THE POLITICS OF SOVEREIGN DEFAULT UNDER FINANCIAL INTEGRATION*

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Abstract

In this paper we study the role of portfolio diversification on optimal default of sovereign debt in a two-country model with large economies that are financially integrated. Financial integration increases the incentives to default not only because part of the defaulted debt is owned by foreigners (the standard redistribution channel), but also because the endogenous macroeconomic cost for the defaulting country is smaller when financial markets are integrated. We show that the sovereign default of one country may be triggered by higher debt (liquidity) issued by other countries. Because the macroeconomic costs of default spill to other countries, creditor countries may find it beneficial ex-post to bail-out debtor countries. Although bailouts create moral hazard problems, they can be welfare improving also ex-ante.

*We would like to thank seminar and conference participants for helpful comments at the Atlanta Fed, Bank of Canada, Claremont McKenna College, Minnesota Workshop in Macroeconomic Theory, Penn State University, Philadelphia Fed, SED meeting in Toulouse, University of California San Diego, University of Georgia, University of Wisconsin, the Stockman Conference at the University of Rochester.
1 Introduction

The last 30 years have been characterized by a dramatic increase in the international integration of financial markets. This allowed governments to ‘export’ their public debt, that is, to borrow from foreign countries. Figure 1 plots the share of public debt for the US and the largest EU countries during the 1997-2010 period and shows that this share has increased substantially during this period.

![Figure 1: Share of public debt held abroad. Left axis for solid line countries. Right axis for dotted line countries. Source: Merler and Pisani-Ferry (2012).](image)

During the same period, countries have also ‘imported’ foreign public debt as domestic residents increased their holdings of securities issued by foreign countries. Figure 2 plots the ownership of debt instruments issued by foreign countries, including foreign government debt, for several countries since 1980. As can be seen from the figure, the ownership of foreign debt has increased dramatically, especially since the mid 1990s.

The two figures illustrate an important trend in global financial markets: the cross-country diversification of financial portfolios. This is a general trend that is not limited to debt instruments but it extends to portfolio investments and FDI. In this paper, however, we focus on debt instruments
and, especially, sovereign debt, because of their role in providing liquidity. The goal is to understand how the international diversification of portfolios affects: (i) the incentive of governments to default on sovereign debt; (ii) the spillover of the macroeconomic costs of default to other countries; (iii) the benefits (ex-post and ex-ante) for the creditor countries to bail-out defaulting countries.

Let’s start with the effect of portfolio diversification on the government choice to default. Of course, if a larger share of sovereign debt is held by foreigners, the incentive to default for the debtor country increases since it redistributes wealth from foreign residents to domestic residents. This mechanism is well recognized in the literature although the study of Broner, Martin and Ventura (2010) challenges its relevance. In this paper, however, we explore a different mechanism through which financial diversification increases the incentive of a country to default. We show that the ‘macroeconomic cost’ of default declines when the country is (internationally) financially diversified.

Why is the macroeconomic cost of default smaller when the country is financially diversified? The central mechanism is the disruption of financial
markets induced by default. When a government defaults on its debt, the holders of government debt incur capital losses. To the extent that financial wealth held by some agents is important for economic decisions, this has a negative effect on aggregate economic activities. Notice that this effect is present independently of whether the country is integrated or operates in a regime of financial autarky. In the latter case, default redistributes wealth between domestic agents. Still, heterogeneity within a country implies that redistribution is not neutral for both economic activity and aggregate welfare (as in D’Erasmo and Mendoza (2016)). When financial markets are integrated (and portfolios diversified), however, domestic residents hold a smaller share of wealth in domestic assets and a larger share in foreign assets. This implies that, when the domestic government defaults, the wealth losses of domestic residents (and, therefore, domestic redistribution) are smaller, which causes a smaller macroeconomic contraction. Then, being the macroeconomic cost smaller, the government has higher incentive to default.

The mechanism described above points out that it is not only the quantity of domestic debt held by foreigners that matters for the choice of a country to default but also the debt issued by foreign countries held by domestic agents. Of course, the quantity of foreign debt held by domestic agents depends on the external supply of foreign debt. This introduces a channel through which the supply of foreign debt affects the incentive of a country to default. More specifically, an increase in the issuance of foreign debt implies that in equilibrium domestic agents hold more of this debt and they are more diversified. Higher diversification then implies that the macroeconomic cost of default is lower, which in turn increases the government incentive to default even if the quantity of domestic debt held by foreigners remains unchanged. This shows that an increase in the stock of debt issued by ‘foreign countries’ (higher international liquidity) could trigger the default of the ‘domestic country’.

The role played by external factors for the choice of a country to default is an important dimension in which our paper differs from a large body of literature. The majority of studies on sovereign default focus on the internal factors that lead a country to default. For example, a sequence of negative productivity or fiscal shocks leads a country to borrow more and, if the economic conditions continue to deteriorate, it becomes optimal or necessary for the country to default. In our paper, instead, we show that the factors that could cause a country to default may not originate domestically. In particular, the debt issued by other countries (higher international liquidity)
may also induce a country to default.

To illustrate the importance of external factors, we consider a model with two large countries: the home country $h$, which is riskier than the foreign country $f$. The issuance of debt and its repayment are chosen optimally by the governments of both countries. Using this model we consider an exogenous change in the debt issued by the ‘foreign’ country and study how this affects the incentive of the ‘home’ country to default. As the debt of country $f$ increases, residents in country $h$ acquire more foreign debt. Thus, the holding of safe, nondefaultable debt increases in country $h$. Consequently, if the home government defaults, domestic agents face a proportionally smaller loss in their financial wealth, which in turn implies that the macroeconomic consequences of default are smaller. This reduces the macroeconomic cost of default in country $h$ and increases the incentive of its government to default.

Greater financial diversification also means that foreigners hold more debt issued by the home country. Therefore, when the home country defaults, the foreign country experiences larger financial losses. In addition to the direct capital losses, the foreign country also experiences a macroeconomic contraction. Therefore, financial diversification creates the conditions for real macroeconomic spillover across countries, which bring us to the second issue studied in the paper, that is, how portfolio diversification affects the international transmission of default to the real sector of nondefaulting countries.

The international spillover has important policy implications: when a country defaults, the other country may have an incentive to bailout the debtor country in order to guarantee the repayment of the debt. In the model a bailout takes the form of a bargaining problem between creditor and debtor countries. The two countries negotiate a financial transfer from the creditor country to the debtor country against a higher repayment of the debt. In those states in which a country has an incentive to default, the ex-post bailout is Pareto improving. However, the anticipation of bailout also encourages the country to borrow more in the first period, which captures the typical moral hazard problem associated with bailout. Despite the moral hazard problem, bailouts may not be inefficient, that is, the ex-ante welfare without bailouts could be lower for both countries.

The possibility that bailouts could be efficient also ex-ante derives from the assumption that countries choose their own debt in period 1 without coordination. This implies that, when a country chooses its debt, it ignores the liquidity benefits that the debt brings to foreigners since part of the debt will be held by residents in the other country. The resulting equilibrium is
then characterized by sub-optimally low issuance of worldwide debt relative to the autarky regime. The anticipation of bailout encourages the more issuance of debt because in the event of default part of the debt will be repaid by the other country, effectively reducing the cost of borrowing. In this way, the anticipation of bailouts partially corrects for the (inefficient) low issuance of public debt induced by the lack of policy coordination.

2 Literature review

This paper builds on a large literature on public debt with incomplete markets. The main role of government debt in our paper is to partially complete the assets market when agents are subject to uninsurable idiosyncratic risk. The mechanism is similar to the one studied in Aiyagari and McGrattan (1998), Golosov and Sargent (2012), and Floden (2001), who study heterogeneous agents models without default. Closer to our paper is Azzimonti, de Francisco, and Quadrini (2014, AFQ henceforth), in which debt is held by agents for consumption smoothing (self-insurance). There are, however, three main departures from this paper. First, our economy is subject to both idiosyncratic and aggregate uncertainty, whereas AFQ considers only idiosyncratic uncertainty. Without aggregate uncertainty sovereign default would never arise in equilibrium. Second, the stock of public debt affects labor markets and hence the aggregate level of production while in AFQ aggregate production was fixed and, therefore, public debt did not have any macroeconomic implications. Finally, and contrary to AFQ, debt can be partially defaulted. Because of the possibility of default, our paper is also related to a growing literature on external sovereign default that builds on Eaton and Gersovitz (1981) (e.g. Aguiar and Amador (2013), Aguiar and Gopinath (2006), Arellano (2008), Cuadra, Sanchez, and Sapriza (2010), Pouzo and Presno (2014), Yue (2010), among others). Aguiar and Amador (2014) and Tomz and Wright (2012) provide recent reviews of this literature.

and Tabellini (1991), instead, focus on the international redistribution of sovereign default.

Our paper is also related to studies that make the cost of default endogenous by assuming that public debt provides liquidity and study the role of secondary markets (see Guembel and Sussman (2009), Broner, Martin, and Ventura (2010), Broner and Ventura (2011), Gennaioli, Martin, and Rossi (2014), Basu (2009), Brutti (2011), and Di Casola and Sichlimiris (2014)). Extending the work of Gennaioli, Martin, and Rossi (2014), some recent papers study the interaction between sovereign debt and domestic financial institutions (e.g. Sosa-Padilla (2012), Bocola (2014), and Perez (2015)). As in our paper, the cost of default is endogenous as it disrupts production and causes a recession. One important difference with our paper, however, is that these studies focus on small open economies which is the mainstream approach in the literature. Our paper, instead, emphasizes the importance of foreign factors by studying large open economies that operate in a globalized market. As emphasized in the introduction, this framework allows us to study how the world supply of financial assets affects the incentive to default and how macroeconomic consequences of default are transmitted to other countries (spillovers). International spillovers allow us to study the optimality of bailouts from the prospective of creditor countries.

Arellano and Bai (2008) also consider an environment in which the choices of debt and default affect other countries. The channel is based on the interest rate change. Our channel of transmission, instead, relies on the degree of portfolio diversification which is important for the international transmission of macroeconomic recessions. Our paper is also related to contributions that study debt restructuring through bargaining as Yan (2010) and Bai and Zhang (2009).

3 The model

We analyze a two-period economy composed of two large countries, ‘home’ and ‘foreign.’ We will use the superscripts $h$ and $f$ to denote, respectively, the home and foreign country. Countries borrow in period 1 and repay the debt or default in period 2.

1See also Borri and Verdelhan (2009), Park (2013), Lizarazo (2013) and Volkan (2013), Pouzo and Presno (2011).

2See Niepelt (2016) and Mihalache (2016) for an alternative renegotiation protocols.
The only difference between the two countries is in the degree of commitment to repay their public debt in period 2. In particular, we assume that with probability $\rho$, country $i \in \{h, f\}$ keeps its commitment to repay in period 2 and with probability $1 - \rho$ it will opportunistically choose whether to default. We denote the commitment state in period 2 by $\xi \in \{\text{Commit}, \text{Not Commit}\}$. One important factor that could affect the commitment of the government is political turnover. Later, we will formalize the change in commitment with political uncertainty in period 2.

