

Efficient Housing Policy Reform

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Abstract

I study efficient housing policy reform in an overlapping generations economy with uninsurable wage risk, incomplete markets, home production, and housing transaction costs. I use a dynamic Mirrlees theory to show that in any efficient allocation housing consumption of every household is implicitly taxed when housing and non-market time are complements in home production. I also use this theory to show that in any efficient allocation homeowners face an implicit tax or an implicit subsidy when they sell their house. Using administrative records for households in the Netherlands, I show that current policy effectively subsidizes housing consumption and taxes households when they buy their house. The average homeowner currently receives an 8 percent subsidy on their housing consumption, and faces a 6 percent transaction tax. I quantify an efficient reform using the calibrated economy under current policy. I find that housing services and non-market time are complements in home production, which translates into an average efficient housing consumption tax of 14 percent. A simple reform, which reduces the transaction tax on buyers from 6 to 2 percent, generates a welfare gain of 2.5 percent of steady-state consumption.

JEL-Codes: D10, D60, H20, R21, R31.

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Introduction

Most high-income economies have adopted a large number of housing policies. Governments levy property taxes, subsidize rents, exempt imputed rents from income taxes, allow for the deduction of home mortgage interest payments for income taxation, and levy taxes on housing transactions. Housing policy affects the cost of housing services and the transaction costs on housing capital, and are naturally accompanied by a debate about aggregate and distributional implications of these policies and of proposed reforms, such as a reduction in the home mortgage interest deduction.

The goal of this paper is to study the reform of housing policy in the Netherlands. I show that housing consumption is effectively subsidized for most households under the current myriad of policies, while in any Pareto efficient allocation housing consumption of every household is implicitly taxed when housing and non-market time are complements in home production activities. Further, while households currently pay a transaction tax when they buy a house, households face an implicit transaction tax or receive an implicit transaction subsidy in any efficient allocation.¹ I find an average effective subsidy on housing consumption for homeowners of 8 percent, while the efficient average tax rate is 14 percent.

I develop my findings in an overlapping generations economy with incomplete markets that incorporates housing services which are produced by illiquid housing capital. Households face idiosyncratic wage risk in the labor market during their working years against which they can self-insure by adjusting their savings and their labor supply. Housing services differ from non-housing consumption because housing services are used together with non-market time in home production activities such as cooking and gardening. Housing capital differs from other forms of savings because housing capital is illiquid due to transaction costs, capturing costs such as real estate broker fees. The complementarity in home production between housing and non-market time, as well as the presence of transaction costs, provides two distinct motives to tax housing, that is, to deviate from the uniform commodity tax prescriptions of [Atkinson and Stiglitz \(1976\)](#) and [Goloso, Kocherlakota, and Tsyvinski \(2003\)](#).

¹I use efficiency to refer to Pareto efficiency, and the term transaction tax to refer to taxes paid when buying or selling a house. In the Netherlands, residential transactions are taxed at 6 percent of the property value. In the United Kingdom, the marginal tax rate on transactions ranges between 0 and 12 percent. In the United States, the transaction tax is levied at the state and local level with state level rates up to 1.5 percent (New Hampshire) and 2 percent (Delaware).

I quantify my findings using administrative records for households in the Netherlands. The government allows for the deduction of home mortgage interest payments for income taxation, subsidizes rents, levies property taxes, exempts housing from asset taxes, and levies a transaction tax on home buyers. Given this prominent role of houses and mortgages in tax policy, the fiscal authority assesses the property value and the outstanding mortgage balance of both rental and owner-occupied housing every year. Combined with individual income records, and employer-provided records on hours worked, I use this data to measure the implied subsidy to housing consumption and to calibrate the complementarity between non-market time and leisure in home production.

I argue for housing policy reform – moving from subsidizing to taxing housing services, and introducing transaction subsidies in some cases – by comparing efficient outcomes with outcomes under current policy. I develop the argument in four steps. First, I study my economy using a dynamic Mirrlees theory to isolate the margins for taxation in any efficient equilibrium with private information on labor productivity. Theory shows the use of implicit transaction taxes and subsidies, and shows that efficient housing consumption taxes critically depend on the complementarity between housing services and non-market time. Second, I analyze my economy from a positive angle, together with administrative records, to measure the effective housing consumption subsidy under current policy. Third, I use this quantitative positive model to estimate the elasticity of substitution between housing services and non-market time in home production. Fourth, I use the estimated model parameters to quantify efficient policy. Step 1 shows the role of implicit transaction subsidies, Step 2 shows that housing consumption is currently subsidized, while Step 1, Step 3 and Step 4 together show that in efficient allocations housing consumption is taxed.

First, I analyze a planning problem to characterize efficient allocations. When housing services and non-market time are complements in home production, I show that every household in any efficient allocation is implicitly taxed on their housing consumption. By taxing housing services, additional leisure is spent in a less desirable house, which provides additional incentives to productive households to work. When housing and non-market time are substitutes, the consumption of housing services is instead subsidized to encourage households to work additional hours.

I also use the dynamic Mirrlees theory to show the use of transaction distortions in efficient allocations. In absence of any distortions homeowners reside in a smaller residence than called for by an efficient plan because

of private concerns over future transaction costs. An efficient allocation therefore calls for transaction taxes that reflect implicit subsidization of housing consumption. Specifically, efficient policy implicitly subsidizes transactions when households sell their house after a negative shock, and implicitly taxes when households sell their house after a positive shock.

Theory thus shows that the consumption of housing is implicitly taxed when housing services and leisure are complements, and shows that households may face a transaction tax or subsidy, in any efficient allocation. Next I measure that housing consumption is subsidized under current policy, while I estimate that housing services and non-market time are complements in home production.

Second, I measure the user's cost of housing capital across households to show that housing consumption is subsidized under current policy. By comparing the user's cost of housing capital under current policy to the user's cost without distortionary taxation, I measure the effective subsidy to housing services under current policy for the distribution of Dutch households. I find an average effective subsidy to homeowners of 7.5 percent, which varies from 3.6 percent for old, low-income households to 14.1 percent for young, high-income households. This effective subsidy is driven by the home mortgage interest deduction and by the exclusion of housing capital from asset income taxation. The home mortgage interest deduction gives an average subsidy of 8.9 percent, and especially benefits young households. The exemption from asset income taxation, on the other hand, subsidizes retirees at 5.6 percent, and provides no benefits for young homeowners.

Third, I use the positive model which incorporates current tax policy to discipline the key parameters. Importantly, I find that housing services and leisure are complements in home production by a "gap"-based indirect inference (Berger and Vavra, 2015). The inferred complementarity between housing and non-market time in home production is based on the estimation of a standard intra-period optimality condition for a home production model without distortionary taxation and transaction costs, which requires households to produce more at home using their low cost input. I use the covariation between wages and home production inputs in the cross section of households to infer an elasticity of substitution between housing and non-market time of 0.95.

Fourth, using the calibrated parameters I conclude that housing consumption is taxed in any efficient allocation by numerically solving for an efficient allocation. Given the calibrated parameters, holding constant the value added consumption tax at 13.4 percent, the average efficient housing consumption tax rate is 13.7

percent. The efficient housing consumption tax rate grows from 13.5 percent at age 25 to 13.8 percent upon retirement. A simple reform, which increases the property tax on buyers to mimicking the efficient 6 to 2 percent as observed in the Netherlands in recent years, generates a welfare gain equivalent to 2.5 percent of steady-state non-housing consumption.

Related literature. The implications of tax policy for housing market outcomes have long been studied. Early work measures the user's cost of housing capital to quantify the implications of effective housing subsidies for housing market outcomes in the United States (Laidler, 1969; Aaron, 1970; Poterba, 1984, 1992). More recent work uses dynamic incomplete market models with heterogeneous households to study the effects of housing policy on housing market outcomes (Gervais, 2002; Chambers, Garriga, and Schlagenhauf, 2009; Floetotto, Kirker, and Stroebel, 2016; Sommer and Sullivan, 2018). These papers generally evaluate policy reforms that reduce the effective housing consumption subsidy for homeowners and find that such reforms increase utilitarian welfare, but are not Pareto improving.

I build on this literature by incorporating home production and by characterizing efficient policy reform. The complementarity in home production between housing services and non-market time provides a motive to distort housing consumption in any efficient allocation. Further, I show households face implicit transaction taxes when they sell their house in efficient allocations. A simple reform, which reduces the transaction tax on buyers from 6 to 2 percent, gives a steady state welfare gain equivalent to 2.5 percent of lifetime consumption. In line with work by Attanasio, Bottazzi, Low, Nesheim, and Wakefield (2012), Besley, Meads, and Surico (2014) and Best and Kleven (2017), this points to sizable distortions due to transaction taxes.

I introduce non-separable preferences between housing services and labor supply by adopting a Beckerian view to home production (Becker, 1965; Ghez and Becker, 1975). Doing so, this paper relates to an extensive literature that studies how home production affects labor supply over the business cycle (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991; McGrattan, Rogerson, and Wright, 1997; Baxter and Jermann, 1999; Aguiar, Hurst, and Karabarbounis, 2013), labor supply over the life-cycle (Rios-Rull, 1993; Aguiar and Hurst, 2005, 2007; Dotsey, Li, and Yang, 2014), and welfare differences across households (Boerma and Karabarbounis, 2019a,b). My contribution is to estimate the elasticity of substitution between housing and non-market time in home production, and to characterize efficient policy for an environment with home production and incomplete asset markets.

By studying efficient housing policy reform for an overlapping generations economy, my paper relates to an extensive literature on efficiency in public finance. [Farhi and Werning \(2013\)](#) and [Golosov, Troshkin, and Tsyvinski \(2016\)](#) study efficient labor income and capital income taxation over the life-cycle in a partial equilibrium framework with skill shocks using a dynamic mechanism design approach. [Stantcheva \(2017\)](#) and [Ndiaye \(2018\)](#) extend their work to endogenize human capital accumulation and retirement, respectively. [Hosseini and Shourideh \(2019\)](#) study efficient reform of labor income and asset taxes for an overlapping generations economy in which skills are deterministic. I introduce housing services consumption and illiquid housing capital to study efficient housing policy reform for an overlapping generations economy with life-cycle skill risk. I incorporate equilibrium responses in house prices by building on [Negishi \(1960\)](#) and [Atkeson and Lucas \(1992, 1995\)](#).

This paper adopts a Mirrlees approach to study policy reform, so the main determinant for distortions is a government's desire to insure households against skill shocks that are not directly observed. Without skill heterogeneity, an efficient allocation is attainable without distorting households' marginal choices. In related work, [Olovsson \(2015\)](#) studies optimal taxation for a representative household using a Ramsey framework with home production. When taxes are necessarily distortionary, and given strong substitutability between market and home services, he finds an optimal tax rate for market services that is well below the tax rate on market consumption. For a heterogeneous household economy with incomplete asset markets, I find that an efficient tax rate for housing services is similar to the tax rate on non-housing consumption because the estimated substitution elasticity between housing services and non-market time is close to one.

In this paper, variable transaction costs, such as seller's fees, and complementarity in home production present two motives to deviate from uniform commodity taxation in the presence of housing. In related work, [Koehne \(2018\)](#) shows that uniform commodity taxation is not efficient in presence of durable purchases with only fixed adjustment costs. Instead, housing services are nondurable in my paper, and variable transaction costs determine the efficient transaction tax.

The remainder of this paper is organized as follows. In [Section 1](#), I lay out the primitive environment. In [Section 2](#), I formulate a planning program to characterize efficient allocations for this environment. I present a characterization of efficient housing taxes in [Section 3](#). After studying forces that determine efficient housing taxes, I turn to study a positive economy by introducing current policy into the primitive environment in

Section 4. The positive economy is calibrated to the Netherlands in Section 5, and is used to quantitatively study policy reform in Section 6. Section 7 concludes.

1 Environment

Demographics. I study an infinite horizon economy populated by overlapping cohorts that live T years. Time is discrete and denoted by $j = 1, 2, \dots$. A cohort is a continuum of households of mass one. Households work the first T_w years and are retired for the remaining years. Household age is denoted by t .

Preferences. Households derive utility from market consumption c , housing services d , and leisure ℓ , and maximize expected utility

$$\mathbb{E} \left(\sum_{t=1}^T \beta^{t-1} u(c_t, d_t, \ell_t) \right),$$

where β is the time discount factor. The period utility function is separable in the utility from consumption $v(c)$, where v is increasing and strictly concave, and the flow utility from home production $h(d, \ell)$,

$$u(c, d, \ell) = v(c) + h(d, \ell). \tag{1}$$

Households have a unit of time each period to spent on work and leisure. When households are retired $\ell = 1$.

To fix ideas, I assume the home production technology is given by a CES aggregator over housing services and non-market time. The home technology is parameterized by the elasticity of substitution, which I denote by σ , and by weight ω . Household preferences over the non-market good are captured by \mathcal{H} , that is,

$$h(d, \ell) = \mathcal{H} \left(\left(\omega d^{\frac{\sigma-1}{\sigma}} + (1-\omega) \ell^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right), \tag{2}$$

where σ and ω are non-negative and constant across households.²

My specification of preferences and the home production technology are a special case of the Beckerian model of home production (Becker, 1965; Ghez and Becker, 1975) where households have preferences over two goods. The first good is non-housing consumption, the second good is produced using housing services

²My model features a single decision maker within each household. Hours worked across spouses are perfect substitutes and in the quantitative analysis I treat ℓ as average leisure time for both spouses.

and time as inputs.³ With $\omega = 0$, I obtain a canonical life-cycle economy, for which the efficient allocation is discussed in [Farhi and Werning \(2013\)](#) and [Golosov, Troshkin, and Tsyvinski \(2016\)](#).

Skill Heterogeneity. Households are heterogenous with respect to their market productivity. The output y that a household produces is the product of their productivity θ and the hours they work $n = 1 - \ell$, that is, $y = \theta n$. Output, consumption and housing services consumption are publicly observed. Households are born at age 0, with ability θ_0 which is distributed following distribution $\pi^0(\theta_0)$, and enter the labor market at age 1. Skills evolve according to a first-order Markov process with age varying distribution function $\pi^t(\theta_t|\theta_{t-1})$ over a fixed set $\Theta \equiv \{\theta_1, \dots, \bar{\theta}_N\}$. The history of the skill shocks is given by θ^t . The probability density function for history $\theta^t \in \Theta^t$ is given by $\pi(\theta^t) \equiv \pi^t(\theta_t|\theta_{t-1}) \dots \pi^1(\theta_1|\theta_0)\pi^0(\theta_0)$. The skill distribution is assumed independent across households within a cohort.

Technology. The economy is endowed with technologies to produce housing services and a general good. The economy is a small open economy with a domestic housing market, meaning that housing services have to be produced domestically. Non-housing goods and services can be traded across countries.

Housing Services. Households obtain housing services by living in a house. Houses differ in the flow services they provide, which is proportional to the house's capital value. One unit of housing capital provides χ units of housing services. The total housing stock can be divided every period into individual houses without cost. The resource constraint for housing services is thus:

$$D_j \leq \chi H_j, \tag{3}$$

where D denotes aggregate housing services and H denotes housing capital.

When a household moves from consuming dwelling d_{j-1} to dwelling d_j there is a technological transaction cost $\Phi(d_j, d_{j-1})$, which captures costs such as real estate broker fees. The transaction cost is differentiable

³The home production literature adopts two classifications of time use. In my baseline model, time is divided between work and non-market time following [Becker \(1965\)](#), [Greenwood and Hercowitz \(1991\)](#), and [Boerma and Karabarbounis \(2019b\)](#). The second approach differentiates non-market time from leisure time ([Benhabib, Rogerson, and Wright, 1991](#); [McGrattan, Rogerson, and Wright, 1997](#); [Baxter and Jermann, 1999](#); [Parente, Rogerson, and Wright, 2000](#); [Karabarbounis, 2014](#); [Boerma and Karabarbounis, 2019a](#)). In Appendix A, I show how the theory extends to this second class of home production models.

in the housing choice, with $\lim_{d_j \rightarrow d} \Phi_1(d_j, d) = 0$ and $\Phi_2(d_j, d) \geq 0$. The aggregate transaction cost for housing services is, with some abuse of notation, denoted by $\Phi_j \equiv \sum_{t, \theta^t} \Phi(d_j(\theta^t), d_{j-1}(\theta^{t-1}))$.

Construction. Time is required to build new houses, in the spirit of Kydland and Prescott (1982). Let $\iota \geq 0$ denote the time periods required to build new houses and let Q_j^H be building projects initiated in period j .⁴ The law of motion for the housing capital stock is then

$$H_{j+1} = H_j + Q_{j+1-\iota}^H. \quad (4)$$

I do not restrict building projects to be nonnegative. For example, (4) allows housing capital to be converted into offices. Without loss, I assume that resources are only allocated to housing projects in their final stage.⁵ In sum, building projects initiated in period j realize in period $j + \iota$ and are financed in period $j + \iota - 1$. Time to build in construction implies that the housing supply is infinitely inelastic in the short run, and infinitely elastic in the long run.

The housing stock depreciates at rate δ^H . Depreciation is exactly offset by required maintenance expenses, in terms of the general good, such that housing capital does not deteriorate with time. Housing investment is the sum of resources allocated to building projects and required maintenance, $I_j^H = Q_{j+1-\iota}^H + \delta^H H_j$.

General Good. The technology for general goods $F(K, Y)$ is homogeneous of degree one in aggregate effective labor Y and business capital K and satisfies the Inada conditions. The general good can be consumed, invested in business capital and housing capital, used to pay government expenditures and transaction costs for housing services Φ_j , or exported to the rest of the world. A small open economy with positive net exports increases its net claims on foreign assets B with return R dictated by the world interest rate. The resource constraint for general goods is thus

$$C_j + I_j^K + I_j^H + G_j + \Phi_j + B_{j+1} \leq F(K_j, Y_j) + RB_j, \quad (5)$$

⁴I model time to build in the housing sector to allow for different construction times for housing capital and business capital. The decision to incorporate time to build on housing rather than on business capital is suggested by the data. In the Netherlands, the production of housing units takes 23 months on average after a building permit is issued (see Section 5), while it takes only 175 days to build a warehouse valued at 50 times income per capita (World Bank's report on Doing Business).

⁵In Kydland and Prescott (1982) resources are allocated to time to build investment projects in all periods between the initial and the final stage, while in this paper resources are allocated only in the final stage. In my environment, these two formulations are isomorphic when the present value resource cost is identical.

where the business capital stock evolves according to $I_j^K = K_{j+1} - (1 - \delta^K)K_j$, where δ^K is the depreciation rate on business capital.

