

A Theory of Wage Rigidity and Unemployment Fluctuations with On-the-Job Search*

Masao Fukui[†]

MIT

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Abstract

I develop a new theory of wage rigidity and unemployment fluctuations. The starting point of my analysis is a generalized version of Burdett and Mortensen's (1998) job ladder model featuring risk-neutral firms, risk-averse workers, and aggregate risk. Because of on-the-job search, my model generates wage rigidity both for incumbent workers, through standard insurance motives, and for new hires, through novel strategic complementarities in wage setting between firms. In contrast to the conventional wisdom in the macro literature, the introduction of on-the-job search implies that: (i) the wage rigidity of incumbent workers, rather than new hires, is the critical determinant of unemployment fluctuations; (ii) fairness considerations in wage setting dampen, rather than amplify, unemployment fluctuations; and (iii) new hire wages are too flexible, rather than too rigid, in the decentralized equilibrium. Quantitatively, the wage rigidity of incumbent workers caused by the insurance motive alone accounts for about one fifth of the unemployment fluctuations observed in the data.

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[†]Email: fukui@mit.edu.

1 Introduction

Does wage rigidity matter for unemployment fluctuations? There is little debate about the fact that the wages of *incumbent* workers are rigid. The conventional view, however, is that this empirically well-documented source of wage rigidity in itself is inconsequential for unemployment fluctuations (Barro, 1977; Pissarides, 2009).¹ The core of the theoretical argument behind this skepticism is that the wages of *new hires*, rather than incumbent workers, are what determine a firm’s marginal cost, and in turn, its hiring incentives.

The starting point of this paper is that the previous argument, as intuitive as it may sound, is at odds with one key feature of labor markets: job-to-job transitions. As Figure 1 shows, such transitions are a pervasive feature of the US labor market, making up more than 40% of new hires.² For firms hiring from a pool of unemployed *and* employed workers, the incentive to create jobs cannot be independent from prevailing incumbent wages. If, in recessions, the wages of incumbent workers do not fall, new jobs have a hard time attracting workers, which in turn discourages job creation.

Motivated by the previous fact, I propose a new theory of wage rigidity and unemployment fluctuations with on-the-job search. Among other things, it implies that: (i) wages of both incumbent workers and new hires are endogenously rigid; (ii) the wage rigidity of incumbent workers, rather than new hires, is the critical determinant of unemployment fluctuations; (iii) fairness considerations in wage setting dampen, rather than amplify, unemployment fluctuations; and (iv) new hire wages are too flexible, rather than too rigid, in the decentralized equilibrium.

Section 2 develops a generalized version of Burdett and Mortensen’s (1998) job ladder model with risk-neutral firms of heterogeneous productivity, risk-averse workers, and aggregate risk. I start with a two-period model to derive a number of sharp qualitative insights. In the first period, firms write state-contingent wage contracts with an exogenous number of incumbent workers to insure against aggregate risk.³ In the second period, aggregate productivity shocks are realized, and firms post vacancies and wages to hire new workers. Without aggregate risk, the model is in the spirit of Burdett and Mortensen (1998). Incumbent firms and poaching firms compete for workers strategically along the job ladders subject to search frictions. While firms can commit to the wage contract, workers cannot: workers search on the job and are free to take an outside offer from other firms. In the presence of aggregate shocks arriving in the second period, the incumbent wage contract plays the role of insurance. Firms need to balance

¹See also Haefke, Sonntag, and van Rens (2013) and Rudanko (2009).

²The US is not an outlier. Engbom (2020) shows that although the US features higher job-to-job transition rates than most European countries, the magnitudes are comparable. Donovan, Lu, and Schoellman (2018) find that developing countries tend to have higher job-to-job transition rates than the US.

³The insurance motive is the most common explanation for incumbent wage rigidity, which goes back at least to Azariadis (1975) and Baily (1974). Therefore, in my model, wage rigidity is an outcome of optimal contracts and does not stem from unexplained inefficiencies.

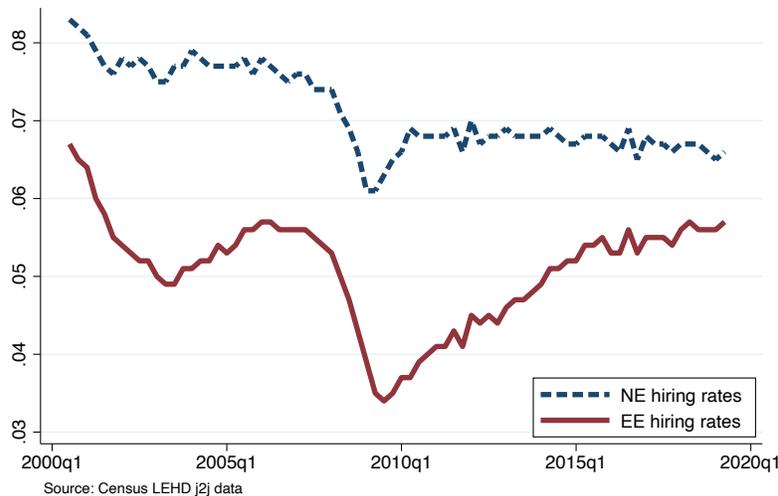


Figure 1: NE and EE hiring rates

Note: Figure 1 shows the NE (non-employment to employment) and EE (employment to employment) hiring rates from 2000-2019. The NE and EE hiring rates refer to the flow of workers from non-employment to employment, and from one employer to another as a fraction of total employment, respectively. Data are from the Census LEHD j2j database.

the provision of insurance and incentivizing the workers to stay with the firm. At the same time, firms also create new jobs to attract workers from a pool of unemployed and employed workers. The wage distributions of incumbent workers and new hires, as well as distribution of vacancy creation, endogenously respond to aggregate shocks as an equilibrium outcome.

Section 3 then characterizes the decentralized equilibrium. Up to a first-order approximation, I show that the equilibrium can be characterized as the solution to a system of ordinary differential equations (ODEs), in which each firm on the job ladder only cares about about the wages and hiring decisions of their neighboring competitors, not the entire distribution. This allows me to derive two main results on wage rigidity and unemployment fluctuations.

The first main result is that wages are endogenously rigid (i.e., they respond less than the aggregate productivity) not only for incumbent workers but also for new hires. The fact that incumbent wages are rigid is intuitive: firms optimally provide some insurance to workers. The fact that new hire wages are also rigid, at least for some firms, is more subtle. Using my ODE characterization of the equilibrium, I show using simple phase diagrams that new hire wages must always feature rigidity at the top of the job ladder. This comes from the fact that at the very top of the job ladder, new hire firms have no incentive to increase wages above what the incumbent firms offer because there would be no additional workers to poach. This extremely strong strategic complementarity spills over toward lower job ladder rungs, and the wages are asymptotically rigid regardless of functional forms or parameter values. This result provides

an explanation for the recent evidence on new hire wage rigidity.⁴

My second main result is that the wage rigidity of incumbent workers, rather than new hires, is the critical determinant of fluctuations in job creation. In fact, in this two-period model, the aggregate response of vacancy creation *only* depends on incumbent wage responses. This implies that despite the fact that my model delivers the endogenous wage rigidity of new hires, it has no consequence on unemployment fluctuations. Moreover, to a first-order approximation, introducing exogenous rigidity in the wages of new hires has no effect on unemployment fluctuations. In this sense, incumbent wage rigidity is a sufficient statistic for unemployment fluctuations regardless of whether or why wages of new hires are rigid.

This result is in contrast to the conventional view that in the textbook Diamond-Mortensen-Pissarides (DMP) models, wage rigidity of new hires is the only source of unemployment volatility. Why are the conclusions strikingly different? My result is the consequence of a combination of two assumptions: on-the-job search, as emphasized earlier, but also wage posting. The presence of on-the-job search implies that the incumbent wage rigidity does affect job creation because it affects the prospective for poaching. Wage posting further implies that any rigidity in the wages of new hires has no first order effect on the profitability of vacancy posting because of the envelope theorem: since firms set the posted wage optimally as a trade-off between hiring more workers and higher costs, any (non-)movement in posted wages has no first order effect on the incentive to create jobs.⁵

As noted earlier, incumbent wage rigidity in my model is not exogenously imposed, but, rather, is derived from a firm's motive to insure workers. This implies that privately optimal risk-sharing contracts between firms and workers drive the unemployment fluctuations. When workers are more risk-averse, the unemployment rate becomes more volatile because firms provide more insurance. This result challenges the consensus in the literature that wage rigidity derived from long-term contracting should not drive unemployment fluctuations in the canonical models of labor markets (Barro, 1977; Rudanko, 2009). Accounting for on-the-job search is crucial for reaching a starkly different conclusion. In fact, I show that in a version of my model without on-the-job search, unemployment volatility is invariant to the workers' risk aversion.

In Section 4, I build on the above insights to consider two extensions of the model. The first one focuses on the introduction of fairness constraints that tie wages of incumbent workers and

⁴Gertler, Huckfeldt, and Trigari (2020), Hazell and Taska (2019), and Grigsby, Hurst, and Yildirmaz (2019) show the rigidity in wages of new hires is comparable to that of incumbent wages.

⁵The importance of new-hire wage rigidity is claimed mainly in the context of the Diamond (1982); Mortensen (1982); Pissarides (1985) models, which not only abstract from on-the-job search but also assume wage bargaining. If wages are bargained, firms would prefer to pay wages as low as possible so long as workers accept the job. Since profits are strictly decreasing in wages, (non-)movements in wages have a first order effect on profits. This brings the issue of whether wage posting or wage-bargaining is a more realistic assumption. Existing survey evidence (Hall and Krueger, 2012; Faberman, Mueller, Şahin, and Topa, 2020) suggests that wage posting is more prevalent, which is consistent with my model.

new hires.⁶ With fairness constraints, firms have to use the same wage to provide insurance for incumbent workers and to attract new hires. Although such constraints are often used in the previous literature to increase the volatility of unemployment, they actually decrease the volatility in my framework. In the models where the wages of new hires are important for the incentive to create jobs, restricting the flexible adjustment in wages of new hires unambiguously amplifies the unemployment fluctuations.⁷ In light of this reasoning, evidence in favor of fairness constraints is often interpreted as the source of amplification in unemployment volatility (e.g., [Bewley, 1999](#)). An overlooked aspect is that such a constraint makes incumbent wages more flexible. If incumbent wage rigidity is important for unemployment fluctuation, as is the case here with on-the-job search, it reduces unemployment volatility. Therefore, the implications of fairness constraints in my model are the opposite of what conventional wisdom would suggest.

The second extension considers the introduction of government-provided insurance. The government makes a transfer to workers during recessions and taxes workers during booms. I show that such public insurance reduces unemployment fluctuations by crowding out firm insurance. Because now that the government provides insurance, incumbent firms need to provide less of it. Consequently, incumbent wages become more flexible, which in turn reduces unemployment volatility. This exercise also clarifies the source of unemployment volatility in my model: it comes from the fact that only incumbent firms can provide insurance to workers — workers cannot write contracts with potential new employers. If workers could write contracts with potential new employers, which is in principle what the government is doing here, the unemployment volatility would disappear.

Section 5 turns to the efficiency of the decentralized equilibrium. I first show that even without aggregate risk, there is a novel source of inefficiency in the economy. As in [Burdett and Mortensen \(1998\)](#), I assume that firms can commit to the wage contract, but workers cannot. Therefore, when firms post vacancies, they do not internalize how their offers affect the outside option of incumbent workers, and in turn, the contracts of incumbent jobs.

First, I show that firms tend to make too aggressive wage offers as long as workers are strictly risk-averse. The planner improves welfare by forcing all firms to offer lower wages. This intervention reduces the consumption dispersion of all workers by reducing its upward potential. As workers prefer smooth consumption profiles, this makes it cheaper for incumbent firms to deliver the same utility to workers, leading to Pareto improvement. Moreover, the externality is larger for more productive firms because their high wage offers contribute most to enlarging workers' consumption dispersion. I next show that, through the same externality,

⁶Such a constraint arises from social norms that workers who perform the same job should be paid the same. The presence of such social norms are documented empirically ([Card, Mas, Moretti, and Saez, 2012](#); [Breza, Kaur, and Shamdasani, 2018](#)).

⁷It has also been common to impose fairness constraints in wage posting models since the seminal work of [Burdett and Mortensen \(1998\)](#). My result clarifies the role played by such constraints within this class of models.

the number of vacancy postings is excessive. Productive firms especially tend to over-create jobs because their vacancies distort incumbent wage contracts the most.

Next, I discuss the efficiency in the presence of aggregate risk. An important implication of my framework is that wage rigidity is not necessarily inefficient because it insulates workers from aggregate risk. In fact, I show that new hire wages are always too flexible relative to the social optimum. This is the case for two reasons. First, competition to attract workers excessively increases the workers' consumption fluctuations. Second, flexibility in new hire wages exacerbates cyclical misallocation. As the wages of incumbent workers respond less than the wages of new hires, workers can flow from more productive firms to less productive firms in booms and reject the offers from more productive firms in recession, manifesting here as misallocation of labor. Forcing new hire wages to respond less improves the allocative efficiency. This is in contrast to the wage rigidity studied in the canonical models of labor markets (e.g., [Hall, 2005](#); [Hall and Milgrom, 2008](#)). There, wages are too rigid, and welfare can be improved by making wages more flexible.

Section 6 concludes by exploring the quantitative importance of the mechanisms described above in a generalized version of the baseline model with continuous time and infinite horizon. Methodologically, I propose a new computational algorithm that starts from the same ODE representation of the decentralized equilibrium as in the baseline two-period model. This allows me to construct equilibria by starting with a guess of the wage that the least productive firms offer, which is the reservation wage, and then to compute recursively the wages along the entire distribution by computationally climbing up the job ladder. Instead of having to solve infinite dimensional fixed point problems, I only need to solve a fixed-point in terms of the sequence of market tightness and reservation wages, which are low dimensional problems. Building on a recent contribution by [Auclert, Bardóczy, Rognlie, and Straub \(2019\)](#), I exploit sequence-space Jacobians to solve this fixed-point, which typically takes less than a few seconds to compute the transition dynamics.

Quantitatively, I find that the wage rigidity of incumbent workers caused by the insurance motive alone generates a 20% dampening of wage responses of new hires and accounts for 20% of the unemployment volatility observed in the data. Contrary to the conventional wisdom, imposing fairness constraints dampens the volatility of unemployment by 70%. Finally, I show that, different to the two-period model, new hire wage rigidity plays a role in unemployment fluctuations, but incumbent wage rigidity remains the dominant source of the fluctuations. Moreover, I show that the type of wage rigidity that matters for unemployment fluctuations in my model is very different from that in the textbook Diamond-Mortensen-Pissarides model. This comes from the fact that the [Burdett and Mortensen \(1998\)](#) model features dynamic competition in the labor market, while such competition is absent in the DMP model.

Related Literature

This paper relates to six strands of the literature. First, it relates to the literature that puts emphasis on the new hire wage rigidity while (implicitly or explicitly) de-emphasizing the role of incumbent wage rigidity; this includes [Barro \(1977\)](#), [Pissarides \(2009\)](#), [Haefke et al. \(2013\)](#), and [Rudanko \(2009\)](#). The latter three papers make a specific point that in the textbook Diamond-Mortensen-Pissarides models, what matters for the incentive to create jobs is the presented discounted value of wage payments to new hires; thus, the response of incumbent wages to aggregate shocks themselves are irrelevant for fluctuations in vacancy creation. As these papers abstract from on-the-job search, they shut down any meaningful interaction between incumbent wages and labor market dynamics. Among them, perhaps the most closely related paper is [Rudanko \(2009\)](#). Like the current paper, she micro-founds the incumbent wage rigidity as risk-neutral firms providing insurance to risk-averse workers. She demonstrates that it barely affects unemployment fluctuations compared with a model with risk-neutral workers. Contrary to [Rudanko's \(2009\)](#) findings, I show that the insurance motive does drive unemployment fluctuations once on-the-job search is taken into account.

Since the emergence of the above papers, subsequent literature has measured and modeled new hire wage rigidity. While [Haefke et al. \(2013\)](#), [Kudlyak \(2014\)](#), and [Basu and House \(2016\)](#) document strong pro-cyclicality of new hire wages, more recent papers, [Gertler, Huckfeldt, and Trigari \(2020\)](#), [Hazell and Taska \(2019\)](#), and [Grigsby et al. \(2019\)](#) have found weak cyclicality. The controversy comes from the difficulty in adjusting worker and job compositions that change over business cycles. In contrast, measuring incumbent wage cyclicality does not suffer from such problems, and there is a widely held consensus that incumbent wages are fairly rigid over business cycles (see [Grigsby, Hurst, and Yildirmaz \(2019\)](#) for the most recent evidence). The implication of my theory is that what is less controversial is what matters the most.

Theoretically, several papers have proposed mechanisms that generate endogenous new hire wage rigidity. In [Menzio and Moen \(2010\)](#), firms can commit to wage contracts to insure incumbent workers, but cannot commit not to fire them. This asymmetric commitment technology implies that firms have an incentive not to lower the wages of new hires to avoid replacing incumbent workers with new hires. Although my model is close in spirit in deriving incumbent wage rigidity from firm insurance, the underlying mechanisms are entirely different. For example, in [Menzio and Moen \(2010\)](#), it is important that a firm that posts a vacancy and a firm with incumbent workers are the same firm, but it is not important in my framework. In [Kennan \(2010\)](#), workers do not ask for higher wages in expansions because they do not know whether the firm's productivity increased or not. I provide another mechanism that relies on strategic complementarity in wage setting. This is a natural mechanism to explore because search friction with on-the-job search implies that firms compete for a worker in an imperfectly

competitive labor market. In this sense, my paper also relates to the recent papers on strategic complementarity in price settings in oligopolistic product markets (Mongey, 2017; Wang and Werning, 2020).

A more popular and simpler way to generate new hire wage rigidity is to impose fairness constraints, together with other assumptions that generate incumbent wage rigidity. Among others, Menzio (2004), Gertler and Trigari (2009), Snell and Thomas (2010), and Rudanko (2019) pursue this approach. They all conclude that such a constraint amplifies unemployment fluctuations because in those models, new hire wage rigidity, rather than incumbent wage rigidity, is the key source of fluctuations. By contrast, I show that with on-the-job search, such a constraint dampens unemployment fluctuations.

There are models in which incumbent wage rigidity matters for unemployment fluctuations. Schoefer (2016) adds financial frictions into Diamond-Mortensen-Pissarides models and shows that incumbent wage rigidity can tighten financial constraints in recessions. Bills, Chang, and Kim (2016) add endogenous effort choice by workers. In their model, if incumbent wages are too high in recessions, incumbent workers provide too much effort, which reduces the value of the additional workforce. Carlsson and Westermark (2016) show that incumbent wage rigidity leads to inefficient job destruction. In contrast to these papers, I provide a simple and empirically well grounded channel that operates through job creation.

The second strand of literature to which this paper relates emphasizes the role of on-the-job search in business cycle dynamics. There are three approaches in this line of research. The first approach adopts directed search and wage posting (competitive search) (Menzio and Shi, 2011; Schaal, 2017; Baley, Figueiredo, and Ulbricht, 2019). The second approach is to assume a random search together with wage bargaining (Lise and Robin, 2017; Moscarini and Postel-Vinay, 2018, 2019; Bilal, Engbom, Mongey, and Violante, 2019). A third approach, which I pursue in this paper, is to assume random search and wage posting in the tradition of Burdett and Mortensen (1998) and Burdett and Coles (2003) (Moscarini and Postel-Vinay, 2013, 2016b; Morales-Jiménez, 2019).⁸ All these papers feature risk-neutral workers, thereby flexible wages, so they do not speak to the issues studied here. Relative to this strand of literature, I introduce risk-averse workers and aggregate risk into Burdett and Mortensen's (1998) models to study the nature and the consequence of wage rigidity.⁹ I build on Burdett-Mortensen model because the model has a well-defined notion of wages.

Several other papers explore alternative mechanisms whereby the presence of on-the-job

⁸While papers using this framework to study long-run wage and employment distribution is abundant (van den Berg and Ridder, 1998; Bontemps, Robin, and den Berg, 1999; Engbom and Moser, 2017; Heise and Porzio, 2019, just to name a few), the literature on transition dynamics is far more scarce. See also Yamaguchi (2010); Bagger, Fontaine, Postel-Vinay, and Robin (2014); Jarosch (2015); Caldwell and Harmon (2019); Karahan, Ozkan, and Song (2019); Engbom (2019), which study the role of on-the-job search in the long-run wage and firm dynamics using the bargaining framework.

⁹Morales-Jiménez (2019) has an extension with exogenous wage rigidity in the form wage adjustment costs à la Rotemberg (1983) but does not separate the role played by rigidity of incumbent workers and new hires.

search amplifies the business cycle through the changes in search efficiency. [Eeckhout and Lindenlaub \(2019\)](#) show that the pro-cyclical job search effort by employed workers leads to self-fulfilling fluctuations in the presence of worker sorting. [Engbom \(2020\)](#) shows that cyclical changes in the composition of employed and unemployed job searchers amplify separation shocks due to greater applications from the latter. I add to this literature by providing a novel mechanism through which the presence of on-the-job search amplifies business cycles.

Third, I build on the long tradition of the literature that micro-founds incumbent wage rigidity as insurance provided by firms. [Azariadis \(1975\)](#) and [Baily \(1974\)](#) are early contributions on this. [Harris and Holmstrom \(1982\)](#) add limited commitment to the workers' side, and this mechanism leads to downward wage rigidity.¹⁰ [Beaudry and DiNardo \(1991\)](#) test its prediction in the data. [Rudanko \(2009\)](#) and [Lamadon \(2016\)](#) embed the mechanism into a search-and-matching labor market. My paper contributes to this literature by demonstrating that such insurance not only explains the wage dynamics, but also increases the volatility of unemployment. Since I focus on the first order approximation around the steady-state, I do not study the non-linear effect such as downward nominal wage rigidity that [Harris and Holmstrom \(1982\)](#) emphasize. However, I conjecture that the non-linear dynamics of my model features downward wage rigidity, which I leave for future work.

Fourth, my paper relates to a series of papers on [Shimer \(2005\)](#) puzzle: i.e., the textbook Diamond-Mortensen-Pissarides models cannot generate unemployment volatility comparable to the data. As mentioned before, many papers rely on new hire wage rigidity (e.g., [Hall, 2005](#); [Dupraz, Nakamura, and Steinsson, 2019](#)). As summarized by [Ljungqvist and Sargent \(2017\)](#), many other solutions rely on increasing the sensitivity of profits to labor productivity by making profits small (e.g., [Hagedorn and Manovskii, 2008](#); [Hall and Milgrom, 2008](#); [Pissarides, 2009](#)). However, [Chodorow-Reich and Karabarbounis \(2016\)](#) criticize these approaches by showing that much of the amplifications disappear if one assumes that the outside options for unemployed workers are equally as cyclical as labor productivity, and they provide evidence for this. In keeping with this evidence, I adopt the assumption that outside options of unemployed workers scale with labor productivity. [Hall \(2017\)](#), [Borovička and Borovičková \(2018\)](#), [Kehoe, Lopez, Midrigan, and Pastorino \(2019\)](#), and [Martellini, Menzio, and Visschers \(2020\)](#) explore whether movements in discount rates or risk premium explain unemployment volatility.¹¹ My contribution to this strand of the literature is that incumbent wage rigidity and on-the-job search, which are uncontroversial features of the data, can help resolve the Shimer puzzle.

Fifth, I build on the recent developments on the computation of the transition dynamics of heterogeneous agent models. It is widely believed that solving the transition dynamics of the Burdett-Mortensen model is challenging because the endogenous distribution enters as a state

¹⁰[Thomas and Worrall \(1988\)](#) further extend this to an environment with two-sided limited commitments.

¹¹See also [Yashiv \(2000\)](#) and [Mukoyama \(2009\)](#).

variable. [Moscarini and Postel-Vinay \(2013\)](#) propose one methodology under a set of restrictive assumptions, and [Morales-Jiménez \(2019\)](#) applies [Reiter \(2009\)](#) approach to approximate the distribution as a low dimensional object. I add to this literature by providing a fast approach to compute the transition dynamics that is accurate to a first order in the size of aggregate shocks, without the need to approximate the distribution. The key idea is that firms do not care about the entire distribution when solving their decision problems. Other than a small number of aggregate variables, such as the market tightness and reservation wages, firms only care about their neighboring competitors. Thus the equilibrium solution boils down to solving a system of ODEs, rather than infinite dimensional fixed point problems. Therefore, I only need to solve for a sequence of aggregate market tightness and reservation wages that is consistent with equilibrium. I apply the sequence-space Jacobian approach by [Auclert et al. \(2019\)](#) in solving the sequence of aggregates to efficiently compute the equilibrium. While [Auclert et al. \(2019\)](#) write “in the model of on-the-job search in [Burdett and Mortensen \(1998\)](#), ..., it is impossible to represent this via a DAG (directed acyclical graph) of feasible dimension,” I show that it can indeed be represented as a low dimensional problem.

Sixth, my paper touches on a growing strand of the literature on theoretical models of monopsony in the labor market ([Manning 2003](#); [Berger, Herkenhoff, and Mongey 2019](#); [Jarosch, Nimczik, and Sorkin 2019](#); [Lamadon, Mogstad, and Setzler 2019](#); [Gouin-Bonenfant 2020](#)). In my model, firms exercise monopsony power because of search frictions. The presence of market power is necessary to study wage rigidity because under perfect competition, wages always respond one-for-one with aggregate productivity. While the literature typically focuses on how the monopsony power shapes the level of wages, I shed light on how the monopsony power shapes the pass-through of aggregate productivity shock through endogenous changes in wage markdowns.

Layout. The rest of the paper is organized as follows: Section 2 describes the basic two-period model. Section 3 provides qualitative insights on why wages are rigid and what this implies for unemployment fluctuations. Section 4 considers two extensions to study the implications of fairness consideration in wage setting and public insurance. Section 5 highlights the inefficiency of the model. Section 6 quantitatively explores the mechanisms by extending the basic model to an infinite horizon and continuous time setup. Section 7 concludes.

2 A Job Ladder Model with Risk Averse Workers and Aggregate Shocks

I start from a two-period model to derive a number of sharp qualitative insights. Later in Section 6, I will turn to the quantification of these results in a continuous time and infinite

horizon version of the model. In this section, I describe the model environment and define equilibrium.

2.1 Preferences and Technology

Consider an economy with two dates, $t = 0, 1$. In the initial period, $t = 0$, the firms and workers write contracts (to be described later). At $t = 1$, all the consumption and production take place. There are two states, $s \in \{h, l\}$ at $t = 1$, with different aggregate productivity, A_s , with $A_h \geq A_l$. The aggregate productivity is revealed at the beginning of $t = 1$. The probability for each state is given by $\pi_s = 1/2$ for $s \in \{h, l\}$. In words, there will be either a boom or recession at $t = 1$ with equal probability.

The economy is populated by two types of agents: a unit mass of workers and a unit mass of firms (or entrepreneurs). Workers consume only at $t = 1$, and their preferences are given by

$$\mathbb{E}u(c_1) \quad \text{with} \quad u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where $\gamma \geq 0$ corresponds to the relative risk aversion. At $t = 0$, workers are initially divided into two groups: a fraction $1 - \mu$ of incumbent workers and a fraction μ of unemployed. Both types of workers search for a job at the beginning of $t = 1$. Unemployed workers meet with a firm with probability λ_s^U , and incumbent workers meet with probability $\lambda_s^E \equiv \zeta \lambda_s^U$, where $\zeta > 0$ is the relative search efficiency of the employed. A worker faces no search cost. When workers end up being unemployed at $t = 1$, they enjoy home production, which produces $A_s b$ amount of consumption goods, where $b > 0$ is a parameter. Here, the outside option of unemployed scales with the aggregate productivity shock, A_s .¹²

Firms consume and produce only at $t = 1$, and they are risk-neutral,

$$\mathbb{E}c_1^e,$$

where c_1^e is the consumption of entrepreneurs. Each firm has access to production technology that is linear in labor,

$$A_s z l,$$

where l is labor and z is the idiosyncratic productivity. The idiosyncratic productivity is a fixed characteristics of a firm. The cross-sectional distribution is continuous and has a bounded support $[\underline{z}, \bar{z}]$ and $\underline{z} \geq b$. Let $G(z)$ and $g(z)$ denote the cumulative and the probability density function, respectively. Each firm z is exogenously endowed with $\ell_0(z)$ amount of employed workers at $t = 0$.

At $t = 1$, firms choose how much vacancy to post, $v_s(z)$, to attract new workers. The vacancy

¹²This is consistent with the evidence documented in [Chodorow-Reich and Karabarbounis \(2016\)](#).

posting is subject to convex cost, $c_s(v; z)$. I assume the cost of vacancy creation, $c_s(v; z)$ takes the form

$$c_s(v; z) = A_s \bar{c}(z) \frac{v^{1+1/\iota}}{1 + 1/\iota}, \quad (1)$$

where $\iota > 0$ corresponds to the elasticity of vacancy creation. The assumption that the cost function scales with the aggregate productivity follows [Blanchard and Galí \(2010\)](#).¹³ This captures the idea that to recruit workers, existing workers must reduce their time devoted to production, which costs a firm lost output. The assumption ensures that the fluctuations in job creation is not driven by differential productivity growth between the output production and recruitment activity.