To understand how default decisions affect the economy, we first describe the politico-equilibrium under autarky. To ease readability, we abstract from the superscript denoting the country type (these will be introduced when presented the case under financial integration). The case with integrated financial markets is described in the next section. This presentation sequence makes clear how financial integration affects the cost of default through portfolio diversification.

In each country there are two types of agents: a measure 1 of workers and a measure 1 of entrepreneurs. The assumption that the number of workers is the same as the number of entrepreneurs is without loss of generality. In the first period, workers and entrepreneurs receive, respectively, the endowments $e_1$ and $a_1$. We can think of $e_1$ and $a_1$ as the wealth of workers and entrepreneurs accumulated up to the end of period 1. Given their wealth, agents make consumption/saving decisions and move to the second period. In period 2 entrepreneurs produce with the input of labor hired from workers. Therefore, production takes place only in period 2. Workers also receive the endowment $e_2$ in period 2.

Workers value consumption and leisure with the utility

$$U(c_1) + \beta U\left(\varphi(c_2, h_2)\right),$$

where $c_1$ and $c_2$ denote consumption in period 1 and 2, respectively, and $h_2$ is the supply of labor in period 2. To simplify the analysis we assume that

$$U(c_t) = \log(c_t) \quad \text{and} \quad \varphi(c_2, h_2) = c_2 - \alpha \frac{h_2^{1 + \frac{1}{\nu}}}{1 + \frac{1}{\nu}}.$$

Workers receive lump-sum transfers $T_t$ from the government each period and are excluded from financial markets (hand-to-mouth). Therefore, con-
sumption in each of the two periods is

\[ c_1 = e_1 + T_1, \]
\[ c_2 = e_2 + w_2 h_2 + T_2, \]

where \( w_2 \) denotes the wage rate earned in period 2. The stark assumption that workers cannot borrow simplifies the exposition but it is not essential. Our results would hold in an environment in which workers have access to financial markets but are subject to a borrowing limit.

The utility of entrepreneurs takes the form

\[ u(d_1) + \beta u(d_2). \]

where \( d_1 \) and \( d_2 \) denote their consumption levels in periods 1 and 2, respectively. Also for entrepreneurs we assume that their utility takes the logarithmic form, that is, \( u(d_t) = \log(d_t) \).

Entrepreneurs produce in the second period using a linear technology

\[ y_2 = A(z_2, \varepsilon_2) l_2, \]

where \( l_2 \) is the input of labor, \( z_2 \) is an aggregate productivity shock, and \( \varepsilon_2 \) is an idiosyncratic productivity shock. We assume that entrepreneurs observe the aggregate state of the economy \( z_2 \) at the beginning of period 2 before hiring workers. However, the idiosyncratic productivity \( \varepsilon_2 \) is observed only after production. This makes the hiring decision risky for entrepreneurs. The importance of this assumption will become clear when we describe the labor market equilibrium.

There is no market for contingent claims and the only assets that entrepreneurs can trade are one-period government bonds. Thus, the idiosyncratic risk cannot be perfectly insured. The budget constraints for entrepreneurs in the first and second periods are, respectively,

\[ d_1 = a_1 - \frac{b_1}{R_1}, \]
\[ d_2 = [A(z_2, \varepsilon_2) - w_2] l_2 + \delta_2 b_1, \]

where \( b_1 \) represents the government bonds purchased in period 1 at price \( 1/R_1 \). As will be described shortly, the government could default on the bonds, in which case the fraction repaid is \( \delta_2 < 1 \). The bond value in period 2 is then \( \delta_2 b_1 \).
The government issues $B_1$ bonds in the first period and distributes all revenues to workers. Thus, the per-worker transfers are equal to

$$T_1 = \frac{B_1}{R_1}. \quad (1)$$

Effectively, the government borrows on behalf of workers. Notice that, since $B_1$ is not restricted to be positive, the government could choose to save. We will focus on parameter values for which $B_1$ is positive but, theoretically, it could be possible for the government to save instead of borrowing.

In period 2 the government repays the debt by taxing workers (or making transfers to them if $B_1 < 0$). However, the government could also choose to partially default by repaying only a fraction $\delta_2 \in [0, 1]$ of the debt. We allow for partial default which is a common feature of the data (see Arellano, Mateos-Planas and Rios-Rull (2013)). Denoting by $\tilde{B}_2 = \delta_2 B_1$ the repayment, the (negative) transfers received by workers in period 2 are

$$T_2 = -\tilde{B}_2. \quad (2)$$

The welfare objective of the government is the weighted sum of the utility of workers and entrepreneurs, that is,

$$(1 - \Psi) \left[ U(c_1) + \beta E_{\xi,z} U(\varphi(c_2, h_2)) \right] + \Psi \left[ u(d_1) + \beta E_{\xi, z, \varepsilon} u(d_2) \right],$$

where $\Psi$ denotes the relative weight assigned to entrepreneurs.

**Uncertainty and timing.** There are three sources of uncertainty: (i) The government commitment $\xi_2$, which is an aggregate shock; (ii) The aggregate productivity $z_2$; and (iii) the idiosyncratic productivity $\varepsilon_2$. Aggregate and idiosyncratic shocks are both realized in period 2. However, while the aggregate shocks $\xi_2$ and $z_2$ are revealed before agents and government make any decisions in period 2, the idiosyncratic shock is revealed after the repayment decision of the government and after the hiring decisions of entrepreneurs. Following is the detailed description of the timing.

**Period 1:**

1. The government chooses debt $B_1$.
2. Entrepreneurs choose savings $b_1$.
3. The interest rate $R_1$ clears the market for government bonds.
Period 2:

1. Government commitment $\xi_2$ and aggregate productivity $z_2$ become public knowledge.

2. The government chooses the debt repayment $\tilde{B}_2 = \delta_2 B_1$. With commitment $\delta_2 = 1$ but without commitment $\delta_2 \leq 1$.

3. Entrepreneurs choose the input of labor $l_2$ and workers choose the supply of labor $h_2$. The wage $w_2$ clears the labor market.

4. The idiosyncratic productivity $\varepsilon_2$ is realized. Production and consumption take place.

3.1 Equilibrium for given policies

We start characterizing the competitive equilibrium for given government policies. To simplify notation, from now on we omit the time subscripts unless they are required to avoid ambiguities. For example, $B$ without subscript denotes the debt issued by the government in period 1 and $\tilde{B} = \delta B$ denotes the repayment of the debt in period 2. If the government repays the debt in full then $\delta = 1$ and $\tilde{B} = B$. The variable $h$ denotes the labor supplied by a worker in period 2. Also in this case we can omit the time subscript because workers supply labor only in period 2. Along the same line, we omit the time subscript for the shocks since they are only realized in period 2.

While government borrowing is chosen in period 1, the repayment is chosen in period 2 after the observation of the aggregate shocks. Denoting by $s = (z, \xi, B)$ the aggregate states in period 2, the debt repayment is denoted by $\tilde{B}(s)$. Being the repayment a function of $s$, it is also contingent on the commitment state $\xi$. To use a compact notation we denote the government policies by $\pi = (B, \tilde{B}(s))$.

The problems solved by workers is

$$\max_{c_1(\pi), c_2(\pi, z), h(\pi, z)} U(c_1(\pi)) + \beta \mathbb{E}_{\xi, z} U\left(\varphi\left(c_2(\pi, z), h(\pi, z)\right)\right)$$

subject to

$$c_1(\pi) = e_1 + \frac{B}{R(\pi)}$$

$$c_2(\pi, z) = e_2 + w(\pi, z)h(\pi, z) - \tilde{B}(s).$$
The problems solved by entrepreneurs is

$$\max_{b(\pi), d_1(\pi), l(\pi, z), d_2(\pi, z, \varepsilon)} \quad u\left(d_1(\pi)\right) + \beta \mathbb{E}_{\xi, z, \varepsilon} u\left(d_2(\pi, z, \varepsilon)\right)$$

subject to

$$d_1(\pi) = a - \frac{b(\pi)}{R(\pi)}$$

$$d_2(\pi, z, \varepsilon) = \left[A(z, \varepsilon) - w(\pi, z)\right] l(\pi, z) + \delta(\pi)b(\pi).$$

**Definition 1** A competitive equilibrium for given policies $\pi = \{B, \tilde{B}(s)\}$ is defined by price functions $R(\pi)$ and $w(\pi, z)$, decision functions for workers, $c_1(\pi)$, $h(\pi, z)$, $c_2(\pi, z)$, and entrepreneurs, $b(\pi)$, $d_1(\pi)$, $l(\pi, z)$, $d_2(\pi, z, \varepsilon)$, such that workers solve problem (3), entrepreneurs solve problem (4), asset and labor markets clear, that is, $b(\pi) = B$ and $l(\pi, z) = h(\pi, z)$.

While the decision of workers reduces to the choice of labor in period 2 (once we use the budget constraints to replace $c_1(\pi)$ and $c_2(\pi)$), the decision of entrepreneurs is more complex. Because of the concavity of the utility function, saving and hiring decisions take into account the risk associated with production. The following lemma characterizes the optimal entrepreneurs’ policies.

**Lemma 2** Let $\phi(\pi, z)$ satisfy the condition $\mathbb{E}_{\varepsilon} \frac{A(z, \varepsilon) - w(\pi, z)}{1 + [A(z, \varepsilon) - w(\pi, z)]\phi(\pi, z)} = 0$. The entrepreneurs’s policies take the form

$$b(\pi) = \frac{a\beta}{1 + \beta} R(\pi),$$

$$d_1(\pi) = \frac{a}{1 + \beta},$$

$$l(\pi, z) = \phi(\pi, z) \delta(\pi)b(\pi),$$

$$d_2(\pi, z, \varepsilon) = \left[1 + \left(A(z, \varepsilon) - w(\pi, z)\right)\phi(\pi, z)\right] \delta(\pi)b(\pi).$$

**Proof.** See Appendix A

Of special interest is the hiring policy of the firm (third equation in the lemma) which depends on the entrepreneur’s wealth $\delta(\pi)b(\pi)$. This is because
labor is risky and when the wealth of the entrepreneurs falls, they take less risk by hiring fewer workers. The aggregation of individual decisions then implies that equilibrium employment and wage depend positively on the debt repayment $\bar{B}(s)$. This is made precise by the following lemma.

**Lemma 3** Suppose that $\phi(\pi, z) > 0$ for all $\bar{B}(s) \geq 0$. Then,

1. The factor $\phi(\pi, z)$ is strictly decreasing in $\bar{B}(s)$;
2. Wages $w(\pi, z)$ and employment $l(\pi, z)$ are increasing in $\bar{B}(s)$.

**Proof.** See Appendix C. ■

Therefore, if in period 2 the government decides to default, that is, $\bar{B}(s) < B$, both employment and wages decline. The central mechanism through which default generates a macroeconomic contraction is by destroying the financial wealth of entrepreneurs. This has two effects. The first effect is to redistribute wealth from entrepreneurs (who hold government debt) to workers (who pay taxes to repay the debt). The assumption that only workers pay taxes is not essential. The mechanism would still operate if taxes were equally paid by workers and entrepreneurs. What matters is that taxes are not proportional to the holding of public debt so that default implies that agents who hold the debt (entrepreneurs) experience a net loss while agents who do not hold the debt (workers) experience a net gain. The second effect, which is a consequence of the first, is to generate a macroeconomic contraction: entrepreneurs end up with lower wealth and, since labor is risky, they hire fewer workers.