2 Efficiency

In this section I study efficiency properties of the economy. I define and characterize efficient allocations, which are necessarily incentive feasible and resource feasible. I first define resource feasibility and incentive feasibility. I then define efficiency and show how to characterize efficient allocations using a planner problem.

Identity and Resource Feasible. A household's identity is its birth year j and its productivity history θ^{t-1} . I use $i \equiv (j, \theta^{t-1})$ to denote a household.⁶ The set of households \mathcal{I} is partitioned into households that are alive in the first period and households born in future periods, $\mathcal{I} \equiv \left\{ \{(t-1, \theta^{t-1})\}_{t=1}^T, \{(j, \theta_0)\}_{j=1}^\infty \right\}$.

An allocation for household i is a sequence of functions that specify non-housing consumption, housing services consumption and labor supply at age $t+v$ given the household's productivity history θ^{t+v} , $x(i) \equiv \{x_{j+v}(\theta^{t+v})\}_{v=0}^{T-t} = \{(c_{j+v}(\theta^{t+v}), d_{j+v}(\theta^{t+v}), y_{j+v}(\theta^{t+v}))\}_{v=0}^{T-t}$. An allocation x specifies an allocation for every household i as well as aggregate quantities:

$$x \equiv \left\{ \{x(i)\}_{\mathcal{I}}, \{(C_j, D_j, Y_j, B_{j+1}, H_{j+1}, K_{j+1}, I_j^H, I_j^K)\}_{j=1}^\infty \right\}.$$

An allocation is resource feasible if and only if the allocation satisfies the resource constraints (3)–(5) in all period j .

Incentive Feasible. Households know their own history θ^t up to age t , and the only possible source of information about this history are reports provided by the household itself.⁷ By the revelation principle I can restrict the reporting space to be the type space without loss of generality. I use $\sigma_t(\theta^t)$ to denote the report that the household plans at date 1 to give about their date t shock when they experience θ^t . A reporting strategy, which specifies a report for every history, is denoted $\sigma \equiv \{\sigma_t(\theta^t)\}_{\Theta^t, t}$. A reporting strategy generates

⁶Every household has only one identity. For every household born in future periods, the productivity history is a singleton, θ_0 , and the household identity is hence (j, θ_0) . In the initial period, households of all ages and productivity histories are alive, which is captured by identities $\{(t-1, \theta^{t-1})\}_{t=1}^T$.

⁷To simplify the exposition I describe incentive compatibility for a household born in the future and I suppress the identity. The corresponding definitions for households that are alive in the initial period naturally follow.

a corresponding report history $\sigma^t(\theta^t) = (\sigma_1(\theta^1), \dots, \sigma_t(\theta^t))$. Denote by Σ the set of reporting strategies. The truthful reporting strategy is such that $\sigma^t(\theta^t) = (\theta_1, \dots, \theta_t)$ for all t and all $\theta^t \in \Theta^t$.

Given a reporting strategy σ , the corresponding household allocation is given by $x^\sigma \equiv \{x_t(\sigma^t(\theta^t))\}_{\Theta^t, t} = \{c_t(\sigma^t(\theta^t)), d_t(\sigma^t(\theta^t)), y_t(\sigma^t(\theta^t))\}_{\Theta^t, t}$. Given a reporting strategy σ and an allocation, the expected lifetime utility is

$$\mathcal{V}(x^\sigma) \equiv \sum_{t=1}^T \sum_{\theta^t} \beta^{t-1} \pi(\theta^t) u(c_t(\sigma^t(\theta^t)), d_t(\sigma^t(\theta^t)), y_t(\sigma^t(\theta^t))/\theta_t). \quad (6)$$

The continuation value after history θ^t , which is denoted by $V^\sigma(\theta^t)$, is given by:

$$V^\sigma(\theta^t) = u(c_t(\sigma^t(\theta^t)), d_t(\sigma^t(\theta^t)), y_t(\sigma^t(\theta^t))/\theta_t) + \beta \sum_{\theta_{t+1}} \pi^{t+1}(\theta_{t+1}|\theta_t) V^\sigma(\theta^{t+1}),$$

for all $t = 1, \dots, T$, with $V^\sigma(\theta^{T+1}) = 0$. The continuation value after history θ^t under a truthful reporting strategy thus solves:

$$V(\theta^t) = u(c_t(\theta^t), d_t(\theta^t), y_t(\theta^t)/\theta_t) + \beta \sum_{\theta_{t+1}} \pi^{t+1}(\theta_{t+1}|\theta_t) V(\theta^{t+1}), \quad (7)$$

for all $t = 1, \dots, T$, with $V(\theta^{T+1}) = 0$.

An allocation is incentive feasible if and only if the truthful reporting strategy is an equilibrium reporting strategy given any history for every household i . An allocation is incentive compatible if and only if for all histories θ^t

$$V(\theta^t) \geq V^\sigma(\theta^t), \quad (8)$$

for all $\sigma \in \Sigma$. The set of incentive compatible allocations for household i is denoted $X_{IC}(i)$. An allocation x is thus incentive feasible if and only if the allocation for household i is incentive compatible for all households $i \in \mathcal{I}$. An allocation is feasible if and only if it is resource feasible and incentive feasible.

Efficiency. An allocation is efficient if and only if there exists no alternative feasible allocation that makes all households weakly better off and some households strictly better off. That is, there exists no alternative feasible allocation \hat{x} such that:

$$\begin{aligned} \mathcal{V}_j(\hat{x}(i); \theta^{t-1}) &\geq \mathcal{V}_j(x(i); \theta^{t-1}) && \forall i \in \mathcal{I} \\ \mathcal{V}_j(\hat{x}(i); \theta^{t-1}) &> \mathcal{V}_j(x(i); \theta^{t-1}) && \text{for some } i \in \mathcal{I}, \end{aligned}$$

I next formulate a planning program and show that this planning problem characterizes efficient allocations.⁸

Planning Problem. Given values $\{\mathcal{V}(i)\}_{\mathcal{I}}$, a capital endowment (B_1, H_1, K_1) , housing allocations in period zero, and a pipeline of building projects $\{Q_{1-v}^H\}_{v=1}^t$, the planning problem is to choose a feasible allocation that maximizes excess resources in the first period such that household values exceed $\mathcal{V}(i)$ for all $i \in \mathcal{I}$. Formally, the planning problem is:

$$\max_x F(K_1, Y_1) + RB_1 - C_1 - I_1^K - I_1^H - G_1 - \Phi_1 - B_2$$

subject to the housing services constraint in every period (3), the law of motion for housing capital (4), the resource constraints for general goods for period $j > 1$ (5), the incentive constraints for all households (8), and the promise keeping constraints for all households $i \in \mathcal{I}$:

$$\mathcal{V}(i) \leq \mathcal{V}_j(x(i); \theta^{t-1}). \tag{9}$$

Theorem 1. Allocation x with corresponding values $\mathcal{V}_j(x(i); \theta^{t-1})$ for all $i \in \mathcal{I}$ is efficient if and only if it solves the planner problem given $\mathcal{V}_j(x(i); \theta^{t-1})$ for all $i \in \mathcal{I}$ with a maximum of zero.

The proof is presented in Appendix B.

Theorem 1 provides a useful characterization of efficient allocations by combining ideas from Atkeson and Lucas (1992, 1995) and Negishi (1960). Atkeson and Lucas (1992, 1995) use prices to decentralize the problem of finding efficient allocations into component planner problems. They prove that an allocation is efficient if the allocation, finite prices, and a distribution of values solve the component planner problems (given prices and initial values) and satisfy the resource constraints. To connect to Negishi (1960), I refer to the allocation, prices, and the distribution of values as a component planner equilibrium. I characterize a component planner equilibrium using a planning formulation similar to Negishi (1960), who characterizes a competitive equilibrium by maximizing a linear social welfare function with appropriate weights subject to resource feasibility.

⁸This definition requires that every household is strictly better off from an ex-ante perspective. It does not require that every household is strictly better off for any realization of future shocks.

3 Efficient Housing Policy

I characterize efficient allocations and study its implications for housing policy.

Component Planner. I study the component planner problem to characterize the solution to the planning problem for a given household. Given a household $i \in \mathcal{I}$ and a value $\mathcal{V}(i)$, the component planner chooses allocation $x(i)$ to maximize excess resources for household i subject to incentive constraints. To simplify the exposition I present the component planner problem of a household born in the future and I suppress notation on its identity.

Given a sequence of multipliers $\{p_j\}_{j=1}^{\infty}$ on the aggregate constraint for housing services (3), I define the excess resources for household i as:

$$\Pi(x(i)) \equiv \sum_{t, \theta^t} \pi(\theta^t) \left(wy(\theta^t) - c(\theta^t) - p_j d(\theta^t) - \Phi(d(\theta^t), d(\theta^{t-1})) \right) / R^{t-1}$$

The component planning problem for household i given value $\mathcal{V}(i)$ is to solve:

$$\Pi(\mathcal{V}(i)) \equiv \max_{x(i)} \Pi(x(i))$$

subject to

$$\mathcal{V}(i) \leq \mathcal{V}(x(i))$$

$$x(i) \in X_{IC}(i)$$

I solve a relaxed version of the component planning problem by using a local downward incentive constraints approach.

Local Downward Incentive Constraints. To solve the component planner problem for household i in a tractable manner, I assume only local downward incentive constraints bind at the solution. Assuming that only local downward incentive constraints bind is a finite types analog for the first-order approach typically adopted in dynamic Mirrlees problems with a continuum of productivity types (Farhi and Werning, 2013; Golosov, Troshkin, and Tsyvinski, 2016; Stantcheva, 2017). I replace the set of incentive compatibility allocations X_{IC} by a superset of allocations satisfying local downward incentive constraints for truthful reporting $X_{LD} \supset X_{IC}$.

Consider a one-shot deviation strategy from truthful reporting for a household with history θ^t . At age t the household reports a lie $l \neq \theta_t$ for a specific realization θ_t and reports truthfully in all other instances.

The one-shot deviation strategy σ^l is thus,

$$\begin{aligned}\sigma_t^l(\theta^{t-1}, \tilde{\theta}) &= \tilde{\theta} && \text{if } \tilde{\theta} \neq \theta_t \\ \sigma_t^l(\theta^{t-1}, \theta_t) &= l.\end{aligned}$$

The continuation value given one-shot deviation strategy σ^l is given by:

$$V^{\sigma^l}(\theta^t) = u(x_t(\theta^{t-1}, l); \theta_t) + \beta \sum \pi^{t+1}(\theta_{t+1}|\theta_t) V^{\sigma^l}(\theta^{t+1}), \quad (10)$$

for all $t = 1, \dots, T$, with $V^{\sigma^l}(\theta^{T+1}) = 0$.

Given a first-order Markov process for labor productivity, there is no difference going forward between a household adopting reporting strategy σ^l with history θ^{t+1} that triggers lie l and a truth-telling household with history $(\theta^{t-1}, l, \theta_{t+1})$. Both households have identical reporting histories so there is no informational difference to distinguish them. Therefore, they necessarily receive the same continuation value:

$$V^{\sigma^l}(\theta^{t+1}) = V(\theta^{t-1}, l, \theta_{t+1}). \quad (11)$$

By the one-shot deviation principle, incentive compatibility is equivalently formulated as:

$$\forall \theta^t \quad V(\theta^t) = \max_l V^{\sigma^l}(\theta^t), \quad (12)$$

for all σ^l . Substituting (10) and (11) into (12), I obtain that $\forall \theta^t$:

$$V(\theta^t) = \max_l u(x_t(\theta^{t-1}, l); \theta_t) + \beta \sum \pi^{t+1}(\theta_{t+1}|\theta_t) V(\theta^{t-1}, l, \theta_{t+1}). \quad (13)$$

This expression gives the following local downward incentive constraint:

$$u(x_t(\theta^t); \theta_t) + \beta \sum \pi^{t+1}(\theta_{t+1}|\theta_t) V(\theta^{t+1}) \geq u(x_t(\theta^{t-1}, \theta_t^-); \theta_t) + \beta \sum \pi^{t+1}(\theta_{t+1}|\theta_t) V(\theta^{t-1}, \theta_t^-, \theta_{t+1}), \quad (14)$$

where θ_t^- is the productivity level right below θ_t . The set of allocations for household i that satisfy the local downward incentive constraints (14) is denoted $X_{LD}(i)$.

The relaxed component planner problem is formulated by replacing the set of constraints that ensure global incentive compatibility in the component planning problem, $X_{IC}(i)$, with the set of constraints that ensure that the allocation satisfies all local downward incentive constraints, $X_{LD}(i)$. I write the relaxed component planner problem as a dynamic program and then characterize its solution.

Recursive Problem. To write the relaxed component planner problem recursively, it is useful to introduce the state variables continuation value $\mathcal{V}(\theta^t)$ and threat value $\tilde{\mathcal{V}}(\theta^t)$,

$$\mathcal{V}(\theta^t) \equiv \sum \pi^{t+1}(\theta_{t+1}|\theta_t)V(\theta^{t+1}) \quad (15)$$

$$\tilde{\mathcal{V}}(\theta^t) \equiv \sum \pi^{t+1}(\theta_{t+1}|\theta_t^+)V(\theta^{t+1}). \quad (16)$$

The continuation value is the expected future value for a truth-telling individual with history θ^t . The threat value is the expected value using the probability distribution for an individual who experienced and reported an identical history until $t - 1$, and who reports θ_t while being one level more skilled θ_t^+ at age t . In other words, the threat value is the continuation value for a one-time local deviation from truthful reporting for an individual with an identical history except for being more skilled at age t . Using these state variables, I rewrite the local downward incentive constraints (14) as:

$$u(x_t(\theta^t); \theta_t) + \beta\mathcal{V}(\theta^t) \geq u(x_t(\theta^{t-1}, \theta_t^-); \theta_t) + \beta\tilde{\mathcal{V}}(\theta^{t-1}, \theta_t^-). \quad (17)$$

Using the continuation value and the threat value, I write the component planning problem recursively,

$$\Pi_t(\mathcal{V}, \tilde{\mathcal{V}}, d, \theta_-) \equiv \max_{x_t(\theta)} \sum \pi^t(\theta|\theta_-) \left(wy_t(\theta) - c_t(\theta) - p_j d_t(\theta) - \Phi(d_t(\theta), d) + \Pi_{t+1}(\mathcal{V}_t(\theta), \tilde{\mathcal{V}}_t(\theta^+), d_t(\theta), \theta) / R \right)$$

where, with some abuse of notation, the choice variable is $x_t(\theta) = \{c_t(\theta), d_t(\theta), y_t(\theta), \mathcal{V}_t(\theta), \tilde{\mathcal{V}}_t(\theta)\}$, and where maximization is subject to (15)–(17):

$$\mathcal{V} = \sum \pi^t(\theta|\theta_-) (u(c_t(\theta), d_t(\theta), y_t(\theta)/\theta) + \beta\mathcal{V}_t(\theta)) \quad (15)$$

$$\tilde{\mathcal{V}} = \sum \pi^t(\theta|\theta_-^+) (u(c_t(\theta), d_t(\theta), y_t(\theta)/\theta) + \beta\mathcal{V}_t(\theta)) \quad (16)$$

$$u(c_t(\theta), d_t(\theta), y_t(\theta)/\theta) + \beta\mathcal{V}_t(\theta) \geq u(c_t(\theta^-), d_t(\theta^-), y_t(\theta^-)/\theta) + \beta\tilde{\mathcal{V}}_t(\theta). \quad (17)$$

This formulation has five state variables: continuation value \mathcal{V} , threat value $\tilde{\mathcal{V}}$, previous housing consumption d , lagged productivity θ_- , and age t . I solve the recursive component planner problem to characterize efficient policies for housing consumption and earnings as well as savings and housing wealth.

A solution to the relaxed component planner problem is a solution to the original component planner problem only if at optimum the local downward incentive constraints (17) are a sufficient condition for global incentive compatibility (8). I verify sufficiency of the local downward incentive constraints in the quantitative

analyses.⁹

Implicit Taxes. Implicit taxes are distortions of households' marginal decisions under an efficient allocation. They provide information about efficient insurance when compared to a benchmark without intervention and similarly provide information about inefficiency in current policy when compared to distortions under current tax policy. In this section I describe four implicit taxes: a labor tax, a savings tax, a housing consumption tax, and a transaction tax.

The implicit housing services tax and the implicit transaction tax are distortions to households' marginal housing decision, and described by

$$p(1 + \tau_d^c(\theta^t)) + \Phi_1(d_t(\theta^t), d_{t-1}(\theta^{t-1})) + \beta \mathbb{E}_t \left((\Phi_2(d_{t+1}(\theta^{t+1}), d_t(\theta^t)) + \tau_d^t(\theta^{t+1})) \frac{u_c(\theta^{t+1})}{u_c(\theta^t)} \right) = \frac{u_d(\theta^t)}{u_c(\theta^t)} \quad (18)$$

where $u_x(\theta^t) \equiv u_x(c_t(\theta^t), d_t(\theta^t), y_t(\theta^t); \theta_t)$ for $x \in \{c, d, y\}$. The implicit housing consumption tax, τ_d^c , is akin to a value-added tax on consumption, while the implicit transaction tax, τ_d^t , appears as a tax paid when household sell their house. Given some implicit taxes, households balance the marginal benefit of housing services with its marginal cost, which also consist of relative price p as well as current marginal transaction costs and changes in expected future transaction costs.

The implicit labor tax distorts between the marginal rate of substitution of consumption for labor and the marginal product of labor w :

$$1 - \tau_{y,t}(\theta^t) \equiv - \frac{u_{y,t}(\theta^t)}{u_{c,t}(\theta^t)} \Big/ w. \quad (19)$$

The implicit savings tax is the distortion in the marginal rate of substitution between current consumption and expected consumption:

$$1 - \tau_{s,t}(\theta^t) \equiv \frac{u_{c,t}(\theta^t)}{\beta R \sum \pi^{t+1}(\theta_{t+1}|\theta_t) u_{c,t+1}(\theta^{t+1})}. \quad (20)$$

Efficient Taxes. I characterize the efficient taxes using the solution to the planner problem. Since efficient taxes depend on productivity history, I describe the efficient distortions as a function of current productivity θ_t , taking as given a history θ^{t-1} . To simplify notation I omit the explicit dependence on the productivity

⁹This approach is common in the dynamic Mirrlees literature (Kapička, 2013; Farhi and Werning, 2013; Golosov, Troshkin, and Tsyvinski, 2016; Stantcheva, 2017).

history, meaning that $x_t(\theta)$ denotes the value of a random variable x given history (θ^{t-1}, θ) and that $x_{t-1} = x_{t-1}(\theta^{t-1})$.

