Automation and Rent Dissipation: Implications for Wages, Inequality, and Productivity*

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Abstract

This paper studies the effects of automation in economies with labor market distortions that generate worker rents—wages above opportunity cost—in some jobs. We show that automation targets high-rent tasks, dissipating rents and amplifying wage losses from automation. It also reduces within-group wage dispersion for exposed groups. Automation-driven rent dissipation is inefficient and reduces (and could even negate) the productivity gains from automation. Using data for the US from 1980 to 2016, we find evidence of sizable rent dissipation and reduced within-group wage dispersion due to automation. Using these estimates and accounting for equilibrium effects, we estimate that automation accounts for 52% of the increase in between-group inequality in the US since 1980, with rent dissipation being responsible for a fifth of this contribution. We also estimate that inefficient rent dissipation offset 60–90% of the productivity gains from automation since 1980.

Keywords: automation, productivity, technology, inequality, wages, rents

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1 Introduction

The US labor market has experienced epochal changes since 1980: not only did inequality increase greatly, but the real wages of workers without a college degree declined or stagnated. While there is no consensus on the causes of this development, the automation of tasks performed by low-education workers appears to have played an important role.[1]

This paper studies the implications of automation for wages, productivity, and welfare in economies with imperfect labor markets, where workers earn rents and are paid wages above their opportunity cost (their next best option). Even though the presence of worker rents is well documented, how automation impacts an economy with noncompetitive elements remains unexplored.[2] Our contribution is to develop a framework to analyze and quantify the effects of automation in the presence of worker rents and to establish that these effects differ from those in competitive models.

We consider an economy where firms allocate tasks to workers of different skills or automate them. Worker rents distort hiring and automation decisions by creating a wedge between the wage firms must pay workers in some tasks and their opportunity cost, for example, because of efficiency wage considerations, constraints on firms cutting wages (due to norms, licensing, or minimum wages), or bargaining.[3] This reduces employment at high-rent tasks and encourages firms to automate these tasks excessively.

Our first contribution is to identify a novel *rent dissipation* mechanism via which automation impacts the labor market: all else equal, new automation technologies target and displace workers from high-rent tasks. Rent dissipation has important implications for within-group wage dispersion, average wages, productivity, and welfare:

1. *Within-group wage dispersion.* By reallocating workers away from high-rent jobs, automation reduces wage dispersion within groups of otherwise-identical workers.

2. *Average group wages.* In competitive labor markets, automation depresses the relative demand for exposed groups of workers via a *displacement effect*—by reducing the share of tasks allocated the. Rent dissipation amplifies wage losses for these groups

[1] See Goldin and Katz (2008), Acemoglu and Autor (2011), and Autor (2019) for overviews of US inequality trends, and Acemoglu and Restrepo (2022) for the role of automation.

[2] The presence of worker rents receives support from the literature on wage losses following job displacement (Ruhm, 1991; Jacobson et al., 1993; Schmieder et al., 2023), wage differentials across jobs (Krueger and Summers, 1988; Katz and Summers, 1989; Card et al., 2018), and wage premia associated with unions and licenses (Kleiner and Krueger, 2013; Gittleman and Kleiner, 2016; Farber et al., 2021).

[3] For ease of exposition, we adopt a reduced-form modeling of rents using wedges, while the Online Supplement provides various micro-foundations that yield the representation we use in the text.

by pushing them away from high-rent tasks and forcing them into lower-wage jobs.

3. *Productivity and welfare.* In competitive labor markets, automation increases TFP by reducing the cost of producing automated tasks. In the presence of distortions, automation generates an additional negative effect on productivity, because rent dissipation is inefficient: the tasks targeted for automation are not the ones where worker wages are high due to scarcity, but due to rents, and as a result, a planner would have preferred to allocate more—rather than less—labor to these tasks. Inefficient rent dissipation makes the net impact of automation on productivity ambiguous.

Our second contribution is to provide reduced-form evidence on rent dissipation. Using US data from 1980 to 2016, we document that the impact of automation on wages within detailed demographic groups takes the distinctive pattern predicted by theory: automation reduces wage dispersion in exposed groups, generating more pronounced wage declines between the 70th and 95th percentiles of the within-group wage distribution than the rest.

In line with the theory, wage losses at the top of exposed groups result from workers being displaced from higher-rent jobs. We document this fact using various proxies of rents proposed in the literature (see Krueger and Summers, 1988; Katz and Summers, 1989). These include: wage differences across industries and occupations (controlling for worker characteristics), wage losses following job displacement, and quit rates, which provide an inverse measure of rents (for workers tend to leave more attractive jobs less frequently). Our reduced-form evidence points to sizable rent dissipation, with wages in automated tasks exceeding worker opportunity cost by 20%-50%, with a central estimate of 35%.

Our third contribution is to quantify the general equilibrium implications of automation for wage levels, productivity and welfare in the presence of labor market distortions. We provide formulas for the change in wages, output, and productivity in terms of the task displacement experienced by demographic groups, the rate of rent dissipation, and cost savings from automation. In equilibrium, the automation of tasks performed by a group of workers impacts others via *ripple effects*. Our formulas summarize these ripples by two matrices: the *propagation matrix*, which encodes information on the strength of direct and indirect competition for tasks between groups of workers, and the *rent-impact matrix*, which encodes information on how task reallocation changes rents across groups.

We compute the equilibrium effects of automation using these formulas, estimates of the propagation and rent-impact matrices, and our estimates of rent dissipation due to automation. According to our results, automation accounts for 52% of the rise in between-group wage inequality since 1980. Of these, 42 percentage points are due to the baseline displacement effects of automation. The remaining 10 percentage points are due to rent dissipation. We also estimate that the impact of automation via costs-savings was to increase TFP by 3% between 1980 and 2016, but the inefficiency of rent dissipation offset 60–90% of these gains. On net, automation is estimated to have increased aggregate TFP by only 0.3–1.3% between 1980 and 2016, and aggregate consumption by just 0.45–1.95%.

Literature: Our main contribution is to develop a framework for studying the effects of automation in labor markets with rents. There are a few works on the interplay between technology and labor market imperfections (e.g., Aghion and Howitt, 1994; Acemoglu, 1997; Caballero and Hammour, 1998; Mortensen and Pissarides, 1998), but these do not study how the impact of automation on wages and productivity is changed by the presence of worker rents, which is our focus here. Recent exceptions include Arnoud (2019) and Leduc and Liu (2022), who explore how the threat of automation affects bargaining.

Our work also contributes to the literature on the determinants of the rise in US wage inequality (e.g., Bound and Johnson, 1992; Katz and Murphy, 1992; Card and Lemieux, 2001). We are closest to papers exploring the effects of automation and lower equipment prices on inequality and wages, including Autor et al. (2003) on the effects of computers on routine tasks, the literature on capital-skill complementarity (for example, Krueger, 1993; Autor et al., 1998; Krusell et al., 2000; Burstein et al., 2019), and our previous work Acemoglu and Restrepo (2022), which modeled and quantified the effects of automation on the US wage structure. While we build on our previous work, neither that work nor any other contributions in literature have studied the interplay between automation technologies and worker rents.

On the theory front, we extend the task model in Acemoglu and Restrepo (2022) to incorporate worker rents.[4] We intentionally use a similar framework to this paper and model rents as resulting from exogenous wedges, as in the recent literature on misallocation (inter alia Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008; Hsieh et al., 2019). This approach maximizes the parallel with the analysis of the impact of automation in a competitive labor market and highlights the differences due to labor market imperfections. We also provide new formulas for the aggregate effects of automation on wages, output, and productivity in economies with labor market distortions. These formulas relate to several

[4]These models, in turn, build on prior works exploring the effects of technologies and trade and task models, including Zeira (1998), Acemoglu and Zilibotti (2001), Acemoglu and Autor (2011), and Acemoglu and Restrepo (2018), as well as Grossman and Rossi-Hansberg 's (2008) model of offshoring.

recent papers exploring how technology affects productivity and welfare in economies with inefficiencies (see Baqaee and Farhi, 2020; Basu et al., 2022; Dávila and Schaab, 2023).

On the empirical front, our main addition to the literature is to study the implications of automation both for between-group and within-group wage inequality. Recent work by Kogan et al. (2021) and Danieli (2024) explores the effects of technological change on within-group inequality, though with little overlap with our approach. We document that groups exposed to automation have seen lower wage growth over time, and that this relative wage decline is more pronounced between the 70th and 95th percentiles of the within-group wage distribution. We also show that this can be explained by a shift away from high-rent jobs. This pattern contrasts with the common view in the literature that inequality has a *fractal* nature (meaning that it has risen at all levels of aggregation, e.g., Katz, 1994), because it is driven by rising demand for skills within as well as between groups (see Acemoglu, 2002). Our theory suggests a more nuanced pattern, where automation can cause an increase in wage inequality between groups and a *decline* within groups impacted by automation.

Finally, our work contributes to the literature on worker rents and their implications for wages and efficiency. This literature has proposed several reasons why workers earn rents, ranging from efficiency wage considerations (Akerlof, 1984; Shapiro and Stiglitz, 1984; Bulow and Summers, 1986), to the use of wages to recruit and retain workers under imperfect information (Stiglitz, 1985), or holdup problems and bargaining (McDonald and Solow, 1981; Grout, 1984; Pissarides, 2000). This literature shows that worker rents can be distortionary and lead to too little employment in high-rent jobs.[5] Recent work by Stansbury and Summers (2020) has also explored the implications of declining worker rents, but emphasizes the erosion of labor market power, rather than automation.

Organization of the paper: Section 2 presents our theoretical framework. Section 3 documents reduced-form evidence in support of the rent dissipation mechanism. Section 4 outlines our approach for estimating the general equilibrium effects of automation. The Appendix includes the main proofs, while the (online) Supplement provides the remaining proofs, extensions, robustness checks and data details.

[5] These distortions can justify second-best policy interventions and imply that technology or trade can have adverse welfare effects. Katz and Summers (1989) pointed out that worker rents call for industrial policy to increase employment in high-rent jobs. The flip side of this claim is that trade can reduce welfare if it shifts employment away from high-rent jobs. This insight goes back to work on immiserizing growth by Bhagwati (1968) and relates to our result that automation can reduce welfare with labor market distortions.

2 Theory: Automation and Labor Market Distortions

This section presents our conceptual framework and derives our main theoretical results. We study a one-sector model and extend our results to a multi-sector one in Section 4.

2.1 Single-Sector Model

Setup: A unique final good y is produced by combining complementary tasks $x \in \mathcal{T}$ (where the set of tasks $\mathcal{T} \subset \mathbb{R}^d$ has mass M). Task quantities, y_x, are aggregated with a constant elasticity of substitution $\lambda \in (0,1)$:[6]

$$y = \left(\frac{1}{M} \int_{\mathcal{T}} (M \cdot y_x)^{\frac{\lambda-1}{\lambda}} \cdot dx \right)^{\lambda/(\lambda-1)}.$$

There is a discrete set of labor types (demographic groups in our application) indexed by g, where $g \in \mathbb{G} = \{1, 2, \ldots, G\}$. Workers in a group share the same productivity across tasks and have different comparative advantages from workers in other groups. Tasks can also be performed using task-specific capital, equipment, or software, denoted by k_x for task x. The total production of task x is therefore

$$y_x = \psi_{kx} \cdot k_x + \sum_g \psi_{gx} \cdot \ell_{gx}.$$

Here, ℓ_{gx} is the amount of labor of type g allocated to task x, while k_x is the amount of task-specific capital used for this task, and ψ_{gx} and ψ_{kx} represent the productivity of different factors in the production of task x and encode their comparative advantages.

There is an inelastic supply ℓ_g of workers of type g to be allocated across tasks, so that

$$\int_{\mathcal{T}} \ell_{gx} \cdot dx \leq \ell_g.$$

We treat task-specific machines, $\{k_x\}_{x \in \mathcal{T}}$, as intermediate goods. They are produced within the same period using the final good at a constant unit cost $1/q_x$. If $q_x = 0$, task x cannot be performed by capital. This implies that total consumption equals net output:

$$c = y - \int_{\mathcal{T}} (k_x / q_x) \cdot dx.$$

[6] Throughout, we let "dx" denote the Lebesgue measure over $\mathbb{R}^d$ and $\int_{\mathcal{T}} f_x \cdot dx$ denote the Lebesgue integral of the function f_x over $\mathcal{T}$. The set $\mathcal{T}$ is assumed measurable.

Labor market distortions and equilibrium: We consider a market equilibrium with labor market distortions modeled using task-specific wedges. A firm performing task x using labor of type g must pay a group- and task-specific wage:

$$w_{gx} = \mu_{gx} \cdot w_g.$$

Here $w_g > 0$ is the *base wage* of group g, and $\mu_{gx} > 0$ is an exogenous wedge that varies across tasks. Workers in group g employed in task x receive the wage w_{gx}, inclusive of the wedge. We treat $\{\mu_{gx}\}_{g \in \mathbb{G}, x \in \mathcal{T}}$ as an attribute of tasks that forces firms to pay workers wages that exceed their pay in other jobs.[7]

The labor market operates as follows: firms take base wages w_g and wedges μ_{gx} as given and decide how many workers from each group to hire for task x at a wage w_{gx}. Workers prefer to be employed at higher-rent jobs, but these are *rationed* in equilibrium: firms only hire workers for each task until the value of the marginal product of labor ($VMPL$) equals the wage, w_{gx}. Workers are assigned at random to tasks until firms' labor demands are satisfied, and the base wage w_g adjusts to ensure all workers are employed.

Formally, a *market equilibrium* is given by a vector of base wages $w = \{w_g\}_{g \in \mathbb{G}}$, output (GDP) y, an allocation of tasks $\{\mathcal{T}_g\}_{g \in \mathbb{G}}, \mathcal{T}_k$, task prices $\{p_x\}_{x \in \mathcal{T}}$, hiring plans $\{\ell_{gx}\}_{gx \in \mathbb{G} \times \mathcal{T}}$, and capital production plans $\{k_x\}_{x \in \mathcal{T}}$ such that:

E1 Tasks prices are equal to the minimum unit cost of performing the relevant task:

$$p_x = \min \left\{ \frac{1}{q_x \cdot \psi_{kx}}, \left\{ \frac{w_g \cdot \mu_{gx}}{\psi_{gx}} \right\}_g \right\}.$$

E2 The allocation of tasks to factors minimizes costs. That is, tasks

$$\mathcal{T}_g = \left\{ x : p_x = \frac{w_g \cdot \mu_{gx}}{\psi_{gx}} \right\}$$

are performed by workers of type g, and tasks

$$\mathcal{T}_k = \left\{ x : p_x = \frac{1}{q_x \cdot \psi_{kx}} \right\}$$

[7] Our notation can capture multiple dimensions of worker rents. A subset of tasks could represent jobs at a firm where workers are unionized. Another subset may represent jobs at an industry or region where wages are artificially high because of licenses. Other tasks could share technological attributes that make monitoring workers challenging and lead to higher efficiency wages. Throughout, we take the wedges $\{\mu_{gx}\}_{g \in \mathbb{G}, x \in \mathcal{T}}$ as given, and our empirical evidence supports the notion that this is a good approximation, though in general some of these wedges can adjust in response to changes in technology or other factors.

are performed by capital.

E3 Task-level demands for labor and capital are given by

$$\ell_{gx} = y \cdot \frac{1}{M} \cdot \psi_{gx}^{\lambda-1} \cdot (\mu_{gx} \cdot w_g)^{-\lambda} \text{ for } x \in \mathcal{T}_g,$$

$$k_x = y \cdot \frac{1}{M} \cdot \psi_{kx}^{\lambda-1} \cdot q_x^{\lambda} \qquad\qquad \text{for } x \in \mathcal{T}_k.$$

E4 The labor market clears for all $g \in \mathbb{G}$, $\int_{\mathcal{T}} \ell_{gx} \cdot dx = \ell_g$. Workers from group g are rationed across tasks in $\mathcal{T}_g$, with ℓ_{gx} assigned to task x. Even though there is rationing, there is no unemployment because base wages adjust to clear labor markets.[8]

E5 The ideal price index condition holds:

$$1 = \left(\frac{1}{M} \int_{\mathcal{T}} p_x^{1-\lambda} \cdot dx\right)^{\frac{1}{1-\lambda}}.$$

In what follows, we normalize wedges so that $\mu_{gx} \geq 1$ and assume that there is a positive mass of tasks for which $\mu_{gx} = 1$ for all g. This normalization implies that base wages can be interpreted as the wage that workers from group g earn in jobs that pay no rents.

We also assume that rents and task productivities are bounded from above and impose the following restrictions on the task space that simplify our exposition:

Assumption 1 (Restrictions on the task space)

- *For each task $x \in \mathcal{T}$, there exists at least one $g \in \mathbb{G}$ such that $\psi_{gx} > 0$.*
- *For each $g \in \mathbb{G}$, the set $\{x \in \mathcal{T}$ st: $\psi_{gx} > 0$, $\psi_{g'x} = 0$ for all $g' \neq g$ and $\psi_{kx} = 0$ or $q_x = 0\}$ has positive measure.*
- *For each $g \in \mathbb{G}$, the integrals $\int_{\{x:\psi_{gx}>0\}} \psi_{gx}^{\lambda-1} \cdot \mu_{gx}^{-\lambda} \cdot dx$ and $\int_{\{\}x:\psi_{gx}>0\}} \psi_{gx}^{\lambda-1} \cdot \mu_{gx}^{1-\lambda} \cdot dx$ are bounded.*
- *Comparative advantage is strict. For any two groups $g \neq g'$ and constants $a, b > 0$, the set of tasks for which $\psi_{gx}/\mu_{gx} = a \cdot \psi_{g'x}/\mu_{g'x}$ and $\psi_{gx}/\mu_{gx} = b \cdot q_x \cdot \psi_{kx}$ has measure zero.*

[8] See Supplement S2 for micro-foundations based on efficiency wages (as in Shapiro and Stiglitz, 1984) or bilateral bargaining (as in Grout, 1984), which yield the formulation of worker rents used here.

This assumption ensures the existence of a unique equilibrium where all workers perform a positive mass of tasks. It also simplifies the characterization of equilibrium by imposing strict comparative advantage, removing any indeterminacy in task allocations. In addition, we adopt the (innocuous) tie-breaking rule that when indifferent, tasks are allocated to capital or the group with the highest index g. Strict comparative advantage implies such ties occur over sets of measure zero.

2.2 Equilibrium with Labor Market Rents

Following Acemoglu and Restrepo (2022), we characterize the equilibrium in terms of task shares. Define the *task shares* of worker group g and capital as

$$\Gamma_g(w) = \frac{1}{M}\int_{\mathcal{T}_g(w)} \psi_{gx}^{\lambda-1}\cdot\mu_{gx}^{-\lambda}\cdot dx \text{ for } g\in\mathbb{G},$$

$$\Gamma_k(w) = \frac{1}{M}\int_{\mathcal{T}_k(w)} (\psi_{kx}\cdot q_x)^{\lambda-1}\cdot dx.$$

The integrals are computed over the set of tasks allocated to worker groups and capital when base wages are w, denoted by $\mathcal{T}_g(w)$ and $\mathcal{T}_k(w)$. Task shares summarize how the value of tasks assigned to workers and capital varies with base wages. Assumption 1 ensures that task shares are bounded, positive, and differentiable (from strict comparative advantage).

In addition, define the *average group rent* earned by g workers in tasks in $\mathcal{T}_g(w)$ as

$$\mu_g(w) = \frac{1}{\Gamma_g(w)}\cdot\int_{\mathcal{T}_g(w)} \psi_{gx}^{\lambda-1}\cdot\mu_{gx}^{1-\lambda}\cdot dx \text{ for } g\in\mathbb{G}.$$

PROPOSITION 1 (EQUILIBRIUM REPRESENTATION) *The market equilibrium exists and is unique. The base wage vector w and output level y solve the equations*

$$w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}}\cdot\Gamma_g(w)^{\frac{1}{\lambda}} \textit{ for } g\in\mathbb{G}, \tag{1}$$

$$1 = \left(\Gamma_k(w) + \sum_g \Gamma_g(w)\cdot\mu_g(w)\cdot w_g^{1-\lambda}\right)^{1/(1-\lambda)}. \tag{2}$$

Equation (1) ensures that base wages clear labor markets. Equation (2) is the ideal price index condition. Equations (1) and (2) are similar to the implications of a standard CES production function combining capital and labor. The key difference is that the CES shares are endogenous and given by task shares, which depend on technology and rents. In what follows, we denote task shares and average group rents in equilibrium by Γ_g and μ_g.

Our next proposition explains how rents distort equilibrium allocations.

PROPOSITION 2 (INEFFICIENCY) *The equilibrium is inefficient:*

- *it features too little employment in high-rent tasks;*
- *it involves inefficient automation of tasks for which*

$$\frac{w_g \cdot \mu_g}{\psi_{gx}} < \frac{1}{\psi_{kx} \cdot q_x} < \frac{w_g \cdot \mu_{gx}}{\psi_{gx}}. \tag{3}$$

Efficiency requires the *VMPL* to be equalized across tasks assigned to workers of a group and the *VMPL* that workers could achieve in automated tasks to be below the *VMPL* in tasks assigned to them. The proposition shows that worker rents distort both margins.[9]

First, rents distort employment decisions: firms hire workers until the *VMPL* equals the task-specific wage $w_g \cdot \mu_{gx}$, and therefore, the *VMPL* in tasks with high rents exceeds the *VMPL* in lower-rent tasks. Consequently, hiring decisions are inefficient and output can be increased by reallocating workers towards higher-rent tasks. This first source of inefficiency depends on the variance of rents across tasks assigned to each group—a common intuition in the missallocation literature (e.g., Hsieh and Klenow, 2009).

The second and more novel inefficiency concerns automation. Tasks for which (3) holds are inefficiently automated: reallocating some of g workers to these tasks from non-automated ones would raise output (per reallocated worker) by $p_x \cdot \psi_{gx} - w_g \cdot \mu_g$—their *VMPL* in task x minus their average *VMPL* in other tasks. These gains are positive whenever (3) holds. Firms inefficiently automate these tasks because the wage they face exceeds the opportunity cost of workers (given by their average *VMPL*). This second source of inefficiency depends on the difference in rents that firms would have paid workers in automated tasks and the rents earned by workers elsewhere in the economy. Notably, it can be present even if there is no dispersion in rents among tasks assigned to workers in equilibrium.

2.3 Effects of Automation

The previous section characterized the equilibrium and established two sources of inefficiency. This section studies how new automation technologies affect wages and productivity.

[9] Besides worker rents, there are no inefficiencies in this economy. The market equilibrium is efficient in the absence of rents (as in Acemoglu and Restrepo, 2022) or when $\mu_{gx} = \mu_g$ for all $g \in \mathbb{G}$ and $x \in \mathcal{T}$.

New automation technologies are represented by an exogenous increase in q_x from zero to a positive level $q'_x > 0$ for tasks in $\mathscr{A}_g^T$ in $\mathcal{T}_g$ across all groups $g \in \mathbb{G}$. The sets $\{\mathscr{A}_g^T\}_{g\in\mathbb{G}}$ are technologically determined and contain tasks that could not be initially automated but are now feasible to automate. For example, advances in robotics in the 1980s and 1990s made it possible to automate industrial tasks such as welding or painting, previously performed by blue-collar workers. The development of enterprise software systems made it feasible to automate clerical tasks performed by clerks and assistants.

We provide formulas for the first-order effects of new automation technologies, obtained by assuming that the sets $\mathscr{A}_g^T$ are "small" and in the interior of $\mathcal{T}_g$.[10] Our formulas characterize the effects of automation in terms of the following objects, all written as functions of wages and employment in the initial equilibrium, $\{\mathscr{A}_g^T\}_{g\in\mathbb{G}}$ and $q'_x > 0$:

- The set of tasks $\mathscr{A}_g$ (as a subset of $\mathscr{A}_g^T$) that firms choose to automate at initial wage levels:
$$\mathscr{A}_g = \left\{ x \in \mathscr{A}_g^T : \frac{w_g \cdot \mu_{gx}}{\psi_{gx}} \geq \frac{1}{q'_x \cdot \psi_{kx}} \right\}.$$
(We focus on the relevant case where the set $\mathscr{A}_g$ has positive measure, so that there is additional automation in equilibrium.)

- The direct task displacement experienced by group g:
$$d\ln\Gamma_g^d = \frac{\int_{\mathscr{A}_g} \ell_{gx} \cdot dx}{\int_{\mathcal{T}_g} \ell_{gx} \cdot dx}.$$
(This measures the reduction in g's task share from the automation of tasks in $\mathscr{A}_g$, and is given by the initial share of group g employment in newly-automated tasks.)

- The average cost savings from automating tasks in $\mathscr{A}_g$,
$$\pi_g = \frac{\int_{\mathscr{A}_g} \ell_{gx} \cdot \mu_{gx} \cdot \pi_{gx} \cdot dx}{\int_{\mathscr{A}_g} \ell_{gx} \cdot \mu_{gx} \cdot dx},$$
where the π_{gx}'s are cost savings from automating task x in $\mathscr{A}_g$,
$$\pi_{gx} = \frac{1}{\lambda - 1}\left[\left(\frac{q'_x \cdot \psi_{kx} \cdot w_g \cdot \mu_{gx}}{\psi_{gx}}\right)^{\lambda-1} - 1\right] \geq 0.$$

[10] Formally, we assume the sets $\mathscr{A}_g^T$ have measure less than ϵ for some small $\epsilon > 0$ and are in the interior of $\mathcal{T}_g$. See the Appendix for the exact definition and the Supplementary Materials for further details and the characterization of the approximation error in these first-order equations.

- The average rent earned by group g workers in newly-automated tasks

$$\mu_{\mathscr{A}_g} = \frac{\int_{\mathscr{A}_g} \ell_{gx} \cdot \mu_{gx} \cdot dx}{\int_{\mathscr{A}_g} \ell_{gx} \cdot dx}.$$

We first take this average rent as given and later provide conditions under which adoption endogenously targets high-rent tasks.

The objects $\langle \{d \ln \Gamma_g\}_{g\in\mathbb{G}}, \{\pi_g\}_{g\in\mathbb{G}}, \{\{\mu_{\mathscr{A}_g}\}_{g\in\mathbb{G}}\} \rangle$ summarize the direct impact of new automation technologies on task shares and average group rents holding wages constant.[11]

In equilibrium, task shares and group rents also change as tasks are reassigned in response to wage changes. We refer to these as *ripple effects*. Figure 1 illustrates the direct and ripple effects. The left panel depicts an equilibrium allocation of tasks to g, g', and capital. The right panel represents new automation technologies in the set $\mathscr{A}_g^T$, and the set of automated tasks in $\mathscr{A}_g \subset \mathscr{A}_g^T$. Following the displacement of g workers from $\mathscr{A}_g$, there is an endogenous reassignment of tasks from other factors towards g, as this group sees a relative wage decline. This reassignment shapes the extent to which the incidence of the displacement effects from automation are shared between g and other groups.