Although the redistributive effect of default is beneficial for workers, the recessionary effect has negative consequences for them (it reduces the demand for labor and, therefore, their wages). Thus, from the perspective of workers, government default implies a trade-off: the benefit is the reduction of taxes and the cost is the reduction of income. From the perspective of entrepreneurs, instead, government default implies only a cost (in addition to losing part of their financial wealth, they also earn lower incomes). The different welfare effects of default on workers and entrepreneurs will be key to understanding the optimal choice of government policies.
3.2 Determination of government policies

To characterize the government’s problem we proceed backward. We first consider the problem solved in period 2. Then, given the optimal policy in period 2, we solve the government’s problem in period 1.

**Government problem in period 2.** In the second period, the government chooses the repayment $\tilde{B}(s)$ given the debt $B$ and the realization of aggregate productivity $z$. If the government remains committed to repay, that is, $\xi = Commit$, then $\tilde{B}(s) = B$. With probability $1 - \rho$, however, the government behaves opportunistically and will default if this improves social welfare. In this case the government makes a take-it-or-leave-it offer to creditors. The repayment offer maximizes the social welfare function which is a weighted sum of the utilities of workers and entrepreneurs. More specifically, it solves the problem

$$
\max_{\tilde{B}(s) \leq B} \left\{ (1 - \Psi)U\left( \varphi(c_2(\pi, z), h(\pi, z)) \right) + \Psi E_{\epsilon} u\left( d_2(\pi, z, \epsilon) \right) \right\}. \tag{5}
$$

Notice that the optimizing repayment $\tilde{B}(s)$ is an element of the policy vector $\pi = (B, \tilde{B}(s))$. Therefore, by changing $\tilde{B}(s)$ the government affects equilibrium consumption and labor.

Consider first the relaxed problem where the repayment is not subject to the constraint $\tilde{B}(s) \leq B$ (or $\delta(\pi) \leq 1$). Assuming that the objective function (5) is strictly concave in $\tilde{B}(s)$, there will be a unique solution to the government problem. The first order condition, derived in Appendix D, takes the form

$$
(1 - \Psi)U'(\varphi(c_2(\pi, z), h(\pi, z))) = \Psi E_{\epsilon} u'(d_2(\pi, z, \epsilon)), \tag{6}
$$

where the prime denotes derivatives.

The first order condition shows that the government equalizes the weighted marginal utility of consumption for workers (net of the dis-utility from working) to the expected marginal utility of consumption for entrepreneurs. Using this condition we derive the following result.

**Proposition 4** Let $e_2 = 0$ and $A(z, \epsilon) = z + \epsilon$. The optimal repayment without commitment ($\xi = Not Commit$) is strictly increasing in the aggregate shock $z$. 

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Proof. See Appendix E □

This result shows that the incentive to default, that is, the incentive to repay a lower value of debt, is higher when the country is in recession. But by defaulting, the government amplifies the recession.

To understand this result we should reconsider the two effects of default described earlier. The first effect is the redistribution of wealth from entrepreneurs to workers. The second is the macroeconomic recession: a lower $\hat{B}(s)$ implies lower entrepreneurs’ wealth which reduces the demand for labor. This is also harmful for workers. However, since the consumption of workers is lower when productivity falls, their marginal utility is higher. From the prospective of the government, this increases the benefit of redistributing wealth toward workers and, therefore, the incentive to default (the first effect). Also, since labor is less productive, the loss of output from distorting the input of labor is smaller (the second effect).

Denote by $B^{\text{max}}(z)$ the ‘unconstrained’ optimal repayment of debt in period 2 without commitment. This is the solution to the government problem (5) without the constraint $\hat{B}(s) \leq B$. This satisfies the first order condition (6). Because the unconstrained repayment is not subject to the constraint $\hat{B}(s) \leq B$, it is independent of $B$ and it only depends on aggregate productivity: whatever the government promised to repay in period 1 becomes irrelevant once we reach period 2.

Figure 3 plots the government indirect utility for two levels of aggregate productivity. The graph is constructed using a specific parametrization of the model. Since the graph is only meant to provide a numerical example, we will postpone the discussion of the parameter values.

The graph plots four curves. The solid lines refer to the case of commitment so that the government cannot default. These two curves have a maximum which is bigger when productivity is high, $z = z_H$. The dotted curves are for the case of no commitment. The reason they become flat is because, if the debt $B$ is bigger than the level in which the welfare function is maximized, the government defaults. Effectively, this brings the debt to the optimal (lower) level and, therefore, beyond this point the welfare function is flat. Notice that the optimizing repayment is bigger when productivity is high, that is, $B^{\text{max}}(z_H) > B^{\text{max}}(z_L)$.

After characterizing the unconstrained optimal repayment, we can now consider the constrained policy which is subject to $\hat{B}(s) \leq B$. Of course, if $B^{\text{max}}(z) \geq B$ the government repays $B$. If $B^{\text{max}}(z) < B$, the government
Figure 3: Second period welfare as a function of $B$. The solid lines, denoted by $W(B,z)$, indicate the social welfare when the government commits to repay the whole debt $B$. The dotted lines, denoted by $V(B,z)$, are the optimized social welfare when the government does not commit to repay the whole debt. Below the optimal repayment, the continuous and dotted lines overlap. Above the optimal repayment, the optimized welfare becomes constant because the government defaults on the debt if this is bigger than the optimal repayment.

repays $B^{\text{max}}(z)$. Therefore, the optimal debt repayment is,

$$B(z, \xi, B) = \begin{cases} 
B^{\text{max}}(z), & \text{if } B^{\text{max}}(z) < B, \xi = \text{Not Commit} \\
B, & \text{otherwise}
\end{cases} 
$$

(7)

**Government problem in period 1.** Because of the particular specification of preferences (log-utility), government policies do not affect consumption in period 1. In fact, Lemma 2 shows that the consumption of entrepreneurs in period 1 is equal to $d_1 = a/(1 + \beta)$ and, therefore, is independent of $B$. In equilibrium, a higher $B$ will be associated with a proportional increase in the interest rate $R(\pi)$ so that entrepreneurial savings $B/R(\pi)$ remain unchanged.

The consumption of workers is $c_1 = c_1 - B/R(\pi)$. But Lemma 2 shows that $B/R(\pi) = a\beta/(1 + \beta)$. Therefore, $c_1$ is also independent of $B$. We can then characterize the government problem in period 1 ignoring the flow of
utility in period 1 and write it as
\[
\max_B \mathbb{E}_{\xi,z} \left\{ (1 - \Psi)U\left( \varphi(c_2(\hat{\pi}, z), h(\hat{\pi}, z)) \right) + \Psi E_u \left( d_2(\hat{\pi}, z, \varepsilon) \right) \right\},
\]
where the policy vector \( \hat{\pi} = (B, B(z, \xi, B)) \) now contains the optimal repayment policy chosen by the government in period 2, as characterized in (7). Even if the government cannot affect welfare in period 1, by choosing the debt today, it affects the optimal repayment and welfare in period 2.

The government objective is a weighted sum of the solid and dotted lines plotted in Figure 3, where the weights are given by the probabilities of different realizations of the aggregate productivity \( z \) and commitment state \( \xi \).

The optimal debt chosen in the first period \( B^* \) satisfies
\[
B^* \in [B^{\text{max}}(z_L), B^{\text{max}}(z_H)].
\]

To understand this result, note that in the extreme case in which country \( i \) has no commitment, \( \rho^i = 0 \), the optimal choice in the first period is \( B^* = B^{\text{max}}(z_H) \). This is the case because \( V(B, z_L) \) is constant for \( B > B^{\text{max}}(z_L) \) whereas \( V(B, z_H) \) is increasing. The government’s objective function given \( \rho^i = 0 \) becomes \( \mathbb{E}_z V(B, z) \), which is increasing up to \( B^{\text{max}}(z_H) \). In period 1, the government is constrained by the default decisions that the government will make in period 2. By choosing the largest possible value of sustainable debt \( B^* = B^{\text{max}}(z_H) \), the government has the option of reducing it in period 2 in the event of bad times (and there is no commitment). Notice that during booms debt is repaid in full \( \delta(B^*, B(B, z_H)) = 1 \), whereas partial defaults are observed in recessions, \( \delta(B^*, B(B, z_L)) < 1 \) (again, provided that the commitment to repay is relaxed). The size of default depends on the relative weights given by the government to workers and entrepreneurs, as well as on the degree of aggregate uncertainty (that is, the distance between \( z_H \) and \( z_L \)). The choice of the first period debt, together with the option to partially default in the second period, attempts to replicate allocations that could be achieved with state-contingent debt (an instrument that, by assumption, is not available to the government).

When \( \rho^i > 0 \), the second period government would repay its debt if \( \xi = \text{Commit} \), regardless of the realization of the productivity shock. In such case, second period welfare is given by \( W(B, z) \), corresponding to the solid lines in the figure. Because these functions are decreasing for any \( B > B^{\text{max}}(z_j) \) for
\[ j = L, H, \] choosing \( B^* = B^{max}(z_H) \) may be sub-optimal. The reason being that debt repayment during recessions is very costly for workers. The first period government, understanding that the second period government may actually repay its debt, has incentives to choose a smaller level of debt than in the environment where \( \rho^i = 0 \) (e.g. when it never commits).

### 4 Financial Integration without renegotiation

We now extend the model with two countries that are financially integrated. The two countries can trade sovereign bonds issued by each country. Labor, however, is immobile. We refer to the first country as ‘home’ country and to the second as ‘foreign’ country.

Denote \( B_h \) the debt issued by the home country. Furthermore, denote by \( B^{hh} \) and \( B^{hf} \) the home debt held, respectively, by entrepreneurs in home and foreign countries. In equilibrium \( B_h = B^{hh} + B^{hf} \). Similarly, the debt issued by the foreign country is denoted by \( B^f \), in part held by entrepreneurs in the home country, \( B^{fh} \), and in part by entrepreneurs in the foreign country, \( B^{ff} \). Therefore, the first superscript indicates the nationality of the government that issued the debt and the second superscript indicates the nationality of the debt holders (entrepreneurs).

The two countries are ex-ante homogeneous except in the degree of commitment to repay the debt. With probability \( \rho^i \) country \( i \in \{h, f\} \) remains committed to repay the debt in period 2 and with probability \( 1 - \rho^i \) it defaults if it finds optimal to do so (that is, if default improves the social welfare of its country). The probability of commitment is known in period one but the commitment status, \( \xi^i \in \{Commit, Not Commit\} \), is revealed only at the beginning of period 2, together with aggregate productivity \( z^i \).

The prices for home and foreign bonds are, respectively, \( 1/R^h \) and \( 1/R^f \). Even though financial markets are perfectly integrated, the prices for home and foreign bonds could differ because of the different probabilities and size of default.

We first consider the case in which there is no renegotiation over debt repayments, analogous to the analysis in the previous sections where the home country makes a take-it-or-leave it offer to its creditors (which are now composed of domestic and foreign entrepreneurs). In the next section, we consider the possibility of renegotiation (bailout) between domestic and foreign governments.
4.1 Equilibrium for given policies

Let’s first define the set of aggregate states in period 2 which are given by
\[ s = (z^h, z^f, \xi^h, \xi^f, B^{hh}, B^{hf}, B^{fh}, B^{ff}). \] Since the two countries are financially integrated, the states include variables related to both countries. The policy variables are \( \pi = (B^h, B^f, B^h(s), B^f(s)) \).

