Examining the Corporate Bond Credit Surface

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Abstract

This paper studies heterogeneity in the cross-section and time-series for the credit spreads of corporate bonds. It uses the Merton (1974) model to compare loan-to-value (LTV) and book-to-firm-value (B/V) as measures of leverage for firm debt. It then gathers data on credit spreads for a large sample of bonds and creates credit surfaces that plot credit spreads at a given time as a function of credit rating and leverage. It verifies predictions of the Merton model using a series of regressions and performs a principal components analysis on movements of the surface. The first principal component reveals that most movements of the surface correspond to a simultaneous shift in level, slope, and convexity, and it is related to changes in volatility. The second principal component reveals that there are moments in which the spreads of riskier firms increase in great excess to the spreads of less risky firms, such as the start of the COVID-19 pandemic. It then finds that Fed programs established around that time were more successful at recovering changes in the first principal component to its pre-pandemic level than changes in the second component, which might be because the Fed declared most high-yield bonds to be ineligible for its facilities. The paper argues that times with large moves in the second principal component are plausible points for the Fed to intervene and support credit to riskier borrowers.

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1 Introduction

The Federal Open Market Committee (FOMC) at the Federal Reserve sets the stance of monetary policy by adjusting the federal funds rate, which is a market rate that banks charge each other on unsecured overnight loans. To a good approximation, the fed funds rate is a riskless interest rate. The FOMC sets the fed funds rate at a level so as to achieve its dual mandate of maximum employment and price stability.

The most significant reason that companies cannot borrow at the riskless rate is that they pose a risk of default, due to which investors demand a "credit spread" on their loans above the riskless rate (Fisher 1959). Merton (1974) provided an early theory on how to price default risk. He models a company's debt as a short put position on the total firm value, which evolves according to a stochastic process characterized by some drift and volatility, struck at the book value of debt. Firms with lower levels of volatility and leverage produce smaller expected amounts of default, and they should thus require lower credit spreads.

Investors have other reasons to charge companies a credit spread in addition to the firm's likelihood of default. Fazzari, Hubbard, and Pederson (1988) argue that outside lenders should charge firms an external finance premium due to asymmetric information, as they cannot fully evaluate a company's investment prospects. Elton et al. (2001) document that taxes and non-diversifiable default risk partially account for spreads. De Jong and Driessen (2005) add liquidity risk as another important consideration.

For these reasons, few firms can borrow at the federal funds rate. Thus, it is not the riskless but rather the risky interest rates that pass through into firm borrowing costs and affect investment spending and employment in the aggregate. The literature clarifies that risky interest rates do not move one-for-one with riskless interest rates. Bernanke and Gertler (1995) find that risky interest rates tend to change in the same direction and with a greater magnitude than riskless rates – in other words, credit spreads move in the same direction as an exogenous shock to riskless rates. They name this important monetary policy transmission channel the "credit channel," and they give two subchannels that explain why it works: the balance sheet and bank lending channels. The balance sheet channel suggests that firm balance sheets become weaker with higher rates, as higher rates tend to reduce asset prices and increase interest expense on floating rate liabilities. The bank lending channel suggests that banks tend to face higher costs as rates rise and depositors move their money from low-earning deposits to higher-earning money-market instruments, which serves to increase bank expenses and decrease banks' desire to supply new loans. These effects together make firms riskier and banks less willing to lend, which together increase the credit spread. Bernanke, Gertler, and Gilchrist (1996) extend that result to show that changes in credit spreads are not homogeneous. Specifically, less creditworthy firms further lose access to credit than more creditworthy firms following policy tightening.

Given this impact of credit on the real economy and the heterogeneity in credit markets, it is startling that the Fed speaks minimally about credit. When the Fed does comment about credit, it is generally about broad indices, such as investment grade and high yield bond spreads. Geanakoplos (2016) argues that the Fed should consider a more nuanced view of credit by monitoring and publishing "credit surfaces" for various types of debt in the economy that plot credit spreads as a function of various explanatory variables. These variables might include FICO score and downpayment in mortgage markets or credit ratings and firm leverage in corporate bond markets. The level, steepness, and convexity of the credit surfaces show the specific state of credit conditions beyond what is reflected in the broad indices. Geanakoplos and Rappoport (2019) take a step in this direction by creating credit surfaces for loans in mortgage, corporate bond, and peer-to-peer lending markets.

This paper extends their work on the corporate bond surface. At a high level, it accomplishes three goals. First, it lays out some theoretical and empirical determinants of credit spreads in the cross section, which underpin the credit surface as a good representation of credit conditions. Second, it makes its primary contribution to the literature, which is its findings of how the corporate bond credit surface tends to move over time. Third, it links these movements to Fed policy. At a more detailed level, the paper is structured as follows.

Section 2 describes the Merton model and sets forth a series of five properties that an ideal leverage measure should satisfy. It shows using simulations that LTV, or loan-to-value, is a theoretically ideal leverage measure according to those properties.

Section 3 describes some practical considerations that make LTV challenging to calculate for real bonds given a lack of data. It presents B/V, or Book Value (of Debt) to Firm Value, as a more practical measure of leverage that satisfies the properties described in the previous section over a narrower but still wide set of parameters. This section also presents an alteration to the Merton model that brings it more in line with real firm debt structures and shows that B/V is a suitable leverage measure in that new model. It suggests that B/V improves on the measure of leverage used by Geanakoplos and Rappoport. It then concludes by suggesting that a good credit surface should factor in credit rating, as well as other variables that explain credit spreads.

Section 4 describes the methodology used to construct the new credit surface, which plots credit spreads as a function of B/V and credit rating. It also includes some summary figures and statistics as well as two snapshots of the surface.

Section 5 finds evidence in the data for the five properties described in Section 2. It uses a pricing model for sample credit spreads that incorporates the B/V measure of leverage, along with credit rating, a proxy for economic activity (PMI), and a proxy for volatility (VIX). The coefficients on the pricing model show that the credit surface is upward sloping and convex in both B/V and Rating. When re-estimating the model over periods of high and low volatility, it emerges that upward shifts in volatility raise, steepen, and increase the convexity of the surface.

Section 6 presents a principal component analysis of movements of the credit surface. It shows that most of the variation in movements of the credit surface can be explained by one eigenvector, which appears to be equivalent to a combination of the level, slope, and convexity of the surface. This finding is interesting because it suggests that credit spreads generally move in the same direction but with differing magnitudes that depend on characteristics of the individual bonds. As predicted by the analysis in previous sections, volatility explains a sizable amount of this first principal component. A large part of the remaining variation can be explained by two other eigenvectors, which correspond to an excess shift in convexity and "wobble" of the surface, respectively. The second principal component is of particular interest, as large upward moves in it can price risky borrowers out of credit markets. It is potentially related to levels of risk aversion among investors.

Section 7 uses the principal components from the previous section to estimate the impact of Fed corporate bond credit facilities (CCFs) on the credit surface. It finds that early pandemic moves in financial markets caused a steepening of the credit surface in excess of a typical upwards move. Announcements of the CCFs lowered the surface but did not fully unwind the excess convexity, which this paper suggests might have been because the Fed opened its facilities primarily to investment grade and not high yields bonds.

Section 8 concludes the paper, discusses next steps that might be worth exploring, and articulates a desire for credit to be studied in and applied with more nuance going forward.

2 LTV as an Ideal Leverage Measure

2.1 Merton Model

In his 1974 paper, Robert Merton proposed a simple way to value the debt of a corporation. He argued that the value of a firm can be split into two securities: debt and equity. The value of the firm evolves according to a stochastic process, and the value of the equity at a certain period is a claim on the residual value of the firm after the debt is paid off.

In the discrete case, one can imagine the firm follows a binomial tree process, in which the firm's total value moves up or down at each time period in a manner characterized by some fixed drift and volatility. The values of the debt and equity in this process can be calculated by finding the payoffs at the final time period and using backward induction. Specifically, at each state, the price of the debt or equity in that state is a weighted average of the prices of the debt or equity in the up and down states that follow. The weights are given by risk-neutral probabilities that are derived from the input parameters.

In the continuous limit, the equity becomes equivalent to a call on the firm value struck at the total book value of debt. The debt is equivalent to a short put position on the firm value struck at the total book value of debt. Merton's simplest model assumes that the firm does not pay coupons on the debt nor dividends on the equity. Using the option-pricing formula derived by Black and Scholes (1973) for that model, he derives the following expression for the price of the firm's debt:

$$\text{Price} = Be^{-rT}\{\Phi\left[h_2(d,\sigma^2 T)\right] + \frac{1}{d}\Phi\left[h_1(d,\sigma^2 T)\right]\}$$

$$h_1(d,\sigma^2 T) = -\left[\frac{1}{2}\sigma^2 T - log(d)\right]/\sigma\sqrt{T} \quad , \quad h_2(d,\sigma^2 T) = -\left[\frac{1}{2}\sigma^2 T + log(d)\right]/\sigma\sqrt{T}$$

In the formula, B is the total book value of the firm's debt, r is the riskless interest rate, T is the time to expiration of the bond, Φ is the normal CDF function, $d \equiv Be^{-rT}/V$ where V is the current firm value, and σ is the volatility of the total firm value. Another important thing to note about the formula is that prices are inversely related to yields to maturity, assuming the firm does not default on its debt. Specifically, since we assume that the bonds are zero coupon bonds, the yield (assuming the firm does not default) can be found via:

$$e^{Yield \cdot T} = B/\text{Price} \rightarrow \text{Yield} = \frac{1}{T}\log\left(B/\text{Price}\right)$$

$$\text{Spread} \equiv \text{Yield} - r = -\frac{1}{T}\log\{\Phi\left[h_2(d,\sigma^2 T)\right] + \frac{1}{d}\Phi\left[h_1(d,\sigma^2 T)\right]\}$$

This formula gives the difference between the yield on the firm's debt and the riskless rate, which is the credit spread on the firm's debt.

2.2 Characteristics of an Ideal Leverage Measure

One important characteristic that affects the quality of a firm's debt is its leverage, which represents the amount of debt that a firm has issued relative to the firm's total value. There are many ways one could measure leverage. This subsection provides five properties that an ideal leverage measure should have, and it explains why. As a note, for the following definitions, the "credit surface" refers to the graph of spreads on a firm's debt versus the leverage measure of the firm, holding other factors constant (i.e. increasing B alone).

Property 1: The credit surface is upward sloping in the leverage measure.

Property 2: The credit surface is convex with respect to the leverage measure.

Property 3: The credit surface shifts upward with a rise in σ.

Property 4: The credit surface steepens with a rise in σ.

Property 5: The credit surface becomes more convex with a rise in σ.

These properties are intuitively desirable in a leverage measure for the following reasons:

Property 1: Spreads should increase with leverage, all else equal, because firms with more leverage have a thinner layer of equity protecting their debt from default.

Property 2: The credit surface should steepen with the leverage measure, all else equal, because the difference in credit quality between a firm with little leverage and moderate leverage is smaller than the difference in credit quality between a firm with a moderate amount of leverage and a significant amount of leverage. The spread should also tend to infinity as a firm issues bonds that promise increasingly more than the firm's current value.

