

Self-Fulfilling Currency Crises: The Role of Interest Rates

By CHRISTIAN HELLWIG, ARIJIT MUKHERJI, AND ALEH TSYVINSKI*

We develop a model of currency crises, in which traders are heterogeneously informed, and interest rates are endogenously determined in a noisy rational expectations equilibrium. In our model, multiple equilibria result from distinct roles an interest rate plays in determining domestic asset market allocations and the devaluation outcome. Except for special cases, this finding is not affected by the introduction of noisy private signals. We conclude that the global games results on equilibrium uniqueness do not apply to market-based models of currency crises. (JEL D84, E43, F32)

It is a commonly held view that financial crises, such as currency crises, bank runs, debt crises, or asset price crashes, may be the result of self-fulfilling expectations and multiple equilibria. For currency crises, this view is formulated in two broad classes of models. The first, pioneered by Maurice Obstfeld (1986), views a devaluation as the outcome of a run on the central bank's stock of foreign reserves; in the second class of models, starting with Obstfeld (1996), devaluations are the result of the central bank's inability or unwillingness to sustain the political or economic costs associated with high interest rates. In both types of environments, domestic asset markets play a central role. In the market equilibrium, an uncovered interest

parity equates the interest premium on domestic assets to the expected currency depreciation. Multiple equilibria arise when there are multiple values for the domestic interest rate premium that are consistent with uncovered interest parity. In one equilibrium, the interest rate tends to be low, reserve losses are small, and there is a low likelihood of a devaluation. In another equilibrium, interest rates are high, reserve losses are large, and there is a high chance of a devaluation. The sudden shifts in financial markets that characterize currency crises are then interpreted as a shift from one equilibrium to another.¹

Building on game-theoretic selection results of Hans Carlsson and Eric E. van Damme (1993), this multiplicity view of crises has recently been challenged by Stephen Morris and Hyun Song Shin (1998), who argue that multiplicity is the consequence of assuming that fundamentals are common knowledge among market participants. Instead, they consider a stylized currency crises model, in which traders observe the relevant fundamentals with small idiosyncratic noise, and show that this leads to the selection of a unique equilibrium.

While their analysis highlights the critical role of the information structure for coordination

* Hellwig: Department of Economics, University of California, Los Angeles, CA 90095 (e-mail: chris@econ.ucla.edu); Mukherji: University of Minnesota; Tsyvinski: Department of Economics, Harvard University, Cambridge, MA 02138 (e-mail: tsyvinski@harvard.edu). This paper originated from an earlier paper written by the late Arijit Mukherji and Aleh Tsyvinski. We thank two anonymous referees for excellent comments and suggestions. We further thank Marios Angeletos, V. V. Chari, Pierre-Olivier Gourinchas, Patrick Kehoe, Stephen Morris, Iván Werning, and especially Andy Atkeson for useful discussions, and seminar audiences at UC-Berkeley, Boston College, Boston University, Cambridge University, Harvard University, London School of Economics, Northwestern University Kellogg School of Management, Penn State, UC-Santa Cruz, Stanford University, UCLA, the Wharton School of the University of Pennsylvania, the Federal Reserve Bank of Minneapolis, the International Monetary Fund, the 1st Christmas Meetings of German Economists Abroad, the 2005 SED meetings, and the 2005 World Congress of the Econometric Society for their comments.

¹ For related analyses of bank runs, debt crises, or asset price crashes, see Douglas W. Diamond and Philip H. Dybvig (1983), Guillermo A. Calvo (1988), Gerard Gennette and Hayne E. Leland (1990), Harold L. Cole and Timothy J. Kehoe (2000), Gadi Barlevy and Pietro Veronesi (2003), and many others.

and multiplicity of equilibria, they also need to assume that the domestic interest rate premium is not market-determined, but exogenously fixed. A currency crisis is then viewed as a coordinated run on foreign reserves, when the value of the domestic currency is out of line with fundamentals. Their model, thus, not only introduces a lack of common knowledge, but also abstracts from an explicit model of domestic asset markets. Moreover, it abstracts from the interest parity condition that appears to be so central to many of the original multiple equilibrium models.

In this paper, we propose a new model of currency crises that allows for informational differences among traders, while at the same time accounting for the market forces of the original models. We use this model to reexamine the respective roles of the information structure and the market characteristics in determining uniqueness versus multiplicity; to do so, we compare the solution of our model when fundamentals are common knowledge with the solution when traders have idiosyncratic, noisy signals. As our main result, we show that when domestic asset markets and interest rates are modeled explicitly, arguments for multiplicity remain valid even in the presence of incomplete, heterogeneous information. We conclude from this that the simple global coordination game studied by Morris and Shin does not fully capture the complex market interactions by which currency crises are characterized.

Specifically, we consider a stylized model of a country's domestic bond market with heterogeneously informed traders, who may either invest domestically or withdraw their funds and invest in dollars. The interest rate on domestic bonds is endogenously determined in a noisy rational expectations equilibrium along the lines of Sanford J. Grossman (1977), Grossman and Joseph E. Stiglitz (1980), and Martin F. Hellwig (1980), with a shock to the domestic bond supply preventing the interest rate from perfectly revealing the state. Our model captures three key features of domestic interest rates that are absent from Morris and Shin's analysis. First, the domestic interest rate responds to the conditions in the domestic bond market, so that, in equilibrium, a

smaller supply of domestic bonds leads to a higher equilibrium interest rate, and a larger loss of foreign reserves by the central bank.² Second, the domestic interest rate may influence the devaluation outcome: a devaluation of the domestic currency may occur either because of large foreign reserve losses, or because of increases in the domestic interest rate (or a combination of both); our model thus embeds both Obstfeld (1986) and Obstfeld (1996) as special cases. Finally, with heterogeneous beliefs, the domestic interest rate serves as an endogenous public signal which aggregates private information.

Our equilibrium characterization augments the optimality conditions for trading strategies and the devaluation outcome by a market-clearing condition that determines interest rates as a function of the underlying fundamentals and supply shocks. Combining these conditions, we arrive at a private information version of the uncovered interest parity condition that determined equilibrium outcomes in the original multiple equilibrium models with common knowledge.

In our model, optimal trading strategies are always uniquely determined. Unlike in Morris and Shin (1998), multiplicity therefore does not originate from a coordination problem; instead, it arises if there are multiple market-clearing interest rates. If traders are sufficiently well informed, an increase in the interest rate may raise the expected devaluation premium by more than the domestic bond return. This im-

² This reflects the idea that an interest rate increase reduces the amount of borrowing in domestic currency and leads to a capital outflow. Although we do not attempt to formally model this positive correlation between domestic interest rates and capital outflows, different motivations may be provided in the context of currency crises models. In Obstfeld (1986), it results from inflationary expectations following a devaluation. In the presence of nominal rigidities, our assumption may also be motivated by real investment behavior and financial constraints, implying that domestic firms are less willing or able to borrow at higher interest rates; this view is put forth, for example, by the literature on "sudden stops" (cf. Calvo, 1998), or in many business cycle models of emerging market economies (see Pablo Andrés Neumeyer and Fabrizio Perri, 2005). Neumeyer and Perri further document a positive correlation between net capital flows and domestic interest rates as a pervasive feature of business cycles in emerging market economies.

plies that optimal trading strategies are non-monotone and lead to a backward-bending demand for domestic bonds. For some realizations of fundamentals and shocks, this generates multiple market-clearing interest rates, and thus multiple equilibria. In one equilibrium, a high interest rate is associated with large reserve losses and a high likelihood of devaluation. Another equilibrium leads to a low interest rate and small reserve losses, and a devaluation becomes unlikely. To complete the description of our results, we sketch the forces behind such nonmonotone asset demands in our two leading cases, when devaluations are triggered, respectively, by high interest rates or by large reserve losses.

When devaluations are triggered solely by high interest rates as in Obstfeld (1996), this nonmonotonicity is immediate and applies identically to the common knowledge and private information versions of our model. Conditional on the interest rate, traders have no reason to coordinate trading strategies and form a conjecture of how the others are likely to trade—their private signal about fundamentals, together with the interest rate, already tell them all they need to know to infer the devaluation outcome. By increasing the cost of sustaining a fixed exchange rate, an increase in the interest rate expands the set of fundamentals for which a devaluation occurs. If the resulting increase in the expected devaluation probability exceeds the corresponding increase in the domestic bond return, the traders will at some point switch their asset holdings from domestic bonds to dollars in response to a small interest rate increase, because they now anticipate that a devaluation is more likely to occur.