**Definition 5** With financially integrated markets, a competitive equilibrium for given policy \( \pi \) is defined by prices \( \{R^i(\pi), w^i(\pi, s)\}_{i \in \{h, f\}} \), decision functions for workers \( \{c^1_i(\pi), h^i(\pi, s), c^2_i(\pi, s)\}_{i \in \{h, f\}} \), decision functions for entrepreneurs \( \{b^hi(\pi), b^fi(\pi), d^1_i(\pi), d^2_i(\pi, s, \varepsilon)\}_{i \in \{h, f\}} \), such that

1. **Workers in country** \( i \in \{h, f\} \) **maximize**

\[
U(c^1_i(\pi)) + \beta \mathbb{E}_s U\left( \varphi(c^2_i(\pi, s), h^i(\pi, s)) \right)
\]

subject to

\[
c^1_i(\pi) = e_1 + \frac{B^i}{R^i(\pi)},
\]

\[
c^2_i(\pi, s) = e_2 + w^i(\pi, s)h^i(\pi, s) - \tilde{B}^i(s).
\]

2. **Entrepreneurs in country** \( i \in \{h, f\} \) **maximize**

\[
\mathcal{U}(d^1_i(\pi)) + \beta \mathbb{E}_{s, \varepsilon}(d^2_i(\pi, s, \varepsilon))
\]

subject to

\[
d^1_i(\pi) = a - \frac{b^{hi}(\pi)}{R^h(\pi)} - \frac{b^{fi}(\pi)}{R^f(\pi)},
\]

\[
d^2_i(\pi, s, \varepsilon) = \left[ A(z^i, \varepsilon) - w^i(\pi, s) \right] l^i(\pi, s) + \delta^h(\pi)b^{hi}(\pi) + \delta^f(\pi)b^{fi}(\pi).
\]

3. **Asset and labor markets clear, that is, for** \( i \in \{h, f\} \)

\[
B^i = b^{hi}(\pi) + b^{fi}(\pi),
\]

\[
l^i(\pi, s) = h^i(\pi, s).
\]
An important difference between autarky and financial integration is that in the latter entrepreneurs hold a portfolio of bonds issued by both home and foreign governments. This has three implications. First, since part of the public debt is held by foreigners, the home government may have a higher incentive to default. Second, the holding of foreign assets in the portfolio of home entrepreneurs (financial diversification) reduces the macroeconomic cost of default for the home country. An implication of this is that, when a foreign country issues more debt, entrepreneurs in the home country hold a larger share of foreign assets, which reduces the macroeconomic cost of defaulting for the home country. This, in turn, rises the incentive of the home country to default. Third, the default of the home country affects employment and output in both countries. In other words, the macroeconomic consequences of sovereign default are exported to other countries (spillover).

While the first implication is common to most of the sovereign default models proposed in the literature, the second and third implications are special features of our model.

We can now provide a characterization of the competitive equilibrium. Since labor is immobile, the optimal choices of workers under financial integration are as in the closed economy. The labor supply is

\[ h^i(\pi, s) = \left( \frac{w^i(\pi, s)}{\alpha} \right)^\nu. \]

The equilibrium wage rate could differ in the two countries depending on the realization of aggregate shocks, which in turn implies differences in employment.

As in the autarky regime, entrepreneurs’ decisions can be characterized in closed form as summarized by the following lemma.

**Lemma 6** Let \( \phi^i(\pi, s) \) satisfy the condition \( \mathbb{E}_z \frac{A(z^i, \varepsilon) - w^i(\pi, s)}{1 + (A(z^i, \varepsilon) - w^i(\pi, s))\phi^i(\pi, s)} = 0. \)

The entrepreneur’s policies in country \( i \) are

\[
\begin{align*}
\phi^i(\pi) & = a \left[ 1 - \theta^h(\pi) - \theta^f(\pi) \right], \\
n^i(\pi) & = \theta^h(\pi) R^h(\pi) a, \\
n^f(\pi) & = \theta^f(\pi) R^f(\pi) a, \\
h^i(\pi, s) & = \phi^i(\pi, s) \left[ \delta^h(\pi) b^hi(\pi) + \delta^f(\pi) b^fi(\pi) \right], \\
n^i_2(\pi, s, \varepsilon) & = 1 + \left( A(z^i, \varepsilon) - w^i(\pi, s) \right) \phi^i(\pi, s) \left[ \delta^h(\pi) b^hi(\pi) + \delta^f(\pi) b^fi(\pi) \right],
\end{align*}
\]
where \( \theta^h(\pi) \) and \( \theta^f(\pi) \) solve

\[
\frac{1 + \beta}{\beta} = \mathbb{E}_s \left[ \frac{1}{\theta^h(\pi) R^h(\pi) + \theta^f(\pi)} \right],
\]

\[
\frac{\beta}{1 + \beta} = \theta^h(\pi) + \theta^f(\pi).
\]

**Proof.** See Appendix F □

Entrepreneurs split their initial wealth \( a \) between current consumption \( d^1_i(\pi) \) and financial assets. Since the fraction saved, \( \theta^h(\pi) + \theta^f(\pi) = \frac{\beta}{1 + \beta} \), does not depend on policies, consumption in the first period is exactly the same as in a closed economy.

The main difference with the autarky equilibrium is that entrepreneurs hold both home and foreign bonds. The fractions of savings allocated by entrepreneurs to home and foreign bonds, \( \theta^h \) and \( \theta^f \), respectively, are independent of their nationality. This means that entrepreneurs in home and foreign countries choose the same composition of portfolio. This derives from the assumption that the two countries are identical in preferences and technology (including the distribution of the idiosyncratic shock) and the fact that entrepreneurs do not pay taxes or receive transfers.³ The second period consumption, on the other hand, could differ due to different realizations of the aggregate shocks \( z^h \) and \( z^f \).

The cross-country diversification of portfolios implies that the sovereign default of one country creates macroeconomic costs for both countries. In fact, because entrepreneurs hold bonds issued by both countries, they all experience a financial loss if one country defaults. This causes a contraction in demand for labor in both countries, which in turn reduces employment \( h^i(\pi, s) \) and wages \( w^i(\pi, s) \) in both countries. Thus, with international financial integration, even if only one country defaults on its sovereign debt, the negative macroeconomic consequences are transmitted to other countries.

Another implication of Lemma 6 is that the consumption of entrepreneurs in period 1 is independent of the borrowing decisions of the governments of the two countries since \( d^h_1 = d^f_1 = a/(1 + \beta) \). This implies that the worldwide consumption of workers in period 1 is independent of government policies.

³If entrepreneurs paid taxes or received transfers, their disposable income would be affected by the realization of \( \xi^i \). Because the probability \( \rho^i \) is country-specific, this would introduce asymmetry in their portfolio decisions.
since \( c_1^h + c_1^f + d_1^h + d_1^f = 2a + 2c_1 \). In other words, the total worldwide consumption of workers and entrepreneurs in period 1 must be equal to their wealth endowment. However, even if total worldwide consumption of workers in period 1 is independent of policies, their distribution among home and foreign workers depends on policies. This is an important difference between the autarky regime and the regime with integrated financial markets: while in autarky the first period welfare is independent of the debt chosen by the government, with financial integration the borrowing decision of a country impacts on the first period welfare of both countries.

### 4.2 Optimal policies

As for the analysis of the autarky regime we characterize the optimal government policies backward starting with the problem solved in period 2. Then, taking as given the optimal government strategy in period 2, we will solve for the optimal policies in period 1.

#### 4.2.1 Optimal policies in period 2

In period 2 the government of country \( i \in \{h, f\} \) chooses repayment \( \tilde{B}^i \) to maximize the weighted sum of utilities of workers and entrepreneurs in country \( i \), analogous to problem (5). Now, however, the optimal choice also depends on the repayment of the other country. The two countries choose their repayment simultaneously and without cooperation through a Nash strategic game. Appendix G shows that, using the conditions that define a competitive equilibrium, the optimization problem of government \( i \) in period 2, given the debt repayment of the other government \( \tilde{B}^{-i} \), can be written as

\[
\max_{\tilde{B}^i \leq B^i} \left\{ (1 - \Psi) \ln \left( \nu w^i (\pi, z^i)^{1+\nu} - \tilde{B}^i \right) + \Psi \ln \left( \frac{\tilde{B}^h + \tilde{B}^f}{2} \right) + \Psi E_\varepsilon \ln \left( 1 + \left[ A(z^i, \varepsilon) - w^i (\pi, z^i) \right] \phi^i (\pi, z^i) \right) \right\}. \tag{8}
\]

Notice that the policy vector is \( \pi = (B^h, B^f, \tilde{B}^h, \tilde{B}^f) \). Therefore, when the government of country \( i \) chooses \( \tilde{B}^i \), it changes one of the variables in this vector taking as given the repayment policy of the other country.
The derivation provided in the appendix uses the equilibrium property for which entrepreneurs in home and foreign countries hold the same portfolio of home and foreign bonds (see Lemma 6). This equilibrium property is captured by the term \((\tilde{B}_h + \tilde{B}_f)/2\) which represents the post-default wealth of entrepreneurs and it is equal to the per-capita worldwide sovereign debt issued by the two countries. Because of portfolio diversification, the welfare function of country \(i\) also depends on the repayment of the other country. Notice that the aggregate productivity of the other country, \(z^{-i}\), is irrelevant for the default decision of country \(i\). The only channel through which the other country affects country \(i\) is through the repayment \(\tilde{B}^{-i}\).

To characterize the optimal repayment policy of country \(i\) (conditional on the repayment policy of the other country), consider first the unconstrained optimization, that is, ignore the constraint \(\tilde{B}_i \leq B_i\). Assuming that the government objective is strictly concave, there will be a unique solution. The first order condition, derived in Appendix H, takes the form

\[ \Psi E_z u'(d_2^{2}(\pi, z^i, \varepsilon)) = (1 - \Psi)U'(\varphi_2^{2}(\pi, z^i)) \Omega_i^{2}(\pi, z^i), \]

where \(\Omega_i^{2}(\pi, z^i)\) is a term that is equal or bigger than 1.

We can see from the first order condition that, if the term \(\Omega_i^{2}(\pi, z^i)\) were equal to 1, then the government would equalize the marginal utilities of the two types of agents, re-scaled by the weights \(\Psi\) and \(1 - \Psi\). This was the optimality condition in the autarky equilibrium. With financial integration, however, \(\Omega_i^{2}(\pi, z^i) \geq 1\). It is as if the government assigns a higher weight to workers when financial markets are integrated. This implies that, keeping everything else equal, the incentive to default is higher compared to autarky.

