Safety Transformation and the Structure of the Financial System *

William Diamond

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Abstract

This paper develops a model of how the financial system is organized to most effectively create safe assets and analyzes its implications for asset prices, capital structure, and macroeconomic policy. In the model, financial intermediaries choose to invest in the lowest risk assets available in order to issue safe securities while minimizing their reliance on equity financing. Although households and intermediaries can trade the same assets, in equilibrium all debt securities are owned by intermediaries since they are low risk, while riskier equities are owned by households. The resulting market segmentation explains the low risk anomaly in equity markets and the credit spread puzzle in debt markets and determines the optimal leverage of the non-financial sector. An increase in the demand for safe assets causes an expansion of the financial sector and extension of riskier credit to the non-financial sector– a subprime boom. Quantitative easing increases the supply of safe assets, leading to a compression of risk premia in debt markets, a deleveraging of the non-financial sector, and an increase in output when monetary policy is constrained. In a quantitative calibration, the segmentation of debt and equity markets is considerably more severe when intermediaries are poorly capitalized.

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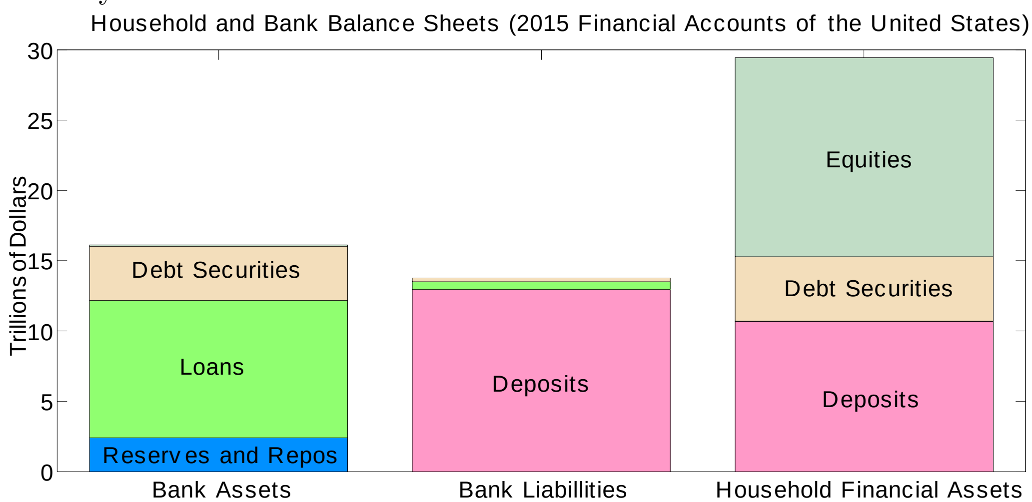
An important role of financial intermediaries is to issue safe liabilities such as bank deposits while holding risky assets such as mortgages and corporate debt. The effectively riskless liabilities created by this "safety transformation" are sold primarily to households, meeting their demand to hold safe, liquid assets. To ensure that their liabilities are riskless, intermediaries must issue enough equity to bear all risk in the portfolio of assets they own. An intermediary's equity capital, and the riskiness of its asset portfolio, determine the quantity of safe liabilities it is able to produce. As shown in the figure below, banks choose a highly levered capital structure and invest almost exclusively in various forms of debt, while equities are held in large quantities by households.



This paper presents a general equilibrium model of how the financial system is organized to perform safety transformation. The model's equilibrium determines (i) the composition of intermediary and household balance sheets, (ii) the leverage of the financial and non-financial sectors, and (iii) the pricing of risk in endogenously segmented debt and equity markets. In addition to its implications for the structure of the financial system, the model provides a framework for understanding the general equilibrium effect of changes in the supply and demand for safe assets. The model implies that an increase in the demand for safe assets replicates many features of the subprime boom, including an expansion of intermediary balance sheets and an increase in non-financial sector leverage. It also implies that quantitative easing policies, by increasing the supply of safe assets, decreases the price of risk in debt markets, causes a deleveraging of the non-financial sector, and stimulates output at the zero lower bound. The model's implications for non-financial sector leverage and the price of risk in bond markets rely crucially on its ability to explain the stylized fact that intermediaries are highly levered and invest in debt while households invest primarily in equities.

Two basic ingredients are at the core of the model. First, households have a demand for riskless assets, obtaining utility directly from holding them. Second, financial intermediaries face costs of issuing external equity. These frictions can respectively be motivated by the use of bank deposits and other safe securities in transactions (Stein 2012, Gorton Pennachi 1990, Clower 1967) and the agency and information frictions which constrain the ability of firms to raise outside equity (Froot Scharfstein Stein 1993, Holmstrom Tirole 1997). A key assumption in the rest of the financial intermediation literature, that households do not have the expertise to hold the assets owned by intermediaries, is absent from the model. This provides a theory of the role of intermediaries in markets for liquid, publicly traded securities which does not require any special ability to monitor or screen borrowers. In particular, this allows the model to be consistent with the large quantity of informationally sensitive equity securities owned by households.

In the model, financial intermediaries buy a portfolio of risky assets to back the safe liabilities that households demand. Because households obtain utility directly from holding riskless securities, they are willing to invest in them at a reduced interest rate, making issuance of these securities attractive to intermediaries. In order to make these claims safe, intermediaries must issue costly equity that will bear all risk in their asset portfolio. In essence, intermediaries can finance themselves with cheap, riskless debt but must issue enough costly equity to ensure the safety of their debt. This model of an intermediary's cost of capital incentivizes it to choose a highly levered capital structure. The intermediary's desire to be highly levered in turn determines which securities it is willing to purchase. Because holding low risk securities allows an intermediary to issue the largest quantity of safe liabilities with a given quantity of equity capital, intermediaries are willing to pay more than households for the lowest risk securities.

The amount of equity issuance required to buy high risk securities makes them unattractive to intermediaries, so high risk securities are bought by households instead.

Because intermediaries will pay more than households for low risk securities but less for high risk securities, the pricing of risk in asset markets is segmented. Low risk securities purchased by intermediaries earn a higher risk-adjusted return than high risk securities purchased by households. This segmentation reflects the fact that the intermediary has a uniquely cheap source of leverage, which increases its willingness to pay for riskless assets, but requires a high price of risk to compensate for its cost of equity issuance. This segmentation is related to the fact that in asset pricing models with leverage constraints (e.g. Frazzini Pedersen 2014, Black 1972), agents who can borrow obtain the best compensation for taking risk by holding levered portfolios of low risk assets. In my model, a financial intermediary, which can borrow cheaply because of the demand for its riskless liabilities, is the agent that can most easily implement levered trading strategies .

This market segmentation is arbitraged by non-financial firms when they choose their capital structure. Each firm chooses its leverage so that its debt is sufficiently low risk that an intermediary is willing to pay for for its debt than a household. In addition, at its optimal leverage, each firm's equity is sufficiently risky that a household is willing to pay more for it than an intermediary. As a result, the firm's total market value is strictly higher than any agent would be willing to pay for the firm's entire stream of profits. As a firm increases its leverage, its debt becomes riskier, and the intermediary who buys the firm's debt charges a higher price of risk than the household who buys the firm's equity. Each firm therefore has a unique optimal leverage, beyond which any further debt issuance increases the riskiness of its debt so much that the firm's total market value decreases.

Because each non-financial firm sells its debt to an intermediary and its equity to a household, debt and equity markets are segmented. Within each asset class, the same marginal investor owns all securities, and there is a unique market price of risk. However, all debt securities are held by financial intermediaries, while all equities are held by households, so the price of risk is strictly greater in the debt market. This provides a rationale for the "credit spread puzzle" (Huang and Huang 2012) in debt markets which finds that credit spreads on corporate bonds are too large to easily reconcile with risk premia in equity markets. This also explains the "low risk anomaly" (Black Jensen Scholes 1972, Baker Bradley Taliaferro 2014) in equity markets, which finds that simple measures of risk (such as CAPM beta) are inconsistent with a high expected return on equities and low risk free rate. The model is able to explain the low risk anomaly because the risk free rate lies strictly below the rate implied by the pricing kernel in equity markets as a result of the demand for safe assets as in (Bansal Coleman 1996).

I apply the model to analyze three macroeconomic issues: the response of the financial system to changes in the supply and demand for safe assets, the transmission mechanism of quantitative easing, and the role of intermediary capital at the zero lower bound. All three issues can be interpreted in a supply and demand framework. The supply of safe assets is a function of the supply of low risk securities and the amount of intermediary capital available to cover the mismatch between low risk assets and riskless liabilities. An increase in safe asset demand reduces interest rates, and the cheap source of riskless debt funding induces intermediaries to expand their balance sheets and provide riskier credit to the non-financial sector. This illustrates how a global demand for U.S. safe assets may have contributed to the subprime boom by reducing yields in debt markets (Bernanke et al. 2011). By decreasing the intermediary's asset-liability mismatch through quantitative easing or increasing intermediary capital available to cover the mismatch, the supply of safe assets is increased. This increased supply raises short term interest rates and reduces the price of credit and duration risk in debt markets. When goods prices are sticky and the zero lower bound is binding as in (Caballero Farhi 2016), a safe asset supply increase stimulates output and compresses risk premia in debt markets while leaving the risk free rate unaffected. This analysis illustrates the macroeconomic importance of a well capitalized financial sector in the market for safe assets, particularly near the zero lower bound.

Relationship to literature Within the financial intermediation literature, this paper is closest to "liability-centric" models such as (Gorton Pennachi 1990, Dang et. al. 2015, Dang et. al. 2016). In this literature, the role of intermediaries is to provide safe, informationally insensitive assets which are demanded by other agents. The key friction in previous papers in this literature is asymmetric information, which constrains the ability of informed and uninformed investors to share risk with each other. More generally, theories of the role of debt on intermediary balance sheets that emphasize its informational insensitivity (going back to Townsend 1979) imply that equity claims are too informationally sensitive to be sold to outsiders.[1] In my model, all investors are equally informed about asset payoffs, but risk sharing is imperfect because the non-financial sector is unable to issue riskless securities. The model explains the role of debt held by intermediaries and equity held by households because this allocation of assets allows the intermediary to issue the most riskless liabilities with a given quantity of equity capital.

Because my model has intermediaries investing in publicly traded debt securities, it speaks to the connection between intermediary capital and the price of risk in securities markets. This places the model within the intermediary asset pricing literature (He Krishnamurthy 2013), where the capitalization of intermediaries plays an important role in determining risk premia. Existing papers within this literature assume that certain risky assets can only be held by (or are more productive in the hands of) intermediaries, who must therefore price the assets in equilibrium. This paper endogenously determines which assets are priced by intermediaries and which are priced by households.[2] Similar to the rest of the literature, risk premia on assets owned by intermediaries are sharply higher when intermediaries are poorly capitalized. However, only relatively safe debt securities are priced by poorly capitalized intermediaries, since intermediaries sell their riskiest assets in a flight to quality when forced to downsize.

The applications of my model are related to the macroeconomic literature on safe asset

[1] As stated in the conclusion of (Townsend 1979), "The model as it stands may contribute to our understand of closely held firms, but it cannot explain the coexistence of publicly held shares and debt."

[2] He and Krishnamurthy 2013 motivate their work by arguing that the pricing kernel for equities and fixed income securities may be different. This feature endogenously arises in my model since intermediaries only choose to hold debt.

shortages (Caballero Krishnamurthy 2009, Bernanke et al 2011, Caballero Farhi 2016). This literature argues that an increasing global demand for safe assets since the 1990s has depressed interest rates, fueled asset bubbles, and eventually pushed the developed world to the zero lower bound. Relative to this literature, my model clarifies how meeting a growing demand for safe assets requires the financial sector to invest in riskier assets and how this induces the non-financial sector to take on more debt. As a result, my model provides one explanation for the growth in household and firm debt issuance during the 2000s subprime boom and how policies such as quantitative easing crowd out intermediary risk taking and non-financial sector leverage.

One additional motivation for this paper is an empirical literature suggesting a disconnect between the pricing of risk in debt and equity markets. (Collin-Dufresne et. al. 2001) shows that changes in credit spreads are difficult to explain using variables that appear in structural models that assume asset markets are integrated. (Gilchrist Zacrajzek 2012) shows that movements in bond risk premia that are unrelated to measures of default risk are highly cyclical and presents evidence that they co-move strongly with bank lending standards and financial sector profitability. (Frazzini Pedersen 2012, 2014) demonstrate that a levered bond portfolio outperforms an unlevered equity portfolio with the same CAPM beta and that low risk assets outperform particularly when the TED spread, a proxy for intermediary distress, is large. In this paper, financial intermediaries invest in a portfolio designed to take advantage of segmentation in risk pricing, and segmented debt and equity markets endogenously emerge in equilibrium.

The rest of the paper is organized as follows. First, I present the baseline two period model which illustrates the main idea. Next I present three applications: the general equilibrium effects of changes in the supply and demand for safe assets, quantitative easing, and the role of intermediary capital at the zero lower bound. I then present a quantitative version of the model in which the segmentation between debt and equity markets is highly nonlinear in intermediary capital. I then conclude. Proofs are in the appendix.

1 Baseline Model

I first present an overview of the model's agents, timing, and frictions and provide a discussion of the key assumptions. Next, I solve each agent's optimization problem in partial equilibrium and discuss their implications. Finally, I solve for the model's unique equilibrium.

Setup The model has two periods ($t = 1, 2$) and three classes of agents: a household, a financial intermediary, and a continuum of non-financial firms indexed by $i \in [0, 1]$. The household's utility is a function of its consumption (c_1, c_2) at times 1 and 2 as well of the quantity of riskless assets d it buys at time 1. It invests its endowment in securities issued by the intermediary and non-financial firms in order to maximize its expected utility. The intermediary is a publicly traded firm which maximizes the market value of its equity. It can invest in the same publicly traded assets available to the household and can issue liabilities backed by the portfolio of assets it purchases. In particular, the intermediary is able to buy risky assets and issue safe liabilities backed by them in order to meet the household's demand for riskless securities. However, all systematic risk on the intermediary's balance sheet must be financed by issuing equity, and the intermediary faces a convex cost $C(e)$ of issuing e units of equity at time 1. Each non-financial firm $i \in [0, 1]$ has exogenous cashflows x_i at time 2 and chooses its capital structure to maximize the total market value of securities it issues. Non-financial firms can only sell debt and equity securities but not arbitrary Arrow-Debreu claims.

At time 1, households consume and all agents participate in markets for securities which pay off at time 2. At time 2, the payoffs of securities are realized and the household consumes the payouts from its assets. An aggregate shock is realized at time 2 to be "good" or "bad". For each non-financial firm $i \in [0, 1]$, an idiosyncratic shock is also realized at time 2 which determines its cashflow x_i. The expected cashflow $E(x_i|good)$ of firm i in the good state is strictly greater than its expected payoff $E(x_i|bad)$ in the bad state. Shocks to the cashflows of non-financial firms are the only source of uncertainty in the model.

Agents trade securities in an asset market at time 1 which pay off at time 2. A collection of securities indexed by $s \in [0,1]$ is available for purchase. Each security s has payoff δ_s at time 2 and is sold for a price p_s at time 1. To map this set of securities to the debt and equity issued by the non-financial sector, let $s = \frac{i}{2}$ refer to firm i's debt and $s = \frac{1}{2} + \frac{i}{2}$ refer to firm i's equity for each $i \in [0,1]$. All assets can be purchased by either the household or the intermediary, but neither agent can short securities.

An important feature of asset markets in this model is that securities cannot be broken into their underlying Arrow-Debreu claims. A security δ_s, which has expected payoff $E(\delta_s|good)$ in the good state and $E(\delta_s|bad)$ in the bad state, is a bundle of both good and bad state claims. The ratio $\frac{E(\delta_s|good)}{E(\delta_s|bad)}$ determines the exposure of security s to systematic risk, and agents can choose to buy securities for which this ratio is high or low. However, it is impossible for an agent who wants only bad state payoffs to avoid buying good state claims as well if all securities have positive expected payoffs in both states.

