

Polarization and delay: uncertainty in reputational bargaining

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Abstract

I show how uncertainty about fundamentals can cause delay in bargaining when agents have reputational concerns. Agents' publicly observable costs of delay change stochastically at some *revelation time*. In addition to rational agents, there are behavioral types committed to many different fixed demands. I show that even when the probability of behavioral types is arbitrarily small, agreement may be delayed until after the revelation time and rational agents may demand almost the entire surplus. If behavioral types can make time-varying demands, however, then the outcome converges to the solution of a complete information alternating offers game.

Keywords: Bargaining, reputation, behavioral types, uncertainty, delay, polarization

1 Introduction

Bargaining often involves agents negotiating when they are uncertain about the future. If an agreement is not reached during some initial period of time, the bargaining environment may change, sometimes favoring one agent in the negotiations and sometimes another. For instance: both trade unions and employers may be uncertain of the costs of a strike to each party until that strike is in progress; firms considering a merger may be uncertain about the impact on each of future regulations or the health of the economy; political parties in a divided legislature may not know which side the press will blame most for shutting down the government until that shutdown occurs; and those same political parties may be uncertain about the outcome of an upcoming Presidential election, with the losing party facing negative press coverage if they subsequently refuse to accept the victor's "mandate".

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Such uncertainty frequently results in inefficient delay to agreement. Trade unions often do go on strike; merger activity falls in times of economic uncertainty; political parties do sometimes shut down the government, and fail to pass pressing legislation during an election year.¹

Standard bargaining models struggle to explain this delay. For instance, if agents make alternating offers in a dollar division task and there is common knowledge of rationality, backward induction implies an immediate agreement. Agents calculate their expected payoffs in all future bargaining environments, before agreeing to a compromise at time zero that reflects both the likelihood of those different futures, and their initial costs of delay.

The main result of this paper shows how relaxing common knowledge of rationality very slightly, by allowing for an arbitrarily small possibility that agents are obstinate, can explain why uncertainty causes delay.

By an obstinate agent, I mean one who is committed to her demand and is unwilling to back down even if the alternative is no agreement. Facing such an opponent a rational person will immediately concede, because delay is costly and cannot secure a better deal. This in turn implies incentives for rational people to *pretend* to be obstinate in the hope that a “tough” reputation will convince a rational opponent to concede.

The model reduces bargaining to the problem of two agents dividing a dollar when there is an infinite horizon. Agents face some initial known costs of delaying agreement (impatience for instance) which apply up to some *revelation time*, R . At the revelation time the state of the world is publicly revealed, which determines agents’ costs of delay from that point forward. There is, therefore, no asymmetric information about costs.

Obstinate agents are modelled as behavioral types (also known as irrational types or commitment types). Following [Abreu and Gul \[2000\]](#) (henceforth AG0) I focus on types committed to fixed demands, they always demand a share $\alpha^i \in [0, 1]$ of the dollar and accept any offer that gives them more than that, and allow for many different types committed to many different (fixed) demands.

It is well known that reputational incentives can result in delay even when there is no uncertainty about the bargaining environment (see for instance AG0). My contribution is to show how reputation and uncertainty interact in perverse ways to dramatically increase delay. Without uncertainty (so that changes in the bargaining environment are predictable), agents must agree almost immediately if the probability that agents are behavioral is sufficiently small. This result is in line with AG0, which considers the stationary infinite horizon bargaining problem. With uncertainty, however, delay may persist.

Indeed, if agents’ face linear flow costs flow costs of delay, each agent has lower costs than her opponent in some state of the world, and the revelation time is not too distant, then as

¹For instance, [Mayhew \[1991\]](#) shows how Congress passes significantly fewer important pieces of important legislation in the two years before a presidential election compared to the two years after an election.

the probability of behavioral types vanishes, so must the probability of agreement before the revelation time. The demands of rational agents polarize in the limit: each (imitates an obstinate type who) demands the entire dollar, and backs down only if it is revealed that she has higher costs than her opponent. The size of inefficiency implied may be as much as half the surplus.

If the cost of delay is merely impatience, then for some parameters similar delay and demand polarization must occur, but for other parameters it will not. Whether or not delay occurs may also depend on which agent has the right/obligation to announce a demand first.

Why does delay occur? Much depends on well known results about reputational bargaining without uncertainty from AGO. First, an agent who is known to be rational must concede immediately to the demand of a possibly behavioral opponent. This means that a rational agent must pretend to be behavioral, and cannot change her initial demand (as this would reveal rationality). Second, when costs of delay are known (after the revelation time R) equilibrium must resemble a war of attrition. At most one agent concedes with positive probability at time R , then each agent concedes at a constant rate to keep that opponent indifferent between conceding at one instant or the next (so concession is proportional to an opponent's cost of delay), and finally both agents reach a probability one reputation at the same time. Third, if reputations at time R converge to zero at the same rate then the agent with higher costs of delay will concede at time R with probability approaching one.

Given the above results, it should be clear that if demands are incompatible and reputations are small shortly before time R , each agent may have strict incentives to wait until the revelation of uncertainty, in the hope that her opponent will be revealed to have higher costs of delay and so concede with high probability.

This is, however, an incomplete partial equilibrium explanation for delay, based on a given pair of conflicting demands assumed to be imitated by rational agents. AGO shows that when there is no uncertainty rational agents make moderate (close to compatible) demands when the probability of behavioral types is small. If this was still the case when there is uncertainty, there would be little incentive to wait until the revelation time, because being conceded to instead of conceding increases an agent's payoff only slightly. The polarization of rational agents' demand choices is thus crucial to the prediction of delay.

The reason for demand polarization might be naively thought to be due to the desire of a rational agent to maximize her (delayed) payoffs in states of the world in which her opponent has high costs. Although part of the answer, this presumes that delay is inevitable. Because of the inefficiency of delay there is always a compromise division, which if agreed immediately would strictly benefit both agents. Moreover, agents *can* develop a reputation for being committed to such a compromise. A compromise is not proposed, however, as it will not be accepted because the act of proposing a compromise decreases an opponent's incentive to accept it by increasing her option value of waiting.

To make this point in a simple example, suppose politicians are negotiating before a (not too distant) Presidential election, which is won by either party with probability half. If party a wins the election, then party b faces higher flow costs than a of delaying agreement, and *visa versa* if party b wins. In equilibrium when the probability of behavioral types is small, each party (imitates a type which) demands the entire dollar, and accepts her opponent's demand only if she loses the election, giving each an expected payoff of approximately $\frac{1}{2} - x_0^i R$ (where x_0^i is party i 's instantaneous flow cost of delay and R is the revelation time). This is worse for both than agreeing a 50/50 compromise division immediately. Why is the compromise not then proposed? Suppose before the election party b demands the entire dollar, but party a proposes the 50/50 compromise instead. This would imply that party b can expect half the surplus if it loses the election, but still the entire surplus if it wins, giving it an expected payoff from waiting of almost $\frac{3}{4} - x_0^b R > \frac{1}{2}$. But in which case, party b still prefers to wait. Delay *is* inevitable.

The demand polarization associated with delay is an interesting feature of the result in its own right. It suggests that the model can help explain not just the difficulty making a deal in Congress during an election year, but also the divergence of parties' platforms ahead of elections, a markedly different prediction compared to the standard median voter model.

Another insight from the model concerns the importance of which agent makes the first demand announcement. As mentioned above, this can affect whether or not delay occurs. Indeed, this prospect implies that *both* agents may prefer that agent a announces her initial demand before b . Even when it does not have implications for the likelihood of delay, the importance of the first move can still be substantial, with first or second mover advantages possible. The order matters because of the strategic nature of demand announcements: after announcing a demand an agent cannot change it without being identified as rational. Previous reputational models have, nonetheless, always shown the order of initial demand announcements to be irrelevant to payoffs when the probability of behavioral types is small.

By allowing for a wide array of behavioral types committed to many different demands, I attempt to avoid the criticism sometimes made of behavioral models, that they simply represent "garbage in, garbage out" (the ability to obtain any desired outcome with the "correct" assumption about types). The above results do, however, require that behavioral types are committed to fixed demands. These types have received special attention in the literature and are the only types considered in AG0. They seem to perhaps offer the best approximation of obstinate agents in bargaining, which I seek to model. In particular, it might seem implausible to believe an agent is genuinely committed to a demand if she was willing to accept a smaller share just moments before.²

I do, however, also extend the model to allow for more "complex" behavioral types whose demands can vary over time and depend on the state of the world. With this richer type space

²Another (ex-post) reason to consider fixed demand types is that they generate model predictions which match real world behavior.

delay disappears and there is no first mover effect as the probability of behavioral types vanishes. Instead rational agents imitate a type committed to what I call the generalized Rubinstein demand, in order to guarantee the associated time zero payoff. The generalized Rubinstein demand is the solution to an alternating offers game without behavioral types for this bargaining environment, as the time between offers becomes arbitrarily small.

The paper most closely related to my prediction of delay with fixed demand types is [Avery and Zemsky \[1994\]](#), in which agents make alternating offers to buy exclusive ownership of an asset whose value follows a multiplicative random walk. In each period the buyer makes a fixed price offer, then the new value of the asset is realized, and only then does the seller decide whether to accept. For some parameters the unique stationary symmetric equilibrium involves the buyer demanding low prices, which are only accepted if the asset falls in value (other equilibria also exist). Similar to my model, the inability to change demands after the state of the world is revealed creates an option value for the seller. In my model, however, the inability to change demands is endogenous rather than exogenous and so is the option value delay, which only emerges because an opponent chooses to concede (the immediate value of a deal does not change). Less related papers in which uncertainty (about proposal rights) can explain bargaining delays include [Fershtman and Seidmann \[1993\]](#) or [Simsek and Yildiz \[2014\]](#).

The generalized Rubinstein convergence result with complex demand behavioral types is in line with the finding in AG0 that outcomes converge to the stationary Rubinstein outcome with fixed demand types in a stationary environment. It is closer still to [Fanning \[2014\]](#) (henceforth F14), which shows convergence to a generalized Rubinstein outcome when there is a stochastic deadline for agreement and behavioral demands may be complex. [Abreu and Pearce \[2007\]](#) (henceforth AP7) was the first paper to consider complex demand types generalizing the results of AG0 in the stationary environment.

The rest of this paper is structured as follows. I describe the model in section 2; prove the existence of a unique equilibrium in section 3; present the main results in section 4; and conclude in section 5. Proofs are included in the Appendix if not given directly.

2 The model

Two agents must agree on how to divide a dollar. There is an infinite horizon. Before some revelation time $R > 0$ agent i discounts payoffs exponentially at rate $r_0^i \geq 0$, and faces linear flow costs of delay $x_0^i \geq 0$, where $r_0^i + x_0^i > 0$ ensures a strict preference for an early deal. A deal agreed at time $t \leq R$ which secured agent i a share $\alpha^i \in [0, 1]$ of the surplus would, therefore, give her a utility of:

$$u^i(\alpha^i, t) = e^{-r_0^i t} \alpha^i - \int_0^t e^{-r_0^i s} x_0^i ds$$

After time R the costs of delay change to rates r_ω^i and x_ω^i when the state of the world is $\omega \in \Omega$, where $r_\omega^i + x_\omega^i > 0$. The state ω is initially unknown to either agent but is distributed according to probability measure p . The state is only revealed to agents at time R , and this is why I call R the revelation time. The assumption that costs of delay change at exactly the same time as the state is revealed simplifies the analysis but is not essential to it.³ For simplicity, I also assume that Ω is finite.

Bargaining takes place in discrete-continuous time. This is an invention of AP7 to allow for continuous time bargaining while avoiding some of the mathematical headaches associated with that (such as openness problems). Time is treated as continuous between (positive) integer times but has a special discrete structure at the integer times so that multiple events can happen there in a sequential order. Each integer time k is divided into the discrete times $k_{-1}, k_0, k_{+1}, k_{+2}, k_{+3}$, where k_l occurs before k_{l+1} . The costs of delay associated with an agreement at k_l are evaluated using the real number k , which means there is no discounting or flow costs incurred across these sequential times. I switch back and forth between referring to time k and time k_l in ways that should be clear from the context. The set of discrete-continuous times is then $DC = \{k_{-1}, k_0, k_{+1}, k_{+2}, [k_{+3}, (k+1)_{-1}] : k \in \mathbb{Z}_+\}$. Without loss of generality, the revelation time R is assumed to be an integer. The state of the world is revealed to agents at time R_0 .

The first integer time is zero. Nothing happens at 0_{-1} or 0_0 . At time 0_{+1} agent 1 announces a bargaining position α^1 . This bargaining position is a measurable function from $DC \times \Omega$ into $[0, 1]$, which details the share which agent 1 intends to demand from any agreement at all times in every state (assuming no earlier agreement has been reached). Because the state is unknown to agents before R_0 , this is constrained to satisfy $\alpha^1(t, \omega) = \alpha^1(t, \hat{\omega})$ for $t \leq R_{-1}$ and all $\omega, \hat{\omega} \in \Omega$. At time 0_{+2} , agent 2 can either immediately accept the offer ($1 - \alpha^1(0_{+1}, \omega)$), or make a counter demand α^2 with similar properties. At any time $t \in [0_{+3}, 1_{-1}]$ either agent can concede to her opponent's existing offer. If both agents concede at exactly the same time then one of the two proposals is implemented with probability half.

Next, consider a typical integer time $k > 0$. At time k_{-1} either agent can concede to her opponent's existing demand. Nothing happens at k_0 , except when $k = R$ in which case the state of the world is revealed. At time k_{+1} one agent, say i , can concede to her opponent j 's existing offer ($1 - \alpha^j(k_{+1}, \omega)$), or announce a (possibly new) bargaining position α^i (which must again map from $DC \times \Omega$ into $[0, 1]$). At time 0_{+2} the other agent j can concede to i 's (possibly new) existing offer ($1 - \alpha^i(k_{+2}, \omega)$) or announce her own (possibly new) bargaining position α^j . At any time $t \in [k_{+3}, (k+1)_{-1}]$ either agent can concede to her opponent's (possibly new) existing offer. The identity of which agent moves at k_{+1} as opposed to k_{+2} is arbitrary for $k \notin \{0, R\}$ but is known to both agents. I assume that agent 1 moves at R_{+1} and agent 2 at R_{+2} .

³Allowing the state to be revealed at time R but having costs change at a later date S will not change the prediction of delay up until time R .

To minimize notation in what follows, I refer to $\alpha^i(t, \omega)$ as $\alpha_0^i(t)$ if $t \leq R_{-1}$ and as $\alpha_{\omega'}^i(t)$ otherwise, where this is without confusion given the constraint on demands to be the same in all states for $t \leq R_{-1}$. I also refer to $\omega' \in \Omega' = \{0\} \cup \Omega$ as the “current” state.

Reputational dynamics comes from a prior probability that agents are behavioral $z^i > 0$. A behavioral type i is identified by a particular bargaining position α^i : a type α^i always demands $\alpha_{\omega'}^i(t)$ at time t when the current state is ω' and accepts any offer greater or equal to that, rejecting smaller offers. In particular if a behavioral type can move at time k_{+1} or k_{+2} she will accept an opponent’s offer if it is compatible with her demand, or otherwise reaffirm her existing bargaining position α^i . The set of behavioral types C^i is finite, and conditional on being behavioral the agent is of type α^i with probability $\pi^i(\alpha^i)$. Special attention is given to behavioral types who are committed to fixed demands, $\alpha_{\omega'}^i(t) = \alpha^i \in [0, 1]$. I call these agents *simple types*.

Simple types are a special case of *complex types* whose demands may change over time and with the state of the world. I nonetheless, impose some restrictions even on complex types. For each current state ω' , either $x_{\omega'}^i > 0$ or $(1 - \alpha_{\omega'}^i(t)) \geq \varepsilon$ for all t and for some $\varepsilon > 0$ (note that this also imposes restrictions on simple types). A complex type’s demand is continuous at integer times if the current state does not change, $\alpha_{\omega'}^i(k_l) = \alpha_{\omega'}^i(k_{+3})$. It is also continuous in expectation when the current state does change at R , that is $\alpha_0^i(R_{-1}) = \mathbb{E}_p \alpha_{\omega'}^i(R_{+3})$. Because of this continuity, I frequently refer to a behavioral demand at integer k in current state ω' as $\alpha_{\omega'}^i(k)$, where this should not cause confusion. Additionally, $\alpha_{\omega'}^i(t)$, is continuously differentiable with respect to t on the intervals $[0_{+3}, R_{-1}]$ and $[R_{+3}, \infty)$, with this derivative represented by $\alpha_{\omega'}^{i'}(t)$. On each interval, a rational agent would optimally concede immediately rather than wait an instant longer if she *knew* her opponent was behavioral; this is precisely captured by the condition: $\alpha_{\omega'}^{i'}(t) + r_{\omega'}^j(1 - \alpha_{\omega'}^i(t)) + x_{\omega'}^j > 0$. Finally, I make a small technical assumption that agent 1’s maximal initial type is incompatible with all types of agent 2, $\max\{\alpha_0^1(0) : \alpha^1 \in C^1\} + \min\{\alpha_0^2(0) : \alpha^2 \in C^2\} > 1$.

A rational agent’s strategy must describe a demand function and concession choice at each k_{+1} or k_{+2} at which she can move, and her concession decision on the intervals $[k_{+3}, (k+1)_{-1}]$, with these choices depending on the previous history of play. Because on the interval $[k_{+3}, (k+1)_{-1}]$ an agent’s action is either “not concede” or “concede” (which ends the game) that decision can be reduced to a stopping time chosen at k_{+3} (with no concession also an option). The only histories which need be considered, therefore, are those at k_{+1} , k_{+2} and k_{+3} for $k \in \mathbb{Z}_+$.

The solution concept is Perfect Bayesian Equilibrium:⁴ an agent’s strategy must be a best response to her opponent’s strategy and her beliefs, where beliefs are defined by Bayes’s rule where possible. Additionally, I restrict beliefs so that following any history inconsistent with agent i being behavioral, j ’s believes i is rational with probability one. This restriction does not

⁴Fudenberg and Tirole [1991] presents a formal definition.

affect the set of equilibrium outcomes, but simplifies the description of some results.

2.1 A preliminary result and discussion

Continuous time bargaining: The (discrete-) continuous time concession game above may seem over stylized. Indeed, if there were no behavioral types it would yield multiple equilibria (although the presence of behavioral types reduces this to a unique prediction).

The main justification for it is the results of AG0. AG0 considers a discrete time model with simple types, and shows that as the time between offers becomes small, all equilibria converge in distribution to a unique continuous time war of attrition equilibrium, regardless of the relative frequency of agents' offers. I jump directly to the continuous time game to allow me to focus on new results.

The reasoning behind the AG0 convergence proof, is that if an agent is ever revealed as rational, and her opponent is possibly behavioral, then she must concede almost immediately if the time between offers is sufficiently small. This means that with two sided incomplete information an agent's strategy effectively reduces to "keep imitating a behavioral type" or "concede". The one-sided reputational result was first shown by Myerson [1991], although the logic is that of the Coase conjecture (Coase [1972]): one sided incomplete information completely determines the outcome.

In my bargaining game, the only time an agent might reveal rationality without conceding is at an integer time through a demand announcement inconsistent with a behavioral type. Offers are effectively arbitrarily frequent in this game, which results in an immediate (not approximate) concession requirement by an agent known to be rational. This is the claim of Lemma 1 below. The proof requires only very minor changes from Lemma 4 in F14 and hence is omitted. It implies that although an agent can change her initial demand announcement at an integer time it is without loss of generality to assume she would never do so, because this would reveal rationality and therefore require her to concede immediately.

Lemma 1. *For the bargaining game with complex behavioral types, there exists some $t' < \infty$ by which a rational agent i must certainly have conceded if agent j 's behavior has been consistent with a behavioral type. Further, if agent i is known to be rational at k_1 but agent j may be behavioral, then agent i must concede before k_{+3} in any equilibrium.*

Demand announcements: The model requires that agents make an announcement about their complete future demand intentions at time zero. If there are only simple behavioral types this is without loss of generality, as initial demands then reveal future behavioral type plans. In fact, even with the complex types above (whose demands don't fall too quickly) future demand announcements are primarily a simplification. In particular, the main result with complex types

(the convergence to the generalized Rubinstein outcome) would still go through even if announcements at integer times only covered the interval $[k_{+3}, (k+1)_{-1}]$.

