Banks, Liquidity Management and Monetary Policy*

Javier Bianchi  
University of Wisconsin and NBER  
Saki Bigio  
Columbia University

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Abstract

We develop a new framework for studying the implementation of monetary policy through the banking sector. Banks are subject to a maturity mismatch problem leading to precautionary holdings of reserves. Through various instruments, monetary policy alters tradeoffs banks face between lending, holding reserves, holding deposits and paying dividends. This translates into the real economy via effects on real interests and lending. We study how these instruments interact with shocks to the volatility in the payments system, bank losses, the demand for loans and with capital requirements. We use a calibrated version of the model to answer, quantitatively, why have banks held onto a substantial increase in reserves while not increasing lending since 2008.

Keywords: Banks, Monetary Policy, Liquidity.

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1 Introduction

The conduct of monetary policy around the world is changing. The last five years have witnessed banking systems experiencing unprecedented losses together with frozen interbank markets. Banks cut back on lending as these events materialized. In an attempt to preserve financial stability and reinvigorate lending, the central banks of the US and Europe have reduced policy rates to almost zero and continuously purchased private loans issued from banks. In response to these unprecedented policy interventions, banks seem to have mostly accumulated central bank reserves without renewing lending as intended by policy.\(^1\) Why and what can central banks do about this remains an open question.

Unsurprisingly, the role of banks in the transmission of monetary policy has been at the centerpiece of policy debates. Unfortunately, there are not many modern macroeconomic models that enable the study of monetary policies implemented through the liquidity management of banks as occurs in practice. In this paper, we present a model that fills this gap. By doing so, we can let our model answer why is it that banks are not lending while parking vast amounts of reserves in their balances. We propose a new theoretical framework that focuses on the institutional details of banking for the implementation of monetary policy. This model enables us to answer a number of theoretical issues. For example, how does the transmission of monetary policy depend on the decisions of commercial banks? What type of shocks can induce banks to hold more reserves and lend less. How will the strength of monetary be affected by this shocks? What is the connection between monetary policy and regulatory capital requirements?\(^2\)

We use the lessons derived from our model to quantitatively investigate why banks are not lending despite all aforementioned policy efforts. Our model is able to validate different hypotheses that are informally discussed in policy and academic circles. Through the lens of the model, we evaluate the plausibility of each of the following:

**Hypothesis 1 - Bank Equity Losses:** The lack of lending responds to optimal behavior by banks given the increase in bank leverage which followed from the large equity losses in 2008.

**Hypothesis 2 - Interbank Uncertainty:** Banks hold more reserves relative to their lending because there is substantial uncertainty about potential costs of accessing the interbank market.

**Hypothesis 3 - Capital Requirements:** The effective and/or expected path of capital requirements are leading banks to hold more reserves and lend less. Central banks are constrained by other regulatory constraints.

**Hypothesis 4 - Weak Demand:** Banks face a weaker effective demand for loans. The lack of demand is caused by the lack of borrowers that meet credit standards or borrowers do not want to borrow.

\(^1\)As is well known, the Bank of Japan had been facing similar issues since the early nineties.

\(^2\)We refer to regulation such as the one put in place through the Dodd-Frank act or the Basel-III committee on bank supervision.
We calibrate our model and fit shocks we can identify and associate them with each hypothesis. We use the properties of the model uncover what shocks can explain why lending is weak while the volume of excess reserves has increased by a factor of 16. Our model suggests that a combination of an early uncertainty about interbank payments and a consequent contraction in loan demand is the most empirically plausible story.

**The Mechanism.** The building block of our model is a liquidity (reserve) management problem. This problem constitutes finding the optimal mix between lending, deposit issuances and holdings of central bank reserves. Banks tradeoff the profit on a loan against potential liquidity risks. Liquidity risks are associated with financial losses that follow when deposits are withdrawn from one bank that lacks sufficient reserves to meet its settlement payment needs. Bank lending reacts to monetary policy because policy instruments alter the tradeoffs in this problem.

Liquidity management is recognized as one of the fundamental problems in banking.\(^3\) When a bank grants a loan, it must create or obtain a liability in the form of a credit line or a demand deposit. Granting a loan is profitable because a higher interest is charged on the loan than what is paid on deposits. However, the trade-off is that more lending relative to a given amount of reserves also increases liquidity risks: when deposits are transferred to another bank, the issuing bank must transfer some asset to settle the transaction. We assume, as occurs in practice, that loans cannot be sold immediately due to various frictions. Hence, the use of central bank reserves is critical to clear settlements after withdrawals. This friction imply that with the increase in deposits that follows from additional lending comes additional liquidity risk. The lower the liquidity ratio of a bank, its deposits-to-reserve ratio, the more likely is a bank to be short of reserves. Banks short of reserves, incur financial losses as they must incur in expensive borrowing from the central bank or other competitors.

We introduce this liquidity management problem into a tractable dynamic general equilibrium model with rational profit-maximizing heterogeneous banks. Bank liquidity management is captured through a portfolio problem with non-linear returns. We show how different instruments operate by altering the incentives banks face to grant loans. Short-run monetary policy effects result from the ability that central banks have to supply reserves or alter market rates. Long-run monetary-policy effects are also present because bank equity returns and the size of the financial sector evolves in response to these policies. An important feature of this channel is that monetary policy will have real effects even when prices are fully flexible. In fact, we show how monetary policy acts like a tax on financial intermediation. Thus, although the model is real, it has effects through the lending channel.

**Implementing Monetary Policy.** The implementation of monetary policy in our model is very rich. In particular, the FED is equipped with the following tools: discount rates, interests on reserves, open-market operations (conventional and unconventional), liquidity facilities and

\(^3\)See Saunders and Cornett (2010).
reserve requirements. All these instruments have a common effect: they can potentially tilt the balance towards more or less lending. Their macroeconomic effects result from changes in lending volumes and interest rates. However, as much as monetary policy can stimulate lending, its power may be altered by exogenous shocks associated with hypotheses 1 through 4. The richness in this set of tools allows us to provide a close description to the FED’s policies during the last five years once we test these hypotheses.

**Testable Implications.** The model delivers a rich set of descriptions for banking and monetary indicators which can be used to validate each hypothesis. For individual banks, it explains the behavior of reserve holdings, lending policies, leverage and dividends. For the banking industry as a whole, it provides descriptions for aggregate lending, interbank lending and excess reserves. It also describes interbank borrowing and lending rates. The model also provides a description for financial indicators for banks such as return on loans, return on equity, banking dividend ratios as well as book and market values for banks. It also has predictions about the size of the financial sector relative to the rest of the economy. At the macroeconomic level, it provides a prediction about the evolution of monetary aggregates, M0, M1 and the money multiplier.

We use this rich set of descriptions to explain the dynamic effects of aggregate outcomes to changes in different monetary policy instruments and financial regulation. Thus, we can use the model to explain the pass-through of policy interest rates and FED balance-sheet operations on these observables. We can also study the effects of exogenous shocks to the volatility of bank withdrawals, losses on equity and shocks that affect the demand for loans.

**Monetary Policy 2008-2013.** The quantitative analysis section tries to explain which of our hypotheses fits best the patterns we have seen in the US data since the 2008-2009 financial crisis. For this, we calibrate our model using our set of observables that include deposit growth rates, the behavior of bank equity, the balance sheet of the FED and its policy rates. We feed the model with sequences of policy variables and observable shocks. We argue that hypothesis 4, that is, a weak demand for loans, has the most likely shock to have been in place and we discuss the implications of this finding. We discuss how we interpret this shock and how this result suggests a potential feedback loan from less lending today to weaker demand for loans tomorrow.

**Organization.** The paper is organized as follows. The following section explains the liquidity management problem through the analysis of a bank’s balance sheet. We then discusses how our model fits with the literature. Section 2, presents a partial equilibrium model of banks that takes a demand for loans as given. Section 3 presents the calibration and empirical analysis. We study the steady state and policy functions in sections 4 and 5. We use this environment to study the effects of deterministic shock paths in section 6. We use the environment to answer questions about monetary policy in the context of the US financial crisis in section 7. A final section of the paper describes how one can extend this model to incorporate richer features very easily.
1.1 Literature Review

There is a tradition in economics that dates at least Bagehot (1999) which stresses the importance of analyzing monetary policy together with financial institutions. A classic attempt to study policy in a model with a full description of households, firms and banks is Gurley and Shaw (1964). With few exceptions, modeling banks was a practice abandoned by macroeconomics for many years. Until the Great Recession, questions about the macroeconomic effects of monetary policy and how this policy is implemented through banks were treated independently.\(^4\)

In the aftermath of the crisis, there have been numerous calls for writing models with an explicit role for banks. The goal is to improve our understanding of the alternative implementations of monetary policy we have seen in recent years.\(^5\) The profession has responded quickly so will undoubtedly do an unfair job surveying the relevant literature. To the best of our knowledge, these new first steps were taken by Gertler and Karadi (2009) and Curdia and Woodford (2009). Those papers study the effects of open market operations in environments where intermediaries face borrowing constraints. These papers provide new insights about the balance-sheet effects of asset purchase programs. These papers model banks without the focus on institutional details that we have here.\(^6\)

In this paper, we focus on very different issues. Banks do not face constraints that follow from agency frictions. Instead, banks issue liabilities deposits which are transferred and become the liabilities of other banks. Their only constraint are given by capital requirements.\(^7\) The importance of this distinction is that here, possible interruptions of monetary policy operate through the interbank market. In the aforementioned paper, FED open-market operations operate by revaluating the balance sheet of firms, here they operate by providing liquidity.

In our model, the random transfer of deposits across banks leads to a potential maturity mismatch problem. The risk of running out of reserves is related to classic illiquidity problems in the classic models of bank runs.\(^8\) In our model, maturity mismatches introduce a value for reserves, but banks are never insolvent. Instead, this mismatch is capturing ideas from the payment systems literature in that reserves exists to settle transactions across banks.\(^9\) Early classic papers that studied static liquidity (reserve) management problem of individual banks are Poole (1968) and

\(^4\)This simplification seemed natural. In the US, banking didn’t seem to matter for macroeconomic performance. For example, The banking industry was among the most stable industries in terms of solvency. More importantly, the pass-through from key policy rates and to lending terms seemed straight.

\(^5\)See for example Woodford (2010) and Mishkin (2011).

\(^6\)Bianchi (2012) studies the optimality of such bailout programs.

\(^7\)This process provides banks an essential role as providers of assets with liquidity services. See Williamson and Wright (2010).

\(^8\)See for example, Diamond and Dybvig (1983), Allen and Gale (1998) or Holmstrom and Tirole (1998). Gertler and Kiyotaki (2013) is a recent model that incorporates bank runs into DSGE models. Illiquidity risks are similar to the frictions found in Gertler and Kiyotaki (2012). In that paper, illiquidity risks follow from roll-over risk and this has effects on for the solvency of a bank. This risk determines the banks lending policy

\(^9\)See in Freeman (1996) and a sequence of related papers which focus on the details of the payment system.
Our model will capture very similar forces as the one described in those papers. The innovation here is adding dynamics and general equilibrium. A theoretical model where reserves emerge as an essential tool for credit creation is Cavalcanti and Andres Erosa (1999). Our paper takes the institutional details of the payments system as given. This simplifies our model and allows us to say more about dynamic effects of shocks the different shocks we study.

The closest modern dynamic macro model to ours is, Brunnermeier and Sannikov (2012). This paper also introduces inside money created by banks and outside money. Outside money plays the same role as inside money as a tool to materialize investment opportunities. We share the spirit of having money as an asset that allows transactions. We differ in that outside money here are central bank reserves which are not used for commercial transactions and play a role as an instrument to hedge illiquidity risks. This maturity mismatch problem explains why monetary policy affects bank lending. Another pair of closely related papers is Williamson (2012) and Rocheteau and Rodriguez-Lopez (2013). Williamson (2012) studies an environment where different assets, among which bank loans stand out, have different properties as mediums of payments. Rocheteau and Rodriguez-Lopez (2013) has a spillover from liquidity needs in a OTC market to the labor market where firms are issuing loans to hire workers. Like us, they use these frameworks to study the liquidity effects of different monetary policy tools. Along that dimension, our model is also related to Stein (2012) and Stein et al. (2013) who study environments where there is an exogenous demand for safe short-term liquid assets with implications for policy. A common feature in all of these papers is that the classic Modigliani-Miller theorem for open-market operations (see Wallace (1981)) is broken.

A model of particular importance for us is Afonso and Lagos (2012). That paper studies the market for FED funds in a matching model. The goal of that paper is to explain how matching frictions in the FED funds market affects the potential costs of being short of reserves. We view our model as a counterpart of theirs in that we study what occurs during the rest of the day taking as given the outcomes in the FED funds market.

Some recent empirical papers documenting liquidity effects following open market operations include Krishnamurthy and Vissing-Jørgensen (2011, 2012). Our model also relates to the very known study of Kashyap and Stein (2000) which documents the effects of monetary policy via the lending channel. We contend that we provide a formalism to many of the arguments in that paper.

Finally, one should note that the approach here is dramatically contrasts with the cashless

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10. There are many textbooks for banking practitioners that deal with this subject. See for example, Saunders and Cornett (2010) for a managerial perspective and Duttweiler (2009) for an operations research perspective.

11. See Kahn and Roberds (2009) for a survey on theoretical and empirical literature on payments. Williamson and Wright (2010) survey theoretical papers within the money search context. Berentsen et al. (2007) introduce credit as competing means of payment to currency also in a money search environment.

12. Another distinction is that it has additional predictions about cross-sectional industry growth for banks. Along that dimension, a model related to this is one is Corbae and D’Erasmo (2013a,b) who also study the industry dynamics of the banking industry.
feature of canonical new-Keynesian models Woodford (1998). Those models require sticky prices in order to induce monetary policy effect. Monetary policy in those models has power because the FED controls nominal interest rates. Our model has real effects despite being real because the FED can alter the incentives to create real balances of inside money. Here, monetary creation by banks is essential. We not only believe our model complements those models for monetary policy analysis. Also, even if one believes that the lending channel is very week, because non-intermediated finance is a close substitute, our model may be useful to identify the shocks may have played a role during the crisis.

2 The Model

The description of our model begins with a partial equilibrium dynamic model of banks. The focus of the section is explaining bank decisions as functions of policy variables. In particular we want to understand the supply of loans as functions of prices and aggregate shocks. We then close the model introducing a real sector which will have a demand for loans.\footnote{We choose this organization of our model because there are many ways of closing the model and we don’t need to impose a particular structure.}

2.1 Environment

Time is discrete and indexed by $t$. There is an infinite horizon. Each period is divided into two stages: a lending stage (l) and a balancing stage (b). The numeraire are dollar deposits. The economy is populated by a continuum of heterogenous banks whose identity is denoted by $z$. Banks face an exogenous demand for loans (for now) and a vector of shocks that we describe later. There is an exogenous deterministic monetary policy determined by the monetary authority which we refer to as the FED.

2.2 Banks

The goal of banks is to maximize dividend payment streams $\{DIV_t\}_{t \geq 0}$. The bank’s preferences over dividend streams are evaluated via an expected utility criterion:

$$
\mathbb{E} \left[ \sum_{t \geq 0} \beta^t U (DIV_t) \right]
$$

where $U (x) \equiv \frac{x^{1-\gamma}}{1-\gamma}$ and $DIV_t$ is the banker’s consumption at date $t$. Banks hold a portfolio of loans, $B_t$, and central bank reserves, $C_t$, as part of their assets and demand deposits, $D_t$, as their liabilities. These are the individual state variables of a bank. We describe some properties of these assets which are very important.
Loans. Loans are a promise to repay the bank \( I_t (1 - \delta) \delta^n \) in period \( t + 1 + n \) for all \( n \geq 0 \), in units of the numeraire.\(^{14}\) Hence, loans constitute long-run assets which are a promise to a geometrically decaying stream of payments as in the Leland model (see Leland and Toft (1996)). We denote by \( B_t \) the total coupon payments at time \( t \), owed to the bank. Using the same sequence of payments, total coupon payments received at any point in time are:

\[
B_t = (1 - \delta)I_{t-1} + (1 - \delta)\delta I_{t-2} + (1 - \delta)\delta^2 I_{t-3} \ldots
\]

This state variable satisfies the following law of motion, \( B_{t+1} = \delta B_t + I_t \), which is a useful recursive representation for the value of all future coupon payments.\(^{15}\) Banks grant new loans \( I_t \) taking the market price \( q_t \) as given.\(^{16}\) When giving out a loan, banks give the borrower demand deposits which amount to \( q^d_t I_t \). Deposits are used by borrowers as means of payments.\(^{17}\) The rest of the loan, the amount \( (1 - q^d_t) I_t \), is a bank’s immediate profits from intermediation which can be used to pay dividends.

An important assumption is that bank loans are illiquid during the balancing stage but liquid during the lending stage. What we mean by this is that loans can be sold to other banks only during the lending stage. The underlying assumption is that banks specialize in their loans perhaps because they have particular expertise on their borrowers, or specialize in certain industries. Loans can be also illiquid due to adverse selection.\(^{18}\) For any of these reasons, banks would need to spend some time to analyze a loan before buying it. This is captured by the potential illiquidity during the lending stage. This assumption can be easily relaxed by allowing sales at a discount at the expense of requiring one additional state variable.\(^{19}\) When loans mature, a bank is payed with deposits. New loans are granted during the lending stage.

Demand Deposits. Behind the scenes, banks have an implicit technology that is enabling transactions between third parties. We have in mind that when granting loans, borrowers receive credit lines which enables them to purchase goods. Deposits are only created when banks provide

\(^{14}\)Payments could begin the period of issuance without, loss of generality. This would make the model collapse to a framework with within-the-period loans. The total coupon payments that a bank will have received by time \( t+T \) from a loan made at \( t \) are \( P_{t+1+T} = (1 - \delta) I_t + (1 - \delta) I_t \delta + (1 - \delta) I_t \delta^2 + \ldots + (1 - \delta) I_t \delta^T \): so \( \lim_{T \to \infty} P_{t+1+T} = I_t \).

\(^{15}\)To see this compute \( B_{t+1} \) as function of \( I_t, I_{t-1} \ldots \) and subtract \( \delta B_t \).

\(^{16}\)This can be easily generalized to allow for some degree of market power. We use the price of the loan for convenience but one can easily go back and forth from prices to interest rates.

\(^{17}\)We explain this explicitly in the appendix.

\(^{18}\)Allowing loans to be sold only during the lending stage is done for convenience. By preventing loans from being sold during the balancing stage, we introduce a form of illiquidity that is essential to having loans being illiquid. Allowing loans to be transferable during the lending stage, is useful to reduce the state space of the model. In particular, we will show that it won’t be necessary to keep track of the composition but only the size of the balance sheet thanks to this assumption.

