Strategic (Dis)Integration*

John Sturm Becko
Princeton

Daniel G. O'Connor
MIT

For latest version, click here.

Abstract

Suppose a country anticipates that it may use trade as a point of leverage in future geopolitical conflicts. How should it develop domestic industries and international trading relationships today in order to strengthen its hand tomorrow? Domestically, we show that the country should abstain from peacetime industrial policies if it can credibly threaten trade taxes as geopolitical punishments during conflict, but not otherwise. Internationally, its peacetime trade policy should promote the accumulation of foreign capital that makes foreign prices—not foreign welfare—more sensitive to trade during conflict. We apply these insights to provide the first quantitative exploration of the US's optimal policies for building geopolitical power vis-à-vis China. The optimal policy promotes US-China trade on both the import and export margins.

*This version: January 2025. First version: March 2024. Author contacts: johnsbecko@gmail.com and doconn@mit.edu. We are grateful for the suggestions and comments of Daron Acemoglu, Rodrigo Adao, Pol Antras, Christopher Clayton, Arnaud Costinot, Farid Farrokhi, Cecile Gaubert, Olivier Kooi, Aaditya Mattoo, Atif Mian, Diego Perez, Steve Redding, Michele Ruta, Jesse Schreger, Ali Shourideh, Martin Stuermer, Bob Staiger, Andre Sztutman, Ivan Werning, Robert Zymek, and conference and seminar participants at the Princeton IES Summer Trade Workshop; NBER SI International Economics and Geopolitics; Harvard; Dartmouth; IMF Research Department; CEPR/Bocconi Double Workshop in Geoeconomics, Geopolitics, and Preventing Conflict Studies; Kiel Institute Workshop on Sanctions Effectiveness; Princeton Macro Faculty Lunch; and MIT Trade Lunch. We would especially like to thank Xiang Ding for sharing data.

1 Introduction

Recent events have prompted countries around the world to reevaluate their economic interdependence. In Europe, the European Union's trade relationship with Russia made it possible to punish Russia's invasion of Ukraine using sanctions. Yet, this relationship's interruption caused economic distress in the EU, prompting "de-risking," "onshoring," and "friendshoring" policies to mitigate future geopolitical risks. Across the Atlantic, the United States passed sweeping legislation promoting its domestic semiconductor industry. It also banned exports of certain chips and prevented US engineers from working in some Chinese industries. While some policymakers justified these actions on narrow economic and security grounds, others have offered explicitly geopolitical rationales about building strategic (in)dependence.

These events bring to the fore classic questions of economic statecraft—i.e., the use of economic tools to achieve foreign policy goals. From a positive perspective, how can we understand countries' geopolitical incentives for economic policies beyond standard trade sanctions? From a normative perspective, how should such policies be designed to accomplish their aims without inflicting undue economic costs?

We shed new light on these questions by embedding a geopolitical game into an otherwise standard neoclassical model of international trade. In the model, countries have preferences over their own economic welfare and each other's geopolitical behavior. For example, the US may prefer that China respect Hong Kong's autonomy, or that Russia abstain from trading arms with North Korea. The economic and geopolitical blocks of the model interact through economic threats—in the form of trade sanctions—that a country can use to influence foreign geopolitical behavior. We study how a country that anticipates making these threats in the future can adopt forward-looking policies in peacetime to increase its leverage.

Our main results characterize the optimal design of two peacetime policy instruments: industrial policy, in the form of domestic capital subsidies, and trade policy, in the form of import tariffs and export taxes. The geopolitical block of our model is structured enough that we can deliver sharp, qualitative results about the geopolitical rationale for using each instrument. In terms of peacetime industrial policy, geopolitics rationalize capital subsidies only for governments with limited ability to threaten trade taxes during conflicts. In terms of peacetime trade policy, geopolitics rationalize taxes with a novel target: the *manipulability* of foreign terms of trade. The economic block of our model is rich enough that we can explore the policy implications of these insights through the lens of a state-of-the-art quantitative trade model. We quantify the US's optimal policies for building geopolitical power vis-à-vis China, finding that they promote both bilateral imports and bilateral exports with China relative to trade with the rest of the world.

The starting point for our analysis is a new, dynamic model of trade and geopolitics. The baseline model contains two time periods—peacetime and conflict—and two countries—Home and Foreign. Each country contains a representative household, goods producer, and capital producer who exchange many goods and many types of capital. In the first period, consumption, capital formation, production, and trade occur in the standard fashion, possibly subject to capital subsidies and/or trade taxes. We assume capital adjusts slowly, so that each country's capital stock during conflict is simply what it formed during peacetime. This assumption is well-suited to types of capital that take a long time to build, like factories, and those which by their very nature must be accumulated ahead of time, like stockpiles.

The model's second period combines economics and geopolitics. First, countries play a game that determines a geopolitical action taken by Foreign and second-period trade taxes imposed by Home. Home makes a trade policy threat—possibly subject to credibility constraints—that specifies its trade taxes as a function of Foreign's geopolitical action. Foreign then chooses a geopolitical action by trading off its geopolitical benefits, which we take as exogenous, against their economic costs, which are endogenously determined by Home's threat. This action and Home's threat together determine Home's realized trade taxes. After the conclusion of this game, households and firms consume, produce, and trade once more. Crucially, they do so using the capital accumulated during peacetime and subject to the trade taxes determined by the game described above.[1]

Within this setting, we study Home's optimal peacetime industrial and trade policies.[2] We do so both in the case where Home can credibly threaten any trade taxes during conflict and in the case where its threats are constrained to trade taxes that would not lower Home welfare beyond a certain level.

Our first main result shows that, when all trade threats are credible, Home's optimal industrial policy is laissez-faire (i.e., capital subsidies are zero). This is counterintuitive since private firms fail to internalize that their capital investments impact Foreign's economic incentive to avoid trade sanctions and, hence, its geopolitical actions. Since private investments have these geopolitical externalities, it is natural to think they should be taxed or subsidized. In fact, such interventions are suboptimal. This reflects that Home's trade threat already balances the geopolitical benefit of influencing Foreign's actions against the economic cost of trade distortions. As a result, whatever marginal changes in Home's trade under conflict are induced by additional Home investments have economic costs that, to first-order, offset

[1]Although this baseline model is intentionally stylized, we provide generalized versions of our results in several extensions. We extend the model to include geopolitical uncertainty, Foreign economic policies, Home geopolitical actions, non-separability of economic and geopolitical preferences, and many countries.

[2]Home's optimal trade threats are closely related to Becko (2024)'s results on optimal sanctions and are not our main focus.

their geopolitical benefits. This finding calls into question the geopolitical rationale for recent industrial policies such as the CHIPS Act.

If industrial policies are an inefficient way to prepare for geopolitical conflict, why are they used in practice? Our next result provides a potential answer by considering the possibility that Home cannot credibly threaten trade taxes that would lower its own economic welfare too much. For example, the EU decided not to impose any immediate sanctions on imports of natural gas from Russia following its invasion of Ukraine because member countries, particularly Germany, feared severe economic fallout (see, e.g., Moll et al. (2023)). We show that in the presence of such a credibility constraint, industrial policy during peacetime emerges as a second-best instrument. The optimal capital subsidies take a simple form, promoting each type of capital in proportion to its rental rate under whichever threatened trade taxes have the highest cost for Home. We argue this result is consistent with recent industrial policies promoting energy independence and semiconductor manufacturing.

The second part of the paper turns to peacetime trade policies. While Home's industrial policies allow it to shape domestic capital accumulation, its trade taxes offer a more direct instrument for targeting changes in Foreign capital. But which changes in Foreign capital serve Home's geopolitical interests? We show these are *not* simply the changes in capital that increase the difference in Foreign's economic utility across its geopolitical actions. This is because part of the difference can be targeted in a less distortionary way using Home's trade threats in the second period. The remainder of this difference—what Home's optimal peacetime trade taxes *do* target—corresponds to changes in Foreign capital that make Foreign's terms of trade more manipulable during conflict. This result's most striking implication is that Home abstains from peacetime trade policy when it cannot make Foreign prices more sensitive to traded quantities. For example, suppose Foreign is a gas exporter who can invest in pipelines that uniformly decrease its unit costs of exporting to Home. Home can subsidize trade to promote these investments and so increase Foreign's gains from trade and hence its economic incentive to choose Home's preferred action. Yet, Home chooses not to because such investments do not make Foreign's export supply curve any less elastic.

Unlike its capital subsidies, Home's optimal peacetime trade policies do not directly depend on the credibility of its trade threats during conflict.[3] Our results therefore establish a sharp contrast between Home's peacetime industrial and trade policies. Industrial policies limit Home's dependence on Foreign to strengthen Home's credibility. Simultaneously, trade policies promote Foreign's dependence on Home, in the sense of making its net import demand less elastic and hence its terms of trade more manipulable.

[3]This is because Home capital affects credibility directly whereas Foreign capital only affects credibility through second-period trade, which Home can target directly using its second-period trade threats.

The final part of the paper uses a dynamic, quantitative trade model to explore our theory's practical implications. To do so, we specialize the many-country extension of our general framework to a parametric model that we then calibrate to data. Our starting point is a standard, multi-sector model with trade in intermediates. We introduce dynamics by incorporating two types of capital. Production capital is sector-specific and lowers firms' variable costs in that sector. Relationship capital is sector-by-origin-specific and lowers the trade costs associated with purchasing inputs from that origin-sector, whether foreign or domestic. These two types of capital offer two different channels through which Home's peacetime trade taxes can make Foreign's terms of trade more manipulable. We calibrate this model in 2017 using data on global trade, tariffs, value-added, and input-output networks. We also incorporate time-varying trade elasticity estimates from Boehm et al. (2020) and country-sector-specific estimates of the sectoral composition of investment from Ding (2021).

We combine our theoretical results with this calibrated model to compute the US's optimal peacetime policies for building geoeconomic power vis-à-vis China. We focus on the case where US trade threats are perfectly credible, which implies it abstains from industrial policy, and so limit our attention to peacetime trade policy. We find that the US's optimal peacetime trade policy fosters Chinese dependence by promoting bilateral trade on both the import and export margins. In both cases, the US's primary motive is to make China's terms of trade more manipulable by shifting its relationship capital investments towards foreign trading partners and away from domestic relationships. While the US's optimal import tariffs are close to uniform across sectors, its optimal export subsidies promote trade more in sectors that China uses primarily for final consumption. Finally, the US imposes more modest trade taxes on other foreign countries to foster China's dependence on third parties.

Related literature In the wake of global supply disruptions triggered by COVID-19 and Western sanctions on Russia, a growing literature has emphasized countries' exposure to global supply chain risk and considered policies such as onshoring or friendshoring (see, e.g., Huang (2017); Elliott et al. (2022); Baldwin et al. (2023)). Relevant to our work, recent theoretical contributions by Traiberman and Rotemberg (2023), Grossman et al. (2023), and Grossman et al. (2024) show how market failures such as learning-by-doing and imperfect competition can lead to excessive economic integration and rationalize re-shoring policies. While we consider similar policy instruments, we intentionally abstract from non-geopolitical market failures by assuming markets are efficient and focusing on the case where the Home planner places the same weight on Home and Foreign economic welfare. This isolates the explicitly geopolitical rationale for policy. Indeed, if Foreign's geopolitical action is exogenous, then laissez faire is optimal in the model. When Foreign's action is endogenous to its economic incentives, a simple Pigouvian tax would implement the first-best allocation and render other policies unnecessary. But—unlike a Pigouvian tax—Home's time-2 trade threats are imperfect: They distort trade and may also be limited in their "firepower." The role of Home's peacetime policies is to help compensate for these imperfections. In this sense, our work relates to the literature on corrective taxation with imperfect instruments (e.g., Diamond (1973)).

In emphasizing the geopolitical rationale for economic policies, we contribute to the literature on trade and conflict. One branch of recent work has emphasized the causal relationships of economic integration on geopolitical conflict and vice-versa (see Martin et al. (2008); Kleinman et al. (2024); Liu and Yang (2024)). Another has studied the impact and design of economic sanctions, with Becko (2024), Bianchi and Sosa-Padilla (2024), and Clayton et al. (2023) studying optimal trade and financial punishments, and Itskhoki and Mukhin (2022), Bachmann et al. (2022), de Souza et al. (2023) and many others quantifying sanctions' actual and potential impacts. Our model connects these two strands by incorporating peacetime capital investments—which shape and are shaped by the risk of geopolitical trade disruptions—and optimal sanctions that countries use to provide geopolitical incentives.

Most closely related is the literature on economic statecraft and the formation of geoeconomic power. Research in this area dates back to at least Hirschman (1945) and Hirschman (1958), and has been developed by numerous contributions in political science including recent work by Blackwill and Harris (2016), Scholvin and Wigell (2018), Farrell and Newman (2019), and Drezner et al. (2021). In economics, this theme has recently been picked up in the contemporaneous papers of Thoenig (2023), Kooi (2024), Alekseev and Lin (2024), and Clayton et al. (2024). Like us, each of these papers considers how pre-conflict economic policies influence the outcome of a during-conflict game that jointly determines geopolitical and economic outcomes.[4] While our frameworks differ in many ways, there are two essential differences that lead to divergent results, particularly for trade policy.[5] First, unlike Thoenig (2023) and Alekseev and Lin (2024), we assume Home's trade has separate trade policy instruments in peacetime and conflict. This is why we find that peacetime trade policies need only target *one component* of the difference in Foreign economic welfare across geopolitical actions: the manipulability of its terms of trade. Second, unlike Kooi (2024) and Clayton et al. (2024), we separate Home's purely geopolitical policy motive from its conventional terms-of-trade motive. This is why we find that peacetime trade policies target the manipulability of foreign prices rather than foreign welfare.

[4] In this sense, we all follow in the footsteps of McLaren (1997)'s model of investments before (purely economic) trade policy bargaining.

[5] In terms of industrial policy, Kooi (2024)'s characterization of peacetime capital subsidies when countries Nash bargain over geopolitical outcomes is similar to our characterization of the case when Home has limited credibility. Nash bargaining and credibility constraints both provide a rationale for Home make investments that shift its welfare under the trade allocations that result from various geopolitical actions.

2 A model of trade and geopolitics

We now present a model of trade and geopolitics. While the baseline environment is stylized along some dimensions, it can be extended significantly, as we describe in Section 2.4.

2.1 Baseline environment

There are two countries, Home ($i = H$) and Foreign ($i = F$), and two periods, peacetime ($t = 1$) and conflict ($t = 2$). Home and Foreign interact both economically and geopolitically.

Economic block The economic block of the model is a two-period neoclassical model of trade with long-lived capital and financial autarky.

The economy contains a finite set of tradable goods $g \in \mathcal{G}$ and a finite set of non-tradable capital varieties $v \in \mathcal{V}$. Capital varieties can represent tangible assets, such as a semiconductor factory or an oil stockpile, or intangible assets, such as a relationships with a supplier. Each country contains three agents: a representative household, a representative goods producer, and a representative capital producer.

The representative household in country i consumes goods $c_{it} = \{c_{itg}\}$ at each time t and experiences utility[6]

$$u_{i1}(c_{i1}) + u_{i2}(c_{i2}) + z_i(a) \tag{1}$$

where a is a geopolitical action that we introduce below. In order to focus our analysis on trade, rather than financial, sanctions, we assume countries are in financial autarky. That is, the Home and Foreign households do not borrow from or lend to one another.[7]

Meanwhile, the representative goods producer in i produces net output $y_{it} = \{y_{itg}\}$ using capital $k^y_{it} = \{k^y_{itv}\}$, subject to a goods production possibilities frontier[8]

$$G_{it}(y_{it}, k^y_{it}) \leq 0$$

During peacetime, the representative capital producer in i produces capital varieties $k_i = \{k_{iv}\}$ using investment $\iota_{i1} = \{\iota_{i1g}\}$ subject to a capital production possibilities frontier[9]

$$\Lambda_i(k_i, \iota_{i1}) \leq 0$$

[6]Factor supply corresponds to negative consumption of some goods.

[7]We explain how financial autarky affects our results in Section 3.2.1, following Theorem 3.

[8]Primary and intermediate inputs correspond to negative net output of some goods.

[9]We require that the model's primitives—i.e., $u_{it}(\cdot)$, $G_{it}(\cdot, \cdot)$, and $\Lambda_i(\cdot, \cdot)$—satisfy basic regularity conditions stated in Appendix A.1.

We assume capital is perfectly durable and cannot be formed during the conflict period, so that the capital stock is the same during conflict and peacetime. Capital producers retain ownership of capital and rent it domestically in both periods.

In each country i and time t, households and firms face goods prices $p_{it} = \{p_{itg}\}$ and capital rental prices $r_{it} = \{r_{itv}\}$. Goods and capital producers rebate their profits to the domestic household.

Finally, at each time t, the Home government levies ad-valorem trade taxes $\tau_t = \{\tau_{tg}\}$ that place a wedge between domestic prices p_{it} and world prices $p_t^w = \{p_{tg}^w\}$.[10] During peacetime, it additionally offers ad-valorem capital subsidies $s_1 = \{s_{1v}\}$ that augment the rental rates paid to capital producers. The government rebates net revenues with a lump-sum transfer to the Home household. We assume the Foreign government is laissez-faire, i.e., uses no capital subsidies or trade taxes.

Geopolitical block The premise of the geopolitical block of the model is that Foreign takes a binary, non-market geopolitical action $a \in \{\underline{a}, \bar{a}\}$ during conflict. For example, Foreign can choose whether or not to claim disputed territory, develop nuclear capabilities, or oppress minority groups. This action has a pure externality on Home utility. While there are no markets that allow Home to buy Foreign geopolitical actions, we allow Home to use trade during conflict as a carrot or stick to influence them.[11]

Formally, we embed a geopolitical game into the second period. First, Home commits to a time-2 trade tax threat $\widetilde{\tau}_2(\cdot)$ that specifies its trade taxes as a function of Foreign's geopolitical action. Second, Foreign chooses its geopolitical action a, balancing exogenous geopolitical preferences against the economic consequences of the resulting Home trade taxes $\tau_2 = \widetilde{\tau}_2(a)$.

We assume that the total utility of the representative agent in each country i additively combines consumption utility—which geopolitical actions affect through Home's trade taxes—with a direct geopolitical utility $z_i(a)$, as in Equation 1. Without loss of generality, we assume Home prefers action $\bar{a}$, i.e., $z_H(\bar{a}) \geq z_H(\underline{a})$. Additive separability implies that Foreign's geopolitical action affects prices only by determining the realization of Home's trade tax threat. This assumption, which we weaken in extensions, rules out the possibility that geopolitical actions affect prices through other channels, for example by increasing demand for weapons or by damaging factories.

[10] For Home imports, a positive trade tax corresponds to a positive import tariff. For Home exports, a positive trade tax corresponds to a positive export subsidy. We allow for infinite trade taxes in order to nest complete embargoes.

[11] In focusing on trade, we abstract from financial sanctions, except to the extent they can be understood as restrictions on the trade of financial goods and services. We do not allow, for example, Home to freeze or seize Foreign assets.

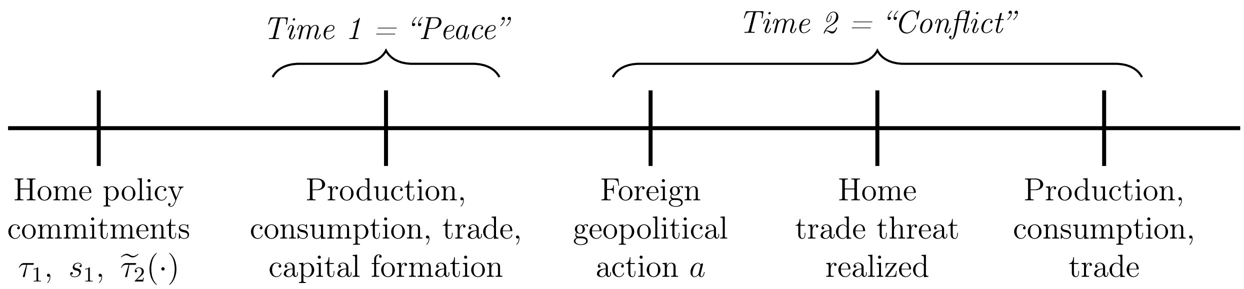

Figure 1: *Timing of model*

Figure 1 summarizes the timing of the model. Note that Home chooses its policies—peacetime trade taxes and subsidies as well as trade taxes threatened under conflict—before the timeline of the model begins. In particular, Home chooses its trade tax threat before the peacetime period and so need not suffer from issues of limited commitment.

2.2 Equilibrium conditions

We now state the model's formal equilibrium conditions. These conditions take as given both Home's peacetime policies τ_1 and s_1 and its conflict trade threat $\widetilde{\tau}_2(\cdot)$. We consider Home's optimal choice of these policies in Section 2.3.

Economic equilibrium conditions Households maximize consumption utility subject to a lifetime budget constraint.

$$\{c_{it}\}_{t=1,2} \in \arg \max_{\{c_t\}_{t=1,2}} \sum_{t=1,2} u_{it}(c_t) \quad \text{s.t.} \quad \sum_{t=1,2} (p_{it} \cdot c_t - I_{it}) \leq 0 \tag{2}$$

where I_{it} is income from domestic profits and lump-sum transfers in i at t, i.e.,

$$I_{it} = p_{it} \cdot (y_{it} - \mathbb{1}_{t=1} \iota_{i1}) + \mathbb{1}_{i=H} (p_t^w \tau_t) \cdot m_{it}$$

where m_{it} denotes the net imports of country i at time t.

Goods producers maximize profits subject to technological feasibility.

$$\{y_{it}, k_{it}^y\} \in \arg \max_{y, k^y} \ p_{it} \cdot y - r_{it} \cdot k^y \quad \text{s.t.} \quad G_{it}(y, k^y) \leq 0 \tag{3}$$

Capital producers maximize profits subject to technological feasibility and facing subsidies.

$$\{k_i, \iota_{i1}\} \in \arg\max_{k,\iota_1} \left(r_{i1}(1 + \mathbb{1}_{i=H} s_1) + r_{i2}\right) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0 \tag{4}$$

Goods markets clear globally and capital markets clear within each country.

$$c_{it} + \mathbb{1}_{t=1} \iota_{t1} = y_{it} + m_{it}, \qquad m_{Ht} + m_{Ft} = 0 \qquad \text{and} \qquad k_i = k^y_{it} \tag{5}$$

There is financial autarky. In other words, trade is balanced within each period.

$$p^w_t \cdot m_{it} = 0 \tag{6}$$

Domestic prices equal world prices augmented by (possibly zero) trade taxes.

$$p_{Ft} = p^w_t \quad \text{and} \quad p_{Ht} = (1 + \tau_t) p^w_t \tag{7}$$

Geopolitical equilibrium conditions Foreign's geopolitical action maximizes the sum of Foreign economic and geopolitical utility, accounting for Home's trade tax threat. To formalize this idea, we assume that, conditional on both countries' capital k_i, Home's trade taxes τ_2 during conflict uniquely determine all equilibrium variables during the conflict period, up to a choice of numeraire.[12] This implies that consumption in each country i is given by a function $\widetilde{c}_{i2}(\tau_2, k_H, k_F)$. Foreign's geopolitical action must then solve

$$a \in \arg\max_{\hat{a} \in \{\underline{a}, \overline{a}\}} \; u_{F2}\Big(\widetilde{c}_{F2}(\widetilde{\tau}_2(\hat{a}), k_H, k_F)\Big) + z_F(\hat{a}) \tag{8}$$

Home's time-2 trade taxes are determined by its trade tax threat and Foreign's geopolitical action.

$$\tau_2 = \widetilde{\tau}_2(a) \tag{9}$$

Definition 1. Given peacetime policies τ_1 and s_1 and conflict trade threats $\widetilde{\tau}_2(\cdot)$, an equilibrium is a profile $\{c_{it}, y_{it}, \iota_{i1}, m_{it}, k^y_{it}, k_i, p^w_{it}, p_{it}, r_{it}, a, \tau_2\}$ satisfying Equations 2–9.

2.3 Planner's problem

We study the problem of a Home planner who sets Home's peacetime trade taxes τ_1, peacetime capital subsidies s_1, and conflict trade threat $\widetilde{\tau}_2(\cdot)$. Importantly, we allow for the Home planner to place possibly non-zero weight λ_F on Foreign economic and geopolitical utility

[12] This assumption holds in most conventional trade models.

(we normalize λ_H to 1). This implies that our results characterize not only the behavior of selfish hegemons but also altruistic "global policemen."

While we assume Home's peacetime policies are unconstrained, we allow for the possibility that some trade threats are unavailable. For example, some threats may not be credible or may violate WTO rules. We let the set of feasible threats depend on both countries' capital k_i, as we have assumed capital and time-2 trade taxes determine all real quantities during conflict. Formally, we denote the set of feasible threats by $\mathcal{T}_2(k_H, k_F)$. The planner's problem is thus

$$\max_{s_1,\tau_1,\widetilde{\tau}_2} \max_{\{c_{it},k_i,a\}} \sum_{i=H,F} \lambda_i \left[u_{i1}(c_{i1}) + u_{i2}(c_{i2}) + z_i(a)\right] \tag{10}$$

$$\text{s.t.} \quad \{c_{it}, k_i, a\} \in \mathcal{E}(s_1, \tau_1, \widetilde{\tau}_2(\cdot)) \quad \text{and} \quad \widetilde{\tau}_2(\cdot) \in \mathcal{T}(k_H, k_F)$$

where $\mathcal{E}(s_1, \tau_1, \widetilde{\tau}_2(\cdot))$ is the set of consumption, capital, and geopolitical actions consistent with the equilibrium conditions given policies s_1, τ_1, and $\widetilde{\tau}_2(\cdot)$.

Definition 2. Policies s_1, τ_1, and $\widetilde{\tau}_2(\cdot)$ are optimal if they solve Equation 10.

Throughout the paper and without further statement, we study optimal policies.

2.4 Extensions

Our main results extend, in some cases with minor modifications, to significantly more general settings. We summarize these extensions below. Section 3.3 discusses each extension in detail and explains how our results apply.

First, we allow for geopolitical uncertainty by assuming that Foreign geopolitical preferences are random and revealed after Home makes its trade threat. This allows punitive trade sanctions to be used along the equilibrium path. Second, we allow both countries to take geopolitical actions and use economic policies. As a result, Foreign can take aggressive economic actions as well as geopolitical actions, and Home can punish Foreign geopolitically (e.g., militarily) as well as economically. Third, we remove the separability between economic and geopolitical preferences by allowing geopolitical actions to affect marginal utilities and production frontiers in both countries. This implies that geopolitical actions can raise demand for certain goods, such as weapons, or damage productive capacity. Fourth, we allow for many countries. This enables us to connect with discussions around bystander countries and map our model to the data.

3 Peacetime industrial and trade policy

This section theoretically characterizes Home's optimal peacetime policies—i.e., its industrial policy in the form of domestic capital subsidies and its trade policy in the form of trade taxes. Later sections apply these theoretical results in order to quantify the US's optimal peacetime geopolitical policies for building economic power vis-à-vis China.

3.1 Optimal industrial policy

We begin by studying Home's industrial policy in the form of domestic capital accumulation subsidies. We first consider the benchmark case in which Home can credibly threaten any time-2 trade taxes in response to Foreign geopolitical actions. We then relax this assumption, focusing on the case where Home is limited to threats with low enough domestic cost.

3.1.1 Industrial policy with unrestricted trade threats

To start, we assume Home has access to the full set of geopolitics-conditional trade tax threats.

Assumption 1. *Home's trade threats are unconstrained, i.e.,*

$$\mathcal{T}_2(k_H, k_F) = \left\{\widetilde{\tau}_2(\cdot) : \{\underline{a}, \overline{a}\} \to [-1, \infty]^{\mathcal{G}}\right\}$$

It is natural to suspect that—even with unconstrained trade threats—Home should use capital subsidies to realign the incentives of atomistic capital producers. Indeed, these capital producers fail to internalize the effect their investments have on the probability of conflict by shifting Foreign economic welfare under any given Home trade taxes.

Simple example To examine this argument more carefully, we consider a simple example. We suppose there are only two goods—a quasilinear numeraire that Home exports to Foreign and another good that Foreign exports to Home. We assume there is a single capital variety, only available in Home, that consists of stockpiles of the Foreign export. And, for simplicity, we focus on the case where Home is indifferent to redistribution of the numeraire between Home and Foreign households.

In this economy, one can represent the second period using a simple supply-and-demand diagram, as shown in the left panel of Figure 2. The diagram shows two different levels of Home imports, one corresponding to Home's more preferred Foreign geopolitical action ("restraint") and the other corresponding to Home's less preferred Foreign geopolitical action ("aggression"). We assume that Home's trade threat is such that only restraint occurs on the equilibrium path, implying that Home threatens its harshest possible sanction—autarky—if Foreign chooses aggression. This explains the value $m_{\text{aggression}} = 0$ on the horizontal axis. We focus on the case where this threat alone is not enough to deter Foreign, implying that Home must subsidize trade if Foreign chooses restraint. Home's ad-valorem import subsidy, $-\tau_2$, places a wedge between the price p_{H2} paid by Home importers and the price p_{F2} received by Foreign exporters, resulting in imported quantity $m_{\text{restraint}}$.[13]

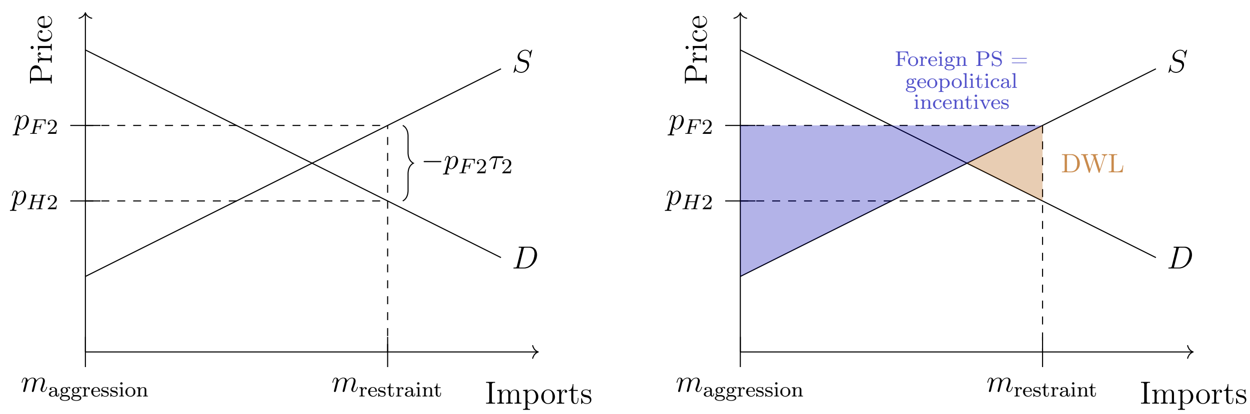

Figure 2: *Second-period equilibrium in a two-good example economy (left panel) with associated welfare gains and losses relative to free trade (right panel).*

The right panel of Figure 2 illustrates the economic trade-off behind Home's choice of trade subsidies. The red region shows the conventional deadweight loss corresponding to Home's distortionary subsidy. The blue region—equal to the difference in Foreign producer surplus between aggression and restraint—shows the geopolitical benefit of Home's subsidy, which is to provide Foreign with an economic incentive to choose restraint. The positive blue region and the negative orange region cannot be directly compared, as the blue region's economic relevance also depends on the Lagrange multiplier on the incentive compatibility constraint associated with implementing Foreign restraint. Note that since we have assumed the planner is indifferent to international redistribution, the diagram does not consider any regions corresponding to transfers between Home and Foreign.

We now return to the question of capital subsidies within the context of this simple example, asking whether peacetime private investment is efficient under the optimal trade tax threats. In the example, capital corresponds to Home stockpiling of Foreign's export, so decreased investment manifests as a upward shift in demand for imports, and vice-versa. Starting from laissez-faire investment, can such demand shifts increase welfare? The left panel of

[13]Without loss of generality, we normalize trade taxes on the numeraire good to zero.

Figure 3 illustrates the two competing welfare effects of a small reduction in Home's stockpile. First—as we have anticipated—the upward shift in Home's demand for Foreign exports increases Foreign's producer surplus under restraint, improving its geopolitical incentives. However, the same shift also generates negative fiscal externalities, reducing welfare by further raising imports that are already distorted upwards by Home's on-path import subsidy.[14] The total welfare impact of reduced investment—i.e., the sum of these two effects—is, at this point, ambiguous, since it depends on the economic value of changes in geopolitical incentives.

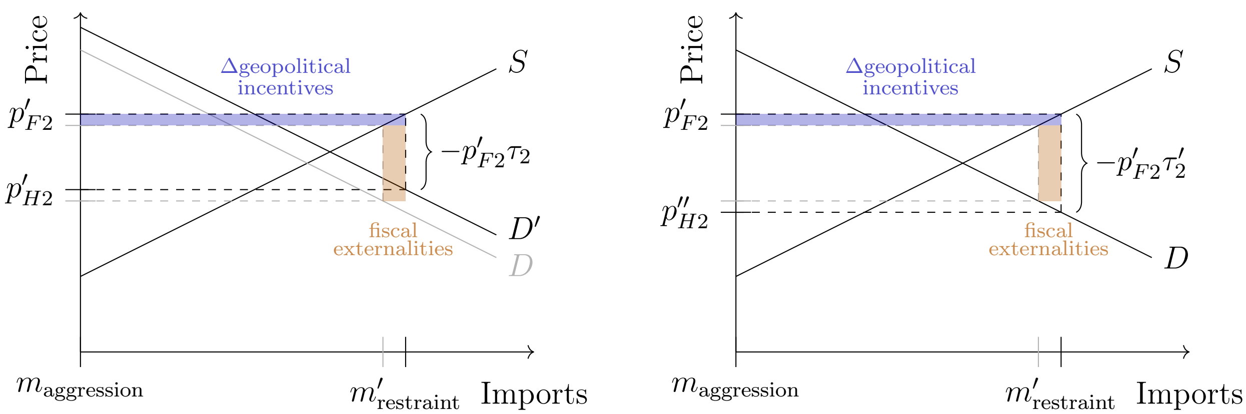

Figure 3: *Welfare impacts of (a) a reduction in Home capital, causing a demand shift (left panel), and (b) an increase in Home import subsidies under restraint (right panel).*

So far, we have argued that it is unclear whether private investment is efficient, since the answer depends on the economic value of changes in Foreign's geopolitical incentives. The key idea behind Theorem 1 is to recognize that this value is implicitly revealed by Home's choice of trade subsidy under restraint. Concretely, consider the first-order condition of Home's planner deciding whether to raise its trade subsidy under restraint. As shown in the right panel of Figure 3, this change in policy generates negative fiscal externalities, but raises Foreign's geopolitical incentives. The optimality of Home's subsidy implies that these two considerations exactly balance one another, which reveals the weight Home must place on marginal changes in Foreign geopolitical incentives. Finally, we can use this same weight to evaluate the net value of a reduction in Home investment. This comparison turns out to be particularly simple because the first-order costs and benefits of a reduction in Home investment are exactly proportional to those of an increase in Home's trade subsidy under restraint, as shown in the figure. In other words, the net benefit of a reduction in Home's

[14]Economic surplus in the market for Foreign's good also grows mechanically, because of the shift up in Home's import demand. We ignore this term because—when Home investment is undistorted—it is exactly offset by a reduction in the value of Home's capital stock and a decrease in investment costs.

capital stock is, to first order, the same as the net benefit of an increase in Home's trade subsidy under restraint, i.e., zero.[15]

General case Our first theoretical result shows that this argument extends to the general case. The logic underlying the result is the same as in the simple example: Home capital subsidies have offsetting geopolitical and economic effects. Any change in Home capital that improves Foreign's geopolitical incentives also raises deadweight losses by an equal and opposite amount. This precise cancellation is not due to an assumption on primitives. Rather, it reflects that Home's optimal trade threat already balances the marginal geopolitical benefits and economic costs of any changes in trade by design.

Theorem 1. *If Assumption 1 holds, then Home capital subsidies are zero, i.e., $s_{1v} = 0$ for all $v \in \mathcal{V}$.*

Another way to understand Theorem 1 is as a manifestation of the "targeting principle." Home's geopolitical goal in subsidizing capital is to correct the externality inherent in Foreign's geopolitical action. But the only way that Home can influence that action is through trade—either actual or threatened. It is therefore more efficient for Home to use trade taxes, which influence geopolitics by distorting trade, than to use capital subsidies, which that influence geopolitics by distorting trade *through changes in Home investment.*[16]

This targeting-principle explanation also explains why Theorem 1 does not require Home to be indifferent to Home-Foreign redistribution, as the simple example assumed. When Home is not indifferent, it may consider using domestic capital subsidies not only to affect Foreign's geopolitical incentives, but also to manipulate its terms of trade. However, it is more efficient to manipulate its terms of trade directly using trade taxes.

3.1.2 Industrial policy with imperfect credibility

We have just provided a condition under which industrial policies—in the form of capital subsidies—are an inefficient way to affect geopolitical outcomes. And yet, policymakers often cite geopolitical considerations to justify capital accumulation policies, such as national petroleum reserves or semiconductor R&D subsidies. What can explain this behavior?

[15] Distorting capital away from the laissez-faire additionally carries a second-order domestic misallocation cost (not shown in the figures) that makes capital subsidies not just redundant with trade taxes, but strictly worse.

[16] Importantly, our assumption that Home consumption utility is defined for any Home net-consumption vector implies that Home can—with some such trade taxes—implement any point on Foreign's time-2 offer curve. This rules out the possibility that Home may want to use capital subsidies in order to increase the maximum possible generosity of the "carrots" it can offer to Foreign through trade subsidies. This assumption could be relaxed as long as Home's maximally generous time-2 trade policy has a high enough domestic cost that Home chooses not to offer it.

One possibility is that Assumption 1 does not apply. That is, countries may lack the ability to credibly threaten all possible trade policies. For example, political economy constraints may prevent them from carrying out punishments with too great a domestic cost. Or the World Trade Organization's non-discrimination principle may prevent them from rewarding a single trading partner for good geopolitical behavior.

We now study Home's optimal capital subsidies in the presence of such constraints on its trade threats. While these constraints could conceivably take many forms, we focus on the case where Home can only threaten trade taxes under which its domestic economic utility is sufficiently high.

Assumption 2. *Home can threaten any trade taxes under which its economic utility is sufficiently high for all Foreign geopolitical actions. I.e., there is some $\bar{U}$ such that*

$$\mathcal{T}_2\left(k_H, k_F\right) = \left\{ \widetilde{\tau}_2(\cdot) : \{\underline{a}, \overline{a}\} \to [-1, \infty]^{\mathcal{G}} \;\middle|\; u_{H2}\Big(\widetilde{c}_{H2}(\widetilde{\tau}_2(\hat{a}), k_H, k_F)\Big) \geq \bar{U} \quad \textit{for} \quad \hat{a} = \underline{a}, \overline{a} \right\}$$

Under Assumption 2, Theorem 1 no longer applies and, in general, optimal capital subsidies can be non-zero. This is because when Home's credibility constraint binds, capital subsidies can move trade in a way that Home's threatened trade taxes cannot. For example, if Home's credibility constraint limits its ability to punish Foreign, then Home can use capital subsidies that shrink Home's comparative advantage and therefore lower Foreign's value from trading with Home. As a result, the targeting principle no longer applies.

How exactly should Home subsidize capital in order to compensate for its constrained trade threats? The answer may at first appear to depend on complex elasticities, such as how much Home capital affects trade and how much trade affects Foreign's geopolitical incentives. However, our next result shows that Home's optimal capital subsidies in fact take a simple form.

In order to state the result we require a notion of would-be rental rates conditional on each geopolitical action a, whether or not a occurs in equilibrium. We define these would-be rental rates using our assumption from Section 2.2 that capital and time-2 trade taxes uniquely determine the time-2 value of all equilibrium variables up to a price constant. Since Home's trade tax threat provides a map from Foreign's geopolitical action to time-2 trade taxes, each Foreign geopolitical action is consistent with a unique-up-to-constant vector of capital rental rates $\{\widetilde{r}_{H2v}(a)\}_{v\in\mathcal{V}}$. We now state the result in terms of these geopolitics-conditional rental rates.

Theorem 2. *If Assumption 2 holds and Home's economic utility is not the same under Foreign's two geopolitical actions, then Home capital subsidies are proportional to capital rental rates conditional on Foreign taking whichever action a_L results in lower Home economic* *utility, i.e.,*

$$r_{H1v}s_{1v} \propto \widetilde{r}_{H2v}(a_L)$$

The simplicity of this result reflects that *conditional on Home's economic utility under each geopolitical action*, trade taxes still target Foreign's geopolitical incentives more efficiently than capital subsidies. Home's planner therefore uses capital subsidies only to shift Home economic utility under each geopolitical action, focusing on the action a_L where the constraint binds. It does this using specific subsidies equal to each variety's marginal economic value under a_L, which is also proportional to the variety's rental rate.

This result has two main implications. First, Home's optimal capital subsidies are positive, since additional capital raises Home welfare given trade and so loosens Home's credibility constraint. A positive subsidy induces firms to internalize this benefit.

Second, Home's subsidies are largest for the whichever capital varieties have the highest rental rates under Home's economically less-preferred Foreign action. In the typical case where Home is worse off when its trade taxes punish Foreign than when they reward it, this suggests Home should subsidize the capital varieties that help it to produce substitutes to Foreign imports, which are scarcer under sanctions. This observation is consistent with the US's strategic reserve of crude oil, which the US net imported until 2020, and its recent subsidies for plants manufacturing semiconductors, which the US net imports.

3.2 Optimal trade policy

Having characterized Home's domestic industrial policies during peacetime, we now turn to its international trade policies. The bulk of our analysis studies the case where Home's trade threats are unconstrained because, as we will show, many constraints on Home's trade threats in the conflict period do not directly affect its peacetime trade taxes. In order to isolate the novel, geopolitical rationale for peacetime trade policy, we focus on the case where Home is indifferent to redistribution between itself and Foreign. We show that one may extend our results to the general case by simply adding conventional terms-of-trade manipulation formulas to our expressions for Home's peacetime trade taxes.

3.2.1 Peacetime trade policy with unrestricted conflict trade threats

We begin our analysis of Home's peacetime trade taxes with a simple observation: The role of these taxes is to influence Foreign capital accumulation. This is a consequence of the fact that, when Home is indifferent to redistribution between itself and Foreign, the only rationale for trade policy is to affect Foreign's geopolitical action. This geopolitical action is determined by Home's time-2 trade threats and capital in both countries. But when Home's trade taxes are unconstrained, peacetime trade has no effect on Home's access to trade tax threats. Moreover, Home can directly determine its own capital stock using domestic capital subsidies. This leaves Foreign capital as the only target of Home's peacetime trade policy.