Property 3: The credit surface should rise with an increase in volatility, all else equal, because volatility is detrimental to debtholders who lose money when the firm defaults and do not share in the upside when the firm performs well.

Property 4: The credit surface should steepen with an increase in volatility, all else equal, because an increase in volatility should affect firms with a little leverage (whose debt is likely safe) less than firms with significant leverage (whose debt is likely risky).

Property 5: The credit surface should become more convex with an increase in volatility, all else equal, because the gap between how an increase in volatility affects a firm with little leverage and moderate leverage is less than the gap between a firm with moderate leverage and significant leverage.

2.3 Loan to Value

Loan to Value (LTV), as defined by Price/V, satisfies all five properties. A proof of those results is outside the scope of this paper, and is currently being created by Rappaport and Geanakoplos. Figure 1 instead presents some simulations that show how spreads relate to LTV and volatility. It was constructed by simulating a Merton model with parameters $T = 1, r = 0, V = 1$ for $d = B/V \in (0, 10)$ and $\sigma \in \{0.2, 0.4, 0.6, 0.8\}$. LTV was calculated as Price/V = Price. The first and second derivatives were computed as slopes of adjacent points in the graph of the spreads and first derivative graphs, respectively. The three graphs together demonstrate all five properties. Property 1 (spreads increase with LTV) is shown on the first graph, Property 2 (spreads are convex with respect with LTV) is shown on the third graph, Property 3 (credit surface rises with volatility) is shown on the first graph, Property 4 (credit surface steepens with volatility) is shown on the second graph, and Property 5 (credit surface becomes more convex with volatility) is shown on the third graph.

In addition, the use of LTV as a leverage measure also fits well with the analysis in Geanakoplos and Fostel (2008), who develop a theory of endogenous leverage cycles that uses an LTV measure for leverage. In their model, a small group of optimistic investors

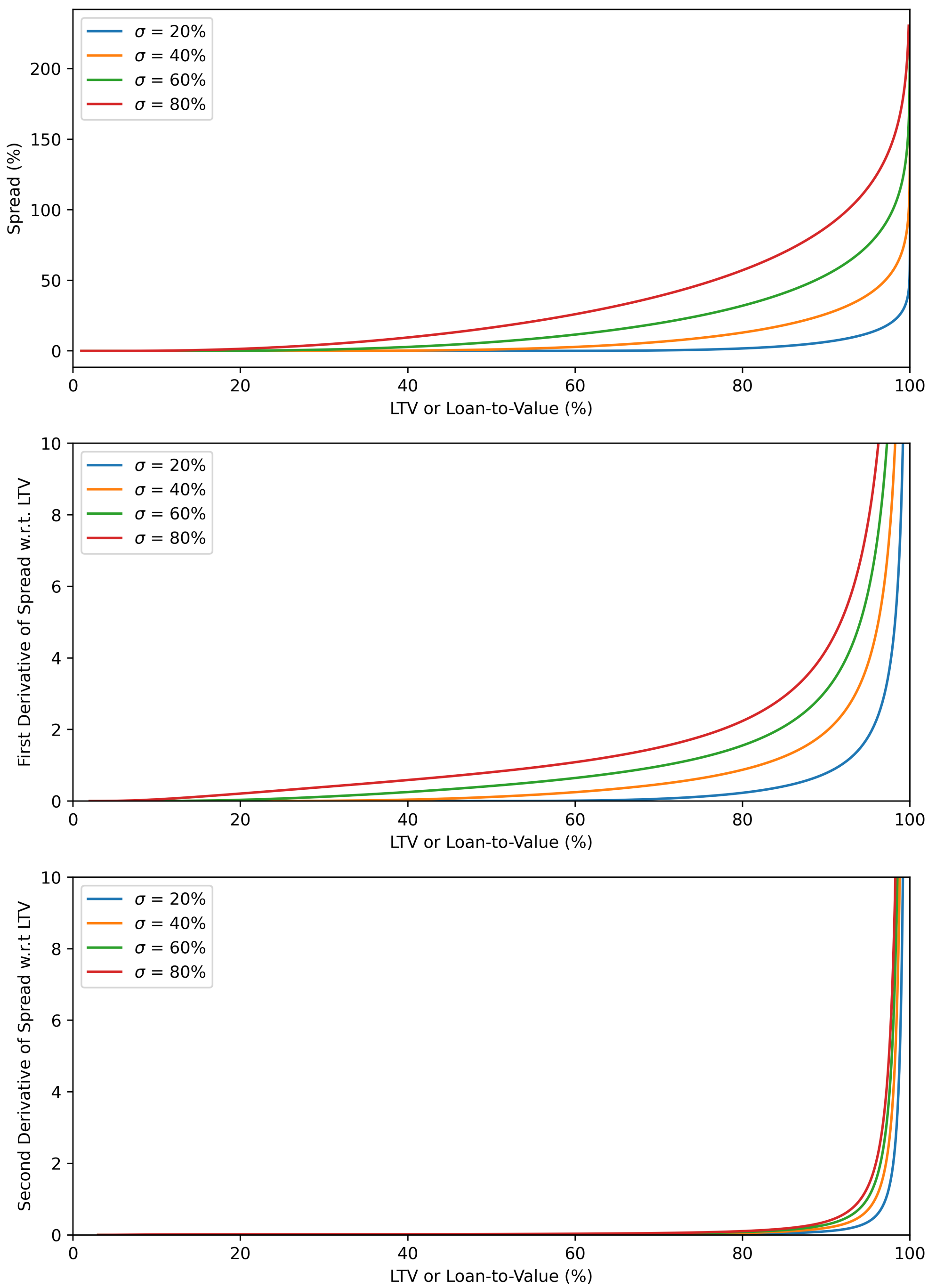


Figure 1: LTV Simulations

buy all of a given asset during times of low volatility, and lenders extend them a significant amount of short-term credit because they are sufficiently collateralized given the low volatility. When volatility rises, however, the lenders pull back, leverage falls, asset prices fall, and the optimistic investors are forced to sell, which triggers a further fall in prices. Their analysis relates leverage to asset prices, and explains an accelerator effect that causes a steepening in the credit surface from volatility that exceeds the mathematical effect derived in Property 2. Their research also explains how a rise in volatility leads to a decrease in the quantity of leverage.

3 B/V as a Practical Leverage Measure

In reality, many firms issue coupon bonds and pay dividends on their equity. These issues are taken care of by Merton (1974). However, the more important consideration for which Merton does not account is that firms' liabilities are not one homogeneous bond with an oft-quoted price. In reality, firms have multiple liabilities that vary across maturity and seniority, and the basic Merton model does not account for that heterogeneity. Many of these liabilities (e.g. bank loans) are seldom traded, so it is challenging to gather market quotes for them. This fact motivates another measure of leverage, B/V or Book Value/Firm Value, since book values of debt are available for most firms. The first subsection discusses how B/V satisfies Properties 1-5 for reasonable (but not all) parameters in the original Merton model. The second subsection introduces a multiple bond variation on the Merton model as presented in Geske (1977) and establishes B/V as a reasonable leverage measure in the new model in comparison to other proposed measures. The third subsection discusses how B/V improves on the leverage measure used in Geanakoplos and Rapppoport (2019), which was Book Debt / EBITDA. The fourth subsection briefly explains why the credit surface should consider credit rating as well.

3.1 B/V Partially Satisfying Properties 1-5

Another measure of leverage, B/V or Book Value/Firm Value, replaces the market value of debt (Price) in LTV with the total amount of debt (Book Value). Book values of debt can be found on firms' balance sheets. Unlike LTV, B/V does not satisfy all five properties of an ideal leverage measure across all parameter choices. However, B/V does satisfy Properties 1 and 3 for all parameters. It satisfies Property 4 for $B/V < 1$. It also satisfies Properties 2 and 5 for small enough values of B/V and σ. Proofs of the parameter ranges for which B/V satisfies Properties 1-4 are given in Appendix 1*, and the range of values for which Properties 2 and 5 hold are shown in Figure 2.

The intersection of the satisfactory parameters across Properties 1-5 is the satisfactory set for Property 5, seeing as it is the most restrictive. Most large firms fall into the satisfactory set of parameters. Bharath and Shumway (2008) find that σ for rated firms ranges generally between 20% (25th percentile) and 60% (75th), and B/V for the sample of bonds used in this paper ranges between 0.14 (25th percentile) and 0.34 (75th).† These values would fall comfortably into the satisfactory range.

3.2 A Merton Model with Two Bonds

Since real firms tend to have multiple liabilities with different seniorities, a more instructive but still simple model involves a firm with two bonds – one senior (book value B_1) and the other junior (book value B_2) – and a residual equity. The firm starts with value V, which evolves until time T. At T, the firm pays out its remaining value first to the senior bondholders, then the junior bondholders, and finally to the equity holders. This model is similar to the one presented in Geske (1977).

The senior bond can be valued like the bond in the original Merton model (note that $d_1 \equiv B_1 e^{-rT}/V$ here). Its price is given by:

$$\text{SB Price} \equiv \text{Senior Bond Price} = B_1 e^{-rT} \cdot \Phi\left[h_2(d_1, \sigma^2 T)\right] + V \cdot \Phi\left[h_1(d_1, \sigma^2 T)\right]$$

*The proof of Property 5 is lengthy and not additionally instructive, so it is omitted.

†B/V is later calculated by dividing the book value of all of the debt by the sum of the book value of debt and the market capitalization of the firm. This is not exactly the same as the B/V here, which uses the sum of the market value of debt and market capitalization of the firm as the denominator, but it is a close approximation.

