Brexit and the Macroeconomic Impact of Trade Policy Uncertainty

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Abstract

The United Kingdom has voted to leave the European Union, but the trade policies that will replace E.U. membership are uncertain, and speculation abounds that this uncertainty will cause immediate harm to the U.K. economy. In this paper, I use a dynamic general equilibrium model with heterogeneous firms, endogenous export participation, and stochastic trade costs to quantify the impact of uncertainty about post-Brexit trade policies. I find that the total consumption-equivalent welfare cost of Brexit for U.K. households is between 0.4–1.2 percent, and that less than a quarter of a percent of this cost is attributable to Brexit uncertainty.

1 Introduction

The United Kingdom voted to leave the European Union on June 23, 2016, but the law that authorized the vote was silent about the trade policies that would replace E.U. membership. The Brexit vote was followed by widespread speculation that uncertainty about future U.K.-E.U. trade policies would cause immediate harm to the U.K. economy, although recent national income accounting and trade data suggest that this harm has yet to materialize. In this paper, I analyze the effects of Brexit on U.K. macroeconomic dynamics and quantify the cost of Brexit uncertainty.

I use a dynamic, stochastic, general equilibrium model of the United Kingdom, the European Union, and the rest of the world to address two quantitative questions about the consequences of Brexit. First, how will departure from the European Union affect the U.K. economy in the short and long run? Second, how will uncertainty about the trade policies that will replace E.U. membership in the future affect the U.K. economy in the present?

The model features three countries, an input-output production structure, heterogeneous firms, and, most importantly, uncertainty about trade costs. Firms make forward-looking decisions about export partic-

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ipation, creating a scope for trade policy uncertainty to affect macroeconomic and trade dynamics. Firms' export participation decisions have an extensive margin, as in Melitz (2003) and Chaney (2008), as well as an intensive margin, as in Arkolakis (2010); firms choose whether to export to a foreign market, and if so, how many customers in that market to serve. Uncertainty can also affect households' decisions about saving and investment. To compute the model's equilibrium I use a novel global method that provides an exact solution, which allows for anaccurate analysis of welfare and the effects of uncertainty.

In my quantitative analysis, I calibrate the model's parameters so that its steady state matches facts about export participation rates that I have computed using two sources of microdata, and an input-output matrix from 2011, when the possibility of Brexit had not yet entered the global consciousness. To assess the overall impact of Brexit, I compare this no-Brexit steady state to an equilibrium in which trade costs follow a stochastic process that captures uncertainty about the outcome of the Brexit referendum and about post-Brexit changes in trade policy. To assess the impact of this uncertainty, I compare the stochastic equilibrium to a set of deterministic equilibria in which model agents have perfect foresight about these events. I find that overall welfare losses from Brexit will be substantial. Consumption-equivalent welfare losses are between 0.4 and 1.2 percent depending on how much post-Brexit trade costs rise. The welfare cost of uncertainty about Brexit is small, however, accounting for less than a quarter of a percent of the overall welfare cost.

In order to assess the impact of Brexit I must specify the set of possible trade policies that could replace E.U. membership in my model. Following Dhingra et al. (2016b,c), I take a parsimonious approach with two possible scenarios. If soft Brexit occurs, the United Kingdom retains tariff-free trade with the European single market through either continued membership in the European Economic Area or bilateral negotiation.[1] If, on the other hand, hard Brexit occurs, the United Kingdom loses single-market access and trades with the European Union according to World Trade Organization rules. In addition to formal tariffs, I incorporate non-tariff trade barriers which I model as iceberg transportation costs. The literature on trade costs has found that non-tariff barriers are often larger than tariffs (Anderson and van Wincoop, 2004; Allen, 2014; Lim, 2016), particularly in the services sector where tariffs are essentially nonexistent. One of the major concerns about Brexit, in fact, centers around whether or not financial services firms will retain passporting rights that enable them to operate in the European Union. I use the estimates of Francois et al. (2013) for non-tariff barriers in E.U. trade with the United States as an upper bound for post-Brexit non-tariff barriers in E.U. trade with the United Kingdom.

In order to analyze the effects of uncertainty about Brexit, I must also model the timing of the Brexit process and the likelihood of each scenario. The first real indication that Brexit might be a possibility arose in January of 2013, when Prime Minister David Cameron promised that he would hold a referendum on European Union membership if his Conservative party was reelected in May of 2015. The Conservatives won reelection and the European Union Referendum Act 2015, which authorized a popular vote on E.U.

[1] Recently, Prime Minister Theresa May has indicated that continued European Economic Area membership is off the table but that membership in a customs union is not. See `https://www.nytimes.com/2017/01/17/world/europe/brexit-theresa-may-uk-eu.html`.

membership, was introduced to the House of Commons shortly thereafter. The bill passed the House of Commons the next month and was approved by the House of Lords in December of 2015. The referendum date was formally announced in February of 2016, and the vote itself took place in June of 2016. Since then, the British government has converged on a March, 2019 target for completion of negotiations with the European Union about post-Brexit policies. I capture this timeline in my model as follows. The economy begins in the no-Brexit steady state in which agents believe trade costs with the European Union will remain at their 2011 levels forever. In 2015 there is an unanticipated shock that initiates a stochastic process for trade costs with the European Union. This process, depicted in figure 2, involves two uncertain events. The first is the Brexit referendum which occurs in 2016, one year after the unanticipated shock. The referendum fails with probability Π_{vote} and passes with probability $1 - \Pi_{vote}$. If the referendum fails, trade costs stay at 2011 levels forever. If the referendum passes, Brexit will occur in 2019 but model agents do not learn which Brexit scenario they will face until this time. The probability of soft Brexit is Π_{brexit} and the probability of hard Brexit is $1 - \Pi_{brexit}$. After the unanticipated shock in 2015, agents have rational expectations about this process. I set Π_{vote}, the probability that the referendum fails, to 75 percent based on prediction market price data. I set Π_{brexit}, the probability of soft Brexit conditional on the referendum's success, to 50 percent. None of my results are sensitive to these transition probabilities, however.

In the long run, Brexit will have a large impact on the U.K. macroeconomy. Depending on which scenario occurs, real GDP will fall by 0.5–1.4 percent, consumption will fall by 0.5–1.3 percent, and trade flows with the remainder of the European Union will fall by 8–45 percent. In the short run, most macroeconomic variables remain close to their no-Brexit steady state values until Brexit occurs in 2019; the announcement of the referendum and the outcome of the vote have little impact on U.K. macroeconomic dynamics. This prediction is consistent with the recent national income accounting and trade data shown in table 1 and figure 1. Once Brexit occurs, though, export participation, trade flows, and macroeconomic variables begin to decline towards their long-run levels. I measure the welfare losses from Brexit using a backward-looking method which conditions on whether Brexit is hard or soft. This method asks U.K. households in each scenario what fraction of their annual consumption they would give up to have remained in the no-Brexit steady state instead. I find consumption-equivalent welfare losses of 0.4 percent and 1.2 percent for soft and hard Brexit, respectively. The present values of these figures are equivalent to about £7,000 and £19,000 per person.

Uncertainty about Brexit in the short run will have little impact on both U.K. macroeconomic dynamics and welfare. I demonstrate this by comparing the stochastic equilibrium described in figure 2 with two perfect-foresight equilibria, one for each possible Brexit scenario, in which households learn immediately after the referendum which scenario will occur. In both the long and short run, macroeconomic dynamics and trade flows in the stochastic equilibrium are virtually identical to their perfect-foresight counterparts. The consumption-equivalent welfare differences between the baseline model and perfect-foresight models are on the order of 1/1,000 of a percent, indicating that uncertainty accounts for about a quarter of a percent of the overall welfare losses from Brexit. The present value of the welfare loss from Brexit uncertainty is less than £50 per person.

I have conducted a wide variety of sensitivity analyses and have found that all of my results are robust. I have analyzed a variety of other theories of export participation dynamics, I have studied a number of alternative Brexit scenarios, and I have experimented with different assumptions about financial markets, sectoral aggregation, and assigned parameter values. None of these sensitivity analyses change my results significantly. In particular, the welfare cost of uncertainty about Brexit is small in all versions of my quantitative analysis.

This paper contributes to several strands of the international trade and macroeconomics literatures. First, it contributes to the literature on the economic consequences of Brexit. A number of recent studies use static models and reduced-form estimations to analyze the impact of Brexit, from increased trade costs and other factors, on U.K. welfare and trade with the European Union (Dhingra et al., 2016b,c; Ebell et al., 2016; Baker et al., 2016). My paper is the first to use a dynamic general equilibrium model to assess the impact of Brexit on the U.K. economy in both the short and long run, and the first to quantify the cost of short-run uncertainty about Brexit. My study is limited, though, to the economic impact of increased post-Brexit trade costs. The United Kingdom stands to benefit from reduced fiscal transfers to the European Union after Brexit, and changes in immigration policy may also affect U.K. households' welfare. Further, leaving the European Union could lower U.K. productivity due to lower foreign direct investment (Dhingra et al., 2016a; Pain and Young, 2004; McGrattan and Waddle, 2017).

More broadly, a number of recent studies analyze the welfare impact of trade reforms in models with capital accumulation and other dynamic adjustment margins (Baldwin, 1992; Bajona and Kehoe, 2010; Dix-Carneiro, 2014; Alessandria et al., 2015; Brooks and Pujolas, 2016). My model features both physical investment and intertemporal trade. My results indicate, though, that these features play minor roles in determining U.K. welfare losses from Brexit. My paper also contributes to the related literature on trade dynamics with heterogeneous firms and endogenous export participation (Alessandria and Choi, 2007; Ruhl, 2008; Alessandria et al., 2013, 2015, 2016; Alessandria and Choi, 2016; Ramanarayanan, 2016). My model builds on these studies by incorporating a novel theory of export participation dynamics (Steinberg, 2018b) into a quantitative, general equilibrium model. The theory incorporates features from both the new exporter dynamics literature, which emphasizes sunk costs of starting to export (Das et al., 2007; Ruhl and Willis, 2017; Alessandria et al., 2015), and the literature on market penetration and the least-traded-products margin (Arkolakis, 2010; Kehoe and Ruhl, 2013; Kehoe et al., 2015).

Finally, my paper contributes to the emerging literature on trade policy uncertainty. In contrast to my finding that the effects of uncertainty about Brexit are small, several studies in this literature have found large effects in other contexts. Pierce and Schott (2016) and Handley and Limão (2013), for example, argue that before China joined the World Trade Organization in 2001, uncertainty about U.S. trade policy towards Chinese goods significantly affected U.S. imports from China and lowered U.S. households' welfare. My analysis, which indicates that uncertainty about Brexit has little macroeconomic impact, highlights the need for further quantitative research in this area.

2 Model

I now develop a dynamic, stochastic, general equilibrium model with three countries: the United Kingdom, the European Union, and the rest of the world. Each country is populated by a representative household and a unit measure of heterogeneous firms. Households work, consume, invest, and save. Firms produce differentiated goods and endogenously accumulate foreign customers in response to changes—or anticipation of possible future changes—in bilateral trade costs.

2.1 Aggregate uncertainty and trade costs

In each period t the model economy experiences an aggregate shock, Z_t, which is drawn from a finite set $\mathcal{Z}_t$. The vector $Z^t = (Z_0, Z_1, \ldots, Z_t)$ denotes a history of aggregate shocks. $\Pi(Z^t)$ is the probability of a given history Z^t. There are two kinds of trade costs, both of which depend on the realization of the aggregate shock: import tariffs, which are rebated lump-sum to households; and iceberg trade costs. $\tau_{i,j}(Z^t)$ is the import tariff on goods produced in country j and sold in country i, and $\xi_{i,j}(Z^t)$ is the iceberg cost of shipping those goods. The process for Z_t is assumed to be non-stationary: the set of possible shocks and the associated probabilities depend on the period as well as the previous shock. This is necessary to capture the nature of the uncertainty about Brexit.

2.2 Households

The representative household in each country $i \in I = \{uk, eu, rw\}$ chooses consumption, $C_i(Z^t)$, investment, $X_i(Z^t)$, and bonds, $B_i(Z^t)$ to maximize lifetime utility,

$$\sum_{t=0}^{\infty} \sum_{Z^t} \beta^t \Pi(Z^t) \frac{C_i(Z_t)^{1-\gamma}}{1-\gamma}, \tag{1}$$

subject to a sequence of budget constraints,

$$P_i(Z^t)(C_i(Z^t) + X_i(Z^t)) + Q(Z^t)B_i(Z^t) = W_i(Z^t)\bar{L}_i + R_i(Z^t)K_i(Z^{t-1}) + B_i(Z^{t-1}) + T_i(Z^t) + D_i(Z^t), \tag{2}$$

a law of motion for capital,

$$K_i(Z^t) = \frac{1}{\varphi}\left[\delta^{1-\varphi}\left(\frac{X_i(Z^t)}{K_i(Z^{t-1})}\right)^{\varphi} - (1-\varphi)\delta\right] K_i(Z^{t-1}) + (1-\delta)K_i(Z^{t-1}), \tag{3}$$

and initial conditions for capital and bonds, $B_i(Z_0)$ and $K_i(Z_0)$. Labor is supplied inelastically. $T_i(Z^t)$ is the lump-sum transfer of tariff revenue from the government and $D_i(Z^t)$ is the aggregate dividend payment from firms in the household's home country. Bonds are not state-contingent; international financial markets are exogenously incomplete.[2]. Bonds are denominated in units of the British consumer price index which is normalized to one without loss of generality. The parameter φ governs the cost of adjusting the capital stock. When $\varphi < 1$, large investments are less effective in augmenting the capital stock as in Eaton et al. (2011b) and Lucas and Prescott (1971).

3 Distributors

Each country i has a unit measure of identical, competitive distributors who combine domestic and imported varieties to produce a nontradable aggregate good that is used for consumption, investment, and intermediate inputs. The aggregation technology has a nested CES structure. The top level takes the standard Armington form,

$$Y_i(Z^t) = \left[\sum_{j \in I} \mu_{i,j} Y_{i,j}(Z^t)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \tag{4}$$

where $Y_i(Z^t)$ is the aggregate good and $Y_{i,j}(Z^t)$ is a bundle of goods purchased from source country j. ζ is the elasticity of substitution between goods from different countries, commonly referred to as the Armington elasticity, and the parameter $\mu_{i,j}$ governs the share of goods from each source country j in the aggregate good. At the bottom level, the source-specific bundles $Y_{i,j}(Z^t)$ are produced by combining differentiated varieties that are produced by monopolistically competitive firms whose behavior I describe below. The price index for each source-specific bundle is $P_{i,j}(Z^t)$ and the elasticity of substitution between varieties from the same source country is θ.

3.1 Firms

Each country i has a unit measure of firms that produce differentiated varieties as in Melitz (2003) and Chaney (2008). Firms can costlessly sell to all domestic distributors but must pay marketing costs to reach foreign distributors as in Arkolakis (2010). The marginal cost of reaching additional foreign customers is decreasing in a firm's current market penetration, inducing firms to penetrate foreign markets gradually over time. The least productive firms, for which the marginal cost of reaching a single foreign distributor exceeds the marginal benefit, do not export at all, while moderately productive firms reach only a few foreign customers. Firms choose how much to market to each foreign destination independently; destinations to which it is more difficult to export have lower export participation rates.

In Steinberg (2018b), I show that this theory of export participation and market penetration dynamics

[2] All results reported in this paper, including the welfare losses associated with uncertainty about Brexit, are robust to alternative assumptions about international financial markets. In section 8 I study a version of the model with financial autarky.

accounts for a wide range of facts that have been documented in the literature about the cross-sectional distribution and life-cycle dynamics of exporters. The theory nests a number of other common theories of export participation as special cases. In section 7 I show that all of these special cases have similar quantitative implications about the cost of Brexit uncertainty.

3.1.1 Production, demand, and profits

Firms are heterogeneous in productivity, a, which is exogenous, constant over a firm's life, and drawn from a distribution $F_i(a)$. A firm with productivity a produces gross output using capital, k, labor, ℓ, and intermediate inputs, m, according to a constant-returns-to-scale, Leontief technology,

$$af(k,\ell,m) = a \min\left\{\frac{k^{\alpha}\ell^{1-\alpha}}{\eta_i}, \frac{m}{1-\eta_i}\right\}, \tag{5}$$

where the parameters α and η_i govern the shares of capital in value added and value added in gross output, respectively.[3]

Firms are also heterogeneous in the fraction of distributors in each foreign market to which they can sell, which is endogenous. Let $D_i = I \setminus \{i\}$ denote the set of country i's export destinations and let n denote a firm's market penetration—the fraction of distributors to which the firm can sell—in a given destination. Conditional on purchasing the firm's product, the demand of a distributor in destination d is a downward-sloping function of the price charged by the firm, p:

$$q_{d,i}(Z^t,p) = \left[(1+\tau_{d,i}(Z^t))^{-\theta} P_{d,i}(Z^t)^{\theta} Y_{d,i}(Z^t)\right] p^{-\theta} \tag{6}$$

The import tariff, $\tau_{d,i}(Z^t)$, enters this expression in the standard way. As I describe in section 4.2.1 below, if hard Brexit occurs, U.K. tariffs on goods from the remainder of the European Union rise, lowering U.K. demand for E.U. firms' products, and vice versa. Total demand for the firm's product in destination d is

$$y_{d,i}(Z^t,n,p) = n q_{d,i}(Z^t,p). \tag{7}$$

Domestic distributors have similar demand functions without tariffs. Firms can sell freely to all domestic distributors, however, so total domestic demand for a firm's product is $y_{i,i}(Z^t,p) \equiv y_{i,i}(Z^t,1,p) = q_{i,i}(Z^t,p)$.

Conditional on market penetration, fims engage in monopolistic competitition, choosing prices and inputs in each market to maximize intratemporal profits. The profits of a firm with productivity a and market penetration n in destination d are given by

$$\pi_{d,i}(Z^t,a,n) = \max_{p,k,\ell,m}\left\{p y_{d,i}(Z^t,n,p) - W_i(Z^t)\ell - R_i(Z^t)k - P_i(Z^t)m\right\}, \tag{8}$$

[3] The literature indicates that value added and intermediates are almost perfectly complementary (Kehoe et al., 2018; Atalay, 2014). This assumption does not affect significantly affect welfare results and delivers more reasonable investment dynamics in the leadup to and immediate aftermath of Brexit.

subject to the resource constraint,

$$y_{d,i}(Z^t, n, p)(1 + \xi_{d,i}(Z^t)) = af(k, \ell, m), \tag{9}$$

which says that the firm must produce an additional $\xi_{d,i}(Z^t)$ units of output in order to deliver one unit of output to its destination because of the iceberg trade cost.[4] The solution to this problem is characterized by the standard constant-markup pricing rule,

$$p_{d,i}(Z^t, a) = (1 + \xi_{d,i}(Z^t))\left(\frac{\theta}{\theta - 1}\right) MC_i(Z^t), \tag{10}$$

where

$$MC_i(Z^t) = \eta_i \left[\left(\frac{R_i(Z^t)}{\alpha}\right)^{\alpha}\left(\frac{W_i(Z^t)}{1-\alpha}\right)^{1-\alpha}\right] + (1 - \eta_i)P_i(Z^t). \tag{11}$$

When Brexit occurs, iceberg trade costs, $\xi_{d,i}(Z^t)$, rise as well as tariffs. This increases the cost of serving E.U. distributors for U.K. firms, leading them to charge E.U. distributors higher prices, and vice versa, further reducing U.K.-E.U. trade. Using this expression, we can write the firm's profits from selling to destination d as $\pi_{d,i}(Z^t, a, n) = \bar{\pi}_{d,i}(Z^t)na^{\theta-1}$, where

$$\bar{\pi}_{d,i}(Z^t) = \left(\frac{1}{\theta}\right)\left(\frac{\theta}{\theta-1}\right)^{1-\theta}\left[(1 + \tau_{d,i}(Z^t))^{-\theta}P_{d,i}(Z^t)^{\theta}Y_{d,i}(Z^t)\right]\left[(1 + \xi_{d,i}(Z^t))MC_i(Z^t)\right]^{1-\theta}. \tag{12}$$

This notation will prove useful in characterizing the optimal behavior of a firm below.