The key question for Home's peacetime trade policy, then, is *what sort* of Foreign capital accumulation it should promote. Since Home's goal is to incentivize Foreign to take Home preferred geopolitical action $\bar{a}$, a natural guess is that Home should promote whatever Foreign capital accumulation most increases the difference in Foreign welfare under Home's time-2 trade taxes following $\overline{a}$ and its trade taxes following $\underline{a}$, i.e.,

$$u_{F2}\Big(\widetilde{c}_{F2}(\widetilde{\tau}_2(\bar{a}), k_H, k_F)\Big) - u_{F2}\Big(\widetilde{c}_{F2}(\widetilde{\tau}_2(\underline{a}), k_H, k_F)\Big) \tag{11}$$

This is a measure of Foreign's gains from trade under Home's preferred geopolitical action $\bar{a}$ relative to its alternative, $\underline{a}$.

Simple example To examine this proposal more carefully, we consider a simple example. As in Section 3.1.1, we assume there are two goods—a quasilinear numeraire that Home exports to Foreign and another good that Foreign exports to Home. We assume there is a single capital variety, only available in Foreign, that shifts Foreign's export supply curve. Figure 2 represents the second period of this model in a simple supply-and-demand diagram. We focus on the case where Home implements its preferred geopolitical action, "restraint" in equilibrium, but cannot do so by only threatening autarky under the alternative action, "aggression", and so offers trade subsidies conditional on restraint.

We begin by supposing that additional Foreign capital causes a uniform downward shift in its export supply curve, as shown in the left panel of Figure 4. Such a uniform shift could capture, for example, the impact of a gas pipeline that reduces transportation costs for each unit of gas Foreign exports. Such a shift has two effects on the Home planner's objective.[17] First, it increases Foreign producer surplus and therefore raises Foreign's economic incentive to choose Home's preferred geopolitical action. The figure shows this change in blue, with solid shading for regions that contribute positively to geopolitical incentives and dotted shading for (smaller) regions that contribute negatively. Second, the supply shift generates negative fiscal externalities by increasing Foreign exports, which are already above their efficient level due to Home's trade subsidies under restraint. The figure shows this change in orange. The total welfare effect of Foreign's capital investment is, at this point, ambiguous, because the sum of these two effects depends on the economic value of changes in Foreign's

[17]Terms-of-trade effects are irrelevant because we assume Home's planner is indifferent to international redistribution. The economic value of increased producer surplus from the supply shift is offset by its cost, since Foreign capital investment is undistorted.

geopolitical incentives.

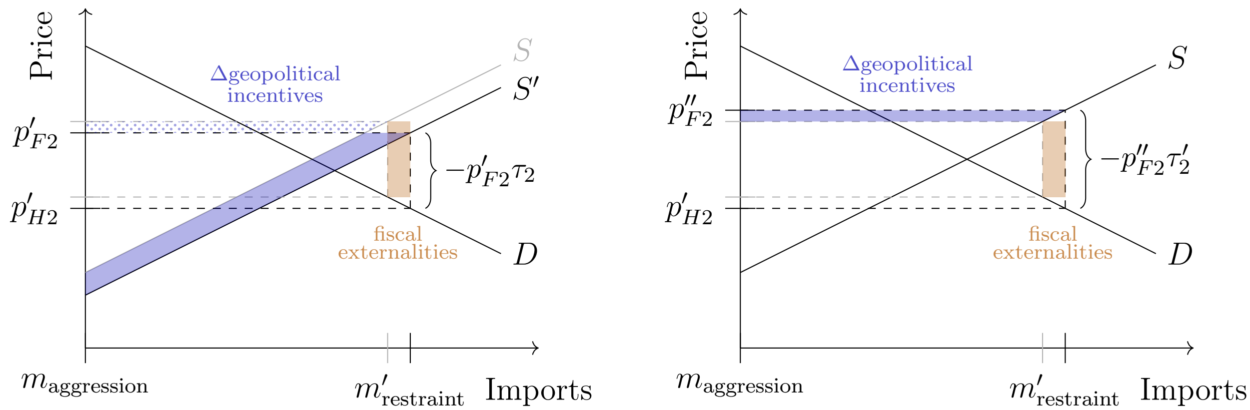

Figure 4: *Welfare impacts of (a) an increase in Foreign capital, causing a supply shift (left panel), and (b) an increase in Home import subsidies under restraint (right panel).*

So far, we have argued it is unclear whether a uniform downward shift in Foreign's export supply curve raises or lowers welfare, since the answer depends on the economic value of changes in Foreign's geopolitical incentives. The key step of our argument is—just as in the case of domestic capital accumulation policies—to infer this value from Home's choice of trade taxes under Foreign restraint. To this end, we return to Foreign's original export supply curve and consider the first-order condition of Home's planner deciding whether to subsidize imports more under restraint. Like a Foreign supply shift, an increase in Home import subsidies raises Foreign producer surplus—and hence Foreign's geopolitical incentives—but also generates negative fiscal externalities, as shown in the right panel of Figure 4. The optimality of Home's second-period trade taxes imply that these two effects exactly offset one another, which reveals the economic value Home places on changes in Foreign's geopolitical incentives.

Finally, we can use the revealed value of geopolitical incentives to compare the costs and benefits of a uniform shift in Foreign's net export supply curve. This comparison is simple because these costs and benefits are proportional to those associated with an increase in Home import subsidies. To see this visually, first note that the rectangles that represent fiscal externalities in Figure 4 have the same height and width in the left and right panels. A simple geometric argument reveals that the change in producer surplus is also the same in the two panels: The two panels have the same initial producer surplus by construction. They have the same final producer surplus because the triangles representing producer surplus have all of the same angles and have the same horizontal length, $m'_{\text{restraint}}$.

We conclude that a change in Foreign capital that uniformly shifts its supply curve has no first-order effect on welfare.[18] Such shifts *do* increase Foreign's gains from trade in the sense of Equation 11. However, they also generate economic costs in the form of fiscal externalities and offer exactly the same trade-off between economic costs and geopolitical benefits as Home's trade threats in the conflict period. Given this, Home prefers to use only its time-2 trade threats, the more direct instrument.

This negative result—that Home does not use peacetime trade taxes to induce uniform shifts in Foreign's export supply curve—paves the way to a positive result: Home *does* use peacetime trade taxes when they induce *non-uniform* shifts in Foreign's export supply curve. This is straightforward to see if one decomposes a non-uniform shift in Foreign's export supply curve into two steps: a uniform shift and a rotation (or more general change of shape) around the final price-quantity point. Figure 5 illustrates this by decomposing a shift from supply curve S to S' into a non-uniform shift from S to S'_{unif} (left panel) and a rotation from S'_{unif} to S' (right panel). The first step has offsetting economic and geopolitical effects on welfare, as discussed above. The second step has no economic effects since by construction it rotates the supply curve around the equilibrium price-quantity point. However, it has a positive impact on Foreign's geopolitical incentives by raising the difference in Foreign's producer surplus between aggression and restraint. Its net effect on welfare therefore positive.

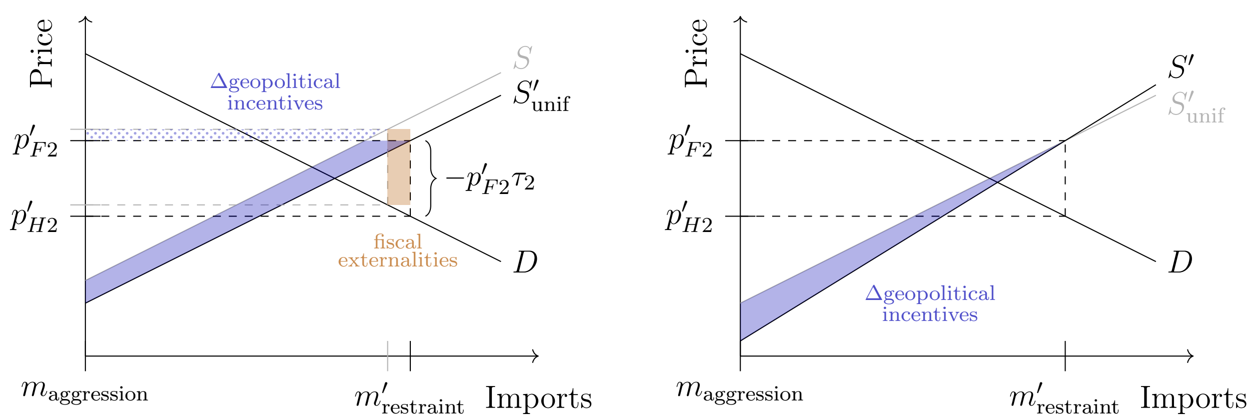

Figure 5: *Decomposition of a non-uniform shift in Foreign's export supply curve into (a) a uniform supply shift and (b) a rotation of the supply curve.*

Exactly which non-uniform changes in Foreign's export supply curve improve welfare? Figure 5 illustrates that it is those that increase the area between S'_{unif} and S'. The size of

[18]Its second order effects (not shown in the figure) are negative because they account for the cost of distorting peacetime trade in order to influence Foreign investment.

the welfare gain is thus, to first order, proportional to

$$\int_{m_{\text{aggression}}}^{m_{\text{restraint}}} \left(\tilde{p}^{S'_{\text{unif}}}(m) - \tilde{p}^{S'}(m)\right) \cdot dm = \int_{m_{\text{aggression}}}^{m_{\text{restraint}}} m \cdot \left(\frac{d\tilde{p}^{S'}(m)}{dm} - \frac{d\tilde{p}^{S}(m)}{dm}\right) \cdot dm \qquad (12)$$

where $\widetilde{p}^S(m)$ is Foreign's inverse supply curve under S and so on. The equality follows from integration by parts and recognizing that, by construction $\widetilde{p}^S$ and $\widetilde{p}^{S'_{\text{unif}}}$ have the same slope. Equation 12 clarifies that a change in Foreign's supply curve raises welfare exactly to the extent it makes supply prices more sensitive to quantities along the path between the quantities traded under aggression and restraint. In this sense, the goal of Home's peacetime trade policy is exactly to make Foreign's export supply curve less elastic and hence its terms of trade more manipulable.

General case Taking stock, our simple example shows that changes in peacetime trade improve welfare if and only if they shift Foreign capital so as to make Foreign's terms of trade more manipulable. Our next formal result establishes that these ideas extend to the general case. Stating this result requires a few additional pieces of notation. First, recall our assumption that capital and trade taxes uniquely determine all time-2 equilibrium variables up to a constant. This implies trade under the more and less preferred geopolitical actions $\bar{a}$ and $\underline{a}$ can be expressed as $\widetilde{m}_{F2}(\widetilde{\tau}_2(\bar{a}), k_H, k_F)$ and $\widetilde{m}_{F2}(\widetilde{\tau}_2(\underline{a}), k_H, k_F)$. Second, we let $\widetilde{p}_{F2}(m, k)$ denote Foreign's inverse net export supply curve that dictates the prices at which it is willing to import or export goods given net imports m and domestic capital k_F.[19] Third, we let $\widetilde{k}_F(m_{F1}, m_{F2})$ denote Foreign's choice of capital as a function of its net imports in the first and second periods.[20]

Theorem 3. *If Assumption 1 holds, then—up to Lerner symmetry—Home's peacetime trade taxes satisfy*

$$\tau_{1g} \;=\; \tau_{1g}^{ToT} \;-\; \kappa \left(\int_{m_{F2}(\underline{a})}^{m_{F2}(\bar{a})} m \cdot \frac{\partial^2 \widetilde{p}_{F2}(m, k_F)}{\partial k_F \, \partial m} \cdot dm \right) \cdot \frac{\partial \widetilde{k}_F(m_{F1}, m_{F2})}{\partial m_{F1g}} \bigg/ p_{F1g} \qquad (13)$$

[19]Formally, we define $\widetilde{p}_{it}(m, k)$ as the derivative with respect to imports of i's time-t Meade utility function,

$$V_{it}(m,k) \equiv \begin{cases} \max_{y,\iota} \; u_{it}(y + m - \iota) \quad \text{s.t.} \quad G_{it}(y,k) \le 0 \quad \text{and} \quad \Lambda_{it}(k,\iota) \le 0, & \text{if } t = 1 \\ \max_{y} \; u_{it}(y+m) \quad \text{s.t.} \quad G_{it}(y,k) \le 0, & \text{if } t = 2 \end{cases}$$

By the first welfare theorem conditional on imports and capital, $V_{it}(m, k)$ is equal to the Foreign consumer's time-2 utility in any equilibrium in which i has net imports m and capital k at time t. Prices are therefore proportional to $V_{it,m}(m, k)$ in any competitive equilibrium. In setting $\widetilde{p}_{it}(m, k) \equiv V_{it,m}(m, k)$, we implicitly normalize i's marginal utility of wealth to one; this is without loss of generality as Foreign prices appear in both the numerator and denominator of Equation 13.

[20]Appendix A.1 provides conditions under which this function is well-defined.

where τ_{1g}^{ToT} denotes Home's optimal trade tax on g for conventional terms-of-trade manipulation, $m_{F2}(\hat{a}) \equiv \widetilde{m}_{F2}(\widetilde{\tau}_2(\overline{a}), k_H, k_F)$ denotes Foreign's net imports under geopolitical action $\hat{a}$, and "$\cdot$" denotes dot products between vectors.[21] *Moreover, if Home is indifferent to redistribution at times* 1 *and* 2*, then* $\tau_{1g}^{ToT} = 0$.[22]

Theorem 3 states that Home uses higher peacetime trade taxes on goods whose trade makes Foreign's net import demand more price-sensitive—or equivalently makes its terms of trade more manipulable. Concretely, fix a good g that Home imports from Foreign. Holding fixed its terms of trade motive, Home applies a larger import tariff on g if reducing Foreign's exports of g (i.e., raising its net imports of g) would induce a change in Foreign capital $\partial \widetilde{k}_F / \partial m_{F1g}$ that increases Foreign's geopolitical incentives beyond what Home can do with time-2 trade taxes. This occurs when the capital change increases the net-import sensitivity $\partial \widetilde{p}_{F2} / \partial m$ of Foreign prices for goods that Foreign net exports—i.e., when it raises the terms-of-trade effects associated with additional trade. Equation 13 indicates that these changes in terms of trade effects must be evaluated at trade quantities along a path between the quantities Foreign trades under the less preferred action $\underline{a}$ and the more preferred action $\bar{a}$.

Compared to existing results on optimal trade policy, the striking feature of this characterization is that it considers a *second derivative* of Foreign prices. Well known expressions for optimal terms-of-trade manipulation taxes state that trade taxes depend on terms-of-trade elasticities (see e.g., Dixit (1985); Becko (2024)). Equation 13 states that peacetime trade taxes do not depend on terms-of-trade elasticities themselves, but rather depend on how these terms-of-trade elasticities *change* with whatever capital adjustments additional trade induces.

Note that to remove Home's terms-of-trade motive, Theorem 3 assumes Home is indifferent to redistribution at *both* time 1 and time 2. Home's indifference to redistribution at time 1 can be thought of as a conventional assumption equivalent to choosing a particular value for λ_F, the Home planner's weight on Foreign utility. Home's indifference to redistribution at time 2 is a less innocuous assumption. Since countries are in financial autarky, their

[21] We provide an explicit expression for τ_{1g}^{ToT} in the proof. To avoid any notational confusion, we note that, letting $\hat{m}_{F2} : [0,1] \to \mathbb{R}^{\mathcal{G}}$ be any smooth path from $\hat{m}_{F2}(0) \equiv m_{F2}(\underline{a})$ to $\hat{m}_{F2}(1) \equiv m_{F2}(\bar{a})$, Equation 3.2.1 can be written as

$$\tau_{1g} = \tau_{1g}^{ToT} - \kappa \sum_{v \in \mathcal{V}} \left(\int_0^1 \sum_{g', g'' \in \mathcal{G}} \hat{m}_{F2g'}(\zeta) \frac{\partial^2 \widetilde{p}_{F2g'}(\hat{m}_{F2}(\zeta), k_F)}{\partial k_{Fv}\, \partial m_{g''}} \hat{m}'_{F2g''}(\zeta) d\zeta \right) \frac{\partial \widetilde{k}_{Fv}(m_{F1}, m_{F2})}{\partial m_{F1g}} \Big/ p_{F1g}$$

[22] Formally, let m_1^* and $m_2^*(\hat{a})$ be bundles of goods, marginal trade in the direction which—at time t and under geopolitical action $\hat{a}$—affects neither (a) Foreign's terms of trade at t and $\hat{a}$ nor (b) Foreign capital. We assume that at time $t = 1$ and at time $t = 2$ and action $\hat{a} = a$, the Home planner is indifferent to marginal redistribution of the corresponding bundle. See the discussion below.

representative consumers need not place the same relative marginal values on goods purchased in times 1 and 2. This implies that, in general, there are social gains from completing missing international financial markets by manipulating terms of trade in favor of whichever country has higher relative marginal utility at a given time. This results in trade taxes that promote Foreign investments in whichever capital varieties disproportionately improve Foreign's terms of trade in the period Foreign would like to save for. Theorem 3 rules this out by focusing on the special case where, although financial markets are incomplete, preferences happen to be such that households have no demand for international borrowing. We unpack this missing-markets component of τ_{g1}^{ToT} in Appendix A.5, but omit it from the main text as the assumptions of Theorem 3 hold in our quantification.

3.2.2 Peacetime trade policy with restricted conflict trade threats

So far, we have characterized Home's optimal peacetime trade taxes under the assumption that Home's time-2 trade threats are unconstrained. We now reconsider peacetime trade policy away from this benchmark.

As in the case of industrial policy, we study a subset of the possible constraints on trade threats. Namely, we focus on constraints for which the set of feasible trade quantities depends only on Home capital.

Assumption 3. *Home can threaten any trade taxes that result in trade quantities within some set that depends only on Home capital. I.e., there exists a function Γ of Home capital for which*

$$\mathcal{T}_2\left(k_H, k_F\right) = \left\{ \widetilde{\tau}_2(\cdot) : \{\underline{a}, \bar{a}\} \to [-1, \infty]^{\mathcal{G}} \;\middle|\; \{\widetilde{m}_{H2}(\widetilde{\tau}_2(\hat{a}), k_H, k_F)\}_{\hat{a}\in\{\underline{a},\bar{a}\}} \in \Gamma(k_H) \right\}$$

where $\widetilde{m}_{H2}(\tau_2, k_H, k_F)$ is the unique value of Home imports consistent with the time-2 equilibrium conditions given trade taxes τ_2 and capital stocks k_H and k_F.

A special case is Assumption 2, under which Home can only threaten trade taxes under which it has sufficiently high economic utility. Concretely, Assumption 2 defines Γ by

$$\Gamma(k_H) \equiv \left\{ \{m_{H2}(\hat{a})\}_{\hat{a}\in\{\underline{a},\bar{a}\}} \mid \forall \hat{a} \in \{\underline{a}, \bar{a}\},\; V_{H2}(m_{H2}(\hat{a}), k_H) \geq \overline{U} \right\}$$

where V_{H2} is Home's time-2 Meade utility (see Footnote 19).

Under this assumption, our characterization of peacetime trade policy extends to restricted trade threats without modification.

Proposition 1. *If Assumption 3 holds, then Home's peacetime trade taxes satisfy Equation 13.*

This result contrasts with our earlier findings on domestic capital subsidies, which depend directly on Home's trade threat constraints. Proposition 1 states that constraints on Home's time-2 trade threats have no direct impact on Home's optimal peacetime trade policy. Such constraints can only affect peacetime trade taxes indirectly, for example by restricting which off-path import quantities $\widetilde{m}_{H2}(\widetilde{\tau}_2(\underline{a}), k_H, k_F)$ Home can credibly threaten.

The irrelevance of time-2 constraints to Home's time-1 trade policy is yet another manifestation of the targeting principle. It is true that Home's peacetime trade policy can help to loosen constraints on its ability to make trade threats during conflict. But under Assumption 3, the only way Home peacetime trade policy loosens these constraints is by inducing changes in the domestic capital stock. Since Home can more directly target changes in capital using its capital accumulation subsidies, it prefers not to distort peacetime trade for this purpose.

Taken together with Theorem 2, Proposition 1 offers a new take on the conundrum of how Home can foster Foreign's dependence without itself becoming dependent on Foreign. On one hand, Proposition 1 states that Home's international trade policies target only Foreign's dependence on Home without concern for how this may affect Home's dependence on Foreign. On the other hand, Theorem 2 states that Home's industrial policies should focus solely on alleviating Home's credibility constraints without regard for how this may affect Foreign's dependence on Home. In short, Home targets Foreign's dependence with its international policy instrument—trade taxes—while at the same time moderating its own dependence using its domestic policy instrument—capital subsidies.

3.3 Extensions

So far, we have characterized Home's optimal peacetime capital subsidies and trade taxes in the baseline model. This model is intentionally stylized along several important dimensions. It features no geopolitical uncertainty, assumes that only Foreign takes geopolitical actions while only Home uses economic policy, abstracts from any direct impact of economic activity on countries' geopolitical choices, and assumes there are only two countries. We now outline extensions of the baseline model that enrich it along each of these dimensions and discuss how they affect our results. Appendix C studies these extensions in detail.

3.3.1 Geopolitical uncertainty

The baseline model is deterministic. This implies that Home never applies trade sanctions (i.e., a "stick") along the equilibrium path, although it may use trade subsidies (a "carrot").[23]

[23] When Foreign's geopolitical preferences are weak enough, Home can control Foreign's geopolitical behavior using only *off-path* threats that are never realized in equilibrium. When Foreign's geopolitical preferences are of intermediate strength, Home's uses trade subsidies (a "carrot") on the equilibrium path but only uses trade

This is at odds with the empirical reality of sanctions.

In Appendix C.1, we consider a variant of the model in which Foreign's geopolitical preferences are random and revealed after Home makes its trade threat. Formally, we assume Foreign's geopolitical utility $z_F(a, \omega)$ depends both on its action a and a state of the world ω. We assume that households have access to perfect domestic insurance markets but maintain the assumption of international financial autarky.[24] Adding geopolitical uncertainty enhances the realism of the model by making it possible for Home to apply punitive sanctions on the equilibrium path. This occurs when ω is such that Foreign has a strong preference for an action Home dislikes. The possibility that Home will have to carry out its sanction threats also provides a rationale for why these threats may be less severe than autarky, even if Home has perfect commitment.

Our results concerning peacetime capital subsidies extend to the case of geopolitical uncertainty with no alterations. The presence of uncertainty does not affect the targeting logic behind either result. Our characterization of peacetime trade policy continues to apply under geopolitical uncertainty, but trade taxes must, in general, account for an additional policy motive. This motive stems from our assumption that Home and Foreign households cannot insure one another against geopolitical shocks due to financial autarky. Home's peacetime trade taxes can help to compensate for this missing market by inducing Foreign capital changes that disproportionately affect Foreign's terms of trade across states of the world. All else equal, these changes improve welfare if they improve Foreign's terms of trade in states of the world where it would run a trade deficit in the presence of perfect international insurance, and vice-versa. As with the missing markets for borrowing and lending discussed below Theorem 3, Home's peacetime trade taxes need not account for missing international insurance markets if preferences are such that households would choose not to trade insurance in equilibrium, for example if they had quasilinear preferences.

3.3.2 Home geopolitical actions and Foreign economic policies

In the baseline model, only Foreign takes geopolitical actions and only Home uses economic policies. This leaves open two questions: First, would Home still use economic policies in the way we have described if it could make military threats in addition to economic ones? Second, how should Home's policies account for Foreign's economic retaliation?

In Appendix C.2, we extend the model to allow Foreign to choose non-zero capital subsidies and trade taxes (in both periods), and to allow Home to choose a geopolitical action. We

taxes (a "stick") off path. When Foreign's geopolitical preferences are strong enough, the cost to Home of influencing its behavior is greater than the benefit, so that Home's trade taxes ignore geopolitics altogether.

[24] We do not allow for international insurance markets because insurance payouts would create an additional source of economic interaction between Home and Foreign beyond trade, which is our focus.

maintain the assumption that Home makes threats and Foreign responds to threats. However, we assume that Foreign's response comprises both its geopolitical action and its time-2 trade taxes, while Home's threat comprises both a trade tax threat and a geopolitical action threat, each of which conditions on Foreign's full response. We relax the assumption that geopolitical actions are binary, requiring only that they are drawn from some finite sets. We assume that Foreign cannot impose prohibitive trade taxes and, for technical convenience, suppose it is restricted to a finite number B of possible threat responses. We revisit Home's optimal peacetime policies in this setting, leaving the optimal Foreign policy for future work.

Once again, Home's peacetime capital subsidies remain as in the baseline model. The only necessary clarification is that when Home's trade threats are constrained according to Assumption 2, its capital subsidies are proportional to rental rates conditional on whatever Foreign response—now inclusive of both geopolitical actions and trade taxes—results in the lowest Home economic utility. Home's peacetime trade taxes are similar to those in the baseline model but with two modifications. First, Foreign's B possible responses to Home's trade threat implies that the planner faces $B-1$ incentive-compatibility constraints. As a result, peacetime trade taxes depend on changes in terms-of-trade elasticities along a $B-1$ different paths, each connecting trade under some off-path action to on-path trade Second, to the extent Home values Foreign welfare, its peacetime trade taxes attempt to correct distortions stemming from Foreign's non-zero capital subsidies and trade taxes.

3.3.3 Inseparability of economics and geopolitics

In the baseline model, geopolitical actions only affect economic outcomes through Home's time-2 trade taxes. This rules out geopolitical actions with direct economic impacts, for example through demand for weapons or the destruction of factories. These issues are relevant to current discussions around trade in dual-use goods and the consequences for semiconductor markets of a Chinese invasion of Taiwan.

In Appendix C.3, we relax the assumption that countries' economic and geopolitical preferences are additively separable. We instead model the household preferences using general time-2 utility functions $u_{i2}(c_{i2}, a_F)$. This allows us to accommodate government purchases required for geopolitical actions (e.g., purchases of military equipment) by subtracting these quantities from household consumption when a certain geopolitical action is taken. We also allow time-2 production to depend on Foreign's geopolitical action using a production possibilities frontier $G_{i2}(y_{i2}, k_i, a_F) \leq 0$.

Allowing for this interaction between economic and geopolitical preferences does not affect our results on Home capital subsidies. Home's peacetime trade taxes continue to depend on trade's impact on Foreign's terms-of-trade manipulability, but now also include an additional term. This term accounts for how the value of trade-induced changes in Foreign capital differs depending Foreign's geopolitical action—ignoring the effect of those actions on trade through Home's trade tax threat. It implies that Home should promote Foreign capital that, all else equal, is more valuable when Foreign takes Home's preferred geopolitical action, and vice-versa. For example, Home may restrict its exports of arms to Foreign to discourage Foreign from building military stockpiles, which are more useful if Foreign goes to war.

3.3.4 Many countries

The baseline model features only two countries. As a result, it does not directly address salient questions concerning the ability of sanctioning and sanctioned countries to substitute toward trade with third parties. It also is not straightforward to map to data.

In Appendix C.4, we extend the model to allow for many countries. For simplicity, we assume that countries other than Home and Foreign are passive in the sense that they do not take geopolitical actions or use capital subsidies or trade taxes. We assume Home sets trade taxes on its bilateral trade on its exports to and import from all other countries.

The presence of such bystander countries does not affect our results on capital subsidies. It affects our results on peacetime trade taxes in two ways. First, the presence of bystander countries alters the workings of the "manipulability" mechanism we have emphasized in the two country case. For one, the trade allocations between which Home seeks to raise Foreign's terms of trade manipulability no longer differ in only their bilateral trade. This is because Home's trade threats can also punish or reward Foreign indirectly by using Home's trade taxes with bystanders to promote or discourage *their* trade with Foreign. Home seeks to raise the manipulability of Foreign's terms of trade with respect to these third-party trade changes. Moreover, Home must accounts for the more complex way that its net imports affect Foreign capital, which with bystanders is not the same as how changes in Foreign net exports affect Foreign capital.

The second way that bystander countries impact our results on peacetime trade policy is by introducing a new qualitative mechanism: Home seeks to move Foreign and bystander capital in ways that—holding fixed Home's own time-2 net imports—move Foreign's time-2 net imports. With two countries, this mechanism is not present because Foreign's net imports are simply Home's net exports; with more than two countries, Home can shift trade between Foreign and bystanders. Which Foreign-bystander trade does Home seek to promote? Intuitively, Home's goal is to loosen the incentive compatibility constraint on Foreign's geopolitical choice. It therefore promotes capital that, under Home's preferred geopolitical action, induces trade changes that improve Foreign's terms of trade and that, under Home's less preferred geopolitical action, induces trade changes that worsen Foreign's terms of trade. While it is conceptually novel, our quantitiative exercise indicates that this mechanism is of secondary importance compared to Foreign's terms-of-trade manipulability.

4 A quantitative model of global interdependence

Having characterized optimal geopolitical policies theoretically, we now attempt to quantify them in the context of US-China relations. We treat the US as Home and China as Foreign, and we assume the US has access to credible trade threats during conflict. Theorem 1 implies that the optimal US capital subsidies are zero in this case, so we focus on trade taxes.

The goal of this section is to formulate and calibrate a parametric trade model that Section 5 then uses to quantify the US's optimal peacetime trade policy. This quantitative model emphasizes two distinct types of capital through which the US can shape Chinese dependence. First, China has sector-specific *production capital* that lowers its variable costs of production. Second, China has sector-and-origin-specific *relationship capital* that lowers its trade costs with each partner (including itself).

4.1 Quantitative model

We now specialize the economic block of the model from the general case studied in Sections 2 and 3 to a parametric model we can tractably map to data. The idea behind our choice of parametric model is to embed capital investment into an otherwise standard static trade model. We start from a standard, many-country, many-sector model with trade in intermediates, similar to Caliendo and Parro (2015). We augment this model assuming that firms use two different types of capital. Firm's capital investment decisions endogenize various productivity shifters in the basic static model.

For ease of reading, we present this parametric model using conventional notation, rather than using the general notation of Section 2.1 to describe the model in terms of a set of utility and production functions $u_{it}(\cdot)$, $G_{it}(\cdot,\cdot)$, and $\Lambda_i(\cdot,\cdot)$. As a result, we—where there is no risk of confusion—slightly abuse notation by indexing some variables differently than in earlier sections of the paper.

Environment There are many countries $i \in \mathcal{I}$ and sectors $n \in \mathcal{N}$. Firms in each country i produce two types of goods in each sector n: First, variety producers in i ship a differentiated variety of n to each destination country j (including $j = i$). Second, domestic composite producers in i use foreign and domestic varieties to produce a domestically usable sector-n composite. Households supply labor and consume domestic composites.

Variety producers For each origin country i and sector n, there is firm that sells i's variety of sector n to each country j. At each time t, this firm produces output q_{ijnt} for each destination j using labor ℓ_{ijnt}, production capital k_{ijnt}, and intermediate inputs of domestic composites $\{x_{in',ijnt}\}_{n'\in\mathcal{N}}$ according to the Cobb-Douglas production function

$$q_{ijnt} = \phi_{ijn}\left(\frac{\ell_{ijnt}}{\theta^{\ell}_{in}}\right)^{\theta^{\ell}_{in}}\left(\frac{k_{ijnt}}{\theta^{k}_{in}}\right)^{\theta^{k}_{in}}\prod_{n'\in\mathcal{N}}\left(\frac{x_{in',ijnt}}{\theta^{x}_{in'n}}\right)^{\theta^{x}_{in'n}} \tag{14}$$

where $\theta^{\ell}_{in}+\theta^{k}_{in}+\sum_{n'\in\mathcal{N}}\theta^{x}_{in'n}=1$. The firm owns a fixed total stock of production capital k_{in} and so, at each time t, must allocate it across destinations in a way that satisfies

$$\sum_{j\in\mathcal{I}} k_{ijnt} \le k_{in} \tag{15}$$

Each such firm chooses its time-invariant capital stock k_{in} at time 1 to maximize discounted profits. Production capital is formed using time-1 investment of domestic composites $\{\iota_{in',in1}\}_{n'\in\mathcal{N}}$ according to the Cobb-Douglas production function

$$k_{in} = \phi_{in}\prod_{n'\in\mathcal{N}}\left(\frac{\iota_{in',in1}}{\theta^{\iota}_{in'n}}\right)^{\theta^{\iota}_{in'n}} \tag{16}$$

where $\sum_{n'\in\mathcal{N}}\theta^{\iota}_{in'n}<1$.

Composite producers For each country j and sector n, there is a firm that converts foreign varieties of n into a domestically usable composite. At each time t, this firm produces output q_{jnt} using relationship capital $\{k_{ijn}\}_{i\in\mathcal{I}}$ and intermediate inputs of foreign and domestic varieties $\{x_{ijn,jnt}\}_{i\in\mathcal{I}}$ according to the CES production function[25]

$$q_{jnt} = \left[\sum_{i\in\mathcal{I}:\ \theta_{ijn}>0}(k_{ijn})^{1/\sigma}(x_{ijn,jnt})^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \tag{17}$$

Each such firm chooses its time-invariant capital stock $\{k_{ijn}\}_{i\in\mathcal{I}}$ at time 1 to maximize discounted profits. Relationship capital formation requires no investment, but must satisfy the constraint

$$\sum_{i\in\mathcal{I}:\ \theta_{ijn}>0}(\theta_{ijn})^{(1-\sigma)/(\bar{\sigma}-\sigma)}\,(k_{ijn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}=1 \tag{18}$$

[25]Recent estimates by Adão et al. (2023) support our implicit assumption that the elasticity of substitution between domestic and foreign varieties is the same as the elasticity of substitution across foreign varieties from different origins.

where $\bar{\sigma} > \sigma$ and $\sum_{i \in \mathcal{I}} \theta_{oin} = 1$. This functional form ensures the firm's demand for intermediates in the long run—i.e., when the firm can adjust relationship capital—is exactly CES with elasticity $\bar{\sigma}$ (see Appendix E.1.1). Our calibration exploits this fact, setting σ and $\bar{\sigma}$ to match estimates of short- and long-run trade elasticities.

Households Each country i contains a representative household that, at each time t, inelastically supplies ℓ_t units of labor and consumes c_{int} units of the domestic composite in each sector n. As in the general model, we assume households are in financial autarky.

At time 1, the household experiences consumption utility

$$\frac{\rho}{\rho-1}\left(\prod_{n \in \mathcal{N}} (c_{in1}/\theta^c_{in})^{\theta^c_{in}}\right)^{(\rho-1)/\rho}$$

where $\sum_{n \in \mathcal{N}} \theta^c_{in} = 1$ and where ρ is the elasticity of inter-temporal substitution.

At time 2, we give households an additional motive for consumption to compensate for the fact that investment demand vanishes in the second period. This allows us to square our two-period model with the data. Concretely, we assume that the household in each country i splits its time-2 consumption $\{c_{in2}\}_{n \in \mathcal{N}}$ into two components: consumption for consumption's sake, $\{c^c_{in2}\}_{n \in \mathcal{N}}$, and "investment-as-consumption", $\{c^\iota_{in2}\}_{n \in \mathcal{N}}$. For the first component, the household experiences utility equal to

$$\beta\frac{\rho}{\rho-1}\left(\prod_{n \in \mathcal{N}} (c^c_{in2}/\theta^c_{in})^{\theta^c_{in}}\right)^{(\rho-1)/\rho}$$

where β is a discount factor. For the second component, we assume the household experiences utility based on the production capital one could produce using the goods its consumes. Motivated by the case where the conflict period is very short compared to peacetime, we assume there is an exogenous marginal value ν_{in} of each unit of country-i, sector-n production capital "formed within the utility." The household therefore experiences investment-as-consumption utility equal to

$$\max_{\{\iota_{in',in2}\}} \sum_{n \in \mathcal{N}} \nu_{in}\phi_{in} \prod_{n' \in \mathcal{N}} \left(\frac{\iota_{in',in2}}{\theta^\iota_{in'n}}\right)^{\theta^\iota_{in'n}}$$
$$\text{s.t.} \sum_{n \in \mathcal{N}} \iota_{in',in2} \leq c^\iota_{in'2}$$

Total consumption utility at time 2 is the sum of these components.

Nesting within general model The parametric model described above is a special case of the many-country extension of the baseline model from Section 2.1. The set of goods $\mathcal{G}$ includes, for each country i, the leisure of country-i households, a domestic composite for each sector n, and a differentiated variety for each sector n and destination j. The set of capital varieties $\mathcal{V}$ includes, for each sector n, one capital variety used in differentiated variety production, and $|\mathcal{I}|$ capital varieties used in composite production, one per origin country.[26] Households' consumption utility $u_{it}(\cdot)$ encompasses both their inelastic labor supply and their consumption preferences over domestic composite goods. The goods production possibilities frontier $G_{it}(\cdot,\cdot)$ describes the possible net output of goods—conditional on capital inputs—implied by the expressions for gross output of goods in Equations 14, 15, and 17. The capital production possibilities frontier $\Lambda_i(\cdot,\cdot)$ describes the possible capital output—conditional on investment—stated in Equations 16 and 18.

4.2 Calibration

We calibrate the model from Section 4.1 to match the world economy in 2017, the last year before the US-China trade war. We summarize our data sources and calibration procedure below; see Appendices D and E.3 for further details.

Data We take $\mathcal{I}$ to be a set of 39 countries that either import or export over 96% of world trade in 2017, plus one rest-of-the-world (RoW) aggregate. We take $\mathcal{N}$ to be a set of 26 sectors determined as the finest common coarsening of those used by Ding (2021) and Adão et al. (2024).

We obtain data on bilateral, pre-tariff, FOB trade flows by sector from the OECD's Inter-Country Input-Output database (ICIO). The ICIO data also provides country-level data on aggregate consumption as well as the sectoral composition of consumption, investment, and sector-specific intermediate expenditures.

We compute bilateral, statutory, ad-valorem-equivalent tariffs at the sectoral level using product-level tariffs imputed by Adão et al. (2024). We average these tariffs across products within each sector, weighting by product-level trade flows from CEPII's BACI data set.

We obtain data on labor shares of value added at the country-sector level from the World Input Output Database. We use values from the most recent year in which this data is available, 2014.

[26] A small adjustment is required to ensure that the problem of composite producers has a solution when firms rent capital (as in Section 2.1) rather than owning capital (as above). Namely, one must replace the relationship capital k_{ijn} that appears in firms' production functions with a rescaled version k_{ijn}/K_{jn}, where K_{jn} is an appropriately chosen homothetic aggregator of $\{k_{ijn}\}_{i\in\mathcal{I}}$. This ensures that the firm's production function has constant returns to scale, but—when Equation 18 holds—it has no other consequences.

Finally, we borrow from Ding (2021) an imputed measure of the sectoral composition of investment spending by firms in each country and sector. We also use his estimates of sector-specific capital depreciation rates and the global risk-free rate of return.

Calibration strategy: economic block There are four types of parameters that we must calibrate: elasticities $\{\sigma, \bar{\sigma}, \rho\}$, preference and production shares $\left\{\theta^{\ell}_{in}, \theta^{k}_{in}, \theta^{x}_{in'n}, \theta_{ijn}, \theta^{\iota}_{in'n}, \theta^{c}_{in}\right\}$, productivity and preference shifters $\{\phi_{ijn}, \phi_{in}, \nu_{in}, \beta\}$ and labor endowments $\{\ell_j\}$.

We set σ and $\bar{\sigma}$ to match the evidence on short- and long-run trade elasticities from Boehm et al. (2020). Following Boehm et al. (2024), we set the short-run elasticity to $\sigma = 1.25$ and the long-run elasticity to $\bar{\sigma} = 2$. We set $\rho = 0.53$, drawing on estimates of China's elasticity of inter-temporal substitution reported in the meta-analysis of Havranek et al. (2015).

We set the preference and production share parameters to match shares in the observed data. Namely, we set $\theta^{x}_{in'n}$ to match intermediate expenditures on n' as a share of revenues among firms in country i and sector n. We set θ^{ℓ}_{in} to match the labor share of value added, given the ratio of value added to revenues implied by $\{\theta^{x}_{in'n}\}_{n' \in \mathcal{N}}$. We set θ^{k}_{in} to account for the remainder of revenues. We set θ_{ijn} to match the share of country j's sector n imports coming from origin i. We set $\theta^{\iota}_{in'n}$ so that (a) the profit share of capital income is consistent with the sectoral rates of capital depreciation and risk-free return, and (b) we match the sectoral composition of investment by firms in country i and sector n. We set θ^{c}_{in} to match the sectoral composition of consumption in country i.

To calibrate productivity and preference shifters, we normalize units of account so that, in the observed equilibrium, wages, marginal private values of capital, and all the prices of all origin-destination-sector goods are equal to one. This normalization pins down the values of ϕ_{ijn} and ϕ_{in}. We set the investment-as-consumption preference shifters ν_{in} so that, under free trade, investment in the first period is equal to consumption-as-investment in the second. In this sense, our calibration is consistent with the idea that, without US trade threats, the economy would be in steady state. We study Home's optimal peacetime policies in the limit where the conflict period is short, i.e., $\beta \to 0$. This choice is consistent with our interpretation of σ as the short-run trade elasticity and assumption that there are no decreasing returns in the value of capital formed inside household preferences at time 2. It also simplifies computation by ensuring that time-2 outcomes do not affect time-1 investments.

Finally, we calibrate the labor endowments ℓ_i so that labor markets clear. We do this by computing the labor demands implied by observed consumption around the world and the various production shares that describe how consumption is produced by workers.

Calibration strategy: geopolitical block In order to compute Home's optimal peacetime trade taxes, we also must take a stance on countries' geopolitical preferences. Geopolitical preferences affect Home's peacetime trade taxes directly, through the Lagrange multiplier κ on the planner's incentive compatibility constraint for Foreign's geopolitical action. They also affect Home's peacetime trade taxes indirectly, by determining the allocations local to which we must evaluate the statistics upon which these taxes depend.

