Information Acquisition, Voting and Trading*

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Abstract

We study acquisition of imperfect signals, voting and trading by shareholders. Opportunities to trade increase the value of information. As long as voting aggregates some information, share price will depend on voting and there are opportunities for signal jamming. This undermines the incentive to vote for the better policy. In equilibrium, both informed and uninformed shareholders extract informational rents when trading. We find that the equilibrium distortions to voting are severe enough that (i) the quality of governance is not increasing in the fraction of shareholders that acquire information and (ii) as the number of shareholders gets large, voting does worse than reliance on a single agent's signal.

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1 Introduction

A central argument for empowering shareholders in corporate decision-making rests on the idea that shareholders will obtain and vote based on relevant information.[1] Many, however, express the concern that the number of shareholders who acquire quality information is insufficient.[2] These concerns are consistent with extant theoretical work on voting. The value of information is mitigated by the likelihood that a shareholder's vote is pivotal, and even ignoring this, there is a natural public good problem as individual shareholders only internalize the impact of decisions on their shares – not all shares.[3] But this perspective assumes that the quality of collective choice through shareholder voting is increasing in the level of information acquisition by shareholders. While it is tempting to draw on intuitions from theories of voting in common values settings, recent work on strategic connections between voting and trading shows that there are strong equilibrium forces pushing against the use of information in voting when trading is also possible.[4] A more complete inquiry is warranted before using the objective of maximizing governance quality as a justification for institutional reforms that enhance shareholder engagement.

In order to flesh out the strategic connections between the level of information acquisition and the quality of firm governance, we develop a model in which a pool of shareholders is heterogeneously informed about the consequences of a firm decision in advance of opportunities to participate in governance and market transactions. A fraction of the shareholders obtain imperfect private signals about which policy is better for firm value, and the remaining shareholders do not learn anything about policy. We find that increasing the fraction of investors who are informed beyond a fairly low threshold harms the equilibrium quality of governance, and the quality of

[1] See for example Hansmann (1988), Jensen and Meckling (1976), Fama and Jensen (1983).

[2] Concerns about the number of investors who passively follow proxy advisors' recommendations to vote have not escaped regulators or scholars. For example, Iliev and Lowry (2015) find about one-quarter of mutual funds entirely vote with the recommendations of proxy advisors.

[3] Persico (2004) considers information acquisition in the standard common values problem with majority rule voting and characterizes the optimal way to handle these sources of inefficiency. Martinelli (2007) considers large population properties of acquisition and aggregation in a common values setting.

[4] Meirowitz and Pi (2022) show that in settings where all voters are partially informed and trading is possible, the level of information aggregation in voting is related to the opportunities to extract informational rents from the market. The quality of voting is quite low when there is noise in the market.

governance will be quite low in every equilibrium regardless of how many shareholders acquire information.

More precisely, if we consider the problem of selecting the fraction of shareholders that acquire information in order to maximize the equilibrium quality of firm governance, we find that the optimal fraction vanishes as the number of shareholders gets large. This is true even without accounting for the cost of acquiring information. Moreover, when the number of shareholders is large, the probability of making the correct choice in equilibrium is bounded by the signal quality available to any one of the shareholders.

The first strategic force to account for is a spillover between trading and voting. If voting aggregates any information and is observed by market participants, then prices may depend on voting choices, and shareholders will sometimes have incentives to distort their voting behavior (relative to sincere voting) in order to create an informational advantage over the market and then extract information rents when trading (Meirowitz and Pi, 2022).[5] Put another way, there is a Shareholder's Dilemma: strategic shareholders must balance the desire to steer policy decisions in ways that they think will enhance the value of the shares they hold when pivotal (this is termed the *Pivotal Effect*) and the desire to create opportunities to gain trading rents when the price is sensitive to what happens in the shareholder meeting (termed the *Signaling Effect*). Thus, the equilibrium level of information revealed in voting is suppressed to a level that balances these often competing effects. We show that as a result of this distortion, the quality of governance is dramatically lower than it would be if all of the information acquired by shareholders were used in voting.

The second strategic force to understand is how the distortion in voting caused by trading opportunity depends on the amount of information that shareholders acquire. It turns out the distortion gets worse as the fraction of informed shareholders increases. Suppose by contradiction that informed voters' strategies do not become less informative as the fraction of shareholders acquiring information increases to 1. There would be two effects: the share price would become increasingly sensitive to the vote tally, and the odds of being pivotal would continue to decrease. These effects would upset the equilibrium condition required for voting to be positively correlated

[5]Interestingly these opportunities to extract information rents apply to shareholders that do not acquire any private information about the firm choice as well as shareholders that do acquire information.

with private information, in which the Pivotal Effect must be at least as strong as the Signaling Effect. So, in equilibrium, as more shareholders buy information, each informed shareholder's vote must be less informative. As a consequence, the distortion in voting is minimized when the fraction of investors that acquire private information is fairly low.

The third strategic force is less subtle and pertains to incentives to acquire information. Not surprisingly, opportunities to extract trading rents increase the value of information above its relevance in selecting the correct policy. So opportunities to trade weakly increase the amount of information that is acquired for any fixed cost.

We study how these three effects balance out. Our main result is that governance is best if the level of information acquisition is just low enough so that everyone who acquires information is willing to vote sincerely in equilibrium. Higher levels of information acquisition lead to less informative voting and result in a lower probability of selecting the optimal policy. Thus, even if one does not account for the direct costs of acquiring information, commentators and regulators might want to rethink blanket statements about the need to encourage more information acquisition in this context.[6] Strikingly, the distortion to voting from these trading opportunities is so severe that with a large number of shareholders, even when the optimal number of investors acquire information, the equilibrium probability of making the correct choice is no better than it would be if the policy choice were delegated to a decision-maker that has received just one of the partially informative signals.

When it comes to thinking about the optimal information cost, an important distinction between our model and work on information acquisition and voting surfaces. Here, shareholders extract trading rents from liquidity in the market, and thus, even though the impact of one vote vanishes, the equilibrium value of information to a shareholder does not. A second novel feature of our model is that shareholders who choose not to acquire information are still able to extract informational rents from

[6] An example of this form of reasoning is a recent argument by SEC Commissioner Daniel M. Gallagher, who writes: *I have grave concerns as to whether investment advisers are indeed truly fulfilling their fiduciary duties when they rely on and follow recommendations from proxy advisory firms. Rote reliance by investment advisers on advice by proxy advisory firms in lieu of performing their own due diligence with respect to proxy votes hardly seems like an effective way of fulfilling their fiduciary duties and furthering their clients' interests. (Source: SEC Website, https://www.sec.gov/news/speech/2013-spch103013dmg)*

voting.[7] Uninformed voters know that their vote did not convey any novel information, whereas the market cannot determine which votes came from shareholders that acquired information and which came from shareholders that did not. Interestingly, because of this opportunity for uninformed shareholders to extract trading rents from their voting behavior, the uninformed voters must introduce noise to voting decisions in equilibrium. This hurts governance, but the uninformed shareholders are happy about doing this.

Extant work has documented two equilibrium channels that limit how much of an impact poorly informed agents have on voting outcomes. Feddersen and Pesendorfer (1997) show that in common values problems, the, suitably named, swing voter's curse drives poorly informed agents to abstain in order to ensure that the decision is made by the agents with more information. Persico (2004) shows that the optimal mechanism for information acquisition and voting will involve balanced sorting by the agents that don't collect information so that the decision is effectively delegated to the better-informed agents. Here, we find that because uninformed shareholders are able to extract informational rents from voting, these channels do not obtain. If the uninformed shareholders are supposed to sort to reduce the noise they create in voting outcomes, there is an incentive to vote opposite to their conjectured strategy and use the fact that the market cannot detect that they deviated. The market then forms an incorrect inference about how the informed voters behaved and the deviator extracts a rent by trading against her vote. The only way to kill this incentive is to maximize the noise introduced by each uninformed voter. On the other hand, if an uninformed shareholder is supposed to abstain, a similar incentive surfaces and she can extract rents from voting and distorting the price. The expected revenue from trading generated by this deviation exceeds the expected impact on the quality of the firm decision. The only way to kill this incentive is for the uninformed shareholders to be voting in equilibrium.

In comparing the quality of governance and average expected utility of shareholders with and without the opportunity to trade, we find that it is unambiguously better to not allow this form of trading. While less information acquisition might occur without trading opportunities, the probability that the correct policy is chosen

[7] This feature has connections with the strategic linkages between actions where an agent can recall that she has previously deviated. This arises in contexts with learning, such as Bhaskar and Nikita (2023) and Halac et al. (2016).

and average shareholder utility are still higher when trading is not possible. However, if the signal that shareholders can acquire is sufficiently informative, then shareholders who acquire information in the equilibrium when trading is allowed would be made worse off by a ban on this form of trading.

Because the strategic spillovers between voting and trading hinge on the ability of investors to use voting as a signal, we consider what happens if transparency in governance is reduced. In a sufficiently opaque setting where the market only observes the policy choice and not the precise voting mandate, much of the friction described above is still present. In this modified setting, the price cannot reflect the precise vote tally, but there are still opportunities to extract informational rents after voting. A shareholder who knows her signal (or that she did not get a signal) and remembers how she voted will form a different assessment of the value of a share than the market maker who only observes which policy obtains majority support. Here, conditional on being pivotal, the preference to vote sincerely is based on the quality of one's own private signal. The benefits of voting against this signal and then trading on this information correspond to the price of a share after the decision is made. In any equilibrium, this price must correspond to the probability that the correct decision is made. Because, in equilibria, the expected payoff from voting for one's signal is weakly higher than the expected payoff from voting against one's signal, the probability of making the correct decision cannot exceed the quality of an agent's own private signal. The logic behind the conclusion that governance quality is low in the institution where the vote tally is not public is easier to get an analytic handle on. For this reason, we establish this result at a level of generality beyond the results from our analysis of the main model. When the distribution of signal qualities and ownership exhibits heterogeneity, the equilibrium probability of selecting the correct policy is no higher than the signal quality of the least informed investor whose equilibrium voting strategy is positively related to her private information.

1.1 Closely Related Literature

A now large literature on voting in common-values problems with asymmetric information builds off the insight that when the voting rule is not well-calibrated to the informational environment, equilibrium behavior may not do a good job aggregating the information held by the voting population (Austen-Smith and Banks, 1996, Feddersen and Pesendorfer, 1997). Bayesian Nash equilibria to voting games typically require that agents are updating not just to their private information but to their private information and a profile (or profiles) of information held by others that would make the agent pivotal given equilibrium strategies.[8] This paper bridges two extensions to this strand of work. The first is work that looks at information acquisition in advance of committee voting. Persico (2004) focuses on characterizing the best equilibrium given a fixed voting rule. Martinelli (2007) focuses on results when the number of voters is large, and Gerardi and Yariv (2008) study the choice of optimal institutions in the voting context.[9]

Focusing on information acquisition in advance of shareholder meetings, Malenko and Malenko (2019) examine the case of a fixed voting rule and show that the sale of a low-quality common signal to all subscribers by a monopolist proxy-advisor can crowd out the acquisition of costlier but conditionally independent private signals. In their model, this hurts governance. The second extension to work on common-values voting is a recent paper that takes the distribution of private information as fixed and homogeneous and adds a trading stage to the classic voting problem (Meirowitz and Pi, 2022). The main finding is that even when the voting rule is well-calibrated to the informational environment, opportunities to extract trading rents from a market create incentive problems in the voting stage, and in equilibrium, voting is not very informative. How this problem is impacted by heterogeneity in shareholder information and how these spillovers between voting and trading impact incentives to acquire information are not studied in these papers.

It is useful to contrast the tension in our model with that in extant work on common values voting where the key friction arises because the voting rule is not well-calibrated to the informational environment. See for example Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1998), Duggan and Martinelli (2001). A common feature of the equilibria to these models is that the level of informativeness of voting will depend on the realized private information that players obtain in ways that offset this mis-calibration of the voting rule and environment. McLennan (1998) shows that this problem is often overcome by optimal equilibria to common values

[8] A large literature builds off this work. Bond and Eraslan (2010), for example consider strategic proposing and Bar-Isaac and Shapiro (2020) examine block holder voting.

[9] Bergemann and Välimäki (2002) study the general mechanism design problem. Both papers establish the tension between efficient acquisition and efficient use of the information in common values settings.

problems. Even with natural restrictions, to say, type symmetric strategies, this may, but doesn't always lead to aggregation problems in the limit as shown in Bhattacharya (2013), Mandler (2012), Duggan and Martinelli (2001), Meirowitz (2002), Acharya and Meirowitz (2017).[10]

Our model is set up to side-step this voting rule calibration issue in order to study a different incentive problem. Because the direct value of a vote hinges on the odds that it is pivotal or decisive, it is not difficult for other considerations to distort voting incentives. We highlight signaling incentives and opportunities to maximize informational advantages from the leakage of information from firm governance to a market. More generally, one may imagine that in many domains where policy choices are based on votes, those empowered to vote have both narrow incentives to influence the policy and broader incentives to influence how observers react to the governance choices. A small number of papers in voting theory and auction theory also exhibit this intuition, but they do not focus on the heterogeneity of information quality or incentives to acquire information; additionally, the signaling incentives are not derived from market participation. Razin (2003) studies voting when the mandate impacts future policy-making by the winner of the election. In a spatial setting, Meirowitz and Shotts (2009) consider a model where the vote count in one period will be used by strategic candidates to forecast the location of the median voter and thus impact candidate platforms in a second election. Atakan and Ekmekci (2014) consider an auction problem where the value of winning depends on post-auction investment, and these investments can depend on information revealed from the auction.

Given the strategic similarities between problems of voting with common values and auctions with interdependent values, it is useful to contrast our results with those on information acquisition in auctions with interdependent values. Bergemann et al. (2009) show that in a generalized Vickrey-Clarke Groves mechanism equilibrium information acquisition exceeds the socially optimal level. In the voting context without trade, the opposite is generally true for the reasons discussed above. In our setting, where most of the rents from information acquisition typically stem from

[10]As seen in Meirowitz and Pi (2022), the limiting forces at work in a model with voting and trading are different. They cause the probability that a vote is informative to converge to $\frac{1}{2}$ at a rate that makes the probability of selecting the correct choice converge to something other than 0 or 1. While this is not typically a feature in extant models of voting in common values with a single election, it is also a key feature in Ahn and Oliveros (2012) where multiple policy decisions are being made.

trading, the flavor of the auction result is restored. With voting and trading, the level of information acquired can be too high given the voting strategies employed. For some levels of information cost, a reduction in information acquisition from the equilibrium level would improve governance quality.

The remainder of the paper is organized as follows. We present the model of information acquisition, voting and trading. We first treat the level of information acquisition as a parameter. This allows one to apply our results to a broader range of questions about the impact of institutional changes that encourage or discourage information acquisition. We then provide a simple but natural extension that endogenizes the level of information acquisition as coming from a binary information acquisition decision by each shareholder at the beginning of the game. A few extensions exploring the robustness of our main results are explored. In particular, we examine whether coordination by uninformed shareholders or abstention can serve to reduce the impact of less informed agents and improve the equilibrium quality of governance; they cannot. We then present results comparing welfare with and without trading and with and without transparency in governance. Proofs of all results not proven in the body appear in the appendix.

2 Model

We consider a firm that has n (an odd number) of shareholders, and each of them has an endowment of 1 share.[11] Shareholders participate in governance by voting, and then they are free to participate in a market by trading shares. We initially assume that a fraction $\frac{k}{n}$ of the shareholders receive private signals prior to the vote. We then explore a natural way to endogenize the acquisition decision in section 3.4.[12]

[11] Assuming that each agent has one vote and one share is largely for analytic convenience. What is important is that there is no controlling shareholder. One can think of shareholders as blockholders, especially in examples with small n. For example, when $n = 21$, one share represents approximately 5% ownership of the firm. This reflects the reality that in a well-diversified market, a firm typically has many minor blockholders but no controlling shareholder. In the sequel, we consider an extension with heterogeneous share endowments. See also, Meirowitz and Pi (2022) where asymmetric examples with blocks are considered.

[12] We choose to present results with k exogenous at first for two reasons. First, these results can be more easily applied to settings where information acquisition is driven by factors not captured in the model. Second, discussions among pundits and regulators on how the presence of information impacts governance are more prevalent than discussions on how information costs impact governance. So, we highlight the results that don't hinge on equilibrium conditions from the acquisition decision before turning to the analysis that involves equilibrium conditions from the acquisition choice.

Governance consists of making a decision $x \in \{0,1\}$. The shareholders face uncertainty about which decision is better for the firm. We denote the underlying state by $\omega \in \{0,1\}$, with the interpretation that if $x = \omega$, each share has value 1, and if $x \neq \omega$, each share has value 0. The common prior is that $Pr(\omega = 1) = Pr(\omega = 0) = \frac{1}{2}$.