In discussing the efficient taxes I emphasize the housing consumption tax and the transaction tax, which are central to this paper. Before discussing efficient housing policy, I describe how the standard taxes, the implicit savings tax and the implicit labor tax, extend to my environment. The derivations are in Appendix C.

Since household preferences are separable, increasing, and strictly concave in non-housing consumption, the inverse Euler equation holds (Rogerson, 1985; Golosov, Kocherlakota, and Tsyvinski, 2003). The solution to the planning problem thus features a positive savings wedge. In my environment, the savings wedge equally applies to housing and financial wealth. To implement this positive savings wedge, policy has to be proof to double deviations as discussed in Kocherlakota (2005), Albanesi and Sleet (2006), and Golosov and Tsyvinski (2006). The labor wedge in my model is similar to Farhi and Werning (2013) and Golosov, Troshkin, and Tsyvinski (2016), and always positive.

Efficient Housing Consumption and Transaction Tax. I discuss the efficient housing consumption tax and the efficient transaction tax. To provide intuition, I discuss the forces that determine how the efficient tax is set. I show that housing services are taxed when housing services and leisure are complements in home production, and that the government efficiently subsidizes or taxes transaction tax depending on households' state when they sell their house. I first present general optimal tax formulas, and then illustrate the main forces using a two stage life-cycle problem with two productivity types.

Efficient taxes on housing services consumption are determined by a static and a dynamic component,

$$\tau_d^c(\theta) = \underbrace{\Delta h_d(d(\theta), 1 - y(\theta)/\theta^+) \frac{I(\theta)}{p\pi(\theta)}}_{\text{static component}} + \underbrace{\beta R \tau_{y,t-1} \frac{\pi_\Sigma(\theta) - \pi_\Sigma^+(\theta)}{\pi(\theta)} \frac{\Delta h_d(d(\theta), 1 - y(\theta)/\theta^+)}{\Delta h_y(d_{t-1}, 1 - y_{t-1}/\theta_{t-1}^+)}}_{\text{dynamic component}}, \quad (21)$$

where $\Delta h_d(d(\theta), 1 - y(\theta)/\theta^+) \equiv h_d(d(\theta), 1 - y(\theta)/\theta^+) - h_d(d(\theta), 1 - y(\theta)/\theta)$, and where $I(\theta)$ is the insurance value at type θ . As is standard in static optimal taxation models, such as Diamond (1998) and Saez (2001), the insurance value is the inverse marginal utility of consumption for households with skills above θ relative to its mean,

$$I(\theta) = \sum_{s=i+1}^N \pi(\theta_s) \frac{1}{v_c(\theta_s)} - (1 - \pi_\Sigma(\theta)) \sum_{s=1}^N \pi(\theta_s) \frac{1}{v_c(\theta_s)}. \quad (22)$$

Since the dynamic component and the insurance value are positive, the home technology determines whether housing consumption is taxed or subsidized.¹⁰

Housing consumption is taxed when housing services and leisure are complementary in home production. Holding constant a household's housing services and labor output, a more productive household enjoys more leisure. When housing services and leisure are complements, a productive household therefore has a higher marginal utility for housing services. By taxing the consumption of housing services, additional leisure time is spent in a less desirable house, which prevents productive households from working inefficiently few hours.

An efficient housing consumption tax balances the distortionary costs for a type θ against the benefit of relaxing the incentive constraints for all types above θ . Relaxing incentive constraints allows the planner to provide more insurance in period t by extracting resources from households more productive than θ , and by distributing these resources across all households with the same history, which is captured by $I(\theta)$. The inverse proportion $1/\pi(\theta)$ captures that the planner is willing to distort a household type more when there are fewer households of this type.

The dynamic component captures that a housing consumption tax in the current period relaxes incentive constraints in prior periods. When the Markov transition matrix for household skills is monotonic (Daley, 1968), implying $\pi_{\Sigma}(\theta) \geq \pi_{\Sigma}^+(\theta)$, a more productive household θ^+ is more likely to be affected by the housing services tax. The planner favors larger absolute distortions, all else equal, to exploit the dynamic incentive effect. The properties of the efficient housing consumption tax are summarized in Proposition 1. The proof is in Appendix C.

Proposition 1. *The housing services wedge is positive if and only if housing services and non-market time are complements in home production.*

Efficient transaction taxes implicitly subsidize households when they sell their residence after a negative shock, meaning that the household's consumption increased, and effectively taxes households when they sell

¹⁰The efficient tax on housing consumption is equivalently written as $\tau_d^c(\theta) = q(\theta^+) \Delta h_d(d(\theta), 1 - y(\theta)/\theta^+)$, where $q(\theta^+)$ is the shadow value of relaxing the incentive constraint for type θ^+ . This formulation of the housing consumption tax, which I derive in Appendix C, directly shows that housing consumption is efficiently taxed or subsidized depending on the complementarity in the home production technology.

their house after receiving a positive shock,

$$\tau_d^t(\theta_{t+1}) = \Phi_2(d(\theta_{t+1}), d(\theta)) \left(\frac{1}{\beta R} - \frac{v_c(c(\theta_{t+1}))}{v_c(c(\theta))} \right). \quad (23)$$

When the selling cost of a house increases in the value of the property that is sold, $\Phi_2 \geq 0$, households pay an implicit transaction tax when their marginal utility from consumption decreases. An efficient allocation provides insurance against transaction costs.

The planner uses transaction taxes to provide insurance against transaction costs when asset markets are incomplete. To understand why, consider the case where housing consumption is also separable from leisure so that the efficient housing consumption tax is zero, and let savings taxes be such that the intertemporal non-housing consumption choice is efficient. The planner's rate of transformation between housing services and non-housing consumption is $p + \Phi_1(d_t(\theta^t), d_{t-1}(\theta^{t-1})) + \frac{1}{R} \sum \pi^{t+1}(\theta_{t+1}|\theta^t) \Phi_2(d_{t+1}(\theta^{t+1}), d_t(\theta^t))$ which reflects that the planner incurs increased transaction costs next period with certainty. Absent any transaction taxes, households substitute at $p + \Phi_1(d_t(\theta^t), d_{t-1}(\theta^{t-1})) + \beta \sum \pi^{t+1}(\theta_{t+1}|\theta^t) \Phi_2(d_{t+1}(\theta^{t+1}), d_t(\theta^t)) \frac{u_c(\theta^{t+1})}{u_c(\theta^t)}$. Households face uncertainty over future transaction costs, and evaluate marginal transaction costs by the corresponding marginal utility of consumption in each state. Relative to the planner, households overweight marginal transaction costs after negative shocks. Efficient transaction taxes correct for this, by subsidizing transaction costs after a negative shock. The efficient transaction tax is summarized by Proposition 2.

Proposition 2. *The efficient transaction tax is positive when households sell their house after a positive shock.*

Illustration with Two Periods and Two Types. I consider a two stage life-cycle to illustrate the motives for efficient taxes. Households are identical in the initial period but either have high productivity, θ_H , or low productivity, θ_L , in the final period, where $\theta_H > \theta_L > 0$. The downward incentive constraint prevents the high productivity household from mimicking the low productivity household.

The housing consumption tax in the final period prevents productive households from pretending to be unproductive. When a productive household pretends to be unproductive the benefit is additional leisure. When housing services and leisure are complements in home production, the increase in leisure is more valuable when the household also enjoys more housing services. To prevent households from pretending to be unproductive, the efficient allocation therefore depresses the consumption of housing services for low

productivity households, which translates into a positive consumption tax. In sum, the efficient allocation discourages the productive households from misreporting by threatening them to enjoy their leisure in a less desirable house. The opposite mechanism applies when housing and leisure are substitutes in home production, which translates into a housing consumption subsidy. When housing services and leisure are neither complements nor substitutes, the housing consumption tax is zero, echoing the uniform commodity tax result of [Atkinson and Stiglitz \(1976\)](#). The sign on the housing consumption tax is determined by the degree of complementary in home production.

The efficient transaction tax is driven by the transaction cost technology and is independent of the home production technology. When choosing their housing services consumption in the initial period, households incorporate how their choice affects their transaction costs in the final period, weighting each state by its respective marginal utility of consumption, e.g. $v_c(c_H)/v_c(c_0)$. The planner insures transaction costs across productivity states by making households internalize the future marginal transaction costs at the marginal utility of consumption in the initial period instead, implying a unit weight for each productivity realization ($v_c(c_0)/v_c(c_0)$). The efficient transaction tax for the high type is thus

$$\tau_d(\theta_H) = \Phi_2(d(\theta_H), d_0) \left(\frac{1}{\beta R} \frac{v_c(c_0)}{v_c(c_H)} - 1 \right)$$

where $c_H > c_0 > c_L$. When households move in the second period they generally face a non-zero transaction tax. Specifically, when households move after a positive shock they efficiently face a positive transaction tax. This argument holds irrespective of the home production technology.

Having characterized the forces that drive efficient taxes, I turn to a positive description of the economy where I introduce current policy and institutions in the Netherlands. I study this economy to measure implicit subsidies on housing consumption under current policy and to infer parameters that quantitatively determine efficient taxes.

4 Positive Economy

I present a quantitative model which I use to estimate preferences and to obtain household-specific levels of lifetime utility under the current housing policy for the Netherlands.

Assets. Households enter each period with savings that can be allocated to three asset classes after observing

their labor productivity. Savings s can be held in the form of financial assets a , housing wealth $h \geq 0$, and mortgages $m \geq 0$. Uncollateralized lending is restricted by the borrowing limit $\underline{s} \leq s$. Mortgages are collateralized loans which can only be held by homeowners ($h > 0$). Households earn net interest r on their financial assets and pay the same interest rate on their mortgage debt.

Households can choose to own or to rent a house. Households can purchase houses with capital levels greater than \underline{h} at a house price p^H per unit of housing capital. This cutoff level is an entry barrier into homeownership for low-income households. Households can otherwise rent houses with capital levels below \underline{h} from landlords at a rental price p per unit. When households choose to be homeowners they incur a required maintenance costs δ^H per unit of housing capital, while landlords incur the maintenance costs on rental units.

Homeowners can finance their house by taking a mortgage, which I model as non-defaultable debt.¹¹ The government dictates lending guidelines to financial intermediaries that limit the size of the mortgage as a function of the value of the property, the household's labor earnings, and its age,

$$m \leq \kappa_t(h, wy). \quad (24)$$

The loan-to-value and loan-to-income requirements are a second entry barrier into homeownership. When households transition from being a homeowner to being a renter they repay their mortgage debt.

Current Housing and Income Tax Policy. The marginal tax rates that vary with income, housing, and mortgage choices are the income tax rate, the rent subsidy, and the marginal tax rate on financial assets. The statutory tax rates are calibrated in Section 5.

The income tax system is progressive and treats homeowners and renters differently. Income \tilde{y} is the sum of labor earnings wy , and imputed rental income $\tau_o p^H h$ minus home mortgage interest expenses rm ,

$$\tilde{y} = wy + \tau_o p^H h - rm, \quad (25)$$

where τ_o is a policy variable that determines the fiscal rent-to-value. A distinguishing feature of the Dutch tax code is that homeowners add part of their imputed rental income to their taxable income.

¹¹I choose to not model bankruptcy given that the average number of bankruptcies in the Netherlands between 2006 and 2014 is only 163 for every 1 million individuals. Data from the American Bankruptcy Institute show that the bankruptcy rate in the United States is 25 times larger in this period (following the approach of Livshits, MacGee, and Tertilt (2010)).

Income is taxed at a progressive marginal rate τ_y which depends on the retirement status. The marginal income tax rate varies across \mathcal{B} income brackets. For example, the marginal income tax rate for workers τ_y^w is given by the piecewise function

$$\tau_y^w = \begin{cases} \tau_{y,1}^w, & 0 \leq \tilde{y} < \underline{b}_1^w \\ \tau_{y,2}^w, & \underline{b}_1^w \leq \tilde{y} < \underline{b}_2^w \\ \vdots & \vdots \\ \tau_{y,\mathcal{B}}^w, & \underline{b}_{\mathcal{B}-1}^w \leq \tilde{y} < \underline{b}_{\mathcal{B}}^w. \end{cases}$$

A household's total income tax, denoted by $T_t^y(\tilde{y})$, is the sum of its income tax across the brackets.

Renters with low levels of labor earnings may qualify for direct rent subsidies. The amount of the subsidy decreases in income below a cutoff level for income and increases in the amount of rent paid below a cutoff. The rent subsidy combines a component that supports a minimum level of housing consumption with a pure subsidy component. The form of the subsidy function, which I denote by $T_t^d(pd, \tilde{y})$, is given in Appendix D.

Finally, financial wealth is taxed. Households pay a tax τ_i on financial assets held in excess of cutoff \underline{a} . The financial wealth tax is captured by $T_t^a(a_t)$.

Household Problem. Households enter every period with accumulated savings s . At the beginning of the period households decide whether to rent or to buy a house and make their portfolio decision across financial assets, housing wealth, and mortgages. I discuss the constraint sets and the decision to buy or own.

Renter's Problem. Renters with net worth s can only hold their savings in the form of financial assets, so $s = a$. Working-age renters with net worth s that receive labor income $wy = \tilde{y}$ pay income taxes $T_t^y(\tilde{y})$ and financial wealth taxes $T_t^a(a)$. Renters can spend their after-tax income on consumption goods, housing services, and transaction costs, and save the remainder, knowing that the portfolio allocation is optimized at the beginning of next period. Renters pay a value-added tax on consumption goods $T^c(c)$, and potentially qualify for a rent subsidy. In summary, the sequential budget constraint for renters is

$$c_t + T^c(c_t) + p_j d_t + T_t^d(p_j d_t, \tilde{y}_t) + \Psi(d_t, d_{t-1}) + s_{t+1} = wy_t - T_t^y(\tilde{y}_t) + R s_t - T_t^a(s_t) + T_t, \quad (26)$$

where T_t is an age-dependent lump-sum transfer.

Renters maximize utility by choosing consumption, housing services, market hours, and savings subject to their budget constraint (26), the borrowing constraint $s_{t+1} \geq \underline{s}$, the time constraint $\ell_t + n_t = 1$, and the size limit for rental units $d \leq \underline{d}$. The constraint set for renters is denoted $\Gamma_t^r(s_t, d_{t-1}, \theta_t)$. At age t , the value of being a renter with net worth s_t , having lived in dwelling d_{t-1} in the prior period, and labor productivity θ_t is

$$V_t^r(s_t, d_{t-1}, \theta_t) = \max_{(c_t, d_t, n_t, s_{t+1})} u(c_t, d_t, \ell_t) + \beta \sum_{\theta_{t+1}} \pi^{t+1}(\theta_{t+1}|\theta_t) V_{t+1}(s_{t+1}, d_t, \theta_{t+1}), \quad (27)$$

subject to $(c_t, d_t, n_t, s_{t+1}) \in \Gamma_t^r(s_t, d_{t-1}, \theta_t)$, and where $V_{t+1}(s_{t+1}, d_t, \theta_{t+1})$ is the value of entering the following period with savings s_{t+1} having lived in dwelling d_t . The future value is determined by the homeownership decision that is made at age $t + 1$ which I discuss below.

Homeowner's Problem. Homeowners hold their net worth as financial assets, housing wealth and mortgages, $s = a + p^H h - m$. Working-age homeowners with taxable income $\tilde{y} = wy + \tau_o p^H h - rm$ pay income taxes $T_t^y(\tilde{y})$ and financial asset taxes $T_t^a(a)$. Homeowners also pay a linear property tax τ_p on the market value of their house and incur required maintenance costs δ^H on their housing capital h . Homeowners allocate their disposable income towards consumption goods, transaction costs and savings. The sequential budget constraint for homeowners is

$$c_t + T^c(c_t) + \Psi(d_t, d_{t-1}) + s_{t+1} = wy_t - T_t^y(\tilde{y}_t) + Ra_t - T_t^a(a_t) + (p_{j+1}^H - \tau_p p_j^H - \delta^H)h_t - Rm_t + T_t, \quad (28)$$

where Rm_t is the gross interest payment on the mortgage.¹²

Homeowners maximize utility by choosing their asset portfolio, consumption, market hours, and savings subject to their budget constraint (28), the portfolio constraint $s = a + p^H h - m$, the borrowing constraint, the time constraint, and the house size restriction for homeowners $d_t \geq \underline{d}$. The constraint set for owners is $\Gamma_t^o(s_t, d_{t-1}, \theta_t)$. The value of being a homeowner is

$$V_t^o(s_t, d_{t-1}, \theta_t) = \max_{(c_t, d_t, n_t, a_t, h_t, m_t, s_{t+1})} u(c_t, d_t, \ell_t) + \beta \sum_{\theta_{t+1}} \pi^{t+1}(\theta_{t+1}|\theta_t) V_{t+1}(s_{t+1}, d_t, \theta_{t+1}), \quad (29)$$

subject to the constraint that $(c_t, d_t, n_t, a_t, h_t, m_t, s_{t+1}) \in \Gamma_t^o(s_t, d_{t-1}, \theta_t)$.

¹²While homeowners cannot shrink their house by choosing not to maintain it, I do allow them to downsize their house without any adjustment cost.

Tenure Choice. At the beginning of every period households make their tenure choice, they decide whether to rent or to buy a house. Households choose to rent a house when the value of being a renter (27) exceeds the value of being an owner (29) given their state,

$$V_t(s_t, d_{t-1}, \theta_t) = \max(V_t^r(s_t, d_{t-1}, \theta_t), V_t^o(s_t, d_{t-1}, \theta_t)). \quad (30)$$

Production. The production side of the economy consists of three types of producers. Rental firms convert housing capital into housing services for renters, construction companies convert the general good into housing capital, and general good producers produce the numeraire good.

Rental firms operate in a competitive market using a technology that transforms one unit of housing capital into χ units of housing services.¹³ Rental firms receive rent p_j per unit of housing services. They borrow funds at interest rate r to buy housing capital at the beginning of the period at unit price p_j^H , incur maintenance costs δ^H and sell their housing capital at the end of the period at unit price p_{j+1}^H . Rental firms also pay property tax τ_p per unit of housing capital and receive a subsidy on interest payments τ_f . In equilibrium, rents are:

$$p_j = \frac{1}{\chi} \left(r(1 - \tau_f) + \tau_p + \hat{\delta}^H - \pi_{j+1}^H \right) p_j^H, \quad (31)$$

where $\hat{\delta}^H \equiv \delta^H / p_H$.