\(^{19}\)There are many reasons why loans may be illiquid. For example, Bolton and Freixas (2009) introduce a differentiated role for different bank liabilities following from asymmetric information. Diamond (1984) and Williamson (1987) introduce specialized monitoring technologies. In Holmstrom and Tirole (1997) moral hazard requires bankers to have a stake on the loans they make.
loans.\textsuperscript{20} This means that banks create an asset (a liability for their borrower) and issuing a liability (an asset for a third party). The borrower may use deposits to purchase goods. The holder of these deposits can, in turn, transfer the funds to others accounts, make payments and so on. Implicitly, bank deposits are playing a role as a medium of exchange. Simultaneously, banks are liable to the holder of those deposits. Demand deposits are the bank’s only form of liability. Deposits are reduced as borrowers make payments to banks as their loans mature.

During the balancing stage, banks face a random deposit-withdrawal shock $w_t$. Withdrawal shocks occur during the lending stage. The process for the stochastic withdrawals satisfies $w_t = \omega_t D_t$, where $\omega \sim F_t (\cdot)$ with support in $(-\infty, 1])$. $F_t$ is an exogenous time-varying distribution for withdrawals. When $w_t$ is positive (negative), the bank looses (receives) deposits for that amount. We think of $w_t$ as capturing the complexity of transactions in the payments system which ultimately lead to randomness in the payments system. This stochastic process seems a natural process since deposits are being constantly used for transactions. A key assumption is that $F_t$ is common across all banks regardless of their size. This makes our model tractable.

For the applications in this paper, we assume that deposits do not leave the banking sector:

**Assumption 1** (Deposit Conservation). Deposits remain within banks: $\int_0^\infty \omega_t F_t (d\omega) = 0$, $\forall t$.

Under the assumption above, we will see, that it is equivalent to saying that there are no withdrawals of reserves from the banking system or otherwise that there are no runs on the system.\textsuperscript{21} Since at the balancing stage, we treat bank loans as perfectly illiquid, when a deposit is transferred from one bank to another, the bank receiving the deposits will request exchange reserves to clear out the deposit transaction.

**Reserves.** Reserves are special assets in that they are always liquid. Banks use reserves to finance deposit transfers. If $\omega_t$ is large, reserves may be insufficient to settle the outflow of deposits. The point at which banks are in need of borrowing is not necessarily zero reserves. In particular, banks may face a reserve deficit if reserves fall short of reserve requirement which is determined by a policy parameter $\rho \in [0, 1]$. We assume that the relevant balance of reserves is the one computed by the end of the balancing stage. Thus, if $\rho D_t \geq C_t$ during the balancing stage, bank’s face a financial cost.

This financial cost deserves some discussion. In practice, when their reserve position falls short (in excess) of meeting the reserves requirements, banks head to the discount (borrowing) window or the interbank market to borrow (lend) the deficit (surplus). These leads to potential financial costs for firms in either side of the market. We capture these cost or benefits of being in either side is determined by the function $\chi$.

\textsuperscript{20}There is an alternative interpretation. Banks can raise deposits and then lend this funds. This will also be a possibility in our model. A bank receiving an inflow of deposits will receive also reserves from other banks to settle the transaction. The banker can then sell reserves and purchase or make new loans. In the aggregate, loan creation is deposit creation.

\textsuperscript{21}We can extend the model easily assuming that the private sector can withdraw deposits in the form of currency. This can be easily incorporated disposing the assumption that deposits do not leave the banking sector.
For the theoretical analysis, it suffices to let $\chi$ be a homogeneous function the reserve deficit. In the quantitative analysis, we use the following functional form for $\chi$: \[\chi_t(x) = \begin{cases} \chi_t x & \text{if } x \leq 0 \\ \frac{\chi_t}{x} & \text{if } x > 0 \end{cases}\]

with the restriction that $\underline{\chi} < \overline{\chi}$. We treat $\chi$ as a policy parameter which behind the scenes captures the dynamics of the FED funds market which, responds to the discount window and the overnight fed-funds market rate chosen by the FED.

In practice, central banks set up lending and borrowing rates overnight in what is called a corridor system. Our interpretation of $\{\chi, \overline{\chi}\}$ is that is an average of repeated interactions in the FED funds market. The values for $\{\chi_t, \overline{\chi}_t\}$ represent the average cost of ending with positive or negative balances considering that banks can borrow from the interbank market with a certain probability or otherwise they must lend or borrow from the FED (see Afonso and Lagos (2012)).

Thus, we treat $\{\chi_t, \overline{\chi}_t\}$ as policy parameters. A natural question we left out of the analysis is why do central banks introduce a distortion in the interbank market. Presumably this has to do with other market imperfections left outside of the analysis.

During the lending stage, banks can trade reserves for deposits between each other at a market price $(1+r_t)$. That is, a bank may choose to acquire $\varphi_t$ in reserves from the market in exchange for taking over some deposits from other banks in amount $(1+r_t)\varphi_t$. Although we assume reserves are bought and sold in exchange for deposits, we have in mind that this market captures a long-term interbank market which is why we associate $r_t$ to long-term LIBOR rates although here its just the spread between the price of deposits and reserves.

**Bank Equity.** The book value of bank equity is the sum of assets minus liabilities: $N_t = B_t + C_t - D_t$. Book value equity evolves according to the realization of bank profits that follow from lending to customers and borrowing from the interbank market. The market value of equity is defined as $E_t = qB_t + C_t(1 + r_t) - D_t$ which will evolve depending on the movements in prices.

Finally, profits are realized during the lending stage as well as the payment of dividends, $DIV_t$. Dividends are paid by issuing deposits to share holders.

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22 Our frameworks allows for a more general specification where $\chi$ is a function of $(\tilde{C}, \tilde{D})$ which is homogenous of degree one. This could capture different constraints allowing for more curvature in the penalty costs. From the banks’ point of view, this is isomorphic to changing the distribution of withdrawal shocks.

23 Afonso and Lagos (2012) provide a formal model for the overnight interbank market. The outcome of their analysis is that banks ending a day with positive balances lend out reserves to banks with negative balances with a certain probability. The balance that is not lend earns interest on deposits at the FED and the fraction that cannot be borrowed pays interests. We could extend our model by allowing the opening of this market with potentially interesting interactions.
2.3 Timing of Events and Laws of Motion

Notation. We use $Z_t$ to denote the value of variable $Z$ during the lending stage and $\tilde{Z}_t$ to denote its value at the beginning of the balancing stage of the same period.

Lending Stage: Banks enter the period during the lending stage with currency $C_t$, a portfolio of loans $B_t$, and a deposits, $D_t$ as their individual states. From the point of view of the bank, the aggregate state includes monetary policy variables, real economic activity observables, and an exogenous demand for loans. This aggregate states are summarized in the vector $X_t$. During the lending stage, banks decide the amount of new loans they want to make, $I_t$, their dividend payments $DIV_t$, and purchases of reserves, $\varphi_t$. Purchases of reserves, $\varphi_t$, occur during the lending stage. Hence, these are different from those that occur during the overnight FED-funds market where reserves are one-for-one with deposits. The price of reserves is $(1 + r_t)$. In practice, banks lend and borrow reserves and do not purchase them. Here, we assume reserves are bought so that we don’t have to keep track of long-term interbank loans. However, we refer to $r_t$ as a long-term LIBOR rate to refer the model closer to the data but we keep in mind that $(1 + r_t)$ is a price more than a lending rate.

Upon a loan banks give a checking account to the borrower, or equivalently a deposit account to whom ever is exchanging a physical good (resources) to the borrower (in exchange for that check). When borrowing (lending) reserves, they issue (are issued) deposits against those assets. Thus, since dividends are paid in the form of bank liabilities, we obtain the following intra-period law of motion for demand deposits:

$$\tilde{D}_t = D_t + qI_t + DIV_t + \varphi_t(1 + r_t) - B_t(1 - \delta).$$

Thus, a bank that begins with $D_t$ as deposits at the beginning of the stage ends with $\tilde{D}_t$ at the end through the following sources. It credits by $qI_t$ the account of his borrower (or whomever he trades with), after a loan of size $I_t$. It also pays dividends to shareholders in amount $DIV_t$. It issues $\varphi_t(1 + r_t)$ liabilities to other banks if it borrows $\varphi_t$ in cash. Finally, $-B_t(1 - \delta)$ deposits are reduced by the payment of previously issued loans.

The evolution of bank reserves is given by the sum of the previous stock plus cash purchases, $\tilde{C}_t = C_t + \varphi_t$. Loans evolve according to the fraction of the original stock that has not matured yet plus the newly issued loans, $\tilde{B}_t = \delta B_t + I_t$. Banks choose $\{I_t, DIV_t, \varphi_t\}_{t \geq 0}$ subject to these laws of motion and some regulatory constraints. Capital requirement constraints impose an upper bound, $\kappa$, on the amount of leverage (marked-to-market) the bank can take so $\tilde{D}_t \leq \kappa \tilde{E}_t$. Liquidity requirements constraints operate similarly and place a lower bound, $\eta$, on reserve holdings per unit of equity so $\tilde{C}_t \geq \eta \tilde{E}_t$. This constraint differs from the monetary policy reserve requirement.

Balancing Stage. During the balancing stage, banks experience $\omega_t$. If by the end of the period, banks do not hold sufficient reserves relative to the required reserves, $\rho \tilde{D}_t$, they incur a
loss of $\chi(\rho D_{t+1} - C_{t+1})$. These losses are financed with deposits. Hence the law of motion for reserves accounts for the withdrawal, $C_{t+1} = \tilde{C}_t - \omega_t \tilde{D}_t$ and $D_{t+1} = \tilde{D}_t(1 - \omega_t) + \chi(\rho D_{t+1} - C_{t+1})$. The latter equation says that reserves by the end of the period is given, by the reserves at beginning of the period minus amount of withdrawn deposits. The second law of motion reflects that the withdrawal reduces the stock of deposits but increases deposits through the interbank market cost function $\chi$.

2.4 Bank Problems

The model can be expressed recursively so from now on, we drop time subscripts. It is understood that prices, government policy variables and aggregate shocks form the aggregate state $X$.

**Lending Stage.** The optimization problem for a bank during the lending stage in recursive form is as follows.

**Problem 1 (Bank Lending Stage)** The bank’s problem during the lending stage is:

$$V^l(C, B, D; X) = \max_{\{I, DIV, \varphi\} \in \mathbb{R}} U(DIV) + \mathbb{E}\left[V^b(\tilde{C}, \tilde{B}, \tilde{D}; \tilde{X})\right]$$

$$\tilde{D} = D + qI + DIV + (1 + r) \varphi - B(1 - \delta) \quad (1)$$

$$\tilde{C} = C + \varphi \quad (2)$$

$$B' = \delta B + I \quad (3)$$

$$\tilde{D} \leq \kappa(q \tilde{B} + (1 + r) \tilde{C} - \tilde{D}), \tilde{D} \geq 0 \quad (4)$$

$$\tilde{C} \geq \eta(q \tilde{B} + (1 + r) \tilde{C} - \tilde{D}), \tilde{C} \geq 0 \quad (5)$$

Banks choose loans $I$, dividends $DIV$, reserve holdings $\varphi$ and to maximize the expected discounted path of dividends subject to the law’s of motions for deposits, reserves and loans, equations (1), (2) and (3). In addition, they must satisfy the policy constraints (4) and (5). On the technical side, the leverage constraints bounds the problem of the banks and thus renders their problem feasible. It prevents a Ponzi-scheme. Moreover, since $\tilde{D} \geq 0$, equity will remain positive at all periods before the $\omega$ shock is realized.\(^{24}\)

During the balancing state, banks make no decisions but rather experience the withdrawal

\(^{24}\)It is important to note that if the bank arrives to a node with negative equity, the problem is not well defined. However, in choosing its policies, it will make decisions such that it is guaranteed that it doesn’t run out of equity. Implicitly, it is assumed that if it violates any constraint, the bank goes bankrupt.
shock \( \omega \). The value function at the balancing stage is:

\[
V^b(\tilde{C}, \tilde{B}, \tilde{D}; \tilde{X}) = \beta \mathbb{E} \left[ V^l(C', B', D'; X')|X \right]
\]

\[
C' = \tilde{C} - \omega \tilde{D}
\]

\[
B' = \tilde{B}
\]

\[
D' = \tilde{D}(1 - \omega) + \chi (\rho D' - C').
\]

Loans remain unchanged during the balancing stage. Instead, \( \omega \tilde{D} \) is transferred to other banks in the form of reserves. This withdrawal also reduces deposits by \( \omega \tilde{D} \) also. Finally, deposits change depending on the penalty faced by banks \( \chi \). We collapse the model into a single period. Since there are no actions between periods, then there’s no need to write-up two Bellman equations. Combining the lending and balancing value functions we obtain the following Bellman equation:

**Problem 2** The bank’s problem during the lending stage is:

\[
V^l(C, B, D) = \max_{\{I, DIV, \tilde{C}, \tilde{D}\} \in \mathbb{R}_+^4} U(DIV) ...
\]

\[
+ \beta \mathbb{E} \left[ V^l(\tilde{C} - \omega' \tilde{D}, \tilde{B}, \tilde{D}(1 - \omega') + \chi (\rho \tilde{D} - (\tilde{C} - \omega' \tilde{D})); X')|X \right]
\]

\[
\tilde{D} = D + qI + DIV_t + (1 + r)\varphi - B(1 - \delta)
\]

\[
\tilde{B} = \delta B + I
\]

\[
\tilde{C} = \varphi + C
\]

\[
\tilde{D} \leq \kappa(q\tilde{B} + \tilde{C} - \tilde{D})
\]

\[
\tilde{C} \geq \eta(q\tilde{B} + (1 + r)\tilde{C} - \tilde{D}).
\]

We characterize the bank’s policies in the following section.

### 2.5 Characterization of Problems

The recursive problem of banks can be characterized through a single state variable. Note that one can substitute for the cash borrowing, \( \varphi_t \), in (2) and \( I \) in equation (3) into to the evolution of deposits, equation (1) to obtain:

\[
\tilde{D} = D + q(\tilde{B} - \delta B) + DIV + (1 + r)(\tilde{C} - C) - B(1 - \delta).
\]

Rearranging terms one obtains:

\[
(1 + r)\tilde{C} + q\tilde{B} + DIV - \tilde{D} = C(1 + r) + q\delta B + B(1 - \delta) - D.
\]
This equation takes the form of a budget constraint for the banker in terms of his wealth. The bankers wealth is composed of the value of cash holdings \( C(1 + r) \), the market price of the illiquid fraction of loans, \( \delta B \), which valued at market price \( q \), the repayment of loans \( B'(1 - \delta) \) minus the stock of issued deposits. Funds can be increased freely by issuing more deposits, \( \hat{D} \). Funds are used to obtain cash for the following period, \( \hat{C} \), to fund new loans \( \hat{B} \) and to pay dividends, \( DIV \).

One can define the marked-to-market equity as, \( E \equiv C(1+r)+q\delta B+B(1-\delta)-D \), corresponding to the right hand side of the bank’s budget constraint. If we ignore the constraints that \( \hat{B} \geq \delta B \), something that we will rule out in equilibrium, we can express the Bellman equation without reference to the composition of the bank’s wealth but only as a function of \( E \). Thus,

**Proposition 1** (Single-State Representation)  
The problem of the bank can be written the following way:

\[
V^l(E) = \max_{\hat{C}, \hat{B}, \hat{D}, DIV \in \mathbb{R}_+^4} u(DIV) + \beta \mathbb{E} \left[ V^l(E') | X \right]
\]

\[
E = q\hat{B} + \hat{C}(1+r) + DIV - \hat{D}
\]

\[
E' = (q'\delta + (1-\delta)) \hat{B} + \hat{C}(1+r') - \hat{D}(1 + r'\omega') - \chi((\rho + \omega) \hat{D} - \hat{C})
\]

\[
\hat{D} \leq \kappa(\hat{B}q + (1+r)\hat{C} - \hat{D})
\]

\[
\hat{C} \geq \eta(\hat{B}q + (1+r)\hat{C} - \hat{D})
\]

This problem is closer to a standard consumption-savings where consumption (dividends here) is financed with holdings of two assets, \( (\hat{B}, \hat{C}) \) and borrowing \( \hat{D} \) subject to leverage and liquidity constraints.\(^{25}\) The continuation value of the value function, has also one argument, \( E' \), which is obtained by substituting the values of \( D', C' \) and \( B' \) as functions of current states. The budget constraint is linear in \( E \) and the objective is homothetic in dividends. Thus, by theorems in Alvarez and Stokey (1998) we have that the solution to this problem exists and is unique and we have that policy functions are linear. One can guess and verify that the objective is:

**Proposition 2** (Homogeneity-\( \gamma \))  
The value function \( V^l(E; X) \) satisfies

\[
V^l(E; X) = v^l(X) E^{1-\gamma}
\]

where \( v^l(X) \) is the value of

\[
\max_{\hat{C}, \hat{B}, \hat{D}, DIV \in \mathbb{R}_+^4} U(div) + \beta \mathbb{E} \left[ v^l(X') | X \right] \mathbb{E}_{\omega'} \left[ (\epsilon')^{1-\gamma} \right]
\]

\(^{25}\)With obvious abuse of notation, \( V \) henceforth denotes the value function in terms of \( E \) rather than \( (C, B, D) \).
subject to,

\[ 1 = q \hat{b} + \hat{c}(1 + r) + div - \hat{d} \]
\[ e' = q' \delta + (1 - \delta) \hat{b} + \hat{c}(1 + r') - \hat{d}(1 + r \omega') - \chi((\rho + \omega) \hat{d} - \hat{c}) \]
\[ \hat{d} \leq \kappa(q \hat{b} + (1 + r) \hat{c} - \hat{d}) \]
\[ \hat{c} \geq \eta(q \hat{b} + (1 + r) \hat{c} - \hat{d}) . \]

Moreover, all the policy functions of \( V^l(E) \) satisfy \( X = xE \). In the expression above, \( \mathbb{E}_{\omega'} \) is the expectation under \( F \).

The fact that the bank’s problem is homogeneous in the bank’s market-value of equity has several implications. First, it implies that two banks with different equity values will be scaled versions of a bank with one unit of equity. This also implies that the distribution of equity is not a state variable, but rather only the aggregate value of equity. Moreover, although there is no invariant distribution for bank equity as the variance of distribution may grow over time, the model does have a prediction about the growth rate of its cross-sectional dispersion. Most importantly, the fact that the market value of equity is the state variable, implies that we can decompose the effects of shocks by into portfolio composition effects and wealth effects.

An additional property of the banker’s problem is that it satisfies a separation theorem whereby the policy function for dividends can be analyzed independently from the policy functions of deposit issuance, cash holdings and loans. Using the principle of optimality, we can break the Bellman equation in two parts. The first will take as given the choice of dividends.

