Matching with Transfers
2015 Koopmans Lecture, Yale University
Part 1: Theory

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Introduction: markets for heterogeneous products

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  - Buyer's utility depends on z (e.g. $\sum x_k z_k$, where $x = (x_1, ..., x_k)$ is buyer-specific)

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- How about marriage?

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  - Models of *competition* (although not necessarily perfect)

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- Applications (among many): intrahousehold allocation (crucial!)

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- 'Tractable General Equilibrium'
- Different models are better suited for some purposes than for others.

Issues related to matching: two examples

Example 1: Assortative matching and inequality

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  - Why did correlation change? Did 'preferences for assortativeness' change? How can we define 'preferences for assortativeness'?
  - How do we compare single-adult households and couples? What about intrahousehold inequality?

Example 2: College premium and the demand for college education

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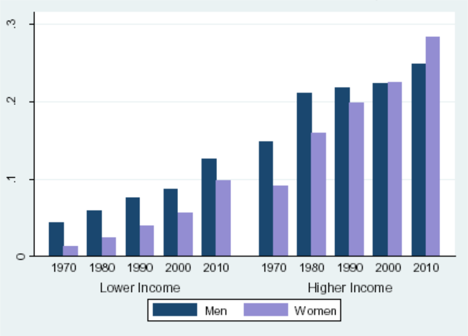
- **Motivation:** remarkable increase in female education, labor supply, incomes worldwide during the last decades.

Figure 3: Fraction of 30- to 34-Year-Olds with College Education, Countries Above Median Per Capita GDP and Below Per Capita GDP, by Sex



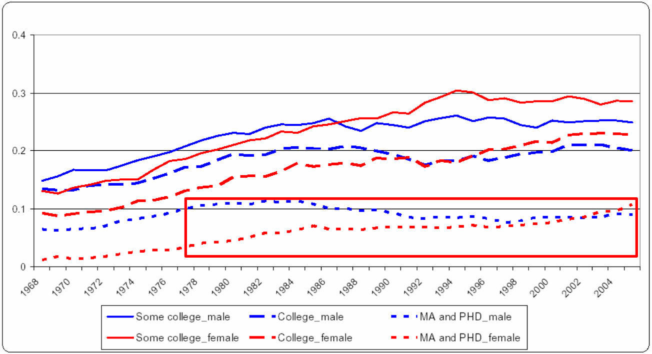
Source: See Figure 1.

Source: Becker-Hubbard-Murphy 2009

Example 2: College premium and the demand for college education

- **In the US:**

Figure 13: Completed Education by Sex, Age 30-40, US 1968-2005



Source: Current Population Surveys.

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- why such different responses by gender?
- impact on intrahousehold allocation?
- impact on household behavior (expenditure, HC investment, etc.)

Roadmap

1. Matching models: general presentation
2. The case of Transferable Utility (TU)
3. Extensions and applications

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   - $\longrightarrow$ requires restrictions on preferences