Construction companies operate a time to build technology in a competitive market. In period $j + 1 - \iota$, a construction company commits to convert general goods into $Q_{j+1-\iota}^h$ units of housing capital using a one-to-one production technology in period j . In period j , general goods are converted into new housing units which are delivered at the end of the period and valued at price p_{j+1}^H . In the first period, the construction companies plan to deliver housing units in period ι . The house price for all periods $j > \iota$ is equal to unity, or equivalently, the supply of houses for all periods $j > \iota$ is perfectly elastic.

General good producers rent capital and hire workers to produce a numeraire good with a Cobb-Douglas technology using business capital and effective labor $F(K, Y)$. Given an interest rate on business capital r^K and a wage rate w , the firm chooses its inputs such that $r^K = F_K(K_j, Y_j)$ and $w = F_Y(K_j, Y_j)$.

¹³In the Netherlands, only 14% of the rental supply is provided by households in 2018. I abstract from direct household rental supply by modeling rental firms providing the rental supply. Chambers, Garriga, and Schlagenhaut (2009) show that households are a prominent supplier of rental units in the United States.

Government. The government collects taxes on consumption, income, property holdings, financial wealth holdings, and housing transactions. Tax revenues finance transfers, government expenditures, rent subsidies, financing subsidies towards rental firms, and interest payments on the government’s outstanding debt. The government issues debt when its expenses exceed its revenues.

Equilibrium. To summarize the model under current policy, I present a formal definition of equilibrium in Appendix E. The housing market is local, and clears in equilibrium.

5 Calibration

The model is calibrated in three steps. In the first step, I calibrate demographic and technology parameters using aggregate data, and I calibrate policy parameters using the descriptions of the responsible government agencies. In the second step, I estimate the skill process using micro data on household wages. In the third step, three preference parameters are calibrated by matching model simulated moments to their empirical counterpart.

Calibration. I calibrate macro-parameters using aggregate data, such as the national income and product accounts, while I calibrate policy parameters using the description by the respective government agency. I use public data from Statistics Netherlands to calibrate the macro-parameters, and policy descriptions by the national tax office, unless stated otherwise. I use data for all years between 2006 and 2014. All amounts are denominated in 2015 euro.

Demographics. Households enter the labor market at 25 and participate for $T_w = 39$ years, which corresponds to the median retirement age of 63. Households live until 77, the median life expectancy for cohorts born between 1946 and 1960, conditional on surviving to age 25. Households enjoy $T_r = 13$ years of retirement. The demographic parameters are summarized in the top panel of Table 1.

Technology. I calibrate the interest rate to the average annual real mortgage rate on outstanding mortgages, which is 3.05 percent.¹⁴ The time discount factor is set so that $\beta R = 1$, or $\beta = 0.97$.

¹⁴Data for the average nominal interest rate on outstanding mortgages is reported by financial intermediaries to the Dutch Central Bank. The interest rate is weighted by the outstanding mortgage balance, and deflated by the consumer price index.

Table 1: Exogenously Calibrated Parameters

Parameter	Value	Data Target	
Demographics			
T	Length of life	53	Median life expectancy of 77
T_r	Retirement age	40	Median retirement age of 63
Technology			
r	Interest rate	0.031	Mean interest rate on mortgage loans
α	Capital share	0.439	Capital income share
δ^K	Depreciation of capital	0.061	Depreciation rate of business capital
δ^H	Depreciation of housing	0.024	Depreciation rate of residential structures
χ	Housing services flow	0.055	Normalization of benchmark user cost, $r + \delta^H$
ι	Time to build	2	Mean building time for new houses
ψ_b	Transaction cost, buyer	0.020	Mean broker fee, buyers
ψ_s	Transaction cost, seller	0.015	Mean broker fee, sellers

Table 1 presents the parameters calibrated exogenously.

The general good is produced by a Cobb-Douglas technology with an output elasticity of business capital, α , set to 0.439, the capital income share in the Netherlands. The depreciation rate of business capital is $\delta^K = 0.061$, which corresponds to the depreciation rate of capital excluding housing, research and development, and software.

Housing services are proportional to the stock of housing capital, which depreciates at rate $\delta^H = 0.024$. The flow services per unit of housing capital, χ , is set such that the rental price in absence of government policy is 1, that is, $\chi = r + \delta^H$. The production of housing units takes 23 months on average after a building permit is issued, which is consistent with $\iota = 2$.¹⁵ I calibrate the transaction cost function to real estate broker fees. The average broker fee for sellers in the Netherlands is equal to about 2 percent of the sales price (Gautier, Siegmann, and van Vuuren, 2018). The mean broker fee for buyers is approximately 1.5 percent of the transaction price. The technological parameters are summarized in the bottom panel of Table 1.

¹⁵Gomme, Kydland, and Rupert (2001) incorporate time to build for business capital into a business cycle model with housing capital, but they do not incorporate time to build in the housing sector. In this paper it takes one year to complete an investment in business capital, in line with Kydland and Prescott (1982) and Gomme, Kydland, and Rupert (2001).

Table 2: Policy Parameters

Policy Instrument		Value
Housing		
τ_r	Transaction tax	0.060
τ_f	Financing subsidy	0.157
τ_o	Imputed rent tax	0.006
τ_p	Property tax	0.001
Other		
τ_c	Consumption tax	0.134
T_b	Retirement benefit	18,140

Table 2 parameterizes affine policy functions. The specification of the nonlinear policy instruments, such as income taxes, asset taxes, rent subsidies, and lending restrictions, is described in the text and presented in Figure 1.

Policy. I parameterize affine policy parameters, which I summarize in Table 2, and describe the nonlinear tax functions.

Buyers pay a transaction tax, τ_t , equal to 6 percent of the property value. The government indirectly subsidizes rental housing by guaranteeing loans of rental firms, which translates into an effective financing subsidy, τ_f , of 15.7 percent.¹⁶ The statutory tax on imputed rent income τ_o is 0.6 percent of the property value. Since property taxes are levied at the local level in the data, I calibrate the model property tax τ_p to the value-weighted average of 0.1 percent.

The sales tax on consumption goods, τ_c , is 13.4 percent, which is the spending weighted average indirect tax on consumption goods. When retired, households receive public pension benefits equal to the minimum wage of full-time workers, or $T_b = 18,140$.

The tax instruments that drive differences in the user's cost across homeowners are income taxes and asset taxes. The first two panels in Figure 1 plot the schedule for each of these nonlinear instruments.

The income tax schedule is progressive with marginal tax brackets of 34, 42 and 52 percent for working

¹⁶Veenstra and van Ommeren (2017) estimate that explicit bailout clauses for Dutch housing corporations reduce their funding costs by 72 basis points. The authors use loan-level data covering approximately 44% of corporations' external funding between 1997 and 2013 to measure the interest rate differential between comparable guaranteed loans and non-guaranteed loans. In 2014, 95% of public housing corporation debt was guaranteed. I model all rental housing as indirectly subsidized.

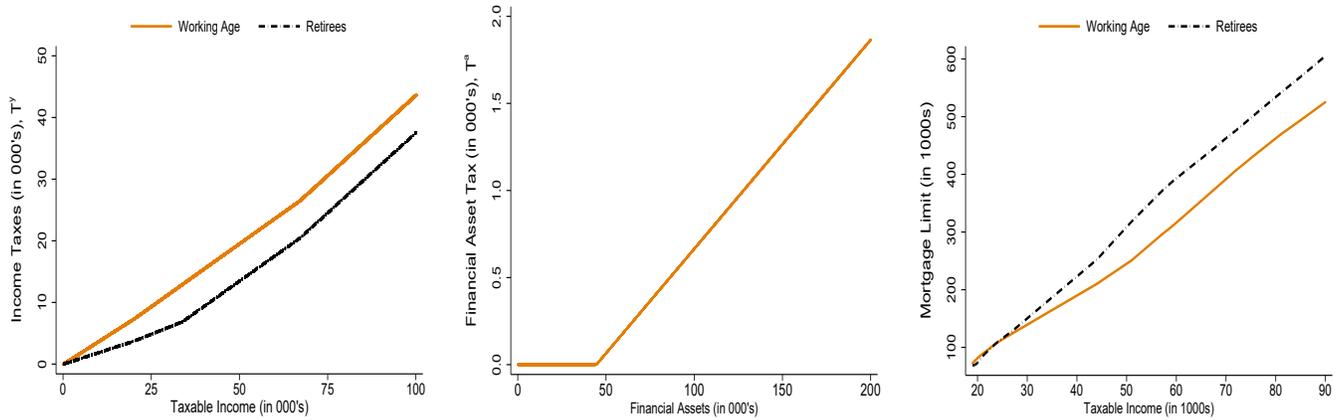


Figure 1: Income Tax, Wealth Tax, and Mortgage Regulation

Figure 1 displays income taxation, wealth taxation, and mortgage regulation in the Netherlands. The left panel shows the income tax schedule for working age households (solid line) and retirees (dashed line). Financial holdings in excess of 46 thousand euro are taxed at 1.2 percent rate (middle). The right-hand panel displays the maximum loan-to-income guidelines that the Dutch government prescribes to financial intermediaries for working age households (solid line) and retirees (dashed line).

age households, with cutoff levels at 20 and 59 thousand euro. The marginal tax rate is reduced for retirees with incomes below 35 thousand euro. The marginal tax rate is 17 percent below 20 thousand euro, and 24 percent below 35 thousand euro. The income tax schedule is shown in the left panel of Figure 1.

Financial wealth in excess of 46 thousand euro is taxed at $\tau_i = 0.012$, a 1.2 percent rate. The government imputes an annual return of 4 percent for financial wealth, which it taxes at a 30 percent rate. The resulting wealth tax schedule is in presented in the middle panel of Figure 1.

The government prescribes guidelines to the financial sector that restrict the extension of home mortgages. The maximum mortgage loan that financial intermediaries can extend is determined by household income. The right-hand panel shows the maximum loan-to-income guideline between 2006 and 2014 for workers and retirees.¹⁷ The extension of mortgages is further limited by a statutory maximum loan-to-value limit of 1.05 that was introduced in 2012. I translate both policies into a single mortgage limit that depends on household age and income as well as the value of the property, as described in (24).¹⁸

¹⁷The mortgage guidelines are written by the National Institute for Budget Information. Starting in 2007, their prescriptions are adopted into a code of conduct for the financial sector. After the financial crisis, the guidelines have been incorporated into a binding legal arrangement. The methodology behind the mortgage guidelines is described in [Warnaar and Bos \(2017\)](#).

¹⁸The mortgage limit is the minimum of the maximum loan-to-income and the maximum loan-to-value. Households effectively

Data. I use linked administrative records between 2006 and 2014 from Statistics Netherlands, the national statistics agency, to measure the user’s cost of homeowners under current policy, to estimate a skill process for different education groups, and to estimate household preferences.

I use a representative subsample of all Dutch households selected by Statistics Netherlands.¹⁹ The sample consists of about 95 thousand households per year, roughly 1.3% of the population of households, covering a total of over 275 thousand individuals. For each household, I integrate data for all household members in a given calendar year. For all analyses, I weight households with the provided sample weights. I consider all households with heads of household above age 25.

Labor Market. Income is measured by employer-provided earnings records. I construct an individual’s annual taxable labor earnings, which includes the employer’s health insurance contribution, by adding all earnings reports within a given calendar year. To construct an hourly wage rate, I divide taxable labor earnings by employer-reported hours worked. Because the model features a single decision maker for each household, I define the household wage rate for married and cohabitating households as the average individual wage rate weighted by the hours worked of each partner. For single households, the individual wage rate is the household wage rate. Household non-market time is given by average individual non-market time which is discretionary time minus individual hours worked. I set an individuals discretionary time equal to 16 hours a day for 365 days.

The measure of educational attainment for each individual is the highest degree they earned. I classify every degree as a low, a medium, or a high level of education. The low education level corresponds to a high school degree or a practical degree, the medium level is a degree from a university of applied sciences, while a high level of education is a university degree. I group households into six education bins, which are unordered pairs of the degree of each partner. Singles are grouped with couples in which both partners have

choose between an annuity mortgage, a linear mortgage, and a balloon mortgage (of at most 50 percent of the property value, as prescribed by the mortgage guidelines). I model the maximum loan-to-value as the maximum outstanding mortgage balance of the three different contracts, under the assumption all households take out a 30 year mortgage at age 35, and extrapolating this function before age 35. By modeling the mortgage limit as a function of age, rather than the time at which the mortgage was extended, I contain the state space of the household problem.

¹⁹Specifically, I use the IPO subsample (Inkomenspanelonderzoek). To simplify the exposition of this section, I omit names of individuals data sets that I link to this sample. If you are interested, contact me for more detail.

obtained the same level of education.

Housing and Assets. To measure housing consumption for each household, I assume that the housing service flow is proportional to the property value of the residence. For both renters and owner-occupiers, I measure housing services consumption using tax assessed property values. The fiscal authority assesses the market value of every property as of January 1 using transaction data of comparable houses. To make property values comparable across time, I deflate property values by the regional house price index.

Households' financial assets and mortgage balances are obtained from a wealth registry which records households' financial position as of January 1. I combine the outstanding mortgage balance with the property value to measure the loan-to-value ratio for every household's primary residence. Financial assets are used to calculate the household's marginal tax rate on financial wealth.

I first use only the labor market data to estimate the household skill process. I then introduce the estimated skill process into the structural model to calibrate household preferences using both labor market and housing data. Finally, I use all data to measure the user's cost of housing for homeowners.

Skill Process. I parameterize the household skill process using estimates obtained outside the model using data on household wages.²⁰ I allow for heterogeneity between education groups in both the life-cycle profile and the idiosyncratic component of wages. For each household education bin I construct the life-cycle profile and I estimate a process for residual wages. To the extent that wages follow different predictable life-cycle profiles across education groups, wage growth is accounted for by a difference in growth profiles rather than by being classified as idiosyncratic risk.

To obtain the life-cycle wage profile and the residual wage, I regress household wages on dummy variables to control for time and age effects within each education group.²¹ The age fixed effects capture the life-cycle profile, the residual is labeled wage risk. Let z_{ijt} be the residual wage for household i at time j with age t . I

²⁰Since every competitive equilibrium is incentive compatible, estimating a productivity process using observed wages is not inconsistent with the assumption that the skill process is not observed by the government.

²¹I estimate the household skill process using stable households, households for which the composition of adults as well as the employment status of the adults is stable over time. When an adult's employment status changes, this is not picked up as household skill risk.

Table 3: Estimated Wage Process Parameters

Education Group	Persistence, ρ		Variation of Innovation, σ_u^2	
	Point Estimate	Confidence Interval	Point Estimate	Confidence Interval
Low, Low	0.9542	(0.9515, 0.9575)	0.0096	(0.0093, 0.0102)
Low, Medium	0.9660	(0.9610, 0.9692)	0.0087	(0.0083, 0.0096)
Low, High	0.9673	(0.9628, 0.9710)	0.0162	(0.0153, 0.0176)
Medium, Medium	0.9570	(0.9536, 0.9612)	0.0099	(0.0091, 0.0103)
Medium, High	0.9616	(0.9520, 0.9782)	0.0109	(0.0082, 0.0124)
High, High	0.9564	(0.9501, 0.9582)	0.0172	(0.0164, 0.0184)

Table 3 shows the estimated wage parameters by education group. The second and third column show the estimates for the persistence of the residual wage, the fourth and fifth column present estimates for the variance of the persistent innovation. The 95 percent confidence intervals are constructed using 1,000 bootstrap samples.

assume residual wages follow a first-order autoregressive process in logs with both persistent and transitory innovations,

$$\log z_{it} = \log \theta_{it} + \varepsilon_{it} \quad i = 1, \dots, N, \quad t = 1, \dots, T,$$

$$\log \theta_{it} = \rho \log \theta_{it-1} + u_{it}$$

where $u_{it} \sim \mathcal{N}(0, \sigma_u^2)$, $\varepsilon_{it} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ and $z_{i0} \sim \mathcal{N}(0, \sigma_{z_0}^2)$, and N is the number of households in the group.

I estimate the parameters that govern the residual wage process using the minimum distance estimator (Chamberlain, 1984). I minimize the distance between empirical moments of the variance-covariance structure for residual wages and their analytical counterpart. Specifically, I target the residual wage variance and the first-order autocovariance at each age. To construct confidence intervals, I estimate the parameters for 1,000 bootstrap samples.

Table 3 shows the estimated persistence ρ , and the variance of the permanent innovation σ_u^2 , for each group. The second and third column present the point estimate and confidence interval for the persistence, the fourth and fifth column present the point estimate and confidence interval for the variance of the persistent innovation. The persistence is similar across groups, ranging from 0.954 for households with lower education to 0.967 for households with a highly educated and a less educated spouse. The standard deviation of the innovation for high education households is 30 percent larger than the standard deviation of the innovation

Table 4: Estimated Parameters

	Parameter	Value	Target	Data	Model
γ	Preference weight on consumption	0.343	Consumption to output ratio	0.64	0.66
ω	Housing share in home production	0.144	Housing share in consumption	0.17	0.16
σ	Elasticity of substitution	0.951	$\text{Cov}(\ell/d, w)/\text{Var}(w)$	-0.44	-0.43

Table 4 presents the preference parameters that are estimated within the model.

for low education households.

Calibration. After calibrating the demographic and technology parameters, and estimating the household skill process, I calibrate household preferences. I assume households' flow utility is of the form

$$u(c, d, l) = \gamma \log c + (1 - \gamma) \log \left(\omega d^{\frac{\sigma-1}{\sigma}} + (1 - \omega) \ell^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (32)$$

that is, I assume both v and \mathcal{H} in equations (1) and (2) given by natural logarithm functions. The flow utility function is parameterized by a Cobb-Douglas weight on non-housing consumption γ , a weight on housing services in the home technology, ω , and the elasticity of substitution between housing services and leisure, σ . The three preferences parameters are chosen to minimize the squared difference between simulated moments from the model and their empirical counterpart. Table 4 displays the three preference parameters and shows that the model well approximates the empirical moments targeted in the calibration.

The preference weight for consumption and the weight on housing services in the home technology are calibrated by targeting aggregate moments. The weight on consumption in preferences targets the aggregate consumption-output ratio, which is 0.642. The weight on housing services in the home technology targets the expenditure share of housing in consumption, which equals 0.174. The measurement of these two moments is discussed in Appendix F.