At the beginning of the game, k of the shareholders receive an imperfect private signal $s_i \in \{0,1\}$ about the underlying state, ω. We assume $Pr(s_i = \omega|\omega) = q > \frac{1}{2}$ and that these signals are conditionally independent. If a shareholder does not receive a signal, we denote her information set with $s_i = \emptyset$. It is convenient to record whether i has received a signal by a_i, with $a_i = 1$ corresponding to receiving a signal and $a_i = 0$ corresponding to not obtaining a signal.

The timing of the game is as follows. In the first period, shareholders cast ballots $v_i \in \{0,1\}$. The publicly available vote tally is denoted by $t = \sum_{i=1}^{n} v_i$. Whichever policy receives more votes is selected,

$$x = \begin{cases} 1 \text{ if } t \geq \frac{n+1}{2} \\ 0 \text{ otherwise.} \end{cases} \tag{1}$$

Note that simple majority rule is well calibrated to this symmetric informational environment, and so without a market, sincere voting $v_i(s_i) = s_i$ can be supported in equilibrium. It is convenient to also describe the tally from shareholders other than i, denoted $t' = \sum_{j \in n - \{i\}} v_j$.

In the second period, after observing the policy x and the vote tally t, a market maker sets a common price $P_x(t)$ for a share. Each trader submits an order $b_i \in \{-1, 0, 1\}$ with the interpretation that $b_i = -1$ denotes selling their share, $b_i = 0$ denotes holding, and $b_i = 1$ denotes buying an additional share. Trades are executed at this common price $P_x(t)$. We assume the price is set to satisfy a standard no-arbitrage condition,

$$P_x(t) = E[1_{x=\omega}|t] = Pr(\omega = x|x, t) \tag{2}$$

where the expectation is taken over a version of the conditional probability that is based on a correct conjecture of the voting strategies (and thus the induced joint distribution of (t, s, ω)). The second equation above is written because we find it convenient to sometimes write $Pr(\omega = x|x, t)$. Note that because shareholders can compute the price based on public information, it does not matter whether we assume that the price is posted before or after orders are submitted.[13] Meirowitz and Pi (2022) extend their model to allow trading orders to impact prices in the presence of noise traders and show that the logic from the posted price model carries over. In particular, as the number of noise traders in the extended model goes to infinity, that model converges to the one with posted prices. As an alternative demonstration of robustness, Meirowitz and Pi (2024) enrich the market structure by following Glosten and Milgrom (1985) so that prices also depend on trading. In particular, a market maker sets bid-ask prices based on trading directions and voting results.

Finally, the state is observed, and the value of the share is realized. One interpretation is that the firm provides a one-time dividend of either 1 or 0 for each share, and the game ends. Thus, at the end of the game, the value of each share is given by

$$v(x,\omega) = \begin{cases} 1 \text{ if } \omega = x \\ 0 \text{ otherwise} \end{cases} \tag{3}$$

and an agent that bought a share obtains payoff $2v(x,\omega) - P_x(t)$, an agent that sold a share receives payoff $P_x(t)$ and an agent that made no trades obtains payoff $v(x,\omega)$.

3 Equilibrium Analysis

Votes may reveal information. They should then impact inferences the market maker draws about the state and influence the price she sets. To capture this possibility, we seek Perfect Bayesian Equilibria. In the information acquisition period, k shareholders acquire private information, while the remaining $n-k$ shareholders do not.[14] We focus on equilibria with type-symmetric voting strategies, in which, during the voting period, each informed shareholder votes for her private signal with probability $m := Pr(v_i = s_i|s_i) \in [\frac{1}{2}, 1]$ for $s_i \in \{0,1\}$. In such an equilibrium, the probability that an informed shareholder's vote is correct conditional on the underlying state is $z := Pr(v_i^I = \omega|\omega) = mq+(1-m)(1-q)$. v_i^I denotes a vote cast by an informed shareholder.

[13]To focus incentives on how information acquisition and governance can depend on the anticipation of optimal trading, we do not explicitly include pre-voting trade. What matters is that at the time of voting, previous market transactions do not fully reveal the private information of the shareholders. The presence of noise traders is sufficient to ensure this feature.

[14]Recall that we first treat k as exogenous and then in the sequel consider a first stage where each shareholder chooses to buy a signal or not at cost, c. We provide bounds on costs to support particular values of k.

Additionally, we focus on equilibria in which the uninformed shareholders all employ the same strategy, voting for each policy with equal probability. We then show that it is not possible to support equilibria in which the uninformed voters use degenerate strategies to split their votes even though this kind of profile would prevent the uninformed voters from introducing noise and hurting governance. We also show that the derived equilibrium path can be supported if abstention is allowed.

We begin analyzing the model by considering sub-forms starting after a vote and revelation of the choice x and tally t.

3.1 Trading

The stock price in the trading period depends on the probability that voting selects the correct policy.[15] Given a selected policy, the price depends on the number of votes for the selected policy. For example, if $z > \frac{1}{2}$, then when $x = 1$, a larger voting tally t implies that more informed shareholders are receiving the signal of 1, and thus the stock price based on a larger t is higher than the stock price based on a smaller t.

The following lemma gives the pricing function as a function of x and t.

Lemma 1 (Stock Price After Voting)**.** *Given the conjectured voting strategy, the price after voting depends on the chosen policy x and voting tally t.*

$$P_x(t) = E[v(x, \omega)|x, t] = \begin{cases} Pr(\omega = 1|t), & \text{if } x = 1 \\ 1 - Pr(\omega = 1|t), & \text{if } x = 0 \end{cases} \tag{4}$$

where

$$Pr(\omega = 1|t) = \frac{Pr(t|\omega = 1)}{Pr(t|\omega = 1) + Pr(t|\omega = 0)}, \tag{5}$$

$$Pr(t|\omega = 1) = \sum_{i=0}^{t} \binom{k}{i} z^i (1-z)^{k-i} \binom{n-k}{t-i} \left(\frac{1}{2}\right)^{n-k}, \tag{6}$$

and

$$Pr(t|\omega = 0) = \sum_{i=0}^{t} \binom{k}{i} (1-z)^i z^{k-i} \binom{n-k}{t-i} \left(\frac{1}{2}\right)^{n-k}. \tag{7}$$

[15] To simplify the model without losing intuitions, we assume that the price does not depend on the number of trading orders. Meirowitz and Pi (2022) show that an informed shareholder's trade-off between voting for the policy she thinks is best and voting against it to maximize trading rents still holds when the pricing function also depends on net trading orders from shareholders and noise traders.

The result is proven in the appendix, but the key idea is that because k votes are correct with probability z and $n-k$ with probability $\frac{1}{2}$, the joint distribution involves a convolution.

In the trading stage, each shareholder buys (sells) one share if her expectation of firm value is higher (lower) than the price. Shareholders may form different beliefs about firm value after the vote tally t is revealed because they have acquired (not acquired) information, they have observed different signals, and they each know how they voted.

Lemma 2 (Trading Strategy). *Given the conjectured voting strategies and mapping $P_x(t)$, at the trading stage each uninformed shareholder buys one share if $x \neq v_i^{\emptyset}$ and sells one share if $x = v_i^{\emptyset}$. Every informed shareholder with $s_i \in \{0, 1\}$ buys one share if $x = s_i$ and sells one share if $x \neq s_i$.*

We use $v_i^{\emptyset}$ to denote the vote cast by an uninformed voter i. Recall that the market cannot determine which votes come from informed shareholders and which come from uninformed shareholders. The proof is in the appendix. Uninformed shareholders bet against their votes. Each uninformed shareholder compares her vote to the chosen policy and then buys (sells) if her vote is different from (the same as) the chosen policy. This is because an uninformed shareholder recognizes that the market will interpret her vote for 1 (0) as evidence in favor of 1 (0) with quality between z and $\frac{1}{2}$, while she knows the vote is based on a coin toss only.

Informed shareholders bet in line with their signal; an informed shareholder compares her private signal to the chosen policy and buys (sells) if her signal is the same as (different from) the chosen policy. This is because an informed shareholder recognizes that her signal is not fully reflected in market prices due to the fact that in equilibrium, voting may not be fully informative. The market takes her vote as having quality between z and $\frac{1}{2}$, but her signal has quality q with $z \leq q$.

3.2 Voting

The vote cast by a shareholder impacts her payoff in two ways. There is a direct effect as it may impact which alternative obtains a majority and is implemented. There is an indirect effect as the vote tally will impact $P_x(t)$, which will impact the shareholder's payoff if she trades. The former effect only happens if the shareholder's vote is pivotal, which occurs if and only if $t' = \frac{n-1}{2}$. The latter effect obtains at every realization of

t', but it has a different magnitude at each realization of t'. In selecting an optimal voting strategy given s_i, a shareholder integrates over the realizations of t' given their signal s_i and the equilibrium strategies being played by other shareholders. Given the symmetry in the informational environment, we can characterize strategies in two steps. First, for the uninformed shareholders, the payoff to either pure strategy is the same if the informed shareholders are employing type symmetric strategies characterized by a mixture m. Thus, from now on, for the uninformed, we focus on the strategy in which they vote for either alternative with probability $\frac{1}{2}$. We show below that other strategies cannot be supported; if one vote (say 1) were more likely for an uninformed shareholder then a deviation (to 0) would be profitable for her. For the informed shareholders, we compare the expected payoff to voting with one's signal vs against one's signal. For convenience, we take the case of a signal of 1. The following intermediate result provides traction on this decision.

Lemma 3 (Signaling Effect and Pivotal Effect)**.** *Take as given the behavior characterized above. Consider an informed shareholder with $s_i = 1$. Her payoffs from voting depend on what we call the Signaling Effect and the Pivotal Effect. In particular,*

$$\begin{aligned} & EU(v_i^1 = 1) - EU(v_i^0 = 0) \\ & = \underbrace{Pr_I(t' = \frac{n-1}{2}|s_i = 1)(2Pr_I(\omega = 1|t' = \frac{n-1}{2}, s_i = 1) - 1)}_{\text{Pivotal Effect}} \\ & - \underbrace{\sum_{t'=0}^{n-1} Pr_I(t'|s_i = 1)(Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t'))}_{\text{Signaling Effect}} \end{aligned} \tag{8}$$

We let v_i^1 (v_i^0) denote a vote based on signal $s_i = 1$ ($s_i = 0$). Pr_I highlights the probability is viewed from the perspective of an informed shareholder. The proof is in the appendix. Since the first term measures the gains from voting for one's signal over voting against one's signal when an informed shareholder i is pivotal, we call it the *"Pivotal Effect"*. The second term sums over realizations of t', the gains from voting against one's signal over voting for one's signal impacting the price and trading to capitalize on this informational advantage. Thus, we call the second term the *"Signaling Effect"*.

In interpreting these expressions, it is useful to recall that for any integer t' between 0 and $n-1$, the conditional probability is the result of a convolution of two random variables, one that corresponds to a vote for the correct or desired policy with probability z and the other that is correct with probability $\frac{1}{2}$. See equations (6) and (7) above.

The pivotal effect is well understood as it drives equilibrium results in extant work on voting in common values problems. Perhaps the most important property for our purposes is that its convergence to 0 is very fast, and this rate is slowed when z approaches $\frac{1}{2}$. The *Signaling Effect* is less well understood as it involves summing over the possible effects that one vote can have on the Bayesian posterior employed by the market maker for all the possible realizations of t.

An informed shareholder's voting strategy depends on which effect dominates. If *Pivotal Effect(z)* $\geq$ *Signaling Effect(z)* when $z = q$, then voting sincerely is a best response for an informed shareholder when the other informed shareholders vote sincerely. However, if *Pivotal Effect(z)* $<$ *Signaling Effect(z)* when $z = q$, sincere voting cannot sustain an equilibrium. Informed shareholders must adopt a mixed voting strategy in equilibrium with $m \in [\frac{1}{2}, 1)$ and $z < q$. Recall that $m := Pr(v_i = s_i|s_i)$ for $s_i \in \{0, 1\}$ is the probability that an informed shareholder votes for her signal, and $z := mq + (1 - m)(1 - q)$ denotes the probability that an informed shareholder's vote is correct. In particular, the equilibrium z is determined by the following indifference condition.

$$\underbrace{\sum_{t'=0}^{n-1} Pr_I(t'|s_i = 1)(Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t'))}_{\text{Signaling Effect}}$$
$$= \underbrace{Pr_I(t' = \frac{n-1}{2}|s_i = 1)(2Pr_I(\omega = 1|t' = \frac{n-1}{2}, s_i = 1) - 1)}_{\text{Pivotal Effect}} \tag{9}$$

Obviously, both the *Signaling Effect* and *Pivotal Effect* are a function of the number of informed shareholders, k. Thus, in equilibrium, k, affects the strength of the *Pivotal Effect* and *Signaling Effect* and thus influences the value of z that satisfies the informed shareholders' equilibrium condition.

We first show the existence of the equilibrium.

Lemma 4 (Existence of the Equilibrium)**.** *Fix k, n, q. For a value of $m \in [\frac{1}{2}, 1]$, there is an equilibrium in which the uninformed shareholders mix, voting for each alternative with equal probability, and informed shareholders vote their signal with probability m.*

The proof and characterization of m and the corresponding value of z are in the appendix. From this characterization, we obtain the following.

Proposition 1 (Equilibrium for Exogenous k)**.** *Fix $q \in (\frac{1}{2}, 1)$,*

1. *Fix n, k. There is an equilibrium that can be described by comparing $\frac{k}{n}$ to a critical threshold $\kappa(n)$ that is characterized in the appendix. In this equilibrium, each uninformed voter randomizes voting for each alternative with probability $\frac{1}{2}$. The voting strategies of the informed shareholders are as follows. If $\frac{k}{n} \leq \kappa(n)$, then each informed shareholder sincerely votes for her signal. If $\frac{k}{n} > \kappa(n)$, each informed shareholder uses a non-degenerate mixed voting strategy characterized in the appendix.*

2. *The critical threshold described above converges to the function (of n):*

$$\lim_{n\to\infty} \kappa(n) = \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}\,erf\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)} \tag{10}$$

3. *For any sequence of games with $\frac{k}{n} \geq \kappa(n)$ eventually, the limiting probability that each informed shareholder's vote is correct converges to the following function of k and n:*

$$z(n,k) = Pr(v_i^I = \omega|\omega) = \frac{1}{2} + \frac{n(2q-1)}{\sqrt{2\pi}k\sqrt{n-1}\,erf\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)} \tag{11}$$

which is decreasing in k. v_i^I denotes a vote cast by an informed shareholder.

The proof is in the appendix. One takeaway is that in order to ensure that informed shareholders vote sincerely, we should avoid having too many informed shareholders ($\frac{k}{n} \leq \kappa(n)$). If the proportion of informed shareholders becomes too high ($\frac{k}{n} > \kappa(n)$), informed shareholders will mix, sometimes voting against their signals. As more shareholders acquire information, the corresponding equilibrium probability that an informed shareholder votes her signal falls, approaching $\frac{1}{2}$. To better understand the logic, it is helpful to consider the following two scenarios.

In the first scenario, we have a small number of informed shareholders and a large number of uninformed shareholders. Since all the uninformed shareholders vote for each alternative with equal probability, the likelihood of a vote being pivotal is relatively high, resulting in a large *Pivotal Effect*. However, because there are only a few informed voters, the probability that a vote is based on an informed shareholder's signal is low, making the price less sensitive to a vote, and hence the *Signaling Effect* is small. In essence, when only a few shareholders acquire information, the *Pivotal Effect* is substantial, while the *Signaling Effect* is limited. Recall that when the *Pivotal Effect* is larger than the *Signaling Effect* when $z = q$, informed shareholders prefer to vote sincerely in equilibrium.

Now, consider the second scenario in which there are many informed shareholders and only a few uninformed shareholders. Suppose, by contradiction, that z does not get smaller as we add more informed shareholders. Then, as $\frac{k}{n}$ increases, because each additional informed voter is not flipping a fair coin while the uninformed voters are and the odds of being pivotal are higher when voters are voting for each alternative with equal probability, the increases of $\frac{k}{n}$ causes the *Pivotal Effect* to diminish. Simultaneously, because informed shareholders cast votes that are correlated with their private information (and thus the state), the price becomes more responsive to t, leading to a larger *Signaling Effect*. In short, as $\frac{k}{n}$ increases, the *Pivotal Effect* shrinks while the *Signaling Effect* grows for any fixed level of z. To maintain a balance between these effects, the probability that each informed shareholder votes her signal must decrease ($z(n,k) \to \frac{1}{2}$) in equilibrium as $\frac{k}{n} \to 1$.

3.3 Information Aggregation

We have observed that in equilibrium, informed shareholders' voting strategies depend on the fraction of informed shareholders. When $\frac{k}{n} \leq \kappa(n)$, informed shareholders vote sincerely, and adding an additional informed shareholder is clearly good for information aggregation; they bring an additional informative signal and vote based on it. However, if $\frac{k}{n} > \kappa(n)$, each informed shareholder votes strategically, and adding an additional informed shareholder decreases the probability that each of the other shareholder's votes matches their private signal. Thus, the increase in the fraction of informed shareholders brings both benefits and costs.

