A Theory of Liquidity and Regulation of Financial Intermediation

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Abstract

This paper studies a Diamond-Dybvig model of financial intermediation providing insurance against unobservable liquidity shocks in the presence of unobservable trades on private markets. We show that in this case competitive equilibria are inefficient. A social planner finds it beneficial to introduce a wedge between the interest rate implicit in optimal allocations and the economy's marginal rate of transformation. This improves risk-sharing by reducing the attractiveness of joint deviations where agents simultaneously misrepresent their type and engage in trades on private markets. We propose a simple implementation of the optimum that imposes a constraint on the portfolio share that financial intermediaries need to invest in short-term assets. In the case of Diamond-Dybvig preferences, the optimal allocation coincides with the unconstrained optimum. For more general preferences, the optimal allocation does not coincide with the unconstrained optimum, and the direction of the policy intervention depends on the nature of the shocks in a manner that we precisely characterize.

Keywords: Optimal Regulations, Financial Intermediation, Optimal Contracts, Market Failures, Mechanism Design.

1 Introduction

A key role of financial intermediaries is to provide insurance against liquidity shocks. Accordingly, the regulation of financial intermediaries is an important concern for central banks and is a frequent topic of debate in the policy-making community. In this paper we answer several important questions. Can markets provide and allocate liquidity insurance efficiently? If not, can we precisely

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identify the origin of the market failure? Can a regulator design a simple policy rule to improve on the allocations provided by competitive markets alone?

Liquidity is a catch-all term referring to several different concepts (see, for example, von Thadden 1999). This paper discusses the desire of agents to insure against liquidity shocks that might affect them in the future. We focus, in particular, on the aggregate amount of resources set aside to satisfy liquidity shocks. In the model, this corresponds to the fraction of savings invested in short term assets, which we refer to as aggregate liquidity. We identify a market failure leading to the underprovision of liquidity. We then show that a simple regulation, working through a general equilibrium channel by lowering long term interest rates, can restore efficiency. The regulatory intervention is justified not by concerns about individual intermediaries but rather by the inadequacy of the aggregate amount of investment in short-term assets in the financial system as a whole.

More specifically, we study a model where financial intermediaries act as providers of insurance against liquidity shocks, in the spirit to Diamond and Dybvig (1983), Jacklin (1987), and Allen and Gale (2004). The Diamond-Dybvig model is an established workhorse for positive and normative analysis of financial intermediation. Its simplicity allows for a precise understanding of the nature of potential market failures and the mechanics of prescribed policy interventions. In this model, some agents receive liquidity shocks that affect their consumption opportunities. Agents who receive high liquidity shocks value early consumption only and derive a higher indirect marginal utility of income.[1]

We impose two informational frictions. Our results are driven by their interactions. The first friction is that liquidity shocks are private information to the agents. Its consequences are well understood. In a model where there is no other friction, an argument similar to that of Prescott and Townsend (1984) or Allen and Gale (2004) can be used to establish that the first welfare theorem holds. The allocations provided by competitive financial intermediaries are constrained efficient. Consequently, this first friction alone does not justify regulating financial intermediation.

The second friction derives from the limits to the observability of consumption: we assume that consumers can borrow and lend to each other on a private market by engaging in hidden side trades.[2] Since the contributions of Allen (1985) and Jacklin (1987), the possibility of agents engaging in hidden side trades has been recognized as an important constraint on risk sharing.[3] This second friction can be interpreted as the case where contracts with financial intermediaries cannot be made exclusive. Arguably, both unobservability of certain financial market transactions and non-exclusivity are becoming more relevant with the increasing sophistication of financial markets. Agents can and do engage in a variety of financial market transactions and routinely deal with several different intermediaries.

We formalize unobservable trades by considering private markets in which agents can trade after they are allocated consumption profiles by either an intermediary or a social planner. Incentive

[1] Our results would carry over to the case of investment opportunity shocks affecting financially constrained firms.

[2] A different interpretation of this friction is non-exclusivity of contracts.

[3] The importance of access to credit markets as a constraint on the optimal program was also emphasized in Chiappori, Macho, Rey, and Salanié (1994).

compatibility together with the possibility of private trades requires the equalization of the present value of resources given to all agents, discounted at the interest rate prevailing on the private market. Efficient liquidity insurance provision, on the other hand, requires redistribution of resources in the present value sense at the interest rate equal to the economy's marginal rate of transformation towards agents with better consumption opportunities – those with a higher marginal utility of income. In the model, this corresponds to agents affected by a liquidity shock: early consumers.

We first define and characterize the competitive equilibrium in the presence of hidden trades. The competitive equilibrium features limited risk sharing – arbitrage among intermediaries makes the interest rate on the private market and the marginal rate of transformation equal. We continue to define and characterize the constrained efficient allocation in the presence of retrading. By affecting the total amount of resources available in each period, the social planner can introduce a wedge between the interest rate prevailing on the private market and the marginal rate of transformation. We show that lowering the interest rate relaxes incentive constraints and improves risk sharing. The intuition is as follows. The planner wants to allocate a higher present value of resources – discounted at the rate of return on the long term asset – to agents affected by a liquidity shock. However, the planner is constrained by the possibility that late consumers will portray themselves as early consumers and save. Lowering the interest rate reduces the return on such deviations and relaxes incentive compatibility constraints. We then analytically characterize the optimal interest rate and show that the constrained efficient allocation with retrading coincides with the constrained efficient allocation without retrading and with the unconstrained "first-best" solution. For the case of Diamond-Dybvig preferences, the social planner can completely negate the frictions imposed by retrading and private information and achieve the unconstrained allocation. This is in stark contrast with the allocation achieved in a competitive equilibrium where the possibility of unobservable trades poses severe constraints on provision of insurance. While the general point that a government intervention can improve on the allocation in a market system with asymmetric information is well known,[4] a contribution of this paper is to provide a clear understanding of the rationale and the direction of the required intervention in the context of a widely used and policy-relevant model.

We propose a simple implementation of the constrained efficient allocation that relies on a natural regulation imposed on financial intermediaries in a competitive market. The regulation takes the form of the imposition of a liquidity floor that stipulates a minimal portfolio share to be held in the short term asset by intermediaries. The liquidity floor increases the amount of the first period aggregate resources and drives the interest rate on the private markets down. We show how the liquidity floor can be chosen to implement the optimal solution. This simple regulation resembles the different forms of reserve requirements imposed on banks. In practice, reserve requirements were mostly developed as an answer to different concerns pertaining to systemic risk or the fear of bank runs. According to our analysis, they also contribute to mitigating the inefficiency that we highlight. The market failure and the required regulation that we consider are novel but are close in spirit

[4] See Hart (1975), Newbery and Stiglitz (1982) and, for the area of financial intermediation, Allen and Gale (2004).

to some arguments that were made in the early stages of financial regulation during the National Banking era, as described in a classical study by Sprague (1910) and in a modern exposition by Chari (1989).

The final part of the paper extends our characterization of the constrained efficient allocation and its implementation to more general, smooth preferences. We show that the structure of Diamond-Dybvig preferences is somewhat special and that the constrained efficient allocations with and without retrading do not necessarily coincide. We then show that depending on the nature of liquidity shocks, the optimal interest rate may be higher or lower than that on the competitive markets, and that the optimal implementation may stipulate either a minimal or a maximal amount of investment in the short term asset. Suppose that agents affected by liquidity shocks, i.e., a desire to consume early, also have better lifetime consumption opportunities, i.e., a higher indirect marginal utility of income. Then the optimal interest rate is lower than the rate of return on the long term asset and the optimal policy is a liquidity floor. The opposite holds when agents hit by a liquidity shock have worse lifetime consumption opportunities.

2 Relation to the literature

This paper builds on a large literature on financial intermediation (Diamond and Dybvig 1983, Jacklin 1987, Bhattacharya and Gale 1987, Hellwig 1994, Diamond 1997, Von Thadden 1999, Caballero and Krishnamurthy 2003, Allen and Gale 2003, 2004[5]). More generally, our paper fits in the literature of optimal allocations with unobservable taste shocks following Atkeson and Lucas (1992).

Our paper is closely related to Jacklin (1987). That paper compares a competitive equilibrium *with* private markets to the social optimum *without* private markets and reaches the conclusion that the prohibition of private markets leads to a Pareto improvement. In our paper, we solve a planning problem with both unobservable types and private markets. In contrast with Jacklin, we do not prohibit private markets to achieve superior or even unconstrained allocations.