Discussion of Assumptions Three of the model's assumptions are particularly important. First, the utility which the household obtains from holding riskless securities (similar to Stein 2012) is a simple, reduced form assumption to create a demand for safe assets. A similar demand could come from a cash in advance constraint (Clower 1967) or from adding an infinitely risk averse agent to the model as in (Caballero Farhi 2016). The key implication of this assumption is a reduction in the risk free rate relative to the pricing of risky assets. This low risk free rate is the defining feature of a demand for safe assets, and it is robust to how the demand is microfounded. One more fundamental explanation for the demand for safe assets follows (Gorton and Pennachi 1990) and (Dang et al 2016), where the informational insensitivity of a riskless asset makes it a useful medium of exchange between asymmetrically informed agents.

Second, the intermediary's cost of external equity issuance constrains its ability to bear risk. The appendix provides a microfoundation for this cost following (Lacker Weinburg 1989), where the intermediary earns rents from its shareholders that are increasing in the amount of risk on its balance sheet. This assumption constrains the size of the intermediary sector, so that it does not buy the entire non-financial sector in order to issue safe securities. A cost of equity issuance or a constraint on the quantity of outside equity that can be issued appears in the vast majority of the literature (e.g. Bernanke Gertler 1989, Holmstrom Tirole 1997, He Krishnamurthy 2013, Brunnermeier Sannikov 2014). Such a cost is necessary for the intermediary to be financially constrained.

Third, the assumption that non-financial firms can only issue debt and equity securities ensures that the non-financial sector is not perfectly able to separate its cashflows into safe and risky tranches. While debt is less risky than equity, all securities issued by non-financial firms are exposed to some degree of systematic risk. Only the intermediary is able to issue riskless securities by issuing enough equity to bear the risk in its asset portfolio. Because the securities of the non-financial sector are risky, it is necessary for the intermediary to bear some risk on its balance sheet. There is no way for the intermediary to buy a riskless portfolio composed of assets issued by the non-financial sector. Perhaps the strongest motivation for this assumption is the empirical fact that firms (and households) issue debt with payments that are not indexed to aggregate states. (Mian Sufi 2008) presents a forceful empirical argument that a lack of indexation in debt contracts played an important role in the 2008-2009 recession, and (Hebert Hartman-Glaser 2016) provides an incomplete contracting framework in which indexed contracts are not traded despite their benefits.

Together, these three assumptions imply that (i) the household demands riskless assets, (ii) securities issued by the non-financial sector are not riskless, and (iii) the intermediary faces a cost of bearing systematic risk on its balance sheet. As a result, my model features a minimal set of frictions for which there is a non-trivial safety transformation role to be played by financial intermediaries. A key assumption which is absent from the model, but appears in essentially the entire financial intermediation literature, is that the intermediary has access to investment opportunities that are not available to households. My emphasis on safety transformation therefore provides a theory that can explain the role intermediaries play in markets for publicly traded securities that households are also able to purchase.

Household's problem The household faces a standard intertemporal consumption problem, except for the fact that it obtains utility directly from its holdings of riskless assets. Given its initial endowment, the household may either consume or invest it in a portfolio of available securities. Risky securities owned by the household are priced by the marginal utility of consumption they provide. The household's special willingness to pay for riskless assets yields an arbitrage opportunity, since the risk free rate lies strictly below that implied by the household's marginal utility of consumption. A trading strategy which exploits this arbitrage opportunity is to buy a portfolio of assets and sell a riskless senior tranche and risky junior tranche backed by the portfolio, similar to the safety transformation intermediaries perform in equilibrium.

The household maximizes expected utility

$$u(c_1) + E[u(c_2)] + v(d). \tag{1}$$

over period 1 consumption c_1, period 2 consumption c_2, and "deposits" d, which are riskless securities owned by the household. u and v are assumed to be strictly increasing, strictly concave, twice continuously differentiable, and satisfy Inada conditions. The household's only choice is how to invest its initial endowment, whose total value is W_H. It may purchase either riskless assets, which yield the direct benefit $v(d)$ as well as a riskless cashflow at period 2, or any other financial security issued by the intermediary or non-financial firms. It cannot sell securities short or borrow in order to invest.

The household's problem is to maximize its expected utility given a deposit rate i_d and prices p_s of securities s which pay the random cashflows δ_s in period 2. Given the interest rate i_d, the price of one unit of deposit at time 1 equals $\frac{1}{1+i_d}$. Consumption at period 2 is the sum of payoffs from deposits and securities $c_2 = \int \delta_s q_H(s)\, ds + d$, where $q_H(s)$ is the quantity of security s purchased by the household. $q_H(s)$ cannot be negative, since short selling is not allowed. The household's problem can be written as

$$\max_{d, q_H(.), c_1} u(c_1) + E\left[u\left(\int_0^1 \delta_s q_H(s)\, ds + d\right)\right] + v(d)$$

$$\text{subject to } c_1 + \frac{d}{1+i_d} + \int_0^1 p_s q_H(s)\, ds = W_H \text{ (budget constraint)},$$

$$q_H(.) \geq 0 \text{ (short sale constraint)}$$

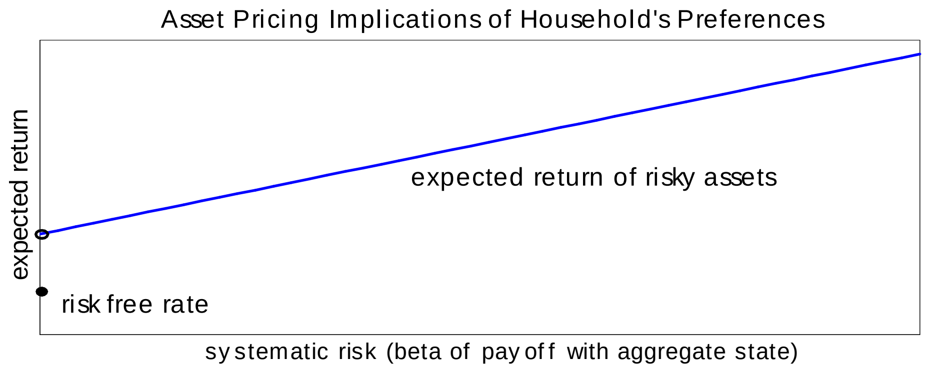
The first order conditions for deposits d (which must be an interior solution since $v'(0) = \infty$) and for the optimal quantity $q_H(s)$ to purchase of security s are

$$u'(c_1) = (1 + i_d)\left(v'(d) + E\left[u'(c_2)\right]\right) \tag{2}$$

$$u'(c_1)\, p_s \geq E\left[u'(c_2)\, \delta_s\right] \tag{3}$$

where inequality 3 becomes an equality if the household holds a positive quantity of security s, $q_H(s) > 0$.

Two features of the household's optimal portfolio choice are notable. First, inequality 3 implies that only securities actually owned by the household must satisfy the consumption Euler equation. If other agents (such as the intermediary) are willing to pay a strictly higher price for an asset than the household, the price will not reflect the household's preferences. This depends crucially on the no short sale constraint, which stops the household from shorting assets it considers overvalued. Second, the extra marginal utility $v'(d)$, reflecting the "safe asset premium" households are willing to pay to hold riskless securities, depresses the risk free rate. The interest rate $i_d = \frac{u'(c_1)}{(v'(d)+Eu'(c_2))} - 1$ implied by the household's optimal behavior would equal the strictly larger rate $\frac{u'(c_1)}{Eu'(c_2)} - 1$ if $v'(d)$ were equal to zero. Safe asset demand leads to a low risk free rate relative to the pricing of other assets owned by the household, as (Krishnamurthy Vissing-Jorgensen 2012) shows empirically in the pricing of treasury securities. This is illustrated in the diagram below.



If all asset prices reflected the household's willingness to pay, the gap between the risk free rate and the pricing of risky assets would be an arbitrage opportunity. Given the set of assets δ_s available for purchase, suppose that a financial intermediary buys $q_I(s)$ units of asset s and sold securities backed by these cashflows to the household. This portfolio pays $\int_0^1 \delta_s q_I(s)\, ds$, equal to $\int_0^1 E(\delta_s|good)\, q_I(s)\, ds$ in the good state and $\int_0^1 E(\delta_s|bad)\, q_I(s)\, ds$. The price of this portfolio is $E\left[\frac{u'(c_2)}{u'(c_1)} \int_0^1 \delta_s q_I(s)\, ds\right]$. If the intermediary sells a riskless security backed by this portfolio paying $\int_0^1 E(\delta_s|bad)\, q_I(s)\, ds$ and a residual claim paying $\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds$ in the good state, the household buys the riskless security at the price $\frac{(v'(d)+Eu'(c_2))}{u'(c_1)} \int_0^1 E(\delta_s|bad)\, q_I(s)\, ds$ and buys the risky residual claim at the price $\frac{1}{2}\frac{u'\left(c_2^{good}\right)}{u'(c_1)} \int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds$. This yields an arbitrage profit of $\frac{v'(d)}{u'(c_1)} \int_0^1 E(\delta_s|bad)\, q_I(s)\, ds$ for the intermediary who constructed this portfolio after subtracting the cost of buying the risky assets δ_s.

This arbitrage trading strategy, selling safe and risky tranches backed by a diversified portfolio of risky assets, is precisely the safety transformation performed by financial intermediaries. The asset pricing implications of safe asset demand therefore already contain a hint of the role intermediaries play in this model. Empirically, intermediaries only perform safety transformation by holding portfolios of low risk assets, staying away from riskier securities such as equities. In the example above it would be most profitable for intermediaries to buy the entire supply of risky assets available in order to issue the largest possible supply $\int_0^1 E(\delta_s|bad)\, q_I(s)\, ds$ of safe securities. The next section develops a model of safety transformation subject to costs of issuing external equity, which explains why intermediaries choose only to invest in low risk assets.

Intermediary's problem The intermediary is a publicly traded firm that maximizes the market value of its net payouts $(\delta_{I,1}, \delta_{I,2})$. The household must own the intermediary's equity in equilibrium (since there are no other buyers). The net payouts $(\delta_{I,1}, \delta_{I,2})$ (which are positive when a dividend is paid and negative when equity is issued) are therefore priced by the household's marginal utility $u'(c_t)$ at $t = 1, 2$, since the first order condition 3 must be an equality. The market value of the intermediary's equity at time 1 is therefore equal to

$$E\left[\frac{u'(c_2)}{u'(c_1)}\delta_{I,2}\right] + \delta_{I,1}. \tag{4}$$

The intermediary raises funds by issuing deposits and equity at time 1, can invest these funds in the same set of assets available to the household for purchase, and must purchase a portfolio which is always sufficient to back its promised payment to depositors. It must pay a cost of $C(e)$ units of the consumption good when it issues equity e at time 1 where $C(e) = 0$ for $e \leq 0$, $C'(0) = 0$, $C'(e) > 0$ for $e > 0$, $C''(e) \geq 0$ for $e \geq 0$. This implies there is no penalty or bonus for paying dividends (since $C(e) = 0$ for $e \leq 0$) but the intermediary faces a strictly convex cost of issuing positive amounts of equity. The appendix presents an agency problem similar to (Lacker Weinburg 1989) which provides a microfoundation for the cost of equity $C(e)$. In this agency problem, the rents earned by the intermediary are costly to its outside shareholders but have no effect on total resources available to be consumed at the time of equity issuance. At time 1, the intermediary's net payout to its shareholders is -1 times the equity it issues, minus the cost of issuing equity $\delta_{I,1} = -(e + C(e))$. At time 2, the intermediary's net payout is the total cashflows from its security portfolio minus the promised payouts to depositors $\delta_{I,2} = \int_0^1 \delta_s q_I(s)\, ds - d$, where $q_I(s)$ is the quantity of security s purchased by the intermediary. The intermediary's problem can therefore be written as

$$\max_{e,d,q_I(.)} E\left[\overbrace{\frac{u'(c_2)}{u'(c_1)}}^{\text{household's pricing kernel}}\overbrace{\left(\int_0^1 \delta_s q_I(s)\,ds - d\right)}^{\text{payout at time 2}} + \overbrace{[-e - C(e)]}^{\text{cost of equity issued at time 1}}\right]$$

$$\text{subject to: } e + \frac{d}{1+i_d} = \int_0^1 p_s q_I(s)\,ds \qquad \text{(budget constraint)}$$

$$\left(\int_0^1 \delta_s q_I(s)\,ds - d\right) \geq 0 \text{ in all states of the world (solvency constraint)}$$

$$q_I(.) \geq 0 \qquad \text{(short sale constraint)}$$

Since all securities have higher expected payoffs in the good than bad state, the solvency constraint only binds in the bad state. It is therefore equivalent to the constraint $\int_0^1 E(\delta_s|bad)\,q_I(s)\,ds - d \geq 0$. Let λ be the Lagrange multiplier on the solvency constraint, so the problem can be written in Lagrangian form (after using the budget constraint to solve for e)

$$\max_{q_I(.),d} Eu'(c_2)\left(\int_0^1 \delta_s q_I(s)\,ds - d\right) - u'(c_1)\left[\int_0^1 p_s q_I(s)\,ds - \frac{d}{1+i_d}\right]$$
$$-u'(c_1)\,C\left(\int_0^1 p_s q_I(s)\,ds - \frac{d}{1+i_d}\right) + \lambda\left(\int_0^1 E(\delta_s|bad)\,q_I(s)\,ds - d\right),$$
subject to $q_I(.) \geq 0$ nonnegative.

The first order conditions are (noting that $e = \int_0^1 p_s q_I(s)\,ds - \frac{d}{1+i_d}$)

$$u'(c_1)(1 + C'(e)) = (1+i_d)(Eu'(c_2) + \lambda) \tag{5}$$

$$u'(c_1)(1 + C'(e))\,p_s \geq Eu'(c_2)\,\delta_s + \lambda E(\delta_s|bad) \tag{6}$$

where the inequality must be an equality for all s for which $q_I(s) > 0$.

The intermediary's ability to issue riskless deposits depends on two separate channels: diversification of idiosyncratic risk, and equity issuance to bear systematic risk. Because the intermediary can issue securities backed by its entire portfolio of assets, it is able to diversify away all idiosyncratic risk in the assets it holds. As a result, it is possible for the intermediary to issue riskless securities even when every asset it owns can have arbitrarily low payoffs. If the intermediary's liabilities were collateralized by individual securities rather than a portfolio, no riskless assets could be issued unless some assets $s \in [0,1]$ have payoffs bounded away from 0. The intermediary's ability to diversify is reflected in the fact that its solvency constraint can be written as $\int_0^1 E\left(\delta_s|bad\right) q_I\left(s\right) ds \geq d$, in which no security's idiosyncratic risk appears. The systematic risk which remains in the intermediary's diversified portfolio is costly for the intermediary. The good state cashflows $\int_0^1 \left[E\left(\delta_s|good\right) - E\left(\delta_s|bad\right)\right] q_I\left(s\right) ds$ on the intermediary's balance sheet after depositors have been paid must be financed by equity issuance.

The intermediary's solvency constraint and cost of equity issuance make it effectively more risk averse to systematic than the household. The intermediary gets a marginal value of $u'\left(c_2^{bad}\right) + \lambda$ from a bad state payoff at time 2 but only $u'\left(c_2^{good}\right)$ from a good state payoff, since only bad state payoffs relax the solvency constraint. At time 1, the intermediary has a marginal value of funds $u'\left(c_1\right)\left(1 + C'\left(e\right)\right)$ which is strictly greater than the marginal value $u'\left(c_1\right)$ of the household whenever $e > 0$. If $\frac{u'\left(c_2^{bad}\right)+\lambda}{u'(c_1)(1+C'(e))} > \frac{u'\left(c_2^{bad}\right)}{u'(c_1)}$ and $\frac{u'\left(c_2^{good}\right)}{u'(c_1)(1+C'(e))} < \frac{u'\left(c_2^{good}\right)}{u'(c_1)}$ (as will be true in equilibrium), the intermediary is willing to pay strictly more at time 1 for bad state payoffs and strictly less for good state payoffs relative to the household.

Asset prices This section uses the optimal portfolio choices of the household and intermediary to characterize asset prices, which have so far been taken as given. It proceeds in two steps. First, I use the portfolio choice inequalities 3and 6for each agent to provide an expression of asset prices. This result shows that equilibrium asset prices are the maximum of the two agents' willingness to pay for the asset. Second, I use the fact that both agents are willing to trade risk free assets at the same interest rate i_d to provide a more economically interpretable asset pricing formula. This result shows how the cost of capital of the financial intermediary, in particular the fact that it can issue riskless assets cheaply but faces costs of issuing external equity, is the fundamental determinant of asset prices.

