Econometrica, Vol. 76, No. 3 (May, 2008), 619–641

NOTES AND COMMENTS

POLITICAL ECONOMY OF MECHANISMS

BY DARON ACEMOGLU, MICHAEL GOLOSOV, AND ALEH TSYVINSKI[1]

We study the provision of dynamic incentives to self-interested politicians who control the allocation of resources in the context of the standard neoclassical growth model. Citizens discipline politicians using elections. We show that the need to provide incentives to the politician in power creates political economy distortions in the structure of production, which resemble aggregate tax distortions. We provide conditions under which the political economy distortions persist or disappear in the long run. If the politicians are as patient as the citizens, the best subgame perfect equilibrium leads to an asymptotic allocation where the aggregate distortions arising from political economy disappear. In contrast, when politicians are less patient than the citizens, political economy distortions remain asymptotically and lead to positive aggregate labor and capital taxes.

KEYWORDS: Dynamic incentives, political economy, taxation.

1. INTRODUCTION

WE INVESTIGATE HOW political economy affects dynamic resource allocation and taxation. As a first step in this direction, we study the dynamics of resource allocation in the electoral accountability model originally developed by Barro (1973) and Ferejohn (1986). In this class of models, politicians decide a range of policies and citizens can vote them out of office if dissatisfied with their performance. We combine this setup with the standard neoclassical growth model. The allocation of resources is indirectly determined by self-interested politicians who have access to a set of unrestricted tax instruments. In contrast to existing analyses of similar models, we model the economic decisions of citizens and the tax decisions of politicians without restricting attention to specific classes of tax policies (such as linear taxes). We then characterize the subgame perfect equilibria that maximize citizens' ex ante utility, which we refer to as *the best sustainable mechanism*(s). Our focus on the best sustainable mechanism is motivated by our interest in understanding how the society might best avoid the distortions created by the presence of self-interested politicians and lack of

[1]We thank Manuel Amador, Marios Angeletos, Abhijit Banerjee, Tamer Basar, Timothy Besley, Olivier Blanchard, Ricardo Caballero, V. V. Chari, Mathias Dewatripont, Emmanuel Farhi, Kenichi Fukushima, Caroline Hoxby, Larry Jones, Patrick Kehoe, Narayana Kocherlakota, Jonathan Levin, David Levine, Guido Lorenzoni, Asuman Ozdaglar, Gerard Padro-i-Miquel, Christopher Phelan, Andrew Postlewaite, Vasiliki Skreta, Robert Townsend, Pierre Yared, Muhamet Yildiz, three anonymous referees, and especially the editor, Eddie Dekel, for useful comments and suggestions. We also thank participants at numerous seminars and conferences for comments, and Georgy Egorov, Laura Feiveson, and Oleg Itskhoki for excellent research assistance. All three authors gratefully acknowledge financial support from the National Science Foundation.

commitment.[2] While the previous literature typically assumes stationary voting rules, we show that the best equilibrium is nonstationary and has qualitatively different implications than stationary equilibria (though the best equilibrium has a very simple structure and is renegotiation-proof).

Our results are closely related to and extend the literature on the dynamic principal–agent problem (see, among others, Harris and Holmstrom (1982), Lazear (1981), Ray (2002)). The most general formulation of dynamic principal–agent problems is provided in Ray (2002). Ray showed that the optimal provision of dynamic incentives induces backloading of payments to the agent. We show that backloading also occurs in our economy (in the absence of capital) in the sense that politicians who remain in power for a long time are rewarded more. The first difference between our work and Ray's is that instead of the principal–agent problem, we analyze political equilibria in a game between citizens and politicians, and we focus on the implications for equilibrium distortions. In addition, our technical results extend those in Ray (2002) in two directions. First, we allow the discount factors of citizens and politicians to differ. When politicians have a lower discount factor, backloading no longer applies and tax distortions remain even in the long run.[3] Second, we analyze a dynamic economy with capital accumulation. The presence of capital introduces an additional state variable and implies that rewards to politicians are not necessarily backloaded even when they have greater discount factors than the citizens. These two differences are important for our focus: politicians are often argued to be more short-sighted than the agents, and the impact of political economy on intertemporal distortions (or on capital taxation) is one of the questions motivating our analysis.

Our paper is also related to and builds on the political economy literature.[4] The main difference between our approach and existing work in this literature is that we neither restrict citizens to stationary electoral policies nor impose exogenous restrictions on tax instruments. This generalized setup enables us to show that political economy distortions persist whenever the politician is less patient than the citizens. In contrast, distortions disappear in the long run when the politician is at least as patient as the citizens. In contrast, as we show below,

[2]Other equilibria will involve more distortions and will not necessarily answer the question of what the best feasible resource allocations are in the presence of political economy distortions.

[3]Ray (2002, p. 567) noted that differences in discount factors may create forces counteracting backloading, but does not offer an analysis of this case. Lehrer and Pauzner (1999) analyzed the related problem of equilibria in repeated games with different discount factors and showed that constrained efficiency requires the more patient player to be rewarded later. In the best sustainable mechanism in our paper, the politician may have backloaded rewards even when he is more impatient than the citizens (see Acemoglu, Golosov, and Tsyvinski (2006)), though, as Theorems 1 and 2 below show, in this case tax/policy distortions never disappear in the long run.

[4]See Acemoglu (2007), Persson and Tabellini (2000), and Besley (2006) for overviews.

when attention is restricted to stationary strategies, these political economy distortions never disappear.[5]

2. MODEL

2.1. *Preferences, Technology, and Equilibrium*

We consider an infinite horizon economy in discrete time, populated by a continuum of measure 1 of identical individuals (citizens). Individual preferences at time $t = 0$ are given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, l_t),$$

where c denotes consumption and l is labor supply. We denote the set of citizens by I and use the subscript i to denote citizens. We impose the standard conditions on U:

ASSUMPTION 1 (Utility): *$U(c, l)$ is twice continuously differentiable with partial derivatives denoted by U_C and U_L, strictly increasing in c, strictly decreasing in l, and jointly concave in c and l. We adopt the normalization $U(0, 0) = 0$. Moreover, $l \in [0, \bar{L}]$.*

The production side of the economy is described by the aggregate production function

$$Y_t = F(K_t, L_t), \tag{1}$$

which is defined inclusive of undepreciated capital (i.e., $F(K_t, L_t) \equiv \tilde{F}(K_t, L_t) + (1 - \theta)K_t$ for some other production function $\tilde{F}(K, L)$ and for some depreciation rate $\theta \in (0, 1)$).

ASSUMPTION 2 (Production Structure): *F is strictly increasing and continuously differentiable in K and L with partial derivatives denoted by F_K and F_L, exhibits constant returns to scale, and satisfies $\lim_{L\to 0} F_L(K, L) = \infty$ for all $K \geq 0$ and $\lim_{K\to\infty} F_K(K, L) < 1$ for all $L \in [0, \bar{L}]$.*

[5]Our work is also related to the growing literature on dynamic political economy. See, among others, Krusell and Rios-Rull (1999), Acemoglu and Robinson (2001), Hassler, Krusell, Storesletten, and Zilibotti (2005), and Battaglini and Coate (2008). In contrast to much of this literature, we focus on subgame perfect equilibria rather than Markovian equilibria. In this respect, our paper is also related to work on sustainable government policy in macro models, which studies the equilibria in a game between citizens and a benevolent government without commitment (e.g., Chari and Kehoe (1990, 1993)).

The condition that $\lim_{K\to\infty} F_K(K, L) < 1$ together with $L \in [0, \bar{L}]$ implies that there is a maximum steady-state level of output that is uniquely defined by $\bar{Y} = F(\bar{Y}, \bar{L}) \in (0, \infty)$. The condition that $\lim_{L\to 0} F_L(K, L) = \infty$ implies that in the absence of distortions there will be positive production.

The allocation of resources is delegated to a politician (ruler). The fundamental political dilemma faced by the society is to ensure that the body to which these powers have been delegated does not use them for its own interests. In the current model, this fundamental dilemma is partly resolved by the control of the politicians via elections.

We assume that there is a large number of potential (and identical) politicians, denoted by the set $\mathcal{I}$. Each politician's utility at time t is given by

$$\sum_{s=0}^{\infty} \delta^s v(x_{t+s}),$$

where x denotes the politician's consumption (rents) and $v:\mathbb{R}_+ \to \mathbb{R}$ is his instantaneous utility function. Notice also that the politician's discount factor, δ, is potentially different from that of the citizens, β. To simplify the analysis, we assume that potential politicians are distinct from the citizens and never engage in production and that once they are replaced they do not have access to capital markets (see footnote 9).