For fixed n, k, q, the expressions that go into $Pr(x = \omega)$ are quite intractable. In Section 6, we consider an extension where t is not public, and there we obtain much cleaner results about information aggregation for arbitrary n. Here, we rely largely on the asymptotic characterization above to characterize the limiting value of $Pr(x = \omega)$

for a sequence of games in which $\frac{k}{n}$ converges to the sequence $\kappa(n)$. In other words, we study the maximal probability of making the correct decision when n is large.

Proposition 2 (Information Aggregation)**.** *Fix $q \in (\frac{1}{2}, 1)$.*

1. *Fix n, $Pr(x = \omega)$ is not monotone on k. When $\frac{k}{n} \leq \kappa(n)$, $Pr(x = \omega)$ increases with k. However, when $\frac{k}{n} > \kappa(n)$, $Pr(x = \omega)$ decreases with k.*

2. *The fraction of informed shareholders that maximizes $Pr(x = \omega)$ is asymptotically given by $\kappa(n)$. Furthermore, in the limit the maximal $Pr^*(x = \omega)$ obtained at $\frac{k}{n} = \kappa(n)$ is*
$$Pr^*(x = \omega) = \Phi\left(\sqrt{\frac{2}{\pi}}(2q - 1)\right), \tag{12}$$
which is strictly smaller than q. In other words, in the limit, $Pr(x = \omega) < q$.

The proof appears in the appendix. Proposition 2 allows us to compare information aggregation in three situations: when everyone acquires a signal, when the optimal fraction of shareholders acquires information, and when the decision is made based on just one signal (e.g., a proxy advisor's public voting recommendation). To show how our result might undermine the regulators' concern that a proxy advisor hurts governance by suppressing information acquisition, we label the third case as everyone passively following an advisor's recommendation. Note here that this decision by one signal (the advisor's recommendation) outperforms equilibrium voting when even the optimal number of signals is acquired. This is because even in the case of the best value of k, the uninformed voters are adding noise to the vote. When the decision is made by exactly one signal, no such noise is present.

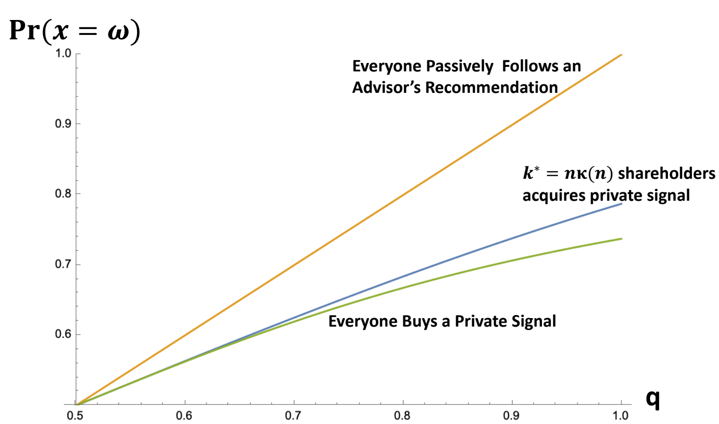


Figure 1: Probability that Voting Selects The Correct Policy Under Three Situations

Figure 1 illustrates that information aggregation is better when everyone votes for the proxy advisor's recommendation than when the optimal fraction of shareholders acquires information (or when everyone acquires a signal). This is because when everyone votes for the proxy advisor's recommendation, there is no distortion in voting strategy caused by trading opportunities. Note that this statement remains robust even if the proxy advisor's signal is less accurate than a private signal. In other words, even if the proxy advisor's recommendation is biased, passively voting for the proxy advisor's recommendation can still lead to more information aggregation than acquiring private signals to vote. The existing literature focuses on the downside of the potential bias in proxy advisors' voting recommendations. However, our results imply that the distortion in voting caused by the post-voting liquidity appears to be a more significant concern.

3.4 Information Acquisition

In this section, we endogenize the number of informed shareholders. Then, we characterize the optimal information cost that can sustain an equilibrium with the optimal level of information acquisition. Our results are most precise when we employ asymptotic methods.

While heterogeneous information acquisition strategies can emerge in a variety of settings, one parsimonious model involves adding an initial stage where each shareholder can choose to acquire the private signal at cost c. Supporting an equilibrium to the larger game in which k shareholders obtain signals in pure strategies and $n-k$ do not buy signals in pure strategies involves characterizing a pair of conditions on c to support investment in acquiring information by k and only k shareholders. In particular, the cost of acquiring a signal must satisfy two conditions: (1) None of the k informed shareholders wants to deviate by becoming uninformed (we denote the payoff to this unilateral deviation by $EU(a_i = 1 \xrightarrow{\text{D}} 0)$). (2) None of the $n-k$ uninformed shareholders want to deviate by becoming informed (the payoff to this deviation is denoted $EU(a_i = 0 \xrightarrow{\text{D}} 1)$). In working through the calculations, one key feature that surfaces is that a shareholder can realize some value from their private signal for any realization of t. That is to say, information provides some benefits even if the shareholder is not pivotal. A second feature worth remembering is that uninformed shareholders are also extracting informational rents from trading, and thus their equilibrium payoff is not just $Pr(x = \omega)$.

Lemma 5 (Information Cost)**.** *Fix q, n, k. There is an equilibrium in which k shareholders buy information, $n - k$ shareholders do not buy information, and the voting and trading strategies are as described above if and only if the cost of information satisfies*

$$\underbrace{EU(a_i = 0 \xrightarrow{D} 1) - EU(a_i = 0)}_{\underline{c}(k)} \leq c \leq \underbrace{EU(a_i = 1) - EU(a_i = 1 \xrightarrow{D} 0)}_{\bar{c}(k)} \tag{13}$$

where the utility differences are defined in the appendix.

The proof is in the appendix. This lemma gives the minimum and maximum costs that can sustain an equilibrium in which exactly k informed shareholders buy information. Specifically, the cost cannot be cheaper than $\underline{c}(k)$; otherwise, an uninformed shareholder would deviate from the equilibrium and buy information. Similarly, the cost cannot be more expensive than $\bar{c}(k)$; otherwise, an informed shareholder would deviate from the equilibrium and not invest in acquiring information.

From the perspective of creating optimal incentives within the class of games analyzed thus far, it is natural to ask what cost maximizes the equilibrium probability that $x = \omega$. We denote this solution by $c^*(n)$. Maximizing $Pr(x = \omega)$ involved a cost that induces $\frac{k}{n} = \kappa(n)$.Note that here we are not looking at welfare maximization, as we do not account for the social cost of information acquisition, kc, and we also do not account for trading rents extracted by the shareholders. Our objective here is narrower: understanding what costs can maximize the quality of governance.

Proposition 3 (Optimal Information Cost)**.** *Fix q. When $n \to \infty$, the cost that can sustain the equilibrium that maximizes $Pr(x = \omega)$ is given by*

$$\lim_{n\to\infty} c^*(n) = (2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right), \tag{14}$$

which is monotonically increasing with q and bounded away from 0.

The proof appears in the appendix. Recall that as the number of shareholders increases, the probability of being pivotal vanishes very quickly. So, without the opportunities to extract trading rents, costs need to vanish quickly in order to support information acquisition by $k^*(n) = \kappa(n)n$ individuals. This is true because $k^*(n)$ goes to infinity (just rather slowly so that the ratio $\kappa(n)$ goes to 0). But because shareholders are extracting trading rents that don't vanish, cost does not need to vanish. This comparison highlights the fact that in this setting where trading and voting are possible, the first-order effects on payoffs are tied to trading, not voting.

Figure 2 plots an informed shareholder's and an uninformed shareholder's expected utility in the equilibrium with the optimal level of information acquisition. It is interesting to note that the uninformed investors are obtaining expected utility above $\frac{1}{2}$, which might be seen as a natural benchmark as it is the expected value of a share if an uninformed decision-maker is relied on to select policy. Second, the informed shareholders are obtaining an expected utility that exceeds 1 when signals are very informative. This captures the fact that they are extracting non-trivial information rents (from traders that are not explicitly modeled but are part of the natural source of liquidity in these kinds of market models.)

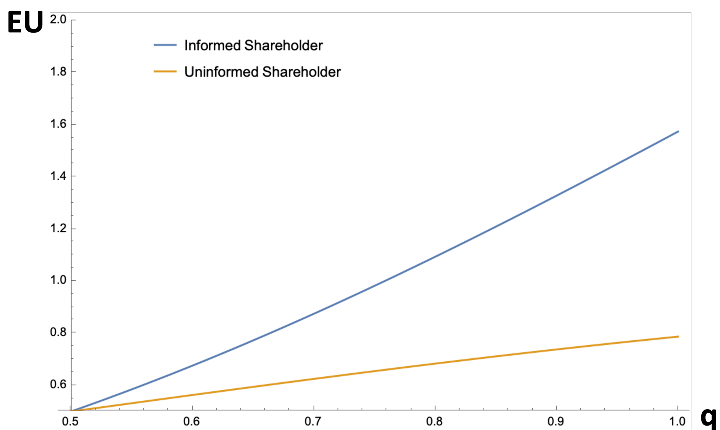


Figure 2: Expected Utility under the Optimal Level of Information Acquisition

4 Robustness

In this section, we examine three features of the equilibrium construction and model specification that may, at first, seem limiting. We rule out the possibility of equilibria where uninformed shareholders use asymmetric pure strategies to split their votes, establish that when abstention is allowed there are equilibria in which the path of play is identical to that characterized above, and demonstrate that relaxing the symmetry of the informational environment in a natural way does not affect our key substantive findings. We first check on the equilibrium selection given the game.

4.1 Deterministically Vote Splitting by Uninformed Voters

We now show that, in fact, the focus on equilibria in which uninformed shareholders vote in mixed strategies is natural. Of course, if we could support sorting by the uninformed voters, information aggregation might be better. But, in a profile in which the uninformed voters use degenerate voting strategies, there are strong incentives to deviate and extract larger informational rents. For convenience, let us assume that k is an odd number and is greater than 1. Suppose by contradiction that there exists an equilibrium in which half of the uninformed shareholders vote for 1, half of the uninformed shareholders vote for 0, and k informed shareholders take symmetric voting strategies (either mixed or pure).[16]

We first show that in this conjectured strategy profile, the uninformed shareholders do not obtain informational rents from the market.

Lemma 6. *In the conjectured profile, uninformed shareholders do not obtain informational rents: their expected payoff from trading is the same as their expected payoff from holding at every information set that is reached.*

The proof appears in the Online Appendix. It's critical to note that the voting strategies in the conjectured profile allow everyone, including informed shareholders, uninformed shareholders, and the market, to infer the number of informed shareholders who vote for policy 1. In particular, since half of the uninformed shareholders always vote for policy 1, everyone knows the number of informed shareholders voting for policy 1, which we denote with t_I. In particular, t_I can be inferred from the relationship $t_I = t - \frac{n-k}{2}$. This eliminates uninformed shareholders' informational rents from trading in the market. But, a unilateral deviation when voting (say from $v_i^\emptyset = 0$ to $v_i^\emptyset = 1$) by an uninformed shareholder leads the market to mistakenly infer the mix of private signals, (t is increased by one, and so the price reflects a belief that one more of the informed shareholders votes for 1). This observation can be used to show that a profitable deviation exists from the conjectured equilibrium profile.

[16] We can also prove a stronger version of the result. In particular, we can demonstrate that there is no equilibrium in which some uninformed shareholders randomly vote, while the rest of the uninformed shareholders deterministically split their votes, and the informed shareholders adopt symmetric voting strategies (either mixed or pure).

Proposition 4. *There are no equilibria in which uninformed shareholders deterministically split their votes.*

The proof is in the Online Appendix. To understand the intuition, let's consider an uninformed shareholder who is expected to vote for 0 in equilibria but deviates and votes for 1 instead. This deviation allows her to mislead the market into believing that there are t_I+1 informed shareholders voting for policy 1. When the chosen policy is 1, she wants to sell because she privately knows that there are only t_I informed shareholder votes for 1, and thus the market is overly optimistic about policy 1. On the other hand, when the chosen policy is 0, she wants to buy because she is slightly more optimistic than the market about policy 0. In the rare case where she happens to be pivotal, the deviation results in the same payoff as in the conjectured equilibrium. In short, the deviation enables the uninformed shareholder to sell the firm when it is over-priced and buy it when it is under-priced. So, the uninformed shareholder can achieve a higher expected payoff by deviating than by remaining in the conjectured equilibrium. Therefore, the conjectured equilibrium with uninformed voters splitting their votes is not an equilibrium. We now consider alternative game forms.

4.2 Allowing Abstention

In this section, we show that the equilibria of the main model mirror an equilibrium to the game when abstention from voting is allowed. In particular, consider the main model but make two changes. We now expand the set of possible voting actions to include abstention, and we define how a tie is resolved. When voting, a shareholder can select from the set $\{0, \emptyset, 1\}$ where $v_i = \emptyset$ is interpreted as an abstention. The outcome is determined by plurality rule from the 0 and 1 votes (with a tie resolved in favor of policy 0). We allow the market to observe the number of votes and the tally t.[17] In order to support an equilibrium with no abstention, we need to specify beliefs at histories in which exactly one voter abstains. We show that one way to do this, which supports the equilibrium, involves the following specification of off-the-path beliefs: when the market observes abstention, they interpret the abstention as the act of a voter that was supposed to vote for 0 in equilibrium. Of course, given the symmetry of the model, if ties are resolved in favor of policy 1, then one can construct

[17] A natural way to think about this is to assume that the market observes the number of votes cast for each alternative and recall that the number of players is common knowledge.

equilibria with supporting beliefs that attribute abstention as the act of someone who was supposed to vote for 1.

Proposition 5. *The equilibrium of the main model characterized above can be augmented to be an equilibrium of the extended model in which abstention is allowed by using the same voting and trading strategies and modifying beliefs in the following way: If one shareholder chooses to abstain and the tally of the* $n-1$ *votes that are cast has* t *votes for 1, the market maker and other agent's beliefs put probability one on the event where* t *out of* n *shareholders were supposed to vote for policy 1. So, the price in the off-equilibrium event is* $P_x(abstention, t) = E[v(x, \omega)|abstention, t, x] = Pr(x = \omega|t)$*, where the last term is exactly the formula from the equilibrium in the baseline model.*

The proof is provided in the Online Appendix, but the key is that with these beliefs, abstention does not create any opportunities to extract additional rents, and it does reduce the opportunity to extract rents present in the current equilibrium. Since the market cannot determine if an abstaining shareholder is informed or uninformed, it interprets the event where t out of $n-1$ shareholders vote for policy 1 and 1 shareholder abstains as occurring from the same realization of private signals that induces the event where t out of n shareholders vote for policy 1 in the equilibrium to the model where abstention is not allowed. This off-path belief renders the deviation unprofitable for both informed and uninformed shareholders.

To better understand this, consider an informed shareholder with $s_i = 0$. For any realization of t, if she chooses to abstain, given the market's off-path belief, the voting results and the price will be the same as if she chooses to vote for policy 0 in equilibrium. Recall that in equilibrium, the informed shareholder's payoff must be equal to $EU(v_i^0 = 0)$. Therefore, the deviation cannot yield a higher payoff than following the equilibrium strategy.

Similarly, consider an uninformed shareholder choosing to abstain. Abstaining will yield the same payoff as voting for 0 in equilibrium. Remember that, in equilibrium, $EU(v_i^\emptyset = 0) = EU(v_i^\emptyset = 1)$. Thus, abstaining is also not profitable for an uninformed shareholder.

Although full characterization of the set of equilibria to the game with abstention is beyond our focus, we can show that type symmetric equilibria in which all informed shareholders vote for the better policy with probability z and all uninformed shareholders abstain cannot be supported.[18]

Proposition 6. *Consider the extension in which abstention is allowed. There is no equilibrium where all uninformed voters abstain from voting ($v_i^{\emptyset} = \emptyset$) and informed shareholders use type symmetric strategies that are correlated with their signal ($z > \frac{1}{2}$).*

The proof is in the Online Appendix. If the $n-k$ shareholders that do not acquire information are supposed to abstain and the k informed shareholders vote, then the uninformed shareholders do not have an informational advantage and do not extract any trading rents on the conjectured equilibrium path. We show that if one of the uninformed shareholders were to deviate and vote, she would get an informational advantage. In particular, she privately knows which policy she votes for. However, the market maker, who observes the voting tally t cast by $k+1$ shareholders, cannot tell how the deviating uninformed shareholder votes unless $t = 0$ or $t = k+1$. Then, she buys if the firm is under-priced and sells if the firm is over-priced. As a result, the deviation from abstaining profitable.