**Proposition 3** (Separation) The value function \( v^l(X) \) solves:

\[
v^l(X) = \max_{div \in \mathbb{R}_+} U(div) + \beta \mathbb{E} \left[ v^l(X') | X \right] \Omega(X)^{1-\gamma} (1 - div)^{1-\gamma}
\]

where \( \Omega(X) \) is the value of the certainty-equivalent portfolio of the bank:

\[
\max_{\delta, \hat{b}, \hat{c} \in \mathbb{R}_+} \mathbb{E}_{\omega'} \left[ (q' \delta + (1 - \delta)) \hat{b} + (1 + r') \hat{c} - \hat{d}(1 + r \omega') - \chi((\rho + \omega) \hat{d} - \hat{c}) \right]^{1-\gamma}
\]

subject to \( 1 = q \hat{b} - \hat{d} + \hat{c}(1 + r) \), \( \hat{d} \leq \kappa(q \hat{b} + \hat{c}(1 + r) - \hat{d}) \) and \( \hat{c} \geq \eta(q \hat{b} + \hat{c}(1 + r) - \hat{d}) \).

The solution to this portfolio problem is isomorphic to the solution to a portfolio in terms of portfolio shares invested in assets of different returns. We obtain this portfolio problem via change of variables that we can later invert to obtain the specific values of \( \hat{c}, \hat{b}, \hat{d} \). Define the following portfolio shares: \( w_b \equiv q \hat{b}, w_c \equiv (1 + r) \hat{c} \) and \( w_d \equiv -\hat{d} \).

**Return on Loans.** Let \( R^B(\delta) \) be there return on a loan given \( \delta \). This return depends on prices and the maturity of the loan and is adjusted for liquidity: \( R^B_t(\delta) \equiv (\delta q_{t+1} + (1 - \delta)) / q_t \).
The return from the maturing portion, \((1 - \delta)\), is \(\frac{1}{q_t}\), is the return from the coupon payment. The rest, \(\delta\) has a return \(q_{t+1}/q_t\), following from the revaluation of the loan.

**Return on Reserves.** The return on the cash and deposit components of the portfolios is determined jointly. These returns depend on the \(\omega\), the withdrawal shock. They can be separated into an independent return and a joint return component that follows from the penalty. The independent return on reserves is \(R^C_t \equiv \left[\frac{1+r_{t+1}}{1+r_t}\right]\). Since \(1 + r_t\) is the relative price of reserves relative to deposits, \(R^C_t\) captures the revaluation component. Note that in a state where \(r_{t+1} = r_t\), reserves have a return equal to one because they yield a constant relative price. Thus, the only return comes from the interest on reserves. When the interest on reserves and the prices change is zero, cash pay no independent return.

**Return on Deposits.** Deposits yield a return (cost) of \(R^D_t (\omega')\), given by \(R^D_t \equiv (1 + r_{t+1}\omega')\). This returns are function of the fraction of withdrawals \(\omega'\). When this deposits leave, the bank losses reserves and deposits. The opportunity cost of loosing those reserves is \(r_{t+1}\). The other term is 1 because deposits bear no interest.

**Portfolio Illiquidity Cost.** Finally, the joint return component is captured by potential cost (or benefit) of running out of reserves. This illiquidity cost is given by,

\[
R^x (w_d, w_c, \omega') \equiv \chi \left((\rho + (1 - \rho)\omega') w_d - \frac{w_c}{(1 + r)}\right).
\]

In this expression, \((\rho + \omega')\) is the sum of the reserve requirement and the withdrawal fraction. The cost the actual amount of cash holdings for the bank are \((w_c / (1 + r))\), the pre-transformation amount of cash. So when the withdrawal and reserves are lower than the amount of cash, the agent has a negative account at the fed which activates the borrowing costs.

**Return on Equity.** Finally, the return on the bank’s equity is given by \(R^E_t (\omega') \equiv R^B_t (\delta) w_b + R^C w_c - R^D_t (\omega') w_d - R^x (w_d, w_c, \omega')\) which is given by the weighted sume of returns minuse the illiquidity cost.

**Portfolio Problem.** Using the returns on the banks assets, we obtain the following liquidity management portfolio problem:

**Proposition 4** (Portfolio) \(\Omega (X)\) solves the following liquidity-management portfolio problem:

\[
\max_{\{w_b, w_d, w_c\}} \left(E_{\omega'} \left[\left(R^B_t (\delta) w_b + R^C w_c - R^D_t (\omega') w_d - R^x (w_d, w_c, \omega')\right)^{1-\gamma}\right]\right)^{1/\gamma}
\]

subject to,

\[
1 = w_b + w_c - w_d
\]

\[
w_d \leq \kappa, w_c \geq \eta, w_d, w_c, w_b \geq 0
\]
The objective of the problem is to maximize the certainty equivalent of \( R^E (\omega') \). However, this portfolio problem is not a standard portfolio problem. It features non-linear returns and has constraints on the portfolio weights. The intuition behind the main mechanism in this paper can be transparently understood by analyzing the strategies from this problem. What should be clear from this problem is that the constraints represent the budget constraint for the bank, the capital requirement and the liquidity requirement. The objective is the same as the objective in \( \Omega (X) \) but expressed in terms of returns to the banks assets. We postpone the discussing its solution to the following section. Solving this problem, yields the solution to the banks policy functions. We can reverse the solution for \( \{ \tilde{c}, \tilde{b}, \tilde{d} \} \) via following formulas: 
\[
\tilde{b} = (1 - \text{div}) \frac{w_b}{q}, \quad \tilde{c} = (1 - \text{div}) \frac{w_c}{(1+r)}
\]

and \( \tilde{d} = -(1 - \text{div}) w_d \).

The separation between the dividend-payment problem and the portfolio problem for the bank implies that the value function is given by:

\[
v^l (X) = \max_{\text{div}} U (\text{div}) + \beta \mathbb{E} [v^l (X') | X] (\Omega^* (X) (1 - \text{div}))^{1-\gamma} .
\]

We can characterize dividends and the value of the bank, without further reference to the choice of deposits, loans or reserves. We have the following proposition.

**Proposition 5 (Portfolio)** Given the solution to \( \Omega^* (X) \), the dividend ratio and value of bank equity are given by:

\[
div (X) = \frac{1}{1 + [\beta \mathbb{E} [v^l (X') | X] \Omega^* (X)]^{1/\gamma}}
\]

and

\[
v^l (X)^{1/\gamma} = 1 + (\beta \Omega^* (X))^{1/\gamma} \mathbb{E} [v^l (X') | X]^{1/\gamma}
\]

for \( \gamma > 0 \). For the risk-neutral case, \( \gamma = 0 \),

\[
div (X) = \begin{cases} 
0 & \text{if } \beta \mathbb{E} [v^l (X') | X] \Omega^* (X) < 1 \\
1 & \text{if } \beta \mathbb{E} [v^l (X') | X] \Omega^* (X) > 0 \\
\in [0,1] & \text{if } \beta \mathbb{E} [v^l (X') | X] \Omega^* (X) = 1 
\end{cases}.
\]

The policy functions by banks will determine a demand for loans and a supply for central bank reserves. This concludes the partial equilibrium description of the bank’s problem. We now describe the actions of the FED.

### 2.6 FED Balance Sheet and Operations

The FED’s balance sheet satisfies the following identity:

\[
\underbrace{D_t^{FED} + B_t^{FED}}_{\text{Assets}} = \underbrace{M0_t + E_t^{FED}}_{\text{Liabilities}} .
\]
In this identity says that $M_0$, the amount of reserves (or high powered money) issued by the FED, $D_{t}^{FED}$ are its holdings of commercial bank deposits, and $B_{t}^{FED}$ are holdings of private loans. The variable $E_{t}^{FED}$ correspond to the FED’s equity. The FED has a monopoly over the supply of reserves, $M_0$ and alters this quantity via open-market operations. The evolution of these state variables for the satisfies the following system:

\[
M_{0t+1} = M_0 + \varphi_t \\
D_{t+1}^{FED} = D_{t}^{FED} + (1 + r) \varphi_t - q_t I_t + \chi_t - \text{trans}_t \\
B_{t+1}^{FED} = \delta B_{t}^{FED} + I_t.
\]

The laws of motion for these state variables are very similar to the laws of motion for banks. Here, $\varphi_t$ represent purchases of deposits from the FED by issuing reserves to commercial banks. With these deposits, the FED can purchase loans $I_t$. The term $\text{trans}_t$ correspond to transfers to the fiscal authority, which is the analogue of FED dividends. We assume these are exogenous to keep the FED’s equity constant in steady state. The term $\chi_t$ represents the FED’s income revenue from the FED funds market or penalties from reserve requirements:

\[
\chi_t = -\int_0^1 \chi (\rho \tilde{D}_t (z) - \tilde{C}_t (z)) dz.
\]

Unconventional Open-Market Operations. Since there are no T-Bills so far, only unconventional monetary operations are available to the FED. In the Appendix we incorporate T-Bills into our model. Thus, unconventional open-market is the purchase of loans without increasing the the deposits held by the FED. These satisfy $(1 + r_t) \varphi_t - q_t I_t = 0$ and where rate of change in the money supply is given by:

\[
\Delta M_0 = \frac{q_t}{(1 + r_t)} I_t.
\]

Open-Market Liquidity Facilities. Liquidity facilities are swap of liabilities of the FED for deposits.

The FED’s Budget Constraint. The FED’s consolidated budget constraint is obtained by substituting the laws of motion of all the variables into the evolution of $M_0$. This yields:

\[
M_{0t+1} = M_0 + \frac{1}{(1 + r_t)} (D_{t+1}^{FED} - D_{t}^{FED} + q_t (B_{t+1}^{FED} - \delta B_{t}^{FED}) - \chi_t + \text{trans}_t)
\]

In equilibrium, the Fed can experience profits or losses (e.g. if they hold bonds, or treasuries they make a return at steady state), or via the charge on penalties to commercial banks, or in turn, via the charge of transfers. Transfers are rebates to the treasury.
2.7 Market Clearing

**Loan Demand.** For now we assume an exogenous demand for loans of the form

\[ q_t = \Theta_t \left( I_t^D \right)^{\epsilon}. \]  

Since, \( q_t \) is the inverse of the interest rate, this demand function is increasing so \( \epsilon > 0 \). In the appendix, we provide a microfoundation for a demand for loans which takes this form of 6 that closes the model. There are many other ways to obtain a similar function. For example, the cash-in-advance model of Lucas and Stokey (1987) can be altered so that the supply of money holdings follows from the deposits issued by our banks. One would need to alter the structure to require a deposit-in-advance constraint. Similarly, Townsend (1980) classical model turnpike model can be altered to allow for credit. Kiyotaki and Moore (2008) derive a similar demand for which to allow for investment opportunities. Also, this demand can be obtained through search frictions as in Lagos and Wright (2005), Williamson (2012) or Rocheteau and Rodriguez-Lopez (2013). We chose our setup as it allows us to obtain a direct static mapping from the quantity of lending to output in a way that is very simple to interpret. Moreover, our setup affects the demand for labor which seems important in these type of models to explain strong movements in output.

We have abstracted from any type of credit risk. This feature is suggested by Stiglitz and Greenwald (2003) Their argument is that monetary policy can have highly non-linear effects because of this feature. Although we are not explicit in the nature of demand shocks here, we have a motivation related with credit rationing.

Market clearing for the loans market requires us to equate \( I_t^D \) to the supply of new loans by banks and the FED. Hence, we have \( I_t^D = B_{t+1} - \delta B_t + B_{t+1}^{FED} - \delta^T B_t^{FED} \).

**Money Market.** Similarly, the market for reserves must clear. Since reserves are not lent outside the banking system equilibrium, requires \( \varphi_t + \varphi_t^{FED} = 0 \). In terms of flow variables \( M_{0t+1} - M_0 = C_{t+1} - C_t \).

**Monetary Aggregates.** Given that the model does not have a role for coins and currency, banks reserves represent the whole of the monetary base. The FED choose its issuance of money at every point in time. Thus, \( M_{0t} = C_t, \forall t \). Deposits, instead correspond to the monetary creation by banks, \( M_{1t} = \int_0^t d_t(z) E_t(z) \, dz \). Hence, similarly to Brunnermeier and Sannikov (2012), the model yields and endogenous money multiplier \( \mu_t = \frac{M_{1t}}{M_{0t}} \).

**Bank Equity Evolution.** The distribution of bank equity is evolving over time according to: \( E_{t+1}(z) = \Psi_t E_t(z) \) where \( \Psi_t(\omega) = \left( \left( \frac{(t^q + (1-t^q))}{q} w^q \right) + w^r + w^r (1 + r' \omega') - \chi (\rho + \omega) w^d - \frac{w^d}{(1+r)} \right) \) is the growth rate of bank equity. This growth rate uses the scale invariance property described in the previous section. The measure of equity holdings at each bank evolves is denoted by \( \Gamma_t \).

Since the model is scale invariant, we just need to keep track of the evolution of average equity,
\[ \int_0^1 E_t(z) \, dz, \] which by independence grows at rate \( \mathbb{E}_\omega[\Psi_t] \). A limiting distribution for \( \Gamma_t \) is not well defined unless one adapts the process for equity growth.

Using this expression and the results from the previous section, we can express the supply of new loans in period \( t \) through:

\[
I_t^S = \tilde{b}_t \int_0^1 E_t(z) \, dz - (1 - \delta) \tilde{b}_{t-1} \int_0^1 E_{t-1}(z) \, dz.
\]

Notice that the supply of new loans, which determines \( q_t \), is the difference between current demand for the stock of loans minus the fraction of previously existing loans that has not matured yet. Similarly, the aggregate demand for central bank reserves is given by,

\[
C_t = \int_0^1 \tilde{c}_t(z) E_t(z) \, dz.
\]

Since this equation holds for every period, including time 0. We have:

\[
C_t = M_{0t} \rightarrow \tilde{c}_t \bar{E}_t = M_{0t}.
\]

with again, a similar feature for \( M_{0t} \).

### 2.8 Equilibrium

The definition of equilibria is now altered to incorporate T-Bills and open market operations.

**Definition.** A competitive equilibrium is a sequence of bank policy rules \( \{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, \tilde{d}_t, \text{div}_t\} \) \( t \geq 0 \), government policies \( \{\rho_t, D_{FED}^t, B_{FED}^t, M_{0t}, \kappa_t, \eta_t, \xi_t, \bar{X}_t\} \) \( t \geq 0 \), bank values \( \{v_t\} \) \( t \geq 0 \), measures of equity distributions \( \{\Gamma_t\} \) \( t \geq 0 \) and prices \( \{q_t, r_t\} \) \( t \geq 0 \), such that: (1) Given price sequences \( \{q_t, r_t\} \) \( t \geq 0 \) and policies \( \{\rho_t, D_{FED}^t, B_{FED}^t, M_{0t}, \kappa_t, \eta_t, \xi_t, \bar{X}_t\} \) \( t \geq 0 \), the policy functions \( \{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, \text{div}_t\} \) \( t \geq 0 \) are solutions to Problem 2. Moreover, \( v_t \) is the value in Proposition 3. (2) The Money Market Clears: \( \tilde{c}_t \bar{E}_t = M_t \). (3) The loan Market Clears: \( I_t^S = \Theta_t^{-1} q_t^\frac{1}{2} \). (4) The measure \( \Gamma_t \) evolves consistently with \( \Psi_t(\omega) \).

**Interest Targets.** In equilibrium, the FED can target a path for interest rates, \( R_t^B(\delta) \) and \( R_t^C \) rather than money aggregates. This path of interest rates is determined by the paths of \( \{r_t, q_t\} \). Such a sequence must satisfy a stationarity condition of bank equity. So in the long-run, the FED is not free to impose any given sequence without exploding its balance sheet. Thus, these policies are subject to limiting condition. Along the transitional dynamics, \( B_{FED}^t \) and \( D_{FED}^t \) can be chosen freely. We are ready to analyze some properties of our model.

**Incorporating T-Bills.** Incorporating Treasury Bills (T-Bills) into our model is quite straightforward. Appendix xxx, discusses two extensions. One in which deposits have the liquidity properties of reserves and the other when they don’t. Although the working of the model is quite similar, this extension is helpful for the quantitative analysis.
2.9 Theoretical Analysis

To gain more intuition, it is convenient to analyze the portfolio problem of the bank derived in Proposition 6.

**Excess Returns in** \( \Omega^* (X) \). Fix any given state \( X \). It is useful to re-write the portfolio problem \( \Omega (X) \) by substituting the weight on loans. We obtain:

\[
\max_{\{w_d, w_c\}} \left( \mathbb{E}_{\omega'} \left[ \begin{pmatrix} R^B & -(R^B - R^C) w_c & (R^B - R^D (\omega')) w_d - R^x (w_d, w_c, \omega') \end{pmatrix} \right]^{1-\gamma} \right) \frac{1}{1-\gamma}
\]

subject to,

\[ w_d \in [0, \kappa] \text{ and } w_c \in [\eta, \infty). \]

The objective in \( \Omega (X) \) clear. If banks hold all their equity on loans, they would obtain \( R^B \) per unit of equity. Issuing an additional deposit yields an arbitrage opportunity when the spread between return on loans and return on deposits is positive: \( (R^B - R^D (\omega')) \). Reserve holdings have the classical opportunity cost of cash, \( (R^B - R^C) \) but they yield the benefit of reducing the exposure to liquidity risk by reducing the expected liquidity cost \( R^x (w_d, w_c, \omega') \).

**Liquidity Premium.** In the context of our model, reserves have a liquidity premium relative to loans. To see this, we can derive the first order conditions of the problem above. The conditions for \( w_c \) and \( w_d \) are respectively

\[
\Omega (X)^{-\gamma} \mathbb{E}_{\omega'} \left[ (R^E)^{-\gamma} \cdot \{ (R^B - R^D (\omega')) - R^x (w_d, w_c, \omega') \} \right] + \kappa = 0
\]

and

\[
\Omega (X)^{-\gamma} \mathbb{E}_{\omega'} \left[ (R^E)^{-\gamma} \cdot \{ (R^B - R^C) + R^x (w_d, w_c, \omega') \} \right] + \eta = 0.
\]

A banks stochastic discount factor is given by

\[
m = \beta (\text{div'} R^E (1 - \text{div}) E)^{-\gamma} / (\text{div})^{-\gamma}.
\]

Thus, multiplying both expressions by \( \beta (\text{div'} (1 - \text{div}) E)^{-\gamma} / (\text{div})^{-\gamma} \) we yields the first order conditions of the original problem:

\[
\mathbb{E}_{\omega'} [m \cdot \{ (R^B - R^D (\omega')) - R^x (w_d, w_c, \omega') \} ] + \kappa = 0 \tag{7}
\]

and

\[
\mathbb{E}_{\omega'} [m \cdot \{ (R^B - R^C) + R^x (w_d, w_c, \omega') \} ] + \eta = 0 \tag{8}
\]

where \( \tilde{\mu}^k \) and \( \tilde{\mu}^\eta \) are renormalized multipliers. We can use these expressions to derive a liquidity premium. Suppose, that the liquidity constraint is not binding so \( \tilde{\mu}^\eta = 0 \). Then, we can reorder (8) and obtain:
\[ \frac{R_B - R^C}{\text{Cash Opportunity Cost}} = -\frac{\mathbb{E}_{\omega'} [m \cdot R^\chi_c (w_d, w_c, \omega')]}{\mathbb{E}_{\omega'} [m]} \]

\[ = \frac{\mathbb{E}_{\omega'} [R^\chi_c (w_d, w_c, \omega')]}{\text{Direct Liquidity Effect}} - \frac{\text{COV}_{\omega'} [m \cdot R^\chi_c (w_d, w_c, \omega')]}{\mathbb{E}_{\omega'} [m]} \cdot \text{Liquidity Risk Premium} \]

This expression is close to standard asset-pricing equations and can be used to obtain measures of the increase in the perceived liquidity risk post crisis. The return on reserves has two terms, a direct return, \( R^C \) and an additional additive term that follows from the reduction in liquidity risk for the bank, \( R^\chi_c (w_d, w_c, \omega') \). The expression above says that the excess return on loans, \( R^B - R^C \), the opportunity cost of holding reserves, equals the additional benefit of holding reserves \( R^\chi_c (w_d, w_c, \omega') \) which is adjusted by the risk-premium associated with the withdrawal shocks. A similar expression can be derived for the spread between loans and deposits:

\[ \frac{R_B - 1}{\text{Arbitrage}} = \frac{R^\chi_d (w_d, w_c, \omega')}{\text{Direct Liquidity Effect}} - \frac{\text{COV}_{\omega'} [m \cdot (R^\chi_c (w_d, w_c, \omega') - R^D (\omega'))]}{\mathbb{E}_{\omega'} [m]} \cdot \text{Liquidity Risk Premium} \]

This expression states that the arbitrage on loans by borrowing with deposits is limited by the direct expected increase in liquidity costs \( \mathbb{E}_{\omega'} [R^\chi_d (w_d, w_c, \omega')] \) adjusted by the liquidity risk premium on deposits.