ASSUMPTION 3 (Politician Utility): *v is twice continuously differentiable, concave, and satisfies $v'(x) > 0$ for all $x \in \mathbb{R}_+$ and $v(0) = 0$. Moreover $\delta \in (0, 1)$.*

The politician in power decides the allocation of resources (or equivalently decides a general set of taxes and transfers). The only restriction on the allocation of resources, in addition to $c_t \geq 0$ and $l_t \in [0, \bar{L}]$, comes from the *participation constraint* of the citizens, which requires that $U(c_t, l_t) \geq 0$ for each t.[6] We denote the three constraints $c_t \geq 0$, $l_t \in [0, \bar{L}]$, and $U(c_t, l_t) \geq 0$ by

(2) $$(c_t, l_t) \in \Lambda \quad \text{for all } t.$$

Since $U(c, l)$ is concave and continuous, Λ is closed and convex (and also nonempty). We use $\operatorname{Int}\Lambda$ to denote the interior of the set Λ, so that $(c_t, l_t) \in \operatorname{Int}\Lambda$ implies that $c_t > 0$, $l_t \in (0, \bar{L})$, and $U(c_t, l_t) > 0$.

We consider the following game. At each time t, the economy starts with a politician $\iota_t \in \mathcal{I}$ in power and a stock of capital inherited from the previous period, K_t. Then:

[6]If the participation constraint $U(c_t, l_t) \geq 0$ is violated for some t, then citizens would supply zero labor at that date and secure utility $U(0, 0) = 0$ without future negative repercussions (see below). However, note that this participation constraint only needs to be satisfied "along the equilibrium path"; the politician can *deviate* and induce an allocation that does not satisfy this constraint.

1. Citizens make labor supply decisions, denoted by $[l_{i,t}]_{i \in I}$, where $l_{i,t} \geq 0$. Output $F(K_t, L_t)$ is produced, where $L_t = \int_{i \in I} l_{i,t}\, di$.

2. The politician chooses the amount of rents $x_t \in \mathbb{R}_+$, a consumption function $\mathbf{c}_t : \mathbb{R}_+ \to \mathbb{R}_+$, which assigns a level of consumption for each level of (current) labor supply, and next period's capital stock $K_{t+1} \in \mathbb{R}_+$, subject to the constraint

$$K_{t+1} \leq F(K_t, L_t) - C_t - x_t,$$

where $C_t = \int_{i \in I} \mathbf{c}_t(l_{i,t})\, di$ is aggregate consumption.[7] We denote a triple $(x_t, \mathbf{c}_t, K_{t+1})$ that is feasible for the politician by $(x_t, \mathbf{c}_t, K_{t+1}) \in \Phi_t$.

3. Elections are held and citizens jointly decide whether to keep the politician or replace him with a new one, $\rho_t \in \{0, 1\}$, where $\rho_t = 1$ denotes replacement.

The important feature here is that even though individuals make their economic decisions independently, they make their political decisions—elections to replace the politician—jointly. This is natural since there is no conflict of interest among the citizens over the replacement decision. Joint political decisions can be achieved by a variety of procedures, including various voting schemes (e.g., Persson and Tabellini (2000)). Here we simply assume that the decision $\rho_t \in \{0, 1\}$ is taken by a randomly chosen citizen.

We assume that at each date there is a public random variable z_t and all agents can condition their strategies on the history of this variable. This will enable us to convexify the value function of the citizens and is discussed in greater detail in the Appendix. Let

$$h^t \equiv \big(K_0, \iota_0, z_0, [l_{i,0}]_{i \in I}, x_0, \mathbf{c}_0, \rho_0, K_1, \dots,$$
$$K_t, \iota_t, z_t, [l_{i,t}]_{i \in I}, x_t, \mathbf{c}_t, \rho_t, K_{t+1}\big)$$

denote the history of the game up to date t and let H^t be the set of all such histories. In the text, to simplify notation we suppress the conditioning on the history of z^t. A *subgame perfect equilibrium* (SPE) is given by labor supply decisions $[l^*_{i,t}]_{i \in I}$ at time t given history h^{t-1}, policy decisions x^*_t, $\mathbf{c}^*_t$, K^*_{t+1} by the politician in power given h^{t-1} and $[l_{i,t}]_{i \in I}$, and electoral decisions by the citizens, ρ^*_t at time t, given history h^{t-1} and $[l_{i,t}]_{i \in I}$, x^*_t, $\mathbf{c}^*_t$, K^*_{t+1} that are best responses to each other for all histories. In addition, we will show below that the SPEs we focus on are "renegotiation-proof." Although the issue of how renegotiation should be handled in dynamic games is not settled and there are many alternative notions in the literature (e.g., Fudenberg and Tirole (1994)), for our purposes the simplest notion of renegotiation-proofness is sufficient. In

[7]One may wish to impose an additional constraint $x_t \leq \eta F(K_t, L_t)$ for some $\eta \in (0, 1)$, so that politician consumption cannot exceed an institutionally imposed limit. This additional constraint does not affect our analysis and qualitative results, and is omitted to reduce notation.

particular, we say that a SPE is *renegotiation-proof* if after any history h^t there does not exist another SPE that can make all active players weakly better off (and some strictly better off), where active players consist of the citizens and the politician who is currently in power.[8] In the present context, this implies that there should not exist an alternative SPE that can make the citizens and the politician in power better off than in the candidate SPE.

We focus on *best SPE*, defined as a SPE that maximizes the utility of the citizens. Consider the constrained optimization problem

$$\text{(3)} \qquad \text{MAX:} \quad \max_{\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

subject to the participation constraint (2), the resource constraint,

$$\text{(4)} \qquad C_t + K_{t+1} + x_t \leq F(K_t, L_t) \quad \text{for all } t,$$

the sustainability constraint for the politician in power,

$$\text{(5)} \qquad w_t \equiv \sum_{s=0}^{\infty} \delta^s v(x_{t+s}) \geq v(F(K_t, L_t)) \quad \text{for all } t,$$

and given the initial capital stock $K_0 > 0$. We have written this program using capital letters, since the consumption and labor supply levels refer both to individual and aggregate quantities. Notice also that in (5) we have defined the expected discounted utility of the politician at time t as w_t. This notation will be used in Theorem 1 below.

The sustainability constraint, (5), requires the equilibrium utility of the politician to be such that he does not wish to choose the maximum level of rents this period, $x_t = F(K_t, L_t)$, which would give him utility $v(F(K_t, L_t))$.[9] We refer to a sequence $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ that is a solution to this problem as a *best sustainable mechanism* (since it implicitly defines a resource allocation mechanism).[10] The constraint, (5), is sufficient to ensure that the politician does not wish to deviate from the mechanism.

[8]Without the qualification "all active players," renegotiation-proofness would be easier to guarantee, since an alternative SPE might make the citizens and the current politician better off, but reduce the utility of some future politician, who becomes less likely to come to power. The definition of renegotiation-proofness here is more demanding and more interesting in the context of political games.

[9]Here we are using the assumption that the politician does not have access to capital markets. If he did, then after deviation he would not consume the entire amount $F(K_t, L_t)$ today, but would invest part of it in the capital market to achieve a smoother consumption profile. When the politician has access to capital markets, a deviation from the implicitly agreed mechanism becomes more attractive and thus (5) becomes more difficult to satisfy, though this does not affect any of our qualitative results.

[10]Conditioning on public histories, this sequence would be written as $\{C_t(z^t), L_t(z^t), K_{t+1}(z^t), x_t(z^t)\}_{t=0}^{\infty}$, since each element would be a function of the history of $z^t \equiv (z_0, \dots, z_t)$.

PROPOSITION 1: *The allocation of resources in the best SPE* (*best sustainable mechanism*) *is identical to the solution of the maximization problem in* (MAX) *and involves no replacement of the initial politician along the equilibrium path*. *Moreover*, *this allocation can be supported as a renegotiation-proof SPE*.

PROOF: First, in view of the concavity of U, no feasible (possibly stochastic) allocation can provide higher ex ante utility to citizens than $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ that is a solution to (MAX). If it did, it would either violate the participation constraint, (2), the resource constraint, (4), or the sustainability constraint, (5), after some history h^t, and would thus not be feasible. Therefore, to prove the proposition it suffices to show that there exists a renegotiation-proof SPE that achieves the solution to (MAX).