There are two reinforcing channels that increase the home country incentive to default. The first channel arises from the redistribution of wealth from foreigners to domestic agents. Because some of the domestic debt is held by foreigners, default redistributes wealth not only from domestic entrepreneurs to domestic workers but also from foreign entrepreneurs to domestic workers. Recall that the portfolio of entrepreneurs is now diversified, with holdings \((\tilde{B}_h + \tilde{B}_f)/2\). The redistribution from foreigners to domestic agents is a channel that is present in many models of sovereign default. But the focus of our

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\(^4\)This implies that the policies \(\pi\) together with the realization of domestic productivity \(z^i\) are sufficient to characterize the equilibrium. Thus, the arguments of all relevant functions are now \(\pi\) and \(z^i\).

\(^5\)In the appendix we show that \(\Omega_i^{2}(\pi, z^i) = \frac{1 - \frac{\partial w_2}{\partial \pi h_2}}{\frac{1}{2} - \frac{\partial w_2}{\partial \pi h_2}} \geq 1\), where \(F = \frac{\tilde{B}_h + \tilde{B}_f}{2}\).
paper is not on this channel. Instead, the focus of our paper is on the second channel, which derives from the fact that default generates lower financial losses for domestic entrepreneurs thanks to their international diversification. Compared to the autarky regime, this implies that the macroeconomic impact of default on the home country is smaller. This mechanism, which is novel in the sovereign default literature, increases the government’s incentive to default.

Denote by \( B^{Max}(z^i, \tilde{B}^{-i}) \) the unconstrained optimal repayment of country \( i \). This can be expressed as a function of the debt repayment of the other country, \( \tilde{B}^{-i} \), and the domestic productivity \( z^i \). Notice that, in general, the unconstrained optimum would also depend on the portfolio composition of domestic entrepreneurs. However, since in the competitive equilibrium home and foreign entrepreneurs choose the same portfolio composition, \( \tilde{B}^{-i} \) and \( z^i \) are sufficient to characterize the optimal repayment of country \( i \). The solution to the constrained problem (8) is \( B^{Max}(z^i, \tilde{B}^{-i}) \) if \( \tilde{B}^{-i} < B^i \) otherwise the solution is \( B^i \).

The Nash equilibrium repayments \( \hat{B}^i \), with \( i \in \{h, f\} \), is defined as

\[
\hat{B}^i = \begin{cases} 
B^{Max}(z^i, \hat{B}^{-i}), & \text{if } B^{Max}(z^i, \hat{B}^{-i}) < B^i, \xi^i = \text{Not Commit} \\
B^i, & \text{otherwise.}
\end{cases}
\]

The Nash solution in period 2 depends on the aggregate productivity in the two countries, \( z^h \) and \( z^f \), the commitment status of the two countries, \( \xi^h \) and \( \xi^f \), and the debt issued by the two countries in period 1, \( B^h \) and \( B^f \). Since entrepreneurs in the two countries choose the same portfolio, that is, \( B^{ih} = B^{if} = \hat{B}^i \), the stocks of debt issued by the two countries, \( B^h \) and \( B^f \), are sufficient statistics for the characterization of the government policies. We can then express the Nash solution as a function of the states \( s = (z^h, z^f, \xi^h, \xi^f, B^h, B^f) \), that is,

\[
\hat{B}^i = B^i(s). \quad (9)
\]

### 4.2.2 Optimal policies in period 1

After characterizing the optimal government policy in period 2, we can now move to period 1 when the governments of the home and foreign countries choose, respectively, \( B^h \) and \( B^f \). Also in period 1 the two countries choose
their debt simultaneously and without cooperation through a Nash game. They maximize their own welfare taking as given the debt issued by the other country as well as the possible equilibrium policies in the second period $B(s)$ as defined in (9).

The second period indirect utility of country $i$ is equal to

$$V^i(s; B) = \left\{ (1 - \Psi) \ln \left( \nu w^i(\pi, z^i)^{1+\nu} - B^i(s) \right) + \Psi \ln \left( \frac{B^h(s) + B^f(s)}{2} \right) + \right.$$  
$$\left. \Psi E_\xi \ln \left( 1 + \left[ A(z^i, \varepsilon) - w^i(\pi, z^i) \right] \phi^i(\pi, z^i) \right) \right\}, \quad (10)$$

where $\pi = \{B^h, B^f, B^h(s), B^f(s)\}$.

We have seen that under autarky, the consumptions of workers and entrepreneurs in period 1 are independent of debt. Because of this, the optimal debt is determined by simply maximizing the expected second period indirect utility. This is no longer the case when financial markets are integrated. This is because the choice of debt in period 1 affects the consumption of workers through the interest rate. The consumption of entrepreneurs in period 1, instead, remains unaffected by the choice of $B^i$ as in the autarky regime. This implies that the government has to consider also the impact that its choice of $B^i$ will have on the utility of workers.

The objective of the government in country $i$ can be written as

$$\max_{B^i} \left\{ (1 - \Psi)U(c^i_1(\pi)) + \beta E_{z, \xi} V^i(s; B) \right\},$$

where we have omitted the utility of entrepreneurs in period 1 since this is unaffected by policies.

Denote by $g^i(B^{-i})$ the best response function of country $i$ to the debt issued by the other country, $B^{-i}$. A Nash equilibrium for the policy game played by the two governments in period 1 is defined as a pair $(\bar{B}^h, \bar{B}^f)$ that satisfies the conditions

$$\bar{B}^h = g^h(\bar{B}^f),$$
$$\bar{B}^f = g^f(\bar{B}^h).$$

Unfortunately, we are unable to derive an analytical characterization of the Nash equilibria. Therefore, we will provide a numerical characterization.
**Numerical example**  For the numerical characterization, we use the following parameter values: $\beta = 0.9825$, $\nu = \alpha = 1$, $a = e_1 = e_2 = 1$. The shocks are independently and identically across country. The idiosyncratic shock $\varepsilon$ is uniformly distributed over the set $\{0.9, 1, 1.1\}$; the aggregate shock $z$ is uniformly distributed over the set $\{0.95, 1.05\}$; the probability of commitment is set $\rho^h = \rho^f = 0.5$. The weight that the planner assigns to workers is $\Psi = 0.5$.

Figure 4 plots the best responses of the two countries in the autarky regime and in the regime with capital mobility as a function of the other country’s debt. In the autarky regime (left panel), the response functions are constant since the equilibrium of one country is not affected by the debt chosen by the other country. With mobility (right panel), instead, the optimal debt depends on the debt chosen by the other country. In particular we observe that the optimal debt of one country decreases as the debt of the other country increases. This is because, when the foreign country issues more debt, domestic entrepreneurs hold more financial wealth in equilibrium (as a results of diversification). Therefore, there is less need for liquidity in the domestic country.

Figure 4: Response functions in period 1 for autarky and mobility. Equilibrium determined by the intersection of the two response functions.

The comparison of the two panels of Figure 4 shows that the equilibrium worldwide supply of debt, $B^h = B^f$, is significantly bigger when the economies are closed than in the regime with financial integration. This happens because, with mobility, part of the debt issued by one country is purchased by entrepreneurs residing in the other country and, therefore, it brings benefits to the other country. But each government does not inter-
nalize the benefits that its debt creates for the other country as a source of liquidity. This implies that each government issues less debt than under autarky. This negative externality results in inefficiently low levels of debt when countries are financially integrated (i.e. too little liquidity).

5 Financial integration with bailouts

So far we have characterized the equilibrium under the assumption that the debt is not renegotiated when a country defaults. In many instances of sovereign default, debt is restructured at the aggregate level (i.e. between governments) and with it there is some form of direct or indirect subsidies. For example, with rescue packages the defaulting countries are able to borrow at rates that are lower than the rates at which they could borrow in the market. In this section we allow for renegotiation and debt restructuring. This seems a natural assumption since both countries could gain from renegotiating the repayment of the debt.

5.1 Second period: Debt renegotiation

In period 2, after observing the realization of $\xi = (\xi^f, \xi^h)$, aggregate productivity $z = (z^h, z^f)$, and given the stocks of home and foreign debt $B = (B^h, B^f)$, one or both countries may default. In this case the two countries might find convenient to negotiate the repayment of the debt.

With renegotiation the two countries bargain the actual debt repayment—which we denote by $P = (P^h, P^f)$—and a transfer $\tau^i$ to country $i$ paid by country $j$, $\tau^i = -\tau^j$.

To characterize the negotiation outcome we use Nash bargaining. This requires the definition of the threat values for the two countries if they do not reach an agreement. In the absence of an agreement the two countries revert to the equilibrium without renegotiation described in the previous section.

Let’s first define some key functions. Given the negotiated policies $P = (P^h, P^f)$ and $\tau = (\tau^h, \tau^f)$, the value for country $i \in \{h, f\}$ in period 2 is

\[ V_i = \frac{1}{2} (P_i^h + P_i^f - \tau_i^h) z_i^h + \frac{1}{2} (P_i^f + \tau_i^f) z_i^f. \]

Because we have not modeled the potential beneficial effects of financial integration, a move from autarky to financially integrated markets results in a welfare loss for both countries in our model. The objective of this paper is not to analyze the trade-off between having an open or a closed economy, but rather to discuss the implications of external forces on the incentives of a country to default.
\[ V^i(\tau, P, z^i) = \left\{ (1 - \Psi) \ln \left( \tilde{w}^i (P, z^i)^{1+\nu} - P^i + \tau^i \right) + \Psi \ln \left( \frac{P^h + P^f}{2} \right) + \right. \]
\[ \left. \Psi \mathbb{E}_x \ln \left( 1 + \left[ A(z^i, \varepsilon) - w^i (P, z^i) \right] \phi^i (P, z^i) \right) \right\}. \] (11)

This equation is analogous to expression (10), but where repayments are now \( P^i \) instead of \( \mathcal{B}^i(s) \) and home workers receive the transfer \( \tau^h \) while foreign workers receive the transfer \( -\tau^h \). Notice that the transfers affect welfare but they do not affect the demand for labor and, therefore, wages. The debt repayments \( P = (P^h, P^f) \), instead, affect employment and wages in addition to welfare.

The above equation illustrates how renegotiation affects the welfare of the two countries. Consider the case in which the home country defaults. For both countries, higher repayment \( P^h \) implies positive effects coming from lower macroeconomic distortions (higher wages for workers and profits for entrepreneurs captured by the first and third terms of equations (11)) and higher repayment to entrepreneurs (captured by the second term). Higher repayments \( P^h \) also imply a cost for the home country due to higher taxes that home workers have to pay (see first term in equation (11)). The transfer \( \tau^h \), instead, is a benefit for the home country since it reduces the tax burden of workers but it is a cost for the foreign country since foreign workers have to pay the transfer (remember that \( \tau^f = -\tau^h \)). Effectively, foreign workers help home workers to repay the debt of the home country. Of course the opposite arises if it is the foreign country that defaults.

The above analysis illustrates why the foreign country may gain from subsidizing the repayment of the home debt: since a higher \( P^h \) increases the welfare of the foreign country by facilitating higher demand of labor, the foreign government may be willing to pay \( \tau^f = -\tau^h \) in order to induce a higher repayment from the home government. Since a higher repayment \( P^h \) has also some positive effects for the home country (in addition to the higher taxes for home workers), the foreign government can convince the home government to repay by ‘partially’ subsidizing the repayment. This will become clear in the numerical example shown below. Before doing so, however, we need to define the bargaining problem formally.