Notice also, that when either of the regulatory requirements is binding, the corresponding euler equations are not satisfied with equality. Potentially, the excess arbitrage and the opportunity cost of reserves are larger. In cases where the capital requirement constraints binds, which often show up in the numerical simulations, monetary policy affects a trade of between lending and holdings of reserves. This point is particularly relevant for the quantitative section.

To make further progress in the analysis, and illustrate these effects more clearly, we study some polar cases that deliver closed form solutions. For the time being, let’s assume that the constraints don’t bind so \( \kappa \to \infty \) and \( \eta = 0 \).

**Polar Case I: Risk-Neutral Banks (\( \gamma = 0 \)).** With risk neutrality monetary policy acts likes a tax on financial intermediation. To see this, assume \( \gamma = 0 \) so that bankers are risk neutral. The objective in the portfolio problem can be written as:

\[ R^B + \max_{\{w_d, w_c\}} (R^B - R^D) w_d - (R^B - R^C) w_c - \mathbb{E}_{\omega'} [R^\chi_c (w_d, w_c)] \]
The expected liquidity cost $\mathbb{E}_\omega [R^x (w_d, w_c)]$ takes the following form:

$$
\mathbb{E}_\omega [R^x (w_d, w_c)] = \bar{\chi} \int_{\bar{\omega}(w_d, w_c)}^{1} (\omega - \bar{\omega}(w_d, w_c)) f(\omega) d\omega - \chi \int_{-\infty}^{\bar{\omega}(w_d, w_c)} (\bar{\omega}(w_d, w_c) - \omega) f(\omega) d\omega.
$$

where $\bar{\omega}(w_d, w_c)$ is the threshold shock that leads the banker to experience a negative balance of reserves (relative to the required reserves). This cutoff can be written in terms of the liquidity ratio of the bank defined by $L \equiv \frac{w_c}{w_d}$. In terms of this ratio, the cutoff is given by:

$$
\bar{\omega}(w_d, w_c) = \bar{\omega}(1, L) = \left( \frac{1}{1+r} - \rho \right).
$$

The liquidity cost $R^x (w_d, w)$ can actually be written also as a linear function of the weight on deposits $\omega_d$ multiplied by a function of the liquidity ratio. This function takes the following form:

$$
\omega_d \tilde{R}^x (1, L) = \omega_d \left( \bar{\chi} \int_{\bar{\omega}(L)}^{1} (\omega - L) f(\omega) d\omega - \chi \int_{-\infty}^{\bar{\omega}^*(L)} (L - \omega) f(\omega) d\omega \right).
$$

Introducing this representation into the original problem, yields a convenient reformulation.

$$
\Omega (X) = R^B + \max_{\omega_d} \omega_d \left( (R^B - R^D) + \max_L \left\{ - (R^B - R^C) L - \tilde{R}^x (1, L) \right\} \right).
$$

This reformulation shows that the bank’s problem is a linear function of the weight. Deposits have a direct return $(R^B - R^D)$, but the banker will choose an optimal amount of liquidity holdings $L$, per unit of deposit that reduces its exposure to the liquidity shocks. The optimal liquidity ratio, weights the cost of obtaining liquidity against the reduction in the expected illiquidity cost.

An expression for the optimal liquidity ratio $L^*$ that maximizes the expression in the curled brackets is provided in the appendix. Once, $L^*$ is obtained, the problem is linear in $\omega_d$ so the solution to the deposit decision would be pinned down entirely by the value of $(R^B - R^D)$ minus the optimal liquidity choice. In the case of risk neutrality, without regulatory constraints, $\omega_d$ solves a linear problem. So either $\omega_d$ is 0, unbounded or the solution indeterminate when the arbitrage on loans equals the expected liquidity cost at the optimal liquidity ratio $L^*$:

$$
\frac{(R^B - R^D)}{\text{Arbitrage on Loans}} = \left( R^B - R^C \right) L^* + R^x (1, L^*).
$$

In equilibrium, this condition must hold implying zero profits. Given that risk-neutrality implies a perfectly elastic, elasticity of intertemporal substitution, banks do not hold equity. This equilibrium condition is affected by monetary policy, but in turn imposes a restriction on the set of equilibria imposed by monetary policy.

**Remark.** Under risk neutrality, $\gamma = 0$, the bank’s portfolio problem may be written as a linear
function of leverage. The return on this leverage is a function of an minimal liquidity ratio cost. Monetary policy introduces a liquidity ratio cost. Thus, monetary policy affects operates like a tax on financial intermediation.

It is easy to see that when the capital requirement constraint is binding, \( w_d = \kappa \), and thus, the optimal choice of \( L \) is independent of \( \kappa \). This implies that monetary policy has effects by altering the liquidity ratio that banks hold, which determines the allocation between loans and reserves for a given leverage.

**Polar Case II: Deterministic withdrawals.** A special case is occurs when \( \Pr(\omega = 0) = 1 \). In this case, there's no uncertainty so there's is no difference between the portfolio decision of a risk-neutral and a risk-averse banker (only the dividend policy changes). Thus, following the steps before, the value of the portfolio problem is:

\[
R^B + \max_{w_d} w_d \left( R^B - R^D \right) + w_d \left( \max_L \left( \bar{\chi} (\rho - L)^+ - \chi (L - \rho)^+ \right) \right).
\]

To have an interior solution for the liquidity ratio, it is required that:

\[
\bar{\chi} > R^B - R^C > -\chi,
\]

since otherwise \( L \) is either 0 or infinity. When this condition is satisfied, the banker sets the liquidity ratio exactly to \( \rho \), the reserve requirement in the model. So without risk, \( \rho \) determines the amount of reserves per unit of loan.

Assuming there is a market for reserves, incorporating this result into the objective yields:

\[
\Omega(X) = R^B + \max_{w_d} w_d \left( (R^B - R^D) - (R^B - R^C) \rho \right).
\]

This objective function shows that monetary policy acts, again, like a tax on loans. In equilibrium under certainty about withdrawals, a finite solution for the volume of deposits is given by:

\[
(R^B - R^D) = (R^B - R^C) \rho.
\]

**Remark.** Without liquidity risk, \( (\omega = 0) \), the bank’s problem is a linear function of the reserve requirements. This requires \( \bar{\chi} > R^B - R^C > -\chi \).

Another thing to note about perfect foresight of bank about withdrawals is that the constraint that \( (R^B - R^D) = (R^B - R^C) \rho \) implies that \( R^C \leq R^D \), for partial reserve requirements. In other words that reserves lose value relative to deposits over time. This result is important. It shows without risk, banks have no precautionary motive for holding reserves. In an equilibrium with stability of interest rates, to have a determinate amount of loans and positive reserves the Central Bank would have to force \( \rho \geq 1 \) to have \( R^C = R^D \). This would force banks to hold as many reserves as deposits. This is known as narrow banking. In this case, lending would not be influenced by the Central Bank and only affected by the amount of equity in the banking system. Alternatively,
reserves would play no role, and \( R^B = R^D \) with infinite leverage. Another possibility is to have \( R^C \leq R^D \), and have \( \rho \leq 1 \), but this possibility leads to a fiscal cost.

If yet again, the capital requirement constraint is binding, \( w_d = \kappa \), and thus the bank’s portfolio is determined. The FED however, has the ability to affect lending via changes in reserve requirements or by setting \( \chi \), in a way that banks ignore the reserve requirements.

**Case III: Zero-Lower Bound.** A zero lower bound is reached when \( \chi = 0 \) (\( \chi \) is the zero function). In this case, all risk is eliminated from the model. In such case reserves are not value for the reduction in the hazard rate of a liquidity cost. In this case, \( \Omega (X) \) becomes a linear program since the solution to this problem is deterministic:

\[
\Omega (X) = \underbrace{R^B}_{\text{Return to Equity}} + \max \_{\{w_d, w_c\}} \underbrace{(R^B - R^C)}_{\text{Cash Opportunity Cost}} w_c + \underbrace{(R^B - R^D)}_{\text{Arbitrage}} w_d
\]

Now, an equilibrium with finite holdings will clearly require: \( R^B = R^C = R^D = 1 \). Any other difference would lead to infinite profits.

This polar example shows that if the Central Bank eliminates all the costs associated with liquidity risk, it has no effect on aggregate lending. The price of reserves \( r \) is pinned down by \( R^C \) and banks are indifferent between holding any mount of reserves. In this case, the amount of lending is entirely determined by the demand for loans at \( q_t = 1 \), the efficient amount of lending. Another way to say this, is that if monetary policy is to have any real effects, it must introduce distortions to the loans market.

**Remark.** If the \( \bar{\chi} = \bar{\chi} = 0 \), monetary policy cannot affect lending.

If capital requirement constraints is present, \( R^B \geq 1 \). If \( R^B = R^C > 1 \), the bank’s portfolio is indeterminate and if \( R^B > R^C \geq 1 \), banks hold no reserves.

### 3 Calibration and Quantitative Analysis

#### 3.1 Dispersion of Deposit Growth

Calibrating our model requires an empirical counterpart for the random-withdrawal process for deposits, \( F_t \). We use information from individual commercial bank Call Reports collected by the Federal Deposit Insurance Corporation (FDIC) to describe the evolution of deposit withdrawals. The Data Call Reports present balance-sheet information for all commercial banks in the US. Publicly available data spans all the quarters from 1990 until 2011. The Appendix, provides more details on how we construct the data we report in this section and other aspects of the data that are relevant for the paper.

We take the stance of calibrating \( F_t \) using information from the volatility of Total Deposits. To justify this choice, we need first to discuss Figure 1. The bars in the figure contain pre-crisis
Figure 1: Cross-Sectional Distribution of Deviation from Cross-Sectional Average Growth Rates sample (2000Q1-2007Q4) information. The solid line that shifts to the left reports Great Recession (post-crisis) (2008Q1-2010Q4) information. The units of observation in Figure 1 correspond to quarter-bank observations on Total Deposits. The histogram plots the empirical frequencies of cross-sectional deviations of the growth rates of each bank quarter from the average growth rate for a given quarter in the sample.

In our model, banks feature the same growth rates of equity as the average bank unless they experience a withdrawal shock. Thus, in the model, a bank showing an increase in deposit growth higher than the mean is a bank experiencing an inflow of deposits, and vice versa. Hence, the deviations from average growth rates have a one-to-one map to the withdrawal shocks. Banks in our model have only one form of liability, demand deposits. In practice, commercial banks have other forms of liabilities that include bonds, long-term deposits (savings deposits) and other variable income securities. For this reason, we must be careful in our choice of the data counterpart of $F_t$. We choose total deposits as our counterpart.
In the Appendix, we show that Total Deposits in the data are substantially less volatily than Demand Deposits. Second, Total Deposits feature a trend which is consistent with the growth of bank liabilities whereas this is not the case for demand deposits. In practice, total deposits may include savings for short periods of time, or may also be removed from a bank at a cost.

Given the substantial variation in the volatility of total deposits observed in Figure 1 we believe this is a relevant measure to capture illiquidity risk. Thus, we use this empirical histogram of quarterly deviations of today deposits, to calibrate $F_t$, the process for withdrawal shocks, non-parametrically. Our model also predicts that the behavior of equity should be perfectly correlated with the behavior of deposits. We report this correlation in the Appendix. We find that the correlation is positive, as suggested by our model significantly lower from one which is reasonable given that equity captures variations in the prices of securities, credit risks and operating costs that we don’t include in the model. The data appendix discusses this point as well as the validity of the growth independence assumption. A final thing to note is that the variation in deposit growth has moved to the left.

### 3.2 Banking Sector Data Moments

Figure 2 plots the evolution of some of key ratios for banks. These moments have a direct mapping to our model. We also use these moments to estimate changes in $\Theta_t$ in the data once we perform the quantitative investigation of our model. A noticeable fact to keep in mind is the increase in the liquidity ratios, together with sharp declines on return on assets, leverage and dividend ratios during the crisis.

### 3.3 Calibration

The values of all parameters are listed in Table 1. We need to assign values to eleven parameters $\{\kappa, \beta, \delta, \gamma, \epsilon, \rho, \eta, r, r^f\}$. We set the capital requirement and the reserve requirement according to standard regulatory measures. In particular, we set $\kappa = 24$, which corresponds to the Tier 1 Capital Ratio, $\rho = 10$ percent. The discount factor is set to 0.99. The risk aversion is set to 1.

The parameter $r$, which captures the opportunity cost of holding cash, is set to 1.5 percent, which is in the range of the historical values for the real Libor rate. We set $\chi$ to be 80 percent of the interest rate so as to match the difference between the Fed Funds and the LIBOR rates. The loan average maturity is set to 2 years, implying a value for $\delta = 0.87$. The value of the loan demand elasticity $\epsilon = 8.0$. For now, we abstract from interest on reserves and the liquidity constraint in the lending stage, i.e, $r^f = 0, \eta = 0$. In addition, we set $\chi^L = 0$. Finally, for the withdrawal we use a non-parametric estimation as described above.
Figure 2: Evolution of Key Ratios for the banking sectors during the last decade.
4 Policy Functions - Prices Given

We start with a partial equilibrium analysis of the model by showing banks policy functions at different prices. Figure 3 reports decisions for cash, loans, dividend, as well as liquidity and leverage ratios, the value of the asset portfolio, liquidity risk, expected returns and expected equity growth. These policies correspond to the solution to the Bellman equation (2) for different values of loan prices $q$.

A first observation that emerge from Figure 3 is that the supply of loans is decreasing in the loan price whereas dividends and cash ratios are increasing in the loan price. As loan prices decrease, loans become relatively more profitable leading banks to keep a lower fraction of its assets in relatively low return assets, i.e., cash. Moreover, banks cut on dividend rate payments to allocate more funds to loan issuances and experience higher equity growth. The exposure to liquidity risk, measured as the expected penalty costs from falling short of cash holdings, is also decreasing in loan prices, reflecting the fact that banks’ asset portfolio becomes relatively more liquid.

Another key observation is that there is a kink in banks’ policies at the value of the loan price where the capital requirement ceases to bind. In particular, when the price of loans is sufficiently high, high profits from intermediation lead banks to reduce deposits to the point in which the capital requirement is not binding. In this region, a decrease in the loan price leads to a sharp increase in lending ratios. To the left of this point, decreases in the loan price lead to a less of a sharper increase in lending rates due to the fact that higher lending needs to be financed with either lower dividend payments or a reduction in cash holdings, which exposes the banks to
more severe effects of withdrawal shocks. In other words, the capital requirement generates an asymmetric response to changes in the loan price depending on whether the capital requirement is binding or not.

Figure 3: Policy function for different Loan Prices

The value of the interest rate plays an important role because it affects the cost of hedging liquidity risk. In fact, a high interest rate increases the opportunity cost of holding cash, and leads to lower lending ratios as figure 4 shows. In turn, this leads to higher dividend rates, lower growth and lower returns on the asset portfolio.

5 Steady State Equilibrium Portfolio

The previous section describe portfolio decisions for an arbitrary loan price. We now analyze the equilibrium portfolio at steady state and investigate the effects of withdrawal shocks over the banks’ balance sheets. The equilibrium portfolio corresponds to the solution of the Bellman equation 1 evaluated at the price that clears the loans market, according to condition 6. As it can be seen from the plot for equity growth in figure 3, the price that clears the market corresponds approximately to \( q = 0.995 \).
The top left panel of Figure 5 shows the value of the risk adjusted portfolio $\Omega_t$ for possible loans and cash weights $w$. The maximum value of the portfolio is reached at $c = 2.4$ and $b = 18.3$, which implies $d = -19.8$ and a binding capital requirement constraint. The optimal portfolio allocations is a result of the trade-off banks face between return and liquidity risk: loans provide a high return whereas cash delivers a lower expected return but delivers the highest payoffs when an adverse withdrawal shock hits the banks.

The value of the return for each asset as a function of the withdrawal shock are shown in panel (d) of Figure 5. For low withdrawal shocks, the return on loans is higher than the return on loans. As the withdrawal shock reaches a value that makes the penalty requirement binding, cash provides extra liquidity benefits and delivers a higher return than loans. The discontinuity in the return for cash occurs at $\omega = 0.05$, the value of the withdrawal shock at which the liquidity penalty becomes strictly positive as illustrated in Panel (e). The return on deposits also fluctuate significantly depending on the withdrawal shock and also have a discontinuity at the value of $\omega$ at which banks start facing penalties from insufficient cash. In fact, the net return on deposits is given by $r\omega$ for values of $\omega$ lower than 0.04, and jump to $(r + \rho \chi)\omega$ for values of $\omega$ higher than 0.04.
The resulting return of the equilibrium portfolio as a function of the withdrawal shock is illustrated in Panel (c). This figure shows that in equilibrium the return of the portfolio is quite sensitive to withdrawal shocks. In fact, the range of portfolio returns can oscillate between 0.94 and 1.05 according to the realization of $\omega$. In equilibrium, the banks that experience positive withdrawal shocks will increase the size of their equity whereas those that experience negative withdrawal shocks will shrink. Finally, panel (f) show the cash and deposit holdings at the end of the period that results from the portfolio choice in the lending stage and the effects of the withdrawal shock in the balancing stage.