Rather than taking a direct stance on the geopolitical primitives of the model—i.e., the geopolitical preferences $z_i(\cdot)$—we calibrate these primitives indirectly to generate a particular value of κ. For simplicity, and as there is no other canonical choice, we study the limit as $\kappa \to 0$ limit. This corresponds to the case where Home's off-path sanction threat is almost strong enough to incentivize Foreign to take its preferred geopolitical action, so that the trade subsidies Home must offer as a "carrot" on-path are very small. Focusing on the $\kappa \to 0$ limit also has the advantage that it implies on-path trade taxes in both periods approach zero. Under our calibration, this ensures there exist welfare weights on each foreign country for which the Home planner is indifferent to redistribution in both periods, as Theorem 3 assumes.[27]

5 Quantifying optimal geopolitical policies

This section quantifies the US's optimal peacetime policies for building geopolitical power vis-à-vis China. We focus on the case of unrestricted trade threats, implying that Home abstains from industrial policies. We compute Home's optimal trade policies by applying our characterization of Home's peacetime trade taxes from Section 3.3.4 to the calibrated model of Section 4.

We now explain the precise exercise we consider and then turn to quantitative results.

5.1 Quantitative exercise

We compute Home's optimal peacetime trade taxes under the assumption it is indifferent to redistribution across all countries in the world. We also assume that all countries other than Home engage in free trade and have no capital subsidies. This approach ensures that there is no rationale for Home's peacetime trade policy other than to build geopolitical influence.

[27]Our calibration of the shifters ν_{in} to investment-as-consumption demand guarantees that, if trade taxes are zero in both periods, then each country's representative households has the same non-investment consumption in both periods. Since β is common across countries, each country has the same ratio of marginal utilities of consumption across time. So countries would not borrow or save even if the model allowed it. This implies there is no "missing markets" rationale for peacetime trade policy; see the discussion following Theorem 3.

Recall that we have calibrated geopolitical preferences in order to study trade taxes in the limit where κ—the Lagrange multiplier on the planner's incentive compatibility constraint for Foreign's geopolitical action—converges to zero. Since peacetime trade taxes converge to zero in this limit (as do on-path time-2 trade taxes), our quantitative exercise makes meaningful predictions only about the *relative* size of Home's trade taxes on different goods, not their absolute magnitudes.

It is important to note that we compute Home's bilateral peacetime trade taxes on all trading partners, not just China. This allows us to quantify how Home may build geopolitical power *indirectly*. For example, the US may opt to tax imports from India so that Indian prices decline, China imports more from India, and China becomes dependent on these imports. Such indirect dependencies are valuable to the extent later US trade taxes can affect India's willingness to trade with China. This is indeed the case, since we assume Home can set different trade taxes by country not only in peacetime but also during conflict. As a result, the US's off-path trade threat can involve both taxes on trade with China and subsidies on trade with bystanders; these subsidies punish China indirectly by raising the price of bystanders' exports. One quirk of our approach is that, in our model, these trade subsidies to bystanders can be made so large as to put China in autarky.[28] The US's off-path trade punishment to China is therefore somewhat extreme. Future work will consider the complementary case where not all US trade threats are credible, ruling out such extreme punishments.

In practical terms, we compute the US's optimal peacetime trade taxes using Appendix Equation 29 for optimal taxes with many countries, into which we substitute numerical derivatives computed using the algorithms described in Appendix E.2. In order to avoid computing the integral that appears in this expression, we take advantage of the observation that

$$\int_0^{m_{F2}^{FT}} m \cdot \frac{\partial^2 \widetilde{p}_{F2}(m, k_F)}{\partial k_F \, \partial m} \cdot dm \;=\; r_{F2}^{FT} - r_{F2}^{A} - m_{F2}^{FT} \frac{\partial \widetilde{p}_{F2}(m_{F2}^{FT}, k_F)}{\partial k_F}$$

where "FT" and "A" denote equilibrium quantities under free trade and autarky, respectively. Finally, as trade taxes are only determined up to Lerner symmetry, we without loss of generality normalize taxes so that Home's revenues from or expenditures on import tariffs are the same as on export taxes. In this sense, Home's trade taxes are equally expansionist or protectionist on the import and export margins.

[28] Technically, this requires that the US have elastic labor supply, so that it is technologically possible for it to produce enough to offer these generous subsidies to bystanders. We implicitly assume that the US household supplies labor elastically but its labor supply preferences are kinked around ℓ_{US}, so that it only supplies a level of labor other than ℓ_{US} in the off-path punishment just described.

5.2 Quantitative results

Figure 6 displays our estimates of Home's optimal peacetime trade taxes for building geopolitical power vis-à-vis China. The left panel displays import taxes while the right panel displays export subsidies. In both panels, blue indicates trade promotion and orange indicates trade discouragement. The absolute scale of values is not important, as (recall) we compute trade taxes up to a constant. Plots normalize values by assigning an absolute value of one to the largest trade tax / subsidy.

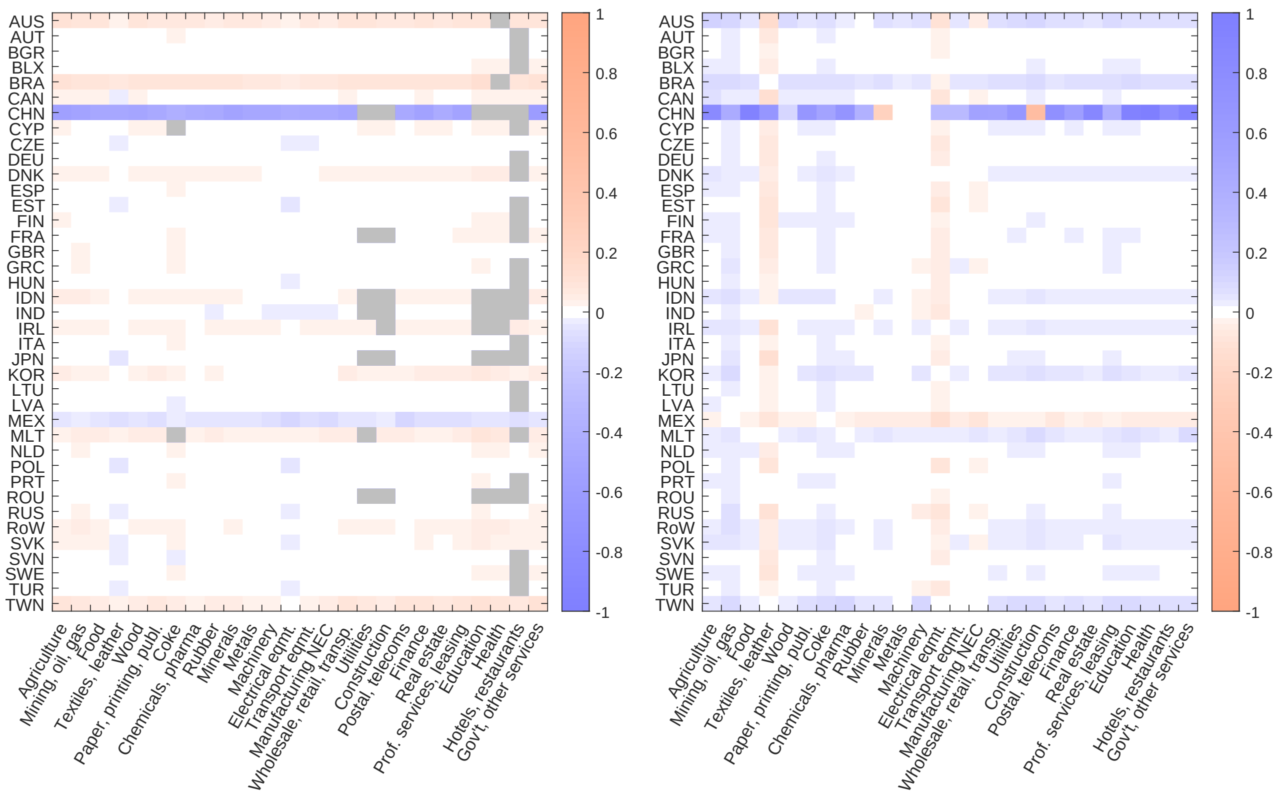

Figure 6: *Optimal peacetime trade taxes for the US. Left panel: import tariffs. Right panel: export subsidies. Gray cells represent origin-sectors that have no exports to the US.*

We divide our discussion of these trade taxes into two parts: taxes on trade with China and taxes on trade with the rest of the world.

Taxes on trade with China The US's optimal peacetime trade taxes use both import subsidies and export subsidies to promote trade with China. This explains the large and predominantly pro-trade values in the China rows of both panels of Figure 6.

What mechanism explains the US's bilateral trade promotion with China? To understand this, we decompose the US's optimal taxes into several additive components. First, with many countries, Home's trade taxes are the sum of a terms-of-trade-elasticity-manipulability term and an adjustment for the presence of many countries (see Section 3.3.4). Second, the manipulability term is the sum of effects that operate through changes in China's relationship capital and effects that operate through changes in its production capital. Third, the relationship capital manipulability term can itself be expressed as a sum over the different trading partners (both foreign and domestic) and sectors in which China forms trading relationships.

We use Shapley decompositions to assess the importance of each of these components in explaining US taxes on trade with China, in the sense of minimizing mean squared prediction error. We find that the US's desire to manipulate China's terms-of-trade elasticities through relationship capital adjustments explains 111% of these bilateral trade taxes. By contrast, the US's desire to manipulate elasticities through production capital and to account for the multi-country adjustment are of little quantitative importance. The US's subsidies on exports to China in each sector are mainly explained by its desire to promote Chinese trading relationships with the US in that sector. The US's subsidies on imports from China in each sector are mainly explained by its desire to promote Chinese trading relationships with third-party countries in other sectors. Intuitively, additional Chinese exports to the US raise the relative price of Chinese goods, which encourages Chinese firms and consumers to import more and invest accordingly. Appendix Figures 9–10 provide detailed Shapley decompositions.

In which sectors does the US particularly promote trade? Its import tariffs are relatively uniform across sectors, ranging from −40% to −58% of the US's largest trade tax. However, its export subsidies vary widely across sectors, ranging from −52% to 100% of the US's largest trade tax. This variation is almost entirely explained by the sectoral composition of China's direct and indirect expenditures on investment goods.[29] As shown in Figure 7, the US subsidizes exports in consumption sectors like food and services but taxes exports in sectors used for investment and the production of investment goods, like construction and minerals. The reason the US promotes trade less in sectors used for investment is that, in our quantitative model, households have different elasticities of demand for investment than for consumption and their investment demand elasticities are less manipulable. The reason the US positively taxes exports in investment sectors—rather than just subsidizing them less—is that Chinese imports from the US crowd out Chinese imports from third-party countries in sectors where Chinese terms-of-trade elasticities are more manipulable.

[29] By "indirect expenditures" on investment goods, we mean expenditures on intermediate inputs used to produce investment goods. For simplicity, we construct this measure while assuming that all investment goods consumed in China are produced using intermediates only from China.

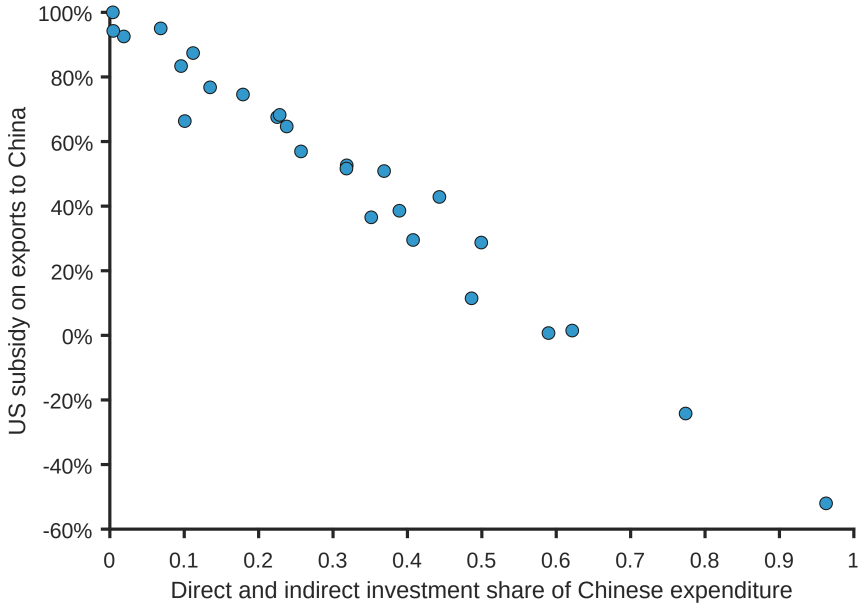

Figure 7: *Variation across sectors in US subsidies on exports to China and the shares of (direct and indirect) investment in Chinese expenditure.*

Taxes on trade with the rest of the world The US's optimal peacetime taxes on trade with countries other than China are much smaller than its bilateral taxes on trade with China. The former have an average magnitude of about 4.3% of the latter. This reflects that changes in trade with the rest of the world carry the same distortionary costs as changes in trade with China, but bring smaller benefits since they impact China only indirectly.

As in the previous section, we decompose the US's optimal peacetime trade taxes with the rest of the world into several channels. Once again, the main qualitative force behind US trade taxes is the US's desire to manipulate China's terms-of-trade elasticities by influencing its relationship capital investments. This channel accounts for 116% of the variation across countries (excluding China) and sectors in US import tariffs and 90% of the variation in US export subsidies. These policies primarily seek to promote Chinese investments in trading relationships with countries other than the US. Appendix Figure 11 provides detailed Shapley decompositions.

Interestingly, the US provides the largest subsidies to imports from country-sectors on which it also levies the largest export taxes. This can be seen by comparing the left and right panels of Figure 6 (and ignoring the China row): The countries (like Mexico) and the sectors (like textiles and electrical equipment) that receive the largest import subsidies in the left panel—i.e. are the most blue—also receive the largest export taxes in the right panel—i.e. are the most red.[30] In other words, the US promotes global scarcity of certain goods—like

[30]See Appendix Figure 12 for a scatterplot of the US's non-China import tariffs against its non-China

those from Mexico, textiles, and electrical equipment—while it promotes global abundance of others—like those from Taiwan and Brazil.

Which countries' goods does the US seek to make scarce, and why? As shown in the left panel of Figure 8, the US particularly promotes global scarcity of goods produced by countries (especially Mexico) that run bilateral trade deficits with China, while it promotes global abundance of goods produced by countries (especially Taiwan) that run bilateral trade surpluses with China. This is consistent with a simple economic mechanism: When a country's goods become scarce, they appreciate in value, encouraging the country to sell fewer exports and buy more imports to balance its current account. For a country like Mexico that buys much more from China than it sells, this results in more trade with China. For a country like Taiwan that sells much more to China than it buys, the reverse is true, so that abundance of its goods—rather than scarcity—promotes trade with China. In both cases, US trade policy promotes Chinese openness, leading China to invest more in relationship capital with third-party countries. Increased Chinese reliance on third-party countries heightens the power of the US's indirect sanctions, which promote US trade with third-party countries in order to reduce third-party trade with China (see Section 5.1).

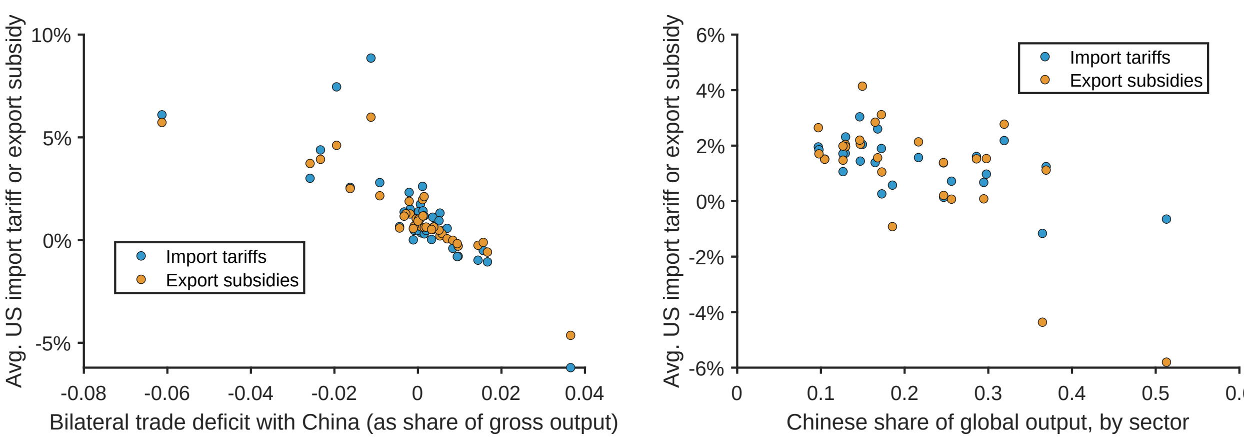

Figure 8: *Left panel: US trade taxes with third-party countries vs. third-party bilateral trade deficits with China. Right panel: US export subsidies with sectors in third-party countries vs. Chinese share of global sectoral output.*

Similarly, which sectors' goods does the US seek to make scarce, and why? As shown in the right panel of Figure 8, the US particularly promotes global scarcity of goods in sectors (like textiles and electrical equipment) for which China produces a large share of global output, while it promotes global abundance of goods in sectors (like mining and many service sectors) for which China produces a small share of global output. This, too, is consistent with a simple

export subsidies.

economic mechanism: The scarcer are goods that China produces, and the more abundant are goods that it does not produce, the more China will trade with third-party countries, leading it to invest more in relationship capital with them. Greater Chinese reliance on third-party countries heightens the power of the US's indirect sanctions.

6 Conclusion

The tides of world trade may be changing. After decades of focus on reducing trade barriers, recent years have seen many countries return to an older priority: economic statecraft. This interest has sparked a wave of policy proposals—from energy stockpiles to friend-shoring—that countries have considered to reduce their dependence on trading partners and, in some cases, promote the reverse.

In this new era, economic theory has lagged behind practice. Our paper attempts to narrow this gap by studying optimal industrial and trade policies in a model that brings together countries' economic and geopolitical decisions. This framework allows us to deliver both qualitative insights about the role of economic policies in addressing geopolitical frictions and quantitative statements about appropriate policies for particular bilateral relationships.

Our analysis leaves many questions unanswered. What policies are appropriate for countries that expect to face sanctions rather than apply them? How do countries' policy motives change in dynamic settings with repeated conflicts over time? Most importantly, what does our understanding of countries' unilateral policies imply for the design of bilateral trade agreements and multilateral organizations such as the WTO? We hope to address these topics in future research.

References

Adão, Rodrigo, John Sturm Becko, Arnaud Costinot, and Dave Donaldson, "Why is Trade Not Free? A Revealed Preference Approach," Technical Report, National Bureau of Economic Research 2023.

_ , _ , _ , and _ , "A world trading system for whom? Evidence from global tariffs," Technical Report 2024.

Alekseev, Maxim and Xinyue Lin, "Trade Policy in the Shadow of Conflict: The Case of Dual-Use Goods," Technical Report 2024.

Bachmann, Ruediger, David Baqaee, Christian Bayer, Moritz Kuhn, Andreas Löschel, Benjamin Moll, Andreas Peichl, Karen Pittel, and Moritz Schularick, "What if? The economic effects for Germany of a stop of energy imports from Russia," Technical Report, ECONtribute Policy Brief 2022.

Baldwin, Richard, Rebecca Freeman, and Angelos Theodorakopoulos, "Hidden exposure: measuring US supply chain reliance," Technical Report, National Bureau of Economic Research 2023.

Becko, John Sturm, "A theory of economic sanctions as terms-of-trade manipulation," *Journal of International Economics*, 2024, p. 103898.

Bianchi, Javier and César Sosa-Padilla, "On wars, sanctions, and sovereign default," *Journal of Monetary Economics*, 2024, *141*, 62–70.

Blackwill, Robert D and Jennifer M Harris, *War by other means: Geoeconomics and statecraft*, Harvard University Press, 2016.

Boehm, Christoph, Andrei A Levchenko, Nitya Pandalai-Nayar, and Hiroshi Toma, "Dynamic Models, New Gains from Trade?," Technical Report, National Bureau of Economic Research 2024.

Boehm, Christoph E, Andrei A Levchenko, and Nitya Pandalai-Nayar, "The long and short (run) of trade elasticities," Technical Report, National Bureau of Economic Research 2020.

Caliendo, Lorenzo and Fernando Parro, "Estimates of the Trade and Welfare Effects of NAFTA," *The Review of Economic Studies*, 2015, *82* (1), 1–44.

Clayton, Christopher, Matteo Maggiori, and Jesse Schreger, "A framework for geoeconomics," Technical Report, National Bureau of Economic Research 2023.

_ , _ , and _ , "A Theory of Economic Coercion and Fragmentation," *Available at SSRN 4767131*, 2024.

de Souza, Gustavo Henrique Bianco, Haishi Li, Yuan Mei, and Naiyuan Hu, "(Trade) war and peace: How to impose international trade sanctions," 2023.

Diamond, Peter A, "Consumption externalities and imperfect corrective pricing," *The Bell Journal of Economics and Management Science*, 1973, pp. 526–538.

Ding, Xiang, "Capital flows in global value chains," Technical Report 2021.

Dixit, Avinash, "Tax policy in open economies," in "Handbook of public economics," Vol. 1, Elsevier, 1985, pp. 313–374.

Drezner, Daniel W, Henry Farrell, and Abraham L Newman, *The uses and abuses of weaponized interdependence*, Brookings Institution Press, 2021.

Elliott, Matthew, Benjamin Golub, and Matthew V Leduc, "Supply network formation and fragility," *American Economic Review*, 2022, *112* (8), 2701–2747.

Farrell, Henry and Abraham L Newman, "Weaponized interdependence: How global economic networks shape state coercion," *International Security*, 2019, *44* (1), 42–79.

Grossman, Gene M, Elhanan Helpman, and Alejandro Sabal, "Optimal Resilience in Multi-Tier Supply Chains," *The Quarterly Journal of Economics*, 2024, p. qjae024.

_ , _ , and Hugo Lhuillier, "Supply chain resilience: Should policy promote international diversification or reshoring?," *Journal of Political Economy*, 2023, *131* (12), 3462–3496.

Havranek, Tomas, Roman Horvath, Zuzana Irsova, and Marek Rusnak, "Cross-country heterogeneity in intertemporal substitution," *Journal of International Economics*, 2015, *96* (1), 100–118.

Hirschman, Albert O, *National power and the structure of foreign trade* 1945.

_ , *The strategy of economic development* 1958.

Huang, Hanwei, "Germs, roads and trade: Theory and evidence on the value of diversification in global sourcing," *Available at SSRN 3095273*, 2017.

Itskhoki, Oleg and Dmitry Mukhin, "Sanctions and the exchange rate," Technical Report, National Bureau of Economic Research 2022.

Kleinman, Benny, Ernest Liu, and Stephen J Redding, "International friends and enemies," *American Economic Journal: Macroeconomics*, 2024, *16* (4), 350–385.

Kooi, Olivier, "Power and Resilience: An Economic Approach to National Security Policy," Technical Report 2024.

Liu, Ernest and David Yang, "International Power," Technical Report 2024.

Martin, Philippe, Thierry Mayer, and Mathias Thoenig, "Make trade not war?," *The Review of Economic Studies*, 2008, *75* (3), 865–900.

McLaren, John, "Size, sunk costs, and Judge Bowker's objection to free trade," *The American Economic Review*, 1997, pp. 400–420.

Moll, Benjamin, Moritz Schularick, and Georg Zachmann, "The power of substitution: The great German gas debate in retrospect," *Brookings Papers on Economic Activity*, 2023.

Scholvin, Sören and Mikael Wigell, "Power politics by economic means: Geoeconomics as an analytical approach and foreign policy practice," *Comparative Strategy*, 2018, *37* (1), 73–84.

Teti, Feodora, "Missing Tariffs, False Imputation, and the Trade Elasticity," 2023.

Thoenig, Mathias, *Trade policy in the shadow of war: A quantitative toolkit for geoeconomics*, Centre for Economic Policy Research, 2023.

Traiberman, Sharon and Martin Rotemberg, "Precautionary protectionism," *Journal of International Economics*, 2023, *145*, 103836.

A Proofs of main results

A.1 Technical assumptions

Our results rely on a few weak technical assumptions. We state these assumptions below and henceforth assume they hold without explicitly referring to them.

First, we impose smoothness, concavity / convexity, and local non-satiation conditions on primitives.

Assumption 4. *The functions $u_{it}(\cdot)$ are three-times differentiable, strictly concave, and locally non-satiated. The functions $G_{it}(\cdot,\cdot)$, and $\Lambda_{i1}(\cdot,\cdot)$ are three-times differentiable and weakly concave, and their negatives are locally non-satiated.*

Second, we assume that at least some good contributes to production to first order. This ensures there is a strictly positive price.

Assumption 5. *There exists a good $g^* \in \mathcal{G}$ for which $G_{it,m_{g^*}}(y,k) < 0$ for all i, t, y, and k.*

Third, we assume that Meade utility functions exist, are sufficiently smooth, and are increasing to first order in imports. These functions are key objects that allow us to combine household and firm optimization in our analysis.

Assumption 6. *The Meade utility functions $V_{it}(\cdot,\cdot)$ exist (i.e., the associated maxima are achieved), are three times differentiable, and satisfy $V_{it,m}(m,k) \gg 0$ for all m, k.*

Fourth, we assume that Foreign's choice of capital conditional on trade is well-defined and sufficiently smooth in trade.

Assumption 7. *For any m_{F1} and m_{F2}, the problem*

$$k_F \in \arg\max_k \; V_{F1}(m_{F1},k) + V_{F2}(m_{F2},k)$$

admits a solution $\widetilde{k}_F(m_{F1},m_{F2})$ (our convexity assumptions ensure this solution is unique). Moreover, $\widetilde{k}_F(m_{F1},m_{F2})$ is differentiable.

A.2 Primal representation of planner's problem

Under the assumptions in Appendix A.1, the planner's problem admits a more tractable primal representation. We state this representation result below and relegate its proof (which is fairly standard but tedious) to Appendix B.2.

To begin, consider the full planner's problem. This is to choose a profile of policies and equilibrium variables

$$\{c_{it}, y_{it}, \iota_{i1}, m_{it}, k^y_{it}, k_i, p^w_{it}, p_{it}, r_{it}, a, s_1, \tau_1, \tau_2, \widetilde{\tau}_2(\cdot)\}$$

that solves

$$\begin{aligned}
\max \quad & \sum_{i=H,F} (\mathbb{1}_{i=H} + \lambda_F \mathbb{1}_{i=F}) \left[u_{i1}(c_{i1}) + u_{i2}(c_{i2}) + v_i(a)\right] \\
\text{s.t.} \quad & \{c_{it}\}_{t=1,2} \in \arg\max_{\{c_t\}_{t=1,2}} \sum_{t=1,2} u_{it}(c_t) \\
& \text{s.t.} \quad \sum_{t=1,2} \left(p_{it} \cdot c_t - p_{it} \cdot (y_{it} - \mathbb{1}_{t=1}\iota_{i1}) - \mathbb{1}_{i=H}(p^w_t \tau_t) \cdot m_{Ht}\right) \leq 0 \\
& \{y_{it}, k^y_{it}\} \in \arg\max_{y, k^y}\; p_{it} \cdot y - r_{it} \cdot k^y \quad \text{s.t.} \quad G_{it}(y, k^y) \leq 0 \\
& \{k_i, \iota_{i1}\} \in \arg\max_{k, \iota_1} (r_{i1}(1 + \mathbb{1}_{i=H} s_1) + r_{i2}) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0 \qquad (19) \\
& c_{it} + \mathbb{1}_{t=1}\iota_{i1} = y_{it} + m_{it}, \qquad m_{Ht} + m_{Ft} = 0, \qquad \text{and} \qquad k_i = k^y_{it} \\
& p^w_t \cdot m_{it} = 0 \\
& p_{Ft} = p^w_t \quad \text{and} \quad p_{Ht} = (1 + \tau_t) p^w_t \\
& u_{F2}\Big(\widetilde{c}_{F2}(\widetilde{\tau}_2(a), k_H, k_F)\Big) + z_F(a) \geq u_{F2}\Big(\widetilde{c}_{F2}(\widetilde{\tau}_2(\neg a), k_H, k_F)\Big) + z_F(\neg a) \\
& \tau_2 = \widetilde{\tau}_2(a) \\
& \widetilde{\tau}_2(\cdot) \in \mathcal{T}(k_H, k_F)
\end{aligned}$$

Lemma 1. *A profile* $\{c_{it}, y_{it}, \iota_{i1}, m_{it}, k^y_{it}, k_i, p^w_{it}, p_{it}, r_{it}, a, s_1, \tau_1, \tau_2, \widetilde{\tau}_2(\cdot)\}$ *solves the planner's problem in Equation 19 if and only if some extended profile*

$$\Big\{c_{it}, y_{it}, \iota_{i1}, m_{it}, k^y_{it}, k_i, p^w_{it}, p_{it}, r_{it}, a, s_1, \tau_1, \tau_2, \widetilde{\tau}_2(\cdot), \gamma_i, \widetilde{c}_{i2}(\cdot), \widetilde{y}_{i2}(\cdot), \widetilde{m}_{i2}(\cdot), \widetilde{k}^y_{i2}(\cdot), \widetilde{p}_{i2}(\cdot), \widetilde{r}_{i2}(\cdot), \widetilde{p}^w_2(\cdot)\Big\}$$

maximizes

$$\begin{aligned}
& V_{H1}(-m_{F1}, k_H) + V_{H2}(-\widetilde{m}_{F2}(a), k_H) + v_H(a) \\
& + \lambda_F \left[V_{F1}(m_{F1}, k_F) + V_{F2}(\widetilde{m}_{F2}(a), k_F) + v_F(a)\right]
\end{aligned}$$

subject to

$$
\begin{aligned}
& k_F = \widetilde{k}_F(m_{F1}, \widetilde{m}_{F2}(a)) \\
& V_{F1,m}(m_{F1}, k_F) \cdot m_{F1} = 0 \\
& V_{F2,m}(\widetilde{m}_{F2}(\hat{a}), k_F) \cdot \widetilde{m}_{F2}(\hat{a}) = 0 \\
& V_{F2}(\widetilde{m}_{F2}(a), k_F) - V_{F2}(\widetilde{m}_{F2}(\neg a), k_F) \geq z_F(\neg a) - z_F(a) \\
& \frac{V_{H2,m}(-\widetilde{m}_{F2}(\cdot), k_H)}{V_{F2,m}(\widetilde{m}_{F2}(\cdot), k_F)} \in \mathcal{T}\,(k_H, k_F)
\end{aligned}
$$

and subject to

$$
\begin{aligned}
& \{y_{i1}, \iota_{i1}\} \in \arg\max_{y,\iota}\; u_{i1}(y + m_{i1} - \iota) \quad s.t. \quad G_{i1}(y, k_i) \leq 0, \quad \Lambda_{i1}(k_i, \iota) \leq 0 \\
& \widetilde{y}_{i2}(\hat{a}) \in \arg\max_{y}\; u_{i2}(y + \widetilde{m}_{i2}(\hat{a})) \quad s.t. \quad G_{i2}(y, k_i) \leq 0 \\
& p_{i1} = \gamma_i V_{i1,m}(m_{i1}, k_i) \qquad and \qquad \widetilde{p}_{i2}(\hat{a}) = \gamma_i V_{i2,m}(\widetilde{m}_{i2}(\hat{a}), k_i) \\
& r_{i1} = -G_{i1,k}(y_{i1}, k_i)/G_{i1,y_{g^*}}(y_{i1}, k_i) p_{i1g^*} \\
& \widetilde{r}_{i2}(\hat{a}) = -G_{i2,k}(\widetilde{y}_{i2}(\hat{a}), k_i)/G_{i2,y_{g^*}}(\widetilde{y}_{i2}(\hat{a}), k_i) \widetilde{p}_{i2g^*}(\hat{a}) \\
& r_{H1} s_1 = -\gamma_H \left[V_{H1,k}(m_{H1}, k_H) + V_{H2,k}(\widetilde{m}_{H2}(a), k_H)\right] \\
& c_{i1} = y_{i1} + m_{i1} - \iota_{i1} \qquad and \qquad \widetilde{c}_{i2}(\hat{a}) = \widetilde{y}_{i2}(\hat{a}) + \widetilde{m}_{i2}(\hat{a}) \\
& m_{F1} = -m_{H1} \qquad and \qquad \widetilde{m}_{H2}(\hat{a}) = -\widetilde{m}_{F2}(\hat{a}) \\
& k^y_{i1} = k_i \qquad and \qquad \widetilde{k}^y_{i2}(\hat{a}) = k_i \\
& p^w_1 = p_{F1} \qquad and \qquad \tau_1 = p_{H1}/p^w_1 - 1 \\
& \widetilde{p}^w_2(\hat{a}) = \widetilde{p}_{F2}(\hat{a}) \qquad and \qquad \widetilde{\tau}_2(\hat{a}) = \widetilde{p}_{H2}(\hat{a})/\widetilde{p}^w_2(\hat{a}) - 1 \\
& c_{i2} = \widetilde{c}_{i2}(a), \qquad y_{i2} = \widetilde{y}_{i2}(a), \qquad m_{i2} = \widetilde{m}_{i2}(a), \qquad k^y_{i2} = \widetilde{k}^y_{i2}(a) \\
& p_{i2} = \widetilde{p}_{i2}(a), \qquad r_{i2} = \widetilde{r}_{i2}(a), \qquad p^w_2 = \widetilde{p}^w_2(a), \qquad \tau_2 = \widetilde{\tau}_2(a)
\end{aligned}
$$

The advantage of the formulation in Lemma 1 is that the objective and the first set of constraints depend only on a reduced set of variables—i.e., m_{F1}, $\widetilde{m}_{F2}(\cdot)$, k_H, k_F, and a—while the second set of constraints is automatically satisfied by for some values of the remaining variables. The second set of constraints allows us to infer the values of other equilibrium variables, including policies, that are consistent with the reduced set of variables.

A.3 Proof of Theorem 1

Consider the planner's problem as formulated in Lemma 1. Note that the objective and the first set of constraints depend only on a reduced set of variables—i.e., m_{F1}, $\widetilde{m}_{F2}(\cdot)$, k_H, k_F, and a—while the second set of constraints is automatically satisfied by for some values of the remaining variables. It follows that, in any profile that solves the planner's problem, this reduced set of variables must maximize the objective subject to only the first set of constraints. That is, it must solve

$$\begin{aligned}
\max_{m_{F1},\widetilde{m}_{F2}(\cdot),k_H,k_F,a} \quad & V_{H1}(-m_{F1},k_H) + V_{H2}(-\widetilde{m}_{F2}(a),k_H) + v_H(a) \\
& + \lambda_F \left[V_{F1}(m_{F1},k_F) + V_{F2}(\widetilde{m}_{F2}(a),k_F) + v_F(a)\right] \\
\text{s.t.} \quad & k_F = \widetilde{k}_F(m_{F1},\widetilde{m}_{F2}(a)) \\
& V_{F1,m}(m_{F1},k_F)\cdot m_{F1} = 0 \\
& V_{F2,m}(\widetilde{m}_{F2}(\hat{a}),k_F)\cdot \widetilde{m}_{F2}(\hat{a}) = 0 \\
& V_{F2}(\widetilde{m}_{F2}(a),k_F) - V_{F2}(\widetilde{m}_{F2}(\neg a),k_F) \geq z_F(\neg a) - z_F(a) \\
& \frac{V_{H2,m}(-\widetilde{m}_{F2}(\cdot),k_H)}{V_{F2,m}(\widetilde{m}_{F2}(\cdot),k_F)} \in \mathcal{T}\left(k_H,k_F\right)
\end{aligned}$$

Under Assumption 1, the final constraint may be dropped. The planner's first-order condition with respect to k_H then implies

$$V_{H1,k}(-m_{F1},k_H) + V_{H2,k}(-\widetilde{m}_{F2}(a),k_H) = 0$$

The second set of constraints in the planner's problem formulation of Lemma 1 then implies that

$$r_{H1}s_1 = 0$$

A.4 Proof of Theorem 2

Consider the planner's problem as formulated in Lemma 1. As these conditions require that $\widetilde{c}_{H2}(\hat{a})$ satisfy the time-2 equilibrium conditions with capital $\{k_i\}$ and trade taxes $\widetilde{\tau}_2(a)$, we have (with a slight abuse of notation) $\widetilde{c}_{H2}(\hat{a}) = \widetilde{c}_{H2}(\widetilde{\tau}_2(\hat{a}),k_H,k_F)$. Moreover, Lemma 3 implies $u_{H2}(\widetilde{c}_{H2}(\hat{a})) = V_{H2}(\widetilde{m}_{H2}(\hat{a}),k_H)$. We may therefore replace the constraint on trade tax threats from Assumption 2 with the condition that

$$V_{H2}(\widetilde{m}_{H2}(\hat{a}),k_H) \geq \bar{U}$$

Note that the planner's problem can—as in the proof of Theorem 1—be divided into (a) an objective and a set of constraints that depend only on m_{F1}, $\widetilde{m}_{F2}(\cdot)$, k_H, k_F, and a, and (b) another set of constraints that is automatically satisfies for some values of the remaining variables. It follows that, in any profile that solves the planner's problem, this reduced set of variables must maximize the objective subject to only the first set of constraints. That is, it must solve

$$\begin{aligned}
\max_{m_{F1},\widetilde{m}_{F2}(\cdot),k_H,k_F,a} \quad & V_{H1}(-m_{F1},k_H)+V_{H2}(-\widetilde{m}_{F2}(a),k_H)+v_H(a) \\
& +\lambda_F\left[V_{F1}(m_{F1},k_F)+V_{F2}(\widetilde{m}_{F2}(a),k_F)+v_F(a)\right] \\
\text{s.t.} \quad & k_F=\widetilde{k}_F(m_{F1},\widetilde{m}_{F2}(a)) \\
& V_{F1,m}(m_{F1},k_F)\cdot m_{F1}=0 \\
& V_{F2,m}(\widetilde{m}_{F2}(\hat{a}),k_F)\cdot \widetilde{m}_{F2}(\hat{a})=0 \\
& V_{F2}(\widetilde{m}_{F2}(a),k_F)-V_{F2}(\widetilde{m}_{F2}(\neg a),k_F)\geq z_F(\neg a)-z_F(a) \\
& V_{H2}(\widetilde{m}_{H2}(\hat{a}),k_H)\geq \bar{U}
\end{aligned}$$

The planner's first-order condition with respect to k_H implies

$$V_{H1,k}(-m_{F1},k_H)+V_{H2,k}(-\widetilde{m}_{F2}(a),k_H)=-\sum_{\hat{a}=\underline{a},\overline{a}}\kappa(\hat{a})V_{H2,k}(\widetilde{m}_{H2}(\hat{a}),k_H)$$

for some $\kappa(\hat{a})\geq 0$. Note that $\kappa(\hat{a})\neq 0$ only when the constraint $V_{H2}(\widetilde{m}_{H2}(\hat{a}),k_H)\geq \bar{U}$ binds, i.e. when $\hat{a}$ is equal to the action a_L that results in the lowest Home economic utility.

The second set of constraints in the planner's problem formulation of Lemma 1 then implies that

$$r_{H1}s_1=\gamma_H\kappa(a_L)V_{H2,k}(\widetilde{m}_{H2}(a_L),k_H)$$

Finally, the definition of $V_{H2}(\cdot,\cdot)$ and the condition for $\widetilde{r}_{H2}(a_L)$ in the second set of constraints implies $\widetilde{r}_{H2}(a_L)\propto V_{H2,k}(\widetilde{m}_{H2}(a_L),k_H)$. This completes the proof.

A.5 Proof of Theorem 3

We begin with the simpler case in which the planner is indifferent to redistribution in both periods. We then turn to the general case.

Special case with indifference to redistribution We begin by following the same steps as the proof of Theorem 1 to show that the variables m_{F1}, $\widetilde{m}_{F2}(\cdot)$, k_H, k_F, and a must solve

$$
\begin{aligned}
\max_{m_{F1},\widetilde{m}_{F2}(\cdot),k_H,k_F,a} \quad & V_{H1}(-m_{F1},k_H)+V_{H2}(-\widetilde{m}_{F2}(a),k_H)+v_H(a) \\
& +\lambda_F\left[V_{F1}(m_{F1},k_F)+V_{F2}(\widetilde{m}_{F2}(a),k_F)+v_F(a)\right] \\
\text{s.t.} \quad & k_F=\widetilde{k}_F(m_{F1},\widetilde{m}_{F2}(a)) \\
& V_{F1,m}(m_{F1},k_F)\cdot m_{F1}=0 \\
& V_{F2,m}(\widetilde{m}_{F2}(\hat{a}),k_F)\cdot \widetilde{m}_{F2}(\hat{a})=0 \\
& V_{F2}(\widetilde{m}_{F2}(a),k_F)-V_{F2}(\widetilde{m}_{F2}(\neg a),k_F)\geq z_F(\neg a)-z_F(a)
\end{aligned}
$$

The remaining equilibrium variables must satisfy the second set of conditions in Lemma 1.