When, instead, a devaluation is triggered by reserve losses, as in Obstfeld (1986), traders do have a coordination motive, since they need to forecast the reserve losses (and hence the other traders' strategies) to forecast the devaluation outcome. Remarkably, the domestic interest rate resolves this coordination problem so that equilibrium trading strategies are uniquely determined. In equilibrium, the domestic interest rate is positively correlated with reserve losses, which allows traders to infer the likely devaluation outcome. If the domestic bond supply is sufficiently responsive to variations in the inter-

est rate, and supply shocks are not too big, this correlation is high enough so that the inferred increase in reserve losses and the devaluation probability exceeds the corresponding increase in domestic returns. As a result, the demand for domestic bonds is backward bending and there are multiple market-clearing interest rates. In contrast, when the domestic bond supply is inelastic and/or shocks are large, this correlation is low, and the inference drawn from the interest rate is so weak that trading strategies are monotone and there is a unique equilibrium.

Why does the introduction of noisy private signals have such dramatic effects on the equilibrium set in Morris and Shin (1998), but little to no effect in our model? Private information does not affect uniqueness versus multiplicity in our model, because multiplicity does not originate from a coordination problem. The relevant source of multiplicity is the nonmonotonicity of optimal trading strategies, which is not affected by the introduction of noisy private signals. When the devaluation outcome depends only on the interest rate, there is no coordination motive in strategies, and it is perhaps not surprising that introducing noisy private signals does not affect the equilibrium set. When, instead, the devaluation outcome depends on reserve losses, there is a coordination motive in trading strategies, but it is not directly relevant for multiplicity, since it is resolved by the interest rate, which endogenously gives the traders a signal about reserve losses.

In contrast, Morris and Shin (1998) represents a special case of our reserve loss model, in which the domestic bond supply is infinitely elastic at a fixed, exogenous interest rate, so that, by design, the interest rate remains completely uninformative. This is the only special case, where observing the interest rate cannot resolve the coordination motive among traders, in which case the change in the information structure from common knowledge to private information has dramatic effects for the equilibrium set. Moreover, the limiting case with infinitely elastic supply at a fixed interest rate is quite different from the case where supply is highly, but not infinitely, elastic, in which case small movements in the interest rate remain informative of the equilibrium loss of foreign reserves.

Related Literature.—Following the original papers of Carlsson and van Damme (1993) and Morris and Shin (1998), several papers have studied the effects of exogenous public information in global coordination games (Christina E. Metz, 2002; Morris and Shin, 2003, 2004) and have shown that this may restore multiplicity under fairly general conditions (Christian Hellwig, 2002). We build on these insights, but depart from their general setup by considering more explicit market structures and modeling the public information endogenously as an interest rate signal. Our paper is related also to George-Marios Angeletos et al. (forthcoming, 2006), who consider the informational effects of costly policy measures (Angeletos et al., 2006) or equilibrium dynamics (Angeletos et al., forthcoming).

Andrew G. Atkeson (2001) discusses the potential problems that the lack of a theory of prices poses for global coordination games.³ Closely related to our paper, Angeletos and Iván Werning (2006) consider a version of Morris and Shin's currency crises game with endogenous information aggregation through the price of a "derivative" asset, or through noisy public signals of aggregate activity; in their model, prices affect the coordination outcome only through the information they provide. They show that equilibrium multiplicity may be restored by the endogenous public signal, provided that private information is sufficiently precise. In this environment, they are the first to show that multiplicity emerges from the equilibrium price function, not from individual actions which are uniquely determined. In our model, the multiplicity occurs within a primary market, in which the interest rate not only aggregates private information, but also has direct effects on the traders' payoffs. This highlights the role of interest rates in determining the ultimate devaluation outcome either directly or indirectly, as it does in the original multiple equilibrium models of currency crises.

Nikola A. Tarashev (2003) analyzes a version of Morris and Shin's currency crises game

with endogenous interest rate determination in a noisy rational expectations equilibrium, in which he concludes that there exists a unique equilibrium. His result appears as a special case of our model, in which the domestic bond supply is inelastic, the impact of the interest rate for the devaluation outcome is small, and supply shocks are large enough to rule out multiple equilibria.

Finally, the possibility of multiple equilibria due to nonmonotone asset demand and supply schedules has also been noted in traditional REE asset pricing models in which coordination problems are absent, such as Genotte and Leland's (1990) analysis of market crashes. In Barlevy and Veronesi (2003), multiplicity arises from the interaction between informed and uninformed traders.

I. Model Description

A. Players, Actions, and Payoffs

We consider an economy populated by a measure one of risk-neutral traders, indexed by $i \in [0, 1]$, and a central bank (CB). Initially, each agent is endowed with one unit of domestic currency. Traders can invest their endowment either in a domestic bond, or they can go to the central bank and exchange the domestic currency one for one for a dollar. The investment in the domestic bond yields a safe market-determined net interest rate r . The return to exchanging the domestic currency for a dollar is determined by whether a devaluation occurs. If there is no devaluation, and the dollar is converted back into domestic currency at the same level, its net return is zero. If, however, the CB decides to abandon the fixed exchange rate, the exchange rate drops to two units of domestic currency for the dollar, and the net return on the dollar is one. These investment returns are summarized in the following table:

	Devaluation	No devaluation
Dollar	1	0
Domestic bond	r	r

³ V. V. Chari and Patrick J. Kehoe (2004) use a noisy rational expectations equilibrium approach to introduce prices in herding models.

B. Devaluation Decision

The central bank's decision to devalue the domestic currency depends on the market-determined domestic interest rate r ; its loss of foreign reserves $A \in [0, 1]$, which measures the total of dollars withdrawn by traders; and an unobserved fundamental θ , which measures the strength of the CB's commitment to maintain a fixed exchange rate. The net value of maintaining the fixed exchange rate is given by $\theta - C(r, A)$, and the central bank will devalue, if and only if

$$(1) \quad \theta \leq C(r, A).$$

The fundamental θ may be interpreted as the value of the peg in the absence of any reserve losses or interest rate increases, and $C(r, A)$ measures the cost of having to defend the exchange rate in the event of high interest rates, or losses of foreign reserves. For our analysis, we will focus on the two polar cases that represent the canonical models of currency crises: $C(r, A) = r$ allows for a scenario, in which the CB is concerned exclusively with high domestic interest rates, such as in Obstfeld (1996). On the other hand, $C(r, A) = A$ represents the case in which a devaluation is purely determined by the CB's loss of foreign reserves. This corresponds to the modeling assumptions in Paul R. Krugman (1979), Robert P. Flood and Peter M. Garber (1984), or Obstfeld (1986).⁴

C. Information Structure and Timing

The currency crisis game has three stages. In stage 1, nature selects $\theta \in \mathbb{R}$ from an improper uniform distribution over the entire real line.⁵ Then, each trader observes an idiosyncratic, private signal about θ , denoted x_i . Conditional on θ , private signals are independent, and identically distributed according to a normal distribution, $x_i \sim \mathcal{N}(\theta, \beta^{-1})$. Let $\Phi(\cdot)$ and $\phi(\cdot)$ denote, respectively, the cdf and pdf of a standard nor-

mal distribution. Then, the cdf of the private signal distribution is given by $\Phi(\sqrt{\beta}(x - \theta))$. We further assume that the Law of Large Numbers applies to the cross-sectional distribution of private signals, so that with probability 1, $\Phi(\sqrt{\beta}(x - \theta))$ also equals the fraction of traders who observe a signal $x_i \leq x$, when the realized fundamental is θ .

In stage 2, the domestic bond market and the central bank open. Traders submit contingent bids $a_i(r) \in [0, 1]$ to the central bank and $d_i(r) = 1 - a_i(r)$ to the domestic bond market. These bid functions $a_i(r)$ and $d_i(r)$ indicate the amount of dollars ($a_i(r)$) and domestic bonds ($d_i(r)$) that a trader wishes to acquire if the market-determined interest rate on domestic bonds is r . The supply of dollars is guaranteed by the central bank. The supply of domestic bonds is exogenously given by $S(s, r)$, a continuous function of the realized interest rate r , and an exogenous supply shock s . $S(s, r)$ is strictly increasing in s and nondecreasing in r , so that an increase in interest rates reduces the available quantity of domestic bonds. The supply shock $s \in \mathbb{R}$ is independent of θ and the private signals, and is normally distributed with mean zero and precision δ , $s \sim \mathcal{N}(0, \delta^{-1})$. Once all bids are submitted and the supply shock is realized, an interest rate r is selected to clear the domestic bond market.

In stage 3, the CB decides whether to maintain the fixed exchange rate after observing θ , r , and the total amount of dollar withdrawals A . This decision follows mechanically from the devaluation rule (1).

D. Strategies and Equilibrium

In stage 2, each trader observes a private signal x_i and submits a bid function $a_i(r)$. We let $a(x_i, r)$ denote the traders' *bidding strategy*, which, conditional on a private signal x_i and interest rate r , indicates the trader's demand for dollars.⁶ Respectively, we denote by $d(x_i, r) = 1 - a(x_i, r)$ the bidding strategy for domestic bonds.

⁴ It is possible to extend our analysis to general cost functions $C(r, A)$ that are nondecreasing in r and A .

⁵ This improper prior assumption is not essential for our results. We could extend our analysis to allow for normal or other proper prior distributions.