Figure 5: Portfolio Choices and Effects of Withdrawal Shocks

6 Transitional Dynamics

This section studies how the economy responds to different shocks. For this purpose, we consider an economy that is at steady state at period $t = 0$ and experience an unanticipated shock. The shocks we consider are equity losses, credit demand shocks, a tightening of capital requirements, uncertainty shocks and rise in funding costs. For each shock, we analyze the transitional dynamics of banking and monetary indicators, as illustrated in Figures 6-14. The superior panel for each figure shows aggregates for equity, lending, cash and the return on loans. The inferior panel shows the ratios for cash-to-equity ratio, loan-to-equity ratio, dividends-to-equity ratio as well as portfolio value, banks’ marginal value and liquidity risk.
6.1 Equity Losses

The first shock we consider is a sudden unexpected decline in bank equity. This shock can be interpreted as capturing an unexpected rise in non-performing loans or losses from other sources of risk. To the extent that equity is the key state variable, the analysis of the transition dynamics to an equity loss would also be relevant to understand the dynamics of the economy in response to other shocks as well.

Figure 6 illustrates the response of the economy to equity losses of 4 percent. The economy experiences on impact a drop in loan prices, i.e. an increase in loan returns, which further reduces the value of equity. Moreover, aggregate lending and cash holdings also decline. Because of the marked-to-market capital requirement constraint, the initial shock causes a fire sale effect as the decline in equity, loan prices and lending mutually reinforce each other. After about 6 years, the economy converges back to the initial steady state.

Let us analyze the transitional dynamics in more detail. What explains the immediate increase in loan returns? Suppose loan returns did not change at all. In this case, banks would keep the same lending ratios and the volume of new loan issuances $E_1 b_1 - E_0 b_0 (1 - \delta)$ would fall. If loan return do not vary, however, the demand for loans would remain unchanged leading to an excess demand for loans. In equilibrium, this means that loan returns need to rise on impact. High return on loans also imply low dividend rates and equity growth. Hence, as the financial sector gradually recovers from the equity losses, the economy experiences a decrease in loan returns until it converges to the same initial steady state.

It is important to notice that the volume of bank lending follows a persistent decline. In fact, the volume of total lending falls on impact following the contraction of the balance sheets and continue to fall after the initial shock. The reason for this persistent decline is due to the long-maturity of loans that introduces a "stickiness" in the behavior of the stock of loans. In particular, the flow of lending can be decomposed into the difference between the new issuances and the repayment of old loans. As the return on loans increase on impact, there is a sharp decline in the demand for new issuances, which is larger than the repayments of previous loans. This explains the initial fall in the stock of loans. As new issuances remain below repayments, the stock of loans continue to decline. Once the stock of loans become low enough, new issuances become larger than loan repayments, and this leads to a monotonic increase in the stock of lending.

Equity losses have other important implications for banks’ liquidity management. In particular, since loans return are high along the transition, banks invest a lower fraction of their equity in cash and invest relatively more in loans. As a result, banks become more exposed to illiquidity risk.

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26 One way to incorporate this explicitly in the model would be to consider an aggregate shock to the default rate on loans.
6.2 Tightening of Capital Requirements

Next, we consider a sudden tightening of capital requirements, i.e., a reduction in $\kappa$.\[27\] This shock can be interpreted as a tightening in regulatory requirements, or alternatively as reflecting a rise in solvency concerns about the banking sector due to standard agency problems.

Figure 7 illustrates the effects of an decrease in $\kappa$ of 30 percent, which is associated with an increase in the capital ratio of 2.5 percent, which is the extra capital buffer imposed by Basel III. We assume that the reduction in capital requirements are permanent and follow a gradual increase, modelled as a deterministic AR(1) process, such that the adjustment in $\kappa$ is almost complete after 5 years.

Before analyzing the transitional dynamics, let us first analyze the effects on the steady state of the economy. A central observation, apparent in Figure 7, is that the new steady state features a higher return on loans. What explains the increase in the return on loans is the fact that the contraction in capital requirements reduces the banks’ loanable funds. As the supply of credit is reduced, this requires a higher return on loans to clear the loans market. Moreover, higher return on loans also imply lower dividend rates as well as lower cash holdings.

What does the transition to the new steady state look like? On impact, that there is a sharp increase in the return on loans that exceed the long-run increase, i.e., there is an overshooting in the increase in the return on loans. The overshooting result is a key prediction of our model. To understand the intuition behind this result, it is useful to consider an alternative transition where the return on loans adjusts immediately to the new steady state. On the supply side, a constant return on loans implies a constant loan-to-equity ratio and because equity is below the new steady state value, this implies that total supply of new lending is lower on impact compared to the new steady state value. On the demand side, a constant return on loans imply a constant demand for loans. Because the economy is in equilibrium at the new steady state, this implies that under the scenario of a constant return on loans, there would be on impact an excess demand for loans. As a result, the return on loans have to experience a sharp increase on impact and then decline towards the new steady state.

The overshooting in the return on loans has implications for the dynamics of banks’ balance sheet positions as well. In particular, lending rates $b_t$ are characterized by a drop on impact followed by a continued drop. This gradual decline in lending rates is consistent with the behavior of loan return. That is, the sharp increase on impact in the return on loans mitigate the contraction in lending rates caused by the tightening of capital requirements. Over time, as the return on loans is reduced, lending rates continue to fall. Total lending $B_t$ follows a relatively similar pattern with the key difference that equity growth along the transition path partially compensates the drop in lending rates. As a result, the stock of loans experience a sharp decline although after a few periods, there is an increase in the stock of loans. In the new steady state, total lending remains

\[27\] Recall that the capital requirement constraint is binding in the initial steady state.
below the initial steady state reflecting the tighter capital requirement constraint.

On the other hand, cash holdings both in absolute value and relative to assets and equity fall more in the short run than in the long run. The asymmetry between the response of reserve and lending rates is due to the general equilibrium effects of the tightening in capital requirements. While the tightening of capital requirements limits the expansion of banks’ balance sheets, the increase in the return on loans generate a reallocation in banks portfolio towards loans. There is another effect at work beyond the price effects on the portfolio decisions. Because banks have lower leverage, this also reduces the liquidity risk premium, leading banks to invest relatively more in loans. Hence, banks have a lower liquidity risk premium and become relatively more exposed to illiquidity risk.

6.3 Bank-Run Probability

The next shock we consider is an increase in uncertainty in the process for withdrawals. In particular, we assume that there is a probability that banks would face a bank-run can lose the entire stock of deposits.\textsuperscript{28} This shock arguably played a role in the US financial crises as confidence in the financial sector plummeted, and is consistent with the dispersion on deposit growth documented in section 3. Figure 8 shows the effects of an increase in the bank-run probability from zero to 10 percent. We assume that this shock follows a deterministic AR(1) process so that the shock dies out after about 2 years.

The introduction of a probability of a bank-run generates a more precautionary behavior by banks as the risk of illiquidity becomes larger. Banks respond by reallocating their portfolio from loans to cash. The decline in the supply of loans generate an increase in loan return, i.e. a decline in loan prices and the value of equity, which tightens the capital requirement constraint and amplifies the initial drop in lending. As explained above, the long-maturity of loans imply that the drop in loans is persistent over time.

The reallocation of assets towards cash also generate a persistent decline in equity. Notice that once the bank-run shock dies out, the banking sector has a level of equity which is below the steady state value. As a result, the return on loans remains above steady state and lending (cash) rates are above (below) steady state.

Finally, the introduction of a bank-run probability generates a decline in dividend rates. This follows again from a precautionary behavior in response to a more uncertain value of banks.

6.4 Credit Demand Shocks

We now study the effects of negative credit demand shock, i.e. a decline $\Theta_t$. This shock can be though as capturing a decline in total factor productivity that reduces the demand of loans by

\textsuperscript{28}To keep the distribution somewhat symmetric, we also assume that with the same probability banks can receive a large inflow of deposits that would double the stock of deposits.
firms. However, a credit demand shock could also have a financial origin, e.g., a decline in the value of firm’s or household collateral that limit their ability to borrow. The explicit modelling of the demand shock is beyond the scope of this paper. Figure 10 illustrates the effects of a temporary decline in credit demand. The shock follows a deterministic AR(1) process that lasts for about 7 years.

The effects of credit demand shocks contrast sharply with the effect of the shocks considered above. In particular, banks respond to a lower demand of loans by paying higher dividends and investing in cash. A negative credit demand shock also generates a rise in loan prices as well as an increase in the value of equity, which also makes the capital requirement less tight.

How does the economy converge to the new steady state? Because the demand shock generate low portfolio returns, banks increase dividend rates and experience a reduction in equity. Moreover, as equity is reduced along the transition, loan prices fall leading to increasing portfolio returns until the steady state. Finally, an important observation is that there is on impact a sharp increase in the reserve rate and in the liquidity ratio. Intuitively, banks respond to the decline in credit demand by reallocating their portfolio towards cash and away from loans.

6.5 Policy Shock 1 - Fed Funds Rate

6.6 Policy Shock 2 - Open Market Operation

7 Monetary Policy during 2008-2013

This section describes the patterns observed for monetary policy in the US during the last 5 years. Then, we introduce a particular sequence of shocks into our model and describe how far does our model go in explaining this facts depending on the shocks that we introduce. We begin summarizing some key facts about monetary policy over the last five years.

7.1 Some Facts

Fact 1: Anomalous Interest Rate Behavior. Figure ?? shows three series. The volatile time series corresponds to the daily FED funds rate. The FED funds rate fluctuates around the corridor determined by the interest on the overnight rate and the borrowing rate, the policy instruments used by the FED to implement its policy. These are the analogue to $\chi_i$ in our model. The dashed line corresponds to the 11-month, LIBOR rate, what we interpret as the analogue of $r_i$ in our model.

What we learn from the figure is that prior to the Great Recession, the 11-month Libor rate used to track the FED Funds rate with a slight spread. This spread wideness during the Great Recession and has remained constantly higher. During the crisis (see the zoom in the bottom panel), the FED Funds market actually leaves the bands, a symptom that the FED may have lost
control over its policy instruments. Instead, the 11-month LIBOR rate spread is several points higher.

**Fact 2: FED Balance-Sheet expansion.** Figure 17 shows the assets held in the Balance Sheet of the US Federal Reserve (FED). The picture shows the vast increase in asset holdings corresponding to different types of open-market operations carried out during the period. The counterpart of this figure is the increase in FED’s liabilities. The various FED programs that explain this increase in the FED’s balance sheet are documented by Adrian et al.

**Fact 3: Excess-Reserve Holdings.** Figure 16 shows that the increment in excess reserves of the banking system. The counterpart to the increment in FED assets in Figure 17 are increases in Federal Reserve (FED) ’s liabilities which in turn, are held in the asset side of banks. Figure 16 shows roughly a 16-fold increment in excess reserves (with only slight increment in required reserves). The fact that the increment is in excess reserves, and not required reserves shows that there has not been an equivalent expansion in lending (or deposit creation) by commercial banks.

**Fact 4: Depressed Lending Activities.** Despite the increment in liquidity (FED funds) in the financial sector, lending actually contracts during the sample period. This is shown in the top left panel of Figure 15. The monetary expansion of the last 4 years has not been met with a proportional increment in lending activities. In turn, bank liabilities have also fallen. This is shown in the top right panel. These figures show themselves through the drop in the M1 money multiplier.

**Fact 5: Equity Losses.** Facts 1-4 are associated with a period of unprecedented market-value equity losses for banks. Figure ?? shows the fall in market-value of bank equity during the period. We interpret this fact as a link between facts 1-4. This list of facts opens some questions that we try to answer with our framework.

**Fact 6: Changing Regulatory Environment.**

### 7.2 Questions

We use our model as a laboratory to investigate how important are hypothesis 1-4 in explaining Facts 1-4 and how these may have potentially affected the ability of monetary policy to induce lending (the lending channel) has been interrupted during the Great Recession. We place our model within a perfect foresight environment because we believe this gives us a transparent understanding of the mechanisms at play, and that this shocks are large persistent events.

In our model, bank equity losses will work by a risk aversion effect. Thus, we ask how much can bank equity losses, Hypothesis 1, explain the pattern above described in the previous section?

An immediate policy response to the crisis was the implementation of a sequence of new bank regulations via the Basel-III agreements. Basel-III tightens $\kappa$ and $\eta$ in our model. Consistent with Hypothesis 2, could tighter bank regulation explain the pattern above?

Fact 1 shows anomalous behaviour for FED funds and LIBOR rates during the period. Hy-
hypothesis 3 leads us to ask, wow much of greater uncertainty in the interbank market explain the facts?

The fourth question concerns Hypothesis 4. In addition to the effects of Hypothesis 1-3, how much of a reduction in loan demand do we need to explain the post recession patterns?

7.3 Our Model’s Answers - A Narrative of the last 5 Years

8 Potential Extensions

We want to discuss before concluding. These extensions will allow a more thorough understanding of monetary transmission highlighted in this model.

Nominal contracts and inflation. We deliberately imposed assumptions in our model so that lending contracts are real. Our model can be enriched to incorporate features that may induce changes in prices without corresponding movements in contracts. The introduction of nominal debt contracts and price movements may help us understand the role of Fisherian debt-deflation phenomena (see Krugman and Eggertson, for example.). Another form of nominal contracts can be introduced in the labor market. These nominal frictions, may allow us evaluate to study the role of the interest rate channel, a balance-sheet channel in tandem with the lending channel we study here.

Credit risk and rationing. When we studied the effects of equity losses, we treated these as unforseen. We believe our model may have additional interesting effects if there are periods where banks are aware, but uncertain, about potential equity losses. Moreover, there is no feedback between lending and default probabilities here. We believe our model may be enriched to allow for credit rationing. One possibility is to introduce a problem of asymmetric information as in Bigio (2010). Alternatively, given the long-maturity of lending, our model can used to explain how the lack of new loans affects the probability of older loans to payback the bank. This would stem from a combination of nominal contracts and long-term lending.

9 Conclusions

The bulk of money-macro models has developed independently from the insights from literature on banking (Diamond and Dybvig (1983)). In doing so, the profession has lacked an explicit modeling of monetary policy through the financial system, and for good reasons. For a long time it did not seem to make much difference, monetary policy seemed to be carried with ease and with stable banks, equity growth, leverage, bank dividends and interest premia. Thus, in absence of crises, the activities of the financial sector can appear irrelevant for long stretches of time. The crisis, revealed that having a model that allows us to study banks in conjunction with banking may be a desired tool in our theoretical toolbox.
This paper has borrowed from the banking literature, to introduce a model where by maturity mismatch between assets and liabilities cause banks to demand reserves. A Central Bank can influence real activity by altering the trade-off in bank lending via various inputs. This model sheds light on what type of shocks may reduce the power of monetary policy. We have used this model to understand the effects of various shocks on the power of monetary policy.
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42


10 Additional Figures
Figure 6: Impulse Response to Equity Losses
Figure 7: Impulse response to permanent gradual increase in capital requirements
Figure 8: Impulse response to temporary increase in the probability of a bank-run
Figure 9: Temporary decrease in Loan Demand (Aggregates)

Figure 10: Impulse response to temporary decrease in loan demand
Figure 11: Temporary decrease in $\chi$ (Aggregates)

Figure 12: Temporary decrease in $\chi$ (Individual)
Figure 13: Temporary decrease in $r$ (Aggregates)

Figure 14: Temporary decrease in $r$ (Individual)
Figure 15: Commercial Bank Assets: 2002-2012. The figure shows several measures of commercial bank lending.
Figure 16: Federal Reserve Assets: 2002-2012. The figure shows the evolution of the Commercial Bank excess (blue) and required (red) reserve holding.
Figure 17: **Federal Reserve Assets: 2002-2012.** The figure shows the expansion in the FED’s asset holdings. The magnitudes are in Millions of US$. 

![Federal Reserve Assets Graph]

53
Figure 18: **Fed Funds Rate 2002-2012.** The figures plot the evolution of the FED Funds rate, the 11-month Libor rate and the overnight lending and borrowing rates.
11 Algorithm for Computing Transitional Dynamics

12 Liquidity Management

**Bank Balance Sheets.** To clarify the ideas in the paper, consider the balance sheet of a bank depicted in the left panel of Figure 19. Banks hold central bank reserves and loans as part of their assets. They hold deposits on demand and equity as liabilities.

When they provide loans, banks are effectively granting credit lines in the form of checks or an electronic account. These credit lines enable borrowers to make payments to purchase goods. After the payment is made, whoever receives those payments, holds those bank liabilities. In turn, the holder can make further payments, and so on. What matters for the bank is that when it provides a loan, it is simultaneously creating a liability. Of course, for every face-value dollar it lends, banks issue less than one-for-one in liabilities. They earn an interest. This increases the bank’s equity. The right panel of Figure 19 shows the balance sheet of a bank after a loan expansion. The accounts show how loan creation is profitable. This balance sheet expansion constitutes the process of monetary creation.

**Liquidity Risk.** A critical feature of our framework is that deposits can be withdrawn immediately. Often, large withdrawals will occur before a loan matures. Hence, when a large withdrawal occurs, deposits are transferred from one bank to another. The receptor bank takes over the liabilities of another bank and will, therefore, request the other bank an assets to settle the transaction. The first bank could, in theory, transfer loans but loans are not liquid.\(^{30}\)

---

\(^{29}\) One other possibility is to raise deposits from another bank. The other bank will transfer reserves. This reserves can be sold to purchase loans.

\(^{30}\) The issuer bank may be a specialist on a particular sector, there may be private information or moral-hazard concerns. Transferring loan may be costly due to transaction costs or the bank may fear losing the client. Another
When reserves are insufficient to meet payments, banks must borrow them from other banks or from the central bank. This, induces a potential illiquidity cost. The left panel of Figure 21 describes this possibility. Suppose there is a withdrawal that exceeds the level of currency reserves at the bank. Banks must rely on costly overnight borrowing to compensate for this withdrawal. These costs induce a reduction in bank equity, and an overall shrinkage of the banks balance sheet.

Comparing equity before the loan expansion in the left panel of Figure 19 with the equity in the right left of Figure 21, we depict the losses that may occur if banks expand there assets too much and a large deposit withdrawal occurs. If withdrawal risks are independent of the size of the balance sheet, the higher the amount of lending, the higher the liquidity risk. To see this, notice that the relative size of reserves to deposits, the bank’s liquidity ratio, falls with the amount of lending (see Figure 20). With less reserves, the bank is forced to use more borrowing to finance the shortage liquidity.

Liquidity risk affects the bank’s decisions to lend because it exposes the bank to more risk. Thus, when providing a loan, banks must take into account how this affects their liquidity ratio and how this increases the risk of incurring in additional costs. In other words, the value of additional lending is its spread minus the potential illiquidity risk. This trade off is clear by comparing the size of equity in Figures 19 and 21. Monetary policy has the power to affect this trade off.