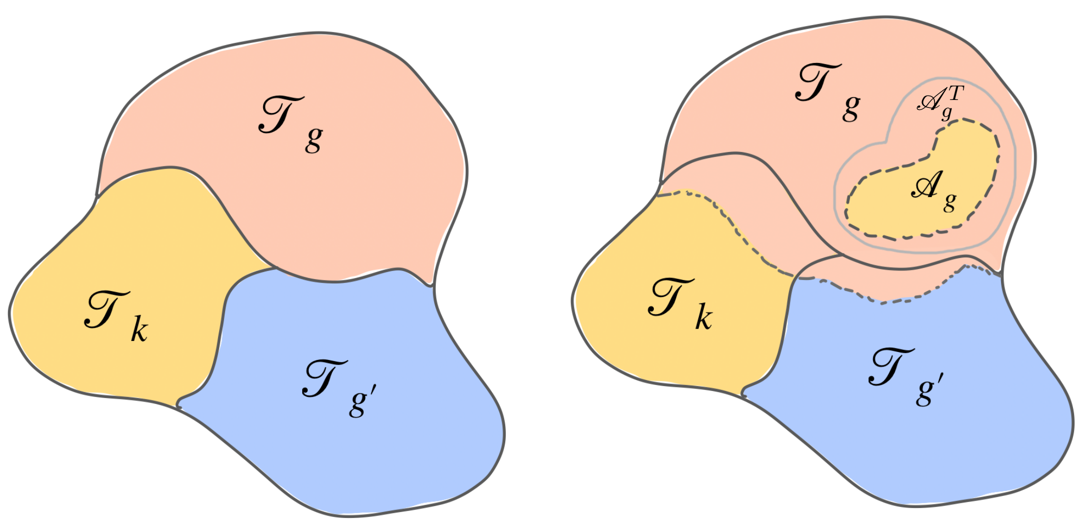

FIGURE 1: THE TASK ALLOCATION AND THE EFFECTS OF AUTOMATION. The left panel provides an example of a task space and an equilibrium allocation of tasks to g, g', and capital. The right panel illustrates the direct task displacement and ripple effects.

[11]These objects can be viewed as *ex ante* sufficient statistics: they are functions of initial wages and allocations (which depend on primitives) and parameters describing the capabilities of new automation technologies (which are taken as exogenous). They do not depend on new equilibrium prices or quantities. This implies that our key equations link changes in prices and quantities to pre-determined objects.

To illustrate the role of direct and ripple effects, differentiate (1) to obtain:

$$d\ln w_g = \frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_g^d + \frac{1}{\lambda}\cdot\sum_{g'}\frac{\partial\ln\Gamma_g(w)}{\partial\ln w_{g'}}\cdot d\ln w_j \quad \text{for } g\in\mathbb{G}.$$

The term $d\ln\Gamma_g^d$ represents the direct task displacement experienced by workers from group g and the third term represents ripple effects—equilibrium task reassignment in response to wage changes. Using these equations to solve for $d\ln w_g$, we derive:

$$d\ln w_g = \frac{1}{\lambda}\cdot\Theta_g\cdot\text{stack}\left(d\ln y - d\ln\Gamma_j^d\right), \qquad \text{with } \Theta = \left(\mathbb{1} - \frac{1}{\lambda}\cdot\mathcal{J}_\Gamma\right)^{-1}.$$

Here $\text{stack}(x_j)$ denotes the column vector $(x_1, x_2, \ldots, x_{\mathbb{G}})$, and $\mathcal{J}_\Gamma$ denotes the $G\times G$ Jacobian with entries $\mathcal{J}_{\Gamma,g,g'} = \partial\ln\Gamma_g(w)/\partial\ln w_{g'}$. As in Acemoglu and Restrepo (2022), we refer to Θ as the *propagation matrix*. Each entry $\theta_{gg'}$ is non-negative and captures the extent to which shocks that reduce the base wage of group g' affect group g via ripple effects.[12]

In our economy with distortions, we also need to keep track of how ripples affect average group rents. This information is summarized by the *rent-impact matrix*:

$$\mathcal{M} = \mathcal{J}_\mu\cdot\left(\mathbb{1} - \frac{1}{\lambda}\cdot\mathcal{J}_\Gamma\right)^{-1},$$

where $\mathcal{J}_\mu$ is a $G\times G$ Jacobian with entries $\mathcal{J}_{\mu,g,g'} = \partial\ln\mu_g(w)/\partial\ln w_{g'}$. This matrix tracks the change in average rents across groups as tasks are reassigned in response to a change in wages. Its entries summarize whether the competition between group g' and g takes place at tasks where g workers earn above-average rents (in which case the entry is positive) or below-average rents (in which case it is negative).

The next proposition provides formulas for the first-order effects of new automation technologies on wages, rents, and productivity.

Proposition 3 (Equilibrium Effects of Automation on Wages and Output) *Consider new automation technologies in small interior sets* $\{\mathscr{A}_g^T\}_{g\in\mathbb{G}}$ *with direct effects* $\left\langle\{d\ln\Gamma_g\}_{g\in\mathbb{G}}, \{\pi_g\}_{g\in\mathbb{G}}, \{\{\mu_{\mathscr{A}_g}\}_{g\in\mathbb{G}}\}\right\rangle$. *The first-order impacts on base wages and output are*

[12] The propagation matrix takes the form of a Leontief inverse because it accumulates successive rounds of reallocation. Acemoglu and Restrepo (2022) show that this inverse exists, has positive entries, and eigenvalues in $[0,1]$. This means that ripple effects play an equalizing role and dampen the direct effects of automation. The entries of the propagation matrix also summarize how substitutable groups of workers are in the aggregate. Recall that different demographic groups are perfect substitutes at the task level, but imperfect substitutes in the aggregate.

given by the solution to the system of equations

$$d\ln w_g = \Theta_g \cdot stack\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_j^d\right) \quad \text{for } g \in \mathbb{G} \tag{4}$$

$$\sum_g s_g \cdot d\ln w_g = \sum_g s_g \cdot d\ln \Gamma_g^d \cdot \frac{\mu_{\mathscr{A}_g}}{\mu_g}\cdot \pi_g, \tag{5}$$

where s_g is the share of g's earnings in output. Moreover, the change in group rents is

$$d\ln \mu_g = \mathcal{M}_g \cdot stack\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_j^d\right) - \left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1\right)\cdot d\ln\Gamma_g^d. \tag{6}$$

and the change in aggregate consumption is $d\ln c = (1/s_L)\cdot d\ln tfp$, where s_L is the labor share in output and $d\ln tfp$ is change in TFP due to new automation technologies:

$$d\ln tfp = \sum_g s_g \cdot d\ln\Gamma_g^d \cdot \frac{\mu_{\mathscr{A}_g}}{\mu_g}\cdot\pi_g + \sum_g s_g \cdot d\ln\mu_g \tag{7}$$

Proof. We provide a sketch of the proof. Equation (4) was derived above. It follows from differentiating the labor-market clearing condition in (1). Equation (5) follows from differentiating the ideal-price index condition in (2), and with (4) pins the change in output.

Define $d\ln tfp = d\ln y - s_K \cdot d\ln k$. With constant returns to scale, we obtain the dual version of Solow's residual: $d\ln tfp = \sum_g s_g \cdot (d\ln w_g + d\ln\mu_g)$. Substituting $\sum_g s_g \cdot d\ln w_g$ from (5) yields (7). The fact that $d\ln c = (1/s_L)\cdot d\ln tfp$ follows from $c = y - k$. ■

Using the formulas in the proposition, we obtain the impact of automation on average group wages $\bar{w}_g = w_g \cdot \mu_g$ as

$$d\ln \bar{w}_g = (\Theta_g + \mathcal{M}_g)\cdot \text{stack}\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_j^d\right) - \left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1\right)\cdot d\ln\Gamma_g^d. \tag{8}$$

To gain intuition for this formula, consider an economy where groups can produce disjoint task sets and capital produces all tasks with $q_x > 0$, so that there are no ripples, the propagation matrix is the identity, and the rent-impact matrix zero.[13] Equation (8) becomes

$$d\ln\bar{w}_g = \underbrace{\frac{1}{\lambda}\cdot d\ln y}_{\text{productivity effect}} - \underbrace{\frac{1}{\lambda}\cdot d\ln\Gamma_g^d}_{\text{displacement effect}} - \underbrace{\left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1\right)\cdot d\ln\Gamma_g^d}_{\text{rent dissipation}}. \tag{9}$$

[13] This holds when (i) for all x, $\psi_{gx} > 0$ implies $\psi_{g'x} = 0$ for all $g' \neq g$; and (ii) for all x, $\psi_{kx} > 0$ implies $\psi_{kx} > \underline{\psi}_k$ for some threshold $\underline{\psi}_k > 0$.

The change in average group wages depends on three effects. The first two operate in models with competitive labor markets, such as Acemoglu and Restrepo (2022), and capture the effects of automation working via labor demand and base wages. These include a positive *productivity effect*, as demand for all workers increase thanks for the expansion of output, and a negative *displacement effect*—automation directly reduces the share of tasks employing workers from group g by $d\ln\Gamma_g^d$ and thus their labor demand.

The third term captures the new *rent dissipation effect*, which amplifies wage losses for exposed groups when $\mu_{\mathscr{A}_g} > \mu_g$, so that automation displaces workers from higher-rent tasks and pushes them into lower-wage jobs. The strength of rent dissipation depends on $\mu_{\mathscr{A}_g}/\mu_g$—how high average rents were in newly-automated tasks relative to the average.

In the general case with ripples in (8), group g's average wages also depend on whether other groups competing with it are displaced by automation. When this competition channel is active, the direct task displacement experienced by other groups also impacts group g via the propagation and rent-impact matrices—the former capturing average competition for tasks and the latter adjusting for whether this competition takes place at higher-rent tasks (in which case there is additional loss of rents for the group in question).

One important difference between the displacement effect and rent dissipation is in their propagation. When automation displaces a group of workers from their tasks, its (relative) base wage declines. This induces firms to reassign marginal tasks to this group, propagating the incidence of the shock to other groups. In contrast, affected groups bear the full incidence of rent dissipation. This is because rent dissipation does not work by reducing base wages via labor demand and thus does not induce further reassignment of tasks. Rather, rent dissipation works by shifting the composition of jobs left to workers. Because it depresses their rents in excess of their base wage, rent dissipation can have sizable impacts on exposed groups that are not dampened by ripple effects.

We next discuss the implications for productivity. In our economy, changes in TFP are proportional to the aggregate change in consumption and the average change in wages paid to workers: $d\ln tfp = s_L \cdot d\ln c = \sum_g s_g \cdot d\ln \bar{w}_g$. We therefore focus on the effects of automation on TFP, which summarize its impact on consumption and average wage levels.

In the absence of ripples, the productivity gains in equation (7) simplify to

$$(10) \qquad d\ln tfp = \underbrace{\sum_g s_g \cdot d\ln\Gamma_g^d \cdot \frac{\mu_{\mathscr{A}_g}}{\mu_g} \cdot \pi_g}_{\text{direct technological gains a-la Hulten}} - \underbrace{\sum_g s_g \cdot \left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1\right) \cdot d\ln\Gamma_g^d}_{\text{changes in allocative efficiency}}.$$

This formula is related to Baqaee and Farhi (2020), who decompose the total effect of new technology in inefficient economies into a direct effect and changes in allocative efficiency. The first term in (10) represents the direct benefits from reducing the cost of producing automated tasks. This term is positive and has the same envelope logic as Hulten's theorem: it is the product of (i) the cost share of automated tasks in output (i.e., their Domar weights), which is $s_g \cdot d\ln\Gamma_g^d \cdot \frac{\mu_{\mathscr{A}_g}}{\mu_g}$, and (ii) cost savings from automation, $\pi_g > 0$. The second term, in turn, reflects changes in *allocative efficiency* due to automation. It captures how changes in the allocation of labor across tasks affect output. In a competitive labor market, this term is zero thanks to the envelope theorem—the initial allocation of labor across tasks already maximized output. This is no longer true in the presence of labor market distortions. In this case, the automation of jobs that pay above-average rents, in the sense that $\mu_{\mathscr{A}_g} > \mu_g$, worsens efficiency because it reallocates workers from tasks where they had a high *VMPL* towards tasks where, on average, their *VMPL* is lower.

A complementary interpretation for the TFP formula identifies the first term in (10) with the value of automation perceived by producers (and passed to consumers, since product markets are competitive). This value is positive and is equal to the surplus of switching from producing tasks in $\mathscr{A}_g$ with labor (at the cost $\frac{w_g \cdot \mu_{gx}}{\psi_{gx}}$) to producing with capital (at the lower cost $\frac{1}{q'_x \cdot \psi_{kx}}$). This private value is not the same as the social value of new automation technologies because of the mismatch between wages paid by firms and the opportunity cost of labor. The second term adjusts for this mismatch: it corrects for the fact that the benefits of automation perceived by producers overstate the social gains when newly-automated tasks used to pay above-average rents.[14]

One important implication of the TFP formula in (10) is that new automation technologies can have a net negative effect on productivity, average real wages, and welfare (measured by aggregate consumption). This occurs when new automation technologies are adopted in tasks where workers earn high rents despite the fact that these technologies generate small cost savings. By contrast, with competitive labor markets, new automation technologies always increase TFP, consumption, and mean wages (even if they can cause the real wages of displaced groups to decline).

Turning to the general case with ripples, the expression for TFP changes in equation (7) illustrates that allocative efficiency can deteriorate due to rent dissipation or because ripple

[14] In our model, rents are unrelated to product market distortions (which are not present in this economy). For this reason, consumers' and firms' valuation of new automation technologies coincide. If labor market rents were related to producer markups, there would be an additional positive welfare effect from automation, because automation would, in this case, increase production in distorted sectors.

effects reallocate workers away from higher-rent tasks, as summarized by $\sum_g s_g \cdot d\ln\mu_g$.

Overall, Proposition 3 highlights the importance of the new rent dissipation mechanism for wages and productivity. The next section provides conditions under which automation endogenously targets higher-rent tasks, generating rent dissipation. Our empirical exercise then estimates the rate of rent dissipation, given by $\mu_{\mathscr{A}_g}/\mu_g - 1$.

2.4 When Does Automation Target High-Rent Tasks?

We now provide sufficient conditions for equilibrium adoption to target higher-rent tasks. A key economic force in our model is that, all else equal, higher rents for workers increase labor costs and encourage automation. This force by itself does not guarantee that newly-automated tasks pay above average rents, because higher-rent tasks may be underrepresented among newly-automatable tasks in $\mathscr{A}_g^T$, or the productivity of capital in these tasks may be low. The next assumption imposes that there is no such "low-rent bias."

Assumption 2 (No Low-Rent Bias) *Let $\bar{F}_g(\mu|\mathcal{S})$ denote the share of group g employment in tasks in $\mathcal{S}$ that pay rents above μ (i.e., one minus the rent cdf). New automation technologies represented by $\{\mathscr{A}_g^T\}_{g\in\mathbb{G}}$ and $\{q'_x\}_{x\in\{\mathscr{A}_g^T\}_{g\in\mathbb{G}}}$ feature no low-rent bias if:*

(i) for all $\mu \geq 1$, $\bar{F}_g(\mu|\mathscr{A}_g^T) \geq \bar{F}_g(\mu|\mathcal{T}_g)$;

(ii) for all $\mu \geq 1$ and $a, \{b_g\}_g > 0$, $\bar{F}_g(\mu|\{x \in \mathscr{A}_g^T : q'_x \cdot \psi_{kx} = a, \{\psi_{gx} = b_g\}_g\}) = \bar{F}_g(\mu|\mathscr{A}_g^T)$.

Part (i) imposes that opportunities for automation are not biased towards low-rent tasks. Part (ii) requires that for tasks that can be automated, workers' comparative advantage relative to capital is unrelated to rent levels. The next proposition shows that (i) and (ii) are sufficient (though not necessary) for the adoption of new automation technologies to target tasks that pay above average rents, generating rent dissipation.[15]

Proposition 4 (Endogenous Targeting of High-Rent Tasks)
Consider new automation technologies represented by $\{\mathscr{A}_g^T\}_{g\in\mathbb{G}}$ and $\{q'_x\}_{x\in\{\mathscr{A}_g^T\}_{g\in\mathbb{G}}}$, and suppose these technologies satisfy Assumption 2 . Then

$$\bar{F}_g(\mu|\mathscr{A}_g) \geq \bar{F}_g(\mu|\mathscr{A}_g^T) \geq \bar{F}_g(\mu|\mathcal{T}_g) \quad \text{for all } \mu > 1.$$

[15] Automation can still affect some groups of workers more than others, as these conditions only need to hold within groups. Note also that Assumption 2 provides sufficient conditions. Rent dissipation can occur more generally, for example, if new automation technologies are biased towards high-rent tasks (and there are natural economic forces for this to be the case) or have only minor bias towards low-rent tasks.

Moreover, when not all tasks in $\mathscr{A}_g^T$ are automated (the set $\mathscr{A}_g^T \setminus \mathscr{A}_g$ has positive measure), the first inequality is strict, and the distribution of rents for group g in newly-automated tasks first-order stochastically dominates the distribution of group g's rents in the economy.

First-order stochastic dominance implies $\mu_{\mathscr{A}_g} > \mu_g$, and so Proposition 4 provides sufficient conditions for automation to generate rent dissipation. Condition (i) in Assumption 2 ensures that $\bar{F}_g(\mu|\mathscr{A}_g^T) \geq \bar{F}_g(\mu|\mathcal{T}_g)$. The key economic force identified above, where high rents encourage automation holding all else equal, coupled with condition (ii) in Assumption 2 ensure that $\bar{F}_g(\mu|\mathscr{A}_g) > \bar{F}_g(\mu|\mathscr{A}_g^T)$. This inequality is strict whenever there is a meaningful adoption margin—meaning that not all tasks in $\mathscr{A}_g^T$ are automated.

The targeting of high-rent tasks also has novel implications for wage dispersion within groups exposed to automation. To show these, assume that each worker performs a single task, so that the within-group distribution of wages is the same as that of rents.

Proposition 5 (Automation and U-Shaped Within-Group Wage Changes) *Suppose that the set of newly-automated tasks $\mathscr{A}_g$ satisfies $\bar{F}_g(\mu|\mathscr{A}_g) > \bar{F}_g(\mu|\mathcal{T}_g)$ for all $\mu > 1$ (Assumption 2 is sufficient for this). Denote by $\ln w_g^p$ the p-th quantile of the distribution of (log) wages in group g and by m_g the initial mass of workers in jobs that pay no rents. The automation of tasks in $\mathscr{A}_g$ shifts the distribution of within-group wages $\ln w_g^p$ as follows:*

- $d \ln w_g^p = 0$ *for* $p \in [0, m_g]$*;*
- $d \ln w_g^p < 0$ *for* $p \in (m_g, 1)$*;*
- $d \ln w_g^p \leq 0$ *for* $p \to 1$*. Moreover, if for all* $\mu > 1$*, there exists* $\delta > 0$ *such that a positive share of at least* δ *tasks with rent* $\mu_{gx} = \mu$ *are in* $\mathcal{T}_g \setminus \mathscr{A}_g^T$*, then* $d \ln w_g^p = 0$ *as* $p \to 1$*.*

Figure 2 illustrates the U-shaped pattern of within-group wage changes outlined in Proposition 5. The lowest wage workers in g are those employed in tasks that pay no rents and thus earn no rents before or after tasks in $\mathscr{A}_g$ are automated. Consequently, their wages change only because the base wage for group g is impacted in equilibrium. Workers in quantiles above m_g are more likely to be employed at automated tasks, generating a further decline in wages $d \ln w_g^p < 0$ for $p \in (m_g, 1)$ due to rent dissipation. At the top we expect one of two scenarios. Either $d \ln w_g^p < 0$ as $p \to 1$, or $d \ln w_g^p = 0$. The second scenario results when there are high-rent jobs assigned to g that cannot be automated.

The results in Proposition 5 contrast with the view that technological progress increased inequality in a fractal way—across all dimensions, including within narrow groups. Our

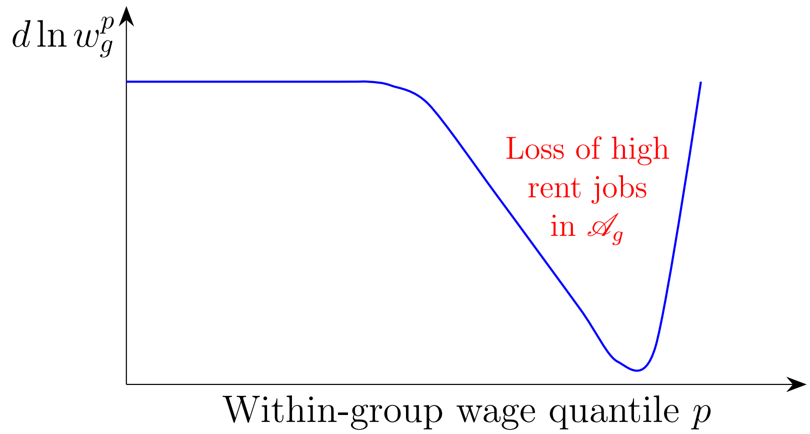

FIGURE 2: PREDICTED CHANGES IN WITHIN-GROUP WAGE QUANTILES DUE TO AUTOMATION.

theory predicts that automation can generate lower within-group inequality for groups exposed to automation, and identifies this outcome as a telltale sign of rent dissipation.

The next section turns to our empirical evidence, focusing in particular on whether the adoption of new automation technologies in the US from 1980 to 2016 targeted above average-rent jobs and generated rent dissipation.

3 Reduced-Form Evidence

This section presents reduced-form evidence on the impact of automation on group-level wages, within-group wage dispersion, and worker rents in the US between 1980 and 2016.

We focus on 500 detailed demographic groups, defined by five education levels, gender, five age groups, five race and ethnicity groups and native/immigrant status. We estimate group-level specifications of the form

$$\text{Change in group } g \text{ outcome 1980–2016} = \beta \cdot \text{task displacement}_g^d + X_g \cdot \gamma + u_g, \tag{11}$$

where task displacement$_g^d$ is the empirical analogue of $d\ln\Gamma_g^d$ and measures the (direct) task displacement due to automation experienced by group g between 1980 and 2016, and X_g includes other covariates, and u_g denotes the residual. In all specifications, we weight groups by their share of US employment in 1980 and report standard errors robust against heteroscedasticity.

Our main outcomes are the change in group average real hourly wages and the changes in the p-th percentile of the within-group wage distribution, $d\ln w_g^p$ for $p = 5, 10, \ldots, 95, 99$. These are computed from the 1980 Census and by pooling 2015-2017 American Community

Survey (ACS) data. We also use the Basic Monthly and Displaced Worker Supplement from the Current Population Survey (CPS) to create proxies for worker rents, as described below.

3.1 Measuring Task Displacement

Our main explanatory variable is the task displacement from new automation technologies between 1980 and 2016. Our strategy for measuring task displacement follows Acemoglu and Restrepo (2022), and makes adjustments to account for rents. It is based on the following assumption.

Assumption 3 (Measurement Assumption) *During 1980–2016, only routine tasks were automated, and within an industry all groups of workers were displaced by automation from routine tasks at a common rate.*

Supplement S1 considers a multi-sector version of our model and shows that, under Assumption 3, the task displacement experienced by a group can be estimated as

$$\text{task displacement}_g^d = \sum_i \frac{\ell_{gi}}{\ell_g} \cdot \text{RCA}_{gi}^{\text{routine}} \cdot \frac{-d\ln s_{\ell i}^d}{(1 + s_{\ell i} \cdot (\lambda - 1) \cdot \pi_i) \cdot \mu_{\mathscr{A}_i}/\mu_i}. \tag{12}$$

This measure is the the product of three terms:

- ℓ_{gi}/ℓ_g represents group g's exposure to industry i. This term accounts for groups' specialization in sectors that introduced new automation technologies.
- $\text{RCA}_{gi}^{\text{routine}}$ is a measure of the revealed comparative advantage of group g in routine jobs in industry i. This term apportions the incidence of automation in the industry based on who performs routine tasks.
- $-d\ln s_{\ell i}^d$ is the percent reduction of the labor share in industry i due to new automation technologies over 1980–2016. In our framework, the rate at which tasks are automated in an industry can be recovered from its labor share decline. The decline is divided by $1 + s_{\ell i} \cdot (\lambda - 1) \cdot \pi_i$ to adjust for the effects of automation on the labor share working via substitution across tasks, and by $\mu_{\mathscr{A}_i}/\mu_i$—the average rent in tasks automated in industry i—to adjust for the effects of automation operating through worker rents.

We measure task displacement using data for 49 industries from the BEA Integrated Industry-Level Production Accounts. We compute employment shares by industry, ℓ_{gi}/ℓ_g, and revealed comparative advantages $\mathrm{RCA}_{gi}^{\mathrm{routine}}$ for the 500 demographic groups from the 1980 Census. We define routine jobs as the 33% occupations with the highest routine content according to O*NET. Following Acemoglu and Restrepo (2022), we estimate $-d\ln s_{\ell i}^{d}$ as the predicted labor share decline from a cross-industry regression of percent labor share changes by industry (from the BEA, from 1987 to 2016 and re-scaled to a 36-year change) against three proxies for automation: the adjusted penetration of industrial robots (from Acemoglu and Restrepo, 2020), the increase in the share of specialized software services in value added, and the increase in the share of dedicated machinery in value added (from the BLS Total Multifactor Productivity Tables). For the adjustment term, we set $\lambda = 0.5$, $\pi_i = 30\%$ and $\mu_{\mathscr{A}_i}/\mu_i = 1.35$ for all industries. These choices are motivated in Section 4.