Let $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ be a solution to (MAX). We next show that it can be supported as a SPE with no politician replacement along the equilibrium path. Introduce the following notation: $h^t = \hat{h}^t$ if $(K_{s+1}(h^s), x_s(h^s)) = (\tilde{K}_{s+1}, \tilde{x}_s)$ and $\mathbf{c}_s(l_{i,s} \mid h^s) = \tilde{C}_s$ for $l_{i,s} = \tilde{L}_s$ and $\mathbf{c}_s(l_{i,s} \mid h^s) = 0$ for $l_{i,s} \neq \tilde{L}_s$ for all $s \leq t$. Consider the strategy profile ρ for the citizens such that $\rho(h^t) = 0$ if $h^t = \hat{h}^t$ and $\rho(h^t) = 1$ if $h^t \neq \hat{h}^t$; that is, citizens replace the politician unless the politician has always chosen a strategy that induces the allocation $\{\tilde{C}_s, \tilde{L}_s, \tilde{K}_{s+1}, \tilde{x}_s\}_{s=0}^{t}$ up to time t. It is a best response for the politician to continue to choose $\{\tilde{C}_s, \tilde{L}_s, \tilde{K}_{s+1}, \tilde{x}_s\}_{s=t}^{\infty}$ after history $h^{t-1} = \hat{h}^{t-1}$ only if

$$\mathbb{E}\left[\sum_{s=0}^{\infty} \delta^s v(\tilde{x}_{t+s}(h^{t+s})) \,\middle|\, h^t\right] \geq \max_{x'_t, \mathbf{c}'_t, K'_{t+1}} \mathbb{E}[v(x'_t) + \delta v_t^c(K'_{t+1}, \mathbf{c}'_t, x'_t) \mid h^t],$$

where $v_t^c(x'_t, \mathbf{c}'_t, K'_{t+1})$ is the politician's continuation value following a deviation to a feasible $(x'_t, \mathbf{c}'_t, K'_{t+1})$. Under the candidate equilibrium strategy, $v^c = 0$ following any deviation; thus the best deviation for the politician is $x'_t = F(\tilde{K}_t, \tilde{L}_t)$, which gives (5). Consequently, (5) is sufficient for the politician not to deviate from $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$.

Next, suppose, to obtain a contradiction, that a solution to (MAX), $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$, can be supported as a SPE with replacement of the initial politician. Consider an alternative allocation $\{\tilde{C}'_t, \tilde{L}'_t, \tilde{K}'_{t+1}, \tilde{x}'_t\}_{t=0}^{\infty}$ such that the initial politician is kept in power along the equilibrium path and receives exactly the same consumption sequence as the new politicians would have received after replacement. Since $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ satisfies (5) for the new politicians at all t, $\{\tilde{C}'_t, \tilde{L}'_t, \tilde{K}'_{t+1}, \tilde{x}'_t\}_{t=0}^{\infty}$ satisfies (5) for all t for the initial politician. Moreover, since $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ must involve at least some positive consumption for the new politicians, $\{\tilde{C}'_t, \tilde{L}'_t, \tilde{K}'_{t+1}, \tilde{x}'_t\}_{t=0}^{\infty}$ yields a higher $t = 0$ utility to the initial politician. Thus, x_0 can be reduced and C_0 can be increased without violating (5), so $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{x}_t\}_{t=0}^{\infty}$ cannot be a solution to (MAX). This yields a contradiction and proves that there is no replacement of the initial politician along the equilibrium path.

We next show that citizens' strategy (in particular, $\rho(h^t) = 1$ if $h^t \neq \hat{h}^t$ and $\rho(h^t) = 0$ if $h^t = \hat{h}^t$) is subgame perfect and the equilibrium characterized here is renegotiation-proof. Let us denote the equilibrium value of the initial politician starting with capital stock K by $w_0(K)$ and denote the maximum feasible value that can be promised to a politician when the capital stock is K by $\bar{w}(K)$ (see the proof of Theorem 2 in the Appendix). Consider the following continuation equilibrium: if $\rho(h^t) = 1$ and $h^t \neq \hat{h}^t$, then the continuation equilibrium is a solution to (MAX), with initial value for the next politician $w' = w_0(K(h^t))$, where $K(h^t)$ is the capital stock after history h^t (that is, after the deviation if there is any). If $\rho(h^t) = 1$ and $h^t = \hat{h}^t$, then the continuation equilibrium is a solution to (MAX), with initial value for the next politician given by $w' = \bar{w}(K(h^t)) \geq w_0(K(h^t))$. Consequently, $\rho(h^t) = 0$ following $h^t = \hat{h}^t$ and $\rho(h^t) = 1$ following $h^t \neq \hat{h}^t$ are best responses for the citizens and are subgame perfect. Moreover, they involve the continuation play of a best SPE; thus the citizens and the politician in power cannot both be made better off. This establishes that the best sustainable mechanism outlined above, which achieves the solution to (MAX), can be supported as a renegotiation-proof SPE. *Q.E.D.*

This proposition enables us to focus on the constrained maximization problem given in (MAX). Moreover, it implies that in the best SPE, the initial politician will be kept in power forever (and that this best SPE is renegotiation-proof). The initial politician is kept in power forever because all politicians are identical and more effective incentives can be provided to a politician when he has a longer planning horizon (i.e., when he expects to remain in power for longer). Naturally, he is only kept in power along the equilibrium path—if he deviates from the implicitly agreed mechanism, he will be replaced.

For future reference, let us define an *undistorted* allocation as a sequence $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ that maximizes (3) without the sustainability constraint (5) (for a given sequence of $\{x_t\}_{t=0}^{\infty}$). An undistorted allocation where $(C_t, L_t) \in \operatorname{Int} \Lambda$ satisfies

$$F_L(K_t, L_t) U_C(C_t, L_t) = -U_L(C_t, L_t), \tag{6}$$

$$U_C(C_t, L_t) = \beta F_K(K_{t+1}, L_{t+1}) U_C(C_{t+1}, L_{t+1}). \tag{7}$$

We say that an allocation $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ features *downward labor distortions* at time t if the left-hand side of (6) is strictly greater than the right-hand side. Similarly, there are *downward intertemporal distortions* when the left-hand side of (7) is strictly less than the right-hand side. Downward distortions imply that there is less labor supply and less capital accumulation than in an undistorted allocation. We will interpret these distortions as corresponding to "aggregate tax distortions," since allocations that involve downward labor and intertemporal distortions can be decentralized by using linear labor and capital taxes.

2.2. *The Best Sustainable Mechanism Without Capital*

Let us start with the economy without capital, so that instead of Assumption 2, we have $Y_t = L_t$. An allocation can now be represented by $\{C_t, L_t, x_t\}$ (thus dropping K_t). An undistorted allocation with $(C_t, L_t) \in \operatorname{Int} \Lambda$ now satisfies $U_C(C_t, L_t) = -U_L(C_t, L_t)$.

We next introduce a sustainability assumption, which ensures that when the maximum amount of utility is given to the politician in every period, this is sufficient to satisfy the sustainability constraint (5). More formally:

ASSUMPTION 4 (Sustainability): *Let* $(\tilde{C}, \tilde{L}) \in \arg\max_{(C,L)\in\Lambda}\{L - C\}$. *Then* $v(\tilde{L} - \tilde{C})/(1-\delta) > v(\tilde{L})$.

The main result of this section is given in the following theorem.

THEOREM 1: *Suppose that* $Y_t = L_t$, *that Assumptions* 1, 3, *and* 4 *hold*, *and that* $U_C(0,0) > U_L(0,0)$. *Then in the best SPE* (*best sustainable mechanism*), *we have*:

1. *There are downward labor distortions at* $t = 0$.
2. *When* $\beta \le \delta$, *the values promised to the politician* $\{w_t\}_{t=0}^{\infty}$ *form a nondecreasing sequence and converge to some* w^*. *Moreover*, $\{C_t, L_t, x_t\}_{t=0}^{\infty}$ *converges to some* (C^*, L^*, x^*), *which satisfies the no-distortion condition* $U_C(C^*, L^*) = -U_L(C^*, L^*)$.
3. *When* $\beta > \delta$, *then there are downward labor distortions even asymptotically. The allocation described above can be supported as a renegotiation-proof SPE.*

See the Appendix for the proof.

Part 1 of the theorem illustrates the additional distortion that arises from the sustainability constraints. As output increases, the sustainability constraint, (5), requires more rents to be given to the politician in power and this increases the effective cost of production for the citizens. The best SPE creates distortions so as to reduce the level of output and thus the rents that have to be paid to the politician.[11]

Part 2 states that as long as $\beta \le \delta$, the economy asymptotically converges to an equilibrium (C^*, L^*, x^*) where there are no aggregate distortions; even

[11]Starting from an undistorted allocation reducing these rents is always beneficial. Loosely speaking, a marginal distortion, reducing labor supply and output by a small amount, creates a "second-order" loss for the citizens, but a "first-order" reduction in the amount of rents that have to be paid to the politician and thus a first-order increase in their consumption and utility.

though there will be rents provided to the politician, these will be financed without introducing distortions. This result is important because it implies that in the long run there will be "efficient" provision of rents to politicians, with the necessary tax revenues raised without distortions (e.g., with lump-sum taxes in a decentralized allocation). This part of the theorem also shows that the (promised) rewards to the politician, given by the sequence $\{w_t\}_{t=0}^{\infty}$, are nondecreasing. Intuitively, current incentives to the politician are provided both by consumption in the current period, x_t, and by consumption in the future represented by the promised value, w_{t+1}. Future consumption by the politician not only relaxes the sustainability constraint in the future, but does so in all prior periods as well. Thus, all else equal, optimal incentives for the politician should be backloaded. As discussed in the Introduction, this intuition for backloading in this political environment is the same as the intuition for backloading in the principal–agent literature (e.g., Ray (2002)).[12]

Part 3 of the theorem states that if the politicians are less patient than the citizens, distortions will never disappear. Since in many realistic political economy models politicians are—or act—more short-sighted than the citizens, this part of the theorem implies that in a number of important cases, political economy considerations will lead to additional distortions that will not disappear even asymptotically. Finally, Theorem 1 also shows that the best SPE can be supported as a renegotiation-proof equilibrium.