⁶ Implicitly we are restricting attention to *symmetric* bidding strategies, in which traders submit identical bidding functions. It is straightforward to rule out equilibria with asymmetric bidding strategies.

Integrating individual bidding strategies over x , we find the total demand for dollars, or equivalently, the CB's reserve losses, as a function of θ and r , denoted $A(\theta, r)$:

(2)

$$A(\theta, r) \equiv \int a(x, r) \sqrt{\beta} \phi(\sqrt{\beta}(x - \theta)) dx.$$

The demand for domestic bonds is then given by $D(\theta, r) \equiv 1 - A(\theta, r)$. Market clearing on the domestic bond market requires that

(3)
$$D(\theta, r) = S(s, r).$$

Therefore, the market-clearing condition characterizes a correspondence $\hat{R}(\theta, s)$, such that for all θ and s , $r \in \hat{R}(\theta, s)$ if and only if r satisfies (3). We will impose assumptions on $S(s, r)$ which guarantee that the set of market-clearing interest rates $\hat{R}(\theta, s)$ is always nonempty. An equilibrium interest rate function $R(\theta, s)$ is then a selection from $\hat{R}(\theta, s)$, that is, a function that assigns a market-clearing interest rate r to each possible realization of θ and s .

Now, consider a trader who observes signal x_i . Let $p(x_i, r)$ denote this trader's belief that a devaluation occurs when the market-clearing interest rate is r . In other words, $p(x_i, r)$ is the posterior probability of a devaluation, conditional on observing a signal x and the market-clearing interest rate being r . A bidding strategy $a(x_i, r)$ is optimal, if and only if

(4)
$$\begin{aligned} a(x_i, r) &= 1, & \text{if } p(x_i, r) > r; \\ a(x_i, r) &\in [0, 1], & \text{if } p(x_i, r) = r; \\ a(x_i, r) &= 0, & \text{if } p(x_i, r) < r. \end{aligned}$$

Condition (4) states that a trader's optimal trading strategy compares the excess return on domestic bonds r to the probability of devaluation $p(x_i, r)$, which here corresponds to the expected depreciation of the domestic currency. For a trader who is exactly indifferent between the two assets, condition (4) can thus be interpreted as an *uncovered interest parity condition*. With heterogeneous information, traders may

have different beliefs about the likelihood of a devaluation, and hence make different portfolio decisions in equilibrium.

As can be seen from (4), r affects optimal bidding strategies through two channels. On the one hand, r determines the payoff of holding the domestic bond. This is captured by the right-hand side of the optimality condition $p(x_i, r) \geq r$. On the other hand, r may affect the trader's expectations about the likelihood of a devaluation. This is captured by the left-hand side of this optimality condition. The ability to submit bids contingent on r enables the traders to take this effect into account in determining optimal bidding strategies.

To complete the description of optimal strategies, we determine the beliefs $p(x_i, r)$. Whenever there exists (θ, s) , s.t. $r = R(\theta, s)$, so that r is observed along the equilibrium path, $p(x_i, r)$ is pinned down by Bayes's Law. These beliefs are consistent with the devaluation outcome, which in equilibrium is determined from the market-clearing interest rate function $R(\theta, s)$ and the aggregate reserve losses by the central bank, $A(\theta, r)$. On the other hand, if $\{(\theta, s) : r = R(\theta, s)\}$ is empty for some r , then r is never realized as a market-clearing interest rate, Bayes's Law no longer determines $p(x_i, r)$, and beliefs are indeterminate. For our leading examples, we will provide a characterization of $p(x_i, r)$ using Bayes's Law on the equilibrium path.

Combining the conditions for optimal bidding strategies, market-clearing and posterior beliefs, we have the following equilibrium definition:

DEFINITION 1: A *Perfect Bayesian Equilibrium* consists of a bidding strategy $a(x_i, r)$, an interest rate function $R(\theta, s)$, a reserve loss function $A(\theta, r)$, and posterior beliefs $p(x_i, r)$, such that (i) $a(x_i, r)$, $A(\theta, r)$, and $R(\theta, s)$ satisfy, respectively, (2), (3), and (4), given beliefs $p(x_i, r)$; and (ii) for all r such that $\{(\theta, s) : r = R(\theta, s)\}$ is nonempty, $p(x_i, r)$ satisfies Bayes's Law.

Below, we focus on a particular class of equilibria in which bidding strategies $a(x_i, r)$ are nonincreasing in the private signals x_i , the information that is conveyed by r can be char-

acterized by a sufficient statistic $z(\theta, s)$, and the equilibrium interest rate function $R(\theta, s)$ is conditioned only on $z(\theta, s)$, not separately on θ and s . For this, we will need to make an additional functional form assumption about the domestic bond supply.

II. Solving the Model

In this section, we explain the main steps that are required to solve our model. First, we restrict attention to equilibria in *monotone strategies*. These are characterized by threshold rules $x^*(r)$ and $\theta^*(r)$ for $r \in (0, 1)$, such that traders demand a dollar, whenever their private signal satisfies $x_i \leq x^*(r)$, and they buy a domestic bond, whenever $x_i > x^*(r)$. A devaluation occurs, if and only if $\theta \leq \theta^*(r)$. These thresholds may be conditioned on the interest rate r .

We first show that threshold strategies are mutually consistent: if the devaluation outcome is characterized by a threshold rule, the traders' optimal strategies are characterized by a threshold rule as well, and vice versa. Suppose, first, that the devaluation outcome is described by an arbitrary threshold rule $\theta^*(r) \in [0, 1]$, so that a devaluation occurs if and only if $\theta \leq \theta^*(r)$. Then, the traders' belief function $p(x_i, r)$ can be rewritten as $p(x_i, r) = \Pr(\theta \leq \theta^*(r) | x_i, r)$. We conjecture (and will verify below) that $p(x_i, r)$ is strictly decreasing in x_i , with $\lim_{x_i \rightarrow \infty} p(x_i, r) = 0$ and $\lim_{x_i \rightarrow -\infty} p(x_i, r) = 1$. Then, for $r \in (0, 1)$, there exists a unique threshold $x^*(r)$, such that

$$(5) \quad p(x^*(r), r) = r.$$

In words, a trader whose private signal is $x^*(r)$ is just indifferent between holding the domestic bond and the dollar. If a trader's signal is higher than the threshold, $x_i > x^*(r)$, he strictly prefers to hold dollars, while a trader whose signal is below the threshold, $x_i < x^*(r)$, strictly prefers to hold the domestic bond. The reverse is also true: given a threshold $x^*(r)$ below which traders acquire the dollar, the total loss of dollar reserves by the central bank is $A(\theta, r) = \Phi(\sqrt{\beta}(x^*(r) - \theta))$, which is decreasing in θ . Therefore, for any r , $\theta - C(r, A(\theta, r))$ is in-

creasing in θ , and there exists a unique $\theta^*(r)$ at which

$$(6) \quad \theta^*(r) = C(r, A(\theta^*(r), r)),$$

where $A(\theta^*(r), r) = \Phi(\sqrt{\beta}(x^*(r) - \theta^*(r)))$, and a devaluation occurs if and only if $\theta \leq \theta^*(r)$. To constitute an equilibrium in monotone strategies, the thresholds $\theta^*(r)$ and $x^*(r)$ must thus jointly solve conditions (5) and (6), given posterior beliefs $p(x, r)$.

To complete the equilibrium characterization, we discuss how the market-clearing interest rate function $R(\theta, s)$ is determined, which in turn enables us to derive the belief function $p(x, r)$, which incorporates the information conveyed in equilibrium by r . If agents use a threshold rule characterized by $x^*(r)$, the demand for dollars is equal to the measure of agents who receive a signal below $x^*(r)$, while the demand for domestic bonds equals the measure of agents whose signal exceeds $x^*(r)$: $D(\theta, r) = 1 - A(\theta, r) = 1 - \Phi(\sqrt{\beta}(x^*(r) - \theta))$. In equilibrium, the market-clearing condition requires that $1 - \Phi(\sqrt{\beta}(x^*(r) - \theta)) = S(s, r)$, for all (θ, s) .

Due to the endogeneity of equilibrium beliefs, complete equilibrium characterizations in noisy REE models of asset markets are often difficult to obtain, unless specific functional form assumptions are made. We now make such an assumption about the functional form of $S(s, r)$ that enables us to solve our model in closed form.

ASSUMPTION 1: $S(s, r) = \Phi(s - \gamma\Phi^{-1}(r))$, and $s \sim \mathcal{N}(0, \delta^{-1})$.

The parameter $\gamma \geq 0$ controls how sensitive the domestic bond supply is to variations in the interest rate. If $\gamma = 0$, supply is inelastic, in which case the quantity of bonds traded in equilibrium does not respond to changes in the interest rate. If, instead, $\gamma > 0$, bond supply is elastic. In this case, an increase in the domestic interest rate will lead to a smaller supply of bonds and, in equilibrium, a larger loss of foreign reserves by the central bank, and this effect becomes stronger the larger is γ . In what

follows, we will refer to γ as the supply elasticity parameter.