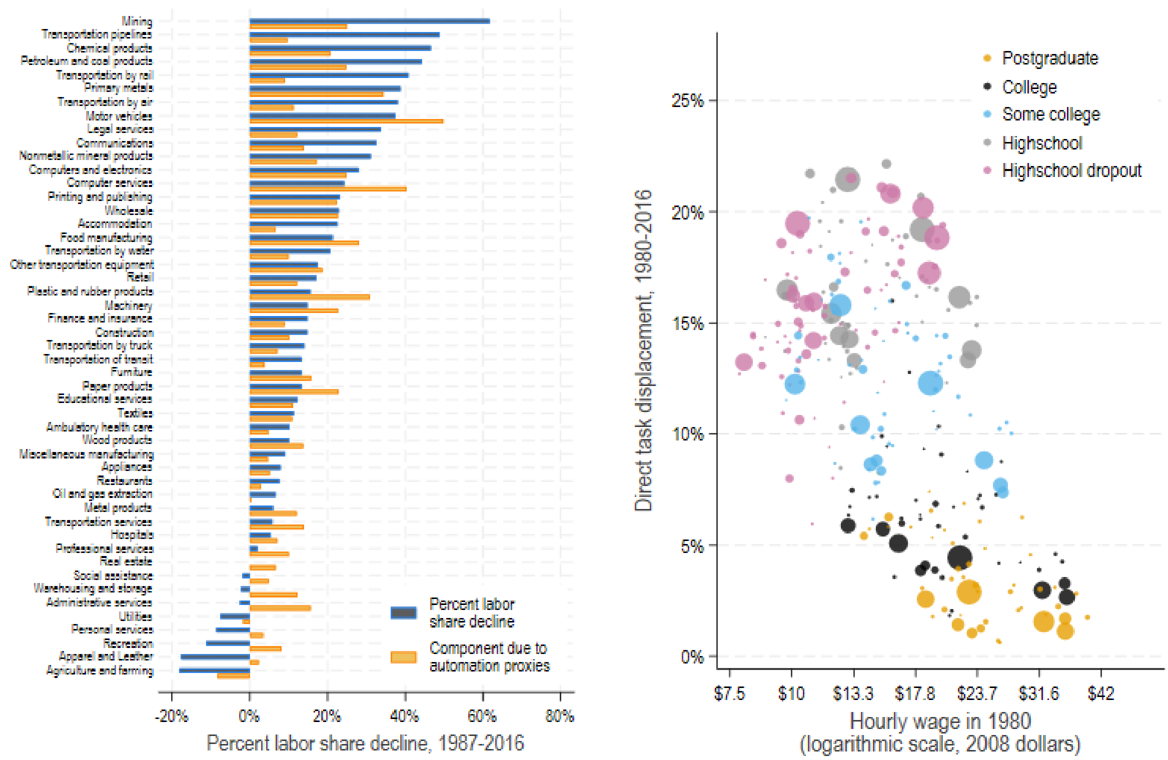

FIGURE 3: DIRECT TASK DISPLACEMENT DUE TO AUTOMATION ACROSS INDUSTRIES AND GROUPS. The left panel plots the labor share decline from 1987 to 2016 (in %) across US industries (positive values correspond to declines). The orange bars denote the component attributed to three proxies of automation. The right panel plots our measure of direct task displacement from equation (12) across 500 demographic groups between 1980–2016.

The left panel of Figure 3 summarizes the industry labor share trends. The blue bars show the observed labor share declines (in percent). The orange bars depict the component attributed to our proxies of automation, which jointly explain 50% of cross-industry changes in labor shares since 1987.

The right panel of Figure 3 depicts our measure of task displacement from automation during 1980–2016 for 500 US demographic groups, plotted against their baseline hourly wages in 1980 in the horizontal axis. Groups with post-college degrees lost few tasks due to automation between 1980 and 2016, while workers in the middle and lower middle of the wage distribution lost 15%–20% of their 1980 tasks to automation.[16]

3.2 Automation and Average Group Wages

As a benchmark, we first explore the reduced-form relationship between automation and average group wages. This involves estimating (11) with the change in group log average wages between 1980 and 2016 as dependent variable and is the analogue of the results in Acemoglu and Restrepo (2022).

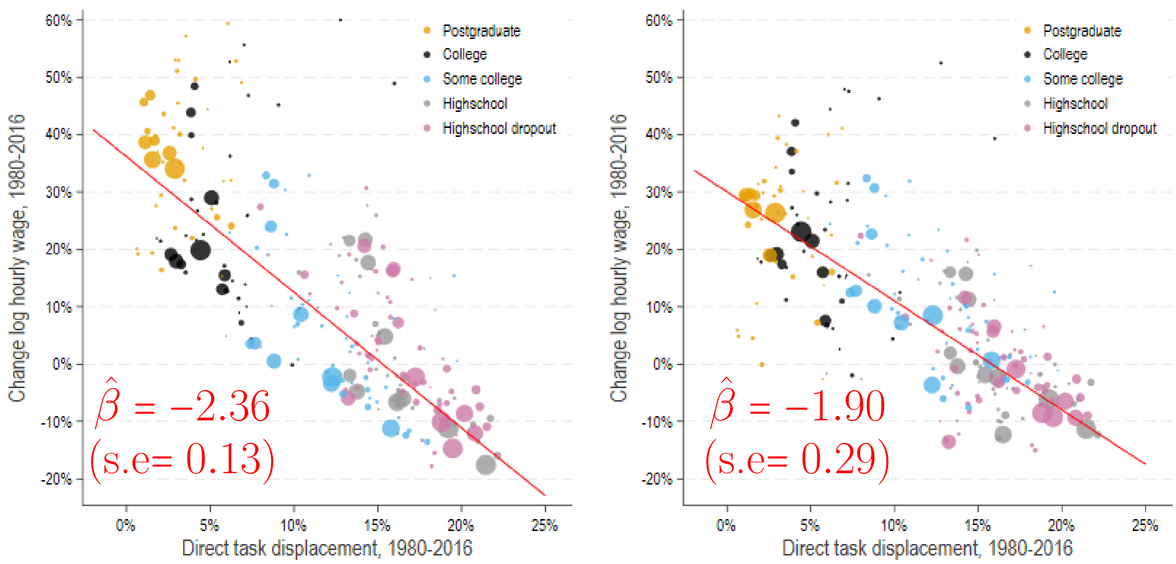

FIGURE 4: REDUCED-FORM RELATIONSHIP BETWEEN AVERAGE GROUP-LEVEL WAGE CHANGES AND TASK DISPLACEMENT. The left panel plots the bivariate relationship between change in group average real wages and task displacement. The right panel partials out the effects of gender and education dummies and sectoral demand and rent shifts.

The left panel in Figure 4 plots the bivariate relationship between changes in average group wages and their exposure to automation. The point estimate indicates that a 10 percentage point increase in task displacement is associated with a 24% reduction in group-level (relative) wages. This single measure of exposure to automation explains 66% of the

[16] The measure in (12) is the same as in Acemoglu and Restrepo (2022), except for the term $\mu_{\mathscr{A}_i}/\mu_i$, since our previous work did not consider the role of rents. There are also minor differences in weights explained in Supplement S3. These adjustments account for the differences in point estimates in this paper.

variation in wage changes between demographic groups in the US since 1980.

The right panel in Figure 4 depicts this relationship when we control for potential determinants of the demand for the labor of a group, including: (i) gender and education dummies, which allow for other forms of skill-biased technological change impacting groups with a college or post-college degree and other technological or social changes impacting the demand for women relative to men; and (ii) changes in sectoral composition affecting labor demand, measured by the following three terms:

$$\text{Sectoral demand shifts}_g = \sum_i \frac{\ell_{gi}}{\ell_g} \cdot \Delta \ln \text{value added}_i,$$

which accounts for exposure to expanding sectors (in value added);

$$\text{Sectoral rent shifts}_g = \sum_i \frac{\ell_{gi}}{\ell_g} \cdot \left(\frac{\bar{w}_{gi}}{\bar{w}_g} - 1 \right) \cdot \Delta \ln \text{value added}_i,$$

which allows financial effects on groups earning above-average rents in expanding sectors (in this expression, $\bar{w}_{gi}/\bar{w}_g$ is the ratio between the average wage earned by a group in industry i and the average group wage).[17] Finally, the third term is the employment shares of groups in manufacturing in 1980, which controls for shocks affecting all US manufacturing workers.

The point estimate for the right panel is now $\hat{\beta} = -1.98$, and implies that a 10 percentage point increase in task displacement from automation is associated with a 20% reduction in average group wages. Differences in exposure to automation continue to account for 53% of the variation in wage changes between US demographic groups since 1980.

Our theory implies that the relationship shown in Figure 4 reflects both direct displacement effects from automation (which reduce the relative base wage of exposed groups) and rent dissipation (which shifts exposed groups towards lower-rent paying jobs). The next section separates their roles and quantifies the importance of rent dissipation.

3.3 Evidence for Rent Dissipation

We first explore the within-group wage changes associated with automation, and document that, in line with the presence of rent dissipation, a sizable share of the wage decline in exposed groups takes place at top percentiles. We then use proxies for rents to show that automation shifts exposed groups of workers away from high rent jobs.

[17] For these covariates, we measure ℓ_{gi}/ℓ_g and $\bar{w}_{gi}/\bar{w}_g$ from the 1980 Census. We also measure the change in value added from the BEA industry accounts for 1987–2016, converted to a 36-year equivalent change.

Automation and within-group wage changes: We estimate a variant of equation (11) with the dependent variable as the change in log wages at the p-th percentile of the within-group wage distribution between 1980 and 2016, $\Delta \ln w_g^p$, for $p = 5, 10, \ldots, 95, 99$. To facilitate the interpretation of our findings, Figure 5 plots the effects of automation at these different percentiles relative to the wage change for the 30th percentile.[18]

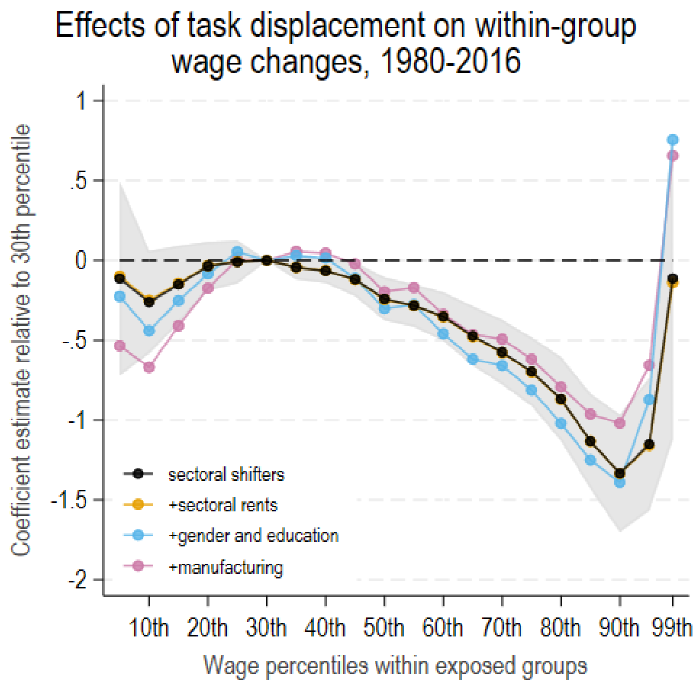

FIGURE 5: REDUCED-FORM RELATIONSHIP BETWEEN WAGE CHANGES ACROSS PERCENTILES OF THE WITHIN-GROUP WAGE DISTRIBUTION AND TASK DISPLACEMENT. The figure plots estimates from a group-level quantile regression of changes in $d \ln w_g^p$ against task displacement for percentiles p ranging from the 5th to the 99th relative to the 30th percentile. Different colors represent estimates from different specifications.

The black line depicts estimates from a specification that controls just for sectoral shifts. The remaining lines add sectoral rent shifts (orange line), gender and education dummies (blue line), and exposure to manufacturing (pink line).

In all specifications, a clear U-shaped pattern is visible. Groups exposed to automation saw a more pronounced decline in wages between the 70th and 95th percentiles of the within-group distribution. There is no differential decline below or around the 30th percentile, and no additional decline at the 99th percentile, consistent with Proposition 5.

[18]This specification is equivalent to an unconditional group-quantile regression, as in Chetverikov et al. (2016), and reveals the impact of automation on within-group wage dispersion. Because the impact on the 30th percentile varies somewhat between specifications, benchmarking to the 30th percentile makes the specifications easier to compare. We show the same figure in levels in Supplement S3.

One concern in interpreting Figure 5 is that minimum wages or other factors preventing low wages from falling further may account for the weaker effects at the bottom. Supplement S3 shows that our results are not impacted by these factors. The U-shaped pattern is robust to controlling for the incidence of the minimum wage or restricting the sample to groups with average real wages above \$13 in 1980. We also show that the results are similar when we control for declining unionization rates.[19]

Measuring rent dissipation from within-group wage changes: The results in Figure 5 motivate our first strategy for estimating the contribution of rent dissipation. In our theory, automation reduces wages at bottom percentiles of the within–group distribution by $d \ln w_g$, since these workers earn no rents. All declines beyond this level are due to the loss of rents, as workers are displaced from high-rent tasks. The pattern in Figure 5 suggests that workers below the 30th percentile in exposed groups earn no rents and are impacted only by changes in base wages, while above the 30th percentile there are worker rents. We can then separate the average wage change in exposed groups $d \ln \bar{w}_g$ into a base wage component, given by the decline at the 30th percentile $\Delta \ln w_g^{30th}$, and a rent dissipation component, given by the additional decline above the 30th percentile $\Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$.

The left panel in Figure 6 depicts estimates of the first component (estimates from equation (11) with $\Delta \ln w_g^{30th}$ as the dependent variable), and the right panel presents estimates of the rent dissipation component, by running the same regression with $\Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$ as the dependent variable. Both specifications partial out gender and education dummies, sectoral demand shifts, rent shifts, and exposure to manufacturing.

The left panel shows that a 10 percentage point higher task displacement is associated with a 15.3% decline in wages at the 30th percentile of exposed groups. The rest of the average wage effect is due to rent dissipation, as depicted in the right panel. A demographic group experiencing a 10 percentage point higher task displacement sees an additional 3.5% decline in wages above the 30th percentile. These numbers imply that a fifth of the overall impact on average wages in Figure 4 are due to rent dissipation, with the rest driven by changes in base wages.

Supplement S3 shows that these results are not sensitive to the use of the 30th percentile is our measure of base wages. The estimates are similar when we use the 20th and 40th. It also presents similar results when the sample is restricted to high-wage groups, when it

[19] A separate concern is that, in low-wage groups, workers between the 70th and 95th percentiles may be more exposed to routine jobs, for example, because these jobs are available to the most skilled workers in these groups. The fact that the pattern applies among higher-wage groups weighs against this concern.

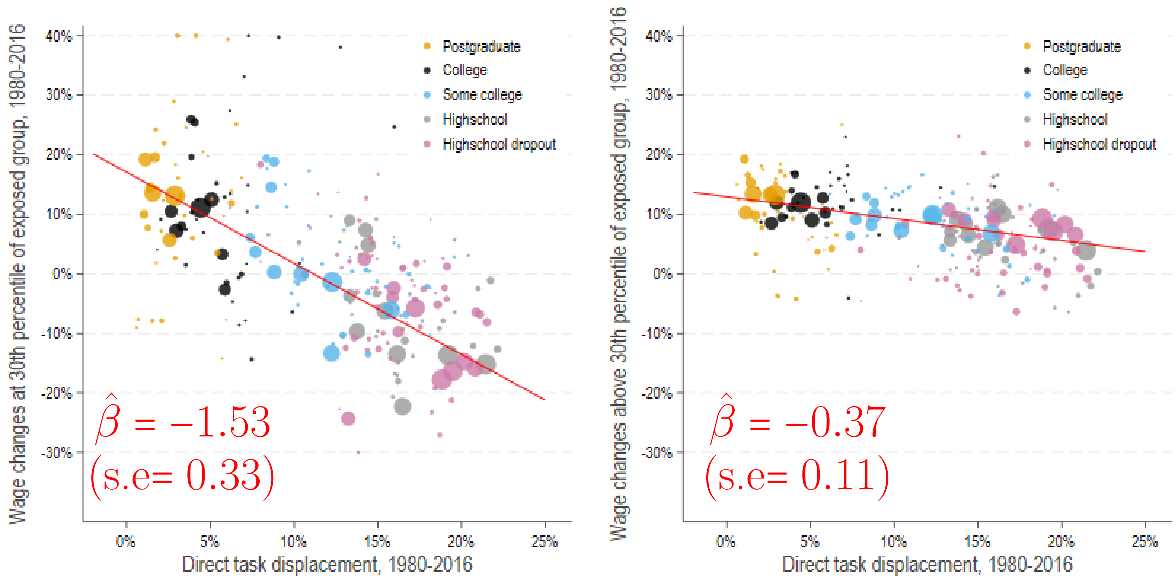

FIGURE 6: REDUCED-FORM RELATIONSHIP BETWEEN RENTS AND TASK DISPLACEMENT. The left panel plots estimates of equation (11) using the change in wages at the 30th percentile of a exposed group as a measure of base wage changes, $\Delta \ln w_g^{30th}$. The right panel plots estimates of equation (11) using the change in wages beyond the 30th percentile of a exposed group as a measure of rent dissipation, namely $\Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$. Both specifications partial out gender and education dummies, sectoral demand and rent shifts, and exposure to manufacturing.

is limited to non-college groups, and when limited to workers with college education.

Measuring rent dissipation using proxies for rents: Our second strategy for estimating the effects of automation via rent dissipation uses time-invariant proxies for rents and traces the change in task composition for groups across jobs with different rents.

A key prediction of our theory is that automation dissipates rents by shifting workers away from high-rent jobs. We measure these shifts in job composition as

$$\Delta \ln \mu_g^{\text{composition}} = \sum_n \text{worker rent proxy}_{gn} \cdot \Delta \ell_{gn},$$

where worker rent proxy$_{gn}$ is one of our time-invariant proxies for rents in job n (defined by industry or industry×occupation) relative to the average job, and $\Delta \ell_{gn}$ is the change in the share of hours worked across jobs by workers from group g between 1980 and 2016. We then estimate equation (11) using $\Delta \ln \mu_g^{\text{composition}}$ as the dependent variable. The coefficient on task displacement gives the extent to which automation shifted exposed groups of workers away from high-rent jobs, and provides direct estimates of its rent dissipation effects.

Our first proxy for rents uses inter-industry and occupation wage differentials in 1980. This approach builds on the wage differentials literature, which documents that such differences are persistent and largely unrelated to observable worker characteristics and job attributes (see Krueger and Summers, 1988; Katz and Summers, 1989). We measure the relative rent paid in an industry and occupation to a worker from group g as $\bar{w}_{gio}/\bar{w}_g$, where $\bar{w}_{gio}$ is the average wage earned by group g in industry i and occupation o in 1980.[20]

The top-left panel in Figure 7 depicts results using this proxy for rents, controlling for our baseline covariates. We find that a 10 percentage point increase in task displacement reduces group rents by 3.9% by reallocating workers away from higher-wage industries and occupations. The similarity of the magnitudes of the estimates in this and the previous strategy suggests that, consistent with our modeling of constant wedges, automation reduces group rents primarily by reallocating workers away from high-rent tasks, and not by reducing rents within jobs.

The remaining panels present results using alternative proxies, aimed at building the case that the relationship in the top-left panel is not due to the loss of compensating differentials or a higher incidence of automation on workers of higher unobserved productivity.

First, we use estimates of wage losses due to displacement as an alternative proxy for rents, based on the idea that displacement losses proxy for rents in previous jobs, while also controlling for differences in worker skills (e.g., Krueger and Summers, 1988; Ruhm, 1991; Jacobson et al., 1993). We compute wage losses using the Displaced Worker Supplement from the CPS, and focus on workers who found a new job after displacement. Because the sample of displaced workers is small, we estimate wage losses by industry and six broad occupations, allow these to vary by gender and education (rather than at the more granular level of 500 demographic groups), and averaged them over 1984–2022.[21]

The top-right panel in Figure 7 depicts our findings using this rent proxy, and controlling for our baseline covariates. We find that a 10 pp increase in task displacement reduces group-level rents by 2% by reallocating workers away from jobs where rents—as measured by wage losses due to displacement—are higher.

[20]This is equivalent to measuring worker rents by a group-specific industry× occupation intercept in a Mincer equation, while controlling for a full set of interactions between age, gender, education, race and birthplace. Our baseline results use wage differentials for the 49 industries in our analysis and 300 detailed Census occupations. Supplement S3 provides robustness checks using wage differentials computed for broader occupational groups or only across industries.

[21]We compute wage losses for workers with at least one year of tenure in their pre-displacement job. A higher tenure threshold reduces the sample of displaced workers, but does not affect our estimates. Supplement S3 also reports robustness checks using a common wage loss, instead of a gender and education-specific wage loss as our proxy for rents.

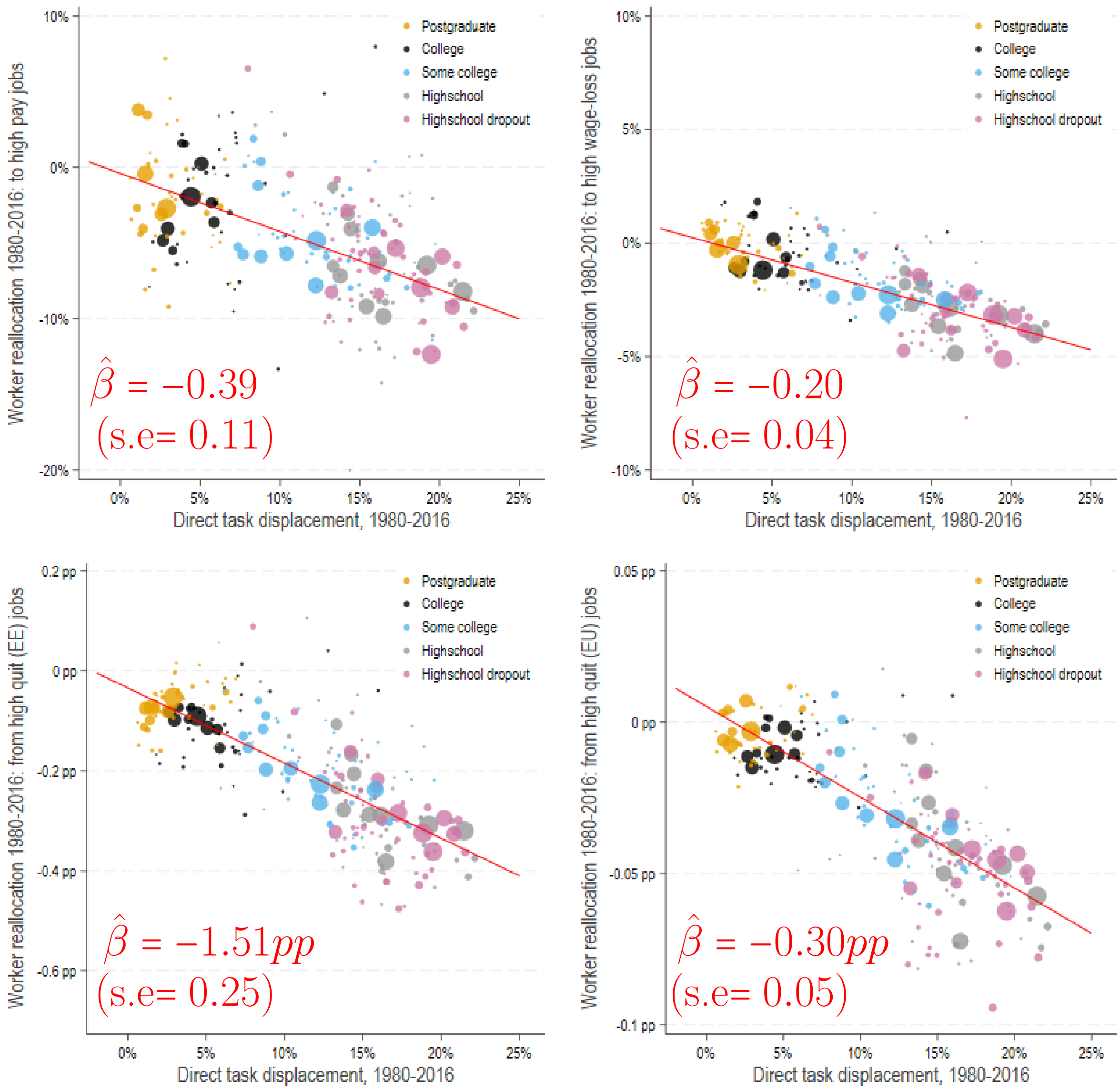

FIGURE 7: REDUCED-FORM RELATIONSHIP BETWEEN RENTS AND TASK DISPLACEMENT. The figures show the reduced-form association between automation and the reallocation of groups across jobs paying different rents. The outcome is measured as $\Delta \ln \mu_g^{\text{reallocation}} = \sum_n \text{worker rent proxy}_{gn} \cdot \Delta \ell_{gn}$, where worker rent proxy$_{gn}$ measures rents paid in job j to group g. The top-left panel presents results using inter-industry and occupation wage differentials in 1980 to proxy for rents. The top-right panel presents results using wage loss from job displacement to proxy for rents (computed from the CPS Displaced Worker Supplement). The bottom-left panel presents results using the employment-to-employment monthly transition rate as an inverse proxy (computed from the Basic Monthly CPS). The bottom-right panel presents results using the voluntary employment-to-unemployment monthly transition rate as an inverse proxy for rents (computed from the Basic Monthly CPS).

Our final two proxies for rents are based on worker quit behavior. The idea is that workers are less likely to quit jobs that pay higher rents (as opposed to jobs that pay higher wages as compensating wage differentials or in return for their greater unobserved skills). We use the Basic Monthly CPS and compute employment-to-employment (EE) and voluntary employment-to-unemployment (EU) monthly transition rates following Fujita et al. (2024). In line with the interpretation above, industries where workers see a 10% larger drop in wages following a job loss have 0.37 pp lower EE rates and 0.065 pp lower voluntary EU rates per month. The EE and EU measures are computed for broader groups defined by gender and education across industries and occupations. The EE measure is averaged over 1994–2023 and the EU measure is average over 1976–2023.[22]

The bottom two panels of Figure 7 depict the estimates using the negative of EE and voluntary EU rates as proxies for rents, controlling once again for baseline covariates. We find that a 10 pp increase in task displacement pushes workers away from jobs with a 0.15 pp lower EE rate and a 0.03 pp lower voluntary EU rate per month. These results suggest that automation displaced workers from jobs that they themselves were less likely to leave. Using the association between wage losses and quit rates reported above, these estimates imply that automation reduced worker wages via rent dissipation by 4%-4.6%, which is of similar magnitude to those obtained from our other proxies.

All proxies for rents in Figure 7 show that there has been a pervasive shift away from high-rent jobs for most groups of US workers since 1980, and identify the automation of high-rent jobs as a plausible driver of this trend. Our regression results indicate that 33% (in the top left panel) to 68% (in the bottom left panel) of these shifts across groups during this period can be explained by differences in exposure to automation.

Taking stock: The results in Figures 5, 6, and 7 support the key implication of our theory—that automation reallocates workers away from higher-rent jobs, generating rent dissipation and compressing within-group wage differences. All our strategies and proxies for rents provide broadly similar estimates for rent dissipation ranging from 0.2 and 0.46, with a central estimate of 0.35. The estimates imply that workers used to earn an average rent of 35% ($\mu_{\mathscr{A}_g}/\mu_g = 1.35$ in the theory) in jobs automated between 1980 and 2016.

[22] We compute average quit rates for the 49 industries in our analysis and 300 detailed occupations. We also partial out time trends in the CPS from these industry×occupation averages. Supplement S3 reports robustness checks where we use average quit rates at different levels of aggregation.

4 Equilibrium Effects of Automation

This section estimates the equilibrium effects of automation in the US between 1980 and 2016, focusing on the role of rent dissipation and its implications. We first extend our model to a multi-sector economy to account for differences in automation and rents across industries. We then derive formulas for the effects of automation and describe how we measure and estimate the objects needed to quantify the effects of automation.

4.1 Multi-Sector Model

Supplement S1 provides the details of the multi-sector economy. The main difference is that it introduces multiple sectors $i \in \mathbb{I}$, each with its own set of tasks $\mathcal{T}_i$. Sectoral outputs are combined into a final good via an aggregator with elasticity of substitution $\eta > 0$.