Let $y_{d,i}(Z^t, a, n) \equiv y_{d,i}(Z^t, n, p_{d,i}(Z^t, a))$ denote the firm's equilibrium exports to destination d, and let $k_{d,i}(Z^t, a, n)$, $\ell_{d,i}(Z^t, a, n)$, and $m_{d,i}(Z^t, a, n)$ denote the firm's demand for factors and intermediate inputs needed to produce these exports. The profits from selling to the domestic market and the associated equilibrium price are similar, with iceberg transportation costs, $\xi_{i,i}(Z^t)$, set to zero. Let $y_{i,i}(Z^t, a)$, $k_{i,i}(Z^t, a)$, $\ell_{i,i}(Z^t, a)$, and $m_{i,i}(Z^t, a)$ denote the firm's domestic sales and its demand for inputs needed to produce this output; they do not depend on market penetration because firms sell to all domestic distributors.

3.1.2 Advertising and market penetration dynamics

The current market penetration of a firm from country i in destination d is a function of the firm's advertising efforts and its market penetration at the beginning of the period. Following Arkolakis (2010), I assume that the fraction of distributors in destination d that see the firm's advertisements, $b_{d,i}$, is a concave function of the number of advertising signals the firm sends, s:

$$b_{d,i}(s) = 1 - [1 - (1 - \lambda)\psi_{d,i}s]^{\frac{1}{1-\lambda}}. \tag{13}$$

[4] The assumption of constant returns to scale implies that we can write a destination-specific resource constraint instead of an overall, firm-level resource constraint.

The parameter $\psi_{d,i}$ governs the efficiency of advertising to destination d and λ governs the degree of diminishing returns. Additionally, I assume that old customers and potential new customers are equally likely to see the firm's advertisements, and that a fraction $\omega_{d,i}$ of old customers who not see advertisements disappear. Combining these assumptions yields a law of motion for the firm's market penetration,

$$n = b_{d,i}(s)(1 - n_-) + [b_{d,i}(s) + (1 - b_{d,i}(s))(1 - \omega_{d,i})]\, n_-, \tag{14}$$

where n_- denotes the firm's market penetration at the beginning of the period. The first term on the right-hand side represents the number of new customers gained by advertising, and the second term represents the number of old customers that are retained. Inverting this law of motion and using equation (13), we can obtain a function for the advertising cost associated with reaching a fraction n of destination d's distributors given an initial market penetration of n_-:

$$\kappa_{d,i}(n, n_-) = \frac{\kappa(n, n_-)}{\psi_{d,i}} \tag{15}$$

where

$$\kappa(n, n_-) = \frac{1}{(1-\lambda)} \left\{ 1 - \left[\frac{1 - n_-}{1 - n_-(1 - \omega_{d,i})} \right]^{1-\lambda} \right\}. \tag{16}$$

The advertising cost is denominated in units of domestic labor.[5]

Finally, I assume that the firm exits exogenously from destination d in two ways. With probability $1 - \phi$, the firm dies and is replaced by a new firm with the same productivity. Newborn firms start with zero market penetration in all export destinations. Conditional on survival, with probability $1 - \chi$ the firm loses all of its customers in a given export destination and must start accumulating customers from scratch in the next period. The second kind of exit occurs independently across destinations; a surviving firm might exit from one export destination but not the other. I incorporate these two forms of exit in order to match bilateral exporter exit rates that I observe in the data, which are high, as well as firm discount rates from the literature, which are low.

3.1.3 Dynamic problem

Because production has constant returns to scale and market penetration dynamics are independent across destinations, we can characterize the firm's market penetration problem for each destination separately. The state variables of the firm's problem are its productivity, a, and its customer base at the beginning of the

[5] In Arkolakis (2010), firms use both domestic and foreign labor to advertise. I assume that advertising uses domestic labor only for simplicity's sake. Real wages do not change significantly across countries after Brexit occurs, though, so this assumption is not quantitatively important.

period, n_-. The value of selling to destination d for a firm in country i is

$$V_{d,i}(Z^t, a, n_-) = \max_{n \in [(1-\omega_{d,i})n_-, 1]} \left\{ \pi_{d,i}(Z^t, a, n) - W_i(Z^t)\kappa_{d,i}(n, n_-) + Q_i(Z^t)\phi \tilde{V}_{d,i}(Z^{t+1}, a, n) \right\}, \tag{17}$$

where

$$\tilde{V}_{d,i}(Z^{t+1}, a, n) = \sum_{Z^{t+1}} \Pi(Z^{t+1}|Z^t) \left[\chi V_{d,i}(Z^{t+1}, a, n) + (1-\chi) V_{d,i}(Z^{t+1}, a, 0) \right] \tag{18}$$

represents the continuation value of a surviving firm. The Euler equation that characterizes the solution to this problem is

$$W_i(Z^t)\kappa_1(n, n_-) \geq \psi_{d,i} \bar{\pi}_{d,i}(Z^t) a^{\theta-1} + Q_i(Z^t)\phi\chi \mathbb{E}\left[W_i(Z^{t+1})\kappa_2(n', n) \right], \tag{19}$$

where n' is the firm's optimal market penetration in the next period. This condition holds with equality if the firm chooses to advertise. I use $h_{d,i}(Z^t, a, n_-)$ to denote the policy function associated with this problem; in the Euler equation above, n' is shorthand for the cumbersome expression $h_{d,i}(Z^{t+1}, a, h_{d,i}(Z^t, a, n_-))$.

I define an entrant as a firm with zero customers at the beginning of the period (either because it is a newborn firm or because it lost all of its customers in the previous period) that chooses to advertise in the current period. The productivity of the marginal entrant, $a^*_{d,i}(Z^t)$, is given by

$$W_i(Z^t)\kappa_1(0,0) = \psi_{d,i} \bar{\pi}_{d,i}(Z^t) a^*_{d,i}(Z^t)^{\theta-1} + Q_i(Z^t)\phi\chi \mathbb{E}\left[W_i(Z^{t+1}\kappa_2(h_{d,i}(Z^{t+1}, a^*_{d,i}(Z^t), 0), 0) \right]. \tag{20}$$

In a steady state in which all trade costs and aggregate variables are constant, any firm that does not enter today will not enter in the future, either. In this case, the entry cutoff can be characterized analytically as

$$a^*_{d,i}(Z^t) = \left[\frac{1}{\bar{\pi}_{d,i}(Z^t)\psi_{d,i}} W_i(Z^t)(1 - Q_i(Z^t)\phi\chi(1-\omega_{d,i}) \right]^{\frac{1}{\theta-1}}. \tag{21}$$

This expression is identical to the entry cutoff in Arkolakis (2010) except for the term $1 - Q_i \omega_{d,i}(\chi_{d,i})$ that represents discounting of future profits and customer base depreciation.

In related work (Steinberg, 2018b), I show that this parsimonious theory of market penetration dynamics is consistent with several key facts that have been documented in the literature about both the cross-sectional distribution and life-cycle dynamics of exporters. As documented by Melitz (2003), only the most productive firms export because the marginal cost of obtaining the first customer, $\kappa_1(0,0)$, is strictly positive. Moreover, the convexity of the advertising cost function (15) implies that export sales are concentrated among large firms (Eaton et al., 2011a; Mayer and Ottaviano, 2008; Bernard et al., 2012) and that small firms have higher trade elasticites (Kehoe et al., 2015; Kehoe and Ruhl, 2013). Finally, new exporters are smaller than incumbents and grow gradually over time (Ruhl and Willis, 2017; Alessandria et al., 2015; Fitzgerald et al., 2016) because the marginal cost of obtaining additional customers is decreasing in the firm's current customer base, i.e., $\kappa_{12}(n, n_-) < 0$. Fitzgerald and Priolo (2018), who find that markups are flat with respect to a firm's tenure in a given market, provide additional empirical support for this theory.

3.2 Aggregation and market clearing

Let $G_{d,i}(Z^t, a, \cdot)$ denote the customer base distribution of firms in country i with productivity a. This distribution evolves according to the law of motion

$$G_{d,i}(Z^{t+1}, a, \mathcal{N}) = \int_{[0,1]} \left[\mathbb{1}_{\{h_{d,i}(Z^t,a,n_-)\in\mathcal{N}\}} \phi\chi + \mathbb{1}_{\{0\in\mathcal{N}\}}(1-\chi\phi) \right] \, \mathrm{d}G_{d,i}(Z^t, a, n_-), \tag{22}$$

where $\mathcal{N}$ denotes a typical subset of the unit interval. With this notation in hand, we can write the bundle of goods produced in country i for destination d, $Y_{d,i}(Z^t)$, as

$$Y_{d,i}(Z^t) = \left[\int_{\mathbb{R}_{++}} \int_{[0,1]} y_{d,i}(Z^t, a, h_{d,i}(Z^t, a, n_-))^{\frac{1-\theta}{\theta}} \, \mathrm{d}G_{d,i}(Z^t, a, n_-) \, \mathrm{d}F_i(a) \right]^{\frac{\theta}{1-\theta}}. \tag{23}$$

Similarly, the price index of this bundle is given by

$$P_{d,i}(Z^t) = \left[\int_{\mathbb{R}_{++}} \int_{[0,1]} (1+\tau_{d,i}(Z^t)) h_{d,i}(Z^t, a, n_-) p_{d,i}(Z^t, a)^{1-\theta} \, \mathrm{d}G_{d,i}(Z^t, a, n_-) \, \mathrm{d}F_i(a) \right]^{\frac{1}{1-\theta}}. \tag{24}$$

The bilateral export participation rate is given by

$$EPR_{d,i}(Z^t) = \int_{\mathbb{R}_{++}} \int_{[0,1]} \mathbb{1}_{\{h_{d,i}(Z^t,a,n_-)>0\}} \, \mathrm{d}G_{d,i}(Z^t, a, n_-) \, \mathrm{d}F_i(a), \tag{25}$$

and the average market penetration rate of exporters is

$$MPR_{d,i}(Z^t) = \int_{\mathbb{R}_{++}} \int_{[0,1]} \mathbb{1}_{\{h_{d,i}(Z^t,a,n_-)>0\}} h_{d,i}(Z^t, a, n_-) \, \mathrm{d}G_{d,i}(Z^t, a, n_-) \, \mathrm{d}F_i(a). \tag{26}$$

There are four market clearing conditions that must be satisfied in equilibrium. First, each country's aggregate output $Y_{i,t}(Z^t)$ must be used for consumption, investment, or intermediate inputs:

$$Y_i(Z^t) = C_i(Z^t) + X_i(Z^t) + \hat{M}_i(Z^t), \tag{27}$$

where $\hat{M}_i(Z^t)$ denotes aggregate demand for intermediate inputs, which can be computed as

$$\hat{M}_i(Z^t) = \int_{\mathbb{R}_{++}} \left\{ \sum_{d\in D_i} \left[\int_{[0,1]} m_{d,i}(Z^t, a, h_{d,i}(Z^t, a, n_-)) \, \mathrm{d}G_{d,i}(Z^t, a, n_-) \right] + k_{i,i}(Z^t, a) \right\} \, \mathrm{d}F_i(a). \tag{28}$$

Aggregate demand for capital, $\hat{K}_i(Z^t)$, and productive labor, $\hat{L}_i(Z^t)$, are calculated analogously. Second and third, factor markets must also clear:

$$K_i(Z^{t-1}) = \hat{K}_i(Z^t); \tag{29}$$

$$\bar{L}_i = \hat{L}_i(Z^t) + \int_{\mathbb{R}_{++}} \int_{[0,1]} \kappa_{d,i}(h_{d,i}(Z^t, n_-), n_-) \, \mathrm{d}G_{d,i}(Z^t, a, n_-) \, \mathrm{d}F_i(a). \tag{30}$$

The second term in the labor market clearing condition (30) represents the labor used in advertising. Finally, the bond market must clear:

$$\sum_{i \in I} B_i(Z^t) = 0. \tag{31}$$

3.3 Equilibrium and computation

An equilibrium is, for each country and all possible histories, a set of:

- aggregate quantities, C_i, X_i, B_i, K_i, T_i, D_i, Y_i, $Y_{i,j}$;
- aggregate prices, W_i, R_i, P_i, $P_{i,j}$;
- intratemporal firm allocations, $y_{d,i}$, $k_{d,i}$, $\ell_{d,i}$, $m_{d,i}$, $y_{i,i}$, $k_{i,i}$, $\ell_{i,i}$, $m_{i,i}$, prices, $p_{i,i}$, $p_{d,i}$, and profits, $\pi_{d,i}$, $\pi_{i,i}$;
- value functions, $V_{d,i}$, and market penetration policy functions, $h_{d,i}$;
- and market penetration distributions, $G_{d,i}$,

that solve the household, distributor, and firm problems, and satisfy the law of motion for market penetration distributions the market clearing conditions. If the aggregate shock Z_t is constant in the long run the model converges to a steady state in which the objects above are constant.

Most dynamic, stochastic, general equilibrium models in macroeconomics and international trade, including those that feature heterogeneous firms like Alessandria and Choi (2007, 2016) and Alessandria et al. (2016), use local methods to approximate the equilibrium near an invariant steady state. In my quantitative exercise, however, there are two steady states to which the equilibrium may converge: one associated with hard Brexit and another with soft Brexit.[6] Moreover, local approximation methods are ill-suited to the analysis of welfare and the effects of uncertainty, both of which take center stage in my study. Instead, I use a global method to solve for the exact equilibrium. The method is similar to that used in Kehoe et al. (2018), Alessandria et al. (2015), and others to solve for transition paths in deterministic models. The presence of uncertainty complicates matters but does not pose an insurmountable barrier as long as the number of possible histories is small, as is the case in my quantitative analysis which I describe in the next section. In brief, if one assumes that the equilibrium converges to a steady state after a finite number of periods, the equilibrium conditions for all possible histories, along with the corresponding equilibrium variables, can be represented by a single nonlinear system that can be solved using standard numerical methods. The appendix contains more details about my solution method.

[6]Strictly speaking, because I allow for unbalanced trade in the long run, steady states depend on net foreign assets, which are endogenous, as well as the trade policy regime (Kehoe et al., 2018). In truth, there is one set of possible steady states for soft Brexit, and another set of possible steady states for hard Brexit.

4 Quantitative analysis

My quantitative analysis proceeds in three steps. The first is to construct a benchmark from which to measure the impact of Brexit. I construct this benchmark during my calibration procedure, in which I set the model's parameters so that its steady state matches macroeconomic and international trade data from 2011. This no-Brexit steady state is a counterfactual that represents the state of the world before Brexit entered the realm of possibility.

Second, I use external data on the costs of E.U. trade with non-U.K. trade partners to construct two possible post-Brexit trade policy regimes: soft Brexit, in which the United Kingdom retains access to the European single market by remaining in the European Economic Area or through bilateral negotiation; and hard Brexit, in which the United Kingdom loses single market access. Each scenario involves two exogenous effects: (i) changes in import tariffs; and (ii) changes in iceberg trade costs. I also specify model agents' perceived probabilities that the Brexit referendum passes, and, conditional on that outcome, that Brexit will be hard or soft. The costs of trade with the rest of the world do not change in either scenario.[7]

Third, I solve for the equilibrium that arises following an unanticipated shock in 2015: Parliament authorizes a referendum on European Union membership in the following year. When the referendum is announced, model agents learn the probability that the referendum will pass and the details and likelihood (conditional on a "leave" vote) of each Brexit scenario. They must wait until 2016 to learn the outcome of the referendum and until 2019 to learn which Brexit scenario they will face. If Brexit occurs in 2019, the economy remains in either hard or soft Brexit forever. Figure 2 illustrates the timing in the Brexit equilibrium.

4.1 Calibrating the no-Brexit steady state

To calibrate the model, I first assign common parameters like the discount factor and elasticities of substitution to standard values. Given these assigned values, I calibrate the remaining parameters to that the model's steady state matches an input-output matrix from 2011, bilateral export participation rates, and several facts other about the distribution and life-cycle dynamics of exporters. The calibrated parameter values are listed in table 2.

4.1.1 Input-output data

I use an input-output matrix from the World Input Output Database (Timmer et al., 2015), henceforth abbreviated as WIOD, to specify production and trade relationships in the no-Brexit steady state. This dataset has been used widely in recent international trade studies including other analyses of Brexit like Dhingra et al. (2016b,c). I use the data from 2011, the last year available in the dataset and several years before Brexit

[7]E.U. regulations prevent the United Kingdom from negotiating free trade agreements with trade partners in the rest of the world while the United Kingdom remains an E.U. member, but once Brexit occurs the United Kingdom will be free to enter into such negotiations. Free-trade agreements with the rest of the world would increase trade with the rest of the world and offset some of the welfare losses caused by the reduction in trade with the European Union.

was considered possible. I aggregate all industries into a single sector and aggregate countries according to the three-country scheme in the model. Panel (a) of table 3 shows the aggregated WIOD data. The first three columns list intermediate inputs, value added, and gross output for each country, while columns 4 through 6 list final demand. All data in the matrix have been normalized so that U.K. GDP is equal to 100.

Trade is unbalanced in the aggregated data, however; the United Kingdom and the rest of the world have trade deficits and the European Union has a trade surplus. In a steady state, in which current accounts are zero, trade imbalances represent interest payments on net foreign assets. A country that has a trade deficit has positive net foreign assets and vice versa, and so treating the raw data as a steady state implies counterfactual net foreign asset positions. To sidestep this issue, I use the RAS procedure (Bacharach, 1965) to construct a similar input-output matrix in which each country's aggregate trade is balanced following Steinberg (2018a). This balanced matrix, which represents the no-Brexit steady state in my quantitative analysis, is shown in panel (b) of table 3. All differences between the balanced matrix and the raw data are minor.

4.1.2 Facts about exporters

In addition to matching aggregate input-output relationships, I also require that my calibrated model match bilateral export participation rates in trade between the United Kingdom, the European Union, and the rest of the world, and facts about the size distribution, exit rates, and growth rates of exporters. I use two sources to calculate these data: the EFIGE dataset (Altomonte and Aquilante, 2012) and the World Bank Exporter Dynamics database (Fernandes et al., 2016, hencefort EDD).

The EFIGE dataset contains detailed survey data on the economic performance of firms in five European countries: France, Germany, Italy, Spain, and the United Kingdom.[8] I use the internationalization section of the survey which asks firms about the regions to which they export. The EDD uses firm-level customs data to construct indicators on the distribution and life-cycle dynamics of exporting firms in a number of developing and developed economies, including six E.U. countries: Belgium, Bulgaria, Estonia, Portugal, Spain, and Sweden.

For U.K. firms, the EFIGE dataset allows me to compute bilateral export participation rates directly. I find that 54.5 percent of U.K. firms export to other E.U. countries, while 41.1 percent of U.K. firms export to the rest of the world. For the other two model countries, I use information from both datasets to compute bilateral export participation rates.[9] For E.U. firms, the EFIGE data allow me to compute the overall export participation rate and the bilateral export participation rate with the rest of the world. I find that 52.0 percent of E.U. firms export, and 71.7 percent of these exporters serve the rest of the world. I augment these data with information from the EDD to infer E.U. firms' export participation rate with the United Kingdom. 11.1

[8] The full EFIGE dataset contains data on Austrian and Hungarian firms as well, but the samples for these countries are small so I exclude them following Piguillem and Rubini (2013)

[9] The EFIGE survey asks firms about the fraction of their exports that go to regions (e.g. the European Union, other European countries, North America, etc.) not individual countries. The appendix contains further details on how I use these two datasets.

percent of E.U. exporters in the EDD serve the United Kingdom, which implies that 5.7 percent of all E.U. firms, including non-exporters, export to this destination. For firms in the rest of the world, I assume that the overall export participation rate is the same as in the European Union (52.0 percent), and use the EDD to infer bilateral export participation rates. 8.3 percent and 21.2 percent of exporters in the rest of the world serve the United Kingdom and the European Union, respectively, which implies that 4.5 percent and 10.7 percent of all firms in the rest of the world export to these destinations.