More general behavioral types: Even the complex behavioral types considered here belong to a restrictive class, essentially continuously differentiable demand functions. This simplifies the analysis considerably. Moreover, the work of AP7 and F14 suggests that an extension to a larger class of types would not change predictions. Those papers allow for types whose demands change discontinuously and in history-contingent ways at integer times, but find that these do not affect results derived with continuously differentiable types so long as agents can announce complete (history contingent) bargaining positions at time zero.⁵

2.2 Strategies and utility

Lemma 1 simplifies the game considerably. The necessity of immediate concession after revealing rationality implies that a rational agent's strategy can be reduced to first deciding (at 0_{+1} or 0_{+2}) which behavioral type to imitate, and second deciding (at time 0_{+3}) at what time, if ever, to concede.⁶ The optimality of an agent's concession time decision at 0_{+3} implies the (sequential) optimality of her concession decision at any later history reached with positive probability. Integer times $k \notin \{0, R\}$ can be ignored entirely.

It is also without loss of generality to assume that the game must end immediately at the first time such that $\alpha_{\omega}^i(t) + \alpha_{\omega}^j(t) \leq 1$, which implies rational agents always concede (weakly) before behavioral types. To see this, notice that behavioral types will always concede at such a time, and rational agent i is guaranteed at least $\frac{1}{2}(\alpha_{\omega}^i(t) + 1 - \alpha_{\omega}^j(t)) \geq \alpha_{\omega}^i(t)$ by conceding immediately. At least one rational agent would therefore lose out if there was any delay to agreement given $r_{\omega}^i + x_{\omega}^i > 0$.

A strategy for agent 1, σ^1 , can therefore be described by a probability distribution μ_1 on the set C^1 , describing the bargaining position adopted by a rational agent and a collection of cumulative distributions $F_{\omega, \alpha^1, \alpha^2}^1$ describing the *total* probability that agent 1 has conceded by time t in state ω when demands are α^1 and α^2 . These map from $[0_{+3}, R_{-1}] \cup \{R_0, R_{+1}, R_{+2}\} \cup [R_{+3}, \infty) \cup \infty$ into $[0, 1]$, where concession at $t = \infty$ means no concession. Because agents do not initially know the state of the world, the distributions are identical for all states for $t \leq R_{-1}$. Recall that agent 2 cannot concede at R_{+1} , agent 1 cannot concede at R_{+2} , and neither can concede at R_0 .

A strategy for agent 2, σ^2 , is similarly made up of $\mu_{\alpha^1}^2$ on $C^2 \cup \{Q\}$ specifying rational 2's counterdemand to α^1 (where Q is acceptance at 0_{+2}) and $F_{\omega, \alpha^1, \alpha^2}^2$ which describes 2's choice of

⁵If behavioral types' demands decrease discontinuously, and do not make complete time zero demand announcements, then the generalized Rubinstein result breaks down. Wolitzky [2011] shows that if tough (generalized Rubinstein) types are initially indistinguishable from soft types (which concede regularly), then rational agents may not want to develop a behavioral reputation at all.

⁶The assumption $\max\{\alpha_0^1(0) \in C^1\} + \min\{\alpha_0^2(0) \in C^2\} > 1$ means that agent 1 always (weakly) prefers to imitate some behavioral type.

concession time in state ω given demands α^1, α^2 . Given μ^1 and $\mu_{\alpha^1}^2$ the conditional (posterior) probability that agents are behavioral after making their demands is given by:

$$\begin{aligned}\bar{z}^1(\alpha^1) &= \frac{z^1 \pi^1(\alpha^1)}{z^1 \pi^1(\alpha^1) + (1 - z^1) \mu^1(\alpha^1)} \\ \bar{z}_{\alpha^1}^2(\alpha^2) &= \frac{z^2 \pi^2(\alpha^2)}{z^2 \pi^2(\alpha^2) + (1 - z^2) \mu_{\alpha^1}^2(\alpha^2)}\end{aligned}\quad (1)$$

Conditional on initial demands α^1, α^2 and an opponent's strategy σ^j , agent i 's expected utility from conceding at time $t \leq R_{-1}$ is:⁷

$$\begin{aligned}U^i(t, \sigma^j | \alpha) &= \int_{s < t} \left(e^{-r_0^i s} \alpha_0^i(s) - \int_0^s e^{-r_0^i v} x_0^i dv \right) dF_{\omega, \alpha^1, \alpha^2}^j(s) \\ &+ (1 - F_{\omega, \alpha^1, \alpha^2}^j(t)) \left(e^{-r_0^i t} (1 - \alpha_0^j(t)) - \int_0^t e^{-r_0^i s} x_0^j ds \right) \\ &+ (F_{\omega, \alpha^1, \alpha^2}^j(t) - F_{\omega, \alpha^1, \alpha^2}^j(t^-)) \left(e^{-r_0^i t} \frac{1}{2} (\alpha_0^i(t) + 1 - \alpha_0^j(t)) - \int_0^t e^{-r_0^i s} x_0^j ds \right)\end{aligned}\quad (2)$$

Conditional on state ω being realized, concession at time $t \geq R_{+1}$ gives an expected utility of:

$$\begin{aligned}U^i(t, \sigma^j | \alpha, \omega) &= \int_{s \leq R_{-1}} \left(e^{-r_0^i s} \alpha_0^i(s) - \int_0^s e^{-r_0^i v} x_0^i dv \right) dF_{\omega}^j(s) \\ &+ \int_{R_0 < s < t} \left(e^{-r_0^i R - r_{\omega}^i(s-R)} \alpha_0^i(s) - \int_0^R e^{-r_0^i v} x_0^i dv - \int_0^R e^{-r_0^i R - r_{\omega}^i(v-R)} x_0^i dv \right) dF_{\omega}^j(s) \\ &+ (1 - F_{\omega, \alpha^1, \alpha^2}^j(t)) \left(e^{-r_0^i R + r_{\omega}^i(t-R)} (1 - \alpha_{\omega}^j(t)) - \int_0^R e^{-r_0^i s} x_0^j ds - \int_0^R e^{-r_0^i R - r_{\omega}^i(s-R)} x_0^j ds \right) \\ &+ (F_{\omega, \alpha^1, \alpha^2}^j(t) - F_{\omega, \alpha^1, \alpha^2}^j(t^-)) \left(e^{-r_0^i R + r_{\omega}^i(t-R)} \frac{1}{2} (\alpha_{\omega}^i(t) + 1 - \alpha_{\omega}^j(t)) - \int_0^R e^{-r_0^i s} x_0^j ds - \int_0^R e^{-r_0^i R - r_{\omega}^i(s-R)} x_0^j ds \right)\end{aligned}\quad (3)$$

A rational agent i 's utility after initial demand announcements is, therefore:

$$U^i(\sigma | \alpha) = \int_{t \leq R_{-1}} U^i(t, \sigma^j | \alpha) dH_{\omega, \alpha^1, \alpha^2}^i(t) + \int_{\Omega} \int_{t \geq R_{+1}} U^i(t, \sigma^j | \alpha, \omega) dH_{\omega, \alpha^1, \alpha^2}^i(t) dp \quad (4)$$

where $H_{\omega, \alpha^1, \alpha^2}^1(t) = \min \left\{ \frac{F_{\omega, \alpha^1, \alpha^2}^1(t)}{1 - \bar{z}^1(\alpha^1)}, 1 \right\}$ describes a rational agent 1's behavior, with a similar

⁷where: $F_{\omega, \alpha^1, \alpha^2}^j(t^-) = \limsup_{s < t} F_{\omega, \alpha^1, \alpha^2}^j(s)$

expression for agent 2. Finally rational agents' expected utilities in the full game are:

$$\begin{aligned}
U^1(\sigma) &= \sum_{\alpha^1} \mu^1(\alpha^1) \left(\alpha_0^1(0) \left((1-z^2)\mu_{\alpha^1}^2(Q) + z^2 \sum_{\alpha_0^2(0) \leq 1-\alpha_0^1(0)} \pi^2(\alpha^2) \right) \right. \\
&\quad \left. + \sum_{\alpha_0^2(0) > 1-\alpha_0^1(0)} U^1(\sigma|\alpha) \left((1-z^2)\mu_{\alpha^1}(\alpha^2) + z^2\pi^2(\alpha^2) \right) \right) \\
U^2(\sigma) &= \sum_{\alpha^1} \left((1-z^1)\mu^1(\alpha^1) + z^1\pi^1(\alpha^1) \right) \\
&\quad \times \left((1-\alpha_0^1(0))\mu_{\alpha^1}^2(Q) + \sum_{\alpha_0^2(0) > 1-\alpha_0^1(0)} U^2(\sigma|\alpha)\mu_{\alpha^1}^2(\alpha^2) \right)
\end{aligned} \tag{5}$$

The equations in 5 are exact analogues of agents' payoffs in AG0. The only difference is that the payoff $U^i(\sigma|\alpha)$ is determined in a slightly different manner, reflecting different continuation games. In what follows I frequently drop the subscripts α^1, α^2 from $F_{\omega, \alpha^1, \alpha^2}^i$ and other variables, when the context is clearly understood, and refer to the equilibrium posterior reputations after demand choices, defined in equation 1, as \bar{z}^i . Finally, because $F_{\omega}^i(t)$ is the same in all states for $t \leq R_{-1}$ I sometimes refer to it as $F_0^i(t)$.

3 Equilibrium

I first consider the continuation game at 0_{+3} following the announcement of a particular pair of initially incompatible demands, $\alpha_0^1(0) + \alpha_0^2(0) > 1$. Lemma 2 shows that any equilibrium for this continuation game must satisfy three conditions. To help with the characterization, define \check{T}_{ω} to be the minimum time such that demands are compatible in state ω or infinity otherwise, that is:

$$\check{T}_{\omega} = \min\{t \leq R_{-1} : \alpha_0^j(t) + \alpha_0^i(t) \leq 1\} \cup \{t \geq R_0 : \alpha_{\omega}^j(t) + \alpha_{\omega}^i(t) \leq 1\} \cup \infty \tag{6}$$

Notice that if $\check{T}_{\omega} \leq R_{-1}$ for some state, then this is true for all states.

Lemma 2. *For a bargaining game with complex behavioral types, in the continuation game at 0_{+3} with incompatible initial demands, $\alpha_0^1(0) + \alpha_0^2(0) > 1$, any equilibrium must satisfy the following properties (where an interval is empty if its upper bound exceeds its lower bound):*

- (i) *There are some times $\dot{T} \leq R_{-1}$ and $T_{\omega}^* < \infty$ ⁸, such that rational agents are indifferent between conceding at any $t \in (0_{+3}, \dot{T}]$ or at $t_{\omega} \in (R_{+3}, T_{\omega}^*]$ in state ω , but do not concede on $(\dot{T}, R_{-1}]$ or (T_{ω}^*, ∞) .*

⁸Where if $T_{\omega}^* \leq R_{-1}$ for some ω then $T_{\omega}^* = \dot{T}$ for all $\hat{\omega}$

(ii) Both agents reach a probability one reputation for being behavioral at time the same time T_ω^* in state ω .⁹

(iii) At most one agent concedes with positive probability at time 0_{+3} , and at most one agent concedes with positive probability at time R in state ω (unless $\check{T}_\omega = R_{-1}$).

Some form of the above lemma is standard in reputational bargaining and follows the proof of Proposition 1 in AG0. The main idea is that concession must be continuous at $t \in (0_{+3}, \check{T})$ and $t_\omega \in (R_{+3}, T_\omega^*)$ because *mass concession*, concession with positive probability, by agent i at t implies that agent j would not rationally concede on $[t - \varepsilon, t]$, but then i would not find it optimal to concede at t . The main difference from AG0 is that without uncertainty, concession is necessarily continuous at all $t > 0$ with no “gaps”. By contrast with uncertainty, it is impossible to rule out mass concession at time R by both agents (in different states of the world), which can mean incentives for rational agents not to concede on some interval $(\check{T}, R_{-1}]$.

By way of an overview of the analysis to come, Figure 1 presents a “typical” equilibrium that obeys the structure outlined in Lemma 2. Assume that behavioral types are simple, so there is no need to worry about demands becoming compatible. Notice that for simple or complex types, Bayes’ rule implies agent i ’s reputation at time t in state ω (if she has not yet conceded) is given by:

$$\bar{z}_\omega^j(t) = \min \left\{ \frac{\bar{z}^j}{1 - F_\omega^j(t)}, 1 \right\}$$

The equilibrium is as follows: at time 0_{+3} agent 2 concedes with positive probability discretely increasing her reputation (if she does not concede). Each agent then concedes continuously (to keep her opponent indifferent between conceding at one instant or the next), which causes reputations to grow continuously until time \check{T} . At that point all remaining agents wait and do not concede on the interval $(\check{T}, R_{-1}]$. The resolution of uncertainty determines agents’ future costs of delay, relatively favoring agent 1 in state a and agent 2 in state b . This means that at time R_{+3} agent 1 concedes with positive probability in state b , discretely increasing her reputation, and agent 2 concedes in state a (nothing happens at R_{+1} or R_{+2}). Following that, each agent concedes continuously (to keep her opponent indifferent between conceding at one instant or the next) until their reputations simultaneously reach probability one at the same time T_ω^* in each state ω . At time \check{T} an agent is indifferent between conceding immediately or waiting in the hope that her opponent will concede at time R_{+3} .

⁹Where this means that if agent i reaches a probability one reputation at R_i then agent j must do so before $T_\omega^* = R_{+3}$

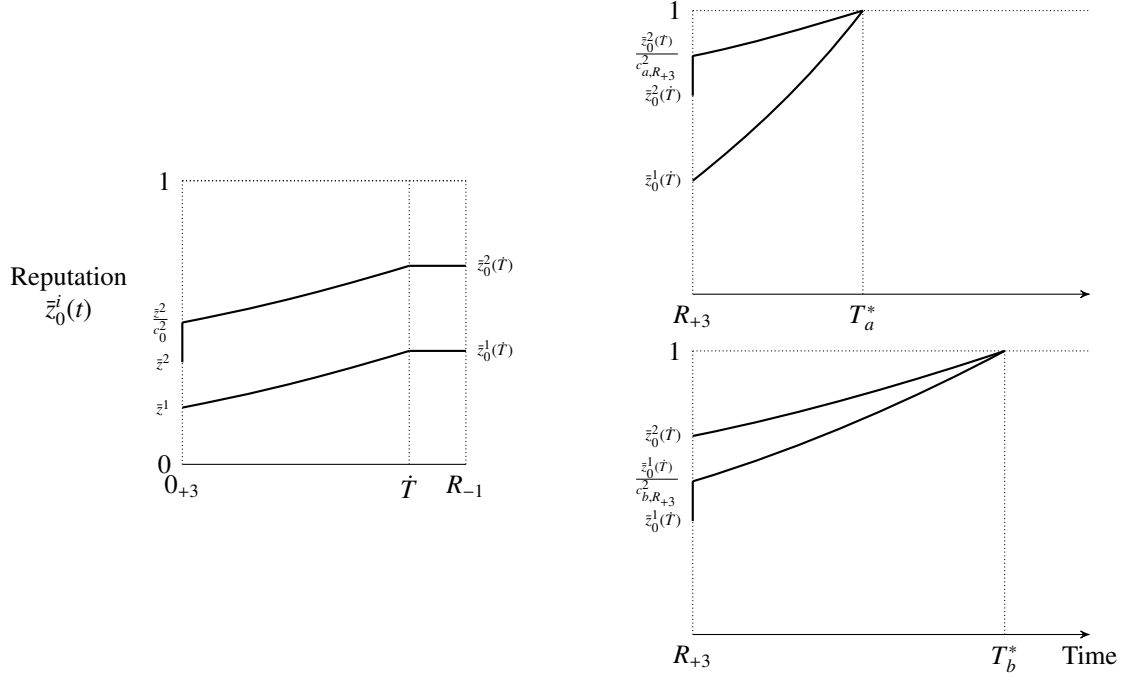


Figure 1. An example of an equilibrium with waiting with $\Omega = \{a, b\}$

After the revelation time

To characterize an equilibrium generally, I work backwards. I first analyze the continuation game at R_{+3} , after the resolution of uncertainty when agents' costs of delay are known, assuming that rational agents have not already conceded with certainty, $F_\omega^j(R_{+2}) < 1 - \bar{z}^j$ (which in particular means $\check{T}_\omega > R_{+3}$). This analysis is almost completely comparable to AG0, and hence I keep the exposition brief.

Lemma 2 implies the unique equilibrium in this continuation game must resemble a war attrition. At time R_{+3} at most one agent can concede to her opponent with positive probability, each agent mixes over concession times to ensure her opponent is indifferent between concession on (R_{+3}, T_ω^*) , and both agents reach a probability one reputation at T_ω^* . For agent i to be indifferent the derivative of equation 3 with respect to t must equal zero, which implies the following concession rate for agent j :

$$h_\omega^j(t) = \frac{f_\omega^j(t)}{1 - F_\omega^j(t)} = \frac{r_\omega^i(1 - \alpha_\omega^j(t)) + \alpha_\omega^{\prime j}(t) + x_\omega^i}{\alpha_\omega^i(t) + \alpha_\omega^j(t) - 1} \quad (7)$$

Agent j 's concession rate depends on agent i 's cost of delaying her own concession (j must compensate i for waiting). For simple types the concession rate is a constant h_ω^j . If agent j concedes continuously at this rate on the interval (s, t) in equilibrium then the probability that

she has *not* conceded by time t , conditional on *not* having conceded by time s is:

$$\frac{1 - F_\omega^j(t)}{1 - F_\omega^j(s)} = \exp\left(-\int_s^t h_\omega^j(v)dv\right)$$

This gradual concession ensures that an agent who does not concede slowly builds reputation. Let $(1 - c_{\omega,R_{+3}}^j)$ be the probability that agent j concedes at time R_{+3} , conditional on not having done so previously, and so $c_{\omega,R_{+3}}^j = \frac{1 - F_\omega^j(R_{+3})}{1 - F_\omega^j(R_{+2})}$. Taking behavior before R_{+3} as given, agent j 's reputation at time $t \in [R_{+3}, T_\omega^*]$ in state ω can be described by:

$$\bar{z}_\omega^j(t) = \frac{\bar{z}_\omega^j}{1 - F_\omega^j(R_{+2})} \frac{1 - F_\omega^j(R_{+2})}{1 - F_\omega^j(R_{+3})} \frac{1 - F_\omega^j(R_{+3})}{1 - F_\omega^j(t)} = \frac{\bar{z}_\omega^j(R_{+2})}{c_{\omega,R_{+3}}^j} \exp\left(\int_R^t h_\omega^j(v)dv\right)$$

Notice that $\exp\left(-\int_R^t h_\omega^j(v)dv\right)$ is continuous and strictly increasing in t and must converge to zero as t approaches \check{T}_ω .¹⁰ This means that even if agent j does not concede with positive probability at time R_{+3} her reputation must eventually reach probability one at some time strictly before \check{T}_ω .

The requirement that both agents reach a probability one reputation at the same time T_ω^* , means that there are two equations, $\bar{z}_\omega^1(T_\omega^*) = \bar{z}_\omega^2(T_\omega^*) = 1$, and two unknowns, T_ω^* and $\frac{c_{\omega,R_{+3}}^i}{c_{\omega,R_{+3}}^j}$ (where the second is a single unknown because at most one agent can concede at time R_{+3}). The intuitive way to solve these equations is to work out which agent would win the ‘‘reputational race’’ to reach a probability one reputation assuming that *neither* agent conceded with positive probability at time R_{+3} . Define agent j 's *exhaustion time in state ω* , $T_{E,\omega}^j < \check{T}_\omega$, as the time she would reach a probability one reputation if she conceded at the equation 7 rate on $(R_{+3}, T_{E,\omega}^j)$ and did not concede at R_{+3} . That is:

$$1 = \bar{z}_\omega^j(R_{+2}) \exp\left(\int_R^{T_{E,\omega}^j} h_\omega^j(s)ds\right)$$

To ensure that both agents reach a probability one reputation at the same time T_ω^* , we must have $T_\omega^* = \min\{T_{E,\omega}^1, T_{E,\omega}^2\}$ and $c_{\omega,R_{+3}}^j$ defined by:

$$c_{\omega,R_{+3}}^j = \bar{z}_\omega^j(R_{+2}) \exp\left(\int_R^{T_\omega^*} h_\omega^j(s)ds\right) \quad (8)$$

Recalling that one of agent i 's optimal strategies is to concede an instant after R_{+3} , implies her

¹⁰The convergence claim may not seem immediate, although it does follow due to bounds on agents' concession rates. Lemma 1 implies the existence of some $t' < \infty$ before which any rational agent j must have conceded when facing a possibly behavioral opponent. Because $\alpha_\omega^j(t)$ is continuous $r_\omega^j(1 - \alpha_\omega^j(t)) + \alpha_\omega^j(t) + x_\omega^j$ is bounded above zero before $\min\{t', \check{T}_\omega\}$, which ensures the convergence.

expected utility in this continuation game is given by:

$$V_{\omega, R_{+3}}^i = (1 - c_{\omega, R_{+3}}^j) \alpha_{\omega}^i(R) + c_{\omega, R_{+3}}^j (1 - \alpha_{\omega}^j(R))$$

This means that agent i does better than immediately accepting her opponent's R_{+3} offer, $(1 - \alpha_{\omega}^j(R))$, if and only if j concedes with positive probability at time R_{+3} . This utility is continuously increasing in $\bar{z}_{\omega}^i(R_{+2})$ and decreasing in $\bar{z}_{\omega}^j(R_{+2})$, strictly for one agent.