4.3 Asymmetric Information Environment

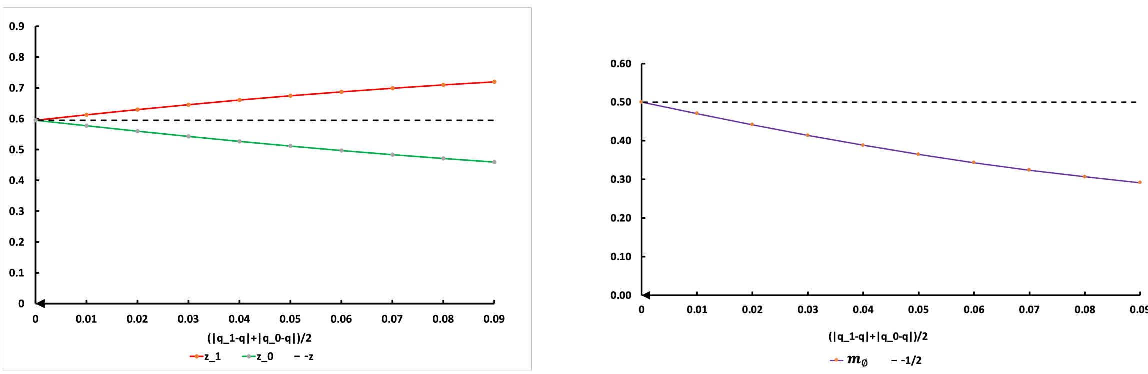
We now show that the symmetry in the informational environment is not essential. Fix n, k and q. In this section, we assume that the information quality of a signal differs across states. In particular, $Pr(s_i = 1|\omega = 1) = q_1$ and $Pr(s_i = 0|\omega = 0) = q_0$, and $q_1 \neq q_0$. The voting strategies conditional on $s_i = 1$, $s_i = 0$, and $s_i = \emptyset$ are denoted with $m_1 =: Pr(v_i^1 = 1)$, $m_0 =: Pr(v_i^0 = 0)$, and $m_{\emptyset} =: Pr(v_i^{\emptyset} = 1)$. Accordingly, we have $z_1 =: Pr(v_i^I = 1|\omega = 1) = q_1 m_1 + (1-q_1)(1-m_0)$ and $z_0 =: Pr(v_i^I = 0|\omega = 0) = q_0 m_0 + (1-q_0)(1-m_1)$. These equilibrium quantities all depend on (q_0, q_1), but for notational ease, we sometimes suppress this dependence. When needed we write $z_0(q_0, q_1)$, $z_1(q_0, q_1)$, and $m_{\emptyset}(q_0, q_1)$.

We consider a sequence of games with q_1, q_0 both converging to q. Let z denote the equilibrium level from the symmetric game with signal quality q. We show that in a sequence of equilibria to this sequence of asymmetric games, z_1 and z_0 converge to z and $m_{\emptyset}$ converges to $\frac{1}{2}$.

[18] We can also prove a stronger version of the result. Specifically, we can demonstrate that there exists no equilibrium where some uninformed shareholders vote randomly while the other uninformed shareholders abstain, and the informed shareholders adopt symmetric voting strategies (either mixed or pure).

Proposition 7. *Fix n, k and q. Consider any sequence of asymmetric games with q_1 and q_0 both converging to q. There exists a δ s.t if max$\{|q_0 - q|, |q_1 - q|\} < \delta$, then there is a type symmetric equilibria to the game which we can describe with equilibrium quantities $z_j(q_0, q_1)$ and $m_\emptyset(q_0, q_1)$. Moreover, $lim_{q_0,q_1 \to q} z_j(q_0, q_1) = z$ for $j = 0, 1$ and $lim_{q_0,q_1 \to q} m_\emptyset(q_0, q_1) = \frac{1}{2}$*

The proof is in the Online Appendix. We illustrate the result with a numeric example. Suppose that $n = 9$, $k = 7$, $q = 0.75$. Figure 3 shows that as $(|q_1 - q| + |q_0 - q|)/2 \to 0$, the equilibrium quantities converge to those for the symmetric game.



(a) z_0 and z_1 converge to z

(b) $m_\emptyset$ converges to $\frac{1}{2}$

Figure 3: As $q_1 \to q$ and $q_0 \to q$, equilibria converge to equilibria of main model

5 Institutional Comparison 1: Is the opportunity to trade good ?

Does the ability to trade improve the equilibrium quality of governance? We know that the possibility of extracting trading rents may increase information acquisition for a fixed cost. However, the opportunity to trade lowers the quality of information aggregation. Following the logic in Persico (2004), we see that in a setting without the opportunity to trade, an optimal equilibrium will have the following form: A fraction $\frac{k}{n}$ of the shareholder acquire a private signal, and they each vote informatively, $v_i^{s_i} = s_i$ for $s_i \in \{0, 1\}$. The remaining $n - k$ shareholders do not buy signals, and then they coordinate to split their votes so as not to influence the outcome. When $n-k$ is even, exactly half of them vote each way, and thus, the outcome comes down to the simple majority rule of the k informed shareholders. If $n - k$ is odd, then one mixes voting for each alternative with probability $\frac{1}{2}$, and $\frac{n-k-1}{2}$ vote for each alternative in pure strategies. Finally, the value k solves a pair of inequalities ensuring that a unilateral deviation by an uninformed shareholder to buy a signal is not valuable enough to offset the cost, c, and a deviation by one of the shareholders that is supposed to buy a signal decreases the expected value of her share by more than c. If c is sufficiently high, $k = 0$, and if c is sufficiently low, $k = n$. A few comments about this equilibrium are in order. First, if abstention were allowed, there would also be an equilibrium in which the $n - k$ uninformed shareholders abstain so as to avoid impacting a decision made by more informed voters, echoing the logic in Feddersen and Pesendorfer (1997). Second, as we saw above, in the model with trade, it is not possible to support an equilibrium where the uninformed split their vote. This is because, from this profile, an uninformed voter would have a strict incentive to deviate in how she votes, as this would provide her with an opportunity to extract information rents when trading. By contrast, when no such trading opportunities exist, an equilibrium where the uninformed split their votes can be supported.

Because the policy choice boils down to majority rule for k votes, each voting correctly with probability q, the probability that the correct policy choice is made corresponds to the probability of getting at least $\frac{k+1}{2}$ ($\frac{k-1}{2}$) successes from odd number (even number) k trials where the odds of success are given by $q > \frac{1}{2}$. We characterize the equilibria that maximize the probability of making the correct decision in the online appendix. Here we highlight the relevant comparison.

Because the probability of making the correct choice in the game with trade is less than q, the comparison in governance quality is stark. As long as c is low enough to support acquisition by at least one shareholder ($k \geq 1$) in the game without trade, this game will support better governance than the corresponding game with trade.

Proposition 8. *Fix n, q, c as long as $c \leq q - \frac{1}{2}$, in the optimal equilibrium to the model without trade, the correct policy is chosen with higher probability than in any equilibrium in the game with trade.*

The proof is in the Online Appendix. The comparisons of utility and shareholder welfare are a bit more subtle. In the game with trade, shareholders are extracting informational rents in the market, whereas in the game without trade, the only source of value for shareholders stems from making the correct choice. A further complication in the comparison is that in the game without trade, as n goes to infinity, the number of shareholders that acquire signals can only go to infinity if the cost vanishes to 0. Martinelli (2007) shows that in a classic common values model without trade, the fraction of individual acquiring information vanishes and the support of costs cannot be bounded away from 0 to allow information aggregation. On the other hand, in the game with trade, supporting a sequence $k(n)$ that goes to infinity is possible, even if the cost doesn't vanish. But as we have seen, governance is maximized when the fraction $\frac{k(n)}{n}$ vanishes at a particular rate.

Despite these complications, the comparison is direct for n large: as we now show, it is better not to have trading. To see this, we consider a fixed q and a sequence $\{n, c_n)\}$ with n going to infinity and compare the limit of average shareholder payoffs in the games with and without trade. We find average shareholder payoffs are higher in the game without trade. For the case where trade is allowed, it is sufficient to focus on a sequence of equilibria with optimal values of $k^*(n)$. If the sequence $\{n, c_n\}$ does not support this, then the payoffs from the model with trade are even lower. Note that, on the one hand, in the game without trade, either $c(n)$ converges to 0 or the fraction of shareholders acquiring signals, $\frac{k(n)}{n}$, converges to 0 or both. Thus, the limiting average expenditure on signals converges to 0. Further, the probability of selecting the correct choice is at least q. Thus, q is a lower bound for the limit of average payoffs in the game without trade. On the other hand, in the game with trade, we know that the optimal sequence $k^*(n)$ will be such that $\frac{k^*(n)}{n}$ converges to 0. So again, the average expenditure on costs vanishes. Moreover, as the fraction of informed shareholders vanishes, the average shareholder payoff converges to the payoff of uninformed shareholders. This payoff is $\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)$, which can be viewed as an upper bound for the limiting average shareholder payoff in the game with trade. But since $\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right) < q$ we obtain the following conclusion

Proposition 9. *Consider a sequence of games indexed by* $\{n, c_n\}$ *with* n *going to infinity. The limiting average payoff from the most informative equilibrium to the game without trade is higher than the limiting average payoff to any convergent sequence of equilibria for the corresponding sequence of games with trade.*

Finally, we point out one potentially important advantage to the institution with trade: some shareholders do better in the game with trade. In particular, if q is high enough, then the informed shareholders do better with trade than without.

Proposition 10. *There exists a value q^* (approximately .76) s.t. if $q \geq q^*$, then there exist sequences of parameters $\{n, c_n\}$ s.t eventually the equilibrium payoffs to shareholders that choose to acquire signals is higher in the game with trade than without.*

The proof is in the Online Appendix. The number of investors that acquire signals will not be the same in the two games, but this result is consequential because it shows that a group of active shareholders would plausibly object to restrictions on trading. Those who acquire information in the game with trade will prefer that equilibrium to any equilibrium of the game without trade. Thus, agents that might seem most engaged and well-informed might be willing to exert resources to challenge institutional changes that prevent this form of trading.

6 Institutional Comparison II: Does hiding t solve the incentive problem?

A key feature of the analysis hinges on the *Signalling Effect*, where changes in the vote tally t impact prices. We have seen that $Pr(x = \omega) < q$ in the limit. One might expect that $Pr(x = \omega)$ could be improved if the vote tally is hidden from the market as this would negate the opportunities to signal jam through voting.

However, in this section, we show that trading distorts voting incentives, even if the vote tally is not disclosed to the market. In fact, as we see here, a very straightforward logic allows us to see that when the tally, t, is hidden so that the market only learns which decision was reached, $Pr(x = \omega) \leq q$ for any n and k.

When t is not observable, the price is set based on the chosen policy x. By applying Bayes' Rule, we have the pricing function

$$P_x = \begin{cases} Pr(\omega = 1 | x = 1) = \sum_{j=\frac{n+1}{2}}^{n} Pr(t = j | \omega = 1), & \text{if } x = 1 \\ Pr(\omega = 0 | x = 0) = \sum_{j=0}^{\frac{n-1}{2}} Pr(t = j | \omega = 0), & \text{if } x = 0 \end{cases} \tag{15}$$

Note that in our symmetric setting $P_1 = P_0$, as the probability that voting makes the correct decision is the same across different states. Put differently, when the tally is not public, the market does not learn anything about firm value, but agents with private information do learn whether the chosen policy is the same as their information. Uninformed shareholders learn whether their vote was in favor or against the chosen policy, and they know they might have been pivotal but didn't bring any information to the decision. It is not difficult to see that the trading strategies are as follows.

Lemma 7. *In equilibrium, each informed shareholder trades according to her private signal; buying (selling) if and only if $x = s_i$ ($x \neq s_i$). On the other hand hand, each uninformed shareholder trades against her vote; buying (selling) if and only if $x \neq v_i^{\emptyset}$ ($x = v_i^{\emptyset}$).*

The proof is in the Online Appendix. To characterize the voting strategies, it is sufficient to observe that when t is hidden from the market, the vote by i is payoff-relevant only if i is pivotal. Conditional on being pivotal, the incremental increase in the value of the share from voting sincerely is q. The value obtained from voting against one's share and selling is the price, which in equilibrium corresponds to the probability that the correct policy is chosen. From this comparison, we obtain the following characterization of voting behavior.

Lemma 8. *In equilibrium, each informed shareholder sincerely votes for her signal if and only if*

$$q \geq \sum_{t=\frac{n+1}{2}}^{n} \sum_{i=0}^{t} \binom{k}{i} q^i (1-q)^{k-i} \binom{n-k}{t-i} \left(\frac{1}{2}\right)^{n-k}. \tag{16}$$

Otherwise, each informed shareholder plays a mixed voting strategy, which is characterized by the following indifference condition, $z =: Pr(v_i^I = \omega|\omega)$ solves

$$q = \sum_{t=\frac{n+1}{2}}^{n} \sum_{i=0}^{t} \binom{k}{i} z^i (1-z)^{k-i} \binom{n-k}{t-i} \left(\frac{1}{2}\right)^{n-k}. \tag{17}$$

Uninformed shareholders randomize with $Pr(v_i^{\emptyset} = 1) = Pr(v_i^{\emptyset} = 0) = \frac{1}{2}$.

The proof is in the Online Appendix.

Proposition 11. *As in the model with t public, to sustain an equilibrium in which informed shareholders vote sincerely, we cannot have too many informed shareholders. In particular, for given n and q, the largest number of informed shareholders that can sustain a sincere-voting equilibrium, $k^*(n)$, is the largest integer satisfying*

$$q \geq \sum_{t=\frac{n+1}{2}}^{n} \sum_{i=0}^{t} \binom{k}{i} q^i (1-q)^{k-i} \binom{n-k}{t-i} \left(\frac{1}{2}\right)^{n-k} \tag{18}$$

The proof is in the Online Appendix. Taking one step back, this characterization provides the following result on governance quality for arbitrary values of the parameters.

Proposition 12. *Fix n and q. When $k \leq k^*(n)$ so that informed shareholders sincerely vote for their signals in equilibria, $Pr(x = \omega) \leq q$. When $k > k^*(n)$ so that informed shareholders take a mixed voting strategy ($z < q$) in equilibria, we have $Pr(x = \omega) = q$.*

Proof. Immediate consequence of Lemma 8. ■

Because the analysis of the model with t private is so much more parsimonious, we are able to easily introduce some substantively relevant asymmetries and still recover the key result on information aggregation. We close with this analysis.

We continue to examine the game in which t is hidden, but extend the result in two ways. We allow different shareholders to own different numbers of shares and receive different quality signals. In particular, let shareholder j own shares V_j and have a signal of quality $q_j = Pr(\omega = s_j | s_j) \in (\frac{1}{2}, 1)$, and we allow for $V_j \neq V_{-j}$ and $q_j \neq q_{-j}$. In addition, $Max\{V_j\} \leq \frac{n-1}{2}$ ensures that no shareholder can solely decide the chosen policy. In the trading stage, shareholder j can buy or sell up to V_j shares. We denote the vector of share endowments and signal qualities as $\mathbf{V}$ and $\mathbf{q}$.

The equilibrium is characterized by a vector m_j of signal contingent mixtures for each informed player, $m_j = Pr(v_j^{s_j} = s_j | s_j)$ for $s_j \in \{0, 1\}$, and as before, the uninformed shareholders are voting for each policy with equal probability, $\frac{1}{2} = Pr(v_j^{\emptyset} = 1) = Pr(v_j^{\emptyset} = 0)$.[19]

Proposition 13. *Fix n, $\mathbf{q}$ and $\mathbf{V}$. In an equilibrium in which $m_j > \frac{1}{2}$ it must be that $P(x = \omega) \leq q_j$. That is to say, the quality of governance by voting is no better than the quality of information available to any informed shareholder whose vote is positively correlated with her signal.*

The proof is in the Online Appendix. How do the forces behind Proposition 12 (and the generalization to 13) compare with the forces behind Proposition 2? When t is not disclosed, a vote has no influence on the price and the chosen policy

[19] In Meirowitz and Pi (2024), we show that (i) with t private, it is also not possible to support equilibria in which the uninformed sort deterministically and (ii) if abstention is allowed, the path of play derived here can still be supported.

unless the vote happens to be pivotal. Thus, we can focus on the pivotal event. In any equilibrium with $z > \frac{1}{2}$, when a voter is pivotal, the expected payoff from voting for her signal and buying must be weakly higher than the expected payoff from voting against and selling. This condition implies that a shareholder's belief that her signal is correct, given her vote being pivotal, must exceed the payoff from selling, $Pr(x = \omega | s_i = x, \text{PIV}) \geq P_x$. In our symmetric environment with majority rule, $Pr(x = \omega | s_i = x, \text{PIV})$ represents just the voter's signal quality, and the price is the probability that the correct choice is made in equilibrium. Thus, $Pr(x = \omega | s_i = x, \text{PIV}) \geq P_x$ means that the quality of governance by voting is no better than the quality of information available to any shareholder that is using her information when voting.