I also use the EDD to compute three other statistics about bilateral trade relationships: the share of exports accounted for by the top 5 percent of exporters, the exit rate, and the growth rate of new exporters relative to incumbents. These statistics do not vary widely across exporting countries or destinations, particularly in developed economies (Fernandes et al., 2016). The average top-5 share is 58.4 percent, the average exit rate is 45.9 percent, and the average growth rate of new exporters relative to incumbents is 13.2 percent.[10] I require that the averages of these statistics for each country in the model match the averages in the EDD.

4.1.3 Assigned parameters

The discount factor, β, is set so that the steady-state real interest rate is 2 percent per year. γ, which governs risk aversion and the elasticity of intertemporal substitution, is set to 2. The depreciation rate, δ, and the capital share, α, are set to 6 percent and one-third, respectively. I follow Alessandria and Choi (2016) and Alessandria et al. (2016) and set θ, the elasticity of substitution between varieties, to 5. φ, the parameter which governs capital adjustment costs, is set to 0.76 as in Steinberg (2016). I set λ, which governs the convexity of the marketing cost function, to one so that $\kappa(n, n_-) = \log((1-n)/(1-n_-(1-\omega_{d,i})))$. I find that this parameter has little effect on the concentration of exports (which is driven primarily by the dispersion of firm productivities) or the growth rate of new exporters relative to incumbents (which is driven primarily by the customer base depreciation rate, $\omega_{d,i}$). None of the results reported in this paper are sensitive to this parameter. Finally, I follow Melitz and Costantini (2007) and set the death probability, $1-\phi$, to 15 percent.

4.1.4 Calibrated parameters

The parameters that govern aggregate production and trade relationships are set directly using the balanced input-output matrix from section 4.1.1. I set the value added shares, η_i, and the Armington shares, $\mu_{i,j}$, so that the data in the matrix satisfy distributors' first-order conditions.[11] I set all bilateral trade costs to zero so that the Armington shares absorb trade costs as well as other sources of home bias. This is without loss of generality given the assumption that tariff revenues are rebated lump-sum to households. Each country's time endowment, $\bar{L}_i$, is set to a fraction $1-\alpha$ of its value added.

[10]These statistics differ in some respects from those reported in other studies like Ruhl and Willis (2017), Alessandria and Choi (2016), and Alessandria et al. (2016) about the characteristics of exporters in overall, not bilateral, trade. Exit rates in bilateral trade are higher than the overall exit rates of exporters and bilateral exports are more concentrated.

[11]As in Kehoe et al. (2018), I choose units so that all steady-state prices are one. This is without loss of generality. See the appendix for more details.

Most of the remaining parameters are calibrated so that the model's steady state matches the facts about exporters listed in section 4.1.2 above. I assume that firms' productivities are distributed lognormally with standard deviation σ_i. I choose these dispersion parameters, together with the marketing efficiency parameters, $\psi_{d,i}$, the probability of exiting a foreign market conditional on survival, χ, and the customer base depreciation rates, $\omega_{d,i}$, so that the model matches bilateral export participation rates, the top 5 share of bilateral exports, the bilateral exit rate of exporters, and the relative growth rate of new exporters in each country.

Given the above assignment of the death rate, $1-\phi$, the exit rate can be targeted directly by setting $1-\chi$ so that the unconditional bilateral exit rate is 45.9 percent as in the EDD data. Each of the other parameters calibrated in this stage of the procedure affects all of the other moments to some degree. Roughly speaking, however, the productivity dispersion parameters control the concentration of exports, the marketing efficiencies control export participation rates,[12] and the customer base depreciation rates control the relative growth rate of new exporters. I find that productivity dispersion and customer base depreciation are similar in all three countries, but marketing efficiencies vary widely. U.K. firms' efficiencies in marketing to the European Union and the rest of the world are similar because similar fractions of U.K. firms export to each of these countries. Conversely, the efficiencies of E.U. and rest of the world firms in marketing to the United Kingdom are low because few of these firms export to this destination. This is consistent with Arkolakis (2010), who finds that fewer firms export to small destinations. Under my calibration strategy, differences in marketing efficiencies across destinations reflect this market-size effect.

One final parameter remains to be calibrated: the Armington elasticity, ζ, which governs the long-run response of aggregate trade flows to changes in prices. I set this parameter so that the model's long-run trade elasticity is 5 (Costinot and Rodríguez-Clare, 2014; Caliendo and Parro, 2015). In the absence of export participation and market penetration dynamics, one could assign a value to this parameter directly to match this target, but in my model it must be calibrated by performing the entire quantitative analysis and analyzing the long-run change in trade flows caused by Brexit. My strategy is as follows: guess a value for ζ, calibrate the other parameters using the approach outlined above and perform the quantative analysis, check the long-run effects of Brexit on trade flows, and update the guess as necessary. The calibrated value of $\zeta = 3.25$ is lower than the target trade elasticity because export participation and market penetration rates change in response to changes in trade costs.

[12] Equation (21) shows that, holding fixed the other parameters, there is a one-for-one relationship between marketing efficiencies and export participation rates. Arkolakis (2010) assumes, with the support of extensive empirical evidence, that advertising is more effective in larger markets. In my model each country has two destinations, and I calibrate the efficiency parameters $\psi_{d,i}$ to match destination-specific export participation rates. The calibrated model is identical to one in which each country has a single (not destination-specific) advertising efficiency parameter and a parameter that governs the market size effect (α in Arkolakis, 2010) that are calibrated to the same targets.

4.2 Brexit scenarios

Having calibrated the model and constructed the no-Brexit steady state, I now describe the details of the two Brexit scenarios and the transition process for the aggregate shock. Table 4 provides a summary of this information.

4.2.1 Tariffs

There are no changes in import tariffs in the soft Brexit scenario because the United Kingdom retains single market access. In the hard Brexit scenario, tariffs are based on three sources of data: the European Union's most-favored-nation (MFN) tariff schedule for 6-digit HS goods industries published by the World Trade Organization (WTO); COMTRADE data on U.K. trade flows for these same industries; and the disaggregated WIOD data from section 4.1.1. First, I use the WTO and COMTRADE data to calculate average MFN tariffs on U.K.-E.U. goods trade. The U.K. tariff on E.U. goods is computed as the average MFN tariff weighted by imports, while the E.U. tariff on U.K. goods is weighted by exports. Second, I multiply these goods-trade tariffs by the goods shares[13] of total U.K. imports from, and exports to, the European Union in the disaggregated WIOD data. This step adjusts tariffs downwards to reflect the fact that the United Kingdom and European Union trade services, on which tariffs are rarely levied, as well as goods.

4.2.2 Iceberg trade costs

To calculate changes in iceberg trade costs, I use the same approach as Dhingra et al. (2016b,c), which is in turn based on Francois et al. (2013)'s estimates of non-tariff barriers in trade between the United States and the European Union. This study reports non-tariff barriers for a set of industries that approximately correspond to the 2-digit ISIC industries in the disaggregated WIOD data,[14], and also reports the fraction of these barriers that could be reduced by policy action. I treat policy-reducible non-tariff barriers in E.U.-U.S.A. trade as worst-case upper bounds for post-Brexit iceberg trade costs in U.K.-E.U. trade. First, I compute average policy-reducible barriers using the WIOD data on U.K.-E.U. trade flows as weights as in section 4.2.1. Second, as in Dhingra et al. (2016b,c), I assume that that iceberg trade costs in the model increase by 25 percent and 75 percent of these averages following soft and hard Brexit, respectively.

4.2.3 Transition probabilities

The aggregate state in the stochastic equilibrium with Brexit follows a non-stationary Markov process. Let Z_{stay} denote the aggregate state associated with European Union membership, and let Z_{soft} and Z_{hard} denote the aggregate states associated with soft and hard Brexit, respectively. We need a fourth pre-Brexit state, Z_{pb}, to which the economy enters after a "leave" vote in the referendum. Trade costs do not rise in the pre-Brexit

[13] I define the goods sector as agriculture, resource extraction, and manufacturing.

[14] Several WIOD industries do not have counterparts in Francois et al. (2013). Many of these industries, such as the sale and maintenance of motor vehicles, are nontraded. See the appendix for more details.

state, but expectations about future trade costs change. The set of possible aggregate states in each period is given by

$$\mathcal{Z}_t = \begin{cases} \{Z_{stay}\} & t < 2016 \\ \{Z_{stay}, Z_{pb}\} & 2016 \leq t \leq 2018 \\ \{Z_{stay}, Z_{soft}, Z_{hard}\} & t \geq 2019 \end{cases} \tag{32}$$

Abusing notation slightly, let $\Pi_t(Z)$ denote the unconditional probability of aggregate state Z in period t, and let $\Pi_t(Z'|Z)$ denote the probability of transitioning from state Z in period $t-1$ to state Z' in period t. Both of these probability functions are time-varying.

Z_{stay} is the only possible state until 2016, the year of the referendum, so $\Pi_t(Z_{stay}) = 1$ for $t < 2016$. Let Π_{vote} denote the probability of a "stay" vote in the referendum. The unconditional probabilities for the aggregate state in 2016 are $\Pi_{2016}(Z_{stay}) = \Pi_{vote}$ and $\Pi_{2016}(Z_{pb}) = 1 - \Pi_{vote}$. If "stay" wins, the economy remains in this state forever: $\Pi_t(Z_{stay}|Z_{stay}) = 1$ for $t > 2016$. If "leave" wins, the economy remains in the pre-Brexit state, Z_{pb}, until Brexit occurs: $\Pi_t(Z_{pb}|Z_{pb}) = 1$ for $2016 < t < 2019$. In 2019, if the economy is in the pre-Brexit state, it switches to either the hard or soft Brexit scenario. Let Π_{brexit} denote the probability of soft Brexit. Then we have $\Pi_{2019}(Z_{soft}|Z_{pb}) = \Pi_{brexit}$ and $\Pi_{2019}(Z_{hard}|Z_{pb}) = 1 - \Pi_{brexit}$. Once this transition has occurred, the economy remains in hard or soft Brexit forever: $\Pi_t(Z_{soft}|Z_{soft}) = \Pi_t(Z_{hard}|P_{hard}) = 1$ for $t > 2019$. Figure 2 provides an illustration of this transition process.

There are two probabilities that we must assign: Π_{vote}, the probability that "stay" prevails in the 2016 referendum, and Π_{brexit}, the probability of soft Brexit conditional on a "leave" vote. Although "leave" won referendum, this outcome was viewed as unlikely by many until the votes began to be tallied. Prediction markets, in fact, reported a 75-percent probability that "stay" would win during the week before the referendum,[15] so I set Π_{vote} to 75 percent. Assigning the probability of soft Brexit, Π_{brexit}, is more problematic. There are no prediction markets that allow bettors to wager on the outcome of Brexit, and there is not yet sufficient post-referendum macroeconomic data to which one could calibrate this parameter using the model. Lacking a solid prior, I assume that hard and soft Brexit are equally likely, which maximizes the entropy of the Brexit outcome. However, as I show in section 6.1, none of my results are sensitive to this choice. Panel (c) of table 4 lists the assigned transition probabilities.

5 Impact of Brexit on the United Kingdom economy

Having described the model and its calibration, I turn now to the quantitative analysis. First, I discuss the model's predictions about Brexit's effects on macroeconomic dynamics and trade flows. Second, I show that these predictions are consistent with data on recent U.K. economic dynamics. Third, I calculate the overall welfare cost of Brexit for U.K. households. Last, I discuss the macroeconomic impact of uncertainty about

[15]See, for example, http://www.slate.com/articles/news_and_politics/moneybox/2016/07/why_political_betting_markets_are_failing.html.

Brexit and calculate the welfare cost of this uncertainty.

5.1 Macroeconomic variables

Figure 3 depicts the impact of Brexit on U.K. macroeconomic variables. The solid blue lines (labeled "Pre-Brexit") depict the trajectories of these variables during 2015–2019, after the referendum is announced—and succeeds—but before Brexit actually takes place. In 2019, the equilibrium path forks. The dashed green and red lines (labeled "Soft" and "Hard") depict the trajectories of macroeconomic variables from 2019 onwards after soft and hard Brexit, respectively. The figures also show the long-run effects of Brexit in each scenario using color-coded bars. I do not plot the counterfactual trajectory in which the referendum fails.

In the long run, real GDP, consumption, and investment fall permanently in both Brexit scenarios. The long-run drops in consumption—0.5 percent and 1.3 percent for soft and hard Brexit, respectively—provide us with back-of-the-envelope measures of U.K. welfare losses from Brexit in each scenario. As we will soon see, these numbers are indeed close to the true welfare losses once transition dynamics into account. In the short run, the effects of Brexit on most macroeconomic variables are muted until Brexit actually occurs in 2019. GDP falls slightly during the pre-Brexit period, but does not drop substantially until Brexit takes place. Investment actually rises slightly in the pre-Brexit period in anticipation of higher future costs. The most pronounced pre-Brexit effects are seen in consumption, which begins to fall when the referendum is announced in 2015 and falls more dramatically in 2016, when the referendum succeeds, even though trade costs do not rise for three more years. This is consistent with permanent income logic: when U.K. households learn that their expected long-run income has fallen, they save to smooth their consumption over time. This behavior causes the United Kingdom to run a trade surplus. Once Brexit occurs in 2019, U.K. households increase their consumption if soft Brexit occurs because their permanent income rises, and if hard Brexit occurs their permanent income, and thus consumption, fall further. In either case, the trade balance reverts towards zero as households have little further incentive to save. In the long run, trade surpluses turn to deficits as households use their accumulated savings to augment consumption.

5.2 Trade flows

Figure 4 illustrates the impact of Brexit on U.K. trade with the European Union. In the long run, imports from the European Union as a fraction of U.K. GDP fall by 10.8 percent for soft Brexit and 44.8 percent for hard Brexit. These two numbers are approximately hardwired by the calibration, which targets a long-run trade elasticity of five. Exports to the European Union fall less than imports in both scenarios, so the bilateral trade balance with the European Union improves. Permanent-income logic, as described above, helps explain this result, but there is a second mechanism at play. Trade costs on shipments from the United Kingdom to the European Union rise less than trade costs on shipments in the other direction, so imports fall more than exports. Barattieri (2014) shows that this same mechanism explains U.S. trade deficits in the 1990s; goods trade liberalized more quickly than services trade during this period, and so U.S. goods immports rose more than U.S. services exports.

In both scenarios, U.K.-E.U. trade does not change significantly during the pre-Brexit period, but falls quickly once Brexit occurs even though export participation takes several more years to adjust. Panels (c) and (d) illustrate the dynamics of export participation rates (the extensive margin of export participation), while panels (e) and (f) illustrate the dynamics of market penetration rates of exporters (the intensive margin of export participation). Like aggregate trade flows, export participation and market penetration rates in U.K.-E.U. trade move little during the pre-Brexit period. There are some small fluctuations in 2016, when "leave" wins the vote, but these fluctuations are minor compared to the long-run effects. Once Brexit occurs, export participation and market penetration fall rates sharply. Along the transition, export participation rates in both countries fall steadily over time, but market penetration rates recover. This recovery is particularly strong for E.U. exporters to the United Kingdom; if soft Brexit occurs, their market penetration rate actually rises in the long run.

The non-monotonic transition dynamics of market penetration rates in U.K.-E.U. trade are driven by two offsetting forces, one of which occurs at the firm level and the other at the cross-sectional level. When trade costs rise, each individual exporter reduces its marketing efforts and its market penetration rate falls steadily over time. However, when the export participation rate falls, the average productivity of remaining exporters rises, and more productive exporters have higher market penetration rates than less productive ones. Consequently, average market penetration rates of exporters fall sharply in the immediate aftermath of Brexit, but as export participation rates fall along the transition to the long run, average exporter productivities rise, leading average market penetration rates to recover. For E.U. exporters to the United Kingdom, these dynamics are particularly pronounced because of the relatively large long-run run decline in their export participation rate. If hard Brexit occurs, for example, the mass of E.U. firms that export to the United Kingdom falls by 37.7 percent (from 5.7 percent to 3.6 percent), while the mass of U.K. firms that export to the European Union only falls by 12.4 percent (from 54.5 percent to 47.8 percent).

As figure 5 shows, Brexit will also affect trade with the rest of the world. Imports from the rest of the world rise after Brexit because these goods are substitutes for those produced in the European Union. Consequently, the rest of the world's export participation and market penetration rates in the United Kingdom rise. U.K. exports to the rest of the world, on the other hand, fall slightly in the long run because Brexit increases U.K. prices, thereby appreciating the U.K. real exchange rate with the rest of the world. U.K. export participation and market penetration rates in the rest of the world change little. Trade with the rest of the world, particularly imports, adjusts more gradually than trade with the European Union because export participation takes longer to adjust. This is because gaining foreign customers requires firms to undertake increasingly costly marketing efforts, while reducing export participation simply requires firms to let their customer bases depreciate. If trade with the rest of the world did not adjust, it is likely that the welfare losses of U.K. households from Brexit would be larger. If, on the other hand, the United Kingdom successfully negotiates free trade deals with countries in the rest of the world after exiting the European Union, as some advocates of Brexit have predicted, trade with the rest of the world could increase more than the model predicts and welfare losses could be lower.

In either Brexit outcome, the rapid adjustment of bilateral trade despite the length of the export participation adjustment process indicates that export participation dynamics play only a small role in driving the aggregate consequences of Brexit. In section 7 below, I confirm this result by analyzing a variety of alternative models of export participation dynamics.

5.3 Comparing the model to data

The national accounts and aggregate trade data that cover the period since the referendum act was introduced to Parliament are shown in table 1 and panels (a) and (b) of figure 1. The pre-Brexit model dynamics are broadly consistent with observed U.K. macroeconomic dynamics during this period. In both model and data, GDP and consumption growth slow when "leave" wins the Brexit vote, the investment rate rises, and aggregate and net trade flows hold steady. The only real discrepancies are an uptick in consumption growth and a decline in trade between the referendum's announcement and the Brexit vote, but these effects are transitory and disappear once the vote occurs. More importantly, all pre-Brexit fluctuations in the data are minor, indicating that anticipation of Brexit has had a small impact on the U.K. macroeconomy. Regardless of their size, the presence of these fluctuations is not evidence of a macroeconomic effect of uncertainty about Brexit. As I show below in section 5.5, similar fluctuations occur in perfect-foresight equilibria in which model agents know which Brexit outcome they will face in advance, indicating that the macroeconomic effects of uncertainty about Brexit are negligible.

The pre-Brexit dynamics of U.K-E.U. bilateral trade flows in the model are also consistent with recent data. Panel (b) of table 1 shows that U.K. goods trade with the European Union has not changed much relative to the size of the U.K. economy since either the Brexit vote was introduced to Parliament or the Brexit vote itself took place.[16] Panel (b) of figure 1 depicts a small decrease in bilateral trade flows immediately after the referendum was introduced, but this drop has been short-lived; trade with the E.U. as a fraction of U.K. GDP was at almost exactly the same level in 2018Q1 as in 2012Q1. There is one caveat to this comparison: when viewed from the perspective of the European Union, instead of the United Kingdom, bilateral trade flows have fallen more dramatically in the data. Panel (c) of figure 1 plots the same bilateral goods trade flows shown in panel (b), converted to Euros and normalized by E.U. GDP instead of U.K. GDP. Euro-denomominated imports from, and exports to, the United Kingdom have clearly fallen since the middle of 2015. This discrepancy is due to the well-known depreciation of the British pound during this period.