Knowing exactly what must happen at time R_{+3} in each state, ties down what must happen at R_{+1} and R_{+2} , and so there is a unique equilibrium for the game at R_0 . Lemma 3, below, shows that if $\alpha_{\omega}^i(R) + \alpha_{\omega}^j(R) > 1$ then it is without loss of generality to assume that neither agent concedes at R_{+1} and R_{+2} (because concession by j at R_l implies no concession by i at R_m for $0 \leq m \leq l$). In states for which $\alpha_{\omega}^i(R) + \alpha_{\omega}^j(R) \leq 1$ agent 1 concedes to the offer $(1 - \alpha_{\omega}^2(R))$ at R_{+1} before agent 2 can concede to $(1 - \alpha_{\omega}^1(R))$ at R_{+2} .

To characterize the equilibrium before the resolution of uncertainty, it is sufficient only to know an agent's expected utility in the R_0 continuation game, $V_{R_0}^i$, which depends continuously on agents' reputations at the start of that continuation game. Regarding notation: I refer to $\bar{z}_{\omega'}(t)$ as the vector of agent i and j 's reputations at time t when the current state is ω' .

Lemma 3. *For a bargaining game with complex behavioral types, in the continuation game at R_0 when behavioral demands have satisfied $\alpha_0^i(t) + \alpha_0^j(t) > 1$ for $t \leq R_{-1}$ and reputations are $\bar{z}_0(R_{-1})$, there is a unique equilibrium. Agent i 's equilibrium continuation payoff, $V_{R_0}^i(\bar{z}_0(R_{-1}))$, is continuously increasing in $\bar{z}_0^i(R_{-1})$ and continuously decreasing in $\bar{z}_0^j(R_{-1})$, strictly for at least one agent. Furthermore, $V_{R_0}^1(\bar{z}_0(R_{-1})) \geq (1 - \alpha_0^2(R))$ and $V_{R_0}^j(\bar{z}_0(R_{-1}), 1) \leq (1 - \alpha_0^j(R))$.*

Before the revelation time

I now proceed backwards to consider bargaining before the resolution of uncertainty treating agents' continuation payoffs at time R_0 as a black box. By Lemma 2, concession is continuous by both agents on some interval $(0_{+3}, \bar{T})$. The concession rate of j which keeps i indifferent to concession on such an interval is found by setting the derivative of equation 2 with respect to t , equal to zero:

$$h_0^j(t) = \frac{f_0^j(t)}{1 - F_0^j(t)} = \frac{r_0^j(1 - \alpha_0^j(t)) + \alpha_0^{j'}(t) + x_0^j}{\alpha_0^i(t) + \alpha_0^j(t) - 1} \quad (9)$$

Again for simple types, this concession rate is a constant, h_0^j . Let $1 - c_0^j = F^j(0)$ be the equilibrium probability that j concedes at time 0_{+3} , then agent j 's reputation at time $t \leq R_{-1}$ is given by:

$$\bar{z}_0^j(t) = \frac{\bar{z}^j}{c_0^j} \exp\left(\int_0^{\min\{t, \check{T}\}} h_0^j(v) dv\right)$$

Lemma 2 allows for two kinds of equilibrium: either both agents reach a probability one reputation by time \check{T} or both agents choose to wait on some interval $(\check{T}, R_{-1}]$ in order to receive the continuation value $V_{R_0}^i$. The first possibility is considered in much the same way as in the continuation game at R_{+3} .

I define agent j 's *initial exhaustion time* $T_E^j\left(\frac{\bar{z}^j}{c_0^j}\right)$ by equation 10. This is the first time less than R that agent j would reach a probability one reputation if she concedes at rate 9 on the interval $(0, T_E^j)$ given a time zero reputation of $\frac{\bar{z}^j}{c_0^j}$ (where $\check{c}_0^j = 1$ is a special case), assuming such a time exists. To assist with this definition let $\check{T} = \min\{\check{T}_\omega : \omega \in \Omega\}$.

$$T_E^j\left(\frac{\bar{z}^j}{c_0^j}\right) = \min\left\{t \geq 0 : \frac{\bar{z}^j}{c_0^j} \exp\left(\int_0^{\min\{t, \check{T}, R\}} h_0^j(s) ds\right) + \mathbb{1}_{[t \geq R]}(t - R) = 1\right\} \quad (10)$$

This initial exhaustion time is continuously increasing in \check{c}_0^j . It may strictly exceed R , although bounds on the numerator of agent j 's concession rate imply that $T_E^j < \check{T}$ if $\check{T} \leq R_{-1}$.

I next define agent j 's *waiting time*, $T_W^j\left(\frac{\bar{z}^j}{c_0^j}, \frac{\bar{z}^i}{c_0^i}\right)$ by equation 11. This is the first time less than R that agent j would wait to receive the R_0 continuation payoff $V_{R_0}^j$ (defined by Lemma 3) assuming agents concede at rate 9 on $(0, T_W^j)$, neither concedes on $(T_W^j, R_{-1}]$, and time zero reputations are $\frac{\bar{z}^j}{c_0^j}$ and $\frac{\bar{z}^i}{c_0^i}$, assuming such a time exists.

$$T_W^j\left(\frac{\bar{z}^j}{c_0^j}, \frac{\bar{z}^i}{c_0^i}\right) = \min\left\{t \geq 0 : e^{-r^j(R-t)} V_{R_0}^j\left(\frac{\bar{z}^j}{c_0^j} \exp\left(\int_0^{\min\{t, T_E^j, R\}} h_0^j(s) ds\right), \frac{\bar{z}^i}{c_0^i} \exp\left(\int_0^{\min\{t, T_E^i, R\}} h_0^i(s) ds\right)\right) - \int_0^{R-t} e^{r^j s} x_0^j ds \geq (1 - \alpha_0^j(\min\{t, R\}))\right\} \quad (11)$$

Remember that $V_{R_0}^j$ is continuously increasing in j 's reputation, continually decreasing in i 's reputation, and is strictly positive. This implies that T_W^j is well defined, and is continuously increasing in \check{c}_0^j and continuously decreasing in \check{c}_0^i . Finally, notice that $V_{R_0}^1 \geq (1 - \alpha_0^2(R))$ implies that $T_W^1 \leq R$, although it is possible that $T_W^2 > R$.

Using these definitions I proceed to define agent j 's *ending time* as $\check{T}^j\left(\frac{\bar{z}^j}{c_0^j}, \frac{\bar{z}^i}{c_0^i}\right) = \min\{T_E^j, T_W^j\}$. Due to the characteristics of its composite parts this ending time is continuously increasing in \check{c}_0^j and continuously decreasing in \check{c}_0^i .

We are now ready to complete the description of an equilibrium. The two possibilities, that both agents reach a probability one reputation at time \check{T} or both choose to wait on some interval

$(\dot{T}, R_{-1}]$, are captured by the single equilibrium condition:

$$\dot{T} = \dot{T}^j \left(\frac{\bar{z}^j}{c_0^j}, \frac{\bar{z}^i}{c_0^i} \right) = \dot{T}^i \left(\frac{\bar{z}^i}{c_0^i}, \frac{\bar{z}^j}{c_0^j} \right)$$

To find such an equilibrium, suppose that neither agent concedes at time zero ($\dot{c}_0^j = \dot{c}_0^i = 1$) and the result is that $\dot{T}^j(\bar{z}) \leq \dot{T}^i(\bar{z})$. By continuously lowering \dot{c}_0^i (increasing time zero concession by agent i) the equilibrium condition must eventually be met. This equilibrium c_0^i clearly exists because of the continuity properties of \dot{T}^j and \dot{T}^i and the facts $\dot{T}^i(1, \bar{z}^j) = T_E^i(1) = 0$, and $\dot{T}^j(\bar{z}^j, 1) \geq 0$. Notice that if agent j reaches a probability one reputation at \dot{T} then so must i, because it is always optimal to concede immediately facing a known behavioral type, and in particular $V_{R_0}^i(\bar{z}_0^i(R_{-1}), 1) \leq (1 - \alpha_0^j(R))$.

By construction this description of behavior satisfies all the requirements laid out in Lemma 2 with incentive compatible strategies for each agent. This equilibrium is also unique, the claim of Proposition 1. The proof is somewhat mechanical, but is based on the idea that given an equilibrium (in which agents wait) increasing \dot{T} without changing time zero concession would shift rational agent i 's concession earlier in time, which would make agent j strictly better off if she chose to wait until R to concede. To ensure incentives to concede on $(0, \dot{T})$ in the new equilibrium, therefore, agent i 's time zero reputation must increase or j 's must decrease. This, however, cannot be true for both agents.

Agent i 's expected payoff in this equilibrium is:

$$U^i(\sigma|\alpha) = (1 - c_0^j)\alpha_0^i(0) + c_0^j \max \left\{ 1 - \alpha_0^j(0), e^{-r_0^i R} V_{R_0}^i \left(\frac{\bar{z}^i}{c_0^i}, \frac{\bar{z}^j}{c_0^j} \right) - \int_0^R e^{r_0^i s} x_0^i ds \right\}$$

Proposition 1. *For a bargaining game with complex behavioral types, in the continuation game at 0_{+3} with reputations following demand choices of \bar{z} , there is a unique equilibrium, with agent i 's equilibrium payoff $U^i(\sigma|\alpha)$ continuously increasing in \bar{z}^i and decreasing in \bar{z}^j , strictly for at least one agent.*

With a unique equilibrium for any time 0_{+3} continuation game with given demands and initial reputations \bar{z} we can now turn to the demand choice game, and allow rational agents to imitate many different behavioral types. This choice must involve agents mixing between different demands so that conditional reputations after demand choices, result in continuation games that guarantee equal payoffs from each imitated demand. The structure of this demand choice game is *identical* to AG0, with payoffs from choices given by the equations in 5. This immediately implies the following proposition with the proof provided by Proposition 2 in AG0.

Proposition 2. *In the bargaining game with complex behavioral types an equilibrium exists. Furthermore, all equilibria yield the same distribution over outcomes.*

4 Almost certainly rational agents

4.1 Characterizing lemmas and definitions

In order to prepare the groundwork for results with a rich set of behavioral types when the prior probability of those types vanish, I first characterize limit outcomes for a single pair of behavioral demands. Again I work backwards and first highlight conditions under which the continuation game at R_{+3} , yields well defined limit payoffs as agents' R_{+2} reputations become vanishingly small. Assume a given pair of imitated demands are incompatible prior to time R_{+3} ($\check{T}_\omega > R_{+3}$). Consider a sequence of continuation games at R_{+3} for which $\bar{z}_{\omega,n}(R_{+2}) \rightarrow 0$ and $L > \frac{\bar{z}_{\omega,n}^1(R_{+2})}{\bar{z}_{\omega,n}^2(R_{+2})} > 1/L$ for some positive constant L .

Recall that both agents must reach a probability one reputation at $T_\omega^* < \check{T}_\omega$ and that $\exp\left(-\int_R^t h_\omega^i(s) ds\right)$ converges to zero if and only if $t \rightarrow \check{T}_\omega$. This means that exhaustion times satisfy $T_{E,\omega}^i \rightarrow \check{T}_\omega$ as $\bar{z}_{\omega,n}^i(R_{+2}) \rightarrow 0$, which in turn implies that $T_\omega^* \rightarrow \check{T}_\omega$. Combining equation 8 for the two agents means that time R_{+3} concession is given by:

$$\frac{c_{\omega,R_{+3}}^i}{c_{\omega,R_{+3}}^j} = \frac{\bar{z}_{\omega,n}^i(R_{+2})}{\bar{z}_{\omega,n}^j(R_{+2})} \exp\left(\int_R^{T_\omega^*} h_\omega^i(s) - h_\omega^j(s) ds\right) \quad (12)$$

I am interested in pairs of demands made by behavioral types for which:

$$\lim_{t \rightarrow \check{T}_\omega} \exp\left(\int_R^t h_\omega^i(s) - h_\omega^j(s) ds\right) \in \{0, \infty\} \quad (13)$$

Suppose the limit in equation 13 is zero, then for the sequence $\bar{z}_{\omega,n}(R_{+2}) \rightarrow 0$ with $L > \frac{\bar{z}_{\omega,n}^1(R_{+2})}{\bar{z}_{\omega,n}^2(R_{+2})} > 1/L$, equation 12 implies $\frac{c_{\omega,R_{+3}}^i}{c_{\omega,R_{+3}}^j} \rightarrow 0$, that is agent i concedes with probability approaching one at R_{+3} . This means that for a sequence of R_{+3} continuation games satisfying the above conditions, there is a well defined limit payoff $\tilde{V}_{\omega,R_{+3}}^i = \lim V_{\omega,R_{+3}}^i \in \{\alpha_\omega^i(R), (1 - \alpha_\omega^j(R))\}$.

Lemma 4 extends the idea of limit continuation payoffs backwards to continuation games starting at time R_0 . To do this we need the following definition: a pair of demands (α^1, α^2) are *generic after the revelation time* if in each state ω they satisfy either time R compatibility, $\alpha_\omega^1(R) + \alpha_\omega^2(R) \leq 1$, or equation 13.

Lemma 4. *Let $E_n = \{z_n^i, \pi^i(\alpha^i), r_{\omega'}^i, x_{\omega'}^i, C^i\}_{i \in \{1,2\}, \omega' \in \Omega}$ be a sequence of bargaining games with complex behavioral types. For a given pair of behavioral demands satisfying $\alpha_0^1(t) + \alpha_0^2(t) > 1$ for $t \leq R_{-1}$ which are generic after the revelation time, suppose that reputations in the R_0 continuation game satisfy $\bar{z}_n(R_{-1}) \rightarrow 0$ and $L > \frac{\bar{z}_n^1(R_{-1})}{\bar{z}_n^2(R_{-1})} > 1/L$ for some positive constant L . Then the sequence of equilibrium continuation payoffs satisfy:*

$$\tilde{V}_{R_0}^i = \lim V_{R_0}^i = \int \alpha_\omega^i(R) \mathbb{1}_{[\omega \in D^i]} + (1 - \alpha_\omega^j(R)) \alpha_\omega^i(R) \mathbb{1}_{[\omega \in D^j]} dp$$

where:

$$D^1 = \Omega \setminus D^2 = \left\{ \omega : \alpha_\omega^1(R) + \alpha_\omega^2(R) > 1, \lim_{t \rightarrow \tilde{T}_\omega} \exp \left(\int_R^t h_\omega^2(s) - h_\omega^1(s) ds \right) = 0 \right\}$$

Notice that for simple types to be generic after the revelation time merely requires that $h_\omega^i \neq h_\omega^j$. This means the set D^i , for which agent i obtains α^i in the limit of R_{+3} continuation games, is simply the set of states in which $h_\omega^i > h_\omega^j$.

Given well defined limit continuation payoffs at R_0 , which do not depend on the exact ratio of agents' reputations, it is possible to similarly characterize agents' limiting payoffs in the 0_{+3} continuation game. For a pair of behavioral demands which satisfy $\alpha_0^1(t) + \alpha_0^2(t) > 1$ for $t \leq R_{-1}$ and are generic after the revelation time, we seek to define a *limit waiting time* \tilde{T}_W^i . This is the minimal time, such that agent i would be willing to wait on the interval $[\tilde{T}_W^i, R]$ for the limit continuation value $\tilde{V}_{R_0}^i$ (defined in Lemma 4) rather than concede immediately, assuming such a time exists. Unlike the waiting times considered previously, limit waiting times can be negative.

$$\tilde{T}_W^i = \min \left\{ t : e^{-r^j(R-t)} \tilde{V}_{R_0}^i - \int_0^{R-t} e^{-r_0^i s} x_0^i ds = (1 - \alpha_0^j(\min\{\max\{t, 0\}, R\})) \right\}$$

Lemma 5 shows that if initial reputations converge to zero and agent i is willing to wait over a longer interval than j for her limit continuation payoff, $\tilde{T}_W^i < \tilde{T}_W^j$, and further j is unwilling to wait from time zero, $\tilde{T}_W^j > 0$, then j must concede at immediately with probability approaching one when agents' initial reputations are small. Furthermore, if both agents are willing to wait from time zero, $\max\{\tilde{T}_W^i, \tilde{T}_W^j\} < 0$, then with probability approaching one there is no agreement before the revelation time.

The idea behind the proof is that if demands are incompatible before R_{-1} and agents' initial reputations converge to zero at the same rate, then so do time R_{-1} reputations, absent time zero concession. This would mean agents' reputations at R_0 satisfy the requirements of Lemma 4 which would imply that agent i has incentives to wait on an interval of $(\tilde{T}_W^i, R_{-1}]$ in the limit. The reason time R_{-1} reputations must converge to zero when initial reputations do is that concession rates on $(0_{+3}, R_{-1}]$ are bounded and hence the total probability of concession on that interval is bounded strictly below 1. To help in the characterization, I define $q(t, \sigma|\alpha)$ as the probability of an agreement before time t conditional on demands α and strategies σ . I similarly define $q(t, \sigma)$ as the probability of agreement before time t in the full game given strategies σ .

Lemma 5. *Let $E_n = \{z_n^i, \pi^i(\alpha^i), r_{\omega'}^i, x_{\omega'}^i, C^i\}_{i \in \{1,2\}, \omega' \in \Omega}$ be a sequence of bargaining games with complex behavioral types. For a given pair of behavioral demands satisfying $\alpha_0^1(t) + \alpha_0^2(t) > 1$ for $t \leq R_{-1}$ which are generic after the revelation time, suppose that reputations after demand*

choices satisfy $\bar{z}_n \rightarrow 0$ and $L > \frac{\bar{z}_n^i}{\bar{z}_n^j} > 1/L$ for some positive constant L . Then the sequence of equilibria satisfies:

$$\begin{aligned} \lim U^i(\sigma|\alpha) &= \alpha_0^i(0) & \lim q(0_{+3}, \sigma|\alpha) &= 1 & \text{if } \tilde{T}_W^j > \max\{\tilde{T}_W^i, 0\} \\ \lim U^i(\sigma|\alpha) &= (1 - \alpha_0^j(0)) & \lim q(0_{+3}, \sigma|\alpha) &= 1 & \text{if } \max\{\tilde{T}_W^j, 0\} < \tilde{T}_W^i \\ \lim U^i(\sigma|\alpha) &= e^{-r_0^i R} \tilde{V}_{R_0}^i - \int_0^R e^{-r_0^i s} x^i ds & \lim q(R_{-1}, \sigma|\alpha) &= 0 & \text{if } \max\{\tilde{T}_W^j, \tilde{T}_W^i\} < 0 \end{aligned}$$

In light of the analysis of Lemma 5, I define a pair of demands (α^1, α^2) to be *generic* if they either 1) satisfy $\alpha_0^1(t) + \alpha_0^2(t) \leq 1$ for some $t \leq R_{-1}$ or 2) are generic after the revelation time and imply either $\max\{\tilde{T}_W^1, 0\} \neq \max\{\tilde{T}_W^2, 0\}$ or $\max\{\tilde{T}_W^1, \tilde{T}_W^2\} < 0$. A type space (C^1, C^2) is *generic* if all demand pairs $(\alpha^1, \alpha^2) \in C^1 \times C^2$ are generic. Focusing on generic type spaces in what follows is not necessary for the analysis, but simplifies it.