In contrast, in the game where the tally is disclosed, if a voter is pivotal, the payoff from selling is not the probability that the correct decision is made in equilibrium. Rather, the payoff is the probability that the correct decision is made conditional on the vote tally being equal to $\frac{n+1}{2}$ or $\frac{n-1}{2}$. $Pr(x = 1 | t = \frac{n+1}{2})$ (or $Pr(x = 0 | t = \frac{n-1}{2})$) is generally less than the probability that the correct decision is made in equilibrium. So, the equilibrium force that requires signal quality to exceed $Pr(x = 1 | t = \frac{n+1}{2})$ (or $Pr(x = 0 | t = \frac{n-1}{2})$) does not necessitate that the probability the correct decision is made is less than the signal quality. However, when t is disclosed, a vote affects the price for any realization of t, so a voter has to consider the *Signaling Effect*. As we have seen, the condition balancing the *Pivotal Effect* and *Signaling Effect* pushes z to be fairly close to $\frac{1}{2}$ at least when $\frac{k}{n}$ approaches 1 and n gets large. Therefore, the equilibrium probability of making the correct decision will also be less than the quality of a signal. But that result is due to balancing *Signaling* and *Pivotal* effects.

7 Discussion

By jointly considering three facets of investor behavior: the acquisition of information about the firms they own, their involvement in the governance of these firms, and their trading behavior, we find that consequential strategic spillovers exist. Opportunities to create trading rents lead to incentives to sometimes vote against the firm's interest. The magnitude (and presence) of this distortion depends on the amount of private information held by shareholders. Because the quality of information aggregation through governance can be worse when more information is available for aggregation, high levels of information acquisition are not necessarily good for firm governance. This argument against a blanket endorsement of ways to increase shareholder information acquisition challenges received wisdom in the literature and the pronouncements of some regulators.

Two additional insights about the broader design of institutions to govern firms surface. First, our findings suggest that the presence of opportunities for shareholders to trade is generally bad for governance, although it might help encourage those governing to be better informed. Second, even though the incentive problems highlighted here stem from signal jamming incentives when market prices depend on voting behavior, the ability of the market to see the vote tally is not central. We show that even if the market only learns the decision and not the voting tally, the probability that the correct decision is made must be quite low. Thus, seemingly easy reductions in transparency do not resolve the problem.

Finally, the analysis highlights an interesting distinction between common values voting problems when trading is and is not possible. Feddersen and Pesendorfer (1997) and Persico (2004) demonstrate a key force in common values problems: equilibrium can push towards efficiency by having the less informed minimize their impact on the collective choice. Here, because of the ability to trade and extract informational rents, these forces are muted. There is too much to be gained by extracting trading rents at the expense of governance quality. Whereas in common values voting problems without trade, it is instructive to think of constructing equilibria to maximize the sum of player utilities (McLennan (1998)), here the possibility of extracting trading rents from market liquidity undermines this logic.

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A Appendix: Proofs

A.1 Proof of Lemma 1: Stock Price

Proof. The expression is obtained by noting the presence of two forms of votes: those from informed voters who vote for the optimal policy with probability z and those from uninformed voters who toss a fair coin. Therefore, we have a convolution of two random variables. Applying Bayes' rule directly yields the result. ■

A.2 Proof of Lemma 2: Trading Strategies

Proof. To ease the notation, we define the function $\theta(t, z, k, n, d)$ as follows

$$\theta(t,z,k,n,d) := \frac{\sum_{i=0}^{t}\binom{k}{i}z^i(1-z)^{k-i}\binom{n-k}{t-i}d}{\sum_{i=0}^{t}\binom{k}{i}z^i(1-z)^{k-i}\binom{n-k}{t-i}d+\sum_{i=0}^{t}\binom{k}{i}z^i(1-z)^{k-i}\binom{n-k}{t-i}(1-d)}$$

Assume, without the loss of generality, that $x = 1$. We first prove that an uninformed shareholder wants to buy if $v_i^{\emptyset} = 0$ and to sell if $v_i^{\emptyset} = 1$. After voting and observing t and $x = 1$, an uninformed shareholder who votes for policy 1 expects share value to be

$$E[v(x,\omega)|t, x = 1, v^{\emptyset} = 1] = \theta(t-1, z, k, n-1, \frac{1}{2})$$

An uninformed shareholder who votes for policy 0 expects share value to be

$$E[v(x,\omega)|t, x = 1, v^{\emptyset} = 0] = \theta(t, z, k, n-1, \frac{1}{2})$$

Since the market does not observe how each voter votes individually, its posterior belief on $\omega = 1$ based on t (and thus the price $P_1(t)$) can be written as

$$P_1(t) = \theta(t-1, z, k, n-1, \frac{1}{2})Pr(v_i^{\emptyset} = 1|t) + \theta(t, z, k, n-1, \frac{1}{2})Pr(v_i^{\emptyset} = 0|t) \quad (19)$$

Intuitively speaking, the market interprets the voting tally t as a convolution of three groups' votes: a single uniformed voter who randomly votes, a group of k informed votes who vote correctly with probability z, a group of $n-k-1$ uninformed voter who randomly vote. Based on t, when the market thinks the uninformed voter votes for 1, she also thinks $t-1$ out of the remaining k informed voters and $n-k-1$ uninformed voters vote for 1 and thus expects the share value to be $\theta(t-1, z, k, n-1, \frac{1}{2})$. When the market believes the uninformed voter votes for 0, she thinks t out of the rest k informed voters and $n-k-1$ uninformed voters vote for 1, and thus her expectation of share value is $\theta(t, z, k, n-1, \frac{1}{2})$. Note that $\theta(t-1, z, k, n-1, \frac{1}{2}) < \theta(t, z, k, n-1, \frac{1}{2})$, which implies that the market is more optimistic on the chosen policy 1 when t out of k informed voters and $n-k-1$ uninformed voters vote for 1 than when $t-1$ out of k informed voters and $n-k-1$ uninformed voters vote for 1. Since $Pr(v_i^{\emptyset}=1|t)+Pr(v_i^{\emptyset}=0|t)=1$, we must have

$$\theta(t-1,z,k,n-1,\frac{1}{2})<P_1(t)<\theta(t,z,k,n-1,\frac{1}{2}), \tag{20}$$

which is equivalent to

$$E[v(x,\omega)|t,x=1,v^{\emptyset}=1])<P_1(t)<E[v(x,\omega)|t,x=1,v^{\emptyset}=0] \tag{21}$$

Thus, an uninformed voter who votes for 1 wants to sell and an uninformed voter who votes for 0 wants to buy when $x=1$.

Now we prove that an informed shareholder who votes for 1 wants to buy if $s_i=1$ and wants to sell if $s_i=0$. An informed shareholder with $s_i=1$ who votes for 1 expects share value to be

$$E[v(x,\omega)|t,x=1,v_i^1=1,s_i=1]=\theta(t-1,z,k-1,n-1,q)$$

An informed shareholder who votes for policy 1 and has $s_i=0$ expects share value to be

$$E[v(x,\omega)|t,x=1,v_i^1=1,s_i=0]=\theta(t-1,z,k-1,n-1,1-q)$$

The price can be written as

$$P_1(t)=\theta(t-1,z,k-1,n-1,q)Pr(v_i^1=1|t)+\theta(t-1,z,k-1,n-1,1-q)Pr(v_i^0=1|t)$$

Note that $\theta(t-1,z,k-1,n-1,q)>\theta(t-1,z,k-1,n-1,1-q)$. Thus, we have

$$E[v(x,\omega)|t,x=1,v_i^1=1,s_i=0]<P_1(t)<E[v(x,\omega)|t,x=1,v_i^1=1,s_i=1]$$

Thus, an informed shareholder who votes for 1 wants to buy if $s_i=1$ and wants to sell if $s_i=0$.

We continue to prove that an informed shareholder who votes for 0 wants to buy if $s_i=1$ and wants to sell if $s_i=0$. If an informed shareholder has $s_i=1$ but votes for 0, her expectation of share value is

$$E[v(x,\omega)|t,x=1,v_i^1=0,s_i=1]=\theta(t,z,k-1,n-1,q)$$

An informed shareholder receives $s_i=0$ and votes for policy 0 expects share value to be

$$E[v(x,\omega)|t,x=1,v_i^0=0,s_i=0]=\theta(t,z,k-1,n-1,1-q)$$

The price can be written as

$$P_1(t) = \theta(t, z, k-1, n-1, q)Pr(v_i^1 = 0|t) + \theta(t, z, k-1, n-1, 1-q)Pr(v_i^0 = 0|t)$$

Since $\theta(t, z, k-1, n-1, 1-q) < \theta(t, z, k-1, n-1, q)$, we have

$$\theta(t, z, k-1, n-1, 1-q) < P_1(t) < \theta(t, z, k-1, n-1, q)$$

It follows that an informed shareholder who votes for 0 wants to buy if $s_i = 1$ and wants to sell if $s_i = 0$.

To conclude, when $x = 1$, an informed shareholder that has signal $s_i = 1$ wants to buy, while an informed shareholder with signal $s_i = 0$ wants to sell. ■

A.3 Proof of Lemma 3: Signaling Effect and Pivotal Effect

Proof. Consider without loss of generality an informed shareholder whose signal is 1. We may write her indifference condition as

$$\begin{aligned}
& EU(v_i^1 = 1) - EU(v_i^1 = 0) \\
&= \sum_{t'=0}^{t'=\frac{n-1}{2}-1} Pr_I(t'|s_i = 1)[P_0(t'+1) - P_0(t')] \\
&+ Pr_I(t' = \frac{n-1}{2}|s_i = 1)[2Pr_I(\omega = 1|s_i = 1, t') - P_1(\frac{n+1}{2}) - P_0(\frac{n-1}{2})] \\
&+ \sum_{t'=\frac{n-1}{2}+1}^{n-1} Pr_I(t'|s_i = 1)[P_1(t') - P_1(t'+1)] \\
&= \underbrace{Pr_I(t' = \frac{n-1}{2}|s_i = 1)(2Pr_I(\omega = 1|t' = \frac{n-1}{2}, s_i = 1) - 1)}_{\text{Pivotal Effect}} \\
&\underbrace{- \sum_{t'=0}^{n-1} Pr_I(t'|s_i = 1)(Pr(\omega = 1|t'+1) - Pr(\omega = 1|t'))}_{\text{Signaling Effect}}
\end{aligned} \tag{22}$$

■

A.4 Proof of Lemma 4: Existence of the Equilibrium

Proof. Note that given the symmetry, each uninformed shareholder obtains the same expected utility from each pure voting strategy, thus mixing is a best response. To prove the existence of the equilibrium, it is sufficient to show that there exists a $z \in (\frac{1}{2}, q]$ such that the LHS of Equation (9) is weakly smaller than the RHS of Equation (9). To see this, first note that the RHS and LHS are continuous in z. Second, when $z = \frac{1}{2}$, the LHS is 0 as the terms involving $Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t') = 0$ when voting is uncorrelated with ω. But the RHS is strictly larger than 0. Thus, we know $LHS < RHS$ when $z = \frac{1}{2}$. Now consider $z = q$. There are two cases. If $LHS > RHS$ at $z = q$, then by continuity and the intermediate value theorem, there is a value of $z < q$ such that $LHS = RHS$ and informed shareholders are indifferent between voting with or against their signal. On the other hand, if $LHS \leq RHS$ at $z = q$, then sincere voting is the unique best response, and we have an equilibrium where the informed shareholders use pure voting strategies. ■

A.5 Proof of Proposition 1: Equilibrium for Exogenous k

Proof. Using the symmetry of the binomial distribution, we can simplify the condition that pins down z to

$$\sum_{t'=0}^{n-1} Pr_I(t'|\omega = 1)(Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t')) \leq Pr_I(t' = \frac{n-1}{2}|\omega = 1)(2q - 1) \tag{23}$$

Dividing both sides by $Pr_I(t' = \frac{n-1}{2}|\omega = 1)$, we have

$$\sum_{t'=0}^{n-1} \frac{Pr_I(t'|\omega = 1)}{Pr_I(t' = \frac{n-1}{2}|\omega = 1)}(Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t')) \leq 2q - 1 \tag{24}$$

Note that conditional on $\omega = 1$, the voting tally t is the convolution of two binomial distributions with different success rates (z and $\frac{1}{2}$). When n is large, this convolution approximates to a normal distribution with the mean of $kz + (n - k)\frac{1}{2}$ and the variance of $kz(1 - z) + (n - k)\frac{1}{4}$. Similarly, conditional on $\omega = 1$ ($\omega = 0$), t' from the perspective of an informed voter is normally distributed with the mean of $(k-1)z+(n-k)\frac{1}{2}$ ($(k-1)(1-z)+(n-k)\frac{1}{2}$) and the variance of $(k-1)z(1-z)+(n-k)\frac{1}{4}$. Thus, the left-hand side of the indifference condition becomes

$$\int_{t'=0}^{n-1} \frac{\phi(t'; \mu_1, \sigma_1)}{\phi(\frac{n-1}{2}; \mu_1, \sigma_1)} \cdot \left(\frac{\phi(t' + 1; \mu_2, \sigma_2)}{\phi(t' + 1; \mu_2, \sigma_2) + \phi(t' + 1; \mu_3, \sigma_3)} - \frac{\phi(t'; \mu_2, \sigma_2)}{\phi(t'; \mu_2, \sigma_2) + \phi(t'; \mu_3, \sigma_3)} \right) dt' \tag{25}$$

where
$\mu_1 = (k - 1)z + (n - k)\frac{1}{2}$, $\sigma_1 = \sqrt{(k - 1)z(1 - z) + (n - k)\frac{1}{4}}$
$\mu_2 = kz + (n - k)\frac{1}{2}$, $\sigma_2 = \sqrt{kz(1 - z) + (n - k)\frac{1}{4}}$

$\mu_3 = k(1-z) + (n-k)\frac{1}{2}$, $\sigma_3 = \sqrt{kz(1-z) + (n-k)\frac{1}{4}}$
We view the equation in the integral symbols as a function of z and denote it as $f(z)$. Taking Taylor expansions of $f(z)$ around $z = \frac{1}{2}$, we have[20]

$$\begin{aligned} f(z) &= f(\frac{1}{2}) + \frac{f'(\frac{1}{2})}{1!})(z-\frac{1}{2}) + \frac{f'(\frac{1}{2})}{2!})(z-\frac{1}{2})^2 + ...\frac{f'(\frac{1}{2})}{j!})(z-\frac{1}{2})^j \\ &= \frac{2k\left(z-\frac{1}{2}\right)e^{-\frac{(n-2t'-1)^2}{2(n-1)}}}{n} - \frac{4\left(z-\frac{1}{2}\right)^2\left((k-1)ke^{-\frac{(n-2t'-1)^2}{2(n-1)}}(n-2t'-1)\right)}{(n-1)n} + ... + O((z-\frac{1}{2})^j) \end{aligned} \tag{26}$$

Integrating $f(z)$ from $t' = 0$ to $t' = n-1$, we find the condition that pins down z becomes

$$\frac{\sqrt{\frac{\pi}{2}}k\sqrt{n-1}(2z-1)\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}{n} \leq 2q-1 \tag{27}$$

It follows that if informed shareholders sincerely vote for their information ($z = q$) in equilibrium, it must be the signaling effect is weakly smaller than the pivotal effect when $z = q$. This implies that

$$\frac{\sqrt{\frac{\pi}{2}}k\sqrt{n-1}(2q-1)\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}{n} \leq 2q-1 \tag{28}$$

which is equivalent to

$$\frac{k}{n} \leq \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)} \tag{29}$$

That is to say, to ensure that all informed shareholders sincerely vote for the information they own in voting ($z = q$), we cannot have too many informed shareholders. In particular, the ratio between the amounts of informed shareholders (k) and the amounts of all shareholders (n) cannot be too high.