We model new automation technologies as an exogenous increase in q_x from zero to q_x' taking place at tasks $\{\mathscr{A}_{gi}^T\}_{i\in\mathbb{I},g\in\mathbb{G}}$ assigned to workers across industries. As before, $\mathscr{A}_{gi}$ denotes the subset of tasks that are actually automated. Additionally, $d\ln\Gamma_{gi}^d$ denotes the direct task displacement for group g in industry i (with $d\ln\Gamma_g^d$ the total task displacement across all industries), π_{gi} denotes the cost savings from automating tasks in $\mathscr{A}_{gi}$, and $\mu_{\mathscr{A}_{gi}}$ is the average rent in these tasks (with $\mu_{\mathscr{A}_g}$ its average across all industries).

Our main result extends Proposition 3 to the multi-sector economy. This extension is the basis for our quantitative exercise.

Proposition 6 (Effects of Automation in Multi-Sector Economy)
Consider new automation technologies in small interior sets $\{\mathscr{A}_{gi}^T\}_{i\in\mathbb{I},g\in\mathbb{G}}$ with direct effects $\left\langle\{d\ln\Gamma_{gi}\}_{i\in\mathbb{I},g\in\mathbb{G}},\{\pi_{gi}\}_{i\in\mathbb{I},g\in\mathbb{G}},\{\{\mu_{\mathscr{A}_{gi}}\}_{i\in\mathbb{I},g\in\mathbb{G}}\}\right\rangle$. The first-order impact on base wages w_g, sectoral prices p_i, and output y are given by the solution to the system of equations:

$$(13)\qquad d\ln w_g = \Theta_g \cdot stack\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_j^d + \frac{1}{\lambda}\cdot\sum_i \frac{\ell_{ij}}{\ell_j}\cdot(\lambda-\eta)\cdot d\ln p_i\right) \quad \textit{for } g\in\mathbb{G}$$

$$(14)\qquad d\ln p_i = \sum_g s_{gi}\cdot d\ln w_g - \sum_g s_{gi}\cdot\frac{\mu_{\mathscr{A}gi}}{\mu_{gi}}\cdot d\ln\Gamma_{gi}^d\cdot\pi_{gi} \quad \textit{for } i\in\mathbb{I}$$

$$(15)\qquad \sum_g s_g\cdot d\ln w_g = \sum_i s_{y_i}\cdot\sum_g s_{gi}\cdot\frac{\mu_{\mathscr{A}gi}}{\mu_{gi}}\cdot d\ln\Gamma_{gi}^d\cdot\pi_{gi},$$

where s_{gi} is the share of group g's earnings in industry i's output and s_{y_i} is the output share *of industry i in GDP. Moreover, the change in group rents is given by*

$$(16)\qquad d\ln\mu_g = \mathcal{M}_g \cdot stack\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_j^d + \frac{1}{\lambda}\cdot\sum_i \frac{\ell_{ij}}{\ell_j}\cdot(\lambda-\eta)\cdot d\ln p_i\right) - \left(\frac{\mu_{\mathscr{A}_g}}{\mu_g}-1\right)\cdot d\ln\Gamma_g^d + \sum_i\left(\frac{\mu_{gi}}{\mu_g}-1\right)\cdot\frac{\ell_{gi}}{\ell_g}\cdot(\lambda-\eta)\cdot d\ln p_i \quad \textit{for } g\in\mathbb{G}$$

and the change in aggregate consumption is $d\ln c = (1/s_L)\cdot d\ln tfp$, where s_L is the economy-wide labor share and $d\ln tfp$ is the contribution of new automation technologies to TFP:

$$(17)\qquad d\ln tfp = \sum_i s_{y_i}\cdot\sum_g s_{gi}\cdot\frac{\mu_{\mathscr{A}gi}}{\mu_{gi}}\cdot d\ln\Gamma_{gi}^d\cdot\pi_{gi} + \sum_g s_g\cdot d\ln\mu_g.$$

From the proposition, we obtain the effect of automation on average group wages as

$$(18)\qquad d\ln\bar{w}_g = (\Theta_g + \mathcal{M}_g)\cdot \text{ stack}\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_j^d + \frac{1}{\lambda}\cdot\sum_i \frac{\ell_{ij}}{\ell_j}\cdot(\lambda-\eta)\cdot d\ln p_i\right) - \left(\frac{\mu_{\mathscr{A}_g}}{\mu_g}-1\right)\cdot d\ln\Gamma_g^d + \sum_i\left(\frac{\mu_{gi}}{\mu_g}-1\right)\cdot\frac{\ell_{gi}}{\ell_g}\cdot(\lambda-\eta)\cdot d\ln p_i.$$

Relative to Proposition 3, these formulas account for the changes in sectoral prices and the induced shifts in sectoral composition due to automation (for example, because automation in one industry reallocates expenditure towards or away from that industry due to price changes). Equation (14) shows that sectoral prices change due to base wage changes (from Shephard's lemma) and decrease in proportion to cost savings from automation. The induced changes in sectoral composition affect base wages by shifting demand across groups (the term $\sum_i \frac{\ell_{ij}}{\ell_j}\cdot(\lambda-\eta)\cdot d\ln p_i$ in 13 and 18) and rents by shifting workers to or away from sectors where they earn above-average rents (the term $\sum_i\left(\frac{\mu_{gi}}{\mu_g}-1\right)\cdot\frac{\ell_{gi}}{\ell_g}\cdot(\lambda-\eta)\cdot d\ln p_i$ in (16) and (18)).

4.2 Measurement and Estimation

The formulas in Proposition 6 allow us to compute the first-order effects of automation on group wages, rents, output, TFP, and welfare in terms of the following objects (all of which can be directly measured or estimated):

(i) the direct task displacement from automation for all demographic groups across industries $d\ln\Gamma_{gi}^d$ and in the aggregate $d\ln\Gamma_g^d$;

(ii) rent dissipation due to automation, $\mu_{\mathscr{A}_{gi}}/\mu_g$ and $\mu_{\mathscr{A}_g}/\mu_g$;

(iii) cost savings from automation, π_{gi};

(iv) the propagation matrix, Θ, and the rent-impact matrix, $\mathcal{M}$;

(v) the elasticities of substitution between tasks, λ, and between sectors, η;

(vi) factor and labor shares by industry at the initial equilibrium.

Because we lack disaggregate data for different types of automation, we assume that the cost savings from the adoption of industrial robots in manufacturing—about 30% in Acemoglu and Restrepo (2020)—applies across the board and set $\pi_{gi} = 30\%$ for all industries and groups. We also assume a common rate of rent dissipation across industries and groups, i.e., $\mu_{\mathscr{A}_{gi}}/\mu_{gi} = 1 + \rho$. Finally, we obtain initial factor and labor shares from BEA industry accounts, and set $\lambda = 0.5$ from Humlum (2020) and $\eta = 0.2$ from Buera et al. (2015).

We next explain how we measure task displacement across sectors, and estimate the rate of rent dissipation ρ and the propagation and rent-impact matrices, Θ, and $\mathcal{M}$.

Measuring task displacement: We use the 500 demographic groups from our reduced-form analysis. We measure task displacement based on industries' labor share decline due to automation, as in the previous section. We then apportion this across groups based on revealed comparative advantage in routine tasks. Specifically, using Assumption 3, the task displacement from automation for group g in industry i can be computed as

$$d \ln \Gamma_{gi}^{d} = \text{RCA}_{gi}^{\text{routine}} \cdot \frac{-d \ln s_{\ell i}^{d}}{1 + s_{\ell i} \cdot (\lambda - 1) \cdot \pi} \cdot \frac{1}{1 + \rho},$$

while the total task displacement across all industries is computed as

$$d \ln \Gamma_{g}^{d} = \sum_{i} \frac{\ell_{gi}}{\ell_g} \cdot d \ln \Gamma_{g}^{d}.$$

This equation is a special case of (12) above, under the assumption that there is a common rent dissipation ρ. We treat ρ as an unknown to be estimated.

Estimating the propagation and rent-impact matrices: The changes in base wages and rents in response to shocks in the multi-sector model are

(19) $$\Delta \ln w_g = \beta_0 - \frac{1}{\lambda} \cdot d\ln \Gamma_g^d + \beta \cdot Z_g + \frac{1}{\lambda} \cdot \mathcal{J}_{\Gamma,g} \cdot \text{stack}(\Delta \ln w_j) + u_g$$

(20) $$\Delta \ln \mu_g = \beta_0^\mu - \rho \cdot d\ln \Gamma_g^d + \beta^\mu \cdot Z_g^\mu + \mathcal{J}_{\mu,g} \cdot \text{stack}(\Delta \ln w_j) + e_g.$$

The constant term β_0 captures common shifts that benefit all workers equally, such as the expansion of GDP, $\Delta \ln y$. Additionally, Z_j denotes observable shocks affecting the demand for workers of group j directly (such as sectoral demand shifts), while Z_j^μ corresponds to these shocks' effects on rents (such as sectoral rent shifts). The terms $\mathcal{J}_{\Gamma,g} \cdot \text{stack}(\Delta \ln w_j)$ and $\mathcal{J}_{\mu,g} \cdot \text{stack}(\Delta \ln w_j)$ account for ripple effects and the endogenous reallocation of tasks. The error term u_g represents unobserved labor demand shocks for demographic group g, while e_g corresponds to any unobserved influences on group g's rents.

We estimate ρ, Θ and $\mathcal{M}$ from the system of equations (19) and (20) via GMM. Following our reduced-form analysis, we measure the change in base wages $d\ln w_g$ by the change at the 30th percentile of the within-group distribution, and take the decline in wages above the 30th percentile of exposed groups as a measure for the change in rents, $d\ln \mu_g$. This procedure requires parameterizing the Jacobians $\mathcal{J}_\Gamma$ and $\mathcal{J}_\mu$, and estimating ρ and the parameters of the Jacobians from the orthogonality conditions

$$d\ln \Gamma_g^d, Z_g, Z_g^\mu \perp u_j, e_j \text{ for all } g, j.$$

and then computing the propagation and rent-impact matrices as $\Theta = \left(\mathbb{1} - \frac{1}{\lambda} \cdot \mathcal{J}_\Gamma\right)^{-1}$ and $\mathcal{M} = \mathcal{J}_\mu \cdot \left(\mathbb{1} - \frac{1}{\lambda}\mathcal{J}_\Gamma\right)^{-1}$ at the estimated parameters.

We parameterize the entries of the Jacobian matrices in terms of job and demographic group-level similarities. For the diagonal terms, we take:

$$\mathcal{J}_{\Gamma,g,g} = (s_g - 1) \cdot \varphi - \sum_n \sum_{g' \neq g} \frac{\ell_{gn}}{\ell_g} \cdot s_{g'n} \cdot \left[\theta + \theta_{\text{job}} \cdot \text{job similarity}_{gg'} + \theta_{\text{edu-age}} \cdot \text{edu-age similarity}_{gg'}\right]$$

and the off-diagonal terms, for $g' \neq g$, are parameterized as:

$$\mathcal{J}_{\Gamma,g,g'} = s_{g'} \cdot \varphi + \sum_n \frac{\ell_{gn}}{\ell_g} \cdot s_{g'n} \cdot \left[\theta + \theta_{\text{job}} \cdot \text{job similarity}_{gg'} + \theta_{\text{edu-age}} \cdot \text{edu-age similarity}_{gg'}\right].$$

Our parameterization assumes that competition between groups for marginal tasks takes place within job categories, denoted by n, and defined in the data at the level of 16 industries and six occupations. The impact of competition from demographic group g' on group g in job category n depends on the importance of this job category for group g, ℓ_{gn}/ℓ_g, and $s_{g'n}$, which captures the share of earnings in job category n accruing to group g', both measured from the 1980 Census. Intuitively, groups with greater which shares should generate more competitive pressure on other groups in the same job category.

The parameters θ, θ_{job}, $\theta_{\text{edu-age}} \geq 0$ represent the extent of competition for marginal tasks across groups. In particular, θ summarizes the competition for tasks common to all workers in a job category,[23] while θ_{job} parameterizes the extent to which competition is more intense for workers performing similar jobs to group g' in the economy as a whole. This is measured by the cosine similarity in job categories performed by groups g' and g in the 1980 Census. Finally, $\theta_{\text{edu-age}}$ parameterizes the extent to which competition for tasks is stronger for workers of similar education and experience, as in Card and Lemieux (2001). This is measured by one minus the difference in experience and education (converted to wages using a Mincer equation) between groups g and g'.

The parameter $\varphi \geq 0$ controls the extent of competition between capital and workers for marginal tasks. Recall that the row sums of the task Jacobian are equal to $-s_k \cdot \varphi$, and thus summarize the extent to which marginal tasks are reallocated towards capital when all base wages increase by a common amount. We calibrate φ externally, exploiting the fact that, with this parameterization, the aggregate elasticity of substitution between capital and labor is $\sigma = \lambda + \varphi$ (capturing the sum of substitution between tasks and substitution between demographic groups in marginal tasks), and set $\varphi = 0.1$ to match estimates of $\sigma = 0.6$ from Oberfield and Raval (2021).

For the rent Jacobian, we follow a similar strategy and parameterize it as

$$\mathcal{J}_{\mu,g,g} = -\sum_n \sum_{g' \neq g} \left(\frac{\bar{w}_{gn}}{\bar{w}_g} - 1 \right) \cdot \frac{\ell_{gn}}{\ell_g} \cdot s_{ng'} \cdot \left[\theta + \theta_{\text{job}} \cdot \text{job similarity}_{gg'} + \theta_{\text{edu-age}} \cdot \text{edu-age similarity}_{gg'} \right],$$

and the off-diagonal terms, for $g' \neq g$, as

$$\mathcal{J}_{\mu,g,g'} = \sum_n \left(\frac{\bar{w}_{gn}}{\bar{w}_g} - 1 \right) \cdot \frac{\ell_{gn}}{\ell_g} \cdot s_{ng'} \cdot \left[\theta + \theta_{\text{job}} \cdot \text{job similarity}_{gg'} + \theta_{\text{edu-age}} \cdot \text{edu-age similarity}_{gg'} \right].$$

The term $\left(\frac{\bar{w}_{gn}}{\bar{w}_g} - 1 \right)$ proxies for rents earned by group g in job category n, and accounts for whether competition from demographic group g' takes place at jobs where g workers earned

[23] This form of common competition can be micro-founded by assuming that worker productivity for all tasks in job category n are drawn from independent Frechet distributions, with a common shape parameter α and different scale parameters across groups. This specification implies $\theta = \alpha + 1 - \lambda$.

above average rents. We compute this from observed wages in the 1980 Census as well. The rows of the rent Jacobian sum to zero, which imposes the restriction that substitution of capital for labor at marginal tasks does not affect group rents on average.

Table S5 in Supplement S3 reports our GMM estimates for ρ and the θ's. Here we summarize the main findings:

- The common rent dissipation coefficient is estimated as $\hat{\rho} = 0.35$ (s.e.=0.12). This estimate aligns with the reduced-form evidence in Section 3.
- The estimated propagation matrix has an average diagonal of 0.34, and the row sum of the off-diagonal terms is about 0.6. This implies that workers from exposed demographic groups bear 36.5% of the incidence of automation, with the rest shifted to other groups via competition for marginal tasks.
- The entries of the estimated rent-impact matrix are small. This implies that indirect effects on group rents via endogenous task reassignment are limited.

4.3 Equilibrium Effects of Automation

This section reports our estimates for the effects of new automation technologies on wages, rents, and TFP in the US between 1980 and 2016 using the formulas from Proposition 6. Table 1 summarizes the estimates in column 2 alongside the data in column 1.

Effects on wages and group rents: Panel A in Table 1 reports our estimates for wages, and Figure 8 depicts the same information by plotting the change in average wages for the 500 demographic groups, computed using equation (18). The vertical axis is for the change in wages during 1980–2016 due to new automation technologies. The horizontal axes in all panels sort groups according to their average hourly wages in 1980.

The panels of Figure 8 show the (cumulative) implications of different economic forces, starting from the productivity effect $(1/\lambda) \cdot d\ln y$ in Panel A. We estimate an expansion in output of 14.4% over 1980–2016 in response to automation, which raises wages by 28.8%. Panel B adds sectoral shifts induced by automation, and plots $(1/\lambda) \cdot d\ln y + (1/\lambda) \cdot \sum_i (\ell_{ij}/\ell_j) \cdot (\lambda - \eta) \cdot d\ln p_i$, and shows that automation generates modest sectoral shifts with limited wage effects.

Panel C adds the direct displacement effects from new automation technologies working through base wages by plotting $(1/\lambda) \cdot d\ln y + (1/\lambda) \cdot \sum_i (\ell_{ij}/\ell_j) \cdot (\lambda - \eta) \cdot d\ln p_i - (1/\lambda) \cdot d\ln \Gamma_g^d$.

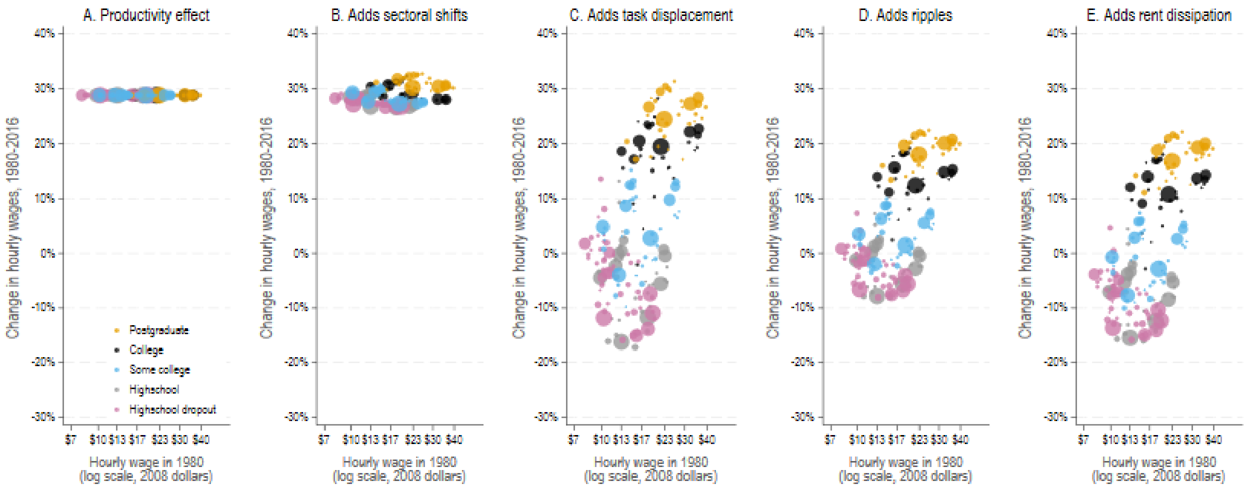

FIGURE 8: WAGE EFFECTS FROM AUTOMATION. The figure plots estimates of the estimated effects of automation on between-group wage changes. In all panels, the horizontal axis gives each group's hourly wage in 1980.

Consistent with the reduced-form evidence, task displacement has sizable effects on the wage structure and generates a decline in the real wage of highly exposed groups, while groups not exposed to automation enjoy real wage gains.

Panel D illustrates the equalizing implications of ripple effects. This panel plots $\Theta_g \cdot$ stack $((1/\lambda) \cdot d\ln y - (1/\lambda) \cdot d\ln \Gamma^d + (1/\lambda) \cdot \sum_i(\ell_{i\cdot}/\ell_j) \cdot (\lambda - \eta) \cdot d\ln p_i)$, and shows a less pronounced response in wage changes across groups than in Panel C. This is because workers directly impacted by automation suffer wage declines and this makes them more effective in competing for marginal tasks previously allocated to other factors. These ripple effects spread two thirds of the incidence of automation across demographic groups.

Panel E adds the change in group rents, $d\ln \mu_g$. It confirms that new automation technologies generated sizable rent dissipation effects during 1980–2016. Most of the decline in rents across groups comes from direct rent dissipation—automation targeting jobs with higher than average rents—rather than the indirect effects working through the rent-impact matrix. As explained in the theory, exposed groups bear the full incidence of rent dissipation. This is why rent dissipation intensifies income losses for these groups, exacerbating the inequality implications of automation.

To further illustrate the contribution of rent dissipation, Figure 9 plots observed group-level wage changes in the vertical axis against the estimated wage changes due to new automation technologies between 1980 and 2016 in the horizontal axis. The left panel plots the effects of new automation technologies via base wages, omitting the contribution of rent dissipation. The right panel plots the full effects including rent dissipation. The change in base wages explains 42% of the observed wage changes, while the estimates on the right, incorporating rent dissipation, explain 52% of observed wage changes. New automation technologies, in total, explain 52% of the rise in between-group wage inequality in the US between 1980 and 2016, with a fifth of this effect being due to rent dissipation.[24]

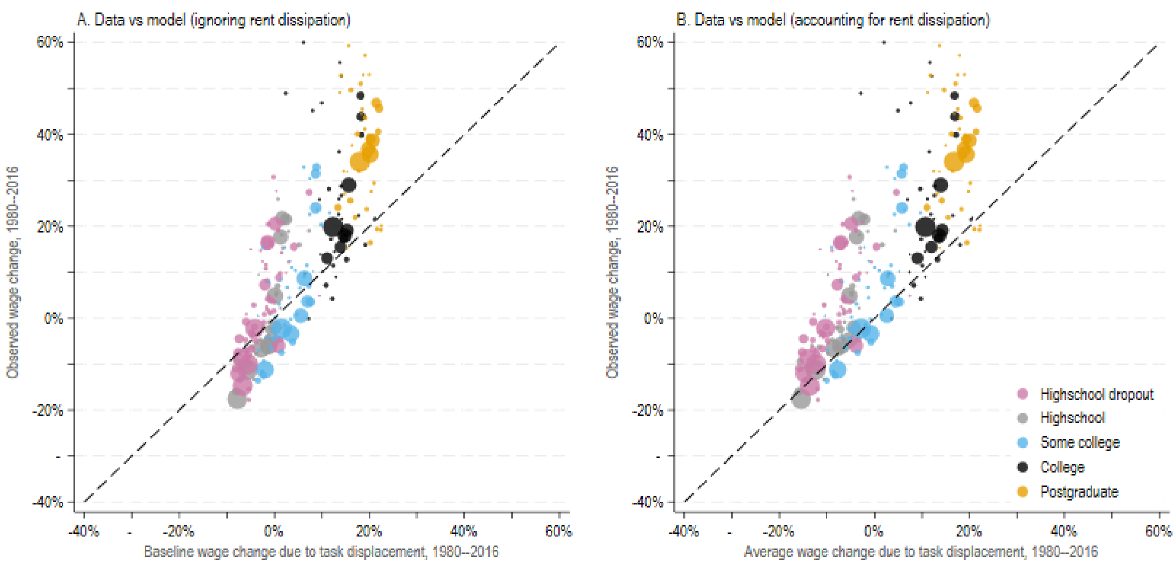

FIGURE 9: WAGE EFFECTS FROM AUTOMATION: MODEL VS DATA. The left panel plots the predicted wage changes, without the effects of changes in rents, against observed wage changes from 1980–2016. The right panel plots the predicted wage changes in our model, incorporating the change in rents, against observed wage changes from 1980–2016.

Rent dissipation is particularly relevant for explaining the lack of real wage growth among low-education groups. The left panel shows several groups below the 45° line, indicating that without rent dissipation, new automation technologies cannot account for the real wage decline seen for various groups from 1980 to 2016. In contrast, in the right panel, where we incorporate the rent dissipation effects, automation accounts for most of the observed real wage declines and stagnation.

As a further illustration, Table 1 reports our estimates of the change in wages and rents for non-college men and women. Without rent dissipation, automation would have led to a 2.4% decline in the real wage of non-college men during 1980–2016 (compared to 6.5% in the data) and a 2.4% increase in the real wage of women. Once rent dissipation is factored in, we estimate an 8% decline for non-college men and a 2.2% decline for non-college women.

[24]The 42% number is lower than the 50% estimate in Acemoglu and Restrepo (2022) from a model with competitive labor markets. This is because our current framework separates out the role of rent dissipation.

Rent dissipation also implies that automation brought essentially no aggregate wage gains in this period. Without rent dissipation, automation would have led to approximately 4.5% average increase in the real wage of US workers during 1980–2016. Once rent dissipation is factored in, we estimate a small 0.5% increase.

Effects on TFP and consumption: Panel B in Table 1 reports our estimates for aggregate quantities and TFP. Our benchmark value for cost savings of $\pi = 30\%$, combined with the extent of automation observed in the data, implies that new automation technologies contributed a 3% increase in TFP to the US economy via cost savings from 1980 to 2016. However, our estimate for $\mu_{\mathscr{A}_g}/\mu_g$ of 35% means that new automation technologies also created inefficient rent dissipation. Our estimates imply that new automation technologies worsened allocative efficiency by about 2.7% during this period. This inefficiency offset 90% of the positive cost savings and led to a net contribution of automation to TFP of 0.3% in total over the time period 1980–2016.[25]

Turning to consumption, and using the formula $d\ln c = (1/s_L) \cdot d\ln tfp$, we estimate that new automation technologies increased aggregate consumption by about 0.46% during 1980–2016. This is smaller than the estimated increase in GDP of 14.4% because the GDP increase reflects greater investment to produce capital equipment. According to our estimates, new automation technologies raised the capital-output ratio—a measure of the share of resources invested—by 27.9%. The increase in investment is in line with BLS data, which indicate a 30% increase in the aggregate capital stock relative to GDP.

4.3.1 Robustness

Table 1 also reports a number of robustness checks. Column 3 reports estimates obtained by assuming a rate of rent dissipation of 25%, as opposed to our baseline estimate of 35%. The impact of rent dissipation on wages and TFP is less pronounced, but it still offsets 70% of the cost savings and average wage gains from new automation technologies.

Column 4 reports estimates obtained by assuming a rate of rent dissipation of 50%. At this rate of rent dissipation, we estimate that automation between 1980 and 2016 would reduce average wages by 0.7%, the wages of non-college men by 9%, and the wages of

[25] US TFP increased by 30% over this time period. Our estimates imply that almost all of this increase was due to other technologies—not due to automation. These other technologies include those that create new tasks for workers or new products, those that provide better information or tools to workers, raising their productivity in tasks already assigned to them, or improvements in capital productivity in tasks that were already automated.

non-college women by 3.3%. We also estimate that the worsening allocative efficiency due to rent dissipation would overwhelm the cost saving gains from automation, reducing aggregate TFP by 0.47% during 1980–2016.

Column 5 reports estimates from a simple specification of the Jacobians where we set $\theta = 1.6 - \lambda$, $\theta_{\text{job}} = \theta_{\text{edu-age}} = 0$ and the rate of rent dissipation equal to 35%. This specification imposes a common elasticity of substitution of 1.6 between groups, as in Katz and Murphy (1992), and leads to very similar similar estimates of the effects of new automation technologies on wages and productivity.

5 Conclusion

This paper developed a framework for studying the effects of automation in labor markets with worker rents—meaning that workers earn above their opportunity cost in some tasks. Our main finding is that the presence of worker rents alter the consequences of automation.