With this assumption, the market-clearing condition can be rewritten as

$$(7) \quad x^*(r) - \frac{\gamma}{\sqrt{\beta}} \Phi^{-1}(r) = z = \theta - \frac{1}{\sqrt{\beta}} s.$$

Equation (7) plays two important roles in our analysis. First, it guarantees market-clearing in the domestic bond market. For a given threshold $x^*(r)$, an interest rate $R(\theta, s) = r$ clears the bond market, if and only if it satisfies (7), for all θ and s . Equation (7) thus characterizes, for given realization of $z = \theta - s/\sqrt{\beta}$ and interest rate r , the signal threshold $x^*(r)$ of the *marginal trader* whose indifference is required for market-clearing.

Second, the market-clearing condition (7) enables us to characterize the information provided by r : the left-hand side of (7) defines z as a function of the interest rate r and the threshold $x^*(r)$, while the right-hand side defines z in terms of the unobservable shocks θ and s . The realization of z can therefore be directly inferred from r and the knowledge of $x^*(r)$. The variable z thus summarizes the information conveyed by r , providing a normally distributed endogenous *public* signal of θ , $z \sim \mathcal{N}(\theta, (\beta\delta)^{-1})$. The precision of this interest rate signal, $\beta\delta$, is increasing in the precision of exogenous private signals, β ; hence, the interest rate serves to aggregate private information. At the same time, bigger shocks in the domestic bond supply (a smaller δ) make r less informative.

At this point, we make a second restriction by focusing on equilibria, in which r is conditioned only on the sufficient statistic z , but not on θ and s separately, so that the same $R(z)$ is selected for any θ, s , s.t. $\theta - s/\sqrt{\beta} = z$. Lemma 1 characterizes the resulting equilibrium beliefs.

LEMMA 1 (Information Aggregation): *Suppose that all other agents follow a threshold rule characterized by $x^*(r)$, and a devaluation occurs, whenever $\theta \leq \theta^*(r)$. Then,*

(i) $R(z)$ is selected from the correspondence $\hat{R}(z)$ of market-clearing interest rates:

$$(8) \quad \hat{R}(z) \equiv \left\{ r \in [0, 1] : z = x^*(r) - \frac{\gamma}{\sqrt{\beta}} \Phi^{-1}(r) \right\}.$$

(ii) If $\{z : r = R(z)\}$ is nonempty, the probability of devaluation $p(x_i, r)$ is given by

$$(9) \quad p(x_i, r) = \Phi\left(\frac{x_i + \delta x^*(r)}{1 + \delta} + \frac{\gamma\delta}{\sqrt{\beta}(1 + \delta)} \Phi^{-1}(r)\right).$$

PROOF:

Part (i) is immediate from (7). For (ii), note that $p(x_i, r) = \Pr(\theta \leq \theta^*(r) | x_i, z(r))$, where $z(r)$ is determined from (7). Since $\theta | x_i, z \sim \mathcal{N}((x_i + \delta z)/(1 + \delta); (\beta + \beta\delta)^{-1})$, $\Pr(\theta \leq \theta^*(r) | x_i, z) = \Phi(\sqrt{\beta + \beta\delta}(\theta^*(r) - (x_i + \delta z)/(1 + \delta)))$, from which (9) follows after substituting for z .

Any monotone strategy equilibrium is thus characterized by an interest rate function $R(z)$, a belief function $p(x_i, r)$, and thresholds $\{x^*(r), \theta^*(r)\}$, s.t. (5) and (6) are satisfied, $p(x_i, r)$ is given by (9) on the equilibrium path, and $R(z)$ is selected from (8), for every realization of z .⁷ This characterization also suggests a simple strategy for solving the model in closed form: substituting (7) and (9) into (5), we determine $x^*(r)$ and $\theta^*(r)$ from (5) and (6). From there, we construct the correspondence $\hat{R}(z)$ of interest rates that are consistent with market-clearing.

As we will show below, in all our examples, there exists a unique solution for the thresholds $\{x^*(r), \theta^*(r)\}$, which is continuous in r . Thus, equilibrium trading strategies are always uniquely determined. If, however, there are realizations of z

⁷ For interest rates that never occur in equilibrium, beliefs and bidding strategies remain undetermined. This freedom to choose out of equilibrium beliefs does not, however, affect optimal behavior on the equilibrium path. Our equilibrium characterization obtains for any selection of beliefs out of equilibrium.

for which $\hat{R}(z)$ admits multiple values of r , it becomes possible to construct multiple market-clearing interest rate functions from $\hat{R}(z)$, and our model admits multiple equilibria.

Now, for a given realization of z , market-clearing requires that the marginal trader who clears the domestic bond market is exactly indifferent between the domestic bond and the dollar. Formally, this requires that $r = p(x^*(r), r)$, where $x^*(r) = z + (\gamma/\sqrt{\beta})\Phi^{-1}(r)$. Substituting this into (9), an interest rate r therefore clears the market ($r \in \hat{R}(z)$), if and only if

$$(10) \quad r = \Phi \left(\sqrt{\beta + \beta\delta} \left(\theta^*(r) - z - \frac{\gamma}{\sqrt{\beta(1 + \delta)}} \Phi^{-1}(r) \right) \right).$$

Equation (10) is the central equilibrium condition of our model that determines the set of market-clearing interest rates for a given realization of z . Formally, it may be interpreted as an uncovered interest parity condition which must hold with equality for the marginal trader. The signal threshold $x^*(r)$ of the marginal trader is set so that the domestic bond market clears, which requires $x^*(r) = z + (\gamma/\sqrt{\beta})\Phi^{-1}(r)$.

Clearly the market-clearing correspondence $\hat{R}(z)$ is always nonempty. To understand under what conditions $\hat{R}(z)$ is single- or multi-valued, we need to discuss how the interest rate r affects (10), the condition which characterizes $\hat{R}(z)$. First, r enters on the left-hand side as the direct payoff from acquiring a domestic bond, measuring the direct *payoff effect* of the interest rate for domestic bonds. All else equal, the payoff effect implies that an increase in r makes domestic bonds more attractive and dollar assets less attractive to the traders. Second, r enters on the right-hand side of (10) through the equilibrium devaluation threshold $\theta^*(r)$, measuring the impact of the interest rate on the devaluation outcome. The sign and magnitude of this *devaluation effect* is ambiguous; as we will discuss below, it actually depends on the specifics of the environment. As can be seen from conditions (9) and (5), these two effects are present for any trader with given signals x and z , not just for the marginal trader.

If the bond supply is not completely inelastic ($\gamma > 0$), a third effect arises from the restriction that the threshold $x^*(r)$ must be consistent with market-clearing. For given state z , an increase in r leads to a reduction in the supply of bonds. In equilibrium, fewer traders buy bonds, and more traders acquire dollars, as r increases. To be consistent with market-clearing, the fact that more traders demand dollars in equilibrium implies that the identity of the marginal trader changes: his expectation about θ must increase, and he must therefore become less optimistic about a devaluation. Formally, for given z , $x^*(r)$ is increasing in r , and so is the marginal trader's posterior expectation of θ , which equals $z + \gamma/[(\sqrt{\beta}(1 + \delta))\Phi^{-1}(r)]$. This *market-clearing effect* of r suggests that an increase in r makes the marginal trader less optimistic about a devaluation, which makes the domestic bond more attractive, reinforcing the direct payoff effect on the left-hand side of condition (10).⁸

In summary, to determine whether our model admits multiple equilibria, we need to compare the payoff, devaluation, and market-clearing effects of r in the equilibrium condition (10). This is done in Lemma 2.

LEMMA 2 (Multiplicity): *Suppose the devaluation threshold $\theta^*(r)$ is continuously differentiable. Then, there exist multiple market-clearing interest rate functions, whenever for some $r \in (0, 1)$,*

$$(11) \quad \frac{d\theta^*}{dr} > \frac{\gamma + \sqrt{1 + \delta}}{\sqrt{\beta}(1 + \delta)} \frac{1}{\phi(\Phi^{-1}(r))}.$$

PROOF:

For any $r \in (0, 1)$, there is at most one realization $z = \hat{z}(r)$, for which r is consistent with (10), and solving (10), we find $\hat{z}(r) = \theta^*(r) - (\gamma + \sqrt{1 + \delta})/[(\sqrt{\beta}(1 + \delta))\Phi^{-1}(r)]$. Therefore, if there exists r' , such that

$$\left. \frac{d\theta^*}{dr} \right|_{r=r'} > \frac{\gamma + \sqrt{1 + \delta}}{\sqrt{\beta}(1 + \delta)} \frac{1}{\phi(\Phi^{-1}(r'))},$$

⁸ For this effect to exist, $\gamma > 0$ is essential. If $\gamma = 0$, the domestic bond supply is inelastic and changes in r have no effect on the quantity of bonds traded in equilibrium.

then $d\hat{z}/dr|_{r=r'} > 0$, and hence $\hat{z}(r' + \varepsilon) > \hat{z}(r') > \hat{z}(r' - \varepsilon)$. Moreover, since $\lim_{r \rightarrow 1} \hat{z}(r) = -\infty$ and $\lim_{r \rightarrow 0} \hat{z}(r) = +\infty$, it follows by continuity of $\hat{z}(r)$ that $\hat{z}(r') = \hat{z}(r'') = \hat{z}(r''')$ for some $r'' \in (r', 1)$ and $r''' \in (0, r')$, so that $z = \hat{z}(r')$ is consistent with multiple market-clearing interest rates. Since $\theta^*(r)$ is continuously differentiable, the same argument applies to any z' within a small open neighborhood of $\hat{z}(r')$. We conclude that $\hat{R}(z)$ is multivalued over an open interval, and hence that there exist multiple market-clearing interest rate functions.