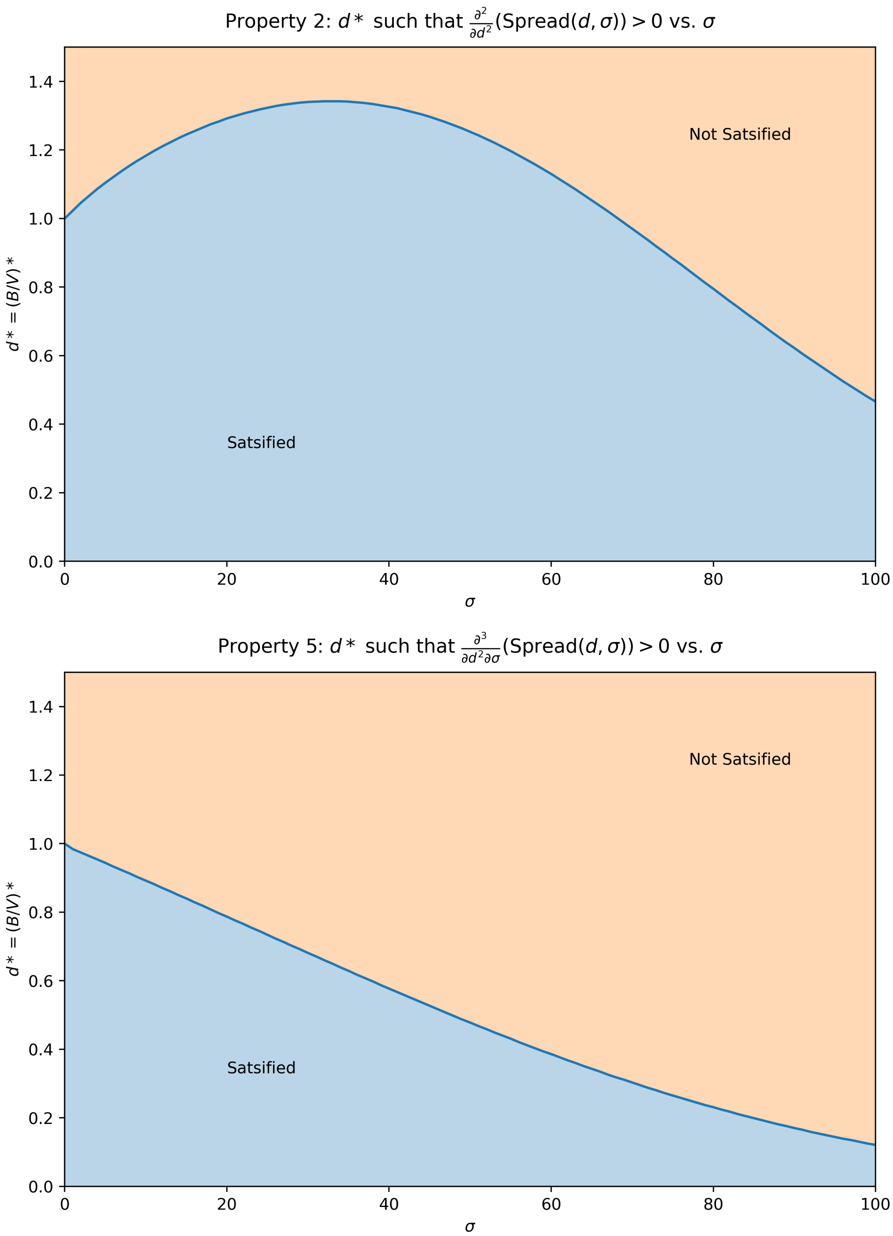


Figure 2: BV Simulations

The equity can be valued like the equity in the original Merton model (note that $d_{1,2} \equiv (B_1 + B_2)e^{-rT}/V$ here). Its price is given by:

$$\text{Equity Price} = V \cdot \Phi\left[-h_1(d_{1,2}, \sigma^2 T)\right] - (B_1 + B_2)e^{-rT} \cdot \Phi\left[h_2(d_{1,2}, \sigma^2 T)\right]$$

The junior bond is the value of the firm net of the senior bond and equity:

$$\text{JB Price} \equiv \text{Junior Bond Price} = V - \text{Senior Bond Price} - \text{Equity Price}$$

As in the original Merton model, it is possible to convert this bond price to a spread by dividing the book value of the bond by its price and taking the logarithm. It is also possible to construct theoretically ideal ($LTV_1 = \text{SB Price}/V$) and practical ($d_1 = B_1/V$) leverage measures for the senior bond. It is challenging to find empirical values for SB Price, however, because senior bonds are traded and quoted less often.

Unlike the Merton model, it is not trivial to construct a LTV measure for the junior bond, which is often traded and quoted. The value of the junior bond, which is the numerator of LTV, would be JB Price. The problem arises in valuing the underlying collateral, which is the denominator. The collateral is no longer the entire value of the firm, as some of the firm value is being used to collateralize the first bond. Some potential leverage measures are listed in the table below. The first measure is the same as the one in 3.1, which is $(B_1 + B_2)/V$. The other measures are JB Price$/(V-$SB Price$)$ and $B_2/(V-B_1)$. These alternate measures make sense at first glance because they use the value of the junior bond as the numerator and residual value of the firm after paying the senior bond as the denominator. The paper compares measures by calculating the spread on the junior bond for a firm over different B_1, B_2 and testing the measures against Property 1: credit spreads increase with leverage.

In specific, the below results are produced by taking a firm with current value $V = 100$, varying B_1, B_2 over a range of values, and letting the firm evolve for $T = 1$ according to a lognormal distribution with parameters $\mu = 0$ and $\sigma = 50$. We also assume $r = 0$. The results are plotted in graphical form in Figure 3 and summarized in Table 1.

In Figure 3, we see that the spread on the junior bond is strictly increasing with respect to B_1 and $B_1 + B_2$. This is expected. As the amount of senior debt increases with the total debt being held fixed, the junior debt becomes riskier per dollar of face value. The same

Table 1: Comparing Leverage Measures

	Spreads	$(B_1 + B_2)/V$	JB Price/(V − SB Price)	$B_2/(V - B_1)$
$B_1 + B_2 \uparrow$ with B_1 fixed	+	+	+	+
$B_1 \uparrow$ with $B_1 + B_2$ fixed	+	0	−	−

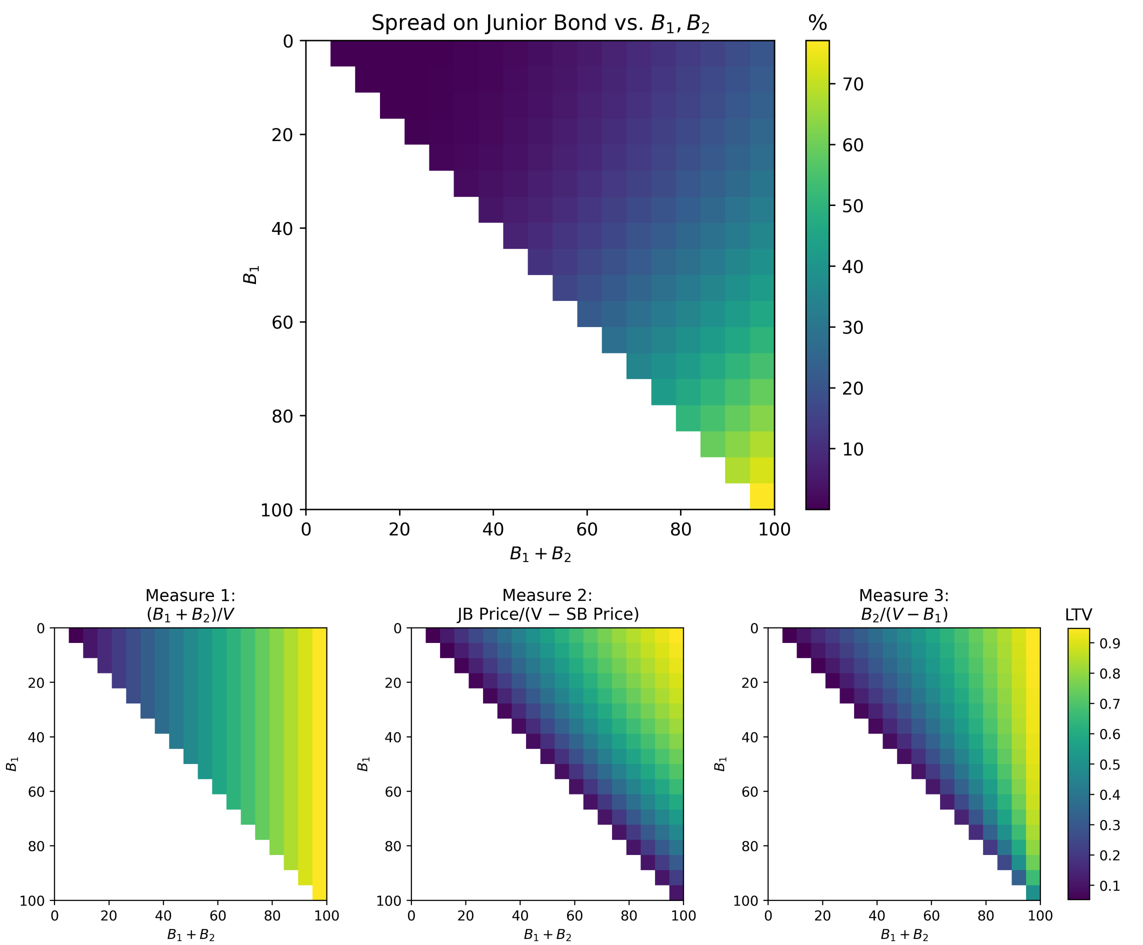


Figure 3: Comparing LTV Measures Against Credit Spreads

holds when the junior debt is increased with the senior debt being held constant.

However, leverage measures 2 and 3 contradict this property. The graphs show that measures 2 and 3 decrease as B_1 increases with $B_1 + B_2$ held fixed even as the credit spread increases. Measure 2 can mathematically be rewritten as JB Price/(JB Price +

Equity Price) $= 1 -$ Equity Price/(JB Price + Equity Price). If $B_1 + B_2$ is held fixed, we know that Equity Price is also held fixed. As B_1 increases, JB Price falls, which implies that measure 2 falls. Measure 3 can mathematically be rewritten as $(B_1 + B_2 - B_1)/(V - B_1) = (B - B_1)/(V - B_1) = 1 - (V - B)/(V - B_1)$ where $B \equiv B_1 + B_2$. This decreases as B_1 rises. Measure 2 also requires the price of senior bonds, which are not generally quoted.

Leverage measure 1 also does not increase with B_1 assuming $B_1 + B_2$ is held fixed (by definition). However, it does not decrease like the other measures. Like the other measures, it increases with $B_1 + B_2$ assuming that B_1 is held fixed. This analysis suggests that this leverage measure, which satisfies Properties 1-5 in the single bond setup and will continue to be used in the remainder of the paper, is a suitable choice.

The analysis also suggests the need to come up with a closed form solution for an LTV measure that satisfies Properties 1-5. Such a solution should ideally only involve the readily observable quantities $B_1, B_2,$ JB Price, and Equity Price so that it can be tested against market data. Creating such a measure would also allow one to compare junior bonds against senior bonds (at least the ones that are traded and quoted). That research is beyond the scope of this paper, however.

3.3 Advantages of B/V Over Book Debt / EBITDA

This leverage measure (B/V) also possesses two advantages over the leverage measure used in Geanakoplos and Rappoport (2019), which was Book Debt / EBITDA.

First, this measure is market-based whereas Book Debt / EBITDA is income statement based. This is an important consideration because market prices change continuously whereas income statements measures change quarterly at the most. Given that credit spreads also change continuously, a market measure of leverage has more potential to reflect high-frequency changes in credit spreads than an income statement measure. Firm values are also forward looking while income is a backward looking measure. Other papers have found benefits in applications from replacing financial statement based measures of leverage with market based ones (Bowman 1980, Maglione 2022).

Second, while not an ideal LTV, this measure takes the approximate form of a loan-to-value. Book Debt / EBITDA takes the form of a debt-to-income ratio instead. A loan-to-value measures the size of the loan relative to the amount of the underlying collateral while a debt-to-income ratio measures the size of the loan relative to the borrower's income that can be applied to pay off the loan. On a theoretical level, income measures just one way a firm can repay its borrowers. Firms have liquidation and acquisition value, and they also have the ability to rollover debt. These various features are not accounted for in EBITDA, but they are in firm value.

3.4 Credit Rating Along with Leverage

While leverage is an important factor in shaping the corporate bond credit surface, it does not capture information about the fundamentals of individual companies. Some businesses naturally require more leverage, such as manufacturing, and the amount of leverage does not uniquely determine the riskiness of a borrower. For this reason, other relevant bond-specific factors should also be taken into account while creating the credit surface. An important factor along these lines is the rating on a bond by ratings agencies. While credit ratings theoretically take leverage into account, they also factor in other information about the expected default of a given borrower. The two factors together should have more explanatory power than either individually, which is shown using empirical data in Section 5.