The threat value for country \( i \) is the value that the country would experience in period 2 if there is no agreement. In this case each country would use
the optimal uncoordinated policy $\mathcal{B}^i(s)$ characterized earlier, and receive no bailout subsidies, $\tau^i = 0$. Thus the threat value is $V^i(s; \mathcal{B})$ defined in (10), that is, the value when both countries implement the optimal policy without renegotiation.

We can then write the bargaining problem as

$$
\max_{\tau^h, P^h, P^f} \left[ V^h(\tau, P, z^h) - V^h(s; \mathcal{B}) \right] \eta \left[ V^f(\tau, P, z^f) - V^f(s; \mathcal{B}) \right]^{1-\eta},
$$

subject to

$$
P^i = B^i, \quad \text{if } \xi^i = \text{Commit}
$$

$$
P^i \leq B^i, \quad \text{if } \xi^i = \text{Not Commit}
$$

where $\eta$ is the relative bargaining power for the home country. As it is standard, the bargaining problem maximizes the weighted product of the net renegotiation surpluses of the negotiating parties. The feasible repayment depends on the commitment of the two countries. Obviously, when both countries commit, $\tau^h = \tau^f = 0$. We denote the solution to the bargaining problem with $\tau = T(s)$ and $P = P(s)$.

To characterize the bargaining solution we resort to the numerical example of the previous sections. Baseline parameters are identical to the ones described in Section 4.2.2. The bargaining weight is set to $\eta = 0.5$. Figure 5 plots the repayment of the debt (total and by the home country) when $\xi^h = \text{Not Commit}$ and $\xi^f = \text{Commit}$; that is, when only the home country does not commit to repay. Total repayment is the fraction $(\tilde{B}^h + \tilde{B}^f)/(B^h + B^f)$ without bailout and $(P^h + P^f)/(B^h + B^f)$ with bailout. The home repayment is the fraction $\tilde{B}^h/B^h$ without bailout and $(P^h - \tau^h)/B^h$ with bailout. Figure 6 depicts the case in which both countries have no commitment to repay, $\xi^h = \xi^f = \text{Not Commit}$.

The two figures show that with bailout the total debt $B^h + B^f$ is fully repaid for relatively low values of debt (that is not the case for large values of debt, not depicted in these figures). Furthermore, higher is the debt of the home country and lower is the fraction $(P^h - \tau^h)/B^h$ repaid by the home country. Although not shown, the repayment of the home country increases with the debt of the foreign country. Essentially, starting with higher debt is similar to having more bargaining power. Therefore, when the foreign country starts with higher debt, the home country repays more. Lastly we observe that the home repayment is higher when the productivity of the home country is high and the productivity of the foreign country is low.
Figure 5: Debt repayments with and w/o commitment when only the home country does not commit. The repayments are plotted as functions of the home debt, when the foreign debt is \( B_f = 0.677 \). ‘Total repay w/o bailout’ is the fraction \( (B_h + B_f)/(B_h + B_f) \) and ‘Total repay with bailout’ is \((P_h + P_f)/(B_h + B_f)\). ‘Home repay w/o bailout’ is the fraction \( B_h/B_h \) and ‘Home repay with bailout’ is the fraction \((P_h - \tau_h)/B_h\). Each line corresponds to different combinations of productivity in the two countries.

5.2 First period: Optimal borrowing

We can now move to the first period where each country chooses the debt \( B^i \) in order to maximize

\[
\max_{B^i} \left\{ (1 - \Psi)U\left(c^i_1(\pi^R)\right) + \beta \mathbb{E}_s V^i\left(T(s), P(s), z^i\right) \right\}.
\]

The policy vector is now \( \pi = \{B, T(s), P(s)\} \). Notice that this problem is similar to the problem studied in the previous section. The only difference is that second period policies are chosen under renegotiation (if at least one country does not commit in period 2). As before, we assume that first period decisions are taken simultaneously and without cooperation by the
Figure 6: Debt repayments with and w/o commitment when both countries do not commit. The repayments are plotted as functions of the home debt, when the foreign debt is $B^f = 0.677$. ‘Total repay w/o bailout’ is the fraction $(\tilde{B}^h + \tilde{B}^f)/(B^h + B^f)$ and ‘Total repay with bailout’ is $(P^h + P^f)/(B^h + B^f)$. ‘Home repay w/o bailout’ is the fraction $\tilde{B}^h/B^h$ and ‘Home repay with bailout’ is the fraction $(P^h - \tau^h)/B^h$. Each line corresponds to different combinations of productivity in the two countries.

governments of the two countries. Therefore, the definition of the (Nash) equilibrium is the same.

The best response functions for the numerical example are plotted Figure 7. The figure also shows the response functions for the environment without bailout and for the autarky case (as shown earlier).

We observe that with bailout the equilibrium debt of the two countries (determined by the intersection of the response functions) is significantly larger than the debt without bailout. This is because, knowing that there is a probability of bailout in period 2, each country has an incentive to borrow more in the first period. Although the debt level with bailout is bigger than in the environment without bailout, it is still smaller than in autarky regime.
In the next subsection we show that, by inducing higher equilibrium debt, the anticipation of bailouts could improve welfare in period 1.

5.3 To bailout or not to bailout?

Although renegotiation is always efficient in period 2, we have seen that its anticipation in period 1 induces higher borrowing and, therefore, higher ex-post incentive to default. It is then natural to ask whether bailouts are also efficient from an ex-ante prospective.

Figure 8 plots the country welfare in period 1 as a function of debt assuming that the foreign debt is equal to home debt. Since the two countries are symmetric and, therefore, in equilibrium they would choose the same debt, these (symmetric) levels of debt represent possible equilibrium outcomes. The curves represent the weighted expected utility in period 1 for
one of the two countries and for the three regimes: autarky, mobility without bailout and mobility with anticipated bailout. The graph also indicates with the vertical lines the equilibrium debt in each of the three regimes. These are determined by the intersection of the best response functions shown in Figure 7.

Figure 8: Social welfare as a function of public debt when both countries choose the same debt in period 1, for three regimes: autarky, mobility without bailout and mobility with bailout. Social welfare is the $\Psi$-weighted utility of workers and entrepreneurs. The vertical lines indicate the debt in the symmetric Nash equilibrium for each of the three regimes.

The first thing to observe is that the welfare with bailout dominates the welfare without bailout for the relevant range of debt. This is also true when we compare the two equilibria (indicated by the first two vertical lines). Therefore, at least for this numerical example, the equilibrium with anticipated bailout brings higher welfare than the equilibrium without bailout.

Why does bailout allow for higher welfare than no bailout? To see why it would be useful to compare the autarky equilibrium with the equilibrium
in which financial markets are integrated but there is no bailout. More debt allows for greater efficiency in period 2. However, with financial integration, when a country issues more debt the efficiency benefits are shared with the other country. Therefore, the incentive to issue debt is lower. In autarky, instead, the country fully internalizes the benefits of issuing its own debt and, therefore, more debt is issued in equilibrium. The result is that the utility in the autarky equilibrium is higher than in the equilibrium with integrated financial markets. Essentially, mobility creates the conditions for the sub-optimal issuance of debt.

Bailouts alleviate this problem. Each country anticipates in period 1 that with some probability the other country will contribute to the repayment of the other country’s debt. Therefore, the incentive to borrow increases, bringing the allocation closer to autarky. Renegotiation acts as a correction mechanism for the positive externality created by the issuance of debt. It makes the effective cost of debt for the issuing country lower since, in the case of default, the other country contributes to its repayment.

5.4 The role of commitment

Countries do not always default in period 2 in this economy. Whether defaults occur in equilibrium depends on the stock of debt each country enters period 2 with, the realization of their productivity shocks, $z = (z^h, z^f)$, and their commitment shocks, $\xi = (\xi^h, \xi^f)$. In this section we examine how the probability of commitment affects the equilibrium debt level chosen in period 1. Although this probability is exogenous in the model, it can be interpreted as the result of political turnover. In particular, in period 2 there is a probability of a new government taking power that assigns a much higher weight to entrepreneurs who, obviously, prefer higher repayment.

In the previous numerical example we have set the probability of commitment for each country to $\rho^i = 50\%$. Now we consider two alternative scenarios with, respectively, a probability of commitment of 60% (high commitment) and 40% (low commitment).

Table 1 reports the equilibrium debt and social welfare for different levels of commitment and for three regimes: Autarky, Mobility w/o bailout

\footnote{Because the utility of entrepreneurs in country $i$ is strictly increasing in debt repayment in period 2, $B^i$, a government assigning weight $\Psi^i = 1$ to entrepreneurs would never find it optimal to default. That is, even if $\xi^i = \text{Not Commit}, B^i(s) = B^i$ regardless of the realization of $s$. Smaller values of $\Psi^i$ would be associated with lower repayment values.}
Table 1: **Equilibrium debt and welfare for different commitment**

<table>
<thead>
<tr>
<th></th>
<th>High commitment ((\rho_h = \rho_f = 0.6))</th>
<th>Baseline ((\rho_h = \rho_f = 0.5))</th>
<th>Low commitment ((\rho_h = \rho_f = 0.4))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Debt  Utility</td>
<td>Debt  Utility</td>
<td>Debt  Utility</td>
</tr>
<tr>
<td>Autarky</td>
<td>0.766 -0.411</td>
<td>0.768 -0.410</td>
<td>0.772 -0.410</td>
</tr>
<tr>
<td>Mobility w/o bailout</td>
<td>0.577 -0.454</td>
<td>0.567 -0.458</td>
<td>0.560 -0.462</td>
</tr>
<tr>
<td>Mobility with bailout</td>
<td>0.671 -0.418</td>
<td>0.693 -0.416</td>
<td>0.718 -0.414</td>
</tr>
</tbody>
</table>

and Mobility with bailout. Looking at the mobility regime with bailout we observe that lower commitment increases equilibrium debt and improves welfare. This is because, with lower commitment there is a higher probability that the debt issued by one country is partially repaid by the other country (through a bailout), which increases the incentive to borrow. This compensates in part for the under-issuance of debt, as the benefits of creating financial assets of one country are shared with the other country. Thus, lower commitment brings the equilibrium closer to autarky where each country internalizes the full benefit of issuing debt.

6 Conclusion

In this paper we have shown that the default of a country on its sovereign debt could be induced by excessive borrowing from other countries if financial markets are integrated. The integration of financial markets increases the incentive to default not only because part of the defaulted debt is owned by foreigners (as widely emphasized in the literature) but also because the ‘endogenous’ macroeconomic cost of default is smaller when the defaulting country is financially integrated.

In our model government debt is held by producers as an insurance instruments. When financial markets are integrated, producers also hold foreign government debt. Therefore, when the domestic government defaults, producers are only partially affected by default with smaller consequences for aggregate production in the domestic country. Furthermore, the higher is
the debt issued by the foreign country, the higher the incentive of the home country to default is since domestic producers hold more foreign debt and, therefore, are more insured. This implies that the sovereign default of a country could be externally induced by the excessive borrowing of foreign countries. From this perspective, the recent debt problems experienced by some European countries can be the result (at least in part) of the increased debt in ‘safe’ industrialized countries since the early 1980s.

