

Aggregate Implications of Innovation Policy

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Abstract

We examine the quantitative impact of changes in innovation policies on aggregate productivity and output in a baseline Klette-Kortum Neo-Schumpeterian growth model. We present simple analytical results isolating the specific features and/or parameters of the model that play the key roles in shaping its quantitative implications for the aggregate impact of innovation policies in the short-, medium- and long-term. We find in our baseline model that permanent changes in innovation policies cannot spur a significant change in aggregate productivity over the medium term horizon (i.e. 20 years) in an economy with a moderate initial growth rate of TFP unless the change in policy leads to a very large change in the innovation rate spurred by a very large change in investment in innovation relative to GDP. Moreover, we show that the medium term dynamics implied by the model are *not* very sensitive to the parameters of the model that determine the model's long run implications. We find that one of the key features of the Klette-Kortum model (and many other Neo-Schumpeterian growth models) that drive these quantitative implications for the medium term is the assumption that there is no *social depreciation of innovation expenditures*.

1 Introduction

Firm's investments in innovation are large relative to GDP and are likely an important factor in accounting for economic growth over time.¹ Many OECD countries use taxes and

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¹There is a wide range of estimates of the scale of firms' investments in innovation. In the new National Income and Product Accounts revised in 2013, private sector investments in intellectual property products

subsidies to encourage these investments in the hope of stimulating economic growth.² To what extent can we change the path of macroeconomic growth over the medium and long term with tax and subsidy policies aimed at encouraging firms to increase their investments in innovation?

We examine this question in the context of a [Klette and Kortum \(2004\)](#) style Neo-Schumpeterian growth model. As described in [Aghion et al. \(2013\)](#), the Klette-Kortum model is a highly influential model that links micro data on firm dynamics to firms' investments in innovation and, in the aggregate, to economic growth in a tractable manner.³ One of the distinguishing features of this model is that there is, potentially, a large gap between the social and the private returns to firms' investments in innovation. As a result, one cannot use standard growth accounting methods for using data on the private returns to these investments to infer the implications for the path of aggregate productivity and output of innovation policy changes that induce a given change in firms' investments in innovation.⁴

We present simple analytical results approximating the impulse responses of the logarithm of aggregate productivity and GDP with respect to a policy-induced (i.e., innovation subsidies) change in the logarithm of firms' spending on innovation relative to GDP. In deriving these results we isolate the specific features and/or parameters of the model that play the key roles in shaping its quantitative implications for the links between changes in innovation policy and the resulting changes in innovation spending, the innovation rate, and the growth of aggregate productivity and GDP at short, medium, and long horizons. We also use these analytical impulse response functions to highlight the features of the model that drive its implications for the socially optimal innovation intensity of the economy. We confirm the utility of these analytical approximations by

were 3.8% of GDP in 2012. Of that amount, Private Research and Development was 1.7% of GDP. The remainder of that expenditure was largely on intellectual property that can be sold such as films and other artistic originals. [Corrado et al. \(2005\)](#) and [Corrado et al. \(2009\)](#) propose a broader measure of firms' investments in innovation, which includes non-scientific R&D, brand equity, firm specific resources, and business investment in computerized information. These broader investments in innovation accounted for roughly 13% of non-farm output in the U.S. in 2005.

²See, for example, Chapter 2 of "Economic Policy Reform: Going for Growth", OECD, 2009.

³Several authors (see, for example, [Akcigit and Kerr 2010](#) and [Lentz and Mortensen 2008](#)) have shown that extended versions of the Klette-Kortum model provide a good fit to many features of micro data on firms.

⁴There is a very large literature that seeks to use standard methods from growth accounting to capitalize firms' investments in innovation and to use the dynamics of that intangible capital aggregate to account for the dynamics of aggregate productivity and output. See, for example, [Griliches, ed \(1987\)](#), [Kendrick \(1994\)](#), [Griliches \(1998\)](#), and [Corrado and Hulten \(2013\)](#). Relatedly, [McGrattan and Prescott \(2012\)](#) use an overlapping generations model augmented to include firms' investments in intangible capital to ask how changes in various tax and transfer policies will impact the accumulation of intangible capital and aggregate productivity and GDP.

comparing them to the numerical solution to the equilibrium transition path following a change in innovation policies in a fully calibrated version of the model.

We find in this baseline model that permanent changes in innovation subsidies cannot spur a significant change in aggregate productivity over the medium term horizon (i.e. 20 years) in an economy with a moderate initial growth rate of TFP unless the change in policy leads to a very large change in the innovation rate spurred by a very large change in firms' investment in innovation relative to GDP.⁵ This finding is not sensitive to changes in parameters that determine the long-run implications of the model and the potential welfare gains that might be achieved from a sustained increase in innovation subsidies. Thus, in this model, if there are large welfare gains to an infinitely-lived consumer from a permanent increase in innovation subsidies, they are achieved only because of changes in the path of consumption that occur in the long run.

We show analytically that one of the key features of the Klette-Kortum model (and nearly all Neo-Schumpeterian growth models) that drive its quantitative implications for the short and medium term is the implicit assumption that there is no *social depreciation of innovation expenditures*. Specifically, in this model, there is private depreciation of past investments in innovation in terms of their impact on firms' profits — firms gain and lose products and profits as they expend resources to innovate upon the products produced by others. In contrast, there is no social depreciation of these investments in terms of their cumulative impact on aggregate productivity — the contribution of past innovation expenditures to aggregate production possibilities never dies out over time.⁶ We show that if one makes the alternative assumption that past innovations experience even moderate social depreciation, that is that aggregate productivity would shrink in the absence of investments in innovation by firms, then the model can produce significantly larger medium-term elasticities of aggregate productivity and GDP to permanent policy-induced changes in the innovation intensity of the economy. In the appendix, we show that in expanding varieties models along the lines of [Luttmer \(2007\)](#) and [Atkeson and](#)

⁵[Comin and Gertler \(2006\)](#) develop a model of medium-term business cycles based on endogenous movements in aggregate productivity that includes adoption, variable markups, and variable factor utilization. They find that with the combination of these factors, their model can account for significant medium-term cyclical productivity dynamics. We see our results as highlighting the endogenous dynamics of aggregate productivity that arise solely from policy-induced variation in the innovation intensity of the economy. [McGrattan and Prescott \(2012\)](#) emphasize how measurement conventions for GDP impact the measurement of aggregate productivity in the face of time variation in the scale of firms' investments in intangible capital. We discuss the role of different NIPA convention methods for shaping the responses of measured productivity and output.

⁶[Corrado and Hulten \(2013\)](#), [Li \(2012\)](#) and [Aizcorbe et al. \(2009\)](#) discuss comprehensive estimates of the depreciation rates of innovation expenditures without distinguishing between measures of the private and social depreciation of these expenditures.

Burstein (2010) there is social depreciation of innovation expenditures due to the exit of incumbent firms, but otherwise these models imply very similar expressions for the dynamics of aggregate productivity as in our baseline Klette-Kortum model. We show that since these models have social depreciation of innovation built-in, they can imply larger medium-term elasticities of aggregate productivity with respect to changes in innovation policies.⁷

The Klette-Kortum model provides a rich and yet tractable model of the birth, growth, and death of firms in which these firm dynamics are driven by incumbent and entrant firms' investments in innovation. One of the principal innovations of the Klette-Kortum model relative to the standard Quality Ladders model introduced by Aghion and Howitt (1992) and Grossman and Helpman (1991) is that it considers investments in innovation by both incumbent and entering firms. We show that, given the same change in innovation subsidies, the two models imply the same change in innovation intensity and aggregate productivity in the long run, up to a first-order approximation. Moreover, if we consider innovation policy changes in the Klette-Kortum model that produce the same transition path for the innovation intensity of the economy as the innovation policy changes considered in a Quality Ladders version of the model,⁸ then the Quality Ladders version of the model will imply a larger change in aggregate productivity up to any finite horizon along the transition path, up to a first-order approximation. We show that this result follows from the assumption in the Klette-Kortum model that incumbent firms have a smaller average cost of innovation than entering firms.

To focus attention on the aggregate implications of innovation policies that change the aggregate innovation intensity of the economy, we maintain a baseline set of assumptions which imply that, while the aggregate level of innovation expenditures may be sub-optimal, there is no misallocation of innovation expenditures across firms in the model economy at the start of the transition following a change in innovation policies. Thus we abstract from the role innovation policies might play in improving the allocation of innovation expenditures across firms and thus raising the aggregate innovation rate without increasing aggregate innovation expenditures. There is a growing literature examining

⁷As we discuss below, the aggregate dynamics implied by our baseline version of the Klette-Kortum model can also be derived in the simpler model studied in Jones (2002). Since the model of Jones (2002) has no social depreciation of innovations, it also implies modest medium-term elasticities of aggregate productivity to changes in the innovation intensity of the economy. Consistent with his results, for longer time horizons under certain parameter configurations, a sustained increase in the innovation intensity of the economy can have a sustained impact on the growth of aggregate productivity.

⁸Considering given changes in the innovation intensity of the economy is reminiscent to Arkolakis et al. (2012), who take changes in trade shares as given when comparing the aggregate implications of alternative trade models.

this possibility, see for example [Acemoglu et al. \(2013\)](#), [Buera and Fattal-Jaef \(2014\)](#), and [Peters \(2013\)](#). We see our results as providing a useful analytical benchmark to which numerical results from richer models can be compared.

The paper is organized as follows. Section 2 describes the model. Section 3 characterizes a balanced growth path. Section 4 presents analytic results on the impact of changes in innovation policy on aggregate outcomes at different horizons. Section 5 discusses the quantitative implications of our analytic results, and Section 6 compares these results to the full numerical solution of our calibrated model. Section 7 concludes. The appendix provides some proofs and other details including the calibration of the model.

2 Model

In this section we describe the physical environment, innovation policies, and the equilibrium.

Physical environment

Time is discrete and labeled $t = 0, 1, 2, \dots$. There are two final goods, the first of which we call the *consumption good* and the second which we call the *research good*. The representative household has preferences over consumption C_t given by $\sum_{t=0}^{\infty} \beta^t L_t \log(C_t/L_t)$, with $\beta \leq 1$, where L_t denotes the population that is constant and normalized to 1, ($L_t = 1$).⁹ Labor can be allocated to current production of the consumption good, L_{pt} , and to research, L_{rt} . The resource constraint requires that labor used in these two activities must sum to the fixed total population that we normalize to 1, that is, $L_{pt} + L_{rt} = L_t = 1$.

Production of the consumption good: The consumption good is produced as a constant elasticity of substitution (CES) aggregate of the output of a continuum of measure one of differentiated intermediate products. Each intermediate good is characterized by an index z that denotes the frontier technology for producing that product. These intermediate goods are then combined to produce the consumption good according to

$$Y_t = A_{pt} \left(\int_z y_t(z)^{(\rho-1)/\rho} dJ_t(z) \right)^{\rho/(\rho-1)}, \quad (1)$$

where $J_t(z)$ denotes the distribution of z across intermediate goods at date t (so $\int_z dJ_t(z) = 1$), A_{pt} denotes a stationary aggregate productivity shock with mean 1 and $\rho \geq 1$.

⁹It is straightforward to extend our analysis to include population growth.

Output of the consumption good, Y_t , is used for two purposes. First, as consumption by the representative household, C_t . Second, as investment in physical (tangible) capital, $K_{t+1} - (1 - d_k) K_t$, where K_t denotes the aggregate physical capital stock and d_k denotes the depreciation rate of physical capital. The resource constraint of the final consumption good is

$$C_t + K_{t+1} - (1 - d_k) K_t = Y_t. \quad (2)$$

Under the national income and product accounting (NIPA) convention that expenditures on innovation are expensed, the quantity in the model corresponding to Gross Domestic Product (GDP) as measured in the data under historical measurement procedures is equal to Y_t .¹⁰

Production of an intermediate good with index z is carried out with physical capital, k , and labor, l , according to

$$y_t(z) = \exp(z_t) k_t(z)^\alpha l_t(z)^{1-\alpha}, \quad (3)$$

where $0 < \alpha < 1$. For each intermediate good, the frontier technology z is owned by an incumbent firm, with exclusive rights to use that technology in production. At the same time, other firms can produce this product with the technology indexed by $z - \Delta_l$. The parameter Δ_l may be interpreted as a parameter of antitrust (or competition) policy for the frontier technology z , as discussed in [Acemoglu \(2009\)](#) (see section 13.1.6).

Each incumbent firm is characterized by the number n of intermediate goods that it produces. Hence, the state variable that characterizes an incumbent firm is a vector of productivities z of length n . Let $G_t(n)$ denote the measure of incumbent firms with n products at time t . The requirement that each product be owned by some firm implies $\sum_{n=1}^{\infty} n G_t(n) = 1$ for all periods t .

It is straightforward to show that in an equilibrium with equal markups across products, aggregate output of the final consumption good, Y_t , is given by

$$Y_t = A_{pt} Z_t (K_t)^\alpha (L_{pt})^{1-\alpha}, \quad (4)$$

where $L_{pt} = \int_z l_t(z) dJ_t(z)$, $K_t = \int_z k_t(z) dJ_t(z)$, and $Z_t = \left[\int_z \exp(z)^{\rho-1} dJ_t(z) \right]^{\frac{1}{\rho-1}}$. Note that Z_t here corresponds to *aggregate productivity* in the production of the final consump-

¹⁰The treatment of expenditures on innovation in the NIPA in the United States is being revised as of the second half of 2013 to include a portion of those expenditures on innovation in measured GDP. If all intangible investments in the model were measured as part of GDP, then measured GDP would be given by $GDP_t = C_t + K_{t+1} - (1 - d_k) K_t + P_{rt} Y_{rt}$, where $P_{rt} Y_{rt}$ denotes intangible investment expenditure, as defined below. We report results on GDP under both measurement procedures.

tion good.

In general, this model-based measure of aggregate productivity, Z_t , does not correspond to measured TFP, which is given by $TFP_t = GDP_t / (K_t^{\tilde{\alpha}} L_t^{1-\tilde{\alpha}})$, where $1 - \tilde{\alpha}$ denotes the share of labor compensation in measured GDP. This adjustment is required because of the expensing of expenditures on innovation (under historical standards for measuring GDP) and because of possible variation over time in the allocation of labor between production and research. The growth rate of this model-based measure of aggregate productivity, however, is equal to the growth rate of measured TFP on a balanced growth path.

Innovation by intermediate goods producing firms: Innovation is conducted by incumbent and entering firms. Incumbent firms and entering firms invest innovative effort to improve upon the technology frontier of a product. A firm that succeeds in innovating on a product raises the frontier from z to $z + \Delta$ and becomes the sole incumbent producer of the product.

The distribution of frontier technologies $J_t(z)$ evolves as follows. Given an initial distribution $J_t(z)$, an independently drawn fraction of products δ_t receive increments to their frontier technologies of size $\Delta \geq \Delta_l$. Hence aggregate productivity grows at the rate g_{zt} , where¹¹

$$\exp(g_{zt}) = \frac{Z_{t+1}}{Z_t} = \left[\delta_t \exp(\Delta)^{\rho-1} + 1 - \delta_t \right]^{\frac{1}{\rho-1}}. \quad (5)$$

We show below that, for the set of policies we consider, it is optimal for every incumbent firm to engage in the same innovative effort per product it owns, independent of the level of z associated with those products. We impose this result here to simplify the notation. Let each firm with n products engage in a total of nq_t units of innovative effort to obtain new products, and let each entering firm engage in one unit of innovative effort to obtain new products. Let M_t be the measure of entrants. Total innovative effort is then given by

$$q_t \sum_{n=1} n G_t(n) + M_t = q_t + M_t.$$

¹¹One can show that this model does not have a stationary distribution of firm sizes in a balanced growth path unless $\rho = 1$. This is because, without this assumption, there is not a stationary distribution of expenditure across products. To ensure a stationary distribution of firm sizes, one can modify the model as follows, without changing its aggregate implications. Assume that at the end of every period t , after production and innovation occur, a measure ζ of those products that did not receive an innovation have their frontier technology z reset to a new level $z' = \log Z_t$. This reset frontier technology is still owned by the same incumbent firm. At the same time as this resetting occurs, the technology freely available to other firms who may choose to produce this good is reset to $\log Z_t - \Delta_l$. The transition law for Z_t is not affected by the reset probability ζ .

Given innovative effort by incumbent and entrant firms, q_t and M_t , firms are matched at random to successful innovations through a matching function. The total measure of products innovated on is given by the matching function

$$\delta_t = m(1, q_t + M_t), \quad (6)$$

where the first argument of $m(\cdot, \cdot)$ denotes the measure of products available to be innovated upon and the second argument denotes the total innovative effort. The function $m(\cdot, \cdot)$ is constant returns to scale and increasing in each argument. In what follows we assume that this function takes the form $m(1, x) = \sigma_0 x^\sigma$, with $\sigma \leq 1$. The case of $\sigma = 1$ corresponds to the case in which the rate at which innovations arrive for an innovating firm is independent of the innovative activity of other firms. The case of $\sigma < 1$ corresponds to congestion frictions in innovative activity.

Innovations are divided up at random among incumbent and entrant firms in proportion to their innovative effort. An entrant firm that engages in one unit of innovative effort obtains a product with probability $\frac{1}{q_t + M_t} \delta_t$. A firm of size n that engages in nq_t units of innovative effort obtains new products according to a Poisson distribution with parameter $n \frac{q_t}{q_t + M_t} \delta_t$.

The research good: We now describe the resource cost of innovative effort. As in [Klette and Kortum \(2004\)](#), a firm with n products has the option of investing $\tilde{c}(qn, n)$ units of the research good to engage in qn units of innovative effort. We assume that $\tilde{c}(\cdot, \cdot)$ is constant returns to scale, so we can rewrite this total investment as $nc(q)$, where $c(\cdot)$ is increasing and convex. In this sense, n indexes an incumbent firm's innovative capacity as well as its number of products. If the firm chooses not to use this technology, then it invests zero units of the research good. Entering firms must invest f units of the research good to engage in one unit of innovative effort. Summing investment across firms and using the fact that $\sum_{n=1} nG_t(n) = 1$ gives the resource constraint for the research good,

$$c(q_t) + fM_t = Y_{rt}. \quad (7)$$

Production of the research good is carried out using research labor L_{rt} according to

$$Y_{rt} = Z_t^{\gamma-1} A_{rt} L_{rt}, \quad (8)$$

where $\gamma \leq 1$. The variable A_{rt} represents the stock of basic scientific knowledge that is assumed to evolve exogenously, growing at a steady rate of g_{A_r} so $A_{rt+1} = \exp(g_{A_r}) A_{rt}$.