On the other hand, if $\frac{k}{n} > \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$, then we have equilibria in which informed shareholders strategically vote with $z < q$. In particular, z is given by

$$\frac{\sqrt{\frac{\pi}{2}}k\sqrt{n-1}(2z-1)\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}{n} = 2q-1 \tag{30}$$

which implies

$$z(n,k) = \frac{1}{2} + \frac{n(2q-1)}{\sqrt{2\pi}k\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)} \tag{31}$$

[20] Note that every term contains $(z-\frac{1}{2})^j$. Recall that $z \in (\frac{1}{2}, q)$. Then, we must have $0 < z - \frac{1}{2} < q - \frac{1}{2} < 1$. Since $0 < z - \frac{1}{2} < 1$, we know that, for any z in $(\frac{1}{2}, q)$, $(z-\frac{1}{2})^j$ must exponentially vanish towards 0 as j increases to ∞.

when $\frac{k}{n} > \kappa(n)$. ■

A.6 Proof of Proposition 2: Information Aggregation

Proof. Using the approximation above, we can say that when n gets large, the probability that the correct policy is chosen converges to

$$Pr(x = \omega) = 1 - \Phi\left(\frac{\frac{n+1}{2} - (kz + (n-k)\frac{1}{2})}{\sqrt{kz(1-z) + (n-k)\frac{1}{4}}}\right) \tag{32}$$

We first consider the case of $\frac{k}{n} \leq \kappa(n)$ in which informed shareholders sincerely vote. Since all informed shareholders sincerely vote ($z = q$), the probability of selecting the correct policy increases as the number of informed increases. To see this, note that

$$\frac{d\ \frac{\frac{n+1}{2} - (kq + (n-k)\frac{1}{2})}{\sqrt{kq(1-q) + (n-k)\frac{1}{4}}}}{d\ k} = \frac{(2q-1)\left(k(1-2q)^2 - 2n + 2q - 1\right)}{2\left(n - k(1-2q)^2\right)^{3/2}} < 0$$

Thus, we know $Pr(x = \omega)$ is increasing in k when $\frac{k}{n} \leq \kappa(n)$.

Then, we consider the case of $\frac{k}{n} > \kappa(n)$. Recall that an informed shareholder takes a mixed strategy, $z = \frac{1}{2} + \frac{n(2q-1)}{\sqrt{2\pi}k\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$ when $\frac{k}{n} > \kappa(n)$. As the number of informed shareholders increases, two effects happen simultaneously. First, as k increases, there are more informative votes ($z > \frac{1}{2}$), which helps information aggregation. Second, as k increases, each informative vote becomes less informative ($\frac{dz}{dk} < 0$), which hurts information aggregation. As a result, the aggregate effect of having more informed shareholders on information aggregation efficiency depends on which effects dominate. After we substitute $z = \frac{1}{2} + \frac{n(2q-1)}{\sqrt{2\pi}k\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$ into $Pr(x = \omega)$, we get

$$Pr(x = \omega) = 1 - \Phi\left(\frac{\sqrt{\pi}\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right) + \sqrt{2}n(1-2q)}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)\sqrt{n\left(\pi - \frac{2n(1-2q)^2}{k(n-1)\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)^2}\right)}}\right) \tag{33}$$

Note that

$$\sqrt{\pi}\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right) + \sqrt{2}n(1-2q) < 0. \tag{34}$$

Then, as k increases, $Pr(\omega = x)$ decreases in the number of informed shareholders.

Overall, when $\frac{k}{n} \leq \kappa(n)$, informed shareholders sincerely vote ($z = q$), and $Pr(x = \omega)$ increases with k. However, when $\frac{k}{n} > \kappa(n)$, informed shareholders strategically vote ($z < q$), and $Pr(x = \omega)$ decreases with k. Thus, $Pr(\omega = x)$ reaches its maximum when $\frac{k}{n} = \kappa(n)$.

To find the maximum $Pr(x = \omega)$, we substitute $k = \frac{n\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$ and $z = q$ into Equation (32).

$$Pr^*(x = \omega) = Pr(x = \omega; \kappa(n)) = 1 - \Phi\left(\frac{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right) + \sqrt{\frac{2}{\pi}}n(1-2q)}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)\sqrt{n - \frac{\sqrt{\frac{2}{\pi}}n(1-2q)^2}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}}}\right) \tag{35}$$

$Pr(x = \omega; \kappa(n))$ denotes the probability that shareholder voting selects the correct policy when the ratio of informed shareholders to all shareholders is exactly $\kappa(n)$.

Since

$$\lim_{n\to\infty} \frac{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right) + \sqrt{\frac{2}{\pi}}n(1-2q)}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)\sqrt{n - \frac{\sqrt{\frac{2}{\pi}}n(1-2q)^2}{\sqrt{n-1}\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}}} = \sqrt{\frac{2}{\pi}}(1-2q), \tag{36}$$

we have

$$\lim_{n\to\infty} Pr(x = \omega; \kappa(n)) = 1 - \Phi\left(\sqrt{\frac{2}{\pi}}(1-2q)\right) = \Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right) \tag{37}$$

, which is strictly smaller than q. ■

A.7 Proof of Lemma 5: Information Cost

Proof. First, we find the condition under which none of the k informed shareholders wants to deviate by becoming uninformed. Consider without the loss of generality an informed shareholder whose signal is $s_i = 1$. Then, her expected payoff in equilibria is equal to

$$\begin{aligned} &EU(a_i = 1) - c \\ &= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t'|s_i = 1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t'|s_i = 1)(2Pr_I(\omega = 1|t', s_i = 1) - P_1(t'+1)) - c \end{aligned} \tag{38}$$

Now we turn to find out the informed shareholders' payoff if she deviates from acquiring information to not acquiring information. Since no particular policy is better than the other policy when shareholder i does not have information about the underlying state, for the purposes of computation, we can conveniently assume her payoff is equal to the payoff from voting for policy 1.

$$EU(a_i = 1 \xrightarrow{\text{D}} 0) = \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t')(2Pr_I(\omega = 0|t') - P_0(t'+1)) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t')P_1(t'+1) \tag{39}$$

The cost that can make the deviation from acquiring information to not acquiring information unprofitable must satisfy

$$c \leq \underbrace{EU(a_i = 1) - EU(a_i = 1 \xrightarrow{\text{D}} 0)}_{\bar{c}(k)} \tag{40}$$

Second, we find the condition under which none of the $n-k$ uninformed shareholders wants to deviate by acquiring a signal. In equilibrium, since each uninformed shareholder is indifferent between $v_i^{\emptyset} = 1$ and $v_i^{\emptyset} = 0$, an uninformed shareholder's expected payoff is equal to

$$EU(a_i = 0) = \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_U(t')(2Pr_U(\omega = 0|t') - P_0(t'+1)) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_U(t')P_1(t'+1) \tag{41}$$

On the other hand, if an uninformed shareholder deviates by acquiring information. Then, suppose without the loss of generality that she gets $s_i = 1$, her expected payoff from the deviation is

$$\begin{aligned}
&EU(a_i = 0 \xrightarrow{\text{D}} 1) - c \\
&= \max\{EU(v_i^1 = 1), EU(v_i^1 = 0)\} - c \\
&= \max\{\sum_{t'=0}^{\frac{n-1}{2}-1} Pr_U(t'|s_i = 1)P_0(t'+1) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_U(t'|s_i = 1)(2Pr_U(\omega = 1|t', s_i = 1) - P_1(t'+1)) \\
&, \sum_{t'=0}^{\frac{n-1}{2}} Pr_U(t'|s_i = 1)P_0(t') + \sum_{t'=\frac{n+1}{2}}^{n-1} Pr_U(t'|s_i = 1)(2Pr_U(\omega = 1|t', s_i = 1) - P_1(t'))\} - c
\end{aligned} \tag{42}$$

To prevent the deviation, we must also have

$$\underbrace{EU(a_i = 0 \xrightarrow{\text{D}} 1) - EU(a_i = 0)}_{\underline{c}(k)} \leq c \tag{43}$$

■

A.8 Proof of Proposition 3: Optimal Information Cost

Proof. Recall that to sustain the equilibrium in which $k^*(n) = \kappa(n)n$ shareholders buy information, the information costs must satisfy

$$\underbrace{EU(a_i = 0 \xrightarrow{\text{D}} 1) - EU(a_i = 0)}_{\underline{c}(k^*(n))} \leq c^*(n) \leq \underbrace{EU(a_i = 1) - EU(a_i = 1 \xrightarrow{\text{D}} 0)}_{\bar{c}(k^*(n))} \tag{44}$$

We show that both $\underline{c}(k^*(n))$ and $\bar{c}(k^*(n))$ converge to $(2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)$ when $n \to \infty$. First, let us focus on $\bar{c}(k^*(n))$ in which we have

$$\begin{aligned}
&\lim_{n\to\infty} \bar{c}(k^*(n)) = \lim_{n\to\infty} EU(a_i = 1) - \lim_{n\to\infty} EU(a_i = 1 \to 0) \\
&= \lim_{n\to\infty} 2q \sum_{t'=\frac{n-1}{2}}^{n-1} Pr_I(t'|\omega = 1) - \lim_{n\to\infty} \sum_{t'=0}^{\frac{n-1}{2}-1} Pr_I(t'|\omega = 0) \\
&= (2q-1) \lim_{n\to\infty} \sum_{t'=\frac{n-1}{2}+1}^{n-1} Pr_I(t'|\omega = 1) + \lim_{n\to\infty} qPr_I(t' = \frac{n-1}{2}|\omega = 1) \\
&= (2q-1) \lim_{n\to\infty} \left(1 - \Phi\left(\frac{\frac{n+1}{2} - ((k^*(n)-1)(1-q) + (n-k^*(n))\frac{1}{2})}{\sqrt{(k^*(n)-1)q(1-q) + (n-k^*(n))\frac{1}{4}}}\right)\right) \\
&= (2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)
\end{aligned} \tag{45}$$

Now we focus on $\underline{c}(k^*(n))$. Similarly, we have

$$\begin{aligned}
&\lim_{n\to\infty} \underline{c}(k^*) = \lim_{n\to\infty} EU(a_i = 0 \to 1) - EU(a_i = 0) \\
&= \lim_{n\to\infty} (2q-1) \sum_{t'=\frac{n-1}{2}+1}^{n-1} Pr_{UI}(t'|\omega = 1) + \lim_{n\to\infty} qPr_{UI}(t' = \frac{n-1}{2}|\omega = 1) \\
&= (2q-1) \lim_{n\to\infty} \left(1 - \Phi\left(\frac{\frac{n+1}{2} - (k^*(n)(1-q) + (n-k^*(n)-1)\frac{1}{2})}{\sqrt{k^*(n)q(1-q) + (n-k^*(n)-1)\frac{1}{4}}}\right)\right) \\
&= (2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)
\end{aligned} \tag{46}$$

Note that

$$\lim_{n\to\infty} \underline{c}(k^*) = \lim_{n\to\infty} \bar{c}(k^*(n)) = (2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right) \tag{47}$$

Recall that $\underline{c}(k^*(n)) \leq c^*(n) \leq \bar{c}(k^*(n))$. Because of the Sandwich Theorem, we have

$$\lim_{n\to\infty} c^*(n) = (2q-1)\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right) \tag{48}$$

, which is increasing in q. ■

B Online Appendix: Proofs of Robustness Checks and Institutional Comparisons

B.1 Proof of Lemma 6

First, note that in the conjectured strategy profile, the price only depends on the voting tally of informed shareholders, t_I. Since the market knows that half of the $n-k$ uninformed shareholders deterministically vote for policy 1, the price is

$$P_x(t) = Pr(x = \omega | t_I = t - \frac{1}{2}(n-k)) \tag{49}$$

Second, since an uninformed shareholder who is expected to vote deterministically also knows that half of the $n-k$ uninformed shareholders deterministically vote for policy 1, her expectation of the firm value conditional on t is

$$E[v(x,\omega)|s_i = \emptyset, t, v_i^{\emptyset}] = Pr(x = \omega | t_I = t - \frac{1}{2}(n-k)) \tag{50}$$

Because $P_x(t) = E[v|s_i = \emptyset, t]$, an uninformed shareholder who deterministically votes in the conjectured equilibria does not have informational advantages over the market.

B.2 Proof of Proposition 4

Proof. Suppose there is an equilibrium in which informed voters use pure strategies in voting. Without loss of generality, we can focus on an uninformed shareholder that deterministically votes for 0 under the profile and consider a unilateral deviation of voting for Policy 1 instead. On the path as well as following the deviation, optimal trading behavior is contingent upon the realization of x and t (the latter is a sufficient statistic for t_I only with no deviation). Since the market does not know that the uninformed shareholder deviates, when t is observed, the market mistakenly concludes that $t_I + 1$ informed shareholders have voted for 1. Recall t_I is the actual number of informed shareholders that voted for 1. Consequently, when $x = 1$, the price is $Pr(\omega = 1|t_I + 1)$. However, the uninformed shareholder privately knows about her deviation (voting for 1), so her expectation of share value is $Pr_U(\omega = 1|t_I)$. She believes the firm is overpriced and thus wants to sell. On the other hand, if $t_I < \frac{k-1}{2}$, then the selected policy is 0. Again, since the market is misled to believe that the number of informed shareholders that have voted for 1 is $t_I + 1$, the market sets the price to be $Pr(\omega = 0|t_I + 1)$. But the uninformed shareholder's expectation of share value is $Pr_U(\omega = 0|t_I)$. Consequently, she thinks the firm is under-priced and wants to buy one share.

There is an off-equilibrium path case. In particular, when all informed shareholders happen to vote for 1 (so that $t_I = k$), following the deviation, the voting tally will be $t = \frac{n-k}{2} + k + 1$. However, in equilibrium, the maximum t that occurs with positive probability is $\frac{n-k}{2} + k$. It is sufficient for us to consider the off-the-path belief following this vote tally that minimizes the potential informational rents associated with this deviation. In particular, we assume that the market believes that this off-the-path history is the result of one uninformed shareholder who should vote for 0 deviating and voting for 1 and everyone else playing the conjectured strategy profile. Any off-the-path belief that involves changing posterior beliefs following this deviation when all informed shareholders have voted for 1 will increase the value of the deviation.

So, under this belief, the uninformed shareholder's expected payoff from the deviation is

$$\begin{aligned} & EU(v_i^{\emptyset} = 0 \xrightarrow{\text{D}} 1) \\ &= \sum_{j=0}^{\frac{k-1}{2}-1} Pr_U(t_I = j)(2Pr(\omega = 0|t_I = j) - Pr(\omega = 0|t_I = j+1)) \\ &+ \sum_{j=\frac{k-1}{2}}^{k-1} Pr_U(t_I = j)Pr(\omega = 1|t_I = j+1) \\ &+ Pr(t_I = k)Pr(\omega = 1|t_I = k) \end{aligned} \tag{51}$$

Recall that in the conjectured equilibria, since the uninformed shareholder does not trade, her expected payoff is

$$EU(v_i^{\emptyset} = 0) = \sum_{j=0}^{k} Pr_U(t_I = j)Pr(\omega = x|t_I = j) \tag{52}$$

Thus, the difference between her expected payoff from the derivation and her expected payoff from the conjectured equilibrium is

$$\begin{aligned} & EU(v_i^{\emptyset} = 0 \xrightarrow{\text{D}} 1) - EU(v_i^{\emptyset} = 0) \\ &= \sum_{j=0}^{\frac{k-1}{2}-1} Pr_U(t_I = j)(Pr(\omega = 0|t_I = j) - Pr(\omega = 0|t_I = j+1)) \\ &+ Pr(t_I = \frac{k-1}{2})(Pr(\omega = 1|t_I = \frac{k-1}{2} + 1) - Pr(\omega = 0|t_I = \frac{k-1}{2})) \\ &+ \sum_{j=\frac{k+1}{2}}^{k-1} Pr(t_I = j)(Pr(\omega = 1|t_I = j+1) - Pr(\omega = 1|t_I = j)) \end{aligned} \tag{53}$$

Note that $Pr(\omega = 1|t_I = \frac{k-1}{2} + 1) = Pr(\omega = 0|t_I = \frac{k-1}{2})$ due to the symmetry of binomial distribution. So the middle term is 0. Because $Pr(\omega = 1|t_I)$ increases with

t_I and $Pr(\omega = 0|t_I)$ decreases with t_I, we know the first and third terms are both strictly positive and thus $EU(v_i^{\emptyset} = 0 \xrightarrow{\text{D}} 1) - EU(v_i^{\emptyset} = 0) > 0$. Thus, the deviation is profitable, and so the conjectured equilibrium with split uninformed votes does not exist. ■

B.3 Proof of Proposition 5

Proof. Note that abstention and then playing the conjectured equilibrium strategy yields the same payoff in this conjectured equilibrium to the extended game as voting for 0 and then playing the equilibrium strategy in the equilibrium to the baseline game. In particular, suppose everyone other than i plays the conjectured equilibrium in the game allowing abstention. Note that if an uninformed shareholder abstains, she wants to buy when $x = 1$ and sell when $x = 0$. It then follows that an uninformed voter's expected payoff from abstention is the same as voting for 0 in equilibrium, $EU(v_i^{\emptyset} = \emptyset) = EU(v_i^{\emptyset} = 0)$ where the former is a payoff in the extended game and the latter is the payoff from the equilibrium to the baseline model. Recall that in equilibrium to the baseline model $EU(v_i^{\emptyset} = 1) = EU(v_i^{\emptyset} = 0)$. Thus, abstention is not profitable for any uninformed voter.