Each vacancy will meet a worker with probability λ_s^F . I have not described whether a firm that posts a vacancy and a firm with incumbent workers are the same firm or not. Because of the constant-returns-to-scale technology, there is no well-defined boundary of firms. Thus, it is not important whether a firm that has an existing workforce and the one that hires new workers are the same firm or not.¹⁴

Finally, the total number of meetings between firms and workers is given by a constant-returns-to-scale matching technology $\mathcal{M}(\tilde{\mu}, V_s)$. The first input to the matching function is the total efficiency unit of search by workers, $\tilde{\mu} \equiv \mu + \zeta(1 - \mu)$. The second input is the total amount of vacancy postings, $V_s \equiv \int v_s(z) dG(z)$. Search is random. When firms meet with a worker, the worker is an unemployed with probability $\chi \equiv \frac{\mu}{\tilde{\mu}}$ and is employed with probability $1 - \chi$. Likewise, when workers meet a firm, the probability that the firm has productivity z is given by $\frac{v_s(z)}{V} g(z)$.

2.2 Contracts and Markets

Firms that have incumbent workers at $t = 0$ write state-contingent wage contracts with workers at $t = 0$. A worker employed by a firm with productivity z is endowed with promised utility $\bar{W}_0(z)$: the firm has to deliver expected utility at least $\bar{W}_0(z)$ through the contract. Although this is an exogenous parameter, one can think of this as an object that is determined in the past when a worker is hired. In fact, this will be the case in the infinite horizon version of my model studied later.

The contract specifies the wage payments in each state $\{w_{0h}(z), w_{0l}(z)\}$. There are two assumptions in the contract. First, workers cannot commit to the contract, so they are free to leave firms when receiving a better offer. Workers are also free to quit and become unemployed. In contrast, I assume firms have full commitment. Second, the contract cannot depend on the outside offers that workers received. A justification for this assumption is that outside offers are

¹³See also [Shimer \(2010\)](#), or more recently [Kehoe et al. \(2019\)](#).

¹⁴It will be important when I later introduce the fairness constraint that exogenously tie incumbent and new hire wages.

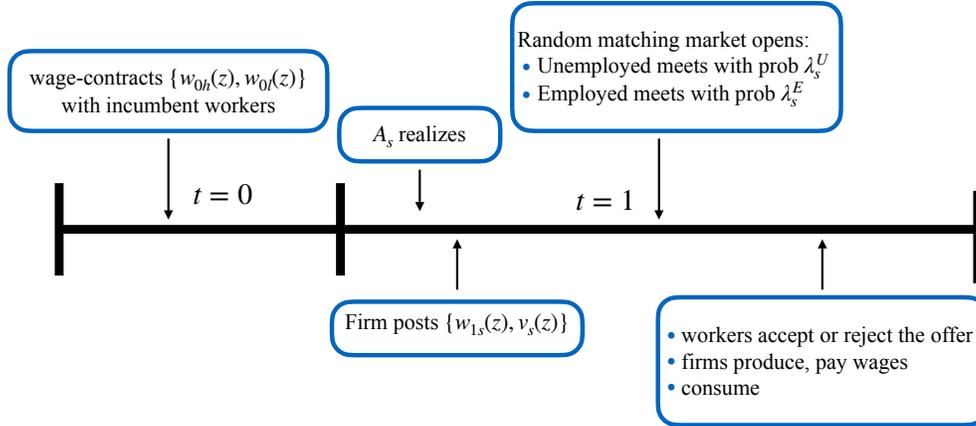


Figure 2: Timing

Note: Figure 2 describes the timing assumption of the model.

not verifiable. These two assumptions are common in the wage posting literature (e.g., [Burdett and Coles, 2003](#)). The risk-neutral firms have an incentive to provide insurance against aggregate productivity shock to risk-averse workers through the contract.

At $t = 1$, when firms post a vacancy after the realization of the aggregate productivity shock, they also post wage, $w_{1s}(z)$. Firms commit to the wage contract; therefore, the offer is a take-it-or-leave-it offer.

Timing. The timing of the model is described in Figure 2. First, incumbent workers and firms write contracts before the realization of aggregate productivity shock. Then, after the observing the aggregate productivity, firms post a vacancy. Next, a matching market opens, and firms and workers meet with each other. Workers either accept or reject the offer, and production and consumption take place.

2.3 Equilibrium Definition

In an equilibrium, incumbent firms at date 0 maximize expected profits taking the wage distribution induced by wage offers by new hire firms at date 1 and the reservation wage of their workers as given, whereas new hire firms at date 1 maximize expected profits taking in the distribution of wage contracts by incumbent firms at date 0 and the reservation wages of workers as given.

Incumbent firms' optimal contracting problem. Incumbent firms take the new hire wage distribution, which I denote as $F_{1s}(w)$, and the meeting probability λ_s^E as given. The incumbent

wage contracts of firms with productivity z solves the following problem:

$$\begin{aligned}
& \max_{\{w_{0h}, w_{0l}\}} \sum_{s \in \{h, l\}} \pi_s (A_s z - w_{0s}) (1 - \lambda_s^E + \lambda_s^E F_{1s}(w_{0s})) \\
\text{s.t. } & \sum_{s \in \{h, l\}} \pi_s \left[(1 - \lambda_s^E) u(w_{0s}) + \lambda_s^E \int \max\{u(w_{0s}), u(\tilde{w})\} dF_{1s}(\tilde{w}) \right] \geq \bar{W}_0(z) \quad (2) \\
& w_{0h} \geq A_h b, \quad w_{0l} \geq A_l b.
\end{aligned}$$

where $\bar{W}_0(z)$ is the promised utility of firm z . The objective function is the expected profits, taking into account the probability of workers being poached. With probability $1 - \lambda_s^E$, a worker does not receive an outside offer, and with probability λ_s^E , s/he receives an offer. If the offer is lower than the current wage, w_{0s} , which happens with probability $F_{1s}(w_{0s})$, s/he find it optimal to stay with the current firm. Otherwise, the worker leaves for the new firm. The constraint guarantees the worker's expected utility, which takes into account that the wage payments in the new firm, is greater than the predetermined promised utility. The constraint $w_s \geq A_s b$ captures the fact that workers can always quit and engage in home production, which firms never find it optimal to let happen.

The key trade-off in this contracting problem is insurance versus incentive. Firms would like to insure workers as the worker's utility is concave, but if firms do too much insurance, firms will not be able to keep workers during good times, while tending to keep workers during bad times. The optimal wage contract strikes a balance between the two.

New hire firms' profit maximization. The new hire firms take the incumbent wage distribution, which I denote as $F_{0s}(w)$, and the meeting probability λ_s^F as given. A firms with productivity z solves the following profit maximization problem:

$$\begin{aligned}
& \max_{v_s, w_{1s}} (A_s z - w_{1s}) \lambda_s^F (\chi + (1 - \chi) F_{0s}(w_{1s})) v_s - c_s(v_s; z), \\
\text{s.t. } & w_{1s} \geq A_s b
\end{aligned} \quad (3)$$

where $\chi \equiv \mu / (\mu + \zeta(1 - \mu))$ is the share of unemployed. Since firms always find it optimal to offer at least $A_s b$ because $z \geq b$ for all z , the unemployed always accept an offer. The remaining fraction $1 - \chi$ of workers are already employed, and they accept the offer with probability $F_{0s}(w_{1s})$. Again, the constraint $w_{1s} \geq A_s b$ captures the fact that the firms always find it optimal to offer wages that at least attract the unemployed. The firm chooses the wage offers and the amount of vacancy to maximize expected profits after observing the aggregate productivity shock, A_s .

The equilibrium definition is as follows:

Definition 1. *Equilibrium consists of incumbent firms' wage contracts, $\{w_{0s}(z)\}$, and new hire firms'*

wage offers and vacancy postings, $\{w_{1s}(z), v_s(z)\}$, associated wage distribution $\{F_{0s}(w), F_{1s}(w)\}$ and meeting probabilities λ_s^E and λ_s^F such that (i) given the entrants' wage distribution F_{1s} and λ_s^E , incumbent wages $\{w_{0s}(z)\}$ solve (2), (ii) given the incumbents' wage distribution F_{0s} and λ_s^F , $\{w_{1s}(z), v_s(z)\}$ solve (3), and (iii) the wage distribution is consistent with the equilibrium wage strategies: $F_{0s}(w) = \frac{1}{1-\mu} \int_{z:w \geq w_{0s}(z)} \ell_0(z) dG(z)$, $F_{1s}(w) = \int_{z:w \geq w_{1s}(z)} (v_s(z)/V_s) dG(z)$; (iv) the meeting probabilities are given by the matching function, $\lambda_s^E = \zeta \frac{\mathcal{M}(\tilde{\mu}, V_s)}{\tilde{\mu}}$ and $\lambda_s^F = \frac{\mathcal{M}(\tilde{\mu}, V_s)}{V_s}$.

Equilibrium is a fixed point in terms of the wage distribution (and matching probabilities). Each individual firm is infinitesimal and takes the wage distribution of competitors as an input to their decision problems. The optimization problems give the wage distribution as an outcome, which has to be consistent with the distribution that firms took as an input.

2.4 Discussion of the main assumptions

The assumption that workers have limited commitment and firms have full commitment has been standard in the literature, which at least goes back to [Harris and Holmstrom \(1982\)](#) or more recently [Lamadon \(2016\)](#). The assumption is realistic. Firms have reputation costs of renegeing the contract, while workers arguably have much less costs in doing so.

The assumption on wage posting also deserves some discussion. This assumption plays important roles in many of my analyses. Another common approach is to use a sequential auction protocol as in [Postel-Vinay and Robin \(2002\)](#). Perhaps, both wage setting protocols co-exist in a real world, but existing empirical evidence suggests wage posting is more prevalent. Survey evidence shows two-thirds of workers do not bargain over wages ([Hall and Krueger, 2012](#); [Faberman et al., 2020](#)). [Faberman et al. \(2020\)](#) also document that counter-offers are rather rare: only 12% of offers that workers received are countered by their employers. Moreover, recent evidence by [Addario, Kline, Saggio, and Sølvssten \(2020\)](#) shows that workers' wages display little dependence on past jobs, contrary to the prediction of sequential auction protocol models, but this fact is consistent with wage posting models.

Another assumption is random search, as opposed to directed search or competitive search ([Moen, 1997](#); [Acemoglu and Shimer, 1999](#); [Menzio and Shi, 2011](#)). Both assumptions are equally common in the literature, and the reality should lie somewhere in between. It is thus an important open question to study wage rigidity in an environment with directed search, which I leave for future work.¹⁵

¹⁵[Bilal et al. \(2019\)](#) argue that the recent evidence on worker and firm flows by [Bagger, Fontaine, Galenianos, and Trapeznikova \(2020\)](#) is consistent with random search but not necessarily with competitive search.

3 Wage Rigidity and Unemployment Fluctuations

This section studies equilibrium wage rigidity and its consequences on unemployment fluctuations. Section 3.1 describes the solution approach. Section 3.2 presents the main result on wage rigidity, and Section 3.3-3.4 study what type of wage rigidity matters for unemployment fluctuations.

3.1 Solution Approach

I first characterize the equilibrium by considering the optimality conditions of firms. The first order necessary condition associated with incumbent firm's optimization problem (2) is

$$\begin{aligned}
 & -(1 - \lambda_s^E + \lambda_s^E F_{1s}(w_{0s}(z))) + (A_s z - w_s(z)) \lambda_s^E F'_{1s}(w_{0s}(z)) \\
 & + \eta(z) \left[(1 - \lambda_s^E) u'(w_{0s}(z)) + \lambda_s^E F_{1s}(w_{0s}(z)) u'(w_{0s}(z)) \right] = 0
 \end{aligned} \tag{4}$$

where $\eta(z)$ is the Lagrangian multiplier on the promise-keeping constraint. The first order conditions associated with new hire firm's optimization problem (3) is

$$(1 - \chi) F'_{0s}(w_{1s}(z)) (A_s z - w_{1s}(z)) - (\chi + (1 - \chi) F_{0s}(w_{1s}(z))) = 0 \tag{5}$$

$$(A_s z - w_{1s}(z)) \lambda_s^F (\chi + (1 - \chi) F_{0s}(w_{1s}(z))) - c'_s(v_s(z); z) = 0. \tag{6}$$

In deriving these conditions, I have assumed that the wage distributions, F_{0s} and F_{1s} , are differentiable. I will later confirm that they are as such in my analysis.

Symmetric equilibrium without aggregate risk. Although I have already simplified the model by focusing two-period model, analyzing the model still poses a challenge. As is clear from the equilibrium definition, the problem involves multiple infinite dimensional objects (incumbent and new hire wage distribution, as well as vacancy distribution in each state). It is intractable not only analytically but even computationally. To the best of my knowledge, there is no efficient algorithm to solve the model non-linearly because the equilibrium does not have a convenient property such as contraction mapping.

To overcome the difficulty, I propose a tractable solution approach. I consider a perturbation of a particular equilibrium with respect to the aggregate risk. I first focus on a particular parametrization that features the following properties: (i) zero aggregate risk, $A_h = A_l$, and (ii) symmetry between incumbent and new hire wages, $w_0(z) = w_1(z) \equiv w(z)$, where I dropped the s subscript as two states are the same. These properties will naturally arise in the steady-state of an infinite horizon setup that I will study later. For this reason, I call this equilibrium as the steady-state equilibrium.

After imposing (i) and (ii), the new hire firms first-order condition (5) becomes

$$(1 - \chi)F'_0(w(z))(Az - w(z)) - (\chi + (1 - \chi)F_0(w(z))) = 0. \quad (7)$$

Here F'_0 is well-defined as there cannot be a mass point in the incumbent wage distribution. If there is a mass point, then one of the incumbent or the new hire firms at the mass point can raise wages by a small amount and discontinuously increase the profits, which contradicts with the optimality of wage setting. Moreover, $w(z)$ is strictly increasing because the objective function (3) is strictly log-supermodular in (z, w) . Because the wages are monotone, it follows that $\hat{F}_0(z) \equiv F_0(w(z)) = \frac{1}{1-\mu} \int^z \ell_0(z) dG(z)$, so $\hat{F}'_0(z) = F'_0(w(z))w'(z)$, where $\hat{F}_0(z)$ is the employment-weighted productivity distribution. Using these expressions, we can rewrite (7) as a single ODE:

$$(1 - \chi)\hat{F}'_0(z)(Az - w(z)) - (\chi + (1 - \chi)\hat{F}_0(z))w'(z) = 0 \quad (8)$$

with the boundary condition $w(\underline{z}) = Ab$ because the least productive firms can only hire from a pool of unemployed. The solution is

$$w(z) = \frac{\chi Ab + (1 - \chi) \int_b^z Az d\hat{F}_0(\tilde{z})}{(\chi + (1 - \chi)\hat{F}_0(z))}, \quad (9)$$

which corresponds to the employment weighted average productivity level conditional on productivity below z . As is standard in [Burdett and Mortensen \(1998\)](#) models, the firms exercise monopsony power, $w(z) < Az$, because of search frictions. Appendix A.1 also shows the second-order condition is satisfied.

Given (9), the optimal vacancy solves

$$(Az - w(z))\lambda^F (\chi + (1 - \chi)F(w(z))) = c'(v(z); z). \quad (10)$$

The meeting probabilities are given by

$$\lambda^F = \frac{1}{V} \mathcal{M}(\tilde{\mu}, V) \quad \lambda^E = \zeta \frac{\mathcal{M}(\tilde{\mu}, V)}{\tilde{\mu}} \quad \text{with } V = \int v(z) dG(z). \quad (11)$$

Finally, I have to guarantee that the incumbent firms find it optimal to offer $w(z)$. I can always guarantee this if the promised utility is appropriately chosen and if the promise-keeping constraint is binding. The promise-keeping constraint is always binding as long as λ^E is small enough, which is the case for sufficiently small ζ . Intuitively speaking, if incumbent firms do not face too tough competition from being poached, they would like to exercise monopsony power to lower wages as much as they can. Then, one can appropriately choose $\bar{W}_0(z)$ so that

the incumbent firms need to offer $w_0(z) = w(z)$. I summarize the discussion as follows:

Lemma 1. *Suppose $A_h = A_l$ and the relative search efficiency of the employed, ζ , is sufficiently small. Then, there exists $\{\bar{W}_0(z)\}$ under which the equilibrium wage strategy is symmetric between incumbent and new hire firms, $w_0(z) = w_1(z) \equiv w(z)$. In such an equilibrium, $\{w(z), v(z), \lambda^F, \lambda^E\}$ are given by (9), (10) and (11), and*

$$\bar{W}_0(z) = (1 - \lambda^E)u(w(z)) + \lambda^E \int \max\{u(w(z)), u(w(\bar{z}))\}(v(\bar{z})/V)dG(\bar{z}).$$

All the proofs are collected in Appendix A. I next turn to the analysis with aggregate risk by taking a first order perturbation around the above symmetric equilibrium.

3.2 Wage Rigidity

I introduce aggregate risk into the economy by assuming $A_h > A_l$. I consider a first order perturbation that is a mean preserving spread around $A_h = A_l \equiv A$, $\ln A_h = \ln A + d \ln A$ and $\ln A_l = \ln A - d \ln A$. I let variables with hat denote the log deviation from the steady-state equilibrium, $\hat{x} \equiv d \ln x$.

3.2.1 Characterization

The following lemma shows that the responses are symmetric between two states:

Lemma 2. *In the presence of small aggregate risk, $\hat{A} > 0$, to a first order, the equilibrium is symmetric between two states: $\hat{w}_{1h}(z) = -\hat{w}_{1l}(z) \equiv \hat{w}_1(z)$, $\hat{w}_{0h}(z) = -\hat{w}_{0l}(z) \equiv \hat{w}_0(z)$, $\hat{v}_h(z) = -\hat{v}_l(z) \equiv \hat{v}(z)$, $\hat{V}_h = -\hat{V}_l \equiv \hat{V}$, $\hat{\lambda}_h^E = -\hat{\lambda}_l^E \equiv \hat{\lambda}^E$, $\hat{\lambda}_h^F = -\hat{\lambda}_l^F \equiv \hat{\lambda}^F$.*

Because the equilibrium conditions are smooth with respect to endogenous variables, the symmetric aggregate productivity shocks induces the symmetric responses. This is useful because we can reduce the number of unknowns by half.

I turn to characterizing the equilibrium responses to the aggregate shock. I first concentrate on the wage responses by assuming vacancies are inelastic. Even in this case, the equilibrium is potentially very complicated because it is an infinite dimensional fixed point problem. New hire firms need to form expectation over the entire incumbent wage distribution and decide where to position their wage rank. Conversely, incumbent firms need to form expectation about entire new hire wage distribution and decide which wage offers they would like to block. These expectations need to be consistent with optimization behaviors. However, it turns out that the equilibrium solution takes a very simple form, as the following lemma shows:

Lemma 3. *Assume vacancy creation is inelastic, $\iota = 0$. In the presence of small aggregate risk, $\hat{A} > 0$, to a first order, the equilibrium incumbent wage responses, $\hat{w}_0(z)$, and new hire wage responses, $\hat{w}_1(z)$,*

solve the following two ODEs:

$$(new\ hire) \quad \hat{w}_1(z) = \theta_{1a}(z)\hat{A} + \underbrace{\theta_{1w}(z)\hat{w}_0(z)}_{\text{competition within a job-ladder rung}} - \underbrace{\theta_{1a}(z)\alpha(z)\frac{w(z)}{w'(z)}\hat{w}'_0(z)}_{\text{competition between job-ladder rungs}} \quad (12)$$

$$(incumbent) \quad \hat{w}_0(z) = \theta_{0a}(z)\hat{A} + \underbrace{\theta_{0w}(z)\hat{w}_1(z)}_{\text{competition within a job-ladder rung}} - \underbrace{\theta_{0a}(z)\alpha(z)\frac{w(z)}{w'(z)}\hat{w}'_1(z)}_{\text{competition between job-ladders rungs}} \quad , \quad (13)$$

with the boundary conditions, $\hat{w}_1(\bar{z}) = \hat{A}$ and $\hat{w}_0(\bar{z}) = \hat{w}_1(\bar{z})$. The coefficient $\alpha(z) \equiv (Az - w(z))/Az$ is the wage markdown and the other coefficients are such that $\theta_{1w}(z) + \theta_{1a}(z) = 1$ and $\theta_{0a}(z) + \theta_{0w}(z) \leq 1$, with equality if workers are risk-neutral, $\gamma = 0$, as shown in Appendix A.3.

The two ODEs come from the log linearization of the first order conditions (4) and (5) and are the best response functions of the firms wage settings. Note that the original best response function of incumbent firm of productivity z depends on F_{1s}, F'_{1s} , which in turn depends on the entire functions of $\{w_{1s}(\bar{z})\}_{\bar{z}}$. The key observation of Lemma 3 is that to a first-order approximation, the best response of incumbent firm of productivity z only depends on, $w_{1s}(z)$ and $w'_{1s}(z)$, not on the entire function $\{w_{1s}(\bar{z})\}_{\bar{z}}$, substantially reducing the dimensionality. To see this, the first order change in cumulative distribution function $F_1(w_0(z))$ is given by

$$dF_1(w_0(z)) = F'_1(w(z))w(z) (\hat{w}_0(z) - \hat{w}_1(z))$$

and the first order change in the density function $F'_1(w_0(z))$ is given by

$$dF'_1(w_0(z)) = F''_1(w(z))w(z)\hat{w}_0(z) - (F''_1(w(z))w(z) + F'_1(w(z)))\hat{w}_1(z) - F'_1(w(z))\frac{w(z)}{w'(z)}\hat{w}'_1(z).$$

That is, the competition remains always local in response to small shocks. Firms do not need to form expectations about the wage offers of firms in significantly different job ladder ranks because they won't affect the labor supply curve. Firms need to only care about how their local competitors will behave.

The term ‘‘competition within a job ladder rung’’ captures how the competitors with exactly the same productivity level affect the labor supply. For example, if the new hire firm with productivity z increases its wages, the incumbent firm with the same productivity z is more likely to be poached. The term ‘‘competition between job ladder rungs’’ captures how the neighboring competitors' wage setting affects the labor supply. For example, if the new hire firm with productivity $z - dz$ increases its wages more than those with productivity $z + dz$, the incumbent firm with productivity z faces more elastic labor supply because there would now be a greater mass of marginal competitors.

The coefficients on (12) and (13) cannot be arbitrary and have theoretical restrictions. First,

$\theta_{1a}(z) > 0$ and $\theta_{1a}(z) + \theta_{1w}(z) = 1$.¹⁶ That is, the new hire firms' problem is homogenous: if the aggregate productivity increases by 1% and the incumbent firms increase wages by 1%, then it is optimal for them to increase wages just by 1%. In contrast, $\theta_{0a}(z) > 0$ and $\theta_{0a}(z) + \theta_{0w}(z) \leq 1$ with strict inequality if and only if $\gamma > 0$. That is, incumbents' overall wage responses are dampened as long as workers are risk-averse. This is intuitive. Because incumbent firms have incentives to insure workers, they do not want to fluctuate wages too much with the aggregate shocks. Moreover, these coefficients can be expressed as a function of steady-state moments, which have a clear data counterpart. The new hire firms' response $\theta_{1a}(z)$ depends only on wage markdown, $\alpha(z)$, and the elasticity of new hire wage density function, $\eta_{F_0}(z)$. These expressions have a natural counterpart in pass-through literature in the context of product price-settings (see [Burstein and Gopinath \(2014\)](#) for a survey). In the context of product price settings, it is well-known that pass-through of costs to product prices mainly depends on the (i) elasticity of demand function and (ii) super-elasticity of the demand function. Here, $\alpha(z)$ captures the former, and $\eta_{F_0}(z)$ captures the latter. In addition to wage markdown and the elasticity of the density function of incumbent firms, the incumbent firms' response depends also on the elasticity of workers' staying probability and the relative risk aversion.

Since the system consists of two ODEs with two unknowns, we need two boundary conditions. The first boundary condition describes what happens at the bottom of the job ladder, $\hat{w}_1(\underline{z}) = \hat{A}$. Because unemployed workers' outside options scale one for one with the aggregate productivity, the least productive firm, which hires only from a pool of unemployed, also needs to move wages one for one with the aggregate productivity. One may wonder why the same boundary condition does not apply for the incumbent firms at the bottom, $z = \underline{z}$. The reason is that the constraint $w_{0s} \geq A_s b$ strictly binds because the firm would like to insure workers as much as possible, and hence they are no longer in the interior solution. Therefore, the bottom boundary of the incumbent firms is at $\hat{w}_0(\underline{z}^+) \equiv \lim_{z \downarrow \underline{z}} \hat{w}_0(z)$, which is free. The second boundary condition describes the behavior at the top of the job ladder, $\hat{w}_0(\bar{z}) = \hat{w}_1(\bar{z})$. It says that both incumbent and new hire firms must find it optimal to offer exactly the same wages at the top of the job ladder. If one firm at the top offers strictly higher wages than the other at the top, it can strictly increase profits by slightly lowering wages because it does not affect the labor supply but yet reduces costs. This extremely strong form of strategic complementarity at the top of the job ladder is at the heart of the analysis that comes next.

3.2.2 Equilibrium Wage Rigidity

Having characterized the equilibrium wage responses as a system of ODEs, I am ready to study their properties.

¹⁶Second-order condition implies $\theta_{1a}(z) > 0$ for all z .

Proposition 1 (Endogenous wage rigidity). *Assume the elasticity of vacancy creation, ι , is sufficiently small. If workers are risk-neutral, $\gamma = 0$, then all wages are flexible, $\hat{w}_1(z) = \hat{w}_0(z) = \hat{A}$ for all z . If workers are risk-averse, $\gamma > 0$, then all incumbent wages are rigid, $\hat{w}_0(z) < \min\{\hat{A}, \hat{w}_1(z)\}$ for all z , and new hire wages are rigid at the top of the job ladder, $\hat{w}_1(z) < \hat{A}$ for z close enough to \bar{z} .*

The proposition states that with risk-neutral workers, wages for both incumbents and new hires are fully flexible. With risk-averse workers, incumbent wages are rigid. Perhaps more surprisingly, new hire wages are also rigid, at least toward the higher end of the job ladder. Let me turn to an explanation for each of the result, assuming $\iota = 0$. By continuity, the results hold if ι is small enough. Throughout, I often impose the elasticity of vacancy creation, ι , is sufficiently small. This is largely a technical assumption. I have not encountered any counter-example even with large ι . Moreover, it has been common to assume relatively small ι in the literature that builds on [Burdett and Mortensen \(1998\)](#) because as $\iota \rightarrow \infty$, the firm size distribution becomes too concentrated to the most productive firm.

As (12) and (13) consist a system of two ODEs, one can draw a phase diagram to explain the proposition, as I do in the left panels of Figure 3a-3c. Let us start with a case of risk-neutral workers, $\gamma = 0$, in the left panel of Figure 3a. I plot $\hat{w}_0(z)$ on a vertical axis and $\hat{w}_1(z)$ on a horizontal axis.¹⁷ A particular point $(\hat{w}_0(z), \hat{w}_1(z))$ in the phase diagram corresponds to a pair of incumbent and new hire wage responses at job ladder (productivity) z . If the point lies inside the gray square, it means that both incumbent and new hire wages respond less than the aggregate productivity, $\hat{w}_0(z) < \hat{A}$ and $\hat{w}_1(z) < \hat{A}$, or in other words, wages are rigid (sticky). In the figure, I draw two lines, each corresponding to the $\hat{w}'_0(z) = 0$ locus and the $\hat{w}'_1(z) = 0$ locus.

When $\gamma = 0$, the two lines have to go through a point (\hat{A}, \hat{A}) . Moreover, the $\hat{w}'_0(z) = 0$ locus needs to have slope greater than one because $\theta_{1w}(z) < 1$, and the $\hat{w}'_1(z) = 0$ locus needs to have a slope less than one because $\theta_{0w}(z) < 1$. One of the boundary conditions states that $\hat{w}_1(\underline{z}) = \hat{A}$, so starting from $z = \underline{z}$, it has to originate from somewhere in the vertical line that goes through (\hat{A}, \hat{A}) . The other boundary condition states that $\hat{w}_1(\bar{z}) = \hat{w}_0(\bar{z})$, so the path needs to end up somewhere in the 45° line. Then, it is immediately clear that the only path that satisfies the two boundary conditions is the one that starts from (\hat{A}, \hat{A}) at $z = \underline{z}$ and stays there until it reaches $z = \bar{z}$. That is, both incumbent and new hire wages are fully flexible. The wage response for each z is depicted in the right panel of Figure 3a. This result is intuitive. With risk-neutral workers, incumbent firms have no incentive to insure workers, so both incumbent and new hire firms' problems are homogenous in aggregate productivity and competitors' wages. Wages just scales up and down with the aggregate productivity.

More interesting cases arise when workers are risk averse, $\gamma > 0$. The left panel of Figure

¹⁷Although the two lines are generically moving around depending on z , I study the phase diagram as if the two loci are unchanged for all z . Qualitative properties are unaffected by this consideration, as long as the coefficients are continuous in z , which is the case here.