4.2 Simple types: no delay without uncertainty

My main results with simple types require that there are many different types imitating many different demands. I say that a simple type space (C^1, C^2) is ε -rich if for each $a \in [0, 1]$ there is some $\alpha^i \in C^i$ such that $|\alpha^i - a| < \varepsilon$ for $i = 1, 2$.

Before considering the model with uncertainty, I first show that without uncertainty delay and first mover effects do not occur as the the probability of behavioral types becomes small.¹¹

Proposition 3 (No uncertainty). *Let $E_n = \{z_n^i, \pi^i(\alpha^i), r_{\omega'}^i, x_{\omega'}^i, C^i\}_{i \in \{1,2\}, \omega' \in \Omega'}$ be a sequence of bargaining games with simple behavioral types and no uncertainty, $|\Omega| = 1$. Suppose that $z_n \rightarrow 0$ and $L > \frac{z_n^1}{z_n^2} > 1/L$ for some positive constant L . Then the sequence of equilibria satisfies $\lim q(0_{+3}, \sigma) = 0$ if the type space is generic, and if the type space is ε -rich then:*

$$\liminf U^i(\sigma) \geq \beta_{\omega}^i - \varepsilon = \max \left\{ 0, \min \left\{ 1, \frac{r_{\omega}^j + x_{\omega}^j - x_{\omega}^i}{r_{\omega}^i + r_{\omega}^j} \right\} \right\} - \varepsilon$$

Proof. Suppose that agents imitate some demands (α^1, α^2) with positive probability in the limit (taking a subsequence if necessary). For this subsequence agents' reputations after demand choices satisfy $\bar{z}_n \rightarrow 0$ and furthermore there exists some positive constant L_2 such that $L_2 > \frac{z_n^1}{z_n^2} > \frac{1}{L_2}$. This means that in the continuation game at time 0_{+3} , the requirements of Lemma 5 regarding initial reputations are satisfied.

For the Proposition's first claim, suppose that demands are generic, which only imposes $h_{\omega}^i \neq h_{\omega}^j$. Without loss of generality, suppose $h_{\omega}^i > h_{\omega}^j$. This implies limit continuation payoffs at R_0 satisfy $\tilde{V}_{R_0}^i = 1 - \tilde{V}_{R_0}^j = \alpha^i$ and therefore $\tilde{T}_W^i < \tilde{T}_W^j = R$. By Lemma 5, therefore, we must have

¹¹When the set of complex types is rich there is no delay with or without uncertainty (see subsection 4.4).

$\lim q(0_{+3}, \sigma|\alpha) = 0$ in this continuation game. As there are only a finite number of demand combinations and this is true for each pair imitated with positive probability in the limit, this proves the first claim.

For the second claim, suppose that the type space is ε -rich, and consider the problem from agent 2's perspective. If agent 1 makes an offer satisfying $(1 - \alpha^1) \geq \beta_\omega^2 - \varepsilon$ then agent 2 can achieve this payoff by accepting immediately. If agent 1 makes a smaller offer with positive limit probability then by imitating a type which demands within ε of β_ω^2 agent 2 ensures that $h_\omega^2 > h_\omega^1$, which means that this pair of demands is generic and furthermore $\tilde{V}_{R_0}^2 = 1 - \tilde{V}_{R_0}^1 = \alpha^2$. In turn, this implies $\tilde{T}_W^2 < \tilde{T}_W^1 = R$ and so by Lemma 5 we must have $\lim U^i(\sigma|\alpha) = \alpha_0^i(0)$ for this 0_{+3} continuation game. The proof for agent 1 is entirely analagous, she can guarantee the limit payoff bound by demanding within ε of β_ω^1 with probability one. \square

4.3 Simple types: delay and first mover effects with uncertainty

Proposition 3 (almost) completely characterizes the outcome of bargaining when the simple type space is sufficiently rich and the probability of behavioral types is small. It is not feasible to characterize the model with uncertainty for all possible parameters in this way. It is, however, possible to do so when agents' costs of delay are only flow costs rather than impatience, that is $r_\omega^i = 0$. The limit outcomes in this case are described in Figure 2.

To more easily compare the effect of the order of agent's initial demand announcements, I refer to agents a and agent b , who are associated with particular costs of delay. To simplify notation I let $p = p(D^a) = 1 - p(D^b)$, where $D^i = \{\omega : x_\omega^i > x_\omega^j\}$ is the set of states in which agent j would concede to agent i 's demand in the limit continuation game at R_{+3} (this is consistent with the definition of D^i in Lemma 4). I assume, with minimal loss of generality, $\frac{p}{x_0^a} > \frac{1-p}{x_0^b}$. This means that agent a has stronger fundamentals in bargaining. For such assumptions Figure 2 shows approximate equilibrium limit outcomes in a rich type space, as R varies. The different values of R are divided into regions (E), (F), (G) and (H). I do not consider the boundary cases $R \in \left\{ \frac{p(1-p)}{x_0^a + x_0^b(1+p)}, \frac{p(1-p)}{x_0^a(1-p) + x_0^b}, \frac{(1-p)}{x_0^b} \right\}$. For each region, the figure lists approximate limit demands, counterdemands, payoffs and the probability of an agreement before time R_{-1} .

Notice in particular that if $p \in (0, 1)$ and the revelation time is not too distant, so that R is in region (E), then agreement is inevitably delayed until after the revelation time with probability approaching one. This is a clear demonstration that uncertainty may create delay, even when there is an arbitrarily small possibility that agents are actually obstinate.

Interestingly, if R is in region (F), delay occurs if and only if agent a must announce her demand first. In this case, both agents get strictly higher payoffs if b announces her demand first. This might lead one to expect agents to agree on a protocol whereby agent b announces first.¹²

¹²Notably if there is an option to be silent initially, and there exist types who delay their initial demand an-

$R :$	0	$\underbrace{\hspace{10em}}_{\substack{\frac{p(1-p)}{x_0^a+x_0^b(1+p)}}}$	$\underbrace{\hspace{10em}}_{\substack{\frac{p(1-p)}{x_0^a(1-p)+x_0^b}}}$	$\underbrace{\hspace{10em}}_{\substack{\frac{(1-p)}{x_0^b}}}$	\rightarrow
		(E)	(F)	(G)	(H)
$\left. \begin{array}{l} a = 1 \\ b = 2 \end{array} \right\}$	α^a	1	1	$\frac{x_0^b R}{1-p}$	1
	α^b	1	1	$\geq (1 - \alpha^a)$	$\geq (1 - \alpha^a)$
	$U^a(\sigma)$	$p - x_0^a R$	$p - x_0^a R$	$\frac{x_0^b R}{1-p}$	1
	$U^b(\sigma)$	$(1-p) - x_0^b R$	$(1-p) - x_0^b R$	$1 - \frac{x_0^b R}{1-p}$	0
	$q(R_{-1}, \sigma)$	0	0	1	1
$\left. \begin{array}{l} a = 2 \\ b = 1 \end{array} \right\}$	α^a	1	$1 - \frac{(x_0^a+x_0^b)R}{p}$	$\frac{x_0^b R}{1-p}$	1
	α^b	1	$\frac{((1-p)x_0^a+x_0^b)R}{p(1-p)}$	1	$\in [0, 1]$
	$U^a(\sigma)$	$p - x_0^a R$	$\frac{(x_0^a+x_0^b)R}{p}$	$\frac{x_0^b R}{1-p}$	1
	$U^b(\sigma)$	$(1-p) - x_0^b R$	$1 - \frac{(x_0^a+x_0^b)R}{p}$	$1 - \frac{x_0^b R}{1-p}$	0
	$q(R_{-1}, \sigma)$	0	1	1	1

Figure 2. Approximate limit outcomes in flow cost model with a rich type space, $p = p(D^a)$, $\frac{p}{x_0^a} > \frac{1-p}{x_0^b}$

Notice that it is only for R in region (F) that the order of demand announcements makes any difference to the limit outcome of bargaining.

The efficiency cost of delay may be large, and indeed may amount to nearly half the bargaining surplus when agent a announces her demand first.¹³ If instead b announces first, then delay may cost up to a fifth of the surplus.¹⁴

Finally, notice that when delay occurs, demands polarize completely, $\alpha^i = 1$ for both agents. Uncertainty drives agents apart, until one or the other must back down completely after the resolution of uncertainty. Rational agents don't just imitate obstinate types, but the most uncompromising of all obstinate types.

The characterization in Figure 2 is proved in Proposition 4. To make it accessible to the general reader, the proof is somewhat informal.

nouncements, as in [Abreu et al. \[2014\]](#), we should expect agents to avoid delay in region (F).

¹³To see this, suppose that agent $a = 1, b = 2$ and R is approximately, but just less than $\frac{p(1-p)}{x_0^a(1-p)+x_0^b}$ and $\frac{p}{x_0^a} \approx \frac{1-p}{x_0^b}$, then $U^a + U^b \approx 1 - \frac{p}{1+p}$ which is close to a half for $p \approx 1$.

¹⁴Let R be approximately, but just less than $\frac{p(1-p)}{x_0^a+x_0^b(1+p)}$ and $\frac{p}{x_0^a} \approx \frac{1-p}{x_0^b}$, then $U^a + U^b \approx 1 - \frac{p(1-p)}{1+p(1-p)}$ which is close to a $\frac{4}{5}$ for $p \approx \frac{1}{2}$.

Proposition 4 (Delay with flow costs). *Let $E_n = \{z_n^i, \pi^i(\alpha^i), r_{\omega'}^i, x_{\omega'}^i, C^i\}_{i \in \{1,2\}, \omega' \in \Omega'}$ be a sequence of bargaining games with a generic simple type space. Suppose that $z_n \rightarrow 0$ and $L > \frac{z_n^1}{z_n^2} > 1/L$ for some positive constant L . Let both agents be patient, $r_{\omega'}^i = 0$, for all $\omega' \in \Omega'$. For agents a and b further let $p = p(D^a) = 1 - p(D^b)$ and $\frac{p}{x_0^a} > \frac{1-p}{x_0^b}$, where $D^i = \{\omega : x_{\omega}^i > x_{\omega}^j\}$. Then for any $\epsilon > 0$ there exists $\varepsilon > 0$ such that if the type space is ε -rich, for all sufficiently large n the probability of delay and equilibrium payoffs are within $\epsilon 0$ of that in Figure 2, also, demands and counterdemands are within $\epsilon > 0$ of that in Figure 2 with probability greater than $1 - \epsilon$.*

Proof. Remember that in the continuation game at time R_{+3} agent i will concede to agent j with probability approaching one (when the probability of behavioral types vanish at the same rate) if and only if $h_{\omega}^i < h_{\omega}^j$. Because agents' only face flow costs of delay, this is equivalent to agent i conceding to j if and only if $x_{\omega}^i > x_{\omega}^j$. This implies that the set of states D^i where j concedes to i in the limit is simply $D^i = \{\omega : x_{\omega}^i > x_{\omega}^j\}$. This simplified form of D^i means that for incompatible demands agents' limit continuation payoffs in the R_0 continuation game (as defined in Lemma 4) are $\tilde{V}_{R_0}^a = 1 - \tilde{V}_{R_0}^b = p\alpha^a + (1-p)(1-\alpha^b)$. This in turn implies that limit waiting times satisfy $\tilde{T}_W^a = R - \frac{p(\alpha^a + \alpha^b - 1)}{x_0^a} < \tilde{T}_W^b = R - \frac{(1-p)(\alpha^a + \alpha^b - 1)}{x_0^b}$, where the inequality follows from the assumption $\frac{p}{x_0^a} > \frac{1-p}{x_0^b}$.

If a given pair of demands is imitated with strictly positive limit probability (taking a subsequence if necessary) then the posteriors after demand choice satisfy the conditions of Lemma 5. For such demands, therefore, if $\tilde{T}_W^b < 0$ then both agents will wait for the resolution of uncertainty, if $\tilde{T}_W^b > 0$ then b concedes to a 's demand with probability approaching 1.

Focus first on the case when agent a moves first, $a = 1, b = 2$. Assume that $a = 1$ imitates a particular demand with positive limit probability. Because b 's payoff from waiting $\tilde{V}_{R_0}^b - x_0^b R$ is increasing in α^b , it is maximised at $(1-p) + p(1-\alpha^a) - x_0^b R$ by $\alpha^b = 1$ (ε close). This limit payoff from waiting is better than immediate concession, $(1-\alpha^a)$, if and only if $\alpha^a > \frac{x_0^b R}{1-p}$ when ε is sufficiently small. Only for this case, therefore, will we have $\tilde{T}_W^b < 0$.

Knowing how b will respond, we can induce agent a 's limit choice by comparing her payoffs which will follow from different demand choices which are assumed to be made with positive limit probability. In particular, if a demanded $\alpha^a < \frac{x_0^b R}{1-p}$ (with positive limit probability) then b would accept her demand with probability approaching 1 in the limit, but demanding any more ensures delay. If $\frac{1-p}{x_0^b} < R$ then a will certainly demand within ε of $\alpha^a = 1$, and this demand will be accepted immediately with probability approaching one in the limit. This characterizes region (H).

For lower R , notice that if a demands more than $\frac{x_0^b R}{1-p}$, b 's counterdemand will be approximately $\alpha^b = 1$ (for ε small) and delay will occur, to give a a payoff of approximately $p\alpha^a - x_0^a R$. This is increasing in α^a , and is therefore maximized at $p - x_0^a R$ by $\alpha^a = 1$. Agent a 's choice, therefore, reduces to demanding and immediately getting (in the limit) within ε of $\frac{x_0^b R}{1-p}$, or getting within ε of $p - x_0^a R$ by demanding as much as possible. The limit payoff associated with waiting is

better whenever $R < \frac{p(1-p)}{x_0^a(1-p)+x_0^b}$ and ε is small. This completes the characterization of regions (E),(F) and (G).

Suppose now b moves first, $a = 2, b = 1$. In this case for any demand made by b with positive limit probability, a can ensure $\tilde{T}_W^b < 0$, and so b 's immediate concession (in the limit), by selecting $\alpha^a < (1 - \alpha^b) + \frac{x_0^b R}{1-p}$. If the right hand side of this equation is larger than 1 then this is satisfied even for a demand of $\alpha^a = 1$, which gives a her best possible limit payoff. In particular, such an outcome must necessarily occur if $\frac{1-p}{x_0^b} < R$. This characterizes the outcome of region (H).

Suppose then that $\frac{1-p}{x_0^b} > R$, and indeed further suppose that b 's demand satisfies $(1 - \alpha^b) + \frac{x_0^b R}{1-p} < 1$. If a 's demand exceeds the left hand side of this equation, delay occurs in the limit. Agent a 's limit payoff with delay is increasing in α^a and so is maximized at $p + (1 - p)(1 - \alpha^b) - x_0^a R$ by demanding approximately $\alpha^a = 1$. This is more than a 's limit payoff from demanding marginally less than $(1 - \alpha^b) + \frac{x_0^b R}{1-p}$ and securing b 's acceptance, if and only if $\alpha^b > \frac{(x_0^a(1-p)+x_0^b)R}{p(1-p)}$, for small ε .

Now, consider b 's initial problem. If she makes a demand $\alpha^b < \frac{(x_0^a(1-p)+x_0^b)R}{p(1-p)}$, then by the argument of the previous two paragraphs a will counterdemand $\alpha^a = \min \left\{ 1, (1 - \alpha^b) + \frac{x_0^b R}{1-p} \right\}$, which b must accept in the limit. This gives b a payoff of approximately $\max \left\{ 0, \alpha^b - \frac{x_0^b R}{1-p} \right\}$, which is increasing in α^b , potentially strictly given that $\frac{1-p}{x_0^b} > R$.

In region (G) $R > \frac{p(1-p)}{(x_0^a(1-p)+x_0^b)R}$ and so $\frac{(x_0^a(1-p)+x_0^b)R}{p(1-p)} > 1 \geq \alpha^b$ for any α^b . This means the outcome in region (G) is certainly for b to demand approximately $\alpha^b = 1$ and for a to counterdemand $\frac{x_0^b R}{1-p}$ which is immediately accepted.

Finally consider, $R < \frac{p(1-p)}{(x_0^a(1-p)+x_0^b)R}$. Given a 's behavior, b 's maximum possible limit payoff from demanding $\alpha^b < \frac{(x_0^a(1-p)+x_0^b)R}{p(1-p)}$ comes from demanding just marginally less than the right hand side of this equation, and drawing a counterdemand of approximately $\alpha^a = 1 - \frac{(x_0^a+x_0^b)R}{p}$, which she accepts immediately. If $\alpha^b > \frac{(x_0^a(1-p)+x_0^b)R}{p(1-p)}$, however, then a counterdemands approximately $\alpha^a = 1$ and the result is delay. Given such a counterdemand, b 's approximate payoff from delay is $(1 - p)\alpha^b - x_0^b R$, which is increasing in α^b , and so maximized at $(1 - p) - x_0^b R$ by a demand of almost $\alpha^b = 1$.

All that remains to do is to compare $\frac{(x_0^a+x_0^b)R}{p}$, the (ε approximate) best payoff from demanding less than $\frac{(x_0^a(1-p)+x_0^b)R}{p(1-p)}$ (resulting in no delay), and $(1 - p) - x_0^b R$, the (ε approximate) best payoff from demanding more than that (delay). Delay is the better option if and only if $R < \frac{p(1-p)}{x_0^a+(1+p)x_0^b}$ for sufficiently small ε . This characterizes outcomes in regions (E) and (F). \square

To help build up intuition for why delay occurs, consider the above model when agent b (the weaker agent) moves first, and suppose that $p = (1 - p) = \frac{1}{2}$. Delay occurs whenever $R <$

$\frac{p(1-p)}{x_0^a + x_0^b(1+p)} = \frac{1}{4x_0^a + 6x_0^b}$, giving agent i an expected payoff of $\frac{1}{2} - x_0^i R$. Both agents would therefore certainly prefer the compromise division $(\frac{1}{2}, \frac{1}{2})$ agreed immediately.

The problem is that neither agent has an incentive to propose this compromise herself, because doing so would put her at a disadvantage in negotiations. Proposing a compromise decreases an agent's option value of waiting, and increases the option value of her opponent.

An agent who proposes a compromise with positive limit probability, must still accept an opponent's aggressive demand in states of the world in which she has higher costs of delay. Her opponent, however, can now concede to a more generous offer when she has high costs of delay. This implies that either the compromise will not secure immediate agreement, or if it does, it will not be on the terms proposed by the compromising agent.

To make this still less abstract, suppose with positive limit probability agent b demands $\alpha^b = 1$ and a counterdemands $\alpha^a = \frac{1}{2}$ instead of $\alpha^a = 1$. This will not only increase agent b 's payoff from immediate concession (from 0 to $\frac{1}{2}$), but also her option value of waiting (from $\frac{1}{2} - x_0^b R$ to $\frac{3}{4} - x_0^b R$). The goal-posts shift. Suddenly, b is no longer willing to accept the compromise immediately but will hold out for more because $\frac{3}{4} - x_0^b R > \frac{1}{2}$ (b accepts the compromise only if uncertainty is resolved and determines that she faces the higher cost of delay). All that agent a would achieve by compromising, therefore, is to increase b 's share in a delayed agreement.

Similar logic holds for agent b 's initial demand. If $R < \frac{1}{4x_0^a + 8x_0^b}$ then demanding $\frac{1}{2}$ (with positive limit probability) means that a will counterdemand $\alpha^a = 1$ and delay will occur anyway. Alternatively, if $R > \frac{1}{4x_0^a + 8x_0^b}$, then a will counterdemand $\alpha^a = \frac{1}{2} + 2x_0^b R$, which b must accept immediately. By only demanding $\frac{1}{2}$ agent b lowers her own option value from waiting, and so her own concession becomes more tempting. Either way, b is worse off than before.

The above Proposition demonstrates the possibility of inefficiency when agents face flow costs of delay. The reputational bargaining literature has typically focussed on the case when agents' costs of delay are due to impatience for a deal $r_{\omega'}^i > 0$. Is the difference important? A first response is that the outcome is robust to small levels of impatience. The proposition will still hold ϵ closely if $r_{\omega'}^i < \epsilon$, for ϵ sufficiently small.

Furthermore, Example 1 shows that delay may occur in the limit with similar demand polarization even if $x_{\omega'}^i = 0$. This example is designed to allow for a fairly simple proof (found in the Appendix) rather than to maximize the inefficiency of delay.