4 Construction of the Credit Surface

This section explains how we created the corporate bond credit surface that will be used to conduct the empirical analyses in the rest of the paper. The process took six steps and solely used data from Refinitiv / LSEG.

1. We downloaded and stored a list of tickers of the current constituents of the Russell

3000, which captures the three thousand largest companies in the United States today by market capitalization.

2. For each of those tickers, we found all of their historical bonds. We then filtered the bonds and kept the ones that satisfied the following criteria: not convertible, not perpetual, fixed coupons, and payments in US Dollars. We also removed bonds belonging to banks because banks have non-traditional capital structures given that deposits make up a large amount of their liabilities. We also only kept bonds that were "unsecured" because unsecured bonds are generally the most liquid and frequently quoted. As an important note, unsecured bonds are still secured by the value of the firm and paid off before equity; they earn their name because they are often the lowest level of debt in the debt structure of firms that issue them.

3. We took that list of bonds and collected monthly data from May 2014* – present and daily data (on trading days) from Jan 2020 – August 2020 on the historical option-adjusted spreads (OAS), credit ratings, amount of long-term debt, and market capitalizations for each bond and firm. We used Moody's credit ratings for individual bonds, although ratings across bonds for most firms were identical. For each month, we used the results as of the last trading day. We converted the ratings into a numerical scale according to the table in Appendix B. For the less frequent data (debt, ratings), we interpolated between entries if there was no data from that month that was available.

4. We aggregated each bond's observations across months so that observations included data for a specific bond's OAS, rating, firm debt load, and firm market cap at a specific time. We then only kept bonds that had at least five separate OAS observations and at least two separate instances of a credit rating from Moody's. We also only kept observations for bonds that had between seven and ten years left to maturity at the

*Refinitiv only stores data on bonds going back ten years. This was a significant limitation, as we could not access observations around the time of the financial crisis.

time of the observation. This step puts the remaining bonds on a level playing field, as bonds with lower maturity tend to have lower credit spreads because there is a generally upward-sloping term structure to credit spreads (Helwege and Turner 1999). We constructed the measure of B/V described in 2.1, as the ratio of the book value of the debt for a bond's issuer at a certain time divided by the sum of the issuer's total book debt and market capitalization. As a note, we would ideally like V to be the sum of the market value of the debt and market cap of the firm, but as mentioned before, the market value of all of the debt is not readily available.

5. We cleaned all the observations according the following steps. In general, the data quality was strong, so the cleaning steps are short. We grouped all observations by bond and dropped any bonds i that contained an observation t such that $\text{Spread}_{i,t} > 2 \cdot \text{Med}\{\text{Spread}_{\cdot,t}\} + 2 \cdot \overline{\text{Spread}_{\cdot,t}}$. These data points likely corresponded to "spikes" in the data series that corresponded to data issues. After that step, we dropped any observations with spreads below 0 or above 1500. These seemed like outliers from a visual inspection of the histogram of spreads across all observations. We also dropped bonds from the company with ticker NAVI, which makes loans and issues a disproportion number of bonds but is not encoded as a bank.

6. We created a 20×20 grid across levels of Rating and B/V ranging from 0 to 95 at each time t, with entries evenly spaced by five units on both axes. To calculate $\widehat{\text{Spread}}_{i,j}$ at a specific point on the grid, we used a Gaussian kernel smoother over all bonds: $\widehat{\text{Spread}}_{i,j} = K^{-1} \cdot \sum_{\text{Bond b}} \text{Spread}_{b,t} \cdot \exp\{-0.3 \times \sqrt{(i - \text{Rating}_{b,t})^2 + (j - \text{B/V}_{b,t})^2}\}$ where $K = \sum_{\text{Bond b}} \exp\{-0.3 \times \sqrt{(i - \text{Rating}_{b,t})^2 + (j - \text{B/V}_{b,t})^2}\}$. The bandwidth of 0.3 was chosen to trade off producing a smooth surface against averaging closer bonds, and there may room for improvement on the selection method. The surfaces were "trimmed" such that $\widehat{\text{Spread}}_{i,j}$ was only kept if there was an observation within 10 units of i, j. This prevents over-extrapolation to regions without observations.

Figure 4 contains plots that describe the sample. Table 2 contains further summary statistics on the bonds in the sample. Figure 5 includes plots of the credit surface from before and after the start of the COVID-19 pandemic.*

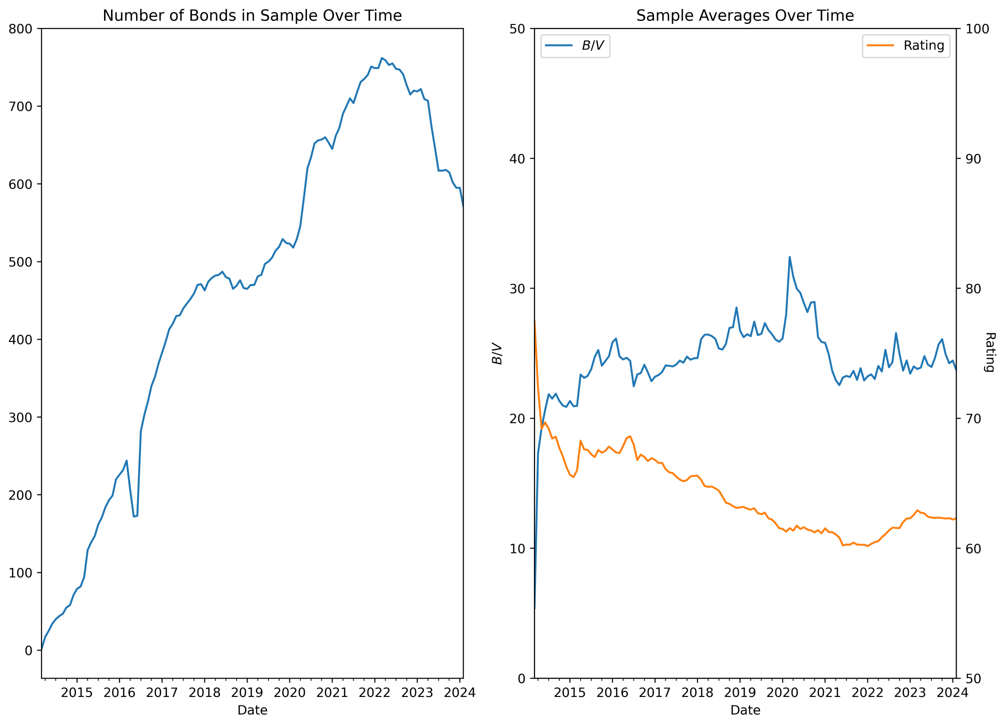
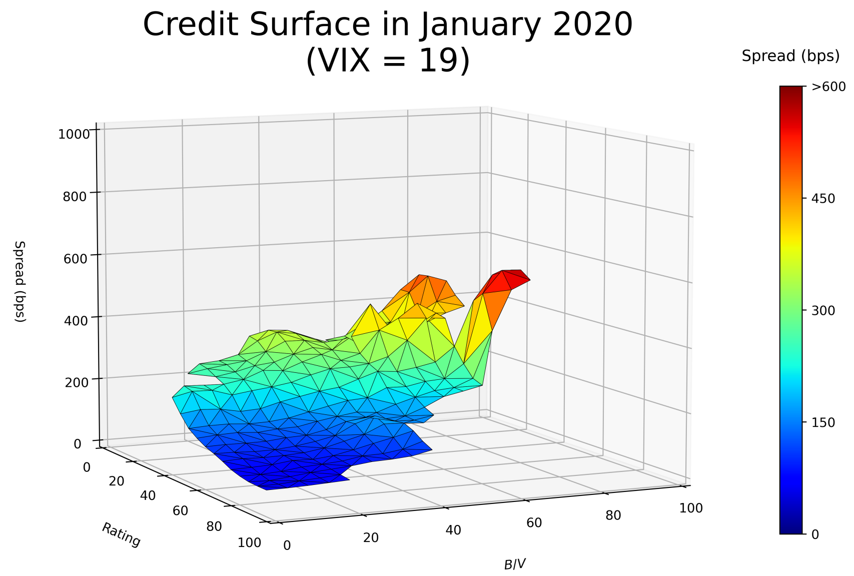


Figure 4: Sample Summary Plots

Table 2: Sample Summary Statistics

	Mean	SD	25th	Median	75th
Time in Sample (y)	2.14	1.05	1.17	2.83	3.00
B/V	25.10	14.84	13.67	21.96	34.40
Rating	62.83	12.77	55.00	65.00	70.00
Spread	158.24	103.58	93.10	131.98	189.90

*The combination of a larger bandwidth (which means more local averaging) and trimming makes the credit surfaces in this paper appear less smooth than the surfaces in Geanakoplos and Rappoport (2019).



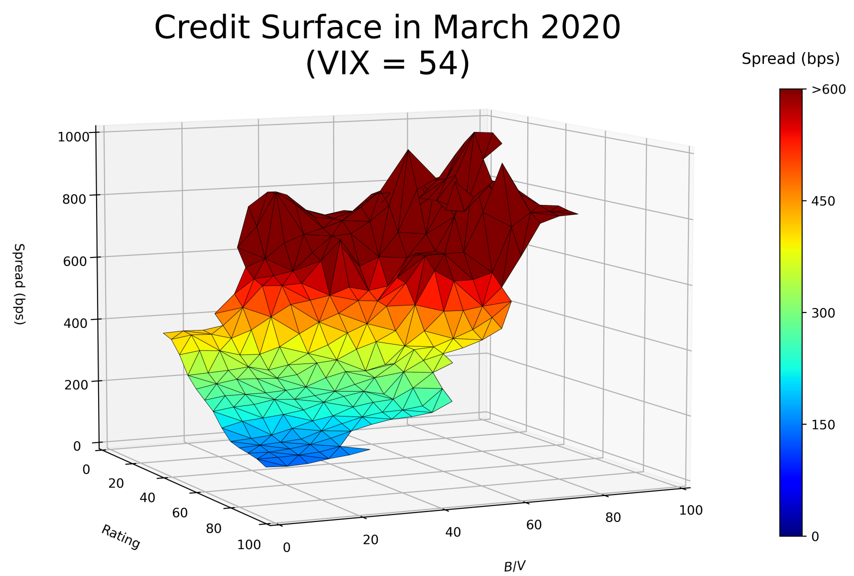


Figure 5: Credit Surfaces

These plots show the credit surface before and after the start of the COVID-19 pandemic. We can see that the credit surface at the end of March 2020 is substantially higher, steeper, and more convex than the surface at the end of January 2020. This is partially explained by the VIX index, which is a proxy for volatility, being much higher in March than January.

5 Testing the Properties of B/V Against the Data

Given our sample of bonds, we can test to see if Properties 1-5 hold in the data. We complete this analysis in five steps. First, we test to see if credit spreads increase and are convex with respect to B/V. Second, we test to see if spreads increase and are convex with respect with Rating. Third, we test if Rating and B/V together provide more information than either individually. Fourth, we test if spreads increase with volatility and reduced economic activity. Fifth, we test if the credit surface steepens and becomes more convex with increased volatility.