We show that automation has targeted higher-rent tasks, which are more expensive for firms to perform using labor and hence more attractive to automate. This has novel implications for within-group inequality, wages, and efficiency:

1. *Within-group wage effects of automation:* Because higher-rent tasks are automated first, automation reduces within-group wage dispersion in exposed groups.

2. *Wage effects of automation:* The impact of automation on group-level wages is amplified, precisely because it targets higher-rent tasks and thus reduces worker rents.

3. *Allocative efficiency:* Automation worsens allocative efficiency because it tends to target higher-rent jobs, which are the ones that were undersupplied before automation (in the sense that these are the tasks where the value marginal product of labor is typically greater). As labor is eliminated from higher-rent tasks, productivity suffers. Consequently, automation may reduce TFP and (utilitarian) welfare, or at the very least, it increases these quantities less than its direct cost saving effects.

The paper also explored the empirical implications of automation in labor markets with worker rents, using both reduced-form and more structural approaches. Our reduced-form econometric work provides support for the rent dissipation mechanism. Most importantly, we find evidence for the distinctive U-shaped pattern of wage changes within groups exposed to automation predicted by our theory. We also find evidence of a shift away from high rent jobs for exposed groups exposed to automation using several distinct proxies for rents.

We complemented the reduced-form evidence with a quantitative exercise that estimates the impact of automation accounting for rent dissipation on aggregates. This exercise suggests that the baseline ("competitive") effects of automation account for 42% of the increase in between-group inequality in the United States since 1980, while rent dissipation adds another 10 percentage points to automation's explanatory power for between-group inequality and is responsible for pushing several demographic groups from stagnant into negative real wage changes. We also estimate that because of worsening allocative efficiency, automation brought small gains in TFP, average wages, and consumption since 1980.

Appendix: Proofs of Propositions 1–5

This appendix proves Propositions 1–5. We first derive the equilibrium conditions in the text and provide a lemma for the Jacobian of task shares that will be used in our proofs.

Preliminaries: This section derives the equilibrium conditions E3 and E4. The production of the final good is perfectly competitive, so $p_x = M^{-\frac{1}{\lambda}} \cdot (y/y_x)^{\frac{1}{\lambda}}$, and

$$y_x = \frac{1}{M} \cdot y \cdot p_x^{-\lambda}. \tag{21}$$

For tasks in $\mathcal{T}_g(w)$, equation (21) implies

$$\ell_{gx} \cdot \psi_{gx} = \frac{1}{M} \cdot y \cdot \left(w_g \cdot \frac{\mu_{gx}}{\psi_{gx}} \right)^{-\lambda},$$

which can be rearranged into E3. For tasks in $\mathcal{T}_k(w)$, equation (21) implies

$$k_x \cdot \psi_{kx} = \frac{1}{M} \cdot y \cdot \left(\frac{1}{q_x \cdot \psi_{kx}} \right)^{-\lambda},$$

which can be rearranged into the capital demand equation in E3. Multiplying equation (21) by p_x and integrating yields

$$y = \int_{\mathcal{T}} p_x \cdot y_x \cdot dx = \frac{1}{M} \cdot y \cdot \int_{\mathcal{T}} p_x^{1-\lambda} \cdot dx.$$

Canceling y on both sides yields the ideal-price index equation E4.

Jacobian lemma: The following lemma will be used in our proofs.

LEMMA 1 *Let* $\Sigma = \mathbb{1} - \frac{1}{\lambda}\frac{\partial \ln \Gamma(w)}{\partial \ln w}$. *For all wage vectors* w, *the matrix* Σ *is non-singular. Moreover,* Σ *is a* $P-$*matrix of the Leontief type (i.e., with non-positive off-diagonal entries) whose inverse* Θ *has all entries non-negative.*

PROOF. Assumption 1 ensures that task shares are a continuous and differentiable function of wages. We now establish the properties of Σ.

First, because $\partial\Gamma_g/\partial w_{g'} \geq 0$ for $g' \neq g$, Σ is a $Z-$matrix (it has negative off diagonals).

Second, Σ has a positive dominant diagonal. This follows from the fact that $\Sigma_{gg} = 1 - \frac{1}{\lambda}\frac{\partial \ln \Gamma_g}{\partial \ln w_g} > 0$, and $\Sigma_{gg} - \sum_{g' \neq g} |\Sigma_{gg'}| = 1 - \sum_{g'} \frac{1}{\lambda}\frac{\partial \ln \Gamma_g}{\partial \ln w_{g'}} > 1$. This last inequality follows because $\sum_{g'} \frac{\partial \ln \Gamma_g}{\partial \ln w_{g'}} \leq 0$: when all wages rise by the same amount, workers lose tasks to capital but do not experience task reallocation among them.

Third, all eigenvalues of Σ have real parts that exceed 1. This follows from Gershgorin's circle theorem: for each eigenvalue e of Σ, we can find a dimension g such that $\|e - \Sigma_{gg}\| < \sum_{g' \neq g} |\Sigma_{gg'}|$. This inequality implies $\Re(e) \in \left[\Sigma_{gg} - \sum_{g' \neq g} |\Sigma_{gg'}|, \Sigma_{gg} + \sum_{g' \neq g} |\Sigma_{gg'}|\right]$. Because $\Sigma_{gg} - \sum_{g' \neq g} |\Sigma_{gg'}| > 1$ for all g, all eigenvalues of Σ have real parts greater than 1.

Fourth, since Σ is a $Z-$matrix whose eigenvalues have positive real part, it is also an $M-$matrix and a $P-$matrix of the Leontief type. The inverse of such matrices exists and has non-negative real entries, $\theta_{gg'} \geq 0$. ■

Proof of Proposition 1. We first show that equilibrium wages and output solve (1) and (2). Aggregating the labor demand equation in E3 over tasks in $\mathcal{T}_g(w)$, we obtain $\ell_g = y \cdot \Gamma_g(w) \cdot w_g^{-\lambda}$. This can be rewritten as the market-clearing condition in (1). Likewise, using the definitions of $\Gamma_g(w)$, $\Gamma_k(w)$, and $\mu_g(w)$, we can re-write E5 as (2).

To show (1) and (2) admit a unique solution, we first show that, given a level for output y, there is a unique set of wages $\{w_g(y)\}_g$ that satisfies the market clearing conditions in (1). We then show there is a unique level of output that satisfies (2) (evaluated at $\{w_g(y)\}_g$).

For the first step, Assumption 1 implies that $\Gamma_g(w)$ lies in a compact set $[\underline{\Gamma}, \bar{\Gamma}]$. The mapping $\mathbb{T} : w \to (\mathbb{T}w_1, \ldots, \mathbb{T}w_G)'$ defined by $\mathbb{T}w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot \Gamma_g(w)^{\frac{1}{\lambda}}$ for $g = 1, 2, \ldots, G$ is a continuous mapping from the compact convex set $\mathbb{X} = \prod_{g=1}^{G}[(y/\ell_g)^{\frac{1}{\lambda}} \cdot \underline{\Gamma}^{\frac{1}{\lambda}}, (y/\ell_g)^{\frac{1}{\lambda}} \cdot \bar{\Gamma}^{\frac{1}{\lambda}}]$ onto itself. The existence of a positive wage vector $\{w_g(y)\}_g$ solving this fixed-point problem follows from Brouwer's fixed point theorem.

We now turn to uniqueness of $\{w_g(y)\}_g$. We can rewrite the system of equations $\{w_g(y)\}_g$ defining $\{w_g(y)\}_g$ in logs as $F(x) = \frac{1}{\lambda} \cdot \text{stack}(\ln y - \ln \ell_j)$, where $x = (\ln w_1, \ldots, \ln w_G)$

and $F(x) = (f_1(x), \ldots, f_G(x))$ with $f_g(x) = x_g - \frac{1}{\lambda} \cdot \ln \Gamma_g(x)$.

The Jacobian of F is given by the M−matrix Σ. Theorem 5 from Gale and Nikaido (1965) shows that the solution to the system $F(x) = a$ is unique if the Jacobian of F is a P-matrix of the Leontief type. The theorem also shows that the unique solution $x(a)$ is increasing in a. As a result, the unique solution to the system of equations in (1) is$\{w_g(y)\}_g$ with $w_g(y)$ strictly increasing in y. We also note that $(y/\ell_g)^{1/\lambda} \cdot \underline{\Gamma}^{1/\lambda} \leq w_g(y) \leq (y/\ell_g)^{1/\lambda} \cdot \bar{\Gamma}^{1/\lambda}$, so that $w_g(y) \to \infty$ as $y \to \infty$, and $w_g(y) \to 0$ as $y \to 0$.

To conclude, we show that there is a unique y that satisfies the ideal-price index equation (2). This condition can be written as $I(y) = 1$, where

$$I(y) = \left(\frac{1}{M} \int_{\mathcal{T}} \left[\min \left\{ \min_g \left\{ w_g(y) \cdot \frac{\mu_{gx}}{\psi_{gx}} \right\}, \frac{1}{q_x \cdot \psi_{kx}} \right\} \right]^{1-\lambda} \cdot dx \right)^{1/(1-\lambda)}.$$

Because wages are strictly increasing in y, $I(y)$ increases in y. Assumption 1 also ensures that a positive mass of tasks must be allocated to labor at any wage level, which implies that $I(y)$ is strictly increasing in y. The function $I(y)$ can be written as $I(y) = \left(\Gamma_k(w(y)) + \sum_g \Gamma_g(w) \cdot \mu_g(w) \cdot w_g(y)^{1-\lambda}\right)^{1/(1-\lambda)}$. As $y \to \infty$, $\Gamma_g(w) \cdot \mu_g(w) \cdot w_g(y)^{1-\lambda} \to \infty$ (since $\Gamma_g(w)$ is bounded from below, $\mu_g(w) \geq 1$, and $\lambda < 1$) and $\Gamma_k(w(y)) \geq 0$. This implies $I(y) \to \infty$. Moreover, as $y \to 0$, $\Gamma_g(w) \cdot \mu_g(w) \cdot w_g(y)^{1-\lambda} \to 0$ (since $\Gamma_g(w)$ and $\mu_g(w)$ are bounded from above and $\lambda < 1$) and $\Gamma_k(w(y)) = 0$ (since, by Assumption 1, all tasks can be produced by at least one type of worker). This implies $I(y) \to 0$.

Because $I(y)$ is strictly increasing in y, there is a unique $y \in (0, \infty)$ for which $I(y) = 1$ and, therefore, a unique equilibrium with wages $w_g = w_g(y)$. The equilibrium wages and the tie-breaking rule for tasks where there is indifference uniquely determine the task allocation.

Our argument for uniqueness also shows that, under Assumption 1, the unique equilibrium features finite output, positive wages, and positive task shares for all workers. Moreover, from $I(y) = 1$, we obtain that, in equilibrium, $1 - \Gamma_k > 0$. ■

Proof of Proposition 2. Starting at the equilibrium allocation, reallocate a mass ϵ of g workers from task x to task x' with x and x' in $\mathcal{T}_g$ and $\mu_{gx'} > \mu_{gx}$. This perturbation raises aggregate output and consumption per reallocated unit of labor by $p_{x'} \cdot \psi_{gx'} - p_x \cdot \psi_{gx} = w_g \cdot [\mu_{gx'} - \mu_{gx}] > 0$. Hence, there is underemployment and high-rents tasks.

We now show that tasks x that satisfy (3) are inefficiently automated. The right-hand side of the inequality implies these tasks are automated in equilibrium. Starting at the equilibrium allocation, reallocate a mass ϵ of g workers drawn proportionally from tasks in

$\mathcal{T}_g$ to task x. This perturbation raises aggregate output and consumption per reallocated unit of labor by $p_x \cdot \psi_{gx} - \int_{\mathcal{T}_g} p_x \cdot \psi_{gx} \cdot \frac{\ell_{gx}}{\ell_g} \cdot dx = \frac{\psi_{gx}}{q_x \cdot \psi_{kx}} - w_g \cdot \mu_g > 0$. ■

We now define the notion of small interior automation shocks.

DEFINITION 1 *Suppose that the task space is a subset of $\mathbb{R}^d$ with $d \geq 2$. An automation shock is small and of order ϵ if:*

- *The d—dimensional sets $\mathscr{A}_g^T$ are measurable and have measure $\mathcal{O}(\epsilon)$.*
- *The $d-1$—dimensional hyper-surfaces $\mathcal{B}_g(w_g) = \left\{x \in \mathscr{A}_g^T : \frac{w_g \cdot \mu_{gx}}{\psi_{gx}} = \frac{1}{q'_x \cdot \psi_{kx}}\right\}$ are smooth, measurable, and have surface area $\mathcal{O}(\epsilon^{1-1/d})$ for all g.*
- *The rate of change in $\mathcal{B}_g(w_g)$ is bounded from above by $\bar{D}$. That is: $D_g(x) = \lim_{h \to 0} \frac{1}{h} \min_{x' \in \mathcal{B}_g(w_g+h)} \|x - x'\| < \bar{D}$ for all $x \in \mathcal{B}_g(w_g)$.*

We also say that an automation shock is interior if $\mathscr{A}_g^T$ is in the interior of $\mathcal{T}_g$.

Our derivations assume we have small interior automation shocks. The requirements that shocks are interior and that the newly-introduced boundary tasks in $\mathcal{B}_g(w_g)$ have surface $\mathcal{O}(\epsilon^{1-1/d})$ and their rate of change is bounded are needed to ensure that advances in automation do not have first-order effects on the task and rent Jacobians.[26]

Proof of Proposition 3. Consider a small interior automation shock in $\mathscr{A}^T$ of order ϵ. For functions over the task space, $F(w)$, we denote by $F^{\mathscr{A}}(w)$ the new function obtained after q_x increases from zero to q'_x in $\mathscr{A}^T$.

Derivation of equation (4): Lemma S1 in the Supplementary Materials shows that

$$(22)\quad d\ln w_g = \frac{1}{\lambda} \cdot d\ln y + \frac{1}{\lambda} \cdot \left(\ln \Gamma_g^{\mathscr{A}}(w) - \ln \Gamma_g(w)\right) + \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w} \cdot \text{stack}(d\ln w) + \mathcal{O}(\epsilon^{2-1/d})$$

This expansion decomposes the effects of wages into the productivity effect, the direct effect of automation on task shares, and the reallocation of tasks in response to wages.

We now approximate $\ln \Gamma_g^{\mathscr{A}}(w) - \ln \Gamma_g(w)$. Let $d\ell_g(x) = \psi_{gx}^{\lambda-1} \cdot \mu_{gx}^{-\lambda} \cdot dx$:

$$\ln \Gamma_g^{\mathscr{A}}(w) - \ln \Gamma_g(w) = \frac{\Gamma_g^{\mathscr{A}}(w) - \Gamma_g(w)}{\Gamma_g(w)} + \mathcal{O}(\epsilon^2) = -\frac{\int_{\mathscr{A}_g} d\ell_g(x)}{\int_{\mathcal{T}_g} d\ell_g(x)} + \mathcal{O}(\epsilon^2) = -d\ln \Gamma_g^d + \mathcal{O}(\epsilon^2).$$

[26] We need the dimension of the task space to be at least two, since otherwise, any nonzero automation shock has a first-order effect on substitution patterns. In Acemoglu and Restrepo (2022), this requirement was not needed because we assumed all tasks for which advances in automation occurred where automated, and so $\mathcal{B}_g(w_g)$ was an empty set. The requirement that advances in automation are interior can be relaxed by imposing bounds on the hyper-surfaces at the intersection of $\mathscr{A}_g^T$ and the initial set of boundary tasks, but this requires additional notation and complicates the proofs.

The first equality follows from an approximation of log changes. The second and third use the definitions of $\Gamma_g^{\mathscr{A}}(w)$ and $d\ln\Gamma_g^d$. Plugging in (22), we obtain

$$d\ln w_g = \frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_g^d + \frac{1}{\lambda}\cdot\frac{\partial\ln\Gamma_g(w)}{\partial\ln w}\cdot \text{stack}(d\ln w) + \mathcal{O}(\epsilon^{2-1/d}).$$

Lemma 1 implies that this system has the unique solution (to a first-order approximation)

$$d\ln w_g = \Theta_g\cdot \text{stack}\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_j^d\right) + \mathcal{O}(\epsilon^{2-1/d}).$$

Derivation of equation (5)*:* We turn to the ideal-price index condition, written as $I(w) = 1$, where we now define $I(w) = \Gamma_k(w) + \sum_g \Gamma_g(w)\cdot\mu_g(w)\cdot w_g^{1-\lambda}$. From Lemma S1, the change in $I(w)$ following a small interior automation shock is

$$dI = I^{\mathscr{A}}(w) - I(w) + I(w)\cdot\frac{\partial\ln I(w)}{\partial\ln w}\cdot d\ln w + \mathcal{O}(\epsilon^{2-1/d}). \tag{23}$$

Note that $\frac{\partial\ln I(w)}{\partial\ln w}\cdot d\ln w$ captures the effect of a change in wages on the cost of producing the final good. Because tasks are allocated in a cost-minimizing way (given wedges), the envelope theorem implies $\frac{\partial\ln I(w)}{\partial\ln w}\cdot d\ln w = (1-\lambda)\cdot\sum_g s_g\cdot d\ln w_g + \mathcal{O}(\epsilon^2)$.

The term $I^{\mathscr{A}}(w) - I(w)$ in (23) captures cost savings from automating tasks in $\mathscr{A}_g$ holding wages constant. We have

$$\begin{aligned}
I^{\mathscr{A}}(w) - I(w) =& \Gamma_k^{\mathscr{A}}(w) - \Gamma_k(w) + \sum_g \Gamma_g^{\mathscr{A}}(w)\cdot\mu_g^{\mathscr{A}}(w)\cdot {w_g}^{1-\lambda} - \sum_g \Gamma_g(w)\cdot\mu_g(w)\cdot {w_g}^{1-\lambda}\\
=& \sum_g \frac{1}{M}\int_{\mathscr{A}_g}\left[(q_x'\cdot\psi_{kx})^{\lambda-1} - (\psi_{gx}/\mu_{gx})^{\lambda-1}\cdot {w_g}^{1-\lambda}\right]\cdot dx\\
=& \sum_g \frac{1}{M}\int_{\mathscr{A}_g}(\psi_{gx}/\mu_{gx})^{\lambda-1}\cdot {w_g}^{1-\lambda}\cdot\left[\left(\frac{q_x'\cdot\psi_{kx}\cdot w_g\cdot\mu_{gx}}{\psi_{gx}}\right)^{\lambda-1} - 1\right]\cdot dx\\
=& \sum_g s_g\cdot d\ln\Gamma_g^d\cdot\frac{\mu_{\mathscr{A}_g}}{\mu_g}\cdot\frac{\frac{1}{M}\int_{\mathscr{A}_g}(\psi_{gx}/\mu_{gx})^{\lambda-1}\cdot\left[\left(\frac{q_x'\cdot\psi_{kx}\cdot w_g\cdot\mu_{gx}}{\psi_{gx}}\right)^{\lambda-1} - 1\right]\cdot dx}{\frac{1}{M}\int_{\mathscr{A}_g}(\psi_{gx}/\mu_{gx})^{\lambda-1}\cdot dx}\\
=& (\lambda-1)\cdot\sum_g s_g\cdot d\ln\Gamma_g^d\cdot\frac{\mu_{\mathscr{A}_g}}{\mu_g}\cdot\pi_g.
\end{aligned}$$

In the last step, we used $\ell_{gx}\cdot\mu_{gx}\propto(\psi_{gx}/\mu_{gx})^{\lambda-1}$ (from E3).

Because in equilibrium $I(w) = 1$, we have $dI = 0$. Equation (23) then implies

$$\sum_g s_g\cdot d\ln w_g = \sum_g s_g\cdot d\ln\Gamma_g^d\cdot\frac{\mu_{\mathscr{A}_g}}{\mu_g}\cdot\pi_g + \mathcal{O}(\epsilon^{2-1/d}).$$

Derivation of equation (6)*:* Lemma S1 implies that

$$d\ln\mu_g = \ln\mu_g^{\mathscr{A}}(w) - \ln\mu_g(w) + \frac{\partial\ln\mu_g(w)}{\partial\ln w}\cdot\text{stack}(d\ln w) + \mathcal{O}(\epsilon^{2-1/d}).$$

We can rewrite the direct effect of automation on rents, $\ln\mu_g^{\mathscr{A}}(w) - \ln\mu_g(w)$, as

$$\begin{aligned}\ln\mu_g^{\mathscr{A}}(w) - \ln\mu_g(w) &= \frac{\mu_g^{\mathscr{A}}(w) - \mu_g(w)}{\mu_g(w)} + \mathcal{O}(\epsilon^2)\\ &= \frac{\dfrac{\mu_g\cdot\int_{\mathcal{T}_g}d\ell_g(x) - \mu_{\mathscr{A}_g}\cdot\int_{\mathscr{A}_g}d\ell_g(x)}{\int_{\mathcal{T}_g}d\ell_g(x) - \int_{\mathscr{A}_g}d\ell_g(x)} - \mu_g}{\mu_g} + \mathcal{O}(\epsilon^2)\\ &= -\left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1\right)\cdot\frac{d\ln\Gamma_g^d}{1 - d\ln\Gamma_g^d} + \mathcal{O}(\epsilon^2)\\ &= -\left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1\right)\cdot d\ln\Gamma_g^d + \mathcal{O}(\epsilon^2).\end{aligned}$$

The first line approximates the change in logs. The second line uses the definition of $\mu_g^{\mathscr{A}}(w)$ and the fact that $\mu_g(w) = \mu_g$. The third line divides by μ_g, cancels terms, and uses the definition of $d\ln\Gamma_g^d$. The last line uses the fact that $\frac{d\ln\Gamma_g^d}{1-d\ln\Gamma_g^d} = d\ln\Gamma_g^d + \mathcal{O}(\epsilon^2)$.

These derivations show that the equilibrium change in rents satisfies

$$d\ln\mu_g = -\left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1\right)\cdot d\ln\Gamma_g^d + \frac{\partial\ln\mu_g(w)}{\partial\ln w}\cdot\text{stack}(d\ln w) + \mathcal{O}(\epsilon^{2-1/d}).$$

Plugging the expression for the change in base wages in (4), this can be expressed as

$$d\ln\mu_g = -\left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1\right)\cdot d\ln\Gamma_g^d + \mathcal{M}_g\cdot\text{stack}\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_j^d\right) + \mathcal{O}(\epsilon^{2-1/d}).$$

Derivation of equation (7)*:* We first prove the dual of the Solow residual. Because all income accrues to capital or labor, we have $y = \sum_g \bar{w}_g\cdot\ell_g + k$. Differentiating yields

$$d\ln y = s_k\cdot d\ln k + \sum_g s_g\cdot d\ln\bar{w}_g \quad \Rightarrow \underbrace{d\ln y - s_k\cdot d\ln k}_{\equiv d\ln tfp} = \sum_g s_g\cdot d\ln\bar{w}_g.$$

The dual of Solow implies $d\ln tfp = \sum_g s_g\cdot d\ln w_g + \sum_g s_g\cdot d\ln\mu_g$. Plugging the formula for $\sum_g s_g\cdot d\ln w_g$ in equation (5) yields equation (7) in the proposition.

Turning to consumption, we have $c = y - k$. Differentiating, $d\ln c = (1/c)\cdot(dy - dk) = (1/s^L)\cdot(d\ln y - (k/y)d\ln k) = (1/s^L)\cdot d\ln tfp$. The last step uses the fact that $s_L = c/y$. ■

Proof of Proposition 4. Let $\Psi = \langle q, \psi_k, \{\psi_g\}_g \rangle$ be the vector of technological attributes of tasks. For any set $\mathcal{S} \subseteq \mathcal{T}$, define the CDF of Ψ in $\mathcal{S}$ by

$$H(\Psi|\mathcal{S}) = \Pr(x : q'_x \leq q, \psi_{kx} \leq \psi_k, \{\psi_{gx} \leq \psi_g\}_g | x \in \mathcal{S}) = \frac{\int_{\mathcal{S} \cap \{x : q'_x \leq q, \psi_{kx} \leq \psi_k, \{\psi_{gx} \leq \psi_g\}_g\}} dx}{\int_{\mathcal{S}} dx},$$

and denote its pdf by $h(\Psi|\mathcal{S})$.