Lemma 2 provides a sufficient condition under which, for given devaluation threshold $\theta^*(r)$, there are multiple market-clearing interest rates. This condition is also necessary for multiplicity within the class of equilibria that we are constructing, that is, if condition (11) is not satisfied for any r , then there exists a unique monotone strategy equilibrium in which the interest rate is conditioned on z only. Condition (11) compares the devaluation effect on the left-hand side against the payoff and market-clearing effects of r on the right-hand side. Multiplicity may result only if the devaluation effect of r is positive and large enough to offset the payoff and market-clearing effects of r , or if $\theta^*(r)$ is locally increasing in r with a slope sufficiently steep. In the next sections, we show how such positive devaluation effects lead to multiple equilibria in the context of our leading examples, $C(r, A) = r$ and $C(r, A) = A$.

III. Devaluation Triggered by Interest Rates

In this section, we discuss the version of our model in which devaluations are driven exclusively by the cost of high interest rates, as in Obstfeld (1996). Formally, we suppose that $C(r, A) = r$, and a devaluation occurs if and only if $\theta \leq r$. We proceed in two steps: first, we examine equilibrium outcomes with common knowledge; then, we examine our model with privately informed traders. By comparing the two environments, we assess the role of the information structure and determine to what extent the insights of the common knowledge environment carry over to the game with private information.

A. Equilibria with Common Knowledge

Suppose that $\theta \in (0, 1]$ is common knowledge among all traders. With a slight abuse of notation, we let $p(\theta, r)$, $A(\theta, r)$, and $D(\theta, r)$ denote, respectively, the traders' beliefs about a devaluation, their optimal bidding strategies, and the resulting aggregate demand for dollars and domestic bonds, for given $\theta \in (0, 1]$ and realized interest rate r .

Clearly, $p(\theta, r) = 1$ whenever $\theta \leq r$, and $p(\theta, r) = 0$ otherwise. Therefore, optimal bidding strategies are characterized as follows: if $r > 1$, the domestic bond strictly dominates the dollar, and individual and aggregate demands for dollars and bonds are $A(\theta, r) = 0$ and $D(\theta, r) = 1$. If $r < 0$, the dollar strictly dominates the domestic bond, so that $A(\theta, r) = 1$ and $D(\theta, r) = 0$. If $r \in (0, 1)$, traders demand dollars ($A(\theta, r) = 1 = 1 - D(\theta, r)$) if and only if $\theta \leq r$, and demand domestic bonds ($A(\theta, r) = 0 = 1 - D(\theta, r)$) otherwise. Finally, when $r = 0$, $\theta > r$, a devaluation does not occur, and traders are indifferent between the dollar and the domestic bond; any $D(\theta, r) \in [0, 1]$ is sustainable as part of the demand schedule for bonds. Similarly, when $r = 1$, a devaluation does occur, and again traders are indifferent between bonds and dollars, so that any $D(\theta, r) \in [0, 1]$ is part of the demand schedule. Figure 1 illustrates the equilibrium in the domestic bond market in a standard demand-and-supply graph, plotting the demand schedule $D(\theta, r)$, as characterized above, and an arbitrary supply function $S(s, r)$. Unless the supply is perfectly elastic at some exogenously given interest rate level r , in which case the supply curve is horizontal, there are two market-clearing interest rates. Under our functional form assumption for $S(s, r)$, both $r = 0$ and $r = 1$ clear the domestic bond market, for any s and $\theta \in (0, 1]$. If $r = 0$, then no devaluation will take place, the central bank will not lose any reserves, and $S(s, 0) = D(\theta, 0) = 1$. On the other hand, if $r = 1$, there will be a devaluation, the reserve losses are equal to 1, and $S(s, 1) = D(\theta, 1) = 0$.

As can be seen from Figure 1, multiplicity of equilibria arises from the existence of multiple market-clearing prices, due to the nonmonotone demand schedule for domestic bonds. This nonmonotonicity arises from the interaction between

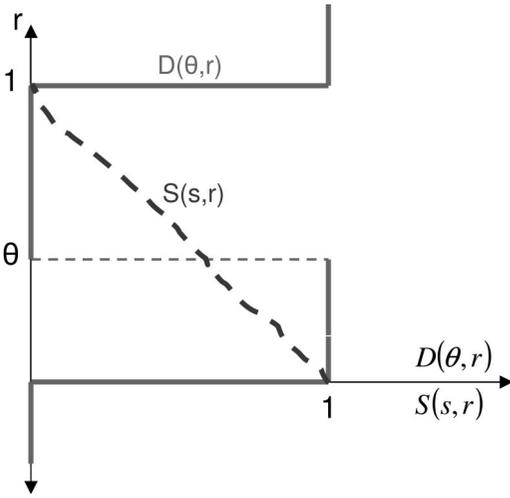


FIGURE 1. DEVALUATION TRIGGERED BY INTEREST RATES
(Common knowledge)

the payoff effect of r for domestic bonds, and the devaluation effect for the return on holding the dollar. While an increase in r continuously raises the return to holding domestic bonds, it also leads to a discrete increase in the payoff to holding the dollar at $r = \theta$, the level at which the interest rate becomes sufficiently high to trigger a devaluation. Therefore, at $r = \theta$, the traders shift their demand to dollar assets, and the demand for domestic bonds drops from 1 to 0. In other words, there are multiple market-clearing interest rates, because the devaluation effect of r locally dominates the payoff effect.⁹

Although this argument delivers multiple equilibria, the underlying logic is quite different from the multiplicity argument in coordination games. When devaluations are driven by interest rates, once bids are conditioned on r , the actions taken by other traders are irrelevant for any given trader's portfolio decision. Therefore, the traders do not have an explicit motive to coordinate bids. In contrast, in Morris and Shin (1998), a coordina-

⁹ Notice that for this result, it is essential that the supply schedule is finitely elastic. If supply is infinitely elastic at some given interest rate r , then the supply determines r , which in turn uniquely determines the devaluation outcome.

tion problem among traders is critical for obtaining multiplicity (under common knowledge) or uniqueness (with private information).

B. Equilibria with Private Information

We next show how the same insights carry over into our market environment with private information. After substituting $C(r, A) = r$ into condition (6), we solve (5) and (6) for the thresholds $\theta^*(r)$ and $x^*(r)$ to find

$$(12) \quad \theta^*(r) = r$$

$$\text{and} \quad x^*(r) = r + \frac{\gamma\delta - \sqrt{1 + \delta}}{\sqrt{\beta}(1 + \delta)} \Phi^{-1}(r).$$

Substituting this solution for $\theta^*(r)$ into the market-clearing correspondence (10), we find the following equilibrium characterization:

PROPOSITION 1: Suppose that $C(r, A) = r$. Then, in any monotone strategy equilibrium,

- (i) for all r s.t. $\{z : r = R(z)\}$ is nonempty, $\theta^*(r)$ and $x^*(r)$ are uniquely characterized by (12).
- (ii) $r \in \hat{R}(z)$, if and only if

$$(13) \quad r = \Phi \left(\sqrt{\beta + \beta\delta} \left(r - z - \frac{\gamma}{\sqrt{\beta}(1 + \delta)} \Phi^{-1}(r) \right) \right).$$

- (iii) There are multiple equilibria, whenever $\sqrt{\beta}(1 + \delta)/(\sqrt{1 + \delta} + \gamma) > \sqrt{2\pi}$.

PROOF:

Parts (i) and (ii) are immediate from preceding arguments. For (iii), note that $d\theta^*/dr = 1$, and therefore the condition of Lemma 2 is satisfied, whenever for some $r \in (0, 1)$,

$$\frac{\gamma + \sqrt{1 + \delta}}{\sqrt{\beta}(1 + \delta)} \frac{1}{\phi(\Phi^{-1}(r))} < 1.$$

Since $[\phi(\Phi^{-1}(r))]^{-1}$ is maximized when $r = 1/2$ and $\Phi^{-1}(r) = 0$, the result follows from

noting that $1/\phi(0) = \sqrt{2\pi}$. This condition is also necessary for multiplicity within the class of equilibria under consideration.