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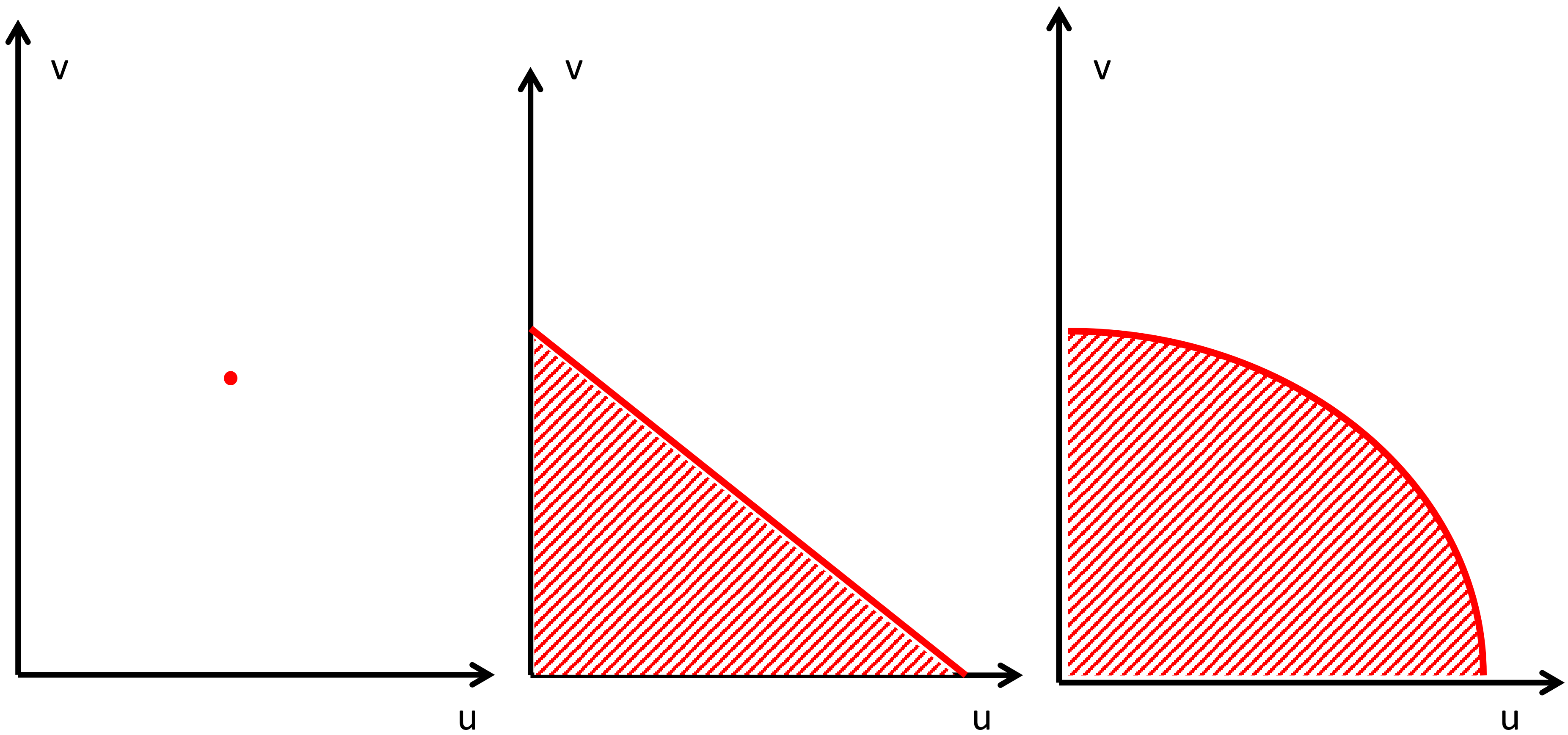

a) NTU   b) TU   c) ITU

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4. Recently: 'general' approaches ('matching with contracts', from Kelso-Crawford to Milgrom-Hatfield-Kominers and friends)
   ... and links with: auction theory, general equilibrium.

Formal structure: Common components

- Compact, separable metric spaces X, Y ('women, men') with *finite* measures F and G. Note that the spaces may be *multidimensional*
- This talk: concentrate on *absolutely continuous* measures.
- Spaces X, Y often 'completed' to allow for singles: $\bar{X} = X \cup \{\emptyset\}$, $\bar{Y} = Y \cup \{\emptyset\}$
- A *matching* defines of a *measure h* on $X \times Y$ (or $\bar{X} \times \bar{Y}$) such that the marginals of h are F and G. Two reasons:
    - allow for randomization
      $\rightarrow$ it is easy to find TU examples (even in one-dimension) where the *unique* stable matching involves randomization
    - emphasize *linearity*
- The matching is *pure* if the support of the measure is included in the graph of some function ϕ
  Translation: matching is *pure* if $y = \phi(x)$ a.e.
  $\rightarrow$ no 'randomization'

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Implications (crucial for empirical implementation)

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- ITU: stable matchings solve

$$u(x) = \max_y \{F(x,y,v(y))\} \text{ and } v(y) = \max_x \{F^{-1}(x,y,u(x))\}$$

for some pair of functions u and v.

Roadmap

1. Matching models: general presentation
2. *The case of Transferable Utility (TU)*
3. Extensions and applications

Transferable Utility (TU)

Definition

A group satisfies TU if there exists monotone transformations of individual utilities such that the Pareto frontier is an hyperplane
$u(x) + v(y) = s(x, y)$ *for all values of prices and income.*

- Two remarks:

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- ... and a question:
  - $\rightarrow$ Consider a model of household behavior: what properties of individual preferences does TU require?

TU and individual preferences

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- Note that: under TU *the group behaves as a single individual* (whose utility is the sum of utilities)

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  - *There exists a dual program, and duality theorem applies*

Duality and optimal transportation (cont.)