Note that the optimization problem above is equivalent to the problem where we subtract and add the trade balance constraint under actions a and $\neg a$, respectively, to the final constraint, resulting in

$$
\begin{aligned}
V_{F2}(\widetilde{m}_{F2}(a),k_F)-V_{F2,m}(\widetilde{m}_{F2}(a),k_F)\cdot\widetilde{m}_{F2}(a) \\
-V_{F2}(\widetilde{m}_{F2}(\neg a),k_F)+V_{F2,m}(\widetilde{m}_{F2}(\neg a),k_F)\cdot\widetilde{m}_{F2}(\neg a)\geq z_F(\neg a)-z_F(a)
\end{aligned}
\tag{20}
$$

Note that

$$
\frac{d}{dm}\left[V_{F2}(m,k_F)-m\cdot V_{F2,m}(m,k_F)\right]=-V_{F2,mm}(m,k_F)\cdot m
$$

So by the fundamental theorem of calculus, Equation 20 is the same as

$$
-\int_{\widetilde{m}_{F2}(\neg a)}^{\widetilde{m}_{F2}(a)} m\cdot V_{F2,mm}(m,k_F)\cdot dm\geq z_F(\neg a)-z_F(a)
$$

Using this equivalent version of the final constraint and substituting for $k_F=\widetilde{k}_F(m_{F1},\widetilde{m}_{F2}(a))$, we arrive at the following version of the planner's problem:

$$
\begin{aligned}
\max_{m_{F1},\widetilde{m}_{F2}(\cdot),k_H,k_F,a} \quad & V_{H1}(-m_{F1},k_H)+V_{H2}(-\widetilde{m}_{F2}(a),k_H)+v_H(a) \\
& +\lambda_F\left[V_{F1}\left(m_{F1},\widetilde{k}_F(m_{F1},\widetilde{m}_{F2}(a))\right)+V_{F2}\left(\widetilde{m}_{F2}(a),\widetilde{k}_F(m_{F1},\widetilde{m}_{F2}(a))\right)+v_F(a)\right] \\
\text{s.t.} \quad & V_{F1,m}\left(m_{F1},\widetilde{k}_F(m_{F1},\widetilde{m}_{F2}(a))\right)\cdot m_{F1}=0 \\
& V_{F2,m}\left(\widetilde{m}_{F2}(\hat{a}),\widetilde{k}_F(m_{F1},\widetilde{m}_{F2}(a))\right)\cdot\widetilde{m}_{F2}(\hat{a})=0 \\
& -\int_{\widetilde{m}_{F2}(\neg a)}^{\widetilde{m}_{F2}(a)} m\cdot V_{F2,mm}\left(m,\widetilde{k}_F(m_{F1},\widetilde{m}_{F2}(a))\right)\cdot dm\geq z_F(\neg a)-z_F(a)
\end{aligned}
$$

The first-order condition with respect to m_{H1g} is

$$
\begin{aligned}
0 = &- V_{H1,m_g} + \lambda_F \left[V_{F1,m_g} + \left(V_{F1,k} + V_{F2,k}(a) \right) \widetilde{k}_{F,m_{F1g}} \right] \\
&+ \kappa_1 \left[V_{F1,m_g} + m_{F1} \cdot V_{F1,mm_g} + m_{F1} \cdot V_{F1,mk} \cdot \widetilde{k}_{F,m_{F1g}} \right] \\
&+ \sum_{\hat{a}=\underline{a},\bar{a}} \widetilde{\kappa}_2(\hat{a}) \widetilde{m}_{F2}(\hat{a}) \cdot V_{F2,mk}(\hat{a}) \cdot \widetilde{k}_{F,m_{F1g}} \\
&- \kappa \left(\int_{\widetilde{m}_{F2}(\neg a)}^{\widetilde{m}_{F2}(a)} m \cdot V_{F2,mmk}\left(m, k_F\right) \cdot dm \right) \cdot \widetilde{k}_{F,m_{F1g}}
\end{aligned}
\tag{21}
$$

where we have omitted and/or simplified the arguments of functions where there is no risk of confusion.

We now simplify this expression in several steps. First, recall that the equilibrium profile must satisfy the second set of conditions in Lemma 1. The condition for k_F implies

$$V_{F1,k}(m_{F1}, k_F) + V_{F2,k}(\widetilde{m}_{F2}(a), k_F) = 0$$

Second, recall our assumption that Home is indifferent to the marginal international redistribution of goods bundles that have no effect on terms of trade and Foreign capital, both at time 1 and under Foreign's on-path geopolitical action a at time 2. Let m^*_{F1}, $m^*_{F2}(\underline{a})$, and $m^*_{F2}(\bar{a})$ be bundles—one for each time and geopolitical action—for which trade in the bundle at its corresponding time and under its corresponding geopolitical action has no effect on Foreign terms of trade or Foreign capital (when trade at other times and under other actions is held fixed).[31] Considering the first-order condition in Equation 21 along the direction of m^*_{F1} implies

$$0 = \underbrace{\cancel{-V_{H1,m} \cdot m^*_{F1} + \lambda_F V_{F1,m} \cdot m^*_{F1}}}_{\text{assumed indifference}} + \kappa_1 V_{F1,m} \cdot m^*_{F1}$$

which implies $\kappa_1 = 0$. Considering the analogous (but not shown) first-order condition with respect to $\widetilde{m}_{F2}(\hat{a})$ along the direction of $\widetilde{m}^*_{F2}(\hat{a})$ implies

$$0 = \widetilde{\kappa}_2(\hat{a}) V_{F2.m}(\hat{a}) \cdot \widetilde{m}^*_{F2}(\hat{a})$$

[31] We assume each such bundle has a non-zero value at world prices. We assume these bundles exist.

which implies $\widetilde{\kappa}_2(\hat{a}) = 0$.[32] Equation 21 therefore simplifies to

$$\frac{V_{H1,m_g}}{V_{F1,m_g}} = \lambda_F \left[1 - \frac{\kappa}{\lambda_F}\left(\int_{\widetilde{m}_{F2}(\neg a)}^{\widetilde{m}_{F2}(a)} m \cdot V_{F2,mmk}\left(m, k_F\right) \cdot dm\right) \cdot \widetilde{k}_{F,m_{F1g}} \Big/ V_{F1,m_g}\right]$$

where note $\lambda_F > 0$ since Home is indifferent to redistribution.

Finally, note that the second set of conditions from Lemma 1 implies

$$V_{F1,m} = (\gamma_F)^{-1} p_{F1} \qquad \text{and} \qquad \frac{V_{H1,m_g}}{V_{F1,m_g}} = \frac{\gamma_F}{\gamma_H}\left(1 + \tau_{1g}\right)$$

for some $\gamma_H, \gamma_F > 0$. In other words, Home's time-1 trade taxes satisfy

$$1 + \tau_{1g} = \frac{\gamma_H \lambda_F}{\gamma_F}\left[1 - \frac{\kappa \gamma_F}{\lambda_F}\left(\int_{\widetilde{m}_{F2}(\neg a)}^{\widetilde{m}_{F2}(a)} m \cdot \widetilde{p}_{F2,mk}\left(m, k_F\right) \cdot dm\right) \cdot \widetilde{k}_{F,m_{F1g}} \Big/ p_{F1g}\right]$$

This completes the proof provided that $a = \bar{a}$. This is without loss of generality because if $a = \underline{a}$ (i.e. if the geopolitical action Home implements on the equilibrium path is the one it dislikes), then the Langrange multiplier κ on Foreign's IC constraint must be zero.

General case without indifference to redistribution Above, we showed that κ_1 and $\widetilde{\kappa}_2(\hat{a})$ are zero if the social planner is indifferent to redistribution at all times and in all states of the world. More generally, they represent the relative social value that the planner places on transfers (of the neutral bundles discussed above) from Foreign to Home. For example

$$\kappa_1 = \frac{V_{H1,m} \cdot m^*_{F1} - \lambda_F V_{F1,m} \cdot m^*_{F1}}{V_{F1,m} \cdot m^*_{F1}} \tag{22}$$

Starting from Equation 21, we now derive a more general expression for Home's time-1 trade taxes when these multipliers are non-zero.

We begin by defining several elasticities. Let $\widetilde{p}_{Ft}(m, k) \equiv V_{Ft,m}(m, k)$ denote Foreign's inverse net import supply curve at time t as a function of its capital. We let Σ_g^{static} denote Foreign's static terms of trade elasticity that captures how time-1 trade changes time-1 terms of trade—both directly through trade and through movements in Foreign capital.

$$\Sigma_g^{\text{static}} \equiv \sum_{g' \in \mathcal{G}} \frac{m_{F1g'}}{\widetilde{p}_{F1g}(m_{F1}, k_F)} \frac{d\widetilde{p}_{F1g'}(m_{F1}, \widetilde{k}_{F1}(m_{F1}, m_{F2}))}{dm_{F1g}}$$

[32] For $\hat{a} = a$, this observation uses the assumption that the planner is indifferent to redistribution between Home and Foreign. For $\hat{a} = \neg a$, we do not need such an assumption because the action is off-path.

We let $\Sigma_g^{\text{dynamic}}(\hat{a})$ denote Foreign's dynamic, action-$\hat{a}$ terms-of-trade elasticity that captures how time-1 trade changes time-2 terms of trade under action $\hat{a}$ through movements in Foreign capital.

$$\Sigma_g^{\text{dynamic}}(\hat{a}) \equiv \sum_{g' \in \mathcal{G}} \frac{\widetilde{m}_{F2g'}(\hat{a})}{\widetilde{p}_{F1g}(m_{F1}, k_F)} \frac{d\widetilde{p}_{F2g}(\widetilde{m}_{F2}(\hat{a}), \widetilde{k}_{F1}(m_{F1}, m_{F2}))}{dm_{F1g}}$$

We let Σ_g^{manip} denote Foreign's incentive for terms of trade manipulation, the term emphasized above.

$$\Sigma_g^{\text{manip}} \equiv \frac{-1}{\widetilde{p}_{F1g}(m_{F1}, k_F)} \left(\int_{\widetilde{m}_{F2}(\underline{a})}^{\widetilde{m}_{F2}(\bar{a})} m \cdot V_{F2,mmk}\left(m, k_F\right) \cdot dm \right) \cdot \frac{\partial \widetilde{k}_{F1}(m_{F1}, m_{F2})}{\partial m_{F1g}}$$

In this notation, rearranging Equation 21 and using Equation 22 and the observation that $V_{H1,m_g}/V_{F1,m_g} = \gamma_H/\gamma_F(1+\tau_{1g})$ implies

$$\begin{aligned}
1+\tau_1 &= \frac{\gamma_F V_{H1,m} \cdot m^*_{F1}}{\gamma_H V_{F1,m} \cdot m^*_{F1}} \left[1 + \tau_{1g}^{ToT} + \hat{\kappa}\Sigma_g^{\text{manip}}\right] \\
\text{where} \quad \tau_{1g}^{ToT} &= \hat{\kappa}_1 \Sigma_g^{\text{static}} + \sum_{\hat{a}=\underline{a},\bar{a}} \hat{\kappa}_2(\hat{a}) \Sigma_g^{\text{dynamic}}(\hat{a}) \\
\hat{\kappa}_1 &\equiv \frac{V_{F1,m} \cdot m^*_{F1}}{V_{H1,m} \cdot m^*_{F1}} \frac{V_{H1,m} \cdot m^*_{F1} - \lambda_F V_{F1,m} \cdot m^*_{F1}}{V_{F1,m} \cdot m^*_{F1}} \\
\hat{\kappa}_2(\hat{a}) &\equiv \frac{V_{F1,m} \cdot m^*_{F1}}{V_{H1,m} \cdot m^*_{F1}} \frac{V_{H2,m}(\hat{a}) \cdot m^*_{F2}(\hat{a}) - \lambda_F V_{F2,m}(\hat{a}) \cdot m^*_{F2}(\hat{a})}{V_{F2,m}(\hat{a}) \cdot m^*_{F2}(\hat{a})} \\
\hat{\kappa} &\equiv \frac{V_{F1,m} \cdot m^*_{F1}}{V_{H1,m} \cdot m^*_{F1}} \kappa
\end{aligned}$$

In words, trade taxes take—up to Lerner symmetry—the same form as when the planner is indifferent to redistribution, plus new terms for static and dynamic terms of trade manipulation. These terms of trade manipulation terms are weighted by the planner's desire to redistribute from Home to Foreign at the allocation that results under each time and geopolitical action.

A.6 Proof of Proposition 1

Consider the planner's problem as formulated in Lemma 1. As these conditions require that $\widetilde{m}_{H2}(\hat{a})$ satisfy the time-2 equilibrium conditions with capital $\{k_i\}$ and trade taxes $\widetilde{\tau}_2(a)$, we have (with a slight abuse of notation) $\widetilde{m}_{H2}(\hat{a}) = \widetilde{m}_{H2}(\widetilde{\tau}_2(\hat{a}), k_H, k_F)$. We may therefore replace the constraint on trade tax threats from Assumption 3 with the condition that

$$\{\widetilde{m}_{H2}(\hat{a})\}_{\hat{a} \in \{\underline{a}, \bar{a}\}} \in \Gamma(k_H)$$

The optimal values of the variables m_{F1}, $\widetilde{m}_{F2}(\cdot)$, k_H, k_F, and a must therefore—analogously to the proof of Theorem 3—solve

$$\begin{aligned}
\max_{m_{F1},\widetilde{m}_{F2}(\cdot),k_H,k_F,a} \quad & V_{H1}(-m_{F1},k_H) + V_{H2}(-\widetilde{m}_{F2}(a),k_H) + \upsilon_H(a) \\
& + \lambda_F \left[V_{F1}(m_{F1},k_F) + V_{F2}(\widetilde{m}_{F2}(a),k_F) + \upsilon_F(a)\right] \\
\text{s.t.} \quad & k_F = \widetilde{k}_F(m_{F1},\widetilde{m}_{F2}(a)) \\
& V_{F1,m}(m_{F1},k_F) \cdot m_{F1} = 0 \\
& V_{F2,m}(\widetilde{m}_{F2}(\hat{a}),k_F) \cdot \widetilde{m}_{F2}(\hat{a}) = 0 \\
& \{\widetilde{m}_{H2}(\hat{a})\}_{\hat{a}\in\{\underline{a},\bar{a}\}} \in \Gamma(k_H)
\end{aligned}$$

The result now follows from the same steps as in the proof of Theorem 3.

B Proofs of supporting results

B.1 Technical lemmas

We now state and prove a few technical lemmas used to prove other supporting results.

First, we show the household problem may be split into an intra-temporal problem plus a condition on prices.

Lemma 2. *For each country i, the conditions*

$$\begin{aligned}
\{c_{it}\}_{t=1,2} \in \arg\max_{\{c_t\}_{t=1,2}} & \sum_{t=1,2} u_{it}(c_t) \\
s.t. \quad & \sum_{t=1,2} \left(p_{it}\cdot c_t - p_{it}\cdot(y_{it} - \mathbb{1}_{t=1}\iota_{i1}) - \mathbb{1}_{i=H}(p_t^w \tau_t)\cdot m_{Ht}\right) \leq 0 \\
c_{it} + \mathbb{1}_{t=1}\iota_{i1} &= y_{it} + m_{it} \\
p_t^w \cdot m_{it} &= 0
\end{aligned}$$

are equivalent the condition that there exist $\gamma_i > 0$ for which

$$\begin{aligned}
c_{it} \in \arg\max_c \ u_{it}(c) \quad s.t. \quad p_{it}\cdot c &\leq p_{it}\cdot(y_{it} - \mathbb{1}_{t=1}\iota_{i1}) + p_{it}\cdot m_{it} \\
p_{it} &= \gamma_i u_{it,c}(c_{it}) \\
c_{it} + \mathbb{1}_{t=1}\iota_{i1} &= y_{it} + m_{it} \\
p_t^w \cdot m_{it} &= 0
\end{aligned}$$

Proof. We prove the forward and reverse directions in turn.

Forward direction: Combining the goods market clearing and trade balance conditions implies

$$p_{it} \cdot c_{it} = p_{it} \cdot (y_{it} - \mathbb{1}_{t=1}\iota_{i1}) + p_{it} \cdot m_{it}$$

Since the provided household optimization condition implies household optimization within each period, we have

$$c_{it} \in \arg\max_{c} \ u_{it}(c) \quad \text{s.t.} \quad p_{it} \cdot c \leq p_{it} \cdot (y_{it} - \mathbb{1}_{t=1}\iota_{i1}) + p_{it} \cdot m_{it}$$

The provided household optimization condition also implies the existence of inverse Lagrange multipliers $\gamma_i > 0$ for which

$$p_{it} = \gamma_i u_{it,c}(c_{it})$$

Reverse direction: The provided trade balance condition, the budget constraints of the intra-temporal household problems, and local non-satiation imply

$$\sum_{t=1,2} (p_{it} \cdot c_t - p_{it} \cdot (y_{it} - \mathbb{1}_{t=1}\iota_{i1}) - \mathbb{1}_{i=H}(p_t^w \tau_t) \cdot m_{Ht}) = 0$$

Combining this observation with the condition on prices and the concavity of household utility implies that, by Lagrangian sufficiency,

$$\begin{aligned} \{c_{it}\}_{t=1,2} \in \arg\max_{\{c_t\}_{t=1,2}} & \sum_{t=1,2} u_{it}(c_t) \\ \text{s.t.} \quad & \sum_{t=1,2} (p_{it} \cdot c_t - p_{it} \cdot (y_{it} - \mathbb{1}_{t=1}\iota_{i1}) - \mathbb{1}_{i=H}(p_t^w \tau_t) \cdot m_{Ht}) \leq 0 \end{aligned}$$

□

Second we show that, taking trade and capital as given, the domestic equilibrium conditions are equivalent to domestic efficiency.

Lemma 3. *Given any time-1 net imports m_{i1} and capital k_i, a time-1 profile $\{c_{i1}, y_{i1}, \iota_{i1}, p_{i1}, r_{i1}\}$*

satisfies the trade-and-capital-conditional intra-temporal equilibrium conditions

$$\begin{aligned}
c_{i1} &\in \arg\max_{c} \; u_{i1}(c) \quad s.t. \quad p_{i1}\cdot c \le p_{i1}\cdot(y_{i1}-\iota_{i1}) + p_{i1}\cdot m_{i1} \\
\{y_{i1}, k^y_{i1}\} &\in \arg\max_{y,k^y} \; p_{i1}\cdot y - r_{i1}\cdot k^y \quad s.t. \quad G_{i1}(y,k^y)\le 0 \\
\iota_{i1} &\in \arg\max_{\iota_1} \; (r_{i1}(1+\mathbb{1}_{i=H}s_1) + r_{i2})\cdot k_i - p_{i1}\cdot \iota_1 \quad s.t. \quad \Lambda_{i1}(k_i,\iota_1)\le 0 \\
c_{i1}+\iota_{i1} &= y_{i1}+m_{i1} \\
k^y_{i1} &= k_i
\end{aligned}$$

if and only if there exist $\gamma_{i1} > 0$ for which

$$\begin{aligned}
\{y_{i1},\iota_{i1}\} &\in \arg\max_{y,\iota} \; u_{i1}(y+m_{i1}-\iota) \quad s.t. \quad G_{i1}(y,k_i)\le 0, \quad \Lambda_{i1}(k_i,\iota)\le 0 \\
p_{i1} &= \gamma_{i1} V_{i1,m}(m_{i1},k_i) \\
r_{i1} &= -G_{i1,k}(y_{i1},k_i)/G_{i1,y_{g^*}}(y_{i1},k_i)p_{i1g^*} \\
c_{i1}+\iota_{i1} &= y_{i1}+m_{i1} \\
k^y_{i1} &= k_i
\end{aligned}$$

Moreover, either set of conditions implies

$$u_{i1}(c_{i1}) = V_{i1}(m_{i1},k_i) \qquad \text{and} \qquad u_{i1,c}(c_{i1}) = V_{i1,m}(m_{i1},k_i)$$

Similarly, given any time-2 net imports m_{i2} and capital k_i, a time-2 profile $\{c_{i2}, y_{i2}, \iota_{i2}, p_{i2}, r_{i2}\}$ satisfies the trade-and-capital-conditional intra-temporal equilibrium conditions

$$\begin{aligned}
c_{i2} &\in \arg\max_{c} \; u_{i2}(c) \quad s.t. \quad p_{i2}\cdot c \le p_{i2}\cdot(y_{i2}-\iota_{i2}) + p_{i2}\cdot m_{i2} \\
\{y_{i2}, k^y_{i2}\} &\in \arg\max_{y,k^y} \; p_{i2}\cdot y - r_{i2}\cdot k^y \quad s.t. \quad G_{i2}(y,k^y)\le 0 \\
c_{i2} &= y_{i2}+m_{i2} \\
k^y_{i2} &= k_i
\end{aligned}$$

if and only if there exist $\gamma_{i2} > 0$ *for which*

$$\begin{aligned}
&y_{i2} \in \arg\max_{y} \ u_{i2}(y + m_{i2}) \quad s.t. \quad G_{i2}(y, k_i) \leq 0 \\
&p_{i2} = \gamma_{i2} V_{i2,m}(m_{i2}, k_i) \\
&r_{i2} = -G_{i2,k}(y_{i2}, k_i)/G_{i2,y_{g^*}}(y_{i2}, k_i) p_{i2g^*} \\
&c_{i2} = y_{i2} + m_{i2} \\
&k^y_{i2} = k_i
\end{aligned}$$

Moreover, either set of conditions implies

$$u_{i2}(c_{i2}) = V_{i2}(m_{i2}, k_i) \qquad \text{and} \qquad u_{i2,c}(c_{i2}) = V_{i2,m}(m_{i2}, k_i)$$

Proof. We prove the time-1 case. The time-2 case is analogous.

Forward direction: The provided time-1 conditions in each country i are the same as the equilibrium conditions in closed economy model with endowment m_{i1} and two competitive firms—one corresponding to goods production and one corresponding to producing a single feasible level of capital output (i.e., k_i) at least cost. The first welfare theorem applies to each of these as-if economies. So production and investment are socially efficient, i.e.,

$$\{y_{i1}, \iota_{i1}\} \in \arg\max_{y,\iota} \ u_{i1}(y + m_{i1} - \iota) \quad \text{s.t.} \quad G_{i1}(y, k_i) \leq 0, \quad \Lambda_{i1}(k_i, \iota) \leq 0 \tag{23}$$

Next, the provided household optimization condition implies there exists $\gamma_{i1} > 0$ for which

$$p_{i1} = \gamma_{i1} u_{i1,c}(c_{i1})$$

Moreover, the definition of $V_{i1}(\cdot, \cdot)$ implies $u_{i1,c}(c) = V_{i1,m}(m_{i1}, k_i)$ (and $u_{i1}(c) = V_{i1}(m_{i1}, k_i)$) if $c = y^*_{i1} - m_{i1} - \iota^*_{i1}$ for y^*_{i1} and ι^*_{i1} the output and investment levels associated with $V_{i1}(\cdot, \cdot)$'s definition. The provided goods market clearing condition and Equation 23 imply this is the case for $c = c_{i1}$, so

$$p_{i1} = \gamma_{i1} V_{i1,m}(m_{i1}, k_i)$$

Finally, the desired condition for r_{i1} follows from first-order conditions of the provided optimality condition for goods producers.

Reverse direction: Consider the same closed economies discussed in the forward direction. The provided conditions for $\{y_{i1}, \iota_{i1}\}$, m_{i1}, and k^y_{i1} imply that the allocation is both technologically feasible and efficient. In each as-if economy, the second welfare theorem applies. So there exist some prices p_{i1}^* and r_{i1}^* for which

$$\begin{aligned} c_{i1} &\in \arg\max_{c}\ u_{i1}(c) \quad \text{s.t.} \quad p_{i1}^* \cdot c \leq p_{i1}^* \cdot (y_{i1} - \iota_{i1}) + p_{i1}^* \cdot m_{i1} \\ \{y_{i1}, k_{i1}^y\} &\in \arg\max_{y,k^y}\ p_{i1}^* \cdot y - r_{i1}^* \cdot k^y \quad \text{s.t.} \quad G_{i1}(y, k^y) \leq 0 \\ \iota_{i1} &\in \arg\max_{\iota_1}\ (r_{i1}^*(1 + \mathbb{1}_{i=H} s_1) + r_{i2}^*) \cdot k_i - p_{i1}^* \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k_i, \iota_1) \leq 0 \end{aligned} \tag{24}$$

It remains to show $p_{i1}^* = p_{i1}$ and $r_{i1}^* = r_{i1}$, up to a price constant. To see this, note that the implied first-order condition for households implies that for some $\gamma_{i1}^* > 0$

$$p_{it}^* = \gamma_{it}^* u_{it}(c_{it})$$

Moreover, the definition of $V_{i1}(\cdot,\cdot)$ implies $u_{i1,c}(c) = V_{i1,m}(m_{i1}, k_i)$ (and $u_{i1}(c) = V_{i1}(m_{i1}, k_i)$) if $c = y_{i1}^* - m_{i1} - \iota_{i1}^*$ for y_{i1}^* and ι_{i1}^* the output and investment levels associated with $V_{i1}(\cdot,\cdot)$'s definition. The provided conditions for y_{i1} and ι_{i1} and goods market clearing imply this is the case for $c = c_{i1}$, so

$$p_{i1}^* = \gamma_{i1}^* V_{i1,m}(m_{i1}, k_i) = \frac{\gamma_{i1}^*}{\gamma_{i1}} p_{i1}$$

Similarly, the implied first-order condition for production firms implies

$$r_{i1}^* = -G_{i1,k}(y_{i1}, k_i)/G_{i1,y_{g^*}}(y_{i1}, k_i) p_{i1g^*}^*$$

so we have $r_{i1}^* = \frac{\gamma_{i1}^*}{\gamma_{i1}} r_{i1}$. We conclude that Equation 24 holds when p_{i1}^* and r_{i1}^* are replaced with p_{i1} and r_{i1}, respectively. $\square$

Third, we provide a simple characterization of the optimality conditions for capital in each country.

Lemma 4. *Suppose that for some $\gamma_i > 0$,*

$$\begin{aligned} p_{i1} &= \gamma_i V_{i1,m}(m_{i1}, k_i) \\ \widetilde{p}_{i2}(a) &= \gamma_i V_{i2,m}(\widetilde{m}_{i2}(a), k_i) \\ r_{i1} &= -G_{i1,k}(y_{i1}, k_i)/G_{i1,y_{g^*}}(y_{i1}, k_i) p_{i1g^*} \\ \widetilde{r}_{i2}(a) &= -G_{i2,k}(\widetilde{y}_{i2}(a), k_i)/G_{i2,y_{g^*}}(\widetilde{y}_{i2}(a), k_i) \widetilde{p}_{i2g^*}(a) \\ \{y_{i1}, \iota_{i1}\} &\in \arg\max_{y,\iota}\ u_{i1}(y + m_{i1} - \iota) \quad s.t. \quad G_{i1}(y, k_i) \leq 0, \quad \Lambda_{i1}(k_i, \iota) \leq 0 \\ \widetilde{y}_{i2}(a) &\in \arg\max_{y}\ u_{i2}(y + \widetilde{m}_{i2}(a)) \quad s.t. \quad G_{i2}(y, k_i) \leq 0 \end{aligned}$$

Then foreign capital k_F satisfies

$$k_i \in \arg\max_{k} \max_{\iota_1} \ (r_{i1}(1 + \mathbb{1}_{i=H}s_1) + \widetilde{r}_{i2}(a)) \cdot k - p_{i1} \cdot \iota_1 \quad s.t. \quad \Lambda_{i1}(k, \iota_1) \leq 0 \tag{25}$$

if and only if

$$\gamma_i \left[V_{i1,k}(m_{i1}, k_i) + V_{i2,k}(\widetilde{m}_{i2}(a), k_i)\right] = \mathbb{1}_{i=H} r_{i1} s_1 \tag{26}$$

Moreover Equation 26 holds for $i = F$ if and only if

$$k_F \in \arg\max_{k} \ V_{F1}(m_{F1}, k_F) + V_{F2}(\widetilde{m}_{F2}(a), k_F)$$

Proof. The final equivalence in the lemma statement is immediate from our convexity assumptions. It therefore suffices to show that Equations 25 and 26 are equivalent.

Using the definition of $V_{it}(\cdot, \cdot)$ and the given conditions for y_{F1}, ι_{F1}, and $\widetilde{y}_{F2}(a)$, Equation 26 is equivalent to the statement that there exists $\varphi^* \geq 0$ for which

$$\begin{aligned}
-\frac{\gamma_i V_{i1,m_{g^*}}(m_{i1}, k_i)}{G_{i1,y_{g^*}}(y_{i1}, k_i)} G_{i1,k}(y_{i1}, k_i) - \gamma_i \varphi^* \Lambda_{i1,k}(k_i, \iota_{i1}) & \\
-\frac{\gamma_i V_{i2,m_{g^*}}(\widetilde{m}_{i2}(a), k_i)}{G_{i2,y_{g^*}}(\widetilde{y}_{i2}(a), k_i)} G_{i2,k}(\widetilde{y}_{i2}(a), k_i) &= -\mathbb{1}_{i=H} r_{i1} s_1 \\
V_{i1,m}(m_{i1}, k_i) &= -\varphi^* \Lambda_{it,\iota}(k_i, \iota_{i1})
\end{aligned} \tag{27}$$

At the same time, using the given conditions for prices, Equation 25 is equivalent to the statement that there exists some $\varphi \geq 0$ and $\hat{\iota}_{i1}$ for which

$$\begin{aligned}
-\frac{\gamma_i V_{i1,m_{g^*}}(m_{i1}, k_i)}{G_{i1,y_{g^*}}(y_{i1}, k_i)} G_{i1,k}(y_{i1}, k_i) - \gamma_i \varphi \Lambda_{i1,k}(k_i, \hat{\iota}_{i1}) & \\
-\frac{\gamma_i V_{i2,m_{g^*}}(\widetilde{m}_{i2}(a), k_i)}{G_{i2,y_{g^*}}(\widetilde{y}_{i2}(a), k_i)} G_{i2,k}(\widetilde{y}_{i2}(a), k_i) &= -\mathbb{1}_{i=H} r_{i1} s_1 \\
V_{i1,m}(m_{i1}, k_i) &= -\varphi \Lambda_{it,\iota}(k_i, \hat{\iota}_{i1}) \\
\Lambda_{it,\iota}(k_i, \hat{\iota}_{i1}) &= 0
\end{aligned} \tag{28}$$

where the condition that $\Lambda_{it,\iota}(k_i, \hat{\iota}_{i1}) = 0$ is by non-satiation. The final two conditions and our convexity assumptions imply

$$\hat{\iota}_{i1} \in \arg\max_{\iota} \ u_{i1}(y + m_{i1} - \iota) \quad \text{s.t.} \quad \Lambda_{i1}(k_i, \iota) \leq 0$$

which ensures $\hat{\iota}_{i1} = \iota_{i1}$. Equations 27 and 28 are therefore equivalent when $\varphi = \varphi^*$. This completes the proof. $\square$

B.2 Proof of Lemma 1

To being, note that by Lemma 2, the planner's problem as stated in Appendix A.2 is equivalent to the problem of choosing a profile $\{c_{it}, y_{it}, \iota_{i1}, m_{it}, k_{it}^y, k_i, p_{it}^w, p_{it}, r_{it}, a, s_1, \tau_1, \tau_2, \widetilde{\tau}_2(\cdot), \gamma_i\}$ that solves the problem in Equation 19 after replacing the condition for $\{c_{it}\}_{t=1,2}$ with

$$\begin{aligned} &c_{it} \in \arg\max_c \; u_{it}(c) \quad \text{s.t.} \quad p_{it} \cdot c \le p_{it} \cdot (y_{it} - \mathbb{1}_{t=1}\iota_{i1}) + p_{it} \cdot m_{it} \\ &p_{it} = \gamma_i u_{it,c}(c_{it}) \end{aligned}$$

Next, recall $\widetilde{c}_{F2}(\tau_2, k_H, k_F)$ is the unique level of consumption consistent with the time-2 equilibrium conditions given trade taxes τ_2 and capital stocks k_H and k_F, or

$$\begin{aligned} c_{i2} &\in \arg\max_c \; u_{i2}(c) \quad \text{s.t.} \quad p_{i2} \cdot c \le p_{i2} \cdot y_{i2} + p_{i2} \cdot m_{i2} \\ \{y_{i2}, k_{i2}^y\} &\in \arg\max_{y,k^y} \; p_{i2} \cdot y - r_{i2} \cdot k^y \quad \text{s.t.} \quad G_{i2}(y, k^y) \le 0 \\ c_{i2} &= y_{i2} + m_{i2}, \qquad m_{H2} + m_{F2} = 0, \qquad \text{and} \qquad k_i = k_{i2}^y \\ p_2^w \cdot m_{i2} &= 0 \\ p_{F2} &= p_2^w \quad \text{and} \quad p_{H2} = (1 + \tau_2) p_2^w \end{aligned}$$

where note we have used trade balance to replace trade taxes revenues $\mathbb{1}_{i=H}(p_2^w \tau_2) \cdot m_{i2}$ with the domestic value of net imports, $p_{i2} \cdot m_{i2}$, in the household budget constraint. We may therefore replace $\widetilde{c}_{F2}(\widetilde{\tau}_2(\hat{a}), k_H, k_F)$ in the equilibrium conditions with a generic function $\widetilde{c}_{F2}(\hat{a})$ provided that we additionally require there to exist functions $\widetilde{c}_{i2}(\cdot)$, $\widetilde{y}_{i2}(\cdot)$, $\widetilde{m}_{i2}(\cdot)$, $\widetilde{k}_{i2}^y(\cdot)$, $\widetilde{p}_{i2}(\cdot)$, $\widetilde{r}_{i2}(\cdot)$, $\widetilde{p}_2^w(\cdot)$ that satisfy the time-2 equilibrium conditions given trade taxes $\widetilde{\tau}_2(\cdot)$ and capital stocks k_H and k_F. In other words, the planner's problem is equivalent to choosing a profile

$$\left\{c_{it}, y_{it}, \iota_{i1}, m_{it}, k_{it}^y, k_i, p_{it}^w, p_{it}, r_{it}, a, s_1, \tau_1, \tau_2, \widetilde{\tau}_2(\cdot), \gamma_i, \widetilde{c}_{i2}(\cdot), \widetilde{y}_{i2}(\cdot), \widetilde{m}_{i2}(\cdot), \widetilde{k}_{i2}^y(\cdot), \widetilde{p}_{i2}(\cdot), \widetilde{r}_{i2}(\cdot), \widetilde{p}_2^w(\cdot)\right\}$$

that maximizes the objective

$$\sum_{i=H,F} (\mathbb{1}_{i=H} + \lambda_F \mathbb{1}_{i=F}) \left[u_{i1}(c_{i1}) + u_{i2}(\widetilde{c}_{i2}(a)) + v_i(a)\right]$$

subject to

$$
\begin{aligned}
& c_{i1} \in \arg\max_{c} \; u_{i1}(c) \quad \text{s.t.} \quad p_{i1} \cdot c \leq p_{i1} \cdot (y_{i1} - \iota_{i1}) + p_{i1} \cdot m_{i1} \\
& \widetilde{c}_{i2}(\hat{a}) \in \arg\max_{c} \; u_{i2}(c) \quad \text{s.t.} \quad \widetilde{p}_{i2}(\hat{a}) \cdot c \leq \widetilde{p}_{i2}(\hat{a}) \cdot \widetilde{y}_{i2}(\hat{a}) + \widetilde{p}_{i2}(\hat{a}) \cdot \widetilde{m}_{i2}(\hat{a}) \\
& p_{i1} = \gamma_i u_{i1,c}(c_{i1}), \qquad \widetilde{p}_{i2}(a) = \gamma_i u_{it,c}(\widetilde{c}_{i2}(a)) \\
& \{y_{i1}, k^y_{i1}\} \in \arg\max_{y,k^y} \; p_{i1} \cdot y - r_{i1} \cdot k^y \quad \text{s.t.} \quad G_{i1}(y, k^y) \leq 0 \\
& \left\{\widetilde{y}_{i2}(\hat{a}), \widetilde{k}^y_{i2}(\hat{a})\right\} \in \arg\max_{y,k^y} \; \widetilde{p}_{i2}(\hat{a}) \cdot y - \widetilde{r}_{i2}(\hat{a}) \cdot k^y \quad \text{s.t.} \quad G_{i2}(y, k^y) \leq 0 \\
& \{k_i, \iota_{i1}\} \in \arg\max_{k,\iota_1} \; (r_{i1}(1 + \mathbb{1}_{i=H} s_1) + \widetilde{r}_{i2}(a)) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0 \\
& c_{i1} + \iota_{i1} = y_{i1} + m_{i1}, \qquad m_{H1} + m_{F1} = 0, \qquad \text{and} \qquad k_i = k^y_{i1} \\
& \widetilde{c}_{i2}(\hat{a}) = \widetilde{y}_{i2}(\hat{a}) + \widetilde{m}_{i2}(\hat{a}), \qquad \widetilde{m}_{H2}(\hat{a}) + \widetilde{m}_{F2}(\hat{a}) = 0, \qquad \text{and} \qquad k_i = \widetilde{k}^y_{i2}(\hat{a}) \\
& p^w_1 \cdot m_{1t} = 0 \\
& \widetilde{p}^w_2(\hat{a}) \cdot \widetilde{m}_{i2}(\hat{a}) = 0 \\
& p_{F1} = p^w_1 \quad \text{and} \quad p_{H1} = (1 + \tau_1) p^w_1 \\
& \widetilde{p}_{F2}(\hat{a}) = \widetilde{p}^w_2(\hat{a}) \quad \text{and} \quad \widetilde{p}_{H2}(\hat{a}) = (1 + \widetilde{\tau}_2(\hat{a})) \widetilde{p}^w_2(\hat{a}) \\
& u_{F2}\Big(\widetilde{c}_{F2}(a)\Big) + z_F(a) \geq u_{F2}\Big(\widetilde{c}_{F2}(\neg a)\Big) + z_F(\neg a) \\
& \tau_2 = \widetilde{\tau}_2(a) \\
& \widetilde{\tau}_2(\cdot) \in \mathcal{T}(k_H, k_F) \\
& c_{i2} = \widetilde{c}_{i2}(a), \qquad y_{i2} = \widetilde{y}_{i2}(a), \qquad m_{i2} = \widetilde{m}_{i2}(a), \qquad k^y_{i2} = \widetilde{k}^y_{i2}(a) \\
& p_{i2} = \widetilde{p}_{i2}(a), \qquad r_{i2} = \widetilde{r}_{i2}(a), \qquad p^w_2 = \widetilde{p}^w_2(a)
\end{aligned}
$$

By the convexity of $\Lambda_{i1}(\cdot, \cdot)$, the condition above for $\{k_i, \iota_{i1}\}$ is equivalent to the two conditions

$$
\begin{aligned}
\iota_{i1} &\in \arg\max_{\iota_1} \; (r_{i1}(1 + \mathbb{1}_{i=H} s_1) + r_{i2}) \cdot k_i - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k_i, \iota_1) \leq 0 \\
k_i &\in \arg\max_{k} \max_{\iota_1} \; (r_{i1}(1 + \mathbb{1}_{i=H} s_1) + r_{i2}) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0
\end{aligned}
$$

Lemma 3 then implies this version of the planner's problem is, in turn, equivalent to the condition that for some γ_{i1} and $\widetilde{\gamma}_{i2}(\cdot)$, the same profile maximizes

$$
\sum_{i=H,F} (\mathbb{1}_{i=H} + \lambda_F \mathbb{1}_{i=F}) \left[V_{i1}(m_{i1}, k_i) + V_{i2}(\widetilde{m}_{i2}(a), k_i) + v_i(a)\right]
$$

subject to

$$
\begin{aligned}
&\{y_{i1}, \iota_{i1}\} \in \arg\max_{y,\iota}\ u_{i1}(y + m_{i1} - \iota) \quad \text{s.t.} \quad G_{i1}(y, k_i) \le 0, \quad \Lambda_{i1}(k_i, \iota) \le 0 \\
&\widetilde{y}_{i2}(\hat{a}) \in \arg\max_{y}\ u_{i2}(y + \widetilde{m}_{i2}(\hat{a})) \quad \text{s.t.} \quad G_{i2}(y, k_i) \le 0 \\
&p_{i1} = \gamma_i V_{i1,m}(m_{i1}, k_i), \qquad \widetilde{p}_{i2}(a) = \gamma_i V_{i2,m}(\widetilde{m}_{i2}(a), k_i) \\
&p_{i1} = \gamma_{i1} V_{i1,m}(m_{i1}, k_i) \\
&\widetilde{p}_{i2}(\hat{a}) = \widetilde{\gamma}_{i2}(\hat{a}) V_{i2,m}(\widetilde{m}_{i2}(\hat{a}), k_i) \\
&r_{i1} = -G_{i1,k}(y_{i1}, k_i)/G_{i1,y_{g^*}}(y_{i1}, k_i) p_{i1g^*} \\
&\widetilde{r}_{i2}(\hat{a}) = -G_{i2,k}(\widetilde{y}_{i2}(\hat{a}), k_i)/G_{i2,y_{g^*}}(\widetilde{y}_{i2}(\hat{a}), k_i) \widetilde{p}_{i2g^*}(\hat{a}) \\
&k_i \in \arg\max_{k} \max_{\iota_1}\ (r_{i1}(1 + \mathbb{1}_{i=H} s_1) + \widetilde{r}_{i2}(a)) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \le 0 \\
&c_{i1} + \iota_{i1} = y_{i1} + m_{i1}, \qquad m_{H1} + m_{F1} = 0, \qquad \text{and} \qquad k_i = k^y_{i1} \\
&\widetilde{c}_{i2}(\hat{a}) = \widetilde{y}_{i2}(\hat{a}) + \widetilde{m}_{i2}(\hat{a}), \qquad \widetilde{m}_{H2}(\hat{a}) + \widetilde{m}_{F2}(\hat{a}) = 0, \qquad \text{and} \qquad k_i = \widetilde{k}^y_{i2}(\hat{a}) \\
&V_{F1,m}(m_{F1}, k_F) \cdot m_{i1} = 0 \\
&V_{F2,m}(\widetilde{m}_{F2}(\hat{a}), k_F) \cdot \widetilde{m}_{i2}(\hat{a}) = 0 \\
&p_{F1} = p^w_1 \quad \text{and} \quad p_{H1} = (1 + \tau_1) p^w_1 \\
&\widetilde{p}_{F2}(\hat{a}) = \widetilde{p}^w_2(\hat{a}) \quad \text{and} \quad \widetilde{p}_{H2}(\hat{a}) = (1 + \widetilde{\tau}_2(\hat{a})) \widetilde{p}^w_2(\hat{a}) \\
&V_{F2}(\widetilde{m}_{F2}(a), k_F) - V_{F2}(\widetilde{m}_{F2}(\neg a), k_F) \ge z_F(\neg a) - z_F(a) \\
&\tau_2 = \widetilde{\tau}_2(a) \\
&\widetilde{\tau}_2(\cdot) \in \mathcal{T}(k_H, k_F) \\
&c_{i2} = \widetilde{c}_{i2}(a), \qquad y_{i2} = \widetilde{y}_{i2}(a), \qquad m_{i2} = \widetilde{m}_{i2}(a), \qquad k^y_{i2} = \widetilde{k}^y_{i2}(a) \\
&p_{i2} = \widetilde{p}_{i2}(a), \qquad r_{i2} = \widetilde{r}_{i2}(a), \qquad p^w_2 = \widetilde{p}^w_2(a)
\end{aligned}
$$

Note that we may equivalently remove the clause "for some γ_{i1} and $\widetilde{\gamma}_{i2}(\cdot)$" and replace

$$
\begin{aligned}
p_{i1} &= \gamma_i V_{i1,m}(m_{i1}, k_i), \qquad \widetilde{p}_{i2}(a) = \gamma_i V_{i2,m}(\widetilde{m}_{i2}(a), k_i) \\
p_{i1} &= \gamma_{i1} V_{i1,m}(m_{i1}, k_i), \qquad \widetilde{p}_{i2}(\hat{a}) = \widetilde{\gamma}_{i2}(\hat{a}) V_{i2,m}(\widetilde{m}_{i2}(\hat{a}), k_i)
\end{aligned}
$$

with

$$
p_{i1} = \gamma_i V_{i1,m}(m_{i1}, k_i) \qquad \text{and} \qquad \widetilde{p}_{i2}(\hat{a}) = \gamma_i V_{i2,m}(\widetilde{m}_{i2}(\hat{a}), k_i)
$$

Given the international goods market clearing, we can freely replace m_{H1} with $-m_{F1}$ and $\widetilde{m}_{H1}(\hat{a})$ with $-\widetilde{m}_{F1}(\hat{a})$. We can similarly drop trade balance for $i = H$. Given the relationships between world prices, domestic prices, and Meade utility functions, we can also replace $\widetilde{\tau}_2(\cdot) \in \mathcal{T}(k_H, k_F)$ with

$$\frac{V_{H2,m}(\widetilde{m}_{H2}(\cdot), k_H)}{V_{F2,m}(\widetilde{m}_{F2}(\cdot), k_F)} \in \mathcal{T}(k_H, k_F)$$

By Lemma 4 and Assumption 7, we can replace the condition for k_F with

$$k_F = \widetilde{k}_F(m_{F1}, \widetilde{m}_{F2}(a))$$

and we can replace the condition for k_H with

$$\gamma_H \left[V_{H1,k}(m_{H1}, k_H) + V_{H2,k}(\widetilde{m}_{H2}(a), k_H)\right] = -r_{H1} s_1$$

Employing these observations and rearranging conditions, we arrive at the desired version of the planner's problem from the statement of the lemma.