To provide an intuition for the proof of the theorem, let us represent the maximization problem in (MAX) recursively (for the special case without capital and ignoring the feasibility constraint on w^+, which is incorporated in the formal proof in the Appendix):

$$V(w) = \max_{(C,L)\in\Lambda, x, w^+} \{U(C, L) + \beta V(w^+)\} \tag{8}$$

subject to

$$C + x \leq L, \tag{9}$$

$$w = v(x) + \delta w^+, \tag{10}$$

$$v(x) + \delta w^+ \geq v(L). \tag{11}$$

[12]Nevertheless, Theorem 1 here and Theorem 2 in the next subsection are not special cases of Ray's results and extend them. First, these theorems cover the case with different discount factors. This is essential for our results regarding the long-run behavior of distortions. Second, with capital as an additional state variable, we will have a dynamic game rather than a repeated game and the backloading result may not necessarily apply (see Theorem 2). Third, as the proof of Theorem 2 illustrates, the equilibrium nature of our problem necessitates an analysis of situations in which allocations converge to the boundary of the feasibility sets and thus requires a different strategy of proof. Finally and least importantly, Ray (2002) made the opposite of Assumption 4 (or Assumption 4′ below).

Here $V(w)$ is the value (discounted lifetime utility) of the citizens when they have promised value w to the politician and w^+ denotes next period's promised value. Constraint (9) imposes the resource constraint (4). Constraint (10) imposes promise keeping, incorporating the fact that the politician will not be replaced. It requires that the promised value w be equal to the sum of the current utility, $v(x)$, and the continuation utility, δw^+. Finally, constraint (11) is the recursive version of the sustainability constraint, (5). Let γ and $\psi \geq 0$ be the Lagrange multipliers on the constraints (10) and (11), respectively. We show in the Appendix that $V(w)$ is concave and differentiable. Furthermore, for the intuitive argument here, suppose that $(C, L) \in \operatorname{Int} \Lambda$. The first-order condition with respect to w^+ and the envelope theorem then imply

$$\text{(12)} \qquad \frac{\beta}{\delta} V'(w^+) = -\gamma - \psi = V'(w) - \psi.$$

Combining the first-order conditions for C and L gives

$$\text{(13)} \qquad U_C(C, L) + U_L(C, L) = \psi v'(L).$$

Equation (13) makes it clear that aggregate distortions are related to the Lagrange multiplier on the sustainability constraint, ψ. Moreover, we must have $\psi > 0$ at $t = 0$, otherwise the politician would receive $w_0 = 0$ initially, which together with (11) would imply $C_t = L_t = 0$ for all t. However, $C_t = L_t = 0$ for all t cannot be a solution when $\psi = 0$ at $t = 0$. Equation (13) then yields $U_C(C, L) + U_L(C, L) > 0$ at $t = 0$.

To obtain the intuition for the second part of Theorem 1, consider the case where $\beta = \delta$ (for the argument for $\beta < \delta$, see the Appendix). Then (12) implies

$$\text{(14)} \qquad V'(w^+) = V'(w) - \psi \leq V'(w).$$

Concavity of the value function $V(\cdot)$ then implies that $w^+ \geq w$, with $w^+ > w$ if $\psi > 0$, and $w^+ = w$ if $\psi = 0$.[13] Therefore, the values promised to the politician form a nondecreasing sequence and converge to some w^*, and (14) implies that ψ must converge to 0. This also implies that $\{C_t, L_t, x_t\}_{t=0}^{\infty}$ converges to some (C^*, L^*, x^*), which satisfies (11) as stated in part 2 of Theorem 1.

This argument breaks down in part 3 of the theorem when $\delta < \beta$, because the politician does not value future rewards sufficiently and the sequence $\{w_t\}_{t=0}^{\infty}$ is not necessarily nondecreasing. In fact, (12) implies that if $\{w_t\}_{t=0}^{\infty}$ converges to some $\hat{w}$, then $\beta V'(\hat{w})/\delta = V'(\hat{w}) - \psi$. Since V' is negative (cf. footnote 13), ψ must be strictly positive in this case and there will necessarily be asymptotic distortions.

[13]Note that from Lemmas 1 and 2 in the Appendix, the derivative V' is negative in the relevant range of values.

2.3. *The Best Sustainable Mechanism With Capital*

We now extend Theorem 1 to an environment with capital, where the production function is given by Assumption 2. We first strengthen the sustainability assumption, Assumption 4. Let us define $\bar{C}$ and $\bar{K}$ such that

$$\bar{C} = \min\{C : (C, \bar{L}) \in \Lambda\} \quad \text{and} \quad \bar{K} = \arg\max_{K \geq 0}\{F(K, \bar{L}) - K - \bar{C}\}. \tag{15}$$

Clearly, $\bar{C}$ is uniquely defined (since $C \geq 0$ and Λ is closed). In view of this and Assumption 2, $\bar{K}$ is also uniquely defined.

ASSUMPTION 4′ (Sustainability With Capital): (1) $\delta v(F(\bar{K}, \bar{L}) - \bar{C} - \bar{K})/(1 - \delta) > v(F(\bar{K}, \bar{L}))$ *and* (2) $\bar{C} + \bar{K} \leq F(0, \bar{L})$.

The first part of Assumption 4′ states that there exists a feasible allocation that delivers sufficient utility to the politician so that the sustainability constraint (5) can be satisfied as a strict inequality.[14] A high discount factor δ is sufficient to ensure that this part of the assumption is satisfied. The second part of the assumption is a technical condition, which guarantees that the equilibrium allocation does not get stuck at some arbitrary capital level, and naturally requires that $F(0, \bar{L}) > 0$. Both parts of this assumption are used only in part 2 of the next theorem to characterize the equilibrium when the utility provided to a politician reaches the boundary of the set of feasible values.

THEOREM 2: *Suppose that Assumptions* 1–3 *and* 4′ *hold. Then in the best SPE*:

1. *There are downward labor distortions at some* $t < \infty$ *and downward intertemporal distortions at* $t - 1$ (*provided that* $t \geq 1$).
2. *When* $\beta \leq \delta$, *the best sustainable mechanism* $\{C_t, K_{t+1}, L_t, x_t\}_{t=0}^{\infty}$ *converges to some* (C^*, K^*, L^*, x^*). *At this allocation, the labor and intertemporal distortions disappear asymptotically, that is*, (6) *and* (7) *hold as* $t \to \infty$.
3. *When* $\beta > \delta$, *there are downward labor and intertemporal distortions*, *even asymptotically*.

The allocation described above can be supported as a renegotiation-proof SPE.

See the Appendix for the proof.

This theorem generalizes the results of Theorem 1 to an environment that is identical to the standard neoclassical growth model. The results are slightly weaker than in Theorem 1. In particular, there may not necessarily be distortions at the initial date, though such distortions will necessarily exist at some date. Perhaps more importantly, expected rewards to the politician are no

[14] This implies that the maximum utility to the politician can be provided without distortions.

longer always increasing. In fact, it is straightforward to construct examples in which the initial capital stock is sufficiently high so that these rewards are decreasing in the best SPE.[15] Also noteworthy is that when $\beta > \delta$, the best SPE not only generates labor distortions, but also intertemporal distortions. These can be thought of as "aggregate capital taxes," since they create a wedge between the marginal product of capital and the ratio of marginal utilities of consumption. Therefore, this model generates a political economy rationale for long-run capital taxation.

2.4. *Stationary Equilibria*

We finally consider the best *stationary* SPE in the economy without capital. With stationary strategies and no capital, x_t has to be constant (conditional on the politician remaining in power). Although a similar result can be stated for the economy with capital, in this case the politician's consumption x would be a function of K, which complicates the analysis. The economy without capital allows us to emphasize the importance of nonstationary SPEs in a clearer fashion.

The previous literature has typically focused on this type of stationary equilibria, in particular assuming that individuals vote "retrospectively" according to some fixed threshold (see, for example, Persson and Tabellini (2000, Chap. 4)).