**Policy Instruments.** In practice, there is a plethora of monetary policy instruments that possibility is securitization, but often this requires complicated structuring to align the incentives of the seller bank. In all this instances, banks rely on reserves to satisfy the needs of the payment system. Naturally, if banks become more cautious for any reason, this has real effects because it may induce real effects in economic activity by decreasing the availability of mediums of exchange in the economy. Assuming that the non-financial sector cannot create this with the same ease as the financial sector.
can be used by central banks to alter the trade off described above. However, most Central Banks rely only on three: conventional open-market operations, discount window lending and reserve requirements. These three tools of monetary policy affect the liquidity management trade-off in different ways, but they all carry the same effect.

Open market operations are large purchase of loans that substitute loans for reserves. In practice, these purchases are transactions on Treasury Bonds (T-Bills) which can be seen as liquid loans. For now let’s abstract from the presence of government paper. In our framework, the exchange of liquid assets for illiquid assets will cause a reduction in the liquidity risk faced by those institutions participating in the bond sale. Since at least part of the banking system will have more reserves, some institution will be inclined to do more lending. Since reserves correspond to M0, and loans are granted by creating deposits, the response by banks corresponds to an endogenous monetary creation.

The discount window operates differently in the model. It affects the financial losses incurred after a withdrawal because banks that lack the reserves will be able to borrow at a lower rate. By reducing financial losses, a reduction in the borrowing rates, banks are more inclined to doing more lending. Finally, reserve requirements operate by determining what is the level of currency after which banks have to start borrowing reserves. Penalties on missing that target are another possible instrument.


13 Data Analysis

1990-2010 Sample Averages. The banking industry underwent a consolidation over the last two decades. Due to the substantial amount of mergers in the industry and measurement error, aggregating balance sheet information directly would be misleading. Hence, we isolates the effects of mergers on the increase in the volatility of different bank liabilities by eliminating observations with observations that lie more than four standard deviations or that exhibit negative entries.

The summary statistics for the quarterly growth rate of the aggregate time series is presented in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.085</td>
<td>771410</td>
</tr>
<tr>
<td>DD</td>
<td>1.029</td>
<td>0.191</td>
<td>778467</td>
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<td>TL</td>
<td>1.022</td>
<td>0.07</td>
<td>773629</td>
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<tr>
<td>TE</td>
<td>1.017</td>
<td>0.083</td>
<td>769077</td>
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<tr>
<td>RTE</td>
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<td>0.097</td>
<td>766806</td>
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<tr>
<td>E</td>
<td>1.018</td>
<td>0.072</td>
<td>774407</td>
</tr>
<tr>
<td>RE</td>
<td>1.018</td>
<td>0.086</td>
<td>769338</td>
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</table>

Table 2: Summary statistics.

The data exhibits very similar patterns for the growth of total deposits and total liabilities. Demand deposits are 2.5 times more volatile than all the deposits. This may respond to a stronger seasonality in this variables. One of the reasons why we use total deposits as our data counterpart for deposits in our model is that it is substantially volatile, the standard deviation is 8.5% per quarter, and it is close to the volatility of total liabilities, 7.0%. Moreover, demand deposits may be exchanged for deposits of longer maturity, which explains why total deposits are less volatile (being a sub-account), but through the lens of our model, this would be as a change in one account for another within the same bank. This can be observed from in the Table 3.

Figure 22 presents the evolution of the growth rates of these time series subtracting the growth rate of the GDP deflator. All the series show a strong seasonal component. The wider curve presents the Hodrick-Prescott filtered series. The trends reveal a decline in the growth rates towards the end of the sample, corresponding to the period of the Great Recession and onwards.
<table>
<thead>
<tr>
<th>Variables</th>
<th>TD</th>
<th>DD</th>
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<th>TE</th>
<th>RTE</th>
<th>E</th>
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<td>TL</td>
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<td>1.000</td>
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<td></td>
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<td>TE</td>
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<td>0.010</td>
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<td>1.000</td>
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<td></td>
</tr>
<tr>
<td>RTE</td>
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<td>0.096</td>
<td>0.858</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
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<td>0.737</td>
<td>0.647</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>RE</td>
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<td>0.024</td>
<td>0.184</td>
<td>0.647</td>
<td>0.777</td>
<td>0.885</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 3: Cross-Sectional correlation for Quarter-Bank observations

![Graphs](Figure 22: Cross-Sectional Average Growth Rates)
For our quantitative analysis, we are particularly interested in the behavior of the series for bank equity. The figure shows a decline in filtered growth rates, but due to the strong volatility in the period, the filtered series does not show a decline in levels. One of the hypotheses that we consider in our quantitative analysis is the decline in equity during the Great Recession. A snapshot of the compounded growth rates reveals a mild decline in the book value of tangible equity even adjusting for Loan and Loss Allowances. Figure 23 shows the pattern. The evolution of the book value may be distorted by other factors such as TARP, and not materialized losses outside the book value. Hence, we will stretch the results and show use a benchmark of 5% for our book value equity losses.

Figure 23: Evolution of RTE during the Great Recession.
Quarterly Cross-Sectional Deviations. Part of the variation in the bank-quarter statistics presented above have a common component, including seasonality, nominal changes in the time series and aggregate trends. To decompose the variation of these liabilities into their common component, we present the summary statistics in terms of deviations of these variables from their quarterly cross-sectional averages. Table 4 presents the results:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.084</td>
<td>771369</td>
</tr>
<tr>
<td>dev DD</td>
<td>0</td>
<td>0.183</td>
<td>777932</td>
</tr>
<tr>
<td>dev TL</td>
<td>0</td>
<td>0.069</td>
<td>773629</td>
</tr>
<tr>
<td>dev TE</td>
<td>0</td>
<td>0.081</td>
<td>769073</td>
</tr>
<tr>
<td>dev RTE</td>
<td>0</td>
<td>0.096</td>
<td>766803</td>
</tr>
<tr>
<td>dev E</td>
<td>0</td>
<td>0.071</td>
<td>774401</td>
</tr>
<tr>
<td>dev RE</td>
<td>0</td>
<td>0.085</td>
<td>769329</td>
</tr>
</tbody>
</table>

Table 4: Summary statistics

One thing one gathers from this table is that most of the variation is preserved even when one subtracts the evolution of aggregate averages. This shows the substantial amount of idiosyncratic volatility among banks. Except for the behavior of demand deposits, the volatility of the liabilities of banks is almost exclusively idiosyncratic. This can be seen from Table 5 which shows the behavior of the correlation in cross-sectional deviations from quarterly means. These correlations are essentially the same as the correlations for historical growth rates implying that the idiosyncratic component is quite high.

The correlation in the data between deviation of tangible equity growth and the deviation in the growth rate of total deposits ranges from 5% to 15% depending on the definition of equity that we use. In the model, this correlation will be very high (though not 1) because deposit volatility is the only source of risk for banks. In practice, banks face other important sources of risks such as loan risk, duration risk and trading risk. This figure however implies that deposit withdrawal risks are non-negligible risks for banks. Figure 1 in the body of the paper, reports the empirical histograms for every quarter-bank growth observation and decomposes the data into two samples pre-crisis (1990Q1-2007Q4) and crisis (2008Q1-2010Q4).
We use the empirical histogram of the quarterly deviations of TD to calibrate $F_t$, the process for withdrawal shocks in our model. In the quantitative analysis, we contrast the behavior of equity volatility that results as an outcome with the corresponding histogram and correlation in the date for this variable.

We also analyze the evolution of the volatility in the variables. This analysis provides the basis for our calibration of the increase in withdrawals shocks during the Great Recession, one of the hypothesis that we test. Figure 24 shows the time series for cross-sectional dispersion in growth rates in all of the series that we study. As the cross-sectional averages these series display a high seasonal component. The Hodrick-Prescott filter of the series reveals non-negligible increases in cross-sectional dispersion, in particular after 2009. The cross-sectional dispersion of all the measures of equity show a 60% increase at the peak of the Great Recession.

Tests for Growth Independence. Our models assumes that the withdrawal process is i.i.d over time and banks. This assumption implies that if we subtract the common growth rates of all the balance sheet variables in our model, the residual should be serially uncorrelated. We test the independence of the deviations-from-means quarterly growth rates using an OLS estimation procedure. We run the deviations in quarterly growth rates from the cross-sectional averages against their lags. The evidence from OLS auto-regressions support the assumption that of time-independent growth. Tables 6 we report coefficients that are statistically identical to zero (with zeros lying within the two standard deviation bounds) for all of the variables of that we study.

14 Evolution of Bank Equity Distribution

Because the economy displays equity growth, equity is unbounded and thus, the support of this measure is the positive real line. Let $\mathcal{B}$ be the Borel $\sigma$-algebra on the positive real line. Then,
Figure 24: Quarterly Cross-Sectional Dispersions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>(Std. Err.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD</td>
<td>0.001</td>
<td>(0.001)</td>
</tr>
<tr>
<td>DD</td>
<td>0.000</td>
<td>(0.001)</td>
</tr>
<tr>
<td>TL</td>
<td>-0.001</td>
<td>(0.001)</td>
</tr>
<tr>
<td>RTE</td>
<td>0.000</td>
<td>(0.001)</td>
</tr>
<tr>
<td>TE</td>
<td>-0.001</td>
<td>(0.001)</td>
</tr>
<tr>
<td>E</td>
<td>-0.001</td>
<td>(0.001)</td>
</tr>
<tr>
<td>RE</td>
<td>0.000</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

Table 6: Autocorrelation Estimates for First Lag of Growth Rates Cross-Sectional Deviations
define as $Q_t(e, E)$ as the probability that an individual bank with current equity $e$ transits to the set $E$ next period. Formally $Q_t : \mathbb{R}_+ \times \mathcal{B} \rightarrow [0, 1]$, and

$$Q(e, E) = \int_{-1}^{1} \mathbb{I} \{ \Psi_t(\omega) e \in E \} F(\omega)$$

where $\mathbb{I}$ is the indicator function of the event in brackets. Then $Q$ is a transition function and the associated $T^*$ operator for the evolution of bank equity is given by:

$$\Gamma_{t+1}(E) = \int_{0}^{1} Q(e, E)\Gamma_{t+1}(e) \, de.$$ 

The distribution of equity is fanning out and the operator is unbounded. Gibrat’s law shows that for $t$ large enough $\Gamma_{t+1}$ is approximated well by a log-normal distribution. Moreover, by introducing more structure into the problem, we could easily obtain a Power law distribution for $\Gamma_{t+1}(E)$. We will use this properties in the calibrated version of the model.

15 Introducing T-Bills

This section describes how we can introduces government bonds (T-Bills) to the analysis. For now, we maintain the assumption that loans to the private sector are riskless. We allow the FED and banks to hold T-Bills in addition to loans in their balance sheets. This analysis helps us understand the distinctions between conventional and unconventional policies through the lens of our model. There are two interpretations regarding the liquidity of T-Bills.

**Liquid T-Bills.** If bonds are as liquid as reserves, T-Bills are close substitutes of reserves. The only differences is for this purpose is that T-Bills cannot be used to meet reserve requirements.

**Illiquid T-Bills.** When bonds cannot be sold during the lending stage, loans and T-Bills become substitutes. In what follows, we assume that bonds bear the same liquidity as loans.

**T-Bill Supply.** We assume that T-Bills are exogenously supplied by the treasury. As loans, T-Bills yield a sequence of coupon payments. Banks demand T-bills and therefore, T-Bills compete with loans to attract lending. The FED, in turn, can purchase bonds participating in the T-Bill market. The supply of T-Bills is isomorphic to the supply of private loans. We denote by $q_t^T$ the relative price of T-Bills to deposits. The supply is given by:

$$q_t^T = \Theta_t^T (t_t^s)^\tau$$

where $t_t^s$ is the flow of T-Bills bought by the financial sector. This form is flexible enough to allow for a fixed supply of T-Bills. We introduce curvature and a random slope, $\Theta_t^T$, to allow for the realistic assumption that the private sector, or the external sectors have an exogenously fluctuating demand for these assets. We assume that T-Bills potentially have a different maturity than regular loans which is a primary source of the spread between both assets. The maturity of
T-Bills follows the same structure, but the parameter that governs this is $\delta^T$.

**Bank’s Problem with Treasury Bills.** When allowing for T-Bills, banks hold T in current bills and purchase $\psi$. The laws of motion for deposits and T-Bills satisfy:

$$
\begin{align*}
\dot{D} & = D + q^B I + DIV + \varphi(1 + r) + q^T \psi + (1 - \delta^T) T \\
\dot{T} & = \delta^T T + \psi
\end{align*}
$$

and the constraints from financial regulation yields:

$$
\begin{align*}
\dot{D} & \leq \kappa(\dot{B} + \dot{T} + \dot{C} - \dot{D}), \dot{D} \geq 0 \\
\dot{C} & \geq \eta(\dot{B} + \dot{T} + \dot{C} - \dot{D}), \dot{C} \geq 0.
\end{align*}
$$

We can consolidate the balance sheet at the lending stage by setting and rearrange terms to obtain:

$$
DIV + qB' + \dot{C}(1 + r) + qT' - \dot{D} = C(1 + r) + q\delta B + B(1 - \delta) + T(1 - \delta^T) + q^T \delta^T T - D.
$$

We can redefine equity on hand as before by allowing for past holdings of T-Bills:

$$
E = C(1 + r) + q\delta B + B(1 - \delta) + T(1 - \delta^T) + q^T \delta^T T - D.
$$

Naturally, there is crowding out effect in this environment since T-Bills and loans compete for bank deposits.

**Return on T-Bills.** We denote by $R_T^T(\delta^T)$ the return on a T-Bill with maturity $\delta^T$. This return depends on prices and the maturity of the loan and is adjusted for liquidity:

$$
R_T^T(\delta^T) \equiv \frac{\delta^T q^{T_{t+1}} + (1 - \delta^T)}{q^T_t}.
$$

Proposition xxx is the analogue of xxxx.

**Proposition 6 (Portfolio)** $\Omega(X)$ is also the solution to the following liquidity-management portfolio problem:

$$
\max_{\{w_b, w_d, w_c\}} \mathbb{E}_{\mathbb{W}} \left[ (R_B^T(\delta) w_b + R_t^T(\delta^T) w_t + R^C w_c - R^D w_d - R^\chi(w_d, w_c))^{1-\gamma} \right]^{\frac{1}{1-\gamma}}
$$

subject to,

$$
\begin{align*}
1 & = w_b + w_t + w_c - w_d \\
w_d & \leq \kappa \left( \frac{w_b}{q} + \frac{w_c}{(1 + r)} - w_d \right), w_d, w_c, w_b \geq 0 \\
\frac{w_c}{(1 + r)} & \geq \eta \left( \frac{w_b}{q} + \frac{w_c}{(1 + r)} - w_d \right).
\end{align*}
$$

**Arbitrage.** Note that for a bank, an investment in a loan or a T-Bill yield is the same. This imposes the following equilibrium condition $R_B^T(\delta) = R_t^T(\delta^T)$. This condition will not hold true once banks face idiosyncratic credit risk on loan investments (as long as they are not sufficiently diversified). Thus, so long as $R_b^T(\delta) = R_t^T(\delta^T)$, we can solve the model using Proposition XXX

65
and work only with the sum \(w_b + w_t\), but not the composition of \(w_b + w_r\) will be determined. Thus, if the FED doesn’t carry any open market operations, the model in this section is isomorphic to the one in the preceding sections except that market clearing conditions differ.

**Conventional Open-Market Operations.** A conventional OMO involves the purchase of T-Bills and the issuance of reserves. This operation is carried out in a tandem of operations which leave the holdings of commercial bank deposits at the FED unchanged. For this, the FED issues M0 to buy bank deposits, but then neutralizes the operation through open market operations via purchases of T-bills. In that case, the FED’s balance sheet satisfies \((1 + r_t) \varphi_t - q_t^T \psi_t = 0\), for positive values of \(t\). Here, T-Bills are purchased in amount, \(\psi_t\) and \(\varphi_t\) are the amount of deposits that neutralize the transaction. Thus, \((1 + r_t) \varphi_t = q_t^T \psi_t\), so the rate of change in the money supply is given by:

\[
\Delta M_0 t = \frac{q_t^T}{(1 + r_t)} \psi_t.
\]

There are four market clearing conditions, one for each possible market. The first is that the flow of T-bills is in equilibrium. This requires \(\psi_t^s = (\Theta_t^T)^{-1} (q_t^T)^{\frac{1}{2}}\). Hence, market clearing requires: \(\psi_t + \psi_t^{FED} = \psi_t^s\). We can express these same equation in terms of the stocks of asset holdings. In this case:

\[
(bond \text{ market}) \ T_{t+1} - \delta T_t + T_{t+1}^{FED} - \delta T_t^{FED} = (\Theta_t^T)^{-1} (q_t^T)^{\frac{1}{2}}
\]

so one can invert this condition and obtain: \(q_t^T = \Theta_t^T (T_{t+1} - \delta T_t + T_{t+1}^{FED} - \delta T_t^{FED})^\tau\).

**16 Derivation of Loan Demand**

We now turn to the real side of the model which is deliberately stylized. The idea is to present an environment where there is a real demand for loans, without complicating the analysis much. We make assumptions such that lending will have real effects despite that the model has no nominal rigidities. Thus, monetary policy carries out real effects in a fully real environment. In terms of the model, the goal here is to obtain a demand for loans like the one in (6).

Thus, the economy is composed of an discrete infinite amount of islands, indexed by \(\tau\). Each island is populated by workers and entrepreneurs. Islands are discovered in period \(\tau\).

**Workers.** Workers choose consumption, labor and deposit holdings. Workers don’t have access to any additional savings technologies. Their period utility is given by:

\[
\max_{\{a_t\}_{t \geq 0}, h_0 \geq 0} \sum_{t=0}^\infty c_t \ - \frac{(h_t)^{1+\nu}}{(1 + \nu)}
\]

where \(h_0\) is the labor supply and \(c\) consumption and \(\nu\) is the inverse of the Frisch elasticity. The worker’s budget constraint in period \(t\) is given by:
where \(d_{t+1}^w\) is the deposits held by the worker in period \(t\), \(w_t\) is the wage and \(p_t\) the price of the good. It is the case that \(d_t^w = 0\), for all \(t \leq \tau\). Workers only work during the period of discovery \(\tau\), so without loss of generality \(h_t = 0\) for \(t \neq \tau\). The only role for workers is to provide the elastic labor-supply schedule. The only reason why they are long-lived is to allow us to talk about maturity in this model. Since deposits must be cleared out slowly, household’s are long-lived. Also, notice that household’s don’t discount consumption over time. This is enough to argue that they are indifferent between consumption in any period so long as the price is constant. This allows us to eliminate any form of price variation in the model together with zero interest on reserves.\(^{32}\)

**Problem 3 (Workers)** *The worker solves:*

\[
U_t^w (\tau, d_t^w) = \max_{c_t, h_t} c_t - \frac{(h_t)^{1+\nu}}{(1+\nu)} + U_{t+1}^w (\tau, d_{t+1}^w)
\]

subject to

\[
d_{t+1}^w + p_t c_t = d_t^w + w_t h_t
\]

with \(d_0^w = 0\), and \(h_t = 0\), for \(t \neq \tau\).