Consider an informed shareholder with $s_i = 0$. Since abstention is effectively the same as voting for 0 in equilibrium to the baseline model, her expected payoff from abstention must be the same as her expected payoff from voting for 0 in equilibrium, $EU(v_i^1 = \emptyset) = EU(v_i^0 = 0)$. Since her expected payoff in equilibrium must be equal to $EU(v_i^0 = 0)$ as $EU(v_i^0 = 0) \geq EU(v_i^0 = 1)$, abstention is not profitable for her. Similarly, consider an informed shareholder with $s_i = 1$. Her expected payoff from abstention satisfies $EU(v_i^1 = \emptyset) = EU(v_i^1 = 0)$. Because her expected payoff in equilibrium must be equal to $EU(v_i^1 = 1)$ as $EU(v_i^1 = 1) \geq EU(v_i^1 = 0)$, it follows that $EU(v_i^1 = 1) \geq EU(v_i^1 = \emptyset)$, and therefore abstention is not profitable to an informed voter with $s_i = 1$ either. ■

B.4 Proof of Proposition 6

Proof. Without loss of generality, we focus on the case where k is odd. Fix n and k. Suppose by contradiction that there is an equilibrium in which the $n - k$ uninformed shareholders abstain and each informed shareholder votes correctly with the probability of $z \in (\frac{1}{2}, q]$.

In such an equilibrium, because uninformed shareholders abstain, the voting tally of all informed votes is observable to the market maker and all shareholders. This implies that for any realization of t, uninformed shareholders' expectation of share value is the same as the market maker's expectation of share value (and thus the price). In other words, when uninformed shareholders abstain, they do not have any informational advantage over the market. So, an uninformed shareholder's expected payoff in the conjectured equilibrium can be written as

$$EU(v_i^{\emptyset} = \emptyset) = \sum_{j=0}^{k} Pr(t_I = j)Pr(\omega = x|t_I = j) \tag{54}$$

where t_I denotes the vote tally from informed voters.

First, observe that if a single uninformed shareholder i deviates and votes, the market maker can detect that the number of votes is $k+1$ instead of k, but unless $t = k+1$ or $t = 0$ the market maker cannot determine how the deviating shareholder voted and thus cannot determine how the k informed shareholders voted. Let $G_x(t)$ denote the off-the-path posterior belief that $\omega = x$ given t votes for 1 from $k+1$ votes cast. Following this single shareholder deviation, the price corresponds to $G_x(t)$.

For any realization of t, since the uninformed shareholder privately knows which policy she votes for, she can infer the voting tally of all informed votes, t_I. For example, if she voted for 1 and observes t out of $k+1$ shareholders (including herself) vote for 1, she knows $t_I = t - 1$. We consider a unilateral deviation in which one uninformed shareholder votes and then for any realization of t, the deviating uninformed shareholder buys if $Pr(x = \omega|t_I) > G_x(t)$, sells if $Pr(x = \omega|t_I) < G_x(t)$, and holds if $Pr(x = \omega|t_I) = G_x(t)$.

We first consider a deviation in which the deviating uninformed shareholder votes for 1 rather than abstaining and then trades using the rule specified above. Note that given the plurality voting rule defined above, a deviation that casts a ballot of 1 will not affect the chosen policy. The difference between her expected payoff from the deviation and her expected payoff from the conjecture equilibrium is

$$\begin{aligned} &EU(v_i^{\emptyset} = \emptyset \xrightarrow{D} 1) - EU(v_i^{\emptyset} = \emptyset) \\ &= \sum_{j=0}^{k} Pr(t_I = j) \left(\max\{2Pr(\omega = x|t_I) - G_x(t_I + 1), G_x(t_I + 1), Pr(\omega = x|t_I)\} - Pr(\omega = x|t_I = j) \right) \end{aligned} \tag{55}$$

which is greater than 0 as long as $G_x(t) \neq Pr(\omega = x|t_I = t-1)$ for some realization(s) of t.

Thus, we have obtained a profitable deviation unless the off-the-path belief has $G(t) = Pr(x = \omega|t_I = t - 1)$ for all realizations of t. To handle the case where the off-the-path belief is such that this equality always holds, we then show that a deviation that involves voting for 0 instead must be profitable.

The difference between the expected utility from voting for 0 and the expected utility from abstention is

$$EU(v_i^\emptyset = \emptyset \xrightarrow{D} 0) - EU(v_i^\emptyset = \emptyset)$$
$$= \sum_{j=0}^{k} Pr(t_I = j)\left(\max\{2Pr(\omega = x|t_I) - G_x(t_I), G_x(t_I), Pr(\omega = x|t_I)\} - Pr(\omega = x|t_I = j)\right). \tag{56}$$

Note that here, in the event that t_I happens to be $\frac{k+1}{2}$, the deviation of voting for 0 changes the policy. But in the pivotal event, when $G_x(t) = Pr(x = \omega|t_I = t-1)$, the payoff from the deviation is weakly higher than the payoff from abstention. To see this, note that when $t_I = \frac{k+1}{2}$, if the uninformed voter does not deviate, the policy will be 1 and her payoff is

$$Pr(\omega = 1|t_I = \frac{k+1}{2})$$

On the other hand, if the uninformed voter votes for 0, the policy will be 0 and her payoff is

$$\max\{2Pr(\omega = 0|t_I = \frac{k+1}{2}) - G_0(\frac{k+1}{2}), G_0(\frac{k+1}{2}), Pr(\omega = 0|t_I = \frac{k+1}{2})\}$$

Since $G_x(t) = Pr(x = \omega|t_I = t-1)$, we have $G_0(\frac{k+1}{2}) = Pr(\omega = 0|t_I = \frac{k-1}{2})$. Because of the symmetry of binomial distribution, we know $Pr(\omega = 0|t_I = \frac{k-1}{2}) = Pr(\omega = 1|t_I = \frac{k+1}{2})$. So, $G_0(\frac{k+1}{2}) = Pr(\omega = 1|t_I = \frac{k+1}{2})$. It follows that

$$\max\{2Pr(\omega = 0|t_I = \frac{k+1}{2}) - G_0(\frac{k+1}{2}), G_0(\frac{k+1}{2}), Pr(\omega = 0|t_I = \frac{k+1}{2})\} \geq Pr(\omega = 1|t_I = \frac{k+1}{2}).$$

When $t_I \neq \frac{k+1}{2}$, the derivation of voting for 0 does not affect the policy. And, if $G_x(t) = Pr(x = \omega|t_I = t-1)$, we have

$$\max\{2Pr(\omega = x|t_I) - G_x(t_I), G_x(t_I), Pr(\omega = x|t_I)\} > Pr(\omega = x|t_I).$$

Thus, we know when $G_x(t) = Pr(x = \omega|t_I = t-1)$, $EU(v_i^\emptyset = \emptyset \xrightarrow{D} 0) - EU(v_i^\emptyset = \emptyset) > 0$

To summarize, as long as $G_x(t) \neq Pr(x = \omega|t_I = t-1)$ for some realization(s) of t, voting for 1 is a profitable deviation. If $G_x(t) = Pr(x = \omega|t_I = t-1)$ for all t, then voting for 0 is a profitable deviation. Thus, either voting for 1 or voting for 0 or both must yield a higher payoff than abstaining. ■

B.5 Proof of Proposition 7

Proof. Start with $q_0 \neq q_1$ and conjecture that a type symmetric equilibrium described by $z_1, z_0, m_\emptyset$ exists. We derive the relevant equilibrium conditions and show that they are continuous in the parameters q_0, q_1 as well as the equilibrium quantities $z_1, z_0, m_\emptyset$. We then show that for some $\delta > 0$ if $|q - q_1| < \delta$ and $|q - q_0| < \delta$ then equilibrium quantities exist. Thus, if q_1, q_0 are close enough to q, a solution $(z_0, z_1, m_\emptyset)$ exists and the solution converges to $(z, z, \frac{1}{2})$. First, we derive the pricing function and equilibrium conditions when $q_1 \neq q_0$. The pricing function becomes

$$P_x(t) = E[v(x,\omega)|x,t] = \begin{cases} Pr(\omega = 1|t), & \text{if } x = 1 \\ 1 - Pr(\omega = 1|t), & \text{if } x = 0 \end{cases} \tag{57}$$

where,

$$Pr(\omega = 1|t) = \frac{Pr(t|\omega = 1)}{Pr(t|\omega = 1) + Pr(t|\omega = 0)} \tag{58}$$

$$Pr(t|\omega = 1) = \sum_{i=0}^{t} \binom{k}{i} z_1^i (1 - z_1)^{k-i} \binom{n-k}{t-i} m_\emptyset^{t-i} (1 - m_\emptyset)^{n-k-t+i} \tag{59}$$

and

$$Pr(t|\omega = 0) = \sum_{i=0}^{t} \binom{k}{i} (1 - z_0)^i z_0^{k-i} \binom{n-k}{t-i} m_\emptyset^{t-i} (1 - m_\emptyset)^{n-k-t+i}. \tag{60}$$

Second, we derive the equilibrium conditions on $(z_0, z_1, m_\emptyset)$. Shareholders' trading strategies are the same as Lemma 2. So, the voting strategies at equilibria are pinned down by the following three equations.

$$\begin{aligned} & EU(v_i^1 = 1) - EU(v_i^1 = 0) \\ &= Pr(t' = \frac{n-1}{2}|s_i = 1)(2Pr(\omega = 1|s_i = 1, t' = \frac{n-1}{2}) - 1) \\ &- \sum_{t'=0}^{n-1} Pr(t'|s_i = 1)(Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t')) \end{aligned} \tag{61}$$

,

$$\begin{aligned} & EU(v_i^0 = 0) - EU(v_i^0 = 1) \\ &= Pr(t' = \frac{n-1}{2}|s_i = 0)(2Pr(\omega = 0|s_i = 0, t' = \frac{n-1}{2}) - 1) \\ &- \sum_{t'=0}^{n-1} Pr(t'|s_i = 0)(Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t')) \end{aligned} \tag{62}$$

, and

$$EU(v_i^{\emptyset}=1)-EU(v_i^{\emptyset}=0)$$
$$=\sum_{t'=0}^{\frac{n-1}{2}-1}Pr_U(t')(2Pr_U(\omega=0|t')-Pr(\omega=0|t'+1))+\sum_{t'=\frac{n-1}{2}}^{n-1}Pr_U(t')Pr(\omega=1|t'+1)$$
$$-\sum_{t'=0}^{\frac{n-1}{2}}Pr_U(t')Pr(\omega=0|t')-\sum_{t'=\frac{n-1}{2}+1}^{n-1}Pr_U(t')(2Pr_U(\omega=1|t')-Pr(\omega=1|t')) \tag{63}$$

Third, we can prove that Equation (61) and Equation (62) both converge to Equation (8) and Equation (63) converges to 0, when $q_1\to q$, $q_0\to q$, $z_0\to z$, $z_1\to z$, and $m_{\emptyset}\to\frac{1}{2}$. To prove this, we show that all components of the above three equations and the pricing function (e.g., $Pr(t'|s_i)$, $Pr(\omega|s_i,t')$, $Pr(\omega|t')$) converge to their corresponding expressions in the maim model where $q_1=q_0=q$. This step involves standard and tedious algebraic manipulations. So, we only exhibit $\lim_{q_1,q_0\to q,z_0,z_1\to z,m_{\emptyset}\to\frac{1}{2}}Pr(t'|s_i=1)$ as an example here. Note that

$$Pr(t'|s_i=1)$$
$$=\frac{Pr(t',s_i=1|\omega=1)Pr(\omega=1)+Pr(t',s_i=1|\omega=0)Pr(\omega=0)}{Pr(s_i=1)}$$
$$=[\sum_{i=0}^{t'}\binom{k-1}{i}z_1^i(1-z_1)^{k-1-i}\binom{n-k}{t-i}m_{\emptyset}^{t'-i}(1-m_{\emptyset})^{n-k-t'+i}q_1$$
$$+\sum_{i=0}^{t'}\binom{k-1}{i}(1-z_0)^i z_0^{k-1-i}\binom{n-k}{t'-i}m_{\emptyset}^{t'-i}(1-m_{\emptyset})^{n-k-t'+i}(1-q_0)]/(q_1+1-q_0) \tag{64}$$

Then,

$$\begin{aligned}&\lim_{q_0,q_1\to q,z_0,z_1\to z,m_{\emptyset}\to\frac{1}{2}}Pr(t'|s_i=1)\\&=\sum_{i=0}^{t'}\binom{k-1}{i}(z)^i(1-z)^{k-1-i}\binom{n-k}{t'-i}(\frac{1}{2})^{n-k}q_1\\&+\sum_{i=0}^{t'}\binom{k-1}{i}(1-z)^i(z)^{k-1-i}\binom{n-k}{t'-i}(\frac{1}{2})^{n-k}(1-q_0)\end{aligned} \tag{65}$$

which is equal to $Pr(t'|s_i=1)$ in the main model where $q_0=q_1=q$. Similarly, we can prove other components of Equation (61) to Equation (63) also converge to their corresponding expressions in the maim model where $q_1=q_0=q$. Thus, the intermediate value theorem arguments employed above can be used to establish that for some $\delta<0$ if $max\{|q-q_0|,|q-q_1|\}<\delta$ solutions $z_0,z_1,m_{\emptyset}$ exist. Moreover, since the conditions are continuous in $q_0, q_1, z_0, z_1, m_\emptyset$ when $q_0 \to q$, $q_1 \to q$, we know that for any selection of the solutions $z_0 \to z$, $z_1 \to z$, and $m_\emptyset \to \frac{1}{2}$. ■

B.6 Proof of Proposition 8

Proof. We first analyze a baseline where trading is impossible and compare the level of information aggregation to that of our the main model with trade.

Lemma 9 (Baseline with No Trading). *1. For fixed n, q, c, the equilibrium that maximizes the probability of making the correct policy choice has the following form: For the largest value of k satisfying*

$$\binom{k}{\frac{k+1}{2}} q^{\frac{k+1}{2}} (1-q)^{\frac{k-1}{2}} (2q-1) \leq c \leq \binom{k-1}{\frac{k-1}{2}} q^{\frac{k-1}{2}} (1-q)^{\frac{k-1}{2}} (2q-1) \tag{66}$$

when k is an odd number, or

$$\begin{aligned} &\left(\frac{1}{2}\binom{k}{\frac{k}{2}} q^{\frac{k}{2}}(1-q)^{\frac{k}{2}} + \frac{1}{2}\binom{k}{\frac{k}{2}-1} q^{\frac{k}{2}-1}(1-q)^{\frac{k}{2}+1}\right)(2q-1) \\ &\leq c \leq \\ &\left(\frac{1}{2}\binom{k-1}{\frac{k}{2}} q^{\frac{k}{2}}(1-q)^{\frac{k}{2}-1} + \frac{1}{2}\binom{k-1}{\frac{k}{2}-1} q^{\frac{k}{2}-1}(1-q)^{\frac{k}{2}}\right)(2q-1) \end{aligned} \tag{67}$$

when k is an even number. a subset of size k of the n shareholders select $a_i = 1$ and $v_i^{s_i} = s_i$ for $s_i \in \{0, 1\}$, and the remaining $n - k$ shareholders select $a_i = 0$ and split their votes. In particular, if $n - k$ is even, then exactly half of these $n - k$ shareholders vote for 0, and exactly half of them vote for 1. If $n - k$ is odd, then one mixes voting for each alternative with probability $\frac{1}{2}$, and $\frac{n-k-1}{2}$ vote for each alternative in pure strategies.

2. *For fixed n, c, q, let k^* denote the equilibrium level of k. When k^* is odd, the correct policy is chosen with probability,*

$$Pr(x = \omega) = \sum_{j=\frac{k^*+1}{2}}^{k^*} \binom{k^*}{j} q^j (1-q)^{k^*-j}$$

When k^* is even, the probability the correct policy is selected is

$$Pr(x = \omega) = \frac{1}{2}\left(\sum_{j=\frac{k^*}{2}}^{k^*} \binom{k^*}{j} q^i (1-q)^{k^*-j} + \sum_{j=\frac{k^*}{2}+1}^{k^*} \binom{k^*}{j} q^i (1-q)^{k^*-j}.\right)$$

3. *$Pr(x=\omega)$ is increasing in k^* and converges to 1 if k^* and n go to infinity.*

Proof. Consider the game where trading is not possible. We construct an equilibrium where informed shareholders vote sincerely and uninformed shareholders use pure strategies to sort, minimizing their impact on the choice. Much of the logic here is not novel to our paper. Because a vote only matters in a pivotal event, it is enough to pin down the voting in any equilibrium by focusing on these events. Note that depending on whether n and k are odd or the pivotal events are slightly different.