The example considers three equally likely states a, b and e .¹⁵ Before the revelation time agents are equally impatient and, therefore, I vary $\delta = e^{-r_0^i R}$ instead of varying R . If state e occurs, then both agents continue to have equal impatience. If state a occurs, however, then agent a becomes more patient than agent b , $3r_a^a = r_a^b$, with the reverse true if state b occurs, $r_b^a = 3r_b^b$. These fundamentals are symmetric, and so the outcome for agents' 1 and 2 are independent

¹⁵Limit delay is impossible with only two states, $x_{\omega'}^i = 0$, and a rich type space.

of whether $a = 1$ or $b = 1$. The limiting outcome for δ close to one (R small) involves agent 1 demanding approximately $\alpha^1 = \frac{5}{8}$, to which agent 2 counterdemands marginally less than $\alpha^2 = \frac{7}{8}$ or $\alpha^2 = \frac{5}{8}$. In either case, agreement is delayed until time R , giving agents' the limiting payoffs $U^1(\sigma) = \delta \frac{11}{24}$ and $U^2(\sigma) = \delta \frac{13}{24}$.

Example 1 (Delay with impatience). *Let $E_n = \{z_n^i, \pi^i(\alpha^i), r_{\omega'}^i, x_{\omega'}^i, C^i\}_{i \in \{1,2\}, \omega' \in \Omega'}$ be a sequence of bargaining games with a generic simple type space. Suppose that $z_n \rightarrow 0$ and $L > \frac{z_n^1}{z_n^2} > 1/L$ for some positive constant L . Let $\Omega = \{a, b, e\}$ with $p(\omega) = \frac{1}{3}$. For agents a and b let neither face flow costs of delay, $x_{\omega'}^a = x_{\omega'}^b = 0$ for all $\omega' \in \Omega'$. Furthermore, let $r_0^a = r_0^b$ with $e^{-r_0^i R} = \delta > \frac{\sqrt{945}-27}{4} \approx 0.935$ as well as $r_e^a = r_e^b$, $3r_a^a = r_a^b$ and $r_b^a = 3r_b^b$. Then whether $a = 1$ or $b = 1$ for any $\epsilon > 0$ there exists $\varepsilon > 0$ such that if the simple type space is ε -rich the sequence of equilibria must satisfy:*

$$\begin{aligned} \lim_{|\alpha^1 - \frac{5}{8}| < \epsilon} \sum \mu(\alpha^1) = 1 & \quad \left| \lim U^1(\sigma) - \delta \frac{11}{24} \right| < \epsilon & \quad \lim q(R_{-1}, \sigma) = 0 \\ \lim_{|\alpha^2 - \frac{5}{8}| < \epsilon, |\alpha^2 - \frac{7}{8}| < \epsilon} \sum \mu(\alpha^1) = 1 & \quad \left| \lim U^2(\sigma) - \delta \frac{13}{24} \right| < \epsilon \end{aligned}$$

In this example, there is an advantage to being the second agent to announce a demand, $\delta \frac{13}{24} > \delta \frac{11}{24}$. The reason for this second mover advantage is that the second agent can “fine tune” her bargaining position, with prior knowledge of her opponent’s action. In particular, she has the option to make a more aggressive demand than agent 1, to which agent 1 will still concede at time R if the state of the world favors agent 2, safe in the knowledge that 1 cannot reoptimize.

A clear first-mover advantage can also emerge. Demonstrating this in a three state example, like that above, is unnecessarily complicated. I therefore present a second example with only two equally likely states a and b . Apart from the absence of state e the parameters are exactly the same as for Example 1. In this example delay does not occur, but for small δ (big R) there is a first mover advantage, and for big δ (small R) there is a second mover advantage. The force which works towards a first mover advantage is that a first mover can stake out an aggressive initial position, which threatens the possibility of delay and wasting payoff for both parties, unless her opponent is willing to make a sufficiently generous counterdemand. By moving first the agent burns her bridges and cannot reoptimize, making it incumbent on her opponent to eliminate inefficiency. This logic is similar to the advantage of a Stackelberg leader. This “commitment” is more powerful when the resources lost in waiting are larger (δ is small).

Example 2 (First and second mover advantage). *Let $E_n = \{z_n^i, \pi^i(\alpha^i), r_{\omega'}^i, x_{\omega'}^i, C^i\}_{i \in \{1,2\}, \omega' \in \Omega'}$ be a sequence of bargaining games with a generic simple type space. Suppose that $z_n \rightarrow 0$ and $L > \frac{z_n^1}{z_n^2} > 1/L$ for some positive constant L . Let $\Omega = \{a, b\}$ with $p(\omega) = \frac{1}{2}$. For agents a and b let neither face flow costs of delay, $x_{\omega'}^a = x_{\omega'}^b = 0$ for all $\omega' \in \Omega'$. Furthermore, let $r_0^a = r_0^b$ with $e^{-r_0^i R} = \delta > \sqrt{15} - 3 \approx 0.873$ as well as $3r_a^a = r_a^b$ and $r_b^a = 3r_b^b$. Then whether $a = 1$ or*

$b = 1$ for any $\epsilon > 0$ there exists $\varepsilon > 0$ such that if the simple type space is ε -rich the sequence of equilibria must satisfy:

$$\begin{aligned} \lim_{|\alpha^1 - \frac{12+5\delta}{18+2\delta}| < \epsilon} \sum \mu(\alpha^1) = 1 & \quad \left| \lim U^1(\sigma) - \frac{18-9\delta}{18+2\delta} \right| < \epsilon \\ \lim_{|\alpha^2 - \frac{11\delta}{18+2\delta}| < \epsilon} \sum \mu(\alpha^1) = 1 & \quad \left| \lim U^2(\sigma) - \frac{11\delta}{18+2\delta} \right| < \epsilon \end{aligned} \quad \lim q(0_{+3}, \sigma) = 1$$

This implies a first mover advantage for $\delta < \frac{9}{10}$, and a second mover advantage otherwise.

The possibility of second mover advantage creates another avenue by which inefficient delay may occur. [Abreu et al. \[2014\]](#) introduce behavioral types which make different fixed demands and also vary in the time those demands are announced. Extending my model to allow such types shows that rational agents may not even choose to announce demands until after the revelation time with probability close to one. Agents do this in order to avoid “committing” to a suboptimal bargaining position. This pattern of behavior seems to square well with some real world examples of bargaining with uncertainty. Parties sometimes don’t even bother to start negotiating until after uncertainty has been eliminated.

4.4 Complex types: the generalized Rubinstein demand

To lay the groundwork for my main result with complex types, I first characterize the generalized Rubinstein for this bargaining environment. This is the solution to an alternating offer game when there are no behavioral types and the time between offers is arbitrarily small.

Consider an alternating offers game with time Δ between offers. Agent 1 can make offers at $\{0, 2\Delta, \dots, R - 2\Delta, R, R + 2\Delta, \dots\}$ and agent 2 at $\{\Delta, 3\Delta, \dots, R - \Delta, R + \Delta, \dots\}$. At time R the state ω is revealed (prior to 1’s offer). Focus first on the game after time R if bargaining has not yet concluded. Let $\alpha_{\omega, \Delta}^i$ and be agent i ’s stationary subgame perfect equilibrium demand when she makes an offer.¹⁶ This must satisfy:

$$\begin{aligned} \alpha_{\omega, \Delta}^i &= \min \left\{ 1 - e^{-r_{\omega}^j \Delta} \alpha_{\omega, \Delta}^j + \int_0^{\Delta} e^{-r_{\omega}^j s} x_{\omega}^j ds, 1 \right\} \\ &= \min \left\{ \max \left\{ \frac{(1 - e^{-r_{\omega}^j \Delta}) + \int_0^{\Delta} e^{-r_{\omega}^j s} x_{\omega}^j ds - e^{-r_{\omega}^j \Delta} \int_0^{\Delta} e^{-r_{\omega}^i s} x_{\omega}^i ds}{1 - e^{-(r_{\omega}^i + r_{\omega}^j) \Delta}}, 1 - e^{-r_{\omega}^j \Delta} + \int_0^{\Delta} e^{-r_{\omega}^j s} x_{\omega}^j ds \right\}, 1 \right\} \end{aligned} \quad (14)$$

Agent 1’s *expected* demand at R before the revelation of uncertainty is then $\alpha_{0, \Delta}^1(R) = \int \alpha_{\omega, \Delta}^1 dp$. The remainder of the game can be solved by backwards induction. At any time $t < R$ at which

¹⁶The standard trick of taking the supremum and infimum of i and j ’s equilibrium payoffs and showing that these must be the same applies here meaning $\alpha_{\omega, \Delta}^i$ is the unique equilibrium demand.

agent i makes an offer her demand must satisfy:

$$\alpha_{0,\Delta}^i(t) = \min \left\{ 1 - e^{-r_0^j \Delta} \alpha_{0,\Delta}^j(t) + \int_0^\Delta e^{-r_0^j s} x_0^j ds, 1 \right\}$$

I define the solution of the above equations as $\Delta \rightarrow 0$ as the generalized Rubinstein bargaining demand. To find it, first take the limit (using l'Hopital's rule) of equation 14 to give:

$$\alpha_\omega^{i*} = \min \left\{ \max \left\{ \frac{r_\omega^j + x_\omega^j - x_\omega^i}{r_\omega^i + r_\omega^j}, 0 \right\}, 1 \right\} \quad (15)$$

Let $\alpha_0^{i*}(R) = \int \alpha_\omega^{i*} dp$. For $t < R$, assume away the constraint $\alpha_{0,\Delta}^i(t) \leq 1$ in equation 15 and substitute in for $\alpha_{0,\Delta}^j(t + \Delta)$ to get a recursive equation for agent i 's demand (labelled $\hat{\alpha}_{0,\Delta}^i(t)$ to emphasize the lack of constraints imposed):

$$\hat{\alpha}_{0,\Delta}^i(t) - e^{-(r_0^i + r_0^j)\Delta} \hat{\alpha}_{0,\Delta}^i(t + 2\Delta) = (1 - e^{-r_0^j \Delta}) + \int_0^\Delta e^{-r_0^j s} x_0^j ds - e^{-r_0^j \Delta} \int_0^\Delta e^{-r_0^i s} x_0^i ds$$

Dividing each side of this equation by Δ and taking the limit (using l'Hopital's rule) when $\Delta \rightarrow 0$ defines a linear ODE:

$$2\hat{\alpha}_0^{i'}(t) = (r_0^i + r_0^j)\hat{\alpha}_0^i(t) - (r_0^j + x_0^j - x_0^i) \quad (16)$$

Solving this ODE using the boundary condition $\hat{\alpha}_0^i(R) = \alpha_0^{i*}(R)$ gives:¹⁷

$$\hat{\alpha}_0^i(t) = \alpha_0^{i*}(R) e^{\frac{r_0^i + r_0^j}{2}(t-R)} + \left(1 - e^{\frac{r_0^i + r_0^j}{2}(t-R)} \right) \frac{r_0^j + x_0^j - x_0^i}{r_0^i + r_0^j} \quad (17)$$

The true limit of the alternating offers demand $\alpha_{0,\Delta}^i(t)$ as Δ becomes small must obey an ODE of the form of equation 16 at all times $t < R$ unless $\alpha_0^{i*}(t) = 0$ and $r_0^j + x_0^j - x_0^i < 0 = \alpha_0^{i*}(t)$ or $\alpha_0^{i*}(t) = 1$ and $r_0^i + x_0^i - x_0^j < 0 = \alpha_0^{i*}(t)$. Verifying that $\hat{\alpha}_0^i(t)$ is either monotonically increasing or decreasing in t means that the generalized Rubinstein demand for $t \leq R-1$ must satisfy:

$$\alpha_0^{i*}(t) = \min \left\{ \max \left\{ \hat{\alpha}_0^i(t), 0 \right\}, 1 \right\}$$

Proposition 5 shows that if there is a type committed to this demand, then rational agents can guarantee the associate time zero payoff as the probability of behavioral types vanishes. The reason is loosely that imitating a generalized Rubinstein type ensures that an opponent always concedes (and hence builds reputation) at a slower rate in the war of attrition, and so that opponent must concede with high probability at time zero to ensure that both agents reach a

¹⁷If $r_0^i = r_0^j = 0$ then $\hat{\alpha}_0^i(t) = \alpha_0^{i*}(R) + (x_0^j - x_0^i)(R - t)$.

probability one reputation at the same time. The proof is similar to Proposition 5 in F14, which explores the link between the reputational and alternating offer models in more depth.

Proposition 5 (Rubinstein). *Let $E_n = \{z_n^i, \pi^i(\alpha^i), r_{\omega'}^i, x_{\omega'}^i, C^i\}_{i \in \{1,2\}, \omega' \in \Omega'}$ be a sequence of bargaining games with complex types. Suppose that $z_n \rightarrow 0$ and $L > \frac{z_n^1}{z_n^2} > 1/L$ for some positive constant L . If there exists a type which committed to the generalized Rubinstein demand, $\alpha^{i*} \in C^i$, then the sequence of equilibria must satisfy:*

$$\liminf U^i(\sigma) \geq \alpha^{i*}(0) \quad (18)$$

5 Conclusion

Uncertainty has a destructive effect on bargaining. Parties fail to reach efficient early agreements despite ample opportunities to do so, and instead wait to see how uncertainty plays out before reaching a deal. I have shown that such behavior is consistent with even an arbitrarily small possibility that bargainers are obstinate. The model also predicts polarization of agents' bargaining positions when delay occurs, with rational agents asking for the entire bargaining surplus.

These conclusions are somewhat depressing. They depend on the assumption that behavioral types are committed to fixed demands. If there is a larger set of behavioral types who make time varying demands, then the game converges to a generalized Rubinstein alternating offer demand. It is certainly worthwhile, therefore, for future work to investigate sources of endogenous motivation for behavioral types, to help understand when commitment to fixed or time varying demands might be expected.

6 Appendix

Proof of Lemma 2

I argue that any equilibrium must satisfy the conditions i)-iii). Let $\{F_\omega^1, F_\omega^2\}_{\omega \in \Omega}$ define an equilibrium in the 0_{+3} continuation game. Define A_ω^i to be the set of times at which it is optimal for rational agent i to concede in state ω given j 's strategy. Optimality conditions only on the information that agent i has at time t in state ω so that concession at $t \leq R_{-1}$ may be ex-ante optimal given j 's strategy but need not be ex-post optimal given that the state is in fact ω . Since I conjecture an equilibrium $A_\omega^i \neq \emptyset$. Recall the definition of \check{T}_ω in equation 6 of the text and define $\tau_\omega^i = \inf\{t : F_\omega^j(t) \geq 1 - \bar{z}^j\}$. Notice that the first claim of Lemma 1 implies that $\tau_\omega^i < \infty$. As argued in the text of section 2 it is without loss of generality to assume rational agents concede (weakly) before behavioral types, so if $\check{T}_\omega < \infty$ then $\tau_\omega^i \leq \check{T}_\omega$. A small clarification, if $\check{T}_\omega = R_0$ then $\tau_\omega^1 \leq R_{+1}$ and $\tau_\omega^2 \leq R_{+2}$.

I first make a series of subclaims:

(a) If $\tau_\omega^i \notin \{R_{-1}, R_{+1}, R_{+2}, R_{+3}\}$ then $\tau_\omega^i = \tau_\omega^j$. If $\tau_\omega^i \in \{R_{-1}, R_{+1}, R_{+2}, R_{+3}\}$ then $\tau_\omega^j \in \{R_{-1}, R_{+1}, R_{+2}, R_{+3}\}$. Given

that rational agents concede before behavioral types this simply says that a rational agent will not delay once she knows that her opponent is behavioral, which is true by assumption. This proves condition ii).

- (b) If F_ω^i jumps at $t \in [0_{+3}, R_{-1}) \cup [R_{+3}, \infty)$ with $t < \check{T}_\omega$ then F_ω^j does not jump at t . If F_ω^i had a jump at t , then because bargaining positions are incompatible prior to \check{T}_ω are continuous and the cost of delay is continuous, agent j would receive a strictly higher utility by conceding an instant after t than by conceding at exactly t .
- (c) If F_ω^i jumps at $t \in (0_{+3}, R_{-1}] \cup (R_{+3}, \infty)$ with $t < \check{T}_\omega$ then j cannot concede with positive probability on the interval $(t - \varepsilon, t)$ for some $\varepsilon > 0$ because doing so would again forgo an expected profit bump from waiting to concede at t (remember if both agents concede at the same time then one of the two divisions is selected with probability $\frac{1}{2}$).
- (d) If F_ω^i jumps at $R_{+3} < \check{T}_\omega$ then F_ω^j does not jump at R_{+3} . Again waiting until an instant after R_{+3} would secure a profit bump. Similarly, if F_ω^2 jumps at $R_{-1} < \check{T}_\omega$ then F_ω^1 is constant on $(R_{-1} - \varepsilon, R_{-1}]$ as waiting to concede at R_{-1} would secure a strict expected profit bump for agent 1.
- (e) If a rational agent j does not concede with positive probability on an interval $(\bar{t}_1, \bar{t}_2]$ with $\bar{t}_2 \leq \check{T}_\omega$ and either $\bar{t}_1 \geq R_{+3}$ or $\bar{t}_2 \leq R_{-1}$, then neither does rational agent i . Conceding at any point on this interval would give agent j a strictly lower payoff than conceding momentarily before that time given the assumption of optimal concession against a know behavioral type.
- (f) For $t \in (0_{+3}, R_{-1}] \cup (R_{+3}, \infty)$ with $t < \check{T}_\omega$ there can be no mass concession by either agent, and so any concession on this interval must be continuous. To see this, notice that by (b) and (c) mass concession by i at $t \neq R_{-1}$ would induce an interval of non-concession by j on $(t - \varepsilon, t]$, for some $\varepsilon > 0$, and so (e) implies there cannot be concession by agent i on this interval, a contradiction. For $t = R_{-1}$ if there is mass concession by agent 2, then by (d) there is no concession by 1 on $(R_{-1} - \varepsilon, R_{-1}]$, which again by (e) entails no concession by 2 on this interval either, a contradiction. If there is mass concession by agent 1 at R_{-1} then because of (c) there is no concession by 2 on $(R_{-1} - \varepsilon, R_{-1})$ and because of the argument of the last sentence, there no mass concession by agent 2 at R_{-1} either. But in this case (e) then implies no concession by 1 on $(R_{-1} - \varepsilon, R_{-1}]$ either, a contradiction. This along with b) and d) proves condition iii) that at most one agent can concede with positive probability at time zero and at time R in state ω unless $\check{T}_\omega = R_{-1}$.
- (g) If $\tau_\omega^i = \check{T}_\omega (\neq R_0)$ then both agents must concede with positive probability on an interval $(\check{T}_\omega - \varepsilon, \check{T}_\omega)$, for all sufficiently small $\varepsilon > 0$. To see this, remember that agents bargaining positions are continuous and so must be exactly compatible at \check{T}_ω , which means conceding at time \check{T}_ω is payoff equivalent to being conceded to. Thus, if the claim were not true, it is without loss of generality to assume that agent j does not concede with positive probability on the interval $(\check{T}_\omega - \varepsilon, \check{T}_\omega]$, which by (e) must induce agent i not to concede on that interval either. More generally, mass concession by j at \check{T}_ω does not affect the continuity of i 's expected payoff from conceding at $t \in (\check{T}_\omega - \varepsilon, \check{T}_\omega]$.
- (h) If $R_{+3} \leq t' < t'' \leq \tau_\omega^i$ then $F_\omega^i(t'') > F_\omega^i(t')$. Suppose not, then let t_ω^{i*} be the supremum of times such that $F_\omega^i(t) = F_\omega^i(t')$, that is $t_\omega^{i*} = \sup\{t : F_\omega^i(t) = F_\omega^i(t')\}$. (g) implies $t_\omega^{i*} < \check{T}_\omega$ and (f) implies there is no mass concession at t_ω^{i*} and so F_ω^i is constant on $(t', t_\omega^{i*}]$. By (e) this must also be true for F_ω^j . But (f) also implies no mass concession by agent j at $t \in [t_\omega^{i*}, t_\omega^{i*} + \varepsilon]$ for small $\varepsilon > 0$, which means conceding at or momentarily after t_ω^{i*} must give agent i a strictly lower utility than conceding an instant after t' . This means t_ω^{i*} cannot be the supremum.
- (i) If $t' < t'' \leq \max\{\tau_\omega^i, R_{-1}\}$ then $F_\omega^i(t'') > F_\omega^i(t')$ or $F_\omega^i(t') = F_\omega^i(R_{-1})$. Suppose not, then define $t^{i*} = \sup\{t \leq R_{-1} : F_\omega^i(t) = F_\omega^i(t')\}$. (g) again implies $t^{i*} < \check{T}_\omega$. If $t^{i*} < R_{-1}$ then by the same arguments as in (h) there is no concession by either agent on $(t', t^{i*}]$ and no mass concession at $t \in [t^{i*}, t^{i*} + \varepsilon]$ for small $\varepsilon > 0$ and so conceding at or just after t^{i*} must give a strictly lower utility than conceding an instant after t' . (f) implies there is no mass concession at R_{-1} , which means that if $t^{i*} = R_{-1}$ then $F_\omega^i(t') = F_\omega^i(R_{-1})$, a contradiction.