Optimal bidding strategies for traders are thus determined for each interest rate r that is observed on the equilibrium path, but multiplicity may arise from the set of market-clearing interest rate functions. This should not appear to be surprising; after all, we already showed that optimal trading strategies under common knowledge were uniquely pinned down by comparing the fundamental θ to the realized interest rate r , but led to a nonmonotone trading strategy.

The same logic applies here. The fundamental θ and the interest rate r uniquely determine the devaluation outcome, with $\theta^*(r) = r$ being the exogenously given devaluation threshold. Given their private signal x_i , and the interest rate, each trader's beliefs about a devaluation, and hence his optimal bidding strategies, are uniquely pinned down. Multiple equilibria result because the devaluation effect locally dominates the payoff and market-clearing effects of r , so that the demand for domestic bonds may be locally decreasing in the interest rate r .

When the supply elasticity is zero, $\gamma = 0$, the market-clearing effect is absent. In this case, once traders are sufficiently well informed, in that the precision of their overall information, $\beta(1 + \delta)$, is sufficiently large, the devaluation effect locally dominates and generates multiple equilibria. This parameter condition is plotted in Figure 2. Uniqueness versus multiplicity depends only on the overall precision of information, not on whether this information results from the traders' exogenous private signals or the interest rate. In particular, note that multiplicity may arise even if δ is close to 0, when the interest rate conveys little or no information about fundamentals.

When the supply elasticity is positive, $\gamma > 0$, the market-clearing effect of r reinforces the direct payoff effect. This expands the set of parameters for which there is a unique equilibrium and shifts the boundary of the uniqueness range to the right, but does not otherwise affect the main conclusions. However, holding the precision parameters β and δ fixed, as the supply elasticity becomes infinite ($\gamma \rightarrow \infty$), the

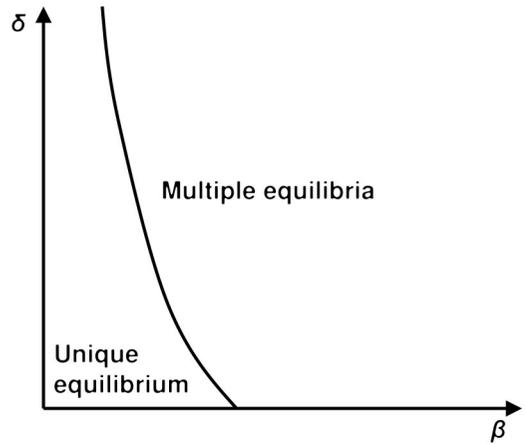


FIGURE 2. DEVALUATION TRIGGERED BY INTEREST RATES
(Uniqueness versus multiplicity)

market-clearing effect becomes so strong that it always outweighs the devaluation effect and thus leads to a unique equilibrium. In this case, the domestic bond supply becomes more and more elastic at an exogenously given interest rate, from which the devaluation outcome is uniquely determined, for each fundamental θ . This connects to our observation from the common knowledge game, where there is always a unique equilibrium, when the domestic bond supply is infinitely elastic at an exogenously given interest rate level.

In summary, the argument for multiplicity put forth by Obstfeld (1996) in models of devaluations triggered by high interest rates applies identically when traders are heterogeneously informed. Given their private information and the interest rate, traders form their beliefs about the likely devaluation outcome, which uniquely determines optimal trading strategies. If traders are sufficiently well informed, the devaluation effect of the interest rate leads to nonmonotone trading strategies: in response to an increase in the domestic interest rate, traders shift their wealth into dollars if they anticipate that the interest rate increase makes a devaluation more likely. As a result, there may be multiple market-clearing interest rates for some fundamental realizations, a high interest rate associated with high likelihood of a devaluation and large loss of foreign

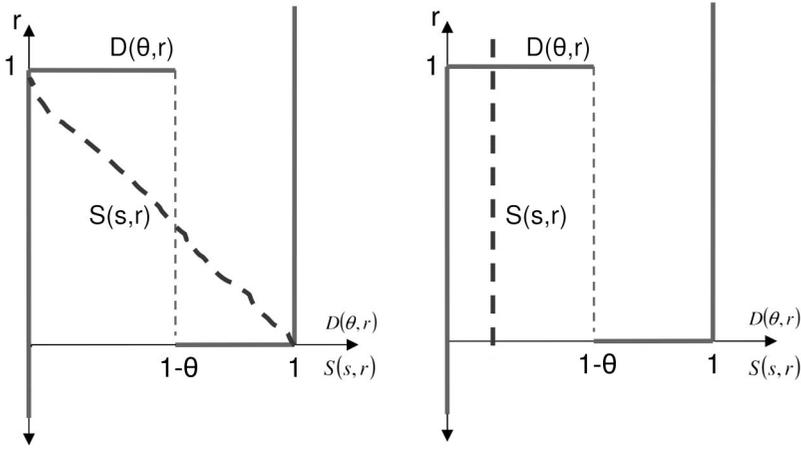


FIGURE 3. DEVALUATION TRIGGERED BY RESERVE LOSSES
(Common knowledge)

reserves by the central bank, and a low interest rate associated with low likelihood of a devaluation and small reserve losses.

IV. Devaluations Triggered by Reserve Losses

In this section, we consider our second leading case, in which devaluations are triggered exclusively by the loss of foreign reserves; a prominent example of such a model is Obstfeld (1986). Formally, we assume $C(r, A) = A$, so that a devaluation occurs if and only if $\theta \leq A$. As before, we split our analysis in two parts. First, we discuss equilibria in the case where $\theta \in (0, 1]$ is common knowledge. Then, we derive the equilibria of our private information economy and compare them to the common knowledge benchmark. We conclude this section by discussing the relation between our results and existing global games models of currency crises in Morris and Shin (1998, 2004), Hellwig (2002), Tarashev (2003), and Angeletos and Werning (2006).

A. Equilibria with Common Knowledge

Suppose that $\theta \in (0, 1]$, s , and $S(s, r)$ are common knowledge. As before, if $r > 1$, agents strictly prefer the domestic bond, so that the aggregate demand for dollars and bonds is $A(\theta, r) = 0$ and $D(\theta, r) = 1$. If $r < 0$, agents strictly

prefer to invest in the dollar, so that $A(\theta, r) = 1$ and $D(\theta, r) = 0$. If $r \in [0, 1]$, however, agents must make an assessment of whether a devaluation is likely to occur to determine optimal bidding strategies. If traders take r as exogenously given without taking into account the fact that the observed r must clear domestic bond markets, then $A(\theta, r) = 0, D(\theta, r) = 1$ and $A(\theta, r) = 1, D(\theta, r) = 0$ are both sustainable as best responses and hence part of the demand schedule for dollar assets and domestic bonds, for any $r \in [0, 1]$: if traders expect no devaluation, they will all demand domestic bonds, in which case the central bank's reserve losses are 0, and a devaluation will indeed not occur. If, instead, the traders do expect a devaluation, they will demand dollars, and the resulting reserve losses of $A(\theta, r) = 1$ will force the devaluation that the traders are expecting. Finally, if $r = 0$, traders remain indifferent between the domestic bond and the dollar, as long as $\theta > A(\theta, r)$; hence, any $D(\theta, r) > 1 - \theta$ can be sustained as part of the demand correspondence. Likewise, if $r = 1$, agents again remain indifferent, as long as $\theta \leq A(\theta, r)$, and hence any $D(\theta, r) \leq 1 - \theta$ is sustainable.

In Figure 3, we plot the resulting demand correspondence for domestic bonds, $D(\theta, r) = 1 - A(\theta, r)$, together with a supply function $S(s, r)$, in a standard demand-and-supply graph. As long as $S(s, r)$ is not completely inelastic, there may be

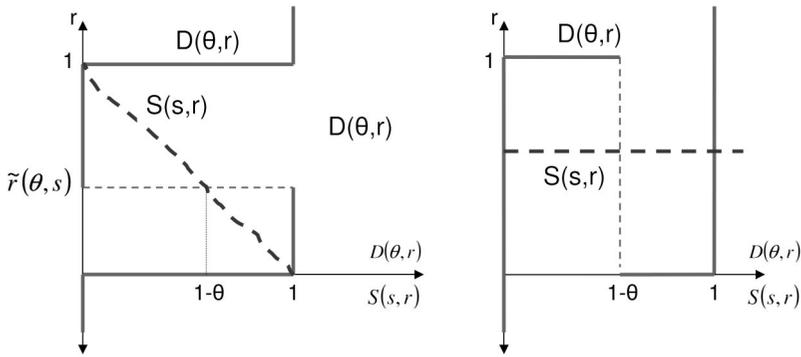


FIGURE 4. DEVALUATION TRIGGERED BY RESERVE LOSSES
(Off equilibrium inference from interest rate)

multiple market-clearing interest rates (left panel). On the other hand, when the domestic bond supply is perfectly inelastic, there exists a unique equilibrium (right panel). The inelastic supply exogenously determines the CB's loss of foreign reserves, and thereby the devaluation outcome. Interest rates adjust to clear the domestic bond market: $r = 1$, if $\theta \leq 1 - S$, when there is a devaluation; and $r = 0$, when $\theta > 1 - S$, and there is no devaluation. The multiplicity result thus relies on the bond supply and reserve losses varying with changes in the interest rate.