Panel (d) of figure 1 illustrates this depreciation by plotting the U.K.'s real exchange rates with the

[16] Recent data on bilateral services trade are not yet available. Goods trade is significantly more volatile than services trade (Kehoe et al., 2018), however, so it is unlikely that U.K. services trade with the European Union has changed much in recent.

European Union and the rest of the world, which have both depreciated by more than 20 percent simce 2015Q2. In the model, U.K. real exchange rates also depreciate during the pre-Brexit period. This follows from Marshall-Lerner logic: permanent income motives drive up the U.K. trade balance, so its real exchange depreciates in equilibrium to compensate. The depreciation in the model is far less than the observed depreciation, but it is widely known that quantitative models have trouble generating the kind of exchange rate volatility that we see in the data; in fact, the seminal study by Obstfeld and Rogoff (2001) identifies this as one of the major puzzles in the field. A version of the model with multiple sectors and import adjustment frictions fares better in generating pre-Brexit depreciation, and sticky wages and exogenous productivity losses further improve the model's performance on this dimension. Delayed nominal exchange rate passthrough may also account for the depreciation of the United Kingdom's real exchange rate in recent quarters; there is growing concern that the pound's recent weakness will soon cause inflation to rise, which could reverse some of the recent real depreciation.

Finally, one might also ask whether the long-run predictions of the model are data-consistent. Brexit has yet to occur, of course, but we can compare the long-run effects of Brexit on U.K. trade flows to a historical analogue: the departure of the United Kingdom from the European Free Trade Area (EFTA).[17] The United Kingdom was a founding member of the EFTA, which began in 1960 as an alternative to the European Economic Community (EEC), the predecessor of the European Union. In 1973, however, the United Kingdom reversed course, exiting the EFTA and joining the EEC instead. Figure 6 shows that in the twenty years following the Uhited Kingdom's exit from the EFTA, the share of U.K. trade with original EFTA members declined sharply, from 32.6 percent in 1972 to 22.2 percent in 1992, a drop of 10.4 percentage points, or 31.9 percent.[18] If hard Brexit occurs in the model, the E.U. share of U.K. trade falls from 47.1 percent to 34.6 percent in the long run, a drop of 12.5 percentage points or 26.5 percent. Thus, the long-run decline in U.K. trade with the EFTA after the United Kingdom's exit from that pact is similar to the long-run decline in U.K. trade with the remainder of the E.U. in the model in the case of hard Brexit.

5.4 Welfare

I measure welfare using a backward-looking method that compares welfare in the no-Brexit steady state to welfare in the history that leads to a particular long-run aggregate state $Z_{lr} \in \{Z_{soft}, Z_{hard}\}$.[19] The welfare loss for soft-Brexit, $\mathcal{W}_i^{soft}$, for example, is given by

$$\frac{U\left((1-\mathcal{W}_i^{soft})C_i^*\right)}{1-\beta} = \sum_{t=0}^{\infty} \beta^t U(C_i(Z_0, Z_1, \dots, Z_{soft})), \tag{33}$$

[17] I thank Nuno Limão for pointing out this example.

[18] I use the EFTA share of U.K. trade in this example, rather than U.K. trade with EFTA as a fraction if U.K. GDP, because trade openness was growing rapidly around the world during this period.

[19] I do not report differences in welfare between the no-Brexit steady state and the equilibrium in which the referendum occurs but does not pass.

where $(Z_0, Z_1, \ldots, Z_{soft})$ is the unique history leading to soft Brexit. The first row in table 5 lists the results of these welfare calculations.

The welfare losses in both Brexit scenarios are close to the long-run decreases in consumption: 0.4 percent for soft Brexit, and 1.2 percent for hard Brexit. These losses are large compared to estimates in the literature of the welfare effects of past trade reforms. Caliendo and Parro (2015), for example, find that U.S. welfare gains from NAFTA were only 0.1 percent, while di Giovanni et al. (2014) find that average country's welfare gain from trade with China is 0.4 percent. On the other hand, Dhingra et al. (2016b,c) predict even larger welfare losses from Brexit than I do, due in part to their assumption that the United Kingdom will miss out on future reductions in intra-E.U. trade costs. The present value of U.K. welfare losses from Brexit are £7,000–£19,000 per person, or, equivalently, 18–49 percent of 2015 U.K. GDP.[20]

5.5 Uncertainty about Brexit

To assess the impact of uncertainty about Brexit, I compare the stochastic Brexit equilibrium depicted in figure 2 with two perfect-foresight equilibria. In the first, model agents learn immediately in 2015 that soft Brexit will occur in 2019, and in the second they learn that hard Brexit will occur instead. The trajectories of trade costs in the perfect-foresight equilibria mirror the realized trade cost trajectories in the soft- and hard-Brexit histories in the stochastic equilibrium, so all differences in outcomes between the stochastic and perfect-foresight versions of the two Brexit scenarios are due solely to uncertainty. The trajectories of the variables of interest in the perfect-foresight equilibria are depicted as dotted lines in teal and orange (labeled "Soft (perf. foresight)" and "Hard (perf. foresight)") in figures 3–5.

The dynamics of macroeconomic variables, trade flows, and real exchange rates in the perfect-foresight equilibria are all similar to the stochastic equilibrium's dynamics. As these figures show, the perfect-foresight trajectories of all variables of interest are close to their stochastic-equilibrium counterparts. During the pre-Brexit period, the trajectories of consumption and other macroeconomic aggregates in the stochastic equilibrium are about halfway between the two perfect-foresight trajectories, but once Brexit occurs the stochastic and perfect-foresight trajectories converge quickly. The same is true for most bilateral trade variables.

There is a pre-Brexit decline in export participation and market penetration in the stochastic equilibrium, particularly in the period immediately preceding Brexit, that does not occur in the stochastic equilibria. This decline is most pronounced for E.U. exporters to the United Kingdom (see panels (d) and (f) of figure 4). The trade policy uncertainty literature suggests that this decline is due to the real option value of waiting until Brexit uncetainty is resolved before paying the costs associated with export participation decisions (Handley and Limão, 2015, 2013; Handley, 2014). The differences between the stochastic-equilibrium and perfect-foresight trajectories of export participation and market penetration rates are small, however, indicating that this mechanism does not impose significant welfare losses.

[20] To compute the present value of consumption-equivalent welfare losses, I first compute the cost, in units of the 2015 U.K. CPI in the model, of purchasing no-Brexit steady-state consumption forever. I then multiply this figure by the ratio of 2015 consumption in the data (£2.17 trillion) and 2015 consumption in the model. From here, the conversion to per-capita or percent-GDP costs is straightforward.

To measure the welfare costs of uncertainty about Brexit I use the same backward-looking method as in section 5.4.[21] For each Brexit scenario, I compare welfare in the perfect-foresight equilibrium with welfare along the history leading to that same scenario in the stochastic equilibrium. This method asks U.K. households, once they learn which Brexit scenario they face in 2019, how much they would have paid to learn that outcome immediately in 2015 instead. This method of measuring welfare losses from Brexit uncertainty is the same as the method used by Handley and Limão (2013). The welfare cost of uncertainty about Brexit is 0.18 percent of the overall welfare cost for soft Brexit, and 0.24 percent for hard Brexit. The present values of these losses are less than £50 per person.

6 Alternative Brexit scenarios

In my baseline quantitative analysis, hard and soft Brexit are assumed to be permanent, equally-likely outcomes that affect tariffs and iceberg trade costs only, and the effects of Brexit on trade costs in each outcome are homogeneous across firms. In this section I analyze several alternative Brexit scenarios in which I explore the importance of these assumptions for my results. Panel (b) of table 5 lists welfare loss calculations for each scenario described below. The results of these exercises indicate that all of these assumptions have little impact on the cost of Brexit uncertainty.

6.1 Probability of hard vs. soft Brexit

Soft and hard Brexit are equally likely in the baseline analysis. This is an ad-hoc choice, so it is important to verify that it has little impact on the results. Here, I consider two alternative scenarios: one in which hard Brexit is more likely ($\Pi_{brexit} = 0.25$), and another in which soft Brexit is more likely ($\Pi_{brexit} = 0.75$).

The overall welfare losses from Brexit are identical in these two alternatives to the baseline losses. The welfare losses from Brexit uncertainty are lower (higher) for soft Brexit in the alternative scenario in which soft Brexit is more (less) likely. The intuition for this result is straightforward: when firms and households believe that hard Brexit is more likely, they make larger adjustments to their market penetration and investment decisions which must be undone if soft Brexit occurs after all. However, the differences between the losses from uncertainty in these scenarios and the baseline results are small, indicating that the probability of soft vs. hard Brexit does not have a significant effect on the cost of Brexit uncertainty.

6.2 Increased market penetration costs instead of/in addition to iceberg costs

I have interpreted non-tariff trade barriers as standard iceberg transportation costs in the baseline analysis. An alternative interpretation is that non-tariff trade barriers manifest as costs that firms must incur to

[21] I have also computed forward-looking measures of welfare which are analogous to risk compensation. These results are similar to the backward-looking measures.

gain foreign customers. Here, I consider an alternative scenario in which Brexit has no impact on iceberg transportation costs, but instead reduces the marketing efficiency parameters, $\psi_{d,i}$.

I choose the post-Brexit values of $\psi_{d,i}$ in this version of the analysis so that the long-run changes in bilateral trade flows in each Brexit outcome are the same as in the baseline version. If soft Brexit occurs, marketing efficiency of U.K. firms in attracting E.U. customers falls by 41 percent, while E.U. firms' marketing efficiency in the United Kingdom falls by 27 percent. If hard Brexit occurs, these marketing efficiency parameters fall by 150 percent and 88 percent, respectively. In this version of the analysis, the overall welfare cost of Brexit is slightly higher than in the baseline and the cost of Brexit uncertainty is almost exactly the same.

I also consider another alternative scenario in which non-tariff barriers rise and marketing efficiencies fall. Here, I use the same numbers for iceberg trade costs as in the baseline analysis in addition to the values listed above for marketing efficiency changes. In this scenario, the overall welfare cost of Brexit is significantly larger than in the baseline analysis. This is to be expected, since trade costs also increase significantly more than in the baseline. U.K. households' overall welfare loss in this scenario is double the baseline figure for soft Brexit and three-quarters higher for hard Brexit. The contribution of Brexit uncertainty to these welfare losses, however, is about the same as in the baseline analysis.

6.3 Additional trade policy uncertainty: reversible Brexit

Brexit is a permanent policy change in the baseline analysis; once soft or hard Brexit occurs, trade policies remain fixed in that regime forever. Precedent, however, suggests that U.K. free trade area membership decisions can be temporary. As discussed in section 5.3 above, the United Kingdom was a founding member of the European Free Trade Area but backed out of the pact in 1973, joining the European Economic Community instead. Here, I introduce an additional source of trade policy uncertainty to my analysis: uncertainty about the permanence of Brexit.

In this alternative scenario, I assume that once Brexit occurs there is a chance that it may be reversed five years later. If reversal occurs, all trade costs fall back to their initial levels from then onward. Consequently, even once Brexit occurs in 2019, model agents remain uncertain about the long-term trade regime in which they will live until 2024. I assume that the chance that Brexit will revert is 50 percent, the same as the probability of soft vs. hard Brexit.

In this version of the analysis there are four equilibrium trajectories for which we must compute welfare losses: permanent soft and hard Brexit, and temporary soft and hard Brexit. The row labeled "Reversible Brexit (permanent)" in panel (b) of table 5 lists the results for the first two trajectories, in which trade costs follow exactly the same paths as in the baseline analysis. The overall welfare losses from a permanent Brexit in this scenario are virtually identical to the baseline results. The losses from Brexit uncertainty are larger, but still no more than about half of a percent of the overall losses.

The row labeled "Reversible Brexit (temporary)" lists the results for the second pair of trajectories, in which Brexit occurs but is reversed after five years. The overall welfare losses from a temporary Brexit are, as one might expect, much lower than the losses from a permanent Brexit; the temporary versions of soft and hard Brexit are each about a tenth as costly as the permanent versions. The losses from Brexit uncertainty when Brexit is temporary are much larger in proportion to the overall losses. Uncertainty accounts for almost two and five percent, respectively, of the overall losses in the temporary versions of soft and hard Brexit. However, the losses from Brexit uncertainty measured in consumption equivalent units are not much different in the temporary-Brexit trajectories than in the baseline permanent-Brexit trajectories. The consumption-equivalent welfare losses from uncertainty are 0.07 (= 0.18*0.44) basis points in the permanent, baseline version of soft Brexit, compared to 0.09 (= 2.30*0.04) basis points in the temporary version of soft Brexit. For hard Brexit, these figures are 0.28 (=0.24*1.18) basis points and 0.08 (=0.71*0.11) basis points, respectively.

The results of this analysis indicate that additional uncertainty about the permanence of Brexit has little impact on the cost of Brexit uncertainty. If Brexit is reversed, the overall welfare losses are small because the long-run losses are negligible. And while the cost of uncertainty is large in relation to these smaller overall losses, the cost of uncertainty in this version of the analysis measured in units of consumption is similar to the cost of uncertainty in the baseline analysis in wihch Brexit is guaranteed to be permanent.

6.4 Additional trade policy uncertainty: firm-level uncertainty

All firms face the same increase in trade costs conditional on the Brexit scenario in the baseline analysis. It is possible, however, that some firms may face significantly larger increases in trade costs than other firms when Brexit occurs. Here, I explore the impact of firm-level uncertainty about post-Brexit trade costs as well as aggregate uncertainty.

To glean information about the extent to which firms could face heterogeneous increases in trade costs when Brexit occurs, I look to the disaggregated E.U. MFN tariff schedule for 6-digit HS industries. For many industries at this level of disaggregation, the E.U. levies no import tariffs at all; the industry at the 25th percentile of the distribution has a tariff of zero. Other industries are taxed signficantly; the average tariff for industries in the top quartile of the the distribution is about 2.5 times the overall average tariff. Further, the tariff distribution is skewed rightward; the average tariff for industries in the middle two quartiles is a little less than three-quarters of the overall average. Certainly, the correspondence between firms and 6-digit HS industries is not one-to-one, but these data indicate that firms are, in fact, likely to face idiosyncratic uncertainty about post-Brexit tariffs as well as aggregate uncertainty.

To capture this idea, in this exercise I assume that when Brexit occurs, firms engaged in trade between the United Kingdom and the European Union draw idiosyncratic trade cost "multiples" that scale up or down the aggregate tariffs, $\tau_{d,i}(Z^t)$, and iceberg costs, $\xi_{d,i}(Z^t)$. The trade cost multiples scale up both forms of trade costs in order to give idiosyncratic trade policy uncertainty the best chance to have a significant macroeconomic impact. Firms learn their multiples at the same time that they learn whether Brexit is soft or hard. Once drawn, these multiples are permanent over the firm's life.[22]

Informed by the data above, I assume the following three-point distribution for trade cost multiples, which are drawn independently from firms' productivities. One quarter of firms draw a multiple of zero; these lucky firms continue to enjoy free trade even after Brexit occurs. One half of firms draw a multiple of 0.75; these firms pay lower trade costs than the overall average. The remaining quarter of firms draw a multiple of 2.5; these unlucky firms face trade cost increases as high as 25 percent (=2.5*6.53 + 2.5*3.58) if hard Brexit occurs. The average trade cost multiple is one, so that the average increase in trade costs is the same as in the baseline analysis.

The overall welfare cost of Brexit for U.K. households is almost exactly the same in this version of the analysis as in the baseline, and the cost of Brexit uncertainty is actually slightly lower. Thus additional uncertainty about firm-level trade costs does not significantly affect the aggregate welfare cost of Brexit uncertainty.

7 Alternative models of export participation dynamics

The model that I have used to evaluate the macroeconomic impact of Brexit uncertainty incorporates a model of exporter behavior that features firm-level dynamics at both the extensive and intensive margins of export participation. My theory of market penetration dynamics nests four special cases in which one or more of these features are absent. Each of these special cases corresponds to a model of exporter behavior that has been studied elsewhere in the literature. In this section, I repeat my quantitative exercise in these special cases to determine whether these features play an important role in determining the welfare cost—or lack thereof—of Brexit uncertainty. I also analyze a fifth alternative model of export participation dynamics with endogenous exit and exporter hysteresis, which my theory does not generate, to determine whether these features could be important.

In each alternative model of exporter behavior described below, I recalibrate all model parameters to match the facts described in section 4.1.4.[23]. Panel (a) of table 5 lists the overall welfare losses from Brexit and the welfare losses from Brexit uncertainty in each of these alternatives. All of the results are similar to the results of the baseline quantitative exercise. This indicates that export participation dynamics at both the extensive and intensive margins, endogenous exit, and exporter hysteresis all play little role in determining the macroeconomic impact of Brexit uncertainty.

[22]When a firm dies post-Brexit in this version of the analysis, a new firm is born with the same trade cost multiple as well as the same productivity.

[23]Strictly speaking, I recalibrate all parameters except for $\omega_{d,i}$, the customer base depreciation rates, which are irrelevant in all of the alternative models. This parameter maps approximately one-to-one with the growth rate of new exporters relative to incumbents, so I drop this moment from the calibration procedure.

7.1 Dynamic sunk cost model

In the first alternative model, I shut down the intensive margin of export participation by setting λ, the parameter that governs diminishing returns in marketing, to zero. In this setup, the marginal cost of reaching additional customers is constant, so any firm for which exporting to a particular destination is profitable serves all customers in that destination. Thus, we can reinterpret the marketing efficiency parameter, $\psi_{d,i}$, as a one-time, sunk cost of entering the export market. This version of the model is, in fact, isomorphic to a simple version of the sunk-cost model of Das et al. (2007) that has been studied extensively in the trade dynamics literature.[24] The problem of a firm in this setting can be expressed as

$$V_{d,i}(Z^t,a,1) = \pi_{d,i}(Z^t,a,1) + Q_i(Z^t)\phi \sum_{Z^{t+1}} \Pi(Z^{t+1}|Z^t)\left[\chi V_{d,i}(Z^{t+1},a,1) + (1-\chi)V_{d,i}(Z^{t+1},a,0)\right], \quad (34)$$

$$V_{d,i}(Z^t,a,0) = \max\left\{V_{d,i}(Z^t,a,1) - W_i(Z^t)\psi_{d,i},\ Q_i(Z^t)\phi \sum_{Z^{t+1}} \Pi(Z^{t+1}|Z^t)V_{d,i}(Z^{t+1},a,0)\right\}, \quad (35)$$

where $V_{d,i}(Z^t,a,1)$ and $V_{d,i}(Z^t,a,0)$ are the values of being an exporter (having a customer base of one) and a non-exporter (a customer base of zero), respectively.

In this version of the model, the overall cost of Brexit is about the same as in the baseline model. The cost of Brexit uncertainty is larger for soft Brexit and smaller for hard Brexit. The differences are small, however, indicating that dynamics of the intensive margin of export participation are not quantitatively important.

7.2 Static market penetration model

In the second alternative model, I shut down firm-level dynamics entirely by setting the survival rate, ϕ, to zero. This alternative features both extensive and intensive margins of export participation but does not feature forward-looking behavior. The firm's problem in this alternative is identical to the static market penetration problem studied by Arkolakis (2010):

$$V_{d,i}(Z^t,a) = \max_{n\in[0,1]} \left\{\pi_{d,i}(Z^t,a,n) - W_i(Z^t)\kappa_{d,i}(n,0)\right\}. \quad (36)$$

The overall cost of Brexit in this version of the model is also the same as in the baseline model. The welfare cost of Brexit uncertainty is lower in both Brexit outcomes, indicating that forward-looking export participation decisions do affec this cost as suggested by the trade policy uncertainty literature. As with the first alternative model, though, the differences are slight, suggesting that these dynamics are not quantitatively important.

[24] Handley and Limão (2013) use exactly this setup to derive analytical results about the effects of trade policy uncertainty on export participation.