PROPOSITION 2: *Consider the environment without capital in Theorem 1, and suppose that Assumptions 1, 3, and 4 hold and that $U_C(0,0) > U_L(0,0)$. Then, in the best stationary SPE, distortions never disappear.*

PROOF: Along a stationary equilibrium path, $x_t = x$ and $L_t = L$ so that

$$\frac{v(x)}{1-\delta} \geq v(L) \tag{16}$$

replaces the sustainability constraint (5). Constraint (16) must bind in all periods with $\psi > 0$, since otherwise the solution to the stationary equivalent of (MAX) would involve $x = 0$ and no distortions. The assumption that $U_C(0,0) > U_L(0,0)$ then implies that in this case $L > 0$, thus $x = 0$ would violate (16). Condition (13), which still applies in this case, then shows that there is a positive distortion on labor in all periods. *Q.E.D.*

This proposition illustrates the role of nonstationary SPE in our analysis. Stationary equilibria do not allow the optimal provision of dynamic incentives

[15]However, it can be shown that with capital, the second partial derivative of the value function of the citizens, $V_w(K, w)$, is nonincreasing (see the Appendix). This provides the appropriate notion of backloading in this generalized economy.

to politicians and imply that political economy distortions never disappear, even when $\beta \leq \delta$.

3. CONCLUDING REMARKS

In this paper, we took a first step toward a political–economic analysis of dynamic resource allocation problems. We focused on economies in which allocation decisions are delegated to self-interested politicians and characterized the best equilibrium (from the viewpoint of the citizens). Political economy considerations lead to a new source of distortions in the allocation of resources (and thus to a new source of distortionary taxation) because of the necessity to satisfy the political sustainability constraints. We provided a full characterization of these distortions and their evolution over time. When politicians are as patient as or more patient than the citizens, these distortions disappear in the long run. The politician in power still receives rents, but these rents are provided without additional distortions. In contrast, when politicians are less patient than the citizens, aggregate distortions remain positive even asymptotically. In this case, there will be asymptotic distortions that resemble positive labor and capital taxes.

The method of analysis presented here can be adapted to analyze the political economy of dynamic taxation in alternative environments. For example, in the dynamic Mirrlees taxation problem, individuals (citizens) also have private information and nonlinear tax schedules have to be incentive compatible to encourage the correct level of labor supply and effort (e.g., Mirrlees (1971), Golosov, Kocherlakota, and Tsyvinski (2003)). Acemoglu, Golosov, and Tsyvinski (2006) showed that in this environment the provision of incentives to politicians can be *separated* from the provision of incentives to individuals. This enables an analysis of this more general environment that is mathematically identical to the one presented here. The main result is that when politicians are as patient as (or more patient than) the citizens, the best SPE involves no distortions in addition to those implied by the individual incentive compatibility constraints and thus the structure of taxation closely resembles that in a standard Mirrlees economy. Resources to compensate the politician are raised in a nondistortionary manner. Instead, when politicians are less patient than the citizens, Mirrleesian taxes must be augmented by additional labor and capital distortions (taxes), even asymptotically.

Dept. of Economics, Massachusetts Institute of Technology, 50 Memorial Drive, Cambridge, MA 02142-1347, U.S.A.; daron@mit.edu,

Dept. of Economics, Massachusetts Institute of Technology, 50 Memorial Drive, Cambridge, MA 02142-1347, U.S.A.; golosov@mit.edu,

and

Economics Department, Harvard University, Cambridge, MA 02138, U.S.A.; tsyvinski@harvard.edu.

Manuscript received April, 2006; final revision received December, 2007.

APPENDIX: TECHNICAL RESULTS AND OMITTED PROOFS

Randomizations and the Properties of the Value Functions

We first formulate the characterization of the best SPE as a recursive program and ensure convexity by using randomizations. The recursive form of (3) is

$$V(K,w)=\max_{C,L,K^+,x,w^+}\{U(C,L)+\beta V(K^+,w^+)\} \tag{A1}$$

subject to

$$C+x+K^+\leq F(K,L), \tag{A2}$$

$$w=v(x)+\delta w^+, \tag{A3}$$

$$v(x)+\delta w^+\geq v(F(K,L)), \tag{A4}$$

$$(C,L)\in\Lambda \quad \text{and} \quad w^+\in\mathbb{W}[K^+], \tag{A5}$$

where $\mathbb{W}[K^+]$ denotes the set of feasible values that can be provided to the politician starting with capital stock K^+. In particular, let us define the maximum utility that can be given to the politician when the capital stock is equal to K_t as

$$\bar{w}(K_t)\equiv\max_{\{C_{t+j},L_{t+j},K_{t+1+j},x_{t+j}\}_{j=0}^{\infty}}\sum_{j=0}^{\infty}\delta^j v(x_{t+j}) \tag{A6}$$

subject to $(C_t,L_t)\in\Lambda$, (4), and (5) for all t. Evidently $\mathbb{W}[K]=[0,\bar{w}(K)]$.

LEMMA 1: *The solution to the maximization problem* (MAX) *starting with the capital stock of* K_0 *is equivalent to the solution to the program* (A1)–(A5) *combined with a choice of initial promised value to the politician*, w_0, *such that* $w_0=\arg\max_{w\in\mathbb{W}[K_0]}V(K_0,w)$.

PROOF: The proof follows from Thomas and Worrall (1990). Clearly any solution to (A1)–(A5) gives a sustainable mechanism. Moreover, the ex ante utility for the citizens from any sustainable mechanism can be obtained as $V(K_0,w)$ from (A1)–(A5) by an argument analogous to the principle of optimality. It then follows that $V(K_{0,}w_0)=\max_{w\in\mathbb{W}[K_0]}V(K_0,w)$ gives the best sustainable mechanism. *Q.E.D.*

The constraint (A4) in the program (A1)–(A5) is not convex and randomizations over the current consumption and the continuation value of the politician may improve the value of the program. This is the reason why we introduced the possibility of conditioning on the (payoff-irrelevant) public history $z^t\equiv(z_0,z_1,\ldots,z_t)$. Define $\mathbf{q}\equiv(C,L,K^+,x,w^+)\in\mathbb{R}^5$ and $\mathcal{C}(w)\equiv\{\mathbf{q}\in$

$\mathbb{R}^5 : (\text{A1})\text{–}(\text{A5})$ are satisfied for given $w\}$, and let $\mathcal{Z}$ be the set of Borel subsets of $\mathcal{C}(w)$. Also define $\mathcal{P}(w)$ as the space of probability measures on $(\mathcal{C}(w), \mathcal{Z})$ and endow it with the weak topology. Incorporating randomization, we can write the recursive formulation as follows:

PROBLEM A1:

$$V(K, w) = \max_{\boldsymbol{\xi} \in \mathcal{P}(w)} \int [U(C, L) + \beta V(K^+, w^+)] \boldsymbol{\xi}(d\mathbf{q}) \tag{A7}$$

subject to

$$C + x + K^+ \leq F(K, L), \quad \boldsymbol{\xi}\text{-almost surely,} \tag{A8}$$

$$v(x) + \delta w^+ \geq v(F(K, L)), \quad \boldsymbol{\xi}\text{-almost surely,} \tag{A9}$$

$$w = \int [v(x) + \delta w^+] \boldsymbol{\xi}(d\mathbf{q}), \tag{A10}$$

$$(C, L) \in \Lambda \quad \text{and} \quad w^+ \in \mathbb{W}[K^+], \quad \boldsymbol{\xi}\text{-almost surely.}$$

The solution to this program will give stochastic sequences $\{x_t(z^t)\}_{t=0}^{\infty}$ and $\{w_t(z^t)\}_{t=0}^{\infty}$.

LEMMA 2: *$V(K, w)$ is concave in w.*

PROOF: Consider w_0 and w_1 and $\boldsymbol{\xi}_0$ and $\boldsymbol{\xi}_1$ that are solutions to the maximization problem, and let $w = (1 - \alpha)w_0 + \alpha w_1$ and $\boldsymbol{\xi}_\alpha = (1 - \alpha)\boldsymbol{\xi}_0 + \alpha \boldsymbol{\xi}_1$ for some $\alpha \in (0, 1)$. Constraints (A8) and (A9) are satisfied for both $\boldsymbol{\xi}_0$ and $\boldsymbol{\xi}_1$, and therefore must be satisfied for $\boldsymbol{\xi}_\alpha$. Constraint (A10) is linear in $\boldsymbol{\xi}$; thus $\boldsymbol{\xi}_\alpha$ also satisfies this constraint. Since the objective function is linear in $\boldsymbol{\xi}_\alpha$, we also have $V(K, (1-\alpha)w_0 + \alpha w_1) \geq (1-\alpha)V(K, w_0) + \alpha V(K, w_1)$, establishing the concavity of V. *Q.E.D.*

The next lemma shows that randomization using only two points is sufficient to achieve the maximum of Problem A1.