From condition (i), it suffices to show $\bar{F}_g(\mu|\mathscr{A}_g) > \bar{F}_g(\mu|\mathscr{A}_g^T)$. For all $\mu \geq 1$, we have

$$\bar{F}_g(\mu|\mathscr{A}_g^T) = \frac{\int_\Psi \int_\mu^\infty \bar{f}_g(u|\Psi, \mathscr{A}_g^T) \cdot h(\Psi|\mathscr{A}_g^T) \cdot du \cdot d\Psi}{\int_\Psi \int_1^\infty \bar{f}_g(u|\Psi, \mathscr{A}_g^T) \cdot h(\Psi|\mathscr{A}_g^T) \cdot du \cdot d\Psi} = \frac{\int_\mu^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du}{\int_1^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du},$$

where the last equality follows from condition (ii). Turning to $\bar{F}_g(\mu|\mathscr{A}_g)$, we have

$$\bar{F}_g(\mu|\mathscr{A}_g) = \frac{\int_\Psi \int_{\max\{\mu, \varrho(\Psi)\}}^\infty \bar{f}_g(u|\Psi, \mathscr{A}_g^T) \cdot h(\Psi|\mathscr{A}_g^T) \cdot du \cdot d\Psi}{\int_\Psi \int_{\max\{1, \varrho(\Psi)\}}^\infty \bar{f}_g(u|\Psi, \mathscr{A}_g^T) \cdot h(\Psi|\mathscr{A}_g^T) \cdot du \cdot d\Psi}.$$

Here, $\varrho(\Psi) = \frac{1}{w_g} \cdot \frac{\psi_g}{q' \cdot \psi_k}$ gives a threshold value for μ_{gx} above which tasks with technological attributes Ψ will be automated. Using condition (ii) we can write this as

$$\bar{F}_g(\mu|\mathscr{A}_g) = \frac{\int_\Psi \int_{\max\{\mu, \varrho(\Psi)\}}^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot h(\Psi|\mathscr{A}_g^T) \cdot du \cdot d\Psi}{\int_\Psi \int_{\max\{1, \varrho(\Psi)\}}^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot h(\Psi|\mathscr{A}_g^T) \cdot du \cdot d\Psi}.$$

We now show that, for all ϱ and $\mu \geq 1$, we have

$$\frac{\int_{\max\{\mu, \varrho\}}^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du}{\int_{\max\{1, \varrho\}}^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du} \geq \frac{\int_\mu^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du}{\int_1^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du} = \bar{F}_g(\mu|\mathscr{A}_g^T), \tag{24}$$

with strict inequality if $\varrho, \mu > 1$. When $\varrho \leq 1$ both sides in (24) are equal to 1. When $\varrho \in (1, \mu]$, (24) becomes

$$\frac{\int_\mu^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du}{\int_\varrho^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du} > \frac{\int_\mu^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du}{\int_1^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du} \Leftrightarrow \int_1^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du > \int_\varrho^\infty \bar{f}_g(u|\mathscr{A}_g^T) \cdot du,$$

which holds as a strict inequality for $\varrho > 1$. Finally, when $\varrho > \mu$, (24) becomes

$$1 > \frac{\int_{\mu}^{\infty} \bar{f}_g(u|\mathscr{A}_g^T) \cdot du}{\int_{1}^{\infty} \bar{f}_g(u|\mathscr{A}_g^T) \cdot du} \Leftrightarrow \int_{1}^{\infty} \bar{f}_g(u|\mathscr{A}_g^T) \cdot du > \int_{\mu}^{\infty} \bar{f}_g(u|\mathscr{A}_g^T) \cdot du,$$

which holds as a strict inequality for $\mu > 1$. To conclude the proof of the proposition, rewrite (24) as $\int_{\max\{\mu,\varrho\}}^{\infty} \bar{f}_g(u|\mathscr{A}_g^T) \cdot du \geq \bar{F}_g(\mu|\mathscr{A}_g^T) \cdot \int_{\max\{1,\varrho\}}^{\infty} \bar{f}_g(u|\mathscr{A}_g^T) \cdot du$. Letting $\varrho = \varrho(\Psi)$ and integrating over $\mathscr{A}_g^T$, we get

$$\begin{aligned}\int_{\Psi} \int_{\max\{\mu,\varrho(\Psi)\}\}}^{\infty} & \bar{f}_g(u|\mathscr{A}_g^T) \cdot h(\Psi|\mathscr{A}_g^T) \cdot du \cdot d\Psi \\ & > \bar{F}_g(\mu|\mathscr{A}_g^T) \cdot \int_{\Psi} \int_{\max\{1,\varrho(\Psi)\}}^{\infty} \bar{f}_g(u|\mathscr{A}_g^T) \cdot h(\Psi|\mathscr{A}_g^T) \cdot du \cdot d\Psi.\end{aligned}$$

The inequality is strict because not all tasks in $\mathscr{A}_g^T$ are automated, which means that $\varrho(\Psi) > 1$ for a positive mass of tasks in $\mathscr{A}_g^T$ for which (24) holds with strict inequality. This inequality can be rearranged as $\bar{F}_g(\mu|\mathscr{A}_g) > \bar{F}_g(\mu|\mathscr{A}_g^T)$. ■

Proof of Proposition 5. The automation of tasks in $\mathscr{A}_g$ shifts the distribution of rents for workers in group g from $\bar{F}_g(\mu|\mathcal{T}_g)$ to $\bar{F}_g(\mu|\mathcal{T}_g \setminus \mathscr{A}_g)$. Consider the old quantile functions for wages (inclusive of rents) in $\mathcal{T}_g$, denoted by $\ln w_g^p$ and the new quantile function for wages (inclusive of rents) in $\mathcal{T}_g \setminus \mathscr{A}_g$, denoted by $\ln w_{g,\text{new}}^p$. Because $\bar{F}_g(\mu|\mathscr{A}_g) > \bar{F}_g(\mu|\mathcal{T}_g)$, we have $\bar{F}_g(\mu|\mathcal{T}_g) > \bar{F}_g(\mu|\mathcal{T}_g \setminus \mathscr{A}_g)$, and the distribution of wages (plus rents) for workers in $\mathcal{T}_g$ dominates, in the first order stochastic sense, their new distribution of wages (plus rents) in $\mathcal{T}_g \setminus \mathscr{A}_g$ (holding w_g constant, as in the proposition).

Below m_g, both quantile functions equal w_g, since the share of workers earning no rents in $\mathcal{T}_g \setminus \mathscr{A}_g$ is greater than or equal to the share of workers earning no rents in $\mathcal{T}_g$. This shows that $d\ln w_g^p = \ln w_{g,\text{new}}^p - \ln w_g^p = 0$ for $p \in [0, m_g]$.

First-order stochastic dominance also implies that the quantile function for wages in $\mathcal{T}_g \setminus \mathscr{A}_g$ is strictly below the quantile function for wages in $\mathcal{T}_g$ for all $\mu > 1$. This shows that $d\ln w_g^p = \ln w_{g,\text{new}}^p - \ln w_g^p < 0$ for all $p \in (m_g, 1)$.

To conclude, suppose that for all $\mu > 1$, there is a positive measure δ of tasks with rent $\mu_{gx} = \mu$ in $\mathcal{T}_g$ that cannot be automated. For $p = 1$, $\ln w_g^p = d\ln w_{g,\text{new}}^p = \ln w_g + \ln \bar{\mu}_g$, where $\bar{\mu}_g$ is the maximum rent earned by group g workers. This is because, by assumption, not all jobs paying a rent $\bar{\mu}_g$ can be automated. This implies $\ln w_g^p = 0$ for $p = 1$. For $p = 1 - \epsilon$, we have a mass ϵ of workers earning a wage above $\ln w_g^{1-\epsilon}$ initially. Of these, a fraction δ workers is still earning a wage above $\ln w_g^p$ after the automation of tasks in

$\mathscr{A}_g$. This implies $\ln w_g^{1-\epsilon} \leq \ln w_{g,\text{new}}^{1-\epsilon\cdot\delta} \leq \ln w_g^{1-\epsilon\cdot\delta}$. Taking limits as $\epsilon \to 0$ and we obtain $\lim_{p\to 1} \ln w_g^p = \lim_{p\to 1} \ln w_{g,\text{new}}^p$, as wanted. ∎

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Table 1: Estimated effects of new automation technology, 1980–2016.

	Data 1980–2016 (1)	Baseline estimates (2)	Low rent dissipation (20%) (3)	High rent dissipation (50%) (4)	Imposing aggregate elasticity of substitution of 1.6 between groups (5)
Panel A. Wage structure:					
Change in average wages, $d\ln\bar{w}_g$	29.15%	0.456%	1.91%	-0.705%	0.497%
Change in base wages, $d\ln w_g$	.	4.54%	4.54%	4.54%	4.54%
Change in group rents, $d\ln\mu_g$	.	-4.08%	-2.63%	-5.24%	-4.04%
Wage changes for non-college men	-6.5%	-8.0%	-6.8%	-9.0%	-6.9%
- base wage changes		-2.4%	-3.2%	-1.7%	-1.3%
- rent dissipation		-5.6%	-3.6%	-7.3%	-5.6%
Wage changes for non-college women	10.6%	-2.2%	-0.8%	-3.3%	-1.6%
- base wage changes		2.4%	2.1%	2.6%	3.0%
- rent dissipation		-4.6%	-2.9%	-5.9%	-4.6%
Between-group wage changes explained					
-due to changes in industry composition		6.72%	7.30%	6.26%	5.92%
-adding direct displacement effects		63.49%	71.16%	57.35%	62.68%
-accounting for ripple effects		41.79%	46.84%	37.75%	35.08%
-accounting for rent dissipation		51.87%	53.38%	50.66%	45.39%
Panel B. Aggregates:					
Change in GDP per capita, $d\ln y$	70%	14.41%	15.90%	13.22%	14.50%
Change in TFP, $d\ln \text{tfp}$	35%	0.304%	1.27%	-0.47%	0.331%
-cost-saving gains		3.02%	3.02%	3.02%	3.02%
-changes in allocative efficiency		-2.72%	-1.75%	-3.49%	-2.69%
-inefficient rent dissipation		-2.70%	-1.73%	-3.47%	-2.7%
Change in consumption, $d\ln c$	70%	0.456%	1.91%	-0.705%	0.497%
Change in k/y ratio	30.0%	27.9%	28.0%	27.8%	28.0%

Notes: This table summarizes the estimated effects of new automation technologies on group wages and aggregates from 1980 to 2016. The estimates are computed using the formulas in Proposition 6. Column 1 reports data for the US for comparison. The wage data is from the US Census and the ACS. The data on GDP and capital are from the BLS. The data on consumption is from the BEA and the data for TFP is from Fernald (2014). Column 2 provides our baseline estimates. Column 3 assumes a lower rate of rent dissipation of 20%. Column 4 assumes a higher rate of rent dissipation of 50%. Column 5 assumes $\theta = 1.6 - \lambda$ and sets $\theta_{\text{job}} = \theta_{\text{edu-age}} = 0$, thus imposing an aggregate elasticity of substitution between worker groups of 1.6.

Supplementary Materials for "Automation and Rent Dissipation"

Daron Acemoglu and Pascual Restrepo

S1 Proofs and details for the multi-sector model.

S1.1 Description of multi-sector model and preliminaries

Description: There are multiple sectors indexed by $i \in \mathbb{I}$ (where $\mathbb{I}$ denotes the set of sectors). Sectoral production functions are given by

$$y_i = \left(\frac{1}{M_i}\int_{\mathcal{T}_i}(M_i \cdot y_x)^{\frac{\lambda-1}{\lambda}} \cdot dx\right)^{\lambda/(\lambda-1)}.$$

The sets of tasks across sectors $\{\mathcal{T}_i\}_{i\in\mathbb{I}}$ are sector-specific, which is without loss of generality since tasks can be relabeled.

Sectoral outputs are combined into a final good that can be used for consumption or to build productive capital. The transformation of sectoral output into the final good y is described by a CES production function with an elasticity of substitution $\eta > 0$:

$$y = \left(\sum_i \alpha_i^{\frac{1}{\eta}} \cdot y_i^{\frac{\eta-1}{\eta}} \cdot dx\right)^{\eta/(\eta-1)}.$$

We denote sectoral prices by $\{p_i\}_{i\in\mathbb{I}}$ and normalize the price of the final good to 1.

The total quantity produced of task x is

$$y_x = \psi_{kx} \cdot k_x + \sum_g \psi_{gx} \cdot \ell_{gx}.$$

Here, ℓ_{gx} is the amount of labor of type g allocated to task x, while k_x is the amount of task-specific capital used for this task.

A fixed supply ℓ_g of workers of type g is allocated across tasks and industries, so that

$$\sum_i \int_{\mathcal{T}_i} \ell_{gx} \cdot dx \leq \ell_g.$$

We treat task-specific capital, $\{k_x\}_{x\in\mathcal{T}}$, as intermediate goods. They are produced within the same period using the final good at a constant unit cost $1/q_x$. If $q_x = 0$, task x cannot be performed by capital. This implies that total consumption equals net output:

$$c = y - \sum_i \int_{\mathcal{T}_i} (k_x/q_x) \cdot dx.$$

As in the single-sector model, we assume there are task specific rents μ_{gx}.

A *market equilibrium* is given by a vector of base wages $\{w_g\}$, output y, sectoral prices p_i, an allocation of tasks $\{\mathcal{T}_{gi}\}_{i,g}, \{\mathcal{T}_{ki}\}_i$, task prices p_x, hiring plans ℓ_{gx}, and capital production plans k_x such that:

E1' Tasks prices equal the minimum unit cost of producing the task

$$p_x = \min\left\{\frac{1}{q_x \cdot \psi_{kx}}, \left\{w_g \cdot \frac{\mu_{gx}}{\psi_{gx}}\right\}_g\right\}.$$

E2' Tasks are allocated in a cost-minimizing way. The set of tasks

$$\mathcal{T}_{gi} = \left\{x \in \mathcal{T}_i : p_x = w_g \cdot \frac{\mu_{gx}}{\psi_{gx}}\right\}$$

will be produced by workers of type g, and the set of tasks

$$\mathcal{T}_{ik} = \left\{x \in \mathcal{T}_i : p_x = \frac{1}{q_x \cdot \psi_{kx}}\right\}$$

will be produced by capital.

E3' Task-level demands for labor and capital are given by

$$\begin{aligned} \ell_{gx} &= y \cdot s_{y_i} \cdot p_i^{\lambda-1} \cdot \frac{1}{M_i} \cdot \psi_{gx}^{\lambda-1} \cdot (\mu_{gx} \cdot w_g)^{-\lambda} && \text{for } x \in \mathcal{T}_{gi}, \\ k_x &= y \cdot s_{y_i} \cdot p_i^{\lambda-1} \cdot \frac{1}{M_i} \cdot \psi_{kx}^{\lambda-1} \cdot q_x^{\lambda} && \text{for } x \in \mathcal{T}_{ki}. \end{aligned}$$

Here $s_{y_i} = \alpha_i \cdot p_i^{1-\eta}$ is the share of industry i in output.

E4' Sectoral prices are given by

$$p_i = \left(\frac{1}{M_i} \int_{\mathcal{T}_i} p_x^{1-\lambda} \cdot dx\right)^{1/(1-\lambda)}$$

E5' The ideal-price index condition holds

$$1 = \left(\sum_i \alpha_i \cdot p_i^{1-\eta}\right)^{1/(1-\eta)}.$$

Preliminaries: We first derive the equilibrium conditions E3' and E4'. The production of the final good is perfectly competitive, and so tasks are priced at their marginal product. This implies $p_x = p_i \cdot M_i^{-\frac{1}{\lambda}} \cdot (y_i/y_x)^{\frac{1}{\lambda}}$, which can be rearranged as

$$y_x = \frac{1}{M_i} \cdot y \cdot s_{y_i} \cdot p_i^{\lambda-1} \cdot p_x^{-\lambda}. \tag{S1}$$

For tasks in $\mathcal{T}_g$, equation (S1) implies

$$\ell_{gx} \cdot \psi_{gx} = \frac{1}{M} \cdot y \cdot s_{y_i} \cdot p_i^{\lambda-1} \left(w_g \cdot \frac{\mu_{gx}}{\psi_{gx}}\right)^{-\lambda},$$

which can be rearranged into the labor demand equation in E3'.

For tasks in $\mathcal{T}_k$, equation (S1) implies

$$k_x \cdot \psi_{kx} = \frac{1}{M} \cdot y \cdot s_{y_i} \cdot p_i^{\lambda-1} \left(\frac{1}{q_x \cdot \psi_{kx}}\right)^{-\lambda},$$

which can be rearranged into the capital demand equation in E3'.

Finally, multiplying equation (S1) by p_x and integrating over $\mathcal{T}_i$ yields

$$s_{y_i} \cdot y = \int_{\mathcal{T}_i} p_x \cdot y_x \cdot dx = \frac{1}{M_i} \cdot s_{y_i} \cdot y \cdot p_i^{\lambda-1} \cdot \int_{\mathcal{T}_i} p_x^{1-\lambda} \cdot dx.$$

Canceling $s_{y_i} \cdot y$ on both sides of this equation yields the sectoral-price index condition E4'.

Equilibrium representation: Before deriving the effects of new automation technology, we extend the representation result in Proposition 1 to the multi-sector economy. As in the single-sector economy, define

$$\Gamma_{gi}(w) = \frac{1}{M_i} \int_{\mathcal{T}_{gi}(w)} \psi_{gx}^{\lambda-1} \cdot \mu_{gx}^{-\lambda} \cdot dx \text{ for each } g \in \mathbb{G} \text{ and } i \in \mathbb{I},$$

$$\Gamma_{ki}(w) = \frac{1}{M_i} \int_{\mathcal{T}_{ki}(w)} (\psi_{kx} \cdot q_x)^{\lambda-1} \cdot dx \text{ for each } i \in \mathbb{I}.$$

The integrals are computed over the set of tasks in industry i allocated to different groups and capital when base wages are w, denoted by $\mathcal{T}_{gi}(w)$ and $\mathcal{T}_{ki}(w)$. In addition, define

$$\mu_{gi}(w) = \frac{1}{\Gamma_{gi}(w)} \cdot \int_{\mathcal{T}_{gi}(w)} \psi_{gx}^{\lambda-1} \cdot \mu_{gx}^{1-\lambda} \cdot dx \text{ for each } g \in \mathbb{G} \text{ and } i \in \mathbb{I}.$$

Proposition S1 *The equilibrium base wages $\{w_g\}$, industry prices $\{p_i\}$, and output y solve the system of equations*

$$w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot \left[\sum_i \alpha_i \cdot p_i^{\lambda-\eta} \cdot \Gamma_{gi}(w)\right]^{\frac{1}{\lambda}}, \tag{S2}$$

$$p_i = \left(\Gamma_{ki}(w) + \sum_g \Gamma_{gi}(w) \cdot \mu_g(w) \cdot w_g^{1-\lambda}\right)^{1/(1-\lambda)}, \tag{S3}$$

$$1 = \left(\sum_i \alpha_i p_i^{1-\eta}\right)^{1/(1-\eta)}. \tag{S4}$$

Proof of Proposition S1. Aggregating the labor demand equation in E3' over tasks in $\mathcal{T}_{gi}$ for all industries, we obtain

$$\ell_g = y \cdot \left[\sum_i \alpha_i \cdot p_i^{\lambda-\eta} \cdot \Gamma_{gi}(w)\right] \cdot w_g^{-\lambda}.$$

This can be rewritten as (S2).

Equation E4' implies that sectoral prices satisfy

$$p_i = \left(\frac{1}{M_i} \int_{\mathcal{T}_i} p_x^{1-\lambda} \cdot dx\right)^{\frac{1}{1-\lambda}} = \left(\Gamma_{ki}(w) + \sum_g \Gamma_{gi}(w) \cdot \mu_{gi}(w) \cdot w_g^{1-\lambda}\right)^{\frac{1}{1-\lambda}}.$$

Finally, equation E5' is the same as (S4). ■

S1.2 Effects of new automation technologies

Before proving Proposition 6, we generalize the definition of a small and interior automation shock to the multisector economy.

Definition 2 *An automation shock is small and of order ϵ if:*

1. *The d—dimensional sets $\mathscr{A}_{gi}^T$ are measurable and have volume $\mathcal{O}(\epsilon)$.*

2. The $d-1$—dimensional hyper-surfaces

$$\mathcal{B}_{gi}(w_g) = \left\{ x \in \mathscr{A}_{gi}^T : \frac{w_g \cdot \mu_{gx}}{\psi_{gx}} = \frac{1}{q'_x \cdot \psi_{kx}} \right\}$$

are smooth, measurable, and have surface area $\mathcal{O}(\epsilon^{1-1/d})$ for all g.

3. The rate of change in $\mathcal{B}_{gi}(w_g)$ is bounded from above by $\bar{D}$. That is:

$$D_{gi}(x) = \lim_{h \to 0} \frac{1}{h} \min_{x' \in \mathcal{B}_{gi}(w_g + h)} \|x - x'\| < \bar{D} \text{ for all } x \in \mathcal{B}_{gi}(w_g).$$

We also say that an automation shock is interior if $\mathscr{A}_{gi}^T$ is in the interior of $\mathcal{T}_{gi}$.

Proof of Proposition 6. Define aggregate task shares at wages w and prices p as

$$\Gamma_g(w,p) = \sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \Gamma_{gi}(w) \text{ for all } g,$$

$$\Gamma_k(w,p) = \sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \Gamma_{ki}(w).$$

Note that $s_{y_i} = \alpha_i \cdot p_i^{1-\eta}$ is a function of sectoral prices.

Consider a small and interior automation shock in $\mathscr{A}_g^T$ of order ϵ. For functions over the task space, $F(w,p)$, denote by $F^{\mathscr{A}}(w,p)$ the new function obtained after q_x increases from zero to q'_x in $\mathscr{A}_g^T$.

Derivation of equation (13): effects on base wages $d \ln w_g$. Lemma S1 shows that we can do a "Taylor expansion" of equation (S2) (in logs) to express the change in equilibrium wages as

$$\text{(S5)} \qquad \begin{aligned} d\ln w_g = \frac{1}{\lambda} \cdot d\ln y + \frac{1}{\lambda} \cdot \left(\ln \Gamma_g^{\mathscr{A}}(w) - \ln \Gamma_g(w) \right) \\ + \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_g(w,p)}{\partial \ln w} \cdot \text{stack}(d\ln w) \\ + \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_g(w,p)}{\partial \ln p} \cdot \text{stack}(d\ln p) + \mathcal{O}(\epsilon^{2-1/d}) \end{aligned}$$

This expansion decomposes the effects of wages into the productivity effect, the direct effect of automation on task shares, the reallocation of tasks in response to wages, and the effect of changes in sectoral prices on task shares.

We now approximate $\ln\Gamma_g^{\mathscr{A}}(w,p)-\ln\Gamma_g(w,p)$. Letting $d\ell_g(x)=\psi_{gx}^{\lambda-1}\cdot\mu_{gx}^{-\lambda}\cdot dx$, we have

$$\begin{aligned}
\ln\Gamma_g^{\mathscr{A}}(w,p)-\ln\Gamma_g(w,p) =& \frac{\Gamma_g^{\mathscr{A}}(w,p)-\Gamma_g(w,p)}{\Gamma_g(w,p)}+\mathcal{O}(\epsilon^2)\\
=& -\frac{\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{\mathscr{A}_{gi}}d\ell_g(x)}{\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{\mathcal{T}_{gi}}d\ell_g(x)}+\mathcal{O}(\epsilon^2)\\
=& -\sum_i\frac{s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{\mathcal{T}_{gi}}d\ell_g(x)}{\sum_{i'}s_{y_{i'}}\cdot p_{i'}^{\lambda-1}\cdot\int_{\mathcal{T}_{gi'}}d\ell_g(x)}\cdot\frac{\int_{\mathscr{A}_{gi}}d\ell_g(x)}{\int_{\mathcal{T}_{gi}}d\ell_g(x)}+\mathcal{O}(\epsilon^2)\\
=& -\sum_i\frac{\ell_{gi}}{\ell_g}\cdot d\ln\Gamma_{gi}^d+\mathcal{O}(\epsilon^2).
\end{aligned}$$

The first-line follows from an approximation of log changes. The second line uses the definition of task shares and of $\Gamma_g^{\mathscr{A}}(w,p)$. The last line is the definition of $d\ln\Gamma_g^d$.

Turning to the effects of sectoral prices on task shares, $\frac{\partial\ln\Gamma_g(w,p)}{\partial\ln p}\cdot\text{stack}(d\ln p)$, we have

$$\frac{\partial\ln\Gamma_g(w,p)}{\partial\ln p}\cdot d\ln p=\sum_i\frac{s_{y_i}\cdot p_i^{\lambda-1}\cdot\Gamma_{gi}}{\sum_{i'}s_{y_{i'}}\cdot p_{i'}^{\lambda-1}\cdot\Gamma_{gi'}}\cdot(\lambda-\eta)\cdot d\ln p_i=\sum_i\frac{\ell_{gi}}{\ell_g}\cdot(\lambda-\eta)\cdot d\ln p_i.$$

Plugging our approximation for $\ln\Gamma_g^{\mathscr{A}}(w,p)-\ln\Gamma_g(w,p)$ and our formula for $\frac{\partial\ln\Gamma_g(w,p)}{\partial\ln p}\cdot\text{stack}(d\ln p)$ into equation (S5), we obtain

$$\text{(S6)}\qquad \begin{aligned}d\ln w_g=&\frac{1}{\lambda}\cdot d\ln y-\frac{1}{\lambda}\cdot d\ln\Gamma_g^d+\frac{1}{\lambda}\cdot\sum_i\frac{\ell_{gi}}{\ell_g}\cdot(\lambda-\eta)\cdot d\ln p_i\\
&+\frac{1}{\lambda}\cdot\frac{\partial\ln\Gamma_g(w)}{\partial\ln w}\cdot\text{stack}(d\ln w)+\mathcal{O}(\epsilon^{2-1/d}).\end{aligned}$$

Lemma 1 implies that this system has the unique solution (to a first-order approximation)

$$\text{(S7)}\qquad d\ln w_g=\Theta_g\cdot\text{stack}\left(\frac{1}{\lambda}\cdot d\ln y-\frac{1}{\lambda}\cdot d\ln\Gamma_j^d+\frac{1}{\lambda}\cdot\sum_i\frac{\ell_{ji}}{\ell_j}\cdot(\lambda-\eta)\cdot d\ln p_i\right)+\mathcal{O}(\epsilon^{2-1/d}).$$

Derivation of equation (14): effects on sectoral prices $d\ln p_i$. Equation (S3) can be written as $p_i^{1-\lambda}=I_i(w)$, where

$$I_i(w)=\Gamma_{ki}(w)+\sum_g\Gamma_{gi}(w)\cdot\mu_{gi}(w)\cdot w_g^{1-\lambda}.$$

Lemma S1 shows that we can expand $I_i(w)$ as

$$dI_i = I_i^{\mathscr{A}}(w) - I_i(w) + I_i(w) \cdot \frac{\partial \ln I_i(w)}{\partial \ln w} \cdot d \ln w + \mathcal{O}(\epsilon^{2-1/d}). \tag{S8}$$

Note that $\frac{\partial \ln I_i(w)}{\partial \ln w} \cdot d \ln w$ captures the effect of a change in wages on the cost of producing the final good at the initial equilibrium allocation. Because tasks are allocated in a cost-minimizing way (given wedges), the envelope theorem implies

$$\frac{\partial \ln I_i(w)}{\partial \ln w} \cdot d \ln w = (1-\lambda) \cdot \sum_g s_{gi} \cdot d \ln w_g + \mathcal{O}(\epsilon^2)$$

The term $I^{\mathscr{A}}(w) - I(w)$ captures the cost saving gains from automating tasks in $\mathscr{A}_{gi}$ holding wages constant. We have

$$\begin{aligned}
I_i^{\mathscr{A}}(w) - I_i(w) =& \Gamma_{ki}^{\mathscr{A}}(w) - \Gamma_{ki}(w) + \sum_g \Gamma_{gi}^{\mathscr{A}}(w) \cdot \mu_{gi}^{\mathscr{A}}(w) \cdot {w_g}^{1-\lambda} - \sum_g \Gamma_{gi}(w) \cdot \mu_{gi}(w) \cdot {w_g}^{1-\lambda} \\
=& \sum_g \left[\frac{1}{M_i} \cdot \int_{\mathscr{A}_{gi}} (q'_x \cdot \psi_{kx})^{\lambda-1} \cdot dx - \frac{1}{M_i} \cdot \int_{\mathscr{A}_{gi}} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot {w_g}^{1-\lambda} dx \cdot \right] \\
=& \sum_g \frac{1}{M_i} \int_{\mathscr{A}_{gi}} \left[(q'_x \cdot \psi_{kx})^{\lambda-1} - (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot {w_g}^{1-\lambda} \right] \cdot dx \\
=& \sum_g \frac{1}{M_i} \int_{\mathscr{A}_{gi}} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot {w_g}^{1-\lambda} \cdot \left[\left(\frac{q'_x \cdot \psi_{kx} \cdot w_g \cdot \mu_{gx}}{\psi_{gx}}\right)^{\lambda-1} - 1 \right] \cdot dx \\
=& I_i(w) \cdot \sum_g s_{gi} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_{gi}} \cdot \frac{\frac{1}{M_i} \int_{\mathscr{A}_{gi}} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot \left[\left(\frac{q'_x \cdot \psi_{kx} \cdot w_g \cdot \mu_{gx}}{\psi_{gx}}\right)^{\lambda-1} - 1 \right] \cdot dx}{\frac{1}{M_i} \int_{\mathscr{A}_{gi}} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot dx} \\
=& I_i(w) \cdot (\lambda - 1) \cdot \sum_g s_{gi} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_{gi}} \cdot \pi_{gi}.
\end{aligned}$$

In the last step, we used the fact that $\ell_{gx} \cdot \mu_{gx} \propto (\psi_{gx}/\mu_{gx})^{\lambda-1}$ (from equilibrium condition E3'), which gives the expression for π_{gi} in the main text. We also used the fact that $s_{gi} \cdot I_i(w) = \Gamma_{gi} \cdot w_g^{1-\lambda}$ (also from E3').