The portfolio choice inequalities 3 of the household and 6 of the intermediary provide a direct characterization of asset prices. Because every security in nonzero supply must be owned by some agent, at least one of these two inequalities must be an equality. This proves the following result.

Proposition 1 *(segmented asset prices) For any asset s in positive supply with payoffs δ_s at time 2, its price at time 1 is the maximum of the willingness to pay of the two agents*

$$p_s = \max\left(\frac{Eu'(c_2)}{u'(c_1)}\delta_s, \left[\frac{Eu'(c_2)\,\delta_s + \lambda E(\delta_s|bad)}{(1+C'(e))\,u'(c_1)}\right]\right). \tag{7}$$

If the two arguments of the max function are not equal, the agent willing to pay a higher price p_s for the payouts δ_s holds the asset's entire supply in equilibrium.

To illustrate the implications of this proposition, it is useful to show how the intermediary's cost of capital determines its willingness to pay for an asset. To do so, note that since both agents are willing to borrow and lend at the risk free rate i_d, so the intermediary's value of a riskless security is equal to the household's in equation 2. Note also that since the realization of $u'(c_2)$ depends only on aggregate shocks, the intermediary's willingness to pay for a security δ_s is the same as one which returns $E(\delta_s|good)$ in the good state and $E(\delta_s|bad)$ in the bad state. Idiosyncratic risk is diversified away and does not earn a risk premium. The intermediary's value of a security paying δ_s can therefore be written as the sum of its value of a security paying $E(\delta_s|bad)$ in all states and a security paying $E(\delta_s|good) - E(\delta_s|bad)$ in the good state. This implies that the price the intermediary is willing to pay to own asset s is equal to

$$\frac{u'\left(c_2^{good}\right)[E(\delta_s|good) - E(\delta_s|bad)]}{2(1+C'(e))\,u'(c_1)} + E(\delta_s|bad)\frac{(Eu'(c_2) + v'(d))}{u'(c_1)} \tag{8}$$

which proves the following corollary. It expresses the price of every asset in terms of the mix of riskless debt and risky equity finance an intermediary would have to raise in order to fund its purchase as part of a diversified asset portfolio.

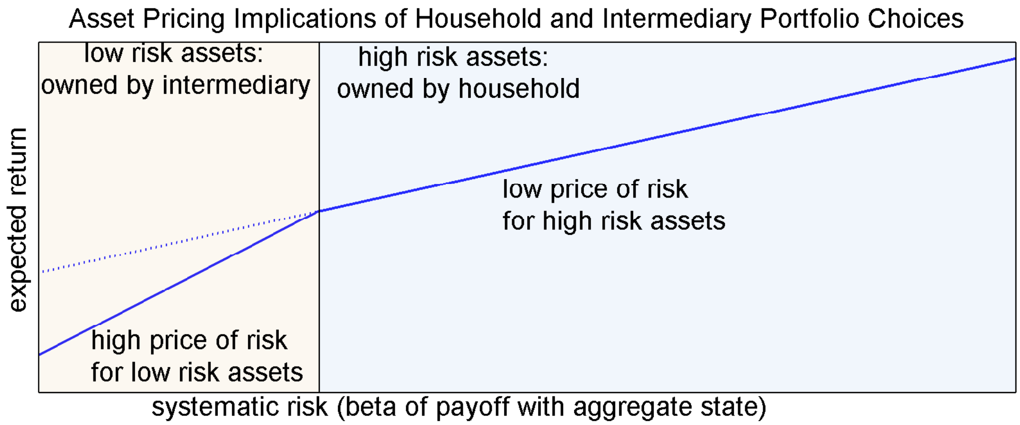
Corollary 2 *(intermediary's cost of capital determines asset prices) Every asset s in positive supply with payoff δ_s has price*

$$p_s = \overbrace{E\left[\frac{u'(c_2)}{u'(c_1)}\delta_s\right]}^{\text{household's willingness to pay for asset}} + \tag{9}$$

$$\max[0, \overbrace{E(\delta_s|bad)}^{\text{safe debt backed by asset}} \overbrace{\frac{v'(d)}{u'(c_1)}}^{\text{safety premium}} - \overbrace{\frac{u'\left(c_2^{good}\right)[E(\delta_s|good) - E(\delta_s|bad)]}{2u'(c_1)}}^{\text{equity required to purchase asset}} \overbrace{\frac{C'(e)}{(1+C'(e))}}^{\text{cost of equity}}] \tag{10}$$

If the expression in the max function is strictly positive, the intermediary values the asset more than the household and owns its entire supply in equilibrium. When $v'(d)$ and $C'(e)$ are both strictly positive, this is the case if and only if $\frac{E(\delta_s|good)}{E(\delta_s|bad)}$ is sufficiently small. It follows that there exists some cutoff $k^* > 1$ such that the intermediary owns all assets with $\frac{E(\delta_s|good)}{E(\delta_s|bad)} < k^*$ and the household owns all assets with $\frac{E(\delta_s|good)}{E(\delta_s|bad)} > k^*$.

This corollary presents a single expression which illustrates how asset prices are determined by the intermediary's cost of capital. First, equilibrium asset prices are the maximum of the willingness to pay of the household and intermediary. Because both agents face short sale constraints, they cannot short assets they consider overvalued and need not have the same willingness to pay for all assets. Second, the difference between the household's and intermediary's willingness to pay for an asset is a function of how much safe debt the asset backs. If an intermediary holds a portfolio which diversifies away idiosyncratic risk, it can issue a total of $E(\delta_s|bad)$ safe securities backed by the random cashflow δ_s. The residual payoff $\delta_s - E(\delta_s|bad)$ has 0 expected payoff in the bad state and an expected payoff of $E(\delta_s|good) - E(\delta_s|bad)$ in the good state. Because only bad state payoffs loosen the intermediary's solvency constraint for issuing riskless liabilities, the claim $E(\delta_s|good) - E(\delta_s|bad)$ which remains after idiosyncratic risk is diversified away must be financed entirely by issuing equity. The "cheapness" of safe debt (measured by $\frac{v'(d)}{u'(c_1)}$) and "costliness" of risky equity (measured by $\frac{C'(e)}{1+C'(e)}$) determine the intermediary's willingness to pay for an asset. The frictions which lead to a violation of Modgliani-Miller for financial intermediaries, biasing them towards issuing riskless debt on a highly levered balance sheet, are reflected in equilibrium asset prices.



As illustrated in the figure above, the portfolio choices of the household and intermediary lead to segmentation between the pricing of low and high systematic risk assets. For assets with $\frac{E(\delta_s|good)}{E(\delta_s|bad)} < k^*$, which are purchased by the intermediary, the price of systematic risk is strictly higher than for assets with $\frac{E(\delta_s|good)}{E(\delta_s|bad)} > k^*$ owned by the household. By the corollary above, increasing $E(\delta_s|good)$ and decreasing $E(\delta_s|bad)$ by one unit changes the price by $\frac{u'\left(c_2^{good}\right)-u'\left(c_2^{bad}\right)}{2u'(c_1)} < 0$ if the household owns asset s. For assets owned by the intermediary, the change in price is $\frac{u'\left(c_2^{good}\right)-u'\left(c_2^{bad}\right)}{2u'(c_1)} - \frac{v'(d)}{u'(c_1)} - \frac{u'\left(c_2^{good}\right)}{u'(c_1)}\frac{C'(e)}{1+C'(e)} < \frac{u'\left(c_2^{good}\right)-u'\left(c_2^{bad}\right)}{2u'(c_1)}$. These differing prices of systematic risk yield a "kinked" securities market line.

This segmentation in risk pricing illustrates the role of intermediaries in asset markets when other investors face borrowing constraints. In models with leverage constraints (e.g. Frazzini Pedersen 2014, Black 1972), high risk assets earn low risk adjusted returns since they are held by risk tolerant agents who cannot use leverage to take risk. Agents who are not borrowing constrained can hold levered portfolios of low risk assets in order to take risk and earn a better risk-return tradeoff. In my model, the natural agent to consider as borrowing unconstrained is a levered financial intermediary such as a bank. Because the riskless liabilities of intermediaries are demanded by households, issuing such liabilities is a uniquely cheap way for intermediaries to borrow. As a result, intermediaries are the marginal holders of low risk assets, earning a more attractive risk-return tradeoff than households. Empirically, financial intermediaries such as banks and insurance companies do in fact hold highly levered portfolios of low risk assets, suggesting this is a natural interpretation of the role such intermediaries play in asset markets.

Non-financial firm's problem This section shows how non-financial firms choose their optimal capital structure. The segmentation in asset prices in the previous section can be arbitraged by non-financial firms through their capital structure choices. As shown above, the intermediary is willing to pay more than the household for all securities which have sufficiently low undiversifiable risk. In order to issue perfectly safe assets which provide utility to households, intermediaries demand low risk assets. To meet this demand for low risk assets, non-financial firms divide their cashflows into a low risk debt security and a high risk equity security, which they respectively sell to the intermediary and to the household. As a result, the cost of capital of the intermediary, which determines its willingness to pay for securities, indirectly determines the cost of capital of the non-financial sector. The Modigliani-Miller violation for intermediaries, which is due to the household's demand for safe assets and the intermediary's cost of external equity, indirectly leads to a Modigliani-Miller violation for all non-financial firms.

Each non-financial firm $i \in [0,1]$ has cashflows x_i at time 2. The cashflows are subject to both aggregate and idiosyncratic shocks. In the good and bad aggregate states, x_i is respectively distributed according to $F(x_i|good)$ and $F(x_i|bad)$. Given the aggregate state, the realized cashflows of non-financial firms are conditionally independent. The only choice of the non-financial firm is the face value of debt D_i that it chooses to issue. The debt of firm i pays $x_i^D = \min(x_i, D_i)$, paying the full face value whenever possible and otherwise paying the firm's entire cashflow x_i. The equity of firm i pays the remaining cashflow after paying off the debt $x_i^E = x_i - \min(x_i, D_i) = \max(x_i - D_i, 0)$.

The problem of the non-financial firm is to maximize the total market value of its debt and equity, taking as given the asset prices implied by the behavior of the household and intermediary. Corollary 2 implies that the sum of the prices of the firm's debt and equity can be written as

$$p_E^i + p_D^i = E\frac{u'(c_2)}{u'(c_1)}x_i + \max\left(0, K_1 E\left(x_i^D|bad\right) - K_2\left[E\left(x_i^D|good\right) - E\left(x_i^D|bad\right)\right]\right) \tag{11}$$
$$+\max\left(0, K_1 E\left(x_i^E|bad\right) - K_2\left[E\left(x_i^E|good\right) - E\left(x_i^E|bad\right)\right]\right)$$

where $K_1 = \frac{v'(d)}{u'(c_1)} > 0$ and $K_2 = \frac{u'\left(c_2^{good}\right)}{2u'(c_1)}\frac{C'(e)}{(1+C'(e))} > 0$. The signs of these two constants reflect the fact that the intermediary is willing to pay more than the household for riskless payoffs but less for payoffs in the good state. If $K_1 = K_2 = 0$, which would occur if household and intermediary had the same willingness to pay for all securities, firm i's total market value would be independent of its capital structure. The fact that $p_E^i + p_D^i$ does depend on the face value of debt D_i illustrates how segmentation in asset markets leads to a violation of Modigliani-Miller for non-financial firms. This is related to (Baker Hoeyer Wurgler 2016), who study the capital structure implications of segmented debt and equity markets and provide supporting empirical evidence for such segmentation.

The firm chooses the face value of debt D_i to maximize its total market value $p_D^i + p_E^i$. If there is some D_i at which the intermediary buys one security issued by the firm and the household buys the other, it must be that $p_E^i + p_D^i$ is strictly greater than either investor's willingness to pay for the firm's total cashflows x_i. The firm will therefore choose such a D_i if one exists. If such a D_i is chosen optimally, it must therefore satisfy the first order condition

$$K_1 \Pr(x_i > D_i|bad) - K_2\left(\Pr(x_i > D_i|good) - \Pr(x_i > D_i|bad)\right) = 0 \tag{12}$$

since $\frac{\partial E\left(x_i^D|H\right)}{\partial D_i} = \frac{\partial E\min(x_i, D_i|H)}{\partial D_i} = \Pr(x_i > D_i|H) = -\frac{\partial E\left(x_i^E|H\right)}{\partial D_i}$ for $H = bad$ and $H = good$. This condition has the interpretation that a security which pays 1 when $x_i > D_i$ and 0 otherwise is of equal value to the household and the intermediary. Because a marginal increase in the face value D_i of debt increases the payout of debt only in states of the world where $x_i > D_i$, it must be the case that this marginal transfer of resources from equity to debt has no effect on firm i's total market value $p_E^i + p_D^i$.

The first order condition 12 uniquely determines the ratio $\frac{\Pr(x_i > D_i|good)}{\Pr(x_i > D_i|bad)}$. For this ratio to determine firm i's capital structure, there must be precisely one D_i for which 12 holds. I therefore impose the following regularity condition.[3]

Condition 3 *(i)* $\frac{\Pr(x_i > D|good)}{\Pr(x_i > D|bad)}$ *is a strictly increasing continuous function of D.*

(ii) $\Pr(x_i > 0|good) = \Pr(x_i > 0|bad) = 1$

(iii) $\lim_{D \to \infty} \frac{\Pr(x_i > D|good)}{\Pr(x_i > D|bad)} = \infty$

Condition 3 implies that as firm i increases its leverage, the systematic risk of its debt increases. As well as providing a unique solution to the first order condition 12 for any positive K_1 and K_2, this condition also implies that

$$\frac{E(\min(x_i, D_i)|good)}{E(\min(x_i, D_i)|bad)} < \frac{\Pr(x_i > D_i|good)}{\Pr(x_i > D_i|bad)} < \frac{E(\max(x_i - D_i, 0)|good)}{E(\max(x_i - D_i, 0)|bad)}.$$

It follows that when D_i is chosen to satisfy 12, firm i's debt has sufficiently low systematic risk that it is bought by the intermediary, while firm i's equity is bought by the household. This verifies that 12 uniquely determine's firm i's optimal capital structure. Plugging in the definitions of K_1 and K_2 yields the following proposition.

Proposition 4 *(optimal non-financial capital structure) If condition 3 is satisfied, the optimal face value of debt D_i for firm i is the unique D_i which solves*

$$\frac{v'(d)}{u'(c_1)} - \frac{u'\left(c_2^{good}\right)}{2u'(c_1)} \frac{C'(e)}{(1 + C'(e))} \left(\frac{\Pr(x_i > D_i|good)}{\Pr(x_i > D_i|bad)} - 1\right) = 0. \tag{13}$$

At this optimal capital structure, firm i's debt is bought by the intermediary and its equity is bought by the household.

