover, $F_{R_d}(l) = F_L(l)$ for $l < D$ and $F_{R_D}(l) = 1$ for $l \geq D$. Therefore, F_{R_I} and F_{R_D} cross exactly once, and hence, the result follows by Theorem 3.A.44 in Shaked and Shanthikumar (point 3.A.59).

(ii) Note that I_C has exactly the same structure as R_D. Hence, the argument above yields $I \leq_{SOSD} I_C$. By assumption, I and R_I are comonotonic random variables. Let $V(I)$ denote the agent's dual utility when he faces lottery given by I, where the dual utility function is *arbitrary*. By the comonotonic additivity of the dual utility (see Yaari 1987), we have

$$V(L) = V(I) + V(R_I) = V(I_C) + V(R_{I_C}).$$

If V represents a risk-averse agent, we obtain that $V(I_C) \geq V(I)$ and hence that $V(R_{I_C}) \leq V(R_I)$. Since, by assumption, $EI = EI_C$, we obtain that $ER_I = ER_{I_C}$. Since the risk-averse Yaari utility V was arbitrary, Theorem 3.A.7 in Shaked and Shanthikumar (due to Chateauneuf, Cohen, and Meilijson 2004) yields that $R_{I_C} \leq_{SOSD} R_I$. ∎

PROOF OF THEOREM 3:

(i) Fix a type θ and consider the term

$$\int_0^{\bar{L}} J(\theta,l)\frac{\partial R(l,\theta)}{\partial l}dl = -\int_0^{\bar{L}} \frac{\partial J(\theta,l)}{\partial l}R(l,\theta)dl$$

that is linear in R. The optimal $R^*(\cdot,\theta)$ must be an extreme point of the feasible set. In particular, $\partial R^*(l,\theta)/\partial l$ exists almost everywhere and equals either zero or one. By the single-crossing assumption, we obtain that a maximum is obtained by setting $\partial R^*(l,\theta)/\partial l = 1$ for $l \leq l^*(\theta)$ and $\partial R^*(l,\theta)/\partial l = 0$ for $l \geq l^*(\theta)$. This yields the extreme point

$$R(l,\theta) = \begin{cases} l, & \text{if } l < l^*(\theta); \\ l^*(\theta), & \text{otherwise.} \end{cases}$$

This is equivalent to setting a deductible $D^*(\theta) = l^*(\theta)$. If the virtual value satisfies the monotonicity condition in the Theorem, then the overall obtained menu $\{D^*(\theta)\}_\theta$ is decreasing in θ. In particular, R is submodular and hence incentive compatible.

(ii) The proof follows as above by first observing that the relevant extreme point satisfies

$$R(l,\theta) = \begin{cases} 0, & \text{if } l < l^*(\theta); \\ l - l^*(\theta), & \text{otherwise;} \end{cases}$$

and hence that

$$\frac{\partial}{\partial l}R(l,\theta) = \begin{cases} 0, & \text{if } l < l^*(\theta); \\ 1, & \text{if } l > l^*(\theta). \end{cases}$$

By the monotonicity assumption, we obtain that $l^*(\theta') \leq l^*(\theta)$ if $\theta' \leq \theta$. In particular, R is submodular and hence incentive compatible.

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