7.3 Static fixed cost model

In the third alternative model, I shut down all firm-level dynamics and the intensive margin of export participation by setting both λ and ϕ to zero. As Arkolakis (2010) shows, this alternative is isomorphic to the widely-studied model of Melitz (2003), in which firms must pay an entry cost each period in order to export. As in the dynamic sunk cost model, in this setting we can interpret the marketing efficiency parameter, $\psi_{d,i}$, as a fixed exporting cost. The firm's problem in this setting is

$$V_{d,i}(Z^t, a) = \max \left\{ \pi_{d,i}(Z^t, a, 1) - W_i(Z^t)\psi_{d,i},\ 0 \right\}. \tag{37}$$

The overall cost of Brexit and the cost of Brexit uncertainty are both about the same in this version of the model as in the static market penetration model. This indicates that modeling the intensive margin of export participation, in either a static or dynamic setup, is not important for measuring the welfare cost of Brexit uncertainty.

7.4 No export costs model

In the fourth alternative model, I shut down both margins of export participation entirely by setting $\psi_{d,i} = 0$. This alternative is isomorphic to a standard Armington model of aggregate trade. All firms export and serve all customers in each foreign market.

Here, the overall cost of Brexit is slightly lower than in the baseline model but the cost of Brexit uncertainty is similar. These results suggest that modeling export participation may be important for accurately measuring the overall impact of Brexit, but confirm that it is not important for measuring the cost of Brexit uncertainty.

7.5 Alessandria-Choi model

In the fifth alternative model, firms pay fixed costs to export that depend on their export status at the beginning of the period. A new exporter must pay a large fixed cost, $\psi_{d,i,0}$, while a continuing exporter pays a small cost, $\psi_{d,i,1}$. Additionally, productivities are independently distributed across time as well as across firms. These features generates endogenous exit and exporter hysteresis. Firms that receive sufficiently bad productivity shocks exit export markets entirely. This model of exporter dynamics, which is not a special case of my theory of export participation dynamics, has been featured in several recent studies of the macroeconomic consequences of trade dynamics (Alessandria and Choi, 2007, 2016; Alessandria et al., 2016) as well as an earlier version of this paper. The firm's problem in this environment is

$$V_{d,i}(Z^t, a, n_-) = \max_{n \in \{0,1\}} \left\{ \pi_{d,i}(Z^t, a, n) - nW_i(Z^t)\psi_{d,i,n_-} + Q_i(Z^t)\phi\tilde{V}_{d,i}(Z^{t+1}, a, n) \right\}, \tag{38}$$

where

$$\tilde{V}_{d,i}(Z^{t+1}, a, n) = \sum_{Z^{t+1}} \Pi(Z^{t+1}|Z^t) \int_{a'} \left[\chi V_{d,i}(Z^{t+1}, a', n) + (1-\chi) V_{d,i}(Z^{t+1}, a', 0)\right] \, \mathrm{d}F_i(a'). \tag{39}$$

In this version of the model, the overall welfare cost of Brexit is between 6 and 9 percent larger than in the baseline version, depending in which Brexit scenario occurs. The cost of Brexit uncertainty is about the same as in the dynamic sunk cost model, indicating that endogenous exit and exporter hysteresis do not play important roles in determining the impact of Brexit uncertainty.

8 Other sensitivity analyses

In addition to the exercises in sections 7–6, I have conducted a wide range of sensitivity analyses in order to explore the importance of other modeling assumptions and assigned parameter values for my results. Panel (c) of table 5 lists the welfare results in these analyses.

8.1 Multiple sectors

In the baseline model there is a single output sector. The literature indicates, however, that the welfare consequences of changes in trade policy may be sensitive to the level of aggregation; multi-sector models often predict larger welfare effects than their single-sector equivalents (Costinot and Rodríguez-Clare, 2014). In the case of Brexit, in particular, modeling trade in services and intermediate inputs could potentially be important because services and intermediates account for significant fractions of U.K.-E.U. trade.

My market penetration dynamics framework is not tractable in a multi-sector setting, so in order to determine whether the overall cost of Brexit or the cost of Brexit uncertainty are sensitive to the level of aggregation I study a multi-sector version of the no export costs model from section 7.4. The multi-sector model features two output sectors—goods and services—and different aggregation technologies for consumption, investment, and intermediate inputs. Buillding on the work of Kehoe et al. (2018) and Eaton et al. (2011b), the model features a rich input-output structure which distinguishes trade in intermediate inputs from trade in final purchases. I calibrate this structure to a two-sector version of the input-output matrix in table 3 constructed from the same WIOD source. I also study a version of the multi-sector model with convex trade adjustment frictions as in Krugman (1986) and Engel and Wang (2011), which exhibits similar gradual trade adjustment dynamics as micro-founded models like Alessandria and Choi (2016), Alessandria et al. (2015), and the baseline model in this paper. The online appendix contains additional details about the multi-sector model, its calibration, and its results.

The overall welfare losses from Brexit are higher in both versions of the multi-sector model as compared to the one-sector baseline. In both versions, the overall welfare losses are 5 percent and 15 percent higher for soft and hard Brexit, respectively. The welfare cost of uncertainty about Brexit is slightly higher in the frictionless multi-sector model and an order of magnitude higher in the version with frictions. Even in the latter version, though, the cost of Brexit uncertainty is at most 2 percent of the overall welfare cost of Brexit.

8.2 Financial autarky

Access to international financial markets allows U.K. households to smooth consumption over time in the baseline model. Much of the international trade literature, by contrast, assumes balanced trade. I have studied an alternative version of my model with financial autarky to determine whether the ability to run trade imbalances is a significant factor in determining the welfare cost of Brexit. The results indicate that it is not: both the overall welfare cost of Brexit and the welfare cost of Brexit uncertainty in the financial autarky model are similar to the baseline results, although the cost of Brexit uncertainty is higher in the financial-autarky version of hard Brexit than in the baseline. In the baseline model, households engage in precautionary saving in the pre-Brexit period to insure against hard Brexit; preventing them from purchasing this insurance makes hard Brexit more painful in the financial autarky model.

8.3 Elasticities

The trade elasticity is a key parameter in calculations of the welfare effects of changes in trade policy because it governs the degree to which households can substitute foreign goods for domestic ones (Arkolakis et al., 2012). The baseline calibration targets a long-run trade elasticity of five, which is common in the international trade literature. The open-economy macro literature, which targets the volatilty of net exports and/or the real exchange rate, typically finds a lower elasticity; Heathcote and Perri (2002), for example, estimate an elasticity of 0.9. In addition to my baseline calibration, I have also analyzed an alternative calibration in which I target a long-run trade elasticity of one. The overall welfare cost of Brexit is almost exactly the same in this calibration as in the baseline, as the decline in substitutability between domestic and foreign goods is offset by a smaller drop in trade flows. The welfare cost of Brexit uncertainty is ten times smaller in this calibration than in the baseline.

Additionally, the baseline calibration sets γ, the coefficient of relative risk aversion, to the standard value of two. This parameter also governs the elasticity of intertemporal substitution. I have analyzed another alternative calibration in which I set γ to five to verify that increasing risk aversion does not substantially affect the results, particularly the cost of uncertainty about Brexit. The overall welfare cost of Brexit in this calibration is the same as in the baseline. The welfare cost of Brexit uncertainty is higher, but is still less than a quarter of one percent of the overall welfare cost.

8.4 Exit rates

The rates at which firms die and exit from the export market affect the degree to which they discount the future in making their export participation and market penetration decisions. In the baseline calibration, firms have a 15 percent chance of dying each period as in Melitz and Costantini (2007), and, conditional on surviving, they have a 30 percent chance of losing access to each of their export markets. Other studies of exporter dynamics, such as Alessandria and Choi (2016) and Alessandria et al. (2016), assume much lower exporter exit rates. I have also analyzed the impact of Brexit under an alternative calibration of my model in which the death rate is zero, so that firms discount the future at the same rate as households (2 percent per year) and have a lower chance of losing access to export markets. The overall welfare cost of Brexit in this calibration is the same as in the baseline calibration, and t5he welfare cost of Brexit uncertainty is marginally lower. This indicates that the rate at which firms discount the future in making their export participation decisions does not have a material impact on the results.

9 Conclusion

In this paper, I have used a model of the United Kingdom and its trade partners to assess the impact of the United Kingdom's impending departure from the European Union—and the impact of uncertainty about what form this departure will take—on trade flows, welfare, and macroeconomic dynamics. The model features two possible outcomes for Brexit: soft, in which the costs of trade with the European Union rise slightly; and hard, in which those costs rise substantially. Forward-looking model agents learn the details of these scenarios in 2015, when the Brexit referendum is announced, but do not know which scenario will occur until Brexit takes place in 2019.

The model predicts that Brexit will have a substantial impact on the U.K. economy, particularly in the long run. Compared to a counterfactual steady state in which Brexit never occurs, trade flows with the European Union will fall by 8–45 percent, consumption will fall by 0.5–1.3 percent, and the present value of U.K. households' welfare losses will amount to £7,000–£19,000 per person. The model also predicts, though, that uncertainty about Brexit will have little macroeconomic impact: perfect-foresight equilibria in which model agents learn immediately which Brexit scenario they will face are virtually identical to the baseline stochastic equilibrium. Consequently, the welfare cost of uncertainty about Brexit is tiny: U.K. households would pay no more than a few dozen pounds per person to avoid this uncertainty. I have shown that my findings hold under a wide range of other models of export participation dynamics and alternative Brexit scenarios.

It is important to point out that this study is limited to an analysis of the increase in trade costs that will occur when the United Kingdom leaves the European Union. Brexit will likely cause other policies to change as well, particularly policies concerning immigration and fiscal benefit transfers. The United Kingdom is likely to benefit from cessation of fiscal benefits because it currently pays more into the benefit pool than it receives. Dhingra et al. (2016b,c) show, though, that the welfare gains from leaving the fiscal benefit system are likely to be small compared to the welfare losses associated with rising trade costs. Further work is needed to assess the impact of policies that restrict migration to and from the United Kingdom, which are likely to have differential effects across population segments. More broadly, the outcome of the Brexit referendum has caused substantial political turmoil which may affect a range of other U.K. economic policies in the future.

When measured against the overall welfare cost of Brexit, I have found that the welfare cost of Brexit uncertainty is likely to be small. In a different context, however, the cost of Brexit uncertainty looks sizeable. My estimates of the cost of Brexit uncertainty have the same order of magnitude as estimates of the welfare cost of business cycles (Lucas, 2003; Imrohoroglu, 2008). In other words, the cost of uncertainty surrounding a one-time Brexit is about the same for a representative household as the cost of unpredictable fluctuations in macroeconomic activity that occur year after year.

References

Alessandria, George and Horag Choi, "Do Sunk Costs of Exporting Matter for Net Export Dynamics?," *The Quarterly Journal of Economics*, 2007, *122* (1), 289–336.

_ and _ , "The Dynamics of the U.S. Trade Balance and Real Exchange Rate: The J Curve and Trade Costs?," Working Paper February 2016.

_ , _ , and Dan Lu, "Trade Integration and the Trade Balance in China," Working Paper July 2016.

_ , _ , and Kim J. Ruhl, "Trade Adjustment and the Welfare Gains from Trade," Working paper 2015.

_ , Sangeeta Pratap, and Vivian Yue, "Export Dynamics in Large Devaluations," Working Papers 062013, Hong Kong Institute for Monetary Research May 2013.

Allen, Treb, "Information Frictions in Trade," *Econometrica*, November 2014, *82*, 2041–2083.

Altomonte, Carlo and Tommaso Aquilante, "The EU-EFIGE/Bruegel-Unicredit Dataset," Working Paper 753, Bruegel 2012.

Anderson, James E. and Eric van Wincoop, "Trade Costs," *Journal of Economic Literature*, September 2004, *42* (3), 691–751.

Arkolakis, Costas, "Market Penetration Costs and the New Consumers Margin in International Trade," *Journal of Political Economy*, 2010, *118* (6), 1151–1199.

_ , Arnaud Costinot, and Andrés Rodríguez-Clare, "New Trade Models, Same Old Gains?," *American Economic Review*, 2012, *102* (1), 94–130.

Atalay, Enghin, "How Important Are Sectoral Shocks?," Working Paper 2014.

Bacharach, Michael, "Estimating Nonnegative Matrices from Marginal Data," *International Economic Review*, September 1965, *6* (3), pp. 294–310.

Bajona, Claustre and Timothy Kehoe, "Trade, Growth, and Convergence in a Dynamic Heckscher-Ohlin Model," *Review of Economic Dynamics*, July 2010, *13* (3), 487–513.

Baker, Jessica, Oriol Carreras, Simon Kirby, Jack Meaning, and Rebecca Piggott, "Modelling events: The short-term economic impact of leaving the EU," *Economic Modelling*, 2016, *58* (C), 339–350.

Baldwin, Richard E, "Measurable Dynamic Gains from Trade," *Journal of Political Economy*, February 1992, *100* (1), 162–74.

Barattieri, Alessandro, "Comparative advantage, service trade, and global imbalances," *Journal of International Economics*, 2014, *92* (1), 1 – 13.

Bems, Rudolfs, "Aggregate Investment Expenditures on Tradable and Nontradable Goods," *Review of Economic Dynamics*, 2008, *11* (4), 852–883.

Bernard, Andrew B., J. Bradford Jensen, Stephen J. Redding, and Peter K. Schott, "The Empirics of Firm Heterogeneity and International Trade," *Annual Review of Economics*, 07 2012, *4* (1), 283–313.

Brooks, Wyatt J. and Pau S. Pujolas, "Capital Accumulation and the Welfare Gains from Trade," Working Paper 2016.

Caliendo, Lorenzo and Fernando Parro, "Estimates of the Trade and Welfare Effects of NAFTA," *Review of Economic Studies*, 2015, *82* (1), 1–44.

Chaney, Thomas, "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, September 2008, *98* (4), 1707–21.

Costinot, Arnaud and Andrés Rodríguez-Clare, "Chapter 4 - Trade Theory with Numbers: Quantifying the Consequences of Globalization," in Elhanan Helpman Gita Gopinath and Kenneth Rogoff, eds., *Handbook of International Economics*, Vol. 4 of Handbook of International Economics, Elsevier, 2014, pp. 197 – 261.

Das, Sanghamitra, Mark J. Roberts, and James R. Tybout, "Market Entry Costs, Producer Heterogeneity, and Export Dynamics," *Econometrica*, 2007, *75* (3), 837–873.

Dhingra, Swati, , Gianmarco Ottaviano, Thomas Sampson, and John van Reenen, "The Impact of Brexit on Foreign Investment in the UK," CEP Brexit Analysis 3, Centre for Economic Performance 2016.

_ , Gianmarco Ottaviano, Thomas Sampson, and John Van Reenen, "The Consequences of Brexit for UK Trade and Living Standards," CEP Brexit Analysis 2, Centre for Economic Performance 2016.

_ , Hanwei Huang, Gianmarco Ottaviano, João Paulo Pessoa, Thomas Sampson, and John Van Reenen, "The Costs and Benefits of Leaving the EU: Trade Effects," CEP Brexit Analysis Technical Paper, Centre for Economic Performance May 2016.

di Giovanni, Julian, Andrei A. Levchenko, and Jing Zhang, "The Global Welfare Impact of China: Trade Integration and Technological Change," *American Economic Journal: Macroeconomics*, 2014, *6* (3), 153–83.

Dix-Carneiro, Rafael, "Trade Liberalization and Labor Market Dynamics," *Econometrica*, 2014, *82* (3), 825–885.

Drozd, Lukasz A. and Jaromir B. Nosal, "Understanding International Prices: Customers as Capital," *American Economic Review*, February 2012, *102* (1), 364–95.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography, and Trade," *Econometrica*, 2002, *70* (5), 1741–1780.

_ , _ , and Francis Kramarz, "An Anatomy of International Trade: Evidence From French Firms," *Econometrica*, September 2011, *79* (5), 1453–1498.

_ , _ , Brent Neiman, and John Romalis, "Trade and the Global Recession," NBER Working Paper 16666,, National Bureau of Economic Research, Inc January 2011.

Ebell, Monique, Ian Hurst, and James Warren, "Modelling the long-run economic impact of leaving the European Union," *Economic Modelling*, 2016, *59*, 196 – 209.

Engel, Charles and Jian Wang, "International trade in durable goods: Understanding volatility, cyclicality, and elasticities," *Journal of International Economics*, January 2011, *83* (1), 37–52.

Fernandes, Ana M., Caroline Freund, and Martha Denisse Pierola, "Exporter Behavior, Country Size, and Stage of Development: Evidence from the Exporter Dynamics Database," *Journal of Development Economics*, 2016, *119*, 121 – 137.

Fitzgerald, Doireann and Anthony Priolo, "How Do Firms Build Market Share?," Working Paper 24794, National Bureau of Economic Research 2018.

_ , Stefanie Haller, and Yaniv Yedid-Levi, "How Exporters Grow," NBER Working Papers 21935, National Bureau of Economic Research, Inc January 2016.

Francois, Joseph, Koen Berden, Saara Tamminen, Martin Thelle, and Paul Wymenga, "Non-Tariff Measures in EU-US Trade and Investment: An Economic Analysis," IIDE Discussion Papers 20090806, Institue for International and Development Economics February 2013.

Handley, Kyle, "Exporting under trade policy uncertainty: Theory and evidence," *Journal of International Economics*, 2014, *94* (1), 50–66.

_ and Nuno Limão, "Policy Uncertainty, Trade and Welfare: Theory and Evidence for China and the U.S," NBER Working Papers 19376, National Bureau of Economic Research, Inc August 2013.

_ and _ , "Trade and Investment under Policy Uncertainty: Theory and Firm Evidence," *American Economic Journal: Economic Policy*, November 2015, *7* (4), 189–222.

Heathcote, Jonathan and Fabrizio Perri, "Financial autarky and international business cycles," *Journal of Monetary Economics*, April 2002, *49* (3), 601–627.

Imrohoroglu, Ayse, "Welfare costs of business cycles," *The New Palgrave Dictionary of Economics*,, 2008.

Kehoe, Timothy J. and Kim J. Ruhl, "How Important Is the New Goods Margin in International Trade?," *Journal of Political Economy*, 2013, *121* (2), 358–392.

_ , Jack Rossbach, and Kim J. Ruhl, "Using the new products margin to predict the industry-level impact of trade reform," *Journal of International Economics*, 2015, *96* (2), 289–297.

_ , Kim J. Ruhl, and Joseph B. Steinberg, "Global Imbalances and Structural Change in the United States," *Journal of Political Economy*, 2018, *126*, 761–796.

Krugman, Paul, "Pricing to Market when the Exchange Rate Changes," NBER Working Papers 1926, National Bureau of Economic Research, Inc May 1986.

Lim, Kevin, "Firm-to-Firm Trade in Sticky Production Networks," Working paper 2016.

Lucas, Robert, "Macroeconomic Priorities," *American Economic Review*, 2003, *93* (1), 1–14.

Lucas, Robert E. and Edward C. Prescott, "Investment Under Uncertainty," *Econometrica*, 1971, *39* (5), 659–681.

Mayer, Thierry and Gianmarco Ottaviano, "The Happy Few: The Internationalisation of European Firms," *Intereconomics: Review of European Economic Policy*, May 2008, *43* (3), 135–148.

McGrattan, Ellen R. and Andrea L. Waddle, "The Impact of Brexit on Foreign Investment and Production," NBER Working Papers 23217 2017.

Melitz, Marc and J. Costantini, "The Dynamics of Firm-Level Adjustment to Trade Liberalization," in E. Helpman, D. Marin, and T. Verdier, eds., *The Organization of Firms in a Global Economy*, Harvard University Press, 2007.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 2003, *71* (6), 1695–1725.

Obstfeld, Maurice and Kenneth Rogoff, "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," in "NBER Macroeconomics Annual 2000, Volume 15," National Bureau of Economic Research, Inc, 2001, pp. 339–412.