C Model extensions

C.1 Geopolitical uncertainty

C.1.1 Extended model

We assume that Foreign's geopolitical preferences randomly determined by a state of the world ω that is revealed to all actors in period 2, after Home's trade threat has been set, but before Foreign takes its geopolitical action. Formally, we model uncertainty by representing all time-2 equilibrium variables as functions of ω. For example, we denote the vector of consumption in i at time 2 by $c_{i2}(\omega)$.

The equilibrium conditions in this augmented model are as follows.

Economic equilibrium conditions Households maximize expected consumption utility subject to a lifetime budget constraint.

$$\begin{aligned} c_{i1}, \{c_{i2}(\omega)\}_{\omega \in \Omega} \in \arg \max_{c_1, \{c_2(\omega)\}} \; & u_{i1}(c_{i1}) + \mathbb{E}[u_{i2}(c_{i2}(\omega))] \\ \text{s.t.} \quad & p_{i1} \cdot c_{i1} - I_{i1} + \mathbb{E}[p_{i2}(\omega) \cdot c_{i2}(\omega) - I_{i2}(\omega)] \leq 0 \end{aligned}$$

where I_{i1} and $I_{i2}(\omega)$ are income from domestic profits and lump-sum transfers, i.e.

$$I_{i1} = p_{i1} \cdot (y_{i1} - \iota_{i1}) + \mathbb{1}_{i=H}(p_1^w \tau_1) \cdot m_{i1}$$
$$I_{i2}(\omega) = p_{i2}(\omega) \cdot y_{i2}(\omega) + \mathbb{1}_{i=H}(p_2^w(\omega)\tau_2(\omega)) \cdot m_{i2}(\omega)$$

where m_{i1} and $m_{i2}(\omega)$ denote the net imports of country i.

Goods producers maximize profits subject to technological feasibility.

$$\{y_{i1}, k_{i1}^y\} \in \arg\max_{y,k^y} \; p_{i1} \cdot y - r_{i1} \cdot k^y \quad \text{s.t.} \quad G_{i1}(y, k^y) \leq 0$$
$$\{y_{i2}(\omega), k_{i2}^y(\omega)\} \in \arg\max_{y,k^y} \; p_{i2}(\omega) \cdot y - r_{i2}(\omega) \cdot k^y \quad \text{s.t.} \quad G_{i2}(y, k^y) \leq 0$$

Capital producers maximize expected profits subject to technological feasibility and facing subsidies.

$$\{k_i, \iota_{i1}\} \in \arg\max_{k,\iota_1} \; (r_{i1}(1 + \mathbb{1}_{i=H}s_1) + \mathbb{E}[r_{i2}(\omega)]) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0$$

Goods markets clear globally and capital markets clear within each country.

$$c_{it} + \iota_{t1} = y_{it} + m_{it}, \qquad m_{H1} + m_{F1} = 0, \qquad k_i = k_{it}^y$$
$$c_{i2}(\omega) = y_{i2}(\omega) + m_{i2}(\omega), \qquad m_{H2}(\omega) + m_{F2}(\omega) = 0, \qquad k_i = k_{i2}^y(\omega)$$

There is financial autarky. In other words, trade is balanced within each period and state of the world.

$$p_1^w \cdot m_{i1} = 0 \qquad \text{and} \qquad p_2^w(\omega) \cdot m_{i2}(\omega) = 0$$

Domestic prices equal world prices augmented by (possibly zero) trade taxes.

$$p_{F1} = p_1^w \quad \text{and} \quad p_{H1} = (1 + \tau_1)p_1^w$$
$$p_{F2}(\omega) = p_2^w(\omega) \quad \text{and} \quad p_{H2}(\omega) = (1 + \tau_2(\omega))p_2^w(\omega)$$

Geopolitical equilibrium conditions Foreign's geopolitical action solves

$$a(\omega) \in \arg\max_{\hat{a} \in \{\underline{a}, \overline{a}\}} \; u_{F2}\Big(\widetilde{c}_{F2}(\widetilde{\tau}_2(\hat{a}), k_H, k_F)\Big) + z_F(\hat{a}; \omega)$$

Home's time-2 trade taxes are determined by its trade tax threat and Foreign's geopolitical action.

$$\tau_2(\omega) = \widetilde{\tau}_2(a(\omega))$$

C.1.2 Extended results

Following the same steps as in the proof of Lemma 1 allows us to represent the planner's problem in its primal form. That is, the planner's problem is equivalent to choosing an extended profile

$$\Big\{c_{i1}, c_{i2}(\cdot), y_{i1}, y_{i2}(\cdot), \iota_{i1}, m_{i1}, m_{i2}(\cdot), k^y_{i1}, k^y_{i2}(\cdot), p^w_{i1}, p^w_{i2}(\cdot), p_{i1}, p_{i2}(\cdot), r_{i1}, r_{i2}(\cdot), a(\cdot),$$
$$s_1, \tau_1, \tau_2(\cdot), \widetilde{\tau}_2(\cdot), \gamma_i, \widetilde{c}_{i2}(\cdot), \widetilde{y}_{i2}(\cdot), \widetilde{m}_{i2}(\cdot), \widetilde{k}^y_{i2}(\cdot), \widetilde{p}_{i2}(\cdot), \widetilde{r}_{i2}(\cdot), \widetilde{p}^w_2(\cdot)\Big\}$$

that maximizes

$$\begin{aligned} &V_{H1}(-m_{F1}, k_H) + \mathbb{E}\left[V_{H2}(-\widetilde{m}_{F2}(a(\omega)), k_H) + v_H(a(\omega))\right] \\ &+\lambda_F\left[V_{F1}(m_{F1}, k_F) + \mathbb{E}\left[V_{F2}(\widetilde{m}_{F2}(a(\omega)), k_F) + v_F(a(\omega))\right]\right] \end{aligned}$$

subject to

$$\begin{aligned} &k_F = \widetilde{k}_F(m_{F1}, \widetilde{m}_{F2}(a(\cdot))) \\ &V_{F1,m}(m_{F1}, k_F)\cdot m_{F1} = 0 \\ &V_{F2,m}(\widetilde{m}_{F2}(\hat{a}), k_F)\cdot \widetilde{m}_{F2}(\hat{a}) = 0 \\ &V_{F2}(\widetilde{m}_{F2}(\bar{a}), k_F) - V_{F2}(\widetilde{m}_{F2}(\underline{a}), k_F) \geq \max_{\omega \text{ s.t. } a(\omega)=\bar{a}} z_F(\underline{a}, \omega) - z_F(\bar{a}, \omega) \\ &\frac{V_{H2,m}(-\widetilde{m}_{F2}(\cdot), k_H)}{V_{F2,m}(\widetilde{m}_{F2}(\cdot), k_F)} \in \mathcal{T}(k_H, k_F) \end{aligned}$$

and subject to

$$\{y_{i1}, \iota_{i1}\} \in \arg\max_{y,\iota} \; u_{i1}(y + m_{i1} - \iota) \quad \text{s.t.} \quad G_{i1}(y, k_i) \leq 0, \quad \Lambda_{i1}(k_i, \iota) \leq 0$$

$$\widetilde{y}_{i2}(\hat{a}) \in \arg\max_{y} \; u_{i2}(y + \widetilde{m}_{i2}(\hat{a})) \quad \text{s.t.} \quad G_{i2}(y, k_i) \leq 0$$

$$p_{i1} = \gamma_i V_{i1,m}(m_{i1}, k_i) \qquad \text{and} \qquad \widetilde{p}_{i2}(\hat{a}) = \gamma_i V_{i2,m}(\widetilde{m}_{i2}(\hat{a}), k_i)$$

$$r_{i1} = -G_{i1,k}(y_{i1}, k_i)/G_{i1,y_{g^*}}(y_{i1}, k_i) p_{i1g^*}$$

$$\widetilde{r}_{i2}(\hat{a}) = -G_{i2,k}(\widetilde{y}_{i2}(\hat{a}), k_i)/G_{i2,y_{g^*}}(\widetilde{y}_{i2}(\hat{a}), k_i) \widetilde{p}_{i2g^*}(\hat{a})$$

$$r_{H1} s_1 = -\gamma_H \left[V_{H1,k}(m_{H1}, k_H) + \mathbb{E}\left[V_{H2,k}(\widetilde{m}_{H2}(a(\omega)), k_H) \right] \right]$$

$$c_{i1} = y_{i1} + m_{i1} - \iota_{i1} \qquad \text{and} \qquad \widetilde{c}_{i2}(\hat{a}) = \widetilde{y}_{i2}(\hat{a}) + \widetilde{m}_{i2}(\hat{a})$$

$$m_{F1} = -m_{H1} \qquad \text{and} \qquad \widetilde{m}_{H2}(\hat{a}) = -\widetilde{m}_{F2}(\hat{a})$$

$$k^y_{i1} = k_i \qquad \text{and} \qquad \widetilde{k}^y_{i2}(\hat{a}) = k_i$$

$$p^w_1 = p_{F1} \qquad \text{and} \qquad \tau_1 = p_{H1}/p^w_1 - 1$$

$$\widetilde{p}^w_2(\hat{a}) = \widetilde{p}_{F2}(\hat{a}) \qquad \text{and} \qquad \widetilde{\tau}_2(\hat{a}) = \widetilde{p}_{H2}(\hat{a})/\widetilde{p}^w_2(\hat{a}) - 1$$

$$c_{i2}(\omega) = \widetilde{c}_{i2}(a(\omega)), \qquad y_{i2}(\omega) = \widetilde{y}_{i2}(a(\omega)), \qquad m_{i2}(\omega) = \widetilde{m}_{i2}(a(\omega)), \qquad k^y_{i2}(\omega) = \widetilde{k}^y_{i2}(a(\omega))$$

$$p_{i2}(\omega) = \widetilde{p}_{i2}(a(\omega)), \qquad r_{i2}(\omega) = \widetilde{r}_{i2}(a(\omega)), \qquad p^w_2(\omega) = \widetilde{p}^w_2(a(\omega)), \qquad \tau_2(\omega) = \widetilde{\tau}_2(a(\omega))$$

Following the same steps as in the proofs of Theorems 1 and 2 reveals that our results on industrial policy are unchanged.

Similarly, following the same steps as in the proofs of Theorem 3 and Proposition 1 reveals that our results on trade policy are unchanged, except for that the function $\widetilde{k}_F(m_{F1}, \widetilde{m}_{F2}(a))$ used in the baseline versions of those results must now be replaced with the version $\widetilde{k}_F(m_{F1}, \widetilde{m}_{F2}(a(\cdot)))$ that accounts for the distribution of Foreign geopolitical actions in the second period. Additionally, we must now assume that the planner is indifferent to redistribution between Home and Foreign under each geopolitical action that occurs with positive probability. When this assumption fails, there is an additional missing-markets motive for policy that corresponds to the missing market for international trade in insurance.

C.2 Home geopolitical actions and Foreign economic policies

C.2.1 Extended model

We assume that both Home and Foreign take geopolitical actions a_H and a_F, and that both use time-1 capital subsidies s_{H1} and s_{F1}, time-1 trade taxes τ_{H1} and τ_{F1}, and time-2 trade taxes τ_{H2} and τ_{F2}. Each country i's geopolitical utility depends on the geopolitical actions of both countries, i.e. it is given by $z_i(a_H, a_F)$. We relax the assumption of binary geopolitical actions, instead assuming each a_i belongs to some general finite set $\mathcal{A}_i$. For technical convenience, we assume that Foreign's joint geopolitical-and-trade-tax actions are restrictd to some finite set $\mathcal{S}_F$. We require at both time 1 and time 2, Foreign tariffs are not prohibitive, i.e. each $\tau_{Ftg} \in (-1, \infty)$. We assume that Home's geopolitical actions and trade taxes are determined by those of Foreign and a joint Home policy threat—encompassing both Home's geopolitical actions and its trade taxes—that we represent with functions $\widetilde{a}_H(\cdot)$ and $\widetilde{\tau}_{H2}(\cdot)$ from $\mathcal{S}_F$ to $\mathcal{A}_H$ and $[-1, \infty]^{\mathcal{G}}$, respectively.

The equilibrium conditions of this extended model are as follows.

Economic equilibrium conditions Households maximize consumption utility subject to a lifetime budget constraint.

$$\{c_{it}\}_{t=1,2} \in \arg\max_{\{c_t\}_{t=1,2}} \sum_{t=1,2} u_{it}(c_t) \quad \text{s.t.} \quad \sum_{t=1,2} (p_{it} \cdot c_t - I_{it}) \leq 0$$

where I_{it} is income from domestic profits and lump-sum transfers in i at t, i.e.,

$$I_{it} = p_{it} \cdot (y_{it} - \mathbb{1}_{t=1} \iota_{i1}) + p_t^w \tau_{it} \cdot m_{it}$$

where m_{it} denotes the net imports of country i at time t.

Goods producers maximize profits subject to technological feasibility.

$$\{y_{it}, k_{it}^y\} \in \arg\max_{y, k^y} \; p_{it} \cdot y - r_{it} \cdot k^y \quad \text{s.t.} \quad G_{it}(y, k^y) \leq 0$$

Capital producers maximize profits subject to technological feasibility and facing subsidies.

$$\{k_i, \iota_{i1}\} \in \arg\max_{k, \iota_1} \; (r_{i1}(1 + s_{i1}) + r_{i2}) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0$$

Goods markets clear globally and capital markets clear within each country.

$$c_{it} + \mathbb{1}_{t=1} \iota_{t1} = y_{it} + m_{it}, \qquad m_{Ht} + m_{Ft} = 0 \qquad \text{and} \qquad k_i = k_{it}^y$$

There is financial autarky. In other words, trade is balanced within each period.

$$p_t^w \cdot m_{it} = 0$$

Domestic prices equal world prices augmented by (possibly zero) trade taxes.

$$p_{it} = (1 + \tau_{it}) p_t^w$$

Geopolitical equilibrium conditions Foreign's geopolitical action and time-2 trade taxes solve

$$(a_F, \tau_{F2}) \in \arg \max_{(\hat{a}_F, \hat{\tau}_{F2}) \in \mathcal{S}_F} \; u_{F2}\Big(\widetilde{c}_{F2}(\widetilde{\tau}_{H2}(\hat{a}_F, \hat{\tau}_{F2}), \hat{\tau}_{F2}, k_H, k_F)\Big) + z_F(\widetilde{a}_H(\hat{a}_F, \hat{\tau}_{F2}), \hat{a}_F)$$

where $\widetilde{c}_{F2}(\tau_{H2}, \tau_{F2}, k_H, k_F)$ is the presumed-unique level of Foreign consumption consistent with trade taxes τ_{i2} and capital k_i.

Home's geopolitical action and its time-2 trade taxes are determined by its geopolitics-and-trade-tax threat and Foreign's geopolitical action and time-2 trade taxes.

$$a_H = \widetilde{a}_H(a_F, \tau_{F2}) \qquad \text{and} \qquad \tau_{H2} = \widetilde{\tau}_{H2}(a_F, \tau_{F2})$$

C.2.2 Extended results

Following the same steps as in the proof of Lemma 1 allows us to represent the planner's problem in its primal form. That is, the planner's problem is equivalent to choosing an extended profile

$$\Big\{c_{i1}, c_{i2}, y_{i1}, y_{i2}, \iota_{i1}, m_{i1}, m_{i2}, k^y_{i1}, k^y_{i2}, p^w_{i1}, p^w_{i2}, p_{i1}, p_{i2}, r_{i1}, r_{i2}, a_i, \widetilde{a}_H(\cdot),$$
$$s_{i1}, \tau_{i1}, \tau_{i2}, \widetilde{\tau}_{H2}(\cdot), \gamma_i, \widetilde{c}_{i2}(\cdot), \widetilde{y}_{i2}(\cdot), \widetilde{m}_{i2}(\cdot), \widetilde{k}^y_{i2}(\cdot), \widetilde{p}_{i2}(\cdot), \widetilde{r}_{i2}(\cdot), \widetilde{p}^w_2(\cdot)\Big\}$$

that maximizes

$$\begin{aligned} &V_{H1}(-m_{F1}, k_H) + V_{H2}(-\widetilde{m}_{F2}(a_F, \tau_{F2}), k_H) + v_H(\widetilde{a}_H(a_F, \tau_{F2}), a_F) \\ &+\lambda_F \left[V_{F1}(m_{F1}, k_F) + V_{F2}(\widetilde{m}_{F2}(a_F, \tau_{F2}), k_F) + v_F(\widetilde{a}_H(a_F, \tau_{F2}), a_F)\right] \end{aligned}$$

subject to

$$
\begin{aligned}
& k_F = \widetilde{k}_F(m_{F1}, \widetilde{m}_{F2}(a_F, \tau_{F2})) \\
& \frac{V_{F1,m}(m_{F1}, k_F)}{1+\tau_{F1}} \cdot m_{F1} = 0 \\
& \frac{V_{F2,m}(\widetilde{m}_{F2}(\hat{a}_F, \hat{\tau}_{F2}), k_F)}{1+\hat{\tau}_{F2}} \cdot \widetilde{m}_{F2}(\hat{a}_F, \hat{\tau}_{F2}) = 0 \\
& V_{F2}(\widetilde{m}_{F2}(a_F, \tau_{F2}), k_F) - V_{F2}(\widetilde{m}_{F2}(\hat{a}_F, \hat{\tau}_{F2}), k_F) \\
& \geq z_F(\widetilde{a}_H(\hat{a}_F, \hat{\tau}_{F2}), \hat{a}_F) - z_F(\widetilde{a}_H(a_F, \tau_{F2}), a_F) \\
& (\widetilde{a}_H(\cdot), \widetilde{\widetilde{\tau}}_{H2}(\cdot)) \in \mathcal{T}(k_H, k_F) \\
& \text{for} \quad \widetilde{\widetilde{\tau}}_{H2}(\hat{a}_F, \hat{\tau}_{F2}) \equiv \frac{V_{H2,m}(-\widetilde{m}_{F2}(\hat{a}_F, \hat{\tau}_{F2}), k_H)}{V_{F2,m}(\widetilde{m}_{F2}(\hat{a}_F, \hat{\tau}_{F2}), k_F)/(1+\hat{\tau}_{F2})} - 1
\end{aligned}
$$

where $\mathcal{T}(k_H, k_F)$ now represents the set of feasible geopolitical-and-trade-tax threats, and subject to

$$
\begin{aligned}
& \{y_{i1}, \iota_{i1}\} \in \arg\max_{y,\iota} \; u_{i1}(y + m_{i1} - \iota) \quad \text{s.t.} \quad G_{i1}(y, k_i) \leq 0, \quad \Lambda_{i1}(k_i, \iota) \leq 0 \\
& \widetilde{y}_{i2}(\hat{a}_F, \hat{\tau}_{F2}) \in \arg\max_{y} \; u_{i2}(y + \widetilde{m}_{i2}(\hat{a}_F, \hat{\tau}_{F2})) \quad \text{s.t.} \quad G_{i2}(y, k_i) \leq 0 \\
& p_{i1} = \gamma_i V_{i1,m}(m_{i1}, k_i) \qquad \text{and} \qquad \widetilde{p}_{i2}(\hat{a}_F, \hat{\tau}_{F2}) = \gamma_i V_{i2,m}(\widetilde{m}_{i2}(\hat{a}_F, \hat{\tau}_{F2}), k_i) \\
& r_{i1} = -G_{i1,k}(y_{i1}, k_i)/G_{i1,y_{g^*}}(y_{i1}, k_i) p_{i1g^*} \\
& \widetilde{r}_{i2}(\hat{a}_F, \hat{\tau}_{F2}) = -G_{i2,k}(\widetilde{y}_{i2}(\hat{a}_F, \hat{\tau}_{F2}), k_i)/G_{i2,y_{g^*}}(\widetilde{y}_{i2}(\hat{a}_F, \hat{\tau}_{F2}), k_i) \widetilde{p}_{i2g^*}(\hat{a}_F, \hat{\tau}_{F2}) \\
& r_{i1} s_{i1} = -\gamma_i \left[V_{H1,k}(m_{i1}, k_i) + V_{i2,k}(\widetilde{m}_{i2}(\hat{a}_F, \hat{\tau}_{F2}), k_i)\right] \\
& c_{i1} = y_{i1} + m_{i1} - \iota_{i1} \qquad \text{and} \qquad \widetilde{c}_{i2}(\hat{a}_F, \hat{\tau}_{F2}) = \widetilde{y}_{i2}(\hat{a}_F, \hat{\tau}_{F2}) + \widetilde{m}_{i2}(\hat{a}_F, \hat{\tau}_{F2}) \\
& m_{F1} = -m_{H1} \qquad \text{and} \qquad \widetilde{m}_{H2}(\hat{a}_F, \hat{\tau}_{F2}) = -\widetilde{m}_{F2}(\hat{a}_F, \hat{\tau}_{F2}) \\
& k^y_{i1} = k_i \qquad \text{and} \qquad \widetilde{k}^y_{i2}(\hat{a}_F, \hat{\tau}_{F2}) = k_i \\
& \tau_{i1} = p_{i1}/p^w_1 - 1 \qquad \text{and} \qquad \widetilde{\tau}_{i2}(\hat{a}) = \widetilde{p}_{i2}(\hat{a}_F, \hat{\tau}_{F2})/\widetilde{p}^w_2(\hat{a}_F, \hat{\tau}_{F2}) - 1 \\
& c_{i2} = \widetilde{c}_{i2}(a_F, \tau_{F2}), \qquad y_{i2} = \widetilde{y}_{i2}(a_F, \tau_{F2}), \qquad m_{i2} = \widetilde{m}_{i2}(a_F, \tau_{F2}), \qquad k^y_{i2} = \widetilde{k}^y_{i2}(a_F, \tau_{F2}) \\
& p_{i2} = \widetilde{p}_{i2}(a_F, \tau_{F2}), \qquad r_{i2} = \widetilde{r}_{i2}(a_F, \tau_{F2}), \qquad p^w_2 = \widetilde{p}^w_2(a_F, \tau_{F2}), \qquad \tau_2 = \widetilde{\tau}_2(a_F, \tau_{F2})
\end{aligned}
$$

Following the same steps as in the proof of Theorem 1 reveals that our results on industrial policy with unconstrainted threats are unchanged. As before, we define Home's set of credible threats as those that keep its its economic utility sufficiently high[33] Following the same steps

[33]That is, a threat is credible provided that

$$
u_{H2}\Big(\widetilde{c}_{H2}(\widetilde{\tau}_{H2}(\hat{a}_F, \hat{\tau}_{F2}), \hat{\tau}_{F2}, k_H, k_F)\Big) \geq \bar{U}
$$

as in the proof of Theorem 2 reveals that our results are unchanged.

Our results on optimal peacetime trade taxes must be modified in a three ways. For simplicity we describe these changes in the case where Home is indifferent to redistribution. First, since we allow for any finite number of Foreign geopolitical-and-economic actions, there are more incentive compatibility constraints. However, each can still be expressed in integral form. Concreretely, we require that for all $(\hat{a}_F, \hat{\tau}_{F2}) \in \mathcal{S}_F$ other than the on-path action (a_F, τ_{F2}),

$$-\int_{\widetilde{m}_{F2}(\hat{a}_F,\hat{\tau}_{F2})}^{\widetilde{m}_{F2}(a_F,\tau_{F2})} m \cdot V_{F2,mm}\left(m, k_F\right) \cdot dm \geq z_F(\widetilde{a}_H(\hat{a}_F, \hat{\tau}_{F2}), \hat{a}_F) - z_F(\widetilde{a}_H(a_F, \tau_{F2}), a_F)$$

Second, changes in Foreign capital generate fiscal externalities that the Home planner must account for to the extent it values Foreign welfare. Finally, to the extent the planner is indifferent to redistribution between Home and Foreign, it cares about the total wedge $(\tau_{H1g} - \tau_{F1g})/(1 + \tau_{F1g})$ distorting trade in each good rather than only Home trade taxes. Following analogous steps to those in the proof of Theorem 3 with these modifications in mind implies that, for some multipliers λ and $\kappa(\hat{a}_F, \hat{\tau}_{F2}) \geq 0$, Home's optimal peacetime trade taxes must satisfy

$$\begin{aligned}\frac{\tau_{H1g} - \tau_{F1g}}{1 + \tau_{F1g}} = \Bigg[& \lambda_F(r_{F1} s_{F1}) \cdot \widetilde{k}_{F,m_{F1g}}/p_{F1g} \\ & - \sum_{(\hat{a}_F,\hat{\tau}_{F2}) \in \mathcal{S}_F \setminus (a_F,\tau_{F2})} \kappa(\hat{a}_F, \hat{\tau}_{F2}) \left(\int_{\widetilde{m}_{F2}(\hat{a}_F,\hat{\tau}_{F2})}^{\widetilde{m}_{F2}(a_F,\tau_{F2})} m \cdot p_{F2,mk}\left(m, k_F\right) \cdot dm \right) \Bigg] \\ & \cdot \widetilde{k}_{F,m_{F1g}}/p_{F1g}\end{aligned}$$

Finally, we consider the case of constrained threats, supposing Home can threaten any geopolitical actions and trade taxes that result in trade quantities within some set that depends only on Home capital. That is, there exists a function Γ of Home capital for which

$$\begin{aligned}\mathcal{T}_2\left(k_H, k_F\right) = \Bigg\{ & (\widetilde{a}_H(\cdot), \widetilde{\tau}_{H2}(\cdot)) : \mathcal{S}_F \to \mathcal{A}_H \times [-1, \infty]^{\mathcal{G}} \\ & \Bigg| \ \{\widetilde{m}_{H2}(\widetilde{\tau}_{H2}(\hat{a}_F, \hat{\tau}_{F2}), \hat{\tau}_{F2}, k_H, k_F)\}_{(\hat{a}_F,\hat{\tau}_{F2}) \in \mathcal{S}_F} \in \Gamma(k_H) \Bigg\}\end{aligned}$$

Following analogous steps to the proof of Proposition 1 imply that (provided the Lagrange multipliers on trade balance constraints remain equal to zero) the above expression for peacetime trade taxes continues to apply when Home's threats are constrained, although these constraints will in general affect the quantities and prices at which one must evaluate that expression.

C.3 Inseparability of economics and geopolitics

C.3.1 Extended model

We assume that time-2 utilities depend on consumption and Foreign's geopolitical action without imposing additive separabiliity, i.e. we assume they are given by some functions $u_{i2}(c_{i2}, a)$. We moreover allow for Foreign's geopolitical action to affect countries' time-2 production possibilities frontiers, which we represent with functions $G_{i2}(y_{i2}, k_{i2}, a)$.

The equilibrium conditions of this extended model are as follows.

Economic equilibrium conditions Households maximize consumption utility subject to a lifetime budget constraint.

$$\{c_{it}\}_{t=1,2} \in \arg\max_{\{c_t\}_{t=1,2}} \; u_{it}(c_1) + u_{i2}(c_2, a) \quad \text{s.t.} \quad \sum_{t=1,2} (p_{it} \cdot c_t - I_{it}) \leq 0$$

where I_{it} is income from domestic profits and lump-sum transfers in i at t, i.e.,

$$I_{it} = p_{it} \cdot (y_{it} - \mathbb{1}_{t=1}\iota_{i1}) + \mathbb{1}_{i=H}(p_t^w \tau_t) \cdot m_{it}$$

where m_{it} denotes the net imports of country i at time t.

Goods producers maximize profits subject to technological feasibility.

$$\{y_{i1}, k_{i1}^y\} \in \arg\max_{y,k^y} \; p_{i1} \cdot y - r_{i1} \cdot k^y \quad \text{s.t.} \quad G_{i1}(y, k^y) \leq 0$$
$$\{y_{i2}, k_{i2}^y\} \in \arg\max_{y,k^y} \; p_{i2} \cdot y - r_{i2} \cdot k^y \quad \text{s.t.} \quad G_{i2}(y, k^y, a) \leq 0$$

Capital producers maximize profits subject to technological feasibility and facing subsidies.

$$\{k_i, \iota_{i1}\} \in \arg\max_{k,\iota_1} \; (r_{i1}(1 + \mathbb{1}_{i=H} s_1) + r_{i2}) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0$$

Goods markets clear globally and capital markets clear within each country.

$$c_{it} + \mathbb{1}_{t=1}\iota_{t1} = y_{it} + m_{it}, \qquad m_{Ht} + m_{Ft} = 0 \qquad \text{and} \qquad k_i = k_{it}^y$$

There is financial autarky. In other words, trade is balanced within each period.

$$p_t^w \cdot m_{it} = 0$$

Domestic prices equal world prices augmented by (possibly zero) trade taxes.

$$p_{Ft} = p_t^w \quad \text{and} \quad p_{Ht} = (1 + \tau_t) p_t^w$$

Geopolitical equilibrium conditions Foreign's geopolitical action solves

$$a \in \arg \max_{\hat{a} \in \{\underline{a}, \overline{a}\}} \; u_{F2}\Big(\widetilde{c}_{F2}(\widetilde{\tau}_2(\hat{a}), k_H, k_F), \hat{a}\Big)$$

Home's time-2 trade taxes are determined by its trade tax threat and Foreign's geopolitical action.

$$\tau_2 = \widetilde{\tau}_2(a)$$

C.3.2 Extended results

In the presence of inseparability between economics and geopolitics, we must define countries' time-2 Meade utilities in a more general way. We define

$$V_{i2}(m, k, a) \equiv \max_y \; u_{i2}(y + m, a) \qquad \text{s.t.} \qquad G_{i2}(y, k, a) \leq 0$$

Using this definition, and following the same steps as in the proof of Lemma 1 allows us to represent the planner's problem in its primal form. That is, the planner's problem is equivalent to choosing an extended profile

$$\Big\{c_{it}, y_{it}, \iota_{i1}, m_{it}, k_{it}^y, k_i, p_{it}^w, p_{it}, r_{it}, a, s_1, \tau_1, \tau_2, \widetilde{\tau}_2(\cdot), \gamma_i, \widetilde{c}_{i2}(\cdot), \widetilde{y}_{i2}(\cdot), \widetilde{m}_{i2}(\cdot), \widetilde{k}_{i2}^y(\cdot), \widetilde{p}_{i2}(\cdot), \widetilde{r}_{i2}(\cdot), \widetilde{p}_2^w(\cdot)\Big\}$$

that maximizes

$$\begin{gathered} V_{H1}(-m_{F1}, k_H) + V_{H2}(-\widetilde{m}_{F2}(a), k_H, a) \\ +\lambda_F \left[V_{F1}(m_{F1}, k_F) + V_{F2}(\widetilde{m}_{F2}(a), k_F, a)\right] \end{gathered}$$

subject to

$$
\begin{aligned}
&k_F = \widetilde{k}_F(m_{F1}, \widetilde{m}_{F2}(a)) \\
&V_{F1,m}(m_{F1}, k_F) \cdot m_{F1} = 0 \\
&V_{F2,m}(\widetilde{m}_{F2}(\hat{a}), k_F, \hat{a}) \cdot \widetilde{m}_{F2}(\hat{a}) = 0 \\
&V_{F2}(\widetilde{m}_{F2}(a), k_F, a) - V_{F2}(\widetilde{m}_{F2}(\neg a), k_F, \neg a) \geq 0 \\
&\frac{V_{H2,m}(-\widetilde{m}_{F2}(\cdot), k_H)}{V_{F2,m}(\widetilde{m}_{F2}(\cdot), k_F)} \in \mathcal{T}(k_H, k_F)
\end{aligned}
$$

and subject to

$$
\begin{aligned}
&\{y_{i1}, \iota_{i1}\} \in \arg\max_{y,\iota}\ u_{i1}(y + m_{i1} - \iota) \quad \text{s.t.} \quad G_{i1}(y, k_i) \leq 0, \quad \Lambda_{i1}(k_i, \iota) \leq 0 \\
&\widetilde{y}_{i2}(\hat{a}) \in \arg\max_{y}\ u_{i2}(y + \widetilde{m}_{i2}(\hat{a}), \hat{a}) \quad \text{s.t.} \quad G_{i2}(y, k_i, \hat{a}) \leq 0 \\
&p_{i1} = \gamma_i V_{i1,m}(m_{i1}, k_i) \qquad \text{and} \qquad \widetilde{p}_{i2}(\hat{a}) = \gamma_i V_{i2,m}(\widetilde{m}_{i2}(\hat{a}), k_i, \hat{a}) \\
&r_{i1} = -G_{i1,k}(y_{i1}, k_i)/G_{i1,y_{g^*}}(y_{i1}, k_i) p_{i1g^*} \\
&\widetilde{r}_{i2}(\hat{a}) = -G_{i2,k}(\widetilde{y}_{i2}(\hat{a}), k_i, \hat{a})/G_{i2,y_{g^*}}(\widetilde{y}_{i2}(\hat{a}), k_i, \hat{a}) \widetilde{p}_{i2g^*}(\hat{a}) \\
&r_{H1} s_1 = -\gamma_H \left[V_{H1,k}(m_{H1}, k_H) + V_{H2,k}(\widetilde{m}_{H2}(a), k_H, a)\right] \\
&c_{i1} = y_{i1} + m_{i1} - \iota_{i1} \qquad \text{and} \qquad \widetilde{c}_{i2}(\hat{a}) = \widetilde{y}_{i2}(\hat{a}) + \widetilde{m}_{i2}(\hat{a}) \\
&m_{F1} = -m_{H1} \qquad \text{and} \qquad \widetilde{m}_{H2}(\hat{a}) = -\widetilde{m}_{F2}(\hat{a}) \\
&k^y_{i1} = k_i \qquad \text{and} \qquad \widetilde{k}^y_{i2}(\hat{a}) = k_i \\
&p^w_1 = p_{F1} \qquad \text{and} \qquad \tau_1 = p_{H1}/p^w_1 - 1 \\
&\widetilde{p}^w_2(\hat{a}) = \widetilde{p}_{F2}(\hat{a}) \qquad \text{and} \qquad \widetilde{\tau}_2(\hat{a}) = \widetilde{p}_{H2}(\hat{a})/\widetilde{p}^w_2(\hat{a}) - 1 \\
&c_{i2} = \widetilde{c}_{i2}(a), \qquad y_{i2} = \widetilde{y}_{i2}(a), \qquad m_{i2} = \widetilde{m}_{i2}(a), \qquad k^y_{i2} = \widetilde{k}^y_{i2}(a) \\
&p_{i2} = \widetilde{p}_{i2}(a), \qquad r_{i2} = \widetilde{r}_{i2}(a), \qquad p^w_2 = \widetilde{p}^w_2(a), \qquad \tau_2 = \widetilde{\tau}_2(a)
\end{aligned}
$$

Following the same steps as in the proofs of Theorems 1 and 2 reveals that our results on industrial policy are unchanged.

Our results on trade policy must be modified somewhat. For simplicity, we focus on the case where Home is indifferent to redistribution. The modification to our results reflects that the incentive compatibility constraint corresponding to Foreign's geopolitical action is no longer separable between Foreign's economic and geopolitical utilities. To isolate this interaction, we note that the IC constraint may nonetheless be written as

$$\begin{aligned}
V_{F2}(\widetilde{m}_{F2}(a), k_F, a) - V_{F2}(\widetilde{m}_{F2}(\neg a), k_F, \neg a) &= V_{F2}(\widetilde{m}_{F2}(a), k_F, a) - V_{F2}(\widetilde{m}_{F2}(\neg a), k_F, a) \\
&\quad + V_{F2}(\widetilde{m}_{F2}(\neg a), k_F, a) - V_{F2}(\widetilde{m}_{F2}(\neg a), k_F, \neg a) \geq 0 \\
&\iff \\
-\int_{\widetilde{m}_{F2}(\neg a)}^{\widetilde{m}_{F2}(a)} m \cdot V_{F2,mm}(m, k_F, a) \cdot dm &+ V_{F2}(\widetilde{m}_{F2}(\neg a), k_F, a) - V_{F2}(\widetilde{m}_{F2}(\neg a), k_F, \neg a) \geq 0
\end{aligned}$$

using analogous steps to those used in the proof of Theorem 3. Using this version of the IC constraint, following analogous steps to those used in the proof of Theorem 3 implies that Home's time-1 trade taxes satisfy

$$\begin{aligned}
\tau_{H1g} = \kappa \Bigg(&-\int_{\widetilde{m}_{F2}(\neg a)}^{\widetilde{m}_{F2}(a)} m \cdot \frac{\partial^2 \widetilde{p}_{F2}(m, k_F, a)}{\partial k_F \, \partial m} \cdot dm \\
&+ \widetilde{r}_{F2}(\widetilde{m}_{F2}(\neg a), k_F, a) - \widetilde{r}_{F2}(\widetilde{m}_{F2}(\neg a), k_F, \neg a) \Bigg) \cdot \frac{\partial \widetilde{k}_F(m_{F1}, m_{F2})}{\partial m_{F1g}} \Bigg/ p_{F1g}
\end{aligned}$$

where $\widetilde{m}_{F2}(\hat{a})$ is shorthand for $\widetilde{m}_{F2}(\widetilde{\tau}_2(\hat{a}), k_H, k_F)$, where $\widetilde{p}_{F2}(m, k_F, \hat{a}) \equiv V_{F2,m}(m, k, \hat{a})$ denotes Foreign prices as a function of imports, capital, and Foreign geopolitical actions, and where $\widetilde{r}_{F2}(m, k, a)$ denotes Foreign rental rates as a function of imports, capital, and Foreign geopolitical actions. Intuitively, this result states that there is are now two geopolitical motives for time-1 trade taxes: They should should promote Foreign capital varieties that (a) make Foreign's terms of trade more sensitive, assuming Foreign takes its on-path geopolitical action, and (b) are disproportionately less valuable when Foreign chooses its off-path geopolitical action, assuming Foreign imports its off-path quantities. The latter is a direct channel that, for example, discourages Home exports of weapons that Foreign can stockpile and investment goods it can use to build weapons factories.

Our results on trade policy with limited credibility change analogously, but without further modifications.

C.4 Many countries

C.4.1 Extended model

We assume the economy contains an any finite number of countries $i \in \mathcal{I}$, including Home and Foreign. Countries i other than Home and Foreign are analogous to Home and Foreign except that they are both economically and geopolitically passive.

The equilibrium conditions of this extended model are as follows.

Economic equilibrium conditions Households maximize consumption utility subject to a lifetime budget constraint.

$$\{c_{it}\}_{t=1,2} \in \arg\max_{\{c_t\}_{t=1,2}} \sum_{t=1,2} u_{it}(c_t) \quad \text{s.t.} \quad \sum_{t=1,2} (p_{it} \cdot c_t - I_{it}) \leq 0$$

where I_{it} is income from domestic profits and lump-sum transfers in i at t, i.e.,

$$I_{it} = p_{it} \cdot (y_{it} - \mathbb{1}_{t=1}\iota_{i1}) + \mathbb{1}_{i=H}(p_t^w \tau_t) \cdot m_{it}$$

where m_{it} denotes the net imports of country i at time t.

Goods producers maximize profits subject to technological feasibility.

$$\{y_{it}, k_{it}^y\} \in \arg\max_{y,k^y} \; p_{it} \cdot y - r_{it} \cdot k^y \quad \text{s.t.} \quad G_{it}(y, k^y) \leq 0$$

Capital producers maximize profits subject to technological feasibility and facing subsidies.

$$\{k_i, \iota_{i1}\} \in \arg\max_{k,\iota_1} \; (r_{i1}(1 + \mathbb{1}_{i=H}s_1) + r_{i2}) \cdot k - p_{i1} \cdot \iota_1 \quad \text{s.t.} \quad \Lambda_{i1}(k, \iota_1) \leq 0$$

Goods markets clear globally and capital markets clear within each country.

$$c_{it} + \mathbb{1}_{t=1}\iota_{t1} = y_{it} + m_{it}, \qquad \sum_{i\in\mathcal{I}} m_{it} = 0 \qquad \text{and} \qquad k_i = k_{it}^y$$

There is financial autarky. In other words, trade is balanced within each period.

$$p_t^w \cdot m_{it} = 0$$

Domestic prices equal world prices augmented by (possibly zero) trade taxes.

$$p_{i\neq H,t} = p_t^w \quad \text{and} \quad p_{Ht} = (1 + \tau_t)p_t^w$$

Geopolitical equilibrium conditions Foreign's geopolitical action solves

$$a \in \arg\max_{\hat{a}\in\{\underline{a},\overline{a}\}} \; u_{F2}\Big(\widetilde{c}_{F2}(\widetilde{\tau}_2(\hat{a}), \{k_i\}_{i\in\mathcal{I}})\Big) + z_F(\hat{a})$$

Home's time-2 trade taxes are determined by its trade tax threat and Foreign's geopolitical action.

$$\tau_2 = \widetilde{\tau}_2(a)$$

C.4.2 Extended results

For simplicity, we restrict attention to the limit where the second period is short compared to the first. We operationalize this assumption by assuming that each country's capital only depends on trade in the first period.