- Dual problem: dual functions $u(x), v(y)$ and solve

$$\min_{u,v} \int_X u(x)\, dF(x) + \int_Y v(y)\, dG(y)$$

under the constraint

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- **Basic result:** *A measure h is associated with a stable matching* (h, u, v) *if and only if it solves the primal problem*
- Proof:

$$\begin{aligned}\int_X u(x)\, dF(x) + \int_Y v(y)\, dG(y) &= \int_{X\times Y} (u(x) + v(y))\, dh(x,y) \\ &\geq \int_{X\times Y} s(x,y)\, dh(x,y)\end{aligned}$$

Duality theorem: equality $\Rightarrow u(x) + v(y) = s(x,y) \qquad h$ - a.e.

Duality and optimal transportation (cont.)

- Corollary: *Let s and $\bar{s}$ be two surplus functions. Assume there exists two functions f and g, mapping R^m to R and R^n to R respectively, such that*

$$s(x, y) = \bar{s}(x, y) + f(x) + g(y)$$

Any stable matching for s is a stable matching for $\bar{s}$ and conversely.

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- Question: can we solve the first equation in y?

Links with hedonic models

- Structure: three sets ('buyers' X, 'sellers' Y, 'products' Z) with measures μ, ν, σ.
- Buyer x: quasi linear preferences $U(x,z) - P(z)$; seller y maximizes profit $P(z) - c(y,z)$
- Equilibrium: price function $P(z)$ that clears markets
- Technically: function P *and* measure α on the product set $X \times Y \times Z$ such that

(i) marginal of α on X (resp. Y) coincides with μ (resp. ν)

(ii) for all (x,y,z) in the support of α,

$$U(x,z) - P(z) = \max_{z' \in K} \left(U(x,z') - P(z') \right)$$
$$\text{and } \; P(z) - c(y,z) = \max_{z' \in K} \left(P(z') - c(y,z') \right).$$

- Note that: $c(y,z)$ does *not* depend on x

Links with hedonic models

- Chiappori, McCann and Nesheim (2010): canonical correspondance between QL hedonic models and matching models under TU.
- Specifically, consider a hedonic model and define surplus:

$$s(x,y) = \max_{z\in Z}(U(x,z) - c(y,z))$$

Let η be the marginal of α over $X \times Y$, $u(x)$ and $v(y)$ by

$$u(x) = \max_{z\in K} U(x,z) - P(z) \text{ and } v(y) = \max_{z\in K} P(z) - c(y,z)$$

Then (η, u, v) defines a stable matching. Conversely, starting from a stable matching (η, u, v), for all (x, y, z) we have:

$$\begin{aligned} u(x) + v(y) &\geq s(x,y) \geq U(x,z) - c(y,z) \text{ therefore} \\ c(y,z) + v(y) &\geq U(x,z) - u(x) \end{aligned}$$

For any z, an equilibrium price is any $P(z)$ such that

$$\inf_{y\in J}\{c(y,z) + v(y)\} \geq P(z) \geq \sup_{x\in I}\{u(x,z) - u(x)\}$$

Supermodularity and assortative matching

- Assume X, Y one-dimensional. Then s is strictly supermodular if whenever $x > x'$ and $y > y'$ then

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- Note that the mapping

$$y \rightarrow \frac{\partial s}{\partial x} \text{ is } \textit{injective}$$

Generalization: the twist condition

- Problem: supermodularity and assortative matching are 1-dimensional
- Generalization ('twist' condition):

Definition

The function $s \in C^1$ satisfies the *twist condition* if, for each fixed $x_0 \in X$ and $y_0 \neq y \in Y$, the mapping

$$x \in X \mapsto \delta(x, y, x_0, y_0) = s(x, y) + s(x_0, y_0) - s(x, y_0) - s(x_0, y)$$

has no critical points.

- Equivalently, for almost all x_0 in X,

$$D_x s\left(x_0, y_1\right) = D_x s\left(x_0, y_2\right) \Rightarrow y_1 = y_2$$

  That is, $y \rightarrow D_x\left(x, y\right)$ is *injective*
- Then the stable matching is *unique* and *pure*

The twist condition

Example 1: Index models

- Definition: there exists $I : \mathbf{R}^n \longrightarrow \mathbf{R}$ and $\sigma : \mathbf{R}^{m+1} \longrightarrow \mathbf{R}$ such that:

$$s(x, y) = \sigma(x, I(y)). \tag{1}$$

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- Both cases: if $D_x\sigma(x, i)$ is injective in i then

$$D_x s(x, y) = D_x\sigma(x, I(y)) \neq D_x\sigma(x, I(y_0)) = D_x s(x, y_0)$$

for any y, y_0 such that $I(y) \neq I(y_0) \to$ Twist!