Pain, Nigel and Garry Young, "The macroeconomic impact of UK withdrawal from the EU," *Economic Modelling*, May 2004, *21* (3), 387–408.

Pierce, Justin R. and Peter K. Schott, "The Surprisingly Swift Decline of US Manufacturing Employment," *American Economic Review*, July 2016, *106* (7), 1632–62.

Piguillem, Facundo and Loris Rubini, "Barriers to Firm Growth in Open Economies," Working paper, EIEF and University of New Hampshire 2013.

Ramanarayanan, Ananth, "Imported Inputs, Irreversibility, and International Trade Dynamics," Working paper 2016.

Ruhl, Kim J., "The International Elasticity Puzzle," Working paper 2008.

_ **and Jonathan L. Willis**, "New Exporter Dynamics," *International Economic Review*, August 2017, *58*, 703–726.

Steinberg, Joseph, "International Portfolio Diversification and the Structure of Global Production," *Review of Economic Dynamics*, 2018, *29*, 195–219.

Steinberg, Joseph B., "On the Source of U.S. Trade Deficits: Global Saving Glut or Domestic Saving Drought?," Working Paper, 2016.

_ , "Export Market Penetration Dynamics," Working paper 2018.

Timmer, Marcel P., Erik Dietzenbacher, Bart Los, Robert Stehrer, and Gaaitzen J. de Vries, "An Illustrated User Guide to the World Input-Output Database: the Case of Global Automotive Production," *Review of International Economics*, 2015, *23* (3), 575–605.

Table 1: Recent U.K. macroeconomic and trade dynamics

Variable	2012Q1–2015Q2	2015Q3–2016Q2	2016Q3–2018Q2
(a) National income accounts			
Real GDP growth (pct. per year)	2.34	1.63	1.59
Consumption growth (pct. per year)	1.67	2.53	1.36
Investment (pct. GDP)	16.23	16.73	17.11
Net exports (pct. GDP)	-1.58	-1.32	-1.39
(b) International trade			
Exports (pct. GDP)	29.03	27.27	29.79
Imports (pct. GDP)	30.61	28.59	31.17
Goods exports to E.U. (pct. GDP)	6.80	5.32	7.22
Goods imports from E.U. (pct. GDP)	10.40	9.27	11.92

Table 2: Calibrated parameter values

Parameter	Meaning	Value	Source or target
(a) Assigned parameters			
β	Discount factor	0.98	Long-run interest rate = 2%
γ	Risk aversion	2.00	Standard
δ	Depreciation rate	0.06	Standard
α	Capital share	0.33	Standard
θ	Elast. of subst. across varieties	5.00	Alessandria et al. (2016)
φ	Capital adjustment cost	0.76	Steinberg (2016)
λ	Marketing cost convexity	1	N/A
$1-\phi$	Death rate	0.15	Melitz and Costantini (2007)
(b) Calibrated parameters			
η_i	Value-added shares	$(0.46, 0.42, 0.40)$	Input-output data
$\bar{L}_i$	Labor endowments	$(66.7, 194, 761)$	Input-output data
$\mu_{uk,j}$	U.K. Armington shares	$(0.84, 0.07, 0.09)$	Input-output data
$\mu_{eu,j}$	E.U. Armington shares	$(0.01, 0.89, 0.10)$	Input-output data
$\mu_{rw,j}$	R.W. Armington shares	$(0.003, 0.024, 0.97)$	Input-output data
σ_i	Productivity distributions	$(0.44, 0.46, 0.49)$	Exporter facts
$\psi_{d,uk}$	U.K. marketing efficiencies	$(0.82, 0.41)$	Exporter facts
$\psi_{d,eu}$	E.U. marketing efficiencies	$(0.02, 0.05)$	Exporter facts
$\psi_{d,rw}$	R.W. marketing efficiencies	$(0.01, 0.01)$	Exporter facts
$\omega_{d,uk}$	U.K. customer base depreciation	$(0.78, 0.78)$	Exporter facts
$\omega_{d,eu}$	E.U. customer base depreciation	$(0.78, 0.78)$	Exporter facts
$\omega_{d,rw}$	R.W. customer base depreciation	$(0.79, 0.79)$	Exporter facts
$1-\chi$	Destination-specific exit rate	0.3	Exporter facts
ζ	Armington elasticity	3.25	Long-run trade elasticity = 5

Table 3: 2011 inter-country input-output matrix (UK GDP = 100)

	Intermediate inputs			Final demand			
	UK	EU	RW	UK	EU	RW	GO
(a) Raw WIOD matrix							
UK	71.4	10.0	10.3	87.8	4.2	5.5	189.2
EU	7.6	500.1	75.2	6.8	530.7	44.5	1,164.9
RW	10.2	72.5	2,346.7	6.0	31.6	2,248.7	4,715.8
VA	100.0	582.3	2,283.6	-	-	-	2,965.9
GO	189.2	1,164.9	4,715.8	100.6	566.6	2,298.7	
(b) Balanced-trade matrix							
UK	71.4	10.3	10.1	87.4	4.5	5.4	189.2
EU	7.3	495.7	70.7	6.4	543.3	41.5	1,164.9
RW	10.5	76.6	2,351.3	6.1	34.5	2,236.7	4,715.8
VA	100.0	582.3	2,283.6	-	-	-	2,965.9
GO	189.2	1,164.9	4,715.8	100.0	582.3	2,283.6	

Table 4: Brexit scenarios: transition probabilities and trade costs

Parameter	Meaning	Value	Source or target
(a) Soft Brexit trade costs			
$\tau_{uk,eu}$	Tariff on U.K. imports from E.U.	0.00%	Not applicable
$\tau_{eu,uk}$	Tariff on E.U. imports from U.K.	0.00%	Not applicable
$\xi_{uk,eu}$	NTB on U.K. imports from E.U.	2.18%	WIOD + Francois et al. (2013)
$\xi_{eu,uk}$	NTB on E.U. imports from U.K.	1.74%	WIOD + Francois et al. (2013)
(b) Hard Brexit trade costs			
$\tau_{uk,eu}$	Tariff on U.K. imports from E.U.	3.58%	COMTRADE + W.T.O.
$\tau_{eu,uk}$	Tariff on E.U. imports from U.K.	2.12%	COMTRADE + W.T.O.
$\xi_{uk,eu}$	NTB on U.K. imports from E.U.	6.53%	WIOD + Francois et al. (2013)
$\xi_{eu,uk}$	NTB on E.U. imports from U.K.	5.21%	WIOD + Francois et al. (2013)
(c) Transition probabilities			
Π_{vote}	Probability of "stay" vote	0.75	Prediction markets
Π_{brexit}	Probability of soft Brexit	0.5	Not applicable

Table 5: U.K. welfare losses from Brexit

	Total (cons. equiv.)		Uncertainty (pct. total)	
Model	Soft	Hard	Soft	Hard
Baseline	0.44	1.18	0.18	0.24
(a) Alternative models				
Dynamic sunk cost	0.45	1.19	0.47	0.11
Static mkt. pen.	0.44	1.17	0.17	0.12
Static fixed cost	0.44	1.17	0.12	0.09
No export costs	0.40	1.04	0.14	0.08
Alessandria-Choi	0.48	1.24	0.39	0.09
(b) Alternative scenarios				
Lower prob. of hard Brexit	0.44	1.18	0.17	0.24
Higher prob. of hard Brexit	0.44	1.18	0.22	0.24
Increased ad. costs	0.49	1.39	0.25	0.23
Increased ad. costs. and NTBs	0.88	2.07	0.22	0.12
Reversible Brexit (permanent)	0.45	1.18	0.52	0.49
Reversible Brexit (temporary)	0.04	0.11	2.30	0.71
Firm-level trade cost uncertainty	0.45	1.20	0.06	0.10
(c) Sensitivity analyses				
Multi-sector	0.46	1.35	0.37	0.18
Multi-sector w/ frictions	0.46	1.33	2.09	1.28
Financial autarky	0.43	1.13	0.14	0.41
Lower Armington elasticity	0.43	1.19	0.38	0.17
Higher risk aversion	0.44	1.18	0.08	0.35
Lower exit rate	0.44	1.18	0.17	0.23

Figure 1: Recent U.K. macroeconomic and trade dynamics

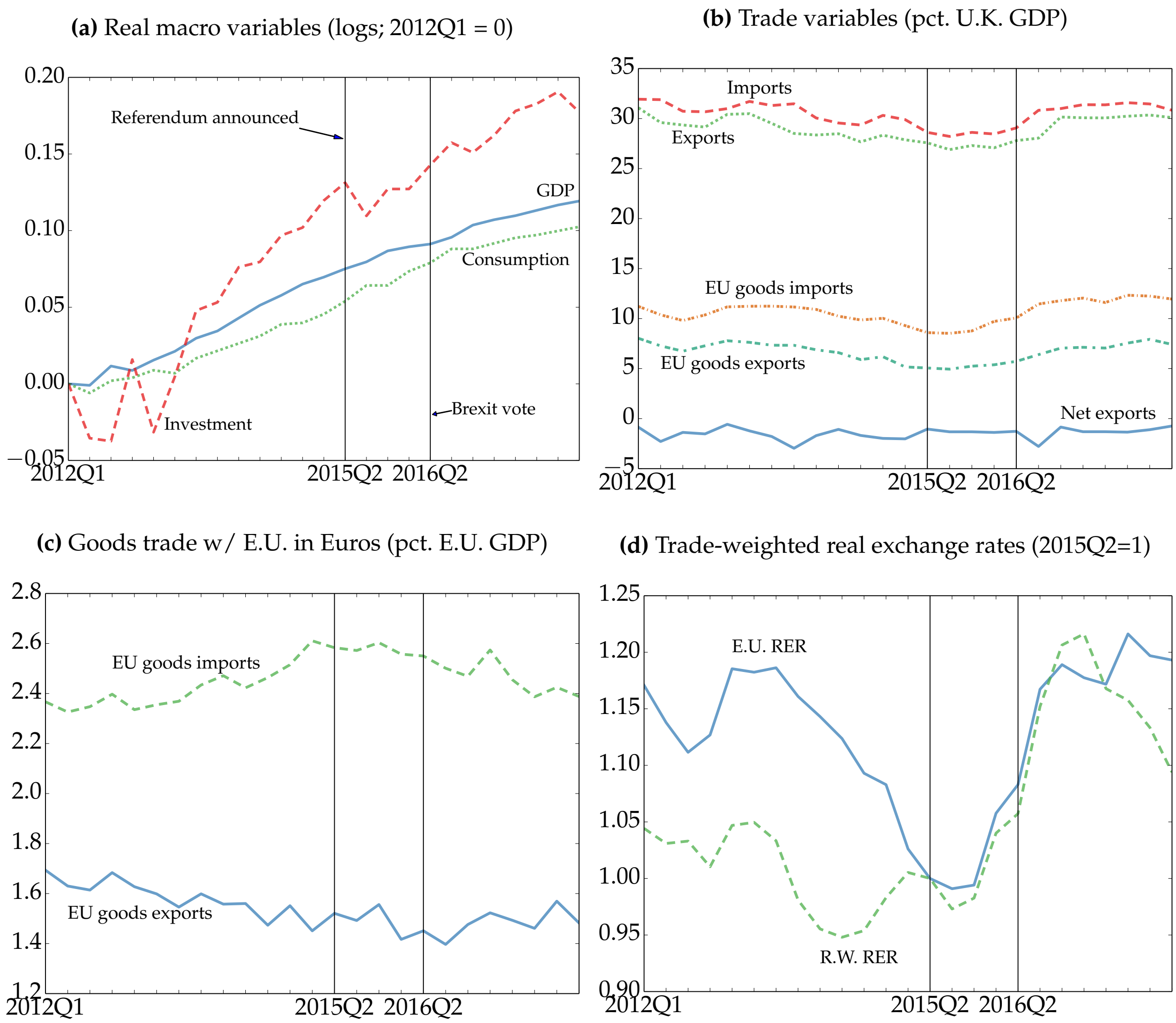


Figure 2: Model timing and uncertainty tree

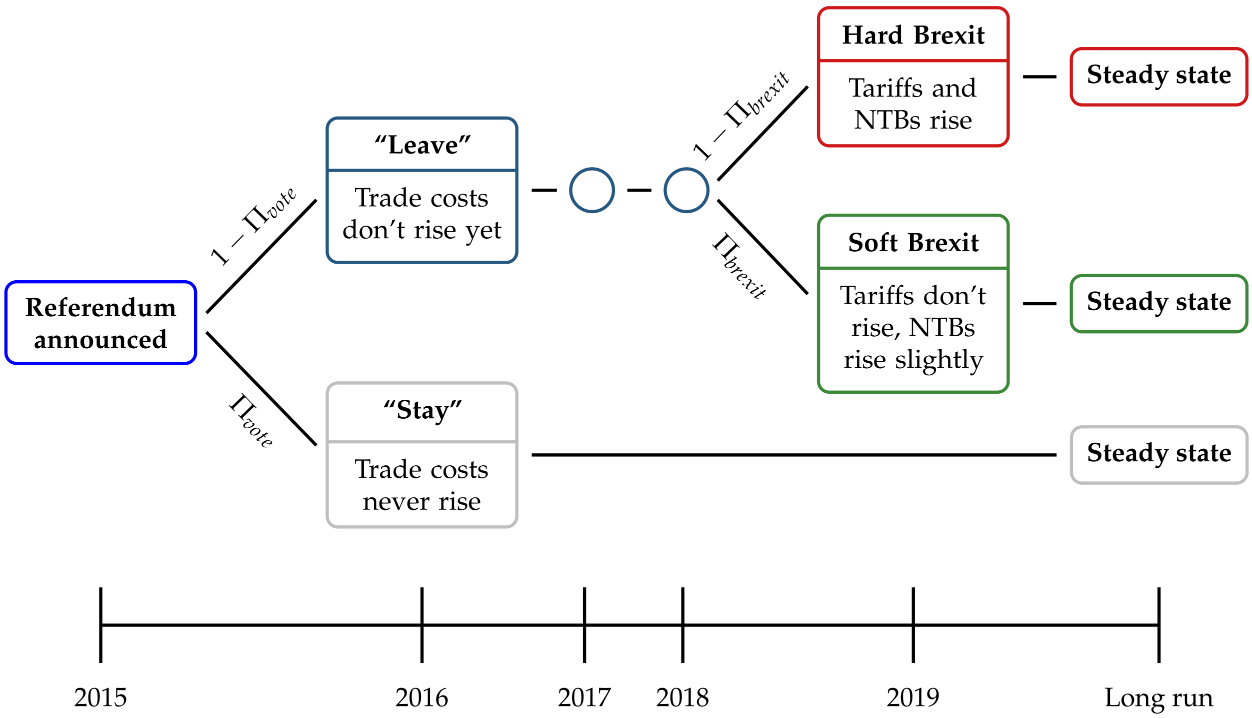


Figure 3: Impact of Brexit on U.K. macro variables

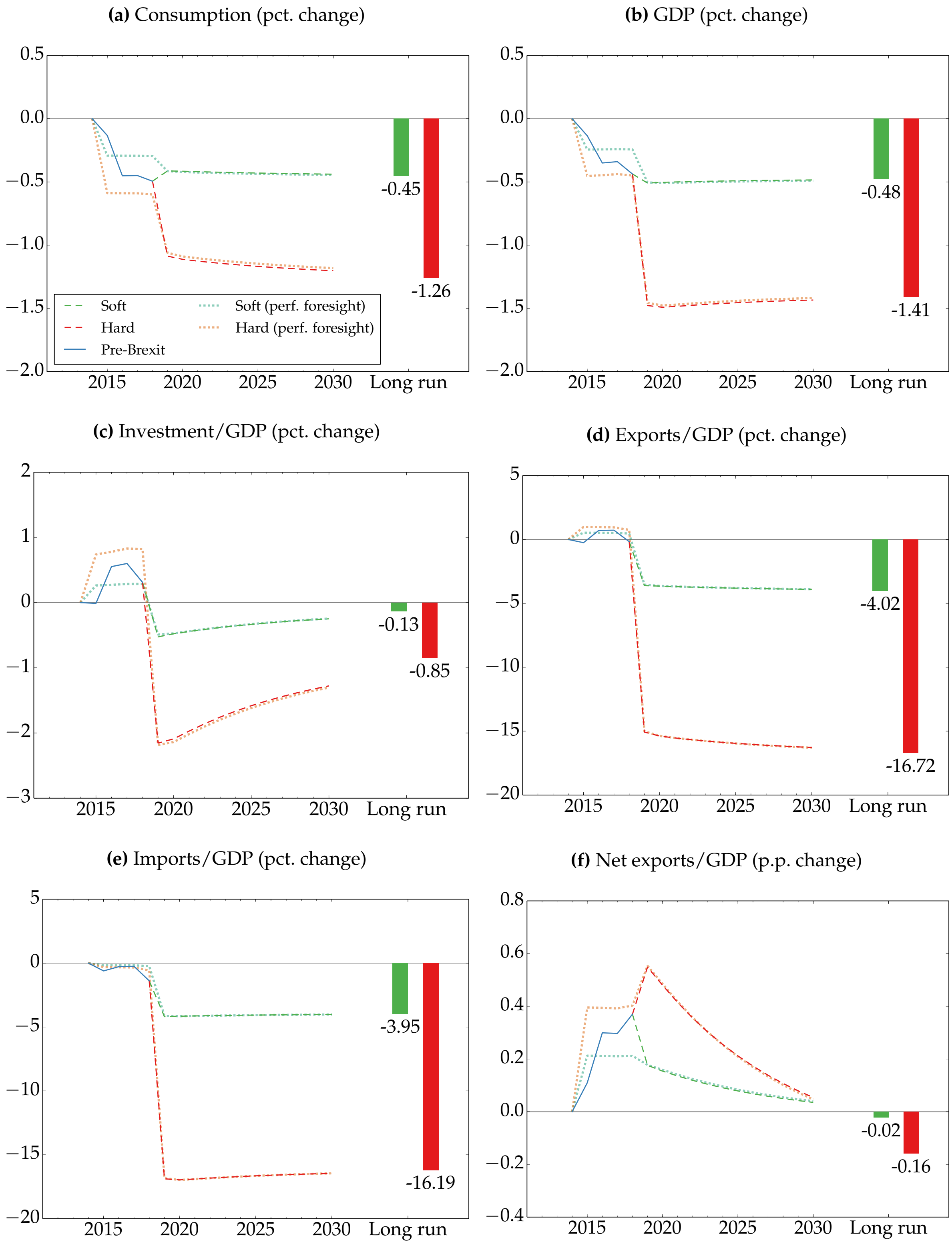
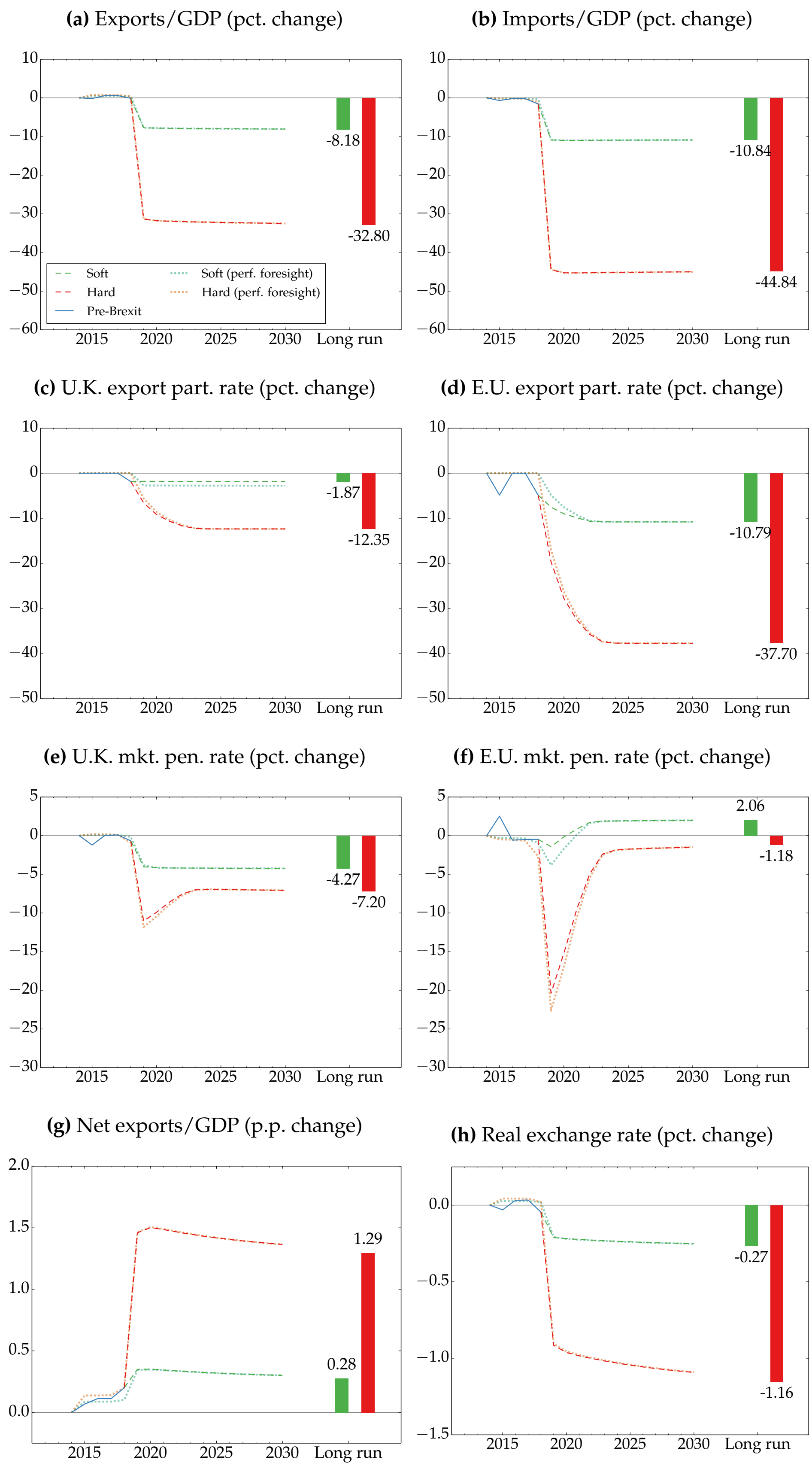