Increases in this stock of scientific knowledge improve the productivity of resources devoted to innovative activity. We interpret A_{rt} as a worldwide stock of scientific knowledge that is freely available for firms to use in innovative activities. The determination of this stock of scientific knowledge is outside the scope of our analysis.¹² In the Appendix we consider an extension in which research production uses both labor and consumption good, as in the lab-equipment model of [Rivera-Batiz and Romer \(1991\)](#).

We interpret the parameter $\gamma \leq 1$ as indexing the extent of *intertemporal knowledge spillovers*, that is the extent to which further innovations become more difficult as the aggregate productivity Z_t grows relative to the stock of scientific knowledge A_{rt} . As γ approaches 1, the resource cost of innovating on the frontier technology becomes independent of Z_t (hence, there are full spillovers). The impact of advances in Z_t on the cost of further innovations is external to any particular firm and hence we call it a spillover. Standard specifications of quality ladders models with fully endogenous growth (including [Klette and Kortum 2004](#)) correspond to the case with full spillovers and $g_{A_r} = 0$.

Policies and Equilibrium

We now describe the decentralization of this economy and define equilibrium with a collection of policies. In this decentralization, we assume that the representative household owns the incumbent firms and the physical capital stock, facing a sequence of budget constraints given by

$$C_t + K_{t+1} = [R_{kt} + (1 - d_k)] K_t + W_t L_t + D_t - E_t,$$

in each period t , where W_t , R_{kt} , D_t , and E_t denote the economy-wide wage, rental rate of physical capital, aggregate dividends paid by firms, and aggregate fiscal expenditures on policies (which are financed by lump-sum taxes collected from the representative household), respectively.

Competitive firms producing the final consumption good choose inputs and output to maximize profits each period subject to (4). By standard arguments, in equilibrium, prices must satisfy $(1 + \tau_s) A_{pt} P_t = [\int_z p_t(z)^{1-\rho} dJ_t(z)]^{\frac{1}{1-\rho}}$, where $p_t(z)$ is the price set for goods with productivity index z , P_t is the price of the final good, and τ_s is a per-unit subsidy

¹²It is common in the theoretical literature to assume that all productivity growth is driven entirely by firms' expenditures on R&D ([Griliches 1979](#), p. 93). As noted in [Corrado et al. \(2011\)](#), this view ignores the productivity-enhancing effects of public infrastructure, the climate for business formation, and the fact that private R&D is not all there is to innovation. We capture all of these other productivity enhancing effects with A_r . Relatedly, [Akcigit et al. \(2013\)](#) considers a growth model that distinguishes between basic and applied research and introduces a public research sector.

on production of the consumption good. This production subsidy can be set to undo the distortions from the intermediate good's markup. We normalize P_t to 1.

Production of the research good is undertaken by competitive firms that take the intertemporal knowledge spillover from innovation as given. Cost minimization in the production of the research good implies that the price of the research good, P_{rt} , is equal to

$$P_{rt} = \frac{Z_t^{1-\gamma}}{A_{rt}} W_t. \quad (9)$$

P_{rt} is the deflator needed to translate changes in innovation expenditure, $P_{rt}Y_{rt}$, into changes in innovation output, Y_{rt} . We define the *innovation intensity of the economy*, s_{rt} , as the ratio of innovation expenditure to the sum of expenditure on consumption and physical capital investment (i.e. the old measure of GDP), that is $s_{rt} = P_{rt}Y_{rt}/GDP_t$.

Intermediate goods producing firms are offered two types of innovation subsidies. First, incumbent firms receive a subsidy to innovation expenditure, which we denote by τ_g . Second, entering firms receive a subsidy to innovation expenditure, which we denote by τ_e . We refer to policies in which $\tau_g = \tau_e$ as *uniform innovation subsidies*.

The variable profits that an incumbent firm earns in period t from production of a product with productivity index z are given by $[p_t(z)y_t(z) - W_t l_t(z) - R_{kt}k_t(z)]$. The incumbent firm that owns this product chooses price and quantity, $p_t(z)$ and $y_t(z)$, to maximize these variable profits subject to the demand for its product and the production function (1). We assume that the gross markup μ charged by the incumbent producer of each product is the minimum of the monopoly markup, $\rho/(\rho-1)$, and the technology gap between the leader and any potential second most productive producer of the good, $\exp(\Delta_l)$. That is $\mu = \min\left\{\frac{\rho}{\rho-1}, \exp(\Delta_l)\right\}$. Variable profits from production can then be written as $\Pi_t \exp(z)^{\rho-1}$, with the constant in variable profits Π_t defined by

$$\Pi_t = \kappa_0 (1 + \tau_s)^\rho A_{pt}^{\rho-1} \left(R_{kt}^\alpha W_t^{1-\alpha} \right)^{1-\rho} Y_t, \quad (10)$$

with $\kappa_0 = \mu^{-\rho}(\mu-1) \left[\alpha^\alpha (1-\alpha)^{1-\alpha} \right]^{\rho-1}$. It is straightforward to show that revenues and employment engaged in production also scale with productivity index z in direct proportion to $\exp(z)^{\rho-1}$. Hence, the equilibrium size of a firm with n products with frontier technologies z_1, \dots, z_n is directly proportional to $\exp(z_1)^{\rho-1} + \dots + \exp(z_n)^{\rho-1}$.

Incumbent firms that use their innovation technology, choose their innovative effort to maximize the value of the firm. For owners of a firm, there are two components to the value of gaining a product through innovation. One comes from the expected discounted stream of profits it earns from selling the product at a markup over marginal cost for as

long as that firm remains the incumbent producer of that product, and the other comes from the contribution that its ownership of the product makes to the firm's innovative capacity.

The expected discounted stream of profits associated with selling a product with productivity index z is given by the solution to the following Bellman equation which takes into account the probability that the firm loses its ownership of this product to another innovating firm

$$V_t(z) = \Pi_t \exp(z)^{\rho-1} + \frac{(1 - \delta_t)}{1 + \bar{r}_t} V_{t+1}(z),$$

where \bar{r}_t denotes the interest rate denominated in the final consumption good, which with log preferences is given by $1 + \bar{r}_t = \beta^{-1} C_{t+1}/C_t$. Integrating this expression across z and using the definition of the aggregate Z_t , we have

$$\int_z V_t(z) dJ_t(z) = V_t Z_t^{\rho-1} = \Pi_t Z_t^{\rho-1} + \frac{(1 - \delta_t)}{1 + \bar{r}_t} V_{t+1} Z_t^{\rho-1}, \quad (11)$$

where $V_t = V_t(0)$.

The second component of the value to a firm of owning a product is given by the contribution of that product to the firm's innovation capacity. We denote by $U_t(n)$ the value of innovative capacity for an incumbent firm with n products. Specifically, $U_t(n)$ corresponds to the expected present value of dividends the incumbent firm expects to earn on products it gains through innovation minus the cost of that innovation. Given that $\tilde{c}(qn, n)$ is constant returns to scale, one can show that this value function can be written as $U_t n$, where U_t is determined by the Bellman equation

$$U_t = \max_{\tilde{q}} - (1 - \tau_g) c(\tilde{q}) P_{rt} + \frac{1}{1 + \bar{r}_t} \frac{\tilde{q}}{q_t + M_t} \delta_t V_{t+1} Z_t^{\rho-1} \exp(\Delta)^{\rho-1} \quad (12)$$

$$+ \frac{1}{1 + \bar{r}_t} \left(\frac{\tilde{q}}{q_t + M_t} \delta_t + 1 - \delta_t \right) U_{t+1}.$$

The first term on the right side indicates the investment required of an incumbent firm to engage in \tilde{q} units of innovative effort per product. The second term indicates the discounted present value of variable profits the firm expects to gain from the innovations that result from this innovative effort. The third term denotes the expected value of the firm's innovative capacity from next period on, taking into account both the gain in products it expects to obtain from its innovative effort (i.e., a firm with n products expects to gain $\frac{\tilde{q}}{q+M} \delta n$ products) and the loss of products it expects as a result of innovative effort from other firms (i.e., a firm with n products expects to lose δn products). Each firm

takes as given the innovative effort of other firms, as given by q_t , M_t , and δ_t . Note that in equilibrium, if the incumbents' innovation technology is used, we must have $U_t \geq 0$. Otherwise, incumbents would choose not to use their innovation technology at all.

The total value of an incumbent firm with n products with frontier technologies z_1, \dots, z_n is the sum of the values over its current products, $\sum_{i=1}^n V_t \exp(z_i)^{\rho-1}$ plus the value of its innovative capacity, $U_t n$. The free entry condition for new firms is given by

$$(1 - \tau_e) P_{rt} f \geq \left(\frac{1}{1 + \bar{r}_t} \frac{\delta_t}{q_t + M_t} \right) \left(V_{t+1} Z_t^{\rho-1} \exp(\Delta)^{\rho-1} + U_{t+1} \right), \quad (13)$$

with this condition holding with equality if the measure of entering firms, M_t , is greater than zero.

The government's aggregate fiscal expenditures on policies in equilibrium are given by

$$E_t = \tau_g c(q_t) P_{rt} + \tau_e f M_t P_{rt} + \tau_s Y_t. \quad (14)$$

Definition of Equilibrium: An equilibrium in this economy is a collection of sequences of aggregate prices $\{\bar{r}_t, P_{rt}, R_{kt}, W_t\}$, prices for intermediate goods $\{p_t(z)\}$, sequences of aggregate quantities $\{Y_t, C_t, L_{pt}, L_{rt}\}$, quantities of the intermediate goods and allocations of physical capital and labor $\{y_t(z), k_t(z), l_t(z)\}$ sequences of $\{\Pi_t\}$, and sequences of firm value functions and innovation decisions $\{V_t, U_t, q_t, M_t\}$ together with distributions of firms and aggregate productivities $\{J_t(z), Z_t\}$ such that, given a set of policies $\{\tau_e, \tau_g, \tau_s\}$, initial stocks $\{A_{r0}, A_{p0}, L_0, K_0\}$, and an initial distribution of productivities $J_0(z)$, households maximize their utility subject to their budget constraint, intermediate good firms maximize profits, all of the feasibility constraints are satisfied, and the distribution of firms evolves as described above.

Key assumptions for our aggregate analysis

In specifying our model, we have made four key assumptions that make our model tractable for analysis of the aggregate implications of changes in innovation policies. With these assumptions, we show that the model allows for aggregation in the sense that, at the margin, changes in the growth rate of aggregate productivity are a function only of changes in firms' aggregate expenditure on innovation and not on the distribution of innovation expenditure across the heterogeneous firms in the economy.

(i) Uniform markups: We have already discussed that the assumption of uniform markups across products allows us to construct the index Z_t of aggregate productivity whose law of motion is determined by the aggregate innovation rate δ_t as shown in expression (5). In the appendix we show that it is sufficient for our simple aggregation to assume that the distribution of markups is constant over time and uncorrelated with product productivity z . In this way, the model can be extended to include cross-section variation in labor revenue productivity.

(ii) And uniform innovation step size: In our specification of the environment and our definition of equilibrium, we assumed that all incumbent firms engage in the same amount of innovative effort per product that they own. This result follows from our assumptions of uniform markups across products and uniform innovation step size Δ across products. To see this, consider the first-order condition from equation (12) for optimal innovative effort per product by incumbent firms, \tilde{q}_t , which is given by

$$(1 - \tau_g) P_{rt} c'(\tilde{q}_t) = \left(\frac{1}{1 + \bar{r}_t} \frac{\delta_t}{q_t + M_t} \right) \left(V_{t+1} Z_t^{\rho-1} \exp(\Delta)^{\rho-1} + U_{t+1} \right). \quad (15)$$

Since none of the terms depend on the incumbent firm's number of products n or the productivities with which the firm can produce those products, we have that $\tilde{q}_t = q_t$ for all products and firms. In the appendix we show that the property that all incumbent firms choose the same q_t extends in an alternative specification of our model in which each product that is innovated on draws a random markup and innovation step size that is independent of the identity of the innovator. In this alternative specification of the model, there is persistent variation in labor revenue productivity and in research intensity (e.g. innovation expenditures relative to revenues) that is not correlated with firm growth (as emphasized in [Klette and Kortum 2004](#)).

(iii) And free entry: If the equilibrium has entering firms, then the zero-profit condition for entry (13) and equation (15) imply that if $M_t > 0$ all incumbent firms innovate at the same rate $\tilde{q}_t = q_t$ per product that they own determined from

$$(1 - \tau_g) c'(\tilde{q}_t) = (1 - \tau_e) f. \quad (16)$$

This result implies that in any equilibrium with positive entry and uniform innovation subsidies, $c'(q) = f$, i.e. the marginal resource cost of innovative effort is equated across firms. We show that this implies that the equilibrium distribution of innovative effort

across firms in this case is *conditionally efficient* in the sense that a social planner would not want to reallocate innovative effort across firms (both entrants and incumbents), holding fixed the level of research output.

(iv) And constant factor shares: Finally, to compute how production of the research good Y_r changes with changes in expenditure on innovation relative to GDP s_r , we make use of the following results about the division of GDP into payments to various factors of production and the relationship of those factor shares to the innovation intensity of the economy and the allocation of labor.

With CES aggregators and Cobb-Douglas production functions, aggregate revenues of intermediate goods firms, $(1 + \tau_s) Y_t$, are split into three components. A share $\frac{\mu-1}{\mu}$ accrues to variable profits from production, $\Pi_t Z_t^{\rho-1} = \frac{\mu-1}{\mu} (1 + \tau_s) Y_t$, a share α/μ is paid to physical capital, $R_{kt} K_t = \frac{\alpha}{\mu} (1 + \tau_s) Y_t$, and a share $(1 - \alpha) / \mu$ is paid as wages to production labor, $W_t L_{pt} = \frac{(1-\alpha)}{\mu} (1 + \tau_s) Y_t$.

With perfect competition in the research sector, $W_t L_{rt} = P_{rt} Y_{rt}$. Using the factor shares above, the allocation of labor between production and research is related to expenditures on the research good by¹³

$$\frac{L_{pt}}{L_{rt}} = \frac{(1 - \alpha) (1 + \tau_s)}{\mu} \frac{1}{s_{rt}}. \quad (17)$$

In the appendix we show that a similar expression holds in a version of the model with random markups across products as discussed above.

3 Balanced growth path

We now characterize the main features of a balanced growth path (BGP) of the equilibrium of our model. We provide additional details of this characterization in the appendix. Our model has two types of BGPs, one with semi-endogenous growth and one with endogenous growth, depending on the parameter γ . If $\gamma < 1$, then our model is a semi-endogenous growth model with the growth rate along the BGP determined by the exogenous growth rate of scientific knowledge g_{A_r} and other parameter values independently of innovation policies, as in [Kortum \(1997\)](#) and [Jones \(2002\)](#). In this case, it is not

¹³Here we are assuming that there is one wage for labor in both production and research. In the Appendix we present an extension in which labor is imperfectly substitutable between production and research as in [Jaimovich and Rebelo \(2012\)](#). The assumption of imperfect substitutability reduces the elasticity of the allocation of labor between production and research with respect to a policy-induced change in the innovation intensity of the economy, resulting in even smaller responses of aggregate productivity and GDP to a given change in the innovation intensity of the economy relative to those in our baseline model.

possible to have fully endogenous growth because such growth would require growth in innovation expenditure in excess of the growth rate of GDP. Ongoing balanced growth can occur only to the extent that exogenous scientific progress reduces the cost of further innovation as aggregate productivity Z grows. In the knife edged case that $\gamma = 1$ and $g_{Ar} = 0$, our model is an endogenous growth model with the growth rate along the BGP determined by firms' investments in innovative activity, as in [Grossman and Helpman \(1991\)](#) and [Klette and Kortum \(2004\)](#).¹⁴

The transition paths of the response of aggregates to policy changes are continuous as γ approaches one. Hence, our analysis of the model's quantitative implications for the response of aggregates in the medium term (20 years) in the semi-endogenous growth case nests the endogenous growth case.

In the semi-endogenous growth case, $\gamma < 1$, the growth rates of aggregates and the interest rate are independent of policies and determined from equations (2), (4), (7), and (8). In particular, the growth rate of aggregate productivity g_z is given by $g_z = g_{Ar}/(1 - \gamma)$. The growth rate of output of the consumption good (and hence consumption, physical capital, and the wage) is given by $g_y = g_z/(1 - \alpha)$, and the rental rate of capital is constant and given by $R_k = \beta^{-1} \exp(g_y) - 1 + d_k$. The rate at which innovations occur, δ , is pinned down from (5). The matching function (6) pins down total innovative effort, $q + M$. Innovation by incumbent firms q is constant and, with positive firm entry, is determined as the solution to (16). Equilibrium entry M is then calculated as a residual. Note that, in the BGP, policies affect the division of innovative effort between incumbents and entering firms, but not the total innovative effort $q + M$ nor the rate of innovation δ .

We focus on parameter values such that on the BGP, new firms enter (so $M > 0$) and incumbents choose to use their innovation technology. To have firm entry in a BGP with innovation by incumbent firms, it must be that the solution for M described above is positive. Incumbent firms find it optimal to use their innovation technology in a BGP with firm entry if and only if at the q that solves equation (16), the post-subsidy average cost of innovation for incumbent firms, $(1 - \tau_g) c(q)/q$, is less than or equal to the post-subsidy average cost of innovation for entrants, $(1 - \tau_e) f$ (we prove this statement in the appendix). This condition is equivalent to the statement $c(q) \leq c'(q)q$ at that q . This holds if $c(q)$ is convex and $c(0) = 0$.

In applying this model to data, we will calibrate the parameters to match a given BGP per capita growth rate of output, g_y . Given a choice of g_y and physical capital share of α , the growth rate of aggregate productivity in the BGP is $g_z = g_y(1 - \alpha)$ independent

¹⁴If $\gamma > 1$, then our model does not have a BGP, as in this case, a constant innovation intensity of the economy leads to an acceleration of the innovation rate as aggregate productivity Z grows.

of γ . This implies that the aggregate innovation rate δ on the BGP is also independent of γ . For a given choice of γ , the growth rate of scientific knowledge consistent with these productivity growth rates is given by $g_{Ar} = (1 - \gamma)g_z$. We assume that g_{Ar} is unmeasured and hence do not make assumptions about this growth rate directly. Instead, we alter this parameter as we vary γ .

4 Aggregate implications of changes in the innovation intensity of the economy: Analytic results

In this section, we derive analytic results regarding the impact of policy-driven changes in the innovation intensity of the economy on aggregate outcomes at different time horizons. These analytical results demonstrate what features of our baseline Klette-Kortum model are key in determining its implications for the aggregate impact of innovation policies. In the next section, we discuss the quantitative implications of our analytical results.