The first four tests take the form of the following four regressions, and their results are displayed in Table 3. Note the significance levels at the bottom of the table.

$$Spread_{i,t} = \alpha + \beta_1 B/V_{i,t} + \beta_2 B/V_{i,t}^2 + \epsilon_{i,t} \quad (1)$$

$$Spread_{i,t} = \alpha + \beta_3 Rating_{i,t} + \beta_4 Rating_{i,t}^2 + \epsilon_{i,t} \quad (2)$$

$$Spread_{i,t} = \alpha + \beta_1 B/V_{i,t} + \beta_2 B/V_{i,t}^2 + \beta_3 Rating_{i,t} + \beta_4 Rating_{i,t}^2 + \epsilon_{i,t} \quad (3)$$

$$Spread_{i,t} = \alpha + \beta_1 B/V_{i,t} + \beta_2 B/V_{i,t}^2 + \beta_3 Rating_{i,t} + \beta_4 Rating_{i,t}^2 + \beta_5 VIX_t + \beta_6 PMI_t + \epsilon_{i,t} \quad (4)$$

This table presents significant conclusions to the four tests.

First, estimated equation (1) shows that credit spreads increase and are convex with respect to B/V. Note that $\partial \widehat{Spread}/\partial(B/V) \approx -0.709 + 2 \times 0.072 \times (B/V)$. This is positive for $B/V > 5$. Also $\partial^2 \widehat{Spread}/\partial(B/V)^2 \approx 2 \times 0.072 = 0.144$. This provides empirical evidence for Properties 1 and 2: that credit spreads should increase and be convex with respect to B/V.

Second, estimated equation (2) shows that credit spreads decrease and are convex with respect to rating. Note that $\partial \widehat{Spread}/\partial(Rating) \approx -15.653 + 2 \times 0.083 \times (B/V)$. This is negative for $Rating < 94$. Also $\partial^2 \widehat{Spread}/\partial Rating^2 \approx 2 \times 0.083 = 0.166$. This confirms our hypothesis in Section 3.4. Ratings agencies conduct fundamental analysis that shine light on companies' default probabilities, so one could argue that ratings should satisfy Properties 1

Table 3: Credit Spread Pricing Models

	Dependent variable: Spread			
	(1)	(2)	(3)	(4)
B/V	-0.709***		-1.637***	-1.341***
	(0.128)		(0.127)	(0.119)
$(B/V)^2$	0.072***		0.061***	0.055***
	(0.002)		(0.002)	(0.002)
Rating		-15.653***	-12.042***	-12.566***
		(0.225)	(0.204)	(0.192)
Rating^2		0.083***	0.062***	0.067***
		(0.002)	(0.001)	(0.001)
VIX				2.092***
				(0.047)
PMI				-3.842***
				(0.053)
const	114.739***	799.388***	648.350***	826.906***
	(1.346)	(7.750)	(6.742)	(7.546)
Observations	55902	55902	55902	55902
R^2	0.338	0.498	0.605	0.666
Adjusted R^2	0.338	0.498	0.605	0.666
Residual Std. Error	84.257	73.392	65.078	59.822
F Statistic	6586.517***	19249.370***	13208.283***	9636.509***
Note:		*p<0.01; ***p<0.001; ***p<0.0001		

and 2 as a measure of risk. While not proved in this paper, it might make sense that ratings satisfy Properties 3-5 as well, and that hypothesis will be tested later in this section.

Third, estimated equation (3) shows that B/V and Rating are jointly more informative ($R^2 = 0.61$) of credit spreads than either individually ($R^2 = 0.34, 0.5$). The signs of the coefficients are again the same as (1) and (2). It is interesting that investors account for both leverage and ratings together when pricing bonds, even though ratings should theoretically incorporate the impact of leverage on firm defaults.

Fourth, estimated equation (4) shows that spreads increase with aggregate stock market volatility and reduced economic activity. The VIX index measures the level of annualized volatility priced into options on the S&P 500 expiring in around thirty days. While stock market volatility does not equal firm volatility, the two are correlated. The PMI, or Purchasing Managers' Index, is a popular diffusion measure of economic activity that is published on a monthly basis and which has been shown to be predictive of real GDP growth (Koenig 2002). Higher PMIs correspond to stronger economic activity. We see that spreads increase with volatility and decrease with stronger economic activity. This provides empirical evidence for Property 3 and reinforces the conclusion from (3) that fundamentals contain information about spreads in addition to the information contained in volatility and leverage.

The fifth test splits the sample in half in terms of monthly volatility, and it takes the form of the following two regressions. The results are displayed in Table 3. Note the significance levels at the bottom of the table.

Table 4: Pricing Model During High and Low Volatility

	Dependent variable: Spread	
	(VIX > 18.1)	(VIX ≤ 18.1)
B/V	-2.169***	-0.198
	(0.162)	(0.169)
$(B/V)^2$	0.069***	0.031***
	(0.003)	(0.003)
PMI	-5.427***	-2.170***
	(0.082)	(0.057)
Rating	-13.199***	-11.337***
	(0.280)	(0.238)
Rating^2	0.067***	0.061***
	(0.002)	(0.002)
VIX	2.654***	0.607***
	(0.078)	(0.130)
const	943.650***	695.986***
	(10.992)	(9.973)
Observations	27970	27932
R^2	0.695	0.642
Adjusted R^2	0.694	0.642
Residual Std. Error	66.378	48.495
F Statistic	5310.583***	5350.921***
Note:	*p<0.01; ***p<0.001; ***p<0.0001	

These results are challenging to grasp in tabular form, so Figure 6 shows them in graphical form. Specifically, the first subplot shows the relationship between $\widehat{Spread}$ and B/V in the

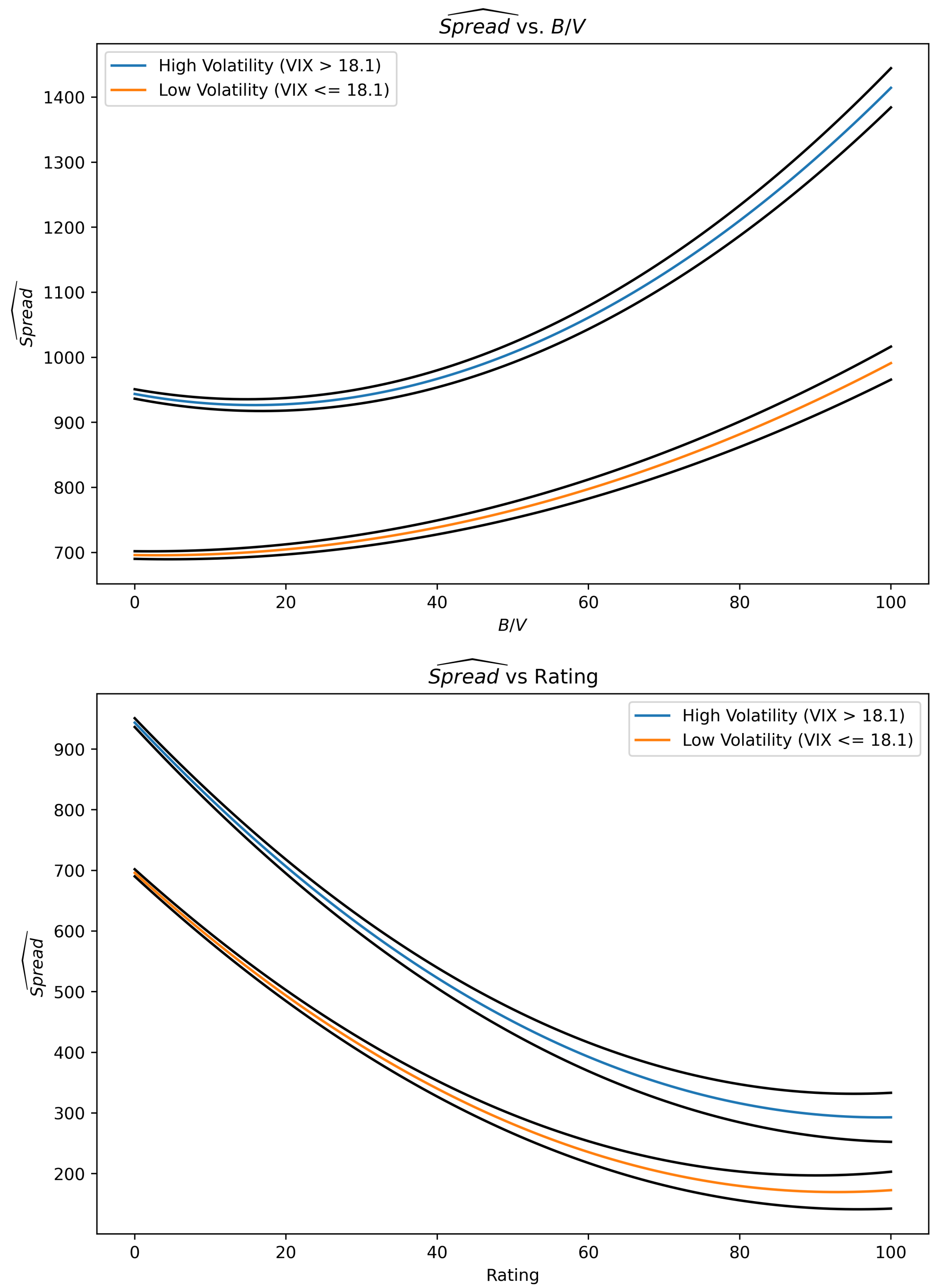


Figure 6: Pricing Models in High and Low Volatility

two volatility settings, letting PMI = Rating = VIX = 0^* to create the plot. The second subplot shows the relationship between $\widehat{Spread}$ and Rating in the two volatility settings,

*While these parameters are not plausible, a change in any of them would only shift the four graphs up or down the same amount. Thus, they have been zero for the purpose of visual convenience.

setting PMI = B/V = VIX = 0 to create the plot. For both subplots, the black bands represent confidence intervals.

The subplots gives visual evidence of Properties 3, 4, and 5 – an increase in volatility raises the credit surface, steepens it, and makes it more convex with respect to B/V. It also interestingly gives evidence that the surface does all three actions with respect to Rating as well. The high volatility setting graph seems to slope downward for low levels of B/V, but that is an artifact of the second degree estimation. The increase in steepness is clear for higher levels of B/V. The properties are more clearly satisfied for Rating.

In summary, this section found that the empirical data bears evidence of Properties 1-5. The properties interestingly hold not only for B/V as a measure of risk, which was theoretically predicted, but also Rating.

6 Movements of the Credit Surface

In the previous section, we looked at factors that affect credit spreads at a specific point in time. In this section, we will look at movements of the credit surface over time. This section and the following one mark the primary contributions of this paper to the literature. This section shows that movements of the credit surface generally follow predictable and consistent shifts of level, slope, and convexity. Outside of those general shifts, the most common moves of the surface significantly increase the surface's convexity. This section also gives a potential driver for each direction of movement and sets up the follow section, which analyzes the Fed's ability to impact movements of the credit surface.