Figure 4: Impact of Brexit on U.K.-E.U. trade



(a) Exports/GDP (pct. change)
(b) Imports/GDP (pct. change)
(c) U.K. export part. rate (pct. change)
(d) E.U. export part. rate (pct. change)
(e) U.K. mkt. pen. rate (pct. change)
(f) E.U. mkt. pen. rate (pct. change)
(g) Net exports/GDP (p.p. change)
(h) Real exchange rate (pct. change)

Figure 5: Impact of Brexit on U.K.-R.W. trade

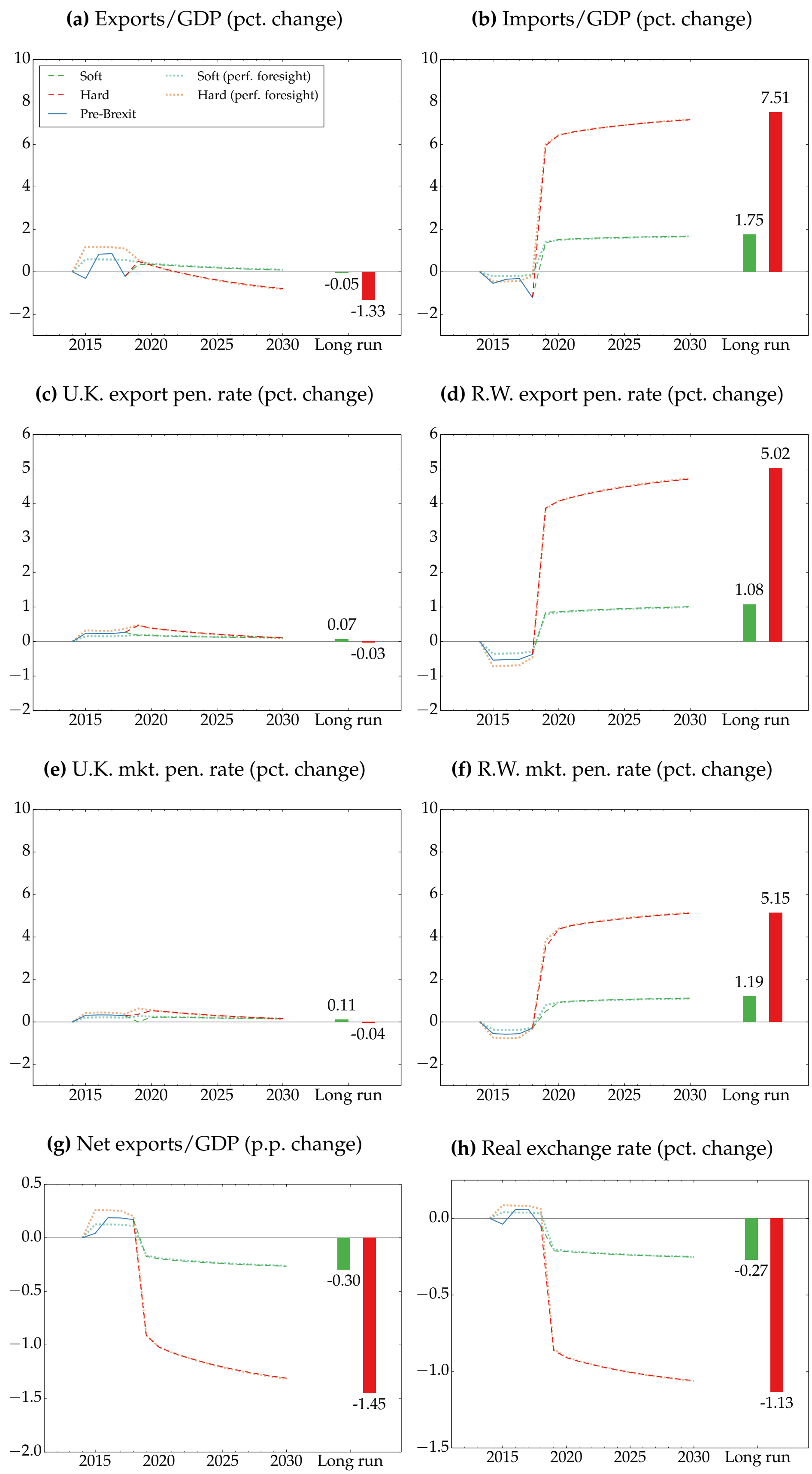
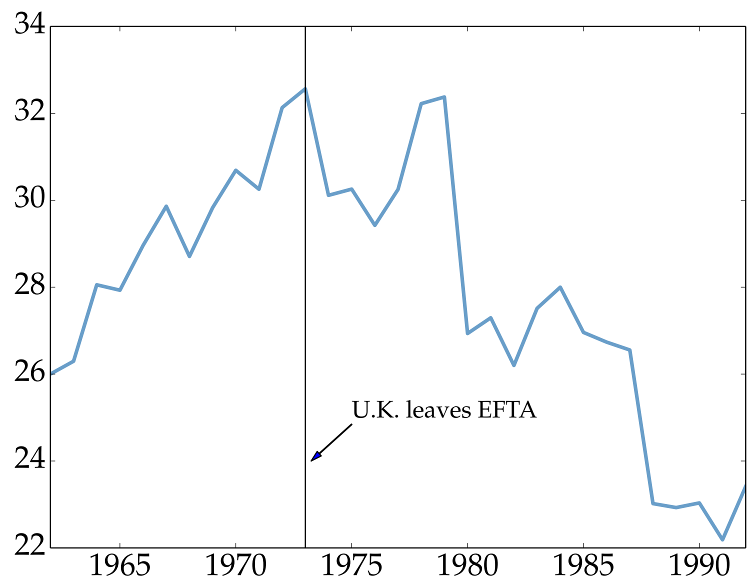


Figure 6: U.K. trade with original EFTA members (pct. total U.K. trade)



Appendix to "Brexit and the Macroeconomic Impact of Trade Policy Uncertainty" (for online publication only)

A Data

This section of the appendix provides additional details on data sources and data processing. All source data are contained in the folder `data` in the online supplement. Python scripts referenced below can be found in the folder `scripts`.

A.1 Recent national accounts and trade data

All of the data reported in table 1 and panels (a)–(c) of figure 1 were extracted from Eurostat. They have been seasonally and calendar adjusted. Filtering methods are well-known to give inaccurate results near the end of time series, so I simply compare the data that cover the period after the referendum act was introduced to Parliament to the data prior to the act's introduction. The first column of the table reports the average quarterly growth rates[25] of key macro variables between 2012Q1 and 2015Q2. I choose 2012Q1 as the starting point for the comparison period to eliminate the effects of recovery from the Great Recession; this is the first quarter in which consumption begins to rise after falling during recession. The second column lists average growth rates between 2015Q3 and 2016Q2, the period between the referendum's introduction to Parliament and the actual vote, and the third column lists the growth rates for 2016Q3–2018Q2, the three quarters for which we have data since the vote. The figure simply plots the raw time series.

The two real exchange rate series shown in panel (d) of figure 1 are computed using CPI and nominal exchange rate series from the IMF's International Financial Statistics Database coupled with the raw WIOD data described in the next subsection. First, I split the non-U.K. countries in the WIOD data into two regions: the European Union and the erest of the world. Second, for each country, I compute a bilateral real exchange rate with the United Kingdom using the CPI and nominal exchange rate data from the IFS. All real bilateral real exchange rates are normalized to one in 2015Q2. Third, for each region I compute the average of the bilateral real exchange rates of the constituent countries, weighted by the U.K.'s total trade flows with that country in the 2011 WIOD data.

All of these steps are performed in the script `recent-data.py`.

A.2 Input-output matrices

Here I describe the construction of the input-output matrices in Table 3. I begin with the 2011 world input-output matrix from the World Input Output Database (Timmer et al., 2015), which contains 40 countries

[25] I report the average values of the investment rate and net exports/GDP instead of growth rates since these variables do not exhibit trend growth.

and 35 industries. I aggregate all countries in the European Union (except for the United Kingdom) into a composite "E.U." country, and the remaining non-U.K. countries into a second "rest of the world" composite. I aggregate all industries into one sector. This yields the matrix in panel (a) of the table. To obtain the balanced matrix in panel (b), I use the RAS method (Bacharach, 1965) to find the most similar matrix in which each country's aggregate trade balance is zero. To apply the method, I add two additional rows for value added, so that each country's value added is in its own row. This allows me to ensure that these value added figures remain unchanged in the procedure. I then impose the restriction in the RAS algorithm that each country's final demand must equal its value added. This implies that net exports must be zero. The balanced matrix is shown in panel (b). Both of these steps are performed in the python script `iomats.py`.

A.3 Exporter facts

In the next two subsections, I describe the processing of the Exporter Dynamics Database and EFIGE data and the calculation of the exporter facts used in the calibration (described in section 4.1.2).

A.3.1 EFIGE Database

The EFIGE dataset (Altomonte and Aquilante, 2012) contains firm-level data for seven European countries on a wide variety of economic performance indicators. Following Piguillem and Rubini (2013), I drop Hungary and Austria due to the small number of observations for these countries, and concentrate on France, Germany, Italy, Spain, and the United Kingdom. I use the following variables:

- `d4`: an indicator of whether a firm is an exporter;
- `d13_1`: the percentage of a firm's exports that go to the 15 core E.U. countries; and
- `d13_2`: the percentage of a firm's exports that go to other E.U. countries.

I compute export participation rates for each country by taking the mean of the variable `d4`. I then report the export participation rate for the United Kingdom and the average export participation rate for non-U.K. countries in the dataset.

For the U.K., I also define a firm as an exporter to the E.U. if `d13_1`+`d13_2` is positive, and define a firm as an exporter to the rest of the world if 1-`d13_1`-`d13_2` is positive. I then compute the bilateral export participation rates conditional on exporting (the analogue of the measure I compute above for non-U.K. countries using the EDD data) as the means of these variables for the subset of U.K. firms that are exporters (firms with `d4`=1).

These steps are performed in the script `efige_expart_facts.py`.

A.3.2 World Bank Exporter Dynamics Database

The World Bank's Exporter Dynamics Database (Fernandes et al., 2016) contains a wide variety of facts about export participation rates, the distribition of exporter sizes, exit rates, and growth rates in 69 countries that are computed from firm-level panel data. These facts are reported for several levels of aggregation across destinations and sectors. I use the following variables from the country-year-destination level data (the `CYD` dataset):

- `A1`: number of firms that export to the given destination;
- `B2ii`: share of exports for by the top 5 percent of exporters;
- `C2`: exit rate of exporters;
- `A11i`: incumbent growth rate; and
- `A12i`: entrant growth rate.

I also use the total number of exporters across all destinations (variable `A1`) and the average number of destinations served by an exporter (`B4i`) from the country-year data (the `CY` dataset).

I split the source countries in the dataset into two regions. There are 7 E.U. countries in the database: Belgium. Bulgaria, Estonia, Norway, Portugal, Spain, and Sweden. I assign the remaining countries into the rest of the world. Similarly, I split destinations into three regions: the U.K., the E.U., and the rest of the world.

To compute bilateral export participation rates (conditional on exporting) I use four steps. First, for each country and year, and destination group (U.K, E.U., or R.W.) I sum the number of exporters for each destination in the group (`A1` in the `CYD` dataset). Second, I divide this sum by the total number of exporters in that country-year (`A1` in the `CY` dataset). Third, for destination regions other than the U.K. (which consists of only one country), I adjust this sum by the average number of destinations served by an exporter (`B4i` from the `CY` dataset) to reflect the fact that many firms export to more than one destination (so that the sum computed in step one could reflect some double counting). These first three steps provide me with an estimate of the propensity of firms in a given source country to export to each region (conditional on being an exporter). Fourth, I compute the averages of these data points for each source region.

To compute the average top 5 share, exit rate, and relative entrant growth rate (the growth rate of entrants minus the growth rate of incumbents), I simply average the relevant variables (`B2ii`, `C2`, `A12i`-`A11i`) in the `CYD` dataset across all source countries, destination countries, and years.

All of these steps are performed in the script `wbedd_expart_facts.py`.

A.4 Tariff and non-tariff barriers

The last two subsections describe the calculation of the post-Brexit trade costs listed in table 4.

A.4.1 Tariffs

To calculate the increases in U.K.-E.U. tariffs for hard Brexit, I use data from two sources:

- W.T.O. data on most-favored-nation tariff rates charged by the European Union on HS6-level products.
- COMTRADE data on HS6-level trade flows between the United Kingdom and the European Union

for 2011.

I combine these two data sources to compute trade-flow-weighted average tariffs for each trade flow direction. The average tariff on U.K. imports from the E.U. is weighted by imports, while the average tariff on E.U. imports from the U.K. is weighted by exports. Then, I multiply these averages by the goods (agriculture, mining, and manufacturing) shares of total imports and exports, since there are no tariffs in services trade. These calculations are shown in the first two rows of Table 4, panel (c). These calculations are perfomed in the script `tariffs.py`.

A.4.2 Non-tariff barriers

The data sources for the non-tariff barrier increases are:

- Francois et al. (2013), who estimate the non-tariff barriers in trade between the European Union and the United States for a subset of ISIC Rev. 2 industries, as well as the fraction of these barriers that are policy-reducible.
- WIOD trade flows between the United Kingdom and the European Union at the ISIC Rev. 2 level (the data from A.2 before aggregating across industries).

First, I calculate the policy-reducible non-tariff barriers in E.U.-U.S.A. trade for each industry by multiplying total barriers by their policy-reducibility fractions. Second, I use a similar approach as in A.3.1 to calculating average barriers in U.K.-E.U. trade, here using the WIOD trade flow data as weights. This is complicated slightly by the fact that the Francois et al. (2013) data map closely, but not exactly, to the ISIC Rev. 2 industries. Table A1 lists the concordance that I use between ISIC and Francois et al. (2013) sectors. In several cases, one ISIC sector maps to multiple Francois et al. (2013) sectors (this is noted in the table with "+" signs). In this case, I calculate the non-tariff barrier for the ISIC sector by taking a simple average of the mapped Francois et al. (2013) sectors. In one case, three ISIC sectors maps to one Francois et al. (2013) sector. In this case, I use the same value for all three ISIC sectors. This calculation is performed in the script `ntb.py`.

B Details on calibration and equilibrium solution method

This section of the appendix provides additional details on the calibration procedure and the numerical method used to solve the model.

B.1 Calibration

Before choosing any parameters, I first add scaling factors, $\bar{Y}_i$ and $\bar{Y}_{i,j}$, to the aggregation technologies which I will calibrate so that all steady-state aggregate prices are one:

$$Y_i(Z^t) = \bar{Y}_i \left[\sum_{j \in I} \left(\mu_{i,j}\right)^{\frac{1}{\zeta}} \left(Y_{i,j}(Z^t)\right)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}$$

$$Y_{i,j}(Z^t) = \bar{Y}_{i,j} \left[\int_{\nu \in N_{i,j}(Z^t)} y_{i,j}(Z^t, \nu)^{\frac{\theta-1}{\theta}} \, \mathrm{d}\nu \right]^{\frac{\theta}{\theta-1}}$$

This is without loss of generality, but it facilitates the calibration procedure because the entries in the input-output matrix represent both real quantities and nominal expenditures (Kehoe et al., 2018).

B.1.1 Assigned parameter values

First, I assign the following parameter values:

- $\beta = 1/1.02$;
- $\delta = 0.06$;
- $\zeta = 3.25$ (this value, which yields a long-run trade elasticity of 5, was determined through manual experimentation);
- $\gamma = 2$;
- $\varphi = 0.8$;
- $\alpha = 1/3$;
- $\theta = 5$;
- $\lambda = 1$;
- $\chi = 0.7$ (this firm parameter, as well as the next one, can be set directly to target the relevant data, there is no need to formally calibrate it);
- and $\phi = 0.85$.

In addition, I set all trade costs to zero so that Armington shares will absorb both subjective home bias and trade costs as discussed in the main text.

B.1.2 Aggregation technology and household parameters

A number of parameters can be set directly from the input-output data. First, I set the value added shares, η_i, as follows:

$$\eta_i = \frac{1 - \left(\frac{\theta M_i^*}{(\theta-1)Y_i^*}\right)}{1 - \left(\frac{\theta M_i^*}{(\theta-1)Y_i^*}\right)\left(1 - \left(\frac{R_i^*}{\alpha}\right)^{\alpha}\left(\frac{W_i^*}{1-\alpha}\right)^{1-\alpha}\right)}$$

As in the main text, stars indicate steady-state equilibrium values, which are taken directly from the input-output matrix in panel (b) of table 3. Second, I calibrate the Armington share parameters, $\mu_{i,j}$, using marginal-product-pricing conditions:

$$\mu_{i,i} = \frac{1}{\sum_{j \in I} \left(Y^*_{i,j} / Y^*_{i,i} \right)};$$
$$\mu_{i,j} = \mu_{i,i} \left(Y^*_{i,j} / Y^*_{i,i} \right).$$

I normalize $\sum_{j \in I} \mu_{i,j} = 1$. Third, I set the top-level scaling factors, $\bar{Y}_i$:

$$\bar{Y}^*_i = \frac{Y^*_i}{\left[\sum_{j \in I} \left(\mu_{i,j} \right)^{\frac{1}{\zeta}} \left(Y^*_{i,j} \right)^{\frac{\zeta - 1}{\zeta}} \right]^{\frac{\zeta}{\zeta - 1}}}.$$

Fourth, I set labor endowments equal to steady state employment: $\bar{L}_i = L^*_i$.