Plugging our formulas for $\frac{\partial \ln I_i(w)}{\partial \ln w} \cdot d \ln w$ and $I^{\mathscr{A}}(w) - I(w)$ in (S8), we obtain

$$d \ln I_i = (\lambda - 1) \cdot \sum_g s_{gi} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_{gi}} \cdot \pi_{gi} + (1-\lambda) \cdot \sum_g s_{gi} \cdot d \ln w_g + \mathcal{O}(\epsilon^{2-1/d}).$$

Using the fact that $(1-\lambda) \cdot d \ln I_i = d \ln p_i$, we can rewrite this as

$$d \ln p_i = \sum_g s_{gi} \cdot d \ln w_g - \sum_g s_{gi} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_{gi}} \cdot \pi_{gi} + \mathcal{O}(\epsilon^{2-1/d}). \tag{S9}$$

Derivation of equation (15): effect on base wage levels. This can be obtained from the ideal-price index condition in equation (S4). The envelope theorem applied to the production of the final good implies

$$0 = \sum_i s_{y_i} \cdot d \ln p_i.$$

Substituting the expression for $d \ln p_i$ in (S9) and rearranging yields

$$\sum_g s_g \cdot d \ln w_g = \sum_i s_{y_i} \cdot \sum_g s_{gi} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_{gi}} \cdot \pi_{gi} + \mathcal{O}(\epsilon^{2-1/d}).$$

Derivation of equation (16): effects on group rents $d \ln \mu_g$. Lemma S1 implies that we can approximate the equilibrium change in group rents as

$$\begin{aligned} d \ln \mu_g = \ln \mu_g^{\mathscr{A}}(w,p) - \ln \mu_g(w,p) + \frac{\partial \ln \mu_g(w,p)}{\partial \ln w} \cdot \text{stack}(d \ln w) \\ + \frac{\partial \ln \mu_g(w,p)}{\partial \ln p} \cdot \text{stack}(d \ln p) + \mathcal{O}(\epsilon^{2-1/d}). \end{aligned}$$

We can rewrite the direct effect of automation on rents, $\ln \mu_g^{\mathscr{A}}(w,p) - \ln \mu_g(w,p)$, as

$$\begin{aligned} &\ln \mu_g^{\mathscr{A}}(w,p) - \ln \mu_g(w,p) \\ &\quad = \frac{\mu_g^{\mathscr{A}}(w,p) - \mu_g(w,p)}{\mu_g(w,p)} + \mathcal{O}(\epsilon^2) \\ &\quad = \frac{\frac{\sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathcal{T}_{gi}} \mu_{gx} \cdot d\ell_g(x) - \sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathscr{A}_{gi}} \mu_{gx} \cdot d\ell_g(x)}{\sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathcal{T}_{gi}} d\ell_g(x) - \sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathscr{A}_{gi}} d\ell_g(x)} - \mu_g}{\mu_g} + \mathcal{O}(\epsilon^2) \\ &\quad = \frac{\frac{\mu_g \cdot \sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathcal{T}_{gi}} d\ell_g(x) - \mu_{\mathscr{A}_g} \cdot \sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathscr{A}_{gi}} d\ell_g(x)}{\sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathcal{T}_{gi}} d\ell_g(x) - \sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathscr{A}_{gi}} d\ell_g(x)} - \mu_g}{\mu_g} + \mathcal{O}(\epsilon^2) \\ &\quad = \frac{\sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathscr{A}_{gi}} d\ell_g(x) - \frac{\mu_{\mathscr{A}_g}}{\mu_g} \cdot \sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathscr{A}_{gi}} d\ell_g(x)}{\sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathcal{T}_{gi}} d\ell_g(x) - \sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathscr{A}_{gi}} d\ell_g(x)} + \mathcal{O}(\epsilon^2) \\ &\quad = -\left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1 \right) \cdot \frac{d \ln \Gamma_g^d}{1 - d \ln \Gamma_g^d} + \mathcal{O}(\epsilon^2) \\ &\quad = -\left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1 \right) \cdot d \ln \Gamma_g^d + \mathcal{O}(\epsilon^2) \end{aligned}$$

The first line approximates the change in logs. The second line uses the definition of $\mu_g^{\mathscr{A}}(w,p)$ and the fact that $\mu_g(w,p) = \mu_g$. The third line uses the definition of average group rents and average group rents at automated jobs, $\mu_{\mathscr{A}_g}$. The fourth line divides by μ_g and cancels terms. The fifth line uses the definition of $d\ln\Gamma_g^d$. The last equality uses the fact that $\frac{d\ln\Gamma_g^d}{1-d\ln\Gamma_g^d} = d\ln\Gamma_g^d + \mathcal{O}(\epsilon^2)$.

We now turn to the effects of sectoral prices on group rents, which can be written as

$$\mu_g(w,p) = \frac{\sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \mu_{gi} \cdot \int_{\mathcal{T}_{gi}} d\ell_g(x)}{\sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathcal{T}_{gi}} d\ell_g(x)}.$$

The effect of sectoral prices on rents is then given by

$$\frac{\partial \ln \mu_g(w,p)}{\partial \ln p} \cdot d\ln p = \sum_i \frac{s_{y_i} \cdot p_i^{\lambda-1} \cdot \mu_{gi} \cdot \int_{\mathcal{T}_{gi}} d\ell_g(x)}{\sum_{i'} s_{y_{i'}} \cdot p_{i'}^{\lambda-1} \cdot \mu_{gi'} \cdot \int_{\mathcal{T}_{gi'}} d\ell_g(x)} \cdot (\lambda-\eta) \cdot d\ln p_i \\ - \sum_i \frac{s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{\mathcal{T}_{gi}} d\ell_g(x)}{\sum_{i'} s_{y_{i'}} \cdot p_{i'}^{\lambda-1} \cdot \int_{\mathcal{T}_{gi'}} d\ell_g(x)} \cdot (\lambda-\eta) \cdot d\ln p_i,$$

which can be rewritten in terms of average group wages and employment as

$$\frac{\partial \ln \mu_g(w,p)}{\partial \ln p} \cdot d\ln p = \sum_i \left(\frac{\mu_{gi}}{\mu_g} - 1\right) \cdot \frac{\ell_{gi}}{\ell_g} \cdot (\lambda-\eta) \cdot d\ln p_i.$$

These derivations show that

$$d\ln \mu_g = -\left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1\right) \cdot d\ln\Gamma_g^d + \sum_i \left(\frac{\mu_{gi}}{\mu_g} - 1\right) \cdot \frac{\ell_{gi}}{\ell_g} \cdot (\lambda-\eta) \cdot d\ln p_i \\ + \frac{\partial \ln \mu_g(w)}{\partial \ln w} \cdot \text{stack}(d\ln w) + \mathcal{O}(\epsilon^{2-1/d}).$$

Using the expression for the change in base wages in (13), we obtain

$$\text{(S10)} \quad d\ln \mu_g = -\left(\frac{\mu_{\mathscr{A}_g}}{\mu_g} - 1\right) \cdot d\ln\Gamma_g^d + \sum_i \left(\frac{\mu_{gi}}{\mu_g} - 1\right) \cdot \frac{\ell_{gi}}{\ell_g} \cdot (\lambda-\eta) \cdot d\ln p_i \\ + \mathcal{M}_g \cdot \text{stack}\left(\frac{1}{\lambda} \cdot d\ln y - \frac{1}{\lambda} \cdot d\ln\Gamma_j^d + \sum_i \frac{\ell_{ji}}{\ell_j} \cdot (\lambda-\eta) \cdot d\ln p_i\right) + \mathcal{O}(\epsilon^{2-1/d}).$$

Derivation of equation (17): effects on TFP, consumption, and mean wages. As in the single-sector economy, the dual version of Solow implies $d\ln tfp = \sum_g s_g \cdot d\ln \bar{w}_g = \sum_g s_g \cdot d\ln w_g + \sum_g s_g \cdot d\ln \mu_g$. Plugging the formula for $\sum_g s_g \cdot d\ln w_g$ in equation (15) yields equation (17) in the proposition.

Turning to consumption, we have $c = y - k$. Differentiating, we get $d\ln c = (1/c) \cdot (dy -$

$dk) = (1/s^L)\cdot(d\ln y-(k/y)d\ln k) = (1/s^L)\cdot d\ln tfp$. Note that $s_L = c/y$ since there is no net capital income in our model, as capital is produced linearly from the final good. ∎

S1.3 Approximation Lemma

A key technical step in the proofs of Proposition 3 and 6 involves the approximation of the effects of automation in three parts: the effects of the automation shock holding prices constant, the effect of prices governed by the Jacobians of task shares with respect to prices, and a small approximation error. This is similar to a first-order Taylor expansion, but instead of considering a change in real arguments, we approximate the effects of a direct change in task allocations generated by automation. The following lemma shows that this expansion provides a valid approximation for "small" automation shocks of order ϵ.

We give a general version of the Lemma that accommodates the multisector economy. Its application to the single-sector economy follows as a corollary.

LEMMA S1 (TAYLOR EXPANSIONS OF FUNCTIONS ON TASK SPACE) *Consider a function of the form*

$$f(w,z) = h\left(\left\{\int_{\mathcal{T}_{gi}(w)} N(\mu_{gx},\psi_{gx})\cdot dx\right\}_{g,i},\left\{\int_{\mathcal{T}_{ki}(w)} N_k(q_x,\psi_{kx})\cdot dx\right\}_{i}, z\right).$$

The sets $\mathcal{T}_{gi}(w)$ and $\mathcal{T}_{ki}(w)$ are defined by E1 and E2, N and N_k are continuous functions of task attributes to $\mathbb{R}^n$ that are bounded in their domains ($\mathcal{T}_{gi}(w)$ and $\mathcal{T}_{ki}(w)$ for all $w>0$), z is a vector of s additional arguments, and h is a continuously differentiable function from $(G+1)\times I\times\mathbb{R}^n+\mathbb{R}^s$ to $\mathbb{R}$.

Let $\mathcal{T}^{\mathscr{A}}_{gi}(w)$ and $\mathcal{T}^{\mathscr{A}}_{ki}(w)$ denote the new task allocation after a small and interior automation shock of order ϵ at wage levels w. Define

$$f^{\mathscr{A}}(w,z) = h\left(\left\{\int_{\mathcal{T}^{\mathscr{A}}_{gi}(w)} N(\mu_{gx},\psi_{gx})\cdot dx\right\}_{g,i},\left\{\int_{\mathcal{T}^{\mathscr{A}}_{ki}(w)} N_k(q'_x,\psi_{kx})\cdot dx\right\}_{i}, z\right),$$

where $q'_x = q_x$ outside $\mathscr{A}^T$.

Suppose that the automation shock changes z and w by dz and dw, both of which are $\mathcal{O}(\epsilon)$. Then the total effect of this shock on f can be approximated as

(S11) $$df = f^{\mathscr{A}}(w,z) - f(w,z) + \frac{\partial f}{\partial w}\cdot dw + \frac{\partial f}{\partial z}\cdot dz + \mathcal{O}(\epsilon^{2-1/d})$$

Proof. Let w and z be the initial equilibrium values of wages and z and $w' = w + dw$ and $z' = z + dz$ the final equilibrium values. The total change in f due to new automation technologies can be written as

$$\begin{aligned} df =& f^{\mathscr{A}}(w', z') - f(w, z) \\ =& f^{\mathscr{A}}(w, z) - f(w, z) + f^{\mathscr{A}}(w', z') - f^{\mathscr{A}}(w, z) \\ =& f^{\mathscr{A}}(w, z) - f(w, z) + \frac{\partial f^{\mathscr{A}}(w, z)}{\partial w} \cdot dw + \frac{\partial f^{\mathscr{A}}(w, z)}{\partial z} \cdot dz + \mathcal{O}(\epsilon^2), \end{aligned}$$

where the last line does a first-order Taylor expansion of $f^{\mathscr{A}}(w', z')$ around (w, z).

We now show that automation in small interior sets does not affect $\frac{\partial f(w,z)}{\partial w}$. Specifically:

$$\frac{\partial f^{\mathscr{A}}(w, z)}{\partial w} = \frac{\partial f(w, z)}{\partial w} + \mathcal{O}(\epsilon^{1-1/d}) \tag{S12}$$

Let $a'_{gi} = \int_{\mathcal{T}^{\mathscr{A}}_{gi}(w)} N(\mu_{gx}, \psi_{gx}) \cdot dx$ and $a_{ji} = \int_{\mathcal{T}_{gi}(w)} N_j(\mu_{jx}, \psi_{jx}) \cdot dx$. We have that $a'_{gi} = a_{gi} + \mathcal{O}(\epsilon)$, since $\mathscr{A}_{gi}(w)$ is of measure $\mathcal{O}(\epsilon)$ and N_g is bounded in its domain.

Let $a'_{ki} = \int_{\mathcal{T}^{\mathscr{A}}_{ki}(w)} N_k(q'_x, \psi_{kx}) \cdot dx$ and $a_{ki} = \int_{\mathcal{T}_{ki}(w)} N_k(q_x, \psi_{kx}) \cdot dx$. We have $a'_{ki} = \int_{\mathcal{T}_{ki}(w)} N_k(q'_x, \psi_{kx}) \cdot dx + \mathcal{O}(\epsilon)$, since $\mathscr{A}^T$ has measure $\mathcal{O}(\epsilon)$ and N_k is bounded in its domain. Moreover, because $q'_x = q_x$ in $\mathcal{T}_{ki}(w)$, we have $a'_{ki} = a_{ki} + \mathcal{O}(\epsilon)$.

We can express the derivatives of $f^{\mathscr{A}}$ with respect to wages as

$$\begin{aligned} \frac{\partial f^{\mathscr{A}}(w, z)}{\partial w} =& \sum_{g,i} \frac{\partial h(\{a'_{gi}\}_{g,i}, \{a'_{ki}\}_i, z)}{\partial a_{gi}} \cdot \frac{\partial}{\partial w} \int_{\mathcal{T}^{\mathscr{A}}_{gi}(w)} N_g(\mu_{gx}, \psi_{gx}) \cdot dx \\ &+ \sum_i \frac{\partial h(\{a'_{gi}\}_{g,i}, \{a'_{ki}\}_i, z)}{\partial a_{ki}} \cdot \frac{\partial}{\partial w} \int_{\mathcal{T}^{\mathscr{A}}_{ki}(w)} N_k(q'_x, \psi_{kx}) \cdot dx. \end{aligned}$$

Using the fact that the derivatives of h are continuous, we can approximate this as

$$\begin{aligned} \frac{\partial f^{\mathscr{A}}(w, z)}{\partial w} =& \sum_{g,i} \frac{\partial h(\{a_{gi}\}_{g,i}, \{a_{ki}\}_i, z)}{\partial a_{gi}} \cdot \frac{\partial}{\partial w} \int_{\mathcal{T}^{\mathscr{A}}_{gi}(w)} N_g(\mu_{gx}, \psi_{gx}) \cdot dx \\ &+ \sum_i \frac{\partial h(\{a_{gi}\}_{g,i}, \{a_{ki}\}_i, z)}{\partial a_{ki}} \cdot \frac{\partial}{\partial w} \int_{\mathcal{T}^{\mathscr{A}}_{ki}(w)} N_k(q'_x, \psi_{kx}) \cdot dx + \mathcal{O}(\epsilon) \end{aligned} \tag{S13}$$

For workers, we approximate the own-wage derivative of integrals over tasks as

$$\begin{aligned}\frac{\partial}{\partial w_g}\int_{\mathcal{T}_{gi}^{\mathscr{A}}(w)} N(\mu_{gx},\psi_{gx})\cdot dx =& \frac{\partial}{\partial w_g}\int_{\mathcal{T}_{gi}(w)} N(\mu_{gx},\psi_{gx})\cdot dx - \frac{\partial}{\partial w_g}\int_{\mathscr{A}_{gi}(w_g)} N(\mu_{gx},\psi_{gx})\cdot dx\\ =& \frac{\partial}{\partial w_g}\int_{\mathcal{T}_{gi}(w)} N(\mu_{gx},\psi_{gx})\cdot dx - \int_{\mathcal{B}_{gi}(w_g)} D_{gi}(\sigma)\cdot N(\mu_{g\sigma},\psi_{g\sigma})\cdot d\sigma\\ =& \frac{\partial}{\partial w_g}\int_{\mathcal{T}_{gi}(w)} N(\mu_{gx},\psi_{gx})\cdot dx + \mathcal{O}(\epsilon^{1-1/d}).\end{aligned}$$

The first-line decomposes the integral over $\mathcal{T}_{gi}^{\mathscr{A}}(w)$ into an integral over $\mathcal{T}_{gi}(w)$ minus an integral over $\mathscr{A}_{gi}(w_g)$ (defined as the set of tasks in $\mathscr{A}_{gi}^T$ automated at a wage w_g). The second line uses the general version of Leibniz integral rule. It replaces a derivative of a volume integral over $\mathscr{A}_{gi}(w_g)$ by an area integral over the boundary tasks $\mathcal{B}_{gi}(w_g)$ (and where $d\sigma$ is the induced Lebesgue measure over the hyper-surface $\mathcal{B}_{gi}(w_g)$). This surface integral is $\mathcal{O}(\epsilon^{1-1/d})$ because D and N are bounded and the surface $\mathcal{B}_{gi}$ has area $\mathcal{O}(\epsilon^{1-1/d})$.

For cross-wage derivatives, we use the fact that the automation shock is interior. This implies that for all $g' \neq g$

$$\frac{\partial}{\partial w_{g'}}\int_{\mathcal{T}_{gi}^{\mathscr{A}}(w)} N(\mu_{gx},\psi_{gx})\cdot dx = \frac{\partial}{\partial w_{g'}}\int_{\mathcal{T}_{gi}(w)} N(\mu_{gx},\psi_{gx})\cdot dx$$

This follows from the fact that changes in other group wages do not affect the task allocation in the interior of $\mathcal{T}_{gi}(w)$.

For capital, we approximate the wage derivative of integrals over tasks as

$$\begin{aligned}\frac{\partial}{\partial w_g}\int_{\mathcal{T}_{ki}^{\mathscr{A}}(w)} N_k(q'_x,\psi_{kx})\cdot dx =& \frac{\partial}{\partial w_g}\int_{\mathcal{T}_{ki}(w)} N_k(q'_x,\psi_{kx})\cdot dx + \frac{\partial}{\partial w_g}\int_{\mathscr{A}_{gi}(w_g)} N_k(q'_x,\psi_{kx})\cdot dx\\ =& \frac{\partial}{\partial w_g}\int_{\mathcal{T}_{ki}(w)} N_k(q'_x,\psi_{kx})\cdot dx + \int_{\mathcal{B}_{gi}(w_g)} D_{gi}(\sigma)\cdot N_k(q'_\sigma,\psi_{g\sigma})\cdot d\sigma\\ =& \frac{\partial}{\partial w_g}\int_{\mathcal{T}_{ki}(w)} N_k(q_x,\psi_{kx})\cdot dx + \mathcal{O}(\epsilon^{1-1/d}).\end{aligned}$$

The first-line decomposes the integral over $\mathcal{T}_{ki}^{\mathscr{A}}(w)$ into an integral over $\mathcal{T}_{gi}(w)$ plus an integral over $\mathscr{A}_{gi}(w_g)$ (defined as the set of tasks in $\mathscr{A}_{gi}^T$ automated at a wage w_g). The second line uses the general version of Leibniz integral rule. It replaces a derivative of a volume integral over $\mathscr{A}_{gi}(w_g)$ by an area integral over the boundary tasks $\mathcal{B}_{gi}(w_g)$ (and where $d\sigma$ is the induced Lebesgue measure over the hyper-surface $\mathcal{B}_{gi}(w_g)$). This surface integral is $\mathcal{O}(\epsilon^{1-1/d})$ because D and N_k are bounded and the surface $\mathcal{B}_{gi}(w_g)$ has area $\mathcal{O}(\epsilon^{1-1/d})$. This last line also replaces q'_x for q_x in the first integral since these interior changes in capital productivity do not affect substitution patterns across initial boundaries.

Plugging the approximations for these wage derivatives in (S13) yields (S12).

To conclude, we show that $\frac{\partial f^{\mathscr{A}}(w,z)}{\partial z} = \frac{\partial f(w,z)}{\partial z} + \mathcal{O}(\epsilon)$. We have

$$\frac{\partial f^{\mathscr{A}}(w,z)}{\partial z} = \frac{\partial h(\{a'_{gi}\}_{g,i}, \{a'_{ki}\}_i, z)}{\partial z} = \frac{\partial h(\{a_{gi}\}_{g,i}, \{a_{ki}\}_i, z)}{\partial z} + \mathcal{O}(\epsilon) = \frac{\partial f(w,z)}{\partial z} + \mathcal{O}(\epsilon).$$

Here we used the fact that the derivatives of h are continuous, and $a'_{gi} = a_{gi} + \mathcal{O}(\epsilon)$ for workers and $a'_{ki} = a_{ki} + \mathcal{O}(\epsilon)$ for capital. ■

Remark: when applying Lemma S1 in our proofs, we use the fact that task productivities and rents are bounded from above, which ensures that the functions N and N_k are also bounded in the relevant domains of integration.

S1.4 Measuring task displacement

This subsection derives the measure of direct task displacement in equation (12) for a multi-sector economy under Assumption 3

Let $\mathcal{R}_{gi}$ denote the set of routine tasks in industry i assigned to group g. Define

$$\Gamma^{\text{routine}}_{gi} = \int_{\mathcal{R}_{gi}} \psi^{\lambda-1}_{xg} \cdot \mu^{-\lambda}_{xg} \cdot dx,$$

as the task share of group g in routine jobs at industry i.

Assumption 3 implies that all routine jobs in industry i are automated at the same rate, so that $d\ln\Gamma^{\text{routine},d}_{gi} = \chi^{\text{routine}}_i$, where $d\ln\Gamma^{\text{routine},d}_{gi}$ is the share of group g employment in routine jobs in $\mathscr{A}_{gi}$ as a fraction of all routine jobs employing g workers in industry i. In addition, the fact that non-routine jobs are not automated implies

$$d\ln\Gamma^d_{gi} = (\ell^{\text{routine}}_{gi}/\ell_{gi}) \cdot \chi^{\text{routine}}_i, \tag{S14}$$

where $\ell^{\text{routine}}_{gi}/\ell_{gi}$ is the share of employment of group g in industry i earned in routine jobs (out of all employment of group g in industry i).

Let's now turn to the labor share in industry i. This is given by

$$s_{\ell i} = \frac{\sum_g \Gamma_{gi} \cdot \mu_{gi} \cdot w^{1-\lambda}_g}{p^{1-\lambda}_i}. \tag{S15}$$

The direct effect of automation on the labor share $s_{\ell i}$ holding wages constant is

$$d \ln s_{\ell i}^d = -\sum_g \frac{s_{gi}}{s_{\ell i}} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_g} - (1-\lambda) \cdot d \ln p_i.$$

Using the formula for $d \ln p_i$ in (S9), we obtain

$$\begin{aligned} d \ln s_{\ell i}^d &= -\sum_g \frac{s_{gi}}{s_{\ell i}} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_{gi}} + (1-\lambda) \cdot \sum_g s_{gi} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_{gi}} \cdot \pi_{gi} \\ &= -\sum_g \frac{s_{gi}}{s_{\ell i}} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_{gi}} \cdot (1 - s_{\ell i} \cdot (1-\lambda) \cdot \pi_{gi}). \end{aligned}$$

Define the average cost-saving gains and average rent dissipation in industry i as

$$\pi_i = \frac{\sum_g s_{gi} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_{gi}} \cdot \pi_{gi}}{\sum_g s_{gi} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_{gi}}}, \qquad 1 + \rho_i = \frac{\sum_g s_{gi} \cdot d \ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathscr{A}_{gi}}}{\mu_{gi}}}{\sum_g s_{gi} \cdot d \ln \Gamma_{gi}^d}.$$

Using these definitions, we can write the change in labor shares as

$$d \ln s_{\ell i}^d = -(1+\rho_i) \cdot (1 - s_{\ell i} \cdot (1-\lambda) \cdot \pi_i) \cdot \sum_g \frac{s_{gi}}{s_{\ell i}} \cdot d \ln \Gamma_{gi}^d.$$

Using equation (S14), we can rewrite the change in the labor share as

$$\text{(S16)} \qquad d \ln s_{\ell i}^d = -(1+\rho_i) \cdot (1 - s_{\ell i} \cdot (1-\lambda) \cdot \pi_i) \cdot \sum_g \frac{s_{gi}}{s_{\ell i}} \cdot (\ell_{gi}^{\text{routine}}/\ell_{gi}) \cdot \chi_i^{\text{routine}}.$$

Using this equation, we can solve for the common rate of automation χ_i^{routine} as

$$\chi_i^{\text{routine}} = \frac{1}{\sum_g \frac{s_{gi}}{s_{\ell i}} \cdot (\ell_{gi}^{\text{routine}}/\ell_{gi})} \cdot \frac{1}{1+\rho_i} \cdot \frac{-d \ln s_{\ell i}^d}{1 - s_{\ell i} \cdot (1-\lambda) \cdot \pi_i}.$$

A second use of equation (S14) then implies

$$d \ln \Gamma_{gi}^d = \text{RCA}_{gi}^{routine} \cdot \frac{1}{1+\rho_i} \cdot \frac{-d \ln s_{\ell i}^d}{1 - s_{\ell i} \cdot (1-\lambda) \cdot \pi_i},$$

where the revealed comparative advantage measure is constructed as

$$\text{(S17)} \qquad \text{RCA}_{gi}^{routine} = \frac{\ell_{gi}^{\text{routine}}/\ell_{gi}}{\sum_{g'} \frac{s_{g'i}}{s_{\ell i}} \cdot (\ell_{g'i}^{\text{routine}}/\ell_{g'i})}.$$

S2 Microfoundations for wedges.

S2.1 Efficiency wage considerations

We consider a static version of an efficiency wage model (i.e. Shapiro and Stiglitz, 1984; Bulow and Summers, 1986).

On the one hand, there is a positive mass of tasks where workers earn a wage w_g and do not have to be monitored or receive extra incentives to work. Workers can always take these jobs freely.

On the other hand, there is a positive mass of tasks where workers need to be monitored and are paid an efficiency wage w_{gx}. In these tasks, workers have two options. They can stick to their duties, produce, and obtain a wage w_{gx}. Or they can shirk. In this case they put no effort on their main job and collect some income $e \cdot w_g$ by moonlighting in the no-rent sector. If not found, they obtain an income $w_{gx} + e \cdot w_g$. However, workers who shirk are detected with probability P_{gx}, fired, and forced to take a job that pays no rents. The no shirking condition is then

$$w_{gx} \geq (1 - P_{gx}) \cdot (w_{gx} + e \cdot w_g) + P_{gx} \cdot w_g.$$

This can be rearranged as

$$w_{gx} = \left(e \cdot \frac{1 - P_{gx}}{P_{gx}} + 1 \right) \cdot w_g.$$

This model thus provides a micro-foundation for wedges $\mu_{gx} = e \cdot \frac{1-P_{gx}}{P_{gx}} + 1$ derived from efficiency wage considerations. Our treatment assumes there are no other contracts that can solve the monitoring problem.