Our paper uses the mechanism design framework and the language of Allen and Gale (2004) to analyze the model of intermediation in the presence of private markets. Our paper shares a common goal with the work of Allen and Gale (2004) in studying whether laissez-faire markets provide and allocate liquidity efficiently. Both papers direct regulation at intermediaries rather than individual consumers. However, we focus on a different mechanism. The result of Allen and Gale (2004) that their equilibrium is inefficient relies on the exogenously imposed incompleteness of markets for trades among intermediaries when there are aggregate shocks. In the absence of incomplete markets for aggregate shocks or in the absence of aggregate shocks, Allen and Gale (2004) conclude that there is no role for regulation. By showing how the planner can manipulate the interest rate on the private markets, we demonstrate that a liquidity requirement can improve upon the competitive equilibrium even when there are incomplete markets for insurance against

[5] For an excellent survey of the literature see Freixas and Rochet (1997) and Gorton and Winton (2002).

aggregate shocks or when there are no aggregate shocks. The characterization of the mechanism through which liquidity requirements affect interest rates and improve upon the market allocation is new to the banking literature.

Holmström and Tirole (1998) provide a theory of liquidity in a model in which intermediaries face borrowing constraints. In their model, a government has an advantage over private markets as it can enforce repayments of borrowed funds while the private lenders cannot. They maintain the assumption of complete markets and show that the availability of government-provided liquidity leads to a Pareto improvement when there is aggregate uncertainty. Lorenzoni (2006) considers a Diamond-Dybvig model of banking with financial markets. His results on the characterization of the optimum are similar to our results for the special case of Diamond-Dybvig setup. However, he maintains a focus on monetary models. Another paper related to our results in the Diamond-Dybvig setup is Caballero and Krishnamurthy (2004). They develop a model of emerging market financial crises in which there is a market for external borrowing and a domestic private market. The domestic market in their model is similar to the private market in our formulation. They show that the equilibrium coincides with the optimal allocation in the presence of private markets. They further show that a range of financial instruments including liquidity requirements and taxes on external borrowing can implement the optimal allocation.

While the focus of this paper is financial intermediation, we also contribute to the literature on optimal policy in the presence of hidden trades.[6] In particular, Golosov and Tsyvinski (2007) study an optimal dynamic Mirrlees taxation model with endogenous private markets. There are two main differences between our paper and their work. The first difference is in the nature of the shocks. In Golosov and Tsyvinski (2007) as in most of models of dynamic taxation (see, e.g., Golosov, Kocherlakota, and Tsyvinski 2003, Golosov, Tsyvinski, and Werning 2006, Kocherlakota 2006, and Farhi and Werning 2007), private information (skill shocks) is dynamic and separable from consumption.[7] In our setup, shocks affect the marginal rate of substitution for consumption and the marginal utility of income. The second difference pertains to the strength of the results that we obtain. Golosov and Tsyvinski (2007) and Bisin et. al. (2001) are able to identify only the direction of a *local* policy change that leads to a Pareto improvement. We characterize the *globally optimal* allocation in the presence of private markets and show that optimal liquidity regulation implements the constrained optimum.

In Diamond (1997), as in our paper, there is more risk sharing among agents of different types than in Jacklin (1987). His result relies on the assumption that some consumers are exogenously restricted from participating in private markets. Unlike that paper, in our model all consumers can participate in markets. An elegant paper by Bisin and Rampini (2006) justifies the institution of bankruptcy in a model of non-exclusive contracts. In their work, borrowers (entrepreneurs) have

[6] See, for example, Arnott and Stiglitz (1986, 1990), Greenwald and Stiglitz (1986), and Hammond (1987). Several recent papers such as Geanakoplos and Polemarchakis (1986) and Bisin et al. (2001) showed, in very general settings, that economies with asymmetric information are inefficient, and argued for Pareto-improving anonymous taxes.

[7] See also Albanesi (2006) for a model of entrepreneurship and financial assets which has elements of unobservable trades.

access to secondary markets. A possibility of default on these secondary contracts decreases returns to hidden borrowing and lending and yields a Pareto improvement.

One justification for reserve requirements in the literature is found in the existence of deposit insurance. The usually given rationale is as follows: deposit insurance encourages risk taking behavior of intermediaries (see, e.g., Merton 1977) which can be controlled by requiring intermediaries to hold adequate levels of liquidity. In this argument, the existence of one potentially suboptimal policy, deposit insurance, justifies another policy – reserve requirements. Typically, however, this literature does not derive deposit insurance as an optimal policy in response to a specified market failure. Moreover, with the exception of Hellman, Murdock, and Stiglitz (2000), the literature does not consider optimal policy in the absence of deposit insurance.

3 Model

We consider a standard model of financial intermediation similar to Diamond and Dybvig (1983) and to Allen and Gale (2004). The economy lasts three periods, $t = 0, 1, 2$. There are two assets (technologies) in the model. The short asset is a storage technology that returns one unit of consumption good at $t+1$ for each unit invested at t. Investment in the long asset has to be done at $t = 0$ to yield $\hat{R} > 1$ units of the consumption good at $t = 2$. Therefore, the time interval from $t = 0$ to $t = 2$ in this model is interpreted, as in the Diamond-Dybvig model, as the time it takes to costlessly liquidate the long-term asset.

The economy is populated by a unit continuum of ex-ante identical agents, or investors. Suppose there are two types of agents denoted by $\theta \in \{0, 1\}$. At $t = 0$, all individuals are (ex-ante) identical and receive an endowment e. At $t = 1$, each consumer gets a draw of his type. With probability $\pi \in (0, 1)$ he is an agent of type $\theta = 0$, and with probability $(1 - \pi)$ he is an agent of type $\theta = 1$; the fraction of agents of each type is therefore π and $1 - \pi$, respectively. We introduce the "baseline" utility function $u : \mathbb{R}^+ \to \mathbb{R}$ and assume that it is twice continuously differentiable, increasing, strictly concave, and satisfies Inada conditions $u'(0) = +\infty$ and $u'(+\infty) = 0$. In terms of the baseline function u, preferences of an agent of type θ are given by utility function $U : \mathbb{R}^+ \times \mathbb{R}^+ \times \{0, 1\} \to \mathbb{R}$, which is assumed to take the form

$$U(c_1, c_2, \theta) = (1 - \theta)u(c_1) + \theta\rho u(c_1 + c_2),$$

where c_1 is agent's consumption in period 1, c_2 is agent's consumption in period 2, and ρ is a constant which is the same for agents of both types. In addition, we assume, as in Diamond and Dybvig (1983), that the coefficient of relative risk aversion is everywhere greater than or equal to 1:

$$\frac{-cu''(c)}{u'(c)} \geq 1 \text{ for all } c > 0, \tag{1}$$

and that $\hat{R}^{-1} < \rho < 1$ (which implies $\rho\hat{R} > 1$).

Agents of type $\theta = 0$ are affected by liquidity shocks, and value consumption in the first period only. Agents of type $\theta = 1$ are indifferent between consuming in the first and the second period. We use these preferences throughout the main body of the paper and, in the the last section, consider a more general class of preferences that demonstrate the somewhat specific properties of the Diamond-Dybvig setup.

A key informational friction is that types of agents are private, i.e., observable only by the agent himself but not by others.

We denote by $\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ an *allocation* of consumption across consumers. An allocation is *feasible* if it satisfies:

$$\pi \left[c_1(0) + \frac{c_2(0)}{\hat{R}} \right] + (1-\pi) \left[c_1(1) + \frac{c_2(1)}{\hat{R}} \right] \leq e. \tag{2}$$

We do not impose a sequential service constraint so there are no bank runs in our model. We also restrict our attention to pure strategies and consider symmetric equilibria (i.e., those in which the strategy of all agents of the same type is the same).

4 Benchmark environment without private markets

In this section, we define and characterize a benchmark economy in which the only friction is unobservability of types. In this environment, agents are given consumption allocations depending on their types. Agents cannot engage in any unobservable transaction, and their consumption is therefore observable.

We start by defining a constrained efficient program, i.e., the problem of the social planner, which we call problem SP^2 or a "second best" problem:

$$\max_{\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}} \pi u(c_1(0)) + (1-\pi)\rho u(c_1(1) + c_2(1)) \tag{3}$$

s.t.

$$\pi \left(c_1(0) + \frac{c_2(0)}{\hat{R}} \right) + (1-\pi) \left(c_1(1) + \frac{c_2(1)}{\hat{R}} \right) \leq e, \tag{4}$$

$$u(c_1(0)) \geq u(c_1(1)), \tag{5}$$

$$u(c_1(1) + c_2(1)) \geq u(c_1(0) + c_2(0)). \tag{6}$$

The planner maximizes expected utility of an agent subject to the feasibility constraint (4) and two incentive compatibility constraints. Constraint (5) ensures that an agent of type $\theta = 0$ does not want to pretend to be an agent of type $\theta = 1$. Constraint (6) ensures that an agent of type $\theta = 1$ does not want to pretend to be an agent of type $\theta = 0$.