This section employs a Principal Components Analysis (PCA) on one month changes in the credit surface over the sample period. It restricts the sample to data from June 2015 – Present, as in earlier periods there were few enough bonds to where an individual bond could solely determine a portion of the surface.

The PCA reconstruction for the changes in the surface can be written as follows, where

$\vec{u}, \vec{v}, \vec{w}$ represent the first, second, and third eigenvectors of the covariance matrix for $\Delta\vec{s}_{1\to T}$. The indexation $(\cdot)_{i,j}$ refers to value $(\cdot)$ at location (i,j) on the surface. $\vec{s}$ refers to the credit spread of the surface.

$$\Delta\vec{s}_t = \begin{bmatrix} \Delta s_{1,1} \\ \Delta s_{1,2} \\ \vdots \\ \Delta s_{1,n} \\ \Delta s_{2,1} \\ \vdots \\ \Delta s_{n,n} \end{bmatrix}_t = A_t \begin{bmatrix} u_{1,1} \\ u_{1,2} \\ \vdots \\ u_{1,n} \\ u_{2,1} \\ \vdots \\ u_{n,n} \end{bmatrix} + B_t \begin{bmatrix} v_{1,1} \\ v_{1,2} \\ \vdots \\ v_{1,n} \\ v_{2,1} \\ \vdots \\ v_{n,n} \end{bmatrix} + C_t \begin{bmatrix} w_{1,1} \\ w_{1,2} \\ \vdots \\ w_{1,n} \\ w_{2,1} \\ \vdots \\ w_{n,n} \end{bmatrix} + \begin{bmatrix} e_{1,1} \\ e_{1,2} \\ \vdots \\ e_{1,n} \\ e_{2,1} \\ \vdots \\ e_{n,n} \end{bmatrix}_t \equiv A_t\vec{u} + B_t\vec{v} + C_t\vec{w} + \vec{e}_t$$

Figure 7 presents the loadings on the first, second, and third eigenvectors. As a note, only parts of the surface in which data was available for all periods were selected as entries of s_t. This is why the eigenvectors do not occupy more of the surface's domain. The eigenvectors intuitively represent the directions that capture that maximum variation in the data. In other words, the surface moves like the first eigenvector most of the time, the second eigenvector most of the remaining time, etc. To be clear, the surface can and often does move in the direction of two eigenvectors at the same time.

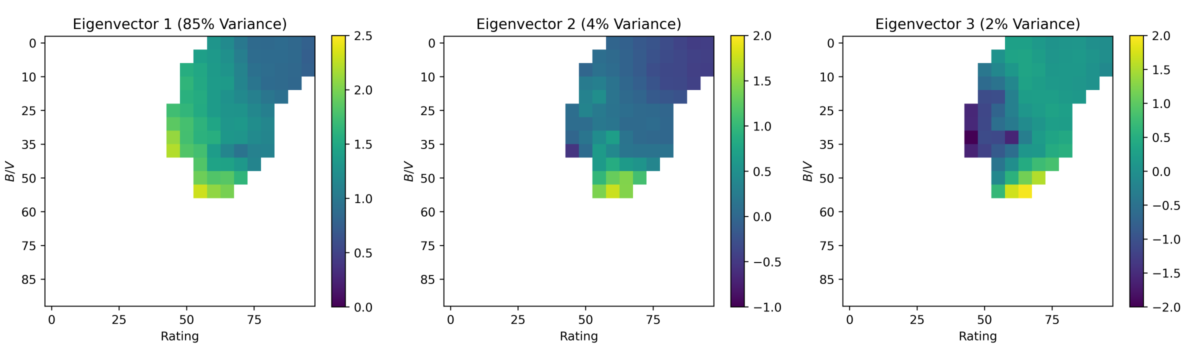


Figure 7: Loadings on the PCA Eigenvectors

The first eigenvector represents a shift up in the level, slope, and convexity of the surface. We can see the level effect because the loadings across the surface are positive, which indicates the surface moves in the same direction. We can see the slope effect because the loadings increase with risk along both the B/V and Rating dimensions. We can see the convexity effect, as an example, because the gap between the loading on the 100 Rating + 0 B/V (0.78) and the 80 Rating + 20 B/V component (1.15) is less than the gap between the 80 Rating + 20 B/V and the 60 Rating + 40 B/V component (1.95).

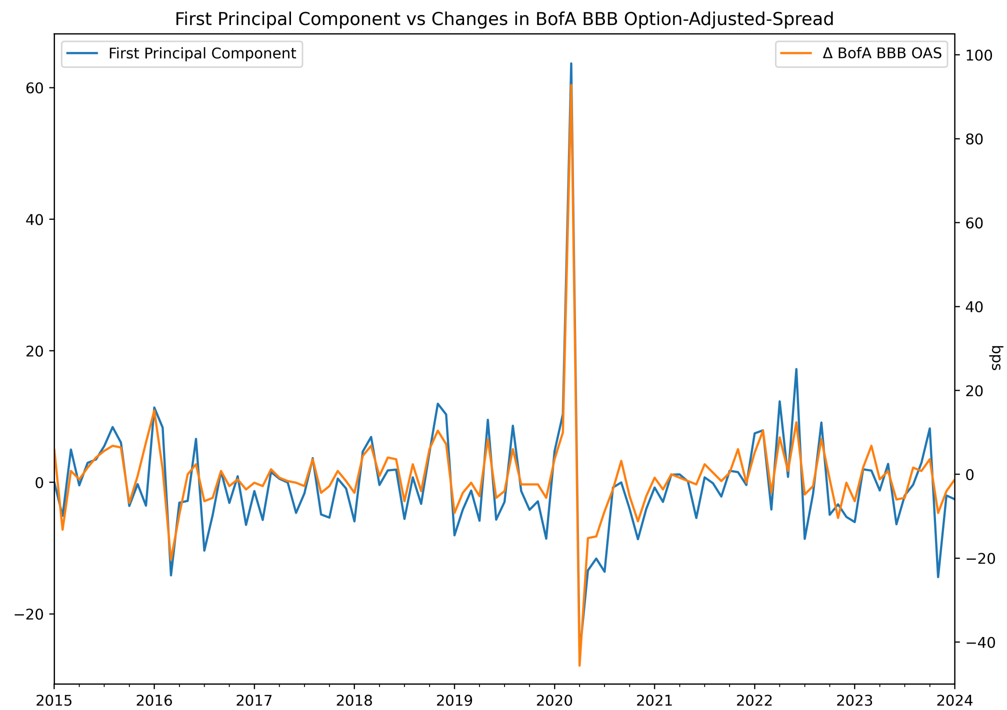


Figure 8: First Principal Component

The first principal component, which corresponds to that eigenvector, correlates significantly ($\rho = 0.94$) to changes in the ICE BofA BBB OAS (Option-Adjusted Spread) index. It also explains 85% of the variation in the movements of the surface, which is highly significant. This relationship is displayed in Figure 8. The loadings on the eigenvector and the magnitude of the explained variance suggest that credit spreads for the most part move together in a predictable way, with the spreads of more risky bonds moving at some $\beta > 1$ with respect to the spreads of safer bonds, where β is an increasing and convex function of both B/V and Rating.

As expected from the results from previous parts, changes in volatility are a major predictor of this first principal component, as changes in volatility theoretically should shift the level, slope, and convexity of the surface. The regression following this paragraph produces an R^2 of 0.29. Other papers that have studied drivers of changes in credit spreads include Collin-Dufresne et al (2001) and Gilchrist and Zakrajsek (2010). Given that this principal component behaves like changes in credit spreads, their analysis can apply to it.*

$$A_t = \beta \Delta VIX_t + e_t$$

The second eigenvector represents an abnormal increase in the convexity of the surface. We can see this because the 100 Rating + 0 B/V loading is slightly negative, the 80 Rating + 20 B/V loading is nearly zero, and the 60 Rating + 40 B/V loading is highly positive.

The second principal component, which corresponds to that eigenvector, explains 4% of the variation in the movements of the surface. This relationship is displayed in Figure 9, with a three month rolling mean used to smooth out some temporary spikes in the series. The two greatest values of the second principal component correspond to late 2018 (1 in the Figure) and the start of the COVID-19 pandemic (2). These values correspond to periods in which credit spreads widened substantially due to a decrease in investor risk appetite. Late 2018 brought concerns over the trade war, signs of a slowing economy, and falling oil prices. March-May 2020 brought concerns over the impact of the pandemic on the global economy. While these spikes were followed by troughs that gradually normalized the moves

*This paper also found that changes in the S&P 500 deliver a higher R^2 than changes in volatility even though the two are highly related ($\rho = -0.78$). This might reflect the fact that stock market conditions contain information about borrower default probabilities in excess of that contained by volatility alone. The impact of stock market changes on credit spreads is related to the balance sheet channel of monetary policy, in which higher asset prices increase collateral values and make firms less risky.

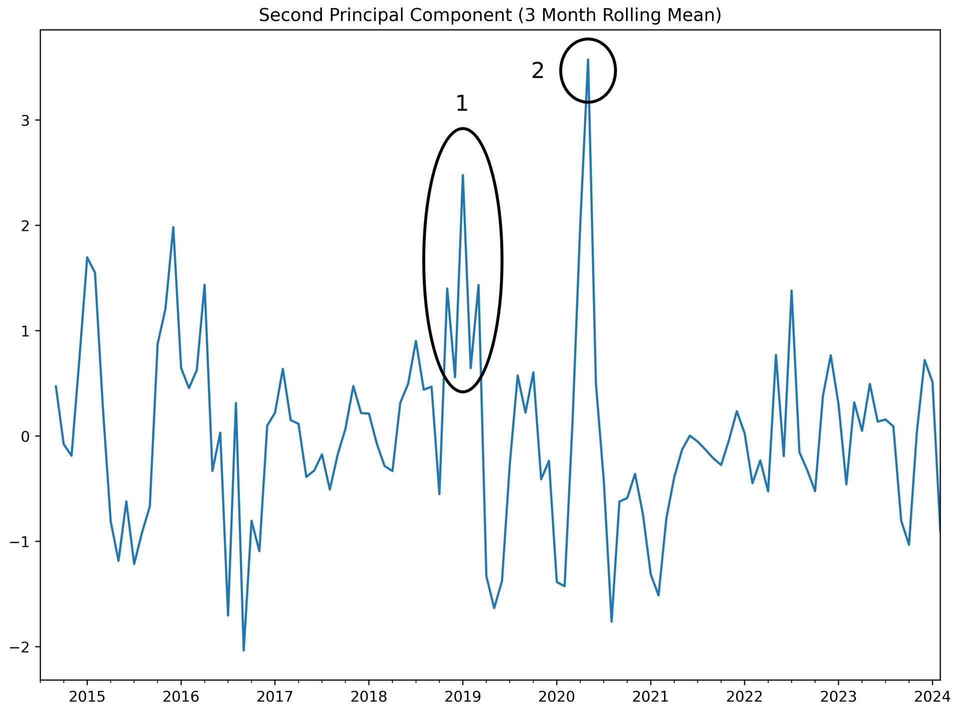


Figure 9: Second Principal Component

in convexity, a full recovery took nearly a year in each case. The Fed has also proved that it can attenuate these spikes, which will be discussed in the next section.