We seek to define functions that specify the net imports of each country as a function of Home's net imports.[34] To this end, we assume that given any time-1 Home net imports m_{H1}, there exists a unique solution (up to a price constant) to the following set of reduced-form equilibrium conditions:

$$0 = m_{H1} + \sum_{i \neq H} m_{i1} \qquad \text{and, for } i \neq H,$$

$$p_1^w \propto V_{i1,m}(m_{i1}, k_i), \qquad 0 = V_{i1,k}(m_{i1}, k_i), \quad \text{and} \quad p_1^w m_{i1} + \frac{p_1^w m_{H1}}{|\mathcal{I}| - 1} = 0$$

When m_{H1} satisfies Home's trade balance constraint, these conditions are implied by the equilibrium conditions; otherwise, they assume Home's deficit or surplus is financed with a uniform transfer to each foreign country. We denote by $\hat{m}_{i1}(m_{H1})$ and $\hat{k}_i(m_{H1})$ each country i's time-1 trade and capital as a function of Home's time-1 trade. For time 2, we similarly assume that, given any time-1 Home net imports m_{H2} and capital $\{k_i\}$, there exists a unique solution (up to a price constant) to the following set of reduced-form equilibrium conditions:

$$0 = m_{H2} + \sum_{i \neq H} m_{i2} \qquad \text{and, for } i \neq H,$$

$$p_2^w \propto V_{i2,m}(m_{i2}, k_i), \quad \text{and} \quad p_2^w m_{i2} + \frac{p_2^w m_{H2}}{|\mathcal{I}| - 1} = 0$$

Taking $k_i = \hat{k}_i(m_{H1})$, we may therefore denote all countries' time-2 trade as a function of Home's time-1 and time-2 trade, i.e. $m_{i2} = \hat{m}_{i2}(m_{H1}, m_{H2})$.

With these tools in hand, we now revisit the planner's problem with many countries. Analogous steps to those in the proof of Lemma 1 imply that the planner's inner primal

[34] With two countries, this was unnecessary, as Foreign net imports were the negative of Home's by trade balance.

problem can be written as[35]

$$
\begin{aligned}
\max_{a,m_{H1},\widetilde{m}_{H2}(\cdot),k_H} \quad & V_{H1}\left(m_{H1},k_H\right)+V_{H2}\left(\widetilde{m}_{H2}(a),k_H\right)+z_H(a) \\
& \sum_{i\neq H}\lambda_i\left[V_{i1}\left(\hat{m}_{i1}(m_{H1}),\hat{k}_i(m_{H1})\right)+V_{i2}\left(\hat{m}_{i2}(m_{H1},\widetilde{m}_{H2}(a)),\hat{k}_i(m_{H1})\right)+z_i(a)\right] \\
& \hat{m}_{F1}(m_{H1})\cdot V_{F1,m}\left(\hat{m}_{F1}(m_{H1}),\hat{k}_F(m_{H1})\right)=0 \\
& \hat{m}_{F2}(m_{H1},\widetilde{m}_{H2}(\hat{a}))\cdot V_{F2,m}\left(\hat{m}_{F2}(m_{H1},\widetilde{m}_{H2}(\hat{a})),\hat{k}_F(m_{H1})\right)=0 \\
& -\int_{\hat{m}_{F2}(m_{H1},\widetilde{m}_{H2}(\neg a))}^{\hat{m}_{F2}(m_{H1},\widetilde{m}_{H2}(a))} m\cdot V_{F2,mm}\left(m,\hat{k}_F(m_{H1})\right)\cdot dm\geq z_F(\neg a)-z_F(a) \\
& \frac{V_{H2,m}(\widetilde{m}_{H2}(\cdot),k_H)}{V_{F2,m}\left(\hat{m}_{F2}(m_{H1},\widetilde{m}_{H2}(\cdot)),\hat{k}_F(m_{H1})\right)}\in\mathcal{T}\left(k_H,\hat{k}_F(m_{H1})\right)
\end{aligned}
$$

Following the same steps as in the proofs of Theorems 1 and 2 reveals that our results on industrial policy are unchanged.

Our results on trade policy must be modified somewhat. For simplicity, we focus on the case where the planner is indifferent to redistribution. To begin, we must expand our notion of the planner's indifference to redistribution across countries. This assumption now has two parts. First, the planner is indifferent—at any time and under any geopolitical action—to the marginal transfers of all goods across foreign countries. Second, it is indifferent to marginal transfers—at any time—between Home and foreign countries of any bundles of goods whose marginal net imports from Home have no effects on any foreign country's terms of trade or capital.[36]We assume such bundles exist and have non-zero world prices at each time and under each geopolitical action.

[35]Note that the construction of $\hat{m}_{F1}(\cdot)$ and $\hat{m}_{F2}(\cdot)$ ensure that the second and third constraints—i.e. Foreign trade balance—also imply trade balance for all countries.

[36]More formally, our assumptions are as follows:

i. For all $i,j\neq H$, $\lambda_i u_{i1,c}(c_{i1})=\lambda_j u_{j1,c}(c_{j1})$ and $\lambda_i u_{i2,c}\left(\widetilde{c}_{i2}(\widetilde{\tau}_2(\hat{a}),\{k_{i'}\})\right)=\lambda_j u_{j2,c}\left(\widetilde{c}_{j2}(\tau_2(\hat{a}),\{k_{i'}\})\right)$ for $\hat{a}=\underline{a},\bar{a}$.

ii. If some bundle m^*_{H1} satisfies, for all $i\neq H$, $\hat{m}_{i1}(m_{H1})\cdot\widetilde{p}_{i1,m}(\hat{m}_{i1}(m_{H1}),k_i)\cdot\hat{m}_{i1,m_{H1}}(m_{H1})\cdot m^*_{H1}=0$ and $\hat{k}_{i,m}(m_{H1})\cdot m^*_{H1}=0$, then $u_{H1,c}(c_{H1})\cdot m^*_{H1}=\sum_{i\neq H}\lambda_i u_{i1,c}(c_{i1})\cdot\hat{m}_{i1,m_{H1}}(m_{H1})\cdot m^*_{H1}$.

iii. If some bundle $m^*_{H2}(a)$ satisfies, for all $i\neq H$, $m_{i2}\cdot\widetilde{p}_{F2,m}(m_{i2},k_F)\cdot\hat{m}_{i2,m_{H2}}(m_{H1},m_{H2})\cdot m^*_{H2}(a)=0$, then $u_{H2,c}(c_{H2})\cdot m^*_{H2}(a)=\sum_{i\neq H}\lambda_i u_{i2,c}(c_{i2})\cdot\hat{m}_{F2,m_{H2}}(m_{H1},m_{H2})\cdot m^*_{H2}(a)$.

The planner's first-order conditions with respect to $\widetilde{m}_{H2}(a)$ and $\widetilde{m}_{H2}(\neg a)$ are

$$\begin{aligned}
0 =& V_{H2,m}(a) + \sum_{i \neq H} \lambda_i V_{i2,m}(a) \cdot \hat{m}_{i2,m_{H2}}(a) \\
&+ \widetilde{\kappa}_2(a) \left[V_{F2,m}(a) + \widetilde{m}_{F2}(a) \cdot V_{F2,mm}(a) \right] \cdot \hat{m}_{F2,m_{H2}}(a) \\
&- \kappa \widetilde{m}_{F2}(a) \cdot V_{F2,mm}(a) \cdot \hat{m}_{F2,m_{H2}}(a)
\end{aligned}$$

and

$$\begin{aligned}
0 =& \widetilde{\kappa}_2(\neg a) \left[V_{F2,m}(\neg a) + \widetilde{m}_{F2}(\neg a) \cdot V_{F2,mm}(\neg a) \right] \cdot \hat{m}_{F2,m_{H2}}(\neg a) \\
&+ \kappa \widetilde{m}_{F2}(a) \cdot V_{F2,mm}(a) \cdot \hat{m}_{F2,m_{H2}}(\neg a)
\end{aligned}$$

Above, we have used the fact that there are no capital subsidies in foreign countries. Considering these first-order conditions in the direction of the neutral bundles $m^*_{H2}(a)$ and $m^*_{H2}(\neg a)$ implies $\widetilde{\kappa}_2(a) = \widetilde{\kappa}_2(\neg a) = 0$. Here we have used the assumption that Home is indifferent to redistribution and the fact that, by construction,

$$1 + \sum_{i \neq H} \hat{m}_{i2,m_{H2}}(a) = 0$$

The planner's first-order condition with respect to m_{H1} is, then,

$$\begin{aligned}
0 =& V_{H1,m} + \sum_{i \neq H} \lambda_i V_{i1,m} \cdot \hat{m}_{i1,m_{H1}} + \cancel{\sum_{i \neq H} \lambda_i V_{i2,m}(a) \cdot \hat{m}_{i2,m_{H1}}} \\
&+ \kappa_1 \left[V_{F1,m} + m_{F1} \cdot V_{F1,mm} + m_{F1} \cdot V_{F1,mk} \right] \cdot \hat{m}_{F1,m_{H1}} \\
&- \kappa \left[\widetilde{m}_{F2}(a) \cdot V_{F2,mm}(a) \cdot \hat{m}_{F2,m_{H1}}(a) - \widetilde{m}_{F2}(\neg a) \cdot V_{F2,mm}(\neg a) \cdot \hat{m}_{F2,m_{H1}}(\neg a) \right] \\
&- \kappa \int_{\widetilde{m}_{F2}(\neg a)}^{\widetilde{m}_{F2}(a)} m \cdot V_{F2,mmk}\left(m, k_F\right) \cdot dm \cdot \hat{k}_{F,m_{H1}}
\end{aligned}$$

The cancellation above reflects that (a) the first order condition with respect to m_{H1} holds $\widetilde{m}_{H2}(a)$ fixed, (b) the construction of $\hat{m}_{i2}(m_{H1}, \widetilde{m}_{H2}(a))$ requires $\widetilde{m}_{H2}(a) + \sum_{i \neq H} \hat{m}_{i2}(m_{H1}, \widetilde{m}_{H2}(a)) = 0$, and (c) the planner is indifferent to redistribution among foreign countries. Considering this first-order condition in the direction of the neutral bundle m^*_{H1} implies $\kappa_1 = 0$—where here we have used the observation that if $\hat{k}_{i,m_{H1}}(m_{H1}) = 0$, then $\hat{m}_{i2,m_{H1}}(m_{H1}, m_{H2}) = 0$ as well. Using these observations and following steps similar to those in the proof of Theorem 3 implies that, up to Lerner symmetry,

$$\tau_{1g} = \widetilde{\kappa}\Bigg[\int_{\widetilde{m}_{F2}(\neg a)}^{\widetilde{m}_{F2}(a)} m \cdot \widetilde{p}_{F2,mk}\,(m, k_F) \cdot dm \cdot \hat{k}_{F,m_{H1}}(m_{H1}) \\ + \widetilde{m}_{F2}(a) \cdot \widetilde{p}_{F2,m}(a) \cdot \hat{m}_{F2,m_{H1}}(a) - \widetilde{m}_{F2}(\neg a) \cdot \widetilde{p}_{F2,m}(\neg a) \cdot \hat{m}_{F2,m_{H1}}(\neg a)\Bigg] \Big/ p^w_{1g} \tag{29}$$

for some $\widetilde{\kappa}$. Above, $\widetilde{m}_{F2}(\hat{a})$ is shorthand for $\widetilde{m}_{F2}(\widetilde{\tau}_2(\hat{a}), \{k_i\})$, $\widetilde{p}_{F2,m}(\hat{a})$ is shorthand for $\widetilde{p}_{F2,m}(\widetilde{m}_{F2}(\hat{a}), k_F)$, and $\hat{m}_{F2,m_{H1}}(\hat{a})$ is shorthand for $\hat{m}_{F2,m_{H1}}(m_{H1}, \widetilde{m}_{H2}(\hat{a}))$.

Intuitively, the first line of Equation 29 captures the same mechanism as we hav emphasized in the two-country case. The only distinction is that it accounts for the fact that Home net imports affect Foreign capital less directly when there are more than two countries. The second line captures a distinct mechanism not present in the two country case. It says that Home should promote time-1 trade in goods that—through capital accumulation in all foreign countries—leads to certain changes in time-2 trade among foreign countries, holding fixed Home's time-2 trade. Such changes are, by goods market clearing, impossible with only two countries. However, this term captures that in the general case, Home (a) promotes time-2 trade under its preferred geopolitical action that improves Foreign's terms of trade and (b) promotes time-2 trade under its less preferred geopolitical action that worsens Foreign's terms of trade.

In our quantitative application, we take advantage of the fact that Equation 29 can equivalently be written as

$$\tau_{1g} = \widetilde{\kappa}\Bigg[(-\widetilde{r}_{F2}(a) + \widetilde{m}_{F2}(a) \cdot \widetilde{p}_{F2,k}(a) + \widetilde{r}_{F2}(\neg a) - \widetilde{m}_{F2}(\neg a) \cdot \widetilde{p}_{F2,k}(\neg a)) \cdot \hat{k}_{F,m_{H1}}(m_{H1}) \\ + \widetilde{m}_{F2}(a) \cdot \widetilde{p}_{F2,m}(a) \cdot \hat{m}_{F2,m_{H1}}(a) - \widetilde{m}_{F2}(\neg a) \cdot \widetilde{p}_{F2,m}(\neg a) \cdot \hat{m}_{F2,m_{H1}}(\neg a)\Bigg] \Big/ p^w_{1g}$$

where $\widetilde{r}_{F2}(\hat{a}) \equiv V_{F2,k}(\widetilde{m}_{F2}(a), k_F)$.

Finally, the argument above continues to apply in the case of constraints on Home's time-2 trade policy that satisfy Assumption 3. This follows from an analogous argument to the proof of Proposition 1.

D Data Appendix

D.1 Data sources

We now describe the data sources used in our calibration.

World Input Output Database We obtain data on value added and labor compensation from the Socio Economic Accounts data in the 2016 release of the World Input Output Database (WIOD). We use only values from the most recent year for which data is available, 2014. This release of WIOD makes data available for a set $\mathcal{I}_{2014}^{WIOD}$ of 43 countries and a set $\mathcal{N}_{2014}^{WIOD}$ of 56 sectors. For each such country i and sector n, we obtain a measure of value added VA_{in}^{WIOD} and labor payments W_{in}^{WIOD}.

OECD Inter-Country Input Output tables We obtain data on flows of intermediate and final goods across countries and sectors from the OECD'S Inter-Country Input Output tables (ICIO), as cleaned by Adão et al. (2024). We use data from only a single year, 2017. We choose this year because it is the last year before the imposition of the Trump tariffs and so arguably provides a more accurate "steady state" representation of the world trading system. The ICIO makes data available for a set $\mathcal{I}^{ICIO+}$ of 76 countries plus a rest-of-the-world aggregate and a set $\mathcal{N}^{ICIO+}$ of 45 sectors. Adão et al. (2024) merge Belgium and Luxembourg, resulting in a set $\mathcal{I}^{ICIO}$ of 75 countries plus a rest-of-the-world aggregate, and they merge the pharmaceutical and chemical sectors, resulting in a set $\mathcal{N}^{ICIO}$ of 44 sectors.

We obtain four variables from the ICIO data. First, for each country $i \in \mathcal{I}^{ICIO}$ and sector $n \in \mathcal{N}^{ICIO}$, we obtain a measure $Y_{in}^{w,ICIO}$ of gross output revenues. Second, for each pair of countries i, j and sector n, we obtain a measure $C_{ijn}^{w,ICIO}$ of final consumption expenditures by households, governments, and non-profit institutions serving households in i on goods in sector n originating in j. Third, for each country i and sector n, we obtain a measure $Inv_{ijn}^{w,ICIO}$ of investment expenditures in i on goods in sector n originating in j. We ignore changes in inventories and direct purchases abroad. Fourth, for each pair of countries i, j and each pair of sectors n, n', we obtain a measure of the intermediate expenditures $Int_{injn'}^{w,ICIO}$ by firms in country i and sector n on goods in sector n' originating in j. All of these measures are at world prices (FOB).

Cleaned CEPII BACI trade data We obtain data on bilateral flows of HS6 goods between countries in 2017 from CEPII's BACI data set, as cleaned by Adão et al. (2024). The BACI makes bilateral trade flow data available for a set $\mathcal{H}^{BACI}$ of 5113 HS6 products between a set of 183 countries. The cleaned data that we borrow from Adão et al. (2024)

aggregates these flows across countries to the level of $\mathcal{I}^{ICIO}$. We thereby obtain, for every pair of countries $i, j \in \mathcal{I}^{ICIO}$ and every HS6 product $h \in \mathcal{H}$, a measure of bilateral trade flows $X_{ijh}^{w,BACI}$ at world prices (FOB). These flows only cover merchandise goods, and they omit "domestic trade", i.e., $X_{iih}^{w,BACI} = 0$.

Cleaned WITS tariff data We obtain data on bilateral tariffs in 2017 from the World Bank's World Integrated Trade Solution (WITS) , as cleaned by Adão et al. (2024). These authors follow a cleaning procedure similar to that proposed by Teti (2023) to address substantial imputation errors in the data provided by the WITS database. They also augment this data from USITC and MAcMap with additional information on discriminatory tariffs applied by a subset of countries. The cleaned data that we borrow from Adão et al. (2024) provides, for all country pairs i, j in $\mathcal{I}^{ICIO}$ and products h in $\mathcal{H}$, a measure $\tau_{ijh}^{m,ABCD}$ of the bilateral, statutory, ad-valorem-equivalent tariffs that j applies to imports of h from i.

Imputed capital network data We borrow estimates of countries' cross-sectoral investment networks from Ding (2021). Ding (2021) draws on a range of data sets to compute these networks in 1997 for a set $\mathcal{I}^{Ding}$ of 40 countries plus a rest-of-the-world aggregate and a set $\mathcal{N}^{Ding}$ of 27 sectors. For all countries i in $\mathcal{I}^{Ding}$ and all sector pairs n, n' in $\mathcal{N}^{Ding}$, we obtain a measure $Inv_{in'n}^{w,Ding}$of the investment expenditures of sector n in country i' on goods produced by sector n' (from all origins o, in FOB world prices), in 1997.

We also use the rates of sector-specific capital depreciation δ_n^{Ding} and the risk-free interest interest rate ($r^{f,Ding} = 0.03$) provided in Ding (2021).

Crosswalks We hand-code several crosswalks from country and sector classifications in the raw data to their finest common coarsenings. For countries we use a set $\mathcal{I}$ of 39 countries plus a rest-of-the-world aggregate; this is the same as $\mathcal{I}^{Ding}$ except for that it merges Belgium and Luxembourg. For sectors we use a set $\mathcal{N}$ of 26 sectors; this is the same as $\mathcal{N}^{Ding}$ except that it merges textile and leather manufacturing. We let $D_{\mathcal{I}}^{ICIO}$, $D_{\mathcal{I}}^{WIOD}$, $D_{\mathcal{I}}^{Ding}$, $D_{\mathcal{N}}^{ICIO}$, and $D_{\mathcal{N}}^{WIOD}$ denote the many-to-one crosswalk matrices that encode this coarsening. For example, $[D_{\mathcal{I}}^{ICIO}]_{ij}$ is an indicator for whether country or country-group i contains country or country-group j. Additionally, we borrow from Adão et al. (2024) a many-to-one crosswalk $D_{\mathcal{N}\mathcal{H}}^{ICIO}$ that assigns each HS6 product in $\mathcal{H}$ to a unique sector in $\mathcal{N}^{ICIO}$. Concretely, its (n, h) value is an indicator for whether sector n contains product h.

D.2 Data cleaning

We now describe the cleaning procedure we apply to the raw data described in Appendix D.1 in order to arrive at the statistics we use for calibration. These are measures of sector-level tariffs $\tau_{ijn}^{m,data}$ and export subsidies $\tau_{ijn}^{x,data}$; shares of country-sector trade flows directed toward each origin-sector, θ_{ijn}^{data}; shares of gross output directed towards labor, $\theta_{in}^{\ell,data}$, capital, $\theta_{in}^{k,data}$, and intermediates from each sector, $\theta_{in'n}^{x,data}$; shares of capital expenditures directed toward investment in capital goods from each sector $\theta_{in'n}^{\iota,data}$; shares of each country's consumption expenditures directed towards goods from each sector, $\theta_{jn}^{c,data}$; and each country's final consumption expenditures C_i^{data}.

Sectoral tariff revenues As a preliminary step, we compute the implied bilateral tariff revenues collected within each ICIO sector, rescaling by the ratio ICIO- to BACI-measured bilateral sectoral trade. We set

$$T_{ijn}^{imp} \equiv \frac{X_{ijn}^{w,ICIO}}{\sum_{h\in\mathcal{H}}[D_{\mathcal{NH}}^{ICIO}]_{nh} X_{ijh}^{w,BACI}} \sum_{h\in\mathcal{H}} [D_{\mathcal{NH}}^{ICIO}]_{nh} \tau_{ijh}^{m,ABCD} X_{ijh}^{w,BACI}$$
$$\text{where} \quad X_{ijn}^{w,ICIO} \equiv C_{ijn}^{w,ICIO} + Inv_{ijn}^{w,ICIO} + \sum_{n'\in\mathcal{N}} Int_{injn'}^{w,ICIO}$$

We set $T_{ijn}^{imp} = 0$ for any (i,j,n) with no BACI-reported trade, i.e., where the denominator above is zero. This includes all service sectors and all domestic trade (i.e., $i = j$).

Aggregation We begin by aggregating all of the raw data sets to their finest common coarsenings, i.e., $\mathcal{I}$ for countries and $\mathcal{N}$ for sectors. We denote the aggregated versions of the measures described in Appendix D.1 by $\bar{VA}_{in}^{WIOD}$, $\bar{W}_{in}^{WIOD}$, $\bar{Y}_{in}^{w,ICIO}$, $\bar{C}_{ijn}^{w,ICIO}$, $\bar{Inv}_{ijn}^{w,ICIO}$, $\bar{Int}_{injn'}^{w,ICIO}$, $\bar{X}_{ijh}^{w,BACI}$, $\bar{\tau}_{ijh}^{m,ABCD}$, and $\bar{Inv}_{in'n}^{w,Ding}$; we also aggregate T_{ijn}^{imp} to a measure $\bar{T}_{ijn}^{imp}$.

As textiles and leather manufacturing have the same depreciate rate according to δ_n^{Ding}, we construct the $|\mathcal{N}|$-length vector $\bar{\delta}_n^{Ding}$ by simply setting this common rate as the depreciation rate for their merged sector. We similarly construct a crosswalk from products in $\mathcal{H}$ to sectors in $\mathcal{N}$, setting $D_{\mathcal{NH}} \equiv D_{\mathcal{N}}^{ICIO} D_{\mathcal{NH}}^{ICIO}$.

Sectoral average trade taxes We set sectoral import tariffs $\tau_{ijn}^{m,data}$ as the ratio of bilateral tariff revenues in a sector above to pre-tariff trade flows.

$$\tau_{ijn}^{m,data} = \bar{T}_{ijn}^{imp} / X_{ijn}^{w,imp}$$
$$\text{where} \quad X_{ijn}^{w,imp} \equiv \bar{C}_{ijn}^{w,ICIO} + \bar{Inv}_{ijn}^{w,ICIO} + \sum_{n'\in\mathcal{N}} \bar{Int}_{injn'}^{w,ICIO}$$

We set $\tau_{ijn}^{m,data} = 0$ for any (i, j, n) with no ICIO-reported trade, i.e., where the denominator above is zero.

We assume that export subsidies are zero, i.e.,[37]

$$\tau_{ijn}^{x,data} \equiv 0$$

Trade shares We next use the adjusted trade flows computed above to compute trade shares across products, origin groups, and origins. We begin by computing aggregate tariff-inclusive flows implied by the BACI data.

$$X_{ijn}^{imp} \equiv (1 + \tau_{ijn}^{m,data}) X_{ijn}^{w,imp}$$

We then compute, for each origin country i, destination country j, and sector n, the share of j's expenditures in n that are directed to i, i.e.,

$$\theta_{ijn} \equiv X_{ijn}^{imp} / \sum_{o \in \mathcal{I}} X_{ojn}^{imp}$$

Tariff-inclusive expenditures We next adjust ICIO consumption, investment, and intermediate spending to account for tariffs. Specifically, we assume that any country's expenditures on goods from a given sector—whether they are consumption, investment, or intermediate expenditures—are sourced from origin countries in the same proportions as one another. Under this assumption, the trade flows computed above indicate how each spending type must be adjusted to account for tariffs. Concretely, we set

$$\begin{aligned} C_{jn}^{imp} &\equiv \frac{\sum_{i \in \mathcal{I}} X_{ijn}^{imp}}{\sum_{i \in \mathcal{I}} X_{ijn}^{w,imp}} \sum_{o \in \mathcal{I}} \bar{C}_{ojn}^{w,ICIO} \\ Inv_{jn}^{imp} &\equiv \frac{\sum_{i \in \mathcal{I}} X_{ijn}^{imp}}{\sum_{i \in \mathcal{I}} X_{ijn}^{w,imp}} \sum_{o \in \mathcal{I}} \bar{Inv}_{ijn}^{w,ICIO} \\ Int_{jnn'}^{imp} &\equiv \frac{\sum_{i \in \mathcal{I}} X_{ijn}^{imp}}{\sum_{i \in \mathcal{I}} X_{ijn}^{w,imp}} \sum_{o \in \mathcal{I}} \bar{Int}_{onjn'}^{w,ICIO} \end{aligned}$$

Intermediate and labor shares of gross output We next compute the share of each country-sector's revenues (i.e., gross output) it spends on intermediate goods from each sector.

[37] Our approach can be easily extended to accommodate non-zero export subsidies, given data on them.

Note that intermediate expenditures include tariff payments, while revenues do not. We set

$$\theta_{in'n}^{x,data} \equiv \frac{Int_{in'n}^{imp}}{\sum_{j \in \mathcal{I}, h \in \mathcal{H}_n} X_{ijh}^{w,imp}}$$

We winsorize the sum of $\theta_{in'n}^{x,data}$ across supplying industries n' at 0.99, rescaling proportionally within (i, n) pairs. This winsorization affects 52 of 1040 total country-sector pairs.

We next compute labor shares of gross output in each country and sector. We begin by computing value added in each country and sector based our aggregated WIOD data.

$$LS_{in}^{data} \equiv \bar{W}_{in}^{WIOD} / \bar{VA}_{in}^{WIOD}$$

We winsorize labor shares at 0.01 and 0.99; this affects the computed labor shares of 42 out of 1040 total country-sector pairs. In the one case where $\bar{VA}_{in}^{WIOD} = 0$ (WIOD reports value added of zero in the Mexican real estate sector), we set LS_{in}^{data} equal to the global aggregate labor share in sector n, i.e.,

$$LS_{in}^{data} \equiv \sum_{j \in \mathcal{I}} \bar{W}_{jn}^{WIOD} / \sum_{j \in \mathcal{I}} \bar{VA}_{jn}^{WIOD}$$

We then compute labor shares of gross output at the country-sector level by combining intermediate shares of gross output with labor shares of value added.

$$\theta_{in}^{\ell,data} \equiv LS_{in} \left(1 - \sum_{n' \in \mathcal{N}} \theta_{in'n}^{data} \right)$$

Level and composition of consumption We compute total final consumption in each country as the sum of sectoral consumption.

$$C_i^{data} = \sum_{n \in \mathcal{N}} C_{in}^{imp}$$

Finally, we compute the share of each sector in each country's consumption basket.

$$\theta_{in}^{c,data} = \frac{C_{in}^{imp}}{C_i^{data}}$$

Directed capital shares We next adapt the methodology of Ding (2021) to compute the share of each country-sector pair's capital expenditures that it directs toward goods from each sector.

First, for each country $i \in \mathcal{I}$ and each pair of sectors $n, n' \in \mathcal{N}$, we impute the investment expenditures—of each sector n in each country i on goods in each sector n'—implied by country i's total investment in n' (as computed above) and sector n's contribution to country i's investment in n' from Ding (2021). I.e.,

$$Inv_{in'n}^{imp} \equiv \frac{\bar{Inv}_{in'n}^{Ding}}{\sum_{n \in \mathcal{N}} \bar{Inv}_{in'n}^{Ding}} Inv_{in'}^{imp}$$

Second, we compute the profits required to rationalize these investments. We do so using the risk-free rate, sector-specific depreciation rates, and the assumption the economy is in steady state. Concretely, we set

$$\Pi_{in'n}^{imp} \equiv \frac{r^{f,Ding}}{\delta_{n'}^{Ding}} Inv_{in'n}^{imp}$$

Third and finally, we compute the share of total payments to capital, at the country-sector level, directed toward investment in each sector. I.e., we set

$$\theta_{in'n}^{\iota,data} \equiv \frac{Inv_{in'n}^{imp}}{\sum_{n'' \in \mathcal{N}} \left(Inv_{in''n}^{imp} + \Pi_{in''n}^{imp} \right)}$$

Because they are based on the assumption the economy is in steady state, the directed capital spending shares $\theta_{in'n}^{\iota,data}$ need not be consistent with observed sectoral investment patterns. To assess the extent of this inconsistency we compare each country's investment spending in each sector according to (a) the tariff-adjusted ICIO data, Inv_{in}^{imp}, and (b) consumption in all countries and the labor, investment, intermediate, and trade shares computed above. We define (b) as

$$\begin{aligned}
\widetilde{Inv}_{in}^{imp} &\equiv \sum_{n' \in \mathcal{N}} \theta_{inn'}^{\iota} (1 - \theta_{in'}^{\ell}) \widetilde{Y}_{in'}^{imp} \\
\text{where} \quad \widetilde{Y}^{imp} &\equiv (\mathrm{id} - \mathcal{M})^{-1} \widetilde{C}^{imp} \\
[\mathcal{M}]_{injn'} &\equiv \frac{X_{ijn}^{w,imp}}{\sum_{o \in \mathcal{I}} X_{ojn}^{imp}} \left(\theta_{jnn'}^{\iota,data} (1 - \theta_{jn'}^{\ell,data} - \sum_{n'' \in \mathcal{N}} \theta_{jnn''}^{data}) + \theta_{jnn'}^{data} \right) \\
[\widetilde{C}^{imp}]_{in} &\equiv \sum_{j \in \mathcal{I}} \frac{X_{ijn}^{w,imp}}{\sum_{o \in \mathcal{I}} X_{ojn}^{imp}} \theta_{jn}^{c,data} C_j^{data}
\end{aligned}$$

Note that $\widetilde{Inv}_{in}^{imp}$ will be equal to investment in our calibrated model under observed tariffs.

The two measures of country-sector investment are broadly consistent. As a global aggregate, investment according to Inv_{in}^{SS} is 102.7% of investment according to $\bar{Inv}_{in}^{ICIO}$. The two measures generate similar patterns of investment across countries and sectors. In logs, their correlation is 98.4%, and the linear regression of $\log(Inv_{in}^{ICIO})$ on $\log(\bar{Inv}_{in}^{ICIO})$ and a constant estimates a slope of 0.975.

E Quantitative appendix

Our focus on the $\beta \to 0$ limit implies that, given any trade taxes, the equilibrium of our dynamic quantitative model is be fully characterized by the equilibria of two static models. In the first model—corresponding to peacetime in the limit of unlikely conflict—capital can adjust and so is essentially an intermediate good. In the second model—corresponding to the conflict period—capital has already been determined and so is a fixed factor.

In both cases, we study a superficially altered version of the quantitative model presented in Section 4.1. First, we assume that production firms rental capital on competitive markets rather than owning it themselves. This aligns our quantitative analysis with our general model. Second,, we assume that it is not relationship capital k_{ijn} that directly enters the production function for sectoral composites but rather normalized-relationship capital k_{ijn}/K_{jn}, where K_{jn} is a homothetic aggregator of $\{k_{ijn}\}_{i\in\mathcal{I}}$. As discussed in Footnote 26, this ensures the problem of sectoral composite producers has a well-defined solution when they rent capital rather than owning it direction. However, it has no other consequences since our assumption on the formation of relationship capital ensures K_{jn} is always equal to one in equilibrium.

E.1 Static model equilibrium conditions

We now state the equilibrium conditions of the static models that describe the peacetime and conflict periods in the $\beta \to 0$ limit.

We let w_i denote wages, r_{in} denote production capital prices, r_{ijn} denote relationship capital prices, p_{ijn} denote origin-destination-sector variety prices, p_{in} denote sectoral composite prices, k_{in} denote production capital, k_{ijn} denote relationship capital, $\iota_{in'n}$ denote sector-directed investment used to produce a country-sector's production capital, τ_{ijn}^m denote ad-valorem import tariffs, and τ_{ijn}^x denote ad-valorem export subsidies. We assume there are no domestic taxes, i.e., $\tau_{iin}^m = \tau_{iin}^x = 0$.

E.1.1 Peacetime equilibrium conditions

In the static model that represents the peacetime period, capital can adjust freely, subject to the investment technology. We now describe the series of equilibrium conditions that determine all endogenous variables, including capital, given trade taxes.

Sectoral composite production with endogenous relationship capital As a preliminary step, we characterize the level of relationship capital that emerges in the equilibrium of the static model. We substitute this level of capital into the production function for sectoral composites, arriving at a new production function that already incorporates equilibrium outcomes in the relationship capital market.

The production function for relationship capital in Section 4.1 implies that for some $\{\kappa_{in}\}$, we have

$$r_{oin} = \kappa_{in} \left(\theta_{oin}\right)^{(1-\sigma)/(\bar{\sigma}-\sigma)} \left(k_{oin}\right)^{(\sigma-1)/(\bar{\sigma}-\sigma)}$$

for all (o,i,n) triplets with $\theta_{oin} > 0$. Combining this rental rate with production function of sectoral composites implies that relationship capital demands satisfy, for some $\{\tilde{\kappa}_{in}\}$,

$$k_{oin} = \tilde{\kappa}_{in} \left(\theta_{oin}\right)^{\sigma/\bar{\sigma}} \left(x_{oin}\right)^{(\bar{\sigma}-\sigma)/\bar{\sigma}}$$

for all (o,i,n) triplets with $\theta_{oin} > 0$, where x_{oin} is the quantity of origin-o destination-i sector-n goods the sector-n composite producer in i uses as an input.

Letting $K_{in} \equiv \left[\sum_{o\in\mathcal{I}:\ \theta_{oin}>0} \left(\theta_{oin}\right)^{(1-\sigma)/(\bar{\sigma}-\sigma)} \left(k_{oin}\right)^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}\right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}$, this implies

$$\frac{k_{oin}}{K_{in}} = \frac{\left(\theta_{oin}\right)^{\sigma/\bar{\sigma}} \left(x_{oin}\right)^{(\bar{\sigma}-\sigma)/\bar{\sigma}}}{\left[\sum_{o'\in\mathcal{I}:\ \theta_{o'in}>0} \left(\theta_{o'in}\right)^{1/\bar{\sigma}} \left(x_{o'in}\right)^{(\bar{\sigma}-1)/\bar{\sigma}}\right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}}$$

or all (o,i,n) triplets with $\theta_{oin} > 0$. Combining this observation with the capital production constraint

$$\sum_{o\in\mathcal{I}:\ \theta_{oin}>0} \left(\theta_{oin}\right)^{(1-\sigma)/(\bar{\sigma}-\sigma)} \left(k_{oin}\right)^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)} = 1$$

implies $K_{in} = 1$, so that

$$k_{oin} = \frac{\left(\theta_{oin}\right)^{\sigma/\bar{\sigma}} \left(x_{oin}\right)^{(\bar{\sigma}-\sigma)/\bar{\sigma}}}{\left[\sum_{o'\in\mathcal{I}:\ \theta_{o'in}>0} \left(\theta_{o'in}\right)^{1/\bar{\sigma}} \left(x_{o'in}\right)^{(\bar{\sigma}-1)/\bar{\sigma}}\right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}} \tag{30}$$

Finally, we substitute back into the production function for origin-destination-product goods. This implies the following after-relationship-capital-optimization production function for sectoral composites

$$q_{in} = \left[\sum_{o \in \mathcal{I}} (\theta_{oin})^{1/\bar{\sigma}} (x_{oin})^{(\bar{\sigma}-1)/\bar{\sigma}} \right]^{\bar{\sigma}/(\bar{\sigma}-1)} \tag{31}$$

Goods prices We now characterize the prices of all goods in terms of wages w_i and rental rates r_{in} of production capital in each country i and sector n. The origin-destination-sector production function in Section 4.1 and the after-relationship-capital-optimization production function for sectoral composites derived above imply

$$p_{ijn} = \frac{1+\tau^m_{ijn}}{1+\tau^x_{ijn}} (\phi_{ijn})^{-1} (w_i)^{\theta^\ell_{in}} (r_{in})^{\theta^k_{in}} \prod_{n' \in \mathcal{N}} (p_{in'})^{\theta^x_{in'n}} \tag{32}$$

$$p_{in} = \left[\sum_{o \in \mathcal{I}} \theta_{oin} (p_{oin})^{1-\bar{\sigma}} \right]^{1/(1-\bar{\sigma})}$$

Spending The expenditures of the country-i household on consumption of sectoral composites, C_i, is equal to household labor income, plus profits, plus trade tax revenues T_i in i.

$$C_i = w_i \ell_i + \sum_{n \in \mathcal{N}} \theta^\pi_{in} r_{in} k_{in} + T_i \tag{33}$$

where $\theta^\pi_{in} \equiv 1 - \sum_{n' \in \mathcal{N}} \theta^\iota_{in'n}$.The utility function in Section 4.1 implies that household expenditure in country i on sectoral composite n is given by

$$C_{in} = \theta^c_{in} C_i \tag{34}$$

Note that capital rental expenditures $r_{in} k_{in}$ can be represented in terms of the revenues of country-i firms on sales of product n to country j, Y_{ijn}, due to the structure of origin-destination-sector good production.

$$r_{in} k_{in} = \theta^k_{in} \sum_{j \in \mathcal{I}} Y_{ijn} \tag{35}$$

We denote aggregate expenditures on country i's sector-n composite by X_{in}. These expenditures include both domestic consumption expenditures and the intermediate input expenditures of domestic firms—both goods and capital producers. The structure of goods and capital production implies

$$X_{in} = C_{in} + \sum_{n' \in \mathcal{N}} \left(\theta^x_{inn'} + \theta^k_{in'} \theta^\iota_{inn'} \right) \sum_{j \in \mathcal{I}} Y_{ijn'} \tag{36}$$

Note these revenues are equal to expenditures X_{ijn} on i's exports of n in j, net of trade taxes.

$$Y_{ijn} = \frac{1+\tau^x_{ijn}}{1+\tau^m_{ijn}} X_{ijn} \tag{37}$$

Next we observe that the origin-destination-sector expenditures X_{ijn} can be written in terms of sectoral composite expenditures, using the structure of after-relationship-capital-optimization production function described in Equation 31.

$$\begin{aligned} X_{ijn} &= \zeta_{ijn} X_{jn} \\ \text{where} \quad \zeta_{ijn} &= \frac{\theta_{ijn}(p_{ijn})^{1-\bar{\sigma}}}{(p_{jn})^{1-\bar{\sigma}}} \end{aligned} \tag{38}$$

This expression also allow us to express trade taxes in terms of sectoral composite expenditures:

$$\begin{aligned} T_i &= \sum_{o\in\mathcal{I}} \sum_{n\in\mathcal{N}} \frac{\tau^m_{oin}}{1+\tau^m_{oin}} X_{oin} - \sum_{j\in\mathcal{I}} \sum_{n\in\mathcal{N}} \frac{\tau^x_{ijn}}{1+\tau^m_{ijn}} X_{ijn} \\ &= \sum_{o\in\mathcal{I}} \sum_{n\in\mathcal{N}} \frac{\tau^m_{oin}}{1+\tau^m_{oin}} \zeta_{oin} X_{in} - \sum_{j\in\mathcal{I}} \sum_{n\in\mathcal{N}} \frac{\tau^x_{ijn}}{1+\tau^m_{ijn}} \zeta_{ijn} X_{jn} \end{aligned} \tag{39}$$

Combining Equations 33–39 implies a linear system for sectoral composite spending X_{in} in terms of prices and exogenous variables.

$$X = [\mathrm{id} - \mathcal{X}]^{-1} X_0 \tag{40}$$

$$\begin{aligned} \text{where } [X]_{in} &= X_{in} \\ [X_0]_{in} &= \theta^c_{in} w_i \ell_i \\ [\mathcal{X}]_{in,jn'} &= \theta^c_{in} \mathbb{1}_{i=j} \sum_{o\in\mathcal{I}} \frac{\tau^m_{oin'}}{1+\tau^m_{oin'}} \zeta_{oin'} \\ &\quad + \left(\left(\theta^x_{inn'} + (\theta^\iota_{inn'} + \theta^c_{in}\theta^\pi_{in'})\theta^k_{in'}\right) \frac{1+\tau^x_{ijn'}}{1+\tau^m_{ijn'}} - \theta^c_{in} \frac{\tau^x_{ijn'}}{1+\tau^m_{ijn'}} \right) \zeta_{ijn'} \end{aligned} \tag{41}$$

Factor markets Finally, we characterize factor demand clearing in terms of aggregate spending on sectoral composites.

The production function for origin-destination-sector goods, together with equations

Equations 37 and 38, implies aggregate labor spending of

$$W_i = \sum_{n \in \mathcal{N}} \theta^{\ell}_{in} \sum_{j \in \mathcal{I}} \frac{1+\tau^x_{ijn}}{1+\tau^m_{ijn}} \zeta_{ijn} X_{jn} \tag{42}$$

Factor market clearing implies

$$w_i \ell_i = W_i \tag{43}$$

The same equations imply capital rental spending on the production capital used in each sector n and country i equal to

$$R_{in} = \theta^k_{in} \sum_{j \in \mathcal{I}} \frac{1+\tau^x_{ijn}}{1+\tau^m_{ijn}} \zeta_{ijn} X_{jn} \tag{44}$$

and the production function for capital implies capital supply that satisfies

$$R_{in} = r_{in} k_{in} = r_{in} \left(\phi_{in} \prod_{n' \in \mathcal{N}} (r_{in}/p_{in'})^{\theta^{\iota}_{in'n}} \right)^{1/\theta^{\pi}_{in}} \tag{45}$$

E.1.2 Conflict equilibrium conditions

In the static model that represents the conflict period, capital is exogenous. We now describe the series of equilibrium conditions that determine all endogenous variables, given capital and trade taxes (which are also exogenous).