In framing the question of how policy-induced changes in the innovation intensity of the economy impact aggregate outcomes at different time horizons, we consider the following thought experiment. Consider an economy that is initially on a BGP with uniform innovation subsidies, $\bar{\tau}_e = \bar{\tau}_g$. As a baseline experiment (which we relax later), consider an unanticipated permanent change in innovation policies to new uniform innovation subsidies $\tau'_e = \tau'_g$ beginning in period $t = 0$ and continuing on for all $t > 0$. We keep the production subsidy τ_s constant.

Given our baseline policy experiment, we first derive the elasticity across BGPs of the innovation intensity of the economy and fiscal expenditures on innovation policies with respect to this policy experiment. We then derive analytical first-order approximations to the change in the transition path and the long run response of aggregate productivity and GDP as a function of this transition path for the innovation intensity of the economy. We then relate the parameters determining the transition dynamics of GDP to those that shape the optimal innovation intensity of the economy. We next consider non-uniform changes in innovation policy. Finally, we compare the implications of our model to those of a standard quality ladders model in which innovation is only carried out by entering firms.

Fiscal impact and innovation intensity across BGPs The following proposition derives the elasticity across BGPs of the innovation intensity of the economy and fiscal expenditures on innovation policies with respect to a uniform change in innovation policies.

Proposition 1. *Consider an economy on a BGP with semi-endogenous growth and positive firm-entry. Suppose that innovation policies change permanently from $\bar{\tau}_g = \bar{\tau}_e$ to $\tau'_g = \tau'_e$. Then, across the old and new BGP the innovation intensity of the economy changes from \bar{s}_r to s'_r , and fiscal expenditures relative to GDP change from $\bar{E}/G\bar{D}P$ to E'/GDP' , with these changes given by*

$$\log s'_r - \log \bar{s}_r = \log(1 - \bar{\tau}_g) - \log(1 - \tau'_g)$$

and

$$\frac{E'}{GDP'} - \frac{\bar{E}}{G\bar{D}P} = s'_r - \bar{s}_r.$$

The proof of this proposition, which is in the appendix, uses the free-entry condition, which implies that the ratio of aggregate variable profits to the post-subsidy price of the research good, $\frac{\Pi Z^{\rho-1}}{(1-\tau_g)P_r}$, as well as output of the research good, Y_r , are both constant across two BGPs with semi-endogenous growth and uniform innovation subsidies. Together with the fact that aggregate profits are a constant share of GDP, we obtain that $s_r (1 - \tau_g)$ is also constant across BGPs, which implies the first expression. The second expression follows from this result and the definition of fiscal expenditures in expression (14).

Proposition 1 implies that in the long-run, our policy experiment will result in a change in the innovation intensity of the economy determined only by the change in the innovation subsidy rate independent of the other parameters of the model. At short and medium horizons, however, this policy will result in a change in the path of the innovation intensity of the economy from $\{\bar{s}_{rt}\}_{t=0}^{\infty}$ (which is constant on the initial BGP) to $\{s'_{rt}\}_{t=0}^{\infty}$ that we will have to solve for numerically. For the remainder of our analytic results, we take this path as given.

Dynamics of aggregate productivity and GDP The following proposition characterizes the transition dynamics of aggregate productivity to a change in the innovation intensity of the economy.

Proposition 2. *Consider an economy on a BGP with uniform innovation subsidies and positive firm-entry. Suppose that at time $t = 0$, an unanticipated, permanent, and uniform change in innovation policies induces a new path for the innovation intensity of the economy given by $\{s'_{rt}\}_{t=0}^{\infty}$. Then the new path for aggregate productivity $\{Z'_t\}_{t=1}^{\infty}$ to a first-order approximation is given by*

$$\log Z'_t - \log \bar{Z}_t = \sum_{k=1}^t \Gamma_k (\log s'_{rt-k} - \log \bar{s}_r) \quad (18)$$

where $\bar{Z}_t = \exp(t\bar{g}_z)\bar{Z}_0$, with

$$\Gamma_1 = \bar{L}_p \frac{\Phi\sigma}{\rho-1} \frac{\exp((\rho-1)\bar{g}_z) - 1}{\exp((\rho-1)\bar{g}_z)} \leq \bar{L}_p \Phi\sigma\bar{g}_z, \quad (19)$$

and

$$\Gamma_{k+1} = \left[1 - (1-\gamma)\frac{\Gamma_1}{\bar{L}_p} \right] \Gamma_k, \quad (20)$$

where \bar{s}_r denotes the innovation intensity, \bar{L}_p the share of labor employed in current production, \bar{g}_z the growth rate of aggregate productivity, and

$$\Phi = \frac{\bar{M}/(\bar{q} + \bar{M})}{f\bar{M}/\bar{Y}_r} \leq 1 \quad (21)$$

the ratio of innovative effort by entrants relative to total innovative effort over research expenditure by entrants relative to total research expenditure, where \bar{M} , \bar{q} , and \bar{Y}_r denote research effort by entrants and incumbents and research output, all on the initial BGP.

Proof. We prove this result by calculating three key elasticities in our model. The first key elasticity is the elasticity of research output Y_r with respect to a change in the innovation intensity of the economy s_r . From equations (8) and (17) we have that, to a first-order approximation,

$$\log Y'_{rt} - \log \bar{Y}_r = \bar{L}_p (\log s'_{rt} - \log \bar{s}_r) - (1-\gamma) (\log Z'_t - \log \bar{Z}_t). \quad (22)$$

Hence, this elasticity is given by \bar{L}_p .

The second key elasticity is the elasticity of the innovation rate with respect to a change in research output. Here from equation (6) we have that, to a first-order approximation,

$$\log \delta'_t - \log \bar{\delta} = \sigma \frac{\bar{M}}{\bar{q} + \bar{M}} (\log M'_t - \log \bar{M}) + \sigma \frac{\bar{q}}{\bar{q} + \bar{M}} (\log q'_t - \log \bar{q}). \quad (23)$$

From equation (7) we have that, to a first order approximation

$$\log Y'_{rt} - \log \bar{Y}_r = \frac{f\bar{M}}{\bar{Y}_r} (\log M'_t - \log \bar{M}) + \frac{c'(\bar{q})\bar{q}}{\bar{Y}_r} (\log q'_t - \log \bar{q}). \quad (24)$$

From equation (16), we have that with a uniform subsidies $\log q'_t = \log \bar{q}$ for all $t \geq 0$. Combining (23) and (24) we have

$$\log \delta'_t - \log \bar{\delta} = \sigma \Phi (\log Y'_{rt} - \log \bar{Y}_r). \quad (25)$$

Hence, this elasticity is given by $\sigma\Phi$.

Finally, the third key elasticity is the elasticity of aggregate productivity growth with respect to the innovation rate. From equation (5) we have that this elasticity is given by

$$\log Z'_{t+1} - \log Z'_t \approx \bar{g}_z + \frac{1}{\rho - 1} \frac{\exp((\rho - 1)\bar{g}_z) - 1}{\exp((\rho - 1)\bar{g}_z)} (\log \delta'_t - \log \bar{\delta}) \quad (26)$$

which is equivalent to

$$\log Z'_{t+1} - \log \bar{Z}_{t+1} \approx \log Z'_t - \log \bar{Z}_t + \frac{1}{\rho - 1} \frac{\exp((\rho - 1)\bar{g}_z) - 1}{\exp((\rho - 1)\bar{g}_z)} (\log \delta'_t - \log \bar{\delta}). \quad (27)$$

Hence, this elasticity is $\frac{1}{\rho - 1} \frac{\exp((\rho - 1)\bar{g}_z) - 1}{\exp((\rho - 1)\bar{g}_z)}$.

Plugging in (22) and (25) into (27) gives

$$\begin{aligned} \log Z'_{t+1} - \log \bar{Z}_{t+1} &= \left[1 - (1 - \gamma) \frac{\Gamma_1}{L_p} \right] (\log Z'_t - \log \bar{Z}_t) \\ &\quad + \Gamma_1 (\log s'_{rt} - \log \bar{s}_r), \end{aligned} \quad (28)$$

which proves the main result.

To show that $\Phi \leq 1$ in equation (21), recall that in any BGP with innovation by both incumbent and entering firms, $(1 - \tau_g)c(\bar{q})/\bar{q} \leq (1 - \tau_e)f$. With uniform innovation policies in that BGP, that implies that $c(\bar{q}) \leq f\bar{q}$. This is sufficient to guarantee that $\Phi \leq 1$. The case in which incumbents do not use their innovation technology is discussed in Proposition 4.

The last step to derive the upper bound on Γ_1 in equation (19) is to show that the elasticity of aggregate productivity growth with respect to the innovation rate, $\frac{1}{\rho - 1} \frac{\exp((\rho - 1)\bar{g}_z) - 1}{\exp((\rho - 1)\bar{g}_z)}$, is bounded above by \bar{g}_z . To do so we use a more general argument (which we apply below when we introduce social depreciation of innovation expenditures and when we discuss other models). We write the transition law for aggregate productivity (5) as

$$\frac{Z_{t+1}}{Z_1} = G(\delta_t), \quad (29)$$

where $\log G(\delta)$ is concave in δ and $\log G(\bar{\delta}) = \bar{g}_z$. Log-linearizing this equation gives

$$\log Z_{t+1} - \log Z_t = \log G(\bar{\delta}) + \frac{G'(\bar{\delta})\bar{\delta}}{G(\bar{\delta})} (\log \delta_t - \log \bar{\delta}).$$

Because G is log-concave, we have that the elasticity of aggregate productivity growth

with respect to the innovation rate is bounded above by

$$\frac{G'(\bar{\delta})\bar{\delta}}{G(\bar{\delta})} \leq \log G(\bar{\delta}) - \log G(0) = \bar{g}_z, \quad (30)$$

where the equality follows from the assumption that $G(0) = 1$.¹⁵ To see how tight this bound is note that by direct calculation,

$$\frac{G'(\bar{\delta})\bar{\delta}}{G(\bar{\delta})} = \frac{1}{\rho - 1} \frac{G(\bar{\delta})^{\rho-1} - G(0)^{\rho-1}}{G(\bar{\delta})^{\rho-1}}. \quad (31)$$

Given $G(\bar{\delta}) = \exp(\bar{g}_z)$ and $G(0) = 1$, we have that for any fixed $\bar{g}_z > 0$ as ρ approaches 1 from above this elasticity approaches \bar{g}_z , while as ρ gets large this elasticity limits to zero. \square

Proposition 2 gives us an analytical expression for the dynamics of aggregate productivity in the transition to a new BGP following an unanticipated, permanent, and uniform change in innovation policies as a function of the transition path for the innovation intensity of the economy that arises in equilibrium as a result of that change in policies. From Proposition 1, we can compute the long-run change in the innovation intensity of the economy that arises as a function of a given permanent and uniform change in innovation policies. In the next corollary, we derive the long-run change in aggregate productivity that corresponds to that change in innovation intensity.

Corollary 1. *Consider a permanent uniform change in innovation policies. Assume that the economy converges to a new BGP with innovation intensity s'_r . Then, the gap in aggregate productivity between the old and new BGP converges, to a first-order approximation, to*

$$\log Z'_t - \log \bar{Z}_t = \frac{\bar{L}_p}{1 - \gamma} (\log s'_r - \log \bar{s}_r) \quad (32)$$

in the semi-endogenous growth case ($\gamma < 1$). In the endogenous growth case ($\gamma = 1$) the gap in aggregate productivity between the old and new BGP is unbounded. The new growth rate of

¹⁵To derive this inequality, we use

$$\frac{G'(\cdot)\delta}{G(\cdot)} \Big|_{\delta=\bar{\delta}} = \frac{\partial \log G(\delta)}{\partial \delta} \Big|_{\delta=\bar{\delta}} \bar{\delta} \leq (\log G(\bar{\delta}) - \log G(\bar{\delta}')) \frac{\bar{\delta}}{\bar{\delta} - \bar{\delta}'}$$

for $0 \leq \bar{\delta}' < \bar{\delta}$, where the inequality follows from concavity of $\log G(\delta)$. With $\bar{\delta}' = 0$, we obtain expression (30).

aggregate productivity to a first-order approximation is given by

$$\log Z'_{t+1} - \log Z'_t = \bar{g}_z + \Gamma_1 (\log s'_{rt} - \log \bar{s}_r). \quad (33)$$

Proof. To derive expression (32) we use equation (22) and the fact that, in response to uniform innovation policies, δ , q , M and hence Y_r remain constant between BGPs. Note that $\frac{\bar{L}_p}{1-\gamma}$ is equal to $\sum_{k=1}^{\infty} \Gamma_k$ in expression (18) if that sum converges. With endogenous growth, $\gamma = 1$, and hence $\Gamma_k = \Gamma_1$ for all k . Equation (33) follows from taking the first difference of equation (18). \square

We next derive the transition path for GDP as a function of the transition paths for aggregate productivity, the innovation intensity of the economy, and the equilibrium rental rate on physical capital.¹⁶

Corollary 2. *The path of GDP corresponding to the policy experiment in Proposition 2 is given, to a first-order approximation, by*

$$\log GDP'_t - \log G\bar{D}P_t = \frac{1}{1-\alpha} (\log Z'_t - \log \bar{Z}_t) - \bar{L}_r (\log s'_{rt} - \log \bar{s}_r) - \frac{\alpha}{1-\alpha} (\log R'_{kt} - \log \bar{R}_k) \quad (34)$$

under the old measurement system in which innovation is expensed, and is given by the above plus $\frac{\bar{s}_r}{1+\bar{s}_r} (\log s'_{rt} - \log \bar{s}_r)$ under a measurement system in which all expenditures on innovation are included in measured GDP.

Proof. We prove this result by taking the log of GDP, which under the old measurement system is equal to the log of Y :

$$\log Y'_t - \log \bar{Y}_t = \log Z'_t - \log \bar{Z}_t + \alpha (\log K'_t - \log \bar{K}_t) + (1-\alpha) (\log L'_{pt} - \log \bar{L}_p)$$

Using $R_{kt} = \frac{\alpha}{\mu} (1 + \tau_s) \frac{Y_t}{K_t}$ and equation (17) we obtain the expression above. To derive the result under the new measure of GDP, we must add in expenditures on research $P_{rt} Y_{rt}$, which we can do by multiplying the old measure of GDP by $(1 + s_{rt})$. \square

Given the result in Corollary 2, it is straightforward to calculate the response of GDP in the long run. Since, with semi-endogenous growth, in the long run the interest rate and rental rate on physical capital return to their levels on the initial BGP (i.e. $\lim_{t \rightarrow \infty} \log R'_{kt} -$

¹⁶The magnitude of the change in the rental rate of physical capital in the transition is related to the equilibrium transition path for the interest rate. From the Euler equation for physical capital, $\log R'_{kt} - \log \bar{R}_k = \frac{\bar{r}}{d_k + \bar{r}} \log (\bar{r}_{t-1} / \bar{r})$ for $t \geq 1$. Solving for the path of the interest rate requires fully solving for the model transition, which we do in Section 6.

$\log \bar{R}_k = 0$), the long run response of GDP is a simple function of the long run responses of aggregate productivity and the innovation intensity of the economy. In particular, from Corollary 1, we have that with $\gamma < 1$, the long run response of GDP is given by

$$\lim_{t \rightarrow \infty} \log GDP'_t - \log G\bar{D}P_t = \left[\frac{1}{1-\alpha} \frac{\bar{L}_p}{1-\gamma} - \bar{L}_r \right] \lim_{t \rightarrow \infty} (\log s'_{rt} - \log \bar{s}_r).$$

Aggregate dynamics and optimal innovation intensity Expression (34) in Corollary 2 is useful for understanding the transition dynamics of measured GDP (corresponding to the resources available for consumption and physical investment). With a permanent increase in the innovation intensity of the economy, GDP falls initially as labor is reallocated from current production to research, and then grows in the transition as the impact of a permanent change in the innovation intensity of the economy on aggregate productivity cumulates over time.

The impact on welfare of a permanent increase in the innovation intensity of the economy clearly depends on the trade-off between the short and long run changes in GDP that result from this increase in innovation expenditures as captured by the parameters Γ_k . To gain intuition for this welfare tradeoff, consider a variation in innovation policies that raises the innovation intensity of the economy only in period $t = 0$, so $s'_{r0} > \bar{s}_r$ and in all other time periods $t \neq 0$, $s'_{rt} = \bar{s}_r$. From Corollary 2, if we ignore changes in the rental rate on physical capital, we have that the log of resources available for consumption and physical investment (i.e. GDP) falls in period $t = 0$ by $\bar{L}_r (\log s'_0 - \log \bar{s}_r)$ and rises in every period $t \geq 1$ by $\frac{\Gamma_t}{1-\alpha} (\log s'_0 - \log \bar{s}_r)$. In the appendix, we show that on the optimal BGP allocation, this perturbation of the path of innovation expenditures has no first order impact on welfare, which is equivalent to the condition that the socially optimal BGP allocation must satisfy

$$\left[\sum_{k=1}^{\infty} \beta^k \frac{\Gamma_k}{1-\alpha} - \bar{L}_r \right] (\log s'_0 - \log \bar{s}_r) = 0$$

for small perturbations of \bar{s}_r . This condition implies that on the optimal BGP allocation,¹⁷

$$s_r^* = (1-\alpha) \frac{L_r^*}{L_p^*} = \frac{\beta \frac{\Gamma_1^*}{L_p^*}}{1-\beta \left[1 - (1-\gamma) \frac{\Gamma_1^*}{L_p^*} \right]}, \quad (35)$$

where s_r^* , L_r^* , and L_p^* are the optimal BGP levels of these variables and Γ_1^* is from equation

¹⁷One can also obtain this expression using the variational argument proposed by Jones and Williams (1998).

(19) evaluated at these optimal quantities. Note from equation (19) that $\frac{\Gamma_1^*}{L^*p}$ is determined by σ, Φ, ρ and \bar{g}_z . Hence, the optimal innovation intensity of the economy is solely determined by these variables, together with β and γ .

From Proposition 1, we have that if we hold the production subsidy τ_s constant, the uniform innovation subsidies $\tau_g^* = \tau_e^*$ needed to implement a change across BGPs from an initial innovation intensity \bar{s}_r and initial subsidies $\bar{\tau}_g = \bar{\tau}_e$ to innovation intensity s_r^* is given by

$$\log(1 - \tau_g^*) = \log(1 - \bar{\tau}_g) + \log \bar{s}_r - \log s_r^*$$

and the corresponding fiscal expenditure on innovation subsidies required is $E^*/GDP^* - \bar{E}/\bar{GDP} = s_r^* - \bar{s}_r$. In the appendix we derive the uniform innovation subsidy and production subsidy to implement the optimal allocation as a function of the parameters of the model.

Non-uniform changes in innovation policies So far we have only considered uniform changes in innovation policies. We now show that if the economy starts on an initial BGP with uniform innovation policies, then the first-order approximation for the transition path of aggregate productivity derived in Proposition 2 does not depend on the particular specification of changes in policies τ_e and τ_g that is used to induce a given path for the innovation intensity of the economy.