The twist condition
Example 2

- Example (Galichon-Salanié 2013, Dupuy-Galichon 2013, Lindenlaub 2015):

$$s(x,y) = f_X(x) + g_Y(y) + \sum_{k=1}^{K} a_k f_k(x_k) g_k(y_k)$$

- Then

$$D_x s(x,y) - D_x s(x,\bar{y}) = \begin{pmatrix} a_1 f_1'(x_1)\left(g_1(y_1) - g_1(\bar{y}_1)\right) \\ \vdots \\ a_K f_K'(x_K)\left(g_K(y_K) - g_K(\bar{y}_K)\right) \end{pmatrix}$$

- If both the fs and the gs are strictly monotonic, then twist; therefore uniqueness and purity
- Moreover, matching such that x_k increases with y_k (Lindenlaub's 'assortative matching')

Intracouple allocation under TU

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- However, with a continuum of agents, *intramatch allocation of welfare is typically pinned down by the equilibrium conditions*
- Known from the outset, but ...
- ... much easier than you would think

Pinning down intracouple allocation under TU

Assume X, Y one dimensional and s supermodular. Then 3 steps

- Step 1: supermodularity implies assortative matching:
  x matched with $y = \psi(x)$ if *the number of women above* x *equals the number of men above* $\psi(x)$

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- Step 1: supermodularity implies assortative matching:
  x matched with $y = \psi(x)$ if *the number of women above x equals the number of men above $\psi(x)$*
- Step 2: Stability implies

$$u(x) = \max_y s(x, y) - v(y)$$

with the max being reached for $y = \psi(x)$.
Therefore

$$u'(x) = \frac{\partial s}{\partial x}(x, \psi(x)) \text{ and } v'(y) = \frac{\partial s}{\partial y}(\phi(y), y)$$

and

$$u(x) = k + \int_0^x \frac{\partial s}{\partial x}(t, \psi(t))\, dt \;,\; v(y) = k' + \int_0^y \frac{\partial s}{\partial y}(\phi(s), s)\, ds$$

$\rightarrow$ Utilities defined up to two additive constants

Pinning down intracouple allocation under TU

- Step 3: pin down the constants
  - Note that
    $$u(x) + v(\psi(x)) = s(x, \psi(x))$$
    which pins down the sum $k + k'$
  - If one gender in excess supply (say women): the 'last married' woman indifferent between marriage and singlehood
  - Note: typically, discontinuity
  - If equal number (knife-edge situation), indeterminate ...
    ... unless corner solutions

Three extensions

- Imperfectly Transferable Utility (ITU)
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Imperfectly Transferable Utility (ITU)

Motivation

- Limitation of TU models: *all Pareto optimums correspond to the same aggregate behavior*
- Therefore, redistributing power between men and women *cannot* impact the structure of expenditures
- 'Collective' literature: important phenomenon

Imperfectly transferable utilities

General case:

- Transfers possible...
- ... but the 'exchange rate' is not constant.
- In practice:

$$u(x) = P(x, y, v(y))$$

with P decreasing in v, usually increasing in x and y.

- Stability:

$$u(x) \geq P(x, y, v(y)) \ \forall x \in X, y \in Y$$

- But: no longer equivalent to a maximization ('total surplus ' not defined).

Imperfectly transferable utility: theory

- Stability

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and equality if marriage probability positive. Hence:

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satisfied for $x = \phi(y)$

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- Knowing ϕ, if $\partial P/\partial y > 0$, v defined up to a constant by:

$$v'(y) = -\frac{\frac{\partial P}{\partial y}(\phi(y), y, v(y))}{\frac{\partial P}{\partial v}(\phi(y), y, v(y))} > 0$$

Assortativity

- 1st OC:

$$H\left(y,\phi\left(y\right)\right)=0\quad\forall y$$

where

$$H\left(y,x\right)=\frac{\partial P}{\partial y}\left(x,y,v\left(y\right)\right)+v'\left(y\right)\frac{\partial P}{\partial v}\left(x,y,v\left(y\right)\right).$$

therefore

$$\frac{\partial H}{\partial y}+\frac{\partial H}{\partial x}\phi'\left(y\right)=0\quad\forall y,$$