Goods prices We begin by characterizing the prices of all goods in terms of the factor prices $\{w_i\}$ and $\{r_{in}\}$ and relationship capital $\{k_{ijn}\}$. The production functions in Section 4.1 imply that goods prices $\{p_{in}\}$ and $\{p_{ijn}\}$ are determined as the solution to the following system of equations.

$$p_{ijn} = \frac{1+\tau^m_{ijn}}{1+\tau^x_{ijn}} (\phi_{ijn})^{-1} (w_i)^{\theta^{\ell}_{in}} (r_{in})^{\theta^k_{in}} \prod_{n' \in \mathcal{N}} (p_{in'})^{\theta^x_{in'n}} \tag{46}$$

$$p_{in} = \left[\sum_{o \in \mathcal{I}:\ \theta_{oin} > 0} (k_{oin}/K_{in}) (p_{oin})^{1-\sigma} \right]^{1/(1-\sigma)}$$

where $K_{in} = \left[\sum_{o \in \mathcal{I}:\ \theta_{oin}>0} (\theta_{oin})^{(1-\sigma)/(\bar{\sigma}-\sigma)} (k_{oin})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)} \right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}$.

Consumer demand We next characterize country-i's household expenditures on sectoral composites, C_{in} , in terms of prices and the Lagrange multiplier ψ_i on the household budget constraint. The two-part utility function presented in Section 4.1 implies that the households' consumption expenditures satisfy

$$C_{in} = \theta^c_{in} P_i \tilde{c}_i + \sum_{n' \in \mathcal{N}} \frac{\theta^\iota_{inn'}}{1 - \theta^\pi_{in'}} P_{in'} \tilde{c}_{in'} \tag{47}$$

$$\begin{aligned} \text{where} \quad \tilde{c}_i &= (\psi_i P_i)^{-\rho} \\ \tilde{c}_{in} &= (\psi_i P_{in} / \tilde{\nu}_{in})^{-1/\theta^\pi_{in}} \\ P_i &= \prod_{n \in \mathcal{N}} (p_{in})^{\theta^c_{in}} \\ P_{in} &= \prod_{n' \in \mathcal{N}} (p_{in'})^{\theta^\iota_{in'n}/(1-\theta^\pi_{in})} \\ \tilde{\nu}_{in} &\equiv \nu_{in} \phi_{in} (1 - \theta^\pi_{in})^{\theta^\pi_{in}} \end{aligned}$$

Spending We next use the above description of consumer demand to characterize total expenditures on each sectoral composite good in terms of prices and budget constraint Lagrange multipliers.

We denote aggregate expenditures on country i's sector-n composite by X_{in} and we denote aggregate expenditures on country i's product n variety by country j by X_{ijn}. Sectoral composite expenditures include both domestic consumption expenditures and the intermediate input expenditures of domestic firms. Letting Y_{ijn} denote the revenues of country-i firms on sales in sector n to country j, the production function in Section 4.1 implies

$$X_{in} = C_{in} + \sum_{j \in \mathcal{I}} \sum_{n' \in \mathcal{N}} \theta^x_{inn'} Y_{ijn'} \tag{48}$$

Note these revenues are equal to expenditures X_{ijn} on i's exports of n in j, net of trade taxes.

$$Y_{ijn} = \frac{1 + \tau^x_{ijn}}{1 + \tau^m_{ijn}} X_{ijn} \tag{49}$$

Next we observe that the origin-destination-product expenditures X_{ijn} can be written in terms of expenditures on sectoral composites, using the structure of production described in

Section 4.1.

$$
\begin{aligned}
X_{ijn} &= \zeta_{ijn} X_{jn} \\
\text{where} \quad \zeta_{ijn} &= \frac{(k_{ijn}/K_{jn})(p_{ijn})^{1-\sigma}}{(p_{jn})^{1-\sigma}}
\end{aligned} \tag{50}
$$

Combining Equations 48–50 implies a linear system for sectoral composite spending X_{in} in terms of prices and exogenous variables.

$$
\begin{aligned}
X &= [\mathrm{id} - \mathcal{X}]^{-1} C \\
\text{where } [X]_{in} &= X_{in} \\
[C]_{in} &= C_{in} \\
[\mathcal{X}]_{in,jn'} &= \theta^x_{inn'} \frac{1+\tau^x_{ijn'}}{1+\tau^m_{ijn'}} \zeta_{ijn'}
\end{aligned} \tag{51}
$$

Lagrange multipliers on household budget constraints We next show how the Lagrange multipliers on household budget constraints are determined by consumption, aggregate spending and factor prices.

Each country i has tax revenues

$$
\begin{aligned}
T_i &= \sum_{o\in\mathcal{I}} \sum_{n\in\mathcal{N}} \frac{\tau^m_{oin}}{1+\tau^m_{oin}} X_{oin} - \sum_{j\in\mathcal{I}} \sum_{n\in\mathcal{N}} \frac{\tau^x_{ijn}}{1+\tau^m_{ijn}} X_{ijn} \\
&= \sum_{o\in\mathcal{I}} \sum_{n\in\mathcal{N}} \frac{\tau^m_{oin}}{1+\tau^m_{oin}} \zeta_{oin} X_{in} - \sum_{j\in\mathcal{I}} \sum_{n\in\mathcal{N}} \frac{\tau^x_{ijn}}{1+\tau^m_{ijn}} \zeta_{ijn} X_{jn}
\end{aligned} \tag{52}
$$

The Lagrange multiplier on each household budget constraint must be such that, given these revenues, the budget constraint holds:

$$
\sum_{n\in\mathcal{N}} C_{in} = w_i \ell_i + \sum_{n\in\mathcal{N}} r_{in} k_{in} + T_i \tag{53}
$$

Factor markets Finally, we characterize factor demand and factor market clearing in terms of prices and aggregate spending on sectoral composites.

The production function in Section 4.1 and Equations 49 and 50 imply the goods producers in each country i have aggregate labor and production capital spending

$$W_i = \sum_{n\in\mathcal{N}} \theta^\ell_{in} \sum_{j\in\mathcal{I}} \frac{1+\tau^x_{ijn}}{1+\tau^m_{ijn}} \zeta_{ijn} X_{jn} \tag{54}$$
$$R_{in} = \theta^k_{in} \sum_{j\in\mathcal{I}} \frac{1+\tau^x_{ijn}}{1+\tau^m_{ijn}} \zeta_{ijn} X_{jn}$$

Factor market clearing implies

$$w_i \ell_i = W_i \tag{55}$$
$$r_{in} k_{in} = R_{in}$$

Any given variety of relationship capital can, in any country, only be allocated to a single sectoral composite producer. However, it will nonetheless be useful to compute the implicit rental rate r_{ijn} associated with each such variety k_{ijn}. The production functions for sectoral composites imply

$$r_{ijn} = \frac{X_{jn}}{k_{ijn}} \frac{1}{\sigma-1} \mathbb{1}_{\theta_{ijn}>0} \left(\frac{(k_{ijn}/K_{jn})(p_{ijn})^{1-\sigma}}{(p_{jn})^{1-\sigma}} - \frac{(\theta_{ijn})^{(1-\sigma)/(\bar{\sigma}-\sigma)} (k_{ijn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}}{(K_{jn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}} \right) \tag{56}$$

Note this can, in general be negative, as relationship capital with one origin detracts from relationship capital with another origin.

E.1.3 Peacetime equilibrium conditions given Home trade

Recall from Appendix C.4 that Home's optimal trade taxes with many countries depend on the sensitivity of (a) Foreign and RoW capital to Home's net imports and (b) Foreign time-2 terms of trade to capital, holding fixed Home's net imports. Toward computing these statistics, we provide an extended set of equilibrium conditions for non-Home equilibrium variables given Home's net imports. As in Appendix C.4, we assume that if Home's fixed net imports result in a trade deficit or surplus, then it finances this deficit or surplus with symmetric international transfers to or from all foreign countries.

We being with the case of peacetime trade.

All of the equilibrium conditions except Equations 33, 38, 40, 42, and 44, still apply for countries other than Home (whose internal equilibrium conditions we need not consider).

These specific equations are replaced by the following alternatives. First, for $i \neq H$, we have

$$C_i = w_i \ell_i + \sum_{n \in \mathcal{N}} \theta_{in}^{\pi} r_{in} k_{in} + T_i + \Gamma_i \tag{57}$$
$$\text{where} \quad \Gamma_i = \frac{1}{|\mathcal{I}| - 1} \sum_{n \in \mathcal{N}} \left[\sum_{d \neq H} \frac{p_{Hdn}}{1 + \tau_{Hdn}^m} m_{Hdn} - \sum_{o \neq H} \frac{p_{oHn}}{1 + \tau_{oHn}^m} m_{oHn} \right]$$

Second, we have

$$X_{ijn} = \begin{cases} \zeta_{ijn} X_{jn} & \text{for } i, j \neq H \\ p_{ijn} m_{ijn} & \text{for } i = H \text{ or } j = H \end{cases} \tag{58}$$
$$\text{where} \quad \zeta_{ijn} = \frac{\theta_{ijn} (p_{ijn})^{1-\bar{\sigma}}}{(p_{jn})^{1-\bar{\sigma}}}$$

Third, following analogous steps to Appendix E.1.1 then implies

$$\begin{aligned} X_{-H} &= \left[\text{id} - \mathcal{X}_{-H} \right]^{-1} X_{-H0} \\ \text{where } [X_{-H}]_{in} &= X_{-H,in} \\ [X_{-H0}]_{in} &= \theta_{in}^c \left(w_i \ell_i + \Gamma_i \right) + \theta_{in}^c \sum_{n' \in \mathcal{N}} \frac{\tau_{Hin'}^m}{1 + \tau_{Hin'}^m} p_{Hin'} m_{Hin'} \\ &\quad + \sum_{n' \in \mathcal{N}} \left(-\theta_{in}^c \frac{\tau_{iHn'}^x}{1 + \tau_{iHn'}^m} + \left(\theta_{inn'} + (\theta_{inn'}^{\iota} + \theta_{in}^c \theta_{in'}^{\pi}) \theta_{in'}^k \right) \frac{1 + \tau_{iHn'}^x}{1 + \tau_{iHn'}^m} \right) p_{iHn'} m_{iHn'} \end{aligned} \tag{59}$$
$$\begin{aligned} [\mathcal{X}_{-H}]_{in,jn'} &= \theta_{in}^c \sum_{o \neq H} \mathbb{1}_{i=j} \frac{\tau_{ojn'}^m}{1 + \tau_{ojn'}^m} \zeta_{ojn'} \\ &\quad + \left(\left(\theta_{inn'}^x + (\theta_{inn'}^{\iota} + \theta_{in}^c \theta_{in'}^{\pi}) \theta_{in'}^k \right) \frac{1 + \tau_{ijn'}^x}{1 + \tau_{ijn'}^m} - \theta_{in}^c \frac{\tau_{ijn'}^x}{1 + \tau_{ijn'}^m} \right) \zeta_{ijn'} \end{aligned}$$

Fourth for $i \neq I$, we have

$$W_i = \sum_{n \in \mathcal{N}} \theta_{in}^{\ell} \sum_{j \in \mathcal{I}} \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} X_{ijn} \tag{60}$$

$$R_{in} = \theta_{in}^k \sum_{j \in \mathcal{I}} \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} X_{ijn} \tag{61}$$

E.1.4 Conflict equilibrium conditions given Home trade

Analogously to Appendix E.1.3, we now provide an extended set of conflict-period equilibrium conditions for non-Home equilibrium variables given Home's net imports.

All of the equilibrium conditions from Appendix E.1.2 continue to apply for countries $i \neq H$, except for Equations 50, 51, and 53. These specific equations are replaced with the following alternatives. First, we have

$$X_{ijn} = \begin{cases} \zeta_{ijn} X_{jn} & \text{for } i, j \neq H \\ p_{ijn} m_{ijn} & \text{for } i = H \text{ or } j = H \end{cases}$$
$$\text{where} \quad \zeta_{ijn} = \frac{(k_{ijn}/K_{jn})(p_{ijn})^{1-\sigma}}{(p_{jn})^{1-\sigma}}$$

Second, following analogous steps to Appendix E.1.2

$$X_{-H} = [\text{id} - \mathcal{X}_{-H}]^{-1} X_{-H0} \tag{62}$$
$$\text{where } [X_{-H}]_{in} = X_{in}$$
$$[X_{-H0}]_{in} = C_{in} + \sum_{n' \in \mathcal{N}} \theta^x_{inn'} \frac{1 + \tau^x_{iHn'}}{1 + \tau^m_{iHn'}} p_{iHn'} m_{iHn'}$$
$$[\mathcal{X}_{-H}]_{in,jn'} = \theta^x_{inn'} \frac{1 + \tau^x_{ijn'}}{1 + \tau^m_{ijn'}} \zeta_{ijn'}$$

Third, for $i \neq H$, we have

$$\sum_{n \in \mathcal{N}} C_{in} = w_i \ell_i + \sum_{n \in \mathcal{N}} r_{in} k_{in} + T_i + \Gamma_i \tag{63}$$
$$\text{where} \quad \Gamma_i = \frac{1}{|\mathcal{I}| - 1} \sum_{n \in \mathcal{N}} \left[\sum_{d \neq H} \frac{p_{Hdn}}{1 + \tau^m_{Hdn}} m_{Hdn} - \sum_{o \neq H} \frac{p_{oHn}}{1 + \tau^m_{oHn}} m_{oHn} \right]$$

E.2 Numerical algorithms for equilibrium computation

This section describes the numerical algorithms we use to solve the quantitative model in the $\beta \to 0$ limit. In this limit, the model is equivalent to the two static models whose equilibrium conditions we have described in Appendix E.1. We therefore provide two algorithms, one for each static model. The first static model takes as inputs all countries' trade taxes. The second static model takes as inputs all countries' trade taxes and also the capital stocks determined by the first.

We also provide a three additional algorithms used to compute the derivatives from Equation 29 that vary Home or Foreign trade. The first simulates the peacetime equilibrium given Home trade. The second simulates the conflict equilibrium given Home trade. The third simulates, for a given country i, the conflict period in country i taking as given i's trade.

In addition to the parameters of the model, these simulations rely on numerical tolerances, a parameter ξ that we use to normalize prices so that nominal world GDP is held constant, and guesses $\{\Psi_i\}$ of the Lagrange multipliers on households budget constraints during conflict.[38]

E.2.1 Numerical algorithm for equilibrium of peacetime static model

We iterate overwages, production capital rental rates, and sectoral composite prices in outer and inner loops indexed by a and b, respectively. We let $\{w_i^a\}$ and $\{r_{in}^a\}$ denote wages and capital rental rates, respectively, in the a^{th} outer loop and let $\left\{p_{in}^{a,b}\right\}$ denote sectoral composite prices in the a^{th} outer loop and b^{th} inner loop. We proceed as follows.

i. At $a = 1$, initialize all $w_i^{a=1} = 1$ and $r_{in}^{a=1} = 1$.

ii. Solve for sectoral composite prices given wages as follows.

   (a) At $b = 1$, initialize all $p_{in}^{a,b=1} = 1$ if $a = 1$ or $p_{in}^{a,b=1} = p_{in}^{a-1,b_{\text{final}}}$ if $a > 1$.

   (b) Define $\widetilde{p}_{in}^{a,b}$ using Equation 32, setting

$$p_{ijn}^{a,b} = \frac{1+\tau_{ijn}^m}{1+\tau_{ijn}^x}(\phi_{ijn})^{-1}(w_i^a)^{\theta_{in}^\ell}(r_{in}^a)^{\theta_{in}^k}\prod_{n'\in\mathcal{N}}(p_{in'}^{a,b})^{\theta_{in'n}^x}$$

$$\widetilde{p}_{in}^{a,b} = \left[\sum_{o\in\mathcal{I}}\theta_{oin}(p_{oin}^{a,b})^{1-\bar{\sigma}}\right]^{1/(1-\bar{\sigma})}$$

   (c) For some numerical parameter $\text{tol}_{\text{inner}}$, test whether

$$\max_{i\in\mathcal{I},n\in\mathcal{N}}|\log\widetilde{p}_{in}^{a,b} - \log p_{in}^{a,b}| < \text{tol}_{\text{inner}}$$

      i. If not, update prices according to

$$p_{in}^{a,b+1} = \exp\left[\log p_{in}^{a,b} + \text{step}_{\text{inner}}\left(\log\widetilde{p}_{in}^{a,b} - \log p_{in}^{a,b}\right)\right]$$

      for some numerical parameter $\text{step}_{\text{inner}} > 0$. Increase b by one and return to step 2.b.

      ii. If so, set $p_{in}^a \equiv p_{in}^{a,b}$, $p_{ijn}^a \equiv p_{ijn}^{a,b}$, and proceed to step 3.

[38] See Appendix E.3 for a discussion of how we calibrate ξ.

iii. Compute sectoral composite spending X^a_{in} according to Equations 38 and 40, setting

$$\begin{aligned}
X^a &= [\mathrm{id} - \mathcal{X}^a]^{-1} X^a_0 \\
\text{where } [X^a]_{in} &= X^a_{in} \\
[X^a_0]_{in} &= \theta^c_{in} w^a_i \ell_i \\
[\mathcal{X}^a]_{in,jn'} &= \mathbb{1}_{i=j} \theta^c_{in} \left(\sum_{o \in \mathcal{I}} \frac{\tau^m_{ojn'}}{1+\tau^m_{ojn'}} \zeta^a_{ojn'} \right) \\
&\quad + \left(\theta^x_{inn'} + (\theta^\iota_{inn'} + \theta^c_{in}\theta^\pi_{in'})\theta^k_{in'}\right) \frac{1+\tau^x_{ijn'}}{1+\tau^m_{ijn'}} \zeta^a_{ijn'} - \theta^c_{in} \frac{\tau^x_{ijn'}}{1+\tau^m_{ijn'}} \zeta^a_{ijn'} \\
\zeta^a_{ijn} &= \frac{\theta_{ijh} (p^a_{ijn})^{1-\bar{\sigma}}}{(p^a_{jn})^{1-\bar{\sigma}}}
\end{aligned}$$

iv. Define $\widetilde{w}^a_i$ and $\tilde{r}^a_{in}$ using Equations 42, 43, 44, and 45, setting

$$\begin{aligned}
\widetilde{w}^a_i &= \frac{1}{\ell_i} \sum_{n \in \mathcal{N}} \theta^\ell_{in} \sum_{j \in \mathcal{I}} Y^a_{ijn} \\
\tilde{r}^a_{in} &= (\phi_{in})^{-1} \prod_{n' \in \mathcal{N}} (p_{in'})^{\theta^\iota_{in'n}} \left(\theta^k_{in} \sum_{j \in \mathcal{I}} Y^a_{ijn} \right)^{\theta^\pi_{in}} \\
\text{where} \quad Y^a_{ijn} &= \frac{1+\tau^x_{ijn}}{1+\tau^m_{ijn}} \zeta^a_{ijn} X^a_{jn}
\end{aligned}$$

v. For some numerical parameter $\mathrm{tol}_{\mathrm{outer}}$, test whether

$$\begin{aligned}
&\max_{i \in \mathcal{I}} |\log \widetilde{w}^a_i - \log w^a_i| < \mathrm{tol}_{\mathrm{outer}} \\
\text{and} \quad &\max_{i \in \mathcal{I}, n \in \mathcal{N}} |\log \widetilde{r}^a_{in} - \log r^a_{in}| < \mathrm{tol}_{\mathrm{outer}}
\end{aligned}$$

(a) If not, update wages and capital rental rates according to

$$\begin{aligned}
w^{a+1}_i &= \frac{\xi}{\sum_{j \in \mathcal{I}} \hat{w}^a_j \ell_j + \sum_{j \in \mathcal{I}, n' \in \mathcal{N}} \hat{r}^a_{jn'} k^a_{jn'}} \hat{w}^a_i \\
r^{a+1}_{in} &= \frac{\xi}{\sum_{j \in \mathcal{I}} \hat{w}^a_j \ell_j + \sum_{j \in \mathcal{I}, n' \in \mathcal{N}} \hat{r}^a_{jn'} k^a_{jn'}} \hat{r}^a_{in} \\
\text{where} \quad \hat{w}^a_i &= \exp\left[\log w^a_i + \mathrm{step}_{\mathrm{outer}} \left(\log \widetilde{w}^a_i - \log w^a_i\right)\right] \\
\hat{r}^a_{in} &= \exp\left[\log r^a_{in} + \mathrm{step}_{\mathrm{outer}} \left(\log \widetilde{r}^a_{in} - \log r^a_{in}\right)\right] \\
k^a_{in} &= \left(\theta^k_{in} \sum_{j \in \mathcal{I}} Y^a_{ijn} \right) / r^a_{in}
\end{aligned}$$

for some numerical parameter $\text{step}_{\text{outer}} > 0$. Increase a by one and return to step 2.

(b) If so, stop and set $w_i = w_i^a$, $r_{in} = r_{in}^a$, $p_{in} \equiv p_{in}^a$, $p_{ijn} \equiv p_{ijn}^a$, $X_{in} \equiv X_{in}^a$, $X_{ijn} \equiv \zeta_{ijn}^a X_{in}^a$, $k_{in} = k_{in}^a$, and—using Equations 30, 36, 37, and the production function for production capital in Section 4.1— set

$$
\begin{aligned}
k_{ijn} &= \begin{cases} \dfrac{(\theta_{ijn})^{\sigma/\bar{\sigma}}(X_{ijn}/p_{ijn})^{(\bar{\sigma}-\sigma)/\bar{\sigma}}}{\left[\sum_{o\in\mathcal{I}:\ \theta_{ojn}>0}(\theta_{ojn})^{1/\bar{\sigma}}(X_{ojn}/p_{ojn})^{(\bar{\sigma}-1)/\bar{\sigma}}\right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}} & \text{if } \theta_{ijn} > 0 \\ 0 & \text{if } \theta_{ijn} = 0 \end{cases} \\
C_i &= \sum_{n\in\mathcal{N}} X_{in} - \sum_{n,n'\in\mathcal{N}} \left(\theta_{inn'}^x + \theta_{in'}^k \theta_{inn'}^\iota\right) \sum_{j\in\mathcal{I}} Y_{ijn'}^a \\
\iota_{in'n} &= \theta_{in'n}^\iota r_{in} k_{in}/p_{in'}
\end{aligned}
$$

Note that setting $k_{ijh} = 0$ when $\theta_{ijh} = 0$ is without loss of generality as k_{ijh} never affects production in such cases.

E.2.2 Numerical algorithm for equilibrium of conflict static model

We iterate over wages, production capital rental rates, sectoral composite prices, and Lagrange multipliers on household budget constraints in one outer and two successive inner loops indexed by a, b, and b', respectively. We let $\{w_i^a\}$ and $\{r_{in}^a\}$ denote wages and production capital rental rates, respectively, in the a^{th} iteration of the outer loop. We let $\left\{p_{in}^{a,b}\right\}$ denote sectoral composite prices in the a^{th} iteration of the outer loop and b^{th} iteration of the first inner loop. We let $\left\{\psi_i^{a,b'}\right\}$ denote budget constraint Lagrange multipliers in the a^{th} iteration of the outer loop and $(b')^{th}$ iteration of the second inner loop. We proceed as follows.

i. At $a = 1$, initialize all $w_i^{a=1} = r_{in}^{a=1} = 1$.

ii. Solve for sectoral composite prices given factor prices as follows.

(a) At $b = 1$, initialize all $p_{in}^{a,b=1} = 1$ if $a = 1$ or $p_{in}^{a,b=1} = p_{in}^{a-1,b_{\text{final}}}$ if $a > 1$.

(b) Define $\widetilde{p}_{in}^{a,b}$ using Equation 46, setting

$$
\begin{aligned}
p_{ijn}^{a,b} &= \frac{1+\tau_{ijn}^m}{1+\tau_{ijn}^x}(\phi_{ijn})^{-1}(w_i^a)^{\theta_{in}^\ell}(r_{in}^a)^{\theta_{in}^k}\prod_{n'\in\mathcal{N}}(p_{in'}^{a,b})^{\theta_{in'n}^x} \\
\widetilde{p}_{in}^{a,b} &= \left[\sum_{o\in\mathcal{I}:\ \theta_{oin}>0}(k_{oin}/K_{in})(p_{oin}^{a,b})^{1-\sigma}\right]^{1/(1-\sigma)} \\
\text{where}\quad K_{in} &= \left[\sum_{o\in\mathcal{I}:\ \theta_{oin}>0}(\theta_{oin})^{(1-\sigma)/(\bar{\sigma}-\sigma)}(k_{oin})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}\right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}
\end{aligned}
$$

(c) For some numerical parameter $\text{tol}_{\text{inner},1}$, test whether

$$\max_{i\in\mathcal{I},n\in\mathcal{N}} |\log \widetilde{p}_{in}^{a,b} - \log p_{in}^{a,b}| < \text{tol}_{\text{inner},1}$$

i. If not, update prices according to

$$p_{in}^{a,b+1} = \exp\left[\log p_{in}^{a,b} + \text{step}_{\text{inner},1}\left(\log \widetilde{p}_{in}^{a,b} - \log p_{in}^{a,b}\right)\right]$$

for some numerical parameter $\text{step}_{\text{inner},1} > 0$. Increase b by one and return to step 2.b.

ii. If so, set $p_{in}^{a} \equiv p_{in}^{a,b}$, $p_{ijn}^{a} \equiv p_{ijn}^{a,b}$, and proceed to step 3.

iii. Compute true-consumption and investment-as-consumption price indices P_i^a and P_{in}^a for $i \neq H$ according to Equation 47, setting

$$P_i^a = \prod_{n\in\mathcal{N}} (p_{in}^a)^{\theta_{in}^c}$$
$$P_{in}^a = \prod_{n'\in\mathcal{N}} (p_{in'}^a)^{\theta_{in'n}^\iota/(1-\theta_{in}^\pi)}$$

Compute the map between consumption and aggregate expenditures on sectoral composites following Equation 51, setting

$$[\mathcal{X}^a]_{in,jn'} = \theta_{inn'}^x \frac{1+\tau_{ijn'}^x}{1+\tau_{ijn'}^m}\zeta_{ijn'}^a$$
$$\text{where} \quad \zeta_{ijn}^a = \frac{(k_{ijn}/K_{jn})(p_{ijn}^a)^{1-\sigma}}{(p_{jn}^a)^{1-\sigma}}$$

iv. Solve for spending on sectoral composites given prices as follows.

(a) At $b' = 1$, initialize all $\psi_i^{a,b'} = \Psi_i$ if $a = 1$ or $\psi_i^{a,b'} = \psi_i^{a-1,b_{final}}$ if $a > 1$.

(b) Compute consumption expenditure according to Equation 47, setting

$$C_{in}^{a,b'} = \theta_{in}^c P_i^a \tilde{c}_i^{a,b'} + \sum_{n'\in\mathcal{N}} \frac{\theta_{inn'}^\iota}{1-\theta_{in'}^\pi} P_{in'}^a \tilde{c}_{in'}^{a,b'}$$
$$\text{where} \quad \tilde{c}_i^{a,b'} = (\psi_i^{a,b'} P_i^a)^{-\rho}$$
$$\tilde{c}_{in}^{a,b'} = \left(\psi_i^{a,b'} P_{in}^a/\tilde{\nu}_{in}\right)^{-1/\theta_{in}^\pi}$$
$$\tilde{\nu}_{in} = \nu_{in}\phi_{in}(1-\theta_{in}^\pi)^{\theta_{in}^\pi}$$

(c) Compute sectoral composite spending $X_{in}^{a,b'}$ according to Equation 50, setting

$$\begin{aligned} X^{a,b'} &= [\mathrm{id} - \mathcal{X}^a]^{-1} C^{a,b'} \\ \text{where } [X^{a,b'}]_{in} &= X_{in}^{a,b'} \\ [C^{a,b'}]_{in} &= C_{in}^{a,b'} \end{aligned}$$

(d) Compute trade tax revenues $T_i^{a,b'}$ according to Equation 50 and 52, setting

$$T_i^{a,b'} = \sum_{o \in \mathcal{I}} \sum_{n \in \mathcal{N}} \frac{\tau_{oin}^m}{1 + \tau_{oin}^m} \zeta_{oin}^a X_{in}^{a,b'} - \sum_{j \in \mathcal{I}} \sum_{n \in \mathcal{N}} \frac{\tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn}^a X_{jn}^{a,b'}$$

(e) Compute households' trade-tax inclusive incomes

$$I_i^{a,b'} = w_i^a \ell_i + \sum_{n \in \mathcal{N}} r_{in}^a k_{in} + T_i^{a,b'}$$

(f) For some numerical parameter $\mathrm{tol}_{\mathrm{inner},2}$, test whether

$$\max_{i \in \mathcal{I}} |\log I_i^{a,b'} - \log \sum_{n \in \mathcal{N}} C_{in}^{a,b'}| < \mathrm{tol}_{\mathrm{inner},2}$$

i. If not, update households' Lagrange multipliers according to

$$\psi_i^{a,b'+1} = \exp\left[\log \psi_i^{a,b'} + \mathrm{step}_{\mathrm{inner},2} \left(\log \sum_{n \in \mathcal{N}} C_{in}^{a,b'} - \log I_i^{a,b'}\right)\right]$$

for some numerical parameter $\mathrm{step}_{\mathrm{inner},2} > 0$. Increase b' by one and return to step 4.b.

ii. If so, set $X_{in}^a \equiv X_{in}^{a,b'}$ and proceed to step 5.

v. Define $\widetilde{w}_i^a$ and $\widetilde{r}_{in}^a$ using Equations 54, and 55, setting

$$\begin{aligned} \widetilde{w}_i^a &= \frac{1}{\ell_i} \sum_{n \in \mathcal{N}} \theta_{in}^\ell \sum_{j \in \mathcal{I}} Y_{ijn}^a \\ \widetilde{r}_{in}^a &= \frac{1}{k_{in}} \theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}^a \\ \text{where} \quad Y_{ijn}^a &= \frac{1 + \tau_{ijn}^x}{1 + \tau_{ijn}^m} \zeta_{ijn}^a X_{jn}^a \end{aligned}$$

vi. For some numerical parameter $\text{tol}_{\text{outer}}$, test whether

$$\max_{i\in\mathcal{I}} \left|\log \widetilde{w}_i^a - \log w_i^a\right| < \text{tol}_{\text{outer}}$$
$$\text{and} \quad \max_{i\in\mathcal{I},n\in\mathcal{N}} \left|\log \widetilde{r}_{in}^a - \log r_{in}^a\right| < \text{tol}_{\text{outer}}$$

(a) If not, update factor prices according to

$$\begin{aligned} w_i^{a+1} &= \frac{\xi}{\sum_{j\in\mathcal{I}} \hat{w}_j^a \ell_j + \sum_{j\in\mathcal{I},n'\in\mathcal{N}} \hat{r}_{jn'}^a k_{jn'}} \hat{w}_i^a \\ r_{in}^{a+1} &= \frac{\xi}{\sum_{j\in\mathcal{I}} \hat{w}_j^a \ell_j + \sum_{j\in\mathcal{I},n'\in\mathcal{N}} \hat{r}_{jn'}^a k_{jn'}} \hat{r}_{in}^a \\ \text{where} \quad \hat{w}_i^a &= \exp\left[\log w_i^a + \text{step}_{\text{outer}} \left(\log \widetilde{w}_i^a - \log w_i^a\right)\right] \\ \hat{r}_{in}^a &= \exp\left[\log r_{in}^a + \text{step}_{\text{outer}} \left(\log \widetilde{r}_{in}^a - \log r_{in}^a\right)\right] \end{aligned}$$

for some numerical parameter $\text{step}_{\text{outer}} > 0$. Increase a by one and return to step 2.

(b) If so, stop and set $w_i = w_i^a$, $r_{in} = r_{in}^a$, $p_{in} \equiv p_{in}^a$, $p_{ijn} \equiv p_{ijn}^a$, $X_{in} \equiv X_{in}^a$, $X_{ijn} \equiv \zeta_{ijn}^a X_{in}^a$, $C_{in} = C_{in}^a$, and—using Equation 56—

$$r_{ijn} = \frac{X_{jn}}{k_{ijn}} \frac{1}{\sigma - 1} \mathbb{1}_{\theta_{ijn}>0} \left(\frac{(k_{ijn}/K_{jn})(p_{ijn})^{1-\sigma}}{(p_{jn})^{1-\sigma}} - \frac{(\theta_{ijn})^{(1-\sigma)/(\bar{\sigma}-\sigma)} (k_{ijn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}}{(K_{jn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}} \right)$$

E.2.3 Numerical algorithm for equilibrium of peacetime period given Home trade quantities

We fix Home's import quantities $\{m_{i\neq H,H,n}\}$ and export quantities $\{m_{H,d\neq H,n}\}$ and solve for the equilibrium in the rest of the world using the equilibrium conditions stated in Appendix E.1.3.

We iterate over wages, production capital rental rates, Home export prices, and sectoral composite prices in outer and inner loops indexed by a and b, respectively. We let $\{w_i^a\}$, $\{r_{in}^a\}$, and $\{p_{H,j\neq H,n}^a\}$ denote wages, production capital rental rates, and Home export prices, respectively, in the a^{th} iteration of the outer loop; and we let $\left\{p_{in}^{a,b}\right\}_{i\neq H}$ denote sectoral composite prices in the a^{th} outer loop and b^{th} inner loop. We proceed as follows.

i. At $a = 1$, initialize all $w_i^{a=1} = 1$, $r_{in}^{a=1} = 1$, and $p_{Hjn}^a = 1$ for all $i, j \neq H$.

ii. Solve for sectoral composite prices given wages as follows.

(a) For $i \neq H$, at $b = 1$, initialize all $p_{in}^{a,b=1} = 1$ if $a = 1$ or $p_{in}^{a,b=1} = p_{in}^{a-1,b_{\text{final}}}$ if $a > 1$.

(b) Define $\widetilde{p}_{in}^{a,b}$ for $i \neq H$ using Equation 32, setting

$$p_{ijn}^{a,b} = \frac{1+\tau_{ijn}^{m}}{1+\tau_{ijn}^{x}}(\phi_{ijn})^{-1}(w_i^a)^{\theta_{in}^{\ell}}(r_{in}^a)^{\theta_{in}^{k}}\prod_{n'\in\mathcal{N}}(p_{in'}^{a,b})^{\theta_{in'n}^{x}}$$

$$\widetilde{p}_{in}^{a,b} = \left[\sum_{o\in\mathcal{I}}\theta_{oin}(p_{oin}^{a,b})^{1-\bar{\sigma}}\right]^{1/(1-\bar{\sigma})}$$

(c) For some numerical parameter $\text{tol}_{\text{inner}}$, test whether

$$\max_{i\neq H, n\in\mathcal{N}} |\log\widetilde{p}_{in}^{a,b} - \log p_{in}^{a,b}| < \text{tol}_{\text{inner}}$$

i. If not, update prices for $i \neq H$ according to

$$p_{in}^{a,b+1} = \exp\left[\log p_{in}^{a,b} + \text{step}_{\text{inner}}\left(\log\widetilde{p}_{in}^{a,b} - \log p_{in}^{a,b}\right)\right]$$

for some numerical parameter $\text{step}_{\text{inner}} > 0$. Increase b by one and return to step 2.b.

ii. If so, set $p_{in}^a \equiv p_{in}^{a,b}$, $p_{ijn}^a \equiv p_{ijn}^{a,b}$, for $i \neq H$ and proceed to step 3.

iii. Compute sectoral composite spending X_{in}^a for $i \neq H$ according to Equations 57, 58 and 59, setting

$$X_{-H}^a = \left[\text{id} - \mathcal{X}_{-H}^a\right]^{-1} X_{-H0}^a$$

where $[X_{-H}^a]_{in} = X_{-H,in}^a$

$$\begin{aligned}
[X_{-H0}^a]_{in} &= \theta_{in}^c\left(w_i^a\ell_i + \Gamma_i^a\right) + \theta_{in}^c\sum_{n'\in\mathcal{N}}\frac{\tau_{Hin'}^m}{1+\tau_{Hin'}^m}p_{Hin'}^a m_{Hin'} \\
&\quad + \theta_{in}^c\sum_{n'\in\mathcal{N}}\left(-\frac{\tau_{iHn'}^x}{1+\tau_{iHn'}^m} + \left(\theta_{inn'} + (\theta_{inn'}^{\iota} + \theta_{in}^c\theta_{in'}^{\pi})\theta_{in'}^k\right)\frac{1+\tau_{iHn'}^x}{1+\tau_{iHn'}^m}\right)p_{iHn'}^a m_{iHn'}^a \\
[\mathcal{X}_{-H}]_{in,jn'} &= \mathbb{1}_{i=j}\theta_{in}^c\sum_{o\neq H}\frac{\tau_{ojn'}^m}{1+\tau_{ojn'}^m}\zeta_{ojn'}^a \\
&\quad + \left(\left(\theta_{inn'}^x + (\theta_{inn'}^{\iota} + \theta_{in}^c\theta_{in'}^{\pi})\theta_{in'}^k\right)\frac{1+\tau_{ijn'}^x}{1+\tau_{ijn'}^m} - \theta_{in}^c\frac{\tau_{ijn'}^x}{1+\tau_{ijn'}^m}\right)\zeta_{ijn'}^a \\
\Gamma_i^a &= \frac{1}{|\mathcal{I}|-1}\sum_{n\in\mathcal{N}}\left[\sum_{d\neq H}\frac{p_{Hdn}^a}{1+\tau_{Hdn}^m}m_{Hdn} - \sum_{o\neq H}\frac{p_{oHn}^a}{1+\tau_{oHn}^m}m_{oHn}\right] \\
\zeta_{ijn}^a &= \frac{\theta_{ijh}(p_{ijn}^a)^{1-\bar{\sigma}}}{(p_{jn}^a)^{1-\bar{\sigma}}}
\end{aligned}$$

iv. Define $\widetilde{w}_i^a$ and $\tilde{r}_{in}^a$ for $i \neq H$ using Equations 58, 61, 61, 44, and 45, setting

$$\widetilde{w}_i^a = \frac{1}{\ell_i} \sum_{n \in \mathcal{N}} \theta_{in}^{\ell} \sum_{j \in \mathcal{I}} Y_{ijn}^a$$

$$\tilde{r}_{in}^a = (\phi_{in})^{-1} \prod_{n' \in \mathcal{N}} (p_{in'})^{\theta_{in'n}^{\iota}} \left(\theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}^a \right)^{\theta_{in}^{\pi}}$$

$$\text{where} \quad Y_{ijn}^a = \begin{cases} \frac{1+\tau_{ijn}^x}{1+\tau_{ijn}^m} \zeta_{ijn}^a X_{jn}^a & \text{for } j \neq H \\ \frac{1+\tau_{ijn}^x}{1+\tau_{ijn}^m} p_{iHn}^a m_{iHn}^a & \text{for } j = H \end{cases}$$

Similarly, for $i \neq H$, set

$$\tilde{p}_{Hin}^a = \frac{1}{m_{Hin}} \zeta_{Hin}^a X_{in}^a$$

Without loss of generality, set

$$\widetilde{w}_H = \widetilde{r}_{Hn} = \widetilde{p}_{HHn} = 1$$

v. For some numerical parameter $\text{tol}_{\text{outer}}$, test whether

$$\max_{i \neq H} \left| \log \widetilde{w}_i^a - \log w_i^a \right| < \text{tol}_{\text{outer}}$$

$$\max_{i \neq H, n \in \mathcal{N}} \left| \log \widetilde{r}_{in}^a - \log r_{in}^a \right| < \text{tol}_{\text{outer}}$$

$$\text{and} \quad \max_{i \neq H, n \in \mathcal{N}} \left| \log \widetilde{p}_{Hin}^a - \log p_{Hin}^a \right| < \text{tol}_{\text{outer}}$$

(a) If not, update wages, capital rental rates, and import prices for $i \neq H$ according to

$$w_i^{a+1} = \frac{\xi}{\sum_{j \neq i} \hat{w}_j^a \ell_j + \sum_{j \neq i, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}^a} \hat{w}_i^a$$

$$r_{in}^{a+1} = \frac{\xi}{\sum_{j \neq i} \hat{w}_j^a \ell_j + \sum_{j \neq i, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}^a} \hat{r}_{in}^a$$

$$p_{Hin}^{a+1} = \frac{\xi}{\sum_{j \neq i} \hat{w}_j^a \ell_j + \sum_{j \neq i, n' \in \mathcal{N}} \hat{r}_{jn'}^a k_{jn'}^a} \hat{p}_{Hin}^a$$

$$\text{where} \quad \hat{w}_i^a = \exp\left[\log w_i^a + \text{step}_{\text{outer}} \left(\log \widetilde{w}_i^a - \log w_i^a\right)\right]$$

$$\hat{r}_{in}^a = \exp\left[\log r_{in}^a + \text{step}_{\text{outer}} \left(\log \widetilde{r}_{in}^a - \log r_{in}^a\right)\right]$$

$$\hat{p}_{Hin}^a = \exp\left[\log p_{Hin}^a + \text{step}_{\text{outer}} \left(\log \widetilde{p}_{Hin}^a - \log p_{Hin}^a\right)\right]$$

$$k_{in}^a = \left(\theta_{in}^k \sum_{j \in \mathcal{I}} Y_{ijn}^a \right) / r_{in}^a$$

for some numerical parameter $\text{step}_{\text{outer}} > 0$. Increase a by one and return to step 2.

(b) If so, stop and, for $i \neq H$, set $w_i = w_i^a$, $r_{in} = r_{in}^a$, $p_{in} \equiv p_{in}^a$, $p_{ijn} \equiv p_{ijn}^a$, $X_{in} \equiv X_{in}^a$, $X_{i,j \neq H,n} \equiv \zeta_{ijn}^a X_{in}^a$, $X_{iHn} \equiv p_{iHn}^a m_{iHn}$, $k_{i \neq H,n} = k_{in}^a$, and—using Equations 30, 36, 37, and the production function for production capital in Section 4.1— set, for $j \neq H$,

$$k_{ijn} = \begin{cases} \frac{(\theta_{ijn})^{\sigma/\bar{\sigma}}(X_{ijn}/p_{ijn})^{(\bar{\sigma}-\sigma)/\bar{\sigma}}}{\left[\sum_{o \in \mathcal{I}:\ \theta_{ojn}>0}(\theta_{ojn})^{1/\bar{\sigma}}(X_{ojn}/p_{ojn})^{(\bar{\sigma}-1)/\bar{\sigma}}\right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}} & \text{if } \theta_{ijn} > 0 \\ 0 & \text{if } \theta_{ijn} = 0 \end{cases}$$

and for $i \neq H$,

$$C_i = \sum_{n \in \mathcal{N}} X_{in} - \sum_{n,n' \in \mathcal{N}} \left(\theta_{inn'}^x + \theta_{in'}^k \theta_{inn'}^\iota\right) \sum_{j \in \mathcal{I}} Y_{ijn'}^a$$

$$\iota_{in'n} = \theta_{in'n}^\iota r_{in} k_{in} / p_{in'}$$

Note that setting $k_{ijh} = 0$ when $\theta_{ijh} = 0$ is without loss of generality as k_{ijh} never affects production in such cases.