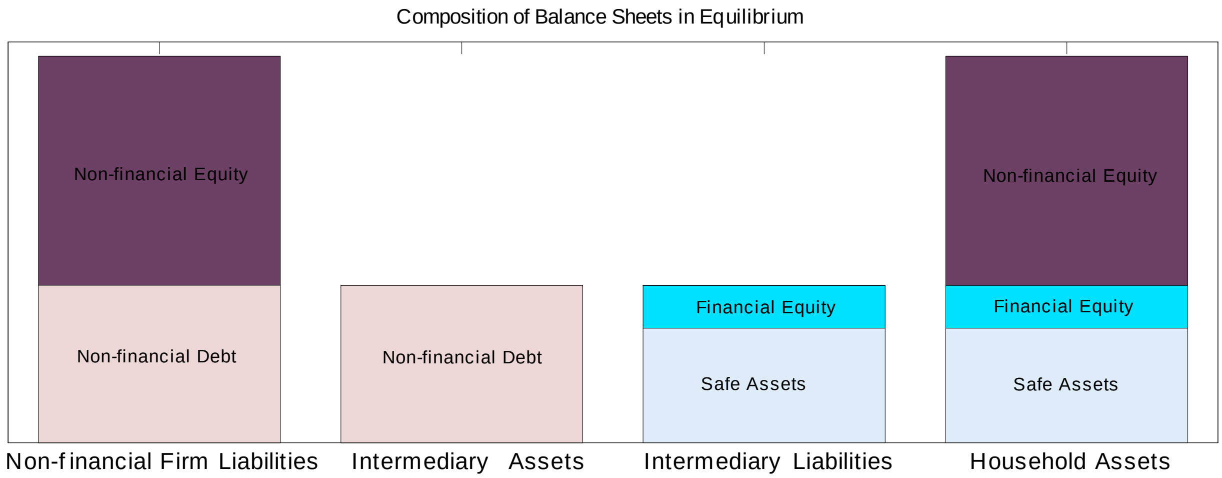
This proposition illustrates how the household's demand for safe assets (measured by $v'(d)$) and the intermediary's cost of equity finance (measured by $\frac{C'(e)}{1+C'(e)}$) jointly determine the op-

[3] Condition 3 (i) is equivalent to the monotone hazard ordering $\frac{f_{\delta_i}(D|good)}{\Pr(\delta_i > D|good)} < \frac{f_{\delta_i}(D|bad)}{\Pr(\delta_i > D|bad)}$ where $f_{\delta_i}(.|H)$ is the conditional density of δ_i given the aggregate state H. Such an assumption appears in (Simsek 2013) to ensure a unique solution in a related leverage determination problem.

timal capital structure of the non-financial sector. As shown in the previous section, the intermediary's cost of capital is reflected in segmented asset prices. The proposition extends this result by showing how the non-financial sector responds to this market segmentation. As a result, the intermediary's ability to issue cheap riskless debt implies that non-financial firms are also able to issue cheap debt as long as its exposure to systematic risk is low enough.

The proposition also provides a simple condition cross-sectional prediction for corporate capital structure. A security which pays $\Pr(x_i > D_i | good)$ in the good state and $\Pr(x_i > D_i | bad)$ in the bad state is of equal value to the household and the intermediary. If the intermediary's willingness to pay for risky securities increases, the non-financial firm will change its capital structure so that the ratio $\frac{\Pr(x_i > D_i | good)}{\Pr(x_i > D_i | bad)}$ increases as well. Under condition 3, this implies an increase in the firm's leverage. In addition, firms for whom $\frac{\Pr(x_i > D_i | good)}{\Pr(x_i > D_i | bad)}$ is greater at a given level of D_i will choose in equilibrium to issue less debt. This implies that firms with more cyclical cashflows chose to be less levered, consistent with the empirical findings of (Schwert and Strebulaev 2015).

A further implication of this proposition, illustrated in the figure below, is that it determines the composition of household, intermediary, and non-financial firm balance sheets. Households invest in the equity of both the financial and non-financial sector and also hold safe assets, since safe assets provide a direct flow of utility. Intermediaries, who supply the safe assets held by households, invest in the debt of the non-financial sector and must issue a buffer of costly equity to bear the risk in their portfolio of debt securities. Non-financial firms sell their debt to intermediaries and equity to households, arbitraging the differing risk premia for low and high risk securities. Note that the fact that equities are held by households while debt securities are held by the intermediary is an endogenous outcome and not directly assumed. Any agent is able to buy any security, but intermediaries are willing to pay more for debt securities and less for equities compared to households.



One final implication of this proposition is that it provides a rationale for the "credit spread puzzle" in debt securities and "low risk anomaly" in equities. Because of the capital structure choices of the non-financial sector, the "kink" in risk premia derived above endogenously leads to segmentation between debt and equity markets, with a greater price of systematic risk in the debt market. As shown in (Huang and Huang 2012), structural credit risk models when calibrated to data from equity markets underestimate the spreads on corporate bonds, referred to as the credit spread puzzle. Such a result can either be interpreted as a failure of existing structural models or taken as evidence that risk is priced more expensively in debt markets than in equity markets, as naturally occurs in my model. The high price of risk in debt markets occurs jointly with a low price of risk in equity markets. This rationalizes the "low risk anomaly" (e.g. Black, Jensen, Scholes 1972, Baker, Bradley, Taliaferro 2014), which finds that for simple measures of risk (such as covariance with returns on an equity market index), risk premia in equity markets are too small to jointly explain a low risk free rate and high expected return on equities. This naturally occurs in my model, since the demand for safe assets depresses the risk free rate. A more direct implication of my segmentation result is that a levered portfolio of bonds outperforms an equally risky unlevered portfolio of equities. This is demonstrated in (Frazzini Pedersen 2012), providing additional evidence for my model's asset pricing implications.

Equilibrium This section characterizes the model's equilibrium, endogenously determining the intermediary's cost of capital, which has been taken as given in the results above. To do so, I must impose the budget and resource constraints which have not been considered in the above results on asset prices and capital structure. Once these constraints are imposed, proposition 4 summarizes all additional information necessary to determine the model's unique equilibrium.

Because $v'(d) > 0$, the intermediary chooses to issue as much safe debt as possible, resulting in a binding solvency constraint. This binding constraint yields two straightforward implications. First, directly from the definition of the intermediary's solvency constraint, the bad state payoff of the intermediary's portfolio equals the quantity of safe debt it issues. Because the intermediary's portfolio is composed entirely of the debt of the non-financial sector as shown in proposition 4, the quantity of riskless assets is

$$d = \int_0^1 E\left(\min\left(x_i, D_i\right) | bad\right) di. \tag{14}$$

In addition, because the entire bad state payoff of the intermediary's portfolio is used to back riskless debt issuance, the intermediary's equity pays out only in the good state. The intermediary's asset portfolio has a good state payoff of $\int_0^1 \left[E\left(\min\left(x_i, D_i\right) | good\right) - E\left(\min\left(x_i, D_i\right) | bad\right)\right] di$ that is not paid to depositors. Assets owned by the intermediary satisfy condition 6 with equality, so the quantity e of equity the intermediary must issue is given by

$$u'\left(c_1\right)\left(1 + C'\left(e\right)\right) e = \frac{1}{2} u'\left(c_2^{good}\right) \int_0^1 \left[E\left(\min\left(x_i, D_i\right) | good\right) - E\left(\min\left(x_i, D_i\right) | bad\right)\right] di. \tag{15}$$

One interpretation of this result is that the systematic risk in the intermediary's asset portfolio (i.e. the mismatch between the risky payoff $\int_0^1 \min\left(x_i, D_i\right) di$ and the intermediary's riskless debt) is financed entirely by equity issuance. These two results combined with the optimal capital structure condition in proposition 4 uniquely determine the equilibrium.

Proposition 5 *(unique equilibrium) The model's unique equilibrium is determined by a face value of debt D_i for each non-financial firm i and quantity e of equity issued by the intermediary which solves the following 2 equations.*

$$\frac{v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)}{u'\left(c_1\right)} - \frac{u'\left(c_2^{good}\right)}{2u'\left(c_1\right)} \frac{C'\left(e\right)}{\left(1+C'\left(e\right)\right)} \left(\frac{\Pr\left(x_i > D_i|good\right)}{\Pr\left(x_i > D_i|bad\right)} - 1\right) = 0. \quad (16)$$

$$\left(1+C'\left(e\right)\right) eu'\left(c_1\right) = \frac{1}{2}u'\left(c_2^{good}\right)\int_0^1 \left[E\left(\min\left(x_i, D_i\right)|good\right) - E\left(\min\left(x_i, D_i\right)|bad\right)\right] di \quad (17)$$

Proof. Note that the ratio $\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)}$ is the same at the optimal debt D_i of each firm i. By condition 3, this single ratio uniquely determines the face values of debt D_i for each firm i. Since the intermediary's portfolio is composed entirely of the debt of non-financial firms, this also determines the portfolio's payoff $\int_0^1 \min\left(x_i, D_i\right) di$. Since $\left[E\left(\min\left(x_i, D_i\right)|good\right) - E\left(\min\left(x_i, D_i\right)|bad\right)\right]$ is nonnegative, there is a unique e which satisfies the intermediary's budget constraint for each value of $\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)} \geq 1$. Plugging this function $e\left(\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)}\right)$ into equation 16 yields a continuous monotone function of $\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)}$ which is positive at $\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)} = 1$ and negative at $\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)}$ sufficiently large. A unique equilibrium therefore exists. ■

This result illustrates in general equilibrium the implications of safety transformation by financial intermediaries. The first equation, taking as given the intermediary's cost of capital (determined by $v'\left(d\right)$ and $C'\left(e\right)$) determines the uniquely optimal capital structure of the non-financial sector by pinning down $\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)}$. Since the debt of the non-financial sector is equal to the asset portfolio of the intermediary, this in turn determines the quantity of riskless claims $\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di$ intermediaries are able to issue and quantity of systematic risk $\int_0^1 \left[E\left(\min\left(x_i, D_i\right)|good\right) - E\left(\min\left(x_i, D_i\right)|bad\right)\right] di$ to which intermediaries are exposed. The second equation, which implies that all systematic risk on intermediary balance sheets must be born by issuing equity, determines just how costly it is for intermediaries to bear the systematic risk to which they are exposed. The ability of intermediaries to issue riskless claims therefore depends both on the quantity of systematic risk they must bear (measured by $\int [E(\min(x_i, D_i)|good) - E(\min(x_i, D_i)|bad)]\, dxi$) and the costs they incur in order to do so (measured by $C'(e)$). As a result, the intermediary's cost of capital (which appears in the first equation) depends on the quantity of risk transformation it must perform (determined by the second), explaining why these two conditions jointly characterize the model's equilibrium.

2 Applications

Supply and Demand for Safe Assets The framework developed in the previous section can be used to understand the general equilibrium effects of changes in the supply and demand for safe assets. Because the model endogenously determines asset prices, intermediary portfolios and leverage, and the capital structure of the non-financial sector, all of these will adjust in order to clear the market for safe assets. I consider three comparative statics: an increase in the marginal utility v' of holding safe assets, the issuance of riskless assets μ backed by lump sum government taxes, and increase in the capital W_I of intermediaries. The first can be interpreted as an overall increase in the demand for safe assets, motivated by the increase in foreign demand for U.S. treasury securities which began after the 1998 Asian financial crisis. The second, issuance of riskless securities by the government, creates a supply of safe assets in addition to those issued by financial sector. The third, increasing intermediary capital, increases the supply of safe assets by increasing the amount of risk the intermediary is able to bear while remaining solvent.

The first comparative static, an increase in the demand for safe assets, is simply an overall increase in the function $v'(d)$ for all d. The effect of this is characterized by applying the implicit function theorem to the system of equations 16 and 17. At a given quantity d of deposits, this leads to a decrease in the risk free rate, providing the intermediary with cheaper debt financing. This incentivizes the intermediary to expand its balance sheet, which requires it to invest in riskier debt than it previously owned. The general equilibrium effects are summarized in the following proposition.

Proposition 6 *(safe asset demand) An increase in the demand for riskless securities, modeled as an increase in the function $v'(d)$ causes:*

1. An increase in the quantity d of riskless securities and intermediary equity issuance e.

2. A reduction in the risk free interest rate and increase in credit spreads, with an overall reduction in borrowing costs for all firms.

3. An increase in the leverage of the non-financial sector

The second comparative static, creating a supply μ of riskless securities backed by lump sum taxes, simply increases the supply of safe assets from the liability d issued by the intermediary to the sum $d+\mu$. This crowds out the intermediary's incentive to perform safety transformation by providing a supply of safe assets that do not lie on the intermediary's balance sheet. For any given quantity d of deposits, the safety premium $v'(d+\mu)$ is decreasing in μ. The effect of this decrease is therefore precisely the opposite of the increase in $v'(d)$ considered in the first comparative static. While the model in the previous section does not explicitly have government debt, riskless government debt can be mapped into the framework above by simply replacing $v'(d)$ with $v'(d+\mu)$.

Proposition 7 *(government debt supply) An increase in the supply μ of riskless securities issued by the government causes*

1. An increase in the quantity $d+\mu$ of total riskless securities, a decrease in riskless securities d issued by the intermediary, and decrease in intermediary equity e.

2. An increase in the risk free interest rate and compression of credit spreads, with an overall increase in borrowing costs for all firms.

3. A decrease in non-financial leverage.

The third comparative static, an increase in intermediary capital, reduces the intermediary's need to issue equity to bear risk. If the intermediary starts with a supply W_I of inside equity, its total equity is $W_I + e$. As a result, its budget constraint (equation 17) becomes

$$(1 + C'(e))(e + W_I)\, u'(c_1) = \frac{1}{2} u'\left(c_2^{good}\right) \int_0^1 \left[E\left(\min\left(x_i, D_i\right)|good\right) - E\left(\min\left(x_i, D_i\right)|bad\right)\right] di$$

while equation 16 determining the non-financial sector's capital structure remains unchanged. In effect, this can be viewed as a shift in the function $C'(.)$, where the marginal cost of equity issuance is $C'(e)$ when $W_I + e$ units of equity have already been issued. As a result, an increase in intermediary capital can be viewed alternatively as a decrease in the intermediary's cost of raising external equity finance. This induces the intermediary to expand its balance sheet and increases the supply of safe assets, since the cost of safety transformation is reduced.

Proposition 8 *(intermediary equity) An increase in intermediary inside equity W_I causes*

1. An increase in the quantity d of riskless securities and decrease in intermediary equity e.

2. An increase in the risk free interest rate and compression of credit spreads. Firms with sufficiently safe debt have an increase in borrowing costs, while firms with sufficiently risky debt have a decrease in borrowing costs.

3. An increase in non-financial leverage.

Proof. Appendix, where expressions are provided. ■

While the proofs of these propositions are somewhat tedious, much of the intuition can be seen in the special case where $C'(e) = C'$ and $v'(d) = v'$ are constant. In this case, the non-financial firm's optimal capital structure condition (equation 13) can be written as

$$1 + \frac{2v'}{u'\left(c_2^{good}\right)} \frac{[1 + C']}{C'} = \frac{\Pr\left(x_i > D_i|good\right)}{\Pr\left(x_i > D_i|bad\right)} \tag{18}$$

This expression determines the face value of debt D_i issued by each non-financial firm as a function of these the constants v' and C'. As v' increases (raising the household's willingness to pay for the intermediary's riskless liabilities) or C' decreases (reducing the cost of equity issuance required for the intermediary to take risk), the optimal quantity of debt D_i of each firm increases. Since the debt of the non-financial sector equals the asset portfolio of the financial sector, this implies that the intermediary must expand its balance sheet. In particular, this expansion requires the intermediary to invest in riskier assets than it previously did, since it already owns the most senior claims on the cashflows issued by non-financial firms. In this sense, my model is qualitatively consistent with the facts from the 2000s subprime boom. As foreigners bought larger amounts of U.S. government debt, the financial sector increased its size and invested in riskier assets in order to meet the growing demand for safe securities as documented in (Bernanke et al 2011). The resulting increase in the leverage of U.S. homeowners has a simple interpretation in terms of my model: intermediaries had to make more loans in order to expand their balance sheets, so households had to borrow more to clear the market.

An additional result, which cannot be seen in the special case above, is the fact that safe security issuance by the government crowds out the risk taking of the financial sector. A special case in which this can be seen is to assume C' is constant but that the household's quantity of riskless securities demanded $d+\mu$ is price inelastic. In this case, we have that if the government issues μ units of riskless securities backed by lump sum taxes, $d = \int_0^1 E\left[\min\left(x_i, D_i\right)|bad\right] di$ must decrease. This implies that by increasing μ, the government can decrease the size of intermediary balance sheets, which must in equilibrium decrease the amount and riskiness of non-financial debt issued. As can be seen from the condition above, this leads to a reduction in v' since $\frac{\Pr(x_i > D_i|good)}{\Pr(x_i > D_i|bad)}$ decreases, resulting in a fall in the risk free rate. Several other recent papers (e.g. Greenwood Hanson Stein 2015, Krishnamurthy Vissing-Jorgensen 2015, Bansal Coleman Lundblad 2011) have models (and empirical evidence) showing that safe security issuance by the government crowds out risk taking in the financial sector when a certain class of risky assets may only be owned by intermediaries. My model extends their results by showing that this leads to a decrease in the non-financial sector's leverage and a compression in credit spreads[4]. This extension relies crucially on the fact that in my model intermediaries are the marginal

[4]Strictly speaking, credit risk premia when C' are constant do not change. This result holds true whenever $C'' > 0$ and can be seen in closed form in the case where the intermediary has a supply of inside equity and cannot issue more.

investor in publicly traded debt securities, illustrating the implications of my model's emphasis on safety transformation by financial intermediaries.