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- 2nd OC:

$$\frac{\partial H}{\partial y}\leq 0 \quad \Leftrightarrow \quad \frac{\partial H}{\partial x}\phi'\left(y\right)\geq 0.$$

or:

$$\left(\frac{\partial^2 P}{\partial x\partial y}\left(\phi\left(y\right),y,v\left(y\right)\right)+v'\left(y\right)\frac{\partial^2 P}{\partial x\partial v}\left(\phi\left(y\right),y,v\left(y\right)\right)\right)\phi'\left(y\right)\geq 0 \ \forall y \tag{3}$$

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$$\max_{L_1, L_2, Q} L_1 Q^{\alpha} + \mu L_2 Q^{\alpha}$$

under

$$Q + w_1 L_1 + w_2 L_2 = \left(w_1 + w_2\right) T$$

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- Then $\mu = w_2 / w_1$, and Pareto frontier:

$$u_1 = -\frac{w_2}{w_1} u_2 + \frac{\alpha}{\left(1+\alpha\right)^2} \frac{\left(w_1 + w_2\right)^2}{w_1} T^2$$

Three extensions

- Imperfectly Transferable Utility (ITU)
- *Multidimensional matching and links with AS models (CMcCP 2015)*
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Matching with different dimensions

- Assume $n < m$
- 'Indifference sets': the same husband y matched with a continuum of potential wives
- In practice:

$$\{x \in X \mid D_y s(x, \bar{y}) = K(\bar{y})\}$$

- If s non degenerate (i.e. if the rank of $D^2_{xy} s = n$) then these sets are submanifolds
- Note that the 'actual' indifference sets depend on the surplus *and* the measures
- Interesting case: $n = 1$

Multi to one dimensional matching ($n = 1$)

- Motivation: 'multidimensional wives' vs 'one-dimensional husbands'
- Crucial notion: *iso-husband curve* (submanifold if s non degenerate)
- Important for two reasons:
  - Theoretical: main outcomes of the matching model; generate testable predictions
  - Empirical: easy to identify (requires specific assumption on the stochastic structure, cf COQ JPE 2009)
- Particular case: index or pseudo index models
- Here:
  - Provide a general method for solving for iso-husband curves
  - If works, then the measure conditions pin down the efficient matching
  - Sufficient condition: *nestedness*

- Potential indifference sets:

$$X_{\bar{y},k} = \{x \in X \mid D_y s(x,\bar{y}) = k\}$$

If s non degenerate, manifold of dimension $m-1$

- Divides X into two pieces: the sublevel set

$$X_{\leq}(y,k) := \{x \in X \mid \frac{\partial s}{\partial y}(x,y) \leq k\}, \tag{4}$$

and its complement $X_{>}(y,k) := X \setminus X_{\leq}(y,k)$.

- For any given $\bar{y}$, choose k such that

$$\mu\left[X_{\leq}(\bar{y},k)\right] = \nu\left[-\infty,\bar{y}\right]$$

- Index model: if $s(x,y) = S(I(x),y)$ then $X_{\leq}(y,k) = \{x \in X \mid I(x) \leq k'\}$ depends on y and k only through k' $\rightarrow$ nested iff twist
- Also true for quasi-index

Multi to one dimensional matching (n = 1)

Construction (cted)

In general: more complicated

- Definition: the model is *nested* if:
  - The sublevel sets $y \in Y \mapsto X_{\leq}(y, k(y))$ depend monotonically on $y \in \mathbf{R}$,
  - Strict inclusion $X_{\leq}(y, k(y)) \subset X_{<}(y', k(y'))$ holding whenever $\nu[(y, y')] > 0$
- Index model: boils down to Spence-Mirrlees. Indeed:
  - The sublevel set $X_{\leq}(y, k)$ does *not* depend on y (depends on k)
  - Monotonicity guaranteed if SM
  - Note that the condition does *not* depend on the measures
- In general: when can we guarantee nestedness?
  - Sufficient conditions involve both the surplus and the measures
  - Nestedness for *all* measures requires quasi-index
  - $\rightarrow$ companion paper