**Entrepreneurs.** Entrepreneurs have utility

\[
\max_{\{c_t \geq 0 \}} \sum_{t=0}^{\infty} c_t
\]

They start their lives with a capital stock of \(k_t\). In addition, the have access to a loan market. Their budget constraint in period \(t\) is given by:

\[
p_t c_t = r_t^e k_t.
\]

**Production Technology.** Production is carried out via \(k\), combined with labor, \(h\), using a Cobb-Douglas technology \(F(k, h) \equiv k^\alpha h^{(1-\alpha)}\) to produce output. Profits are \(AF(k, h) - wh\). Workers are hired from an elastic supply schedule, \(w = (h^w)^\nu\) determined from the worker’s problem.

Before production, an entrepreneur hires an amount of labor promising to pay \(wl\). It is possible that the entrepreneur reneges on this promise and defaults on his entire payroll. This enforcement problem is not present between banks and firms. In particular, banks can fully enforce their loan contracts. The lack of commitment between firms and workers induces a financial need by firms. Firms can borrow from banks to finance payroll, issuing a liability to the banking sector, whereas

\(^{32}\)The model could be modified to allow workers to save. With GHH preferences, this would not alter the labor supply. However, the model would require and additional state variable but the substance of the model would not change. GHH are commonly used to prevent counterfactual contractions in the labor supply.
banking institutions take liabilities in the form of deposit accounts with their workers.

Production is distributed over time via the following relationship. \( y_t = (1 - \delta) \delta^{1 - \tau} R^e_\tau \) where \( R^e_\tau \) is the production per entrepreneur in island \( \tau \).

This delivers a problem for banks similar to Christiano and Eichenbaum (1992). This problem is given by:

**Problem 4 (Producer)** The entrepreneur solves:

\[
R^e_\tau (k; q^l_t, w_t) = \max_{k, h} Ak^{\alpha} h^{1-\alpha} - w_t h - (l - q^l_t l)
\]

subject to:

\[
(1 - \text{tax}) w_t h \leq q^l_t l
\]

In this problem, \( q^l \) is the amount of deposits (or credit) available to the firm. The firm uses this credit is used to finance the payroll of the firm. The tax allows us to introduce a labor market distortion that affects the real demand for loans. The solution to this problem is given by:

**Proposition 7 (Loan Demand)** In equilibrium:

\[
q^l_t \equiv ((1 - \tau) A (1 - \alpha) k^{\alpha}) \left( \frac{\nu + 1}{1 - \alpha} \right) l \frac{\alpha(1 + 2\nu)}{(1 - \alpha)^2}
\]

so \( \epsilon = -\frac{\alpha(1 + 2\nu)}{(1 - \alpha)^2} \) and \( \Theta \equiv ((1 - \tau) A (1 - \alpha) k^{\alpha}) \). In addition, \( p_t = 1 \) without loss of generality.

**Equilibrium with Real Sector.** A competitive equilibrium with the real sector is, a partial equilibrium where the loan demand is given by Proposition 7.

17 Proofs

17.1 Proof of Propositions 1, 2 and 3

This section provides a proof of the optimal policies described in Section 3.4. The proof of Proposition 1 is straight forward by noticing that once \( E \) is determined, the banker does not care how he came-up with those resources. The proof of Propositions 2 and 3 is presented jointly and the strategy is guess and verify. Let \( X \) be the aggregate state. We guess the following.

\[
V (E; X) = v (X) E^{1-\gamma} \]

where \( v (X) \) is the slope of the value function, a function of the aggregate state that will be solved for implicitly. Policy functions are given by: \( DIV (E; X) = div (X) E \), \( \tilde{B} (E; X) = \tilde{b} (E; X) E \), \( \tilde{D} (E; X) = \tilde{d} (E; X) E \) and \( \tilde{C} (E; X) = \tilde{c} (E; X) E \).

17.1.1 Proof of Proposition 2

Given the conjecture for functional form of the value function, the value function satisfies:
\[ V(E;X) = \max_{DIV,\tilde{B},\tilde{D}} U(DIV) + \beta \mathbb{E} \left[ v(X') (E')^{1-\gamma} \right] |X] \]

Budget Constraint: \[ E = q\tilde{B} + \tilde{C}(1 + r) + DIV - \tilde{D} \]

Evolution of Equity: \[ E' = (q' \delta + (1 - \delta)) \tilde{B} + \tilde{C} (1 + r_c) (1 + r') - \tilde{D}(1 + (1 + r_c)r'\omega') - \chi((\rho + \omega) \tilde{D}) \]

Capital Requirement: \[ \tilde{D} \leq \kappa(\tilde{B}q + \tilde{C}(1 + r) - \tilde{D}) \]

Liquidity Requirement: \[ \tilde{C} \geq \eta(\tilde{B} + \tilde{C} - \tilde{D}) \]

where the form of the continuation value follows from our guess. We can express all of the constraints in the problem as linear constraints in the ratios of \( E \). Dividing all of the constraints by \( E \), we obtain:

\[
1 = div + q\tilde{b} + (1 + r)\tilde{c} - \tilde{d} \\
E'/E = (q' \delta + (1 - \delta)) \tilde{b} + \tilde{c} (1 + r_c) (1 + r') - \tilde{d}(1 + (1 + r_c)r'\omega') - \chi((\rho + \omega) \tilde{d} - \tilde{c}) \\
\tilde{d} \leq \kappa(\tilde{B}q + \tilde{C}(1 + r) - \tilde{D}) \\
\tilde{c} \geq \eta(\tilde{b} + \tilde{c} - \tilde{d})
\]

where \( div = DIV/E, \tilde{b} = \tilde{B}/E, \tilde{c} = \tilde{C}/E \) and \( \tilde{d} = \tilde{D}/E \). Since, \( E \) is given at the time of the decisions of \( B,C,D \) and \( DIV \), we can express the value function in terms of choice of these ratios. Substituting the evolution of \( E' \) into the objective function, we obtain:

\[
V(E;X) = \max_{div,\tilde{c},\tilde{d}} U(divE) + \beta \mathbb{E} \left[ v(X') (R(\omega, X', X') E')^{1-\gamma} \right] |X] \\
1 = div + q\tilde{b} + (1 + r)\tilde{c} - \tilde{d} \\
\tilde{d} \leq \kappa(\tilde{B}q + \tilde{C}(1 + r) - \tilde{D}) \\
\tilde{c} \geq \eta(\tilde{b} + \tilde{c} - \tilde{d})
\]

where we use the fact that \( E' \) can be written as:

\[ E' = R(\omega, X, X') E \]

where \( R(\omega, X, X') \) is the realized return to the bank’s equity and defined by:

\[
R(\omega, X, X') \equiv (q(X') \delta + (1 - \delta)) \tilde{b} + (1 + r(X'))\tilde{c} - (1 + r(X')\omega')\tilde{d} - \chi((\rho + \omega) \tilde{d} - \tilde{c})
\]

We can do this factorization for \( E \) because the evolution of equity on hand is linear in all the term where prices appear. Moreover, it is also linear in the penalty \( \chi \) also. To see this, observe
that $\chi\left((\rho + \omega) \tilde{D} - \tilde{C}\right) = \chi\left((\rho + \omega) \tilde{d} - \tilde{c}\right) E$ by definition of $\{\tilde{d}, \tilde{c}\}$. Since, $E \geq 0$ always, we have that

$$(\rho + \omega) \tilde{D} - \tilde{C} \leq 0 \iff (\rho + \omega) \tilde{d} - \tilde{c} \leq 0.$$ 

Thus, by definition of $\chi$,

$$\chi((\rho + \omega) \tilde{D} - \tilde{C}) = \begin{cases} E \chi((\rho + \omega) \tilde{d} - \tilde{c}) & \text{if } (\rho + \omega) \tilde{d} - \tilde{c} \leq 0 \\ E \chi((\rho + \omega) \tilde{d} - \tilde{c}) & \text{if } (\rho + \omega) \tilde{d} - \tilde{c} > 0 \end{cases}.$$ 

Hence, the evolution of $R(\omega, X, X')$ is a function of the portfolio ratios $b, c$ and $d$ but not of the level of $E$. With this properties, one is allowed to factor out, $E^{1-\gamma}$ from the objective because it is a constant when decisions are made. Thus, the value function may be written as:

$$V(E; X) = E^{1-\gamma} \left[ \max_{\text{div,} \tilde{c}, \tilde{b}, \tilde{d}} U(\text{div}) + \beta E \left[ \tilde{v}(X') R(\omega, X, X')^{1-\gamma} | X \right] \right]$$

(10)

$$1 = \text{div} + q\tilde{b} + (1 + r)\tilde{c} - \tilde{d}$$

$$\tilde{d} \leq \kappa(Bq + \tilde{C}(1 + r) - \tilde{D})$$

$$\tilde{c} \geq \eta(b + \tilde{c} - \tilde{d}).$$

Then, let an arbitrary $\tilde{v}(X)$ be the solution to:

$$\tilde{v}(X) = \max_{\text{div,} \tilde{c}, \tilde{b}, \tilde{d}} U(\text{div}) + \beta E \left[ \tilde{v}(X') R(\omega, X, X')^{1-\gamma} | X \right]$$

$$1 = \text{div} + q\tilde{b} + (1 + r)\tilde{c} - \tilde{d}$$

$$\tilde{d} \leq \kappa(Bq + \tilde{C}(1 + r) - \tilde{D})$$

$$\tilde{c} \geq \eta(b + \tilde{c} - \tilde{d}).$$

We now shows that if $\tilde{v}(X)$ exists, $v(X) = \tilde{v}(X)$ verifies the guess to our Bellman equation. Substituting $v(X)$ for the particular choice of $\tilde{v}(X)$ in (10) allows us to write $V(E; X) = \tilde{v}(X) E^{1-\gamma}$. Note this is true because maximizing over $\text{div, } \tilde{c}, \tilde{b}, \tilde{d}$ yields a value of $\tilde{v}(X)$. Since, this also shows that $\text{div, } \tilde{c}, \tilde{b}, \tilde{d}$ and independent of $E$, and $DIV = \text{div} E, \tilde{B} = \tilde{b} E, \tilde{C} = \tilde{c} E$ and $\tilde{D} = \tilde{d} E$.

17.1.2 Proof of Proposition 3

We now solve for $\tilde{v}(X)$ and show that the value of equity on hand can be written as a consumption-savings problem and an independent portfolio problem. To do so, let $\tilde{b}, \tilde{c}$ and $\tilde{d}$ be the fraction of
loans, reserves and deposits invested by the bank after paying dividends. To simplify notation, we suppress the reference to the aggregate state $X$. These functions are mathematically defined as:

$$\dot{b} \equiv \frac{\bar{b}}{1 - \text{div}}, \dot{c} \equiv \frac{\bar{c}}{1 - \text{div}} \text{ and } \dot{d} \equiv \frac{\bar{d}}{1 - \text{div}}.$$  

and collecting terms we obtain:

$$\text{div} + (1 - \text{div}) \left( q \dot{b} + (1 + r) \dot{c} - \dot{d} \right) = 1.$$  

Thus, the resource constraint for the bank can also be written as,

$$q \dot{b} + (1 + r) \dot{c} - \dot{d} = 1.$$  

Similarly, we can multiply the capital and liquidity constraints by $(1 - \text{div})$ and express the constraints in:

$$\dot{d} \leq \kappa (b + c - d)$$  
$$\dot{c} \geq \eta (b + c - d).$$  

To make further progress, we employ the principle of optimality and solve for $\{\dot{b}, \dot{c}, \dot{d}\}$, assuming we already know $\text{div}$. Let’s assume an arbitrary $\text{div}^\circ$ as the optimal choice in $v(X)$. Then $v(X)$ also satisfies:

$$v(X) = \max_{b, c, d} U(\text{div}^\circ) + (1 - \text{div}^\circ)^{1-\gamma} \beta \mathbb{E} \left[ v(X') R(\omega, X, X')^{1-\gamma} \right] X$$

subject to:

$$1 = q \dot{b} + \dot{c}(1 + r) - \dot{d}$$  
$$\dot{d} \leq \kappa (b + c - d)$$  
$$\dot{c} \geq \eta (b + c - d).$$  

We note that $R(\omega, X, X') = (q(X') \delta + (1 - \delta)) \dot{b} + (1 + r(X')) \dot{c} - (1 + r(X') \omega') \dot{d} - \chi((\rho + \omega) \dot{d} - \dot{c})(1 - \text{div}^\circ)$. Since $R(\omega, X, X')$ only enters in the continuation utility, then, $\{b, c, d\}$ must solve:

$$\max_{b, c, d} \mathbb{E} \left[ v(X') (q(X') \delta + (1 - \delta)) \dot{b} + (1 + r(X')) \dot{c} - \dot{d}(1 + r(X') \omega') - \chi((\rho + \omega) \dot{d} - \dot{c})^{1-\gamma})X \right].$$
subject to

\[ 1 = q \dot{b} + c(1 + r) - \dot{d} \]
\[ \dot{d} \leq \kappa (b + c - \dot{d}) \]
\[ \dot{c} \geq \eta (b + c - \dot{d}) \]

if it is part of a solution, otherwise there is a better solution to \( v(X) \).

When, \( X' \) is deterministic \( v(X') \) is known at stage \( X \). In such cases, we can factor out this problem out of the max. However, we must be careful with the sign of \( v(X') \) since the max operator switches to a min operator when we change signs. We will show that when \( \gamma > 1 \), \( v(X') \) is negative, so we need to minimize term inside the brackets. To prevent changing the max operator to a min operator, we use the certainty equivalent operator. Thus, \( \{ \dot{b}, \dot{c}, \dot{d} \} \) are solutions to:

\[ \Omega(X) = \max_{\dot{b}, \dot{c}, \dot{d}} \mathbb{E}_\omega \left[ (q(X') \delta + (1 - \delta)) \dot{b} + (1 + r(X')) \dot{c} - \dot{d}(1 + r(X') \omega') - \chi((\rho + \omega) \dot{d} - \dot{c}) \right]^{1 - \gamma} \]

subject to

\[ 1 = q \dot{b} + c(1 + r) - \dot{d} \]
\[ \dot{d} \leq \kappa (b + c - \dot{d}) \]
\[ \dot{c} \geq \eta (b + c - \dot{d}) \].

When, \( (1 - \gamma) < 0 \), the solution to \( \Omega(X) \) will be equivalent to minimizing the objective. For \( \gamma \to 1 \), the objective becomes:

\[ \Omega(X) = \exp \{ \mathbb{E}_\omega [\log (R(\omega, X, X'))] \} \].

Note that we can only do this separation when \( X \) is deterministic because otherwise we need to account for the correlation between \( v(X') \) and \( R(\omega, X, X') \). However, for now we assume the problem is deterministic. Since, the solution to \( \Omega(X) \) is the same for any \( div \), and not just the optimal, \( div^0 \). The objective can be written as

\[ v(X) = \max_{div} U(div) + (1 - div)^{1 - \gamma} \beta \mathbb{E} [v(X') \Omega(X)^{1 - \gamma}|X] \],

which is the formulation in Proposition 3. Proposition 5 shows an explicit solution for \( div \) and \( v(X) \).
17.2 Equivalence in Problems

We now show that using our guess for policy functions we can recover the consumption savings problem above. In the original problem, the first order conditions for \( \tilde{b} \) is:

\[
\left( \tilde{b} \right): q(X) U'(\text{div}(X) ) E = \beta \left( \mathbb{E} \left[ v(X) u'(E') \frac{\partial E'}{\partial \tilde{b}} \right] \right) + \mu_\kappa(X) E^{1-\gamma} \kappa - \mu_\eta(X) E^{1-\gamma} \eta. \tag{11}
\]

Now, we know from \( \text{xxx} \) that \( \frac{\partial E'}{\partial \tilde{b}} \) is:

\[
(q(X') \delta + (1 - \delta)) E.
\]

Hence, the first order condition becomes:

\[
q(X) U'(\text{div}(X) ) E^{1-\gamma} = \beta \mathbb{E} \left[ v(X) u'(R(\omega, X, X') E) (q(X') \delta + (1 - \delta)) E \right] + \mu_\kappa(X) E^{1-\gamma} \kappa - \mu_\eta(X) E^{1-\gamma} \eta,
\]

and simplifying \( E \) this equation further more, we obtain:

\[
q(X) U'(\text{div}(X) ) = \beta \mathbb{E} \left[ v(X) R(\omega, X, X')^{-\gamma} (q(X') \delta + (1 - \delta)) \right] + \mu_\kappa(X) \kappa - \mu_\eta(X) \eta.
\]

Similarly, the first order condition for \( \tilde{c}(X) \) yields,

\[
(\tilde{c}): (1+r(X))U'(\text{div}(X) ) = \beta \mathbb{E} \left[ v(X) R(\omega, X, X')^{-\gamma} \frac{\partial R(\omega, X, X')}{\partial \tilde{c}} \right] + \mu_\kappa(X) \kappa - \mu_\eta(X) (\eta - 1), \tag{12}
\]

and a similar expression can be found for \( \tilde{d}(X) \) :

\[
\left( \tilde{d} \right): U'(\text{div}(X) ) = \beta \mathbb{E} \left[ v(X) R(\omega, X, X')^{-\gamma} \frac{\partial R(\omega, X, X')}{\partial \tilde{d}} \right] + \mu_\kappa(X) (1 - \kappa) + \mu_\eta(X) \eta. \tag{13}
\]

Note that 11, 12 and 13 are independent of \( E \). One can multiply equation 11 by \( \dot{b} \), equation 12 by \( \dot{c} \) and equation 13 by \( \dot{d} \), and obtain:

\[
U'(\text{div}) = (1 - \text{div})^{1-\gamma} \beta \mathbb{E} \left[ v(X) \Omega(X)^{1-\gamma} \right]
\]

To see this, note that we are using the definitions

\[
\mu_\kappa(X) \left[ \kappa(\dot{b}(E; X) + \dot{c}(E; X) - \dot{d}(E; X)) - \dot{d}(E; X) \right] = 0
\]

73
and
\[ \mu_\eta(X) \left[ \bar{c}(E; X) - \eta \bar{b}(E; X) + \bar{c}(E; X) - \bar{d}(E; X) \right] = 0 \]
and that \( q\bar{b} + (1 + r)\bar{c} - \bar{d} = 1 \). Since, \( U'(\text{div}) = (1 - \text{div})^{-\gamma} \beta \mathbb{E} [v(X) \Omega(X)^{1-\gamma}] \) is the same first order condition for the for the problem in Proposition 5, we know that the optimal dividend choice satisfies the conditions in that problem also.