S2.2 Bargaining models

Consider a one-shot model where firms must make an investment to create a position before matching with a worker, as in Grout (1984).

A firm producing task x can create ℓ_{gx} positions for workers of type g. Creating each position takes up $\kappa \in (0, 1)$ units of labor, which implies that the total amount of labor available for production is $\ell_{gx} \cdot (1 - \kappa)$. The firm must pay this cost in advance, which implies that once workers are matched to their positions, there is a surplus to bargain over.

The firm obtains a surplus of $p_x \cdot \psi_{gx} - w_{gx}$ if the negotiation succeeds and 0 otherwise. The worker obtains a surplus of w_{gx} if the negotiation succeeds and w_g otherwise. As before, we assume that there is a positive mass of jobs that pay no rents at which workers can always access. The wage w_{gx} is determined by Nash bargaining, with workers' bargaining power given by $\beta_{gx} \in (0, 1-\kappa)$.

Lemma S2 (Representation result) *The equilibrium of the bargaining economy coincides with that of our baseline model by taking $\widetilde{\psi}_{gx} = \psi_{gx} \cdot (1-\kappa)$ and $\mu_{gx} = \frac{(1-\kappa)\cdot(1-\beta_{gx})}{1-\kappa-\beta_{gx}} \geq 1$.*

Proof. Free entry for firms implies

$$(1-\beta_{gx}) \cdot (p_x \cdot \psi_{gx} - w_g) \leq \kappa \cdot p_x \cdot \psi_{gx}.$$

This can be written as

$$p_x \leq w_g \cdot \frac{\mu_{gx}}{\psi_{gx} \cdot (1-\kappa)},$$

which coincides with E1 and E2 for $\widetilde{\psi}_{gx} = \psi_{gx} \cdot (1-\kappa)$. Thus, the bargaining model gives the same rule for allocating tasks across workers and capital than our baseline model with exogenous wedges.

Moreover market clearing for task $x \in \mathcal{T}_g$ requires

$$\psi_{gx} \cdot (1-\kappa) \cdot \ell_{gx} = y \cdot \frac{1}{M} \cdot (\psi_{gx} \cdot (1-\kappa))^{\lambda} \cdot (\mu_{gx} \cdot w_g)^{-\lambda}$$

which coincides with E3 for $\widetilde{\psi}_{gx} = \psi_{gx} \cdot (1-\kappa)$. Thus, the bargaining model gives the same allocation of labor by tasks as our baseline model with exogenous wedges.

Turning to wages paid to workers, we have

$$w_{gx} = \beta_{gx} \cdot p_x \cdot \psi_{gx} + (1-\beta_{gx}) \cdot w_g = \mu_{gx} \cdot w_g.$$

This implies the bargaining model gives the same wage payments by task as our baseline model with exogenous wedges. ∎

S3 Empirical Details.

S3.1 Data sources and details

Our main data sources are the same used in Acemoglu and Restrepo (2022), and we refer readers to this paper for details. This paper brings in new proxies for rents, described in detail below.

Wage differentials: our first proxy for job-specific rents is the inter-industry and occupation wage differentials. As explained in the text, we compute this using 1980 Census data as $\bar{w}_{gio}/\bar{w}_g$. In this expression, $\bar{w}_{gio}$ is the average wage earned by group g in industry i and occupation o in 1980. This is computed for the 49 industries in our analysis and 300 detailed Census occupations.

Wage losses from job displacement: our second proxy for job-specific rents is the industry-specific wage loss from job displacement, computed separately for workers by gender and education. As explained in the text, we compute this using the CPS Displaced Worker Supplement as the average change in (log) wages before and after a displacement event. We restrict the sample of displaced workers to those with at least a year of tenure before the displacement episode who have since then found a new job. The resulting sample contains 37,355 displaced workers, observed between 1984 and 2022. We winsorize previous job and current job wages from below at 100 dollars per week and we also winsorize the change in log wages at the 5th and 95th percentiles to avoid outliers.

We compute the average wage loss for these workers for the 49 industries in our analysis and in sic broad occupations. We allow these to vary by worker education (grouping only college vs non-college groups) and gender. We do not compute this measure by detailed worker demographic characteristics because the resulting cells would be too small.

Quit rates: our last proxy for job-specific rents is the (inverse of the) monthly quit rate from jobs in an industry and occupation. This is computed from the panel component of the Basic Monthly CPS.

We consider two forms of quits. First, we compute the EE rate, following the cleaning procedure in Fujita et al. (2024). EE transitions are identified using a new question added to the CPS in 1994, and so this measure is only available for 1994–2023. As pointed out in Fujita et al. (2024), there is a growing share of workers reporting “not knowing” if they still work for the same employer, which we treat as missing observations.

Second, we compute voluntary EU transitions per month. For this, we consider a separation into unemployment as voluntary if workers report that the reason for being unemployed is that they are “job leavers”. This measure is available for the 1976–2023 period. For our analysis, we average EE and voluntary EU rates over these years and by industry, occupation, and group (only in terms of gender and for college vs non-college workers). We use the 49 industries in our analysis and 300 detailed Census occupations, which we match to the CPS.

S3.2 Main estimates reported in paper

Table S1 provides a summary of the reduced-form evidence in the paper. The panels report estimates of the reduced form equation (11) for different outcome variables. The specification in column 1 reports a bivariate regression. The specification in column 2 adds sectoral demand shifts, gender and education dummies as covariates. The specification in column 3 adds sectoral rent shifts as a covariate. The specification in column 4 adds the share of employment in manufacturing from the 1980 Census as a covariate.

Table S1: Summary of reduced-form evidence, 1980-2016.

	(1)	(2)	(3)	(4)
	Panel A. Dependent variable: percent change in group average wage, $\Delta \ln \bar{w}_g$			
Direct task displacement	-2.36	-2.06	-2.06	-1.90
	(0.13)	(0.25)	(0.27)	(0.29)
R^2 for task displacement	0.67	0.58	0.58	0.54
Observations	500	500	500	500
	Panel B. Dependent variable: wage change above 30th percentile, $\Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$			
Direct task displacement	-0.35	-0.53	-0.50	-0.37
	(0.06)	(0.13)	(0.11)	(0.11)
R^2 for task displacement	0.23	0.34	0.32	0.24
Observations	500	500	500	500
	Panel C. Dependent variable: wage changes at 30th percentile, $\Delta \ln w_g^{30th}$			
Direct task displacement	-2.01	-1.53	-1.55	-1.53
	(0.14)	(0.29)	(0.30)	(0.33)
R^2 for task displacement	0.57	0.43	0.44	0.44
Observations	500	500	500	500
	Panel D. Dependent variable: change in group rents due to reallocation—wage differentials from Census			
Direct task displacement	-0.46	-0.36	-0.35	-0.39
	(0.06)	(0.10)	(0.11)	(0.11)
R^2 for task displacement	0.39	0.31	0.30	0.33
Observations	496	496	496	496
	Panel E. Dependent variable: change in group rents due to reallocation—wage loss due to displacement from CPS			
Direct task displacement	-0.17	-0.19	-0.18	-0.20
	(0.02)	(0.03)	(0.03)	(0.04)
R^2 for task displacement	0.47	0.53	0.51	0.56
Observations	500	500	500	500
	Panel F. Dependent variable: change in group rents due to reallocation—(minus) EE rates from CPS			
Direct task displacement	-1.276	-1.273	-1.332	-1.506
	(0.085)	(0.278)	(0.255)	(0.253)
R^2 for task displacement	0.58	0.57	0.60	0.68
Observations	500	500	500	500
	Panel G. Dependent variable: change in group rents due to reallocation—(minus) voluntary EU rates from CPS			
Direct task displacement	-0.223	-0.292	-0.286	-0.300
	(0.023)	(0.043)	(0.045)	(0.050)
R^2 for task displacement	0.43	0.56	0.55	0.58
Observations	500	500	500	500
Covariates:				
Education and gender		✓	✓	✓
Sectoral demand shifters		✓	✓	✓
Sectoral rent shifters			✓	✓
Manufacturing share				✓

Notes: This table presents estimates of the reduced-form relationship between the direct task displacement due to new automation technology from 1980 to 2016 and various group-level outcomes. The sample includes 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is indicated in the panel headers. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroscedasticity are reported in parentheses.

S3.3 Robustness checks

Acemoglu and Restrepo (2022) report several robustness checks for the reduced-form relationship between group average wages and the task displacement due to automation. Here, we provide robustness checks for the relationship between automation and within-group wage dispersion and rents, which is the novel empirical aspect in this paper.

Automation and within-group wage changes: We first asses the robustness of the U-shape pattern in Figure 5. Figure S1 reports estimates by percentile in levels and not relative to the 30th percentile as in the main text.

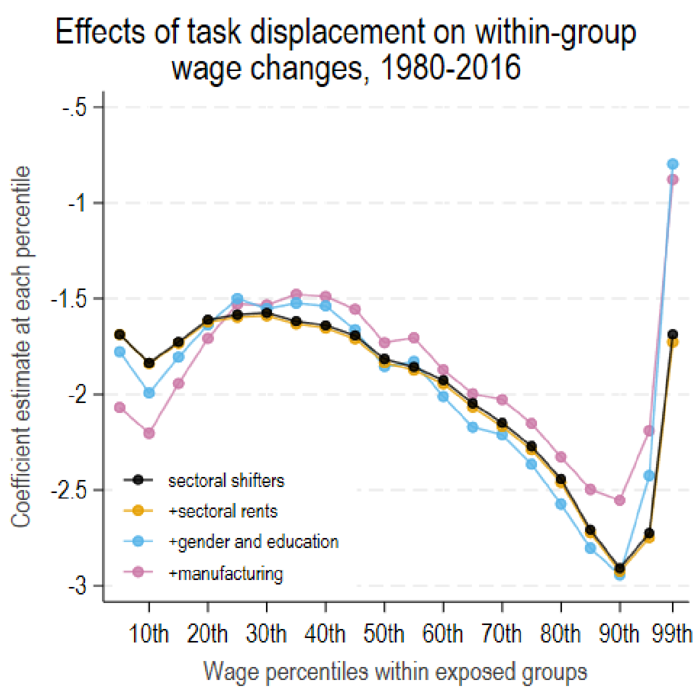

Figure S1: Reduced-form relationship between wage changes across percentiles of the within-group wage distribution and task displacement. the left panel plots estimates from a group quantile regression of changes in $d \ln w_g^p$ against task displacement for percentiles p ranging from the 5th to the 99th. The lines provide estimates for different specifications. The right panel excludes worker groups with an average hourly wage below \$13 dollars in 1980. This panel reports estimates relative to the 30th percentile.

Figure S2 shows the U-shape pattern is robust to including additional controls or restricting the sample to high-wage groups. The left panel reports estimates constraining the estimation sample to groups with an average real wage in 1980 above \$13 dollars. For these groups, wage changes become flat below the 30th percentile. We continue to see a clear

U-shape pattern of withing-group wage changes, with a more pronounced decline between the 30th and 95th percentiles. The right panel shows that this pattern is also robust to controlling for the incidence of the minimum wage and declining unionization rates across industries. For the minimum wage, we control for the share of workers in each group earning an hourly wage below 3.1 dollars in 1980 (the Federal minimum wage). For unionization, we control for groups' exposure to industries with declining unionization rates. These are computed from the CPS as in Acemoglu and Restrepo (2022).

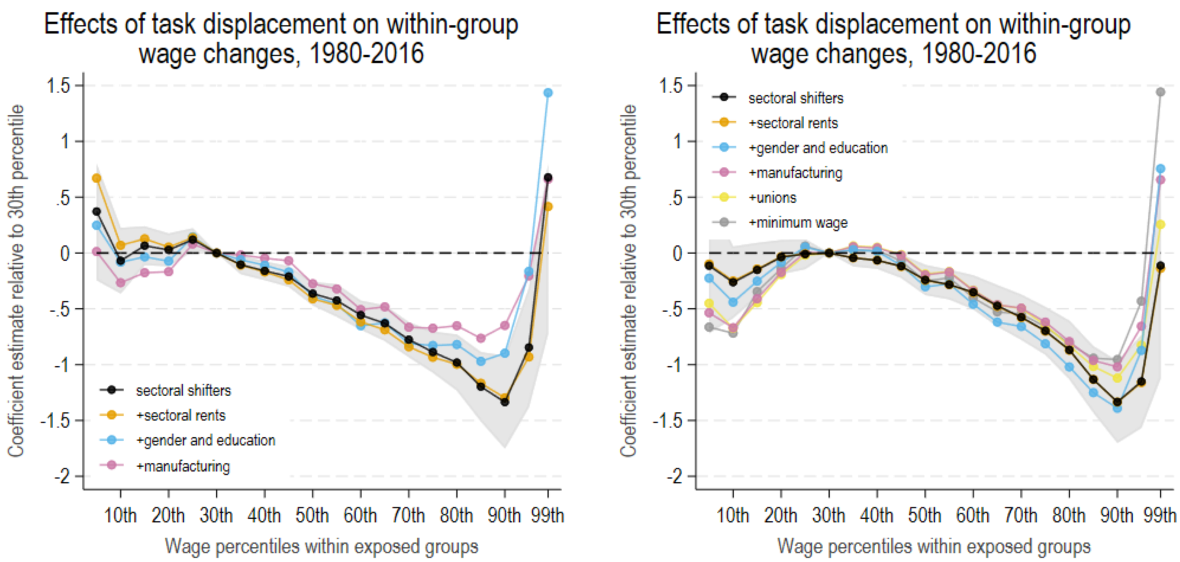

FIGURE S2: REDUCED-FORM RELATIONSHIP BETWEEN WAGE CHANGES ACROSS PERCENTILES OF THE WITHIN-GROUP WAGE DISTRIBUTION AND TASK DISPLACEMENT. The figure plots estimates from a group-level quantile regression of changes in $d\ln w_g^p$ against task displacement for percentiles p ranging from the 5th to the 99th relative to the 30th percentile. Different colors represent estimates from different specifications.

Measuring rent dissipation from within-group wage changes: Table S2 provides estimates of the decline in wages within exposed groups above their 20th and 40th percentiles. The table also reports these estimates for groups with an average wage above $13 in 1980, no college degree, or a college degree, respectively. These estimates point to a rent dissipation of 19–40% in jobs automated during 1980–2016 by new automation technology.

Table S2: Robustness checks: inferring rent dissipation from within-group wage changes.

	(1)	(2)	(3)	(4)
	Panel A. Dependent variable: wage change above 20th percentile, $\Delta \ln \bar{w}_g - \Delta \ln w_g^{20th}$			
Direct task displacement	-0.42	-0.45	-0.42	-0.19
	(0.08)	(0.20)	(0.18)	(0.16)
R^2 for task displacement	0.20	0.21	0.20	0.09
Observations	500	500	500	500
	Panel B. Dependent variable: wage change above 40th percentile, $\Delta \ln \bar{w}_g - \Delta \ln w_g^{40th}$			
Direct task displacement	-0.31	-0.56	-0.52	-0.41
	(0.07)	(0.15)	(0.13)	(0.11)
R^2 for task displacement	0.21	0.38	0.35	0.28
Observations	500	500	500	500
	Panel C. Dependent variable: wage change above 30th percentile, $\Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$—only groups with average wage above $13 in 1980			
Direct task displacement	-0.25	-0.30	-0.31	-0.27
	(0.07)	(0.14)	(0.12)	(0.14)
R^2 for task displacement	0.16	0.19	0.20	0.18
Observations	364	364	364	364
	Panel D. Dependent variable: wage change above 30th percentile, $\Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$—includes only groups without a college degree			
Direct task displacement	-0.19	-0.54	-0.47	-0.32
	(0.10)	(0.15)	(0.13)	(0.13)
R^2 for task displacement	0.04	0.10	0.09	0.06
Observations	300	300	300	300
	Panel E. Dependent variable: wage change above 30th percentile, $\Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$—includes only groups with a college degree			
Direct task displacement	-0.63	-0.35	-0.37	-0.38
	(0.22)	(0.28)	(0.33)	(0.34)
R^2 for task displacement	0.12	0.07	0.07	0.07
Observations	200	200	200	200
Covariates:				
Education and gender		✓	✓	✓
Sectoral demand shifters		✓	✓	✓
Sectoral rent shifters			✓	✓
Manufacturing share				✓

Notes: This table presents estimates of the reduced-form relationship between the direct task displacement due to new automation technology from 1980 to 2016 and various group-level outcomes. The sample includes 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is indicated in the panel headers. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroscedasticity are reported in parentheses.

Measuring rent dissipation using proxies for rents: Table S3 and S4 provide robustness checks for our measures of reallocation away from high rent jobs. For these tests, we focus on a specification that includes all our baseline covariates and change the way we measure rents across specifications.

Panel A provides robustness checks for our measure of rent reallocation based on inter-industry and occupation wage differentials. Column 1 reports our baseline estimates. Column 2 uses a measure of wage differentials that partials out differences in wages across states. This accounts for the possibility that jobs are located in areas with different costs of living, which would generate wage variation in the form of compensating differentials. Column 3 uses a broader occupational grouping with 6 occupations in total (instead of the 300 in our baseline). Column 4 also uses this broader occupational grouping and partials out regional wage differences. Column 5 computes a common inter-industry and occupation wage differential averaging across all groups. Column 6 uses this common differential and also partials out regional wage differences.

Panel B provides robustness checks for our measure of rent reallocation based on wage losses from displacement. Column 1 reports our baseline estimates. Column 2 uses a common wage-loss measure averaging across all workers, and not only across workers of the same gender and education of a group. Column 3 also uses a common wage-loss measure averaging across all workers, but controls for the observable characteristics of displaced workers when computing these losses. For these columns, we compute wage losses for the 49 industries in our analysis and six broad occupational groups. Columns 4–6 report similar specifications but now compute wage losses at the industry level.

Panels C and D provide robustness checks for our measure of rent reallocation based on (the inverse of) monthly quit rates. Column 1 reports our baseline estimates. Column 2 uses a common quit rate averaging across all workers, and not only across workers of the same gender and education of a group. Here we only control for differences in EE rates and voluntary EU rates over time, to account for trends in EE and EUV rates. Column 3 also uses a common wage loss measure averaging across all workers, but controls for the observable characteristics of displaced workers across industries and occupations when computing monthly transition rates. Columns 4–6 report similar specifications but now compute wage losses by industry and broad occupational group. For these we use the 49 industries in our analysis and 6 occupational groups.

Table S3: Robustness checks: reallocation away from high-rent jobs of exposed workers (I)

	Panel A. Dependent variable: reallocation to high rent jobs, measured by wage differentials in 1980 Census					
	Baseline	Partial out state differences	Broader occupational groups	Broader occupational groups and partial out state differences	Common wage differences	Common wage differences and partial out state differences
	(1)	(2)	(3)	(4)	(5)	(6)
Direct task displacement	-0.385	-0.368	-0.380	-0.363	-0.505	-0.488
	(0.108)	(0.104)	(0.095)	(0.091)	(0.093)	(0.088)
R^2 for task displacement	0.33	0.32	0.37	0.36	0.51	0.52
Observations	496	496	499	499	500	500

	Panel B. Dependent variable: reallocation to high rent jobs, measured by wage loss from job separation in CPS					
	Wage losses by industry, broad occupation, gender, and education	Common wage losses by industry and broad occupation	Common wage losses by industry and broad occupation (adjusted for demographics)	Wage losses by industry, gender, and education	Common wage losses by industry	Common wage losses by industry (adjusted for demographics)
	(1)	(2)	(3)	(4)	(5)	(6)
Direct task displacement	-0.198	-0.227	-0.229	-0.183	-0.190	-0.203
	(0.038)	(0.031)	(0.032)	(0.034)	(0.028)	(0.029)
R^2 for task displacement	0.56	0.71	0.67	0.53	0.66	0.68
Observations	500	500	500	500	500	500

Notes: This table presents estimates of the reduced-form relationship between the direct task displacement due to new automation technology from 1980 to 2016 and the reallocation of exposed groups away from high-rent jobs. The sample includes 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is indicated in the panel headers. The different columns present results varying the measurement of rents across jobs. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroscedasticity are reported in parentheses. All specifications control for sectoral demand shifters, sectoral rent shifters, gender and education dummies, and the share of employment in manufacturing in 1980.

Table S4: Robustness checks: reallocation away from high-rent jobs of exposed workers (II)

	Panel C. Dependent variable: reallocation to high rent jobs, measured by (minus) EE rates from CPS					
	Quit rates by industry, occupation, gender, and education	Common quit rate by industry and occupation	Common quit rate by industry and occupation (adjusted by demographics)	Quit rates by industry, broad occupation, gender, and education	Common quit rate by industry and broad occupation	Common quit rate by industry and broad occupation (adjusted by demographics)
	(1)	(2)	(3)	(4)	(5)	(6)
Direct task displacement	-1.506	-2.152	-1.849	-1.506	-1.645	-1.457
	(0.253)	(0.305)	(0.301)	(0.253)	(0.292)	(0.283)
R^2 for task displacement	0.68	0.66	0.58	0.68	0.65	0.60
Observations	500	498	498	500	500	500

	Panel D. Dependent variable: reallocation to high rent jobs, measured by (minus) voluntary EU rates from CPS					
	Quit rates by industry, occupation, gender, and education	Common quit rate by industry and occupation	Common quit rate by industry and occupation (adjusted by demographics)	Quit rates by industry, broad occupation, gender, and education	Common quit rate by industry and broad occupation	Common quit rate by industry and broad occupation (adjusted by demographics)
	(1)	(2)	(3)	(4)	(5)	(6)
Direct task displacement	-0.300	-0.382	-0.354	-0.300	-0.305	-0.282
	(0.050)	(0.061)	(0.057)	(0.050)	(0.048)	(0.044)
R^2 for task displacement	0.58	0.65	0.63	0.58	0.68	0.68
Observations	500	500	500	500	500	500

Notes: This table presents estimates of the reduced-form relationship between the direct task displacement due to new automation technology from 1980 to 2016 and the reallocation of exposed groups away from high-rent jobs. The sample includes 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is indicated in the panel headers. The different columns present results varying the measurement of rents across jobs. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroscedasticity are reported in parentheses. All specifications control for sectoral demand shifters, sectoral rent shifters, gender and education dummies, and the share of employment in manufacturing in 1980.

S3.4 Estimates of the propagation and rent impact matrices

The estimation assumes the parameterization of the task-share and rent Jacobians in Section 4.2. Setting $\varphi = \sigma - \lambda$, we can rewrite (19) and (20) as

$$\sigma \Delta \ln w_g + d \ln \Gamma_g^d = \tilde{\beta}_0 + \tilde{\beta} \cdot Z_g + \sum_{g' \neq g} \theta_{gg'} \cdot (\Delta \ln w_{g'} - d \ln w_g) + u_g$$

$$\Delta \ln \mu_g = \beta_0^\mu - \rho \cdot d \ln \Gamma_g^d + \beta^\mu \cdot Z_g^\mu + \sum_{g' \neq g} \theta_{gg'}^\mu \cdot (\Delta \ln w_{g'} - d \ln w_g) + e_g.$$

Here, $\tilde{\beta}_0 = \lambda \beta_0 + \varphi \cdot \sum_{g'} s_{g'} \cdot \Delta \ln w_g$, $\tilde{\beta} = \lambda \beta$ and the spillover terms are given by

$$\begin{aligned}
\theta_{gg'} = & \sum_n \cdot \frac{\ell_{gn}}{\ell_g} \cdot s_{ng'} \cdot \theta \\
& + \sum_n \frac{\ell_{gn}}{\ell_g} \cdot s_{ng'} \cdot \theta_{\text{job}} \cdot \text{job similarity}_{gg'} \\
& + \sum_n \frac{\ell_{gn}}{\ell_g} \cdot s_{ng'} \cdot \theta_{\text{edu-age}} \cdot \text{edu-age similarity}_{gg'} \\
\theta_{gg'}^\mu = & \sum_n \left(\frac{\bar{w}_{gn}}{\bar{w}_g} - 1 \right) \cdot \frac{\ell_{gn}}{\ell_g} \cdot s_{ng'} \cdot \theta \\
& + \sum_n \left(\frac{\bar{w}_{gn}}{\bar{w}_g} - 1 \right) \cdot \frac{\ell_{gn}}{\ell_g} \cdot s_{ng'} \cdot \theta_{\text{job}} \cdot \text{job similarity}_{gg'} \\
& + \sum_n \left(\frac{\bar{w}_{gn}}{\bar{w}_g} - 1 \right) \cdot \frac{\ell_{gn}}{\ell_g} \cdot s_{ng'} \cdot \theta_{\text{edu-age}} \cdot \text{edu-age similarity}_{gg'}
\end{aligned}$$

Table S5 reports estimates of ρ, θ, θ_{simj}, and $\theta_{\text{edu-age}}$ obtained from estimating this system of equations via GMM. Columns 1–3 allow for ripple effects separately along each of the dimensions considered. Column 4 reports our baseline estimates. Column 5 report estimates imposing the restriction θ, θ_{simj}, $\theta_{\text{edu-age}} \geq 0$. In all specifications, the covariates for the wage equation are: education and gender dummies, sectoral demand shifts, and the share of employment in manufacturing in 1980. The covariates for the rent equation are: education and gender dummies, sectoral rent shifts, and the share of employment in manufacturing in 1980.

Table S5: GMM estimates of the parametric task share and rent Jacobians.

	(1)	(2)	(3)	(4)	(5)
Rent dissipation	0.409	0.422	0.394	0.355	0.405
	(0.137)	(0.140)	(0.134)	(0.119)	(0.127)
Ripples, θ	1.536			-0.476	
	(0.647)			(1.076)	
Ripples, θ_{job}		2.986		1.861	2.301
		(1.390)		(1.686)	(1.677)
Ripples, $\theta_{\text{edu-age}}$			1.967	0.714	0.684
			(0.867)	(1.039)	(0.815)
Joint significance spillovers (p-value)				0.28	0.07
Observations	500	500	500	500	500
Covariates in wage equation (19):					
Education and gender, sectoral demand shifters, manufacturing	✓	✓	✓	✓	✓
Covariates in rent equation (20):					
Education and gender, sectoral rent shifters, manufacturing	✓	✓	✓	✓	✓

Notes: This table presents GMM estimates of equations (19) and (20). The estimation assumes the parameterization of the task-share and rent Jacobians in Section 4.2. Columns 1–3 allow for ripple effects separately along each of the dimensions considered. Column 4 reports our baseline estimates. Column 5 report estimates imposing the restriction θ, θ_{simj}, $\theta_{\text{edu-age}} \geq 0$. Standard errors robust against heteroscedasticity are given in parenthesis. The table also reports a test for the joint significance of ripples.