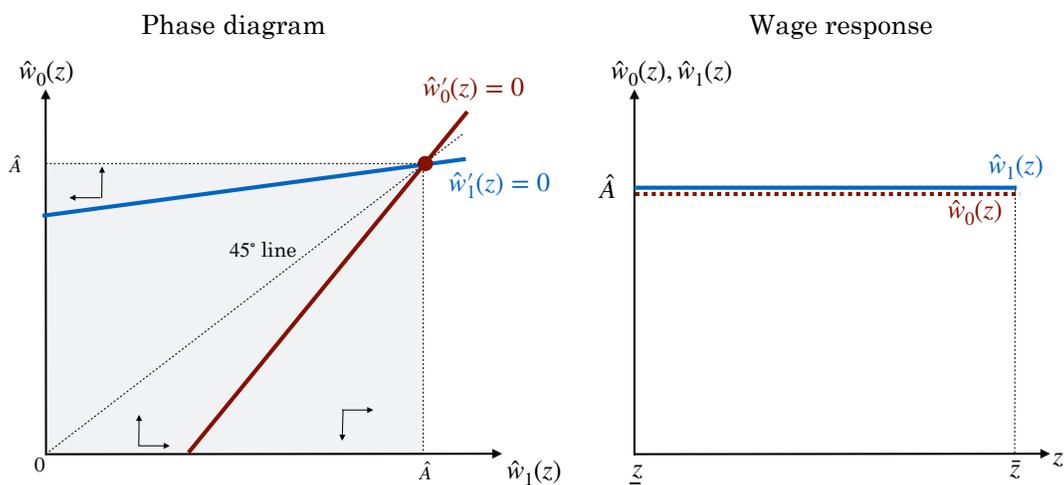


Figure 3a: $\gamma = 0$

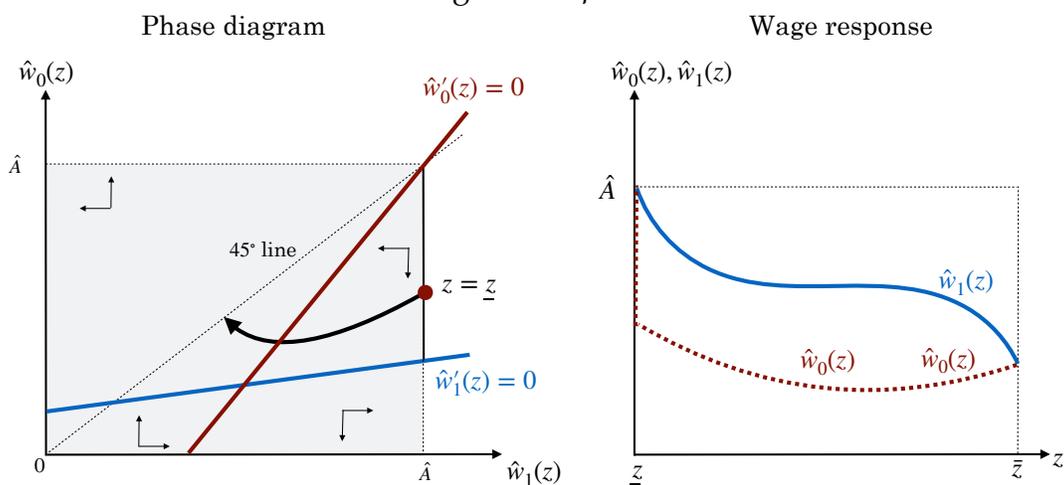


Figure 3b: $\gamma > 0$ and $\theta_{1w} > 0$

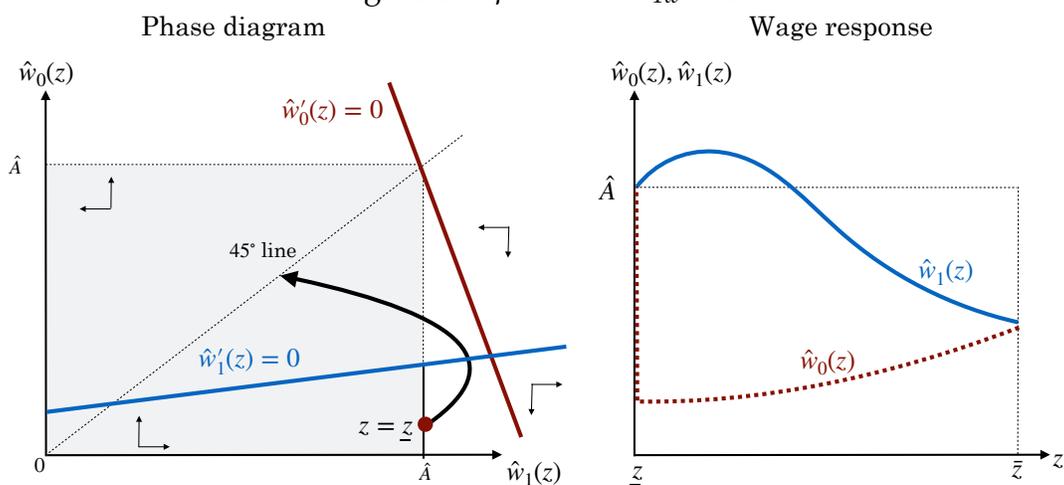


Figure 3c: $\gamma > 0$ and $\theta_{1w} < 0$

Note: The left panels of Figure 3a-3c show the phase diagrams. The right panels show the wage responses for each z .

3b shows the phase diagram with $\gamma > 0$ and typical parametrization $\theta_{1w} > 0$. With $\gamma > 0$, the $\hat{w}'_1(z) = 0$ locus uniformly shifts downward compared with $\gamma = 0$. With $\theta_{1w} > 0$, the $\hat{w}'_1(z) = 0$ locus is upward sloping. A path that satisfy the boundary conditions are drawn as a black line: it needs to start with $\hat{w}_1(\underline{z}) = \hat{A}$ and end up on the 45° line. In this case, since the entire path lies inside the gray square, both incumbent and new hire wages are rigid. The wage response at each job ladder is drawn in the right panel of Figure 3b. Incumbent wages are unresponsive throughout the job ladder. In contrast, new hire wages become less and less responsive as we look at a higher job ladder rank, eventually reaching the same rigidity at the top of the job ladder.

However, Figure 3b is not the only possibility. Suppose $\theta_{1w} < 0$. Then the $\hat{w}'_0(z) = 0$ locus is negatively sloped, as shown in the left panel of Figure 3c. In this case, the path that satisfies boundary condition could be the one depicted as a black line. The right panel of Figure 3c shows the corresponding wage response at each job ladder rank. In this case, new hire wages respond more than the aggregate productivity at the lower end of the job ladder, but incumbent wages respond less for the entire job ladder.

The result that incumbent wages are always rigid for $\gamma > 0$ is not surprising. As firms have an incentive to insure workers, they respond less than the aggregate productivity. The reason why new hire wages are also always rigid at the higher end of the job ladder comes from the strategic complementarity in wage setting. job ladder models feature a very extreme form of strategic complementarity at the top, $z = \bar{z}$: no firm wants to set wages strictly above the competitor's one. This strategic complementarity spills over from the top to lower job ladder ranks. If incumbent and new hire firms at the very top, $z = \bar{z}$, set exactly the same wages, firms at a slightly lower-rank, $z = \bar{z} - dz$, also set the similar wages. The reason why the new hire wages can overshoot at the lower-end of the job ladder is as follows. The case $\theta_{1w} < 0$ happens when

$$\eta_{F_0}(z) \geq \frac{1 - 2\alpha(z)}{\alpha(z)},$$

$\eta_{F_0}(z) = \frac{d \ln F'_0(w(z))}{d \ln w}$ is the elasticity of density of wage distributions. This says the elasticity of incumbent wage density function is large enough (but not too large as $\theta_{1a} > 0$ requires $\eta_{F_0}(z) < \frac{2-2\alpha(z)}{\alpha(z)}$). Intuitively speaking, when this is the case, the new hire firms can poach a lot more workers if they increase wages slightly more than the incumbent firms. Therefore they have an incentive to become aggressive in making high wage offers if incumbent wages are not responsive — that is, new hire firms' wage setting is strategic substitutes with respect to incumbents'.

It is worth noting that on-the-job search was the key for new hire wages to feature any kind of rigidity. Without on-the-job search, new hire firms find it optimal to offer outside options of the unemployed, $w_{1s}(z) = A_s b$, so new hire wages are fully flexible, $\hat{w}_{1s}(z) = \hat{A}$. It is precisely the competition for employed workers through which incumbent wage rigidity spills over to

new hire wage rigidity.

Relationship to recent evidence on new hire wage rigidity. My result above shows that the two empirically well grounded assumptions, (i) incumbent wage rigidity and (ii) on-the-job search, naturally leads to endogenous new hire wage rigidity, especially at the top of the job ladder. This result provides an explanation for the recently documented empirical evidence. While [Haefke et al. \(2013\)](#) or [Kudlyak \(2014\)](#) originally documented that new hire wages are substantially more cyclical than incumbent wages, more recent evidence that carefully adjusts for the job compositions ([Gertler et al., 2020](#); [Hazell and Taska, 2019](#); [Grigsby et al., 2019](#)) shows that new hire wages are much less cyclical than previously thought. However, we tend to lack a theoretical understanding of the underlying mechanisms without imposing ad-hoc constraints on wage setting. My model provides a natural explanation for this. Although there are some other theories of endogenous new hire wage rigidity ([Menzio and Moen, 2010](#); [Kennan, 2010](#)), a distinguishing feature of my theory is that it predicts that new hire wages should feature more rigidity at the higher job ladder rank. Consistently with this prediction, [Bloesch and Taska \(2019\)](#) use the data from online vacancies to document that posted wages are much less cyclical for high-wage jobs than low-wage jobs.

Non-strategic incumbent firms. The mechanism that generates endogenous new hire wage rigidity comes from strategic complementarity in wage setting. It relies on the fact *both* incumbent firms and new hire firms are acting strategically what wages to offer workers. If one has a view that the reason why incumbent wages are rigid is because of the cost of changing wages or other institutional constraints, then it might not be realistic to think incumbent firms are acting strategically. Here, I will argue that new hire wages are (asymptotically) rigid even if incumbent firms are non-strategic.

Suppose incumbent firms mechanically fix wages due to some costs of changing wages or other constraints, $\hat{w}_0(z) = 0$ for all z .¹⁸ Then, from (12), new hire wage responses are given by

$$\hat{w}_1(z) = \theta_{1a}(z)\hat{A}.$$

The key question here is whether $\theta_{1a}(z) < 1$ or not. If $\theta_{1a}(z) < 1$, then new hire wages become rigid whenever incumbent firms cannot adjust wages. The following proposition shows that this is indeed always the case toward the higher end of the job ladder:

Proposition 1' (Endogenous wage rigidity with non-strategic incumbent firms). *Assume the distribution of z is such that $\bar{z} \rightarrow \infty$ with finite variance. If incumbent firms have exogenously fixed wages, $\hat{w}_0(z) = 0$, then new hire wages are rigid at the top of the job ladder, $\hat{w}_1(z) < \hat{A}$ for z close enough to \bar{z} .*

¹⁸The argument goes through for any constant $C < \hat{A}$ with $\hat{w}_0(z) = C$.

Therefore, regardless of incumbent firms being strategic or not, the job-ladder model robustly predict that there should be endogenous new hire wage rigidity toward the top of the job-ladder. However, the underlying mechanism here is distinct from the one with the strategic incumbent firms. Proposition 1' comes from the fact that very productive firms are shielded from competition in the labor market. The degree of competition in this class of model is determined by the number of neighboring competitors. Since very productive firms have fewer of them, their monopsony power is high. As firms become more monopsonistic, their wage offers are increasingly tied to the workers outside options, $w_0(z)$, which is fixed here. Therefore, their wage offers become disconnected from the marginal product of labor. As such, wages are not responsive to aggregate productivity changes. In fact, I can show

$$\lim_{z \rightarrow \infty} \theta_{1a}(z) = 0,$$

which means new hire wages become completely rigid for very productive firms when incumbent firms fix wages.

The fact that very productive firms are insulated from competition in the labor market is a common feature of [Burdett and Mortensen \(1998\)](#) models. Recently, [Gouin-Bonenfant \(2020\)](#) exploits this insight to study the implications for labor shares. I exploit the same insight but shed light on the implications for wage rigidity.

3.3 Incumbent Wage Rigidity Drives Unemployment Fluctuations

In Section 3.2, I have shown the model generates both incumbent and new hire wage rigidity, but which wage rigidity is important for unemployment fluctuations? [Pissarides \(2009\)](#) makes a strong argument that *only* new hire wage rigidity matters for job creation. In what follows, I challenge his conclusion.

I present two results in sequence:

Proposition 2 (Incumbent wage rigidity as sufficient statistic). *Aggregate vacancy, V_s , is a function only of incumbent wage distribution, $\{w_{0s}(z)\}$. To a first order approximation, firm-level and aggregate vacancy responses are given by*

$$\hat{v}(z) = \iota \left[\frac{1 - \alpha(z)}{\alpha(z)} (\hat{A} - \hat{w}_0(z)) + \hat{\lambda}^F \right], \quad (14)$$

$$\hat{V} = \frac{\iota}{1 + \iota(1 - \kappa)} \mathbb{E}_v \left[\frac{1 - \alpha(z)}{\alpha(z)} (\hat{A} - \hat{w}_0(z)) \right], \quad (15)$$

where $\kappa \equiv \frac{d \ln \mathcal{M}(\tilde{\mu}, V)}{d \ln V}$ is the elasticity of the matching function with respect to vacancy, and $\mathbb{E}_v[x(z)] \equiv \int x(z)(v(z)/V)dG(z)$ denotes the vacancy-weighted average of a given variable.

The above result shows incumbent wages are sufficient statistics for unemployment fluctuations. It comes from the fact that since wages of new hires, $\{w_1(z)\}$, are optimally chosen, the profit from vacancy posting is not a function of $\{w_1(z)\}$. Consequently, while my model delivers new hire wage rigidity endogenously, such rigidity in itself has no consequence on unemployment fluctuations. The following result shows imposing further rigidity in new hire wages has no consequence either.

Proposition 2' (Incumbent wage rigidity as sufficient statistic with constrained new hire wages). *Assume new hires wage changes are exogenously given by $\hat{w}_1(z) = \hat{w}_1^{exo}(z)$ for some $\hat{w}_1^{exo}(z)$. To a first order approximation, the firm-level and the aggregate level vacancy responses are still given by (14) and (15).*

While the result comes from the linearization of (6), the proposition is a striking result. It says that incumbent wage rigidity is the *only* source of fluctuations in job creation, and any form of rigidity in the wage of new hires, no matter whether the rigidity is endogenously derived or exogenously imposed, has no consequence on unemployment fluctuations. In other words, incumbent wage rigidity is a “sufficient statistic” for unemployment fluctuations. The result is precisely the opposite from what the conventional wisdom would suggest.

First, why does incumbent wage rigidity matter for job creation? It is because incumbent wage rigidity affects the prospect of poaching. If incumbent wages do not fall when the aggregate productivity falls, then incumbent workers are better paid relative to the overall economic condition. Under this situation, potential new employers have a hard time attracting incumbent workers. This reduces the return from the posting vacancy, in turn reducing job creation. Second, why does wage rigidity of new hires not matter for job creation? It is because of envelope theorem. Without shocks to the aggregate productivity, new hire firms were optimally setting wages to maximize profits, facing trade-off of paying higher labor costs and attracting more workers. Therefore, any first order (non-)response of their wages has no effect on profits from posting a vacancy, and in turn, on job creation.

The fact that rigidity of incumbents matters is very robust; the fact that it is the only rigidity that matters, so that new wage rigidity does not matter, is less robust to extension of the model. It is always the case that a firm is not affected by its own rigidity of the wage, but there are potentially general equilibrium effects from the wage of others. The result here is stark because of the two-period assumption. I explore the robustness of the result in the context of quantitative infinite-horizon model in Section 6. Although new hire wage rigidity matters there, the quantitative magnitudes are small, and I find the incumbent wage rigidity still remains the dominant source of unemployment fluctuations.

Relationship to Pissarides (2009). The fact that incumbent wage rigidity does matter for job creation comes from the presence of on-the-job search. The fact that new hire wage rigidity does

not matter for job creation comes from the assumption of wage posting. These two assumptions shape the backbone of [Burdett and Mortensen's \(1998\)](#) model. [Pissarides \(2009\)](#) and many others obtained the opposite conclusion because the argument is based on the DMP model. The DMP model has been popularly used to study the business cycle dynamics of unemployment due to its tractable nature, but this class of model assumes wage-bargaining and no on-the-job search.

To clarify the difference, consider an alternative version of my model with two modifications. First, let us assume there is no on-the-job search, $\zeta = 0$. Second, assume wages are bargained for instead of posted. With these assumptions, any wage $w_1(z) \in [Ab, Az]$ generates positive gains from trade between unemployed and firms with productivity z . Therefore, wages can be anywhere in the bargaining set, $[Ab, Az]$, as in [Hall \(2005\)](#). Starting from steady-state value of $w_1(z)$, suppose the wage responses are given by $\hat{w}_1^{exo}(z)$, as before. The rest of the models are unchanged.

Appendix [A.7](#) shows that with these assumptions, to a first order, the firm-level vacancy response is given by

$$\hat{v}(z) = \iota \left[\frac{1 - \alpha(z)}{\alpha(z)} (\hat{A} - \hat{w}_1(z)) + \hat{\lambda}^F \right], \quad (16)$$

and the aggregate level response is

$$\hat{V} = \frac{\iota}{1 + \iota(1 - \kappa)} \mathbb{E}_v \left[\frac{1 - \alpha(z)}{\alpha(z)} (\hat{A} - \hat{w}_1(z)) \right]. \quad (17)$$

These expressions echo [Pissarides \(2009\)](#) that new hire wages are the only source of fluctuations in job creation. Equation (17) is also a version of [Ljungqvist and Sargent's \(2017\)](#) formula incorporating firm heterogeneity and the finite elasticity of vacancy creation. By comparing (17) with (15), one again sees the striking contrast between the two. The two expressions only differ in terms of whether it is new hire or incumbent wages that enter the job creation equation. Expression (17) does not depend on incumbent wages because by abstracting from on-the-job search, it mechanically shuts down any interaction between incumbent workers and labor market dynamics. Expression (17) does depend on new hire wages because firms are not optimizing what wages to offer. With wage-bargaining, firms would prefer to pay as low of wages as possible so long as workers accept the job. This implies that new hire wage rigidity does have a first order effect.

Given that a different set of assumptions deliver strikingly different implications of wage rigidity, the natural question to ask is which assumptions are empirically relevant. The prevalence of on-the-job search is hard to deny. As mentioned in the introduction, 40-50% of new hires are employer-to-employer transitions. The assumption of wage posting is more controversial, but as discussed in Section [2.2](#), the available evidence is more supportive of wage posting than wage bargaining.

Employer-to-Employer (EE) transition rates. Equation (15) immediately implies that the UE (unemployment to employment) transition rate is unaffected by new hire wage rigidity because log-deviation in the UE rate is simply

$$\widehat{UE} \equiv \hat{\lambda}^U = \kappa \hat{V}.$$

In contrast, the EE transition rate is affected by the new hire wage rigidity. The EE rate is defined as

$$EE_s = \lambda_s^E \int (1 - F_{1s}(w_{0s}(\tilde{z}))) \ell_0(\tilde{z}) dG(\tilde{z})$$

because workers employed in firm z move to new employers whenever they receive better wage offers, which happens with probability $1 - F_{1s}(w_{0s}(z))$. The log deviation in the EE rate is

$$\widehat{EE} = \frac{\lambda^E}{EE} \int F_1'(w_0(\tilde{z})) w(\tilde{z}) [\hat{w}_1(\tilde{z}) - \hat{w}_0(\tilde{z})] \ell_0(\tilde{z}) dG(\tilde{z}) + (\text{terms unrelated to } \hat{w}_1^{exo}(\tilde{z})) \quad (18)$$

This expression implies that relative rigidity in incumbent and new hire wages matter for the EE rate, and the EE rate responds more when new hire wages are more flexible relative to incumbent wages. Intuitively speaking, when new hire wages respond more than the incumbent wages, new hire firms can poach more workers. In fact, a firm poaches workers from other firms with higher productivity, causing a misallocation of workers. Therefore, although new hire wage rigidity is irrelevant to the UE rate, it matters a lot for the EE rate.

Since $\hat{w}_1(z) \geq \hat{w}_0(z)$ in equilibrium as we saw in Proposition 1, equation (18) implies that EE rate is strongly amplified relative to the case with flexible wages, $\hat{w}_1(z) = \hat{w}_0(z) = \hat{A}$. This fact is consistent with the evidence documented in [Haltiwanger, Hyatt, Kahn, and McEntarfer \(2018\)](#). They show that the firm wage ladder is strongly procyclical, meaning the number of workers who climb up the job-ladder collapses in recessions.¹⁹ My theory provides a natural explanation of this.

3.3.1 Beyond First Order Approximation

Non-optimizing new hire wages in the steady-state. The reason why new hire wage rigidity does not affect job creation is because of the envelope theorem. Then, it is natural to think that if new hire wages are not optimized in the steady-state, they start to matter for unemployment fluctuations. I will show that although new hire wages matter, the way it matters is more subtle than one may think. Holding the incumbent wage distribution the same as (9), suppose the

¹⁹See also [Barlevy \(2002\)](#), [Mukoyama \(2014\)](#), and [Nakamura, Nakamura, Phong, and Steinsson \(2019\)](#) for related evidence.

new hire wage distribution in the steady-state equilibrium is given by $w_1^{non}(z) \neq w(z)$. Let

$$\tau_1(z) \equiv \frac{(1-\chi)F'(w_1^{non}(z))w_1^{non}(z)}{(\chi + (1-\chi)F(w_1^{non}(z)))} - \frac{w_1^{non}(z)}{Az - w_1^{non}(z)}$$

denote the wedge of the optimality condition for the wage setting of new hire firms. I consider a small deviation of $w_1^{non}(z)$ from $w(z)$ so that $\tau_1(z) < 0$ if $w_1^{non}(z) > w(z)$, $\tau_1(z) = 0$ if $w_1^{non}(z) = w(z)$, and $\tau_1(z) > 0$ if $w_1^{non}(z) < w(z)$. Let $\hat{w}_1^{exo}(z) \equiv \ln w_{1s}(z) - \ln w_1^{exo}(z)$ denote the arbitrary constraint on new hire wage setting expressed as a log-deviation from the non-optimized ones. Linearizing the optimality condition for vacancy creation gives

$$\hat{v}(z) = \iota \left[\hat{\lambda}^F + \frac{1-\alpha(z)}{\alpha(z)} (\hat{A} - \hat{w}_0^{exo}(z)) + \tau_1(z) \hat{w}_1^{exo} \right].$$

This expression immediately tells us new hire wage rigidity now matters for unemployment fluctuations. Suppose $\hat{A} > 0$. The more rigidity in new hire wages (lower \hat{w}_1^{exo}) implies amplification of job creation if $\tau_1(z) < 0$, but it implies dampening of job creation if $\tau_1(z) > 0$. Intuitively speaking, when the initial wage is located in the increasing part of the profit function, the fact that wages cannot increase will decrease the profit from the vacancy posting. This dampens job creation in response to the positive shock to the aggregate productivity. In contrast, when the initial wage is located in a decreasing part of the profit function, job creation is amplified. Therefore, whether new hire wage rigidity amplifies unemployment fluctuation or not is generally ambiguous.

Second-order approximation. Another consideration is to study higher-order effects. Applying the second-order approximation to the optimality condition for vacancy creation at the firm-level, one can write

$$\frac{d^2 \log v(z)}{d \log A^2} = v_1 \frac{d^2 \log w_1^{exo}(z)}{d \log A^2} + v_2 \frac{d \log w_0^{exo}(z)}{d \log A} \frac{d \log w_1^{exo}(z)}{d \log A} + \left(\text{terms unrelated to } \frac{d \log w_1^{exo}(z)}{d \log A} \right)$$

where $v_1 \equiv \iota \frac{(1-\alpha(z))}{\alpha(z)^2} [2(1-\alpha(z)) - \alpha(z)\eta_{F_0}(z)] < 0$ and $v_2 \equiv -\iota \frac{(1-\alpha(z))}{\alpha(z)} \left\{ \eta_{F_0}(z) - \frac{(1-\alpha(z))}{\alpha(z)} \right\}$. Not surprisingly, new hire wage rigidity has a second-order effect on job creation, but does new hire wage rigidity amplify job creation? Not necessarily. The first term implies that relative to the case without rigidity, $\frac{d \log w_1^{exo}(z)}{d \log A} > 0$, if we impose rigid new hire wages, $\frac{d \log w_1^{exo}(z)}{d \log A} = 0$, fluctuation in job creation is amplified in response to a negative shock, but it is dampened in response to a positive shock. The sign of the second term is generally ambiguous. Therefore, incorporating new hire wage rigidity in this environment does not necessarily amplify job creation, even to a second-order.

3.4 Firm Insurance Drives Unemployment Fluctuations

In Section 3.3, I studied the implications of an arbitrary form of wage rigidity on vacancy creation, but I also showed that in Section 3.2, my model endogenously generates wage rigidity as an equilibrium outcome. Now, I connect the two to study the unemployment fluctuations arising from equilibrium wage rigidity. Extending Lemma 3, the first order equilibrium responses with endogenous vacancy creation $\{\hat{w}_1(z), \hat{w}_0(z), \hat{v}(z), \hat{V}, \hat{\lambda}^E, \hat{\lambda}^F\}$ solve

$$\begin{aligned}\hat{w}_1(z) &= \theta_{1a}(z)\hat{A} + \theta_{1w}(z)\hat{w}_0(z) - \theta_{1a}(z)\alpha(z)\frac{w(z)}{w'(z)}\hat{w}'_0(z) \\ \hat{w}_0(z) &= \theta_{0a}(z)\hat{A} + \theta_{0w}(z)\hat{w}_1(z) - \theta_{0a}(z)\alpha(z)\frac{w(z)}{w'(z)}\hat{w}'_1(z) + \theta_{0a}(z)\alpha(z)\{1 - \theta_{\lambda,p}(z)\}\hat{\lambda}^E \\ &\quad + \theta_{0a}(z)\alpha(z)\theta_{\lambda,r}(z)(\hat{V}(z) - \hat{V}) + \theta_{0a}(z)\alpha(z)(\hat{v}(z) - \hat{V}) \\ \hat{v}(z) &= \iota \left[\frac{1 - \alpha(z)}{\alpha(z)}(\hat{A} - \hat{w}_0(z)) + \hat{\lambda}^F \right],\end{aligned}\tag{19}$$

with $\hat{\lambda}^F = (\kappa - 1)\hat{V}$, $\hat{\lambda}^E = \kappa\hat{V}$, and $\hat{V} = \frac{1}{\bar{v}} \int v(z)\hat{v}(z)dG(z)$, where $\hat{V}(z) \equiv \frac{\int^z v(\bar{z})\hat{v}(\bar{z})dG(\bar{z})}{\int^z v(\bar{z})dG(\bar{z})}$ is the log change of the cumulative amount of vacancies, $\theta_{\lambda,p}(z) \equiv \frac{\lambda^E(1-F(w(z)))}{1-\lambda^E+\lambda^EF(w(z))}$ is the share of workers who meet with other firms and are poached, and similarly $\theta_{\lambda,r}(z) \equiv \frac{\lambda^EF(w)}{1-\lambda^E+\lambda^EF(w)}$ is the share of workers who meet with other firms but reject the offer. The boundary conditions remain the same: $\hat{w}_1(\underline{z}) = \hat{A}$ and $\hat{w}_0(\underline{z}) = \hat{w}_1(\underline{z})$.

Compared with Lemma 3, the vacancy responses enter into the best response function of incumbent wage settings. For example, if there is a positive response of aggregate vacancy, $\hat{\lambda}^E > 0$, all else equal, incumbent firms have the incentive to raise wages to prevent workers from leaving.

The following proposition shows that the firm insurance does drive unemployment fluctuations.²⁰

Proposition 3 (Risk-aversion and unemployment fluctuations). *If workers are risk-neutral, $\gamma = 0$, there is no unemployment fluctuation.*

As we move to risk-averse workers, $\gamma > 0$, the economy exhibits unemployment fluctuation, $\hat{V} > 0$. Furthermore, one can show that the maximum unemployment fluctuation occurs in the limit where workers are infinitely risk-averse, $\gamma \rightarrow \infty$:

$$\lim_{\gamma \rightarrow \infty} \hat{V} \rightarrow \frac{\iota}{1 + \iota(1 - \kappa)} \mathbb{E}_v \left[\frac{1 - \alpha(z)}{\alpha(z)} \right] \hat{A} > 0.$$

²⁰The assumption that ι is not too large is largely a technical assumption. If ι is too large, I cannot exclude the possibility that incumbent firms raise wages more than the productivity increases. However, I never found such a numerical example.