Corollary 3. *Consider an economy on an initial BGP as in Proposition 2 that experiences a change in innovation policies at time $t = 0$ that is unanticipated but not uniform, i.e. with $\tau_e^t \neq \tau_g^t$. Let the new path for the innovation intensity of the economy given by $\{s_{rt}^t\}_{t=0}^\infty$. Then the new path for aggregate productivity and GDP to a first order approximation are given as in Proposition 2 and Corollary 2.*

Proof. In deriving the elasticity of the innovation rate with respect to a change in research output in Proposition 2 we used the fact that with uniform changes in innovation policies, $\log q_t^t - \log \bar{q} = 0$. With non-uniform changes in innovation policies, $\log q_t^t - \log \bar{q} \neq 0$. Combining (23) and (24) we have that, up to a first order approximation,

$$\log \delta_t^t - \log \bar{\delta} = \sigma \Phi \left(\log Y_{rt}^t - \log \bar{Y}_r \right) + \sigma \frac{\bar{q}}{\bar{q} + \bar{M}} \left(1 - \frac{c'(\bar{q})}{f} \right) (\log q_t^t - \log \bar{q}).$$

Starting with uniform innovation policies, $\bar{\tau}_e = \bar{\tau}_g$, equation (16) implies that $c'(\bar{q}) = f$, so expression (25) still holds. The other two key elasticities do not depend on $\log q_t^t - \log \bar{q}$, so they are also unchanged. The response of GDP is calculated using the same steps as in Corollary 2. \square

Comparing models with and without innovation by incumbent firms So far, we have considered a Klette-Kortum style model that includes innovation by incumbent and entering firms. The standard Quality Ladders model introduced by [Aghion and Howitt \(1992\)](#) and [Grossman and Helpman \(1991\)](#) features innovation only by entering firms. Here we compare the aggregate implications of changes in innovation policies in these two types of models.

We can nest the Quality Ladders model in our framework by assuming that $c(0)$ is sufficiently high so that $c(\bar{q}) > f\bar{q}$. In this case, incumbent firms will choose not to use their innovation technology. The equations characterizing equilibrium in this case have $q_t = 0$ and $c(q_t) = 0$ in equations (6), (7), and $U_t = 0$ in equation (13) above. By direct calculation, the parameter Φ defined in Proposition 2 is equal to one in this case.

To compare the aggregate implications of these models, consider a Quality Ladders version of the model and a Klette-Kortum version of the model that share the same parameter values for $\gamma, \sigma, \alpha, \mu, g_{Ar}$, and the same production subsidy τ_s . Let the initial innovation subsidies in the Klette-Kortum version of the model be uniform ($\tau_g = \tau_e$). Let the initial innovation subsidies in the Quality Ladders version of the model be chosen such that the innovation intensity of this economy (and hence the allocation of labor between production and research) on the initial BGP is the same as for the Klette-Kortum version of the model on its initial BGP.

We then have the following proposition regarding the impact of uniform changes in policies in the long run for these two versions of the model.

Proposition 3. *Consider a Klette-Kortum model and a Quality Ladders model that are calibrated equivalently as described above and have semi-endogenous growth ($\gamma < 1$). Assume an unanticipated, permanent, and uniform change of policies in the two models such that the change in the log of the subsidy to entering firms, $\log(1 - \tau_e)$, is the same. Then both models produce the same change in the innovation intensity of the economy, aggregate productivity, GDP, and fiscal expenditures on innovation subsidies relative to GDP as a result of this change in policies from the old BGP to the new BGP.*

Proof. This result follows from the fact that the long-run change in aggregate productivity, GDP and fiscal expenditures derived above is independent of the value of Φ . \square

It is more difficult to make comparisons between these two models' implications for the transition path from the initial BGP to the new BGP. This is because the equilibrium path of the innovation intensity in the transition $\{s'_{rt}\}$ may differ between the two models. We do not have analytical results regarding this transition path. However, we can compare the implications of the two models for $\{\Gamma_k\}$ that determine elasticities of aggregate

productivity and GDP with respect to the innovation intensity of the economy. We do so in the following Proposition.

Proposition 4. *Consider a Klette-Kortum and a Quality Ladders version of the model calibrated equivalently as described above. Then the elasticities $\{\Gamma_k^{KK}\}$ in the Klette-Kortum version of the model are related to the elasticities $\{\Gamma_k^{QL}\}$ in the Quality Ladders version of the model defined in proposition 2 as follows. For any $T \geq 1$,*

$$\sum_{k=1}^T \Gamma_k^{KK} \leq \sum_{k=1}^T \Gamma_k^{QL}.$$

With semi-endogenous growth, $\gamma < 1$, as $T \rightarrow \infty$

$$\sum_{k=1}^{\infty} \Gamma_k^{KK} = \sum_{k=1}^{\infty} \Gamma_k^{QL}$$

while with endogenous growth ($\gamma = 1$), $\Gamma_k^{KK} = \Gamma_1^{KK} \leq \Gamma_1^{QL} = \Gamma_k^{QL}$ for all $k \geq 1$.

Proof. For both models, we have,

$$\sum_{k=1}^T \Gamma_k = \frac{\bar{L}_p}{1-\gamma} \left[1 - \left(1 - (1-\gamma) \frac{\Gamma_1}{\bar{L}_p} \right)^T \right].$$

The result follows from the observation that $\Phi = 1$ in the Quality Ladders model and $\Phi \leq 1$ in the Klette Kortum model, so that $\Gamma_1^{KK} \leq \Gamma_1^{QL}$. \square

What this result implies is that if we consider innovation policy changes in the Klette-Kortum model that produce the same transition path for the innovation intensity of the economy $\{s'_{rt}\}$ as the innovation policy changes considered in the Quality Ladders version of the model, then the Quality Ladders version of the model will imply a larger change in aggregate productivity up to any period T along the transition path, to a first-order approximation. In the long-run, as $T \rightarrow \infty$, with semi-endogenous growth, the two models deliver the same response of aggregate productivity and GDP. Likewise, with endogenous growth, for every T , the response of the growth rate will be larger in the Quality Ladders version of the model than in the Klette-Kortum version of the model, again to a first-order approximation.

Note from Corollary 3 that the result in Proposition 4 does not depend on the assumption that the policy change in the Klette Kortum version of the model is uniform. Innovation policy changes that are not uniform in that version of the model will impact

the innovation decisions of incumbents. To a first-order approximation, holding fixed a transition path for the innovation intensity of the economy, changes in the innovation decisions of incumbent firms do not impact the aggregate implications of that model.

5 Quantitative implications of analytical results

In this section, we illustrate with a particular numerical example (based on the full calibration of the model discussed in Section 6 and the Appendix) the use of our simple analytical results for quantitative analysis of the aggregate implications of changes in innovation policies in our model. In Section 6, we assess the accuracy of the approximations that we make here by numerically computing the equilibrium of the fully specified model.

Consider a calibration of our model in which the time period is set to one year and the growth rate of aggregate productivity on the initial BGP is $\bar{g}_z = .0125$, or 1.25%, so that (for our choice of α) the growth rate of output per worker is 2%, similar to that experienced in the United States over the postwar period. We set $\rho = 4$ and assume that the initial innovation intensity of the economy is $\bar{s}_r = .11$, similar to the levels estimated by Corrado et al. (2009) for the United States over the last few years. In the full calibration of the model described in the appendix, this innovation intensity of the economy corresponds to a share of labor employed in current production of $\bar{L}_p = .833$ and we calibrate the innovation cost function for incumbent firms, $c(q)$, such that the parameter $\Phi = .966$ is close to its maximum value of one. We assume that there is no congestion in the mapping from innovation effort to the innovation rate, and hence we have σ equal to its maximum value of one. With these assumptions, the elasticity of aggregate productivity with respect to changes in the innovation intensity of the economy is given by $\Gamma_1 = .01$, very close to its upper bound in equation (19).¹⁸ Moreover, this value of Γ_1 is close to its largest feasible value equal to $\bar{g}_z = .0125$. Hence, we regard this numerical example as useful in illustrating close to a best case scenario for finding large responses of aggregate productivity to changes in the innovation intensity of the economy and high sensitivity of these responses to the degree of intertemporal knowledge spillovers.

The only other parameter of the model that we need to specify to apply our analytical results is the intertemporal knowledge spillover parameter γ which, together with \bar{L}_p and Γ_1 determine the decay rate of the elasticities Γ_k . We consider three alternative values of the intertemporal knowledge spillover parameter γ . The first case we call the *high spillover* case. In it we set $\gamma \rightarrow 1$. The second case we call the *medium spillover* case, and in it we set

¹⁸Recall from equation (19) that as ρ approaches 1, a value that is commonly used in the literature, Γ_1 approaches the upper bound of $\bar{L}_p \Phi \sigma \bar{g}_z = 0.01$. With $\rho = 4$, we have $\Gamma_1 = .00987$.

$\gamma = 0$. The third case we call the *low spillover* case, and in it we set $\gamma = -2$.

Consider an unanticipated, uniform, and permanent increase in innovation subsidies that, in the long run requires an increase in fiscal expenditure on these policies equal to 3 percent of GDP. Note that this is a large change in fiscal expenditures on innovation subsidies, roughly equal to the total revenue collected from corporate profit taxes relative to GDP in 2007. From Proposition 1, we have that, in the long-run, these policies raise the innovation intensity of the economy to $s'_r = \bar{s}_r + .03 = .14$, so that $\log s'_r - \log \bar{s}_r = .24$, and that the increase in subsidy rates needed to achieve this change in the innovation intensity of the economy is given by $\log(1 - \bar{\tau}_g) - \log(1 - \tau'_g) = .24$.

We illustrate our first order approximation to the dynamics of the transition for our model economy in Figure 1. In constructing this figure, as shown in Panel A, we assume that on this transition path, the physical capital to output ratio is constant at its BGP level. As shown in Panel B, we also assume that that the innovation intensity of the economy jumps to its new BGP level immediately, i.e. $\log s'_{rt} - \log \bar{s}_r = .24$ for all $t \geq 0$. We do so to illustrate the quantitative implications of the model given a specified path for the innovation intensity of the economy. In the next section, we compute the full model to examine the actual transition path for the innovation intensity and physical capital to output ratio of the economy given our specific policy experiment to evaluate the usefulness of these approximations. We now use this example to discuss four main quantitative implications of our analytical results.

(i) Intertemporal knowledge spillovers and long run elasticities In the bottom left and right panels of Figure 1 (panels E and F) we show the transition paths for aggregate productivity and GDP, both as ratios to their levels on the initial BGP for the first 100 years of the transition. From Corollary 1, we have that the response of aggregate productivity in the long-run is very sensitive to the choice of intertemporal knowledge spillover parameter γ . As $\gamma \rightarrow 1$, the response of aggregate productivity becomes infinite because the growth rate of aggregate productivity is approximately $\Gamma_1(\log s'_r - \log \bar{s}_r) = 0.0024$ (24 basis points) above its initial level permanently (in the limit). In contrast, if $\gamma = 0$, then the long run response of aggregate productivity relative to its initial BGP path is $\bar{L}_p \times .24 = .2$ so productivity is up only 20% relative to its level on the initial BGP in the long run. More striking, if $\gamma = -2$, then aggregate productivity rises only 6.7% relative to its initial BGP path in the long run. Using the formula for the dynamics of GDP under the assumption that the physical capital to output ratio remains constant obtained in Corollary 2, we see that the response of GDP in the long run is also very sensitive to the choice of intertemporal knowledge spillover parameter.

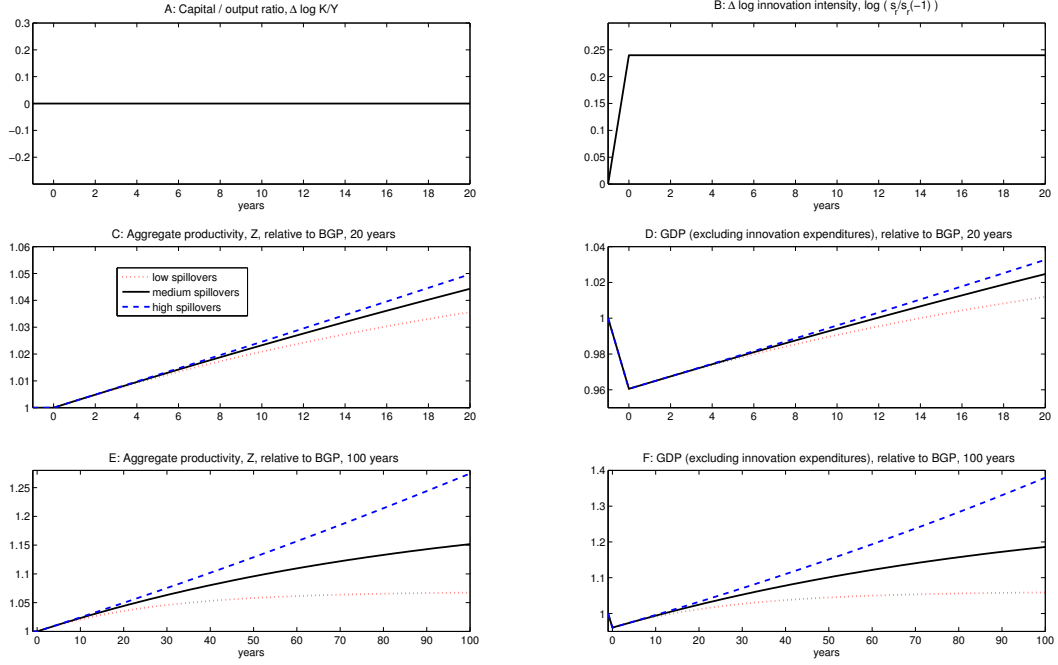


Figure 1: Approximate Transition Path to a Permanent Increase in Innovation Intensity

(ii) Small impact effect Now consider the response of aggregate productivity at the start of the transition. In the middle left and right panels of Figure 1 (panels C and D), we zoom in on the transition of aggregate productivity and GDP for the first 20 years of the transition. Here we see that in the first year of the transition, aggregate productivity rises only a little relative to the initial BGP path (only $\Gamma_1 \times 0.24 = 0.0024$ independent of γ). This implication of the model follows from the bound on the elasticity $\Gamma_1 \leq \bar{g}_z$ obtained in Proposition 2.

(iii) Intertemporal knowledge spillovers and medium-term productivity dynamics Now consider the response of aggregate productivity over the medium term (20 years) shown in Panel C of Figure 1. Recall that the elasticity of the cumulative response of aggregate productivity to a permanent change in the innovation intensity of the economy over 20 years is given by $\sum_{k=1}^{20} \Gamma_k$ for the alternative choices of γ . In the first case, as $\gamma \rightarrow 1$, we have $\sum_{k=1}^{20} \Gamma_k \rightarrow 20 \times \Gamma_1 = .20$ and a response of aggregate productivity at year $t = 20$ of the transition of $\log Z'_{20} - \log \bar{Z}_{20} = .20 \times .24 = 0.048$. In the second case, with $\gamma = 0$, we have $\sum_{k=1}^{20} \Gamma_k = .18$ and $\log Z'_{20} - \log \bar{Z}_{20} = 0.043$. Finally in the third case, with $\gamma = -2$, we have $\sum_{k=1}^{20} \Gamma_k = .145$ and $\log Z'_{20} - \log \bar{Z}_{20} = 0.034$. We see, then, that the model's predictions for the response of aggregate productivity to a given permanent change in

the innovation intensity of the economy 20 years into the transition are not particularly sensitive to choices of the intertemporal knowledge spillover parameter γ in comparison to the strong dependence of the model's long run predictions for aggregate productivity on this parameter.

(iv) Output gains are in the long run Next consider the transition path for GDP exclusive of innovation expenditures (resources available for consumption and physical investment) shown in panel D of Figure 1. In that figure, we see that GDP falls considerably on impact, regains its initial level in roughly 10 years, and is only modestly above its initial level in 20 years. Moreover, we see that the path of GDP in these first 20 years is not particularly sensitive to the choice of the knowledge spillover parameter γ . Recall from Proposition 2 and Corollary 2 that, holding fixed the rental rate of physical capital, the elasticity of GDP at horizon t with respect to a permanent change in the innovation intensity of the economy is given by

$$\frac{\log GDP'_t - \log G\bar{D}P_t}{\log s'_r - \log \bar{s}_r} = \left(\sum_{k=1}^t \frac{\Gamma_k}{1 - \alpha} - \bar{L}_r \right).$$

In our calibration, this term is equal to $-\bar{L}_r = -.167$ on impact at $t = 0$. With $\gamma \rightarrow 1$, this elasticity approaches .133 at $t = 20$, while with $\gamma = -2$, it is .051 at $t = 20$. In our particular policy experiment, since the log change in the innovation intensity of the economy is .24, the log change in GDP excluding innovation expenditures is 0.032 at $t = 20$ with $\gamma \rightarrow 1$ and only 0.012 at $t = 20$ with $\gamma = -2$. Recall that to convert these results to implications for GDP inclusive of innovation expenditure one must multiply the level of GDP exclusive of these expenditures by $(1 + s_{rt})$.

This result that our model's implications for the medium term elasticity of aggregate productivity and GDP with respect to a permanent change in innovation policies is relatively small for a wide range of values of the intertemporal knowledge spillover parameter γ does not imply that the current equilibrium level of innovation expenditures is close to optimal. In fact, from equation (35), we have that, for a wide range of values of consumers' discount factor β and of the intertemporal knowledge spillover parameters γ , our model implies that the optimal BGP innovation intensity of the economy is higher than the initial level we have assumed. Specifically, using our parameter values above, we have $\Gamma_1^*/L_p^* = \sigma \Phi \frac{1}{\rho-1} \frac{\exp((\rho-1)\bar{g}_z)-1}{\exp((\rho-1)\bar{g}_z)} = 0.012$. Equation (35) then implies that with $\beta = .99$ (the real interest rate is one percentage point higher than the growth rate) and $\gamma = .99$ (close to endogenous growth), the optimal innovation intensity of the economy is $s_r^* = 1.18$, that is, innovation expenditures should exceed expenditure on consump-

tion and physical investment combined by 18%. On the conservative side, with impatient consumers ($\beta = .96$) and low intertemporal knowledge spillovers ($\gamma = -2$), $s_r^* = 0.155$, a figure that is not too far from the measures of investments in intangible capital relative to GDP calculated by [Corrado et al. \(2009\)](#).

Clearly, our model's implications for the optimal innovation intensity of the economy are highly sensitive to assumptions about consumers' patience β and the level of intertemporal knowledge spillovers γ . Moreover, the model's implications for the optimal innovation intensity of the economy are not particularly sensitive to other parameters, given that we know that the term Γ_1/\bar{L}_p must be bounded by the value of \bar{g}_z that we match in calibrating the model.