LEMMA 3: *There exists $\boldsymbol{\xi} \in \mathcal{P}(w)$ that achieves $V(K, w)$ with randomization between two points, $(C_0, L_0, K_0^+, x_0, w_0^+)$ and $(C_1, L_1, K_1^+, x_1, w_1^+)$ with probabilities ξ_0 and $1 - \xi_0$.*

PROOF: To achieve convexity, we only need the constraint set to be convex. The constraint set here is $\mathcal{C}(w) \in \mathbb{R}^5$. From Caratheodory's theorem (e.g., Proposition 1.3.1 in Bertsekas, Nedic, and Ozdaglar (2003, pp. 37–38)), the convex hull of $\mathcal{C}(w)$ can be achieved with six points (see Acemoglu, Golosov, and Tsyvinski (2006)). *Q.E.D.*

Suppose, to obtain a contradiction, that there are more than two points with positive probability. We consider the case of three points (the same argument applies to any finite number of points). Suppose that randomization occurs between $(C_0, L_0, K_0^+, x_0, w_0^+)$, $(C_1, L_1, K_1^+, x_1, w_1^+)$, and $(C_2, L_2, K_2^+, x_2, w_2^+)$ with probabilities $\xi_0, \xi_1, \xi_2 > 0$. Suppose without loss of generality that $v(x_0) + \delta w_0^+ \leq v(x_2) + \delta w_2^+ \leq v(x_1) + \delta w_1^+$ and let $\alpha \in [0, 1]$ be such that $v(x_2) + \delta w_2^+ = \alpha[v(x_0) + \delta w_0^+] + (1 - \alpha)[v(x_1) + \delta w_1^+]$. Suppose first

$$U(C_2, L_2) + \beta V(K_2^+, w_2^+) > \alpha[U(C_0, L_0) + \beta V(K_0^+, w_0^+)] + (1 - \alpha)[U(C_1, L_1) + \beta V(K_1^+, w_1^+)].$$

Then $\hat{\boldsymbol{\xi}} \in \mathcal{P}(w)$ assigning probability $\hat{\xi}_2 = 1$ to $(C_2, L_2, K_2^+, x_2, w_2^+)$ is feasible and yields higher utility than the original randomization, yielding a contradiction. Next suppose

$$U(C_2, L_2) + \beta V(K_2^+, w_2^+) < \alpha[U(C_0, L_0) + \beta V(K_0^+, w_0^+)] + (1 - \alpha)[U(C_1, L_1) + \beta V(K_1^+, w_1^+)],$$

and consider an alternative $\hat{\boldsymbol{\xi}} \in \mathcal{P}(w)$ assigning probability $\xi_0 + \alpha\xi_2$ to $(C_0, L_0, K_0^+, x_0, w_0^+)$ and probability $\xi_1 + (1 - \alpha)\xi_2$ to $(C_1, L_1, K_1^+, x_1, w_1^+)$, which is again feasible and gives a higher utility than the original randomization, once again yielding a contradiction. Therefore, any $\hat{\boldsymbol{\xi}} \in \mathcal{P}(w)$ must satisfy

$$U(C_2, L_2) + \beta V(K_2^+, w_2^+) = \alpha[U(C_0, L_0) + \beta V(K_0^+, w_0^+)] + (1 - \alpha)[U(C_1, L_1) + \beta V(K_1^+, w_1^+)].$$

But then the optimum can be achieved by simply randomizing between $(C_0, L_0, K_0^+, x_0, w_0^+)$ and $(C_1, L_1, K_1^+, x_1, w_1^+)$ with respective probabilities $\xi_0 + \alpha\xi_2$ and $\xi_1 + (1 - \alpha)\xi_2$.

Lemma 3 implies that we can focus on randomizations between two points and can take aggregate public history to be of the form $z^t \in \{0, 1\}^t$. Let us then denote the solutions for any w by $(C_i(w), L_i(w), K_i^+(w), x_i(w), w_i'(w), \xi_i(w)$ for $i \in \{0, 1\})$, naturally with $\xi_0(w) + \xi_1(w) = 1$. Rewrite Problem A1 in the equivalent form:

PROBLEM A2:

$$V(K, w) = \max_{\{\xi_i, K_i^+, C_i, L_i, x_i, w_i^+\}_{i=0,1}} \sum_{i=0,1} \xi_i[U(C_i, L_i) + \beta V(K_i^+, w_i^+)] \tag{A11}$$

subject to

$$C_i + x_i + K_i^+ \leq F(K, L_i) \quad \text{for} \quad i = 0, 1, \tag{A12}$$

$$v(x_i) + \delta w_i^+ \geq v(F(K, L_i)) \quad \text{for} \quad i = 0, 1, \tag{A13}$$

$$w = \sum_{i=0,1} \xi_i [v(x_i) + \delta w_i^+], \tag{A14}$$

$$(C_i, L_i) \in \Lambda \quad \text{and} \quad w_i^+ \in \mathbb{W}[K_i^+] \quad \text{for} \quad i = 0, 1. \tag{A15}$$

LEMMA 4: *$V(K, w)$ is differentiable in w and K.*

PROOF: The proof builds on the approach by Benveniste and Scheinkman (1979). We provide the proof of differentiability in w. The proof for K is similar. Fix an arbitrary $w_0 \in \operatorname{Int} \mathbb{W}[K]$. From Lemma 3, in Problem A1 when $w = w_0$, the optimal value can be achieved by randomizing between $(\bar{C}_i(w_0), \bar{L}_i(w_0), \bar{K}_i^+(w_0), \bar{x}_i(w_0), \bar{w}_i^+(w_0))$ with probabilities $\bar{\xi}_i(w_0)$ for $i = 0, 1$, where $\bar{w}_i^+(w_0) \in \mathbb{W}[\bar{K}_i^+(w_0)]$. By hypothesis, $(\bar{C}_i(w_0), \bar{L}_i(w_0)) \in \operatorname{Int} \Lambda$ for $i = 0, 1$. Define $(\hat{x}_0(w), \hat{x}_1(w))$ such that $w = v(\hat{x}_i(w)) + \delta \bar{w}_i^+(w_0)$ and $\hat{C}_i(K, w) \equiv F(K, \bar{L}_i(w_0)) - \bar{K}_i^+(w_0) - \hat{x}_i(w)$. Clearly, $\hat{C}_i(K, w)$ is concave and differentiable in w (since $\hat{x}_i(w)$ is convex and differentiable in w). For w in a sufficiently small neighborhood of w_0, we know that $(\hat{C}_i(K, w), \bar{L}_i(w_0)) \in \operatorname{Int} \Lambda$. Now consider the function

$$\begin{aligned} &Q(K, w) \\ &\quad = \sum_{i=0,1} \bar{\xi}_i(w_0) \big[U(\hat{C}_i(K, w), \bar{L}_i(w_0)) + \beta V(\bar{K}_i^+(w_0), \bar{w}_i^+(w_0)) \big]. \end{aligned}$$

Note that $V(K^+, w^+)$, w^+, and K^+ are held constant at $V(\bar{K}_i^+(w_0), \bar{w}_i^+(w_0))$, $\bar{w}_i^+(w_0)$, and $\bar{K}_i^+(w_0)$ for $i = 0, 1$. Therefore, $Q(K, w)$ is concave in w (because U is concave in C and $\hat{C}_i(K, w)$ is concave in w) and differentiable in w (because U is differentiable in C, $\hat{C}_i(K, w)$ is differentiable in w, and $V(\bar{K}_i^+(w_0), \bar{w}_i^+(w_0))$ does not depend on w). Moreover, by construction $(\hat{C}_i(K, w), (\bar{K}_i^+(w_0), \bar{L}_i(w_0), \hat{x}_i(w), \bar{w}_i^+(w_0)))$ for $i = 0, 1$ satisfy (A12)–(A15), and since $V(K, w)$ is the maximum value of (A11) subject to (A12)–(A15), we have

$$Q(K, w) \leq V(K, w) \quad \text{and} \quad Q(K, w_0) = V(K, w_0). \tag{A16}$$

From Lemma 2, $V(K, w_0)$ is concave in w_0 and therefore $-V$ is convex. If f is convex, there exists a closed, convex, and nonempty set ∂f such that for all $\nu \in \partial f$ and any x and x', we have $f(x') - f(x) \geq \nu(x' - x)$ (see Bertsekas, Nedic, and Ozdaglar (2003, Chap. 4)). Let $\partial V^-(K, w)$ be the set of subdifferentials of $-V$, that is, all $-\nu$ such that $-V(K, \hat{w}) + V(K, w) \geq -\nu(\hat{w} - w)$. By definition, $\partial V^-(K, w)$ is a closed, convex, and nonempty set. Consequently, for any subgradient $-\nu$ in $\partial V^-(w_0)$, we have

$$\nu(w - w_0) \geq V(K, w) - V(K, w_0) \geq Q(K, w) - Q(K, w_0),$$

where the first inequality is by the definition of a subgradient and the second follows from (A16). This implies that $-\nu$ is also a subgradient of $-Q(K, w_0)$.

But since $Q(K, w_0)$ is differentiable, $-\nu$ must be unique; therefore, $V(K, w_0)$ is also differentiable. *Q.E.D.*

Proofs of Theorems 1 and 2

We start with the proof of Theorem 2, since some of the results in Theorem 1 will be obtained as corollaries. In the remainder of the Appendix, we reduce notation by suppressing the stochastic nature of the sequences of values or allocations whenever this will cause no confusion.