We can also define an unconstrained optimum that we call SP^1 in which there is no private information – the "first best" program. That program differs from the problem SP^2 in that the incentive compatibility constraints (5) and (6) are omitted.

As noted by Diamond and Dybvig (1983), the incentive compatibility constraints are not binding at the optimum of (3). In other words, solutions to problems SP^1 and SP^2 coincide. The following Theorem establishes this formally.

Theorem 1 *Solutions to problems SP^1 and SP^2 coincide, and are fully characterized by*

$$c_2(0) = c_1(1) = 0, \tag{7}$$

$$u'(c_1(0)) = \rho \hat{R} u'(c_2(1)), \tag{8}$$

$$\pi c_1(0) + (1-\pi)\frac{c_2(1)}{\hat{R}} = e. \tag{9}$$

Moreover, $c_1(0) > e$ and $c_2(1) < \hat{R}e$.

Proof. In the Appendix. ■

The planner redistributes resources to consumers of type $\theta = 0$ who are given a higher present value of consumption than the value of their endowment. Late consumers, those with $\theta = 1$, receive consumption that is less than the present value of their endowment.

We can also define a competitive equilibrium problem in which there is a continuum of intermediaries providing insurance to agents. The intermediaries are subject to the same constraint as the social planner and do not observe the types of agents. We omit a formal definition here. Note however that a version of the first welfare theorem would hold here as shown by Prescott and Townsend (1984) and Allen and Gale (2004): the competitive equilibrium allocations would coincide with the solution to the problem SP^2. The key to this result is that consumption is observable – agents cannot engage in unobservable trades.

5 Private markets

The allocations described in the previous section may not be achieved if agents can engage in private transactions. Allen (1985) and Jacklin (1987) were the first to point out that the possibility of such trades may restrict risk sharing across agents. In this section, we first formally describe how to model unobservable consumption. This formalization will be central to defining and characterizing both competitive equilibria and constrained efficient allocations with private markets.

Consider an environment in which all consumers have access to a market in which they can trade assets among themselves unobservably.[8] Formally, suppose that consumers are offered a menu of contracts $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$. A consumer treats the contract and the equilibrium interest rate R on the private market as given and chooses his optimal reporting strategy θ' that determines his endowment of consumption $(c_1(\theta'), c_2(\theta'))$. Unlike in the environment without private markets,

[8] All our analysis is easily extended to the case in which agents can trade not only among themselves but also with other intermediaries. This case would bring this model closer to an interpretation as an environment of non-exclusive contracts. A key assumption that allows us to extend our results to that case is that portfolios of the intermediaries (investment in short and long assets) are observable while transactions with individual consumers are not. Our choice of modeling side trades as private markets allows us to economize on notation without affecting the substance of the results.

the actual after-trade consumption (x_1, x_2) may differ from the consumption specified in the contract, since it is impossible to preclude a consumer from borrowing and lending a certain amount s on the private market.[9] Given a menu of consumption allocations $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ and an interest rate R, an agent of type θ solves:

$$\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta) = \max_{x_1,x_2,s,\theta'} U(x_1, x_2; \theta), \tag{10}$$

subject to:

$$x_1 + s = c_1\left(\theta'\right), \tag{11}$$

$$x_2 = c_2\left(\theta'\right) + Rs. \tag{12}$$

In what follows, we define $x_1(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $x_2(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $s(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $\theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$ as a solution to problem (10).

We now formally define an equilibrium in the private market.

Definition 1 *An equilibrium in the private market given the profile of endowments $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ consists of interest rate R and, for each agent of type θ: allocations $x_1(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $x_2(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, trades $s(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, and choices of reported types $\theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$ such that*

(i) $x_1(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $x_2(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $s(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $\theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$ constitute a solution to problem (10);

(ii) the feasibility constraints on the private market are satisfied for $\forall t = 1, 2$:

$$\begin{aligned} &\pi x_t(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 0) + (1-\pi)\, x_t(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 1) \\ \leq\ &\pi c_t\left(\theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 0)\right) + (1-\pi)\, c_t\left(\theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 1)\right). \end{aligned} \tag{13}$$

6 Competitive equilibrium with private markets CE^3

In this section, we formally describe competitive equilibria and show how risk sharing is hindered by the possibility of agents engaging in unobservable trades in private markets.

Consider a market with a continuum of intermediaries. We assume throughout the paper that all activities at the intermediary level are observable. In period 0, before the realization of idiosyncratic shocks, consumers deposit their initial endowment with an intermediary. The intermediary provides a menu of consumption allocations $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$. In the presence of private markets, intermediaries need to take into account, in addition to unobservable types, that consumers are able to engage in transactions in the private market. Contracts are offered competitively, and there is free entry for intermediaries. Therefore, each consumers sign a contract with the intermediary who promises the highest ex-ante expected utility. We denote the equilibrium utility of a consumer by $\underline{U}$.

[9] It can be shown that a consumer trades only a risk free security (Golosov and Tsyvinski 2007).

We assume that intermediaries can trade bonds b among themselves. Without aggregate uncertainty the market for these trades is very simple. We denote by q the price of a bond b in period $t = 1$ that pays one unit of consumption good in period 2. All intermediaries take this price as given. They also pay dividends d_1, d_2 to its owners.[10]

It is important to note that intermediaries take the interest rate on the private market R as given. The maximization problem of the intermediary that faces intertemporal price q, interest rate on the private market R, and reservation utility of consumers $\underline{U}$ is

$$\max_{\{c_1(\theta),c_2(\theta)\}_{\theta\in\{0,1\}},(d_1,d_2),b} d_1 + \frac{d_2}{\hat{R}} + qb - \frac{b}{\hat{R}} \tag{14}$$

s.t.

$$\pi\left(c_1(0) + \frac{c_2(0)}{\hat{R}}\right) + (1-\pi)\left(c_1(1) + \frac{c_2(1)}{\hat{R}}\right) + d_1 + \frac{d_2}{\hat{R}} + qb - \frac{b}{\hat{R}} \leq e, \tag{15}$$

$$\theta = \theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta),\ \forall\theta, \tag{16}$$

$$\pi\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 0) + (1-\pi)\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 1) \geq \underline{U}\ . \tag{17}$$

The first constraint in the intermediary's problem is the budget constraint. The second constraint is incentive compatibility that states that, given the profile of consumptions $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ and the possibility to borrow or lend at an interest rate R, consumers choose to truthfully reveal their types, i.e. the true type θ is a solution to the problem (10); we can restrict the intermediaries to truth-telling mechanisms because the Revelation Principle applies. The last constraint states that the intermediary cannot offer a contract which delivers a lower expected utility than the equilibrium utility $\underline{U}$ from the contracts offered by other intermediaries. In equilibrium, all intermediaries act identically and make zero profits. The definition of the competitive equilibrium is then as follows.

Definition 2 *A competitive equilibrium with private markets, CE^3, is a set of allocations $\{c_1^*(\theta), c_2^*(\theta)\}_{\theta\in\{0,1\}}$, a price q^*, dividends $\{d_1^*, d_2^*\}$, bond trades b^*, utility $\underline{U}^*$, and the interest rate on the private market R^* such that*

(i) each intermediary chooses $\{\{c_1^*(\theta), c_2^*(\theta)\}_{\theta\in\{0,1\}}, \{d_1^*, d_2^*\}, b^*\}$ to solve problem (14) taking q^*, R^*, and $\underline{U}^*$ as given;

(ii) consumers choose the contract of an intermediary that offers them the highest ex-ante utility;

(iii) the aggregate feasibility constraint (2) holds;

(iv) the private market, given the menus $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$, is in an equilibrium of Definition 1, and R is an equilibrium interest rate on the private market;

(v) intermediaries make zero profits;

(vi) bond markets clear: $b = 0$.

It is easy to see that the interest rate on the markets for trades among intermediaries must be

[10] Since intermediaries make zero profits in equilibrum, we do not formally specify how these dividends are distributed.

equal to the return on the production technology, so that $1/q = \hat{R}$. We now present a lemma that shows that the incentive compatibility constraints (16) can be expressed in a simple form: the net present value of resources allocated to each type must be equalized when discounted at the market interest rate R. The proof is simple. If the present values are not equated across types, an agent would pretend to claim a type that gives a higher present value of allocations and engage in trades on the private markets to achieve its desired consumption allocation.