### 17.3 Proof of Proposition 5

Taking first order conditions we obtain:
\[
\text{div} = \beta \mathbb{E} [v'(X') | X] \Omega^*(X)^{-(1-\gamma)/\gamma} (1 - \text{div})
\]
and therefore one obtains:
\[
\text{div} = \frac{1}{1 + \left[ \beta \mathbb{E} [v'(X') | X] (\Omega^*(X))^{1-\gamma} \right]^{1/\gamma}}.
\]

Substituting this expression for dividends, one obtains a functional equation for the value function:
\[
v'(X) = \frac{1}{\left( 1 + \left[ \beta \mathbb{E} [v'(X') | X] (\Omega^*(X))^{1-\gamma} \right]^{1/\gamma} \right)^{1-\gamma}} + \beta \mathbb{E} [v'(X') | X] (\Omega^*(X))^{1-\gamma} \left[ \frac{\beta \mathbb{E} [v'(X') | X] (\Omega^*(X))^{1-\gamma}^{\frac{1}{\gamma}}} {\left( 1 + \left[ \beta \mathbb{E} [v'(X') | X] (\Omega^*(X))^{1-\gamma} \right]^{1/\gamma} \right)^{1-\gamma}} \right]^{1-\gamma}
\]
\[
= \left( 1 + \left[ \beta \mathbb{E} [v'(X') | X] (\Omega^*(X))^{1-\gamma} \right]^{1/\gamma} \right)^{1-\gamma} \left[ \frac{\beta \mathbb{E} [v'(X') | X] (\Omega^*(X))^{1-\gamma}^{\frac{1}{\gamma}}} {\left( 1 + \left[ \beta \mathbb{E} [v'(X') | X] (\Omega^*(X))^{1-\gamma} \right]^{1/\gamma} \right)^{1-\gamma}} \right]^{1-\gamma}
\]
\[
= \left( 1 + \left[ \beta \mathbb{E} [v'(X') | X] (\Omega^*(X))^{1-\gamma} \right]^{\frac{1}{\gamma}} \right) \left( 1 + \left[ \beta \mathbb{E} [v'(X') | X] (\Omega^*(X))^{1-\gamma} \right]^{\frac{1}{\gamma}} \right)^{1-\gamma}
\]
\[
= \left( 1 + \left[ \beta \mathbb{E} [v'(X') | X] (\Omega^*(X))^{1-\gamma} \right]^{\frac{1}{\gamma}} \right) \gamma.
\]

Therefore, we obtain the following map:
\[
v'(X)^{\frac{1}{\gamma}} = 1 + (\beta \Omega^*(X))^{\frac{1}{\gamma}} \mathbb{E} [v'(X') | X]^{\frac{1}{\gamma}}.
\]
Performing a change of variables, and denoting \( v(X) \equiv v'(X)^{\frac{1}{\gamma}} \), we have the following functional
equation:

\[ u(X) = 1 + \left( \beta (\Omega^*(X))^{1-\gamma} \right)^{\frac{1}{\gamma}} \mathbb{E} \left[ u(X') \right]^{\frac{1}{\gamma}}. \]

We can treat the right hand side of this functional equation as an operator. This operator will be a contraction depending on the values of \( \left( \beta (\Omega^*(X))^{1-\gamma} \right)^{\frac{1}{\gamma}} \). Theorems in Alvarez and Stokey guarantee that this operator satisfies the dynamic programming arguments.

In a non-stochastic steady state we obtain:

\[ u'(X) = \left( \frac{1}{1 - \left( \beta (\Omega^*(X))^{1-\gamma} \right)^{\frac{1}{\gamma}}} \right)^{\gamma}. \]

### 18 Analytic Cases

#### 18.1 Risk Neutrality

**Optimal Liquidity Ratio Under Risk Neutrality.** For any given \( w_d \), the optimality condition for \( L \) is the same. We have the following first order condition:

\[ \frac{\partial \tilde{R}^x (1, L)}{\partial L} \leq (R^B - R^C) \text{ with equality if } L > 0. \]

The optimal liquidity ratio \( L^* \), can be obtained by

\[ (R^B - R^C) = \left( \tilde{\chi} \int_{\tilde{\omega}(L^*)}^{1} f(\omega) d\omega - \chi \int_{-\infty}^{\tilde{\omega}(L^*)} f(\omega) d\omega \right) \]

\[ = \tilde{\chi} \left( 1 - F(\tilde{\omega}(L^*)) \right) - \chi F(\tilde{\omega}(L^*)) \]

\[ = \tilde{\chi} \left( 1 - F \left( \frac{L^*}{1+r} - \rho \right) \right) - \chi F \left( \frac{L^*}{1+r} - \rho \right) \]

or is zero if the solution does not exist. The condition requires that the right hand side, which is a convex combination of \( \tilde{\chi} \) and \( \chi \) to be equal to \( (R^B - R^C) \). To have an interior solution, the problem requires that:

\[ \tilde{\chi} > R^B - R^C > -\chi. \]

The intuition is that for any value of \( w_d \), the choice of \( L \) must equate the \( R^B - R^C \) cost of cash to the reduction in the hazard of paying a penalty. To have an interior solution, it has to be the case that \( \tilde{\chi} > R^B - R^C \) since otherwise the bank would hold no reserves. The only reason why its willing to forgo the opportunity cost of not making a loan, \( R^B - R^C \), is to reduce the likelihood of paying \( \tilde{\chi} \). If \( -\chi > R^B - R^C \), we are at the opposite extreme where the bank is willing to buy as many the reserves as possible because holding reserves is a better arbitrage opportunity than lending.
Let’s now use the solution of the optimal liquidity ratio, $L^*$, which is interior. To back out
the cost of liquidity for the banker, we substitute this optimal value and recompute the expected
liquidity cost at the optimum. The value of the objective for the banker as a function of $L^*$ is:

$$
\Omega (X) = R^B + \max_{w_d} w_d \left( (R^B - R^D) + \max_L \left\{ - (R^B - R^C) L^* - \bar{R}^\chi (L^*) \right\} \right)
$$

This polar example shows the key trade off in the model. Holding reserves is costly because the bank forgoes a unit of arbitrage, $(R^B - R^C)$. However, holding reserves reduces the risk of a liquidity cost, $(R^B - R^C)$. When choosing the optimal size for deposits $\omega_d$, the banker incorporates into this decision for leverage, the amount of reserves it will hold, $L^*$. Thus, through the optimized liquidity ratio, monetary policy acts like a tax on financial intermediation measured by the cost of liquidity holdings $(R^B - R^C) L^* - \mathbb{E}_{\omega^d} [R^\chi (1, L^*)]$.

**Interaction with Regulatory Constraints. Incomplete Section.** There are more interesting interactions when regulatory constraints are in place:

subject to:

\[
\begin{align*}
    w_d & \leq \kappa \left( \frac{1}{q} - \left( \frac{1}{q} - \frac{1}{1 + r} \right) w_c + \left( \frac{1}{q} - 1 \right) w_d \right), \quad w_d, w_c \geq 0 \\
    w_c & \geq \eta \left( \frac{1}{q} - \left( \frac{1}{q} - \frac{1}{1 + r} \right) w_c + \left( \frac{1}{q} - 1 \right) w_d \right)
\end{align*}
\]

There are several cases of interest.

When constraints are non-trivial, the constraint set has the following properties:

\[
\begin{align*}
    w_d \leq \frac{\kappa}{1 - \left( \frac{1}{q} - 1 \right) \kappa} \left( \frac{1}{q} - \left( \frac{1}{q} - \frac{1}{1 + r} \right) w_c \right) \quad \text{and non-binding if} \quad \left( \frac{1}{q} - 1 \right) \kappa \geq 1 \quad \text{or} \quad \left( \frac{1}{q} < \frac{1}{1 + r} \right)
\end{align*}
\]

and

\[
\begin{align*}
    \left( \frac{1}{q} - 1 \right) w_d \leq -\eta \frac{1}{q} + \left( \eta \left( \frac{1}{q} - \frac{1}{1 + r} \right) + 1 \right) w_c \quad \text{and non-binding if} \quad \left( \frac{1}{q} - \frac{1}{1 + r} \right) > 0.
\end{align*}
\]

Thus, $w_c$ will be chosen so to maximize the size of the constraint set, if the constraint set is non-empty. This would be given by:

\[
\bar{w}^d (w_c) = \min \left( \frac{\kappa}{1 - \left( \frac{1}{q} - 1 \right) \kappa} \left( \frac{1}{q} - \left( \frac{1}{q} - \frac{1}{1 + r} \right) w_c \right), -\eta \frac{1}{q} + \left( \eta \left( \frac{1}{q} - \frac{1}{1 + r} \right) + 1 \right) \right)
\]

This amount becomes:

\[
\mathbb{E}_{\omega^d} [w_d R^\chi (L^*)] = -w_d \left( \mathbb{E}_{\omega^d} [\omega | \omega > \bar{\omega} (L^*)] (1 - F (\bar{\omega} (L^*))) \right) - \mathbb{E}_{\omega^d} [\omega | \omega < \bar{\omega} (L^*)] F (\bar{\omega} (L^*))
\]

76
and since reserves have no cost:

\[ \bar{w}_c = \max_{w_c} \bar{w}^d(w_c) \]

which is the solution to that maximizes the constraint set. If \( R^B = R^C \), reserves have no cost. Then, \( w_c = \bar{w}_c \) without loss of generality. If instead reserves have a cost, if reserves have a cost, \( R^B > R^C \), the banker will compare between the benefit of relaxing the the constraint and not. Thus,

\[ (R^B - R^C) \leq \frac{\partial \bar{w}^d(w_c)}{\partial w_c} (R^B - R^D) \]

A first thing to note is that if \( \left( \frac{1}{q} - 1 \right) \kappa \geq 1 \) and \( w_c = 0 \), the capital requirement constraint is never binding. Thus, if \( (R^B - R^D) > 0 \), \( w_d \) is infinite. If \( R^B = R^D \) it is indeterminate and \( R^B - R^D < 0 \), \( w_d = 0 \). This would mean that prices adjust. Hence, in equilibrium \( \left( \frac{1}{q} - 1 \right) \kappa < 1 \), then \( w_d \) is binding and equal to the solution of:

\[ w_d = \frac{\kappa}{1 - \left( \frac{1}{q} - 1 \right) \kappa} \left( \frac{1}{q} - \left( \frac{1}{q} - \frac{1}{(1 + r)} \right) w_c \right). \]

Thus, the solution of the program depends on whether \( w_c \) relaxes the constraint. If \( \frac{1}{q} > \frac{1}{(1+r)} \), then \( w_c \) constraints the problem furthermore. Thus, it will be made as small as possible so that it the constraint bind. Once again, when regulatory constraints are in place, monetary policy can affect real activity but is constrained by the equilibrium conditions imposed by financial regulation.

With regulatory constraints in place, the choice of \( \omega^d \) will be constrained by the capital requirement constraint or the liquidity requirement constraints. The decision of \( L^* \) and \( \omega^d \) cannot be analyzed sequentially as both terms enter in the constraint. However, there is a clear trade off. The banker will want to increase his \( \omega^d \), provided its profitable to do so. Obtaining liquidity funds has an equity reduction cost so it will have an effect on the constraint. Thus, when constraints are binding there is an additional trade-off between increasing returns at the expense of increased liquidity exposure.

**Remark.** Assume there’s no financial regulation in place, so that \( \kappa \) is finite and \( \eta \geq 0 \). Monetary policy can affect the mix between \( \omega^d \) and \( \omega^c \) by affecting the cost of liquidity but its ability to affect outcomes is constrained by financial regulation.

### 18.1.1 Computing Steady-States

The analysis in the section above looks a the optimal portfolio decisions. This section studies the equilibrium properties in steady state. We break the problem into different cases. We substitute the \( t \) subscripts for \( ss \) to refer to a given steady state. An important thing to note is that in steady, prices for won’t change but there may be a law of motion for \( M_0 \) and \( E \). In a steady state,
the value of returns is given by:

\[ R_{ss}^B (\delta) \equiv \delta + \frac{(1-\delta)}{q_{ss}} \]

\[ R_{ss}^C \equiv 1 + r_{c,ss} \]

\[ R_{ss}^D \equiv (1 + (1 + r_{c,ss}) r_{ss}) \]

and

\[ R^\chi (w_d, w_c) \equiv \chi \left( (\rho + \omega) w_d - \frac{w_c}{1+r} \right). \]

18.2 Monetary Policy Power

Risk-Neutral Probabilities. Let \( \hat{E}_m \) be the expectation under the risk-neutral probability measure consistent with \( m \). Then, we obtain expression for the liquidity premium in the model. Ignoring the multipliers, and rearranging to obtain:

\[ \hat{E}_\xi [R^B] = \hat{E}_\xi [R^D (\omega')] + \hat{E}_\xi [R^\chi_d (w_d, w_c, \omega')] \]

and

\[ \hat{E}_\xi [R^C (\omega')] = \hat{E}_\xi [R^D (\omega')] + \hat{E}_\xi [R^\chi_d (w_d, w_c, \omega')] + \hat{E}_\xi [R^\chi_c (w_d, w_c, \omega')] \]

which since \( \hat{E}_\xi [R^\chi_c (w_d, w_c, \omega')] < 0 \), reflects the presence of a liquidity insurance premium:

\[ \hat{E}_\xi [R^B] - \hat{E}_\xi [R^C (\omega')] = -\hat{E}_\xi [R^\chi_c (w_d, w_c, \omega')]. \quad (15) \]

so by independence:

\[ R^B - R^C = -\frac{\mathbb{E}_{\omega'} [\xi \cdot R^\chi_c (w_d, w_c, \omega')]}{\mathbb{E}_{\omega'} [\xi]]. \quad (16) \]

\[ = \mathbb{E}_{\omega'} [R^\chi_c (w_d, w_c, \omega')] - \frac{COV (R^\chi_c (w_d, w_c, \omega'), \xi)}{\mathbb{E}_{\omega'} [\xi]]. \quad (17) \]

Risk-Neutral Banks. Thus, in a steady state, \( L^* \) in (14) solves:

\[ \left( \delta + \frac{(1-\delta)}{q_{ss}} - (1 + r_{c,ss}) \right) = \bar{\chi} (1 - F(L^*)) - \chi F(L^*). \]

An unconstrained solution requires a solution of the equation above to obtain a value for \( L^* (q_{ss}, r_{ss}, \rho, \bar{\chi}, \chi) \) and in order to have a finite value for \( \omega^d \), the following must also hold:

\[ \delta + \frac{(1-\delta)}{q_{ss}} = \left( \delta + \frac{(1-\delta)}{q_{ss}} - (1 + r_{c,ss}) \right) L^* + \chi \mathbb{E}_{\omega} [\omega \mid \omega > L^*] (1 - F(L^*)) - \bar{\chi} \mathbb{E}_{\omega} [\omega \mid \omega < L^*] F(L^*). \]
Substituting the solution of $L^* (q_{ss}, r_{ss}, \rho, \bar{\chi}, \chi)$ in the equation above yields an implicit solution of $q_{ss}$ as a function of policy parameters $\{r_{ss}, \rho, \bar{\chi}, \chi\}$. Thus, the Central Bank has the ability to choose $q_{ss}$, and consequently the amount of loans in steady state with the restriction that $q_{ss} \in (0, 1]$, and $\bar{\chi} > \delta + \frac{(1-\delta)}{q_{ss}} - (1 + r_{c,ss}) > -\chi$. We can say something more strong. Since the banker earns zero expected profits by the marginal deposit, his expected return to the portfolio is:

$$\Omega_{ss} = \delta + \frac{(1-\delta)}{q_{ss}}.$$ 

Under risk neutrality, positive equity holdings would require to solve:

$$1 \leq \beta \Omega_{ss},$$

since otherwise bankers would pay dividends immediately. This condition imposes a constraint on the set of steady state values of $q_{ss}$ as positive equity is a required to have positive loans. Without this condition, leverage would have to be equal to infinity but this would violate the constraint of non-negative equity. If $\beta \Omega_{ss} < 1$, banks would retain equity and $E_t$ would grow indefinitely with leverage decreasing over time.

When constraints are active, $(\kappa, \eta)$ taking non-degenerate values, $L^*$ and $\omega^d$ will fall within the constraint set imposed by financial regulation. The analysis above shows that at least the leverage constraint would be binding for implied values of $q_{ss}$ such that the sequence of equity is shrinking. In such case, the quantity of loans will be determined by the constraints imposed by financial regulation.

**Monetary-Policy Power.** The first two polar cases show that monetary policy acts like a tax. In the third one, there is no monetary policy. However, a monetary policy $(M_0t, \rho, \bar{\chi}, \chi)$ faces some constraints. Under risk-neutrality (case I), the Central Bank can affect the cost of lending, and the amount of loans by increasing the costs of obtaining liquid funds. As noted above, under risk neutrality, there is no role for bank equity. To talk about $\omega^d$ and $L$ we must use a limit argument where there is some minimal, non-binding amount of equity.

Under risk-neutrality, monetary policy has partial control over lending in this economy. At $t$, the system of equilibrium conditions is:

$$\begin{align*}
(R^B_t - R^C_t) &= \bar{\chi}_t \left(1 - F_t \left(\frac{L^*_t}{(1 + r_t)} - \rho_t\right)\right) - \bar{\chi}_t F_t \left(\frac{L^*_t}{(1 + r_t)} - \rho_t\right) \\
B_t &= D_t = (L^*_t)^{-1} M_0 t \\
q_t &= \Theta_t \left(B_t - (1 - \delta) B_{t-1}\right) \\
R_t^B &= 1 + (1 - \delta) \frac{q_{t+1}}{q_t} \\
R_t^B &= 1 + (R^B_t - R^C_t) L^*_t + R^X (L^*_t) 
\end{align*}$$

subject to:
\[ \bar{\chi} > R^B_t - R^C_t > -\chi_t. \]

Monetary policy can achieve any sequence of \( \{B_t\} \) in the set of sequences that satisfies the constraints above. We are interested in understanding this problem furthermore to determine what shocks can reduce the set of equilibrium outcomes. A follow up version of this paper should provide a more detailed discussion.

**Remark.** Under risk neutrality, \( \gamma = 0 \), monetary policy affects the supply of loans indirectly by affecting the liquidity ratio of banks \( L^* \) through its policy choice \( (M^0_t, \rho, \bar{\chi}, \chi) \). Shocks to \( (\Theta_t, F_t) \) affect the set of equilibrium outcomes.

**Conjecture.** Negative shocks to the demand for \( \Theta_t \) reduce the set of possible outcomes for \( B_t \). However, monetary policy can undo the effects of withdrawal volatility shocks to \( F_t \).

**Interaction between Monetary Policy and Regulatory Constraints.** Another shock that affects the set of outcomes is the effect of regulatory constraint \( \kappa \). We have yet to study the effects of such shocks on the set of monetary policy. With the bank regulation in place, the equilibrium conditions are altered because the constraint imposes a shadow value on various objects. So far, we can only conjecture the results.

**Conjecture.** Negative shocks to the demand for \( \kappa_t \) reduce the set of possible outcomes for \( B_t \). Thus, the effects of monetary policy are affected by the capital requirements. However, the effects of \( \eta_t \) can be undone by monetary policy.