PROOF OF THEOREM 2: Since V is differentiable from Lemma 4 and concave from Lemma 2, the first-order conditions are necessary and sufficient for the maximization (A11). Assigning multipliers $\lambda_i \xi_i$ to the constraints (A12), $\psi_i \xi_i$ to (A13), and γ to (A14), and denoting the derivative of $V(K, w)$ with respect to w by $V_w(K, w)$, we have

$$\beta \xi_0 V_w(K_0^+, w_0^+) + \delta \psi_0 \xi_0 + \delta \gamma \xi_0 \leq 0,$$
$$\beta \xi_1 V_w(K_1^+, w_1^+) + \delta \psi_1 \xi_1 + \delta \gamma \xi_1 \leq 0,$$

with both equations holding as equality for $w_i^+ \in \operatorname{Int} \mathbb{W}[K_i^+]$. Therefore,

$$\frac{\beta}{\delta} V_w(K_i^+, w_i^+) \leq -\gamma - \psi_i, \tag{A17}$$

again with equality for $w_i^+ \in \operatorname{Int} \mathbb{W}[K_i^+]$. Moreover, since V is differentiable,

$$V_w(K, w) \geq -\gamma, \tag{A18}$$

again with equality for $w \in \operatorname{Int} \mathbb{W}[K_i^+]$. Combining the first-order conditions for C_i, L_i, and K_i^+, we have that for $(C_i, L_i) \in \operatorname{Int} \Lambda$,

$$\begin{aligned} &F_L(K, L_i) U_C(C_i, L_i) + U_L(C_i, L_i) \\ &\quad = \psi_i v'(F(K, L_i)) F_L(K, L_i) \quad \text{for} \quad i = 0, 1, \end{aligned} \tag{A19}$$

$$\begin{aligned} &\beta \sum_{j \in \{0,1\}} \xi_j^+ F_K(K_i^+, L_j^+) \big[U_C(C_j^+, L_j^+) + \psi_j^+ v'(F(K_i^+, L_j^+)) \big] \\ &\quad = U_C(C_i, L_i) \quad \text{for} \quad i = 0, 1. \end{aligned} \tag{A20}$$

Part 1: Suppose, to obtain a contradiction, that (A13) is slack for all t and $i = 0, 1$. Then the solution to (A11) involves $x_{i,t} = 0$ for all t and $i = 0, 1$, and thus $w_0 = 0$. Assumptions 1 and 2 imply that without any distortions, $F(K_0, L_0) > 0$; thus the politician can deviate to $x_0 = F(K_0, L_0) > 0$ and increase his utility, yielding a contradiction. Therefore, (A13) must bind at some t and i with $\psi_{i,t} > 0$. Then (A19) implies that there will be downward labor distortions at that t, and (A20) implies that there will be downward intertemporal distortions at $t - 1$.

Part 2: Fix some $w \in \text{Int}\,\mathbb{W}$. Since $\beta \leq \delta$ and $V_w(K, w) \leq 0$, (A17) implies

$$V_w(K_i^+, w_i^+) \leq -\gamma - \psi_i \quad \text{for} \quad i = 0, 1.$$

Combining this with (A18) and $\psi_i \geq 0$ yields

$$V_w(K, w) \geq V_w(K_i^+, w_i^+) \quad \text{for} \quad i = 0, 1.$$

This implies that $\{V_w(K_t, w_t)\}_{t=0}^{\infty}$ is a nonincreasing (stochastic) sequence (in the sense that every realization of V_w at time t is no less than its value at $t-1$) and necessarily converges on the extended real line. There are therefore *three* cases to consider.

CASE 1: $\{V_w(K_t, w_t)\}_{t=0}^{\infty}$ converges to some $V_w > -\infty$ and for all convergent subsequences $\{w_{t_n}, K_{t_n}\}_{t=0}^{\infty}$ of $\{w_t, K_t\}_{t=0}^{\infty}$, we have that K_{t_n} converges to some K_∞ and w_{t_n} converges to some $w_\infty \in \text{Int}[0, \bar{w}(K_\infty)]$. This is only possible if the associated subsequence of multipliers $\{\psi_{t_n}\}_{t=0}^{\infty}$ converges to 0. Equations (A19) and (A20) then imply the desired result.

CASE 2: $\{V_w(K_t, w_t)\}_{t=0}^{\infty}$ converges to some $V_w > -\infty$ and $\{w_t\}_{t=0}^{\infty}$ has a subsequence converging to $w_\infty \in \text{Bd}[0, \bar{w}(K_\infty)]$ (where Bd denotes boundary). We now establish two lemmas that show that distortions also disappear in this case. Recall that $\bar{w}(K_t)$ denotes the maximum value that can be given to the politician starting with capital stock K_t. The next lemma states that if we reach the upper boundary of $\mathbb{W}[K_t] \equiv [0, \bar{w}(K_t)]$ at some t, we will always remain at the upper boundary of future $\mathbb{W}[K_t]$s.

LEMMA 5: *Let* $\{C_{t+j}^*, L_{t+j}^*, K_{t+1+j}^*, x_{t+j}^*\}_{t=0}^{\infty}$ *be the solution to the problem* (A6) *and recall that* $\bar{w}(K_t^*) = \sum_{j=0}^{\infty} \delta^j v(x_{t+j}^*)$. *If* $w_{t'} = \bar{w}(K_{t'})$ *for some* t', *then* $w_t = \bar{w}(K_t)$ *for all* $t \geq t'$.

PROOF: Suppose to obtain a contradiction that the last statement is not true. Then there exists some feasible sequence $\{C_{t+j}, L_{t+j}, K_{t+1+j}, x_{t+j}\}_{j=0}^{\infty}$ and $K_{t+1+j^*} = K_{t+1+j^*}^*$ for some $j^* > 0$ such that $\sum_{s=0}^{\infty} \delta^s v(x_{t+j^*+s}) > \sum_{s=0}^{\infty} \delta^s v(x_{t+j^*+s}^*)$. Now form the sequence $(\tilde{C}_{t+j}, \tilde{L}_{t+j}, \tilde{K}_{t+1+j}, \tilde{x}_{t+j}) = (C_{t+j}^*, L_{t+j}^*, K_{t+1+j}^*, x_{t+j}^*)$ for all $j < j^*$ and $(\tilde{C}_{t+j}, \tilde{L}_{t+j}, \tilde{K}_{t+1+j}, \tilde{x}_{t+j}) = (C_{t+j}, L_{t+j}, K_{t+1+j}, x_{t+j})$ for all $j \geq j^*$. This new sequence is feasible in view of the fact that $K_{t+1+j^*} = K_{t+1+j^*}^*$, and it gives value

$$\begin{aligned}\tilde{w}(K_t) &= \sum_{s=0}^{j^*} \delta^s v(x_{t+j+s}^*) + \delta^{j^*} \sum_{s=0}^{\infty} \delta^s v(\tilde{x}_{t+j^*+s}) \\ &> \sum_{s=0}^{j^*} \delta^s v(x_{t+j+s}^*) + \delta^{j^*} \sum_{s=0}^{\infty} \delta^s v(x_{t+j^*+s}^*) = \bar{w}(K_t^*),\end{aligned}$$

yielding a contradiction and establishing the lemma. *Q.E.D.*

LEMMA 6: *Suppose that Assumption 4′ holds and that* $w_{t'} = \bar{w}(K_{t'})$ *for some* $t' \geq 0$. *Then* $w_t > v(F(K_t, L_t))$ *for all* $t \geq t'$.

PROOF: Suppose that $w_{t'} = \bar{w}(K_{t'})$ for some t'. Then Lemma 5 implies that $w_t = \bar{w}(K_t)$ for all $t \geq t'$. Now to obtain a contradiction, suppose that at some $t \geq t'$ we have $w_t = v(F(K_t, L_t))$. By the second part of Assumption 4′, a feasible variation is as follows: $L_{t+s} = \bar{L}$ and $C_{t+s} = \bar{C}$ for all $s \geq 0$; $K_{t+s} = \bar{K}$ for all $s \geq 1$; and $x_t = F(K_t, \bar{L}) - \bar{C} - \bar{K}$ and $x_{t+s} = F(\bar{K}, \bar{L}) - \bar{C} - \bar{K}$ for all $s \geq 1$.

First suppose $F(K_t, L_t) \leq F(\bar{K}, \bar{L})$. Then this variation gives the politician value

$$\begin{aligned} w' &= v(F(K_t, \bar{L}) - \bar{C} - \bar{K}) + \frac{\delta}{1-\delta} v(F(\bar{K}, \bar{L}) - \bar{C} - \bar{K}) \\ &\geq \frac{\delta}{1-\delta} v(F(\bar{K}, \bar{L}) - \bar{C} - \bar{K}) > v(F(\bar{K}, \bar{L})), \end{aligned}$$

where the first inequality exploits the first part of Assumption 4′ and the last inequality uses $F(K_t, L_t) \leq F(\bar{K}, \bar{L})$. This yields the desired contradiction.