To estimate the elasticity of substitution between housing services and leisure in home production, I use the model without distortionary taxation and transaction costs as an auxiliary model. Specifically, I use the optimality conditions for housing consumption and leisure in the misspecified model to derive the regression equation

$$\log \left(\frac{\ell}{d} \right) = -\sigma \log \left(\frac{\omega}{1 - \omega} \right) - \sigma \log \left(\frac{w}{p} \right), \quad (33)$$

where w is the household wage and p is the user’s cost of housing capital in absence of distortionary taxes (34).²² I separately estimate (33) using actual data and simulated data. I choose the elasticity of substitution in the model such that the regression coefficient implied by the model is as close as possible to the regression coefficient in the data.²³

The elasticity of substitution between housing and leisure in the home production technology is identified by the covariation in the home production input ratio with the opportunity cost of time, similar to [Rupert, Rogerson, and Wright \(1995\)](#) and [Aguiar and Hurst \(2007\)](#). The opportunity cost of time in my framework is the household wage rate. In absence of transaction costs and distortionary taxation, housing consumption increases one for one with wages without changing hours when the home technology is Cobb-Douglas ($\sigma = 1$). When housing services and non-market time are complements ($\sigma < 1$), the input mix decreases less than one for one with wage changes. Transaction costs change this relationship. As wages increase, households may not increase their housing consumption because of transaction costs, leading to a bias in the regression coefficient. In sum, the regression coefficient only indirectly informs the elasticity of substitution.

To align the administrative micro data and the data generated by the unitary household model, I measure non-market time ℓ in the micro data as average leisure time for adult household members, and the household wage rate as the annual hours-weighted average of hours worked by adult members. To obtain an individual measure of housing services consumption, I regress the value of the household’s residence on dummy variables for the number of adults in the household.

I find that with an elasticity of substitution of $\sigma = 0.951$ the model matches the regression coefficient of -0.44 in the micro data. While the regression coefficient naively suggest a strong complementarity between housing services and leisure, this naive estimate is biased downward due to presence of adjustment costs. My estimate is similar to the assumed elasticity of substitution of one in the business cycle analysis by in

²²Note that I do not require v and \mathcal{H} to be natural logarithm functions to obtain this optimality condition.

²³The identification is akin to the “gap” based indirect inference used by [Berger and Vavra \(2015\)](#). [Berger and Vavra \(2015\)](#) minimize the gap between optimal consumption of durables if a household pays a fixed adjustment cost relative to actual durable consumption. To construct this gap in their data, they use a model-generated mapping from observables to the optimal choice after incurring fixed adjustment costs to impute the optimal choice. My “gap” is similar, yet more direct. I minimize the gap between the optimal ratio of home production inputs when a household does not face transaction costs and distortionary taxes relative to the observed home production input ratio. My model-generated mapping from my observable, the household wage, to the optimal choice is given by (33).

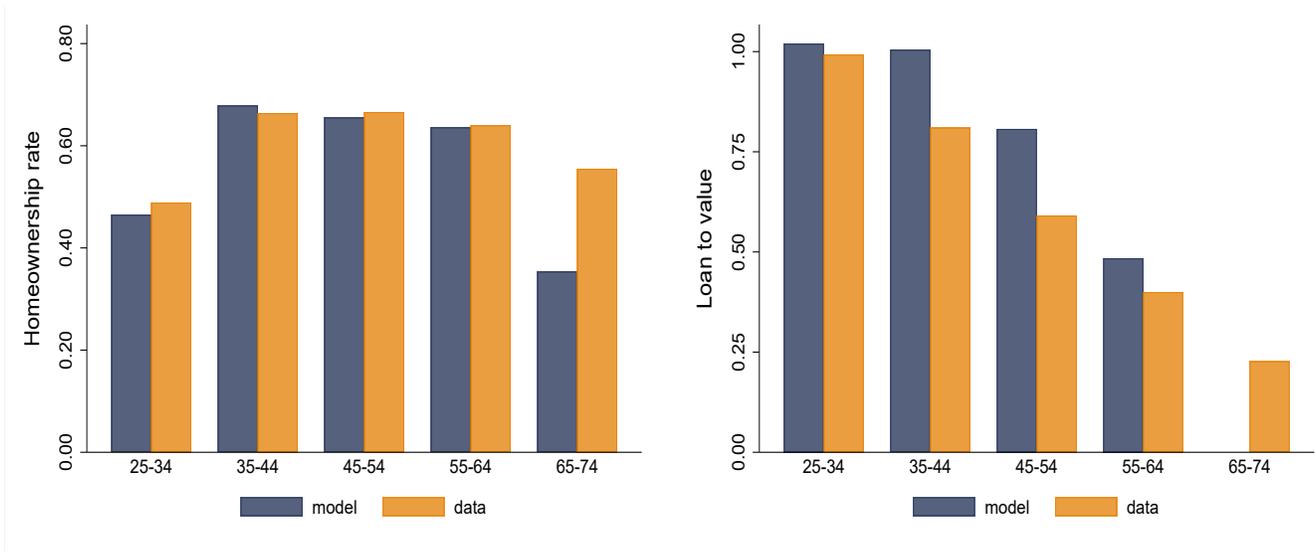


Figure 2: Homeownership and Loan to Value

Figure 2 compares the homeownership rate and the loan-to-value ratio by household age in the model to the data. The left-hand panel compares homeownership in the data (in orange) to homeownership in the model (in blue). The right-hand panel compares the loan-to-value ratio in model and data.

Greenwood and Hercowitz (1991).

Model Validation. Before using the model to study policy reform, I compare the model’s predictions to a set of predictions that were not explicitly targeted in the calibration.

In Figure 2, I compare the homeownership rate and the loan-to-value ratio by household age in the model to the data. The left panel shows that homeownership increases between age 25 and age 45 and decreases in retirement. The loan-to-income and loan-to-value requirement restrict homeownership early in life. The minimum house size to own and transaction costs act as an entry barrier into homeownership throughout life. While the model matches the homeownership rate of households until retirement, it predicts a lower homeownership rate for retirees. Since households do not have preferences for leaving a bequest, and live until age 77 with certainty, they consume all savings in retirement. To smooth non-housing consumption households eventually sell their house and move to a rental unit, depressing the rate of homeownership for retirees in the model.

The right-hand panel shows the loan-to-value ratio in the model and the data. In the model, homeowners

take out the maximum mortgage loan given their income and the value of their property. When they satisfy the income requirements, households take out the maximum size of their mortgage given the property value and their age. The household loan-to-value ratio by age reflects the maximum loan-to-value requirement as described in Footnote 18. In the model, households are only required to pay off their outstanding mortgage balance after age 35. In the data, households start reducing their outstanding mortgage balance earlier, and faster (for example, under a linear mortgage contract) explaining the gap between loan-to-value ratio in the data and the model.

6 Quantitative Results

I quantify efficient policy reform by comparing efficient housing taxes with effective housing subsidies under current policy. I measure that the average homeowner receives an effective housing subsidy of 7.7 percent, which is declining in age. In contrast, I find an efficient average housing tax of 13.8 percent, which is almost constant with age, by computing the efficient allocation for the calibrated economy. Finally, I use the insights from the efficient allocation to inform simple policy reforms.

User’s Cost. To measure the effective subsidy on housing consumption under current policy I evaluate the user’s cost of housing capital. User costs measure the marginal cost of housing services and are proportional to the static housing services wedge in an economy with proportional taxes on non-housing consumption. I measure the effective subsidy on housing consumption under current policy by comparing the user cost under current policy to the user cost in the absence of distortionary policy.²⁴

Absent distortionary policy, households and firms can borrow at interest rate r to buy housing capital, on which they incur maintenance cost δ^H , and which they can sell with a capital gain π^H . The user cost for a laissez-faire economy, which I denote p^l , is therefore

$$p^l = r + \hat{\delta}^H - \pi^H, \tag{34}$$

where $\hat{\delta}^H \equiv \delta^H/p_H$. The laissez-faire user cost increases in the cost of capital and the maintenance cost, and

²⁴In Appendix G, I derive the user’s cost for renters and homeowners from their budget constraints. This user cost approach is similar to Laidler (1969), Aaron (1970), Dougherty and Van Order (1982), Poterba (1984, 1992), Himmelberg, Mayer, and Sinai (2005), and Poterba and Sinai (2008).

decreases with the capital gain. Given the calibrated values for the cost of capital and the depreciation rate of housing in Table 1, and an average real housing capital gain of minus 2.8 percent per year, I calculate a benchmark user's cost of 8.3 percent. For a property of 250 thousand euro, this implies a laissez-faire monthly rent of 1,725.

Homeowners. Housing policy changes the user cost of homeowners by reducing their financing costs and by increasing their expenses. The reduction in the borrowing cost depends on the fraction of the property financed with debt, which I denote by \varkappa_i , where the subscript i indicates variation across households i . To the extent that a property is mortgage-financed, the borrowing cost reduces due to the deductibility of mortgage interest payments from taxable income. The value of the home mortgage interest deduction depends on the household's marginal income tax rate $\hat{\tau}_{yi}$.²⁵ To the extent that a property is equity-financed, the borrowing cost is reduced due to the exclusion of housing capital from financial assets, which face a marginal rate $\hat{\tau}_{ai}$. The expenses on housing services increase due to property tax τ_p , which is not deductible for income taxation, and increase due to the imputation of rental income into taxable income. Fraction τ_o of imputed rental income is treated as taxable income, and thus faces a marginal tax rate $\hat{\tau}_{yi}$. Combining the reduction in borrowing costs and the increase in expenditures, the user cost for homeowners amounts to

$$p^o = r - \pi^H + \hat{\delta}^H - \hat{\tau}_{yi}r\varkappa_i - \hat{\tau}_{ai}(1 - \varkappa_i) + \tau_p + \hat{\tau}_{yi}\tau_o. \quad (35)$$

When households reduce their mortgage balance, they increase their user cost by $\hat{\tau}_{yi}r - \hat{\tau}_{ai}$. The effective subsidy for homeowners is given by $p^o/p^l - 1$.

Renters. The user cost of renters is reduced by both direct and indirect subsidies. The user cost for renters increases to the extent that indirect taxes increase the market rental rate and decreases due to direct housing subsidies, $\hat{\tau}_{di}$. By the expression for the market rental rate (31), rents increase due to property taxes, and decrease due to financing subsidies towards rental firms, τ_f . Hence, the user cost for renters is

$$p^r = (r(1 - \tau_f) - \pi^H + \hat{\delta}^H + \tau_p)(1 + \hat{\tau}_{di}), \quad (36)$$

which decreases in the marginal housing subsidy ($\hat{\tau}_{di} < 0$), and the financing subsidies towards rental firms.

²⁵The marginal income tax rate, $\hat{\tau}_{yi}$, is shorthand notation for the marginal income tax rate faced by household i evaluated at their after-tax income level $\tilde{y}_i = wy_i + \tau_o p_j^H h_i - rm_i$, that is, $\hat{\tau}_{yi} = T_{1t}^y(\tilde{y})$.

Table 5: House Values and Mortgage Balances

Household Income (in thousand euro)						
Age of head	< 60	60–80	80–120	120–200	> 200	All
<i>Panel A: Renters</i>						
Rental Property Value (in thousand euro)						
25–35	152.5	168.8	188.5	220.9	–	160.1
35–50	158.4	174.8	197.9	251.4	402.3	170.1
50–65	161.1	175.5	191.4	221.3	323.0	172.1
> 65	278.5	213.1	246.0	286.9	477.1	274.2
All	194.2	177.2	192.7	237.8	375.3	197.2
<i>Panel B: Owners</i>						
Property Value (in thousand euro)						
25–35	167.1	185.1	210.8	256.5	321.6	190.6
35–50	212.7	223.5	255.5	324.9	433.4	255.3
50–65	233.6	245.8	269.4	325.7	425.0	274.4
> 65	255.0	314.0	348.9	395.4	507.2	285.7
All	223.0	229.5	260.0	325.4	431.4	255.4
<i>Panel B: Owners</i>						
Loan-to-value ratio						
25–35	1.00	1.03	1.04	1.05	1.03	1.03
35–50	0.75	0.75	0.76	0.80	0.84	0.77
50–65	0.42	0.50	0.51	0.52	0.56	0.49
> 65	0.20	0.24	0.29	0.33	0.42	0.22
All	0.56	0.70	0.69	0.69	0.72	0.64

Table 5 summarizes main housing variables for all households in the Netherlands for owners and renters.

The effective subsidy for renters is $p^r/p^l - 1$. Given the policy calibration in Table 2, renters that do not receive direct housing subsidies, receive an effective subsidy of 4.6 percent.

Effective Subsidy. Whether current policy implies an effective subsidy or tax on homeowners is a quantitative question. The user cost expressions, (35) and (36), show that the effective subsidy varies in the cross section due to variation in marginal tax rates on income $\hat{\tau}_{yi}$, assets $\hat{\tau}_{ai}$, as well as variation in the loan-to-value ratio α_i . Quantitatively, the effective subsidy varies significantly between age and income groups, with young households financing homeownership through debt and high income households facing higher marginal tax

rates.

Using tax records, I calculate the user's cost for the cross section of homeowners in the Netherlands. For homeowners, I use the variation in marginal tax rates on income, assets, and imputed rental income, as well as variation in the loan-to-value ratio. For every homeowner, in every time period, I evaluate (35). All variables indexed by i are household-specific, real house price inflation is specific to geographic regions, while all other parameters are common across households.

Table 6 shows the user cost for homeowners between 2006 and 2014, averaged by age and income groups. The table shows that housing services are significantly subsidized under current policy with strong variation across age and income groups. Panel A shows the total subsidy, the bottom panels separately display the contribution of the home mortgage interest deduction and the exemption of wealth taxes to the total subsidy.

The average effective housing subsidy for homeowners amounts to 7.7 percent. The average values range from 10.8 percent for young homeowners to 4.8 percent for old homeowners. The variation within age groups is driven by progressive income taxation given that leverage ratios are relatively constant within each age group. The subsidy increases with household income, reflecting that marginal tax rates on income and assets increase in their base. The subsidy decreases in age as homeowners reduce their mortgage balance while the benefit from the mortgage interest deduction exceeds the benefit of the exemption from wealth taxation.

Panel B and C show that housing is strongly subsidized through the home mortgage interest deduction and the exclusion of housing capital from wealth taxation. Young homeowners are subsidized through the home mortgage interest deduction, which increases with income due to the progressive income tax schedule. Since they hold a small amount of financial assets, which thus face a zero marginal rate, young homeowners do not benefit from the exclusion of housing from wealth taxation. For old homeowners the opposite is true. Old homeowners are mostly subsidized through the exclusion of housing from wealth taxation.

Table 6 suggests significant heterogeneity in the effects of policy reform. All else equal, Panel B suggests that eliminating the home mortgage interest deduction increases the average user cost by 9.2 percent. This increase would be particularly strong for young homeowners with large mortgages. Eliminating the exclusion of housing from wealth taxation mostly affects old homeowners.

In sum, the user's cost shows that housing consumption of renters and homeowners is subsidized across the age and income distribution. To assess whether the current subsidies are efficient, I compare the current

Table 6: Effective Subsidy for Homeowners

	Household Income					
Age of head	< 60	60–80	80–120	120–200	> 200	All
<i>Panel A</i> Effective Subsidy (in percent)						
25–35	9.8	10.7	11.6	12.6	14.1	10.8
35–50	7.1	7.4	8.3	10.2	11.4	8.2
50–65	5.1	6.0	6.8	8.2	10.0	6.7
> 65	3.6	6.4	7.2	7.7	9.5	4.7
All	6.1	7.6	8.1	9.4	11.0	7.5
<i>Panel B</i> Home Mortgage Interest Deduction						
25–35	13.5	14.5	15.5	17.0	18.8	14.6
35–50	9.9	10.2	10.8	12.6	14.4	10.9
50–65	5.6	7.0	7.5	8.3	9.5	7.2
> 65	1.5	2.2	3.1	3.8	6.6	2.0
All	7.1	9.5	10.0	10.8	12.3	8.9
<i>Panel C</i> Exclusion from Wealth Taxation						
25–35	0.1	0.1	0.1	0.1	-0.1	0.1
35–50	0.8	1.0	1.3	1.8	1.9	1.2
50–65	3.2	2.9	3.3	4.2	5.0	3.5
> 65	4.7	7.2	7.4	7.4	7.4	5.6
All	2.4	1.8	2.1	2.9	3.3	2.4

Table 6 presents the effective housing subsidy for homeowners under status quo housing policy. Panel A summarizes the effective subsidy for various income and age groups. Panel B and Panel C respectively account for the effect of the home mortgage interest deduction and the exclusion of housing from wealth taxation.

policy to efficient policy.

Efficient Reform. To understand whether the current user’s cost is close to efficient, I contrast the housing consumption wedge under current policy against the housing consumption wedge under an efficient reform. I use the estimated preference parameters and wage processes, and the technology parameters to calibrate

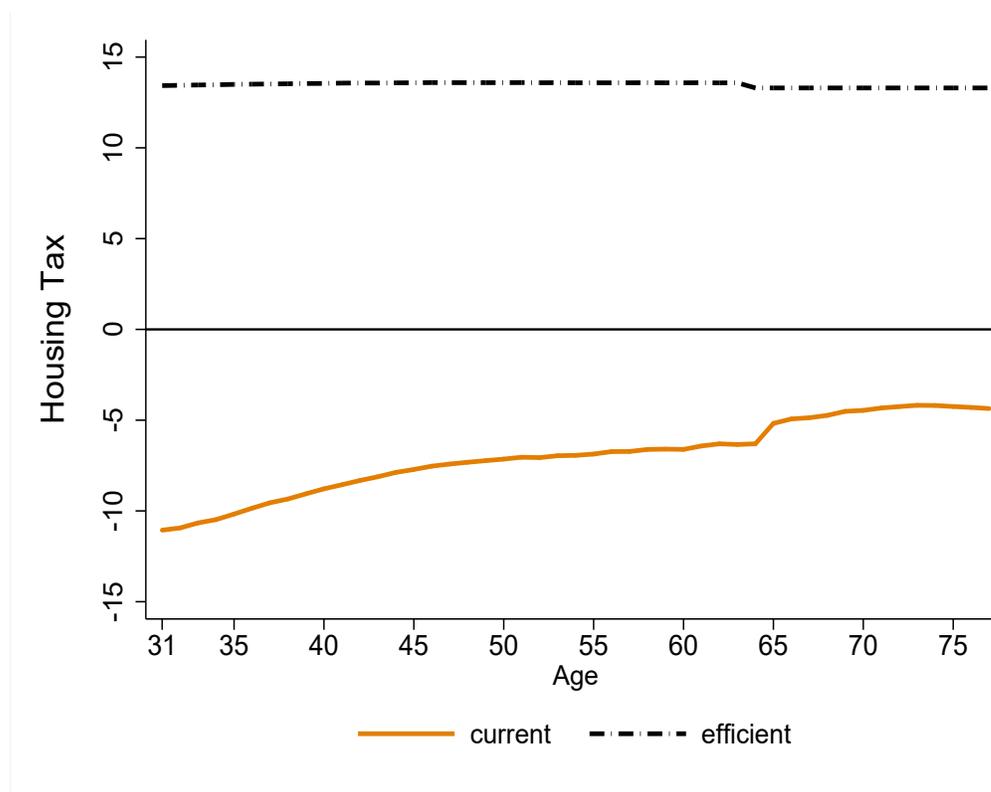


Figure 3: Efficient and Current Housing Policy

Figure 3 displays the average measured housing consumption subsidy under current policy (orange solid line) against an average efficient housing consumption tax (black dashed line) by household age.

the component planner’s problem discussed in Section 3. I discuss the numerical algorithm in Appendix H.²⁶

Figure 3 shows that the efficient housing consumption wedge significantly differs from the user’s cost under current policy. The orange solid line displays the current user’s cost by household age as given by the data underlying Panel A in Table 6, while the black dashed line reports an average efficient housing consumption tax, which I obtain by evaluating a solution to the component planning problem. The average user’s cost under current policy decreases from 12 percent to 5 percent over the life cycle, while the average efficient housing consumption tax is almost constant over the life-cycle, increasing from 13.5 percent to 13.8

²⁶The numerical work of Farhi and Werning (2013), Golosov, Troshkin, and Tsyvinski (2016), Stantcheva (2017) and Ndiaye (2018) importantly relies on a random walk specification for the skill process and a preference specification which ensures that the recursive formulation of the component planning problem scales with the previous skill realization. In my paper, the estimated skill process does not follow a random walk and the general home production preferences do not scale with the previous skill realization. In Appendix H, I describe the algorithm that I use in detail.