First, we focus on the pivotal event for the case where k is odd. Consider a shareholder who is expected to buy a signal in equilibrium. If she gets signal 1, she will vote for policy 1, and the policy 1 will be implemented. Without the signal, she would randomly vote, and thus, each policy is chosen with a half probability.[21] Thus, the value of $s_i = 1$ to her is

$$\begin{aligned}&Pr_I(\omega=1|PIV,s_i=1)-[\frac{1}{2}Pr_I(\omega=1|PIV,s_i=1)+\frac{1}{2}Pr_I(\omega=0|PIV,s_i=1)]\\&=\frac{1}{2}[Pr_I(\omega=1|PIV,s_i=1)-Pr_I(\omega=0|PIV,s_i=1)]\end{aligned}\tag{68}$$

Similarly, if she gets signal 0, the value of $s_i = 0$ to her is

$$\begin{aligned}&Pr_I(\omega=0|PIV,s_i=0)-[\frac{1}{2}Pr_I(\omega=1|PIV,s_i=0)+\frac{1}{2}Pr_I(\omega=0|PIV,s_i=0)]\\&=\frac{1}{2}[Pr_I(\omega=1|PIV,s_i=0)-Pr_I(\omega=0|PIV,s_i=0)]\end{aligned}\tag{69}$$

Then, the value of acquiring a signal is

$$\begin{aligned}&Pr(s_i=1)Pr_I(PIV|s_i=1)\frac{1}{2}[Pr_I(\omega=1|PIV,s_i=1)-Pr_I(\omega=0|PIV,s_i=1)]\\&+Pr(s_i=0)Pr_I(PIV|s_i=0)\frac{1}{2}[Pr_I(\omega=1|PIV,s_i=0)-Pr_I(\omega=0|PIV,s_i=0)]\\&=\binom{k-1}{\frac{k-1}{2}}q^{\frac{k-1}{2}}(1-q)^{\frac{k-1}{2}}(2q-1)\end{aligned}\tag{70}$$

To ensure the informed shareholder does not deviate from the equilibrium, we must have

$$c\leq\binom{k-1}{\frac{k-1}{2}}q^{\frac{k-1}{2}}(1-q)^{\frac{k-1}{2}}(2q-1)\tag{71}$$

Now, consider a shareholder who is expected to be uninformed and vote for 1 in equilibrium. If she deviates to $a_i = 1$ and receives signal 1, she will vote for 1. Since she will vote for 1 even if she does not buy a signal, her expected value of $s_i = 1$ is

[21] Other deviation strategies, such as purely voting for 0, purely voting 0, or mixing with different probabilities, yield the same expected payoff.

zero. If she gets signal $s_i = 0$, she will vote for 0. Then, in the event of being pivotal ($\frac{k+1}{2}$ informed shareholders vote for policy 1), the value of the signal $s_i = 0$ is

$$Pr_U(\omega = 0|PIV, s_i = 0) - Pr_U(\omega = 1|PIV, s_i = 0) \tag{72}$$

Thus, the value of acquiring a signal for a shareholder who is expected to be uninformed and vote for 1 is

$$\begin{aligned}&Pr(s_i = 0)Pr_U(PIV|s_i = 0)[Pr_U(\omega = 0|PIV, s_i = 0) - Pr_U(\omega = 1|PIV, s_i = 0)]\\&= \binom{k}{\frac{k+1}{2}} q^{\frac{k+1}{2}}(1-q)^{\frac{k-1}{2}}(2q-1)\end{aligned} \tag{73}$$

Note that the value of acquiring a signal for a shareholder who is expected to be uninformed and vote for 0 is also $\binom{k}{\frac{k+1}{2}} q^{\frac{k+1}{2}}(1-q)^{\frac{k-1}{2}}(2q-1)$.

Thus, to ensure an uninformed shareholder does not want to deviate and acquire a signal, the information cost must satisfy

$$\binom{k}{\frac{k+1}{2}} q^{\frac{k+1}{2}}(1-q)^{\frac{k-1}{2}}(2q-1) \le c \tag{74}$$

To conclude, to sustain the equilibrium in which exactly k (an odd number) shareholders acquire information, the information cost c must satisfy

$$\binom{k}{\frac{k+1}{2}} q^{\frac{k+1}{2}}(1-q)^{\frac{k-1}{2}}(2q-1) \le c \le \binom{k-1}{\frac{k-1}{2}} q^{\frac{k-1}{2}}(1-q)^{\frac{k-1}{2}}(2q-1) \tag{75}$$

Now we analyze the case that k is an even number. We denote the vote by the uninformed voter who randomly votes with v_r.

First, consider a shareholder who is expected to buy a signal in equilibrium. If she does not acquire a signal, she could simply vote for 0.[22] Then, the expected value of signal $s_i = 0$ is zero for her. On the other hand, if she gets $s_i = 1$, she will vote for 1. The value of acquiring a signal to her is

$$\begin{aligned}&Pr(s_i = 1, v_r = 0)Pr_I(PIV|s_i = 1, v_r = 0)\\&\times [Pr_I(\omega = 1|s_i = 1, PIV, v_r = 0) - Pr_I(\omega = 0|s_i = 1, PIV, v_r = 0)]\\&+ Pr(s_i = 1, v_r = 1)Pr_I(PIV|s_i = 1, v_r = 1)\\&\times [Pr_I(\omega = 1|s_i = 1, PIV, v_r = 1) - Pr_I(\omega = 0|s_i = 1, PIV, v_r = 1)]\\&= \frac{1}{2}\binom{k-1}{\frac{k}{2}} q^{\frac{k}{2}}(1-q)^{\frac{k}{2}-1}(2q-1) + \frac{1}{2}\binom{k-1}{\frac{k}{2}-1} q^{\frac{k}{2}-1}(1-q)^{\frac{k}{2}}(2q-1)\\&= \left(\frac{1}{2}\binom{k-1}{\frac{k}{2}} q^{\frac{k}{2}}(1-q)^{\frac{k}{2}-1} + \frac{1}{2}\binom{k-1}{\frac{k}{2}-1} q^{\frac{k}{2}-1}(1-q)^{\frac{k}{2}}\right)(2q-1)\end{aligned} \tag{76}$$

[22] All other deviation strategies will lead to the same expected payoff.

Thus, in order for an informed shareholder to not deviate, the information cost c needs to satisfy

$$c \leq \left(\frac{1}{2}\binom{k-1}{\frac{k}{2}} q^{\frac{k}{2}}(1-q)^{\frac{k}{2}-1} + \frac{1}{2}\binom{k-1}{\frac{k}{2}-1} q^{\frac{k}{2}-1}(1-q)^{\frac{k}{2}} \right)(2q-1) \tag{77}$$

Now, consider a shareholder who is expected to be uninformed and vote for 0. Consider the pivotal event. If she buys a signal and gets signal $s_i = 0$, she will vote for policy 0, which is the same strategy as if she is uninformed. Thus, the expected value of $s_i = 0$ is zero for her. On the other hand, if she gets signal $s_i = 1$, she will vote for policy 1. Overall, the value of acquiring a signal to her is

$$\begin{aligned}
&Pr(s_i = 1, v_r = 0) Pr_U(PIV|s_i = 1, v_r = 0)\\
&\times [Pr_U(\omega = 1|s_i = 1, PIV, v_r = 0) - Pr_U(\omega = 0|s_i = 1, PIV, v_r = 0)]\\
&+ Pr(s_i = 1, v_r = 1) Pr_U(PIV|s_i = 1, v_r = 1)\\
&\times [Pr_U(\omega = 1|s_i = 1, PIV, v_r = 1) - Pr_U(\omega = 0|s_i = 1, PIV, v_r = 1)]\\
&= \frac{1}{2}\binom{k}{\frac{k}{2}} q^{\frac{k}{2}}(1-q)^{\frac{k}{2}}(2q-1) + \frac{1}{2}\binom{k}{\frac{k}{2}-1} q^{\frac{k}{2}-1}(1-q)^{\frac{k}{2}+1}(2q-1)\\
&= \left(\frac{1}{2}\binom{k}{\frac{k}{2}} q^{\frac{k}{2}}(1-q)^{\frac{k}{2}} + \frac{1}{2}\binom{k}{\frac{k}{2}-1} q^{\frac{k}{2}-1}(1-q)^{\frac{k}{2}+1} \right)(2q-1)
\end{aligned} \tag{78}$$

Note that the value of acquiring a signal to a shareholder who is expected to be uninformed and vote for 1 and a shareholder who is expected to be uninformed and randomly vote is also $\left(\frac{1}{2}\binom{k}{\frac{k}{2}} q^{\frac{k}{2}}(1-q)^{\frac{k}{2}} + \frac{1}{2}\binom{k}{\frac{k}{2}-1} q^{\frac{k}{2}-1}(1-q)^{\frac{k}{2}+1} \right)(2q-1)$

Hence, to support an equilibrium in which exactly k (an even number) shareholders acquire information, the information cost c needs to satisfy

$$\begin{aligned}
&\left(\frac{1}{2}\binom{k}{\frac{k}{2}} q^{\frac{k}{2}}(1-q)^{\frac{k}{2}} + \frac{1}{2}\binom{k}{\frac{k}{2}-1} q^{\frac{k}{2}-1}(1-q)^{\frac{k}{2}+1} \right)(2q-1)\\
&\leq c \leq\\
&\left(\frac{1}{2}\binom{k-1}{\frac{k}{2}} q^{\frac{k}{2}}(1-q)^{\frac{k}{2}-1} + \frac{1}{2}\binom{k-1}{\frac{k}{2}-1} q^{\frac{k}{2}-1}(1-q)^{\frac{k}{2}} \right)(2q-1)
\end{aligned} \tag{79}$$

Finally, for each ω, the quantity $Pr(t|\omega)$ is given from a Binomial and with $z = q > \frac{1}{2}$, $Pr(t \geq \frac{n+1}{2}|\omega = 1)$ and $Pr(t < \frac{n+1}{2}|\omega = 0)$ are known to converge to 1. ■

Now, we can prove the statement of Proposition 7.

Because $Pr(x = \omega; \kappa(n))$ is increasing in n and $\lim_{n\to\infty} Pr(x = \omega; \kappa(n)) = \Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right) < q$, we know

$$Pr(x = \omega; \kappa(n)) < q, \quad \textit{for any } n$$

When $k = 1$, in the optimal equilibrium of the game without trading, $Pr(x = \omega; \textit{no trading}) = q$. When there is no trading, to sustain the optimal equilibrium with $k \geq 1$, the information cost c must satisfy

$$EU(a_i = 1, k = 1) - EU(a_i = 1 \xrightarrow{\text{D}} 0, k = 1) = q - c - \frac{1}{2} \geq 0 \tag{80}$$

, which is equivalent to

$$c \leq q - \frac{1}{2}. \tag{81}$$

Furthermore, as k increases, $Pr(x = \omega; \textit{no trading})$ also increases. So, we have

$$Pr(x = \omega; \textit{no trading}) > Pr(x = \omega; \textit{trading}) \quad \textit{as long as } c \leq q - \frac{1}{2}.$$

■

B.7 Proof of Proposition 10

Proof. From the proof of Proposition 3, we know that when $\frac{k}{n} = \kappa(n)$, each informed shareholder's expected payoff without considering information cost is $2q\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)$, while each uninformed shareholder's expected payoff is $\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right)$. In contrast, for any sequence with k going to infinity in the game without trading, in equilibrium each informed shareholder's expected payoff without considering information cost converges to 1, and each uninformed shareholder's expected payoff also converges to 1. Note that the CDF function

$$\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right) < 1 \quad \textit{for any } q \in (\frac{1}{2}, 1)$$

So, an uninformed shareholder's expected payoff is strictly higher when trading is not possible.

Note that

$$\frac{d(2q\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right) - 1)}{dq} > 0 \tag{82}$$

and

$$\lim_{q \to 1} 2q\Phi\left(\sqrt{\frac{2}{\pi}}(2q-1)\right) = 2\Phi\left(\sqrt{\frac{2}{\pi}}\right) > 1 \tag{83}$$

Thus, there is a unique q^* such that when $q > q^*$, each informed shareholder's expected payoff is higher when there is trading opportunity. But when $q < q^*$, each informed shareholder's expected payoff is high when trading is impossible. The q^* is determined by

$$2q^*\Phi\left(\sqrt{\frac{2}{\pi}}(2q^*-1)\right)-1=0 \tag{84}$$

which implies that $q^* \approx 0.76$ ■

B.8 Proof of Lemma 7

Proof. Suppose without the loss of generality that an informed shareholder receives $s_i = 1$. When $x = 1$, her expectation of the share value is $Pr_I(\omega = 1|x = 1, s_i = 1)$, while the price is $P_1 = Pr(\omega = 1|x = 1)$. $Pr_I(\omega = 1|x = 1, s_i = 1) > Pr(\omega = 1|x = 1)$ immediately follows from Bayes' rule. Thus, she wants to buy. When $x = 0$, she expects the share value to be $Pr_I(\omega = 0|x = 0, s_i = 1)$, while the price is $P_0 = Pr(\omega = 0|x = 0)$. Since $Pr_I(\omega = 0|x = 0, s_i = 1) < Pr(\omega = 0|x = 0)$, she wants to sell. Similarly, consider an uninformed shareholder who votes for 1. When $x = 1$, her expectation of share value is smaller than the price, $Pr_U(\omega = 1|v_i^\emptyset = 1, x = 1) < Pr(\omega = 1|x = 1)$ as a result of Bayes' rule. So, she wants to sell one share. If $x = 0$, since $Pr_U(\omega = 0|v_i^\emptyset = 1, x = 0) > Pr(\omega = 1|x = 0)$, the uninformed shareholder wants to buy one share if the chosen policy is different from her vote. ■

B.9 Proof of Lemma 8

Proof. Consider an informed shareholder whose signal is 1. The difference between her expected utility from voting for 1 and her expected utility from voting for 0 is

$$\begin{aligned}&EU(v_i^1=1)-EU(v_i^1=0)\\&=Pr(PIV|s_i=1)(2Pr(\omega=1|PIV,s_i=1)-P_1-P_0)\\&=Pr(PIV|s_i=1)(2q-2Pr(x=\omega))\end{aligned} \tag{85}$$

To sustain an equilibrium in which informed shareholders sincerely vote, it must be $EU(v_i^1 = 1) - EU(v_i^1 = 0) \geq 0$, which implies that $q \geq Pr(x = \omega)$. Similarly, to sustain an equilibrium in which informed shareholders take a mixed voting strategy, it must be $EU(v_i^1 = 1) - EU(v_i^1 = 0) = 0$, which implies that $q = Pr(x = \omega)$.

Finally, we verify that $Pr(v_i^\emptyset = 1) = Pr(v_i^\emptyset = 0) = \frac{1}{2}$ can sustain the equilibrium. For an uninformed shareholder, the difference between her expected utility from voting for 1 and her expected utility from voting for 0 is

$$\begin{aligned}&EU(v_i^{\emptyset}=1)-EU(v_i^{\emptyset}=0)\\&=\sum_{t'=0}^{\frac{n-1}{2}-1}Pr_U(t')(2Pr_U(\omega=0|t')-P_0)+\sum_{t'=\frac{n-1}{2}}^{n-1}Pr_U(t')P_1\\&-\left(\sum_{t'=0}^{\frac{n-1}{2}}Pr_U(t')P_0+\sum_{t'=\frac{n+1}{2}}^{n-1}Pr_U(t')(2Pr_U(\omega=1|t')-P_1)\right)\end{aligned}\tag{86}$$

Using the fact that $P_1 = P_0$ and the symmetry of binomial distribution and substituting $Pr(v_i^{\emptyset} = 1) = Pr(v_i^{\emptyset} = 0) = \frac{1}{2}$, the above equation is simplified to

$$Pr_U(t' = \frac{n-1}{2})(P_1 - P_0)\tag{87}$$

, which is equal to 0. ■

B.10 Proof of Proposition 11

Proof. The right-hand side of inequality (18) is equal to $Pr(x = \omega)$, which is increasing in k. And, when $k = 0$, $Pr(x = \omega) = (\frac{1}{2})^n < q$. When $k = n$, $Pr(x = \omega) = \sum_{t=\frac{n+1}{2}}^{n} \binom{n}{t} q^t (1-q)^{n-t} > q$. Thus, $k^*(n)$ exists and is unique. ■

B.11 Proof of Proposition 13

Proof. First, note that the trading strategies of informed shareholders are the same as in Lemma 2. This is because Bayes' rule implies that $Pr(x = \omega|x, s_j) \geq Pr(x = \omega|x)(Pr(x = \omega|x, s_j) \leq Pr(x = \omega|x))$ when $s_j = x$ ($s_j \neq x$). Suppose without the loss of generality that $s_j = 1$. Then, the difference between the expected payoff from voting for 1 and the expected payoff from voting against 1 is

$$\begin{aligned}&EU(v_j^1=1)-EU(v_j^1=0)\\&=Pr(PIV|s_j=1)(2V_jPr(x=1|PIV,s_j=1)-V_jP_1-V_jP_0)\end{aligned}\tag{88}$$

Since $Pr(x = 1|PIV, s_j = 1) = q_j$ and $P_1 = P_0 = Pr(x = \omega)$, the condition $EU(v_j^1 = 1) - EU(v_j^1 = 0) \geq 0$ implies that

$$q_j \geq Pr(x = \omega)\tag{89}$$

■