Next suppose that $F(K_t, L_t) > F(\bar{K}, \bar{L})$ (which naturally implies that $K_t > \bar{K}$). Then the above variation gives the politician value

$$\begin{aligned} w' &= v(F(K_t, \bar{L}) - \bar{C} - \bar{K}) + \frac{\delta}{1-\delta} v(F(\bar{K}, \bar{L}) - \bar{C} - \bar{K}) \\ &> v(F(K_t, \bar{L}) - F(\bar{K}, \bar{L})) + v(F(\bar{K}, \bar{L})) \\ &\geq v(F(K_t, \bar{L})) \geq v(F(K_t, L_t)), \end{aligned}$$

where the first inequality uses the first part of Assumption 4′ and that $F(\bar{K}, \bar{L}) > \bar{C} + \bar{K}$ (again from Assumption 4′). The second inequality follows from the fact that for a concave function $f(x) \geq 0$, $f(x) \leq f(x-y) + f(y)$ for $y \leq x$, and the final inequality uses $L_t \leq \bar{L}$. The string of inequalities again leads to a contradiction, establishing that $w_t > v(F(K_t, L_t))$ for all $t \geq t'$. *Q.E.D.*

Consequently, even if $\{w_t\}_{t=0}^{\infty}$ converges to $w_\infty \in \mathrm{Bd}[0, \bar{w}(K_\infty)]$, constraint (5) will ultimately become slack, so that $\psi_t \to 0$ and the desired result follows.

CASE 3: $\{V_w(K_t, w_t)\}_{t=0}^{\infty} \to -\infty$ and there exists a subsequence of $\{w_t\}_{t=0}^{\infty}$ converging to some $w_\infty \in \mathrm{Int}[0, \bar{w}(K_\infty)]$. This implies that either $\{\gamma_t\}_{t=0}^{\infty} \to \infty$ or $\{\psi_t\}_{t=0}^{\infty} \to \infty$. Then the first-order condition $v'(x_t) = \lambda_t / (\gamma_t + \psi_t)$ implies that either $x_t = \infty$ or $\lambda_t = \infty$. The former is impossible in view of the resource constraint (since $Y_t \leq \bar{Y} < \infty$ for all t). The latter would imply that

$U_C(C_t, L_t) \to \infty$. Since U is concave, $U_C \to \infty$ is only possible when $C_t \to 0$. Since $(C, L) \in \Lambda$, this implies $L_t \to 0$, and from Assumption 2, $x_t \to 0$ and thus $w_t \to \bar{w}(K_\infty)$, which is in this case equal to 0. However, Lemma 6 implies that the best SPE with $w_t \to \bar{w}(K_\infty)$ cannot involve $w_t \to 0$; thus this case can be ruled out and this establishes Part 2.

Part 3: Suppose that $\beta > \delta$. If $\{V_w(K_t, w_t)\}_{t=0}^\infty$ converges to some V_w, then (A17) and (A18) imply that $\psi_t \to \psi_\infty > 0$. Then equations (A19) and (A20) immediately imply that the asymptotic allocation is distorted downward. Next, suppose that $\{V_w(K_t, w_t)\}_{t=0}^\infty$ does not converge. Nevertheless, it has a convergent subsequence (which may converge to $-\infty$, but this is ruled out by the same argument as in the previous part). Equations (A17) and (A18) then imply that the multipliers associated with this subsequence satisfy $\psi_{i,t_n} \to \psi_{\infty,n} > 0$. This establishes that $\limsup[F_L(K, L_i)U_C(C_i, L_i) + U_L(C_i, L_i)] > 0$ and $\limsup[\beta F_K(K^+, L_i^+)U_C(C_i^+, L_i^+) - U_C(C, L)] > 0$. Consequently, distortions do not disappear asymptotically.

Finally, renegotiation-proofness follows from the last part of the proof of Proposition 1, completing the proof. *Q.E.D.*

PROOF OF THEOREM 1: The main results in this theorem follow as corollaries of the equivalent results from Theorem 2. We thus only prove the three differences from that theorem. First, there are downward distortions at $t = 0$ (instead of at some $t < \infty$). Since there is no capital, if the sustainability constraint (5) were slack at $t = 0$, it would remain so at all future dates, implying that $x_t = 0$ for all t, and thus $w_0 = 0$. But $U_C(0, 0) > U_L(0, 0)$ implies that in the absence of distortions, $L_0 > 0$, so that deviating and setting $x_0 = L_0 > 0$ would be a profitable deviation for the politician. This yields a contradiction and establishes that there must be downward labor distortions at $t = 0$.

Second, the sequence of $\{V_w(w_t)\}$s is nonincreasing. This, combined with the concavity of V, implies that $\{w_t\}_{t=0}^\infty$ is nondecreasing and thus converges to some $w^* \in [0, \infty]$.

Finally, Assumption 4 implies that $v(\tilde{L} - \tilde{C})/(1 - \delta) > v(\tilde{L})$ and ensures that constraint (5) is slack when w_t converges to $w^* \le v(\tilde{L} - \tilde{C})/(1 - \delta)$. *Q.E.D.*

REFERENCES

ACEMOGLU, D. (2007): "Modeling Inefficient Institutions," in *Advances in Economics and Econometrics: Theory and Applications*, ed. by R. Blundell, W. Newey, and T. Persson. Cambridge, U.K.: Cambridge University Press, 341–380. [620]

ACEMOGLU, D., AND J. A. ROBINSON (2001): "A Theory of Political Transition," *American Economic Review*, 91, 1369–1401. [621]

ACEMOGLU, D., M. GOLOSOV, AND A. TSYVINSKI (2006): "Markets versus Governments: Political Economy of Mechanisms," Working Paper 12224, NBER. [620,632,634]

BARRO, R. (1973): "The Control of Politicians: An Economic Model," *Public Choice*, 14, 19–42. [619]

BATTAGLINI, M., AND S. COATE (2008): "A Dynamic Theory of Spending, Taxation, and Debt," *American Economic Review* (forthcoming). [621]

BENVENISTE, L., AND J. SCHEINKMAN (1979): "On the Differentiability of the Value Function in Dynamic Models of Economics," *Econometrica*, 47, 727–732. [636]

BERTSEKAS, D., A. NEDIC, AND A. OZDAGLAR (2003): *Convex Analysis and Optimization*, Boston: Athena Scientific. [634,636]

BESLEY, T. (2006): *Principled Agents? The Political Economy of Good Government*, London: Oxford University Press. [620]

CHARI, V. V., AND P. KEHOE (1990): "Sustainable Plans," *Journal of Political Economy*, 94, 783–802. [621]

——— (1993): "Sustainable Plans and Mutual Default," *Review of Economic Studies*, 60, 175–195. [621]

FEREJOHN, J. (1986): "Incumbent Performance and Electoral Control," *Public Choice*, 50, 5–25. [619]

FUDENBERG, D., AND J. TIROLE (1994): *Game Theory*. Cambridge, MA: MIT Press. [623]

GOLOSOV, M., N. KOCHERLAKOTA, AND A. TSYVINSKI (2003): "Optimal Indirect and Capital Taxation," *Review of Economic Studies*, 70, 569–587. [632]

HARRIS, M., AND B. HOLMSTROM (1982): "A Theory of Wage Dynamics," *Review of Economic Studies*, 49, 315–333. [620]

HASSLER, J., P. KRUSELL, K. STORESLETTEN, AND F. ZILIBOTTI (2005): "The Dynamics of Government: A Positive Analysis," Mimeo, Stockholm. [621]

KRUSELL, P., AND J.-V. RIOS-RULL (1999): "On the Size of Government: Political Economy in the Neoclassical Growth Model," *American Economic Review*, 89, 1156–1181. [621]

LAZEAR, E. (1981): "Agency, Earnings Profiles, Productivity, and Hours Restrictions," *American Economic Review*, 71, 606–620. [620]

LEHRER, E., AND A. PAUZNER (1999): "Repeated Games With Differential Time Preferences," *Econometrica*, 67, 393–412. [620]

MIRRLEES, J. A. (1971): "An Exploration in the Theory of Optimal Income Taxation," *Review of Economic Studies*, 38, 175–208. [632]

PERSSON, T., AND G. TABELLINI (2000): *Political Economics: Explaining Economic Policy*. Cambridge, MA: MIT Press. [620,623,631]

RAY, D. (2002): "Time Structure of Self-Enforcing Agreements," *Econometrica*, 70, 547–582. [620,628]

THOMAS, J., AND T. WORRALL (1990): "Income Fluctuations and Asymmetric Information: An Example of Repeated Principle–Agent Problem," *Journal of Economic Theory*, 51, 367–390. [633]