Figure 4 illustrates the result by plotting wages (left-top), vacancy (right-top), the UE rate (left-bottom) and the EE rates (right-bottom) against the relative risk aversion, γ . First, the model generates no unemployment fluctuations with risk-neutral workers, $\gamma = 0$. As we have already seen, with risk-neutrality, incumbent wages move one for one with the aggregate productivity. Since the cost of vacancy also scales with the aggregate productivity, the profitability from a vacancy posting is unchanged.²¹ This serves as a useful benchmark. As soon as we move away from risk-neutral workers, incumbent firms start to insure workers, which generate incumbent wage rigidity. This rigidity tends to spill over to new hire wages, as depicted in the left-top panel of Figure 4. Because we have already seen that the incumbent wage rigidity drives the fluctuation in vacancy creation, the response of vacancy increases with γ (the right-top panel of Figure 4), reaching the limit described in the proposition as $\gamma \rightarrow \infty$. Moreover, the model predicts the EE rate to substantially respond more than the UE rate because the fact that new hire wages respond more than the incumbent wages make poaching easier, as we have already discussed.

The fact that firm insurance solely drives unemployment fluctuation is in stark contrast to the arguments made in Barro (1977) and Rudanko (2009). Both papers point out that long-term contracts between firms and workers do not contribute to unemployment fluctuations. Here, on-the-job search was crucial to reach the opposite conclusion. The following proposition formally illustrates the importance of on-the-job search:

Proposition 4 (On-the-job-search and unemployment fluctuations). *If there is no on-the-job search, $\zeta = 0$, there is no unemployment fluctuation.*

Abstracting from on-the-job search shuts down any interaction between incumbent wages and labor market dynamics. Since the hiring pool only consists of unemployed and their outside option scales with the aggregate productivity, the return from vacancy posting is invariant to the aggregate productivity. In this case, as explained in Rudanko (2009) and Pissarides (2009), the incentive to create jobs is disconnected from the incumbent wage rigidity, no matter how rigid they are.

It is worth emphasizing wage rigidity and unemployment fluctuation in my model solely come from optimal contracting problems between firms and workers. The theory differs from existing models of wage rigidity and unemployment fluctuations in the following two senses: First, it does not rely on any unexplained inefficiencies such as the ad-hoc cost of changing wages, and, thus, immune to Barro's (1977) critique that wage rigidity should not interfere with mutually beneficial contracts. Second, it does not rely on an arbitrary choice of the wage setting rule in the bargaining set (Hall, 2005). The degree of wage rigidity and unemployment fluctuations in my model are disciplined by the structural parameters, such as workers' risk

²¹The same benchmark case appears in Blanchard and Galí (2010) and Kehoe et al. (2019).

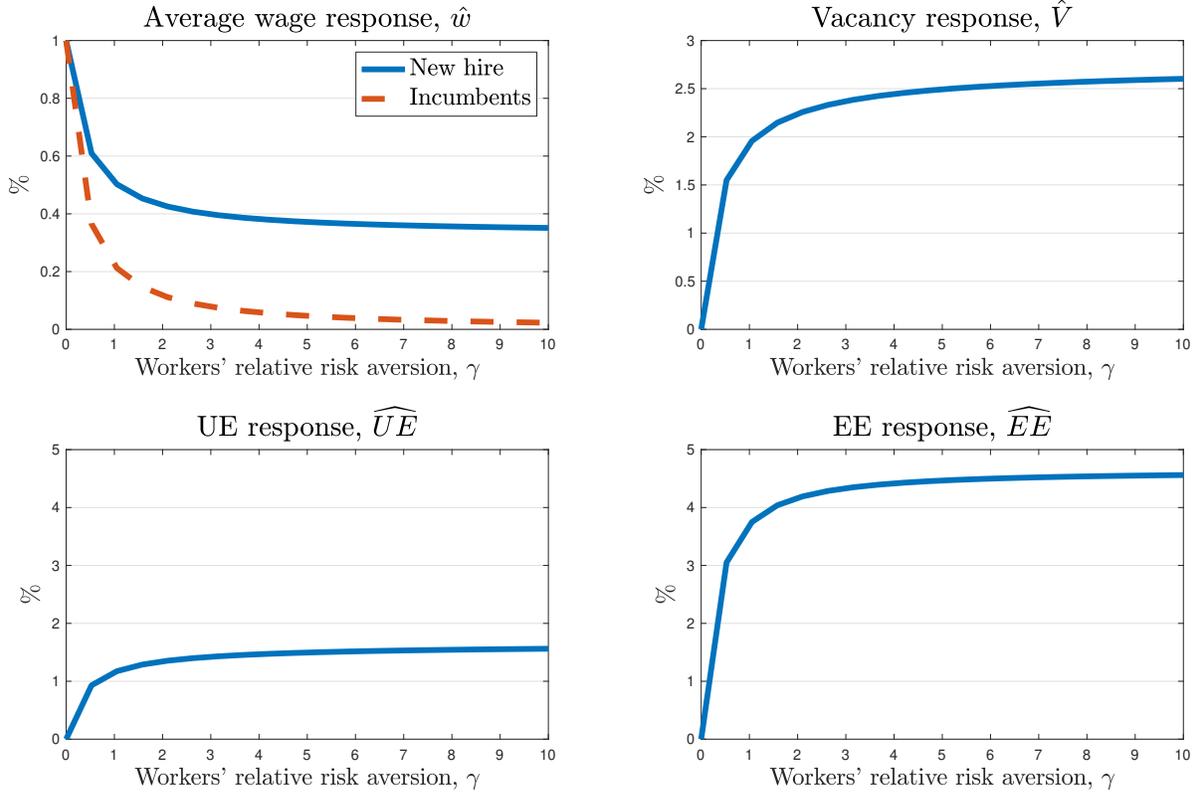


Figure 4: Workers' risk aversion and wage and unemployment fluctuations

Note: The figure shows a numerical example of the responses to a 1% aggregate productivity shock for each value of γ . All the reported values are log deviations from the steady-state. The parameter values are $\mu = 0.06, \zeta = 0.2, \mathcal{M}(\bar{\mu}, V) = \bar{\mu}^{1-\kappa} V^\kappa, \kappa = 0.6, \iota = 1, b = 1, \bar{c}(z) = z^{\bar{c}}, \bar{c} = 10, G(z) = 1 - z^{-\alpha}, \alpha = 5$.

aversion. In contrast, models of wage rigidity in DMP tradition lack such discipline, so they cannot speak to the questions of how wage rigidity changes with counterfactual policies, for example.

Propositions 3 and 4 continue to hold, without qualifications, in the quantitative infinite horizon model that I study later in Section 6.

4 Extensions: Internal Firm Fairness and Public Insurance

Building on the insights that I derived in Section 3, I consider two comparative statics in the model. The first exercise studies the effect of imposing fairness constraint within a firm, which prevents firms to discriminate incumbent workers and new hires. The second exercise considers the effect of public insurance.

4.1 Fairness Constraints Dampen Unemployment Fluctuations

In the baseline model, I have assumed that incumbent wages and new hire wages can be set separately in an unconstrained manner. However, in practice, if incumbent workers and new hires belong to the same firm, it might be difficult to discriminate wages due to fairness concerns.²² It is often argued that such a constraint amplifies unemployment fluctuations by making new hire wages more rigid. This idea at least goes back to [Bewley \(1999\)](#), and has been formalized later in several papers ([Snell and Thomas, 2010](#); [Rudanko, 2019](#); [Menzio, 2004](#); [Gertler and Trigari, 2009](#)).²³ Because conventional wisdom says that new hire wage rigidity is the source of amplification, it is natural to expect that any constraint that prevents the flexible adjustment of new hire wages would amplify unemployment fluctuations. However, I will argue that these implications are reversed in my model. The key idea is that fairness constraints make new hire wages more rigid, but incumbent wages more flexible. As incumbent wage rigidity is the source of amplification in my model, the constraint dampens unemployment fluctuations.

I assume that the boundary of firms are such that each productivity z corresponds to a single firm.²⁴ Then, I impose that the firms cannot discriminate new hire wages and incumbent wages due to fairness concerns or other social norms, $w_{0s}(z) = w_{1s}(z)$. Each firm z solves the following problem:

$$\begin{aligned}
 & \max_{w_{0s}, w_{1s}, v_s} \sum_{s \in \{h, l\}} \pi_s \left[(A_s z - w_{0s})(1 - \lambda_s^E + \lambda_s^E F_{1s}(w_{0s})) \ell_0(z) \right. \\
 & \quad \left. + \lambda_s^F v_s (\chi + (1 - \chi) F_{0s}(w_{1s})) (A_s z - w_{1s}) - c_s(v_s; z) \right] \\
 & \text{s.t.} \quad \sum_{ss \in \{h, l\}} \pi_s \left[(1 - \lambda_s^E) u(w_{0s}) + \lambda_s^E \int \max\{u(w_{0s}), u(\tilde{w})\} dF_{1s}(\tilde{w}) \right] \geq \bar{W}_0(z), \\
 & \quad w_{0s} \geq A_s b, \quad w_{1s} \geq A_s b, \\
 & \quad w_{0s} = w_{1s}.
 \end{aligned}$$

Therefore, firms maximize the weighted average of profits from new hires (the first term) and incumbents (the second term) while delivering the promised utility to incumbent workers. I again consider a perturbation around a symmetric steady-state equilibrium. In the steady-state, the fairness constraint, $w_{0s} = w_{1s}$, is not binding because incumbent workers and new hires are offered the same wages anyway. With shocks, the constraint is binding because incumbent and new hire firms now face different incentives to set wages. Let $\hat{w}(z) \equiv \hat{w}_0(z) = \hat{w}_1(z)$ denote the firm-level wage responses. The following lemma characterizes the response to the aggregate

²²The presence of such social norms are documented empirically ([Card et al., 2012](#); [Breza et al., 2018](#)).

²³Besides the business cycle literature, it has been common to impose fairness (equal treatment) constraints in wage posting models since [Burdett and Mortensen \(1998\)](#).

²⁴Or one can think that all firms with the same productivity are symmetric.

productivity shock:

With fairness constraints, first order equilibrium responses, $\{\hat{w}(z), \hat{v}(z), \hat{V}, \hat{\lambda}^E, \hat{\lambda}^F\}$ solve

$$\begin{aligned}\hat{w}(z) &= \theta_a^{eq}(z) d \ln A_s - \theta_a^{eq}(z) \alpha(z) \frac{w(z)}{w'(z)} \hat{w}'(z) + \theta_a^{eq}(z) \varphi(z) \alpha(z) \{1 - \theta_{\lambda,p}(z)\} \hat{\lambda}^E \\ &\quad + \theta_a^{eq}(z) \varphi(z) \alpha(z) \theta_{\lambda,r}(z) (\hat{V}(z) - \hat{V}) + \theta_a^{eq}(z) \varphi(z) \alpha(z) (\hat{v}(z) - \hat{V}), \\ \hat{v}(z) &= \iota \left[\frac{1 - \alpha(z)}{\alpha(z)} (\hat{A} - \hat{w}_0(z)) + \hat{\lambda}^F \right],\end{aligned}$$

$\hat{\lambda}^F = (\kappa - 1)\hat{V}$, $\hat{\lambda}^E = \kappa\hat{V}$, and $\hat{V} = \frac{1}{V} \int v(z) \hat{v}(z) dG(z)$ with the boundary condition $\hat{w}(\underline{z}) = \hat{A}$, where $\theta_a^{eq}(z) \equiv \frac{1}{1 + \gamma \varphi(z) \theta_\lambda(z)}$ and $\varphi(z) \equiv \frac{\lambda^E F_1'(w(z)) \ell_0(z)}{\lambda^F v(z) (1 - \chi) F_0'(w(z)) + \lambda^E F_1'(w(z)) \ell_0(z)}$.

As one might expect, with fairness constraints, the best responses are the weighted average of the best responses of (12) and (13) after imposing $\hat{w}_0(z) = \hat{w}_1(z)$. The following result is immediate from the above expressions:

Proposition 5 (Fairness constraints and wage rigidity). *Assume the elasticity of vacancy creation, ι , is sufficiently small. Fairness constraints raise the flexibility of incumbent wages at the bottom of the job-ladder, $\hat{w}(z) > \hat{w}_0(z)$ for z close enough to \underline{z} .*

The result says near the bottom of the job ladder, incumbent wages become more flexible, which comes from the boundary condition at the bottom. This is intuitive, as incumbent wages not only serve as insurance but also need to attract new workers. While I cannot prove that this holds globally, the wage rigidity at the bottom of the job ladder plays a dominant role for unemployment fluctuations. This is because low-productivity firms have a low surplus (low $\alpha(z)$), so their vacancies are particularly more sensitive to wage rigidity (see equation (16)).

Figure 5 shows a numerical example of how incorporating fairness constraints affects labor market fluctuations. The left top panel shows the responses of wages. As one would expect, the fairness constraint increases the flexibility of incumbent wages, and reduces new hire wage flexibility for most of the range of γ . Because incumbent wage rigidity is the sole driver of vacancy fluctuations, the fairness constraints dampen the vacancy response, as the right top panel shows. This is in stark contrast to conventional wisdom. The bottom two panels show the effect on the EE and the UE rates. Notably, fairness constraint dampens the EE responses much more than the UE response. This comes from the fact that with fairness constraints, the term highlighted in (18) is zero. Because wages are strictly increasing in productivity z , workers always flow from less productive firms to more productive firms. Therefore, in contrast to the case without fairness constraints, there is no cyclical misallocation.

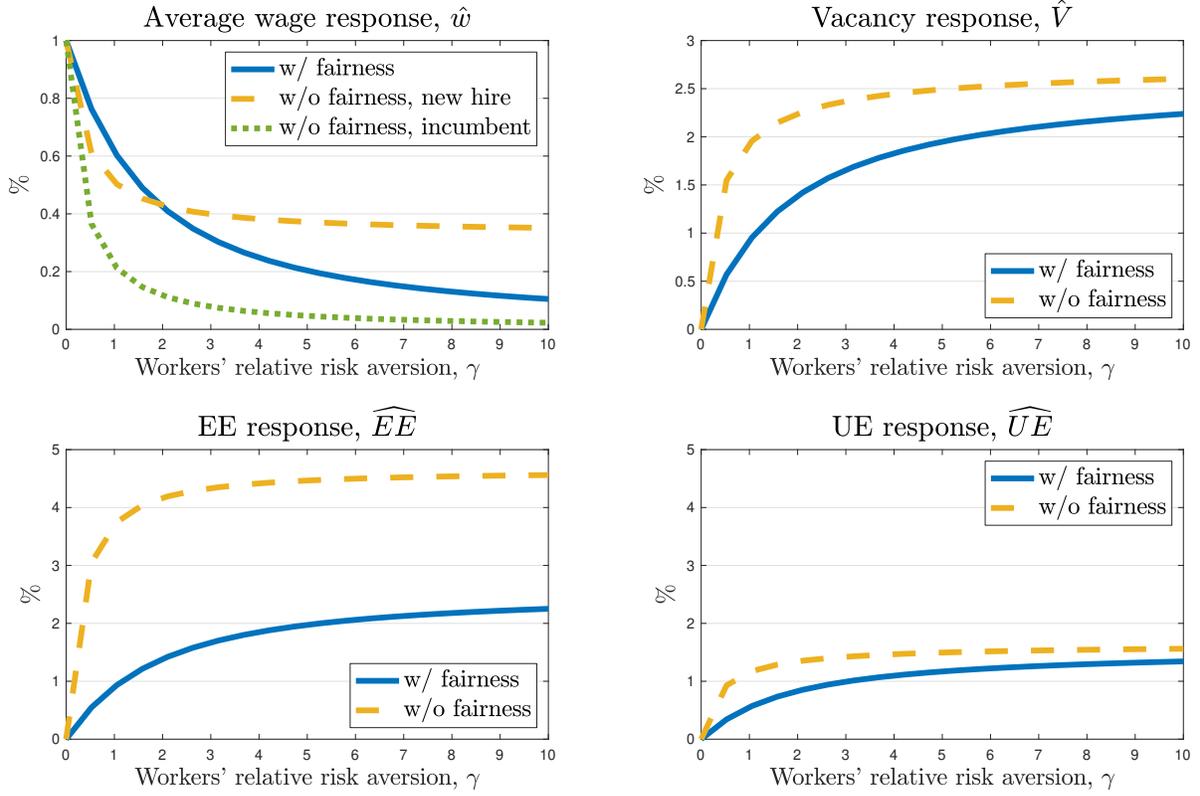


Figure 5: The impact of fairness constraints on labor market fluctuations

Note: The figure shows a numerical example of the responses to a 1% aggregate productivity shock for each value of γ . All the reported values are log deviations from the steady-state. The parameter values are the same as Figure 4.

4.2 Government-provided Insurance Dampens Unemployment Fluctuations

The source of unemployment fluctuations in my model comes from firm insurance. What if the government could also provide insurance to workers? Suppose the government imposes lump-sum taxes/transfers, T_s , in state s to all workers (including the unemployed), financed via a lump-sum tax from entrepreneurs. Under this assumption, the only modification is the promise-keeping constraint of incumbent firms:

$$\sum_{ss \in \{h,l\}} \pi_s \left[(1 - \lambda_s^E) u(w_{0s} + T_s) + \lambda_s^E \int \max\{u(w_{0s} + T_s), u(\bar{w} + T_s)\} dF_{1s}(\bar{w}) \right] \geq \bar{W}_0(z).$$

Because income taxes are unconditional, workers' job mobility decisions are not affected. The transfers are revenue-neutral in expectation, $\sum_s \pi_s T_s = 0$. I assume that in a steady-state, $T = 0$, and $dT_l = -dT_h \equiv dT > 0$ under small aggregate risk. This means the government transfers money in recessions and taxes in booms. The linearized best response of incumbent firms now

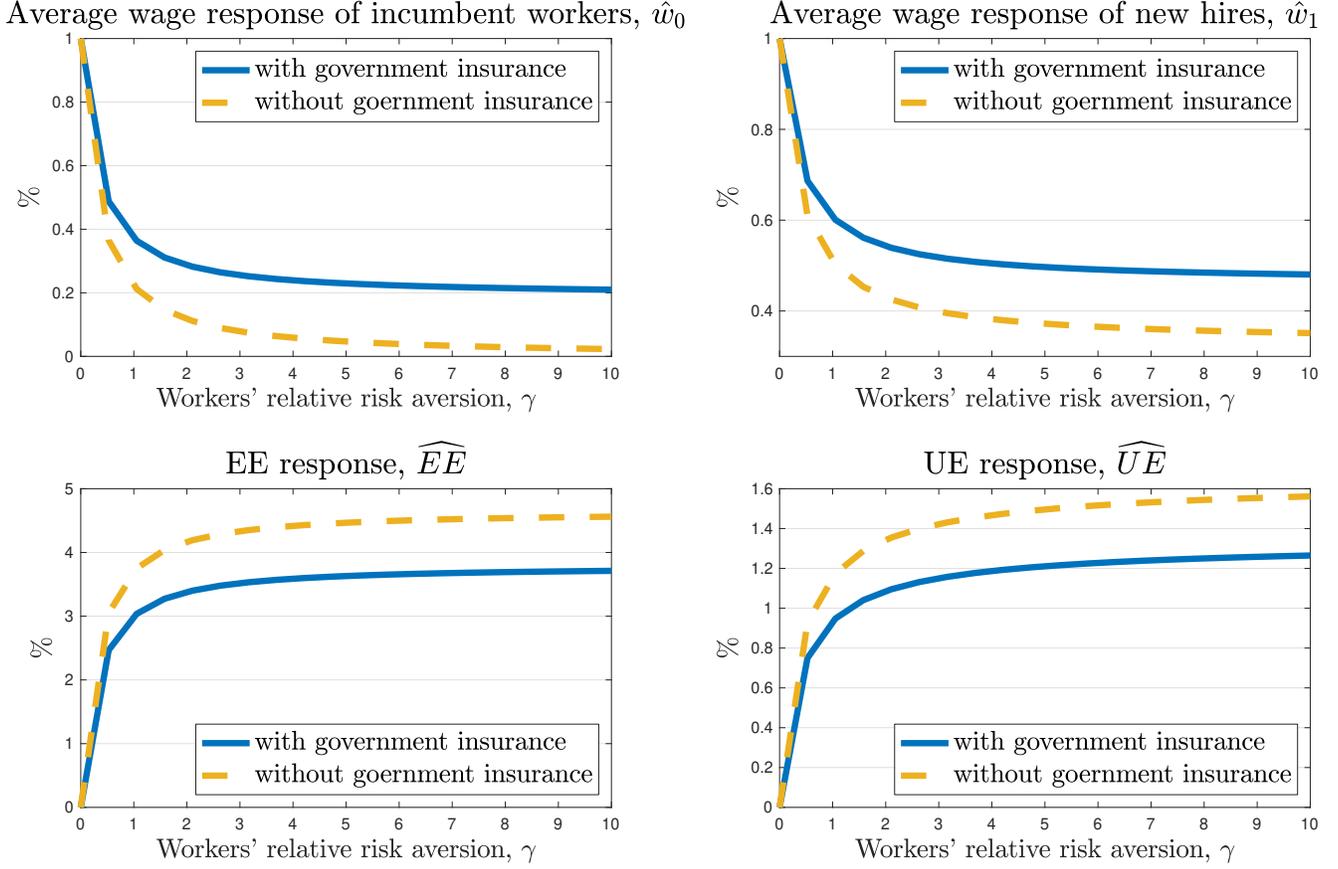


Figure 6: Labor market response with and without government insurance

Note: Figure 6 shows a numerical example of the responses to a 1% aggregate productivity shock for each value of γ . All the reported values are log-deviation from the steady-state. The cyclicity of the transfer is set to $dT = 0.2$. The remaining parameter values are the same as Figure 4.

become

$$\hat{w}_0(z) = \theta_{0a}(z)\hat{A} + \underbrace{\gamma\omega_2(z)\theta_{0a}(z)}_{\text{public insurance effect } (\geq 0)} \frac{1}{w(z)} dT + \theta_{0w}(z)\hat{w}_1(z) - \theta_{0a}(z)\alpha(z) \frac{w(z)}{w'(z)} \hat{w}'_1(z) \quad (20)$$

$$+ \theta_{0a}(z)\alpha(z) \{1 - \theta_{\lambda,p}(z)\} \hat{\lambda}^E + \theta_{0a}(z)\alpha(z)\theta_{\lambda,r}(z) (\hat{V}(z) - \hat{V}) + \theta_{0a}(z)\alpha(z) (\hat{v}(z) - \hat{V}),$$

where $\omega_2(z) > 0$ is defined in Appendix A.3. All the other equilibrium conditions are unchanged.

The following result is immediate:

Proposition 6 (Public insurance and unemployment fluctuations). *If workers are risk-neutral, $\gamma = 0$, public insurance has no effect on equilibrium.*

Moving to the case with risk-averse workers, $\gamma > 0$, we can see from equation (20) that, holding everything else constant, public insurance makes incumbent wages more flexible. The

intuition is that firms now do not need to provide insurance as much as before because the government partially substitutes for it. If incumbent wages become more flexible, this dampens unemployment fluctuations. Figure 6 shows a numerical example by making a comparison with and without government insurance. I set $dT = 0.2$. The left panel confirms that incumbent wages become more flexible with government insurance. The middle panel shows that through strategic complementarity, the public insurance also increases the flexibility of new hire wages. Finally, the right panel shows that the fluctuations in vacancy creation are dampened.

While it is an incentive for firms to provide insurance to incumbent workers which drives unemployment fluctuations, providing more insurance to *all* workers reduces unemployment fluctuations. The crucial market failure in my model is that workers are allowed to write contracts *only* with their current employers. If they could write contracts with potential new hire firms, which is in principle what the government is trying to do here, the unemployment fluctuations would disappear. Note that if private agents do not anticipate that the government-provided insurance, there is no effect on unemployment fluctuations. Any ex-post tax on labor income that is imposed after the realization of aggregate productivity does not affect the firm's nor the worker's behavior. Therefore, it is precisely the ex-ante role of public insurance that crowds out firm insurance, which in turn reduces unemployment volatility.

5 Inefficiencies: Steady States and Response to Shocks

So far, we have focused on the positive implications of the model. What are the normative implications? I first ask whether the steady-state equilibrium is efficient or not. Then, I study whether equilibrium responses to the aggregate shock is efficient. The spirit of exercise is to consider whether a small perturbation of the equilibrium achieves Pareto improvement or not. If it does, it implies the equilibrium is constrained inefficient. Although a more satisfactory and interesting analysis is to derive the optimal policy, I leave this for the future followup work.

5.1 Steady-state Inefficiencies

Let us start from the welfare properties of the steady-state equilibrium. While [Gautier, Teulings, and Vuuren \(2010\)](#) and [Cai \(2020\)](#) study the efficiency property of the Burnett-Mortensen model with risk-neutral workers, to the best of my knowledge, the efficiency property with risk-averse workers has not been studied before. I consider a planner who can directly intervene to perturb (i) the wage offers; (ii) the amount of vacancies; and (iii) unemployment benefit, $\{\delta w_0(z), \delta w_1(z), \delta v(z), \delta b\}$, where δx denotes the marginal changes in x . I assume the unemployment benefit is financed via a lump-sum tax on firms.

Inefficient wage offers. First, fixing $\delta v(z) = 0$, consider a small reduction in new hire firm's wage, $\delta w_1(z) < 0$, for some z , combined with $\{\delta w_0(z), \delta b\}$, which would leave workers indifferent. This exercise is meant to isolate whether the wage offers in equilibrium is efficient or not by shutting down the vacancy margin. Can such an intervention improve welfare? The following proposition shows the answer is yes if workers are risk-averse:

Proposition 7 (New hire wages are too high). *Consider the steady-state equilibrium. For any z , there exists a feasible perturbation featuring the same amount of vacancies at all firms, but strictly lower wages for new hires at firm z , $\delta w_1(z) < 0$, that yield a Pareto improvement if and only if workers are strictly risk-averse, $\gamma > 0$.*

Therefore, new hire firms make too aggressive wage offers, which is more true for more productive firms. In what follows, I sketch the proof. As workers have to be indifferent, the perturbation must satisfy

$$\begin{aligned} \left((1 - \lambda^E) + \lambda^E F_1(w(\tilde{z})) \right) u'(w(\tilde{z})) dw_0(\tilde{z}) + \mathbb{I}(z > \tilde{z}) \lambda^E u'(w(z)) (v(z)/V) g(z) \delta w_1(z) &= 0 \\ (1 - \lambda^U) u'(Ab) A \delta b + \lambda^U u'(w(z)) (v(z)/V) g(z) \delta w_1(z) &= 0 \end{aligned}$$

for all $\tilde{z} \in [\underline{z}, \bar{z}]$. The question is whether such perturbation can raise the net total surplus (total firms' profits), which is given by

$$\begin{aligned} \mathcal{F} &= \int \left[(Az - w_0(\tilde{z})) (1 - \lambda^E + \lambda^E F_1(w_0(\tilde{z}))) \ell_0(\tilde{z}) \right] dG(\tilde{z}) \\ &\quad + \int \left[v(\tilde{z}) \lambda^F (\chi + (1 - \chi) F_0(w_1(\tilde{z}))) (Az - w_1(\tilde{z})) - c(v(\tilde{z}); \tilde{z}) \right] dG(\tilde{z}) + \mu (1 - \lambda^U) Ab \end{aligned}$$

where $\tilde{F}(w) \equiv \chi + (1 - \chi) F_0(w)$. As long as workers are strictly risk-averse, $\gamma > 0$, the answer is yes:

$$\begin{aligned} d\mathcal{F} &= -v(z) \lambda^F \left[\chi \left(1 - \frac{u'(w(z))}{u'(Ab)} \right) + (1 - \chi) \int^z \left(1 - \frac{u'(w(z))}{u'(w(\tilde{z}))} \right) \frac{\ell_0(\tilde{z})}{1 - \mu} dG(\tilde{z}) \right] g(z) \delta w_1(z) \\ &> 0 \end{aligned}$$

where the last inequality follows from $\delta w_1(z) > 0$ and $u'' < 0$. This implies that we can Pareto improve welfare by forcing a new hire firm to slightly lower the wage offer. The reason is that new hire firms do not internalize their contribution to the idiosyncratic income risks. If new hire firms lower wage offers, workers' consumption dispersion goes down, and it becomes cheaper for incumbent firms to deliver the promised utility if the utility function is concave. Of course, such an intervention potentially creates a form of misallocation because workers now accept an offer from less productive firms. However, as there is no misallocation in the steady-state, such consideration has no first order effect on welfare and only has a second-order

effect. Moreover, we see that the term inside parenthesis is strictly increasing in z , which means that the externality is greater for more productive firms. The reason is that, because workers only accept a better wage offer, the contribution to the income risk is greater for high-paying productive firms.