- (j) If F_ω^j is continuous at t , then $U^i(s, \sigma^j|\alpha)$ and $U^i(s, \sigma^j|\alpha, \omega)$ as defined in the text are continuous at $s = t$. This follows immediately from their definition.

Given (e), (h) and (i) it follows that there exists some $\hat{T} \leq R_{-1}$ such that $A_\omega^i \cap (\hat{T}, R_{-1}] = \emptyset$, but A_ω^i is dense in $[0, \hat{T}]$ and in $[R_{+3}, \tau_\omega^i]$ whenever respectively $\hat{T} > 0$ or $\tau_\omega^i > R_{+3}$. From (f) and (g) we may assume F_ω^j is continuous on $(0, \hat{T})$ and (R_{+3}, τ_ω^i) and hence by (j) and (g) $U^i(s, \sigma^j|\alpha)$ is continuous on $(0, \hat{T})$ and $U^i(s, \sigma^j|\alpha, \omega)$ is continuous on $(R_{+3}, \tau_\omega^i]$. This implies that $A_\omega^i \supseteq (0, \hat{T}] \cup (R_{+3}, \tau_\omega^i]$, which in turn implies that $U^i(s, \sigma^j|\alpha)$ and $U^i(s, \sigma^j|\alpha, \omega)$ are constant on those intervals. This proves condition i).

Proof of Lemma 3

I first argue that if $\alpha_\omega^i(R) + \alpha_\omega^j(R) > 1$ in state ω then it is without loss of generality to assume that agents do not concede at R_{+1} or R_{+2} . Consider, the game at R_{+2} with initial reputations $\bar{z}_\omega(R_{+1})$. Let $(1 - c_{\omega, R_{+1}}^i)$ be the conditional probability that agent i concedes at R_{+i} . First observe that agent 2 will not concede at R_{+2} if agent 1 is expected to concede at R_{+3} . If $c_{\omega, R_{+3}}^1 = 1$, however, then $\bar{z}_\omega^1(R_{-1})$ pins down T_ω^* in the R_{+3} continuation game with the condition $\bar{z}_\omega^1(R_{+1}) \exp\left(\int_R^{T_\omega^*} h_\omega^1(s) ds\right) = 1$. But if that is so the total probability of concession by agent 2 at R_{+2} and R_{+3} , is uniquely determined to ensure that she reaches a probability one reputation at T_ω^* , $c_{\omega, R_{+2}}^2 c_{\omega, R_{+3}}^2 = \bar{z}_\omega^2(R_{+1}) \exp\left(\int_R^{T_\omega^*} h_\omega^2(s) ds\right)$.

An entirely analogous argument works at R_{+1} when initial reputations are $\bar{z}_\omega(R_0)$ to show that $c_{\omega, R_{+1}}^1 c_{\omega, R_{+3}}^1$ is uniquely determined, and is equal to one if $c_{\omega, R_{+2}}^2 c_{\omega, R_{+3}}^2 < 1$. It is therefore, without loss of generality to assume that agent don't concede at R_{+1} or R_{+2} when $\alpha_\omega^i(R) + \alpha_\omega^j(R) > 1$. Assuming no mass concession at R_{+1} or R_{+2} the unique equilibrium's continuation payoff in state ω at R_{+1} equals the continuation payoff at R_{+3} , $V_{\omega, R_{+3}}^i(\bar{z}_\omega(R_0))$, and this is continuously increasing in $\bar{z}_\omega^i(R_0)$ and decreasing in $\bar{z}_\omega^j(R_0)$, with both strict for at least one agent.

Alternatively, in the case $\alpha_\omega^i(R) + \alpha_\omega^j(R) < 1$, then it is clear that agent 1 would concede immediately at R_{+1} to prevent her opponent conceding at R_{+2} .

Expected payoffs at R_0 are simply the average of payoffs at R_{+1} . This can thus be summarized as:

$$V_{R_0}^i(\bar{z}_0(R_{-1})) = \int (1 - c_\omega^j) \alpha_\omega^i(R) + c_\omega^j (1 - \alpha_\omega^j(R)) dp \quad (19)$$

where $c_\omega^i = c_{\omega, R_{+1}}^i c_{\omega, R_{+3}}^i$ is uniquely determined in state ω if $\alpha_\omega^i(R) + \alpha_\omega^j(R) > 1$ and $(1 - c_\omega^2) = c_\omega^1 = 1$ if $\alpha_\omega^i(R) + \alpha_\omega^j(R) \leq 1$.

If demands are incompatible at R_{-1} then certainly in some states $\alpha_\omega^i(R) + \alpha_\omega^j(R) > 1$ because demands are continuous in expectation. This means that $V_{R_0}^i(\bar{z}_0(R_{-1}))$ is continuously increasing in $\bar{z}_0^i(R_{-1})$ and decreasing in $\bar{z}_0^j(R_{-1})$ with this strict for at least one agent. The continuity of demands in expectation also implies $V_{R_0}^1(\bar{z}_0(R_{-1})) \geq 1 - \alpha_0^2(R)$ as agent 1 can always concede in every state at R_{+1} . Finally, if agent j is certainly behavioral then an agent i can never obtain a continuation payoff of more than $1 - \alpha_\omega^j(R)$ in any state (at R_{+i} or R_{+3}). The continuity of demands in expectation, therefore, implies $V_{R_0}^i(\bar{z}_0^i(R_{-1}), 1) \leq 1 - \alpha_0^j(R)$.

Proof of Proposition 1

The existence of an equilibrium follows from arguments in the text. First I claim that, given \hat{T} , c_0^i is unique. To see this, first suppose that agents reach a probability one reputation at \hat{T} . Assume $c_0^j = 1$ (as it must for one agent). Decreasing c_0^i would result in agent i reaching a probability one reputation strictly before \hat{T} . Increasing c_0^i would mean having $z^i(\hat{T}) < 1$. Both imply contradictions to agent j reaching a probability one reputation at \hat{T} .

Suppose then that the equilibrium with waiting, so that $\bar{z}_0^j(\hat{T}) < 1$ for $i = 1, 2$ then as detailed in the proof of Lemma 3 $V_{R_0}^i(\bar{z}_0(R_{-1}))$ is strictly increasing in $\bar{z}_0^i(R_{-1})$ and decreasing in $\bar{z}_0^j(R_{-1})$ for some agent, and indeed for

both agents if $\hat{T} < R_{-1}$. Increasing an arbitrary agent i 's time zero concession without changing $\hat{T} < R_{-1}$ would strictly increase $z^i(R_{-1})$ implying strict incentives for agent i to wait on some interval $[\hat{T} - \varepsilon, R_{-1}]$ for some $\varepsilon > 0$ if $\hat{T} < R_{-1}$, a contradiction. Similarly if $\hat{T} = R_{-1}$ then increasing agent i 's time zero concession would either again imply that agent i has strict incentives to wait on some interval $[R_{-1} - \varepsilon, R_{-1}]$ or agent j would strictly prefer to concede at R_{-1} rather than wait for her R_0 continuation payoff, again a contradiction.

For there to be multiple equilibria, therefore, there must be multiple \hat{T} . It is immediately clear that there cannot be two different equilibria, with different \hat{T} such that both agents reach a probability one reputation at \hat{T} . At least one of the equilibria, therefore, must involve agents waiting on the interval $[\hat{T}, R_{-1})$ (if the only waiting equilibrium had $\hat{T} = R_{-1}$ it must be unique).

Suppose that agents concede at time zero with probabilities $(1 - \hat{c}_0^i)$, concede at the equilibrium rates $h_0^i(t)$ on the interval $(0, y]$, do not concede on the interval $(y, R_{-1}]$ but play the appropriate continuation game from time R_0 onwards. Consider the expected utility conditional on no concession at time 0_{+3} (i.e. from the perspective of time 0^+) of an agent i who concedes an instant after R_{+3} if $\alpha_\omega^1(R) + \alpha_\omega^2(R) > 1$ and concedes at R_{+i} if $\alpha_\omega^1(R) + \alpha_\omega^2(R) \leq 1$ assuming an opponent's equilibrium play in the R_{+3} continuation game. This utility is given by:

$$U_{W,0^+}^i(y, \hat{c}_0^i, \hat{c}_0^j) = \int_0^y \left(h_0^j(s) \exp\left(-\int_0^s h_0^j(v) dv\right) \left(e^{-r_0^i s} \alpha_0^i(s) - \int_0^s e^{-r_0^i v} x_0^i dv \right) \right) ds \\ + \exp\left(-\int_0^y h_0^j(v) dv\right) \left(e^{-r_0^i R} V_{R_0}^i(\bar{z}_0(y)) - \int_0^R e^{-r_0^i s} x_0^i ds \right)$$

where the continuation payoff, $V_{R_0}^i(\bar{z}_0(y))$, has the form:

$$V_{R_0}^i\left(\frac{\bar{z}^i}{\hat{c}_0^i} \exp\left(\int_0^y h_0^i(v) dv\right), \frac{\bar{z}^j}{\hat{c}_0^j} \exp\left(\int_0^y h_0^j(v) dv\right)\right) = \int \alpha_\omega^i(R) - \hat{c}_\omega^j(\alpha_\omega^j(R) + \alpha_\omega^i(R) - 1) dp$$

with $\hat{c}_\omega^1 = (1 - \hat{c}_\omega^2) = 1$ if $\alpha_\omega^j(R) + \alpha_\omega^i(R) \leq 1$ and otherwise \hat{c}_ω^i jointly determined with \hat{T}_ω to ensure both agents reach a probability one reputation at the same time:

$$\hat{c}_\omega^j = \frac{\bar{z}^j}{\hat{c}_0^j} \exp\left(\int_0^y h_0^j(s) ds + \int_R^{\hat{T}_\omega} h_\omega^j(s) ds\right) \quad (20)$$

Suppose that there is a waiting equilibrium with $\hat{T} \in [0_{+3}, R_{-1})$. In this equilibrium it must be that $U_{W,0^+}^i(\hat{T}, c_0^i, c_0^j) \geq (1 - \alpha_0^j(0))$ where c_0^i and c_0^j are the appropriate equilibrium values of time zero concession. I first show that $U_{W,0^+}^i$ is increasing in y above \hat{T} , that is, its partial derivative is positive. Notice that \hat{T}_ω and \hat{c}_ω^j are implicit functions of y here and are continuously differentiable with respect to y almost everywhere on $[\hat{T}, R_{-1}]$ and in particular are differentiable at \hat{T} . Let $\frac{\partial \hat{T}_\omega}{\partial y} = \hat{T}'_\omega$ and $\frac{\partial \hat{c}_\omega^j}{\partial y} = \hat{c}'_\omega^j$ then:

$$\frac{\partial U_{W,0^+}^i(y, \hat{c}_0^i, \hat{c}_0^j)}{\partial y} = \exp\left(-\int_0^y h_0^j(v) dv\right) \left(-e^{-r_0^i R} \int \hat{c}'_\omega^j (\alpha_\omega^i(R) + \alpha_\omega^j(R) - 1) dp \right. \\ \left. + h_0^j(y) \left(e^{-r_0^i y} \alpha_0^i(y) - e^{-r_0^i R} \int \alpha_\omega^i(R) - \hat{c}_\omega^j (\alpha_\omega^j(R) + \alpha_\omega^i(R) - 1) dp + \int_y^R e^{-r_0^i s} x_0^i ds \right) \right) \quad (21)$$

where equation 20 implies that \hat{c}'_ω^j , for states in which $\alpha_\omega^i(R) + \alpha_\omega^j(R) > 1$, is given by:

$$\hat{c}'_\omega^j = \begin{cases} \hat{c}_\omega^j (h_0^j(y) + h_\omega^j(\hat{T}_\omega) \hat{T}'_\omega) = \beta \text{ if } \frac{\hat{c}_\omega^j}{\hat{c}_0^j} < 1 \text{ or both } \frac{\hat{c}_\omega^j}{\hat{c}_0^j} = 1 \text{ and } \beta < 0 \\ 0 \text{ otherwise} \end{cases}$$

with \hat{T}'_ω given by:

$$\hat{T}'_\omega = \begin{cases} -\frac{h'_0(y)}{h'_\omega(\hat{T}_\omega)} & \text{if } \frac{\hat{c}_\omega^j}{c_\omega^j} < 1 \\ -\frac{h'_0(y)}{h'_\omega(\hat{T}_\omega)} & \text{if } \frac{\hat{c}_\omega^j}{c_\omega^j} < 1 \\ \min\left\{-\frac{h'_0(y)}{h'_\omega(\hat{T}_\omega)}, -\frac{h'_0(y)}{h'_\omega(\hat{T}_\omega)}\right\} & \text{otherwise} \end{cases}$$

Let the set of states where either $\hat{c}_\omega^j \neq 0$ or $c_\omega^j \in (0, 1)$ be labeled ϕ^j and the set of states for which $\alpha_\omega^j(R) + \alpha_\omega^i(R) > 1$ be labelled ψ (notice that if $\omega \in \phi^j$ then $\omega \in \psi$). To show that the y partial derivative of $U_{W,0^+}^i$ is positive when evaluating at $y = \hat{T}$, $\hat{c}_0^i = c_0^i$, $\hat{c}_0^j = c_0^j$, suppose not, then:

$$\begin{aligned} h_0^j(\hat{T})e^{-r_0^j \hat{T}} \alpha_0^j(\hat{T}) &\leq h_0^j(\hat{T}) \left(e^{-r_0^j R} \int \alpha_\omega^j(R) - c_\omega^j (\alpha_\omega^j(R) + \alpha_\omega^i(R) - 1) dp - \int_T^R e^{-r_0^j s} x_0^j ds \right) \\ &\quad + e^{-r_0^j R} \int \hat{c}_\omega^j (\alpha_\omega^j(R) + \alpha_\omega^i(R) - 1) dp \\ &= h_0^j(\hat{T}) \left(e^{-r_0^j R} \int \alpha_\omega^j(R) - c_\omega^j (\alpha_\omega^j(R) + \alpha_\omega^i(R) - 1) dp - \int_T^R e^{-r_0^j s} x_0^j ds \right) \\ &\quad + e^{-r_0^j R} \int (\alpha_\omega^i(R) + \alpha_\omega^j(R) - 1) \hat{c}_\omega^j \left(h_0^j(\hat{T}) - \frac{h_0^i(\hat{T}) h_\omega^j(T_\omega^*)}{h_\omega^i(T_\omega^*)} \right) \mathbb{1}_{[\omega \in \phi^j]} dp \\ &= h_0^j(\hat{T}) \left(e^{-r_0^j R} \int \alpha_\omega^j(R) - c_\omega^j \mathbb{1}_{[\omega \notin \phi^j]} (\alpha_\omega^j(R) + \alpha_\omega^i(R) - 1) dp - \int_T^R e^{-r_0^j s} x_0^j ds \right) \\ &\quad - e^{-r_0^j R} \int (\alpha_\omega^i(R) + \alpha_\omega^j(R) - 1) \hat{c}_\omega^j \frac{h_0^i(\hat{T}) h_\omega^j(T_\omega^*)}{h_\omega^i(T_\omega^*)} \mathbb{1}_{[\omega \in \phi^j]} dp \end{aligned}$$

The inequality follows from the supposition that partial derivative is (weakly) negative. The first equality substitutes in for \hat{c}_ω^j for $\omega \in \phi^j$ (elsewhere the derivative is certainly zero). The final equality is obtained by adding and subtracting $h_0^j(\hat{T})e^{-r_0^j R} \int c_\omega^j \mathbb{1}_{[\omega \in \phi^j]} (\alpha_\omega^j(R) + \alpha_\omega^i(R) - 1) dp$. The second line of the final equality is certainly negative. This implies that the first line of the final equality alone must be greater than $h_0^j(\hat{T})e^{-r_0^j \hat{T}} \alpha_0^j(\hat{T})$. Cancelling common factors and separating the states for which $\omega \notin \phi^j$ into those where $\omega \notin \psi$ and those where $\omega \in \psi \setminus \phi^j$ (for which $c_\omega^j = 1$) implies:

$$\alpha_0^j(\hat{T}) < e^{-r_0^j(R-\hat{T})} \int \alpha_\omega^j(R) - \left(\mathbb{1}_{[\omega \in \psi \setminus \phi^j]} + c_\omega^j \mathbb{1}_{[\omega \notin \psi]} \right) (\alpha_\omega^j(R) + \alpha_\omega^i(R) - 1) dp - \int_0^{R-\hat{T}} e^{-r_0^j s} x_0^j ds \quad (22)$$

For there to be an equilibrium with \hat{T} , c_0^i , c_0^j it must be that agent j (weakly) prefers to wait until after R_0 , rather than concede immediately at time \hat{T} :

$$(1 - \alpha_0^j(\hat{T})) \leq e^{-r_0^j(R-\hat{T})} \int (1 - \alpha_\omega^j(R)) + (1 - c_\omega^j) \left(\mathbb{1}_{[\omega \in \psi \setminus \phi^j]} + \mathbb{1}_{[\omega \notin \psi]} \right) (\alpha_\omega^j(R) + \alpha_\omega^i(R) - 1) dp - \int_0^{R-\hat{T}} e^{-r_0^j s} x_0^j ds$$

where this imposes the fact that $(1 - c_\omega^j) = 0$ for $\omega \in \phi^j$. Adding this inequality to equation 22 and imposing the fact of $c_\omega^i + c_\omega^j = 1$ if $\omega \notin \psi$ gives:

$$1 < e^{-\min(r_0^i, r_0^j)(R-\hat{T})} \int 1 - \hat{c}_\omega^j(\hat{T}) \mathbb{1}_{[\omega \in \psi \setminus \phi^j]} (\alpha_\omega^i(R) + \alpha_\omega^j(R) - 1) dp - \int_0^{R-\hat{T}} e^{-r_0^j s} c_0^j + e^{-r_0^i s} x_0^i ds$$

This presents a contradiction, however, as the right hand side is certainly less than 1. This contradiction implies that the partial derivative of $U_{W,0^+}^i$ is strictly positive when evaluated at \hat{T} , c_0^i , c_0^j .

Knowing this, define $\hat{c}_0^j(y)$ and $\hat{c}_0^i(y)$ on $[\hat{T}, R-2]$ to ensure $U_{W,0^+}^i(y, \hat{c}_0^i(y), \hat{c}_0^j(y)) = (1 - \alpha_0^j(0))$. This is the (unique) level of time of time zero concession that would make agent i indifferent between conceding at time 0^+ or waiting until after time R_0 to concede. It is clear that $\frac{\hat{c}_0^i(y)}{\hat{c}_0^j(y)}$ is continuous and the previous argument shows that $\frac{\hat{c}_0^i(y)}{\hat{c}_0^j(y)} > \frac{c_0^i}{c_0^j} =$

for y slightly larger than \tilde{T} . Either agent i must concede less at time zero or agent j must concede more to reduce i 's continuation value from waiting until after R_0 . For such y it must be that agent j strictly prefers to wait, $U_{W,0^+}^j(y, \hat{c}_0^j(y), \hat{c}_0^i(y)) > (1 - \alpha_0^i(0))$.