B.1.3 Firm parameters

The remaining parameters are calibrated by solving a system of equations. These parameters are:

- productivity dispersion, σ_i, for $i \in I$;
- scaling factors, $\bar{Y}_{i,j}$, for $i, j \in I \times I$;
- marketing efficiency costs, $\psi_{d,i}$, for $i \in I, d \in D_i$;
- customer base depreciation rates, $\omega_{d,i}$, for $i \in I, d \in D_i$;

I solve for values of these parameters that jointly satisfy the following conditions:

- for each country i and destination d, the export participation rate is matches the bilateral export participation rates computed in section 4.1.2;
- for each country i, the share of exports accounted for by the top 5 percent of exporters is 58.4 percent;
- for each country i and destination d, the size of the average growth rate of a new exporter is 13.2 percent higher than the average growth rate of the average incumbent exporter;
- and for each pair i, j, the price of the bilateral aggregate, $Y^*_{i,j}$, is one.

While these parameter values must be solved jointly, each one loosely maps to one of the conditions. The dispersions, σ_i, control the top 5 share. The marketing costs, $\psi_{d,i}$, control export participation rates. The depreciation rates, $\omega_{d,i}$, control the relative growth rates of new entrants. And the scaling factors, $\bar{Y}_{i,j}$, control bilateral aggregate prices.

B.2 Solution method

Typically, DSGE models are solved by linearizing the equilibrium conditions around an invariant, deterministic steady state (higher-order approximations are necessary for analyzing the effects of aggregate uncertainty and making welfare comparisons). The stochastic structure in my model is not amenable to this approach; the process for the aggregate state is non-stationary and there are three long-run steady states, each of which is selected endogenously.[26] Instead, I use a global method that provides an exact, not approximated, solution.

The basic approach is the same as in Kehoe et al. (2018) and Alessandria et al. (2015): assume that the equilibrium converges to a steady state after a finite number T of periods, then solve the resulting finite system of equilibrium variables and equations using the standard Newton-Raphson method. These papers study deterministic models, however. The presence of aggregate uncertainty in my paper complicates the application of this approach but does not prevent it because the number of possible histories is finite.

The computational procedure is further complicated by the need to solve the dynamic program of an exporter. I discretize the firm's state space into a square grid with 300 productivity nodes and 50 market penetration nodes. None of the results are sensitive to the fineness of this grid. For each history Z^t and productivity node a, I use the endogenous grid method to obtain the market penetration policy function of an incumbent exporter and standard 1-dimensional optimization to obtain the policy function of a potential entrant (a firm with zero market penetration at the beginning of the period).

To reduce the size of the solution space, I solve for many of the equilibrium variables analytically as functions of "target" variables which I use in the solver. For example, consumption is implied by gross output, investment, and intermediate inputs through the aggregate market clearing condition (24). The equilibrium variables that are used in the solver are:

- wages, $\{W_i(Z^t)\}_{t=0}^T$;
- bilateral prices, $\{P_{i,j}(Z^t)\}_{t=0}^T$;
- bond prices, $\{Q(Z^t)\}_{t=0}^T$;
- rental rates, $\{R_i(Z^t)\}_{t=0}^T$;
- gross output, $\{Y_i(Z^t)\}_{t=0}^T$;
- and investment, $\{X_i(Z^t)\}_{t=0}^{T-1}$.

The equilibrium equations that are used in the solve are

- numeraire normalization, $P_{uk}(Z^t) = 1$, for $t \leq T$;
- balance of payments for $i = uk, eu$ and $t \leq T$;
- labor market clearing for $i \in I$ and $t \leq T$;
- capital market clearing for $i \in I$ and $t \leq T$;
- Euler equations for $i \in I$ and $t < T$;
- and market clearing for bilateral trade, $Y_{i,j}(Z^t) = \int y_{i,j}(Z^t, \nu)$, for $i, j \in I \times I$ and $t \leq T$.

The program to solve the model is written in C. It can be found in the folder "quanal/dyn_mkt_pen" in the online supplement. The Alessandria-Choi model is in the folder "quanal/fixed_costs" and the multisector model is in the folder "quanal/multisector." Please note that I have compiled these programs in Linux and

[26] As mentioned in the main text, even holding trade costs fixed there is a continuum of possible steady states because I allow for unbalanced trade.

linked to BLAS and LAPACK routines in the Intel MKL library. If you do not have acccess to this library, you can use alternative libraries instead (e.g. Atlas, GSL). If you are using Windows and need help, please contact me for assistance.

C Multi-sector model

In this section of the appendix, I describe the multi-sector version of the model briefly mentioned in the main text. I also explain the calibration of the multi-sector model.

Each country in the multi-sector model has two sectors, goods ($s = 1$) and services ($s = 2$). As in Armington, gross output is differentiated by source country and sector but homogeneous across firms within each country-sector pair.[27] International trade is conducted by intermediaries that aggregate purchases of domestic and foreign gross output into artificial composites, which are then sold to domestic households consumption and investment, and to firms for intermediate inputs. Households have the same preferences and solve almost the same maximization problem as in the one-sector model. The only difference is that households choose investment for each sector separately, and sectoral capital stocks follow a law of motion with adjustment costs as in equation (3) in the main text.

C.1 Production and demand system

C.1.1 International trade

In each country i and sector s, competitive distributors intermediate trade in intermediate inputs and final expenditures separately. The intermediate composite, $M_{i,s}(Z^t)$, is an Armington aggregate of sector-s intermediates from all source countries, $M_{i,s,j}(Z^t),\ j \in I$:

$$M_{i,s}(Z^t) = \left\{ \sum_{j \in I} \left(\mu_{i,s,j}\right)^{\frac{1}{\zeta_{i,s}}} \left(M_{i,s,j}(Z^t)\right)^{\frac{\zeta_{i,s}-1}{\zeta_{i,s}}} \right\}^{\frac{\zeta_{i,s}}{\zeta_{i,s}-1}}.$$

The final expenditure composite in country i, $F_{i,s}(Z^t)$, is given by

$$F_{i,s}(Z^t) = \left\{ \sum_{j \in I} \left(\theta_{i,s,j}\right)^{\frac{1}{\sigma_{i,s}}} \left(F_{i,s,j}(Z^t)\right)^{\frac{\sigma_{i,s}-1}{\sigma_{i,s}}} \right\}^{\frac{\sigma_{i,s}}{\sigma_{i,s}-1}}.$$

The elasticities of substitution vary across countries, sectors, and uses. Later, I will calibrate them to match WIOD trade flow data and elasticity estimates from Caliendo and Parro (2015).

[27] The exporter dymamics framework of Alessandria and Choi (2007) used in the one-sector model is not tractable here. The Armington specification is numerically tractable and consistent with theoretical and quantitative findings in the literature regarding the macroeconomic effects of trade frictions. Adding static firm heterogeneity as in Eaton and Kortum (2002) and other "new" trade models would yield similar results (Arkolakis et al., 2012; Eaton et al., 2011b).

As in the one-sector model, there are two kinds of trade costs: formal import tariffs and non-tariff iceberg trade costs. Each type of trade cost varies by destination country, sector, source country, use. $\tau^m_{i,s,j}(Z^t)$ and $\tau^f_{i,s,j}(Z^t)$ denote country i's tariffs on intermediate and final imports from country j's s-sector, respectively. All tariff revenues are rebated to households as lump-sum transfers. Similarly, $\xi^m_{i,s,j}(Z^t)$ and $\xi^f_{i,s,j}(Z^t)$ denote country i's non-tariff iceberg costs of intermediate and final imports from country j's s-sector.

C.1.2 Gross output

Gross output of country i's sector s, $Y_{i,s}(z^t)$, is produced by competitive firms using value added, $V_{i,s}(z^t)$, and intermediate inputs of goods and services purchased from distributors, $M^d_{i,s,1}(z^t)$ and $M^d_{i,s,2}(z^t)$, according to a Leontief technology:

$$Y_{i,s}(Z^t) = \min\left\{\frac{V_{i,s}(Z^t)}{\eta_{i,s,v}}, \frac{M^d_{i,s,2}(Z^t)}{\eta_{i,s,1}}, \frac{M^d_{i,s,2}(Z^t)}{\eta_{i,s,2}}\right\},$$

Value added is produced using capital, $K^d_{i,s}(Z^t)$, and labor, $L^d_{i,s}(Z^t)$, according to the usual Cobb-Douglas function.

$$V_{i,s}(Z^t) = (K^d_{i,s}(Z^t))^{\alpha_{i,s}}(L^d_{i,s}(Z^t))^{1-\alpha_{i,s}}.$$

I use the superscript d to distinguish firms' demand for factors and intermediates from factor supply, which is chosen by households, and intermediate supply, which is chosen by distributors.

C.1.3 Final demand

Each country i's aggregate consumption basket is a CES aggregate of retail goods and services:

$$C_i(Z^t) = \left\{(\epsilon_{i,1})^{\frac{1}{\rho}}\left(C_{i,1}(Z^t)\right)^{\frac{\rho-1}{\rho}} + (\epsilon_{i,2})^{\frac{1}{\rho}}\left(C_{i,2}(Z^t)\right)^{\frac{\rho-1}{\rho}}\right\}^{\frac{\rho}{\rho-1}}.$$

Following Bems (2008), aggregate investment in each country i is a Cobb-Douglas aggregate of inputs purchased from goods and services retailers:

$$X_i(Z^t) = (X_{i,1}(Z^t))^{\varepsilon_{i,1}}(X_{i,2}(Z^t))^{\varepsilon_{i,2}}$$

C.1.4 Market clearing

The market clearing conditions in the multi-sector model are:

$$
\begin{aligned}
Y_{i,s}(Z^t) &= \sum_{j\in J}\left(M_{i,s,j}(Z^t)+F_{i,s,j}(Z^t)\right),\ \forall i\in I,\ \forall s\in S\\
M_{i,s}(Z^t) &= \sum_{r\in S} M^d_{i,r,s}(Z^t), \forall i\in I,\ \forall s\in S\\
F_{i,s}(Z^t) &= C_{i,s}(Z^t)+X_{i,s}(Z^t),\ \forall i\in I,\ \forall s\in S\\
X_i(Z^t) &= \sum_{s\in S} X^d_{i,s}(Z^t),\ \forall i\\
K_{i,s}(Z^t) &= K^d_{i,s}(Z^t),\ \forall i,s\\
\bar{L}_i(Z^t) &= \sum_{s\in S} L^d_{i,s}(Z^t),\ \forall i\\
0 &= \sum_{i\in I} B_i(Z^t)
\end{aligned}
$$

C.2 Calibration

As in the one-sector model, I first assign elasticities of substitution and other common parameters, then calibrate remaining parameters so that the steady-state replicates an input-output matrix.

C.2.1 Assigned parameters

Assigned parameters, like the discount factor and the capital share, which have the same meaning in the multi-sector model as they do in the one-sector model, are set to the same values as in the main text.

There are several new elasticities, however. I follow Kehoe et al. (2018) and use Atalay (2014)'s estimate of 0.65 for the elasticity of substitution between goods and services in consumption, ρ. To set the Armington elasticities — which are also trade elasticities since this model has no extensive margin — I refer to Caliendo and Parro (2015), who estimate trade elasticities for the same 2-digit ISIC industries that comprise the goods sector in the input-output matrix described below. For each country i, I set the intermediate and final goods trade elasticities, $\zeta_{i,1}$ and $\sigma_{i,1}$, to averages of the Caliendo and Parro (2015) estimates, weighted by these industries shares' in country i's total intermediate goods imports and total final goods imports, respectively. Intermediate goods trade elasticities range from 6.6 to 7.6, while final goods trade elasticities range from 4.4 to 5.3. All countries have higher elasticities for intermediates than for final use. For the services sector, I follow Costinot and Rodríguez-Clare (2014) and set the intermediate and final services trade elasticities, ζ_{i2} and σ_{i2}, all to 5, the average of the Caliendo and Parro (2015) estimates. Note that the aggregate trade elasticities are close to 5 in the multi-sector model.

C.2.2 Parameters calibrated to input-output data

The remaining parameters are calibrated so that the no-Brexit steady state replicates a two-sector version of the input-output matrix constructed from the 2011 WIOD data. The goods sector includes agriculture, mining, and all manufacturing industries (2-digit ISIC codes 37 and lower). The services sector includes all other industries. The two-sector matrix is shown in table A3. With this matrix and the elasticities and other externally-calibrated parameters in hand, we can calibrate the remaining parameters using the model's equilibrium conditions as in the one-sector model.

C.3 Post-Brexit trade costs

To calculate the increases in tariffs and non-tariff barriers after Brexit in the multi-sector model I use the same approach as in the one-sector model, but perform the computations for each sector separately. For tariffs in hard Brexit, the goods-sector tariffs are the figures from A.3.1 prior to scaling by the goods shares of total trade flows; there are no tariffs for the services sector. For non-tariff barriers, I perform the computation separately by sector and use, since the WIOD data that are used as weights distinguish between intermediate and final trade. The results are in table A4.

C.4 Version with import adjustment frictions

The baseline multi-sector model has been calibrated using trade elasticity estimates from Caliendo and Parro (2015), which are in line with other estimates in the literature of trade flows' sensitivity to price changes in the long run. As Ruhl (2008) points out, in the short run trade flows typically respond less to price changes than long-run elasticity estimates would dictate; Heathcote and Perri (2002) estimate an elasticity of 0.9 using HP-filtered data. Recent theoretical work on trade dynamics and endogenous trade elasticities emphasizes fixed costs of importing or exporting as used in the one-sector model (Alessandria and Choi, 2007; Ruhl, 2008; Alessandria et al., 2013, 2015; Ramanarayanan, 2016), source-specific durable goods (Engel and Wang, 2011), destination-specific marketing capital (Drozd and Nosal, 2012), and firm-to-firm relationship stickiness (Lim, 2016). Incorporating these sorts of features into the mult-sector model is not computationally feasible, so I take a similar approach to Engel and Wang (2011) and add convex costs of adjusting import quantities.

In this version of the model, distributors must pay costs to adjust the quantities of inputs they import from other countries. The total adjustment cost paid by sector-s distributors in country i for intermediate trade is

$$\sum_{j\in I\setminus i}\left[\frac{\varphi_m}{2}\left(\frac{M_{i,s,j}(Z^t)}{M_{i,s,j}(Z^{t-1})}-1\right)^2\right].$$

The parameter φ_m governs the size of the adjustment costs, which are paid in units of labor. Decreasing inputs from one foreign country in favor of increasing inputs from another incurs two adjustment costs;

substituting inputs from the rest of the world for inputs from the European Union after Brexit is particularly costly. There is a similar adjustment cost for trade in final expenditures. Distributors solve dynamic problems in this version of the model, choosing inputs to maximize the expected present value of dividends.

In contrast to the adjustment-cost model suggested by Krugman (1986) and studied in Drozd and Nosal (2012), in which producers pay costs to adjust export quantities, this model delivers time-varying trade elasticities. In this exercise, I calibrate the adjustment cost parameters φ_m and φ_f so that the average short-term trade elasticity for the United Kingdom, measured over the one-year period 2019–2020 immediately after Brexit implementation, is 1, the standard value in the international business cycle literature.

Table A1: Concordance between ISIC Rev 2. and Francois et al. (2013) sectors

ISIC code	ISIC sector name	Francois et al. (2013) sector name
AtB	Agriculture, Hunting, Forestry and Fishing	–
C	Mining and Quarrying	–
15t16	Food, Beverages and Tobacco	Food & beverages
17t18	Textiles and Textile Products	Textiles
19	Leather, Leather and Footwear	–
20	Wood and Products of Wood and Cork	Wood & paper products
21t22	Pulp, Paper, Paper , Printing and Publishing	Wood & paper products
23	Coke, Refined Petroleum and Nuclear Fuel	–
24	Chemicals and Chemical Products	Chemicals + Cosmetics + Pharmaceuticals
25	Rubber and Plastics	–
26	Other Non-Metallic Mineral	–
27t28	Basic Metals and Fabricated Metal	Metals
29	Machinery, Nec	Machinery
30t33	Electrical and Optical Equipment	Electronics + OICE + Biotech. + Medical equip.
34t35	Transport Equipment	Aerospace & Space + Automotive
36t37	Manufacturing, Nec; Recycling	–
E	Electricity, Gas and Water Supply	–
F	Construction	Construction
50	Sale, Maint. and Repair of Motor Vehicles	–
51	Wholesale Trade and Commission Trade	–
52	Retail Trade, Except Motor Vehicles	–
H	Hotels and Restaurants	Travel Services
60	Inland Transport	Transport Services
61	Water Transport	Transport Services
62	Air Transport	Transport Services
63	Other Transport Activities	–
64	Post and Telecommunications	Communication Services
J	Financial Intermediation	Financial Services + Insurance Services
70	Real Estate Activities	–
71t74	Renting of M&Eq, Other Business Activities	ICT + Other Business Services
L	Public Admin and Defence	–
M	Education	–
N	Health and Social Work	–
O	Other Community and Personal Services	Personal & Recreational Services
P	Private Households with Employed Persons	–

Table A2: Assigned parameters in multisector model

Parameter	Meaning	Value	Source or target
(a) Trade elasticities			Literature + WIOD
$\zeta_{uk,s}$	UK intermediates	(7.6,5.0)	
$\zeta_{eu,s}$	EU intermediates	(7.5,5.0)	
$\zeta_{rw,s}$	ROW intermediates	(6.6,5.0)	
$\sigma_{uk,s}$	UK final	(4.8,5.0)	
$\sigma_{eu,s}$	EU final	(4.4,5.0)	
$\sigma_{rw,s}$	ROW final	(5.3,5.0)	
(b) Other parameters			
ρ	Consumption elasticity	0.65	Atalay (2014)
β	Discount factor	0.98	2% long-run interest rate
γ	Risk aversion	2.0	Standard
α	capital share	0.33	Standard
δ	Depreciation rate	0.06	Standard
φ_k	Capital adjustment cost convexity	0.76	Steinberg (2016)

Table A3: 2011 world input-output table in multi-sector model

		Intermediate inputs						Final demand			
		UK		EU		ROW					
		Goods	Services	Goods	Services	Goods	Services	UK	EU	ROW	GO
UK	Goods	2.68	2.96	2.12	0.77	1.79	1.11	4.00	1.95	2.25	39.27
	Services	4.61	27.46	0.56	1.83	0.62	1.93	42.41	0.28	0.67	160.73
EU	Goods	1.77	1.41	62.49	29.10	16.11	8.38	3.24	56.30	19.68	396.98
	Services	0.10	0.74	48.01	124.72	5.11	10.13	0.33	224.21	3.84	834.37
ROW	Goods	1.87	1.61	18.16	6.71	474.06	211.32	2.58	14.24	288.49	2,038.10
	Services	0.29	1.64	4.63	8.81	163.50	391.38	0.60	2.48	900.01	2,946.68
VA		19.63	80.37	198.49	417.19	1,019.05	1,473.34	-	-	-	3,208.07
GO		39.27	160.73	396.98	834.37	2,038.10	2,946.68	106.34	598.92	2,429.88	9,551.27

Table A4: Brexit scenarios in multisector model

Exogenous change	Soft Brexit	Hard Brexit
(a) Tariffs on goods trade		
Imports from E.U.	0.00	4.23
Exports to E.U.	0.00	3.29
(b) Non-tariff barriers (imports from E.U.)		
Intermediate goods	1.76	5.23
Intermediate services	0.98	2.94
Final goods	3.08	9.24
Final services	0.38	1.13
(c) Non-tariff barriers (exports to E.U.)		
Intermediate goods	1.49	4.47
Intermediate services	1.44	4.32
Final goods	2.62	7.85
Final services	1.065	3.20