Inefficient vacancy creation. Next, I ask whether the vacancy creation is efficient or not. To focus on vacancy margin, I fix $\delta w_1(z) = 0$. Then, I consider a perturbation $\delta v(z)$, $\{\delta w_0(\tilde{z})\}$ and δb that leave workers indifferent and see whether such a perturbation can raise the total net surplus. The following expression characterizes the effect of such perturbation on the total net surplus:

$$\begin{aligned}
 d\mathcal{F} = & \overbrace{\lambda^F \int^z \left(\frac{u(w(z)) - u(w(\tilde{z}))}{u'(w(\tilde{z}))} - [w(z) - w(\tilde{z})] \right) dP(\tilde{z}) \delta v(z)}^{\text{idiosyncratic income risk externality } (\leq 0)} \\
 + & \underbrace{(\kappa - 1)\lambda^F \int \int^{\tilde{z}} \left[(A\tilde{z} - w(\tilde{z})) - (w(\tilde{z}) - w(\zeta)) + \frac{u(w(\tilde{z})) - u(w(\zeta))}{u'(\tilde{z})} \right] dP(\zeta) \frac{v(\tilde{z})}{V} dG(\tilde{z}) g(z) \delta v(z)}_{\text{congestion externality } (\leq 0)},
 \end{aligned} \tag{21}$$

where $\kappa \equiv \frac{d \ln \mathcal{M}}{d \ln V}$ is the elasticity of matching function with respect to vacancy, and $P(z) \equiv \chi \mathbb{I}(z > \underline{z}) + (1 - \chi) \int^z \frac{1}{1-\mu} \ell_0(\tilde{z}) dG(\tilde{z})$ is the search-efficiency weighted cumulative employment distribution (including unemployed).

The expression shows that the welfare effect of reducing the vacancy of a particular firm z can be decomposed into two (generically) non-zero terms. The first term, which I label as idiosyncratic income risk externality, captures that firms do not internalize their contribution to the worker's income risk when they create jobs. One can immediately see that if workers are risk-neutral, this term is zero. With risk-averse workers, this term is negative because more job creation increases the workers upward income risk, which makes it costlier for incumbent firms to deliver the promised utility. Welfare can be improved if the planner forces firms to reduce job creation. Moreover, the absolute size of this term is increasing in z because they contributed the most to enlarging workers' consumption dispersion. This implies productive firms tend to be too large relative to the social optimum. In contrast to this, many theories predict productive firms are too small compared with the social optimum. Under search and matching frictions, [Acemoglu \(2001\)](#) shows that unproductive firms create too many jobs relative to productive ones because they do not internalize that they crowd out more productive matches. [Golosov, Maziero, and Menzio \(2013\)](#) show also in the context of a frictional labor market that too few workers seek jobs in productive firms because of search risk. Under an oligopsonistic labor market, productive firms hire too few workers (e.g., [Berger et al., 2019](#)) due to labor market power.

The second term, which I label the congestion externality, is relatively more standard ([Gau-](#)

tier et al., 2010; Cai, 2020). This term is zero when the elasticity of the matching function with respect to vacancy is one, $\kappa = 1$. Since the wage posting model can be interpreted as firms having all the bargaining power, this condition is analogous to Hosios'(1990) condition. As we move toward $\kappa < 1$, this term becomes negative. Firms do not internalize that their job creation will congest the market and lower meeting probabilities of other firms. Welfare can be improved by reducing job creation. Notably, this term does not depend on z , so the externality is the same for all firms.

I summarize the above discussion as follows:

Proposition 8 (Too many vacancies are created, especially in more productive firms). *Consider the steady-state equilibrium. For any z , there exists a feasible perturbation featuring the same new hire wages at all firms, but strictly lower vacancies at firm z , $\delta v(z) < 0$, that yield a Pareto improvement if and only if workers are risk-averse, $\gamma > 0$, or Hosios condition fails, $\kappa < 1$. Furthermore, for a given change in vacancies δv , the change in social surplus is increasing with the productivity of firm z .*

5.2 Inefficient Wage Flexibility under Aggregate Risk

So far, I have focused on the efficiency in the steady-state. The natural next question is whether the equilibrium response to aggregate shocks is efficient or not. In particular, are wages too flexible or too rigid in equilibrium? Because wage rigidity in my model is fully micro-founded as an optimal contracting problems, these questions are well defined. This is in contrast to many existing models of wage rigidity, in which rigidity is imposed exogenously.

Inefficient new hire wage flexibility. I consider a planner who can directly intervene to perturb new hire wages in each state, but cannot control vacancies. Let me first concentrate on the efficiency of new hire wage settings by imposing $\delta w_{0h}(z) = \delta w_{0l}(z) = 0$ for all z . Take a particular firm z_s in each state s , and consider a perturbation, $\delta w_{1h}(z_s), \delta w_{1l}(z_s)$. For such a perturbation to leave workers indifferent, they must satisfy

$$\begin{aligned} & \mathbb{I}(w_{0h}(\tilde{z}) \leq w_{1h}(z_h)) \lambda_h^E u'(w_{1h}(z_h)) dw_{1h}(z_h) \frac{v_s(z_h)}{V} g(z_h) \\ & + \mathbb{I}(w_{0l}(\tilde{z}) \leq w_{1l}(z_l)) \lambda_l^E u'(w_{1l}(z_l)) dw_{1l}(z_l) \frac{v_s(z_l)}{V} g(z_l) = 0 \end{aligned} \quad (22)$$

for all \tilde{z} . Note that (22) implies that unemployed workers are also left indifferent, because $\lambda_s^E = \zeta \lambda_s^U$. I focus on the non-trivial case with $w_{0h}(\tilde{z}) \leq w_{1h}(z_h)$ and $w_{0l}(\tilde{z}) \leq w_{1l}(z_l)$ for all \tilde{z} . This implies that there exists \tilde{z} such that $w_{0h}(\tilde{z}) = w_{1h}(z_h)$ and $w_{0l}(\tilde{z}) = w_{1l}(z_l)$. Since we have already learned that the vacancies are unaffected by any small movement in new hire wages, so a term that involve $\delta v_s(z)$ does not show up in the above expression. The question is whether

such perturbation can raise the expected net total surplus, which is given by

$$\mathcal{F} = \sum_{s=l,h} \pi_s \left\{ \int \left[(A_s \tilde{z} - w_{0s}(\tilde{z})) (1 - \lambda_s^E + \lambda_s^E F_{1s}(w_{0s}(\tilde{z}))) \ell_0(\tilde{z}) \right] dG(\tilde{z}) \right. \\ \left. + \int \left[v(\tilde{z}) \lambda_s^F (\chi + (1 - \chi) F_0(w_{1s}(\tilde{z}))) (A_s \tilde{z} - w_{1s}(\tilde{z})) - c_s(v(\tilde{z}); \tilde{z}) \right] dG(\tilde{z}) + \mu (1 - \lambda_s^U) A_s b \right\}.$$

The changes in net surplus can be computed as

$$d\mathcal{F} = \overbrace{\sum_{s=l,h} \left[A_s (z_s - \tilde{z}) \lambda_s^E F'_{1s}(w_{1s}(z_s)) \ell_0(\tilde{z}) \right] \delta w_{1s}(z_s)}^{\text{misallocation}} \\ + \underbrace{\left[\mu \lambda_s^U + \lambda_s^E \int_{\tilde{z}} \ell_0(\tilde{z}) dG(\tilde{z}) \right] \left(\frac{u'(w_{0l}(\tilde{z}))}{u'(w_{0h}(\tilde{z}))} - 1 \right) \frac{v_l(z_l)}{V_l} g(z_l) \delta w_{1l}(z_l)}_{\text{aggregate risk-sharing}}.$$

The first term, which I labeled the misallocation, comes from the fact that there is cyclical misallocation in my model. As we have seen in Proposition 3, if workers are risk-averse, incumbent firms' wages respond less than the new hire firms with the same productivity. Therefore, workers can flow toward a less productive firm in booms, and reject the offer from a more productive firms in recessions. That is, $z_s - \tilde{z}$ is negative for $s = h$ and positive for $s = l$. If the planner forces new hire firms to respond less to the aggregate productivity, this will alleviate the misallocation. The second term, which I labeled the aggregate income risk-sharing, comes from the fact that new hire firms do not internalize their contribution to the aggregate income risk. As is the case with idiosyncratic income risk externality, if the planner could force new hire firms to be less aggressive, this would alleviate the limited commitment friction of the contract between incumbent firms and workers. As long as u is strictly concave, this term is positive for $\delta w_{1h}(z_h) < 0$ and $\delta w_{1l}(z_l) > 0$. This leads me to conclude:

Proposition 9 (New hire wages are too flexible). *Consider the equilibrium with aggregate risk. Assume the elasticity of vacancy creation, ι , is sufficiently small. There exists a perturbation featuring lower new hire wages in some firms in the high state, $\delta w_{1h}(z_h) < 0$, and higher new higher wages in the low state, $\delta w_{1l}(z_l) > 0$, that yields a Pareto improvement if and only if workers are risk averse, $\gamma > 0$.*

That is, the new hire wages are too flexible in equilibrium. Despite my model generates endogenous new hire wage rigidity, the planner improves the welfare by making new hire wages even more rigid. We have seen that more new hire wage rigidity has no consequence for unemployment fluctuations, but it still improves welfare.

Incumbent wage flexibility. Although it is of great interest to understand whether incumbent wages are too rigid or too flexible in equilibrium, I can only analytically study this for a special

case. The complication arises in taking into account endogenous responses of job creation associated with the changes in incumbent wages. When vacancies are inelastic, $\iota \rightarrow 0$, I can shut off this channel. Appendix B shows that incumbent wages are also too flexible in equilibrium. Intuitively speaking, incumbent firms move around wages with aggregate productivity in order to block poaching from new hire firms. However, such competition is business stealing. If firms could collectively focus on insuring workers, this would raise the welfare.

6 Quantitative Exploration

To quantify the mechanisms, I extend the previous two-period model to a infinite horizon model in continuous time. I first describe the environment where there is no aggregate shock and consider the one-time unanticipated aggregate shock.

6.1 From Two-period to Infinite Horizon

Preferences and technology. The economy is populated by a mass of workers and a mass of entrepreneurs. Workers are risk-averse with preferences

$$W_0 = \int_0^\infty e^{-\rho t} u(c_{wt}) dt,$$

where $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ as before, and entrepreneurs are risk neutral,

$$\int_0^\infty e^{-\rho t} c_{kt} dt.$$

The flow value of the unemployed is Ab , where A is the aggregate productivity. Firms operate the linear production technology, $y = Azl$, and z is the firm's permanent productivity distributed according to the cumulative density function $G(z)$.

Unemployed and employed workers meet with a firm with Poisson intensity λ_t^U and $\lambda_t^E \equiv \zeta \lambda_t^U$, respectively. Employed workers exogenously separate with firms with Poisson intensity δ . Similarly to Moscarini and Postel-Vinay (2016a) and Gertler et al. (2020), I also introduce reallocation shock with intensity κ . When hit by the reallocation shock, the worker is forced to move to another firm with the same productivity and inherits the same wage contracts. This is meant to capture the fact that not all job-to-job transitions are for climbing up the job ladders and appear for various other reasons (e.g. spousal moving). This assumption is not solely for realism. Without reallocation shock, the employer-to-employer transition rate observed in the data implies that firms face strong competition with each other. This makes it difficult to match the observed degree of wage cyclicality in the data.

Firms post a vacancy with convex cost, $Ac(v; z)$, which scales with the aggregate labor productivity, as before. This assumption ensures balanced growth, in which the permanent changes in the aggregate productivity, A , leave the long-run unemployment rate unchanged. Each vacancy meets with a worker with intensity λ_t^F . As before, among the workers firms meet with, fraction $\chi_t = \frac{\mu_t}{\mu_t + \zeta(1 - \mu_t)}$ is unemployed and the remaining fraction $1 - \chi_t$ is employed. The total number of meetings between firms and workers is given by a CRS matching technology $\mathcal{M}(\tilde{\mu}_t, V_t)$.

Contracts and markets. Firms compete for workers by posting wage contracts, w . I assume a wage contract can only depend on labor productivity, Az , and thus excluding the possibility of wage-tenure contracts studied in [Burdett and Coles \(2003\)](#). In principle, firms would like to make wages contingent on tenure to backload the incentives. Although studying such full dynamic contracts would be interesting, I believe such consideration is largely orthogonal to my focus: aggregate risk sharing.²⁵ Moreover, [Burdett and Coles \(2010\)](#) show that wage-tenure contracts with heterogenous firms massively complicates the analysis. For example, the equilibrium wage distribution need not necessarily be smooth.

The contract specifies the utility that firms deliver to workers at each point in time rather than the path of the wages. While this assumption is innocuous under perfect foresight equilibrium, it matters when hit by an unanticipated shock. Under the assumption that utility is specified in the contract, there is room for rewriting the contracts to adjust wages. Finally, as before, workers cannot commit to the contracts, and the possibility of counter-offers are excluded. With constant productivity and constant wage contracts, workers accept the arriving wage offers if and only if the offer is higher than the current wage.

Equilibrium objects. The unemployment rate, μ_t , evolves according to

$$\partial_t \mu_t = \delta(1 - \mu_t) - \mu_t \lambda_t^U, \quad (23)$$

where $\chi_t \equiv \frac{\mu_t}{\mu_t + \zeta(1 - \mu_t)}$ and $\partial_t y_t \equiv \frac{\partial y_t}{\partial t}$ are the short-hand notation for the time derivative for any y_t . Let $P_t(w)$ denote the employment weighted wage distribution function. It follows the following law of motion:

$$\partial_t P_t(w) = -\delta P_t(w) - \lambda_t^E (1 - F_t(w)) P_t(w) + \frac{1}{1 - \mu_t} \mu_t \lambda_t^U F_t(w). \quad (24)$$

²⁵One can justify my assumption if I let workers' elasticity of intertemporal substitution to be zero. Then, as [Burdett and Coles \(2003\)](#) showed, the optimal wage-tenure contract features a constant wage throughout the tenure.

The value of a firm with productivity z per unit of employee that offers wage w satisfies²⁶

$$\rho J_t(w, z) = A_t z - w - (\delta + \kappa + \lambda_t^E (1 - F_t(w))) J_t(w, z) + \partial_t J_t(w, z).$$

The firm choose what wages to offer and how much vacancies to post at time t by solving

$$\{w_t(z), v_t(z)\} \in \arg \max_{w, v} v \lambda_t^E Q_t(w) J_t(w, z) - A_t c(v; z), \quad (25)$$

where $Q(w) \equiv \chi_t \mathbb{I}(w \geq \underline{w}_t) + (1 - \chi_t) P_t(w)$ and \underline{w} is the reservation wage for unemployed. The a worker's value function with wage w , $W(w)$, satisfies

$$\rho W_t(w) = u(w_t) + \delta \{U_t - W_t(w)\} + \lambda_t^E \int \max \{0, W(\tilde{w}) - W(w_t)\} dF_t(\tilde{w}) + \partial_t W_t(w), \quad (26)$$

where the value of unemployment, U_t , is given by

$$\rho U_t = u(A_t b) + \lambda_t^U \int \max \{0, W_t(\tilde{w}) - U_t\} dF_t(\tilde{w}) + \partial_t U_t. \quad (27)$$

The reservation wage for the unemployed, \underline{w}_t , must be such that workers are indifferent between being employed and unemployed $W_t(\underline{w}_t) = U_t$.

Appendix E.1 defines the perfect-foresight equilibrium and characterizes the steady state of this economy with $A_t = A$.

Transition Dynamics in Response to Aggregate Shocks. As in the two-period model, I consider the following experiment. Before $t \leq 0$, the economy is in its steady-state. At $t = 0$, the economy experiences an unanticipated one-time increase in the variance of the aggregate productivity. The aggregate productivity for $t > 0$ is given by

$$\ln A_t = \begin{cases} \ln A_h = \ln A + d \ln A & \text{with probability } \pi_h \equiv 1/2 \\ \ln A_l = \ln A - d \ln A & \text{with probability } \pi_l \equiv 1/2 \end{cases}.$$

That is, the aggregate productivity is either permanently high or low for $t > 0$. The focus on permanent productivity shocks has been common in the search and matching literature (e.g., [Ljungqvist and Sargent, 2017](#); [Kehoe et al., 2019](#)), and it is empirically reasonable given the high persistence on the productivity process.

Once firms and workers anticipate the aggregate risk, firms that already hire incumbent

²⁶I assumed away the possibility of endogenous separation (or exits). In principle, firms would like to fire workers if $J(w, z) < 0$. However, since firms at the exist threshold, $z = \underline{z}$, employ zero workers, even if I allow for the possibility of endogenous separation, there is no first order effect on the equilibrium outcomes.

workers (re)write a state-contingent wage contracts at $t = 0$ that solve

$$\begin{aligned} & \max_{\{w_{0s}^{inc}\}} \sum_{s \in \{h,l\}} \pi_s J_{0s}(w_{0s}, z) \\ \text{s.t.} \quad & \sum_{s \in \{h,l\}} \pi_s W_{0s}(w_{0s}^{inc}) \geq W(w(z)), \end{aligned}$$

where $s = h$ and $s = l$ denote the state with high and low productivity, respectively. In other words, firms offer a state-contingent wage that promise at least the same expected utility to a worker as before to maximize its expected profits. This is because I made an assumption that contracts are written in terms of the utility to be delivered. The optimal incumbent wage responses $w_{0s}^{inc}(z)$ satisfy the following first order condition:

$$\partial_w J_{0s}(w_{0s}^{inc}(z), z) + \eta(z) W'_{0s}(w_{0s}^{inc}(z)) = 0, \quad (28)$$

where $\eta(z)$ is the Lagrangian-multiplier on the promise-keeping constraint. After $t > 0$ onwards, given the incumbent wages as initial conditions, the equilibrium follows the perfect foresight path described above.

It is again worth noting that without aggregate risk-sharing (i.e., risk-neutral workers) or there is no on-the-job search, the economy exhibits no fluctuation in unemployment. Formally, Propositions 3 and 4 continue to hold, as established in Appendix A.14. This ensures that it is precisely the complementarity between risk-sharing and on-the-job search that drives nontrivial labor market dynamics in my model. In what follows, I explore this complementarity by assuming $\gamma > 0$ and $\zeta > 0$.

6.2 New Solution Method

As is well-known, solving the transition dynamics of the wage posting model has been considered as a challenge because of the need to keep track of the endogenous evolution of distribution. I develop a general and efficient computational approach to solve the transition dynamics of a wide class of wage posting job ladder models. Throughout, I focus on first order responses, which is crucial for my approach.

The key idea behind the computational algorithm is the same as how I solved the two-period model. Although it is widely believed that one needs to keep track of the path of wage and employment distribution to compute the equilibrium, I argue this is not the case. As long as we focus on the first order response, no firm cares about the entire distribution per se. Firms only care about the wages and vacancies of their neighbors. That is, best responses can be still represented as a system of ODEs in the infinite horizon model.

Figure 7 shows the DAG (directed acyclical graph) representation of the equilibrium, fol-

that it is entirely possible for their methodology to cover wage posting models.

Among others, two key advantages of my approach are worth emphasizing. First, the computation is extremely efficient. It typically takes less than a second to compute the transition dynamics with a small number of grid points. Even with a large number of grid points, it only takes 1-5 seconds. While I do not pursue here, the efficiency of computation enables one to fully estimate the model using business cycle moments. Second, my approach does not require approximation of the distribution. An alternative approach to solve the first order transition dynamics of [Burdett and Mortensen \(1998\)](#) is to use [Reiter's \(2009\)](#) method to approximate the wage distribution with a few parameters, as done by [Morales-Jiménez \(2019\)](#). However, the accuracy of such approximation is not known because there has been no approach to solve this without an approximation.²⁸ This is especially concerning because the best response of firms depend on the entire shape of the wage distribution, as I showed in Lemma 3.²⁹

There is also an alternative approach by [Moscarini and Postel-Vinay \(2016b\)](#) that solves the non-linear dynamics. However, their methodology relies on a fairly restrictive set of assumptions. For example, the workers need to be risk-neutral; therefore, cannot be applied to my model. Appendix C describes the solution method in more details.

6.3 Calibration

Functional form assumptions. The vacancy cost function is parametrized as the iso-elastic function, $c(v; z) \equiv \bar{c}z^{\iota} \frac{v^{1+\iota}}{1+\iota}$. The matching function is assumed to be a Cobb-Douglas form, $\mathcal{M}(\tilde{\mu}, V) = \bar{m}\tilde{\mu}^{1-\kappa}V^{\kappa}$. The firm's productivity distribution is parametrized as a Pareto distribution, $G(z) = 1 - (b/z)^{\Lambda}$ with $\Lambda > 1$ being the tail parameter.

Parameter values. Table 1 describes the calibration. The time frequency is monthly. I first set the elasticity of the matching function to $\kappa = 0.6$ following [Blanchard and Diamond \(1989\)](#). The discount rate is set to match the 5% annual interest rate, $\rho = 0.004$. The separation rate is set to 1.6%, $\delta = 0.016$, corresponding to the average of the BLS labor status flow from employed to unemployed over the period of 1990-2019. I also set the matching efficiency parameter, $\bar{m} = 0.1$, which is a normalization. The elasticity of vacancy creation is set to $\iota = 1$, following [Kaas and Kircher \(2015\)](#) and [Gertler and Trigari \(2009\)](#), and I normalize $b \equiv 1$. The tail parameter of productivity is set so that the standard deviation of log productivity is 0.2, which corresponds to the lower end of the value reported in [Foster, Grim, Haltiwanger, and Wolf \(2016\)](#). I set κ so that half of job changers experience wage increases, which follows [Gertler et al. \(2020\)](#) and [Moscarini and Postel-Vinay \(2016a\)](#).

take the full distribution of wages as an input to their decision problem, and it is impossible to represent this via a DAG of feasible dimension." To the contrary, I show it is possible, as I do in Figure 7.

²⁸Now, one can use my approach to test the accuracy of this approach.

²⁹In contrast, in Bewly-Hugget-Aiyaragari models, only the mean of the (asset) distribution matters for interest rates and wages.

Parameter	Description	Value	Source/Target
Panel A. Externally assigned			
κ	Elasticity of matching function	0.6	Blanchard and Diamond (1989)
ρ	Discount rate	0.004	5% annual interest rate
δ	Separation rate	0.016	1.6% EU rate
ι	Elasticity of vacancy cost function	1	Kaas and Kircher (2015)
Λ	Pareto tail of productivity distribution	5	S.d. of log productivity 0.2
\varkappa	Reallocation shock	0.0075	Share of EE with wage increase 1/2
\bar{m}	Matching efficiency	0.1	Normalization
b	Outside option of unemployed	1	Normalization
Panel B. Internally calibrated parameters			
\bar{c}	Vacancy cost parameter	0.035	Unemployment rate 6%
ι_z	Vacancy cost parameter	8	Aggregate profit share 14%
ζ	Relative efficiency of on-the-job search	0.08	1.5% EE rate
γ	Relative risk aversion	15	Wage volatility relative to output 38%

Table 1: Parametrization

Note: Table 1 describes the choice of the parameter values and their sources or targeted moments. Panel A shows the parameters exogenously assigned. Panel B shows the parameters that are internally calibrated to match the data moments.

I choose $\{\bar{c}, \iota_z, \zeta, \gamma\}$ to match the (i) steady-state unemployment rate of 6%; (ii) monthly job-to-job transition rate of 1.5%; (iii) the aggregate profit share of 14% reported by Gutiérrez and Philippon (2018) for the US; and (iv) the relative standard deviation of average real wage growth to the real output growth of 0.38. I use the real output in the non-farm business sector as a measure of the real output, and average hourly earnings of production and nonsupervisory employees deflated by PCE as a measure of the real wage. Both series are obtained from BLS. I explicitly target the profit share because this is the key determinant of fluctuations in job creation for the given level of wage rigidity, as can be seen from equation (15) (see also Ljungqvist and Sargent (2017)). To match the relative volatility of real wages, the model requires a relative risk aversion of 15. This value is higher than the most macro models, but is fairly consistent with the value used in the finance literature. The reason why I need a relatively high risk aversion is that the presence of on-the-job search implies that between firm competition acts as a strong force in preventing effective risk-sharing. This is in contrast to Rudanko (2009). She uses a model with risk-sharing but without on-the-job search and shows that the model tends to deliver too rigid wages compared with the data.

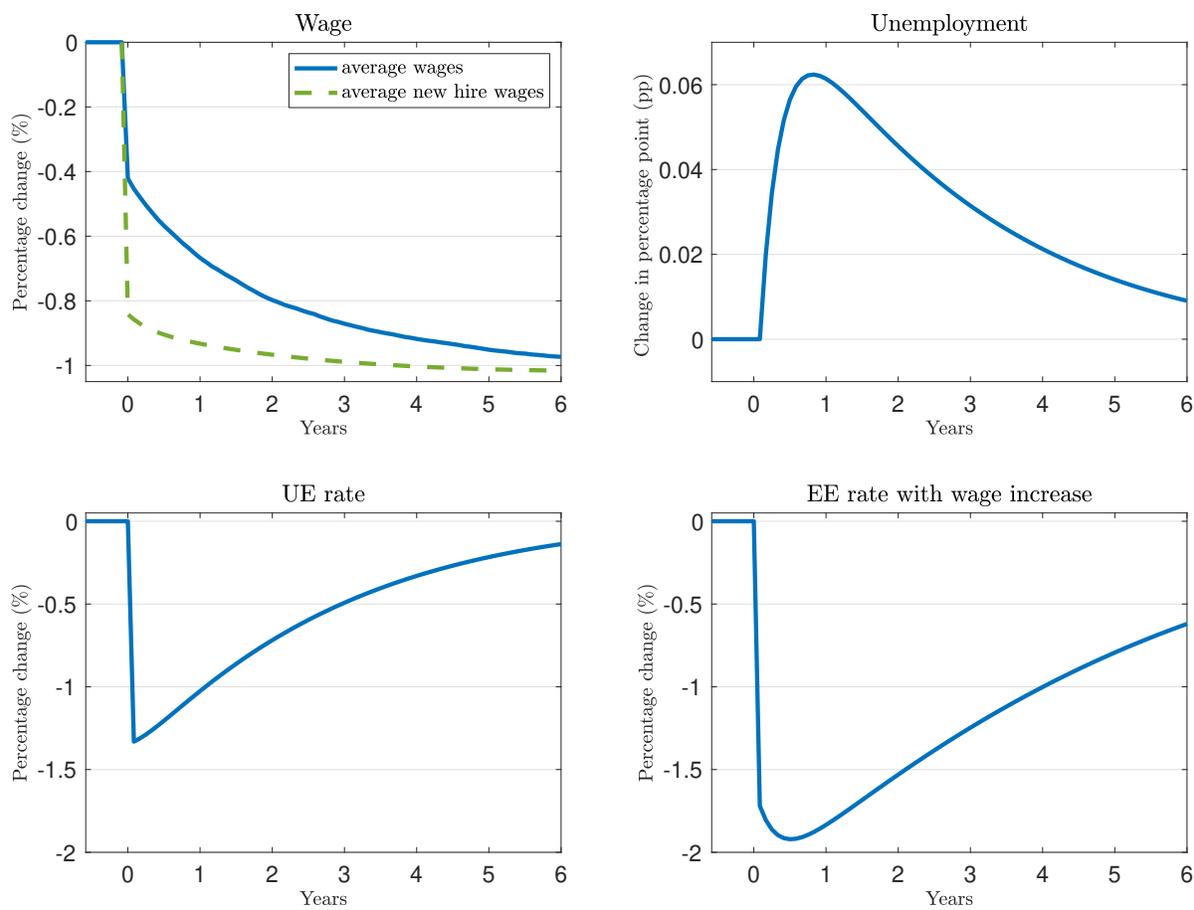


Figure 8: Impulse response to the realization of a 1% negative productivity shock

Note: Figure 8 shows the impulse response of the economy to the realization of a 1% negative productivity shock.

6.4 Results

Figure 8 shows the impulse response function for the realization of a 1% negative productivity shock. The left-top panel shows the dynamics of average wages in the blue solid line and the average new hire wages in the dashed green line. The average wage is substantially sluggish mainly because firm insurance induces a muted incumbent wage response. The new hire wage response is also dampened through the strategic complementarity highlighted in the two-period model. Compared with the fully flexible case, the initial new hire wage response is dampened by 15-20%. The magnitude is smaller than the two-period model because when new hire firms decide on their wage offers, this not only takes into account the competition with incumbent firms but also with the subsequent new hire firms. Since wages need to eventually adjust fully in the long-run, subsequent new hire firms offer more flexible wages. To prevent being poached by those firms, current firms have an incentive to offer more flexible wages than the two-period model.

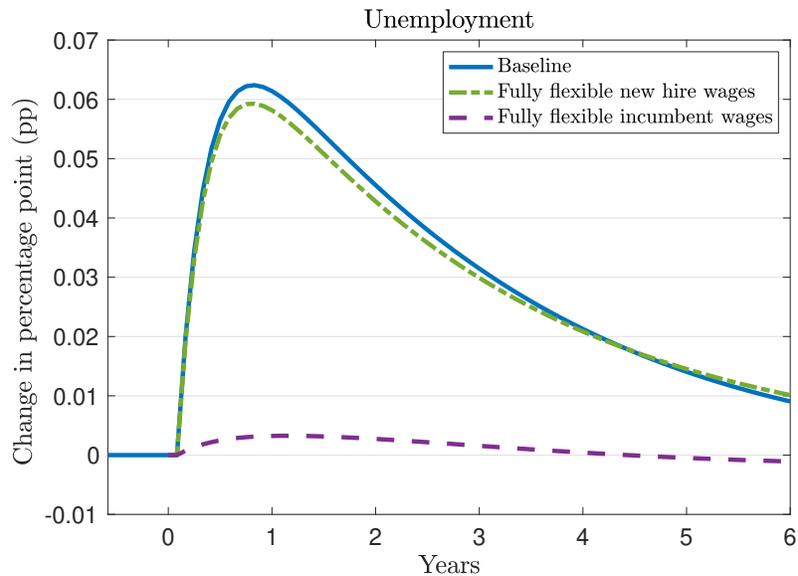


Figure 9: Decomposition of the unemployment response

Note: Figure 9 shows the decomposition of the impulse response of the unemployment rate. The green dash-dotted line assumes new hire wages respond one for one with the aggregate productivity, holding the incumbent wages the same as the baseline response. The purple dashed line assumes the incumbent wages respond one for one with the aggregate productivity, holding the new hire wages the same as the baseline response.