Sensitivity to alternative parameter choices Our characterization of our model's dynamics using simple first order approximations is useful for highlighting which features of the model are important in determining its quantitative implications and for understanding quantitatively the sensitivity of these implications with respect to these model features. For example, consider the impact of calibrating the model to a higher level of productivity growth \bar{g}_z on the initial BGP. If we double this initial level of productivity growth, holding other parameters fixed (except for $g_{A,r}$, which we modify to be consistent with this higher growth rate), then from expression (19) the upper bound on the elasticity of aggregate productivity on impact Γ_1 also doubles. The change in the exact value of Γ_1 depends on the value of ρ : for ρ close to 1, Γ_1 also doubles; as ρ approaches infinity, the value of Γ_1 becomes insensitive to \bar{g}_z . With $\rho = 4$, doubling \bar{g}_z from .0125 to .025 raises Γ_1 from 0.01 to .019. In the case of endogenous growth, $\gamma = 1$, the 20-year elasticity of aggregate productivity $\sum_{k=1}^{20} \Gamma_k$ increases from .2 to .39. In the semi-endogenous growth case, $\gamma < 1$, the terms Γ_k decay more quickly, so, in the case with low intertemporal knowledge spillovers $\gamma = -2$ that we considered, this medium term elasticity rises from .143 to .212. Hence in this case, the elasticities of productivity implied by the model are larger and the medium term implications of the model are more sensitive to changes in the knowledge spillover parameter γ .

Likewise, consider the sensitivity of our results to the calibration of the innovation intensity of the economy \bar{s}_r on the initial BGP. We considered a policy experiment in which innovation subsidies are increased in a uniform manner resulting in the long run in a change of fiscal expenditures on these policies of 3 percent of GDP. From Proposition 1, we have that the corresponding long-run change in the innovation intensity of the economy $s' - \bar{s}_r$ is also 3 percent of GDP. If we calibrate the model to a lower initial value of \bar{s}_r on the initial GDP, then, mechanically, our model implies that this policy experiment

results in a larger change in the *log* of the innovation intensity of the economy. Thus, keeping the model elasticities Γ_k unchanged, the magnitude of the response of aggregate productivity to this policy experiment will be larger. For example, if we had assumed an initial innovation intensity of the economy of 5 percent in line with the new measures of intangible investment relative to Private Business Output in the NIPA in recent years, then our policy experiment would have increased the log of the innovation intensity of the economy by $\log .08 - \log .05 = .47$ rather than $.24$.

Next consider the implications of assuming larger differences between the average cost of innovation for incumbent and entering firms ($c(q)$ versus $c'(q)q$). Changes in this assumption impact the parameter Φ . In our calibration, we have set $\Phi = .966$ close to its maximum value of one. As discussed in Proposition 4, if we choose a smaller value of Φ , the elasticity of aggregate productivity with respect to a permanent change in the innovation intensity of the economy is reduced at every horizon, while the implications of the model for the long run are unchanged.

Finally, in assuming that $\sigma \leq 1$, we have abstracted from contemporaneous positive spillovers in research effort. Instead, if one assumes that there are increasing contemporaneous returns in the mapping from firms' research effort to the aggregate innovation rate, one could model these by assuming $\sigma > 1$. The impact of this change in assumptions on our model elasticities Γ_k is straightforward. Assuming a value of $\sigma > 1$ raises Γ_1 and the elasticity of aggregate productivity to a permanent change in the innovation intensity of the economy at any horizon in the same manner that assuming a lower value of Φ reduces these elasticities.¹⁹

Social depreciation of innovation expenditures A striking implication of Proposition 2 is that we are able to derive an upper bound on the elasticity of the growth of aggregate productivity with respect to the innovation intensity of the economy, Γ_1 , without specifying many of the underlying parameters of the model. What is key to determining the magnitude of that bound, however, is the implicit assumption that is made in specifying the model regarding the level of aggregate productivity growth that would occur if output of the research good were to be set to zero — a quantity that we term *the social depreciation of innovation expenditures*. Note that in our model there is private depreciation of innovation expenditures. In particular, individual firms perceive that they will lose the profits associated with any successful innovation at the rate δ_t determined by the innovation expenditures of other firms. But society does not lose the benefits of this past

¹⁹See, for example, Bloom et al. (2013) for estimates of contemporaneous technological spillovers using U.S. detailed firm level data.

innovation as captured in the productivity aggregate Z_t .

To see the importance of this implicit assumption regarding the social depreciation of innovation built in to our Neo-Schumpeterian model, consider equation (30) from the proof of Proposition 2. In that equation, from the log concavity of $G(\delta)$, defined as $\frac{Z_{t+1}}{Z_t} = G(\delta_t)$, we had the bound on the elasticity of aggregate productivity with respect to the innovation rate of

$$\frac{\log Z'_{t+1}/Z'_t - \bar{g}_z}{\log \delta'_t - \log \bar{\delta}} = \frac{G'(\bar{\delta})\bar{\delta}}{G(\bar{\delta})} \leq \log G(\bar{\delta}) - \log G(0) = \bar{g}_z - \log G(0).$$

Mechanically, as long as the transition law for aggregate productivity is log concave in the innovation rate, calibration of the transition law at the BGP innovation rate and at a zero innovation rate is sufficient to bound the elasticity of aggregate productivity with respect to the innovation rate at the BGP. The assumption that aggregate productivity remains constant with an innovation rate of zero (i.e. $G(0) = 1$) gave us the bound on this elasticity of \bar{g}_z .

Consider the implications of the alternative assumption that aggregate productivity would decline if the innovation rate were zero. Specifically, let $\frac{Z_{t+1}}{Z_t} = G(0) < 1$ be the gross growth rate of aggregate productivity if the innovation rate is equal to zero. Then the elasticity of aggregate productivity with respect to the innovation rate is bounded by a number larger than \bar{g}_z . Moreover, by direct calculation of this elasticity using the specific functional form for $G(\delta)$ in our baseline Klette-Kortum model (see equation (31)) we see that with social depreciation of innovation this elasticity is in fact larger. For example, if we assume that aggregate productivity would decline by 3.5% if the innovation rate was zero ($G(0) = 0.965$) then with $\rho = 4$ and $G(\bar{\delta}) = 1.0125$, this elasticity is 0.044 (and the upper bound above is 0.048) rather than 0.0123 (and an upper bound of 0.0125). Thus, with a moderate social depreciation rate of innovation of 3.5% per year, we obtain an elasticity of aggregate productivity growth with respect to the innovation rate that is roughly three and a half times larger than what we obtain in the absence of social depreciation.

With this assumption, together with our initial calibration of σ, Φ , and \bar{L}_p , we get $\Gamma_1 = .036$ rather than the value of $\Gamma_1 = .01$ that we obtained under the assumption of no social depreciation of innovation. This implies that with high intertemporal knowledge spillovers ($\gamma \rightarrow 1$) the medium term elasticity of aggregate productivity with respect to a permanent change in the innovation intensity of the economy is $\sum_{k=1}^{20} \Gamma_k = .72$ rather than 0.2 with no social depreciation of innovation. With low intertemporal knowledge spillovers ($\gamma = -2$), this elasticity is $\sum_{k=1}^{20} \Gamma_k = .26$ rather than .14 with no social depreci-

ation of innovation. On the other hand, $\sum_{k=1}^{\infty} \Gamma_k = \frac{L_p}{1-\gamma}$, which determines the long-term change (across BGPs) in aggregates, is independent of the social rate of depreciation.

These results imply that in an economy with a moderate initial TFP growth rate and with a modest social depreciation of innovation, the medium term elasticity of aggregate productivity with respect to a permanent change in the innovation intensity of the economy is both significantly larger than it is with no social depreciation of innovation and much more sensitive to the magnitude of intertemporal knowledge spillovers. In particular, with significant social depreciation of innovation and high intertemporal knowledge spillovers, we have from Corollary 2 that measured GDP exclusive of innovation expenditures rises relative to its initial BGP path after only 5 years of transition following a permanent increase in the innovation intensity of the economy instead of after more than 10 years as shown in panel D of Figure 1.

Note as well that the faster transition dynamics for measured GDP that one obtains when one assumes that there is social depreciation of innovation expenditures also has normative implications. Specifically, with social depreciation of innovation expenditures, Γ_1/L_p rises (holding fixed all other parameters and policies) and hence from equation (35), the optimal innovation intensity of the economy also rises.

The importance of social depreciation of innovation in determining an upper bound on the elasticity of productivity growth with respect to output of the research good that we derive is not specific to the Quality Ladders/Klette Kortum type model considered here. For example, it applies as well to the growth model in Jones (2002) in which the growth of aggregate productivity can be written directly as a function of output of the research good Y_{rt}

$$\frac{Z_{t+1}}{Z_t} = H(Y_{rt}),$$

where $\log H$ is an increasing and concave function of research output.²⁰ Here, the elasticity of aggregate productivity growth with respect to research output is given by

$$\frac{\log Z'_{t+1}/Z'_t - \bar{g}_z}{\log Y_{rt} - \log \bar{Y}_r} = \frac{H'(\bar{Y}_r)\bar{Y}_r}{H(\bar{Y}_r)} \leq \log H(\bar{Y}_r) - \log H(0),$$

where the inequality follows from the concavity of $\log H$ with respect to research output. In Jones (2002), it is assumed that there is no social depreciation of innovation, so $H(0) = 1$ and this elasticity is bounded above by \bar{g}_z . On the other hand, if one assumes that there is social depreciation of innovation, i.e. $H(0) < 1$, this elasticity can be larger.

²⁰The transition law for aggregate productivity is given in equation (5) in Jones (2002). We interpret the expression on the right hand side of that equation as output of the research good.

Similar arguments can be applied to models of firm dynamics with decreasing returns to scale, e.g. [Hopenhayn \(1992\)](#), or love-for-variety, e.g. [Melitz \(2003\)](#) and [Luttmer \(2007\)](#). In the appendix we characterize the dynamics of aggregate productivity growth in response to a policy-induced change in the innovation intensity of the economy in the context of two expanding-varieties models and obtain expressions very similar to those in [Section 4](#). The magnitudes of the elasticities of aggregate productivity in these models can be larger than those in our baseline Klette-Kortum model because these models imply that there can be social depreciation of innovation. In the first model that we consider, only entering firms invest resources in innovation, similar to the model in [Luttmer \(2012\)](#). The social depreciation of innovation is defined as the growth rate of aggregate productivity if there was no entry of new firms. Aggregate productivity falls when there is no entry of new firms if the loss of productivity due to the exit of incumbent firms exceeds the gains in productivity due to the growth of surviving incumbents. In the second model that we consider, both entering and incumbent firms invest resources in innovation, similar to the model in [Atkeson and Burstein \(2010\)](#). The social depreciation of innovation is defined as the growth rate of aggregate productivity if there was no entry of new firms and no expenditure in innovation by incumbent firms. In this case the social depreciation of innovation can be larger than in the model with no innovation by incumbents.

6 Full numerical solution

In the previous two sections we have presented an analytical partial characterization of the transition dynamics of our model, up to a first-order approximation, following a permanent change in innovation policies. In that approximation, we took as given the transition path for the innovation intensity of the economy and the ratio of physical capital to output. In this section we use a calibrated version of the model to solve numerically (as described in the appendix) for the full transition dynamics of the economy following a permanent change in innovation policies. We use this solution to evaluate the usefulness of our analytical approximation.

In our analytical results, we saw that the dynamics of aggregate productivity in the model given a path for the innovation intensity of the economy are shaped by very few parameters. In contrast, to solve the model numerically we must choose all the model's parameters. We discuss in detail our procedure for doing so in the appendix. Our calibration procedure targets data from the US's NIPA supplemented with aggregate data from [Corrado et al. \(2009\)](#) on firms' investments in intangible capital. In this calibration, the values for \bar{g}_z , \bar{s}_r , \bar{L}_p , Φ and σ , as well as the change in innovation policies, are the same

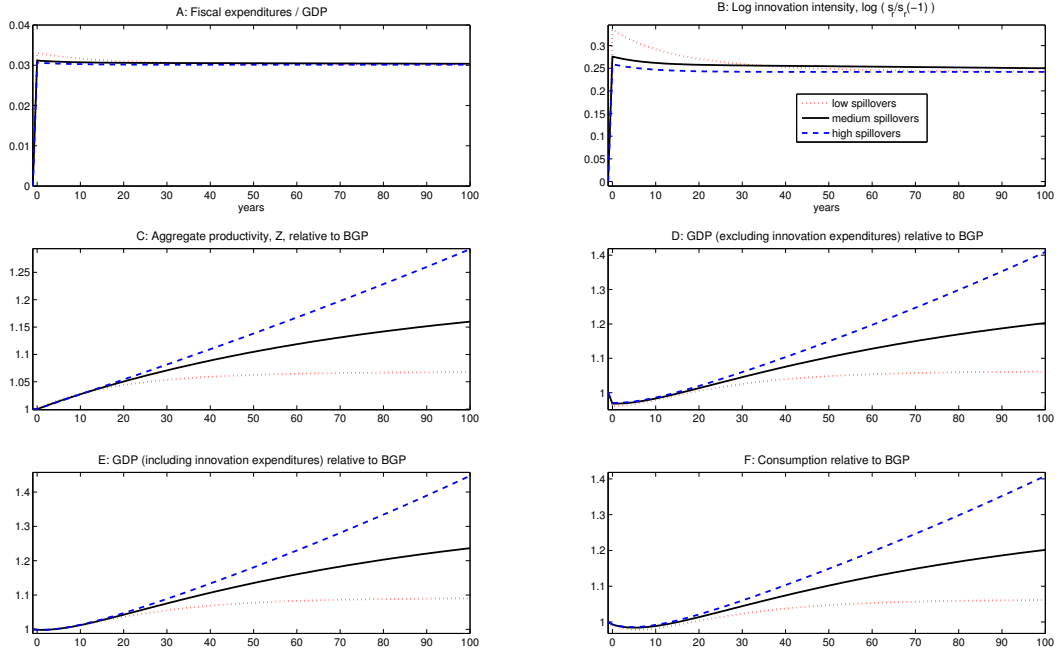


Figure 2: 100-year Transition Dynamics to a Permanent Increase in Innovation Intensity, Full numerical solution

as those used in Section 5. We report results from this experiment in Figures 2 and 3 for versions of our baseline calibrated model with a high, medium, and low intertemporal knowledge spillover as in section 5.

Long run dynamics We first consider the transition dynamics of our model economy over the first 100 years. In Panel A of Figure 2 we show the response of fiscal expenditures as a percent of GDP over the first 100 years of the transition to the new BGP. We see that for each value of intertemporal spillovers that we consider, the change in these fiscal expenditures are very close to their long run value of 3% throughout the transition, independent of the value of intertemporal knowledge spillovers. Thus we see little intertemporal variation in the fiscal impact of the innovation policies that we consider over the long term.

The long run change in the innovation intensity of the economy, s_r , that results from this innovation subsidy is equal to .24 in log terms and is independent of the value of knowledge spillovers. In panel B of Figure 2, we show the dynamics of $\log s_{rt}$ over the first 100 years of the transition. Here we see that there are mild intertemporal substitution effects in innovation expenditures in that this innovation intensity rises by a bit more

than .24 in log terms in the early phase of the transition, particularly for the case with low intertemporal knowledge spillovers. The intertemporal substitution of innovation expenditures shown in panel B is more pronounced with low γ because in this case the price of the research good is expected to rise quickly during the transition.

In panel C of Figure 2, we show the response of the level of aggregate productivity during the first 100 years of the transition for our three values of γ . Likewise, in panel D of Figure 2, we show the response of measured GDP (the resources available for consumption and physical investment) during the first 100 years of the transition. The results shown in these panel are quite similar to those for aggregate productivity and measured GDP over the first 100 years shown in Panels E and F of Figure 1. As we will see below, the impact of intertemporal substitution in the path for the innovation intensity of the economy on the transition path for aggregate productivity and of changes in the physical capital to output ratio on measured GDP appear primarily in the initial phase of the transition. In panel E of Figure 2, we show the transition path for measured GDP inclusive of innovation expenditures over the first 100 years. We see that this alternative measure of GDP does not fall substantially on impact as does the measure of GDP excluding innovation expenditures shown in panel D.

In panel F of Figure 2, we show the transition for consumption over the first 100 years. We see that consumption falls initially (independent of γ) and then rises after a decade or more, where the magnitude of the rise is quite sensitive to γ . As a consequence of these differences in the response of consumption in the very long run, we find that the welfare implications of an increase in innovation subsidies are quite sensitive to the degree of intertemporal knowledge spillovers that we assume. We find in the low spillover case that the equivalent variation in consumption is 3.9%, in the medium spillover case it is 13.9%, and in the high spillover case it is 41.1%. Thus, with large intertemporal knowledge spillovers, we find very large welfare gains from the innovation subsidies in this experiment. The differences in the welfare implications of innovation policy changes across the high and low intertemporal knowledge spillover economies arise as a result of the growing differences in the paths of consumption past the medium term horizon of 20 years in combination with our choice of $\beta = .99$ for the consumer's discount factor (so that the real interest rate is 1 percent higher than the growth rate).

Medium Term Dynamics In Figure 3, we consider the evolution of aggregates during the first 20 years of the transition. We zoom in on this first phase of the transition to better evaluate the quality of our approximation.

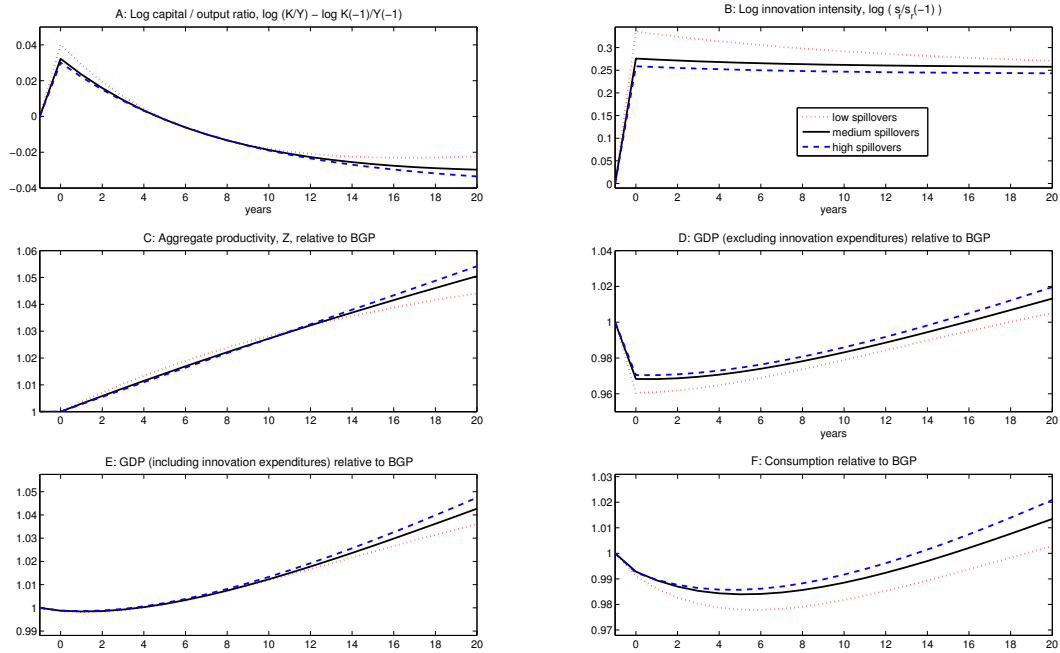


Figure 3: 20-year Transition Dynamics to a Permanent Increase in Innovation Intensity, Full Numerical Solution

In panel A of Figure 3, we show the evolution of the log of the physical capital to output ratio over the first 20 years of the transition. This transition path corresponds to the negative of the transition path for the log of the rental rate on physical capital. In panel B of Figure 3, we show the path of the log of the innovation intensity of the economy. Clearly, there are dynamics of these ratios that we ignored in our analytical approximation shown in panels A and B of Figure 1.