Quantitative Easing This section extends the model above into a 3 period setting in order to study the effect of changes in government debt maturity, motivated by the Federal Reserve's use of quantitative easing in which short duration bank reserves were issued in order to purchase longer duration debt securities. The debt of non-financial firms and the government can either be short duration (paying off at time 2) or long duration (paying off at time 3), and financial intermediaries can choose to take on both credit and duration risk in their portfolio choice. The basic insight connecting this extension to the model above is that the intermediary's ability to remain solvent after a bad shock depends both on its exposure to credit and duration risk. As a result, shortening the duration of government debt behaves very similarly to an increase in the supply of government debt in the 2 period model, since duration risk is taken off of the intermediary's balance sheet. The good or bad aggregate shock in the 2 period model impacts cashflows at both times 2 and 3, and for tractability I assume that $\frac{u'(c_3)}{u'(c_2)}$ is held constant across realizations of the shock.

Time runs from $t = 1, 2, 3$. To preserve the structure of the 2 period model derived above, all aggregate uncertainty is realized at time 2.

Households have preferences

$$E\left(u\left(c_1\right) + \beta_2 u\left(c_2\right) + \beta_2 \beta_3 u\left(c_3\right)\right) + v\left(d\right)$$

where c_t denotes consumption at time t and d denotes the set of riskless securities owned at time 1 which pay off at time 2. Households are subject to the same frictions as above. At time 2, β_3 is realized to either equal β_{low} or $\beta_{high} > \beta_{low}$ with probability $\frac{1}{2}$ independently of the good or bad state aggregate shock. This is a simple approach to inducing randomness in the risk free rate at time 2 so that the price of long term debt fluctuates. The portfolio choice first order conditions at time 1 are analogous to those above

$$\begin{aligned} u'(c_1)\,p_{s,1} &\geq \beta_2 E u'(c_2)\,(p_{s,2} + \delta_{s,2}) \\ u'(c_1) &= (1 + i_{d,1})\,(\beta_2 E u'(c_2) + v'(d)), \end{aligned}$$

but in addition at time 2 they must satisfy

$$u'(c_2) = (1 + i_{d,2})\,\beta_3 u'(c_3)$$

where $i_{d,2}$ is the risk free rate at time 2. Since there is no aggregate risk at time 3, $(1 + i_{d,2})$ is the expected return on all assets from time 2 to time 3. It follows that for every asset, its price at time 2 is $p_{s,2} = \frac{1}{1+i_{d,s}} E_2 \delta_{s,3} = \beta_3 \frac{u'(c_3)}{u'(c_2)} E_2 \delta_{s,3}$ where E_2 is the conditional expectation given all information known at time 2.

The intermediary is identical as above, buying securities at time 1 in order to create riskless claims which pay off at time 2. It faces the same budget, solvency, and short sale constraints as in the two period model. Securities owned at time two are worth the sum of their dividend payments $\delta_{s,2}$ and their market prices $p_{s,2}$, so the intermediary's willingness to pay for an asset depends on its total value $\delta_{s,2} + p_{s,2}$ at time 2. Similar to the 2 period model above it solves the problem

$$\begin{aligned} & \max_{e,d,q_I(.)} E \frac{u'(c_2)}{u'(c_1)} \left(\left[\int (\delta_{s,2} + p_{s,2})\, q_I(s)\, ds - d \right] \right) + [-e - C(e)] \\ \text{subject to } e + \frac{d}{1+i_d} &= \int_0^1 p_{s,1} q_I(s)\, ds \text{ (budget constraint)} \\ \left(\int_0^1 (\delta_{s,2} + p_{s,2})\, q_I(s)\, ds - d \right) &\geq 0 \text{ in all states of the world (solvency constraint)} \\ & q_I(.) \text{ nonnegative (short sale constraint)} \end{aligned}$$

To characterize the intermediary's optimal behavior, I first determine in which state its solvency constraint binds. Note that the intermediary's solvency constraint can only bind when the payoff its portfolio $\int_0^1 (\delta_{s,2} + p_{s,2})\, q_I(s)\, ds$ is the smallest. To determine which when this occurs, note that since $p_{s,2} = \beta_3 \frac{u'(c_3)}{u'(c_2)} E_2 \delta_{s,3}$, the payoff of the intermediary's portfolio can be written as $\int_0^1 \left(\delta_{s,2} + \beta_3 \frac{u'(c_3)}{u'(c_2)} E_2 \delta_{s,3}\right) q_I(s)\, ds$. Since all assets have lower expected returns after the bad credit event, it follows that the intermediary's solvency constraint can be written as $\int_0^1 \left(E(\delta_{s,2}|bad) + \beta_3^{low} \frac{u'(c_3)}{u'(c_2)} E(\delta_{s,3}|bad)\right) q_I(s)\, ds - d \geq 0$, since the solvency constraint only binds when $\beta_3 = \beta_3^{low}$ and the bad credit event occurs. It follows that the intermediary's first order conditions can be written as

$$\begin{aligned} u'(c_1)(1 + C'(e))\, p_{s,1} &\geq \beta_2 E u'(c_2)(p_{s,2} + \delta_{s,2}) + \lambda \left(E(\delta_{s,2}|bad) + \beta_3^{low} \frac{u'(c_3)}{u'(c_2)} E(\delta_{s,3}|bad)\right) \\ (1 + C'(e))\, u'(c_1) &= (1 + i_{d,1}) \left[\beta_2 E u'(c_2) + \lambda\right], \end{aligned}$$

where λ is the multiplier on the solvency constraint. These first order conditions reflect the fact that the intermediary finds payoffs in the worst state of the world (where both β_3 is low and credit risk is bad) especially valuable, since only these payoffs increase its ability to issue riskless securities demanded by households.

As in the 2 period model, imposing that both agents are willing to buy a riskless security at time 1 at the same interest rate i_d can be used to remove the Lagrange multiplier λ from the above expressions. The difference in the two agents' willingness to pay for a security with payoffs $(\delta_{s,2}, \delta_{s,3})$ is

$$\begin{aligned} & v'(d) \left[E(\delta_{s,2}|bad) + \beta_3^{low} \frac{u'(c_3)}{u'(c_2)} E(\delta_{s,3}|bad)\right] - \frac{C'(e)}{1 + C'(e)} * \\ & E u'(c_2) \left[\delta_{s,2} + \beta_3 \frac{u'(c_3)}{u'(c_2)} \delta_{s,3} - E(\delta_{s,2}|bad) + \beta_3^{low} \frac{u'(c_3)}{u'(c_2)} E(\delta_{s,3}|bad)\right]. \end{aligned}$$

Given the willingness to pay of the household and intermediary for assets, the non-financial firms choose the maturity $(D_{i,2}, D_{i,3})$ of their debt. For simplicity, I consider the case where the firm's debt at time t is backed only by its cashflows at time t.[5] Under this assumption, the firm's debt pays $\min(x_{i,2}, D_{i,2})$ at time 2 and $\min(x_{i,3}, D_{i,3})$ at the 3. By an identical argument to that characterizing the firm's optimal capital structure in the 2 period model, the firm's debt issuance $(D_{i,2}, D_{i,3})$ is characterized by

$$\begin{aligned} v'(d) - \frac{C'(e)}{1+C'(e)} \frac{u'\left(c_2^{good}\right)}{2} \left[\frac{[\Pr(x_{i,2} > D_{i,2}|good)]}{[\Pr(x_{i,2} > D_{i,2}|bad)]} - 1\right] &= \\ v'(d)\left[\frac{u'(c_3)}{u'(c_2)} \beta_3^{low} \Pr(x_{i,3} > D_{i,3}|bad)\right] - \frac{C'(e)}{1+C'(e)} * \\ Eu'(c_3)\left[\beta_3 \{x_{i,3} > D_{i,3}\} - \beta_3^{low} \Pr(x_{i,3} > D_{i,3}|bad)\right] = 0. \end{aligned}$$

where $x_{i,2}$ and $x_{i,3}$ are assumed to satisfy the same regularity condition 3 as in the 2 period model.

Together with the intermediary's budget constraint,

$$(1 + C'(e))\, eu'(c_1) = \beta Eu'(c_2) \int_0^1 \left[\begin{array}{c} [\min(x_{i,2}, D_{i,2}) - E[\min(x_{i,2}, D_{i,2})|bad]] \\ +\frac{u'(c_3)}{u'(c_2)}\left[\beta_3 \min(x_{i,3}, D_{i,3}) - \beta_3^{low} E(\min(x_{i,3}, D_{i,3})|bad)\right] \end{array}\right] di$$

this characterization of the choice of corporate debt maturity determines the model's unique equilibrium. This system of equations allows me to prove the following proposition, illustrating the effects of quantitative easing.

Proposition 9 *(quantitative easing) If the government issues short term debt in order to repurchase either long term debt or risky assets:*

(i) The safe asset premium $v'(d)$ decreases, so the risk free rate $i_{d,1}$ at time 1 increases.

(ii) Credit and duration risk premia decrease.

(iii) The issuance of both short and long term debt by the non-financial sector decreases.

[5] An alternative and more realistic model of corporate debt payout would be to allow the firm to borrow against the value of its equity at time 2 in order to avoid default subject to the constraint that new issuance must be junior to existing debt.

Proof. Appendix. ■

The basic intuition for the effects of quantitative easing is that both duration and credit risk held on intermediary balance sheets must be financed by costly equity issuance. As a result, decreasing the amount of duration risk the intermediary must taken in order to issue a given quantity of riskless securities is analogous to an increase in the overall supply of safe securities issued by the government and has similar effects. This can be seen from the fact that if the intermediary purchases a riskless bond paying off at time 3, it backs $\beta_3^{low}\frac{u'(c_3)}{u'(c_2)}$ units of short term riskless debt issuance, while the bound is worth $\beta_3^{high}\frac{u'(c_3)}{u'(c_2)}$ if $\beta_3 = \beta_3^{high}$ is realized at time 2. The remaining payoff $\left(\beta_3^{high} - \beta_3^{low}\right)\frac{u'(c_3)}{u'(c_2)}$ which occurs when $\beta_3 = \beta_3^{high}$ must therefore be financed by equity issuance by the intermediary. As a result, riskless payoffs in period 3 held by the intermediary must be financed by a mix of debt and equity issuance, while short term riskless assets can be held with 100% leverage. Shortening the duration of government debt therefore reduces the intermediary's need to issue equity, similar to an overall increase in the supply of government issued safe assets. As in the two period model, this crowds out the intermediary's incentive to take risk and compresses risk premia in credit markets.

One advantage of my approach to the effects of quantitative easing over existing papers is that no assumptions about the "moneyness" of long term debt are required. Several papers (e.g. Greenwood Hanson Stein 2015, Williamson 2016) proceed under the assumptions that government debt of various maturities provide non-pecuniary services as either a source of pledgeable collateral or money-like transaction services but that long term debt is less useful in this regard. Under such an assumption, an aggregate liquidity shortage is mechanically decreased by a shortening of the duration of government debt. In my setting, the amount of "money services" provided by a long term bond is a function of how much short term riskless debt it is able to back when held as part of a diversified portfolio. A useful implication of my modelling approach is that the duration risk premium for government debt has a natural interpretation in terms of the risk premium intermediaries require in order to borrow short and lend long. Such a duration premium exists because intermediaries endogenously choose to expose themselves to interest rate risk. This provides a rationale for theories of the term structure which emphasize the balance sheets of arbitrageurs (Vayanos Vila 2009) The market price of duration risk decreases as either credit or duration risk is taken off of intermediary balance sheets, illustrating how quantitative easing can flatten the yield curve and compress credit spreads.

The Zero Lower Bound This section embeds the two period model developed above into

a simple framework with nominal rigidities in order to study the role of financial intermediaries at the zero lower bound. To maintain tractability, I make the extreme assumption that goods prices are perfectly rigid, following the original liquidity trap analysis of (Krugman 1998). Given this price rigidity, I assume that the central bank sets the interest rate i_d subject to the zero lower bound constraint $i_d \geq 0$ which binds since households will swap riskless bonds for cash when interest rates are negative. The household's optimality condition for investing in riskless securities

$$u'(c_1) = (1 + i_d)\left[E\left(u'(c_2) + v'(d)\right)\right]$$

determines consumption at time 1 as a function of c_2, d, and i_d. At time 1, the total supply of consumption resources available need not be consumed, since the goods market is in disequilibrium. Whenever the real interest rate i_d which would occur in a flexible price equilibrium (the natural rate) is negative, the binding zero lower bound constraint reduces c_1 to a quantity strictly less than the amount of output available.

When the natural is negative and nominal rates are zero, output at time 1 can be stimulated by reducing the premium $v'(d)$ on riskless securities. This can be seen from the fact that when i_d is fixed, $u''(c_1)\frac{\partial c_1}{\partial d} = v''(d)$ and u'' has the same sign as v''. As a result, increasing the supply of safe assets stimulates demand and output. This result is similar to the findings in (Caballero Farhi 2016). Because my framework connects the supply of riskless assets d to the risk taking capacity of the financial sector, my model allows me to extend this existing work by considering aggregate demand effects of policies which focus on the balance sheets of financial intermediaries.

To analyze the effects of policy at the zero lower bound, I first develop a tractable version of the model's equilibrium conditions which allows for output at time 1 to be demand constrained. The equilibrium conditions which characterize the flexible price version of the model's equilibrium still hold, but they no longer characterize the equilibrium since c_1 is no longer given. Using the household's optimality condition for investing in riskless assets to substitute for $u'(c_1)$ and setting $i_d = 0$ yields the following system of equations

$$v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right) + \frac{u'\left(c_2^{good}\right)}{2}\left[\frac{1}{\left(1+C'(e)\right)} - 1\right]\left(\frac{\Pr\left(x_i > D_i|good\right)}{\Pr\left(x_i > D_i|bad\right)} - 1\right) = 0.$$

$$\begin{aligned} & \left(1+C'(e)\right) e\left[Eu'(c_2) + v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)\right] \\ = & \frac{1}{2}u'\left(c_2^{good}\right)\int_0^1 \left[E\left(\min\left(x_i, D_i\right)|good\right) - E\left(\min\left(x_i, D_i\right)|bad\right)\right] di \end{aligned}$$

which characterizes equilibrium with the economy stuck at the zero lower bound. As above increasing the demand for safe assets or reducing the supply increases the premium $v'(d)$ on safe securities determined in equilibrium. However, because the interest rate i_d cannot move, this implies that $u'(c_1)$ must increase in order to ensure asset markets clear. This yields a loss in period one output c_1 due to a shortage of demand.

Proposition 10 *(zero lower bound) Output at time 1 is decreasing in the demand for safe assets and is increasing in the supply of government issued safe assets and in intermediary capital.*

Proof. Appendix. ■

3 Dynamic model

In order to understand quantitatively the connection between financial intermediaries and the pricing of credit risk, I embed the above framework in an infinite horizon model. As above, intermediaries are the marginal holders of debt while households are the marginal holders of equity, since a portfolio of debt securities is the safest portfolio an intermediary can buy. One key new feature appears quantitatively- the degree of segmentation between debt and equity markets is highly nonlinear in intermediary capitalization. In states of the world where intermediaries are well capitalized, asset prices in debt and equity markets imply roughly the same risk-return tradeoff. When intermediaries are poorly capitalized, debt and equity markets are very segmented, and large arbitrage opportunities exist for any agent who is able to trade both in the stock and the bond market. This is consistent with the fact that pricing anomalies such as the CDS-bond basis and covered interest parity violations tend to be largest in periods of financial distress. One direct implication of this time varying segmentation between stock and bond markets is that structural credit risk models, which attempt to predict credit spreads using information from equity markets, should perform particularly poorly when intermediaries are poorly capitalized.