We have also considered the possibility of debt renegotiation, which can be interpreted as a ‘bailout’ policy. We have shown that the anticipation of a bailout increases the indebtedness of both countries. However, the higher indebtedness is not necessarily welfare reducing. The reason being that the higher level of sustainable debt corrects for an externality that emerges when financial markets are integrated. The externality arises because each country ignores the benefits that public debt has for the other country in terms of liquidity to entrepreneurs. When bailouts are possible, countries anticipate that with some probability the other country will contribute to the repayment of their own debt, effectively reducing the cost of borrowing, and creating the conditions for higher liquidity. This corrects the under-issuance of debt and makes the equilibrium more efficient. Therefore, bailouts could be Pareto efficient not only ex-post but also ex-ante.
7 Appendix

A Proof of Lemma 2

The FOC of the entrepreneur maximize the problem stated in eq. (4) are

\[ l(\pi, z) : \quad E_\varepsilon \left[ u' \left( d_2(\pi, z, \varepsilon) \right) \left( A(z, \varepsilon) - w(\pi, z) \right) \right] = 0, \tag{12} \]

and

\[ b(\pi) : \quad u' \left( d_1(\pi) \right) = \beta E_{\xi, z, \varepsilon} \left[ u' \left( d_2(\pi, z, \varepsilon) \right) \delta(\pi) R(\pi) \right]. \tag{13} \]

Guess policy functions to be

\[ l(\pi, z) = \phi(\pi, z)b(\pi)\delta(\pi) \]

\[ b(\pi) = \gamma R(\pi)a, \]

where \( \{\phi(\pi, z)\}_{z, \xi} \) and \( \gamma \) are unknown. These imply that \( d_1 = a(1 - \gamma) \) and \( d_2(\pi, z, \varepsilon) = b\delta(\pi)[1 + \phi(\pi, z)(A(z, \varepsilon) - w(\pi, z))]. \) Using eq. (13) and the two guesses, we obtain

\[ \beta \frac{1 - \gamma}{\gamma} E_{\xi, \varepsilon, z} \left[ \frac{1}{1 + \phi(\pi, z)[A(z, \varepsilon) - w(\pi, z)]} \right] = 1. \tag{14} \]

Replacing the guesses in eq. (12) we get

\[ E_\varepsilon \left[ \frac{A(z, \varepsilon) - w(\pi, z)}{1 + \phi(\pi, z)[A(z, \varepsilon) - w(\pi, z)]} \right] = 0. \]

This equation implicitly defines \( \phi(\pi, z). \) Multiply each side of it by \( \phi(\pi, z), \) then subtract 1 from both sides and reorganize to obtain

\[ E_\varepsilon \left[ \frac{1}{1 + \phi(\pi, z)[A(z, \varepsilon) - w(\pi, z)]} \right] = 1. \]

Taking expectations with respect to \( \xi, \varepsilon, z, \) yields

\[ E_{\xi, \varepsilon, z} \left[ \frac{1}{1 + \phi(\pi, z)[A(z, \varepsilon) - w(\pi, z)]} \right] = 1. \]

Replacing the last expression into eq. (14) and simplifying yields \( \gamma = \frac{\beta}{1+\beta}. \) Replacing this back into the guess for \( b(\pi), \) we obtain \( b(\pi) = \frac{\beta}{1+\beta} R(\pi)a. \) The expressions for \( d_1 \) and \( d_2(\pi, z, \varepsilon) \) in the Lemma are obtained by replacing the results into the entrepreneurs’ budget constraints. QED
B Competitive equilibrium for given policies

The interest rate is given by

\[ R(\pi) = \frac{(1 - \beta)B}{\beta a}. \]  (15)

The wage rate \( w(\pi, z) \) and the labor demand factor \( \phi(\pi, z) \) are implicitly determined by the equations

\[ w(\pi, z) = \alpha \left( \phi(\pi, z) \tilde{B}(s) \right)^{\frac{1}{\beta}}, \]  (16)

\[ E_\varepsilon \left\{ \frac{A(z, \varepsilon) - w(\pi, z)}{1 + [A(z, \varepsilon) - w(\pi, z)] \phi(\pi, z)} \right\} = 0. \]  (17)

Aggregate labor is given by

\[ l(\pi, z) = \phi(\pi, z) \tilde{B}(s) = h(\pi, z). \]  (18)

Finally, consumption is determined by the equations

\[ c_1 = c_1 + \frac{\beta a}{(1 + \beta)}, \]

\[ \varphi \left( c_2(\pi, z), h(\pi, z) \right) = \left( \frac{\alpha \nu}{1 + \nu} \right) h(\pi, z)^{1 + \frac{1}{\nu}} + \left( \frac{\alpha \nu}{1 + \nu} \right) w(\pi, z)^{1 + \nu} + e_2 - \tilde{B}(s), \]

\[ d_1 = \frac{a}{1 + \beta}, \]

\[ d_2(\pi, z, \varepsilon) = \left[ 1 + \left( A(z, \varepsilon) - w(\pi, z) \right) \phi(\pi, z) \right] \tilde{B}(s). \]

C Proof of Lemma 3

Replace eq. (16) into eq. (17) to obtain

\[ E_\varepsilon \frac{A(z, \varepsilon) - \alpha \left( \phi \tilde{B} \right)^{1/\nu}}{1 + \left[ A(z, \varepsilon) - \alpha \left( \phi \tilde{B} \right)^{1/\nu} \right] \phi} = 0, \]
where dependence on the aggregate state \( z \) has been omitted to ease readability. This can be written more compactly as the following implicit function

\[
F(\phi, \tilde{B}) \equiv E_\epsilon \left\{ \left[ A(z, \varepsilon) - \alpha \left( \phi \tilde{B} \right)^{1/\nu} \right]^{-1} + \phi \right\}^{-1} = 0.
\]

Using the implicit function theorem,

\[
\frac{\partial \phi}{\partial \tilde{B}} = -\frac{\partial F/\partial \tilde{B}}{\partial F/\partial \phi} = -\frac{E_\epsilon G^{-2}(A(z, \varepsilon) - w)^{-2} \frac{w}{\nu B}}{E_\epsilon G^{-2} \left[ (A(z, \varepsilon) - w)^{-2} \frac{w}{\nu \phi} + 1 \right]} < 0
\]

since \( G \equiv \left[ A(z, \varepsilon) - \alpha \left( \phi \tilde{B} \right)^{1/\nu} \right]^{-1} + \phi > 0 \). This establishes the first result.

Differentiate eq.(16) to obtain \( \frac{\partial w(\pi, z)}{\partial \tilde{B}} \). After some algebraic manipulations,

\[
\frac{\partial w}{\partial \tilde{B}} = \frac{1}{\nu} \tilde{B} \frac{\partial \phi}{\partial \tilde{B}} + 1
\]

where \( \frac{\tilde{B} \partial \phi}{\phi \partial \tilde{B}} \leq 0 \) is the elasticity of the entrepreneurs’ labor share \( \phi \) with respect to \( \tilde{B} \). We will show that wages are increasing in \( \tilde{B} \) by contradiction. Suppose \( \frac{\partial w}{\partial \tilde{B}} < 0 \). Since \( \tilde{B} \geq 0 \) (by assumption), it must be the case that

\[
\frac{\tilde{B} \partial \phi}{\phi \partial \tilde{B}} < -1.
\]

Alternatively,

\[
\frac{E_\epsilon G^{-2}(A(z, \varepsilon) - w)^{-2} \frac{w}{\nu}}{E_\epsilon G^{-2} \left[ (A(z, \varepsilon) - w)^{-2} \frac{w}{\nu \phi} + \phi \right]} > 1.
\]

But this would imply that \( E_\epsilon G^{-2} \phi < 0 \), a contradiction.

Finally, using the fact that \( H = l = \left( \frac{w}{\alpha} \right)^\nu \), we can show that

\[
\frac{\partial H}{\partial \tilde{B}} = \frac{\nu H}{w} \frac{\partial w}{\partial \tilde{B}} \geq 0.
\]

QED
D Optimality condition under autarky

The first order condition of the relaxed problem (that is, ignoring the constraint $\tilde{B} \leq B$) is

$$\Psi \mathbb{E}_\varepsilon u'(d_2) \left[ -\frac{\partial w}{\partial B} h + 1 + (A - w) \frac{\partial h}{\partial B} \right] + (19)$$

From the optimality condition of agents, we know that

$$\mathbb{E}_\varepsilon u'(d_2)(A - w) = 0$$

and

$$U'(\varphi(c_2, h_2)) \left[ \frac{\partial \varphi}{\partial c_2} w + \frac{\partial \varphi}{\partial h_2} \right] = 0$$

which allows us to simplify terms involving $\frac{\partial h}{\partial B}$. Replacing these in eq. (D) and simplifying, we obtain

$$\left( 1 - \frac{\partial w}{\partial B} h \right) \left( \Psi \mathbb{E}_\varepsilon u'(d_2) - (1 - \Psi) U'(\varphi(c_2, h_2)) \frac{\partial \varphi}{\partial h_2} \right) = 0.$$  

The result follows from the fact that the first term in parenthesis is nonzero. QED

E Proof of Proposition 4

Replacing $c_2$ and $d_2$ in eq. (6), defining $\psi = \frac{1 - \Psi}{\Psi}$, and rearranging, we obtain

$$\mathbb{E}_\varepsilon \frac{1}{1 + (A(z, \varepsilon) - w(\pi, z)) \phi(\pi, z)} = \psi \frac{\tilde{B}}{\tilde{\nu} w(\pi, z)^{1+\nu} - \tilde{B}}.$$  

The left hand side is equal to 1 from the optimality condition of entrepreneurs. Hence,

$$\tilde{\nu} w(\pi, z)^{1+\nu} - \tilde{B} = \psi \tilde{B}. \Rightarrow w(\pi, z) = \left[ \frac{\tilde{B}(1 + \psi)}{\tilde{\nu}} \right]^{\frac{1}{1+\nu}}.$$  

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Equating this to eq. (16) and simplifying delivers

$$\phi(\tilde{B}) = \frac{\phi_0}{\tilde{B}^{1+\nu}} \quad \text{where} \quad \phi_0 = \frac{1}{\alpha} \left[ 1 + \psi^\nu \right]^{\frac{1}{1+\nu}}. \quad (20)$$

Wages become

$$w(\tilde{B}) = \omega_0 \tilde{B}^{1+\nu} \quad \text{where} \quad \omega_0 = (\alpha \phi_0)^\frac{1}{\nu}. \quad (21)$$

Replacing eq. (20) and (21) into eq. (17), we obtain an implicit function $\tilde{B}(z)$

$$F(\tilde{B}, z) = E_x \left\{ \frac{1}{1 + [A(z, \varepsilon) - w(\tilde{B})] \phi(\tilde{B})} \right\} - 1 = 0 \quad (22)$$

We can obtain $\partial \tilde{B} / \partial z$ using the implicit function theorem:

$$\frac{\partial \tilde{B}}{\partial z} = -\frac{\partial F(\tilde{B}, z)}{\partial z} \frac{\partial \tilde{B}}{\partial F(\tilde{B}, z)} \quad (23)$$

where

$$\frac{\partial F(\tilde{B}, z)}{\partial z} = -E_x [1 + [A(z, \varepsilon) - w] \phi]^{-1} \phi < 0 \quad (24)$$

using that $A(z, \varepsilon) = z + \varepsilon$ to replace $\partial A / \partial z = 1$, and

$$\frac{F(\tilde{B}, z)}{\partial \tilde{B}} = -E_x [1 + [A(z, \varepsilon) - w] \phi]^{-1} \left[ -\phi \frac{\partial w}{\partial \tilde{B}} + [A(z, \varepsilon) - w] \frac{\partial \phi}{\partial \tilde{B}} \right].$$

Using eqs. (20) and (21), we obtain

$$\frac{\partial w}{\partial \tilde{B}} = \frac{1}{1 + \nu} \frac{w}{\tilde{B}} \quad \text{and} \quad \frac{\partial \phi}{\partial \tilde{B}} = -\frac{1}{1 + \nu} \frac{\phi}{\tilde{B}}.$$

Replacing these equations in $\frac{F(\tilde{B}, z)}{\partial \tilde{B}}$ above, we have

$$\frac{F(\tilde{B}, z)}{\partial \tilde{B}} = E_x [1 + [A(z, \varepsilon) - w] \phi]^{-1} \frac{A(z, \varepsilon) \phi}{(1 + \nu) \tilde{B}} > 0. \quad (25)$$

Using eq. (24) and eq. (25) in eq. (23) establishes the result.