Lemma 1 *An allocation satisfies the incentive compatibility constraint (16) if and only if*

$$c_1(0) + \frac{c_2(0)}{R} = c_1(1) + \frac{c_2(1)}{R}. \tag{18}$$

Proof. In text above. ■

Let us rewrite the problem of the intermediary in a more tractable form by considering its dual, simplifying the incentive compatibility constraint using Lemma 1 and the fact that $d_1 + \frac{d_2}{\hat{R}} = 0$ and $b = 0$, since we are interested in symmetric allocations only:

$$\max_{\{c_1(\theta),c_2(\theta)\}_{\theta\in\{0,1\}}} \pi\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 0) + (1-\pi)\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 1), \tag{19}$$

s.t. (18) and

$$\pi\left(c_1(0) + \frac{c_2(0)}{\hat{R}}\right) + (1-\pi)\left(c_1(1) + \frac{c_2(1)}{\hat{R}}\right) \leq e. \tag{20}$$

We now argue that $R = \hat{R}$, otherwise, arbitrage opportunities are created. For example, suppose that $R < \hat{R}$, i.e. the interest rate on the private market is lower than $\hat{R}$. An intermediary then chooses to invest only in the long asset (and therefore only offers contracts paying $\pi c_2(0) + (1-\pi) c_2(1) = \hat{R}e$ in period $t = 2$) and sets investment in the short asset to be equal to zero (paying $\pi c_1(0) + (1-\pi) c_1(1) = 0$ in period $t = 1$). Since consumers care only about the present value of the contract because private market exists, they will buy this contract. In period $t = 1$, after the types are realized, agents of type $\theta = 0$ would borrow on the private market at the interest rate R while agents of type $\theta = 1$ would not be able to supply first period good. Hence, the market clearing condition would not hold, and we conclude that that $R < \hat{R}$ cannot be an equilibrium interest rate. Analogously, we rule out $R > \hat{R}$. The only candidate equilibrium interest rate is $R = \hat{R}$ so that intermediaries do not engage in arbitrage.

At interest rate $\hat{R}$, the intermediary is indifferent between investing in short asset and long asset. However, since $\hat{R} > 1$, consumers of type $\theta = 1$ would only demand second period goods on the private market, while consumers of type $\theta = 0$ would demand first period goods only. Incentive compatibility, given Lemma 1 with $R = \hat{R}$, implies that competing intermediaries would deliver goods of present value e to each of consumers. Consequently, market clearing condition requires that there are πe units of first period good and $(1-\pi)\hat{R}e$ units of second period good available on the market; consumers with $\theta = 0$ consume first period goods only while consumers $\theta = 1$ consume second period ones. We summarize this reasoning in the following proposition.

Proposition 1 *Let R^* denote equilibrium price on the private market corresponding to the competitive equilibrium in Definition 2. Then $R^* = \hat{R}$. Moreover,*

$$\begin{aligned} c_1(1) &= e, \qquad c_2(0) = 0, \\ c_1(1) &= 0, \qquad c_2(1) = \hat{R}e. \end{aligned}$$

Proof. In text above. ■

This proposition states that risk sharing is severely limited in a competitive equilibrium with side trades. Arbitrage among competing intermediaries forces the equilibrium interest rate on the private market to be equal to the return on the long run asset $\hat{R}$. Then, as in Jacklin (1987) and Allen and Gale (2004), the present values of consumption entitlements (evaluated at $\hat{R}$) are equated across consumers of different types:

$$c_1(0) + \frac{c_2(0)}{\hat{R}} = c_1(1) + \frac{c_2(1)}{\hat{R}}.$$

7 Constrained efficient allocation with private markets

In this section, we define and characterize the constrained efficient problem with private markets. We call the program SP^3 or the "third best" program. Consider a social planner that cannot observe or shut down trades on private markets and cannot observe agents' types. The difference with the problem SP^2 is that, in addition to the private information faced by SP^2, planner SP^3 faces constraints that agents may trade on the private market. The social planner SP^3 chooses the allocation $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ that maximizes the ex ante utility of consumers. The revelation principle shows that, without loss of generality, the social planner can offer a contract $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ so that all consumers choose to report their types truthfully to the planner and not to trade on the private market.

Formally, the constrained efficient allocation $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ is the solution to the problem SP^3 given by:

$$\max_{\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}} \pi U(c_1(0), c_2(0); 0) + (1-\pi) U(c_1(1), c_2(1); 1), \tag{21}$$

s.t.

$$\pi\left(c_1(0) + \frac{c_2(0)}{\hat{R}}\right) + (1-\pi)\left(c_1(1) + \frac{c_2(1)}{\hat{R}}\right) \le e, \tag{22}$$

$$U(c_1(\theta), c_2(\theta); \theta) \ge \tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R; \theta)\ \forall\theta, \tag{23}$$

where R is an equilibrium interest rate on the private market, given the profile of endowments $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ according to Definition 1.

We now show that choosing consumption allocations in the constrained efficient problem (21) is equivalent to the problem of a planner choosing an interest rate R on the private market and allocating the same income (present value of consumption allocations) I to agents of different types. The planner can introduce a wedge between the interest rate R on the private market and the rate of return on the long run asset $\hat{R}$. The incentive compatibility constraint again presents itself as a requirement that the same present value of resources I is allocated across agents of different types. Therefore, the planner effectively has only two instruments: an income I and an interest rate R.

Formally, we proceed as follows. Let:

$$V(I,R;\theta) = \max_{x_1,x_2} U(x_1,x_2;\theta) \tag{24}$$

subject to

$$x_1 + \frac{x_2}{R} \leq I, \tag{25}$$

be the ex post indirect utility of an agent of type θ if her income is I, and the interest rate on the private market is R. Denote the solutions to this problem (uncompensated demands) by $x_1^u(I,R;\theta)$ and $x_2^u(I,R;\theta)$.

Consider the problem of a social planner who chooses the interest rate R and income I to maximize the expected indirect utility of agents subject to feasibility constraints.

$$\max_{I,R} \pi V(I,R;0) + (1-\pi)\, V(I,R;1) \tag{26}$$

subject to

$$\pi \left(x_1^u(I,R;0) + \frac{x_2^u(I,R;0)}{\hat{R}} \right) + (1-\pi) \left(x_1^u(I,R;1) + \frac{x_2^u(I,R;1)}{\hat{R}} \right) \leq e, \tag{27}$$

where $x_1^u(I,R;\theta), x_2^u(I,R;\theta)$ are defined above as solutions to (24).

We now prove the equivalence of the problem (21) and the problem (26).

Lemma 2 *Let I^* and R^* be solutions to (26), and $\{x_1^u(I^*,R^*;\theta), x_2^u(I^*,R^*;\theta)\}_{\theta\in\{0,1\}}$ be solutions to (24) given I^* and R^*. Then $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ defined by*

$$c_t(\theta) = x_t^u(I^*,R^*;\theta),\ \forall\theta \in \{0,1\}\ \ \forall t \in \{1,2\} \tag{28}$$

are solutions to problem (21). Conversely, if $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ solves problem (21), then there exist I^* and R^* which solve (26) *if $\{x_1^u(I^*,R^*;\theta), x_2^u(I^*,R^*;\theta)\}_{\theta\in\{0,1\}}$ are given by* (28)*, and such that $\{x_1^u(I^*,R^*;\theta), x_2^u(I^*,R^*;\theta)\}_{\theta\in\{0,1\}}$ solve (24) for $I = I^*$ and $R = R^*$.*

Proof. In the appendix. ■

The above lemma reduces the dimensionality of the problem. The planner chooses only two variables: the interest rate R and income I. Using this lemma, we can now provide a characterization of the constrained efficient allocation.

Theorem 2 *Solutions to the constrained efficient problem with private markets, SP^3, constrained efficient problem without private markets, SP^2, and informationally unconstrained problem, SP^1, coincide. Moreover, the interest rate R^* on the private market corresponding to the solution of SP^3 is such that $R^* \in (1, \rho\hat{R}]$. If $u(c) = \log(c)$, then $R^* = \rho\hat{R}$.*

Proof. In the appendix. ■

This theorem is one of the central results of the paper: a social planner, even in the presence of hidden trades, can achieve allocations superior to the ones achieved by competitive markets. Moreover, we fully characterize the constrained efficient allocation and show that for the case of Diamond-Dybvig preferences, it coincides with the unconstrained, full information optimum, SP^1. The intuition for the result is that lowering the interest rate relaxes the incentive compatibility constraints. Consider a relevant deviation in the model. An agent of type $\theta = 1$ wants to claim to be an agent of type $\theta = 0$ and then save the allocation $c_1(0)$ at the private market interest rate R^*. An interest rate on the private markets $R^* < \hat{R}$ reduces the profitability of this deviation. In the case of Diamond-Dybvig preferences, lowering the interest rates allows perfect screening of the different types and achieves not only the constrained efficient allocation SP^3 but also the unconstrained optimum SP^1. Note that the manipulation of the equilibrium interest rate by the planner is indirect and happens through the general equilibrium effect of changing the profile of endowments. The planner can increase the amount of investment in the short asset (amount of allocations paid in the first period) and correspondingly reduce the amount of investment in the long asset (amount of allocations paid in the second period). Lemma 2 showed that such a manipulation of endowments induces the desired change of the interest rate in the private market.