Table 7: Simple Policy Reform

Implemented policies	Δc	Δf_h
Reduce transaction tax from 6 percent to 2 percent	2.48	2.03
Equalize mortgage interest deductability	0.23	0.30
Alternative proposals		
Increase imputed rent tax to 2%	0.18	0.35

Table 7 presents the steady-state welfare outcomes of simple policy reforms. The second column shows the lifetime non-housing consumption equivalent gain of simple policy reform; the third column displays the change in the homeownership rate.

percent.

The efficient user cost is about 14 percent because the estimated elasticity of substitution between housing and leisure only implies a small complementarity. When the elasticity of substitution between housing and leisure equals one, the uniform commodity tax prescription of [Atkinson and Stiglitz \(1976\)](#) applies and so the efficient housing consumption wedge is equal to zero for every household in any efficient allocation. Holding constant the tax rate on non-housing consumption, the efficient housing consumption wedge equals zero only if the effective housing services tax is equal to the 13.4 percent tax on non-housing consumption. Quantitatively, I find that the difference from uniform commodity taxation is small, with an average efficient housing consumption wedge of 14 percent.

Simple Reforms. The implementation of an efficient reform requires cohort-specific and history-dependent taxes. I also use the model to simulate simple policy reforms to evaluate the long-run implications of policy reforms that were implemented by the Dutch government, and to design alternative simple reforms informed by the efficient reform.

Implemented Reform. In recent years, the government reduced both the transaction tax and the deductibility of home mortgage interest expenses. In 2011, during the recession, the transaction tax was lowered from 6 to 2 percent to spur the housing market. In 2014, the government started to reduce the deductibility of home mortgage payments. Specifically, it reduces the maximum rate at which mortgage interest payments can be deducted from 52 percent, the top income tax rate, to 37 percent, the lowest marginal income tax rate for workers, by 2023. The previous analysis indicates that these reforms move the effective tax rates on housing

closer to the efficient tax rates. I use the model to evaluate the long-run effects of these policy changes on homeownership and household welfare by comparing steady states before and after policy changes. When I conduct these policy experiments, I hold constant the level of government debt and adjust the intercept of the income tax schedule to balance the government's budget.

Table 7 shows long-run consequences of simple policy reform. The second column shows the lifetime non-housing consumption equivalent gain of simple reform, while the third column displays the change in the homeownership rate. The first row shows that the welfare gain due to lowering the transaction tax is equal to 2.48% of lifetime non-housing consumption as households increase their consumption of housing services. A low transaction tax reduces the barrier to entry into homeownership with a small loss on public revenues. Transaction tax revenues on a given transaction fall, but this revenue loss is offset by increased property tax revenues as households live in larger houses, and increased transactions.

The second row of Table 7 shows that the reform of the mortgage interest deduction only generates a small increase in household welfare. Reducing the deductibility of mortgage interest expenses for high-income households increases welfare by 0.23 percent of lifetime consumption and slightly increases homeownership. This reform reduces the tax expenditure on high-income households, which is redistributed to households as a lump-sum transfer. The reduction in the home mortgage interest deduction hardly affects the decision rent or own or households that were previously homeowners, but allows marginal households to become homeowners.

Alternative Reform. I use the expression for the effective homeowner subsidy (35), together with the efficient average tax rate of 13.8 percent, to inform alternative policy reform. I approximate efficient average housing taxes by changing current tax parameters. I vary the imputed rent tax τ_o from 0.6 to 2 percent which only affects the housing wedge of homeowners.

The third row of Table 7 shows the consequences of increasing the imputed rent tax from 0.6 to 2 percent. The mechanism is similar to the reduction of the mortgage interest deduction. Increasing the imputed rent tax increases the user's cost for homeowners and tax revenues. The increased cost hardly affects the decision rent or own for the original homeowners, but the increased transfer allows some households close to the margin to become homeowners.

7 Conclusion

In the study efficient reform of housing policy in an overlapping generations economy with uninsurable wage risk, incomplete asset markets, home production, and housing transaction costs.

I use a dynamic Mirrlees theory to show that in any efficient allocation housing consumption of every household is taxed when housing consumption and non-market time are complements in home production. By taxing housing services additional non-market time is spent in a less desirable dwelling, which provides incentives to productive households to produce. I also use this theory to show that in any efficient allocation homeowners do not pay a transaction tax when they buy their house, but pay a tax or receive a subsidy when they sell their house. Specifically, the government subsidizes households when they sell their house after a bad skill realization, and taxes households when they sell their house after a good skill realization in order to prevent households from residing in a small residence because of private concerns over future transaction costs.

Using administrative records for all households in the Netherlands, I show that current policy effectively subsidizes housing consumption and taxes households when they buy their house. The average homeowner currently receives an 8 percent subsidy on their housing consumption, which decreases from 11 percent to 5 percent over the lifecycle, and faces a 6 percent transaction tax.

I quantify an efficient reform using the calibrated economy under current policy. I find that housing and non-market time are complements in home production, which translates into an average efficient housing consumption tax of 14 percent, which is almost constant over the lifecycle. A simple reform, which reduces the transaction tax from 6 to 2 percent, generates a welfare gain of 2.5 percent of steady-state consumption.

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Efficient Housing Policy Reform

Appendix

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A Extensions of the Theory

In the main text I analyze a Beckerian framework in which goods and non-market time are inputs in the production of commodities that enter into household utility. In this appendix I show how the insights from the baseline analysis extend to a framework where time spent working in the market and at home directly enter into the household utility function as in [Gronau \(1986\)](#).

To show how the main insights extend, consider an economy with a market good c , a home commodity produced using housing services and non-market time $h(d, h_N)$, and leisure time $\ell = 1 - h_M - h_N$. I assume the planner observes the allocation of consumption c , housing d , and labor supply y , but does not observe household skill θ , or time allocated to home production h_N . Households have preferences over market goods, the home commodity, and leisure. Preferences are continuous, strictly concave, and separable with respect to market consumption, and the home production technology is continuous and concave.

Given an allocation of market goods, housing, and effective labor supply (c, d, y) , household type θ chooses their non-market time $h_N \in [0, 1 - y/\theta]$ to maximize utility. By the maximum theorem, the value function is strictly concave, and the solution h_N is a continuous function in the allocation of housing services and effective labor supply (d, y) . In sum, the value function is

$$v(c, d, y; \theta) = \max_{h_N} u(c, h(d, h_N), \ell) = u(c) + \tilde{h}(d, y). \tag{A.1}$$

Given [\(A.1\)](#), the analysis in the main text carries over to the framework where time spent working in the market and at home directly enter into the utility with the understanding that the specification of the home technology differs.

B Proof to Theorem 1

Proof. I show both directions by contradiction.

\Rightarrow If an allocation x is efficient it solves the planner problem given $\mathcal{V}_j(x(j, \theta^{t-1}); \theta^{t-1})$ for all $i \in \mathcal{I}$ with a maximand of zero. Suppose x does not solve the planner problem and let \hat{x} denote a solution to the planner problem. Because x is feasible, the allocation \hat{x} generates strictly excess resources in the first period. Construct an alternative allocation \tilde{x} identical to \hat{x} but increase initial consumption such that the ICs are satisfied. The allocation \tilde{x} strictly Pareto dominates x , which is a contradiction.

\Leftarrow If an allocation x solves the planner problem given $\mathcal{V}_j(x(j, \theta^{t-1}); \theta^{t-1})$ for all $i \in \mathcal{I}$ with a zero maximand, then it is efficient. Suppose that x is not efficient, then there exists an alternative feasible allocation \hat{x} such that all households are better off, with some household i strictly better off. Since allocation \hat{x} is feasible and delivers at least $\mathcal{V}_j(x(j, \theta^{t-1}); \theta^{t-1})$ for all $i \in \mathcal{I}$, \hat{x} is a candidate solution to the planner problem. Construct an alternative allocation \tilde{x} , which is equal to \hat{x} but equally reduce initial consumption for household i that is strictly better off under \hat{x} (such that the ICs are satisfied). Alternative allocation \tilde{x} is feasible and generates excess resources in the initial period. This contradicts that x is a solution to the planner problem. \blacksquare

C Derivation Wedges

I characterize the efficient labor and housing services wedge using the optimality conditions to the component planner problem. Recall that the component planner chooses $x_t(\theta) = \{c_t(\theta), d_t(\theta), y_t(\theta), \mathcal{V}_t(\theta), \tilde{\mathcal{V}}_t(\theta)\}$ to solve

$$\Pi_t(\mathcal{V}, \tilde{\mathcal{V}}, d, \theta_-) \equiv \max_{x_t(\theta)} \sum \pi^t(\theta|\theta_-) \left(w y_t(\theta) - c_t(\theta) - p_j d_t(\theta) - \Psi(d_t(\theta), d) + \Pi_{t+1}(\mathcal{V}_t(\theta), \tilde{\mathcal{V}}_t(\theta^+), d_t(\theta), \theta) / R \right)$$

where maximization is subject to (15)–(17):

$$\mathcal{V} = \sum \pi^t(\theta|\theta_-) (v(c_t(\theta)) + h(d_t(\theta), y_t(\theta)/\theta) + \beta \mathcal{V}_t(\theta)) \quad (15)$$

$$\tilde{\mathcal{V}} = \sum \pi^t(\theta|\theta^+) (v(c_t(\theta)) + h(d_t(\theta), y_t(\theta)/\theta) + \beta \mathcal{V}_t(\theta)) \quad (16)$$

$$v(c_t(\theta)) + h(d_t(\theta), y_t(\theta)/\theta) + \beta \mathcal{V}_t(\theta) \geq v(c_t(\theta^-)) + h(d_t(\theta^-), y_t(\theta^-)/\theta) + \beta \tilde{\mathcal{V}}_t(\theta), \quad (17)$$

where I use that preferences are separable in consumption (1).

I denote the multiplier on the promise keeping constraint (15) by ν_t , the multiplier on the threat-keeping constraint (16) by μ_t , and multipliers on the downward incentive constraints (17) by $q_t(\theta_i)$, where θ_i denotes

productivity realization $\theta_i \in (\theta_1, \dots, \theta_N)$. The optimality conditions to the component planner problem for consumption, housing services, and effective hours are

$$[c_t(\theta_i)] \quad \pi^t(\theta_i|\theta_-) = v_c(c_t(\theta_i)) (\nu_t \pi^t(\theta_i|\theta_-) - \mu_t \pi^t(\theta_i|\theta_-^+) + q_t(\theta_i) - q_t(\theta_{i+1})) \quad (\text{A.2})$$

$$[d_t(\theta_i)] \quad \pi^t(\theta_i|\theta_-) p_j = -\pi^t(\theta_i|\theta_-) \Psi_1(d_t(\theta_i), d) + h_d(d_t(\theta_i), y_t(\theta_i)/\theta_i) (\nu_t \pi^t(\theta_i|\theta_-) - \mu_t \pi^t(\theta_i|\theta_-^+) + q_t(\theta_i)) \\ - h_d(d_t(\theta_i), y_t(\theta_i)/\theta_{i+1}) q_t(\theta_{i+1}) + \pi^t(\theta_i|\theta_-) \Pi_{t+1,3}(\mathcal{V}_t(\theta_i), \tilde{\mathcal{V}}_t(\theta_{i+1}), d_t(\theta_i), \theta_i)/R \quad (\text{A.3})$$

$$[y_t(\theta_i)] \quad \pi^t(\theta_i|\theta_-) w = -h_y(d_t(\theta_i), y_t(\theta_i)/\theta_i) (\nu_t \pi^t(\theta_i|\theta_-) - \mu_t \pi^t(\theta_i|\theta_-^+) + q_t(\theta_i)) \\ + h_y(d_t(\theta_i), y_t(\theta_i)/\theta_{i+1}) q_t(\theta_{i+1}). \quad (\text{A.4})$$

The optimality conditions for promised utility $\mathcal{V}_t(\theta_i)$ and threat utility $\tilde{\mathcal{V}}_t(\theta_i)$ are

$$[\mathcal{V}_t(\theta_i)] \quad 0 = \pi^t(\theta_i|\theta_-) \Pi_{t+1,1}(\mathcal{V}_t(\theta_i), \tilde{\mathcal{V}}_t(\theta_{i+1}), d_t(\theta_i), \theta_i) + \beta R (\nu_t \pi^t(\theta_i|\theta_-) - \mu_t \pi^t(\theta_i|\theta_-^+) + q_t(\theta_i)) \quad (\text{A.5})$$

$$[\tilde{\mathcal{V}}_t(\theta_i)] \quad 0 = \pi^t(\theta_i|\theta_-) \Pi_{t+1,2}(\mathcal{V}_t(\theta_{i-1}), \tilde{\mathcal{V}}_t(\theta_i), d_t(\theta_{i-1}), \theta_{i-1}) - \beta R q_t(\theta_i). \quad (\text{A.6})$$

The envelope conditions are

$$\Pi_{t,1}(\mathcal{V}, \tilde{\mathcal{V}}, d, \theta_-) = -\nu_t \quad (\text{A.7})$$

$$\Pi_{t,2}(\mathcal{V}, \tilde{\mathcal{V}}, d, \theta_-) = \mu_t \quad (\text{A.8})$$

$$\Pi_{t,3}(\mathcal{V}, \tilde{\mathcal{V}}, d, \theta_-) = -\sum \pi^t(\theta|\theta_-) \Psi_2(d_t(\theta), d). \quad (\text{A.9})$$

It costs more resources to deliver a high promised utility, or excess resources decrease in the promised value, (A.7). It is cheap to stay below a high threat utility, or excess resources increase in the threat value, (A.8). Past housing services consumption decreases excess resources to the extent that current adjustment costs increase (A.9).

Housing Services Wedge and Labor Wedge. I obtain the housing services wedge and the labor wedge by manipulating the optimality conditions. I omit age script t when this does not cause confusion, and I use x_i to denote $x(\theta_i)$ for $x \in \{c, d, y\}$ and π_i and π_i^+ to abbreviate the conditional probability mass functions. The cumulative conditional probability mass function is abbreviated by $\pi_{\Sigma,i}$ and $\pi_{\Sigma,i}^+$.

Labor Wedge. To derive the labor wedge, I substitute the optimality condition for consumption (A.2) into

the optimality condition for effective labor supply (A.4) to write

$$w = -\frac{h_y(d_i, y_i/\theta_i)}{v_c(c_i)} + \Delta h_y(d_i, y_i/\theta_{i+1}) \frac{q_{i+1}}{\pi_i}, \quad (\text{A.10})$$

where $\Delta h_y(d_i, y_i/\theta_{i+1})$ denotes the first difference in labor productivity. The efficient labor wedge is the distortion between the marginal rate of substitution of consumption for labor and the marginal product of labor (19), which thus satisfies

$$\tau_y = \Delta h_y(d_i, y_i/\theta_{i+1}) \frac{q_{i+1}}{w\pi_i}. \quad (\text{A.11})$$

To simplify the labor wedge I note $q_{i+1} = -\sum_{s=i+1}^N (q_{s+1} - q_s)$, where the difference between consecutive multipliers follows by rearranging the optimality condition for consumption (A.2),

$$q_{s+1} - q_s = (\nu - \mu)\pi_s - \mu(\pi_s^+ - \pi_s) - \pi_s \frac{1}{v_c(c_s)}. \quad (\text{A.12})$$

Summing equation (A.12) over all labor productivity states, and by noting that $q_1 = q_{N+1} = 0$, this implies

$$\sum \pi_i \frac{1}{v_c(c_i)} = \nu - \mu. \quad (\text{A.13})$$

To further characterize the labor wedge, I use the optimality condition for the threat value (A.6), the envelope condition for the threat value (A.8), and the expression for the labor wedge in (A.11), to write μ as

$$\mu = \beta R w \frac{\tau_{y,-}}{\Delta h_y(d_-, y_-/\theta_-^+)}. \quad (\text{A.14})$$

The labor wedge is characterized by substituting (A.12), (A.13), and (A.14) into (A.11),

$$\tau_y = \Delta h_y(d_i, y_i/\theta_{i+1}) \frac{I_i}{w\pi_i} + \beta R \tau_{y,t-1} \frac{\pi_{\Sigma,i} - \pi_{\Sigma,i}^+}{\pi_i} \frac{\Delta h_y(d_i, y_i/\theta_{i+1})}{\Delta h_y(d_-, y_-/\theta_-^+)},$$

where I_i is the insurance value (22). The labor wedge is the analog of the labor wedge in Golosov, Troshkin, and Tsyvinski (2016) for an economy with home production.