Suppose then that $\frac{\hat{c}_0^j(y)}{\hat{c}_0^i(y)}$ and $U_{W,0^+}^j(y, \hat{c}_0^j(y), \hat{c}_0^i(y))$ are strictly increasing for $y \in [\tilde{T}, t']$ for some $t' \in (\tilde{T}, R_{-1})$ but that is not true for $y \in [t', R_{-1}]$. The same sequence of arguments above implies that partial derivative of $U_{W,0^+}^i$ with respect to y is strictly positive at $y = t'$, $\hat{c}_0^i = \hat{c}_0^i(y)$, $\hat{c}_0^j = \hat{c}_0^j(y)$, which implies a contradiction. This shows that $U_{W,0^+}^j(y, \hat{c}_0^j(y), \hat{c}_0^i(y)) > (1 - \alpha_0^i(0))$ for $y \in (\tilde{T}, R_{-1}]$ which means there cannot be a second equilibrium with agents waiting on $(y, R_{-1}]$ with $y > \tilde{T}$ as one of the two agents would have strict incentives not to concede on $(0, y)$.

Because rational j cannot get more than $1 - \alpha_0^i(R)$ in the R_0 continuation game when facing a behavioral opponent ($V_{R_0}^j(\bar{z}_0^j(R_{-1}), 1) \leq 1 - \alpha_0^i(R)$), the fact that $U_{W,0^+}^j(y, \hat{c}_0^j(y), \hat{c}_0^i(y))$ is strictly increasing in y for $y \geq \tilde{T}$ implies that some rational types of agent i must always remain to concede after R_0 , and so in particular $\bar{z}^i \leq \exp\left(-\int_0^R h_0^i(s)ds\right) < 1$. Because agent i was arbitrarily selected the same arguments can be applied to her opponent j , and so $\bar{z}^j \leq \exp\left(-\int_0^R h_0^j(s)ds\right) < 1$, implying there is not an equilibrium where agents reach a probability one reputation before R_{-1} .

Finally, the second claim about agent i 's equilibrium utility increasing in \bar{z}^i and decreasing in \bar{z}^j , strictly for at least one agent, follows immediately from the continuity properties of \tilde{T}^i and \tilde{T}^j .

Proof of Lemma 4

Let ψ be the set of states for which $\alpha_\omega^1(R) + \alpha_\omega^2(R) > 1$, as in the proof of Proposition 1. Notice that if $\omega \in D^i \setminus \psi$ then necessarily $i = 2$ (if demands are compatible then agent 1 concedes first). Because it is without loss of generality to assume that neither agent concedes at R_{+1} or R_{+2} if $\alpha_\omega^1(R) + \alpha_\omega^2(R) > 1$ (see Lemma 3), by arguments in the text about the outcome of the limit of R_{+3} continuation games when $\bar{z}_n(R_{+2}) \rightarrow 0$ and $L > \frac{\bar{z}_n^1(R_{+2})}{\bar{z}_n^2(R_{+2})} > 1/L$ we have:

$$\begin{aligned} \tilde{V}_{R_0}^i &= \lim V_{R_0}^i(\bar{z}_n(R_0)) = \int \mathbb{1}_{[\omega \notin \psi]} V_{\omega, R_{+1}}^i + \mathbb{1}_{[\omega \in \psi]} \lim V_{\omega, R_{+3}}^i(\bar{z}_n(R_0)) ds \\ &= \int \mathbb{1}_{[\omega \in D^i \setminus \psi]} \alpha_\omega^i(R) + \mathbb{1}_{[\omega \notin D^i \cup \psi]} (1 - \alpha_\omega^j(R)) + \mathbb{1}_{[\omega \in D^i \cap \psi]} \alpha_\omega^i(R) + \mathbb{1}_{[\omega \in \psi \setminus D^i]} (1 - \alpha_\omega^j(R)) dp \\ &= \int \alpha_\omega^i(R) \mathbb{1}_{[\omega \in D^i]} + (1 - \alpha_\omega^j(R)) \alpha_\omega^i(R) \mathbb{1}_{[\omega \in D^j]} dp \end{aligned}$$

Proof of Lemma 5

For $\tilde{T}_W^j > \max\{\tilde{T}_W^i, 0\}$, first notice that when $\alpha_0^i(t) + \alpha_0^j(t) > 1$ for $t \leq R$ we have $\exp\left(-\int_0^R h_0^i(s)ds\right) \geq L^i > 0$ for some constant L^i . This implies $\bar{z}_n^i \exp\left(\int_0^t h_0^i(s)ds\right) \rightarrow 0$ for all $t \leq R$ and $\frac{L^i}{L^j} > \frac{\bar{z}_n^i}{\bar{z}_n^j} \exp\left(\int_0^t h_0^i(s) - h_0^j(s)ds\right) > \frac{L^i}{L^j}$. For any $\varepsilon > 0$, therefore, $\left|V_{R_0}^i\left(\bar{z}_n^i \exp\left(\int_0^t h_0^i(s)ds\right), \bar{z}_n^j \exp\left(\int_0^t h_0^j(s)ds\right)\right) - \tilde{V}_{R_0}^i\right| < \varepsilon$ for sufficiently large n for all $t \in [0, R]$. This implies that $|\tilde{T}^i(\bar{z}_n) - \max\{0, \tilde{T}_W^i\}| < \varepsilon$ for large n and so $\tilde{T}_W^j > \max\{\tilde{T}_W^i, 0\}$ then implies that $\tilde{T}^i(\bar{z}_n) > \tilde{T}^j(\bar{z}_n)$ for sufficiently large n and so agent j must make mass concession at time zero.

I claim that c_0^j converges to zero. Suppose not, then for some subsequence there is a bound with $c_0^j > L_2 > 0$ for all n and some constant L_2 . This must imply time R_0 reputations converge to zero at the same rate, and so continuation payoffs at R_0 still approach $\tilde{V}_{R_0}^i$. That is, $\left|V_{R_0}^i\left(\bar{z}_n^i \exp\left(\int_0^t h_0^i(s)ds\right), \frac{\bar{z}_n^j}{L_2} \exp\left(\int_0^t h_0^j(s)ds\right)\right) - \tilde{V}_{R_0}^i\right| < \varepsilon$ for all $t \in [0, R]$ for sufficiently large n . This, however, implies $\tilde{T}^i\left(\bar{z}_n^i, \frac{\bar{z}_n^j}{c_0^j}\right) > \tilde{T}^j\left(\frac{\bar{z}_n^j}{c_0^j}, \bar{z}_n^i\right)$ for sufficiently large n , a contradiction.

The second case of $\max\{\tilde{T}_W^j, 0\} < \tilde{T}_W^i$ is merely complimentary to the first.

In the third case of $\max\{\tilde{T}_W^i, \tilde{T}_W^j\} < 0$, again absent time zero concession $V_{R_0}^i$ converges to $\tilde{V}_{R_0}^i$. By the definition of \tilde{T}_W^i and \tilde{T}_W^j there is an equilibrium where both agents strictly prefer not to concede on $[0_{+3}, R_{-1}]$ for sufficiently large n . Remembering that the equilibrium is unique we are done.

Proof of Example 1

First consider the continuation game at time R_{+3} . Suppose that $(1 - \alpha^a) = k(1 - \alpha^b)$. If $k > 3$ then $h_\omega^a > h_\omega^b$ for all ω ; if $k \in (1, 3)$ then $h_\omega^a > h_\omega^b$ if and only if $\omega \in \{a, e\}$; if $k \in (\frac{1}{3}, 1)$ then $h_\omega^a > h_\omega^b$ if and only if $\omega = a$; if $k < \frac{1}{3}$ then $h_\omega^a < h_\omega^b$ for all ω . Remember, agent b concedes to agent a at R_{+3} in state ω with probability approaching one as the probability of behavioral types vanish (at the same rate) if and only if $h_\omega^a > h_\omega^b$. Notice that fundamentals are symmetric for agents a and b . Hence, if $(1 - \alpha^2) = m(1 - \alpha^1)$ this implies limit expected continuation payoffs $\tilde{V}_{R_0}^2 = 1 - \tilde{V}_{R_0}^1$ for the R_0 continuation game are: α^2 if $m > 3$; $\frac{1}{3}(2\alpha^2 + (1 - \alpha^1))$ if $m \in (1, 3)$; $\frac{1}{3}(\alpha^2 + 2(1 - \alpha^1))$ if $m \in (\frac{1}{3}, 1)$ and $(1 - \alpha^1)$ if $m < \frac{1}{3}$. These regions for m are labelled (E), (F), (G), and (H) respectively, and are displayed graphically on the left of Figure 3.

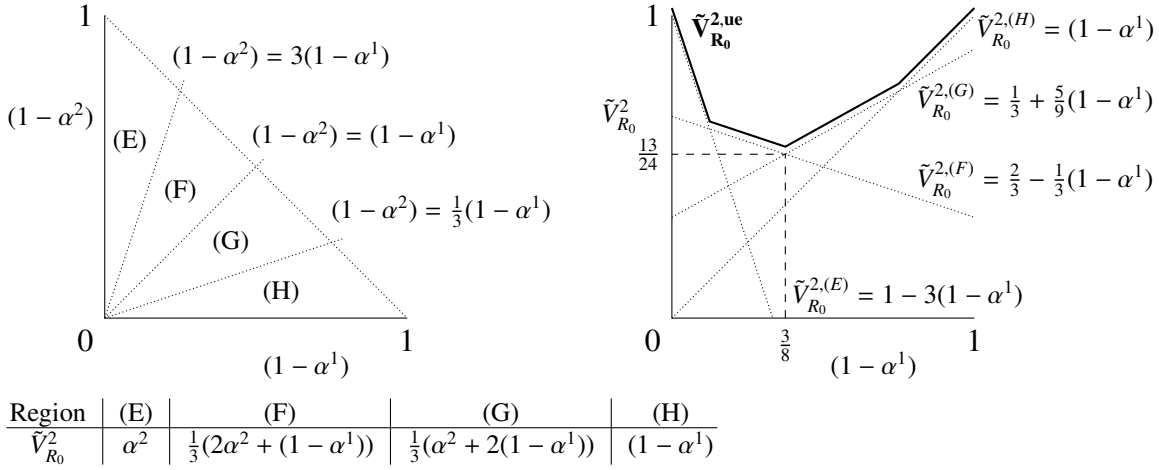


Figure 3. Left: How continuation values depends on offers. Right: Boundary continuation values.

Suppose agent 1 announced an offer $(1 - \alpha^1)$ with positive probability in the limit (taking a subsequence if necessary). Let $\tilde{V}_{R_0}^{2,(E)}$ be the limit R_0 continuation payoff for agent 2 associated with her maximum possible counter-demand, such that $(1 - \alpha^1, 1 - \alpha^2)$ is in region (E), that is, $\alpha^2 < 1 - 3(1 - \alpha^1)$. If the type space is ε rich, for ε small, then this is approximately $\tilde{V}_{R_0}^{2,(E)} = \alpha^2 = 1 - 3(1 - \alpha^1)$. Doing similarly for region (F), agent 2's approximate maximal region (F) demand is $\alpha^2 = 1 - (1 - \alpha^1)$, implying $\tilde{V}_{R_0}^{2,(F)} = \frac{2}{3} - \frac{1}{3}(1 - \alpha^1)$. For region (G) 2's approximate maximal demand is $\alpha^2 = 1 - \frac{1}{3}(1 - \alpha^1)$ which gives $\tilde{V}_{R_0}^{2,(G)} = \frac{1}{3} + \frac{5}{9}(1 - \alpha^1)$. Finally, for region (H) we always have $\tilde{V}_{R_0}^{2,(H)} = 1 - \alpha^1$. These values are illustrated by dotted lines in right hand diagram of Figure 3. The upper envelope of these values is defined by $\tilde{V}_{R_0}^{2,ue} = \max\{\tilde{V}_{R_0}^{2,(E)}, \tilde{V}_{R_0}^{2,(F)}, \tilde{V}_{R_0}^{2,(G)}, \tilde{V}_{R_0}^{2,(H)}\}$, which is marked by a bold line.

I next claim that if $(1 - \alpha^i) > (1 - \alpha^j)$ then limit waiting times satisfy $\tilde{T}_W^i < \tilde{T}_W^j$. If $(1 - \alpha^i) = k(1 - \alpha^j)$ for $k > 3$ then $\tilde{V}_{R_0}^j = (1 - \alpha^i)$ and so clearly $\tilde{T}_W^i < \tilde{T}_W^j = R$. Consider, therefore the intermediate case with $k \in (1, 3)$. For such demands let $d^i(\delta')$ and $d^j(\delta')$ be defined as:

$$d^i(\delta') = \frac{\delta'}{3}(2\alpha^i + (1 - \alpha^j)) - (1 - \alpha^j)$$

$$d^j(\delta') = \frac{\delta'}{3}(\alpha^j + 2(1 - \alpha^i)) - (1 - \alpha^i)$$

Notice that $d^i(\delta')$ is the utility difference between waiting to get the discounted payoff $\delta' \tilde{V}_{R_0}^i$ and conceding immediately to get $(1 - \alpha^j)$, for an arbitrary discount factor δ' . I claim that $d^i(\delta') > d^j(\delta')$ for $\delta' \in [0, 1]$. If this is

known then $\tilde{T}_W^i < \tilde{T}_W^j$ follows. To see this let $\delta^j = e^{-r_0^j(R-\tilde{T}_W^j)}$. By the definition of \tilde{T}_W^j we have $d^j(\delta^j) = 0$. By $d^i(\delta^i) > d^j(\delta^i)$ we therefore have $0 < d^i(\delta^i)$, and so a smaller $\delta^i < \delta^j$ (implying a smaller $\tilde{T}_W^i < \tilde{T}_W^j$) is required to make i indifferent between waiting and conceding immediately (remembering that $r_0^i = r_0^j$).

To prove that $d^i(\delta^i) > d^j(\delta^i)$ consider $d^i(\delta^i) - d^j(\delta^i)$ in the equation below:

$$d^i(\delta^i) - d^j(\delta^i) = (\alpha^j - \alpha^i) + \frac{\delta^i}{3}(4\alpha^i - 1 - 2\alpha^j)$$

Suppose $4\alpha^i - 1 - 2\alpha^j > 0$ then certainly $d^i(\delta^i) - d^j(\delta^i) > 0$ because $\alpha^j > \alpha^i$. So suppose instead $4\alpha^i - 1 - 2\alpha^j \leq 0$ then $d^i(\delta^i) - d^j(\delta^i)$ is minimized by choosing $\delta^i = 1$. That minimum is therefore achieved at $d^i(\delta^i) - d^j(\delta^i) = \frac{1}{3}(\alpha^i + \alpha^j - 1) > 0$, which proves the claim.

Suppose then that agent 1 made an offer $(1 - \alpha^1) < \frac{1}{2}$ with positive limit probability. If agent 2 were to choose some counteroffer with positive limit probability, which caused agent 1 to concede with probability approaching one, that offer must satisfy $\tilde{T}_W^1 > \max\{\tilde{T}_W^2, 0\}$. This in turn implies that either $(1 - \alpha^2) > 3(1 - \alpha^1)$ or $(1 - \alpha^2) > (1 - \alpha^1)$ and $(1 - \alpha^2) > \frac{\delta\alpha^1}{3-2\delta}$. Of the offers, which cause agent 1 to concede, agent 2 would certainly choose the smallest possible. Define, therefore, $U^{2,win} = 1 - \min\{3(1 - \alpha^1), \max\{(1 - \alpha^1), \frac{\delta\alpha^1}{3-2\delta}\}\}$ if $(1 - \alpha^1) < \frac{1}{2}$ and 0 otherwise. This is agent 2's approximate best limit payoff (in a rich type space) consistent with agent 1 conceding at time zero with probability approaching one after initially offering $(1 - \alpha^1)$ with positive limit probability.

If agent 2 does not make a counterdemand that results in agent 1 conceding immediately with probability approaching one, then either she herself must concede immediately, payoff $(1 - \alpha^1)$, or both agents wait until time R , payoff $\delta\tilde{V}_{R_0}^2$ (with 2's waiting payoff maximised at $\tilde{V}_{R_0}^{2,ue}$). And so let $\hat{U}^2 = \max\{U^{2,win}, \delta\tilde{V}_{R_0}^{2,ue}, (1 - \alpha^1)\}$. This must be approximately agent i 's payoff for large n if agent 1 offered $(1 - \alpha^1)$ with positive limit probability. Like its arguments this is a function of $(1 - \alpha^1)$. Those three arguments are plotted on the left hand side of Figure 4. This reveals that if $(1 - \alpha^1) < \frac{3-2\delta}{9-\delta}$ then $\hat{U}^2 = U^{2,win} = 1 - 3(1 - \alpha^1)$ with approximately $\alpha^2 = \hat{U}^2$. If $(1 - \alpha^1) \in (\frac{3-2\delta}{9-\delta}, \frac{3\delta}{9-5\delta})$ then $\hat{U}^2 = \delta\tilde{V}_{R_0}^{2,ue}$, there is delay with probability close to one, and if $(1 - \alpha^1) < \frac{3}{8}$ then approximately $\alpha^2 = \alpha^1$, but if $(1 - \alpha^1) > \frac{3}{8}$ then approximately $\alpha^2 = 1 - \frac{1}{3}(1 - \alpha^1)$. Finally if $(1 - \alpha^1) > \frac{3\delta}{9-5\delta}$ then $U^{2,win} = (1 - \alpha^1)$. It can be verified that $\delta > \frac{\sqrt{945}-27}{4}$ is exactly the condition needed to ensure that $U^{2,win}$ intersects $\delta\tilde{V}_{R_0}^{2,ue}$ only once.

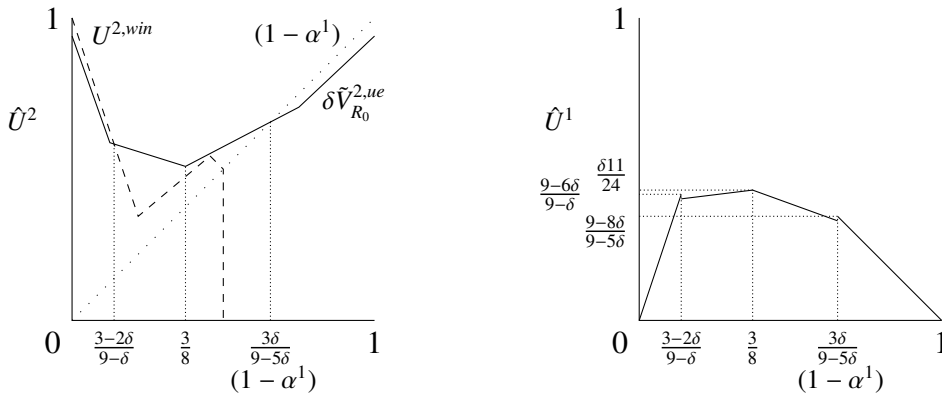


Figure 4. Left: Finding agent 2's best response. Right: Payoffs for agent 1.

Knowing this we are ready to characterize agent 1's limit demand choice. Let $\hat{U}^1 = \delta - \hat{U}^2$ for $(1 - \alpha^1) \in (\frac{3-2\delta}{9-\delta}, \frac{3\delta}{9-5\delta})$ and $\hat{U}^1 = 1 - \hat{U}^2$ elsewhere. This represents agent 1's approximate payoff for large n if she imitated a type offering $(1 - \alpha^1)$ with positive limit probability, after incorporating the best response of agent 2. This is plotted on the right hand side of Figure 4.

If $(1 - \alpha^1) < \frac{3-2\delta}{9-\delta}$ then $\hat{U}^1 = 3(1 - \alpha^1)$, which reaches a maximum of almost $\frac{9-6\delta}{9-\delta}$. If $(1 - \alpha^1) \in (\frac{3-2\delta}{9-\delta}, \frac{3}{8})$ then

$\hat{U}^1 = \frac{\delta}{3}(1 + (1 - \alpha^1))$, which achieves a maximum of almost $\frac{\delta 11}{24}$. If $(1 - \alpha^1) \in (\frac{3}{8}, \frac{3\delta}{9-5\delta})$ then $\hat{U}^1 = \frac{\delta}{9}(6 - 5(1 - \alpha^1))$, which a maximum of almost $\frac{\delta 11}{24}$. If $(1 - \alpha^1) > \frac{3\delta}{9-5\delta}$ then $\hat{U}^1 = \alpha^1$, which achieves a maximum of almost $\frac{9-8\delta}{9-5\delta}$.