E.2.4 Numerical algorithm for equilibrium of conflict period given Home trade quantities

We fix Home's import quantities $\{m_{i \neq H,H,n}\}$ and export quantities $\{m_{H,d \neq H,n}\}$ and solve for the equilibrium in the rest of the world using the equilibrium conditions stated in Appendix E.1.4.

We iterate over wages, production capital rental rates, Home export prices, sectoral composite prices, and Lagrange multipliers on household budget constraints in one outer and two successive inner loops indexed by a, b, and b', respectively. We let $\{w_i^a\}$, $\{r_{in}^a\}$, and $\left\{p_{H,j\neq H,n}^a\right\}$ denote wages, production capital rental rates, and Home export prices, respectively, in the a^{th} iteration of the outer loop. We let $\left\{p_{in}^{a,b}\right\}$ denote sectoral composite prices in the a^{th} iteration of the outer loop and b^{th} iteration of the first inner loop. We let $\left\{\psi_i^{a,b'}\right\}$ denote budget constraint Lagrange multipliers in the a^{th} iteration of the outer loop and $(b')^{th}$ iteration of the second inner loop. We proceed as follows.

i. At $a = 1$, initialize all $w_i^{a=1} = 1$, $r_{in}^{a=1} = 1$, and $p_{Hjn}^a = 1$ for all $i, j \neq H$.

ii. Solve for sectoral composite prices given factor prices as follows.

(a) For $i \neq H$, at $b = 1$, initialize all $p_{in}^{a,b=1} = 1$ if $a = 1$ or $p_{in}^{a,b=1} = p_{in}^{a-1,b_{\text{final}}}$ if $a > 1$.

(b) Define $\widetilde{p}_{in}^{a,b}$ for $i \neq H$ using Equation 46, setting

$$
\begin{aligned}
p_{ijn}^{a,b} &= \frac{1+\tau_{ijn}^m}{1+\tau_{ijn}^x} (\phi_{ijn})^{-1} (w_i^a)^{\theta_{in}^\ell} (r_{in}^a)^{\theta_{in}^k} \prod_{n' \in \mathcal{N}} (p_{in'}^{a,b})^{\theta_{in'n}^x} \\
\widetilde{p}_{in}^{a,b} &= \left[\sum_{o \in \mathcal{I}:\ \theta_{oin} > 0} (k_{oin}/K_{in}) (p_{oin}^{a,b})^{1-\sigma} \right]^{1/(1-\sigma)} \\
\text{where} \quad K_{in} &= \left[\sum_{o \in \mathcal{I}:\ \theta_{oin} > 0} (\theta_{oin})^{(1-\sigma)/(\bar{\sigma}-\sigma)} \, (k_{oin})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)} \right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}
\end{aligned}
$$

(c) For some numerical parameter $\text{tol}_{\text{inner},1}$, test whether

$$
\max_{i \neq H, n \in \mathcal{N}} |\log \widetilde{p}_{in}^{a,b} - \log p_{in}^{a,b}| < \text{tol}_{\text{inner},1}
$$

i. If not, update prices for $i \neq H$ according to

$$
p_{in}^{a,b+1} = \exp \left[\log p_{in}^{a,b} + \text{step}_{\text{inner},1} \left(\log \widetilde{p}_{in}^{a,b} - \log p_{in}^{a,b} \right) \right]
$$

for some numerical parameter $\text{step}_{\text{inner},1} > 0$. Increase b by one and return to step 2.b.

ii. If so, set $p_{in}^a \equiv p_{in}^{a,b}$, $p_{ijn}^a \equiv p_{ijn}^{a,b}$, for $i \neq H$ and proceed to step 3.

iii. Compute true-consumption and investment-as-consumption price indices P_i^a and P_{in}^a according to Equation 47, setting

$$
\begin{aligned}
P_i^a &= \prod_{n \in \mathcal{N}} (p_{in}^a)^{\theta_{in}^c} \\
P_{in}^a &= \prod_{n' \in \mathcal{N}} (p_{in'}^a)^{\theta_{in'n}^\iota/(1-\theta_{in}^\pi)}
\end{aligned}
$$

Compute the map between consumption and aggregate expenditures on sectoral composites by non-Home countries following Equation 51, setting

$$
\begin{aligned}
[\mathcal{X}_{-H}^a]_{in,jn'} &= \theta_{inn'}^x \frac{1+\tau_{ijn'}^x}{1+\tau_{ijn'}^m} \zeta_{ijn'}^a \\
\text{where} \quad \zeta_{ijn}^a &= \frac{(k_{ijn}/K_{jn})(p_{ijn}^a)^{1-\sigma}}{(p_{jn}^a)^{1-\sigma}}
\end{aligned}
$$

for $i, j \neq H$. And compute international transfers Γ_i^a following Equation 63

$$\Gamma_i^a = \frac{1}{|\mathcal{I}|-1} \sum_{n \in \mathcal{N}} \left[\sum_{d \neq H} \frac{p_{Hdn}^a}{1+\tau_{Hdn}^m} m_{Hdn} - \sum_{o \neq H} \frac{p_{oHn}^a}{1+\tau_{oHn}^m} m_{oHn} \right]$$

iv. Solve for spending on sectoral composites given prices as follows.

(a) At $b' = 1$, for all $i \neq H$, initialize all $\psi_i^{a,b'} = \Psi_i$ if $a = 1$ or $\psi_i^{a,b'} = \psi_i^{a-1,b_{final}}$ if $a > 1$.

(b) Compute consumption expenditure for $i \neq H$ according to Equation 47, setting

$$\begin{aligned} C_{in}^{a,b'} &= \theta_{in}^c P_i^a \tilde{c}_i^{a,b'} + \sum_{n' \in \mathcal{N}} \frac{\theta_{inn'}^\iota}{1-\theta_{in'}^\pi} P_{in'}^a \tilde{c}_{in'}^{a,b'} \\ \text{where} \quad \tilde{c}_i^{a,b'} &= (\psi_i^{a,b'} P_i^a)^{-\rho} \\ \tilde{c}_{in}^{a,b'} &= \left(\psi_i^{a,b'} P_{in}^a / \tilde{\nu}_{in} \right)^{-1/\theta_{in}^\pi} \\ \tilde{\nu}_{in} &= \nu_{in} \phi_{in} (1-\theta_{in}^\pi)^{\theta_{in}^\pi} \end{aligned}$$

(c) Compute sectoral composite spending $X_{in}^{a,b'}$ for $i \neq H$ according to Equation 62, setting

$$\begin{aligned} X_{-H}^{a,b'} &= \left[\text{id} - \mathcal{X}_{-H}^a \right]^{-1} X_{-H0}^{a,b'} \\ \text{where } [X_{-H}^{a,b'}]_{in} &= X_{in}^{a,b'} \\ [X_{-H0}^{a,b'}]_{in} &= C_{in}^{a,b'} + \sum_{n' \in \mathcal{N}} \theta_{inn'}^x \frac{1+\tau_{iHn'}^x}{1+\tau_{iHn'}^m} p_{iHn'}^a m_{iHn'} \end{aligned}$$

(d) Compute trade tax revenues $T_i^{a,b'}$ for $i \neq H$ according to Equation 50 and 52, setting

$$\begin{aligned} T_i^{a,b'} = & \sum_{n \in \mathcal{N}} \frac{\tau_{Hin}^m}{1+\tau_{Hin}^m} p_{Hin}^a m_{Hin} + \sum_{o \neq H} \sum_{n \in \mathcal{N}} \frac{\tau_{oin}^m}{1+\tau_{oin}^m} \zeta_{oin}^a X_{in}^{a,b'} \\ & - \sum_{n \in \mathcal{N}} \frac{\tau_{iHn}^x}{1+\tau_{iHn}^m} p_{iHn}^a m_{iHn} - \sum_{j \neq H} \sum_{n \in \mathcal{N}} \frac{\tau_{ijn}^x}{1+\tau_{ijn}^m} \zeta_{ijn}^a X_{jn}^{a,b'} \end{aligned}$$

(e) Compute households' trade-tax inclusive incomes for $i \neq H$ (as in Equation 63)

$$I_i^{a,b'} = w_i^a \ell_i + \sum_{n \in \mathcal{N}} r_{in}^a k_{in} + T_i^{a,b'} + \Gamma_i^a$$

(f) For some numerical parameter $\text{tol}_{\text{inner},2}$, test whether

$$\max_{i \neq H} |\log I_i^{a,b'} - \log \sum_{n \in \mathcal{N}} C_{in}^{a,b'}| < \text{tol}_{\text{inner},2}$$

i. If not, update households' Lagrange multipliers for $i \neq H$ according to

$$\psi_i^{a,b'+1} = \exp\left[\log \psi_i^{a,b'} + \text{step}_{\text{inner},2}\left(\log \sum_{n \in \mathcal{N}} C_{in}^{a,b'} - \log I_i^{a,b'}\right)\right]$$

for some numerical parameter $\text{step}_{\text{inner},2} > 0$. Increase b' by one and return to step 4.b.

ii. If so, set $X_{in}^a \equiv X_{in}^{a,b'}$ for $i \neq H$ and proceed to step 5.

v. Define $\widetilde{w}_i^a$ and $\widetilde{r}_{in}^a$ for $i \neq H$ using Equations 54, and 55, setting

$$\widetilde{w}_i^a = \frac{1}{\ell_i} \sum_{n \in \mathcal{N}} \theta_{in}^{\ell} \sum_{j \in \mathcal{I}} Y_{ijn}^a$$

$$\widetilde{r}_{in}^a = \frac{1}{k_{in}} \theta_{in}^{k} \sum_{j \in \mathcal{I}} Y_{ijn}^a$$

$$\text{where} \quad Y_{ijn}^a = \begin{cases} \frac{1+\tau_{ijn}^x}{1+\tau_{ijn}^m} \zeta_{ijn}^a X_{jn}^a & \text{for } j \neq H \\ \frac{1+\tau_{ijn}^x}{1+\tau_{ijn}^m} p_{iHn}^a m_{iHn}^a & \text{for } j = H \end{cases}$$

Similarly, for $i \neq H$, set

$$\widetilde{p}_{Hin}^a = \frac{1}{m_{Hin}} \zeta_{Hin}^a X_{in}^a$$

Without loss of generality, set

$$\widetilde{w}_H = \widetilde{r}_{Hn} = \widetilde{p}_{HHn} = 1$$

vi. For some numerical parameter $\text{tol}_{\text{outer}}$, test whether

$$\max_{i \neq H} |\log \widetilde{w}_i^a - \log w_i^a| < \text{tol}_{\text{outer}}$$

$$\max_{i \neq H, n \in \mathcal{N}} |\log \widetilde{r}_{in}^a - \log r_{in}^a| < \text{tol}_{\text{outer}}$$

$$\text{and} \quad \max_{i \neq H, n \in \mathcal{N}} |\log \widetilde{p}_{Hin}^a - \log p_{Hin}^a| < \text{tol}_{\text{outer}}$$

(a) If not, update factor prices according to

$$\begin{aligned}
w_i^{a+1} &= \frac{\xi}{\sum_{j\neq i}\hat{w}_j^a \ell_j + \sum_{j\neq i, n'\in\mathcal{N}} \hat{r}_{jn'}^a k_{jn'}^a}\hat{w}_i^a \\
r_{in}^{a+1} &= \frac{\xi}{\sum_{j\neq i}\hat{w}_j^a \ell_j + \sum_{j\neq i, n'\in\mathcal{N}} \hat{r}_{jn'}^a k_{jn'}^a}\hat{r}_{in}^a \\
p_{Hin}^{a+1} &= \frac{\xi}{\sum_{j\neq i}\hat{w}_j^a \ell_j + \sum_{j\neq i, n'\in\mathcal{N}} \hat{r}_{jn'}^a k_{jn'}^a}\hat{p}_{Hin}^a \\
\text{where}\quad \hat{w}_i^a &= \exp\left[\log w_i^a + \text{step}_{\text{outer}}\left(\log \widetilde{w}_i^a - \log w_i^a\right)\right] \\
\hat{r}_{in}^a &= \exp\left[\log r_{in}^a + \text{step}_{\text{outer}}\left(\log \widetilde{r}_{in}^a - \log r_{in}^a\right)\right] \\
\hat{p}_{Hin}^a &= \exp\left[\log p_{Hin}^a + \text{step}_{\text{outer}}\left(\log \widetilde{p}_{Hin}^a - \log p_{Hin}^a\right)\right]
\end{aligned}$$

for some numerical parameter $\text{step}_{\text{outer}} > 0$. Increase a by one and return to step 2.

(b) If so, stop and, for $i \neq H$, set $w_i = w_i^a$, $r_{in} = r_{in}^a$, $p_{in} \equiv p_{in}^a$, $p_{ijn} \equiv p_{ijn}^a$, $X_{i\neq H,n} \equiv X_{in}^a$, $X_{i,j\neq H,n} \equiv \zeta_{ijn}^a X_{in}^a$, $X_{iHn} \equiv p_{iHn}^a m_{iHn}$, and—using Equation 56—set

$$r_{ijn} = \frac{X_{jn}}{k_{ijn}}\frac{1}{\sigma-1}\mathbb{1}_{\theta_{ijn}>0}\left(\frac{(k_{ijn}/K_{jn})(p_{ijn})^{1-\sigma}}{(p_{jn})^{1-\sigma}} - \frac{(\theta_{ijn})^{(1-\sigma)/(\bar{\sigma}-\sigma)}(k_{ijn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}}{(K_{jn})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}}\right)$$

for $j \neq H$.

E.2.5 Numerical algorithm for equilibrium of conflict period given own trade quantities

We now fix a country i with import quantities $\{m_{o\neq i,i,n}\}$, export quantities $\{m_{i,d\neq i,n}\}$, production capital $\{k_{in}\}$, and relationship capital $\{k_{ijn}\}$. We assume these imports and exports are simply an endowment of the representative consumer (a negative endowment in the case of exports) and solve for the equilibrium in the conflict period taking them as given.

Our algorithm iterates over wages, production capital rental rates, imported good prices, sectoral composite prices, and Lagrange multipliers on the household budget constraint in one outer and two successive inner loops indexed by a, b, and b', respectively. We let w_i^a, $\{r_{in}^a\}$, and $\left\{p_{o\neq i,i,n}^a\right\}$ denote wages, production capital rental rates, and imported good prices, respectively, in the a^{th} iteration of the outer loop. We let $\left\{p_{in}^{a,b}\right\}$ denote sectoral composite prices in the a^{th} iteration of the outer loop and b^{th} iteration of the first inner loop. We let $\psi_i^{a,b'}$ denote the budget constraint Lagrange multiplier in the a^{th} iteration of the outer loop and $(b')^{th}$ iteration of the second inner loop. Although variables are indexed by "i", we compute them only for the fixed country i of interest.

i. At $a = 1$, initialize all $w_i^{a=1} = r_{in}^{a=1} = p_{o\neq i,i,n}^{a=1} = 1$.

ii. Solve for sectoral composite prices given factor and import prices as follows.

(a) At $b = 1$, initialize all $p_{in}^{a,b=1} = 1$ if $a = 1$ or $p_{in}^{a,b=1} = p_{in}^{a-1,b_{\text{final}}}$ if $a > 1$.

(b) Define $\widetilde{p}_{in}^{a,b}$ using Equation 46, setting

$$p_{iin}^{a,b} = (\phi_{iin})^{-1}(w_i^a)^{\theta_{is}^{\ell}}(r_{in}^a)^{\theta_{is}^k}\prod_{n'\in\mathcal{N}}(p_{in'}^{a,b})^{\theta_{in'n}^x}$$

$$\widetilde{p}_{in}^{a,b} = \left[\mathbb{1}_{\theta_{iin}>0}(k_{iin}/K_{in})(p_{iin}^{a,b})^{1-\sigma} + \sum_{o\neq i:\ \theta_{oin}>0}(k_{oin}/K_{in})(p_{oin}^a)^{1-\sigma}\right]^{1/(1-\sigma)}$$

$$\text{where}\quad K_{in} = \left[\sum_{o\in\mathcal{I}:\ \theta_{oin}>0}(\theta_{oin})^{(1-\sigma)/(\bar{\sigma}-\sigma)}\,(k_{oin})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}\right]^{(\bar{\sigma}-\sigma)/(\bar{\sigma}-1)}$$

(c) For some numerical parameter $\text{tol}_{\text{inner},1}$, test whether

$$\max_{n\in\mathcal{N}}|\log\widetilde{p}_{in}^{a,b} - \log p_{in}^{a,b}| < \text{tol}_{\text{inner},1}$$

i. If not, update prices according to

$$p_{in}^{a,b+1} = \exp\left[\log p_{in}^{a,b} + \text{step}_{\text{inner},1}\left(\log\widetilde{p}_{in}^{a,b} - \log p_{in}^{a,b}\right)\right]$$

for some numerical parameter $\text{step}_{\text{inner},1} > 0$. Increase b by one and return to step 2.b.

ii. If so, set $p_{in}^a \equiv p_{in}^{a,b}$, $p_{iin}^a \equiv p_{iin}^{a,b}$, and proceed to step 3.

iii. Compute true-consumption and investment-as-consumption price indices P_i^a and P_{in}^a according to Equation 47, setting

$$P_i^a = \prod_{n\in\mathcal{N}}(p_{in}^a)^{\theta_{in}^c}$$

$$P_{in}^a = \prod_{n'\in\mathcal{N}}(p_{in'}^a)^{\theta_{in'n}^{\iota}/(1-\theta_{in}^{\pi})}$$

iv. Solve for spending on sectoral composites given prices as follows.

(a) At $b' = 1$, initialize all $\psi_i^{a,b'} = \Psi_i$ if $a = 1$ or $\psi_i^{a,b'} = \psi_i^{a-1,b_{final}}$ if $a > 1$.

(b) Compute consumption expenditure according to Equation 47, setting

$$\begin{aligned} C_{in}^{a,b'} &= \theta_{in}^{c} P_i^a \tilde{c}_i^{a,b'} + \sum_{n' \in \mathcal{N}} \frac{\theta_{inn'}^{\iota}}{1-\theta_{in'}^{\pi}} P_{in'}^a \tilde{c}_{in'}^{a,b'} \\ \text{where} \quad \tilde{c}_i^{a,b'} &= (\psi_i^{a,b'} P_i^a)^{-\rho} \\ \tilde{c}_{in}^{a,b'} &= \left(\psi_i^{a,b'} P_{in}^a / \tilde{\nu}_{in}\right)^{-1/\theta_{in}^{\pi}} \\ \tilde{\nu}_{in} &= \nu_{in} \phi_{in} (1-\theta_{in}^{\pi})^{\theta_{in}^{\pi}} \end{aligned}$$

(c) Compute the representative household's income inclusive of the value of its endowment of imports less the cost—implied by Equation 46—of producing its negative endowment of exports

$$I_i^{a,b'} = w_i^a \ell_i + \sum_{n \in \mathcal{N}} r_{in}^a k_{in} + \sum_{j \neq i} \sum_{n \in \mathcal{N}} \left[p_{jin}^a m_{jin} - p_{iin}^a \frac{\phi_{iin}}{\phi_{ijn}} m_{ijn} \right]$$

(d) For some numerical parameter $\text{tol}_{\text{inner},2}$, test whether

$$|\log I_i^{a,b'} - \log \sum_{n \in \mathcal{N}} C_{in}^{a,b'}| < \text{tol}_{\text{inner},2}$$

i. If not, update the household's Lagrange multiplier according to

$$\psi_i^{a,b'+1} = \exp\left[\log \psi_i^{a,b'} + \text{step}_{\text{inner},2} \left(\log \sum_{n \in \mathcal{N}} C_{in}^{a,b'} - \log I_i^{a,b'} \right) \right]$$

for some numerical parameter $\text{step}_{\text{inner},2} > 0$. Increase b by one and return to step 4.b.

ii. If so, set $C_{in}^a \equiv C_{in}^{a,b'}$ and proceed to step 5.

v. Compute sectoral composite spending X_{in}^a according to Equations 48–50 and the observation that producing an export $m_{i,j \neq i,n}$ requires production costs of $\phi_{iin} p_{iin} m_{ijn} / \phi_{ijn}$

in sector n and country i. Namely, set

$$\begin{aligned}
X^{a,b'} &= [\mathrm{id} - \mathcal{X}^a]^{-1} X_0^{a,b'} \\
\text{where } [X^{a,b'}]_{in} &= X_{in}^{a,b'} \\
[X_0^{a,b'}]_{in} &= C_{in}^{a,b'} + \sum_{j \neq i} \sum_{n' \in \mathcal{N}} \theta^x_{inn'} \frac{\phi_{iin'}}{\phi_{ijn'}} p_{iin'} m_{ijn'} \\
[\mathcal{X}^a]_{n,n'} &= \theta^x_{inn'} \zeta^a_{iin'} \\
\text{where} \quad \zeta^a_{oin} &= \frac{(k_{oin}/K_{in})(p^a_{oin})^{1-\sigma}}{(p^a_{in})^{1-\sigma}}
\end{aligned}$$

vi. Define $\widetilde{w}_i^a$, $\widetilde{r}_{in}^a$, and $\tilde{p}^a_{o\neq i,i,n}$ using Equations 54, 55, and 50, setting

$$\begin{aligned}
\widetilde{w}_i^a &= \frac{1}{\ell_i} \sum_{n \in \mathcal{N}} \theta^\ell_{in} \sum_{j \in \mathcal{I}} Y^a_{ijn} \\
\widetilde{r}_{in}^a &= \frac{1}{k_{in}} \theta^k_{in} \sum_{j \in \mathcal{I}} Y^a_{ijn} \\
\tilde{p}^a_{oin} &= \frac{1}{m_{oin}} \zeta^a_{oin} X^a_{in} \\
\text{where} \quad Y^a_{ijn} &= \begin{cases} \zeta^a_{iin} X^a_{in} & \text{if } i = j \\ \frac{\phi_{iin}}{\phi_{ijn}} p^a_{iin} m_{ijn} & \text{if } i \neq j \end{cases}
\end{aligned}$$

vii. For some numerical parameter $\mathrm{tol}_{\mathrm{outer}}$, test whether

$$\begin{aligned}
|\log \widetilde{w}_i^a - \log w_i^a| &< \mathrm{tol}_{\mathrm{outer}} \\
\text{and} \quad \max_{n \in \mathcal{N}} |\log \widetilde{r}_{in}^a - \log r_{in}^a| &< \mathrm{tol}_{\mathrm{outer}} \\
\text{and} \quad \max_{o \neq i, n \in \mathcal{N}} |\log \tilde{p}^a_{oin} - \log p^a_{oin}| &< \mathrm{tol}_{\mathrm{outer}}
\end{aligned}$$

(a) If not, update factor and import prices according to

$$\begin{aligned}
w_i^{a+1} &= \frac{\ell_i + \sum_{n' \in \mathcal{N}} k_{in'}}{\hat{w}_i^a \ell_i + \sum_{n' \in \mathcal{N}} \hat{r}_{in'}^a k_{in'}} \hat{w}_i^a \\
r_{in}^{a+1} &= \frac{\ell_i + \sum_{n' \in \mathcal{N}} k_{in'}}{\hat{w}_i^a \ell_i + \sum_{n' \in \mathcal{N}} \hat{r}_{in'}^a k_{in'}} \hat{r}_{in}^a \\
p_{o \neq i,i,n}^{a+1} &= \frac{\ell_i + \sum_{n' \in \mathcal{N}} k_{in'}}{\hat{w}_i^a \ell_i + \sum_{n' \in \mathcal{N}} \hat{r}_{in'}^a k_{in'}} \hat{p}_{oin}^a \\
\text{where} \quad \hat{w}_i^a &= \exp\left[\log w_i^a + \text{step}_{\text{outer}} \left(\log \widetilde{w}_i^a - \log w_i^a\right)\right] \\
\hat{r}_{in}^a &= \exp\left[\log r_{in}^a + \text{step}_{\text{outer}} \left(\log \widetilde{r}_{in}^a - \log r_{in}^a\right)\right] \\
\hat{p}_{oin}^a &= \exp\left[\log p_{oin}^a + \text{step}_{\text{outer}} \left(\log \tilde{p}_{oin}^a - \log p_{oin}^a\right)\right]
\end{aligned}$$

for some numerical parameter $\text{step}_{\text{outer}} > 0$. Increase a by one and return to step 2.

(b) If so, stop and set $w_i = w_i^a$, $r_{in} = r_{in}^a$, $p_{in} \equiv p_{in}^a$, $p_{o \neq i,i,n}^w \equiv p_{oin}^a$, $p_{i,j \neq i,n}^w \equiv \frac{\phi_{iin}}{\phi_{ijn}} p_{iin}^a$ $X_{in} \equiv X_{in}^a$, $C_{in} \equiv C_{in}^a$, and—using Equation 56—

$$r_{oin} = \frac{X_{in}}{k_{oin}} \frac{1}{\sigma - 1} \mathbb{1}_{\theta_{oin} > 0} \left(\frac{(k_{oin}/K_{in})(p_{oin})^{1-\sigma}}{(p_{in})^{1-\sigma}} - \frac{(\theta_{oin})^{(1-\sigma)/(\bar{\sigma}-\sigma)} (k_{oin})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}}{(K_{in})^{(\bar{\sigma}-1)/(\bar{\sigma}-\sigma)}} \right)$$

Note that the world prices implicitly assume country i sets its trade taxes to zero.

E.3 Model calibration

This appendix details the model calibration procedure outlined in Section E.3. The calibration takes as inputs cleaned 2017 data on and imputations of tariffs $\tau_{ijn}^{m,data}$ and export subsidies $\tau_{ijn}^{x,data}$; trade shares θ_{ijn}^{data}; labor, capital, and intermediate shares of gross output, $\theta_{in}^{\ell,data}$, $\theta_{in}^{k,data}$, and $\theta_{in'n}^{x,data}$; directed investment shares of capital income, $\theta_{in'n}^{\iota,data}$; consumption shares $\theta_{in}^{c,data}$; and final consumption expenditures C_i^{data}. See Sections D.1 and D.2 for a description of how we construct these variables.

We calibrate the elasticities $\{\sigma, \bar{\sigma}, \rho\}$, preference and production shifters $\{\theta_{in}^c, \nu_{in}, \theta_{in}^\ell, \theta_{in}^k, \theta_{in'n}^x, \theta_{in'n}^\iota, \theta_{ijn}, \phi_{ijn}, \phi_{in}\}$, and labor endowments $\{\ell_j\}$ under two assumptions. First, the data we observe corresponds to the peacetime period in the $\beta \to 0$ limit. This implies that the static equilibrium conditions described in Appendix E.1.1 apply. Second, under free trade, the economy is stationary in the sense that sectoral composite prices are the same in both periods, up to a constant, and first-period consumption and investment spending coincide with second-period consumption-as-consumption and consumption-as-investment spending, up to the same constant.

Our calibration normalizes the units of account of origin-destination-sector goods, labor, and production capital in each country so that $p_{ijn} = w_i = r_{in} = 1$ in the observed peacetime equilibrium for all $i, j \in \mathcal{I}$ and $n \in \mathcal{N}$. It then follows from Equation 32 that the prices of sectoral composites are also equal to one. We then calibrate parameters as follows.

Finally, our calibration saves the data-implied level of nominal world GDP, ξ, and Lagrange multipliers on household budget constraints Ψ_i. Our simulations use ξ to normalize prices so that nominal world GDP is held constant. They use Ψ_i to form a starting guess for the Lagrange multiplier on households' budget constraint in the conflict period.

Elasticities We set σ and $\bar{\sigma}$ to match the evidence on short- and long-run trade elasticities from Boehm et al. (2020). Following Boehm et al. (2024), we set the short-run elasticity to $\sigma = 1.25$ and the long-run elasticity to $\bar{\sigma} = 2$. We set $\rho = 0.53$, the mean estimate for China's elasticity of inter-temporal substitution reported in the meta-analysis of Havranek et al. (2015).

Preference and production shares Since all prices are equal to one, we simply set all production and preference share parameters to their empirical analogs in the 2017 data. Concretely, we set

$$\begin{aligned} \theta^{c}_{in} = \theta^{c,data}_{in}, \quad \theta^{\ell}_{in} = \theta^{\ell,data}_{in}, \quad \theta^{k}_{in} = \theta^{k,data}_{in} \\ \theta^{x}_{in'n} = \theta^{x,data}_{in'n}, \quad \theta^{\iota}_{in'n} = \theta^{\iota,data}_{in'n}, \quad \theta_{ijn} = \theta^{data}_{ijn} \end{aligned}$$

Labor endowments We set countries' labor endowments to the levels required to produce the final consumption vector we observe in the data. To this end, we first compute the vector of country-sector gross outputs using following expression for revenues in terms of consumption, based Equation 37, the price normalization, Equation 38, Equation 36, and Equation 34.

$$Y_{in} = \sum_{j \in \mathcal{I}} \frac{1 + \tau^{x}_{ijn}}{1 + \tau^{m}_{ijn}} \theta_{ijn} \left(\theta_{jn} C_j + \sum_{n' \in \mathcal{N}} \left(\theta^{x}_{jnn'} + \theta^{k}_{jn'} \theta^{\iota}_{jnn'} \right) Y_{jn'} \right)$$

Inverting this expression implies a matrix expression for Y_{in}.

$$
\begin{aligned}
Y &= (\mathrm{id} - \mathcal{M})^{-1}\widetilde{C} \\
\text{where} \quad [\mathcal{M}]_{injn'} &\equiv \frac{1+\tau^x_{ijn}}{1+\tau^m_{ijn}}\theta_{ijn}\left(\theta^x_{jnn'} + \theta^k_{jn'}\theta^\iota_{jnn'}\right) \\
[\widetilde{C}]_{in} &\equiv \sum_{j\in\mathcal{I}} \frac{1+\tau^x_{ijn}}{1+\tau^m_{ijn}}\theta_{ijn}\theta^c_{jn}C_j
\end{aligned}
$$

We define Y^{data}_{in} by

$$
Y^{data} \equiv (\mathrm{id} - \mathcal{M})^{-1}\widetilde{C}
$$

when $\mathcal{M}$ is computed using $\tau^x_{ijn} = \tau^{x,data}_{ijn}$ and $\tau^m_{ijn} = \tau^{m,data}_{ijn}$, and when $\widetilde{C}$ is computed at $C_j = C^{data}_j$. Finally, we set labor endowments to meet the labor demand implied by this level of gross output.

$$
\ell_i = \sum_{n\in\mathcal{N}} \theta^\ell_{in} Y^{data}_{in}
$$

Nominal world GDP We compute nominal world GDP as

$$
\xi = \sum_{i\in\mathcal{I},n\in\mathcal{N}} (\theta^\ell_{in} + \theta^k_{in})Y^{data}_{in}
$$

Production shifters We set the production shifters ϕ_{ijn} and ϕ_{in} to be consistent with our price normalization.

Since all prices are one, Equation 32 implies

$$
\phi_{ijn} = (1+\tau^{m,data}_{ijn})/(1+\tau^{x,data}_{ijn})
$$

for all $i, j \in \mathcal{I}$ and $n \in \mathcal{N}$.

To calibrate ϕ_{in} we first compute the level of capital bills R^{data}_{in} implied by the revenues Y^{data}_{in} consistent with the rest of our calibration, namely

$$
R^{data}_{in} \equiv \theta^k_{in} \sum_{j\in\mathcal{I}} \frac{1+\tau^{x,data}_{ijn}}{1+\tau^{m,data}_{ijn}}\theta_{ijn}\left(\theta_{jn}C^{data}_j + \sum_{n'\in\mathcal{N}}\left(\theta^x_{jnn'} + \theta^k_{jn'}\theta^\iota_{jnn'}\right)Y^{data}_{jn'}\right)
$$

The price normalization and Equation 45 together imply that

$$
\phi_{in} = (R^{data}_{in})^{\theta^\pi_{in}}
$$

Investment-as-consumption preferences Recall we assume that, under free trade, sectoral composite prices are the same in both periods, up to a constant, and first-period consumption and investment spending coincide with second-period consumption-as-consumption and consumption-as-investment spending, up to the same constant.

We therefore simulate the peacetime economy under free trade using the values of all parameters calibrated above. This implies values for sectoral composite prices p_{in}^{FT}, consumption spending C_i^{FT} and investment spending K_{in}^{FT} by the producers of production capital for each sector. Under our time-invariance assumption, Equation 47 implies

$$\begin{aligned} \tilde{\nu}_{in} &= \frac{(P_{in}^{FT})^{1-\theta_{in}^{\pi}}}{(P_i^{FT})^{(\rho-1)/\rho}} \frac{(K_{in}^{FT})^{\theta_{in}^{\pi}}}{(C_i^{FT})^{1/\rho}} \\ \text{where} \quad P_i^{FT} &\equiv \prod_{n\in\mathcal{N}} (p_{in}^{FT})^{\theta_{in}^{c}} \\ P_{in}^{FT} &\equiv \prod_{n'\in\mathcal{N}} (p_{in'}^{FT})^{\theta_{in'n}^{\iota}/(1-\theta_{in}^{\pi})} \end{aligned}$$

for $\tilde{\nu}_{in} \equiv \nu_{in}\phi_{in}(1-\theta_{in}^{\pi})^{\theta_{in}^{\pi}}$. We accordingly set

$$\nu_{in} \equiv \frac{\tilde{\nu}_{in}}{\phi_{in}(1-\theta_{in}^{\pi})^{\theta_{in}^{\pi}}}$$

Lagrange multipliers on household budget constraints Using Equation 47, we set Ψ_i to the implied Lagrange multiplier on the household budget constraint in the simulation of peacetime under free trade, i.e.,

$$\Psi_i \equiv \left(\frac{C_i^{FT}}{(P_i^{FT})^{1-\rho}}\right)^{-1/\rho}$$

F Additional figures

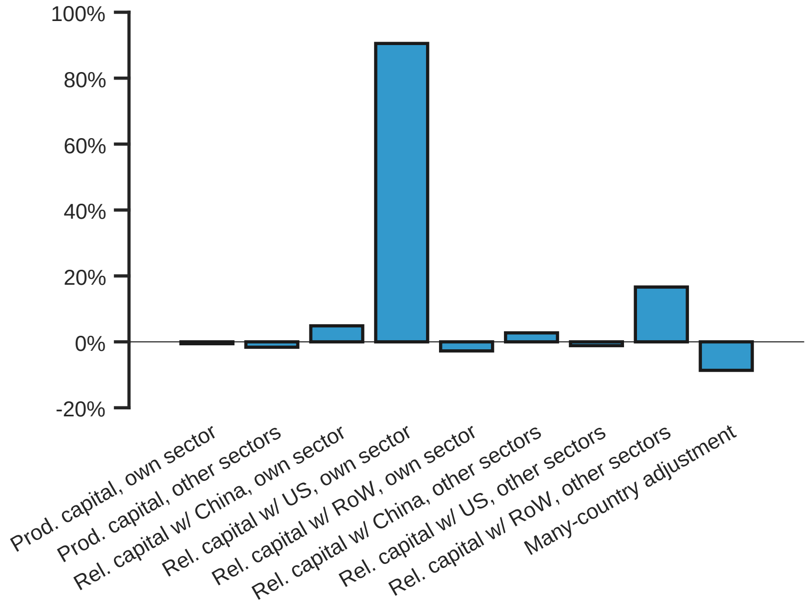

Figure 9: *Shapley decomposition of* $1 - MSE(\hat{x}_i, x_i)/Var(x_i)$*, where* x_i *represents actual US trade taxes (import tariffs and export taxes) on different country-sector pairs* i *and where* $\hat{x}_i$ *is US trade taxes predicted by a subset of taxes' additive components. The first eight components correspond to changes in the manipulability of China's terms of trade due to adjustments in its capital of various types with various trading partners. "Own sector" refers to changes in capital in the same sector as a trade tax applies to; "other sectors" is the complement.*

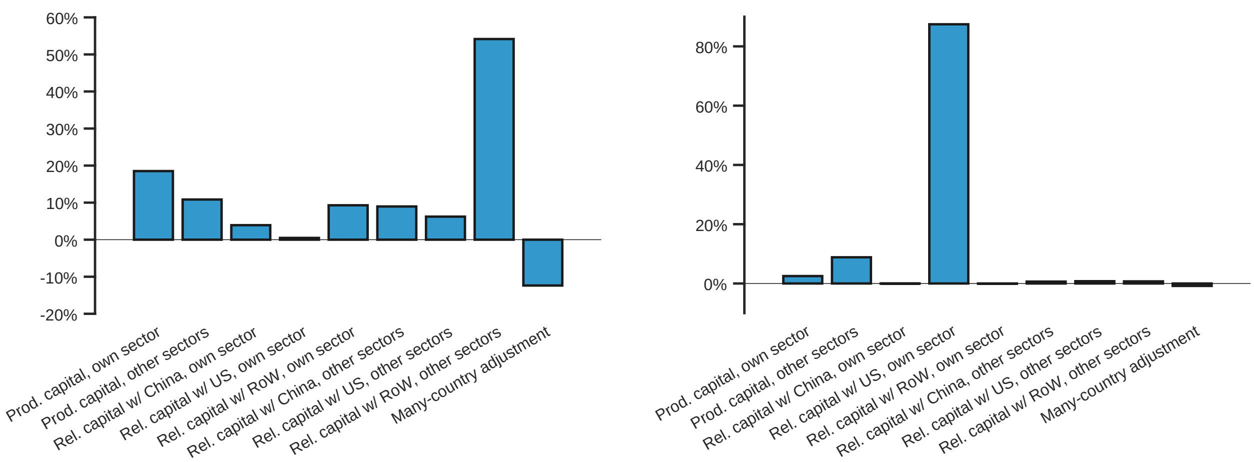

Figure 10: *Left panel: Shapley decomposition of variance in US import tariffs across Chinese sectors. Right panel: Shapley decomposition of variance in US export subsidies across Chinese sectors. The first eight components correspond to changes in the manipulability of China's terms of trade due to adjustments in its capital of various types with various trading partners. "Own sector" refers to changes in capital in the same sector as a trade tax applies to; "other sectors" is the complement.*

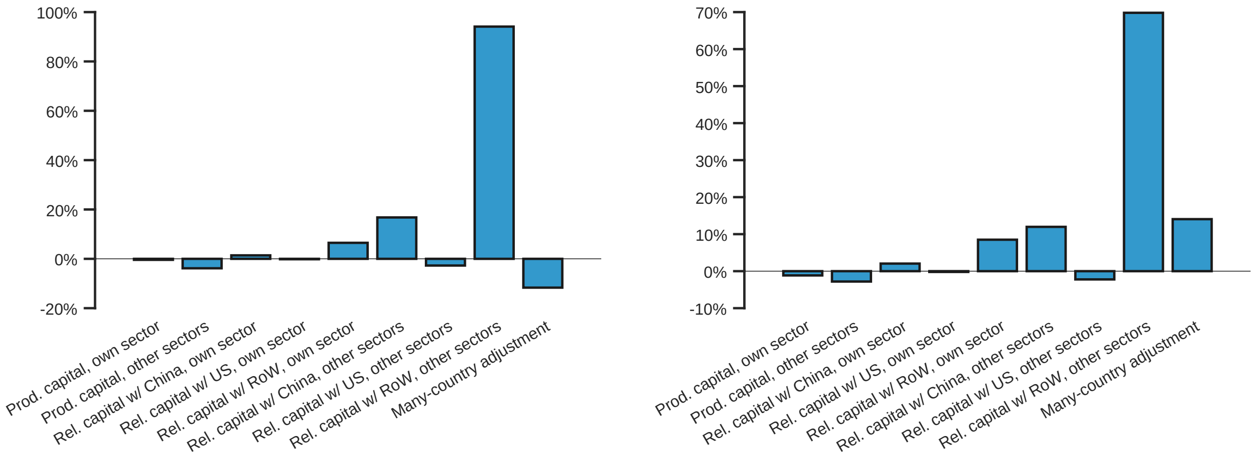

Figure 11: *Left panel: Shapley decomposition of variance in US import tariffs on non-Chinese country-sector pairs. Right panel: Shapley decomposition of variance in US export subsidies on non-Chinese country-sector pairs. The first eight components correspond to changes in the manipulability of China's terms of trade due to adjustments in its capital of various types with various trading partners. "Own sector" refers to changes in capital in the same sector as a trade tax applies to; "other sectors" is the complement.*

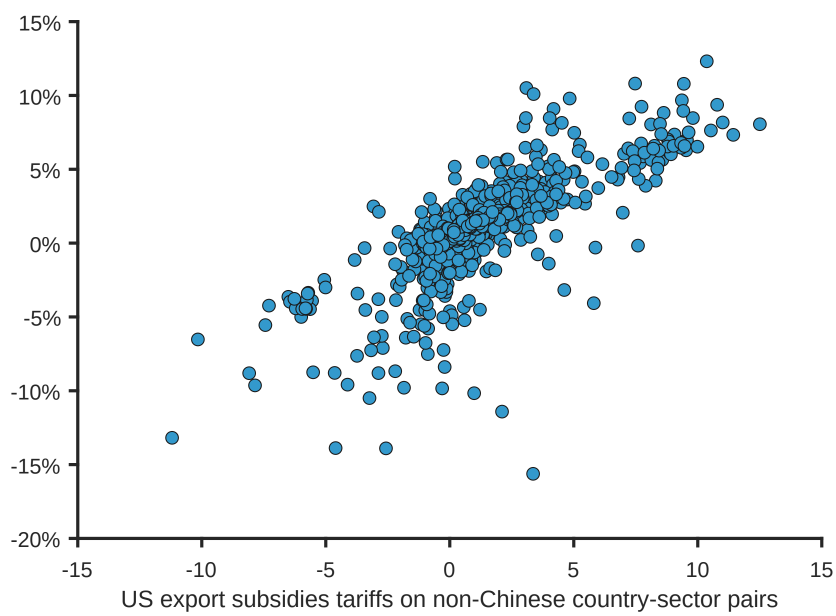

Figure 12: *Variation across sectors in US subsidies on exports to China and the shares of (direct and indirect) investment in Chinese expenditure.*