More generally, lowering the interest rate benefits agents who value consumption in the first period more. If these agents also have a higher marginal utility of income – as is the case for Diamond-Dybvig preferences – this leads to an improvement in the provision of liquidity insurance and in the ex ante welfare. Recall that the unobservability of agents' types and possibility of trades require that agents of various types receive the same present value of consumption evaluated at the private market interest rate R:

$$c_1(0) + \frac{c_2(0)}{R} = c_1(1) + \frac{c_2(1)}{R}.$$

However, the amount of resources evaluated at the real rate of return may differ across agents

$$c_1(\theta) + \frac{c_2(\theta)}{\hat{R}} \neq c_1(\theta') + \frac{c_2(\theta')}{\hat{R}}.$$

In our case, the change in the interest rate transfer resources to the agent $\theta = 0$ affected by a liquidity shock, who is marginally more valuable to the regulator.

What can we conclude from Theorem 2? Intermediaries in the competitive equilibrium provide limited risk sharing. The planner can improve upon the competitive equilibrium and in fact achieve the unconstrained optimum. In the next section we show how the imposition of a simple regulation on financial intermediaries in a competitive equilibrium can implement the constrained efficient allocation.

8 Implementing constrained efficient allocations – liquidity requirements

In this section we show that there exists an intervention – *a liquidity floor* – that implements the constrained efficient allocation SP^3.

A liquidity requirement is a constraint imposed on all intermediaries, i.e., a constraint on the problem (14) that requires that investment in the short asset (payments to the consumers in the first period) for any intermediary should be higher than a level i

$$\pi c_1(0) + (1-\pi) c_1(1) \geq i. \tag{29}$$

An attractive feature of a liquidity requirements is that it does not require a regulator to observe individual contracts $c_1(\theta)$ – only the aggregate portfolio allocation of the intermediaries needs to be observed.

We now intuitively describe the effects that a binding liquidity requirement has on the interest rate on private markets. Let $\hat{X}$ be the investment in the short asset that arises in a competitive equilibrium as in Definition 2. Suppose that a liquidity floor i is set higher than the amount of the first period claims provided by competitive markets:

$$i \geq \hat{X}.$$

When a liquidity floor is imposed, the aggregate endowment in the first period is equal to i rather than $\hat{X}$. Private trading markets in which agents participate after receiving their allocation from the intermediaries are an exchange economy: at the aggregate level, no resources can be transferred at this stage from one period to the next. The liquidity floor increases the aggregate endowment of the first period good in the private market (and, correspondingly, decreases the aggregate endowment of the second period good) and, therefore, has a general equilibrium effect in indirectly lowering the interest rate R below $\hat{R}$.

In the absence of regulations, the interest rate R on the private market is equal to $\hat{R}$. As we showed in Proposition 1, any difference between R and $\hat{R}$ would be arbitraged away by intermediaries. Imposing a liquidity floor lowers the interest rate and implements the constrained efficient allocation by putting a limits on this arbitrage.

Proposition 2 *Let the liquidity floor i^* defined in (29) be given by*

$$i^* = \pi I^*, \tag{30}$$

where I^* is the solution to (26). Then competitive equilibrium allocation specified in Definition 2 with the imposed liquidity floor (formally, an additional constraint (29)*) coincides with the constrained efficient allocation SP^3.*

Proof. In the Appendix ■

This proposition is important as it specifies a simple regulation that implements the optimum. Note that this regulation does not prohibit private markets. Rather, it affects the investments and holding of assets by financial intermediaries. In general, deriving implementations of constrained efficient allocations is a difficult task in environments where private trades are possible. An abstract treatment of a related problem is given in Bisin et al. (2001) who show that, in a general class of environments with anonymous markets, taxes can achieve Pareto improvements. The difference with our setup is that they do not define the constrained efficient problem SP^3 but rather show that a local linear tax can improve upon the market allocation. Golosov and Tsyvinski (2007) study a dynamic model of optimal taxation and define the optimal program similar to our SP^3. They also show that a linear tax on savings may locally improve upon the competitive equilibrium allocation.

9 Some historical background

In this section we argue that some elements of our model can be connected to the debates that were taking place in the period of the National Banking System in the United States (1863-1913). We follow the discussion of the classic work by Sprague (1910a, 1910b) and the modern exposition and interpretation by Chari (1998). Sprague and Chari are mostly interested in banking crises and panics while we focus on liquidity provision more generally. However, some of their arguments identify frictions that resemble the ones that we are emphasizing.

The National Banking System of reserves was a three-tier structure: regional banks (the first tier), designated banks in reserve cities (the second tier), and designated banks in New York City (the third tier). The different tiers of the banking system were subject to different reserve requirements. Banks in the first and second tiers could decide whether to hold their reserves in cash or to deposit them in institutions of the upper tier. As a result, lower tier banks had an incentive to deposit their reserves in New York City banks rather than holding liquid assets or cash. In effect, the reserves of the lower tier banks deposited in New York City were loans and did not contribute to the overall amount of reserves in the system.

At the time, the demand for withdrawals fluctuated with the quality of the crops and was hard to predict. In other words, liquidity shocks were prevalent. The National Banking System experienced several major banking crises. Many commentators argued that these crises were in part due to the insufficient amount of aggregate reserves in the form liquid assets set aside by the financial system. Sprague (1910a, pp. 96-97), commenting on the crisis of 1873, wrote that "The aggregate [reserves] held by all national banks of the United States does not finally much exceed 10 per cent of their direct liabilities." This amount was much lower than the statutory requirement.

The blame was put on the practice of paying interest on the reserves deposited to the New York City banks. Sprague writes "But this practice of paying interest on bankers' deposits, as it now obtains, has other and more far-reaching consequences. It is an important cause of the failure to maintain a reserve of lending power in periods of business activity and the fundamental cause of the failure" (1910b) and "The abandonment of the practice of paying interest upon deposits will remove a great inducement to divide ... reserves between cash in hand and deposits in cities" (Sprague 1910a, p. 97). In other words, interest rates on reserves in New York banks were too high, crowding out liquid assets.[11]

In response to the crisis of 1873, the New York Clearing House Association was created. Its main purpose was to improve the allocation of liquidity by allowing banks to draw on each other's reserves. However, financial innovation progressively undermined the role of the Clearing House Association. In particular, the rise of trust companies in the beginning of 1900s contributed significantly to the severe crisis of 1907 (Moen and Tallman 1992). The trust companies accounted for a significant amount of assets – nearly as much as banks – and had very small reserve requirements: they did not fall under the banking regulations and were not part of the New York Clearing House Association. These trusts, however, were engaged in significant transactions with the banks that were members of the Association, and member banks often used trusts to circumvent reserve requirements (Sprague, 1910a, p.227).

These episodes illustrate several features of financial intermediation that are also at play in our model. First, aggregate liquidity – and not only concerns about the liquidity or solvency of any particular individual intermediary – matters. Second, high interest rates inefficiently divert resources from low-return liquid assets. Finally, side trades and financial innovation can severely undermine financial regulations. A lesson for our times is that an efficient regulation should have a wide scope and cover a variety of financial institutions – for example, mutual funds and hedge funds.

10 General preferences

The analysis of the previous sections characterized constrained efficient allocations in the presence of private markets and showed that a liquidity floor implements the optimal allocation in the case of Diamond-Dybvig preferences. Moreover, the constrained efficient allocations with and without private markets coincide. In this section, we consider a more general specification of preferences. We show that the form of preferences matters for the form of the optimal regulation. Also, the ability of agents to engage in trades may lead to constrained efficient allocations inferior to those without trades. We then provide an analytical characterization of this more general model.

[11] See also the address of George S. Coe, a prominent financier of that time, to the New York Clearing House Association discussing why individual banks have an incentive to underinvest in the assets with short term maturity (Sprague 1910a, pp. 377-378).