From Proposition 2, we have that the intertemporal substitution in the path for the innovation intensity of the economy shown in Panel B of Figure 3 impacts the model-implied transition for aggregate productivity. By comparing the approximated path for aggregate productivity shown in panel C of Figure 1 to the fully simulated path shown in panel C of Figure 3, we can see that the approximation is fairly accurate despite the dynamics of the innovation intensity of the economy shown in Panel B. In particular, the log of aggregate productivity relative to the initial BGP path in year 20 in the fully simulated path is .053 (versus 0.048 based on the approximation) with $\gamma \rightarrow 1$, .049 (versus .043) with $\gamma = 0$, and .043 (versus .035) with $\gamma = -2$.

From expression (34), we have that the full dynamics of GDP are impacted both by the dynamics of the innovation intensity of the economy and of the ratio of physical capital to

output. On impact, these two factors have opposite effects on GDP — the initial increase in the ratio of physical capital to output raise GDP while the initial increase in the innovation intensity of the economy above its new long-run level lowers GDP. By comparing the approximated path for GDP shown in panel D of Figure 1 to the fully simulated path shown in panel D of Figure 3, we again can see that the approximation is fairly accurate despite the fact that we ignored the dynamics in the innovation intensity of the economy and the capital-output ratio.

In panels E and F of Figure 3, we show the path of GDP inclusive of innovation expenditures and of consumption during the first 20 years of the transition. We see here that consumption falls initially for at least 10 years during the transition indicating that the potentially large welfare gains are achieved only in the long run.

Figure 3 shows that a large and persistent increase in innovation subsidies has a relatively small impact on aggregate productivity and GDP over a 20 year horizon, and the response of aggregates does not vary much with the extent of intertemporal knowledge spillovers assumed in the model.

These results suggest that it would be hard to verify whether innovation policies yield large output and welfare gains using medium term data on the response of aggregates to changes in innovation policies. We illustrate this point in Figure 4. In that figure we show results obtained from simulating the response of aggregates in our model to our baseline increase in innovation subsidies in a version of our model with Hicks neutral AR1 productivity shocks A_{pt} with a persistence of 0.9 and an annual standard deviation of 2%. We introduce these shocks as a proxy for business cycle shocks around the BGP. We show histograms generated from 3000 simulations of the model for the first 20 years of the transition. The units on the horizontal axis show the log of the ratio of detrended GDP at the end of the 20th year of transition to initial GDP and the vertical axis shows the frequency of the corresponding outcome for GDP. In panel A of the figure, we show results for GDP excluding innovation expenditures. In panel B, we show results for GDP including innovation expenditures. The red bars show results for the model with low intertemporal knowledge spillovers and the blue bars show the results with high spillovers. Clearly, in each panel, the distribution represented by the blue bars is slightly to the right of that represented by the red bars. The histograms in panel B are shifted to the right relative to those in panel A reflecting the fact that GDP including innovation expenditures has a higher elasticity of changes in the innovation intensity of the economy. But it is also clear in each panel that, using either measure of GDP, that it would be very hard to distinguish the degree of intertemporal knowledge spillovers (and, hence, the long term effects from this innovation subsidy) in aggregate time series data even if we had the benefit of a true

policy experiment.

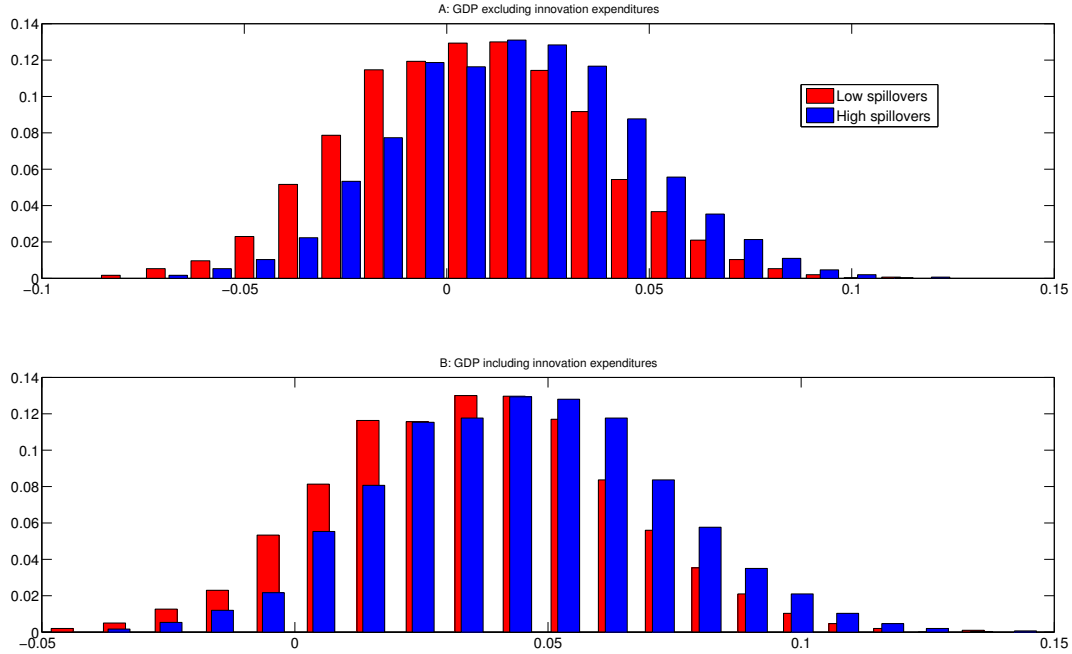


Figure 4: Histogram 20-year Increase in GDP to a Permanent Increase in Innovation Intensity Including Productivity Shocks

Non-uniform changes in innovation policies Up to this point, in our quantitative results, we have considered policy experiments in which the economy starts on a BGP with uniform innovation subsidies and transitions to a new BGP with new uniform innovation subsidies. We use the assumption that the economy initially has uniform innovation subsidies to isolate the aggregate implications of changes in innovation policies that work through changing the overall innovation intensity of the economy rather than changing the allocation of research expenditure across firms. In Corollary 3 we see that if the economy starts with uniform innovation subsidies, to a first order approximation it does not matter whether the new innovation subsidies are uniform or not. In the appendix, we present the full transition dynamics of the model to evaluate whether there are important second order effects that arise when large non-uniform changes in innovation policies are considered. We consider permanent and unanticipated increases in the innovation subsidy to incumbents τ_g only that result in a long run increase in the innovation intensity of the economy from $\bar{s}_r = .11$ to $s'_r = .14$. (In the long run, this subsidy requires fiscal expenditures of 3.3% relative to GDP rather than 3% under our baseline experiment with

uniform innovation policies). We show these dynamics for economies with two different curvatures of the incumbents' innovation cost function, $c(q)$, which determines the elasticity of innovation effort, q , with respect to the incentives to innovate. We consider an inelastic case in which the share of innovative effort by incumbents, $q/(q + M)$, across the old and new BGP is constant at our baseline level of 75%, and an elastic case in which the share of innovative effort by incumbents increases significantly across old and new BGPs from 75% to 96% (implying a substantial fall in the share of employment in new firms). We find in both cases that our first order approximation is fairly accurate in that the dynamics of aggregate productivity and measured GDP are not much different than those that we found with uniform changes in innovation policies.

7 Conclusion

In this paper, we have derived a simple first-order approximation to the transition dynamics of aggregate productivity and GDP in response to a permanent policy-induced change in the innovation intensity of the economy implied by a baseline Klette-Kortum model. Our results highlight the tight connection between the model's quantitative implications for the transition dynamics for aggregate productivity and GDP following a sustained change in innovation policies and the implicit assumption one makes regarding the social depreciation of innovation in specifying the model. If one assumes that there is no social depreciation of innovation, then in using this model to analyze changes in innovation policies in advanced economies (with relatively slow baseline growth rates of productivity), one is driven to the conclusion that changes in innovation policies would only pay off in terms of sustained increases in productivity, output, and consumption after 10 years or more, regardless of the other parameters of the model. Under this assumption, such policy changes could lead to substantial welfare gains only if consumers are patient and willing to wait for boost in consumption several decades in exchange for a reduction in consumption today. In contrast, if one assumes that there is substantial social depreciation of innovation, then one opens up the possibility that a change in innovation policies could have a significantly larger payoff in the medium term in terms of increased productivity, output, and consumption. These results indicate the importance of improving our modeling and measurement of the social rather than private depreciation of innovation.

The analytical approach that we have taken here to characterizing our model's transition dynamics is also useful for examining related questions such as the implications of changes in the patent rate and/or changes in the firm startup rate as measured by the

share of labor employed in young firms for the dynamics of aggregate productivity. See, for example [Kogan et al. \(2012\)](#) for a study of patent based measures of the innovation rate and [Decker et al. \(2013\)](#) for a study of changes in the firms' startup rate in the U.S. In choosing a model to analyze these questions, implicit assumptions about the social depreciation of innovation should also play an important role in shaping the quantitative results of the analysis.

Finally, under our baseline assumptions, we have abstracted from the productivity and welfare gains that might be achieved by reallocating a given level of investment in innovation across heterogeneous firms. Under alternative settings in which there are first-order gains from such a reallocation, one may wish to consider a whole range of policies aimed at reallocating innovation expenditures across firms. The research challenge here is to find reliable metrics for evaluating which firms should be doing more investment in innovation and which should be doing less. Regardless, it is likely that in such extensions of our baseline model, the social depreciation of innovation will still be an important parameter shaping the quantitative results.

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A Additional details of BGP, proof of Proposition, and solving transition dynamics 1

In the body of the paper we described the BGP growth rates and innovation rates. Here we describe how to solve for other aggregates.

Rescaled Bellman equations: To start, it is useful to present rescaled Bellman equations describing the value of firms. Defining $v_t = \frac{V_t Z_t^{\rho-1}}{P_{rt}}$, $u_t = \frac{U_t}{P_{rt}}$, we have (without imposing BGP),

$$v_t = \frac{\Pi_t Z_t^{\rho-1}}{P_{rt}} + \frac{(1 - \delta_t)}{1 + r_t} \frac{v_{t+1}}{\exp((\rho - 1) g_{zt})}, \quad (36)$$

$$u_t = - (1 - \tau_g) c(q_t) + \frac{1}{1 + r_t} \frac{1}{\exp((\rho - 1) g_{zt})} \frac{q_t}{q_t + M_t} \delta_t v_{t+1} \exp(\Delta)^{\rho-1} + \frac{1}{1 + r_t} \left(\frac{q_t}{q_t + M_t} \delta_t + 1 - \delta_t \right) u_{t+1}. \quad (37)$$

and the free entry condition is

$$(1 - \tau_e) f \geq \left(\frac{1}{1 + r_t} \frac{\delta_t}{q_t + M_t} \right) \left(\frac{v_{t+1}}{\exp((\rho - 1) g_{zt})} \exp(\Delta)^{\rho-1} + u_{t+1} \right), \quad (38)$$

where $1 + r_t = (1 + \bar{r}_t) \frac{P_{rt}}{P_{rt+1}}$. In a BGP, the constant in rescaled profits $\Pi Z^{\rho-1}/P_r$, and values $V Z^{\rho-1}/P_r$, U/P_r are constant over time (as well as r , g_z and δ as discussed in the body of the paper).

Solving other BGP aggregates: The constant $\Pi Z/P_r$ is pinned down from the free entry condition as follows. Manipulating equations (36) and (37) and imposing BGP with positive entry, we can write the free-entry condition (38) as

$$(1 - \tau_e) f M = \frac{1}{1 + \frac{r}{\delta} \left(\frac{q}{M} + 1 \right)} \left[\bar{\zeta}_g \frac{\Pi Z^{\rho-1}}{P_r} - (1 - \tau_g) c(q) \right], \quad (39)$$

$$\text{where } \bar{\zeta}_g = \frac{r + \delta}{r \exp((\rho - 1) g_z) / \exp(\Delta)^{\rho-1} + \delta} \geq 1$$

and $r = \beta^{-1} - 1$ in the BGP (since in the BGP, P_{rt} falls at a rate g_y). Given the value of $\Pi Z^{\rho-1}/P_r$ that solves this equation, the allocation of labor L_p/L_r is determined as a function of parameters using

$$\frac{L_p}{L_r} = \left(\frac{1 - \alpha}{\mu - 1} \right) \frac{\Pi Z^{\rho-1}}{P_r} \frac{1}{c(q) + fM}. \quad (40)$$

When $\gamma < 1$, the growth rate g_z is determined as described in the text (and hence also δ , q and M). The level of aggregate productivity Z , for a given current value of scientific knowledge A_r , is determined using equation (8). When $\gamma = 1$, one can use the same procedure but instead of solving for Z (the BGP level of Z , given A_r , is not pinned down), one must solve for the growth rate g_z (and the corresponding values of δ , q and M). The innovation intensity of the economy, s_r , is calculated as a function of parameters using expression (17). Finally, we solve for aggregate output, Y , using (4), the stock of physical capital, K_t , using the factor shares of physical capital and production-labor, and consumption, C_t , using (2).

Active innovation by incumbents: We now discuss the condition stated in Section (3) that in a BGP if incumbents use their innovation technology (i.e. $u \geq 0$) then $(1 - \tau_g) c(q) / q \leq (1 - \tau_e) f$ at the level of q that solves (16). The condition $u \geq 0$ (in a BGP) implies

$$-(1 - \tau_g) \frac{c(q)}{q} + \frac{1}{1+r} \frac{1}{q+M} \delta \left[v \exp(\Delta)^{\rho-1} + u \right] \geq 0.$$

Combining this inequality with the inequality from the free-entry condition, (38), we have

$$(1 - \tau_g) \frac{c(q)}{q} \leq \frac{1}{1+r} \frac{1}{q+M} \delta \left[\frac{v}{\exp((\rho-1)g_z)} \exp(\Delta)^{\rho-1} + u \right] \leq (1 - \tau_e) f$$

which proves our result.

Proof of Proposition 1: Under the assumption of semi-endogenous growth ($\gamma < 1$), δ , g_z , r , and ξ_g are constant between BGPs. Under uniform innovation policies and firm-entry, M , q and Y_r are constant between BGPs. By equation (39), $\frac{\Pi Z^{\rho-1}}{P_r} \frac{1}{1-\tau_g}$ must be constant between BGPs. Using $\Pi_t Z_t^{\rho-1} = \frac{\mu-1}{\mu} (1 + \tau_s) GDP_t$ and the fact that Y_r is constant between BGPs, $\frac{1}{s_r(1-\tau_g)} \frac{(\mu-1)(1+\tau_s)}{\mu}$ must be unchanged between BGPs. With τ_s constant between BGPs, we immediately obtain the first result of the proposition. The second result follows from the fact that with uniform innovation policies we have $\frac{E}{GDP} = \tau_g s_r + \tau_s$, combined with the previous result that $s_r (1 - \tau_g)$ is constant between BGPs.

Solving transition dynamics In Sections 4 and 5 section we approximated the aggregate transition taking as given the transition path for the innovation intensity of the economy and the ratio of physical capital to output. To solve for the path of these two variables for a given change in policies or other parameters, we solve the model numerically. Specifically, we solve for the path of Z_t , K_t , v_t , and u_t using the four following Euler equations: (36), (37), (38) and $R_{kt} = d_k + \bar{r}_t$ where $\bar{r}_t = \frac{1}{\beta} \frac{C_{t+1}}{C_t} - 1$. Recall that q_t is solved for using equation (16) assuming that there is positive firm entry (which must be checked). Given a path of Z_t and K_t we can solve for all other equilibrium outcomes using static equations. We solve for the 4 Euler equations using either standard

linearization methods or a shooting algorithm, and we obtain very similar results.

B Calibration

Here we describe the calibration of the model that we use in Sections 5 and 6.

Policies: τ_s, τ_g, τ_e . Given our analytical results, we assume that subsidies to innovation on the BGP in the data are uniform, so that $\tau_e = \tau_g$. As a baseline calibration, we set all policies to zero, but we do indicate in the calibration formulas below where these policies enter.

Interest rate minus growth rate: In our model, on a BGP, the gap between the consumption interest rate and the growth rate of consumption is determined by the consumers' discount factor β . We set this gap to 0.01, which implies $\beta = 1/1.01$. Given this choice of discount factor, on the BGP, the model interest rate in terms of the research good is given by $r = 0.01$. We set the growth rate of consumption to $g_Y = .02$, which implies a consumption interest rate of $\bar{r} = 0.03$. This calibration of r is consistent with the difference between the interest rate and the consumption growth rate in [McGrattan and Prescott \(2012\)](#).

Final Consumption Good Production Function: The production function for the final consumption good is parameterized by ρ , which controls the elasticity of the residual demand curve faced by intermediate goods producers. In our baseline calibration we set $\rho = 4$. This elasticity establishes an upper bound on the markup μ that can be chosen.

Equilibrium Markup: Defining NIPA profits to intangible capital as $GDP_t - R_{kt}K_t - W_tL_t$, the share of these profits in GDP, denoted by π_t , is given by

$$\pi_t = (1 + \tau_s) \left(1 - \frac{1}{\mu} \right) - s_{rt} - \tau_s. \quad (41)$$

From equation (41), the choice of the markup μ is disciplined by data on the innovation intensity of the economy, s_r , and the share of NIPA profits paid to intangible capital relative to GDP. In our baseline calibration, we target a share of NIPA profits paid to intangible capital in GDP, π , of 1% from [McGrattan and Prescott \(2005\)](#) and an innovation intensity of the economy, s_r , of 11% similar to the levels estimated by [Corrado et al. \(2009\)](#) for the United States over the last few years. This implies a markup of 13.6%, $\mu = 1.136$.