The key reason why the segmentation between debt and equity markets is highly nonlinear in intermediary capitalization is that the intermediary's solvency constraint is only occasionally binding. In "crisis times", an intermediary at time t buys just enough assets to back its deposits at time $t+1$ given the worst possible shock, satisfying the solvency constraint with a minimum of capital. However, in "normal times" intermediaries will hold strictly more assets than are needed to remain solvent, insuring against the possibility of future solvency constraints. As can be seen in my quantitative results below, crisis times only occur when intermediary capital is sufficiently low. When intermediary capital reaches the point when the solvency constraint starts to bind, risk premia in debt markets become extremely sensitive to further decreases in intermediary capital, while they are not so sensitive in normal times.

To illustrate the dynamic implications of the mechanisms studied in the two period model, I adhere as closely as possible to the existing literature. In particular, my model of financial intermediaries follows (Gertler Kiyotaki 2010, Gertler Karadi 2011), who provide a microfoundation for forcing intermediaries to pay a fraction of their wealth as dividends each period. The fact that intermediaries are forced to pay dividends is important to ensure the model has a nontrivial steady state, since otherwise intermediaries will gradually save their way out of financial constraints via retained earnings. Other than this detail, my intermediaries have the same features as those in the two period model. In particular, they face solvency constraints which restrict the quantity of deposits they can issue, short sale constraints on the securities they purchase, and costs of issuing outside equity. For tractability, I choose functional forms for these various frictions so that only the ratio of intermediary wealth to output, rather than each variable separately, is the unique state variable that must be tracked to solve the model.

To solve the model, I adapt some tools from the literature on general equilibrium asset pricing in incomplete markets. My solution technique for a Markovian equilibrium is based on the recursive procedure of (Dumas Lyasoff 2012), which uses time varying welfare weights as state variables in a problem with multiple households. While there is only one household in my model, the intermediary's shadow value of internal funds behaves similarly to a welfare weight. In a complete market, this shadow value cannot vary over time, while in my setting it is strictly decreasing and convex in intermediary wealth. While (Dumas Lyasoff 2012) does not consider short sale or solvency constraints, the duality techniques of (Cvitanic Karatzas 1992) for portfolio constraints are easily adapted so that the method allows for my model's constraints on portfolio choice. The literature on the asset pricing implications of solvency constraints (Chien Lustig 2010, Gottardi Kubler 2015), where time varying welfare weights are used heavily, focuses on the case where arbitrary Arrow-Debreu securities can be traded. My model therefore broadens the set of questions to which these tools can be applied and draws connections between this literature and financial intermediation.

Model details Time is infinite and runs from $t = 1, ...\infty$. At each period, a binary shock s_t is realized to be "good" or "bad". A set of securities indexed by $k \in [0, 1]$ are available for trade at time t which pay $\delta_k = 1$ after the good shock or $\delta_k = k$ after the bad shock in supply $q_t(k)$. The total payoff of these securities $y_t = \int_0^1 \delta_k q_{t-1}(k)\,dk$ equals the total output of the economy at time t.

Households maximize expected utility $E\sum_{t=0}^{\infty}\beta^t(u(c_t)+v(d_t))$ over consumption c_t and deposits d_t. At time t, they have wealth W_t which can be consumed or invested. They can invest either in a set of securities indexed by $k \in [0,1]$ at price $p_{k,t}$ or in deposits which yield the interest rate $i_{d,t}$ as well as the extra utility $v(d_t)$ incurred directly from holding them. This yields the budget constraint $W_t = c_t + \int_0^1 p_{k,t}q_{C,t}(k)\,dk + \frac{d_t}{1+i_{d,t}}$. The household's wealth at time $t+1$ is the sum of payoffs from securities it owns plus deposits $W_{t+1} = \int_0^1 \delta_{k,t+1}q_{H,t}(k)\,dk + d_t$. The household's problem can be written as

$$\max_{\left\{e_t,d_t,q_{H,t}(.)\right\}_{t=1}^{\infty}} E\sum_{t=0}^{\infty}\beta^t(u(c_t)+v(d_t)) \text{ subject to}$$

$$\begin{aligned}
c_t &= W_t - \int_0^1 p_{k,t}q_{H,t}(k)\,dk - \frac{d_t}{1+i_{d,t}} \\
W_{t+1} &= \int_0^1 \delta_{k,t+1}q_{H,t}(k)\,dk + d_t \\
& \quad q_{H,t}(.) \text{ nonnegative}
\end{aligned}$$

In my numerical simulations, I use the functional forms

$$u(c_t) = c_t^{1-\gamma}$$

Similar to the two period model above, the household has first order conditions

$$\begin{aligned}
u'(c_t)\,p_{k,t} &\geq \beta E_t u'(c_{t+1})\,\delta_{k,t+1} \\
u'(c_t) &= (1+i_{d,t})\left[\beta E_t u'(c_{t+1}) + v'(d_t)\right]
\end{aligned}$$

where any asset k held in positive quantity at time t must satisfy its first order condition with equality. The interpretation of these first order conditions is identical to that in the two period model. Any asset k held in positive quantity by the household must satisfy the consumption Euler equation. However, deposits, because they yield the extra marginal utility $v'(d_t)$, trade at a strictly lower interest rate $i_{d,t}$ than the risk free rate implied by the household's marginal utility of consumption. As above, this provides the intermediary with a cheap source of riskless debt finance.

Intermediaries are publicly traded firms who maximize shareholder value, like in the two period model. At time t, they start with wealth W_t^I and can raise funds by issuing additional equity at a cost $C(e_t)$ or by issuing deposits at the going rate $i_{d,t}$. These funds can be invested in publicly traded securities. As described above, intermediaries must pay a minimum dividend of γW_t^I at time t. They also face a solvency constraint like in the two period model which requires the portfolio they buy at time t to always be able to back their promised deposit liabilities d_t. Their problem can be written as

$$
\begin{aligned}
& \max_{\{e_t, d_t, q_{I,t}(.)\}_{t=1}^{\infty}} E \sum_t \beta^t u'(c_t)\left[-e_t - C(e_t) + \gamma_t\right] \\
& \text{subject to } W_t^I + e_t + \frac{d_t}{1+i_{d,t}} = \int_0^1 p_{k,t} q_{I,t}(k)\, dk + \gamma_t, \quad \text{(period budget constraint)} \\
& W_{t+1}^I = \int_0^1 \delta_{k,t+1} q_{I,t}(k)\, dk - d_t \text{ (definition of wealth)} \\
& \left(\int_0^1 \delta_{k,t+1} q_{I,t}(k)\, dk - d_t\right) \geq 0 \text{ with probability 1 (solvency constraint)} \\
\gamma_t \geq \; & \gamma W_t^I \text{ (dividend payout constraint)} \\
& q_{I,t}(.) \geq 0 \text{ (short sale constraint).}
\end{aligned}
$$

To characterize the intermediary's optimal behavior, note first that it will only voluntarily pay dividends when $C'(e_t) = 0$, in which case it could choose $e_t < 0$ instead of $\gamma_t > 0$. It is without loss of generality to therefore assume $\gamma_t = \gamma W_t^I$. As in the two period model, let λ_t be the multiplier on the solvency constraint at time t. This constraint only binds after bad shocks, so the problem can be written in Lagrangian form as

$$\max_{\{e_t,d_t,q_{I,t}(.)\}_{t=1}^{\infty}} E\sum_t \beta^t u'(c_t)\left[-e_t - C(e_t) + \gamma W_t^I + \lambda_t\left(\int_0^1 kq_{I,t}(k)\,dk - d_t\right)\right]$$

$$\text{subject to } W_t^I(1-\gamma) + e_t + \frac{d_t}{1+i_{d,t}} = \int_0^1 p_{k,t}q_{I,t}(k)\,dk, \text{ (period budget constraint)}$$

$$W_{t+1}^I = \int_0^1 (p_{k,t+1}+\delta_{k,t+1})\,q_{I,t}(k)\,dk - d_t \text{ (definition of wealth)}$$

$q_I(.)$ nonnegative (short sale constraint).

The first order conditions are

$$\begin{aligned} u'(c_t)(1+C'(e_t))\,p_{k,t} &\geq \beta E_t\left[\gamma(1+C'(e_{t+1})) + (1-\gamma)\right]u'(c_{t+1})\,\delta_{k,t+1} \\ &\quad +\lambda_t k \\ u'(c_t)(1+C'(e_t)) &= (1+i_d)\left(\beta E_t\left[\gamma(1+C'(e_{t+1})) + (1-\gamma)\right]u'(c_{t+1}) + \lambda_{s,t}\right) \end{aligned}$$

where the inequality for the purchase of asset k must be an equality if the asset is held in positive quantity, $q_{I,t}(k) > 0$. The multiplier λ_t on the intermediary's solvency constraint at time t satisfies the complementary slackness condition

$$\lambda_t\left(\int_0^1 kq_{I,t}(k)\,dk - d_t\right) = 0$$

which ensures it is only nonzero when the solvency constraint binds.

To understand these first order conditions, note that at time t, the intermediary's value of internal funds is $(1+C'(e_t))$, since this is the marginal cost of raising an extra unit of equity. If these newly raised funds are invested in securities, only a fraction γ of the proceeds can be re-invested at time $t+1$. The remaining fraction $(1-\gamma)$ must be paid out as dividends. The intermediary's marginal value of cashflows received at time $t+1$ is therefore $\gamma(1+C'(e_{t+1})) + (1-\gamma)$, since $1+C'(e_{t+1})$ is its marginal value of internal funds at time $t+1$. If the intermediary chooses to pay more than γW_{t+1} in dividends, it must be the case that $C'(e_{t+1}) = 0$, so $1 = \gamma(1 + C'(e_{t+1})) + (1-\gamma) = 1 + C'(e_{t+1})$ is still a valid expression for the marginal value of payoffs at time $t+1$.

In equilibrium, because intermediaries choose to issue more equity when poorly capitalized, their value of internal funds is higher when their capital is low. This endogenously increases the intermediary's risk aversion above that of its shareholders. In addition to providing funds at time $t+1$, securities purchased at time t also loosen the intermediary's time t solvency constraint if it is binding. Since the intermediary can only issue deposits up to the worst possible realization of its portfolio $\int_0^1 kq_{I,t}(k)\,dk$, only the payoffs and resale of securities in this bad state are able to loosen the solvency constraint. The value of bad state payoffs to loosen the intermediary's solvency constraint is a second mechanism which adds to the intermediary's endogenous risk aversion, and in states of the world where the solvency constraint binds this channel can be quantitatively much more important. To summarize, the intermediary's optimal behavior implies that $\frac{u'(c_{t+1})[\gamma(1+C'(e_{t+1}))+(1-\gamma)]+\lambda_{s,t}\{s_{t+1}=bad\}}{u'(c_t)[1+C'(e_t)]}$ is a valid pricing kernel for all asset k purchased by the intermediary and for deposits.

The optimal behavior of the household and intermediary yield a characterization of equilibrium which is similar to the two period model. As above, since both agents participate in the deposit market, they must trade deposits at the same interest rate. Also, a cutoff asset k_t^* exists for which both agents are willing to pay the same price. Safer assets with $k > k_t^*$ are held at time t by the intermediary while riskier assets with $k < k_t^*$ are held by the household. Finally, the intermediary's budget must be sufficient to purchase all assets with $k > k_t^*$. These three equilibrium conditions uniquely characterize the two period model's equilibrium, while in the infinite horizon model they jointly define an operator whose fixed point is an equilibrium.[6] To see this, suppose that in a Markovian equilibrium, the intermediary's shadow value of internal funds is the function $\lambda(W, c)$. The fact that both agents invest in or issue deposits at the rate $i_{d,t}$ at time t implies

[6]While I do not formally prove that the intermediary's portfolio can be charecterized by a cutoff asset, it is possible to check ex-post after solving the model numerically if this is indeed the case. My numerical solution passes this test.

$$\begin{aligned}\frac{1}{1+i_{d,t}} &= \beta E_t \frac{u'(c_{t+1})+v'(d_t)}{u'(c_t)} = \beta E_t \frac{u'(c_{t+1})\left[\gamma\left(1+C'(e_{t+1})\right)+(1-\gamma)\right]+\lambda_{s,t}}{u'(c_t)\left(1+C'(e_t)\right)} \\ &= \beta E_t \frac{u'(c_{t+1})\left[\gamma\lambda\left(W_{t+1},c_{t+1}\right)+(1-\gamma)\right]+\lambda_s\left(W_t,c_t\right)}{u'(c_t)\lambda\left(W_t,c_t\right)}.\end{aligned}$$

Similarly, if both agents are willing to pay the same price for asset k_t^* at time t, then

$$\begin{aligned}u'(c_t)\,p_{k_t^*,t} &= \beta E_t u'(c_{t+1})\left(p_{k_t^*,t+1}+\delta_{k_t^*,t+1}\right) \\ &= \beta E_t \frac{\left[\gamma\left(1+C'(e_{t+1})\right)+(1-\gamma)\right]u'(c_{t+1})}{\left(1+C'(e_t)\right)}\left(p_{k_t^*,t+1}+\delta_{k_t^*,t+1}\right)+\lambda_{s,t}\min_t\left(p_{k_t^*,t+1}+\delta_{k_t^*,t+1}\right) \\ &= \beta E_t \frac{\left[\gamma\lambda\left(W_{t+1},c_{t+1}\right)+(1-\gamma)\right]u'(c_{t+1})\left(p_{k_t^*}\left(W_{t+1},c_{t+1}\right)+\delta_{k_t^*,t+1}\left(W_{t+1},c_{t+1}\right)\right)}{\lambda\left(W_t,c_t\right)} \\ &\quad +\beta\lambda_{s,t}\left(W_t,c_t\right)k_t^*\end{aligned}$$

Finally, since $\beta\frac{\gamma(1+C'(e_{t+1}))+(1-\gamma)}{(1+C'(e_t))} = \beta\frac{[\gamma\lambda(W_{t+1},c_{t+1})+(1-\gamma)]}{\lambda(W_t,c_t)}$ is the pricing kernel for assets owned by the intermediary at time t if the solvency constraint is not binding, the intermediary's budget constraint can be written as

$$\beta E_t \frac{u'(c_{t+1})}{u'(c_t)}\frac{\left[\gamma\lambda\left(W_{t+1},c_{t+1}\right)+(1-\gamma)\right]}{\lambda\left(W_t,c_t\right)}\left(\int_{k^*}^{1}\left(p_{k_t^*,t+1}+\delta_{k_t^*,t+1}\right)dk-d_t\right)=\left[W_t+e_t\right]$$

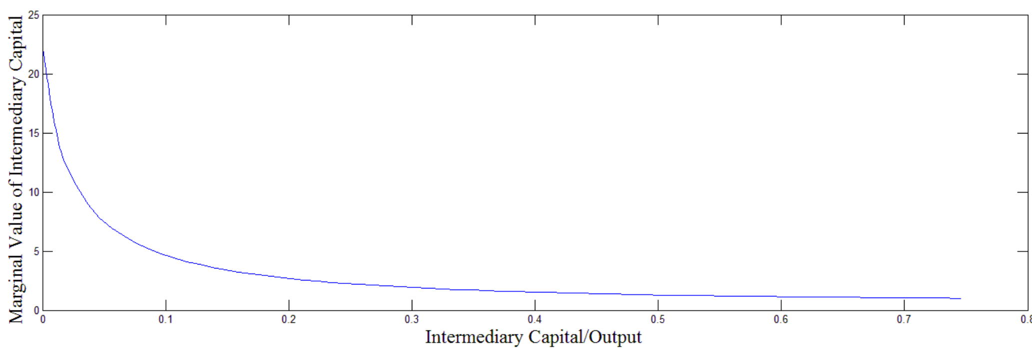
The complementary slackness condition $\lambda_t \min_t\left(\int_{k^*}^{1}\left(p_{k_t^*,t+1}+\delta_{k_t^*,t+1}\right)dk-d_t\right)=0$ implies that this expression for the intermediary's budget constraint is also valid when the solvency constraint does bind, because the portfolio $\left(\int_{k^*}^{1}\left(p_{k_t^*,t+1}+\delta_{k_t^*,t+1}\right)dk-d_t\right)$ has zero payoffs in the worst possible realization of shocks at time $t+1$, and only payoffs in this state loosen the solvency constraint.