The efficient labor wedge is positive and balances the distortionary costs for type θ against the benefit of relaxing incentive constraints for all types above θ . By relaxing period t incentive constraints, a planner can provide additional insurance using resources extracted from households more productive than type θ . This insurance value of relaxing incentive constraint is given by I_i . The dynamic component captures that

efficient labor wedges at age t relax incentive constraints at prior ages to the extent that decisions of a more productive household at a prior age are more likely to be distorted going forward.

Housing Services Wedge. To derive the housing wedge, I substitute the optimality condition for consumption (A.2), and the envelope condition for housing services (A.9), into the optimality condition for housing services consumption (A.3) to obtain

$$p_j + \Phi_1(d_i, d) = \frac{h_d(d_i, y_i/\theta_i)}{v_c(c_i)} - \Delta h_d(d_i, y_i/\theta_{i+1}) \frac{q_{i+1}}{\pi_i} - \sum \pi(\hat{\theta}|\theta_i) \Phi_2(d_{t+1}(\hat{\theta}), d_i) / R.$$

The efficient housing services wedge (18) is thus

$$\tau_d = \frac{1}{p_j} \Delta h_d(d_i, y_i/\theta_{i+1}) \frac{q_{i+1}}{\pi_i} + \frac{1}{p_j} \sum \pi(\hat{\theta}|\theta_i) \Phi_2(d_{t+1}(\hat{\theta}), d_i) \left(\frac{1}{R} - \beta \frac{v_c(c_{t+1}(\hat{\theta}))}{v_c(c_i)} \right), \quad (\text{A.15})$$

which by substituting (A.12), (A.13), and (A.14) is equivalent to,

$$\begin{aligned} \tau_d &= \Delta h_d(d_i, y_i/\theta_{i+1}) \frac{I_i}{\pi_i p_j} + \beta R \tau_{y,t-1} \frac{\pi_{\Sigma,i} - \pi_{\Sigma,i}^+}{\pi_i} \frac{w}{p_j} \frac{\Delta h_d(d_i, y_i/\theta_{i+1})}{\Delta h_y(d_-, y_-/\theta_-^+)} \\ &+ \frac{1}{p_j R} \sum \pi(\hat{\theta}|\theta_i) \Phi_2(d_{t+1}(\hat{\theta}), d_i) \left(1 - \beta R \frac{v_c(c_{t+1}(\hat{\theta}))}{v_c(c_i)} \right) \end{aligned} \quad (\text{A.16})$$

Housing Capital and Business Capital Wedge. I characterize the efficient distortion on savings using a variational argument. The savings distortion applies to both business capital and housing capital and is obtained from the inverse Euler equation.

Consider an allocation $x(i)$ that solves the component planner problem for household i and fix a history θ^t . Consider the perturbed allocation $x^\delta(i) = (c^\delta(i), d^\delta(i), y^\delta(i))$, where the index $\delta > 0$ denotes the amount utility is decreased by at age $t + 1$,

$$\begin{aligned} v(\underbrace{c(\theta^{t+1}) - \varepsilon(c(\theta^{t+1}), \delta)}_{\equiv c^\delta(\theta^{t+1})}) &= v(c(\theta^{t+1})) - \delta \\ v(\underbrace{c(\theta^t) + \varepsilon(c(\theta^t), \delta)}_{\equiv c^\delta(\theta^t)}) &= v(c(\theta^t)) + \beta \delta, \end{aligned}$$

with $(d^\delta(\theta^s), y^\delta(\theta^s)) = (d(\theta^s), y(\theta^s))$ for age t and age $t + 1$. For every other history, the perturbed allocation is identical to the component planner solution.

The promise keeping constraint and the incentive constraints are satisfied under the perturbed allocation $x^\delta(i)$. The perturbed allocation increases utility at age t by $\beta\delta$ and decreases utility at $t+1$ by δ for histories passing through θ^t . Due to discounting the promise keeping constraint and the incentive constraints are both satisfied.

For small $\delta > 0$, I have $\varepsilon(c(\theta^{t+1}), \delta) = \delta/v_c(c(\theta^{t+1}))$ and $\varepsilon(c(\theta^t), \delta) = \beta\delta/v_c(c(\theta^t))$, and hence the change in excess resources given by

$$\pi(\theta^t) \left(\left(\frac{1}{R} \right)^{t-1} \frac{\beta\delta}{v_c(c(\theta^t))} - \left(\frac{1}{R} \right)^t \sum \pi^{t+1}(\theta_{t+1}|\theta_t) \frac{\delta}{v_c(c(\theta^{t+1}))} \right). \quad (\text{A.17})$$

At the solution to the component planner problem such a perturbation does not generate excess resources. In other words, the derivative of excess resources with respect to δ equals zero at the solution, which gives the inverse Euler equation

$$\frac{1}{v_c(c(\theta^t))} = \frac{1}{\beta R} \sum \pi^{t+1}(\theta_{t+1}|\theta_t) \frac{1}{v_c(c(\theta^{t+1}))}. \quad (\text{A.18})$$

Given the definition of the savings wedge (20), the efficient intertemporal distortion is

$$1 - \tau_s(\theta^t) = \frac{\left(\sum \pi^{t+1}(\theta_{t+1}|\theta_t) (v_c(c(\theta^{t+1})))^{-1} \right)^{-1}}{\sum \pi^{t+1}(\theta_{t+1}|\theta_t) v_c(c(\theta^{t+1}))}. \quad (\text{A.19})$$

Because the utility from consumption v is strictly concave, the savings wedge is positive.

D Rent Subsidy

I provide the formula for the rent subsidy. The marginal subsidy varies across \mathcal{D} rent brackets. Households with income below \underline{y} and rent expenses below $p\underline{d}_{\mathcal{D}}$ qualify for a subsidy. The subsidy combines a component supporting a housing services consumption floor with a pure subsidy component.

The first component of the subsidy supports a minimum level of housing services consumption \underline{d}_2 . To support this level of housing services consumption, the government fully subsidizes the gap between $p\underline{d}_2$ and households' sustainable rent expenditures. Households' sustainable rent expenditures are calculated using a second-order polynomial in income \tilde{y} , with a minimum of $p\underline{d}_1 \leq p\underline{d}_2$. Working-age households receive $p\underline{d}_2 - \max(\alpha_2^w \tilde{y}^2 + \alpha_1^w \tilde{y}, p\underline{d}_1)$ if this amount is positive. Retirees receive $\max(p\underline{d}_2 - \max(\alpha_2^r \tilde{y}^2 + \alpha_1^r \tilde{y}, d_{\min}), 0)$.

Above the targeted minimum level of housing services consumption, the government subsidizes rents up to $p\underline{d}_3 \geq p\underline{d}_2$ at rate τ_d . Low income households pay $1 - \tau_d$ out of their pocket on rent between $p\underline{d}_2$ and $p\underline{d}_3$.

Equivalently, low income households receive an additional $\tau_d \min(\max(pd - pd_2, 0), pd_3 - pd_2)$. In sum, the formula for the rent subsidy is:

$$T_t^d(pd, \tilde{y}) = \max(pd_2 - \max(\alpha_2^w \tilde{y}^2 + \alpha_1^w \tilde{y}, d_{\min}), 0) + \tau_d \min(\max(pd - pd_2, 0), pd_3 - pd_2).$$

Note that the formula for the rent subsidy depends on households' retirement status through the households' sustainable rent expenditures.

E Definition of Equilibrium

Given a government expenditures sequence $\{G_j\}$, government policy, planned housing projects $\{P_{1-v}^H\}_{v=1}^t$, an initial savings distribution $\{s_1(i)\}_{\mathcal{I}}$, and initial assets $\{(B_0, H_0, K_0)\}$, an equilibrium consists of a price sequence $\{(w_j, r_j, p_j, p_j^H)\}$ and allocation $x^e \equiv \left\{ \{x^e(i)\}_{\mathcal{I}}, \{(A_j, B_j, C_j, D_j, H_j, K_j, M_j, S_j, Y_j)\}_{j=1}^{\infty} \right\}$, where the equilibrium allocation for individual $i \in \mathcal{I}$ is

$$x^e(i) = \left\{ (a_{j+v}(\theta^{t+v}), c_{j+v}(\theta^{t+v}), d_{j+v}(\theta^{t+v}), h_{j+v}(\theta^{t+v}), m_{j+v}(\theta^{t+v}), s_{j+v}(\theta^{t+v}), y_{j+v}(\theta^{t+v})) \right\}_{v=0}^{T-t},$$

such that:

1. Allocation functions $\{a_t, c_t, d_t, h_t, m_t, y_t, s_t\}$ solve the household maximization problem
2. Aggregate quantities are consistent with individual decision rules

$$A_j = \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) a_j(\theta^t)$$

$$H_j^o = \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) h_j(\theta^t)$$

$$C_j = \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) c_j(\theta^t)$$

$$M_j = \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) m_j(\theta^t)$$

$$D_j = \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) d_j(\theta^t)$$

$$S_j = \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) s_j(\theta^t)$$

$$Y_j = \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) y_j(\theta^t)$$

3. Factor prices are consistent with the firm maximization problem

$$r_j = F_K(K_j, Y_j) - \delta^k$$

$$w_j = F_Y(K_j, Y_j)$$

4. The rental price of housing, p_j , is consistent with the rental firm's maximization problem
5. The house price, p_j^H , is consistent with the construction firm's maximization problem
6. The goods market and local housing market clear every period

$$C_j + I_j^K + I_j^H + G_j + \Phi_j + B_{j+1} = F(K_j, Y_j) + RB_j$$

$$D_j = \chi H_j$$

where $I_j^K = K_{j+1} - (1 - \delta^K)K_j$, $I_j^H = P_{j+1}^H + \delta^H H_j$.

7. The government budget constraint is satisfied, and $\lim_{j \rightarrow \infty} B_j / R^j \in [0, \infty)$.

Steady State Characterization. Given the equilibrium definition, I characterize a steady state.

1. By the firm's problem, the interest rate pins down the capital-labor ratio and the wage. The problem of the construction firm determines the house price, $p^H = 1$, and the landlord problem pins down rental price p .
2. Given prices and government policy, the household problem gives solution $\{a_t, c_t, d_t, h_t, m_t, y_t, s_t\}$.
3. Use solution to household problem to obtain aggregate quantities $(A, B, C, D, H, H^o, M, Y, S, \Phi, K)$.
4. Total private savings is S , and the domestic housing stock is $H = D/\chi = H^o + H^r$. Domestic savings are the sum of private and public savings, and the domestic capital stock is the sum of business capital and housing capital. Given net foreign assets, public savings are determined.

F Aggregate Data

I use data from the national income and product accounts to measure the aggregate capital income share, the consumption-output ratio, and the expenditure share of housing in total consumption. In Section 5, I use these moments to calibrate the capital share in production, α , the preference weight for consumption, γ , and the weight of housing services in home production, ω . I construct the moments using data that are publicly available through Statistics Netherlands' Statline.

National Income and Product Accounts. I measure the capital share using the income accounts and the consumption-output ratio using the national product accounts. In Table A.1, I split national income

Table A.1: National Income Product Accounts, 2006–2014

Total Adjusted Income	1.000
Labor Income	0.561
Compensation of Employees	0.501
Wages and Salary	0.395
Supplements to Wages and Salary	0.106
70% of proprietors' income	0.060
Capital Income	0.439
Profits	0.165
30% of proprietors' income	0.026
Indirect Business Taxes	0.102
<i>Sales Tax</i>	0.098
Consumption of Fixed Capital	0.168
Consumer Durable Depreciation	0.041
Imputed Capital Services	0.035
Consumer Durable Services	0.011
Government Capital Services	0.024

Table A.1 provides headline statistics for national income following the income approach. Author's calculations using data from Statistics Netherlands.

between labor income and capital income. Labor income includes the compensation of employees and 70% of proprietors' income, while all other forms of income are categorized as capital income.

Capital income is adjusted to align my model with the data. First, I subtract sales taxes to measure production at producer prices rather than consumer prices. Second, I impute capital services for consumer durables and government capital. The imputed services are assumed to be 4% of the current-cost net stock of consumer durables and government fixed assets. Finally, I impute depreciation of consumer durables. Because the depreciation rate of consumer durables is not available for the Netherlands, I assume the depreciation rate is equal to 5% which is the corresponding rate for the United States as calculated in [McGrattan and Prescott \(2017\)](#). I find a capital income share of 0.439, which is the value I choose for α .

On the production side, shown in Table A.2, I also adjust for sales taxes, capital services, and consumer

Table A.2: National Income Product Accounts, 2006-2014

Total Adjusted Product	1.000
Consumption	0.642
Personal consumption expenditures	0.462
<i>Less:</i> Consumer durable goods	0.055
<i>Less:</i> Imputed sales tax, nondurables and services	0.087
<i>Plus:</i> Imputed capital services, durables	0.011
Government consumption expenditures, nondefense	0.246
<i>Plus:</i> Imputed capital services, government capital	0.024
Consumer durable depreciation	0.041
Tangible investments	0.343
Gross private domestic investments	0.166
Consumer durable goods	0.055
<i>Less:</i> Imputed sales tax, durables	0.011
Government gross investment	0.040
Net exports of goods and services	0.093
Defense spending	0.013

Table A.2 provides headline statistics for national income following the product approach. Author's calculations using data from Statistics Netherlands.

durables depreciation. I assume that sales taxes primarily fall on personal consumption expenditures, and I allocate proportionally to durable goods, non-durable goods and services. Non-durable goods and services are consumption while durable goods are a tangible investment. Imputed capital services increase aggregate consumption, the sum of personal and government consumption from the national accounts. Consumption of consumer durables depreciates the outstanding stock, which motivates me to classify consumer durables depreciation as consumption. The consumption-output ratio equals 0.642. I use this number to calibrate the preference weight on consumption.

Expenditure Share on Housing. To calibrate the share on housing services in the home technology ω , I measure the expenditure share on housing as a fraction of total consumption. In ??, I show the expenditure share on housing between 1995 and 2015. The expenditure share on housing is relatively stable at 16 percent

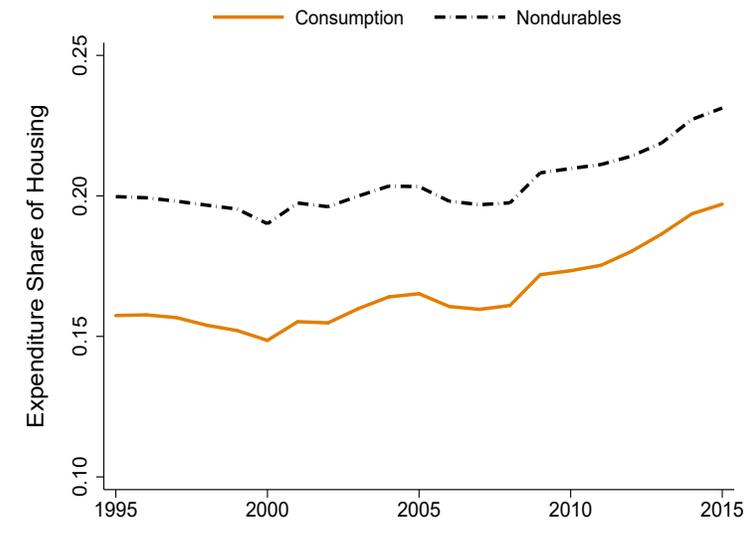


Figure A.1: Expenditure Share on Housing

Figure A.1 displays the expenditure share on housing in the Netherlands between 1995 and 2015. The solid orange line shows the expenditures on housing services in proportion to total consumption, the black dashed line shows the expenditures on housing services in proportion to the consumption of nondurables.

until the beginning of the housing crisis, but equals 18 percent on average after 2008. I target the average expenditure share between 2006 and 2014 of 17.4 percent.²⁷

G Household Problem

In this appendix I derive the user cost for renters and homeowners and I characterize the solution to the household problem to obtain the estimation equation.

User Cost. The user cost, the cost of a marginal unit of housing, is obtained by differentiating the budget constraint with respect to housing capital. I assume that the household does not incur a transaction cost on the marginal unit of housing capital.

Homeowners. To derive the homeowner's user cost, it is useful to rewrite their budget constraint, (28), by

²⁷The expenditure share in the Netherlands is close to the expenditure share on housing in the United States. Piazzesi and Schneider (2016) report a mean housing share of 17.8 percent between 1959 and 2014.

adding and subtracting the gross market return on the investment in their house, $Rp^H h_t$. By recalling that homeowners hold their savings as financial assets, housing wealth and mortgages, $s_t = a_t + p^H h_t - m_t$, and by recalling the definition of before-tax income (25), I write

$$c_t + T_t^c(c_t) + \Psi(d_t, d_{t-1}) + s_{t+1} = wy_t - T_t^y(wy_t + b_t + \tau_o p^H h_t - rm_t) + Rs_t - T_t^a(s_t - p^H h_t + m_t) + T_t + (\Delta p^H - \tau_p p^H - \delta^H - p^H r) h_t.$$

I calculate the user cost for homeowners holding constant the fraction of the property that is debt-financed, which I denote $\varkappa \equiv m/(p^H h)$. Furthermore, I denote the marginal income tax rate by $\hat{\tau}_y \equiv T_{1,t}^y(wy_t + b_t + \tau_o p^H h_t - rm_t)$ and the marginal tax rate on wealth by $\hat{\tau}_a \equiv T_{1,t}^a(s_t - p^H h_t - m_t)$. Hence, the user cost for homeowners is

$$p_j^o = r + \tau_p + \delta^H - \pi_{j+1}^H - \hat{\tau}_y r \varkappa + \hat{\tau}_y \tau_o - \hat{\tau}_a (1 - \varkappa). \quad (35)$$

Renters. I obtain the user cost for renters by using their budget constraint (26), and the market price for rental services (31). Using the budget constraint,

$$c_t + T^c(c_t) + pd_t + T_t^d(pd_t, \tilde{y}_t) + \Psi(d_t, d_{t-1}) + s_{t+1} = \tilde{y}_t - T_t^y(\tilde{y}) + Rs_t - T_t^a(Rs_t) + T_t,$$

the marginal cost of housing services for non-moving renters is,

$$p^r = (r(1 - \tau_r) - \pi^H + \hat{\tau}_p + \delta^H)(1 + \hat{\tau}_d), \quad (36)$$

where $\hat{\tau}_d \equiv T_{1,t}^d(pd_t, \tilde{y}_t)$.

H Computation Component Planning Problem

I discuss the numerical approach to solving the planner problem. I scale the program to obtain a state space that is stable across ages, and transform the problem to a multiplier grid. To simplify notation, I omit age script t when this does not cause confusion. Further, I use x_i to denote $x(\theta_i)$ for $x \in \{c, d, y, \mathcal{V}, \tilde{\mathcal{V}}\}$ and π_i and π_i^+ to abbreviate the conditional probability mass functions.