10.1 Setup

In this section, we consider a more general model of financial intermediation. There is now a continuum of possible types. We denote the preference shock by $\theta \in \Theta = [\theta_L, \theta_H] \subset [0,1]$, where $\theta_L < \theta_H$. At $t = 1$, each consumer gets an i.i.d. draw of his type from a distribution with c.d.f. $F(\theta)$. We assume that the "law of large numbers" holds, and that the cross-sectional distribution of types is the same as the probability distribution F. One can, therefore, interpret $F(\theta)$ as the share of agents with types below θ. Investors' preferences are represented by a utility function $u(c_1, c_2; \theta)$, where c_t denotes consumption at date $t = 1$, 2. The utility function $u(\cdot, \cdot; \theta)$ is assumed to be concave, increasing, and continuous for every type θ. We also assume the following single crossing property.

Assumption 1 (Single crossing): $\frac{\partial}{\partial\theta}\left(\frac{\partial u/\partial c_2}{\partial u/\partial c_1}\right) > 0.$

Specifically, we focus on three types of preferences which we use to study discount factor shocks, liquidity shocks, and valuation-neutral shocks. Let $\hat{u}(\cdot)$ be concave, increasing, and continuous.

Example 1 *Discount factor shocks:* $u(c_1, c_2; \theta) = \hat{u}(c_1) + \theta\hat{u}(c_2)$.

The first feature of these preferences is that an agent with a higher θ shock has a higher marginal utility of consumption in the second period. The second feature of these preferences is that an agent with higher θ has higher lifetime marginal utility of income.

Example 2 *Liquidity shocks:* $u(c_1, c_2; \theta) = \frac{1}{\theta}\hat{u}(c_1) + \hat{u}(c_2)$.

In this case, a low θ shock increases marginal utility of consumption in the first period. The second feature of these preferences is that an agent with lower θ has a higher lifetime marginal utility of income than an agent with higher θ.[12] These preferences are a straightforward generalization of the Diamond-Dybvig setup.

Example 3 *Valuation-neutral shocks: Let* $\hat{u}(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$ *and*

$$u(c_1, c_2; \theta) = \frac{1-\theta}{\left[\theta^{1/\sigma} + (1-\theta)^{1/\sigma}\hat{R}^{\frac{1-\sigma}{\sigma}}\right]^{\sigma}}\hat{u}(c_1) + \frac{\theta}{\left[\theta^{1/\sigma} + (1-\theta)^{1/\sigma}\hat{R}^{\frac{1-\sigma}{\sigma}}\right]^{\sigma}}\hat{u}(c_2). \tag{31}$$

If $\hat{u}(c) = \log(c)$, *then*

$$u(c_1, c_2; \theta) = (1-\theta)\hat{u}(c_1) + \theta\hat{u}(c_2).$$

[12] A natural question arises whether uility specification of liquidity shocks $\frac{1}{\theta}\hat{u}(c_1) + \hat{u}(c_2)$ is a renormalization of the discount shocks $\hat{u}(c_1) + \theta\hat{u}(c_2)$, and that by dividing utility in the case of discount shocks by θ we would arrive to the model with liquidity shocks. It is true that both of preferences have the same marginal rates of substitution. However, the preferences are different in the direction of marginal utility of income. In the case of liquidity shocks, it is low θ that gives an agent a higher marginal utility of income. In the case of discount factor shocks, it is exactly the opposite – higher θ leads to higher lifetime marginal utility of income.

In this case, agents differ in marginal utility of consumption across periods, but all agents have the same marginal value of income. Note that in the case of the log utility, there is no need to normalize preferences by $\hat{R}$, and valuation-neutral preferences do not depend on technology.

10.2 Characterization

Many of the definitions and results of the previous sections immediately apply in this generalized setup. We omit the formalism for the cases where results derived above directly generalize. We refer the reader to the working paper version for the more detailed analysis. Specifically, the definition of the equilibrium in private markets given by Definition 1 immediately extends to this section. Let R be the interest rate on the private market. The analysis of the competitive equilibrium with private markets in this environment is a direct extension of Definition 2. As in Proposition 1, we conclude that in a competitive equilibrium, redistribution is limited, as the present values of consumption allocations are equated for all consumers at the interest rate $\hat{R}$.

We can now extend the definition of the constrained efficient problem with private markets, SP^3 :

$$\max_{\{c_1(\theta),c_2(\theta)\}_{\theta\in\Theta}} \int_\Theta u\left(c_1\left(\theta\right), c_2\left(\theta\right); \theta\right) dF\left(\theta\right), \tag{32}$$

s.t.

$$\int_\Theta \left(c_1\left(\theta\right) + \frac{c_2\left(\theta\right)}{\hat{R}}\right) dF\left(\theta\right) \leq e, \tag{33}$$

$$u\left(c_1\left(\theta\right), c_2\left(\theta\right); \theta\right) \geq \tilde{V}(\{c_1\left(\theta\right), c_2\left(\theta\right)\}_{\theta\in\Theta}, R; \theta), \forall\theta, \theta', \tag{34}$$

where R constitutes an equilibrium on the private market, given the profile of endowments $\{c_1\left(\theta\right), c_2\left(\theta\right)\}_{\theta\in\Theta}$ (the corresponding problem of a consumer of type θ, which defines $\tilde{V}(\{c_1\left(\theta\right), c_2\left(\theta\right)\}_{\theta\in\Theta}, R; \theta)$, is the same as (10)).

As a next step we define a relevant notion of liquidity requirement. Here, it is more general and can take the form of either a liquidity floor or a liquidity cap. Formally, a liquidity requirement is a constraint imposed on all intermediaries, that requires that investment in the short asset for any intermediary should be higher (lower) than a certain level i:[13]

$$\left(\int_\Theta c_1\left(\theta\right) dF\left(\theta\right)\right) \begin{array}{c}\geq \\ \leq\end{array} i. \tag{35}$$

We call a liquidity requirement a *liquidity cap* if (35) is imposed with a less than or equal to sign. A liquidity cap stipulates the maximal amount of the short asset that an intermediary can hold. We call a liquidity requirement a *liquidity floor* if (35) is imposed with a greater or equal sign.

We introduce the following notation. First, $x_t^c(V\left(I, R, \theta\right), R, \theta)$ denotes the compensated demand of agents of type θ in period $t \in \{1, 2\}$ which solves problem (24). We use I and R indices

[13] Formally, we define a problem that is a generalization of the problem of an intermediary (14) and impose the liquidity requirement as an additional constraint as we did for Proposition 2.

to denote partial derivatives with respect to income I and interest rate R, respectively.

Assumption 2. *For all* (I, R),

$$\int_{\Theta} x^c_{2,R}(V(I, R, \theta), R, \theta) dF_\theta + \operatorname{Cov}\left\{x^u_{2,I}(I, R; \theta), \frac{x^u_2(I, R; \theta)}{R^2}\right\} > 0. \tag{36}$$

The lemma that follows presents two natural cases in which Assumption 2 holds.

Lemma 3 *Assumption* 2 *holds under the conditions that follow.*

1. *The function* $u(x_1; x_2, \theta)$ *is homothetic of degree* 1 *with respect to* (x_1, x_2).
2. *The variance of the shocks is small. Consider a family of distributions* $\{F^\gamma\}$ *indexed by* $0 \le \gamma \le 1$ *with support in* $[\theta_L, \theta_H]$. *Suppose that* $F^\gamma(\theta)$ *is continuous in* (θ, γ) *and* $\lim_{\gamma \to 0} \sigma_{F^\gamma} = 0$ *where* σ_{F^γ} *is the variance of* F^γ. *Then there exists* $0 < \bar{\gamma} < 1$ *such that for all* $0 \le \gamma \le \bar{\gamma}$, *Assumption* 2 *holds.*

Proof. In the Appendix. ■

The theorem that follows characterizes the constrained efficient allocation SP^3 and determines the form of the liquidity requirement that implements this allocation for different forms of these more general preferences.

Theorem 3 *Suppose Assumption 2 holds. Let* R^* *be the interest rate on the private market in the solution to the constrained efficient problem* SP^3.

1. *If preferences are of the discount shock form as in example* 1, *then* $R^* > \hat{R}$. *Competitive markets are inefficient, and the liquidity requirement that implements* SP^3 *is a liquidity cap.*
2. *If preferences are of liquidity shock form as in example* 2, *then* $R^* < \hat{R}$. *Competitive markets are inefficient, and the liquidity requirement that implements* SP^3 *is a liquidity floor.*
3. *If preferences are valuation-neutral shocks, then* $R^* = \hat{R}$. *No regulations are needed. Moreover, the solution to the optimal problem* SP^2 *without private markets, constrained efficient solution with private markets* SP^3, *and the solution to the competitive equilibrium with private markets* CE^3 *coincide.*

Proof. In the Appendix. ■

Theorem 3 provides a characterization of the constrained efficient allocation and the form of liquidity adequacy requirement implementing it. We show how the structure of the implementation depends on agents' preferences. The proof of the theorem also derives characterization of the interest rate R^* in terms of an easily interpretable wedge.