In the reserve-loss model, traders face an explicit coordination motive as they must form a conjecture about each other's trading strategies to predict the resulting reserve losses. The discussion above was based on the premise that all traders form such conjectures by assuming that all the other traders behave optimally, and when the resulting strategies are inconsistent with market-clearing, the traders discard the market-clearing hypothesis. Instead, one might assume that traders form their conjectures about the other traders' strategies on the basis of market-clearing, without requiring necessarily that the implied strategies are optimal. The difference between these two approaches matters only out of equilibrium, when either optimality or market-clearing must be violated, and it might therefore appear purely semantic at this point. However, the idea that the observation of the interest rate r , together with the market-clearing condition, can be used to form a

consistent conjecture of the equilibrium reserve losses, thus resolving the coordination problem in the market, turns out to be central for understanding behavior *on the equilibrium path*, when we go to the private information version of our model.

To see how traders might use r to form a conjecture about the other traders' strategies and aggregate reserve losses, notice that market-clearing implies $S(s, r) = D(\theta, r) = 1 - A(\theta, r)$. If the domestic bond supply is positively, but finitely, elastic, the observation of r enables traders to infer the central bank's equilibrium loss of foreign reserves $A(\theta, r)$, which thus enables them to predict the devaluation outcome. Let $\tilde{r}(\theta, s)$ denote the interest rate level for which $1 - \theta = S(s, \tilde{r})$. Whenever $r \geq \tilde{r}(\theta, s)$, $A(\theta, r) = 1 - S(s, r) \geq \theta$, so that traders expect a devaluation. Whenever $r < \tilde{r}(\theta, s)$, $A(\theta, r) = 1 - S(s, r) < \theta$ and traders anticipate no devaluation. In each case, the resulting demand for domestic bonds is again uniquely pinned down by comparing r to the resulting devaluation premium, 0 or 1; if $\tilde{r}(\theta, s) \in (0, 1)$, it is again backward bending. We sketch this case in Figure 4 (left panel).

With this conjecture, multiplicity once again arises because of a nonmonotonicity in optimal trading strategies: the fact that r must be consistent with market-clearing allows traders to infer the central bank's reserve losses and hence the devaluation outcome. A higher r indicates a smaller supply of domestic bonds, and larger loss of foreign reserves. At $r = \tilde{r}(\theta, s)$, a small increase in r

leads traders to the conclusion that the implied reserve losses become sufficiently large to trigger a devaluation; in response, they shift their bidding strategies from domestic bonds back into dollars. This nonmonotonicity in trading behavior gives rise to multiple market-clearing interest rates.

On the other hand, if the domestic bond supply is infinitely elastic at exogenously given r , any quantity of domestic bonds is consistent with market-clearing, so r provides no information about the reserve losses A (Figure 4, right panel). In that case, $A(\theta, r) = 1$ and $A(\theta, r) = 0$ are both consistent with best-response behavior and market-clearing, implying multiple equilibria due to an explicit coordination problem, as discussed above. The case with infinite supply elasticity differs from our model with finite supply elasticity, because r remains completely uninformative of the central bank's reserve losses. This, however, is exactly the case on which Morris and Shin (1998) focus their analysis to show that the introduction of noise in the observation of θ leads to the selection of a unique equilibrium.

B. Equilibria with Private Information

We now return to our model with noisy private signals. Using $C(r, A) = A$, the conditions for the devaluation threshold $\theta^*(r)$ and the indifference condition for $x^*(r)$ can be written as

$$(14) \quad \theta^*(r) = \Phi(\sqrt{\beta}(x^*(r) - \theta^*(r)));$$

$$(15) \quad r = \Phi(\sqrt{\beta + \beta\delta}(\theta^*(r) - x^*(r)) + \frac{\gamma\delta}{\sqrt{1 + \delta}}\Phi^{-1}(r)).$$

Solving this pair of equations for $\{\theta^*(r), x^*(r)\}$, we find the following unique solution:

$$(16) \quad \theta^*(r) = \Phi\left(\frac{\gamma\delta - \sqrt{1 + \delta}}{1 + \delta}\Phi^{-1}(r)\right)$$

and $x^*(r) = \theta^*(r) + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\theta^*(r)).$

This leads to the following equilibrium characterization:

PROPOSITION 2: *Suppose that $C(r, A) = A$. Then, in any monotone strategy equilibrium:*

- (i) *For all r s.t. $\{z : r = R(z)\}$ is nonempty, $\theta^*(r)$ and $x^*(r)$ are uniquely characterized by (16).*
- (ii) *For given z , $r \in \hat{R}(z)$ if and only if*

$$(17) \quad r = \Phi\left(\sqrt{\beta + \beta\delta}\left(\Phi\left(\frac{\gamma\delta - \sqrt{1 + \delta}}{1 + \delta}\Phi^{-1}(r)\right) - z - \frac{\gamma}{\sqrt{\beta(1 + \delta)}}\Phi^{-1}(r)\right)\right).$$

- (iii) *There are multiple equilibria whenever*

$$\frac{\sqrt{\beta(1 + \delta)}}{\sqrt{1 + \delta + \gamma}} \frac{\gamma\delta - \sqrt{1 + \delta}}{1 + \delta} > \sqrt{2\pi}.$$

PROOF:

Steps (i) and (ii) are again immediate. For (iii), applying Lemma 2, and noting that

$$\frac{d\theta^*}{dr} = \frac{\gamma\delta - \sqrt{1 + \delta}}{1 + \delta} \phi \times \left(\Phi\left(\frac{\gamma\delta - \sqrt{1 + \delta}}{1 + \delta}\Phi^{-1}(r)\right)\right) \frac{1}{\phi(\Phi^{-1}(r))},$$

there exist multiple equilibria whenever

$$\frac{\gamma\delta - \sqrt{1 + \delta}}{1 + \delta} \phi\left(\Phi\left(\frac{\gamma\delta - \sqrt{1 + \delta}}{1 + \delta}\Phi^{-1}(r)\right)\right) > \frac{\gamma + \sqrt{1 + \delta}}{\sqrt{\beta(1 + \delta)}}.$$

Since the left-hand side reaches a maximum at $r = 1/2$, this condition is satisfied when-

$$\frac{\sqrt{\beta(1 + \delta)}}{\sqrt{1 + \delta + \gamma}} \frac{\gamma\delta - \sqrt{1 + \delta}}{1 + \delta} > 1/\phi(0) = \sqrt{2\pi}.$$

As before, this condition is also necessary for multiplicity within the class of equilibria under consideration.

Once again, optimal trading strategies and the devaluation outcome are uniquely determined for any interest rate r , but there may be multiple market-clearing interest rate functions. As we already discussed in the common knowledge context, this arises because, in equilibrium, the observation of r enables traders to make a noisy inference of the other traders' actions through the market-clearing condition; r thus serves as an endogenous public signal of the foreign reserve losses A , which traders can use to forecast the devaluation outcome and hence uniquely determine optimal trading strategies.

The possibility of multiple equilibria then depends on whether the devaluation threshold $\theta^*(r)$ is increasing or decreasing in r . Both scenarios are possible, and which one materializes depends on the supply elasticity parameter γ and the variance of the supply shocks δ^{-1} . If $\gamma\delta \leq \sqrt{1 + \delta}$, $\theta^*(r)$ is decreasing in r , the devaluation effect of r is negative and reinforces the payoff and market-clearing effects, which implies that there is a unique market-clearing interest rate, irrespective of β . On the other hand, if $\gamma\delta > \sqrt{1 + \delta}$, $\theta^*(r)$ is increasing in r , the devaluation effect is positive and counteracts the payoff and market-clearing effects, which gives rise to multiple market-clearing interest rates once β is sufficiently large. We plot this uniqueness condition in Figure 5.