Consider the profit maximization problem in state $(\mathcal{V}_-, \tilde{\mathcal{V}}_-, d_-, \theta_-, t)$:

$$\Pi_t(\mathcal{V}_-, \tilde{\mathcal{V}}_-, d_-, \theta_-) \equiv \max \sum \pi_i \left(w_j y_{it} - c_{it} - p_j d_{it} - \Phi(d_{it}, d_-) + \Pi_{t+1}(\mathcal{V}_{it}, \tilde{\mathcal{V}}_{i+1t}, d_{it}, \theta_i) / R_j \right)$$

where the choice variable is $x_{it} = \{c_{it}, d_{it}, y_{it}, \mathcal{V}_{it}, \tilde{\mathcal{V}}_{it}\}$, and maximization is subject to:

$$u(c_{it}, d_{it}, y_{it}/(\theta_i \zeta_t)) + \beta \mathcal{V}_{it} = u(c_{i-1t}, d_{i-1t}, y_{i-1t}/(\theta_i \zeta_t)) + \beta \tilde{\mathcal{V}}_{it} \quad \forall i = 2, \dots, N$$

$$\mathcal{V}_- = \sum_i \pi_i \left(u(c_{it}, d_{it}, y_{it}/(\theta_i \zeta_t)) + \beta \mathcal{V}_{it} \right)$$

$$\tilde{\mathcal{V}}_- = \sum_i \pi_i^+ \left(u(c_{it}, d_{it}, y_{it}/(\theta_i \zeta_t)) + \beta \mathcal{V}_{it} \right).$$

The deterministic age profile of productivity is captured by ζ_t .

To solve the life-cycle program, I ensure that the promised utility and the threat utility lie on a time-invariant grid by scaling remaining lifetime values by the geometric sum of current and future discount factors. I use $\beta_t \equiv 1 + \beta + \dots + \beta^{T-t}$ to denote the geometric sum of current and future discount factors at time t . The transformation ensures that promised utility and threat utility are measured in per period units rather than as remaining lifetime values. Formally, I define the scaled promised value by $\hat{\mathcal{V}}_{it} \equiv \mathcal{V}_{it}/\beta_{t+1}$ and the scaled threat value by $\hat{\tilde{\mathcal{V}}}_{it} \equiv \tilde{\mathcal{V}}_{it}/\beta_{t+1}$. These definitions imply $\hat{\mathcal{V}}_- = \mathcal{V}_-/\beta_t$ and $\hat{\tilde{\mathcal{V}}}_- = \tilde{\mathcal{V}}_-/\beta_t$. I scale the objective function in the same way, or $\hat{\Pi}_t(\hat{\mathcal{V}}_-, \hat{\tilde{\mathcal{V}}}_-, d_-, \theta_-) \equiv \Pi_t(\hat{\mathcal{V}}_-, \hat{\tilde{\mathcal{V}}}_-, d_-, \theta_-)/\beta_t$.

Claim 1. The component planner problem is equivalent to the following scaled program:

$$\hat{\Pi}_t(\hat{\mathcal{V}}_-, \hat{\tilde{\mathcal{V}}}_-, d_-, \theta_-) \equiv \max \sum \pi_i \left(\frac{1}{\beta_t} (w_j y_{it} - c_{it} - p_j d_{it} - \Phi(d_{it}, d_-)) + \frac{\beta_{t+1}}{\beta_t} \hat{\Pi}_{t+1}(\hat{\mathcal{V}}_{it}, \hat{\tilde{\mathcal{V}}}_{i+1t}, d_{it}, \theta_i) / R_j \right),$$

where the choice variable is $x_{it} = \{c_{it}, d_{it}, y_{it}, \hat{\mathcal{V}}_{it}, \hat{\tilde{\mathcal{V}}}_{it}\}$, and maximization is subject to:

$$\frac{1}{\beta_t} u(c_{it}, d_{it}, y_{it}/(\theta_i \zeta_t)) + \beta \frac{\beta_{t+1}}{\beta_t} \hat{\mathcal{V}}_{it} = \frac{1}{\beta_t} u(c_{i-1t}, d_{i-1t}, y_{i-1t}/(\theta_i \zeta_t)) + \beta \frac{\beta_{t+1}}{\beta_t} \hat{\tilde{\mathcal{V}}}_{it} \quad \forall i = 2, \dots, N \quad (q_i) \quad (\text{A.20})$$

$$\hat{\mathcal{V}}_- = \sum_i \pi_i \left(\frac{1}{\beta_t} u(c_{it}, d_{it}, y_{it}/(\theta_i \zeta_t)) + \beta \frac{\beta_{t+1}}{\beta_t} \hat{\mathcal{V}}_{it} \right) \quad (\nu_t) \quad (\text{A.21})$$

$$\hat{\tilde{\mathcal{V}}}_- = \sum_i \pi_i^+ \left(\frac{1}{\beta_t} u(c_{it}, d_{it}, y_{it}/(\theta_i \zeta_t)) + \beta \frac{\beta_{t+1}}{\beta_t} \hat{\mathcal{V}}_{it} \right) \quad (\mu_t) \quad (\text{A.22})$$

Proof. Equivalence follows by dividing the objective function and all constraints of the component planner problem by β_t , and by multiplying and dividing by β_{t+1} the choices for the promised and the threat values, and the period $t + 1$ objective function. ■

I characterize the solution to the program through its first-order optimality conditions. The optimality

conditions with respect to consumption, housing services, and output are:

$$\pi_i = v_c(c_{it}) \left(\nu_t \pi_i - \mu_t \pi_i^+ + q_i - q_{i+1} \right) \quad (\text{A.23})$$

$$w_j \pi_i = h_\ell(d_{it}, \ell_{it}) \frac{1}{\theta_i \hat{\zeta}_t} \left(\nu_t \pi_i - \mu_t \pi_i^+ + q_i \right) - h_\ell(d_{it}, \ell_{it}^+) q_{i+1} \frac{1}{\theta_{i+1} \hat{\zeta}_t} \quad (\text{A.24})$$

$$p_j \pi_i = -\pi_i \Phi_1(d_{it}, d_-) + h_d(d_{it}, \ell_{it}) \left(\nu_t \pi_i - \mu_t \pi_i^+ + q_i \right) - h_d(d_{it}, \ell_{it}^+) q_{i+1} \\ + \beta_{t+1} \pi_i \hat{\Pi}_{t+1,3}(\hat{\mathcal{V}}_{it}, \hat{\mathcal{V}}_{i+1t}, d_{it}, \theta_i) / R_j. \quad (\text{A.25})$$

The optimality conditions for the promise utility and the threat utility are:

$$0 = \pi_i \hat{\Pi}_{t+1,1}(\hat{\mathcal{V}}_{it}, \hat{\mathcal{V}}_{i+1t}, d_{it}, \theta_i) + \beta R_j (\nu_t \pi_i - \mu_t \pi_i^+ + q_i)$$

$$0 = \beta R_j q_i - \pi_{i-1} \hat{\Pi}_{t+1,\tilde{y}}(\hat{\mathcal{V}}_{i-1t}, \hat{\mathcal{V}}_{it}, d_{i-1t}, \theta_{i-1}).$$

Before I characterize the solution to the dynamic program over the life-cycle, I rewrite some optimality conditions in ways that are useful. First, I write the optimality condition for consumption as:

$$\pi_i \left(\frac{1}{u_c(c_{it})} - \nu_t \right) = q_i - q_{i+1} - \mu_t \pi_i^+, \quad (\text{A.26})$$

Summing over all states at age t , and realizing that $q_1 = 0$ because the lowest type cannot pretend to be less productive, I obtain a restriction on the sum of the multipliers,

$$\sum \pi_i \frac{1}{u_c(c_{it})} = \nu_t - \mu_t. \quad (\text{A.27})$$

Furthermore, it is useful to write the envelope conditions for promised utility and threat utility as:

$$\hat{\Pi}_{t,1}(\hat{\mathcal{V}}_-, \hat{\mathcal{V}}_-, d_-, \theta_-) = -\nu_t$$

$$\hat{\Pi}_{t,2}(\hat{\mathcal{V}}_-, \hat{\mathcal{V}}_-, d_-, \theta_-) = \mu_t.$$

I use the envelope conditions to eliminate the derivatives of the value function for the promised utility and the threat utility in the system of equations by incorporating the choice variables

$$-\nu_{it+1} = \hat{\Pi}_{t+1,1}(\hat{\mathcal{V}}_{it}, \hat{\mathcal{V}}_{i+1t}, d_{it}, \theta_i)$$

$$\mu_{it+1} = \hat{\Pi}_{t+1,2}(\hat{\mathcal{V}}_{it}, \hat{\mathcal{V}}_{i+1t}, d_{it}, \theta_i).$$

As a result, the optimality conditions for the promised utility and the threat utility are written as:

$$\nu_{it+1} = \beta R_j (\nu_t \pi_i - \mu_t \pi_i^+ + q_i) / \pi_i \quad (\text{A.28})$$

$$\mu_{i-1t+1} = \beta R_j q_i / \pi_{i-1}. \quad (\text{A.29})$$

I use this observation to solve the system of optimality conditions given states (ν_t, μ_t, d_-) instead of states $(\hat{\mathcal{V}}_-, \hat{\mathcal{V}}_-, d_-)$. I note that (A.24), (A.25), (A.26), (A.28), (A.29), and the local incentive constraints (A.20) form a system of $6N - 2$ equations and unknowns given state (ν_t, μ_t, d_-) . I use the equations to characterize $6N - 2$ unknowns $\{\{(c_{it}, d_{it}, y_{it}, \nu_{it+1})\}_{i=1}^N, \{(\mu_{it+1}, q_{i+1})\}_{i=1}^{N-1}\}$. After characterizing the unknowns, I evaluate the value of the profit function, and residually determine the implied promised and threat value by using the promise keeping condition (A.21) and the threat-keeping condition (A.22).

Final Work Period with Retirement. In the final work period no threat values are chosen since there is no difference in the productivity distribution next period which the government can exploit to distinguish productivity differences today. As a result, the incentive constraints feature only promised values. For period $t = T_W$, the planner problem is thus

$$\hat{\Pi}_t(\hat{\mathcal{V}}_-, \hat{\mathcal{V}}_-, d_-, \theta_-) \equiv \max \sum \pi_i \left(\frac{1}{\beta_t} (w_j y_{it} - c_{it} - p_j d_{it} - \Phi(d_{it}, d_-)) + \frac{\beta_{t+1}}{\beta_t} \hat{\Pi}_{t+1}(\hat{\mathcal{V}}_{it}, d_{it}, \theta_i) / R_j \right),$$

where maximization is subject to

$$\frac{1}{\beta_t} u(c_{it}, d_{it}, y_{it} / (\theta_i \zeta_t)) + \beta \frac{\beta_{t+1}}{\beta_t} \hat{\mathcal{V}}_{it} = \frac{1}{\beta_t} u(c_{i-1t}, d_{i-1t}, y_{i-1t} / (\theta_i \zeta_t)) + \beta \frac{\beta_{t+1}}{\beta_t} \hat{\mathcal{V}}_{i-1t},$$

the promise keeping condition (A.21) and the threat-keeping constraint (A.22). In this case, the first-order conditions are given by (A.23), (A.24), (A.25),

$$\nu_{it+1} = (\beta R_j (\nu_t \pi_i - \mu_t \pi_i^+) + q_i - q_{i+1}) / \pi_i, \quad (\text{A.30})$$

the promise keeping condition (A.21), threat keeping condition (A.22), and the incentive constraints (A.20).

I compute the solution using the Newton-Raphson method over $2N - 1$ variables. I provide a guess for the bottom $N - 1$ elements of the consumption allocation c_{T_W} and I guess the housing services consumption vector d_{T_W} . Given guess $\{c_{iT_W}\}_{i=1}^{N-1}$ and states (ν_{T_W}, μ_{T_W}) , (A.27) generates consumption at the top c_{NT_W} . Given consumption c_{T_W} , and state (ν_{T_W}, μ_{T_W}) , I solve for q using the optimality condition for consumption

(A.26), with $q_1 = 0$. Given q , I use the optimality condition with respect to the promised utility (A.30) to obtain the state for retirement ν_{iT_W+1} . Given housing services consumption d_{T_W} , and the state for retirement ν_{iT_W+1} , I obtain the implied promised value and profit function from equations (A.21) and (A.22).

I observe from the first-order optimality conditions that the high productivity type's marginal decisions are undistorted as $q_{N+1} = 0$, implying $\frac{h_\ell(d_{NT_W}, \ell_{NT_W})}{h_d(d_{NT_W}, \ell_{NT_W})} = \theta_N \zeta_{T_W} w / \tilde{p}$, and identifying ℓ_{NT_W} . Given all future values, I generate $\{\ell_{iT_W}, \ell_{iT_W}^+\}_{i=1}^{N-1}$ by backward iteration using the local incentive constraints

$$v(c_{it}) + h(d_{it}, \ell_{it}) + \beta\beta_{t+1}\hat{\mathcal{V}}_{it} = v(c_{i-1t}) + h(d_{i-1t}, \ell_{i-1t}^+) + \beta\beta_{t+1}\hat{\mathcal{V}}_{i-1t}.$$

I generated $3N$ unknowns, $\{c_{NT_W}, \{y_{iT_W}, \nu_{iT_W+1}\}_{i=1}^N, \{q_{i+1}\}_{i=1}^{N-1}\}$, which leaves $2N - 1$ residual equations. I iterate until convergence using the optimality conditions for housing services (A.25), and the bottom $N - 1$ optimality conditions for output (A.24). The promise-keeping condition (A.21) and threat-keeping constraint (A.22) are used to residually determine the promised value $\hat{\mathcal{V}}_-$, and the threat value $\hat{\mathcal{V}}_-$.

Intermediate Period. I solve the program at each point in the state space (ν_t, μ_t, d_{t-1}) , taking as given the value function for the next period.

The equations that characterize the solution are the optimality conditions with respect to consumption (A.23), output (A.24), housing services (A.25), promised utility (A.28) and threat utility (A.29), the promise keeping condition, the threat keeping condition, and the local downward ICs. I compute the solution using the Newton-Raphson method over $2N - 1$ variables. I guess the first $N - 1$ elements of c_t and the allocation of housing services d_t . The guess is the solution at the state (ν_t, μ_t, d_{t-1}) in the following period. Given a guess $\{c_{it}\}_{i=1}^{N-1}$ and a state (ν_t, μ_t, d_{t-1}) , (A.27) generates consumption at the top c_{Nt} . Given consumption c_t , and a state (ν_t, μ_t, d_{t-1}) , I solve for q using the optimality condition for consumption (A.26), with $q_1 = 0$. Given d and q , we use the first-order condition with respect to the promised utility (A.28) and the threat utility (A.29) to obtain next period's states. Given the state for next period $(\nu_{it}, \mu_{it}, d_{it})$, and a realization for labor productivity θ_i , I use the implied promised value and threat value from the value function to obtain $(\hat{\mathcal{V}}_{it}, \hat{\mathcal{V}}_{it}, \theta_i)$.

We observe from the first-order conditions that the highest type's marginal decisions are undistorted (as $q_t(\theta_{N+1}) = 0$), or equivalently, $\vartheta_\ell(\tilde{\ell}_t(\theta_N)) = \theta_N \gamma / \tilde{c}_t(\theta_N)$. Given $\tilde{\ell}_t(\theta_N)$ and all the future values, we generate

$(\tilde{\ell}_t(\theta_i), \tilde{\ell}_t^+(\theta_i))_{i=1}^{N-1}$ by backward iteration using the incentive compatibility constraints:

$$\begin{aligned} \gamma \log \tilde{c}_t(\theta_i) + \vartheta \left(\tilde{\ell}_t(\theta_i) \right) + \beta \beta_{t+1} \hat{\mathcal{V}}_t^s(\theta_i) &= \gamma \log \tilde{c}_t(\theta_{i-1}) + \vartheta \left(\tilde{\ell}_t^+(\theta_{i-1}) \right) + \beta \beta_{t+1} \hat{\mathcal{V}}_t^s(\theta_i) \\ \implies \vartheta \left(\tilde{\ell}_t^+(\theta_{i-1}) \right) &= u(\tilde{c}_t(\theta_i)) + \vartheta \left(\tilde{\ell}_t(\theta_i) \right) + \beta \beta_{t+1} \hat{\mathcal{V}}_t^s(\theta_i) - u(\tilde{c}_t(\theta_{i-1})) - \beta \beta_{t+1} \hat{\mathcal{V}}_t^s(\theta_i). \end{aligned}$$

We have generated $5N-2$ unknowns. This leaves $N-1$ residual equations, the first $N-1$ optimality conditions for output, which we iterate until convergence. The promise-keeping condition and threat-keeping constraint are used to residually determine the promised value $\hat{\mathcal{V}}_-$, and the threat value $\hat{\mathcal{V}}_-$.

I observe from the first-order optimality conditions that the high productivity type's marginal decisions are undistorted as $q_{N+1} = 0$, implying $\frac{h_\ell(d_{Nt}, \ell_{Nt})}{h_d(d_{Nt}, \ell_{Nt})} = \theta_N \zeta_t w / \tilde{p}$, and identifying ℓ_{Nt} . Given all future values, I generate $\{\ell_{it}, \ell_{it}^+\}_{i=1}^{N-1}$ by backward iteration using the local incentive constraints

$$v(c_{it}) + h(d_{it}, \ell_{it}) + \beta \beta_{t+1} \hat{\mathcal{V}}_{it} = v(c_{i-1t}) + h(d_{i-1t}, \ell_{i-1t}^+) + \beta \beta_{t+1} \hat{\mathcal{V}}_{it}.$$

I generated $4N-1$ unknowns, $\{c_{Nt}, \{y_{it}, \nu_{it+1}\}_{i=1}^N, \{(\mu_{it+1}, q_{i+1})\}_{i=1}^{N-1}\}$, which leaves $2N-1$ residual equations. I iterate until convergence using the first-order conditions for housing services (A.25), and the bottom $N-1$ optimality conditions for output (A.24). The promise-keeping condition (A.21) and threat-keeping constraint (A.22) are used to residually determine the promised value $\hat{\mathcal{V}}_-$, and the threat value $\hat{\mathcal{V}}_-$.

First Period. The algorithm in the first period is identical to the algorithm in the middle period, where in the initial period $\mu_1 = 0$, and there are no adjustment costs.