The right-top panel shows the response of unemployment rate. Note that with risk-neutral workers, $\gamma = 0$, there should not be any response of unemployment rate. As in two-period model, as soon as we move away toward risk-averse workers, the model does generate unemployment fluctuations, which is in stark contrast to the conventional wisdom that long-term contracts should not contribute to unemployment fluctuations. The bottom left and the bottom right panels show the UE rate and the EE rate with a wage increase, respectively. EE rate with a wage increase declines more sharply and recovers more slowly than the UE rate. This collapse in the number of workers who climb up the job ladder is consistent with the fact documented in [Haltiwanger et al. \(2018\)](#). They show that the firm wage ladder is strongly procyclical, and my theory provides a natural explanation of this.³⁰

Decomposing unemployment response. The model generates a sluggish adjustment in both incumbent and new hire wages as well as volatility in unemployment. It is then natural to ask what drives unemployment fluctuation: is it incumbent wage rigidity or new hire wage rigidity? The answer to this question was stark in the two-period model, but it is not in an infinite horizon model. In the infinite horizon model, sluggish adjustments in future new hire wages lowers the probability of being poached in the future for the current firms, which raises

³⁰See also [Barlevy \(2002\)](#), [Mukoyama \(2014\)](#), and [Nakamura et al. \(2019\)](#) for related evidence.

Moments	Model	Data
Panel A. Relative s.d. to real output growth		
average real wage growth	0.38	0.38
UE rate	1.36	6.97
log vacancy	7.36	37.9
unemployment rate	0.30	2.23
constant separation unemployment rate	0.30	1.61
Panel B. Autocorrelation		
real wage growth	0.10	0.18
UE rate	0.97	0.96
unemployment rate	0.99	0.99
log vacancy	0.95	0.98
Panel C. Correlation with unemployment		
real wage growth	-0.20	-0.13
UE rate	-0.96	-0.83
log vacancy	-0.93	-0.52

Table 2: Business cycle moments

Note: Table 2 shows the business cycle moments in the model and in the data. The real output measure is the real output in nonfarm business sector from BLS. The real wage is average hourly earnings of production and nonsupervisory employees deflated by PCE, also from BLS. The constant separation unemployment rate assumes the EU rate is constant at $\delta = 1.6\%$. Vacancy data comes from the composite Help-Wanted index by [Barnichon \(2010\)](#).

the incentive to create jobs.

To shed light on this issue, I exogenously change each of the incumbent wage and the path of new hire wages separately and simulate the model. In the first experiment, I force all the incumbent firms to adjust wage one for one with productivity holding the path of new hire wages fixed. In the second experiment, I force all the new hire wages to adjust one for one with productivity, holding incumbent wages fixed. Figure 9 shows the response of unemployment under each counterfactual scenario. One can immediately see that most of the unemployment response disappears if incumbent wages are flexible. In contrast, the response is barely affected even if new hire wages are fully flexible. This decomposition shows that my results are indeed driven by incumbent wage rigidity.

Business cycle moments. Table 2 compares the business cycle moment of the model to the data. By design, the model matches the standard deviation of real wage growth. The model generates roughly 20% of the volatility in the UE rate and vacancy. Since the volatility in the unemployment rate not only comes from the UE rate but also fluctuations in separation rate, which I abstract from, my model generates volatility in unemployment rate smaller than 20% of the data. To make a fair comparison, I construct a time-series of the unemployment rate

that assumes constant separation rate, following [Shimer \(2012\)](#). Specifically, the adjusted unemployment rate is given by $u_{t+1}^{adj} = \delta(1 - u_t^{adj}) + (1 - UE_t)u_t^{adj}$, where $\delta = 1.6\%$ and UE_t is the UE rate taken from the data. The model explains roughly 20% of volatility of this variable.

The magnitude of volatility is relatively small compared with the data. This comes from two reasons. The first reason is relatively standard. I have chosen parameters so that the aggregate profit share is 15%, which is consistent with the data. As is well-known, search and matching models require low surplus (profit share) to generate amplifications ([Ljungqvist and Sargent, 2017](#)). This is true in my model, as equation (15) crucially depends on $\alpha(z)$, the profit share. Using a standard DMP model with a representative firm, [Hall \(2005\)](#), [Hagedorn and Manovskii \(2008\)](#), and others have been able to generate realistic volatility in labor market because they have assumed that the profit share is less than 5%. It is difficult for the wage posting job ladder model to deliver such low profit share with reasonable heterogeneity in firm productivity. Highly productive firms are profitable, so they become large in size, raising the aggregate profit share. Since these channels are well understood and not my focus, I do not pursue an approach to engineer my model to generate a low profit share.

Second reason comes from the type of wage rigidity that matters for unemployment fluctuations in my model. As I will shortly explain in detail in [Section 6.5](#), my model gives an important role not only to incumbent wage rigidity but also to a full dynamic response of wages: how much the wages will adjust over the very long horizon. Since wages will be fully flexible in the long-run (after 4-5 years in my model), this tends to diminish the amplification of the model.

Fairness constraints. I revisit the question of whether the fairness constraint amplifies or dampens unemployment fluctuations by using this quantitative model. This is interesting also from the perspective of the literature. The literature that uses the wage posting job ladder model to study business cycle almost always imposed fairness constraints, following the tradition of [Burdett and Mortensen \(1998\)](#) (e.g., [Moscarini and Postel-Vinay, 2013](#); [Morales-Jiménez, 2019](#)). It is worth clarifying the role that such a constraint was playing in these papers.

I impose a restriction that firms cannot discriminate wages across employees. Firms commit to a sequence of wage payments $\{w_s\}$ that delivers W_t of the expected lifetime utility to the workers employed at the firm. Workers accept the job that offers a higher value. I delegate the detail description of the environment to [Appendix E.2](#).

[Figure 10](#) shows the impulse response with fairness constraints. The top left panel shows the wage response, and as one would expect, the wage response lies in the middle of new hire and average wages in the baseline model. The right-top panel shows that fairness constraints indeed dampens unemployment response by around 70%. This has two implications. First, the common practice of imposing fairness constraints in the wage posting job ladder model tends to worsen Shimer puzzle. Second, while some papers argue that fairness constraints amplify unemployment fluctuations, the implications are reversed once we take into account on-the-job

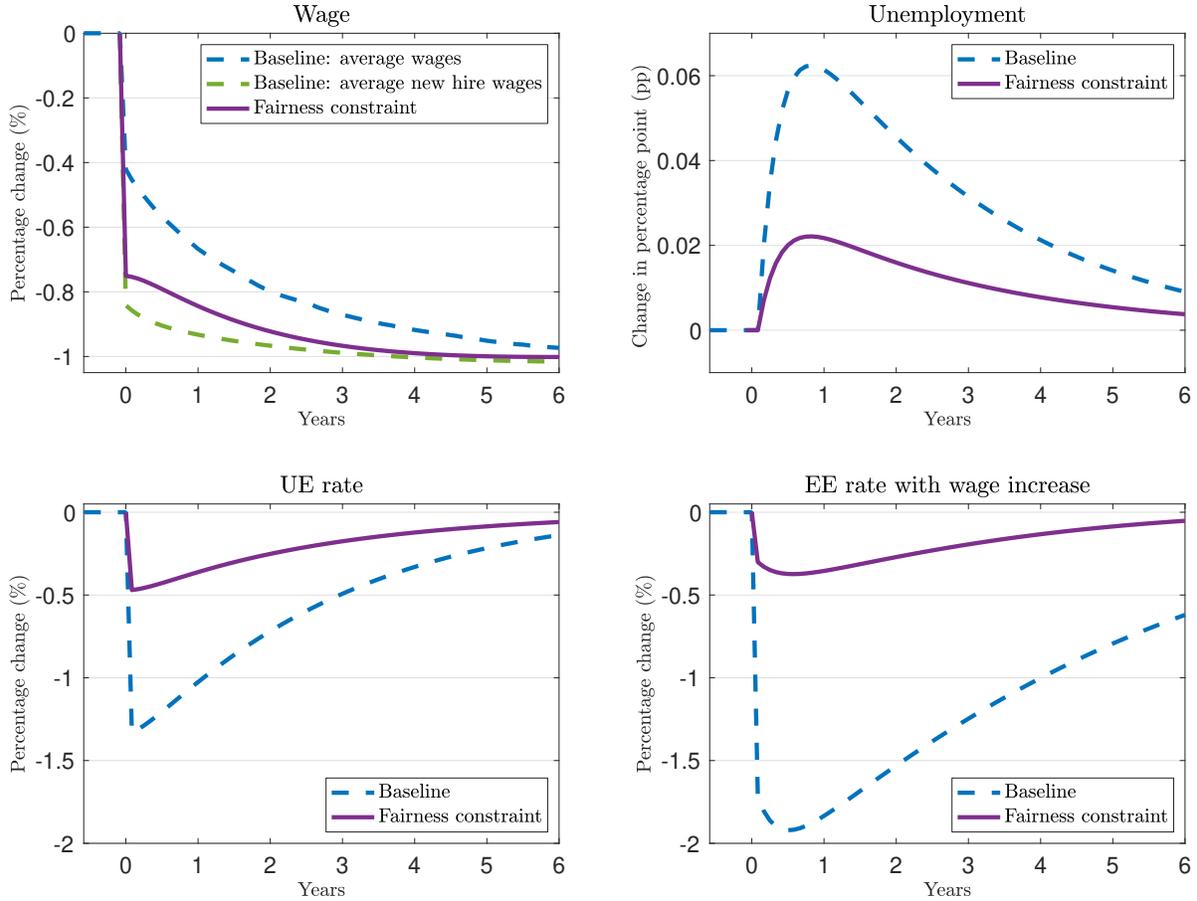


Figure 10: Impulse response with fairness constraints

Note: Figure 10 shows the impulse response of the economy to the realization of a 1% negative productivity shock with and without fairness constraints.

search. The bottom two panels show the response of the UE and the EE rate. As in the two-period model, fairness constraints dampen the EE response much more than the UE response.

6.5 Which Wage Rigidity Matters?

In this infinite horizon model, what type of wage rigidity is relevant for the incentive to create jobs? The answer to this question helps us understand the above simulation results. I explain it using the notion of how much job values are sensitive to wage changes at each point in time.

Let us focus on the baseline infinite horizon model without fairness constraints. I consider its discrete time approximation where a period corresponds to a month. The value of job with productivity z and a wage contract w is

$$J_t(w, z) = Az - w + (1 - (\hat{\rho} + \lambda_{t+1}^E(1 - F_{t+1}(w))))J_{t+1},$$

where $\hat{\rho} \equiv \rho + \delta + \kappa$. The optimality condition for vacancy creation at $t = 1$ is

$$\lambda_1^F Q_1(w) J_1(w, z) = Ac'(v).$$

I consider the following thought experiment: Before $t = 1$, the economy is in a steady-state. Suppose at $t = 1$, there is exogenous increase in A combined with the arbitrary changes in wage distribution, $\{w_t(z)\}_{t,z}$, (including its own wage), with $w_0(z)$ being the incumbent wages of firm z . How do the changes in the wage distribution affect the value of job creation? The exercise is the partial equilibrium, so I fix all other variables (λ_t^F, λ_t^E , and vacancies) fixed. Therefore we are interested in $\mathcal{E}_{1,t}(z, \tilde{z}) \equiv \frac{\partial \ln(Q_1(w(z))J_1(w(z), z))}{\partial \ln w_t(\tilde{z})}$. First, it is straightforward to see $\mathcal{E}_{1,t}(z, \tilde{z}) = 0$ for $z \neq \tilde{z}$: small wage changes of infra-marginal competitors do not affect the job value. Second,

$$\sum_{s=0}^{\infty} \mathcal{E}_{1,s}(z, z) = -\frac{1 - \alpha(z)}{\alpha(z)},$$

where $\alpha(z) \equiv (Az - w(z))/Az$ is the profit share. I define

$$weight_t \equiv \int \frac{\mathcal{E}_{1,t}(z, z)}{\sum_{s=0}^{\infty} \mathcal{E}_{1,s}(z, z)} (v(z)/V) dG(z), \quad (29)$$

which captures how much the value of job is sensitive to the wage changes at each point in time after integrated using vacancy as density.

The blue bar in Figure 11 shows the weight in the baseline model. First, it places 35% of the weight on incumbent wages ($t = 0$). Second, it places 0% of the weight on contemporaneous wage changes. Third, the weight is spread over the entire period with each having 3-4%. The first result indicates that the incumbent wage rigidity is the most important determinant of job creation. The second result comes from the envelope theorem as in the two-period model. The third result comes from the fact that firms face a constant threat of being poached and that higher wage offers in the future makes this possibility more likely. All these results are consequence of dynamic competition in the labor market. Firms that post a job today not only compete with incumbent firms to poach workers, but also with future poaching firm.

The red bar in Figure 11 shows the same object in DMP model. In stark contrast, DMP models put 100% weight on contemporaneous wage. The reason is that the labor market competition is completely absent in this class of models. The value of a job in DMP model is

$$J_t^{DMP}(w, z) = Az - w + (1 - (\rho + \delta))J_{t+1}^{DMP},$$

and the optimal vacancy creation is

$$\lambda_1^F J_1^{DMP}(w, z) = Ac'(v),$$

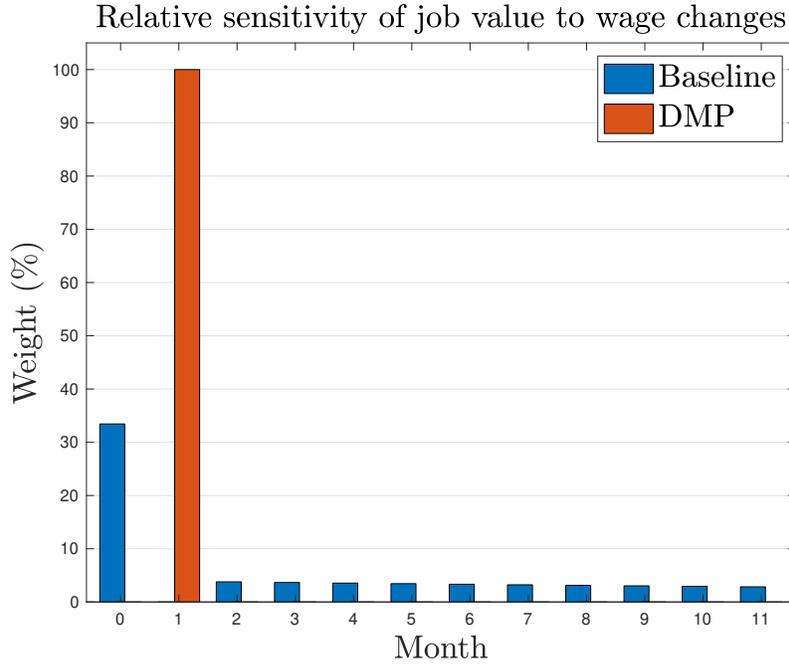


Figure 11: Relative sensitivity of job value to wage changes in each point in time

Note: Figure 11 plots $weight_t$, defined in (29), which captures the relative sensitivity of job value to wages in each point in time. Month 1 corresponds to the sensitivity to the new wages when the job is created. Month 0 corresponds to the sensitivity to incumbent wages. Month $t > 1$ corresponds to the sensitivity to wage offer at month t .

which does not depend on any other wages than its own wage, $\mathcal{E}_{1,s}(z, z) = 0$ for all $s \neq 1$. One can also compute that $\mathcal{E}_{1,s}(z, z) = -\frac{1-\alpha(z)}{\alpha(z)}$, so that $\sum_{s=0}^{\infty} \mathcal{E}_{1,s}(z, z) = -\frac{1-\alpha(z)}{\alpha(z)}$, which implies that the total response is the same with the baseline model, but the importance of the wage rigidity in each point in time completely differ.

This has an implication for measuring the theoretically relevant notion of wage rigidity. Since [Bils \(1985\)](#), it has been common to estimate *contemporaneous* wage rigidity, which is the contemporaneous correlation between wage changes and unemployment rate. While this is theoretically well grounded from the viewpoint of DMP model, it is not if one believes in the wage posting model with on-the-job search. As Figure 11 shows, the theory implies that we need to measure *intertemporal* wage rigidity, which consists of (i) incumbent wage rigidity, and (ii) how the wages at time s respond to the aggregate shock at time $t < s$. This comes from the fact that in this class of the job ladder model, labor market competition is inherently dynamic. Firms that intend to create jobs today need to compete with incumbent firms and future jobs. Consequently, those competitors' wage responses become the important determinant of job creation.

7 Conclusion

Is incumbent wage rigidity important for unemployment fluctuations? Conventional wisdom says no. My paper says yes by arguing that the key missing piece in the conventional view is on-the-job search. Models of wage rigidity have been abstracting from on-the-job search, thereby mechanically shutting down any meaningful interaction between incumbent wages and labor market dynamics. I showed that once we take into account on-the-job search in an environment where firms insure incumbent workers, (i) both new hire and incumbent wages are endogenously rigid; (ii) but only the latter form of wage rigidity is the key determinant for unemployment fluctuations.

I operationalize the idea using a generalized version of [Burdett and Mortensen's \(1998\)](#) job ladder model featuring risk-neutral firms, risk-averse workers, and aggregate risk. Besides the main messages, I showed a number of other results such as the fact that fairness constraints and public insurance dampen unemployment fluctuations, and the novel source of inefficiency makes wages too flexible in equilibrium. Overall, I believe my theory provides a useful starting point in rethinking the nature and the consequence of wage rigidity in an arguably more realistic labor market model than the canonical DMP model.

I conclude by discussing several avenues for future research. First, my model features wage rigidity that is symmetric between booms and recessions, because of the first order approximation. I conjecture that my model will feature downward wage rigidity with a higher order approximation, through the mechanism of [Harris and Holmstrom \(1982\)](#). Since downward wage rigidity is the pervasive feature of the data, it would be promising to study its interaction with the labor market dynamics in a micro-founded manner. Second, while it has been common to assume an exogenously incomplete market in the heterogenous household literature, my model features an endogenously incomplete market through firm insurance subject to limited commitment frictions. It would be interesting to add consumption and saving decisions in my model to study the interaction between aggregate demand, equilibrium wage rigidity, and labor market dynamics. Third, while I mostly focused on theoretical aspects, my theory provides a new angle for looking at the data. For example, it would be fruitful to look into the relationship between the prevalence of on-the-job search, wage rigidity, and employment fluctuations at various levels of disaggregation.

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Appendix

A Proofs

A.1 Proof of Lemma 1

The derivation of (9) and (10) are provided in the main text. We need to show that the second-order condition for the new hire firms

$$F_0''(w(z))(Az - w(z)) - 2F_0'(w(z)) < 0$$

is satisfied. Totally differentiating (7) gives

$$F_0''(w(z))(Az - w(z)) = -F_0'(w(z))A\frac{1}{w'(z)} + 2F_0'(w(z)).$$

Therefore the second-order condition is

$$\begin{aligned} F_0''(w(z))(Az - w(z)) - 2F_0'(w(z)) &= -F_0'(w(z))A\frac{1}{w'(z)} \\ &< 0 \end{aligned}$$

since $F'(w(z)) > 0$ and $w'(z) > 0$.

Now consider incumbent firms. We have to guarantee that the promise-keeping constraint is binding. It is enough to impose the following assumption.

Assumption 1. *Parameters are such that $\Pi_0(w; z) \equiv (Az - w)(1 - \lambda^E + \lambda^E F_1(w))$ is decreasing in w for all z , where $F_1(w) \equiv \int_{w \geq w(z)} v(\tilde{z})/VdG(\tilde{z})$, and $w(z), v(z), \lambda^E, V$ are given by (9), (10) and (11).*

The assumption is always satisfied as long as λ^E is small enough. In fact, if the cost of vacancy is such that the vacancy is constantly proportional to employment, $v(z) = \bar{v}\ell_0(z)$,³¹ it is sufficient to have $\lambda^E < 1 - \chi$. Empirically, the share of employer-to-employer transitions among new hires is 40%, which implies $1 - \chi = 0.4$, while employer-to-employer transition rate at the quarterly frequency is around 5%, which implies $\lambda^E \approx 0.1$.³² Therefore the assumption is arguably natural to impose. Under Assumption 1, equation (4) implies that the constraint must

³¹This is empirically reasonable. Davis, Faberman, and Haltiwanger (2013) document that vacancy rate ($v(z)/\ell_0(z)$) is uncorrelated with firm-size measures.

³²Under the constant vacancy rate, $v(z)/\ell_0(z)$, the half of workers transition to new employer conditional on meetings.

be binding, $\eta(z) > 0$. As the worker's utility is strictly increasing in $w_0(z)$, if

$$\bar{W}_0(z) = (1 - \lambda^E)u(w(z)) + \lambda^E \int \max\{u(w(z)), u(w(\tilde{z}))\}(v(\tilde{z})/V)dG(\tilde{z}),$$

the incumbent firms have to set $w_0(z) = w(z)$. These complete that the equilibrium has the the properties claimed.

A.2 Proof of Lemma 2

The set of equilibrium conditions are

$$\begin{aligned} (1 - \chi)F'_{0s}(w_{1s}(z))(A_s z - w_s(z)) - (\chi + (1 - \chi)F_{0s}(w_{1s}(z))) &= 0 \\ -(1 - \lambda_s^E + \lambda_s^E F_{1s}(w_{0s}(z))) + (A_s z - w_s(z))\lambda_s^E F'_{1s}(w_{0s}(z)) & \\ + \eta(z) \left[(1 - \lambda_s^E)u'(w_{0s}(z)) + \lambda_s^E F_{1s}(w_{0s}(z))u'(w_{0s}(z)) \right] &= 0 \\ \sum_{s \in \{h,l\}} \pi_s \left[(1 - \lambda_s^E)u(w_{0s}(z)) + \lambda_s^E \int \max\{u(w_{0s}(z)), u(\tilde{w})\}dF_{1s}(\tilde{w}) \right] &= \bar{W}_0(z) \\ (A_s z - w_{1s}(z))\lambda_s^F (\chi + (1 - \chi)F_{0s}(w_{1s}(z))) &= c'_s(v_s(z); z) \\ \lambda_s^F = \frac{1}{V} \mathcal{M}(\tilde{\mu}, V), \quad \lambda^E = \zeta \frac{\mathcal{M}(\tilde{\mu}, V)}{\tilde{\mu}} &\text{ with } V = \int v(z)dG(z) \end{aligned}$$

Applying the generalized implicit function theorem (Luenberger, 1969) jointly to $\{w_{0s}(z), w_{1s}(z), v_s(z), V_s, \dots\}$ with respect to $d \ln A$, we have

$$\begin{aligned}
\frac{d \ln w_{1s}(z)}{d \ln A} &= \theta_{1a}(z)(\mathbb{I}(s = h) - \mathbb{I}(s = l)) + \theta_{1w}(z) \frac{d \ln w_{0s}(z)}{d \ln A} - \theta_{1a}(z) \alpha(z) \frac{w(z)}{w'(z)} \frac{d}{dz} \frac{d \ln w_{0s}(z)}{d \ln A} \\
\frac{d \ln w_{0s}(z)}{d \ln A} &= \theta_{0a}(z)(\mathbb{I}(s = h) - \mathbb{I}(s = l)) + \theta_{0w}(z) \frac{d \ln w_{1s}(z)}{d \ln A} - \theta_{0a}(z) \alpha(z) \frac{w(z)}{w'(z)} \frac{d}{dz} \frac{d \ln w_{1s}(z)}{d \ln A} \\
&\quad + \theta_{0a}(z) \alpha(z) \{1 - \theta_{\lambda,p}(z)\} \frac{d \ln \lambda_h^E}{d \ln A} \\
&\quad + \theta_{0a}(z) \alpha(z) \theta_{\lambda,r}(z) \left(\frac{1}{\int^z v(\tilde{z}) dG(\tilde{z})} \int \frac{dv_s(\tilde{z})}{d \ln A} dG(\tilde{z}) - \frac{1}{V} \frac{dV_h}{d \ln A} \right) \\
&\quad + \theta_{0a}(z) \alpha(z) \left(\frac{1}{v(z)} dv_h(z) - \frac{1}{V} dV_h \right) \\
&\quad + \theta_{0a}(z) \left[(1 - \lambda^E) u'(w_0(z)) + \lambda^E F_1(w_0(z)) u'(w_0(z)) \right] \frac{d\eta(z)}{d \ln A} \\
0 &= (1 - \lambda^E + \lambda^E F_1(w(z))) u'(w(z)) w(z) \sum_s \frac{d \ln w_{0s}(z)}{d \ln A} \\
&\quad + \int \sum_s \frac{d \ln w_{1s}(\tilde{z})}{d \ln A} (v(\tilde{z})/V) dG(\tilde{z}) \\
&\quad + \lambda^E \left(\int \max\{u(w(z)), u(\tilde{w})\} dF_1(\tilde{w}) - u(w(z)) \right) \frac{d \ln \lambda_s^E}{d \ln A} \\
&\quad + \lambda^E \int_z u(w(\tilde{z})) \sum_s \left(\frac{1}{V} \frac{dv_s(\tilde{z})}{d \ln A} - \frac{v(\tilde{z})}{V^2} \frac{dV_s}{d \ln A} \right) dG(\tilde{z}) \\
\frac{1}{v(z)} \frac{dv_s(z)}{d \ln A} &= \iota \left[\frac{d \ln \lambda_s^F}{d \ln A} + \frac{1 - \alpha(z)}{\alpha(z)} \left(\mathbb{I}(s = h) - \mathbb{I}(s = l) - \frac{d \ln w_{0s}(z)}{d \ln A} \right) \right] \\
\frac{d \ln \lambda_s^F}{d \ln A} &= (\kappa - 1) \frac{1}{V} \frac{dV_s}{d \ln A} \\
\frac{d \ln \lambda_s^E}{d \ln A} &= \kappa \frac{1}{V} \frac{dV_s}{d \ln A} \\
\frac{dV}{d \ln A} &= \int \frac{dv_s(z)}{d \ln A} dG(z),
\end{aligned}$$

where I used the assumption that $\pi_s = 1/2$. One can see that all the endogenous variables enter symmetrically between two states. We also know that at the top, wage offers must be symmetric

Lemma 4. *To a first order, $\frac{d \ln w_{1h}(\bar{z})}{d \ln A} = \frac{d \ln w_{0h}(\bar{z})}{d \ln A} = -\frac{d \ln w_{1l}(\bar{z})}{d \ln A} = -\frac{d \ln w_{0l}(\bar{z})}{d \ln A}$*

Proof. I proceed in four steps.

Step 1: $w_{1s}(\bar{z}) \leq w_{0s}(\bar{z})$ for $s \in \{h, l\}$ in equilibrium. Suppose not: $w_{1s}(\bar{z}) > w_{0s}(\bar{z})$ holds in equilibrium. Then the new hire firms can strictly increase profits by slightly lowering the wage (no change in labor supply, but lower costs). This is a contradiction.

Step 2: to a first order, $\frac{d \ln w_{0h}(\bar{z})}{d \ln A} = -\frac{d \ln w_{0l}(\bar{z})}{d \ln A}$. This is implied by the promise-keeping constraint at the top:

$$\sum_{s \in \{h,l\}} \pi_s u(w_{0s}(\bar{z})) = \bar{W}_0(\bar{z}) \quad (30)$$

because the first step implies that there cannot be higher wage offers than $w_{0s}(\bar{z})$ in equilibrium.

Step 3: $w_{0h}(\bar{z}) = w_{0l}(\bar{z})$. Suppose to the contrary $w_{0h}(\bar{z}) > w_{0l}(\bar{z})$. Then by slightly reducing $w_{0h}(\bar{z})$ and raising $w_{0l}(\bar{z})$ by the same amount will (i) weakly increases the labor supply, and (ii) relaxes the constraint $\sum_{s \in \{h,l\}} \pi_s u(w_{0s}(\bar{z})) \geq \bar{W}_0(\bar{z})$. This is a contradiction that $w_{0h}(\bar{z})$ was optimally set. Combined with Claim 1, $w_{1h}(\bar{z}) = w_{0h}(\bar{z})$.