To understand when $\theta^*(r)$ is increasing in r , consider the conditions characterizing optimal trading strategies and the devaluation threshold, (14) and (15). For given $\theta^*(r)$ and $x^*(r)$, an increase in r raises both the payoff to the domestic bond and the marginal trader's expectation about a devaluation, since for given $x^*(r)$, a higher r is associated with a lower value of the public signal z . Intuitively, r serves as an endogenous signal not only of θ but also of the foreign reserve losses, with a higher r being indicative of a smaller supply of domestic bonds and a larger loss of foreign reserves. The parameter condition on γ and δ determines whether the payoff effect or the effect on the likelihood of a devaluation dominates, and hence whether at the margin traders become more or less aggressive as r increases. This con-

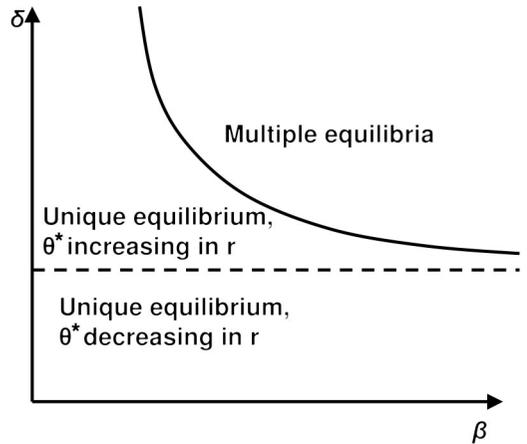


FIGURE 5. DEVALUATION TRIGGERED BY RESERVE LOSSES
(Uniqueness versus multiplicity)

dition relates the strength of the devaluation effect to the informativeness of the interest rate r as a signal of equilibrium reserve losses: if $\gamma\delta < \sqrt{1 + \delta}$, the supply elasticity γ is low, and shocks are large (δ is small). In this case, the interest rate has little impact on the realized bond supply, carries little information about the likely reserve losses, and hence has little influence on the trader's beliefs about a devaluation. On the other hand, if $\gamma\delta > \sqrt{1 + \delta}$, the bond supply is sufficiently elastic and shocks are not too big, so that changes in r affect the realized bond supply, and become informative of the central bank's equilibrium reserve losses. The information carried by r then leads to nonmonotone trading behavior, whereby traders optimally choose to withdraw dollars once the interest rate (and therefore the implied reserve losses) are sufficiently high. This in turn generates multiple market-clearing interest rates and multiple equilibria.

To summarize, information transmitted by the interest rate about the equilibrium demand for bonds and the central bank's foreign reserve losses plays a critical role here, because it enables traders to resolve the coordination problem that is present in the reserve loss model. Multiple equilibria then arise when the information about reserve losses is sufficiently precise to generate nonmonotone trading behavior and multiple market-clearing interest rates. The in-

tuition for this result once again parallels our discussion in the common knowledge case: through the market-clearing condition, a higher interest rate leads traders to infer larger losses of foreign reserves by the central bank, making a devaluation more likely.¹⁰ If the domestic bond supply is sufficiently elastic and supply shocks are not too large, increases in the interest rate lead to a positive devaluation effect, which may counteract the direct payoff and market-clearing effects to generate multiple market-clearing interest rates for certain realizations of z .

C. Discussion

We conclude this section by discussing the relation between our results and the results obtained by Morris and Shin (1998), as well as other global games models of currency crises in which devaluations are driven by reserve losses.

To apply the global games equilibrium selection results, Morris and Shin (1998) consider an environment in which r is exogenously fixed. Their analysis is a special case of our reserve loss model, in which supply is infinitely elastic at an exogenously given interest rate r ; r then does not convey any information in equilibrium about fundamentals, and in the unique equilibrium, optimal strategies are characterized by a threshold $x^{MS}(r)$ and the devaluation threshold $\theta^{MS}(r)$, which must satisfy the following pair of equations: $r = \Pr(\theta \leq \theta^{MS} | x^{MS}) = \Phi(\sqrt{\beta}(\theta^{MS} - x^{MS}))$ and $\theta^{MS} = \Phi(\sqrt{\beta}(x^{MS} - \theta^{MS}))$. In the unique equilibrium, $\theta^{MS}(r) = 1 - r$.

As we already discussed above, for the case with common knowledge, when r is exogenously fixed, traders are unable to draw any inference from r and therefore face an explicit coordination problem. This leads to very different equilibrium bidding strategies than the case where supply is highly, but not infinitely, elastic and inference from r remains possible using the market-clearing condition.

An even stronger contrast arises when we compare the special case of Morris and Shin (1998) with our model with highly but not

perfectly elastic supply. Taking the limit of $\sqrt{\beta}(\gamma\delta - \sqrt{1 + \delta})/(\gamma + \sqrt{1 + \delta})$ as $\gamma \rightarrow \infty$, there exists a unique equilibrium if and only if $\sqrt{\beta}\delta \leq \sqrt{2\pi}$; multiple equilibria necessarily arise once β is sufficiently high. As γ becomes larger and larger, fluctuations in r become smaller and smaller, and r converges to an exogenously given limit. This, however, does not imply that r loses its value as a signal: as the supply elasticity becomes large, even vanishingly small fluctuations of r remain informative of changes in the supply of domestic bonds and the amount of foreign reserve losses.

This limit characterization as $\gamma \rightarrow \infty$ has the following interpretation. Consider the global games model with an exogenously fixed interest rate (or infinitely elastic supply), but add an exogenous public signal $y \sim \mathcal{N}(\theta, \eta^{-1})$ with mean θ and precision η . The main result of the global games literature with exogenous public and private information (Morris and Shin, 2003, 2004; Hellwig, 2002) shows that there exists a unique equilibrium if and only if $\eta/\sqrt{\beta} \leq \sqrt{2\pi}$, that is, if the precision of private signals is sufficiently high relative to the public signal precision. Once we replace the precision of the exogenous public signal η with the endogenous precision of the interest rate $\beta\delta$, we find that our limit condition for multiplicity exactly replicates the condition with exogenous information. This results because interest rate fluctuations vanish and r no longer has any direct payoff implications in the limit as the supply elasticity becomes infinite. The interest rate then affects trading strategies only through the information it conveys in equilibrium.

This limiting case also mirrors the analysis in Angeletos and Werning's (2006) model of information aggregation through a derivative asset market. In their model, traders may trade a derivative asset prior to the currency crises game, which is modeled as in Morris and Shin. Since the derivative price serves only to aggregate information, but has no effects on payoffs in the coordination game, uniqueness versus multiplicity is determined purely by the extent to which the price provides public information, as characterized by the uniqueness condition given above. Information aggregation then overturns the Morris-Shin limit uniqueness result and leads to multiplicity when exogenous

¹⁰ Notice that in the model with devaluations driven by interest rates, traders did not need to make such indirect inference, since there was a direct link from the interest rate to the devaluation decision.

private information and the endogenous public signal are both sufficiently precise.

Since $\sqrt{\beta}(\gamma\delta - \sqrt{1 + \delta})/(\gamma + \sqrt{1 + \delta}) \leq \sqrt{\beta\delta}$, the condition for multiplicity becomes more stringent away from the limit. When supply is finitely elastic, or $\gamma < \infty$, the payoff effects associated with fluctuations in r mitigate the informational effects, and these payoff effects become larger as the domestic bond supply becomes less and less elastic. When the supply elasticity parameter γ becomes sufficiently small, there is a unique equilibrium. If $\gamma = 0$, the bond supply is perfectly inelastic and given by $\Phi(s)$, the loss of foreign reserves is $1 - \Phi(s)$, and a devaluation occurs, if and only if $\theta \leq 1 - \Phi(s)$. Just as in the common knowledge game, the devaluation outcome is then uniquely determined by the exogenous fundamentals and the shocks to the domestic bond supply, and the interest rate merely adjusts to clear the domestic bond market. This case underlies Tarashev's (2003) argument for equilibrium uniqueness.¹¹

V. Conclusion

In this paper, we have studied a stylized currency crises model with heterogeneously informed traders and interest rate determination in a noisy rational expectations equilibrium. Our analysis shows that contrary to what is suggested by the global games literature, multiple equilibria in market-based models of currency crises result from specific features of the market environment that are present, irrespective of the information structure. Fundamentally, this is due to the observation that multiplicity in currency crises environments does not result from an explicit coordination motive among traders, but because optimal trading strategies lead to non-monotone demand schedules for domestic assets and multiple market-clearing interest rates. This, however, is not captured by a

¹¹ Strictly speaking, his model also allows for direct effects of r on the devaluation outcome. These effects, however, are bounded in magnitude and complemented by a lower bound on the size of supply shocks, so that his uniqueness result effectively relies on the inelastic supply when reserve losses matter, and on the large noise generated by supply shocks to prevent the interest rate motive from generating multiplicity.

stylized global coordination game, which abstracts from the role of interest rates and markets.

Our paper provides a first step toward integrating insights from the global games literature into market-based models of currency crises. Although it provides a more explicit model of financial markets, it remains stylized in many respects, thus suggesting several avenues for future research. For example, we did not model the supply of domestic bonds, the central bank objectives, or the devaluation premium from first principles. A first extension may therefore be to model these features explicitly in the context of a private information model. Second, our model may be used as a building block to understand the effects of policy interventions in currency crises, when such interventions have informational as well as allocative effects. Angeletos et al. (2006) provide a step in this direction, but one would like to examine to what extent their analysis applies to more realistic market settings. Third, one may consider how equilibrium outcomes are affected by richer informational environments that allow, for example, for public information disclosures. We leave an analysis of these questions for future work.

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