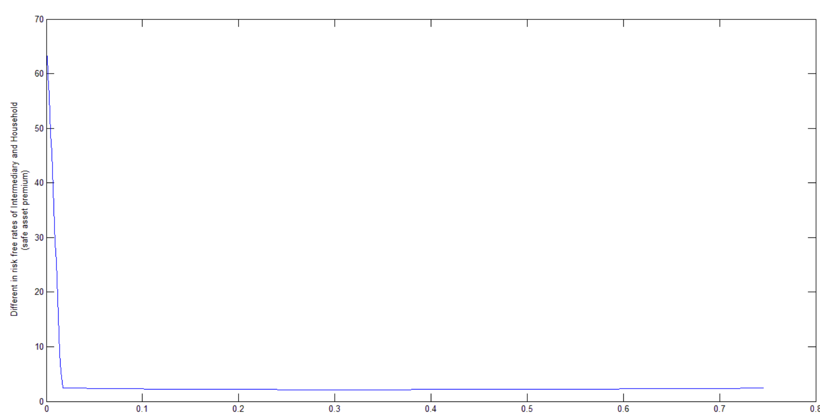
To solve for a Markov equilibrium of the dynamic model, I must find some function $\lambda(W,c)$ which satisfies the above three equilibrium conditions at every possible initial value of (W,c). For tractability, I choose functional forms so that the function $\lambda(W,c)$ depends only on one state variable rather than two. As is common in the literature, the intermediary wealth to output ratio $\frac{W}{c}$ is a valid state variable if $\lambda(W,c)$ is a homogenous function. For this to be the case, I choose the functional forms

$$
\begin{aligned}
u(c_t) &= c_t^{1-\gamma}; \gamma = 3 \\
v(d_t) &= d_t^{1-\gamma} \\
C(e_t) &= 100\left(\frac{e_t}{y_t}\right)^2 \\
s_t(k) &= y_t \\
\gamma &= .1 \\
\frac{y_{t+1}}{y_t} &= 1.1 \text{ if } s_{t+1} = \text{good}, .5 \text{ if } s_{t+1} = \text{bad}
\end{aligned}
$$

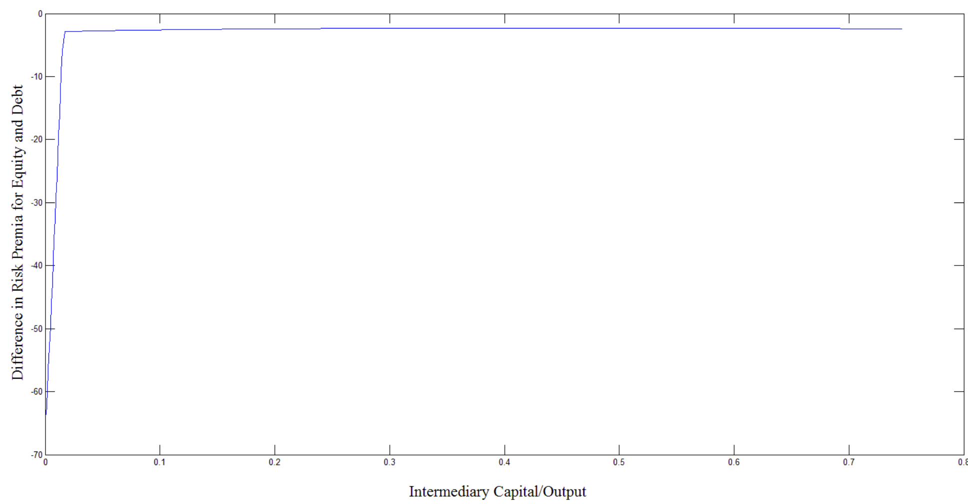
To solve the infinite horizon model numerically, I solve a finite horizon approximation and send the number of periods to infinity. By construction, such a solution cannot have rational bubbles, which may or may not exist in other Markov equilibria of the model. The procedure for solving the model is to iteratively construct functions $\lambda_n(W,c)$ which maps the intermediary's wealth and the economy's output to the intermediary's shadow value of internal funds with n periods remaining, starting with $\lambda_1(W,c) = 1$. If $\|\lambda_n - \lambda_{n+1}\|_\infty$ gets small as n increases, this approximates a solution to the infinite horizon model.

My numerical results show a strong nonlinearity in the degree of segmentation between debt and equity markets. The first plot, shows the intermediary's shadow value of internal funds as a function of its capitalization. This shows that the intermediary's value of internal funds is decreasing and convex in intermediary capital. However, the following two plots show a much more severe nonlinearity in the asset pricing implications of my model, which is due to the binding of the intermediary's solvency constraint. My second plot shows the safe asset premium, which is the difference between the risk free rate and what it would be without the utility of holding safe assets $v'(d)$. The kink point in the plot at which it grows steeply as intermediary capital decreases is precisely the point at which the intermediary's solvency constraint first begins to bind. The third plot shows the difference between the intermediary and household's willingness to pay for a security which pays 1 after a good shock and -1 after a bad shock, and is a measure of the differing risk premia charges by the two agents.[7] This measure of differing risk prices in low and high risk assets (or equivalently debt and equity securities as explained above) also decreases sharply in intermediary capital below the point where the solvency constraint begins to bind.





[7] Any security which pays more in the good than bad state is a convex combination of this security and a riskless asset.



Conclusion This paper develops a general equilibrium model of safety transformation, the issuance of safe liabilities backed by a portfolio of risky assets. These safe liabilities issued by financial intermediaries meet the demands of households to hold safe assets. In the model, the structure of the financial system is organized to provide these safe assets while minimizing financial intermediaries' need to issue costly equity. As a result, financial intermediaries choose to be highly levered and invest exclusively in low risk debt securities, while riskier equities are held by households. The resulting segmentation of debt and equity markets provides an arbitrage opportunity that the non-financial sector exploits when choosing its capital structure. A key difference between this framework and the rest of the intermediation literature is that households and intermediaries have access to the same investment opportunities. The model provides a framework for understanding the role of intermediaries in publicly traded securities markets and provides a risk based explanation for the role of debt on intermediary balance sheets.

The model provides a framework for understanding how the financial system responds in general equilibrium to changes in the supply and demand for safe assets. It explains how a growing demand for safe assets may have contributed to the subprime boom, causing an increasing in non-financial sector leverage and an expansion of intermediary balance sheets. In addition, the model clarifies how policies such as quantitative easing that increase the supply of safe assets crowd out the incentive for the financial sector to perform safety transformation and lead to a compression of risk premia in debt markets. Looking forward, a key question is to understand how the private and social benefits of safety transformation by the financial sector may or may not be aligned and to understand the implications for financial regulation.

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4 Appendix

Proof of propositions 6-8: Recall that the model's equilibrium can be characterized by the system of equations

$$v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right) + \frac{u'\left(c_2^{good}\right)}{2}\left[\frac{1}{\left(1 + C'\left(e\right)\right)} - 1\right]\left(\frac{\Pr\left(x_i > D_i|good\right)}{\Pr\left(x_i > D_i|bad\right)} - 1\right) = 0$$

$$\left(1 + C'\left(e\right)\right) eu'\left(c_1\right) - \frac{1}{2}u'\left(c_2^{good}\right)\int_0^1 \left[E\left(\min\left(x_i, D_i\right)|good\right) - E\left(\min\left(x_i, D_i\right)|bad\right)\right] di = 0$$

Define a function $M_1\left(e, r\right)$ to be the value of the first expression for a given value of e and $r = \frac{\Pr(x_i > D_i|good)}{\Pr(x_i > D_i|bad)}$ and

$M_2\left(e, r\right)$ to be the value of the second expression. Equilibrium occurs when $M\left(e, r\right) = \left(M_1\left(e, r\right), M_2\left(e, r\right)\right) = (0, 0)$

The Jacobian of M is

$$\frac{\partial M_1}{\partial r} = v''\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right) \frac{\partial \int_0^1 E(\min(x_i,D_i)|bad)di}{\partial r} - \frac{u'\left(c_2^{good}\right)}{2} \frac{C'(e)}{(1+C'(e))} < 0$$

$$\frac{\partial M_1}{\partial e} = \frac{-C''(e)}{2(1+C'(e))^2} u'\left(c_2^{good}\right)\left(\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)} - 1\right) < 0$$

$$\frac{\partial M_2}{\partial r} = -\frac{1}{2}u'\left(c_2^{good}\right) \frac{\partial}{\partial r}\int_0^1 \left[E\left(\min\left(x_i, D_i\right)|good\right) - E\left(\min\left(x_i, D_i\right)|bad\right)\right] di < 0$$

$$\frac{\partial M_2}{\partial e} = u'\left(c_1\right)\left(1 + C'\left(e\right) + C''\left(e\right)e\right) > 0$$

where $\frac{\partial \int_0^1 E(\min(x_i,D_i)|bad)di}{\partial r} = \int_0^1 \frac{\Pr(x_i>D_i|bad)}{\frac{\partial}{\partial D_i}\left(\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)}\right)} di$ and

$\frac{\partial}{\partial r}\int_0^1 \left[E\left(\min\left(x_i, D_i\right)|good\right) - E\left(\min\left(x_i, D_i\right)|bad\right)\right] di = \int_0^1 \frac{\Pr(x_i>D_i|good) - \Pr(x_i>D_i|bad)}{\frac{\partial}{\partial D_i}\left(\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)}\right)} di$. The inverse of the Jacobian is

$$\begin{bmatrix} \frac{\partial M_1}{\partial r} & \frac{\partial M_1}{\partial e} \\ \frac{\partial M_2}{\partial r} & \frac{\partial M_2}{\partial e} \end{bmatrix}^{-1} = \frac{1}{\frac{\partial M_1}{\partial r}\frac{\partial M_2}{\partial e} - \frac{\partial M_1}{\partial e}\frac{\partial M_2}{\partial r}} \begin{bmatrix} \frac{\partial M_2}{\partial e} & -\frac{\partial M_1}{\partial e} \\ -\frac{\partial M_2}{\partial r} & \frac{\partial M_1}{\partial r} \end{bmatrix}$$ whose elements have signs $\begin{bmatrix} - & - \\ - & + \end{bmatrix}$. I use this result to now characterize the three comparative statics stated in the proposition

(i) Increased demand for safe assets

If the demand function v' for safe assets is exogenously increased, by the implicit function theorem

$$\begin{bmatrix} \frac{\partial r}{\partial v'} \\ \frac{\partial e}{\partial v'} \end{bmatrix} = -\frac{1}{\frac{\partial M_1}{\partial r}\frac{\partial M_2}{\partial e} - \frac{\partial M_1}{\partial e}\frac{\partial M_2}{\partial r}} \begin{bmatrix} \frac{\partial M_2}{\partial e} & -\frac{\partial M_1}{\partial e} \\ -\frac{\partial M_2}{\partial r} & \frac{\partial M_1}{\partial r} \end{bmatrix} \circ \begin{bmatrix} \frac{\partial M_1}{\partial v'} \\ \frac{\partial M_2}{\partial v'} \end{bmatrix}$$

where $\frac{\partial M_1}{\partial v'} = 1$ and $\frac{\partial M_2}{\partial v'} = 0$, so

$$\begin{bmatrix} \frac{\partial r}{\partial v'} \\ \frac{\partial e}{\partial v'} \end{bmatrix} = -\frac{1}{\frac{\partial M_1}{\partial r}\frac{\partial M_2}{\partial e} - \frac{\partial M_1}{\partial e}\frac{\partial M_2}{\partial r}} \frac{1}{u'(c_1)} \begin{bmatrix} u'\left(c_1\right)\left(1 + C'\left(e\right) + C''\left(e\right)e\right) \\ \frac{1}{2}u'\left(c_2^{good}\right) \frac{\partial}{\partial r}\int_0^1 \left[E\left(\min\left(x_i, D_i\right)|good\right) - E\left(\min\left(x_i, D_i\right)|bad\right)\right] di \end{bmatrix}$$

so $\frac{\partial r}{\partial v'} > 0$ and $\frac{\partial e}{\partial v'} > 0$, so the intermediary expands its balance sheet and issues more equity. In addition, note that the change in the equilibrium level of $v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)$ must be equal to

$-\frac{u'\left(c_2^{good}\right)}{2}\left[\frac{\partial e}{\partial v'}\frac{\partial}{\partial e}\frac{1}{(1+C'(e))}\left(\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)} - 1\right) - \frac{C'(e)}{(1+C'(e))}\frac{\partial r}{\partial v'}\right]$, which is positive since $\frac{\partial r}{\partial v'} > 0$ and $\frac{\partial e}{\partial v'} > 0$.

Since $v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)$ increases, the risk free rate falls and the aggregate quantity d of riskless securities issued increases. Since r increases, the leverage of each nonfinancial firm also increases.

(ii) Increased government supply of safe assets

If the supply μ of riskless government debt is exogenously increased, for a given value of r, note that

$$\begin{bmatrix} \frac{\partial M_1}{\partial \mu} \\ \frac{\partial M_2}{\partial \mu} \end{bmatrix} = \begin{bmatrix} v''\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right) \\ 0 \end{bmatrix} = v''\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right) \begin{bmatrix} \frac{\partial M_1}{\partial v'} \\ \frac{\partial M_2}{\partial v'} \end{bmatrix}$$

so $\begin{bmatrix} \frac{\partial r}{\partial \mu} \\ \frac{\partial e}{\partial \mu} \end{bmatrix} = v''\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right) \begin{bmatrix} \frac{\partial r}{\partial v'} \\ \frac{\partial e}{\partial v'} \end{bmatrix}$.

It follows that $\frac{\partial r}{\partial \mu} < 0$ and $\frac{\partial e}{\partial \mu} < 0$ since $v'' < 0$. The change in the equilibrium value of $v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)$ is equal to

$$-\frac{u'\left(c_2^{good}\right)}{2}\left[\frac{\partial e}{\partial \mu}\frac{\partial}{\partial e}\frac{1}{(1+C'(e))}\left(\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)}-1\right)-\frac{C'(e)}{1+C'(e)}\frac{\partial r}{\partial \mu}\right] < 0.$$

It follows that the risk free rate increases since $v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right)\right)$ decreases. The quantity of riskless liabilities issued by the financial sector decreases, but the total supply including the government supply increases. By the same arguments as in part (i), credit spreads compress and the non-financial sector decreases its leverage.

(iii) Increase in intermediary capital

Finally, suppose the quantity W of intermediary capital before equity e is raised is increased.

We have that $\frac{\partial M_1}{\partial e} = 0$ and $\frac{\partial M_2}{\partial e} = \left(1 + C'\left(e\right)\right) u'\left(c_1\right)$ so

$$\begin{bmatrix} \frac{\partial r}{\partial W} \\ \frac{\partial e}{\partial W} \end{bmatrix} = -\frac{1}{\frac{\partial M_1}{\partial r}\frac{\partial M_2}{\partial e}-\frac{\partial M_1}{\partial e}\frac{\partial M_2}{\partial r}} \begin{bmatrix} \frac{\partial M_2}{\partial e} & -\frac{\partial M_1}{\partial e} \\ -\frac{\partial M_2}{\partial r} & \frac{\partial M_1}{\partial r} \end{bmatrix} \circ \begin{bmatrix} 0 \\ \left(1 + C'\left(e\right)\right) u'\left(c_1\right) \end{bmatrix}$$

$$= -\frac{(1+C'(e))u'(c_1)}{\frac{\partial M_1}{\partial r}\frac{\partial M_2}{\partial e}-\frac{\partial M_1}{\partial e}\frac{\partial M_2}{\partial r}} \begin{bmatrix} \frac{C''(e)}{2(1+C'(e))^2}u'\left(c_2^{good}\right)\left(\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)}-1\right) \\ v''\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)\frac{\partial\int_0^1 E(\min(x_i,D_i)|bad)di}{\partial r}-\frac{u'\left(c_2^{good}\right)}{2}\frac{C'(e)}{(1+C'(e))} \end{bmatrix}$$

which implies $\frac{\partial r}{\partial W} > 0$ and $\frac{\partial e}{\partial W} < 0$. Since $v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)$ decreases because $\frac{\partial r}{\partial W} > 0$, the risk free rate increases. Since

$\frac{\partial e}{\partial W} < 0$, credit spreads compress. The leverage of the non-financial sector increases since $\frac{\partial r}{\partial W} > 0$.