We now provide a discussion of these examples to highlight the rationale and the direction of the intervention in Theorem 3. In the case of liquidity shocks (Example 2), the planner wants to allocate a higher present value of resources to agents with lower θ as they have higher marginal lifetime utility of income. An agent with $\theta' > \theta$ wants to engage in the following deviation: pretend to be an agent with a lower type θ and save on the private market. An interest rate $R < \hat{R}$ discourages such deviation and relaxes the incentive compatibility constraint. In the case of discount factor shocks (Example 1), the direction of the deviation is reverse: to pretend to be an agent of higher θ and to borrow on the private market. Increasing the interest rate on the private market to $R > \hat{R}$ discourages such deviations.[14]

The case of Diamond-Dybvig preferences is conceptually close to the case of liquidity shocks preferences in this section. The direction of the relevant deviation and the ability to affect agents with a lower interest rate directly generalize to the more general preferences. The important difference between these two cases is that, in general, solutions to the constrained efficient problem with trades, SP^3, and constrained efficient problem without trades, SP^2 (or problem without private information, SP^1) do not coincide. For Diamond-Dybvig preferences, the solutions to SP^1, SP^2, and SP^3 coincide. The reason for that is that agents of type $\theta = 0$ and $\theta = 1$ have very different marginal rates of substitution. In the case of $\theta = 0$, the marginal rate of substitution between period $t = 1$ and $t = 2$ is zero, and in case of $\theta = 1$, the marginal rate of substitution is one. Such differences in preferences allow the social planner to perfectly screen the agents of different types. In general, this is not the case, and the solutions to constrained efficient programs SP^2 and SP^3 would not coincide.

We now provide another simple case for which a closed form solution may be derived. Consider the case of liquidity shocks and assume that utility takes the form:

$$u\left(c_1, c_2; \theta\right) = \frac{1}{\theta} c_1 + \log c_2.^{15}$$

Assume that the distribution of shocks $F\left(\theta\right)$ is log-normal with $\left(\mu, \sigma\right)$. Then one can show that the optimal interest rate is given by:

$$R^* = \hat{R} e^{-\sigma^2}.$$

This closed form solution provides us with an additional insight. As the variance of the shocks σ increases and the informational friction increases, the optimal interest rate decreases.

[14] The liquidity cap may appear a somewhat unrealistic requirement. One can, however, argue that it mimics an implicit subsidy to investment. Such subsidies are prevalent, especially, in the context of encouraging small businesses (for example, through the Small Business Administration in the USA). One can draw a parallel between agents in our model who experience liquidity shocks and a model where entrepreneurs face investment risk shocks.

[15] These preferences do not strictly fall in our specification of liquidity shocks preferences, $u\left(c_1, c_2; \theta\right) = \frac{1}{\theta} \hat{u}\left(c_1\right) + \hat{u}\left(c_2\right)$, because the baseline utility functions corresponding to the first and second period consumption differ. We use these preferences nevertheless as they admit a simple closed form solution.

11 Other extensions

In this section we consider three extensions of the model described above: introducing aggregate shocks, modeling an environment with idiosyncratic shocks to financial intermediaries, and considering direct access to intertemporal technology by agents, as well as possibility of heterogenous financial intermediaries.

11.1 Aggregate shocks

It is easy to extend the model to the case in which the economy experiences aggregate shocks to e and $\hat{R}$ that are known in period $t = 0$. Suppose that there are N aggregate states $\eta = \{1, 2, .., N\}$ and the state is observable. We denote the probability of these states by $\mu(\eta)$. We notice that it is technologically impossible for the society to transfer resources across states. Therefore, the problem with aggregate shocks can be reduced to solving N independent problems described in case without aggregate shocks and is, essentially, a comparative statics exercise with respect to the aggregate shock.

11.2 Idiosyncratic shocks to intermediaries and interbank markets

In this section we discuss an extension of the model to the case in which intermediaries experience idiosyncratic observable shocks. We show that if there are complete interbank markets then this model reduces to the case described in previous sections in which all intermediaries are identical. The intuition for this result is simple: in period 0, intermediaries can trade bonds with the payoff contingent on the shocks realized in period 1. We now illustrate the result for the case with no aggregate uncertainty in which intermediaries face return shocks.

Formally, we proceed as follows. At time $t = 1$, an intermediary can face a rate of return shock $n \in \{1, ..., N\}$ with probability η_n under which the return on the long asset is $\hat{R}(n)$. We assume that there is no aggregate uncertainty and that

$$\sum_{n=1}^{N} \eta_n \hat{R}(n) = \hat{R},$$

$$\sum_{n=1}^{N} \eta_n = 1.$$

At time 0 there are interbank markets in which intermediaries trade N Arrow-Debreu securities. The price of each security is q_n. The security pays 1 if state n occurs and 0 otherwise. Prices q_n are determined by a market clearing condition. It is immediate to see that intermediaries choose to fully insure themselves at $t = 0$ against idiosyncratic shocks. The problem of each intermediary then reduces to the case of no idiosyncratic shocks described above.

11.3 Direct access to technology

Another variation of our setup is the case in which some agents have access to technology that yields $\hat{R}$ directly without the need for financial intermediaries while other agents need an intermediary to access the technology. If we modified our assumption that all activities at the level of intermediaries are observable and instead supposed that the regulator could observe the aggregate investment in the technology yielding $\hat{R}$, then our results would also hold. The constrained optimum in that model would be implemented by a tax on returns to investment of those who can access the technology and by a liquidity adequacy requirement on the financial intermediaries providing liquidity insurance for agents who cannot access the technology.

11.4 Heterogeneity of financial intermediaries

The model abstracts from heterogeneity of financial intermediaries in terms of their size, and from the fact that, in practice, financial intermediaries hold assets of different risk. It is easy to modify the setup to consider intermediaries of different sizes. One can extend the model and show that a liquidity requirement in a ratio form, stipulating the proportion of funds invested in a liquid (short-term) asset would implement the optimal allocation.

If markets are complete as in the discussion above, financial intermediaries holding assets of different risk engage in transactions among themselves to insure against that risk. If there is an additional friction, for example, incompleteness of markets for risk sharing among intermediaries, the size of the liquidity adequacy requirements may need to be modified to take into account such friction. In reality, there are a variety of liquidity regulations already imposed on banks, mainly to control risks. Such regulations would probably perform some of the roles of the social planner predicted by our model as they increase the aggregate investment in the liquid (short-term) asset provided by the markets.

12 Conclusion

The theoretical mechanism of this paper addresses a critique of the financial intermediation literature: retrading puts significant limitations on the provision of insurance against liquidity shocks. We showed that a social planner can significantly improve the provision of insurance against liquidity shocks even if agents can trade privately. Indeed, in the case of the widely used Diamond-Dybvig model, the social planner can achieve the first best. A simple intervention – liquidity requirement – can implement the constrained efficient allocation. The simplicity of the Diamond-Dybvig model allows for a transparent characterization of the market failure that we analyze and of the direction of the intervention needed to correct it.

Currently, other regulations – for example, those aimed at controlling the risks taken by financial intermediaries – are already in place and contribute to alleviating the inefficiencies that we are describing. Understanding the precise interaction between prudential and liquidity regulations is an important topic of research.

13 Appendix

13.1 Proof of Theorem 1

First, we prove that solution to problem SP^1 is characterized by (7), (8), and (9). Indeed, $c_2(0) > 0$ cannot be optimal because agents of type 0 do not value consumption in period $t = 2$, while if $c_1(1) = z > 0$, then changing the allocation to $c_1'(1) = 1$ and $c_2'(1) = c_2(1) + \hat{R}z$ would increase the maximand in (3) while satisfying the feasibility constraint (4). Now (8) is obtained in the standard way as the first-order condition for the problem given (7), and the feasibility constraint may be rewritten as (9). To verify that this allocation is incentive compatible, and thus a solution to problem SP^2, we only need to check that $c_2(1) \geq c_1(0)$, which follows from $\rho\hat{R} > 1$. This proves that (6) holds, while (5) is trivial, given $c_1(1) = 0$).

Let us now show that for the solution to (3) we necessarily have $c_1(0) > e$ and $c_2(1) < \hat{R}e$. To see this, assume for a moment that $c_1(0) \leq e$; this implies $c_2(1) \geq \hat{R}e$ because of (9) (similarly, assuming $c_2(1) \geq \hat{R}e$ would imply $c_1(0) \leq e$). One can prove that if function u satisfies (1) and $z_1 < z_2$, then

$$\frac{u'(z_1)}{u'(z_2)} \geq \frac{z_2}{z_1}. \tag{37}$$

The easiest way to see this is to rewrite (1) as

$$-\frac{u''(c)}{u'(c)} \geq \frac{1}{c},$$

which is equivalent to

$$-\left(\log u'(c)\right)' \geq (\log c)'.$$

Integrating this from z_1 to z_2, we get

$$-\left(\log u'(z_2) - \log u'(z_1)\right) \geq \log z_2 - \log z_1,$$

and after taking the exponent this becomes (37).