Notice that \hat{U}^1 jumps down at $(1 - \alpha^1) = \frac{3-2\delta}{9-\delta}$ and jumps up at $(1 - \alpha^1) = \frac{3\delta}{9-5\delta}$ depending on whether there is delay or not. Nonetheless, is readily verified that $\frac{\delta 11}{24} > \max\{\frac{9-6\delta}{9-\delta}, \frac{9-8\delta}{9-5\delta}\}$ for $\delta > \frac{\sqrt{945}-27}{4}$. This shows that the only demands which agent 1 will make with positive probability in the limit are those sufficiently close to $\alpha^1 = \frac{5}{8}$. This will cause agent 2 to only make counterdemands with positive limit probability of approximately $\alpha^2 = \frac{5}{8}$ or $\alpha^2 = \frac{7}{8}$. In either case there is delay until time R with probability approaching one. This in turn implies that for large n equilibrium payoffs must be approximately $U^1(\sigma) = \delta \frac{11}{24}$ and $U^2(\sigma) = \delta \frac{13}{24}$.

Finally, I argue that well defined limit payoffs exist. Notice first that $\lim(U^1(\sigma) + U^2(\sigma)) = \delta$ as for all demands imitated with positive limit probability delay up to R is certain but agreement at R is also certain (as demands are generic). Second, suppose there were two subsequences $k(n)$ and $l(n)$ which led to different limit payoffs: $\lim U^1_{k(n)} > \lim U^1_{l(n)}$ and take any of agent 1's strategies imitated with positive probability in the limit of $k(n)$. If agent 1 imitated this demand with probability one in the $l(n)$ subsequence then her limit payoff would be at least $\lim U^1_{k(n)}$, because limit payoffs in a R_{+3} continuation game reached with positive limit probability do not depend on the exact ratio of agents' reputations.

Proof of Example 2

The proof of this example is very similar to that of Example 1 and I refer to arguments presented there. As there, fundamentals are symmetric for agents a and b , and so if $(1 - \alpha^2) = m(1 - \alpha^1)$ then agent 2's limit expected continuation payoff, $\tilde{V}_{R_0}^2$, is α^2 if $m > 3$; $\frac{1}{2}(\alpha^2 + (1 - \alpha^1))$ if $m \in (\frac{1}{3}, 3)$; and $(1 - \alpha^1)$ if $m < \frac{1}{3}$. These regions for m are labelled (E), (F) and (G) respectively, and are displayed graphically on the left of Figure 5.

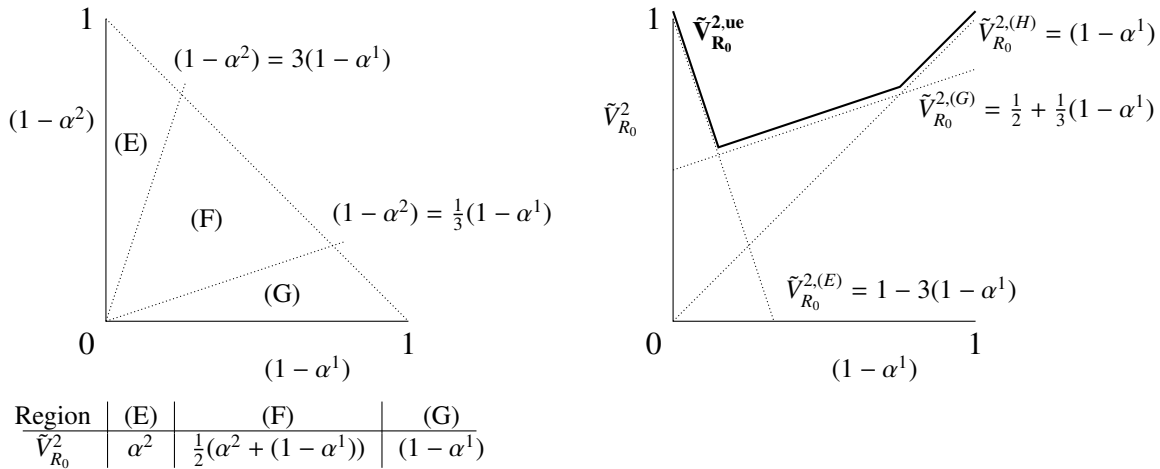


Figure 5. Left: How continuation values depends on offers. Right: Boundary continuation values.

Suppose agent 1 announced an offer $(1 - \alpha^1)$ with positive probability in the limit (considering a subsequence if necessary). Let $\tilde{V}_{R_0}^{2,(K)}$ be the limit continuation payoff for agent 2 associated with her maximum possible counterdemand, such that $(1 - \alpha^1, 1 - \alpha^2)$ is in region (K). This implies $\tilde{V}_{R_0}^{2,(E)} = 1 - 3(1 - \alpha^1)$, $\tilde{V}_{R_0}^{2,(F)} = \frac{1}{2} + \frac{1}{3}(1 - \alpha^1)$ and $\tilde{V}_{R_0}^{2,(G)} = 1 - \alpha^1$. These values are illustrated by dotted lines in right hand diagram of Figure 5. The upper envelope of these values, $\tilde{V}_{R_0}^{2,ue} = \max\{\tilde{V}_{R_0}^{2,(E)}, \tilde{V}_{R_0}^{2,(F)}, \tilde{V}_{R_0}^{2,(G)}\}$, is marked by a bold line.

I next show that if $(1 - \alpha^i) > (1 - \alpha^j)$ then limit waiting times satisfy $\tilde{T}_W^i < \tilde{T}_W^j$. If $(1 - \alpha^i) = k(1 - \alpha^j)$ for $k > 3$ then $\tilde{V}_{R_0}^i = (1 - \alpha^i)$ and so clearly $\tilde{T}_W^i < \tilde{T}_W^j = R$. Consider, therefore the intermediate case with $k \in (1, 3)$. For

such demands let $d^l(\delta')$ and be defined as:

$$d^l(\delta') = \frac{\delta'}{2}(\alpha^l + (1 - \alpha^m)) - (1 - \alpha^m)$$

For agent l facing opponent m , $d^l(\delta')$ is the utility difference between waiting to get the discounted payoff $\delta' \tilde{V}_{R_0}^l$ and conceding immediately to get $(1 - \alpha^m)$, for an arbitrary discount factor δ' . It is easy to see that $d^i(\delta') > d^j(\delta')$ for $\delta' \in [0, 1)$ because $d^i(\delta') - d^j(\delta') = (\alpha^j - \alpha^i)(1 - \delta')$ and by assumption $\alpha^j > \alpha^i$. Knowing this, $\tilde{T}_W^i < \tilde{T}_W^j$ follows by the same argument as in Example 1 (define $\delta^j = e^{-r_0^j(R - \tilde{T}_W^j)}$, then $d^j(\delta^j) = 0 < d^i(\delta^j)$).

Suppose agent 1 made an offer $(1 - \alpha^1) < \frac{1}{2}$ with positive limit probability. If agent 2 were to choose some counteroffer with positive limit probability, which caused agent 1 to concede with probability approaching one, that offer must satisfy $\tilde{T}_W^1 > \max\{\tilde{T}_W^2, 0\}$. This in turn implies that either $(1 - \alpha^2) > 3(1 - \alpha^1)$ or $(1 - \alpha^2) > (1 - \alpha^1)$ and $(1 - \alpha^2) > \frac{\delta\alpha^1}{2-\delta}$. Define, therefore, $U^{2,win} = 1 - \min\{3(1 - \alpha^1), \max\{(1 - \alpha^1), \frac{\delta\alpha^1}{2-\delta}\}\}$ if $(1 - \alpha^1) < \frac{1}{2}$ and 0 otherwise. As before, this is agent 2's approximate best payoff (for a rich type space) consistent with agent 1 conceding at time zero with probability approaching one after initially offering $(1 - \alpha^1)$ with positive limit probability.

If agent 2 does not make a counterdemand that results in agent 1 conceding immediately with probability approaching one, then either she herself must concede immediately, payoff $(1 - \alpha^1)$, or both agents wait until time R , payoff $\delta \tilde{V}_{R_0}^2$ (with 2's waiting payoff maximised at $\tilde{V}_{R_0}^{2,ue}$). Let $\hat{U}^2 = \max\{U^{2,win}, \delta \tilde{V}_{R_0}^{2,ue}, (1 - \alpha^1)\}$. As before, this must be approximately agent i 's payoff for large n if agent 1 were to offer $(1 - \alpha^1)$ with positive limit probability. The three arguments of this function are plotted on the left hand side of Figure 6. This reveals that if $(1 - \alpha^1) < \frac{3-2\delta}{9-\delta}$ then $\hat{U}^2 = U^{2,win} = \alpha^2 = 1 - 3(1 - \alpha^1)$. If $(1 - \alpha^1) \in (\frac{3-2\delta}{9-\delta}, \frac{3\delta}{9-5\delta})$ then $\hat{U}^2 = \delta \tilde{V}_{R_0}^{2,ue}$, where if $(1 - \alpha^1) < \frac{3}{8}$ then approximately $\alpha^2 = \alpha^1$ and if $(1 - \alpha^1) > \frac{3}{8}$ then approximately $\alpha^2 = 1 - \frac{1}{3}(1 - \alpha^1)$. Finally, if $(1 - \alpha^1) > \frac{3\delta}{9-5\delta}$ then $\hat{U}^2 = (1 - \alpha^1)$. It can be verified that $\delta > \sqrt{15} - 3$ is exactly the condition needed to ensure that $U^{2,win}$ intersects $\delta \tilde{V}_{R_0}^{2,ue}$ only once.

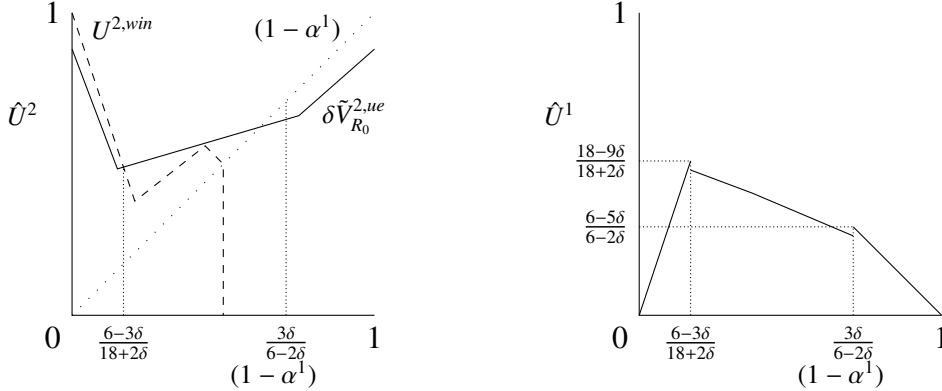


Figure 6. Left: Finding agent 2's best response. Right: Payoffs for agent 1.

Knowing this we are ready to characterize agent 1's choice. Let $\hat{U}^1 = \delta - \hat{U}^2$ for $(1 - \alpha^1) \in (\frac{6-3\delta}{18+2\delta}, \frac{3\delta}{6-2\delta})$ and $\hat{U}^1 = 1 - \hat{U}^2$ elsewhere. This is agent 1's approximate payoff for large n if she imitated a type offering $(1 - \alpha^1)$ with positive limit probability, after incorporating the best response of agent 2. This is plotted on the right hand side of Figure 6.

If $(1 - \alpha^1) < \frac{3-2\delta}{9-\delta}$ then $\hat{U}^1 = 3(1 - \alpha^1)$, which reaches a maximum of almost $\frac{18-9\delta}{18+2\delta}$. If $(1 - \alpha^1) \in (\frac{6-3\delta}{18+2\delta}, \frac{3\delta}{6-2\delta})$ then $\hat{U}^1 = \frac{\delta}{6}(3 - 2(1 - \alpha^1))$, which achieves a maximum of almost $\frac{18-9\delta}{18+2\delta} - (1 - \delta)$. Finally if $(1 - \alpha^1) > \frac{3\delta}{6-2\delta}$ then $\hat{U}^1 = \alpha^1$ which achieves a maximum of almost $\frac{6-5\delta}{6-2\delta}$.

It is readily verified that \hat{U}^1 achieves its global maximum of $\frac{18-9\delta}{18+2\delta}$ for an offer slightly smaller than $(1 - \alpha^1) =$

$\frac{6-3\delta}{18+2\delta}$. This shows that the only demands which agent 1 will make with positive probability in the limit are those sufficiently close to $\alpha^1 = \frac{12+5\delta}{18+2\delta}$, to which agent 2 will counterdemand approximately $\frac{11\delta}{18+2\delta}$. Agent 1 will then accept immediately with probability approaching one. This in turn implies that for large n equilibrium payoffs must be approximately $U^1(\sigma) = \frac{18-9\delta}{18+2\delta}$ and $U^2(\sigma) = \frac{11\delta}{18+2\delta}$. Payoffs reach a well defined limit for the same reasons as in Example 1.

Proof of Proposition 5

Consider the situation of agent 2. If agent 1 makes the demand $\alpha_0^1(0) \leq (1 - \alpha_0^{2*}(0))$ then agent 2 may obtain the payoff bound by accepting that demand. Consider then the possibility that agent 1 makes the demand $\alpha_0^1(0) > (1 - \alpha_0^{2*}(0))$ with positive probability in the limit (considering a subsequence if necessary), but agent 2 imitates the demand α^{2*} with probability one. This implies that reputations after demand choices satisfy $\bar{z}_n \rightarrow 0$ and $L_2 \geq \frac{1}{\bar{z}_n}$ for large n and for some positive constant L_2 .

Recall the definition of \check{T}_ω in equation 6 of the text. I first consider the case in which $\check{T}_\omega \leq R_{-1}$. This implies that both agents must reach a probability one reputation at $\check{T} < \check{T}_\omega$ but also $\check{T} \rightarrow \check{T}_\omega$ (because agent i 's initial exhaustion time is defined by $\exp\left(-\int_0^{\check{T}_E^i} h_0^i(s) ds\right) = \bar{z}^i$, and so $\bar{z}_n^i \rightarrow 0$ implies $\check{T}_E^i \rightarrow \check{T}_\omega$). Define the difference between agents' bargaining positions to be $l_0(t) = \alpha_0^1(t) + \alpha_0^2(t) - 1$ and so $(1 - \alpha_0^1(t)) = \alpha_0^2(t) - l_0(t)$ and $\alpha_0^1(t) = l_0'(t) - \alpha_0^2(t)$. Equilibrium requires:

$$\begin{aligned} \frac{c_0^1}{c_0^2} &= \frac{\bar{z}^1}{\bar{z}^2} \exp\left(\int_0^{\check{T}} h_0^1(s) - h_0^2(s) ds\right) \\ &= \frac{\bar{z}^1}{\bar{z}^2} \exp\left(-\int_0^{\check{T}} \frac{(1 - \alpha_0^{2*}(s))r_0^1 + \alpha_0^{2*}(s) + x_0^1 - (1 - \alpha_0^1(s))r_0^2 - \alpha_0^1(s) - x_0^2 ds}{\alpha_0^1(s) + \alpha_0^2(s) - 1}\right) \\ &= \frac{\bar{z}^1}{\bar{z}^2} \exp\left(-\int_0^{\check{T}} \frac{2\alpha_0^{2*}(s) - \alpha_0^{2*}(s)(r_0^1 + r_0^2) + r_0^1 + x_0^1 - x_0^2}{l_0(s)} - \frac{l_0^1(s) ds}{l_0(s)} + r_0^2 ds\right) \\ &\leq \frac{\bar{z}^1}{\bar{z}^2} \exp\left(-\int_0^{\check{T}} -\frac{l_0^1(s) ds}{l_0(s)} + r_0^2 ds\right) \\ &\leq L_2 \frac{e^{-r_0^2 \check{T}} l_0(\check{T})}{l_0(0)} \end{aligned}$$

The second line follows by first expressing hazard rates in terms of primitives. The third line simply substitutes expressions involving $\alpha_0^1(t)$ for those involving $l_0(t)$ and $\alpha_0^2(t)$. The fourth line follows because either $\alpha_0^{2*}(s) = \hat{\alpha}_0^{2*}(s)$, as defined by equations 17 which means the cancellation is exact, or $\alpha_0^{2*} = 1$, which as argued in the text, implies $r_0^2 + x_0^2 - x_0^1 < 0 = \alpha^{2*}(t)$. Notice that if $\alpha_0^{2*}(s) = 0$ and $r_0^1 + x_0^1 - x_0^2 < 0$ then $\alpha_0^{2*}(0) = 0$, which contradicts $\alpha^1(0) > 1 - \alpha_0^{2*}(0)$. The fifth line then follows from integration as well as using the bound $\frac{\bar{z}^1}{\bar{z}^2} < L_2$. Because $\check{T} \rightarrow \check{T}_\omega$ as $\bar{z} \rightarrow 0$ and $l_0(\check{T}_\omega) = 0$, this fifth line then ensures that $c_0^1 \rightarrow 0$.

Moving onto the case of $\alpha_0^j(t) + \alpha_0^i(t) > 1$ for $t \leq R_{-1}$. Consider the continuation game at R_{+3} in state ω if $\alpha_\omega^j(R) + \alpha_\omega^i(R) > 1$, $\bar{z}_{n,\omega}^i(R_{+2}) \rightarrow 0$ and $L_3 > \frac{\bar{z}_{n,\omega}^i(R_{+2})}{\bar{z}_{n,\omega}^j(R_{+2})} > \frac{1}{L_3}$ for some positive constant L_3 . Exhaustion times now imply that $T_\omega^* \rightarrow \check{T}_\omega$.

Define the difference between agents' positions as $l_\omega(t) = \alpha_\omega^i(t) + \alpha_\omega^j(t) - 1$. Using exactly the same steps as

previously we have:

$$\begin{aligned} \frac{c_{\omega,R_{+3}}^1}{c_{\omega,R_{+3}}^2} &= \frac{\bar{z}_{\omega}^1(R_{+2})}{\bar{z}_{\omega}^2(R_{+2})} \exp\left(-\int_R^{T_{\omega}^*} \frac{2\alpha_{\omega}^{2*}(s) - \alpha_{\omega}^{2*}(s)(r_{\omega}^1 + r_{\omega}^2) + r_{\omega}^1 + x_{\omega}^1 - x_{\omega}^2}{l_{\omega}(s)} - \frac{l_{\omega}^1(s)ds}{l_{\omega}(s)} + r_{\omega}^2 ds\right) \\ &\leq \frac{\bar{z}_{\omega}^1(R_{+2})}{\bar{z}_{\omega}^2(R_{+2})} \exp\left(-\int_R^{T_{\omega}^*} -\frac{l_{\omega}^1(s)ds}{l_{\omega}(s)} + r_{\omega}^2 ds\right) \\ &\leq L_3 \frac{e^{-r_{\omega}^2(T_{\omega}^* - R)} l_{\omega}(T_{\omega}^*)}{l_{\omega}(R)} \end{aligned}$$

where the first inequality follows from the fact that if $\alpha_{\omega}^{2*} > 0$ then $2\alpha_{\omega}^{2*}(t) = 0 \geq \alpha_{\omega}^{2*}(r_{\omega}^1 + r_{\omega}^2) - r_{\omega}^1 - x_{\omega}^1 + x_{\omega}^2$ and the second from integration and the bound $\frac{\bar{z}_{\omega}^1(R_{+2})}{\bar{z}_{\omega}^2(R_{+2})} \leq L_3$. The final line must converge to zero as $T_{\omega}^* \rightarrow \check{T}_{\omega}$ because $l_{\omega}(T_{\omega}^*) \rightarrow 0$ if $\check{T}_{\omega} < \infty$ and $e^{-r_{\omega}^2 T_{\omega}^*} \rightarrow 0$ otherwise. This ensures that $c_{\omega}^1(R_{+3}) \rightarrow 0$.

If agent 1 must concede with probability approaching one in every state for which $\alpha_{\omega}^i(t) + \alpha_{\omega}^j(t) > 1$, then limit continuation payoffs as defined by Lemma 4 are $\tilde{V}_{R_0}^2 \geq \alpha_0^{2*}(R)$ and $\tilde{V}_{R_0}^1 \leq (1 - \alpha_0^{2*}(R))$. This implies that $\tilde{T}_W^2 < R = \tilde{T}_W^1$. Finally, by invoking Lemma 5 agent 2 can obtain a limiting payoff of at least $\alpha_0^{2*}(0)$ given that $\frac{\bar{z}_b^1}{\bar{z}_b^2} < L_3$ and $\bar{z}_b \rightarrow 0$.

This holds for any demands made with positive probability by agent 1 and because there are only a finite set of behavioral types this proves the Proposition for agent 2. By entirely similar logic, agent 1 can secure the limiting payoff $\alpha_0^{1*}(0)$ by initially imitating the Rubinstein type α^{1*} with probability one.

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