Step 4: $\frac{d \ln w_{0l}(\bar{z})}{d \ln A} = \frac{d \ln w_{1l}(\bar{z})}{d \ln A}$. Suppose to the contrary that $\frac{d \ln w_{0l}(\bar{z})}{d \ln A} > \frac{d \ln w_{1l}(\bar{z})}{d \ln A}$. Then consider a perturbation of incumbent firms strategy that changes $\frac{d \ln w_{0l}(\bar{z})}{d \ln A}$ by $\Delta w_{0l}(\bar{z}) < 0$ and changes $\frac{d \ln w_{1l}(\bar{z})}{d \ln A}$ by $\Delta w_{0h}(\bar{z}) > 0$, with $\Delta w_{0l}(\bar{z}) = -\Delta w_{0h}(\bar{z})$. Around a symmetric steady-state, this does not impact worker's welfare to a first order, and therefore does not affect the constraint (30):

$$\begin{aligned} \sum_{s \in \{h,l\}} \pi_s u(w_{0s}(\bar{z})) &= \pi_s u'(w(\bar{z})) w(\bar{z}) (\Delta w_{0l}(\bar{z}) + \Delta w_{0h}(\bar{z})) \\ &= 0. \end{aligned}$$

However, this has the first order increase in labor supply:

$$\Delta(\text{labor supply}) = \underbrace{0 \times \Delta \hat{w}_{0l}(\bar{z})}_{\text{because there was no mass in the neighborhood}} + \underbrace{F'(w(\bar{z})) \times \Delta \hat{w}_{0h}(\bar{z})}_{\text{because there is a mass of competitors (from Step 3)}},$$

which in turn implies that this has a first order increase in profits. This is a contradiction that $\frac{d \ln w_{0l}(\bar{z})}{d \ln A}$ was optimum.

From Step 2, 3 and 4, we confirm

$$\frac{d \ln w_{1h}(\bar{z})}{d \ln A} = \frac{d \ln w_{0h}(\bar{z})}{d \ln A} = -\frac{d \ln w_{1l}(\bar{z})}{d \ln A} = -\frac{d \ln w_{0l}(\bar{z})}{d \ln A}.$$

□

Therefore Lemma 4 implies that at the boundary, $z = \bar{z}$, wage responses must be symmetric between two states. Then given all the coefficients in the system of ODEs enter symmetrically between two states, any solution has to be symmetric as well.

A.3 Proof of Lemma 3

Linearizing the new hire firms' FOC (5) gives

$$(1 - \chi)F'_{0s}(w_{1s}(z))(A_s z - w_s(z)) - (\chi + (1 - \chi)F_{0s}(w_{1s}(z))) = 0$$

$$w(z) [-F''_0(w(z))(Az - w(z)) + 2F'_0(w(z))] \hat{w}_{1s} \quad (31)$$

$$= F'_0(w(z))Az\hat{A} + (Az - w(z))\partial_{\mathbf{w}_0}F_{0s}(w(z)) - \partial_{\mathbf{w}_0}F_{0s}(w(z)), \quad (32)$$

where $\partial_{\mathbf{w}_0}$ denote the partial derivative with respect to entire distribution of $\{w_0(z)\}$. Using the fact that

$$\partial_{\mathbf{w}_0}F_{0s}(w_{0s}(z)) = -F'_0(w(z))w(z)\hat{w}_{0s}(z)$$

$$\partial_{\mathbf{w}_0}F'_{0s}(w_{0s}(z)) = \partial_{\mathbf{w}_0} \left(\frac{\ell_0(z)g(z)}{w'_{0s}(z)} \right)$$

$$= \partial_{\mathbf{w}_0} \left(\frac{\ell_0(z)g(z)}{w_{0s}(z) \frac{d \ln w_{0s}(z)}{dz} \Big|_{z=\zeta_{0s}(w)}} \right) \quad (\text{where } \zeta_{0s} \text{ is the inverse function of } w_{0s})$$

$$= -\frac{\ell_0(z)g(z)}{w(z) \frac{d \ln w(z)}{dz}} d \ln w_{0s}(z) - \frac{\partial}{\partial z} \left(\frac{\ell_0(z)g(z)}{w(z) \frac{d \ln w(z)}{dz}} \right) \zeta'_0(w_0(z))w_0(z) d \ln w_{0s}(z)$$

$$- \frac{\ell_0(z)g(z)}{w(z) \frac{d \ln w(z)}{dz}} \frac{1}{\frac{d \ln w(z)}{dz}} d \frac{d \ln w_{0s}(z)}{dz}$$

$$= -F'_0(w(z))d \ln w_{0s}(z) - \frac{\partial}{\partial z} \left(\frac{\ell_0(z)g(z)}{w(z) \frac{d \ln w(z)}{dz}} \right) \frac{1}{w'(z)} w(z) d \ln w_0(z)$$

$$- F'_0(w(z)) \frac{w(z)}{w'(z)} d \frac{d \ln w_{0s}(z)}{dz}$$

$$= -F'_0(w(z))d \ln w_{0s}(z) - F''_0(w(z))w(z)d \ln w_0(z) - F'_0(w(z)) \frac{w(z)}{w'(z)} \hat{w}'_0(z),$$

on can rewrite (31) as

$$[2(1 - \alpha(z)) - \alpha(z)\eta_{F_0}(z)] \hat{w}_1(z) = \hat{A} + [2(1 - \alpha(z)) - \alpha(z)\eta_{F_0}(z) - 1] \hat{w}_0(z) - \alpha(z) \frac{w(z)}{w'(z)} \hat{w}'_0(z).$$

Rearranging, one obtains (12).

Similarly, linearizing incumbent firms' FOC (4),

$$\begin{aligned}
& -\lambda^E F'(w(z))w(z)\hat{w}_{0s}(z) - \lambda^E \partial_{\mathbf{w}_1} F_{1s}(w) + Az\lambda^E F'(w)\hat{A}_s \\
& + (A_s z - w_s)\lambda^E F''(w)w\hat{w}_{0s}(z) + (A_s z - w_s)\lambda^E \partial_{\mathbf{w}_1} F'_{1s}(w_{0s}) \\
& + \eta(1 - \lambda^E)u''(w)w\hat{w}_{0s}(z) + \eta\lambda^E F(w)u''(w)w\hat{w}_{0s}(z) + \eta\lambda^E f(w)u'(w)\hat{w}_{0s}(z) \\
& + \eta\lambda^E u'(w_s)\partial_{\mathbf{w}_1} F_{1s}(w_{0s}) + d\eta(z) \left[(1 - \lambda^E)u'(w_s) + \lambda^E F(w_s)u'(w_s) \right] = 0.
\end{aligned}$$

One can use symmetry from Lemma 2 to eliminate $d\eta(z)$:

$$\begin{aligned}
& -\lambda^E F'(w(z))w(z)\hat{w}_0(z) - \lambda^E \partial_{\mathbf{w}_1} F_{1s}(w) + Az\lambda^E F'(w)\hat{A} \\
& + (Az - w(z))\lambda^E F''(w)w\hat{w}_0(z) + (Az - w(z))\lambda^E \partial_{\mathbf{w}_1} F'_{1s}(w_{0s}) \\
& + \eta(z)(1 - \lambda^E)u''(w(z))w\hat{w}_0(z) + \eta(z)\lambda^E F(w)u''(w(z))w\hat{w}_0(z) + \eta(z)\lambda^E F'_1(w(z))u'(w(z))\hat{w}_0(z) \\
& + \eta(z)\lambda^E u'(w(z))\partial_{\mathbf{w}_1} F_{1s}(w_{0s}(z)) = 0.
\end{aligned}$$

Using

$$\begin{aligned}
\partial_{\mathbf{w}_1} F_{1s}(w_{0s}(z)) &= -F'_1(w(z))w(z)\hat{w}_{1s}(z) \\
\partial_{\mathbf{w}_1} F'_{1s}(w_{0s}(z)) &= -F'_1(w(z))d \ln w_{1s}(z) - F''_1(w(z))w(z)d \ln w_1(z) - F'_1(w(z))\frac{w(z)}{w'(z)}\hat{w}'_1(z)
\end{aligned}$$

and the Lagrangin multipliers at the steady-state, $\eta(z) = \frac{(1 - \lambda^E + \lambda^E F(w(z))) + (Az - w(z))\lambda^E F'(w(z))}{u'(w(z))[(1 - \lambda^E) + \lambda^E F(w(z))]}$ and rearranging, one obtains

$$\begin{aligned}
& \left[2(1 - \alpha(z)) - \alpha(z)\eta_{F_1}(z) - \{(1 - \alpha(z)) + \alpha(z)\eta_\lambda(z)\} + \frac{\gamma}{\eta_\lambda(z)} \{(1 - \alpha(z)) + \alpha(z)\eta_\lambda(z)\} \right] \hat{w}_0(z) \\
& = \hat{A} + [2(1 - \alpha(z)) - \alpha(z)\eta_{F_1}(z) - 1 - \{(1 - \alpha(z)) + \alpha(z)\eta_\lambda(z)\}] \hat{w}_1(z) - \alpha(z)\frac{w(z)}{w'(z)}\frac{d}{dz}\hat{w}'_1(z).
\end{aligned}$$

Define

$$\begin{aligned}
\omega_1(z) &\equiv 2(1 - \alpha(z)) - \alpha(z)\eta_{F_1}(z) - \{(1 - \alpha(z)) + \alpha(z)\eta_\lambda(z)\} \\
\omega_2(z) &\equiv \frac{1}{\eta_\lambda(z)} \{(1 - \alpha(z)) + \alpha(z)\eta_\lambda(z)\},
\end{aligned}$$

where $\eta_{F_0}(z) = \frac{d \ln F'_0(w(z))}{d \ln w}$ and $\eta_{F_1}(z) = \frac{d \ln F'_1(w(z))}{d \ln w}$ are the elasticity of density of wage distributions, $\eta_\lambda(z) \equiv \frac{d \ln(1 - \lambda^E + \lambda^E F_1(w))}{d \ln w}$ is the elasticity of worker's staying probability, and γ is the relative risk aversion of workers. Then we obtain (13).

Now, I turn to the boundary conditions. Lemma 4 shows the boundary condition at the top is $\hat{w}_0(\bar{z}) = \hat{w}_1(\bar{z})$. Regarding the bottom, it must be the case that either $\hat{w}_1(\underline{z}) = \hat{A}$ or

$\hat{w}_0(\underline{z}) = \hat{A}$ as an interior solution. Because of the constraint that wages must be higher than the outside option of being unemployed, $\hat{w}_1(\underline{z}) \geq \hat{A}$ and $\hat{w}_0(\underline{z}) \geq \hat{A}$. Suppose that $\hat{w}_1(\underline{z}) > \hat{A}$ and $\hat{w}_0(\underline{z}) > \hat{A}$. Then one of the firms offering lower wages can lower wages without affecting the labor supply, contradicting to the optimality. If $\hat{w}_1(\underline{z}) = \hat{A}$ at an interior, then it must be the constraint $\hat{w}_0(\underline{z}) \geq \hat{A}$ must be (weakly) binding (not at an interior solution) because if $\hat{w}_0(\underline{z}) > \hat{A}$, then it would contradict the presumption that $\hat{w}_1(\underline{z}) = \hat{A}$ was an interior solution. In this case, the boundary of the incumbent firms is $w_0(\underline{z} + dz)$ and by continuity of wage strategy, it must be $w_0(\underline{z} + dz) \leq \hat{A}$. Similarly, $\hat{w}_0(\underline{z}) = \hat{A}$ at an interior solution, then $\hat{w}_1(\underline{z} + dz) \leq \hat{A}$. Finally, I claim that $\hat{w}_1(\underline{z}) = \hat{A}$ is the appropriate boundary condition, and $\hat{w}_0(\underline{z} + dz)$ is a free jump variable for the bottom of incumbent firms. Suppose to the contrary that $\hat{w}_0(\underline{z}) = \hat{A}$ is the boundary condition, then the system of ODEs imply that for any $\gamma > 0$, $\hat{w}_1(z) < \hat{A}$ and $\hat{w}_0(z) > \hat{A}$ for all z . To prove this, starting from $\hat{w}_0(\underline{z}) = \hat{A}$ and $\hat{w}_1(\underline{z} + dz) \in [0, \hat{A}]$, $\hat{w}'_0(z) > 0$ at $\hat{w}(z) = \hat{A}$ for any $\hat{w}_1(z) \in [0, \hat{A}]$ and $\hat{w}'_1(z) < 0$ at $\hat{w}_1(z) = \hat{A}$ for all $\hat{w}_0 \geq \hat{A}$. Therefore the path needs to feature $\hat{w}_1(z) < \hat{A}$ and $\hat{w}_0(z) > \hat{A}$ for all z , but then it would never be able to satisfy the boundary condition at the top, $\hat{w}_1(\bar{z}) = \hat{w}_0(\bar{z})$, a contradiction. Therefore $\hat{w}_1(\underline{z}) = \hat{A}$ is an appropriate boundary condition.

A.4 Proof of Proposition 1

Since all the coefficients on linear ODEs are continuous in z , there must exist a unique solution. Part (i) follows from the fact $\theta_{1a}(z) + \theta_{1w}(z) = 1$ and $\theta_{0a}(z) + \theta_{0w}(z) = 1$. Then one can easily verify $\hat{w}_1(z) = \hat{w}_0(z) = \hat{A}$ is a unique solution that satisfy boundary conditions.

In order to prove part (ii), consider whether there exists ζ such that $\hat{w}_0(\zeta) > \hat{A}$. There can potentially be two such cases. First case is $(\hat{w}_0(\zeta), \hat{w}_1(\zeta))$ with $\hat{w}_0(\zeta) > \hat{A}$ and $\hat{w}_1(\zeta) \leq \hat{A}$. Then starting from such a point, it is not possible to satisfy the boundary condition at the top. This is because $\hat{w}'_0(z) > 0$ at $\hat{w}(z) = \hat{A}$ for any $\hat{w}_1(z) \in [0, \hat{A}]$ and $\hat{w}'_1(z) < 0$ at $\hat{w}_1(z) = \hat{A}$ for all $\hat{w}_0 \geq \hat{A}$, and therefore the path features $\hat{w}_1(z) < \hat{A}$ and $\hat{w}_0(z) > \hat{A}$ for all $z > \zeta$.

Second case is $(\hat{w}_0(\zeta), \hat{w}_1(\zeta))$ with $\hat{w}_0(\zeta) > \hat{A}$ and $\hat{w}_1(\zeta) > \hat{A}$, but such a point is never reached. Starting from $w_1(\underline{z}) = \hat{A}$ and $\hat{w}_0(\underline{z} + dz) \in [0, \hat{A}]$, in order to reach such a point, it needs to go through either (i) $\hat{w}_0(z) = \hat{A}$ and $\hat{w}_1(z) \in [0, \hat{A}]$ or (ii) $\hat{w}_0(z) = \hat{A}$ and $\hat{w}_1(z) > \hat{A}$. Case (i) is already excluded from the previous paragraph. Case (ii) is also not possible because $\hat{w}'_0(z) < 0$ and $\hat{w}'_1(z) < 0$ at $\hat{w}_0(z) = \hat{A}$ and $\hat{w}_1(z) > \hat{A}$. These arguments complete the proof that $\hat{w}_0(z) < \hat{A}$.

In order to prove $\hat{w}_0(z) < \hat{w}_1(z)$, suppose to the contrary that there exists ζ such that $\hat{w}_0(\zeta) > \hat{w}_1(\zeta)$. It is always true that in such a region, $\hat{w}'_0(\zeta) > 0$. Then there can be potentially two cases: (i) $\hat{w}'_1(\zeta) < 0$ or (ii) $\hat{w}'_1(\zeta) > 0$. In the former case, it would never be able to satisfy the boundary condition at the top. The latter case is never reached.

Lastly, since the path needs to end up with $\hat{w}_0(\bar{z}) < \hat{A}$ and $\hat{w}_1(\bar{z}) = \hat{w}_0(\bar{z})$, and the path is

continuous, it is immediate to see that there must exist \bar{z} such that $\hat{w}_1(z) < \hat{A}$ for $z > \bar{z}$.

A.5 Proof of Proposition 1'

Note that

$$\theta_{1a}(z) = 2(1 - \alpha(z)) - \alpha(z)\eta_{F_0}(w(z)).$$

Totally differentiating (7) with respect to z gives

$$2(1 - \alpha(z)) - \alpha(z)\eta_{F_0}(w(z)) = \frac{w(z)}{w'(z)z}.$$

As $z \rightarrow \infty$, $w(z) \rightarrow \chi Ab + (1 - \chi) \int_b^\infty A\bar{z}d\hat{F}(\bar{z})$. From (8), we have

$$\begin{aligned} w'(z)z &= \frac{1}{(\chi + (1 - \chi)\hat{F}_0(z))} (1 - \chi)\hat{F}'_0(z)(Az - w(z))z \\ &\leq \frac{1}{(\chi + (1 - \chi)\hat{F}_0(z))} (1 - \chi)\hat{F}'_0(z)Az^2 \end{aligned}$$

Taking the limit, $z \rightarrow \infty$,

$$\begin{aligned} \lim_{z \rightarrow \infty} w'(z)z &\leq \lim_{z \rightarrow \infty} \frac{1}{(\chi + (1 - \chi)\hat{F}_0(z))} (1 - \chi)\hat{F}'_0(z)Az^2 \\ &= 0 \end{aligned}$$

where the last inequality follows from the assumption of finite variance. Therefore

$$\begin{aligned} \lim_{z \rightarrow \infty} \theta_{1a}(z) &= \frac{w'(z)z}{w(z)} \\ &= 0, \end{aligned}$$

which completes the proof.

A.6 Proof of Proposition 2

The optimality condition for vacancy creation is

$$(A_s z - w_{1s}(z))\lambda_s^F (\chi + (1 - \chi)F_{0s}(w_{1s}(z))) = A_s \bar{c}(z)(v_s(z))^{1/\iota}.$$

Taking log derivative,

$$\begin{aligned} \hat{\lambda}^F + \frac{1}{\alpha(z)} \hat{A} + \underbrace{\left(\frac{(1-\chi)F'_0(w(z))w(z)}{(\chi+(1-\chi)F_0(w(z)))} - \frac{w(z)}{Az-w(z)} \right)}_{=0 \text{ (from FOC of wages)}} \hat{w}_1^{exo}(z) \\ - \frac{(1-\chi)F'(w(z))w(z)}{(\chi+(1-\chi)F(w(z)))} \hat{w}_0^{exo}(z) = \hat{A} + \frac{1}{\iota} \hat{\vartheta}(z) \\ \Leftrightarrow \hat{\lambda}^F + \frac{1-\alpha(z)}{\alpha(z)} (\hat{A} - \hat{w}_0^{exo}(z)) = \frac{1}{\iota} \hat{\vartheta}(z), \end{aligned}$$

which is the firm-level vacancy response. To derive the aggregate response, note $\hat{\lambda}^F = (\kappa - 1)\hat{V}$. After multiplying both sides by $v(z)/V$ and adding up for all z , we obtain the aggregate response.

A.7 Vacancy response in DMP models

Without on-the-job search and with wage bargaining, the optimality condition for vacancy creation is

$$(A_s z - w_{1s}(z)) \lambda_s^F = A_s \bar{c}(z) (v_s(z))^{1/\iota}.$$

Taking log-derivative,

$$\frac{1-\alpha(z)}{\alpha(z)} (\hat{A} - \hat{w}_1(z)) + \hat{\lambda}_s^F = \frac{1}{\iota} \hat{\vartheta}(z).$$

To derive the aggregate response, note $\hat{\lambda}^F = (\kappa - 1)\hat{V}$. After multiplying both sides by $v(z)/V$ and adding up for all z , we obtain the aggregate response.

A.8 Derivations of equilibrium conditions with endogenous vacancy

The incumbent firm's FOC is

$$-(1 - \lambda_s^E + \lambda_s^E F_{1s}(w_{0s})) + (A_s z - w_s) \lambda_s^E F'_{1s}(w_{0s}) + \eta \left[(1 - \lambda_s^E) u'(w_{0s}) + \lambda_s^E F_{1s}(w_{0s}) u'(w_{0s}) \right] = 0.$$

Linearizing using symmetry,

$$\begin{aligned}
& \left[2(1 - \alpha(z)) - \alpha(z)\eta_{F_1}(z) - \{(1 - \alpha(z)) + \alpha(z)\eta_\lambda(z)\} + \frac{\gamma}{\eta_\lambda(z)} \{(1 - \alpha(z)) + \alpha(z)\eta_\lambda(z)\} \right] \hat{w}_0(z) \\
&= \hat{A} + [2(1 - \alpha(z)) - \alpha(z)\eta_{F_1}(z) - 1 - \{(1 - \alpha(z)) + \alpha(z)\eta_\lambda(z)\}] \hat{w}_1(z) - \alpha(z) \frac{w(z)}{w'(z)} \frac{d}{dz} \hat{w}'_1(z) \\
& \quad \frac{1}{AzF'(w)} \left\{ (1 - F_1(w(z))) + (Az - w(z))F'_1(w(z)) - \left(1 + (Az - w(z)) \frac{1}{w(z)} \eta_\lambda(z) \right) (1 - F_1(w(z))) \right\} \hat{\lambda} \\
& \quad \left\{ \alpha(z) \frac{1}{w(z)} \eta_\lambda(z) \right\} d \left(\frac{\bar{V}(z)}{V} \right) \\
& \quad \alpha(z) \frac{1}{v(z)/V} d \left(\frac{v(z)}{V} \right),
\end{aligned}$$

which I can rewrite further to obtain

$$\begin{aligned}
& \left[2(1 - \alpha(z)) - \alpha(z)\eta_{F_1}(z) - \{(1 - \alpha(z)) + \alpha(z)\eta_\lambda(z)\} + \frac{\gamma}{\eta_\lambda(z)} \{(1 - \alpha(z)) + \alpha(z)\eta_\lambda(z)\} \right] \hat{w}_0(z) \\
&= \hat{A} + [2(1 - \alpha(z)) - \alpha(z)\eta_{F_1}(z) - 1 - \{(1 - \alpha(z)) + \alpha(z)\eta_\lambda(z)\}] \hat{w}_1(z) - \alpha(z) \frac{w(z)}{w'(z)} \frac{d}{dz} \hat{w}'_1(z) \\
& \quad + \alpha(z) \left\{ 1 - \frac{\lambda^E (1 - F(w))}{1 - \lambda^E + \lambda^E F(w)} \right\} \hat{\lambda}^E \\
& \quad \left\{ \alpha(z) \frac{\lambda^E F(w)}{1 - \lambda^E + \lambda^E F(w)} \right\} d \left(\frac{\bar{V}(z)}{V} \right) \\
& \quad \alpha(z) (\hat{v}(z) - \hat{V}),
\end{aligned}$$

which is the one in the lemma. To complete the proof that boundary conditions are unchanged, note that if

$$\begin{aligned}
& \theta_{0a}(z) \hat{A} + \theta_{0w}(z) \hat{A} \\
& + \underbrace{\theta_{0a}(z) \alpha(z) \{1 - \theta_{\lambda,p}(z)\} \hat{\lambda}^E + \theta_{0a}(z) \alpha(z) \theta_{\lambda,r}(z) (\hat{V}(z) - \hat{V}) + \theta_{0a}(z) \alpha(z) (\hat{v}(z) - \hat{V})}_{\equiv B} < \hat{A},
\end{aligned}$$

then the same argument as in Lemma 3 applies because it only relied on the fact that $\hat{w}'_1(z) = 0$ locus in the phase diagram shifts downward. I can always guarantee this if ι is small enough, as $\lim_{\iota \rightarrow 0} B = 0$.

A.9 Proof of Proposition 3

We can verify that $\hat{w}_0(z) = \hat{A}$, $\hat{w}_1(z) = \hat{A}$, $\hat{v}(z) = 0$, and $\hat{V} = 0$ are the solutions the ODEs with $\gamma = 0$. The proposition follows because there is a unique solution. Next, as $\gamma \rightarrow \infty$, $\hat{w}_0(z) \rightarrow 0$ almost everywhere. Then combined with Proposition 2, we have the claim.

A.10 Proof of Proposition 4

As the new hire firms only hire from unemployed, the wage offer to unemployed is $w_{1s}(z) = A_s b$. Then the vacancy creation condition is

$$\lambda_s^F (A_s z - A_s b) = A_s \bar{c}(v)^{1/\iota},$$

in which A_s cancel out. Therefore vacancy is a constant, which implies the unemployment rate is a constant in response to the shock to aggregate productivity.

A.11 Proof of Proposition 4.1

The first order condition with binding fairness constraint is

$$\begin{aligned} & \lambda_s^F v(z) \left[(1 - \chi) F'_{0s}(w_s(z)) (A_s z - w_s(z)) - (\chi + (1 - \chi) F_{0s}(w_s(z))) \right] \\ & - \ell_0(z) \left[(1 - \lambda_s^E + \lambda_s^E F_{1s}(w_{0s}(z))) + (A_s z - w_s(z)) \lambda_s^E F'_{1s}(w_s(z)) \right. \\ & \left. + \eta(z) \left[(1 - \lambda_s^E) u'(w_s(z)) + \lambda_s^E F_{1s}(w_s(z)) u'(w_s(z)) \right] \right] = 0 \end{aligned}$$

Taking the first order approximation, we have

$$\begin{aligned} & \lambda^F v (1 - \chi) F'_0(w) \left[\hat{A} - \hat{w}(z) - \alpha(z) \frac{w(z)}{w'(z)} \hat{w}'(z) \right] \\ & + \lambda^E F'(w) \ell(z) \left[\hat{A} - \hat{w}(z) - \alpha(z) \frac{w(z)}{w'(z)} \hat{w}'(z) - \gamma \theta_\lambda(z) \hat{w}(z) \right. \\ & \left. \alpha(z) \{1 - \theta_{\lambda,p}(z)\} \hat{\lambda}^E + \alpha(z) \theta_{\lambda,r}(z) (\hat{V}(z) - \hat{V}) + \alpha(z) (\hat{v}(z) - \hat{V}) \right] = 0. \end{aligned}$$

Rearranging, we obtain the expression in the proposition.