Proof of Proposition 9 As noted in the main text, equilibrium is characterized by a solution

to

$$v'(d)+\left(\frac{1}{1+C'(e)}-1\right)\frac{u'\left(c_2^{good}\right)}{2}\left[\frac{[\Pr(x_{i,2}>D_{i,2}|good)]}{[\Pr(x_{i,2}>D_{i,2}|bad)]}-1\right] =$$
$$v'(d)\left[\frac{u'(c_3)}{u'(c_2)}\beta_3^{low}\Pr(x_{i,3}>D_{i,3}|bad)\right]-\frac{C'(e)}{1+C'(e)}*$$
$$Eu'(c_3)\left[\beta_3\{x_{i,3}>D_{i,3}\}-\beta_3^{low}\Pr(x_{i,3}>D_{i,3}|bad)\right]=0.$$

and

$$(1+C'(e))\,eu'(c_1)=\beta Eu'(c_2)\int_0^1\left[\begin{array}{c}[\min(x_{i,2},D_{i,2})-E[\min(x_{i,2},D_{i,2})|bad]]\\ +\frac{u'(c_3)}{u'(c_2)}\left[\beta_3\min(x_{i,3},D_{i,3})-\beta_3^{low}E(\min(x_{i,3},D_{i,3})|bad)\right]\end{array}\right]di$$

The value of these 3 expressions can be computed from knowing the ratio $r=\frac{[\Pr(x_{i,2}>D_{i,2}|good)]}{2[\Pr(x_{i,2}>D_{i,2}|bad)]}=\frac{Eu'(c_3)[\beta_3\{x_{i,3}>D_{i,3}\}]}{\beta_3^{low}\Pr(x_{i,3}>D_{i,3}|bad)}$ determining the debt of each firm and the quantity e of equity issued by the intermediary. The map $N(r,e)$ from r and e to values of

$$v'\left(\int_0^1\left(E[\min(x_{i,2},D_{i,2})|bad]+\beta_3^{low}E(\min(x_{i,3},D_{i,3})|bad)\right)di\right)$$
$$+\left(\frac{1}{1+C'(e)}-1\right)\frac{u'\left(c_2^{good}\right)}{2}\left[\frac{[\Pr(x_{i,2}>D_{i,2}|good)]}{[\Pr(x_{i,2}>D_{i,2}|bad)]}-1\right]$$

and

$$(1+C'(e))\,eu'(c_1)-\beta Eu'(c_2)\int_0^1\left[\begin{array}{c}[\min(x_{i,2},D_{i,2})-E[\min(x_{i,2},D_{i,2})|bad]]\\ +\frac{u'(c_3)}{u'(c_2)}\left[\beta_3\min(x_{i,3},D_{i,3})-\beta_3^{low}E(\min(x_{i,3},D_{i,3})|bad)\right]\end{array}\right]di$$

characterizes equilibrium at $N(r,e)=0$. N has Jacobian

$\frac{\partial N_1}{\partial r}=v''\left(\int_0^1\left(E[\min(x_{i,2},D_{i,2})|bad]+\beta_3^{low}E(\min(x_{i,3},D_{i,3})|bad)\right)di\right)*[$

$\frac{\partial}{\partial r}\int_0^1 \left(E\left[\min\left(x_{i,2}, D_{i,2}\right)|bad\right] + \beta_3^{low} E\left(\min\left(x_{i,3}, D_{i,3}\right)|bad\right)\right) di] - \frac{C'(e)}{1+C'(e)}\frac{u'\left(c_2^{good}\right)}{2} < 0$

$\frac{\partial N_1}{\partial e} = \frac{-C''(e)}{2(1+C'(e))^2} u'\left(c_2^{good}\right)\left(\frac{\Pr(x_i>D_i|good)}{\Pr(x_i>D_i|bad)} - 1\right) < 0$

$\frac{\partial N_2}{\partial r} = -\beta E u'(c_2) \frac{\partial}{\partial r}\int_0^1 \begin{bmatrix} \left[\min\left(x_{i,2}, D_{i,2}\right) - E\left[\min\left(x_{i,2}, D_{i,2}\right)|bad\right]\right] \\ +\frac{u'(c_3)}{u'(c_2)}\left[\beta_3 \min\left(x_{i,3}, D_{i,3}\right) - \beta_3^{low} E\left(\min\left(x_{i,3}, D_{i,3}\right)|bad\right)\right] \end{bmatrix} di < 0$

$\frac{\partial N_2}{\partial e} = u'(c_1)\left(1 + C'(e) + eC''(e)\right) > 0$

The effects of quantitative easing, by the implicit function theorem, solve $\begin{bmatrix} \frac{\partial r}{\partial Q} \\ \frac{\partial e}{\partial Q} \end{bmatrix} = -\frac{1}{\frac{\partial N_1}{\partial r}\frac{\partial N_2}{\partial e} - \frac{\partial N_1}{\partial e}\frac{\partial N_2}{\partial r}} \begin{bmatrix} \frac{\partial N_2}{\partial e} & -\frac{\partial N_1}{\partial e} \\ -\frac{\partial N_2}{\partial r} & \frac{\partial N_1}{\partial r} \end{bmatrix} \circ \begin{bmatrix} \frac{\partial N_1}{\partial Q} \\ \frac{\partial N_2}{\partial Q} \end{bmatrix}$ where $\begin{bmatrix} \frac{\partial N_1}{\partial Q} \\ \frac{\partial N_2}{\partial Q} \end{bmatrix}$ is the partial equilibrium effect of an increasing in the quantity Q of riskless government debt issued to purchase riskless long duration assets, holding fixed r and e. To compute $\frac{\partial N_1}{\partial Q}$ and $\frac{\partial N_2}{\partial Q}$ first note that issuing one unit of short duration riskless debt at time 1 buys a face value F of long duration riskless debt where $F\beta_3^{low} < 1$ and $F\beta_3^{high} > 1$,

It follows that $\frac{\partial N_1}{\partial Q} < 0$ and $\frac{\partial N_2}{\partial Q} > 0$. This implies that $\frac{\partial r}{\partial Q} < 0$ and $\frac{\partial e}{\partial Q} < 0$. This implies that credit and duration risk premia are compressed since $\frac{\partial e}{\partial Q} < 0$, and the rest of the proposition follows directly.

4.0.1 Proof of proposition 10

As shown in the main text, with nominal rigidities at the zero lower bound, equilibrium is characterized by the solution to

$$v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right) + \frac{u'\left(c_2^{good}\right)}{2}\left[\frac{1}{\left(1 + C'(e)\right)} - 1\right]\left(\frac{\Pr\left(x_i > D_i|good\right)}{\Pr\left(x_i > D_i|bad\right)} - 1\right) = 0$$

$$\begin{aligned} &\left(1 + C'(e)\right) e\left(Eu'(c_2) + v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)\right) \\ -&\frac{1}{2}u'\left(c_2^{good}\right)\int_0^1 \left[E\left(\min\left(x_i, D_i\right)|good\right) - E\left(\min\left(x_i, D_i\right)|bad\right)\right] di = 0 \end{aligned}$$

If I define M to be the map from r and e (where $r = \frac{\Pr(x_i > D_i | good)}{\Pr(x_i > D_i | bad)}$) the value of these two expressions, M has Jacobian

$$\frac{\partial M_1}{\partial r} = \frac{v''\left(\int_0^1 E(\min(x_i, D_i)|bad)di\right)}{u'(c_1)} \frac{\partial \int_0^1 E(\min(x_i, D_i)|bad)di}{\partial r} + \frac{u'\left(c_2^{good}\right)}{2u'(c_1)} \left[\frac{1}{(1+C'(e))} - 1\right] < 0$$

$$\frac{\partial M_1}{\partial e} = \frac{-C''(e)}{(1+C'(e))^2} \frac{u'\left(c_2^{good}\right)}{2u'(c_1)} \left(\frac{\Pr(x_i > D_i | good)}{\Pr(x_i > D_i | bad)} - 1\right) < 0$$

$$\frac{\partial M_2}{\partial r} = -\frac{1}{2} u'\left(c_2^{good}\right) \frac{\partial}{\partial r} \int_0^1 \left[E\left(\min\left(x_i, D_i\right)|good\right) - E\left(\min\left(x_i, D_i\right)|bad\right)\right] di$$

$$+ \left(1 + C'\left(e\right)\right) e \frac{v''\left(\int_0^1 E(\min(x_i, D_i)|bad)di\right)}{u'(c_1)} \frac{\partial \int_0^1 E(\min(x_i, D_i)|bad)di}{\partial r} < 0$$

$$\frac{\partial M_2}{\partial e} = u'\left(c_1\right)\left(1 + C'\left(e\right) + C''\left(e\right)\left(e + W\right)\right) > 0$$

For any perturbation μ of the problem above,

$$\begin{bmatrix} \frac{\partial r}{\partial \mu} \\ \frac{\partial e}{\partial \mu} \end{bmatrix} = -\frac{1}{\frac{\partial M_1}{\partial r}\frac{\partial M_2}{\partial e} - \frac{\partial M_1}{\partial e}\frac{\partial M_2}{\partial r}} \begin{bmatrix} \frac{\partial M_2}{\partial e} & -\frac{\partial M_1}{\partial e} \\ -\frac{\partial M_2}{\partial r} & \frac{\partial M_1}{\partial r} \end{bmatrix} \circ \begin{bmatrix} \frac{\partial M_1}{\partial \mu} \\ \frac{\partial M_2}{\partial \mu} \end{bmatrix}$$

Increasing intermediary capital W has $\begin{bmatrix} \frac{\partial M_1}{\partial W} \\ \frac{\partial M_2}{\partial W} \end{bmatrix}$

$$= \begin{bmatrix} 0 \\ \left(1 + C'\left(e\right)\right)\left(Eu'\left(c_2\right) + v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)\right) \end{bmatrix}$$

It follows that $\frac{\partial r}{\partial W} > 0$, so $v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)$ decreases, and thus $u'\left(c_1\right)$ increases since

$u'\left(c_1\right) = Eu'\left(c_2\right) + v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)$. Increasing the demand v' for safe assets yields

$\begin{bmatrix} \frac{\partial M_1}{\partial v'} \\ \frac{\partial M_2}{\partial v'} \end{bmatrix} = \begin{bmatrix} 1 \\ \left(1 + C'\left(e\right)\right) e \end{bmatrix}$. The change in the equilibrium value of $v'\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right)$ must equal

$-\frac{u'\left(c_2^{good}\right)}{2}\left[\frac{\partial e}{\partial v'}\frac{\partial}{\partial e}\frac{1}{(1+C'(e))}\left(\frac{\Pr(x_i > D_i|good)}{\Pr(x_i > D_i|bad)} - 1\right) + \left[\frac{1}{(1+C'(e))} - 1\right]\frac{\partial r}{\partial v'}\right]$, which is positive since $\frac{\partial r}{\partial v'} > 0$ and $\frac{\partial e}{\partial v'} > 0$.

It follows that $u'\left(c_1\right)$ increases, so c_1 decreases, reducing output.

Finally, an increase in the supply μ of government issued safe assets has

$\begin{bmatrix} \frac{\partial M_1}{\partial \mu} \\ \frac{\partial M_2}{\partial \mu} \end{bmatrix} = v''\left(\int_0^1 E\left(\min\left(x_i, D_i\right)|bad\right) di\right) \begin{bmatrix} \frac{\partial M_1}{\partial v'} \\ \frac{\partial M_2}{\partial v'} \end{bmatrix}$, so c_1 increases with the supply of government issued safe assets.

A microfoundation for the intermediary's costly external finance constraint At time 1, the household divides into a consumer and an intermediary, as in (Gertler Kiyotaki 2009). Each agent maximizes the value of resources it returns to the household at time 2. The consumer is given an endowment W_1 of wealth which it can either consume or invest as described in the main paper. The intermediary initially has no wealth, but it can buy portfolios of assets which back securities it can sell to the investor.

The intermediary faces an agency problem when selling securities to the household, since the cashflows on the intermediary's balance sheet are observable but not verifiable. If at time 2 the intermediary reports a smaller payout than actually occurs, it obtains private benefits from diverting the remaining cashflows. It must therefore be incentive compatible for the intermediary to accurately report its cashflows, which implies that it must receive a payment greater than its private benefits of diversion.

The intermediary purchases a portfolio $q_I(s)$ of securities, which pays $\int_0^1 E(\delta_s|good)\, q_I(s)\, ds$ in the good state and $\int_0^1 E(\delta_s|bad)\, q_I(s)\, ds$ in the bad state. The intermediary also issues securities which pay a total of δ_I^{good} in the good state and δ_I^{bad} in the bad state. The remaining payoffs $\int_0^1 E(\delta_s|good)\, q_I(s)\, ds - \delta_I^{good}$ and $\int_0^1 E(\delta_s|bad)\, q_I(s)\, ds - \delta_I^{bad}$ go to the intermediary in each state of the world. If the intermediary reports the bad state when the good state occurs, it loses the payment $\left(\int_0^1 E(\delta_s|good)\, q_I(s)\, ds - \delta_I^{good}\right) - \left(\int_0^1 E(\delta_s|bad)\, q_I(s)\, ds - \delta_I^{bad}\right)$ but can divert the resources $\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds$ from which it gets the payoff $M\left(\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds\right)$ for some constant $M < 1$. The IC constraint for reporting the good state is therefore

$$\begin{aligned}
&\left(\int_0^1 E(\delta_s|good)\, q_I(s)\, ds - \delta_I^{good}\right) - \left(\int_0^1 E(\delta_s|bad)\, q_I(s)\, ds - \delta_I^{bad}\right) \\
&\geq M\left(\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds\right).
\end{aligned}$$

Given this good state IC constraint, I now solve for the optimal contract satisfying it and show that the bad state IC constraint is also satisfied. To minimize the payment to the intermediary subject to the IC constraint, note that $\delta_I^{bad} = \int_0^1 E(\delta_s|bad)\, q_I(s)\, ds$ so the IC constraint becomes $\left(\int_0^1 E(\delta_s|good)\, q_I(s)\, ds - \delta_I^{good}\right) \geq M\left(\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds\right)$. The payout δ_I^{good} is minimized when this is satisfied with equality. $\delta_I^{good} = \int_0^1 E(\delta_s|good)\, q_I(s)\, ds - M\left(\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds\right)$. Given the portfolio $q_I(s)$, the maximal incentive compatible payout the intermediary can make is $\delta_I^{bad} = \int_0^1 E(\delta_s|bad)\, q_I(s)\, ds$ and $\delta_I^{good} = \int_0^1 E(\delta_s|good)\, q_I(s)\, ds - M\left(\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds\right)$. For this choice of $\left(\delta_I^{bad}, \delta_I^{good}\right)$, if the intermediary reports the good state when the bad state occurs, it receives a negative payment and has no resources to divert since $\delta_I^{good} > \delta_I^{bad} = \int_0^1 E(\delta_s|bad)\, q_I(s)\, ds$, so all IC constraints are satisfied.

Given these total payouts, the intermediary optimally issues a riskless debt claim paying $\int_0^1 E(\delta_s|bad)\, q_I(s)\, ds$ in both states of the world and an equity claim paying

$$\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds - M\left(\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds\right)$$

in the good state and 0 in the bad state in order to provide the maximum of riskless assets.

The intermediary's budget constraint given this constraint can therefore be written as

$$u'(c_1)\, e = \frac{1}{2} u'\left(c_2^{good}\right)(1-M)\left(\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds\right)$$

and its willingness to pay for a security with payoff δ_s can be written as

$$E\frac{u'(c_2)}{u'(c_1)}\delta_s + v'(d)\, E(\delta_s|bad) + \frac{1}{2} u'\left(c_2^{good}\right)(1-M)\left[E(\delta_s|good) - E(\delta_s|bad)\right]$$

which is equivalent to the intermediary's costly external finance problem in the main text

where $1 - M = \frac{1}{1+C'(e)}$. In order to reproduce an arbitrary cost of equity issuance $C(e)$, replace $M\left(\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds\right)$ by the function

$$M\left(\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds, e\right) = \frac{1}{1+C'(e)} \int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds.$$

In this formulation, the marginal agency cost of risk on the intermediary's balance sheet depends on $\frac{1}{1+C'(e)}$ rather than solely on $\int_0^1 [E(\delta_s|good) - E(\delta_s|bad)]\, q_I(s)\, ds$.