Aggregate Production of the Final Consumption Good: Aggregate production of the final consumption good in equation (4) is parameterized by α . We set $\alpha = 0.37$ to match the observation that the share of rental payments to physical capital on the BGP, given by $(1 + \tau_s) \frac{\alpha}{\mu}$, is equal to 0.33. With this choice of α , we also have that the share of labor compensation (including production and research) in GDP is given by $(1 + \tau_s) \frac{1-\alpha}{\mu} + s_r = 0.66$. The rest of GDP corresponds to profits paid to intangible capital, $\pi = 1\%$.

The Allocation of Labor: The equilibrium allocation of labor between production and research is pinned down in equation (17) by the choices of parameters above and our calibrated innovation

intensity of the economy, s_r . In our baseline calibration, with $s_r = .11$, we have $L_p = .833$.

BGP Growth Rates for Scientific Knowledge and Aggregate Productivity: Given our calibration of per capita GDP growth of 2% and our physical capital share of α , we calibrate the growth rate of aggregate productivity in the BGP, \bar{g}_z , to 1.25%. For a given choice of γ , the growth rate of scientific knowledge consistent with these productivity growth rates is given by $g_{Ar} = (1 - \gamma) \bar{g}_z$. We do not make assumptions about this growth rate directly since we do not observe it. Instead, we alter this parameter as we vary γ .

Innovation Step Size and BGP Innovation Rate: Our choices of innovation step size Δ and the BGP innovation rate δ must be consistent with the BGP growth rate of aggregate productivity for intermediate firms, g_z given in equation 5. We must also have that the innovation step size exceeds the markup ($\Delta \geq \mu$) to ensure that an incumbent firm has a technological advantage over its latent competitor consistent with its assumed markup. In our baseline calibration, we set $\Delta = \Delta_l = \mu$. Given our choice of elasticity ρ and the implied value of \bar{g}_z , from equation (5), we get then get $\delta = 0.08$ in our baseline calibration.

Share of innovative effort by entering firms relative to their share of research expenditure: Under the assumptions on policies made above, the ratio $\Phi = \frac{\bar{M}/(\bar{q} + \bar{M})}{f\bar{M}/\bar{Y}_r}$ is already determined by parameters that we have previously calibrated (r , δ , and $\bar{\zeta}_g$), and by the choice of targets for s_r and π that have been already discussed. To see this, combining the free-entry condition (39) and the following expression for π/s_r under $\tau_s = 0$,

$$\frac{\pi}{s_r} = \frac{\frac{\Pi Z^{\rho-1}}{fMP_r} - \left(\frac{c(q)}{fM} + 1\right)}{\left(\frac{c(q)}{fM} + 1\right)}$$

we obtain

$$\frac{c(q)}{fM} = \frac{\frac{1-\tau_e}{1-\tau_g} \left(1 + \frac{r}{\delta} \left(\frac{q}{M} + 1\right)\right) - \frac{\bar{\zeta}_g}{1-\tau_g} \left(1 + \frac{\pi}{s_r}\right)}{\frac{\bar{\zeta}_g}{1-\tau_g} \left(1 + \frac{\pi}{s_r}\right) - 1}.$$

Setting $\tau_g = \tau_e = 0$,

$$\Phi = \frac{\frac{M}{q+M}}{\frac{fM}{c(q)+fM}} = \frac{\frac{c(q)}{fM} + 1}{\frac{q}{M} + 1} = \frac{\frac{r}{\delta}}{\bar{\zeta}_g \left(1 + \frac{\pi}{s_r}\right) - 1}.$$

In our baseline calibration, we have $\Phi = 0.966$ so this fraction is close to its upper bound of one, which means that the average cost of innovation by incumbents is close to the marginal cost of innovation by incumbents.

Incumbents' innovation cost function:

Our analytic results showed that $c(q)$ does not matter for the first order aggregate effects of changes in innovation policies (starting from uniform innovation policies) except through Φ . For the numerical evaluation of our results, we set $c(q) = \bar{c} + \phi_0 q^\phi$. When considering uniform

changes in innovation policies, we do not need to specify the curvature parameter ϕ , because incumbents do not change their innovation effort q (Moreover, according to Corollary 3, this parameter does not matter to a first-order approximation even when considering a policy experiment in which the subsidies to innovation are not uniform). We choose \bar{c} to target the share of profits in GDP, π , discussed above. We choose the parameter ϕ_0 so that the level of q in the initial BGP (from equation (16)) implies a share of employment of entering firms of 3%. In the model, the share of employment by entering firms is given by

$$s_{new} = \frac{\bar{M}}{\bar{q} + \bar{M}} \frac{\exp((\rho - 1) \bar{g}_z) - (1 - \bar{\delta})}{\exp((\rho - 1) \bar{g}_z)}.$$

This choice of parameters implies that incumbents account for roughly 75% of total innovative effort. Our baseline results do not depend on the choice of s_{new} as long as we have positive entry and positive innovation by incumbents. In any calibration of our model, one must check that incumbents actually want to use their innovation technology (i.e. $U > 0$). This is the case in our calibration.

The Intertemporal Knowledge Spillover Parameter γ : Our procedure for calibrating the model to a given BGP does not discipline the parameter γ . In our policy experiments below, we consider three alternative values of γ corresponding to a *low spillover case* ($\gamma = -2$), a *medium spillover case* ($\gamma = 0$) and a *high spillover case* (γ approaching one).

Other parameters: One can show that the parameters f , σ_0 , and A_r at time 0 can all be normalized to 1 without affecting our results. We abstract from contemporaneous congestion effects in innovative effort by setting $\sigma = 1$. We do this to give our model the best chance of generating fast transition dynamics.

C Non-uniform changes in innovation policies

We now solve the full transition dynamics of the model to evaluate whether there are important second order effects that arise when large non-uniform changes in innovation policies are considered. We consider permanent and unanticipated increases in the innovation subsidy to incumbents τ_g only that result in a long run increase in the innovation intensity of the economy from $\bar{s}_r = .11$ to $s'_r = .14$. (In the long run, this subsidy requires fiscal expenditures of 3.3% relative to GDP rather than 3% under our baseline experiment with uniform innovation policies). We show these dynamics for economies with two different curvatures of the incumbents' innovation cost function, $c(q)$, which determines the elasticity of innovation effort, q , with respect to the incentives to innovate. We consider an inelastic case in which the share of innovative effort by incumbents, $q/(q + M)$, across the old and new BGP is constant at our baseline level of 75%, and an elastic case in which the share of innovative effort by incumbents increases significantly across old and new BGPs from 75% to 96% (implying a substantial fall in the share of employment in new firms).

We set $\gamma = 0$ in all versions of the economy that we consider here. We show results in Figure 5 for the evolution over the first 20 years of the log of the physical capital output ratio, the innovation intensity of the economy, aggregate productivity, GDP exclusive of innovation expenditures, GDP including innovation expenditures, and consumption. In each panel of the figure, we show results from the baseline transition with uniform policies together with the results with inelastic and elastic innovative effort by incumbent firms. Note that the different responses of aggregates between the economies with the non-uniform policy change and inelastic and elastic innovative effort by incumbents arise as a result of second order terms: to a first order approximation, these responses should be the same. The different responses of aggregates between the baseline economy with a uniform policy change (so that equilibrium q does not change) and the economy with a non-uniform policy change and inelastic innovative effort by incumbent firms (so, again, q does not change) arise as a result of the different paths for the innovation intensity of the economy due to differences in the intertemporal substitution of innovation expenditure along the transition. We find in both cases that our first order approximation is fairly accurate in that the dynamics of aggregate productivity and measured GDP are not much different than those that we found with uniform changes in innovation policies.

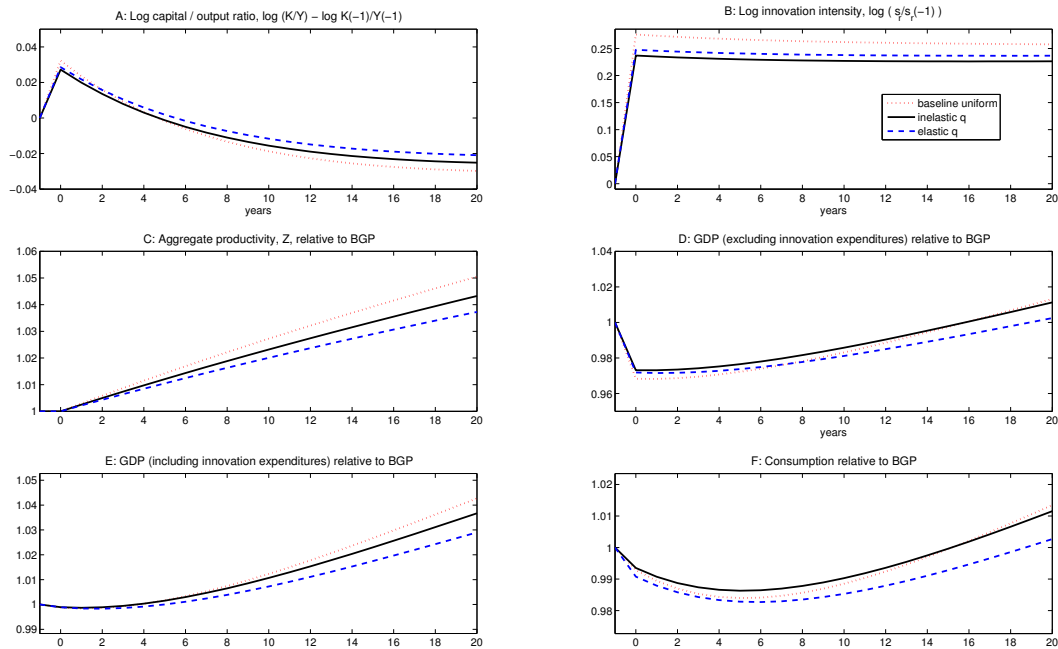


Figure 5: 20-year Transition Dynamics to a Uniform and Non-Uniform Innovation Policy, Medium Knowledge Spillover, Full Numerical Solution

D Optimal allocations and optimal innovation policy

We now characterize the aggregate allocation of labor in the BGP of the social optimum (under the assumption of semi-endogenous growth and positive firm-entry) and the policy that implements this allocation in the BGP of the equilibrium in the following proposition. We allow for social depreciation of innovations.

Proposition 5. *If the social optimum allocation has a BGP with firm-entry and semi-endogenous growth ($\gamma < 1$), then that optimal BGP corresponds to the equilibrium BGP with a subsidy to the production of the consumption good given by $\tau_s^* = \mu - 1$, and a uniform subsidy to innovation by incumbents and entrants given by*

$$\tau_g^* = \tau_e^* = 1 - (\mu - 1) \zeta_g \frac{\frac{r}{\sigma\Phi} \frac{(\rho-1)\exp((\rho-1)g_z)}{\exp((\rho-1)g_z) - \zeta} + 1 - \gamma}{\frac{r}{\delta\Phi} + 1}. \quad (42)$$

These policies result in an aggregate allocation of labor in the BGP given by

$$\frac{L_r^*}{L_p^*} = \frac{1}{(1-\alpha)} \frac{\frac{\beta}{\rho-1} \frac{\exp((\rho-1)g_z) - \zeta}{\exp((\rho-1)g_z)} \Phi\sigma}{\left(1 - \beta + (1-\gamma) \frac{\beta}{\rho-1} \frac{\exp((\rho-1)g_z) - \zeta}{\exp((\rho-1)g_z)} \Phi\sigma\right)} \quad (43)$$

where g_z , $r = \frac{1-\beta}{\beta}$, Φ and ζ_g are all functions of parameters and independent of policies when subsidies to innovation are uniform ($\tau_g = \tau_e$), and $\zeta = G(0)$ indexes the social depreciation of innovations.

Note that using equation (17) with $\tau_s = \mu - 1$, since, $\frac{\Gamma_1^*}{L_p^*} = \frac{1}{\rho-1} \frac{\exp((\rho-1)g_z) - \zeta}{\exp((\rho-1)g_z)} \Phi\sigma$, we obtain expression(35) for the optimal innovation intensity of the economy in the text that we derived using a simple variational argument. As $r \rightarrow 0$, the optimal policies and allocation simplify to $\tau_g^* = \tau_e^* = 1 - (\mu - 1)(1 - \gamma)$, and $\frac{L_r^*}{L_p^*} = \frac{1}{(1-\alpha)(1-\gamma)}$.

We now prove the proposition.

Proof:

The result that the optimal q is common across products follows from the fact that q enters linearly into the matching function and the cost of innovation per product, $c(q)$, is strictly convex. Given a common q across all products, the planner's problem is

$$\max_{\{Z_{t+1}, L_{pt}, K_{t+1}, M_t, q_t\}} \sum_{t=0}^{\infty} \beta^t \log \left(A_{pt} Z_t K_t^\alpha L_{pt}^{1-\alpha} + (1 - d_k) K_t - K_{t+1} \right)$$

subject the two following constraints

$$\begin{aligned} -\mu_t \beta^t \left\{ \frac{Z_{t+1}}{Z_t} - \left[m (M_t + q_t) \left(\exp(\Delta)^{\rho-1} - \zeta \right) + \zeta \right]^{\frac{1}{\rho-1}} \right\} \\ -v_t \beta^t \left\{ f M_t + c(q_t) - A_{rt} Z_t^{\gamma-1} (1 - L_{pt}) \right\} \end{aligned}$$

Inspecting this maximization problem, it is clear that the optimal level of q is the solution to (16) under uniform innovation subsidies, $\tau_g^* = \tau_e^*$. The FOC w.r.t K_{t+1} is

$$\frac{1}{C_t} = \beta \frac{\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - d_k}{C_{t+1}}. \quad (44)$$

Given that the private equilibrium return to capital is $R_{kt} = \frac{\alpha}{\mu} (1 + \tau_s) \frac{Y_t}{K_t}$, it follows that $\tau_s^* = \mu - 1$. The F.O.C. w.r.t L_{pt} is

$$\frac{(1 - \alpha) Y_t}{C_t} \frac{Y_t}{L_{pt}} = v_t \frac{Y_{rt}}{1 - L_{pt}}. \quad (45)$$

The F.O.C. w.r.t Z_{t+1} is

$$\frac{1}{Z_t} \mu_t - \mu_{t+1} \beta \frac{Z_{t+2}}{Z_{t+1}^2} = \beta \frac{1}{C_{t+1}} \frac{Y_{t+1}}{Z_{t+1}} + \beta (\gamma - 1) v_{t+1} \frac{Y_{rt+1}}{Z_{t+1}} \quad (46)$$

The F.O.C. w.r.t M_t is

$$\frac{1}{\rho - 1} \mu_t \left(\frac{Z_{t+1}}{Z_t} \right) \frac{m'_t}{\left[m_t \left(\exp(\Delta)^{\rho-1} - \zeta \right) + \zeta \right]} \left(\exp(\Delta)^{\rho-1} - \zeta \right) = v_t f \quad (47)$$

where m' is the derivative of $m(\cdot)$. Combining (46) and (47),

$$\frac{Z_{t+1}}{Z_t} \mu_t - \mu_{t+1} \beta \frac{Z_{t+2}}{Z_{t+1}} = \beta \frac{Y_{t+1}}{C_{t+1}} + \beta \left(\frac{\gamma - 1}{\rho - 1} \right) \mu_{t+1} \left(\frac{Z_{t+2}}{Z_{t+1}} \right) \frac{m'_{t+1} \left(\exp(\Delta)^{\rho-1} - \zeta \right)}{\left[m_{t+1} \left(\exp(\Delta)^{\rho-1} - \zeta \right) + \zeta \right]} \frac{Y_{rt+1}}{f}$$

which, in a BGP, can be written as

$$\mu \exp(g_z) \left(1 - \beta + \beta \left(\frac{1 - \gamma}{\rho - 1} \right) \frac{m' \left(\exp(\Delta)^{\rho-1} - \zeta \right) \frac{Y_r}{f}}{\left[m \left(\exp(\Delta)^{\rho-1} - \zeta \right) + \zeta \right]} \right) = \beta \frac{Y}{C}$$

Combining (45) and (47) in the BGP and using $m = \sigma_0 (q + M)^\sigma$ we obtain expression (43). Finally, the innovation is set so that the socially optimal labor allocation in the BGP equals the equilibrium allocation of labor in the BGP given by

$$\frac{L_p}{1 - L_p} = \left(\frac{1 - \alpha}{\mu - 1} \right) \frac{\Pi Z^{\rho-1} / P_r}{Y_r} = (1 - \tau_g) \left(\frac{1 - \alpha}{\mu - 1} \right) \frac{\frac{r}{\delta} + \Phi}{\zeta_g \Phi}$$

where the first equality follows from (40) and the second is obtained using (39).

E Model extension: Occupation choice

Suppose that workers draw a productivity x to work in the research sector, where x is drawn from a Pareto with minimum 1 and slope coefficient $\eta > 1$. There are two wages, W_{pt} and W_{rt} . For the marginal agent,

$$\bar{x}_t W_{rt} = W_{pt}$$

Given that the minimum value of x is 1, any interior equilibrium with positive production requires $W_{rt} \leq W_{pt}$. The aggregate supplies of production and research labor are (having normalized the labor force to 1),

$$L_{pt} = F(\bar{x}_t) = 1 - \bar{x}_t^{-\eta}$$

$$L_{rt} = \int_{\bar{x}_t}^{\infty} x f(x) dx = \frac{\eta}{\eta - 1} \bar{x}_t^{1-\eta}.$$

The equilibrium allocation of labor is determined by

$$\frac{W_{pt} L_{pt}}{W_{rt} L_{rt}} = \frac{(1 - \alpha)(1 + \tau_s)}{\mu} \frac{1}{s_{rt}}$$

and

$$\frac{L_{pt}}{L_{rt}} = \frac{\eta - 1}{\eta} \frac{1 - \left(\frac{W_{pt}}{W_{rt}}\right)^{-\eta}}{\left(\frac{W_{pt}}{W_{rt}}\right)^{1-\eta}}$$

Note that as η goes to infinity, W_{pt}/W_{rt} must converge to 1 in order for L_{pt}/L_{rt} to be finite. It is also straightforward to derive the following expression for the elasticity of the aggregate allocation of labor with respect to the innovation intensity of the economy:

$$\Delta \log \frac{L_{pt}}{L_{rt}} = - \frac{(\eta - 1) \left(1 + \frac{W_r L_r}{W_p L_p}\right)}{(\eta - 1) \left(1 + \frac{W_r L_r}{W_p L_p}\right) + 1} \Delta \log s_{rt}.$$

When η converges to 1 (high worker heterogeneity), the elasticity of L_{pt}/L_{rt} converges to 0. When η converges to infinity (no worker heterogeneity), the elasticity of L_{pt}/L_{rt} converges to the one in our baseline model.