The third eigenvector represents a "wobble" in the surface in which spreads on high B/V bonds increase/decrease and spreads on bonds with low ratings decrease/increase. We can see this because the 40 Rating loadings are negative, the 55 B/V loadings are positive, and the remaining loadings are near zero. The third principal component, which corresponds to that eigenvector, explains 2% of the variation in the movements of the surface. There was not a clearly identified pattern to that principal component.

7 The Fed and the Credit Surface

The Fed has historically avoided taking on credit risk, as it is precluded in doing so by the Federal Reserve Act outside of "unusual and exigent circumstances." It has taken this stance to protect its independence. Losses on loans, which are primarily made to financial institutions, give an appearance of the Fed "bailing out" financial institutions at a cost to the American public. Following the passing of the Dodd-Frank Act, the Fed also needs to seek approval of the Treasury secretary before making loans on which it might take losses, and the Treasury has offered equity in recent risky loans to absorb those potential losses.

This philosophy also makes economic sense most of the time, when supply and demand for risk and capital determine credit spreads across the economy. Shifts in these quantities cause the credit surface to move predictably most of the time, with the previous section finding that spreads of riskier bonds move with some $\beta > 1$ to safer bonds. The literature has specifically examined how traditional monetary policy affects credit spreads. Gertler and Karadi (2015) use a VAR to estimate that exogenous tightening of monetary policy increases credit spreads for over six months. Javadi et al (2017) find that rate cuts reduce credit spreads more than rate hikes increase spreads. Anderson and Cesa-Bianchi (2020) also find that shocks to monetary policy affect firms with high levels of leverage more than firms with low levels of leverage.

During periods such as the start of the COVID-19 pandemic, however, the surface can steepen and become more convex by an amount greatly exceeding its usual movements, and these dislocations can persist for many months. During this time, risky firms face more challenges in securing borrowing, which dampens their output and hiring.

The Fed has the ability to attenuate these excess shifts, and it exercised that authority in March and April of 2020. In response to the large financial market moves early in the month, which caused investment grade spreads to rise above 5% and high yield spreads to rise above 10%, the Fed stepped in with force. On March 15th, the Fed cut the federal funds rate to zero and introduced a substantial amount of quantitative easing (QE). On March 23rd, the

Fed expanded its QE program and other lending facilities. It also announced the creation of the Primary Market Corporate Credit Facility (PMCCF) and Secondary Market Corporate Credit Facility (SMCCF). Through the PMCCF and SMCCF, the Fed was able to buy bonds of investment grade companies directly from them and through the market, respectively. Through the SMCCF, it could also buy shares of investment grade bond exchange-traded funds (ETFs). On April 9th, the Fed expanded both facilities to include companies that were downgraded from investment grade after the start of the pandemic, and it said it would buy some high yield bond ETFs. The CCFs became operational in June. Ultimately, the Fed did not make any loans through the PMCCF, and it purchased a relatively small quantity of $14 billion of assets through the SMCCF. The mere announcements of both CCFs had a dramatic effect on credit spreads, however, as they reduced investment grade spreads by over 200 bps and high yield spreads by over 300 bps by the end of April (Boyarchenko 2022).

The SMCCF is particularly significant because it is the Fed's first time supporting the corporate bond market directly by taking on credit risk. In other words, it is the Fed's first program that directly targets the corporate bond credit surface rather than the market for riskless debt, which one can theoretically approximate as the safest corner of the credit surface.[*] A few papers have studied the impact of the SMCCF on credit spreads. Nozawa and Qiu (2021) find that both SMCCF announcements significantly reduced credit spreads, and their impact was heterogeneous across credit ratings. Gilchrist et al (2024) find that beyond an effect on credit spreads broadly, the program further lowered the spreads of bonds that were eligible for the program relative to bonds that were not. Both papers attribute some of the program's effect to a lowering of default risk and decrease in investor risk aversion.

The analysis in this chapter extends the analysis in the literature by understanding the Fed's impact on credit spreads around the time of the two major SMCCF announcements through the first two principal components derived in the previous section.

[*] In practice, the safest corner of the credit surface generally does not have a spread of zero. This is because banks have regulations that disincentivize them from holding nearly riskless corporate bonds relative to fully riskless government bonds.

Figure 10 shows the evolution of the two principal components from January – August 2020. The data is smoothed with a rolling mean for visualization purposes. March 23rd and April 9th, the dates of the two announcements relevant to the SMCCF, are pointed out.

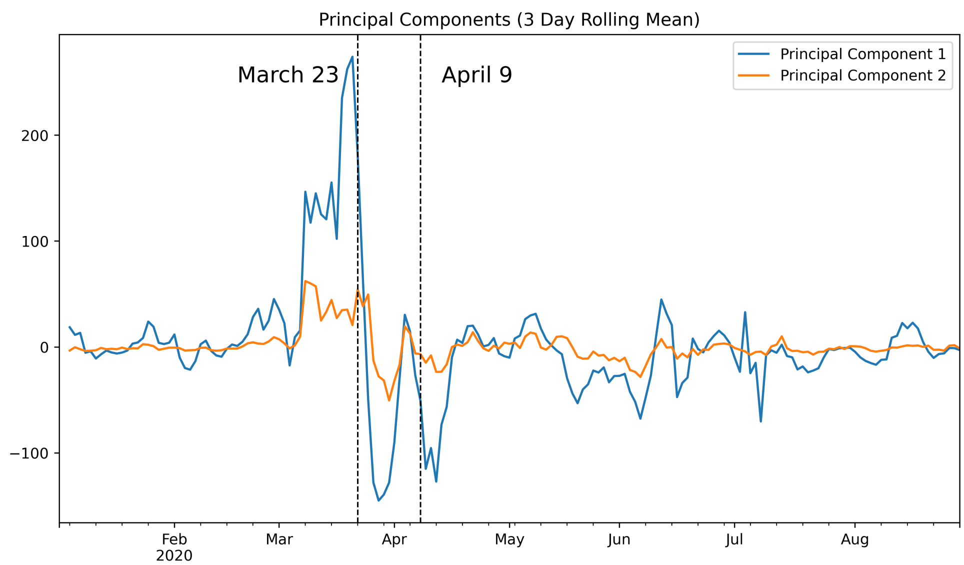


Figure 10: Principal Components from Jan – August 2020

In the plot, we can see with higher-frequency data the peak in the second principal component corresponding to the start of the COVID-19 pandemic, which was described in the previous section. We can see that the both announcements had a drastic effect on the two principal components. The first principal component roughly corresponds to a shift in the level, slope, and convexity of the credit surface, so its negative values suggest that the Fed announcements eased credit conditions for all borrowers. The second principal component roughly corresponds to a shift in excess convexity of the surface, so its negative values suggest that the SMCCF announcements further eased credit conditions for riskier borrowers.

However, the two principal components did not move down in the same amount. We can better see the differences in their reactions to the announcements via Figure 11.

The credit surface moved downwards quickly in the direction of the first eigenvector following the first announcement while it took a few more days to move downwards in the

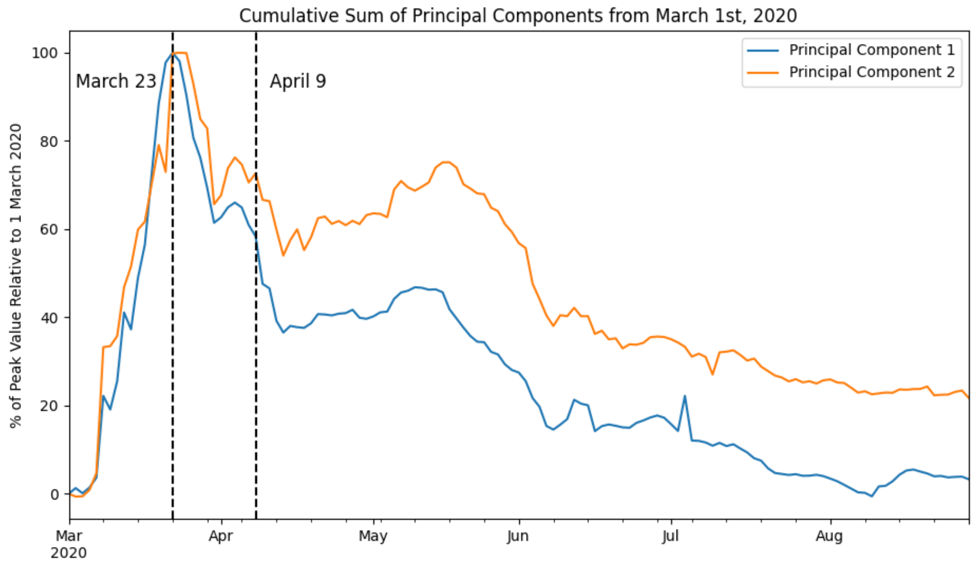


Figure 11: Cumulative Sum of Principal Components from Jan – August 2020

direction of the second eigenvector. This result reinforces Nozawa and Qiu's finding that safer credit spreads (which are partially reflected in the first principal component) moved quickly following the first announcement while riskier spreads (which are more isolated in the second principal component) took more time. This heterogeneity likely stemmed in part from the Fed deciding on March 23rd to only buy investment grade bonds, which suggests that the Fed's choice of purchases give it control over specific parts of the surface. This hypothesis is backed by a finding from Gilchrist et al (2024) that the SMCCF announcement produced different changes in the spreads of eligible vs. ineligible investment grade bonds.

By April 9th, the second principal component had retraced less than 30% of its total move while the first principal component had retraced over 40%. Following the April 9th announcement, the gap between the recoveries widened quickly from 10% to nearly 20%.* While this jump might not be expected given that the Fed opened the SMCCF to high yield bonds that had recently been downgraded from investment grade, it can potentially be

*To be clear, the cumulative changes in both principal components were falling over the majority of this period. The recovery "gap" described refers to the difference in the amount that the two principal components fell in percentage terms.

reconciled given that some market participants were expecting for the Fed to step in even more forcefully and provide more support to the high yield market. The recovery gap rose to over 30% in mid-May and eventually fell back to around 20% by the end of August.

Some of the long-lasting difference in the recoveries between the two principal components is due to market participants pricing in higher default probabilities to more levered and less creditworthy borrowers. That adjustment would tend to increase the second principal component, as it would push up spreads of the riskier firms relative to the safer firms in excess of movements of the first eigenvector. However, as mentioned, another contributor to the difference might stem from the Fed's decision to only purchase investment grade bonds. Over 15% of the total recovery gap is accounted for by differences in the three trading days following the two announcements.

More careful testing of that second hypothesis, while beyond the scope of this paper, would reveal how influential the Fed is in determining atypical movements of the credit surface. If the Fed is truly influential, which the literature currently suggests, it might want to consider taking a more active role in considering the credit surface when setting monetary policy. Assuming monetary policy and economic conditions are moving the credit surface in its traditional direction, the Fed may be content without intervening. However, during future moments that feature atypical upward shifts of the credit surface, such as in late 2018 and the start of the COVID-19 pandemic, the Fed may wish to step in with credit facilities that target the riskier portion of the credit surface. The Fed can attenuate those atypical rises. Such an intervention would also incentivize borrowing and investment, which would boost both employment and inflation, hence serving the Fed's dual mandate.