QED
F Proof of Lemma 6

The entrepreneurs' maximization problem is

$$\max x_i \ln d_1^i(\pi) + \beta E_{s,\varepsilon} \ln d_2^i(\pi, s, \varepsilon)$$

$$d_1^i(\pi) = a - \frac{b^{hi}}{R^h(\pi)} - \frac{b^{fi}}{R^f(\pi)}$$

$$d_2^i(\pi, s, \varepsilon) = (A(z^i, \varepsilon) - w^i(\pi, s))l^i(\pi, s) + b^{hi}\delta^h(s) + b^{fi}\delta^f(s),$$

where \( x_i = \{d_1^i, d_2^i, l^i, b^{fi}, b^{hi}\} \) is their set of choices.

Their FOC are

$$\frac{1}{d_1^i(\pi)} \frac{1}{R^h(\pi)} = \beta E_{s,\pi,\varepsilon} \frac{\delta^h(s)}{d_2^i(\pi, s, \varepsilon)} \quad (26)$$

$$\frac{1}{d_1^i(\pi)} \frac{1}{R^f(\pi)} = \beta E_{s,\pi,\varepsilon} \frac{\delta^f(s)}{d_2^i(\pi, s, \varepsilon)} \quad (27)$$

$$E_{\varepsilon} \frac{A(z^i, \varepsilon) - w^i(\pi, s)}{d_2^i(\pi, s, \varepsilon)} = 0 \quad (28)$$

From eqs. (26) and (27) we obtain

$$R^h(\pi) = \eta R^f(\pi) \quad \text{where} \quad \eta = \frac{E_{s,\pi,\varepsilon} \frac{\delta^f(s)}{d_2^i(\pi, s, \varepsilon)}}{E_{s,\pi,\varepsilon} \frac{\delta^h(s)}{d_2^i(\pi, s, \varepsilon)}}.$$

Guess the following

$$b^{hi}(\pi) = \theta^{hi}(\pi)R^h(\pi)a$$

$$b^{fi}(\pi) = \theta^{fi}(\pi)R^f(\pi)a$$

$$l_i(\pi) = \phi^i(\pi, s)[b^{hi}(\pi)\delta^h(s) + b^{fi}(\pi)\delta^f(s)]$$

Under that guess (and abstracting from arguments to simplify notation)

$$d_1^i(\pi) = a(1 - \theta^{hi}(\pi) - \theta^{fi}(\pi))$$
\[ d_2^i(\pi) = \left( [A(z^i, \varepsilon) - w^i(\pi, s)]\phi^i(\pi, s) + 1 \right) \left[ b^{hi}(\pi)\delta^h(s) + b^{fi}(\pi)\delta^f(s) \right], \]

Moreover,

\[ b^{hi}(\pi)\delta^h(s) + b^{fi}(\pi)\delta^f(s) = a[\theta^{hi}(\pi)R^h(\pi)\delta^h(s) + \theta^{fi}(\pi)R^f(\pi)\delta^f(s)] \]

Replacing the equations above in eq. (28) and using the fact that \( b^{hi}(\pi)\delta^h(s) + b^{fi}(\pi)\delta^f(s) \) is independent of \( \varepsilon \) we obtain

\[ E_\varepsilon \frac{A(z^i, \varepsilon) - w^i(\pi, s)}{[A(z^i, \varepsilon) - w^i(\pi, s)]\phi^i(\pi, s) + 1} = 0 \]

Multiplying by \( \phi^i(\pi, s) \) and subtracting 1 from both sides, we get

\[ E_\varepsilon \frac{1}{[A(z^i, \varepsilon) - w^i(\pi, s)]\phi^i(\pi, s) + 1} = 1 \] (29)

Replacing the guesses in eq. (27)

\[ \frac{1}{1 - \theta^{fi}(\pi) - \theta^{hi}(\pi)} = \beta E_{s,\pi} \left\{ \frac{R^f(\pi)\delta^f(s)}{\theta^{hi}(\pi)R^h(\pi)\delta^h(s) + \theta^{fi}(\pi)R^f(\pi)\delta^f(s)} \right\} \]

From eq. 29, we know that for each \( \{s, \pi\} \), the term involving \( E_\varepsilon \) is equal to 1. Using the fact that \( R^h(\pi) = \eta R^f(\pi) \),

\[ \frac{1}{1 - \theta^{fi}(\pi) - \theta^{hi}(\pi)} = \beta E_{s,\pi} \left[ \frac{1}{\theta^{hi}(\pi)\eta\delta^h(s)/\delta^f(s) + \theta^{fi}(\pi)} \right] \] (30)

Replace the guesses into eq. (26), and follow the same steps to obtain

\[ \frac{1}{1 - \theta^{fi}(\pi) - \theta^{hi}(\pi)} = \beta E_{s,\pi} \left[ \frac{\eta\delta^h(s)/\delta^f(s)}{\theta^{hi}(\pi)\eta\delta^h(s)/\delta^f(s) + \theta^{fi}(\pi)} \right] \] (31)

Multiply both sides of eq. (30) by \( \theta^{fi}(\pi) \), and both sides of eq. (31) by \( \theta^{hi}(\pi) \), and add the resulting expressions. This delivers,

\[ \theta^{hi}(\pi) + \theta^{fi}(\pi) = \frac{\beta}{1 + \beta} \] (32)

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Substituting into eq. (30) and remembering that $\eta = R^h(\pi)/R^f(\pi)$ we obtain

$$\frac{1 + \beta}{\beta} = E_{s,\pi} \left[ \frac{1}{\theta^{hi}(\pi) \frac{\delta h(s) R^h(\pi)}{\delta f(s) R^f(\pi)} + \theta^{fi}(\pi)} \right]$$

(33)

QED

G Derivation of government’s objective under FI

Let $\tilde{B}^i(s) = B^i \delta^i(s)$. In the second period, country $i$’s government solves a problem like eq. (5), where allocations satisfy the conditions established by Lemma 6. Since all entrepreneurs are identical in $t = 1$, $b^{ii} + b^{ij} = B^i$. This implies that

$$b^{ii}(\pi) = b^{ij}(\pi) = \theta^i(\pi) R^i(\pi) a \Rightarrow b'(\pi) = \frac{B^i}{2},$$

for $i, j \in \{h, f\}$. Using the results from Lemma 6 and noticing that $\theta^i(\pi)$ is country-independent, we get

$$R^h(\pi) = \frac{B^h}{2a \theta^h(\pi)} \quad \text{and} \quad R^f(\pi) = \frac{B^f}{2a \theta^f(\pi)}.$$

We can use $R^h(\pi) = \eta R^f(\pi)$ to obtain

$$\eta = \frac{B^h \theta^f(\pi)}{B^f \theta^f(\pi)}$$

Replacing this into eq. (33) and simplifying, we get

$$\theta^f(\pi) = \frac{\beta}{1 + \beta} E_{s,\pi} \left[ \frac{1}{1 + \frac{B^h}{B^f}} \right]$$

(34)

Replacing this into eq. (32), we obtain $\theta^h(\pi)$. Hence, the financial wealth of both home and foreign entrepreneurs is $(\tilde{B}^h + \tilde{B}^f)/2$. 43
Equating the aggregate supply of labor to the aggregate demand for labor we obtain

\[ h^i(\pi, s) = \left[ \frac{w^i(\pi, s)}{\alpha} \right]^\nu = \phi^i(\pi, s) \left[ \frac{\tilde{B}^h + \tilde{B}^f}{2} \right]. \]

Replacing \( b^h(\pi) \) and \( b^f(\pi) \) in the labor market equilibrium condition, delivers two equations determining \( w^i \) and \( \phi^i \),

\[ w^i(\pi, z^i) = \alpha \left( h^i(\pi, z^i) \right)^{1/\nu} E^\varepsilon \frac{A(z^i, \varepsilon) - w^i(\pi, z^i)}{A(z^i, \varepsilon) - w^i(\pi, z^i)\phi^i(\pi, z^i) + 1} = 0, \]

with \( h^i(\pi, z^i) = \phi^i(\pi, z^i) \left[ \frac{\tilde{B}^h + \tilde{B}^f}{2} \right] \). The wage rate and, therefore, the factor that determines the demand of labor depend on country \( i \)'s productivity \( z^i \) and wealth of home entrepreneurs after government default \( (\tilde{B}^h + \tilde{B}^f)/2 \). Therefore, we will denote the wage as \( w^i(\pi, z^i) \) and the labor demand factor as \( \phi^i(\pi, z^i) \). Replacing these results in entrepreneurs’ and workers’ budget constraints delivers eq. (8). QED

H Optimality condition under Financial Integration

When \( \xi^i = \text{Not Commit} \), country \( i \)'s first order condition of the relaxed problem (that is, ignoring the constraint \( \tilde{B}^i \leq B^i \)) is

\[
\Psi E^\varepsilon u'(d_2^i) \left[ -\frac{\partial w^i}{\partial B^i} h^i + 1 \right] + (1 - \Psi) U'(\varphi(c_2^i, h_2^i)) \left[ \frac{\partial \varphi}{\partial c_2^i} \left[ \frac{\partial w^i}{\partial B^i} h^i - 1 \right] + \frac{\partial \varphi}{\partial h_2^i} \frac{\partial h^i}{\partial B^i} \right] = 0.
\]

We can use the optimality condition of entrepreneurs and workers to further simplify this expression using steps similar to those under autarky (see Appendix D). The equation is reduced to

\[
\Psi E^\varepsilon u'(d_2^i) \left[ -\frac{\partial w^i}{\partial B^i} h^i + 1 \right] + (1 - \Psi) U'(\varphi(c_2^i, h_2^i)) \frac{\partial \varphi}{\partial c_2^i} \left[ \frac{\partial w^i}{\partial B^i} h^i - 1 \right] = 0.
\]
Letting $F = \frac{B_h + B_f}{2}$ and defining $\Omega^i(\pi, z^i) = \frac{1 - \frac{\partial^2 h}{\partial \pi^i \partial \pi^i} h}{\frac{\partial}{\partial \pi^i} h} \geq 1$ and re-arranging this equation delivers the optimality condition in the text.

QED
References