A.12 Proof of Proposition 8

In order to leave workers indifferent, the set of perturbation must satisfy

$$\begin{aligned} & \left((1 - \lambda^E) + \lambda^E F_1(w_0(\tilde{z})) \right) u'(w_0(\tilde{z})) dw_0(\tilde{z}) \\ & + \mathbb{I}(z < \tilde{z}) u(w_1(\tilde{z})) g(z) (\lambda^E / V) dv(z) + \mathbb{I}(z > \tilde{z}) u(w_1(z)) g(z) (\lambda^E / V) dv(z) \\ & - u(w_0(\tilde{z})) \lambda^E \int_z v(\tilde{z}) dG(\tilde{z}) \frac{\partial(\lambda^E / V)}{\partial v(z)} dv(z) + \int_z u(w_1(\zeta)) v(\zeta) g(\zeta) d\zeta \frac{\partial(\lambda^E / V)}{\partial v(z)} dv(z) = 0 \\ & (1 - \lambda^U) u'(Ab) Adb + u(w_1(z)) g(z) (\lambda^E / V) dv(z) + \int_z u(w_1(\zeta)) (v(\zeta) / V) g(\zeta) d\zeta \frac{\partial(\lambda^U / V)}{\partial v(z)} dv(z) = 0 \end{aligned}$$

The effect on net total surplus is

$$\begin{aligned}
& - \int (A\tilde{z} - w(\tilde{z})) \left[\lambda^E \int_{\tilde{z}} v(\zeta) dG(\zeta) \right] dG(\tilde{z}) \\
& - \int (A\tilde{z} - w(\tilde{z})) \left[\lambda^E \mathbb{I}(z > \tilde{z}) \right] dG(\tilde{z})
\end{aligned}$$

$$\begin{aligned}
d\mathcal{F} &= -c'(v(z))g(z) \\
&+ \int \left\{ -(A\tilde{z} - w(\tilde{z})) \int_{\tilde{z}} v(\zeta) dG(\zeta) \frac{\partial \lambda^E / V}{\partial v(z)} + (A\tilde{z} - w(\tilde{z})) \lambda^E \mathbb{I}(z < \tilde{z}) \right\} \ell_0(\tilde{z}) dG(\tilde{z}) \\
&+ \lambda^F (\chi + (1 - \chi) F_0(w_1(z))) (Az - w_1(z)) \\
&+ \int \left\{ v(\tilde{z}) (\chi + (1 - \chi) F_0(w_1(\tilde{z}))) (Az - w_1(\tilde{z})) \frac{\partial \lambda^F}{\partial v(z)} \right\} dG(\tilde{z}) \\
&- \mu (Ab - Ab) \int_{\tilde{z}} v(\zeta) dG(\zeta) \frac{\partial \lambda^U / V}{\partial v(z)} \\
&- \lambda^E \int^z (A\tilde{z} - w(\tilde{z})) \ell_0(\tilde{z}) dG(\tilde{z}) dv(z) - \mu \lambda^U (Ab - Ab) dv(z) \\
&- \int \left\{ (1 - \lambda^E + \lambda^E F_1(w_0(\tilde{z}))) d w_0(\tilde{z}) \right\} \ell_0(\tilde{z}) dG(\tilde{z}) - \mu (1 - \lambda^U) db \\
&= - \underbrace{\int \left\{ (A\tilde{z} - w(\tilde{z})) \int_{\tilde{z}} v(\zeta) dG(\zeta) \frac{\partial \lambda^E / V}{\partial v(z)} + \mu Ab \int_{\tilde{z}} v(\zeta) dG(\zeta) \frac{\partial \lambda^U / V}{\partial v(z)} \right\} \ell_0(\tilde{z}) dG(\tilde{z})}_{(1)} \\
&+ \underbrace{\int \left\{ v(\tilde{z}) (\chi + (1 - \chi) F_0(w_1(\tilde{z}))) (Az - w_1(\tilde{z})) \frac{\partial \lambda^F}{\partial v(z)} \right\} dG(\tilde{z})}_{(2)} \\
&- \underbrace{\lambda^E \int^z (A\tilde{z} - w(\tilde{z})) \ell_0(\tilde{z}) dG(\tilde{z}) dv(z) - \mu \lambda^U (Ab - Ab) dv(z)}_{(3)} \\
&- \underbrace{\int \left\{ (1 - \lambda^E + \lambda^E F_1(w_0(\tilde{z}))) d w_0(\tilde{z}) \right\} \ell_0(\tilde{z}) dG(\tilde{z}) - \mu (1 - \lambda^U) db}_{(4)},
\end{aligned}$$

where I used FOC of vacancy creation in the last equality. The first line (1) is

$$\begin{aligned}
& - \int \left\{ (A\tilde{z} - w(\tilde{z})) \int_{\tilde{z}} v(\zeta) dG(\zeta) \frac{\partial \lambda^E / V}{\partial v(z)} + \mu Ab \int_{\tilde{z}} v(\zeta) dG(\zeta) \frac{\partial \lambda^U / V}{\partial v(z)} \right\} \ell_0(\tilde{z}) dG(\tilde{z}) \\
&= - \left(\int \left[w(\tilde{z}) P_0(\tilde{z}) - \int^{\tilde{z}} w(\zeta) dP_0(\zeta) \right] v(\tilde{z}) dG(\tilde{z}) \right) \frac{\partial (\mathcal{M} / V)}{\partial v(z)}
\end{aligned}$$

where $P_0(z) \equiv \chi \mathbb{I}(z > \underline{z}) + (1 - \chi) \frac{1}{1-\mu} \int^z \ell_0(\tilde{z}) dG(\tilde{z})$ is the cumulative employment distribution, and I used the expression of steady-state $w(z)$. The second line (2) is

$$\begin{aligned} & \int \left\{ v(\tilde{z}) (\chi + (1 - \chi) F_0(w_1(\tilde{z}))) (A\tilde{z} - w_1(\tilde{z})) \frac{\partial \lambda^F}{\partial v(z)} \right\} dG(\tilde{z}) \\ &= \int \{ v(\zeta) (A\zeta - w(\zeta)) P(\zeta) \} dG(\zeta) \frac{\partial \mathcal{M}/V}{\partial v(z)} \end{aligned}$$

The third line is

$$\begin{aligned} & - \lambda^E \int^z (A\tilde{z} - w(\tilde{z})) \ell_0(\tilde{z}) dG(\tilde{z}) dv(z) - \mu \lambda^U A b dv(z) \\ &= - \lambda^F \int^z [w(z) - w(\tilde{z})] dP_0(\tilde{z}) dv(z) \end{aligned}$$

The fourth line is

$$\begin{aligned} & - \int \left\{ (1 - \lambda^E + \lambda^E F_1(w_0(\tilde{z}))) dw_0(\tilde{z}) \right\} \ell_0(\tilde{z}) dG(\tilde{z}) - \mu (1 - \lambda^U) db \\ &= \left(- \int \int^{\tilde{z}} \frac{u(w(\zeta))}{u'(w(\tilde{z}))} dP(\zeta) v(\tilde{z}) dG(\tilde{z}) + \int \left\{ v(\zeta) \frac{u'(w(\zeta))}{u'(w(\tilde{z}))} P(\zeta) \right\} dG(\zeta) \right) \frac{\partial \mathcal{M}/V}{\partial v(z)} dv(z) \\ & \quad + \lambda^F \left(\int^z \frac{1}{u'(w(\tilde{z}))} u(w(z)) - \frac{1}{u'(w(\tilde{z}))} u(w(\tilde{z})) dP(\tilde{z}) \right) dv(z) \end{aligned}$$

Combining, we obtain the desired expression.

$$\begin{aligned} d\mathcal{F} &= \int \left((A\tilde{z} - w(\tilde{z})) P(\tilde{z}) - \int^{\tilde{z}} \left[(w(\tilde{z}) - w(\zeta)) - \frac{1}{u'(w(\tilde{z}))} (u(w(\tilde{z})) - u(w(\zeta))) \right] dP(\zeta) \right) v(\tilde{z}) dG(\tilde{z}) \frac{\partial (\mathcal{M}/V)}{\partial v(z)} \\ & \quad + \lambda^F \int^z \left(\frac{1}{u'(w(\tilde{z}))} [u(w(z)) - u(w(\tilde{z}))] - [w(z) - w(\tilde{z})] \right) dP(\tilde{z}) \end{aligned}$$

A.13 Proof of Proposition 9

Provided in the main text.

A.14 Neutrality Result in Infinite Horizon setup

Assume workers are risk-neutral. Let $w(z) = A\bar{w}(z)$. Then we can guess and verify that in the steady-state, all the value functions are homogenous in A :

$$W(w) = AW(\bar{w}), \quad U = A\bar{U}, \quad J(w, z) = AJ(\bar{w}, z).$$

Now consider aggregate risk with $\{A_h, A_l\}$. Since the cost of vacancy scales with A , if $\{w(z), v(z)\}$ is a steady-state equilibrium with A , $\{A_s/Aw(z), v(z)\}$ for $s = h, l$ is also an equilibrium with

no transition dynamics. Finally, (28) and the promise keeping constraint are satisfied as value functions are homogenous in A . Therefore we have that wages scale with aggregate productivity and no changes in employment and vacancy distribution. Without on-the-job search, wages are concentrated at $w(z) = Ab$ for all z . Therefore again, the value functions scale with A .

B Inefficiently flexible incumbent wages

I assume $\iota = 0$ and consider a small perturbation of incumbent wages of a particular firm z , $dw_{0h}(z)$ and $dw_{0l}(z)$. In order to leave workers indifferent, such perturbation must satisfy For incumbents,

$$\sum_s \pi_s \left(1 - \lambda^E + \lambda^E F_{1s}(w_{0s}(z)) \right) u'(w_{0s}(z)) dw_{0s}(z) = 0 \quad .$$

Then changes in net total surplus can be computed as

$$\begin{aligned} d\mathcal{F} &= -v(z)\lambda^F(1-\chi)F'_{0h}(w_{0h}(z))(A_h w_{1h}^{-1}(w_{0h}(z)) - w_{0h}(z))dw_{0h}(z) \\ &\quad - v(z)\lambda^F(1-\chi)F'_{0l}(w_{0l}(z))(A_l w_{1l}^{-1}(w_{0l}(z)) - w_{0l}(z))dw_{0l}(z) \\ &= \underbrace{-v(\tilde{z})\lambda_h^F(\chi + (1-\chi)F_{0h}(w_{0h}(z))) \left(1 - \frac{u'(w_{0h}(z)) (1 - \lambda_h^E + \lambda_h^E F_{1h}(w_{0h}(z)))}{u'(w_{0l}(z)) (1 - \lambda_l^E + \lambda_l^E F_{1l}(w_{0l}(z)))} \right)}_{<0} dw_{0h}(z), \end{aligned}$$

where I used new hire firm's FOC in the last equality. Therefore $dw_{0h}(z) < 0$ and $dw_{0l}(z) > 0$ improves welfare. We thus conclude

Proposition 10. *Assume $\iota \rightarrow 0$. Consider the equilibrium with aggregate risk. There exists a small perturbation of new hire wages $dw_{0h}(z), dw_{0l}(z)$ with $dw_{0h}(\tilde{z}) < 0$ and $dw_{0l}(\tilde{z}) > 0$ that yield Pareto improvement.*

C Details of solution method in infinite horizon model

I first log-linearize all the optimality condition. The first order approximation of the firm's value function that hired a worker at time τ is given by

$$\begin{aligned} \rho_f dJ_t(w, z) &= Azd \ln A_t - wd \ln w - \lambda^E(1 - F(w))J(w, z)d \ln \lambda_t^E \\ &\quad + \lambda^E J(w, z)dF_t(w) - (\delta + \lambda_t^E(1 - F_t(w)))dJ_t(w, z) + \partial_t dJ_t(w, z), \end{aligned}$$

where

$$dF_t(w) = -F'(w)wd \ln w_t(z) + \frac{1}{V}d\bar{V}_t(z) - \frac{1}{V^2}dV_t,$$

and $d\bar{V}_t(z) \equiv \int^z dV(\bar{z})dG(\bar{z})$. The response of $\partial_w J_t(w, z)$ is

$$\begin{aligned} \rho_f d\partial_w J_t(w, z) &= -\lambda^E F'(w) \partial_w J(w, z) d \ln w_t(z) \\ &\quad + \left\{ \lambda_t^E (1 - F(w)) \partial_w J(w, z) + \lambda^E F'(w) J(w, z) \right\} d \ln \lambda_t^E \\ &\quad + \lambda^E J(w, z) dF_t(w) + \lambda^E J(w, z) dF'_t(w) - (\delta + \lambda_t^E (1 - F_t(w))) d\partial_w J(w, z) \\ &\quad + \lambda_t^E F'_t(w) dJ(w, z) + \partial_t d\partial_w J_t(w, z), \end{aligned}$$

where

$$\begin{aligned} dF'_t(w) &= -F'(w) d \ln w_t(z) - F''(w) w d \ln w_t(z) - F'(w) \frac{w(z)}{w'(z)} \frac{d}{dz} (d \ln w_t(z)) \\ &\quad + F'(w) \left\{ \frac{1}{v(z)g(z)} \frac{d}{dz} (d\bar{V}_t(z)) - \frac{1}{V} dV_t \right\} \end{aligned}$$

The response of employment distribution is

$$\begin{aligned} \partial_t dP_t(w) &= -\left(\delta + \lambda_t^E (1 - F_t(w)) \right) dP_t(w) - \lambda^E (1 - F(w)) P_t(w) d \ln \lambda_t^E \\ &\quad + \left(\lambda^E P(w) + \frac{1}{1 - \mu} \mu \lambda^U \right) dF_t(w) + \frac{1}{(1 - \mu)^2} \lambda^U F(w) d\mu_t + \delta F(w) d \ln \lambda_t^U \end{aligned}$$

and the response of $P'_t(w)$ is

$$\begin{aligned} \partial_t dP'_t(w) &= -\left(\delta + \lambda_t^E (1 - F_t(w)) \right) dP'_t(w) + \left(F'(w) P_t(w) - \lambda^E (1 - F(w)) P'_t(w) \right) d \ln \lambda_t^E \\ &\quad + \lambda^E F'(w) dP_t(w) + \lambda^E P'(w) dF_t(w) + \left(\lambda^E P(w) + \delta \right) dF'_t(w) \\ &\quad + \frac{1}{(1 - \mu)^2} \lambda^U F'(w) d\mu_t + \delta F'(w) d \ln \lambda_t^U. \end{aligned}$$

The unemployment rate follows

$$\partial_t d\mu_t = -(\lambda^U + \delta) d\mu_t - \lambda^U \mu d \ln \lambda_t^U,$$

and the matching function implies

$$d \ln \lambda_t^U = d \ln \lambda_t^E = -\kappa d \ln \tilde{\mu}_t + \kappa d \ln V_t.$$

Therefore the first order response of new hire wages solve

$$\begin{aligned} Q(w) d\partial_w J_t(w, z) + (1 - \chi) \partial_w J(w, z) dP_t(w) + (1 - \chi) J(w, z) dP'_t(w) \\ + (1 - \chi) P'(w) dJ_t(w, z) + ((1 - P(w)) \partial_w J(w, z) - P'(w) J(w, z)) d\chi_t = 0, \end{aligned} \tag{33}$$

where $d\chi_t \equiv \left\{ \frac{\zeta}{(\zeta(1-\mu_t)+\mu_t)^2} \right\} d\mu_t$. As in the two-period model, the above expression only depends on a few number of variables, $\{d \ln w_t(z), dV_t, d\bar{V}_t(z), \frac{d}{dz} d \ln w_t(z), \frac{d}{dz} d\bar{V}_t(z), d\mu_t, d \ln \lambda_t\}$. Crucially, it does not depend on the wage distribution, and only depends on the wages of the neighboring competitors. The first order approximation of the optimality condition for vacancy creation is

$$\lambda^F \hat{P}_t(w) J(w, z) d \ln \lambda_t^F + \lambda^F J(w, z) dQ_t(w) + \lambda^F Q(w) dJ_t(w, z) = Ac''(v(z)) \frac{1}{g(z)} \frac{d}{dz} (d\bar{V}_t(z)). \quad (34)$$

The incumbent's wage response is

$$d\partial_w J_0(w, z) - \frac{\partial_w J(w, z)}{W'(w)} dW'_0(w) = 0, \quad (35)$$

where $dW'_t(w)$ is given by

$$\rho_w dW'_t(w) = u'(w_t) d \ln w - \delta dW'_t(w) - \lambda_t^E (1 - F_t(w)) dW'_t(w) + \lambda^E W'(w) dF_t(w) + \partial_t dW'_t(w).$$

The boundary conditions are

$$\begin{aligned} d \ln w_t(z) &= d \ln \underline{w}_t \\ d \ln w_0^{inc}(\bar{z}) &= d \ln w_0(\bar{z}). \end{aligned}$$

I solve the transition dynamics using the following algorithm. First, I guess the sequence of two aggregates: $\{d \ln \underline{w}_t, dV_t\}$. Given these two aggregates, one can immediately compute $\{d\mu_t, d\chi_t, d \ln \lambda_t^E, \partial_t \ln \lambda_t^U, d \ln \lambda_t^F\}$ using the matching function. Then I solve a system of linear ODEs, (33), (34) and (35) to obtain $\{d \ln w_t(z), d\bar{V}_t(z)\}$. I iterate over the guess of $\{d \ln \underline{w}_t, dV_t\}$ until I have $W_t(\underline{w}_t) = U_t$ and $d\bar{V}_t(\bar{z}) = dV$. In practice, I simply invert the matrix to find equilibrium $\{d \ln \underline{w}_t, dV_t\}$. This takes less than a second to compute the transition dynamics. Following [Auclert et al. \(2019\)](#), figure 7 shows the directed acyclical graph (DAG) representation of the first order responses of the economy.

With fairness constraint. Let ω_{1t} and ω_{2t} denote the Lagrangian multipliers on the constraint (38) and (39), respectively. The optimality conditions of firm's problem are

$$\partial_t \ell_t = -(\delta + \lambda_t^E (1 - F_t(W_t))) \ell_t + v \lambda_t^F (\chi_t + (1 - \chi_t) P_t^{eq}(W_t))$$

$$\rho_w W_t = u(w_t) + \delta \{U_t - W_t\} + \lambda_t^E \int \max \{0, \tilde{W} - W_t\} dF_t(\tilde{W}) + \partial_t W_t,$$

$$-\ell - \omega_2 u'(w) = 0$$

$$(Az - w) - \omega_1 \left(\delta + \lambda_t^E (1 - F_t(W_t)) \right) = \rho_f \omega_1 - \dot{\omega}_1$$

$$\omega_1 \left(\lambda_t^E F'(W) \ell + v \lambda^F (1 - \chi_t) P'(W) \right) + \omega_2 \left\{ \rho_w + \delta + \lambda^E (1 - F_t(W)) \right\} = \rho_f \omega_2 - \dot{\omega}_2$$

$$\lambda_t^F (\chi_t + (1 - \chi_t) P_t^{eq}(W_t)) \omega_{1t} = c'(v).$$

The initial condition W_0 is pinned down by the risk-sharing condition:

$$\omega_1 \left(\lambda_t^E F'(W_0) \ell + v \lambda^F (1 - \chi_t) P'(W_0) \right) + \omega_2 + \eta = 0,$$

where η is the Lagrangian multiplier constraint on the promise-keeping constraint.

As before, linearizing the equilibrium conditions to obtain the system of linear ordinary differential equations.

$$\begin{aligned} & \left(\lambda_t^E F'(W) \ell + v \lambda^F (1 - \chi_t) P'(W) \right) u'(w) d\omega_1 + \omega_1 \left(\lambda_t^E F'(W) \ell + v \lambda^F (1 - \chi_t) P'(W) \right) u''(w) w d \ln w \\ & + u'(w) \omega_1 \lambda_t^E \ell dF'(W) + u'(w) \omega_1 \lambda_t^E \ell F'(W) d \ln \lambda_t^E + u'(w) \omega_1 \lambda_t^E F'(W) d\ell \\ & + \lambda^F (1 - \chi_t) P'(W) u'(w) \omega_1 dv + v \lambda^F (1 - \chi_t) P'(W) u'(w) \omega_1 d \ln \lambda^F \\ & + v \lambda^F (1 - \chi_t) u'(w) dP'(W) - v \lambda^F \omega_1 u'(w) P'(W) d\chi \\ & - \left\{ \rho_w + \delta + \lambda^E (1 - F_t(W)) \right\} d\ell - \left\{ \lambda^E (1 - F_t(W)) \right\} \ell d \ln \lambda^E - \gamma \partial_t d \ln w_t = 0 \end{aligned}$$

$$\begin{aligned} \partial_t d\ell_t &= -(\delta + \lambda_t^E (1 - F_t(W_t))) d\ell_t + v \lambda_t^F (1 - P_t(W_t)) d\chi_t + \lambda_t^E \ell_t d \frac{J(z)}{V} \\ &+ (\chi_t + (1 - \chi_t) P_t(W_t)) \lambda_t^F dv - \lambda_t^E (1 - F_t(W_t)) \ell_t d \ln \lambda_t^E + (\chi_t + (1 - \chi_t) P_t(W_t)) \lambda_t^F v d \ln \lambda_t^F \\ &+ v \lambda_t^F (1 - \chi_t) dP_t(W_t) \end{aligned}$$

$$\begin{aligned} \partial_t dW &= u'(w) d \ln w - (\rho + \lambda^E + \delta) dW_{t+\Delta} + \delta d\bar{U}_{t+\Delta} + \lambda^E \int \max\{d\tilde{W}_t, dW_t\} dF^{eq}(\tilde{W}_t) \\ &+ \lambda^E \left[-W(w_s) + \int \max\{\tilde{W}, W\} dF^{eq}(\tilde{W}) \right] d \ln \lambda^E + \lambda^E \int \max\{W(\tilde{z}), W\} d\hat{F}'(\tilde{z}) d\tilde{z} \end{aligned}$$

$$\begin{aligned} d\omega_{1,t} &= Az d \ln A_t - w d \ln w - (\rho + \delta + \lambda^E (1 - F^{eq}(W))) d\omega_{1,t} - \omega_1 \lambda_t^E (1 - F_t(W_t)) d \ln \lambda_t^E + \omega_1 \lambda_t^E d\hat{F}(z) \\ &\left(\lambda_t^E F'(W_0) \ell + v \lambda^F (1 - \chi_t) P'(W_0) \right) d\omega_1 + \omega_1 d \left(\lambda_t^E \ell d + v \lambda^F (1 - \chi_t) P'(W_0) \right) + d\omega_2 + d\eta = 0, \end{aligned}$$

and the rest of the equilibrium conditions are unchanged from the one without fairness constraint.

D Importance of New hire wage rigidity in other environments

D.1 Competitive search

Consider the following model with competitive search (Moen, 1997; Acemoglu and Shimer, 1999) without on-the-job search. The model is static that follows Wright, Kircher, Julien, and Guerrieri (2019). Each firm with productivity z posts wage w , and workers see all the wages and direct their search. Let $q(w)$ denote the unemployment-to-vacancy ratio in the sub-market with wage w . Then $\lambda^F(w) \equiv \mathcal{M}(q(w), 1)$ denote the meeting probability of a firm when the firm posts w , and $\lambda^U(w) \equiv \frac{1}{q(w)}\mathcal{M}(q(w), 1)$ is the meeting probability of unemployed. Unemployed workers earn Ab . Workers must be indifferent across sub-markets:

$$\begin{aligned}\bar{U} &= \lambda^U(w)w + (1 - \lambda^U(w))Ab \\ &= \frac{1}{q(w)}\mathcal{M}(q(w), 1)w + \frac{1}{q(w)}(1 - \mathcal{M}(q(w), 1))Ab,\end{aligned}$$

which defines $q(w)$ implicitly.

If they can, then firms set wages so as to

$$w^* = \arg \max_w \Pi(w; A, z) = \mathcal{M}(q(w), 1)(Az - w).$$

The optimal amount of vacancy creation for a given wage w is that

$$v^*(w; A, z) = \arg \max_v \Pi(w; A, z)v - c(v; A, z).$$

Then by envelope theorem, if the firm was setting the wage optimally, there is no first order effect of wages on profits:

$$\frac{\partial \Pi(w^*, A)}{\partial w} = 0,$$

which implies that vacancy is unaffected by wage as well

$$\frac{\partial v^*(w^*; A, z)}{\partial w} = 0.$$

In this environment, what matters for the vacancy creation of a particular firm is not whether that firm can adjust wages or not, but that whether other firms can adjust wages. How the wages of other firms determined? In the baseline competitive search environment, there is a perfectly elastic free-entry, $c'' = 0$. If those entrants can freely choose wages, then there cannot be any equilibrium new hire wage rigidity. Similar results hold in the context of price-setting, as studied by Bilbiie (2020). One can work with new hire wage rigidity with inelastic entry, but this also kills the tractability of competitive search.

E Quantitative Infinite-Horizon Setup

E.1 Perfect-foresight Equilibrium and Steady-state Characterization without Fairness Constraints

The wage offer distribution is

$$F_t(w) = \frac{1}{V_t} \int_{z:w(z) \leq w} v_t(z) dG(z). \quad (36)$$

The meeting probabilities are

$$\lambda_t^U = \frac{1}{\tilde{\mu}_t} \mathcal{M}(\tilde{\mu}_t, V_t), \quad \lambda_t^E = \zeta \lambda_t^U, \quad \lambda_t^F = \frac{1}{V_t} \mathcal{M}(\tilde{\mu}_t, V_t), \quad \text{where } V_t \equiv \int v_t(z) dG(z). \quad (37)$$

Equilibrium definition is as follows:

Definition 2. *Equilibrium with constant aggregate productivity consists of a sequence of $\{w_t(z), v_t(z)\}$, $\{P_t(w), F_t(w), \underline{w}_t, \mu_t\}$, $\{\lambda_t^U, \lambda_t^E, \lambda_t^F\}$ such that (i) given $\{P_t(w), F_t(w), \lambda_t^U, \lambda_t^E, \lambda_t^F, \underline{w}_t\}$, $\{w_t(z), v_t(z)\}$ solve (25); (ii) the reservation wages satisfy $W_t(\underline{w}_t) = U_t$, where U_t and W_t are given by (26) and (27), respectively; (iii) the unemployment, the wage employment distribution, $P_t(w)$, and the wage offer distribution, $F_t(w)$, satisfy (23), (24), and (36), respectively; and (iv) meeting probabilities are given by (37).*

The steady-state unemployment rate is given by $\mu = \frac{\delta}{\delta + \lambda^U}$. The steady-state employment weighted wage distributions are

$$P(w) = \frac{\delta F(w)}{\delta + \lambda^E(1 - F(w))}, \quad Q(w) = \frac{\delta}{\delta + \lambda_t^E(1 - F_t(w))}.$$

The firm's Bellman equation in the steady-state is

$$J(w, z) = \frac{Az - w}{\rho_f + \delta + \kappa + \lambda^E(1 - F(w))}.$$

Using the above expressions, one can rewrite firms' FOCs as

$$\lambda^E F'(w) \frac{Az - w(z)}{\delta + \lambda^E(1 - F(w(z)))} + \lambda^E F'(w) \frac{Az - w(z)}{\rho + \delta + \kappa + \lambda^E(1 - F(w(z)))} = 1$$

and

$$\lambda^F Q(w(z)) J(w(z), z) = Ac'(v(z)).$$

Because firms' profits are log-supermodular in (w, z) , wages are increasing in firm's productivity. Therefore, as in [Burdett and Mortensen \(1998\)](#), the steady-state equilibrium is rank-

preserving in a sense that workers always move toward more productive firms. Defining $\hat{F}(z) \equiv F(w(z)) = \frac{1}{V} \int^z v(\tilde{z}) dG(\tilde{z})$ and $\hat{Q}(z) \equiv Q(w(z))$, we can write the steady-state equilibrium wage and vacancy distribution $\{w(z), \hat{F}(z)\}$ as the solution to the following system of ODEs:

$$\lambda^E \hat{F}'(z) \frac{Az - w(z)}{\delta + \lambda_t^E (1 - \hat{F}(z))} + \lambda^E \hat{F}'(z) \frac{Az - w(z)}{\rho + \delta + \kappa + \lambda_t^E (1 - \hat{F}(z))} = w'(z)$$

$$\lambda^F \hat{Q}(z) J(w(z), z) = Ac' (V \hat{F}'(z) / g(z))$$

with the boundary conditions $w(\underline{z}) = \underline{w}$, where \underline{w} satisfies $W(\underline{w}) = U$, and $\hat{F}(\underline{z}) = 0$. One still needs to solve fixed point in terms of aggregate vacancy, V , because meeting probabilities, λ^F and λ^U , and unemployment rate μ , using (37). Note that in the steady-state, workers' risk aversion, γ , plays no role.

Given the wage and vacancy distributions, workers value function in the steady-state are given by

$$\rho W(w) = u(w) + \delta \{U - W(w)\} + \lambda^E \int \max \{0, W(\tilde{w}) - W(w)\} dF(\tilde{w})$$

$$\rho U = u(Ab) + \lambda^U \int \max \{0, W(\tilde{w}) - U\} dF(\tilde{w}).$$

E.2 Environment with Fairness Constraints

I impose a restriction that firms cannot discriminate wages across employees. Firms commit to a sequence of wage payments $\{w_s\}$ that delivers W_t of the expected lifetime utility to the workers employed at the firm. Workers accept the job that offers a higher value. Let $F_t^{eq}(W) \equiv \frac{1}{V_t} \int_{z: W_t(w_t(z)) \leq W} v_t(z) dG(z)$ denote the cumulative distribution function of the offer distribution. The employment distribution of worker value P_t^{eq} evolves in an analogous manner as in (24):

$$\partial_t P_t^{eq}(W) = -\delta P_t^{eq}(W) - \lambda_t^E (1 - F_t^{eq}(W)) P_t^{eq}(W) + \frac{1}{1 - \mu_t} \mu_t \lambda_t^U F_t^{eq}(W).$$

The employment in a particular firm z evolves according to

$$\partial_t \ell_t = -(\delta + \lambda_t^E (1 - F_t^{eq}(W_t))) \ell_t + v \lambda_t^F (\chi_t + (1 - \chi_t) P_t^{eq}(W_t)) \quad (38)$$

For a given W_0 , a firm chooses its wage policy and vacancies to maximize profits

$$\Pi(W_0; z) \equiv \max_{\{W_t, w_t, v_t\}} \int e^{-\rho t} \{(Az_t - w_t) \ell_t - c(v_t)\} dt$$

subject to (38) and the worker's Bellman equation:

$$\rho_w W_t = u(w_t) + \delta \{U_t - W_t\} + \lambda_t^E \int \max \{0, \tilde{W} - W_t\} dF_t^{eq}(\tilde{W}) + \partial_t W_t. \quad (39)$$

The rest of the models are unchanged from before. Appendix E.2 characterizes the steady-state of this economy.

Again, I consider the following dynamics. At $t = 0$, the economy is at the steady-state. Then there is a news that the aggregate productivity could be permanently high or low in the following periods. Firms insure workers by writing state contingent wage contracts that deliver the expected utility that is at least as large as promised in the steady-state:

$$\begin{aligned} \max_{\{W_t^s\}} \quad & \sum_{s \in \{h,l\}} \pi_s \Pi(W_0^s; z) \\ \text{s.t.} \quad & \sum_s \pi_s W_t^s \geq W(z). \end{aligned}$$

Given the initial W_0^s , the economy follows the perfect foresight equilibrium described above. Differently from before, wages are not fixed during the tenure period. Rather, firms offer the same time-varying wages to both incumbent workers and new hires.

As before, the steady-state is rank-preserving: more productive firms offer higher wage (values) to workers. The steady-state wages $w(z)$ and wage offer distribution $\hat{F}(z) \equiv \int^z v(\tilde{z}) / V dG(\tilde{z})$ solve the following system of ODEs:

$$\begin{aligned} 2\lambda^E \hat{F}'(z) \frac{Az - w(z)}{\delta + \lambda^E (1 - \hat{F}'(z))} &= 1 \\ \lambda^F \hat{Q}(z) J(w(z), z) &= Ac'(V\hat{F}'(z)/g(z)). \end{aligned}$$

Compared with the model without equal treatments, we can immediately see that as $\rho \rightarrow 0$, the steady-state equilibrium coincide. I focus on a symmetric steady-state, in which firms with the same productivity employ the same number of workers. Let $W(z)$ denote the utility that a firm with productivity z promises to workers in the steady-state.