Competitive version of Rochet-Choné

- Model:
  - n-dimensional space of products $z = (z_1, ..., z_n) \in Z \subset \mathbf{R}_+^n$;
  - n-dimensional space of buyers (measure μ):
    $x = (x_1, ..., x_n) \in X \subset \mathbf{R}_+^n$. $\rightarrow$ utility $U(x, z) - P(z)$ where
    $U(x, z) = \sum_{i=1}^n x_i z_i$,
  - One dimensional space of producers (measure ν); profit
    $P(z) - c(y, z)$, where

$$c(y, z) = \frac{1}{2y} \sum_{i=1}^{n} z_i^2$$

  - Either each producer produces one good (real estate), or constant returns to scale
  - $\rightarrow$ Rochet-Choné with competitive producers
  - Producers heterogeneity is not crucial (ν could be Dirac), but competition is.

Competitive version of Rochet-Choné

Resolution:

- Surplus:

$$s(x, y) = \max_{z \in Z} \left(\sum_{i=1}^{n} x_i z_i - \frac{1}{2y} \sum_{i=1}^{n} z_i^2 \right)$$

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  and S satisfies Spence-Mirrlees!
- Consequence: existence, uniqueness, and purity ('assortative matching')

- μ uniform (normalized to have total mass 1) on the quarter disk

$$\left\{(x_1, x_2) \mid x_1^2 + x_2^2 \leq 1, x_1 \geq 0, x_2 \geq 0\right\}$$

- ν uniform on $[1, 2]$.
- Optimal matching:

$$F(x) = |x|^2 + 1$$

- Agent x then buys the product z such that:

$$z_i = x_i \left(\sum_{k=1}^{n} x_k^2 + 1 \right), \quad i = 1, \ldots, n.$$

- Note: *no bunching*

Competitive version of Rochet-Choné

- Utilities:

$$\frac{\partial u}{\partial x_i}(x) = \frac{\partial s}{\partial x_i}(x, F(x)) = x_i \left(1 + \sum_{k=1}^{n} x_k^2\right)$$

which yields

$$u(x) = A + \frac{1}{2}\sum_{i=1}^{n} x_i^2 + \frac{1}{4}\left(\sum_{i=1}^{n} x_i^2\right)^2$$

- Similarly

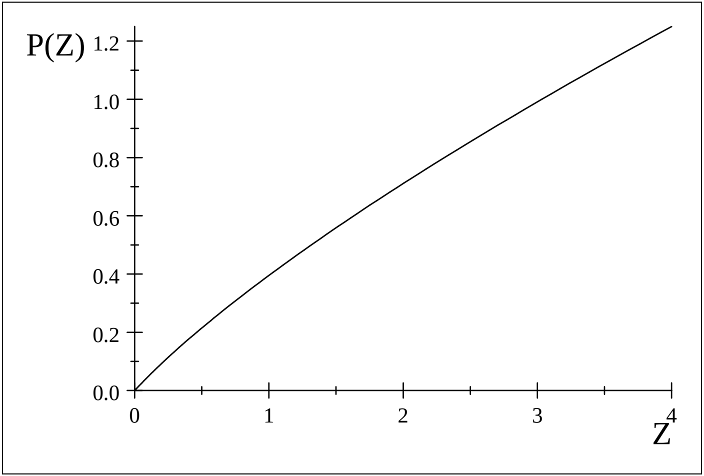
$$v(y) = B + \frac{(y-1)^2}{4}$$

and $A + B = 0$; assume $A = B = 0$ (least productive producer makes zero profit)

- Price: if $Z = \sum_{i=1}^{n} z_i^2$ then

$$(Z, P(Z)) = \left(y^2(y-1), \frac{1}{4}(3y-1)(y-1)\right)$$

Competitive version of Rochet-Choné



Competitive R-C: pricing schedule

Competitive version of Rochet-Choné

Conclusions:

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  - General property
- Model of competition under adverse selection $\longrightarrow$ matching approach provides a natural definition of an equilibrium in such a framework.
- A crucial remark, however, is that the model is characterized by its *private value* nature, since the producer's profit is not directly related to the identity of the consumer buying its product (it only depends on the characteristics of the product and its price)
  $\longrightarrow$ different from common value (e.g. RS)

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    - may influence investment behavior → may explain the puzzle

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    - may influence investment behavior → may explain the puzzle
    - but a structural model is needed!

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  3. Which one is correct?
     → **None**: the investment is typically *efficient*

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  - The fictitious game is much easier to simulate (matching $\rightarrow$ linear programming)