8 Conclusion

In review, this paper motivated, created, and studied the credit surface for corporate bonds. It introduced the Merton model to price the debt of a firm. It used the Merton model to compare leverage measures according to five properties that an ideal leverage measure might want to satisfy. It found that LTV is an ideal leverage measure according to those properties, but due to data access issues, settled on B/V as a practical but suitable leverage measure. It then collected data on credit spreads for a variety of bonds over the past ten years and created credit surfaces. It verified using a series of panel regressions that the theoretical properties of B/V hold in the sample. It then performed a principal components analysis on movements of the surface and found that most movements (the first principal component) corresponded to a simultaneous shift in level, slope, and convexity. The next largest direction of variation (the second principal component) corresponded to a substantial increase in the spreads of riskier firms relative to less risky firms. It identified two periods in which this second principal component peaked, and it categorized those periods as times of decreased investor risk appetite. It finally looked at the credit surface's response to Fed programs established at the beginning of the COVID-19 pandemic that directly supported the corporate bond market. It found that the programs, which only purchased bonds of companies that were investment grade prior to the pandemic, were (in conjunction with other Fed programs) more successful at recovering changes in the first principal component to its pre-pandemic level than changes in the second component. This result suggests that the Fed can control parts of the credit surface, and the paper argues that the Fed should consider expanding a future program to riskier bonds so that it can attenuate future second principal component moves that price riskier borrowers out of the market.

There are a variety of questions that we would have liked to explore but could not due to time constraints, which we can split into theoretical and empirical directions.

On the theoretical side, we would have liked to derive a closed-form LTV measure for the two bond alteration on the Merton model. We would have liked to expand that model to one in which firms have debt maturing at different periods and decide each time debt comes due whether to pay it down with internal cash, sell equity, or borrow.* We also would have liked to further explore the literature on arguments as to whether periods of market stress that shut out riskier borrowers benefits or harms the aggregate economy in the long run.

On the empirical side, we would have liked to verify the predictions of those LTV measures with empirical data. We would have also liked to find a dataset that has bond prices and ratings going back to before the Global Financial Crisis. We would have liked to find constituents of the Russell 3000 for each of the years of our sample, so we could include bonds of companies that have since exited the index. We would have liked to account for individual firm volatility rather than using a broad equity volatility measure. We would have liked to include quantities of issuance in addition to the spreads. We would have liked to more clearly identify which economic or market variables correlate with the second principal component. We would have liked to isolate the effect on movements of the second principal component during the pandemic that came from the Fed not making more high yield bonds eligible for its credit facilities. We also would have liked to study the transmission of exogenous shocks to the corporate bond credit surface into the real economy.

This paper concludes by voicing a hope that future research will continue to develop nuance in the academic view of credit. Reducing credit to a single "spread" is often a poor approximation. Rather, this paper shows that credit is full of large but important and understandable heterogeneity. It also hopes that the advances in theory will make their way to policy. While the Fed has understandable reasons to stay away from credit, this paper shows that it has the ability to affect the distribution of credit should it choose to do. The paper also shows that there are moments where shifts in credit surfaces are large enough such that intervention merits a closer look. In those times, as it has under extreme circumstances in the past, the Fed should dare to explore.

*This paper did look into a binomial tree model for a multi-period setting and found promising results, but it stopped short of finding a closed-form LTV measure for a multi-period, multi-bond setting.

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A Proofs of B/V Satisfying Properties 1-4

For $T = 1, r = 0, V = 1$, we know $B = d$ and:

$$\text{Spread} = -\log(\text{Price}/B) = -\log(\text{Price}/d)$$

$$\text{Price}/B = \Phi\left[-\frac{1}{2}\sigma - \frac{log(d)}{\sigma}\right] + \frac{1}{d}\Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right] = \text{Price}/d$$

Property 1:

$$\frac{\partial \text{Spread}}{\partial d} > 0$$

Proof of Property 1:

$$\begin{aligned}
\frac{\partial \text{Spread}}{\partial d} &= -\frac{d}{\text{Price}} \times \frac{\partial}{\partial d}\left(\Phi\left[-\frac{1}{2}\sigma - \frac{log(d)}{\sigma}\right] + \frac{1}{d}\Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right]\right) \\
&= -\frac{d}{\text{Price}} \times \{\frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma - \frac{log(d)}{\sigma}\right)^2\right] \times -\frac{1}{d\sigma} \\
&\quad + \frac{1}{d} \times \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right)^2\right] \times \frac{1}{d\sigma} \\
&\quad - \frac{1}{d^2} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right]\} \\
&\equiv -\frac{d}{\text{Price}} \times \{A + B + C\}
\end{aligned}$$

Here we note that:

$$\frac{A}{B} = \frac{\frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma - \frac{log(d)}{\sigma}\right)^2\right] \times -\frac{1}{d\sigma}}{\frac{1}{d} \times \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right)^2\right] \times \frac{1}{d\sigma}} = -\frac{d}{\exp\{\log\, d\}} = -1$$

This implies that $A + B = 0$. Since d, $\text{Price}, \Phi(\cdot) > 0$:

$$\frac{\partial \text{Spread}}{\partial d} = -\frac{d}{\text{Price}} \times C = \frac{1}{d \times \text{Price}} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right] > 0$$

Property 2:

$$\frac{\partial^2 \text{Spread}}{\partial d^2} > 0$$

Proof of Property 2:

$$\frac{\partial^2 \text{Spread}}{\partial d^2} = \frac{\partial}{\partial d}\left(\frac{1}{d \times \text{Price}}\right) \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right] + \frac{1}{d \times \text{Price}} \times \frac{\partial}{\partial d}\left(\Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right]\right)$$

$$\begin{aligned}
\frac{\partial}{\partial d}\left(\frac{1}{d \times \text{Price}}\right) &= \frac{\partial}{\partial d}\left(\frac{1}{d^2} \times \exp\{\text{Spread}\}\right) \\
&= -\frac{2}{d^3} \times \exp\{\text{Spread}\} + \frac{1}{d^2} \times \exp\{\text{Spread}\} \times \frac{\partial \text{Spread}}{\partial d} \\
&= -\frac{2}{d^3} \times \frac{d}{\text{Price}} + \frac{1}{d^2} \times \frac{d}{\text{Price}} \times \frac{1}{d \times \text{Price}} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right] \\
&= -\frac{2}{d^2 \times \text{Price}} + \frac{1}{d^2 \times \text{Price}^2} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right]
\end{aligned}$$

$$\frac{\partial}{\partial d}\left(\Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right]\right) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right)^2\right] \times \frac{1}{d\sigma}$$

$$\begin{aligned}
\frac{\partial^2 \text{Spread}}{\partial d^2} = &-\frac{2}{d^2 \times \text{Price}} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right] + \frac{1}{d^2 \times \text{Price}^2} \times \Phi^2\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right] \\
&+ \frac{1}{d \times \text{Price}} \times \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right)^2\right] \times \frac{1}{d\sigma}
\end{aligned}$$

Property 3:

$$\frac{\partial \text{Spread}}{\partial \sigma} > 0$$

Proof of Property 3:

$$
\begin{aligned}
\frac{\partial \text{Spread}}{\partial \sigma} &= -\frac{d}{\text{Price}} \times \frac{\partial}{\partial \sigma}\left(\Phi\left[-\frac{1}{2}\sigma - \frac{log(d)}{\sigma}\right] + \frac{1}{d}\Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right]\right) \\
&= -\frac{d}{\text{Price}} \times \{\frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma - \frac{log(d)}{\sigma}\right)^2\right] \times \left(-\frac{1}{2} + \frac{log(d)}{\sigma^2}\right) \\
&\quad + \frac{1}{d} \times \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right)^2\right] \times \left(-\frac{1}{2} - \frac{log(d)}{\sigma^2}\right)\} \\
&= -\frac{d}{\text{Price}} \times \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right)^2\right] \\
&\quad \times \{\frac{1}{d} \times \left(-\frac{1}{2} + \frac{log(d)}{\sigma^2}\right) + \frac{1}{d} \times \left(-\frac{1}{2} - \frac{log(d)}{\sigma^2}\right)\} \\
&= \frac{1}{\text{Price}} \times \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right)^2\right] \\
&> 0
\end{aligned}
$$

Property 4:

$$\frac{\partial^2 \text{Spread}}{\partial d \partial \sigma} > 0$$

Proof of Property 4:

$$\begin{aligned}
\frac{\partial^2 \text{Spread}}{\partial d \partial \sigma} &= \frac{\partial}{\partial \sigma}\left(\frac{1}{d \times \text{Price}} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right]\right) \\
&= \frac{\partial}{\partial \sigma}\left(\frac{1}{d^2} \times \exp\{\text{Spread}\} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right]\right) \\
&= \frac{1}{d^2} \times \frac{\partial}{\partial \sigma}\left(\exp\{\text{Spread}\} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right]\right) \\
&= \frac{1}{d^2} \times \{\exp\{\text{Spread}\} \times \frac{1}{\text{Price}} \times \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right)^2\right] \\
&\quad \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right] + \exp\{\text{Spread}\} \times \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right)^2\right] \\
&\quad \times \left(-\frac{1}{2} - \frac{log(d)}{\sigma^2}\right)\} \\
&= \frac{1}{d^2} \times \exp\{\text{Spread}\} \times \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right)^2\right] \\
&\quad \times \{\frac{1}{\text{Price}} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right] + \left(-\frac{1}{2} - \frac{log(d)}{\sigma^2}\right)\}
\end{aligned}$$

Note that:

$$\frac{1}{d^2} \times \exp\{\text{Spread}\} \times \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right)^2\right] > 0$$

Also for $d = 1$:

$$\frac{1}{\text{Price}} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right] + \left(-\frac{1}{2} - \frac{log(d)}{\sigma^2}\right) = 0$$

From here, it can be shown that for $d < 1$:

$$\frac{1}{\text{Price}} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right] + \left(-\frac{1}{2} - \frac{log(d)}{\sigma^2}\right) > 0 \to \frac{\partial^2 \text{Spread}}{\partial d \partial \sigma} > 0$$

For $d > 1$, however, we get the counter-intuitive result that:

$$\frac{1}{\text{Price}} \times \Phi\left[-\frac{1}{2}\sigma + \frac{log(d)}{\sigma}\right] + \left(-\frac{1}{2} - \frac{log(d)}{\sigma^2}\right) < 0 \to \frac{\partial^2 \text{Spread}}{\partial d \partial \sigma} < 0$$

B Ratings to Numerical Scores (Majnoni 1999)

Moody's Rating	Score
Aaa	100
Aa1	95
Aa2	90
Aa3	85
A1	80
A2	75
A3	70
Baa1	65
Baa2	60
Baa3	55
Ba1	50
Ba2	45
Ba3	40
B1	35
B2	30
B3	25
Caa1	20
Caa2	15
Caa3	10
Caa	5
Below	0

Majnoni, G., Ferri, G., & Liu, L.G. (1999). How the proposed Basel guidelines on rating-agency assessments would affect developing countries. The World Bank.