F Model Extension: Random Markups and Innovation Step Sizes

Here we show that we can alter the assumption that markups are constant across products in our baseline model to allow for variation in measured labor productivity across firms. We can

do so without substantially changing our main results. We can also alter the assumption that the innovation step size is constant without changing our main results. We do so as follows.

Suppose now that every time a product is innovated, a new markup is drawn from a distribution H_μ and a new innovation step size is drawn from a distribution H_Δ . The markup and step size draws are independent over time and do not depend on the productivity level z of the product that gets an innovation nor on the identity of the innovator. Note that if we were to follow [Klette and Kortum \(2004\)](#) and [Lentz and Mortensen \(2008\)](#) in having the markup or innovation step size depend on the identity of the firm, then our results do not fully generalize because there are first order gains from reallocating innovative effort across firms in equilibrium.

We now summarize the key changes that result from these assumptions. The expression (4) for aggregate output is replaced by

$$Y_t = \frac{[\int \mu^{1-\rho} dH(\mu)]^{\frac{\rho}{\rho-1}}}{\int \mu^{-\rho} dH(\mu)} Z_t K_t^\alpha L_{pt}^{1-\alpha}.$$

Given the independence between the markup and z , we can write the constant in aggregate profits as in expression (10), where now the constant κ_0 is defined as

$$\kappa_0 = \left[\alpha^\alpha (1-\alpha)^{1-\alpha} \right]^{\rho-1} \int \mu^{-\rho} (\mu-1) dH_\mu(\mu).$$

The shares of aggregate revenues, $(1 + \tau_s) Y_t$, accruing to variable profits, physical capital, and production labor are $1 - \int \mu^{-1} dH_\mu(\mu)$, $\alpha \int \mu^{-1} dH_\mu(\mu)$ and $(1 - \alpha) \int \mu^{-1} dH_\mu(\mu)$, respectively. Similarly, given the independence between the innovation step size and z , we can define an effective step size

$$\exp(\tilde{\Delta})^{\rho-1} = \int \exp(\Delta)^{\rho-1} dH_\Delta(\Delta)$$

which replaces $\exp(\Delta)^{\rho-1}$ in the law for motion of aggregate productivity (5), in the value function (12) and in the first-order condition for \tilde{q}_t , (12). Therefore, all incumbent firms engage in the same amount of innovative effort per product that they own, $\tilde{q}_t = q_t$. Given these changes, it is straightforward to show that the model's implications for aggregate transition dynamics that we discuss in the paper do not change.

Under this extension, the model generates persistent variation in measured labor productivity across firms and in research intensity (defined as innovation expenditures relative to revenues). In particular firms that have, on average, products with higher markups have higher measured labor productivity. Moreover, since all firms choose the same level of innovative effort per product, measured labor productivity and research intensity are not correlated with firm growth in terms of its number of products (as emphasized in [Klette and Kortum 2004](#)).

G Model Extension: Goods and Labor used as inputs in research

We consider an extension in which research production uses both labor and consumption good, as in the lab-equipment model of [Rivera-Batiz and Romer \(1991\)](#), and discuss the central changes to our analytic results. Specifically, the production of the research good is given by

$$Y_{rt} = A_{rt} Z_t^{\gamma-1} L_{rt}^\lambda X_t^{1-\lambda},$$

and the resource constraint of the final consumption good is

$$C_t + K_{t+1} - (1 - d_k) K_t + X_t = Y_t.$$

Given this production technology, the BGP growth rate of aggregate productivity g_z is given by $g_z = \frac{g_{A_r}}{\theta}$, where $\theta = 1 - \gamma - \frac{1-\lambda}{1-\alpha}$. The condition for semi-endogenous growth is $\theta > 0$. The knife-edge condition for endogenous growth is $\theta = 0$ and $g_{A_r} = 0$, which can hold even if $\gamma < 1$.

Revenues from the production of the research good are divided as follows

$$W_t L_{rt} = \lambda P_{rt} Y_{rt}, \text{ and } X_t = (1 - \lambda) P_{rt} Y_{rt}. \quad (48)$$

The allocation of labor between production and research is related to the innovation intensity of the economy by

$$\frac{L_{pt}}{L_{rt}} = \frac{(1 - \alpha) (1 + \tau_s)}{\lambda \mu} \frac{1}{s_{rt}} \frac{Y_t}{GDP_t'}, \quad (49)$$

where $GDP_t = Y_t - X_t = C_t + K_{t+1} - (1 - d_k) K_t$ when innovation expenditures are excluded in GDP. Factor payments are a constant shares of Y_t .

Our analytical results need to be modified for two reasons. First, the role that the term $1 - \gamma$ played in shaping the dynamics of the economy is now played by θ . Second, several of our analytical elasticities need to be modified by the ratio of GDP to Y , which is equal to $\frac{GDP_t}{Y_t} = (1 + (1 - \lambda)s_{rt})^{-1} \leq 1$. These two changes lead to the following two changes in our results.

The results in proposition (1) are now given by

$$\log \frac{s'_r}{1 + (1 - \lambda) s'_r} - \log \frac{\bar{s}_r}{1 + (1 - \lambda) \bar{s}_r} = \log(1 - \bar{\tau}_g) - \log(1 - \tau'_g)$$

and

$$s'_r \frac{GDP'}{Y'} - \bar{s}_r \frac{G\bar{D}P}{\bar{Y}} = \frac{E'}{GDP'} \frac{GDP'}{Y'} - \frac{\bar{E}}{G\bar{D}P} \frac{G\bar{D}P}{\bar{Y}}.$$

In Proposition 2, the elasticity of research output Y_r with respect to a change in the innovation

intensity of the economy s_r is now given by

$$\log Y'_{rt} - \log \bar{Y}_r = \bar{L}_p \frac{G\bar{D}P}{\bar{Y}} (\log s'_{rt} - \log \bar{s}_r) - \theta (\log Z'_t - \log \bar{Z}_t) - \frac{(1-\lambda)\alpha}{1-\alpha} (\log R'_{kt} - \log \bar{R}_k)$$

The third term in the right hand side reflects the change in research output Y_{rt} that result from changes in Y_t relative to K_t when $\lambda < 1$. Abstracting from movements in R_k we have

$$\Gamma_1 = \bar{L}_p \frac{G\bar{D}P}{\bar{Y}} \frac{\Phi\sigma}{\rho-1} \frac{\exp((\rho-1)\bar{g}_z) - 1}{\exp((\rho-1)\bar{g}_z)} \leq \bar{g}_z, \quad (50)$$

and

$$\Gamma_{k+1} = \left[1 - \theta \frac{\Gamma_1}{\bar{L}_p} \right] \Gamma_k, \quad (51)$$

The result in corollary 1 is now stated as

$$\log Z'_t - \log \bar{Z}_t = \frac{\bar{L}_p \frac{G\bar{D}P}{\bar{Y}}}{\theta} (\log s'_r - \log \bar{s}_r)$$

Finally, the result in corollary 2 is now adjusted to account for the change in GDP/Y ,

$$\log \frac{GDP_t}{Y_t} - \log \frac{G\bar{D}P}{\bar{Y}} = - \left(1 - \frac{GDP}{Y} \right) (\log s'_r - \log \bar{s}_r).$$

H Aggregate dynamics in expanding varieties models

We consider here versions of expanding varieties models with firm dynamics. These model differs from a Neo-Schumpeterian model in that here, each firm is identified with a distinct product for as long as it operates. The variety of products available to consumers grows when a new firm enters and shrinks when an existing firm dies. In the first version that we consider, only entering firms require an investment in the research good and incumbent firms grow according to an exogenously given process. This version of the model is a simplified version of one of the models considered in [Luttmer \(2007\)](#) and [Luttmer \(2012\)](#). In the second version of the model, both entering and incumbent firms invest units of the research good. This is a simplified version of the model considered in [Atkeson and Burstein \(2010\)](#).

Only entering firms invest At each point in time there is a continuum of firms. Each firm produces and sells monopolistically (at a constant markup) a single differentiated good. Firms (and their products) exit at the exogenous rate δ_f .²¹ The production function is given by (3) and the corresponding aggregate production function is given by (4), with aggregate productivity $Z_t =$

²¹For simplicity, we abstract from fixed costs of production of the form considered in [Melitz \(2003\)](#) that generate endogenous exit and variation in the innovation rate across incumbent firms that are close to the exit threshold.

$[\int_z \exp(z)^{\rho-1} dJ_t(z)]^{\frac{1}{\rho-1}}$ where the measure of firms $\int_z dJ_t(z)$ can expand or contract depending on the entry and exit of firms rather than being fixed at 1. Conditional on survival, an incumbent firm's productivity z increases by a step size Δ_z with a fixed probability \bar{q} and decreases by $-\Delta_z$ with probability $1 - \bar{q}$. Creating a new firm in period t requires an investment of f units of the research good. That firm starts production of a new differentiated good in period $t + 1$ with productivity \bar{z}_{t+1} , which grows exogenously at a rate $g_z = \bar{z}_{t+1} - \bar{z}_t$. Let M_t denote the measure of entering firms at time t . The research good is produced with labor according to $Y_{rt} = A_{rt} Z_t^\gamma L_{rt}$ and the resource constraint for the research good is $fM_t = Y_{rt}$. It is convenient to define $\tilde{Y}_{rt} = Y_{rt} \left(\frac{\exp(\bar{z}_t)}{Z_t}\right)^{\rho-1}$, which in a BGP with firm entry is constant at $\tilde{Y}_{rt} = \tilde{Y}_r$.

Under these assumptions, we can write the law of motion of aggregate productivity as

$$\left(\frac{Z_{t+1}}{Z_t}\right) = H(\tilde{Y}_{rt}) = \left[H(0)^{\rho-1} + \frac{\tilde{Y}_{rt}}{f} \exp(g_z)^{\rho-1} \right]^{\frac{1}{\rho-1}},$$

where $H(0)^{\rho-1} = (1 - \delta_f) [\bar{q} \exp(\Delta_z)^{\rho-1} + (1 - \bar{q}) \exp(-\Delta_z)^{\rho-1}]$. In a BGP with positive entry, the growth rate of aggregate productivity is $\bar{g}_z = \frac{g_{A_r} + (\rho-1)g_z}{\theta}$, where $\theta = \rho - 1 - \gamma \geq 0$.

We now compute and bound the elasticity of aggregate productivity growth with respect to the innovation intensity of the economy. We do so in two steps. In the first, we compute the elasticity of aggregate productivity growth with respect to normalized research output \tilde{Y}_{rt} . In the second step, we compute the elasticity of normalized research output \tilde{Y}_{rt} with respect to the innovation intensity of the economy. By direct calculation, the first elasticity is

$$\frac{H'(\tilde{Y}_r)}{H(\tilde{Y}_r)} \tilde{Y}_r \Big|_{\tilde{Y}_r = \tilde{Y}_r} = \frac{1}{\rho-1} \left(\frac{H(\tilde{Y}_r)^{\rho-1} - H(0)^{\rho-1}}{H(\tilde{Y}_r)^{\rho-1}} \right) \leq \log H(\tilde{Y}_r) - \log H(0).$$

Note first that, as before, $H(\tilde{Y}_r)$ is calibrated to equal the exponent of the BGP growth rate of aggregate productivity, $\exp(\bar{g}_z)$. The term $H(0)$ is no longer equal to one as in our baseline Neo-Schumpeterian model, but instead it is determined by the exit rate of incumbent firms together with the growth rate of those incumbents that survive. One can show that in this model, to ensure a stationary firm size distribution with positive firm entry, it is necessary to have $H(0) < 1$. Moreover, in this model the term that determines the elasticity above, $\frac{H(\tilde{Y}_r)^{\rho-1} - H(0)^{\rho-1}}{H(\tilde{Y}_r)^{\rho-1}}$, is equal to the share of employment by entering firms in the BGP.

To compute the elasticity of normalized research output with respect to the innovation intensity of the economy, we use the fact that $\tilde{Y}_{rt} = A_{rt} \exp(\bar{z}_t)^{\rho-1} Z_t^{-\theta} L_{rt}$ and that equation (17) in our baseline model still applies.

Using these results, we can derive the analog of equation (28) in Proposition 2. In an economy that is initially on a BGP with positive firm-entry, an unanticipated, permanent change in innovation policies (i.e. subsidy to firm entry) that induces a new path for the innovation intensity of the economy given by $\{s'_{rt}\}_{t=0}^\infty$, the new path for aggregate productivity $\{Z'_t\}_{t=1}^\infty$ to a first-order

approximation is given by

$$\log \left(\frac{Z'_{t+1}}{Z'_t} \right) - \bar{g}_z = \left(\frac{\exp(\bar{g}_z)^{\rho-1} - H(0)^{\rho-1}}{\exp(\bar{g}_z)^{\rho-1}} \right) \frac{1}{\rho-1} [\bar{L}_p (\log s'_{rt} - \log \bar{s}_r) - \theta (\log Z'_t - \log \bar{Z}_t)]. \quad (52)$$

Both entering firms and incumbent firms invest We now extend the previous model and assume that incumbent firms can invest resources to increase the productivity of the product they own. A firm owning a product with productivity z that invests $\exp\left(\frac{z}{\bar{z}_t}\right)^{\rho-1} c(q - q_l)$ units of the research good in the current period t has a probability $q \geq q_l$ that its productivity z increases by a step size Δ_z and a probability $1 - q$ that productivity decreases by Δ_z , where $c(\cdot)$ is increasing and convex with $c(0) = 0$. The assumption that the cost of innovation falls with the productivity of entrants ensures that incumbent firms innovate in the long run.

Under these assumptions it is straightforward to show that all continuing firms choose the same level of q , independent of their productivity z (given our assumption that the cost scales up with $\exp(z)^{\rho-1}$). In an equilibrium with positive entry, q_t is determined from the following condition:

$$(1 - \tau_g) c'(\tilde{q}_t) = \frac{(1 - \tau_e) f(1 - \delta_f)}{\exp(\bar{g}_z)^{\rho-1}} [\exp(\Delta_z)^{\rho-1} - \exp(-\Delta_z)^{\rho-1}]. \quad (53)$$

where $\tilde{q}_t = q_t - q_l$. The law of motion for aggregate productivity is

$$\left(\frac{Z_{t+1}}{Z_t} \right) = H(\tilde{M}_t, \tilde{q}_t) = \left[H(0,0)^{\rho-1} + \exp(\bar{g}_z)^{\rho-1} \tilde{M}_t + (1 - \delta_f) \tilde{q}_t [\exp(\Delta_z)^{\rho-1} - \exp(-\Delta_z)^{\rho-1}] \right]^{\frac{1}{\rho-1}} \quad (54)$$

where $\tilde{M}_t = \left(\frac{\exp(\bar{z}_t)}{Z_t}\right)^{\rho-1} M_t$, and $H(0,0)^{\rho-1} = (1 - \delta_f) [q_l \exp(\Delta_z)^{\rho-1} + (1 - q_l) \exp(-\Delta_z)^{\rho-1}]$ is the growth rate of aggregate productivity when $M_t = 0$ and $q_t = q_l$ (that is, when aggregate innovation expenditure is zero). Note that this transition law for aggregate productivity is a function of both M_t and q_t and not simply a function of output of the research good Y_{rt} as in the previous model. As we show below, we will be able to derive a meaningful elasticity of aggregate productivity with respect to the innovation intensity of the economy starting from an initial BGP with uniform innovation policies as we did in our baseline Klette-Kortum model. The resource constraint for the research good is

$$\tilde{Y}_{rt} = A_{rt} \exp(\bar{z}_t)^{\rho-1} Z_t^{-\theta} L_{rt} = \tilde{M}_t f + c(\tilde{q}_t). \quad (55)$$

We now compute the elasticity of aggregate productivity growth with respect to normalized research output starting in an initial equilibrium with positive firm entry and uniform innovation

subsidies (i.e. $\tau_g = \tau_e$). We first have

$$\frac{dH(\tilde{M}, \tilde{q})}{d\tilde{Y}_r} = \left[\frac{\partial H(\tilde{M}, \tilde{q})}{\partial \tilde{M}} \frac{d\tilde{M}}{d\tilde{Y}_r} + \frac{\partial H(\tilde{M}, \tilde{q})}{\partial \tilde{q}} \frac{d\tilde{q}}{d\tilde{Y}_r} \right] = \frac{1}{\rho - 1} \frac{\exp(\bar{g}_z)^{\rho-1}}{fH(\tilde{M}, \tilde{q})^{\rho-2}}$$

where we used equations (53) and (55), which implies

$$\frac{\frac{dH(\tilde{M}, \tilde{q})}{H(\tilde{M}, \tilde{q})}}{\frac{d\tilde{Y}_r}{\tilde{Y}_r}} = \frac{1}{\rho - 1} \frac{\tilde{Y}_r \exp(\bar{g}_z)^{\rho-1}}{fH(\tilde{M}, \tilde{q})^{\rho-1}}.$$

Using equations (54), (53) and (55) in a BGP with uniform innovation policies and positive firm entry, we can re-write this elasticity as

$$\frac{\frac{dH(\tilde{M}, \tilde{q})}{H(\tilde{M}, \tilde{q})}}{\frac{d\tilde{Y}_r}{\tilde{Y}_r}} = \frac{\Phi}{\rho - 1} \frac{H(\tilde{M}, \tilde{q})^{\rho-1} - H(0, 0)^{\rho-1}}{H(\tilde{M}, \tilde{q})^{\rho-1}},$$

with

$$\Phi = \frac{\tilde{Y}_r}{\tilde{Y}_r + c'(\tilde{q})\tilde{q} - c(\tilde{q})} < 1$$

The last inequality follows from the convexity of $c(\cdot)$ together with the assumption that $c(0) = 0$. Note that in our baseline Klette-Kortum, Φ can also be written in this way.

We can obtain the elasticity of normalized research output with respect to the innovation intensity of the economy following the same steps as above. We obtain dynamics for aggregate productivity in response to policy-induced (uniform or non-uniform) changes in the innovation intensity of the economy given by

$$\log\left(\frac{Z'_{t+1}}{Z'_t}\right) - \bar{g}_z = \left(\frac{\exp(\bar{g}_z)^{\rho-1} - H(0, 0)^{\rho-1}}{\exp(\bar{g}_z)^{\rho-1}}\right) \frac{\Phi}{\rho - 1} [\bar{L}_p (\log s'_{rt} - \log \bar{s}_r) - \theta (\log Z'_t - \log \bar{Z}_t)].$$

In this model, assigning a value to $H(0, 0)$ would require knowing what the exit rate is and what the growth rate of productivity for incumbent firms is if these firms set their innovation spending to zero.