Now, taking $z_1 = c_1(0) \leq e < \hat{R}e \leq c_2(1) = z_2$, we get, using (8)

$$\rho\hat{R} = \frac{u'(c_1(0))}{u'(c_2(1))} \geq \frac{c_2(1)}{c_1(0)} \geq \frac{\hat{R}e}{e} = \hat{R},$$

which is impossible since $\rho < 1$. This contradiction proves that $c_1(0) > e$ and $c_2(1) < \hat{R}e$.

13.2 Proof of Lemma 2

The proof of equivalence consists of 2 steps. First, we take a solution $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ to problem (21) and show that it may be implemented for some I and R satisfying (27), in the sense that $(c_1(\theta), c_2(\theta))$ would solve (24) for any θ. Second, we take I^* and R^* that solve problem

(26) and show that in that case, $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ given by $c_t(\theta) = x_t^u(I, R; \theta)$ are feasible in problem (21).

Take any solution $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ to problem (21) and let R be the equilibrium interest rate for the private market given these endowments. Note that the incentive compatibility constraint (23) can be rewritten as

$$U(c_1(\theta), c_2(\theta); \theta) \geq \tilde{V}(c_1(\theta') + \frac{c_2(\theta')}{R}, R; \theta) \text{ for all } \theta, \theta',$$

meaning that agent of type θ does not get more than utility from endowment by pretending to be of type θ' and then trading in private market. By (24), this implies

$$\tilde{V}(c_1(\theta) + \frac{c_2(\theta)}{R}, R; \theta) \geq \tilde{V}(c_1(\theta') + \frac{c_2(\theta')}{R}, R; \theta) \text{ for all } \theta, \theta'.$$

Since $\tilde{V}$ is strictly increasing in its first argument, this is equivalent to

$$c_1(\theta) + \frac{c_2(\theta)}{R} = \max_{\theta'}\left(c_1(\theta') + \frac{c_2(\theta')}{R}\right) \text{ for all } \theta, \theta'.$$

Consequently,

$$c_1(0) + \frac{c_2(0)}{R} = c_1(1) + \frac{c_2(1)}{R};$$

denote this value by I. Let us prove that for (I, R), $(c_1(\theta), c_2(\theta))$ solve (24) for any θ; this would automatically imply (27) if we take $x_t^u(I, R; \theta) = c_t(\theta)$ for all t, θ, because $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ satisfies (22). But $(c_1(\theta), c_2(\theta))$ constitute a solution (10) and, moreover, these are still a solution under the additional constraint $\theta' = \theta$ which is implicit in (24). This proves that any solution $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ to problem (21) may be implemented through the appropriate choice of I and R; note that the values of maximands in (21) and (26) are equal for these parameter values, hence, the maximum in problem (26) is at least as large as one in problem (21) (recall that $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ is a solution to problem (21)).

Conversely, suppose that (I^*, R^*) solve problem (26); let $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ be given by $c_t(\theta) = x_t^u(I, R; \theta)$, where $\{x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta)\}_{\theta\in\{0,1\}}$ are solutions to (24) for (I^*, R^*). We need to check that $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ is feasible for problem (26), i.e., satisfies (22), (23), and (**??**). Clearly, (22) immediately follows from (27). Note that since (25) is binding, we must have

$$c_1(0) + \frac{c_2(0)}{R} = c_1(1) + \frac{c_2(1)}{R} = I^*,$$

meaning that

$$\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R^*, \theta) = V(I^*, R^*; \theta) \text{ for all } \theta,$$

because with income I^* for both types and in the presence of private markets a consumer does as well by reporting truthfully $\theta' = \theta$ as he would if he reported $\theta' \neq \theta$ in (10). Now (23) follows from the fact that $(x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta))$ solves (24); (**??**) follows from the same fact, combined with the uniqueness of solution to (24) (in terms of allocations; reported type may not be determined uniquely because the agent may be indifferent, which is, for instance, true in this case) due to its convexity. We have proved that if (I^*, R^*) solve problem (26), then the induced allocation are feasible in the social planner's problem (21). Again, note that for these parameter values the maximands are equal, which implies that the maximum in problem (21) is at least as large as one in problem (26).

We have proved that the maximums in problems (21) and (26) coincide. Consequently, any $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ that solves problem (21) induces (I^*, R^*) which solve (26) and which is such that (28) holds, and vice versa. This proves the equivalence of problems. We have proved Lemma 2.

13.3 Proof of Theorem 2

We proceed as follows. We show that there exist (I^*, R^*) for which $\{x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta)\}_{\theta\in\{0,1\}}$, found as solutions to (24), coincide with the solution to problem SP^2 (and thus problem SP^1, since the solutions to those coincide). By Lemma 2, if we define $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ by (28), conditions (22), (23), and (**??**) will be satisfied, provided that (27) is satisfied. This means that the solution to problems SP^1 and SP^2 is feasible for problem SP^3. But this implies that it is also a solution to SP^3, because SP^3 is obtained from SP^1 by imposing additional constraints. Given that solution to SP^1 is unique, we conclude that so is solution to SP^3, in particular, income I and interest rate R are determined uniquely.

Define (I^*, R^*) as solutions the following system of equations:

$$\begin{aligned} u'(I) &= \rho\hat{R}u'(RI), && (38)\\ I &= \frac{e}{\pi + (1-\pi)\frac{R}{\hat{R}}}. && (39) \end{aligned}$$

In particular, R^* is a solution to

$$\frac{u'\left(\frac{e}{\pi+(1-\pi)\frac{R}{\hat{R}}}\right)}{u'\left(\frac{Re}{\pi+(1-\pi)\frac{R}{\hat{R}}}\right)} - \rho\hat{R} = 0. \tag{40}$$

Denote by $f(R)$ the left-hand side of (40). Note that $f(R)$ is increasing and $f(1) = 1 - \rho\hat{R} < 0$, so that the solution to (40) involves $R^* > 1$. Since the coefficient of relative risk aversion is everywhere greater than 1 and $R^* > 1$, we can apply (37) to obtain

$$\frac{u'\left(\frac{e}{\pi+(1-\pi)\frac{R^*}{\hat{R}}}\right)}{u'\left(\frac{R^*e}{\pi+(1-\pi)\frac{R^*}{\hat{R}}}\right)} \geq R;$$

this show that $R^* \leq \rho\hat{R}$. Note that if $u(c) = \log(c)$, we have $R^* = \rho\hat{R}$. We have shown that $R^* \in (1, \rho\hat{R}]$. Moreover, this argument establishes existence and uniqueness of (I^*, R^*): indeed, as we just showed R^* is uniquely determined by (40), and then I^* is uniquely determined by (39).

Now let us solve (24) for (I^*, R^*) given by (38) and (39). First, take $\theta = 0$; then (24) becomes

$$\max_{x_1, x_2} u\left(x_1\right)$$

s.t. (25); the solution is, obviously, $x_1^u\left(I^*, R^*, 0\right) = I^*$, $x_2^u\left(I^*, R^*, 0\right) = 0$. Now take $\theta = 1$; the problem becomes

$$\max_{x_1, x_2} \theta\rho u\left(x_1 + x_2\right)$$

s.t. (25); since $R^* > 1$, the solution is $x_1^u\left(I^*, R^*, 1\right) = R^* I^*$, $x_2^u\left(I^*, R^*, 1\right) = 0$. Let us check that (8) and (9) are satisfied (for $c_t\left(\theta\right) = x_t^u(I^*, R^*; \theta)\ \forall t, \theta$). Indeed,

$$u'\left(c_1\left(0\right)\right) = u'\left(I^*\right) = \rho\hat{R}u'\left(R^* I^*\right) = \rho\hat{R}u'\left(c_2\left(1\right)\right);$$

we used (38) here and, similarly,

$$\pi c_1\left(0\right) + (1-\pi)\frac{c_2\left(1\right)}{\hat{R}} = \pi I^* + (1-\pi)\frac{R^* I^*}{\hat{R}} = I^*\left(\pi + (1-\pi)\frac{R^*}{\hat{R}}\right) = e,$$

because (I^*, R^*) solves (39). This proves that solution $\{x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta)\}_{\theta\in\{0,1\}}$ of problem (24) satisfies (7), (8), and (9), and therefore is a solution to SP^1 and SP^2. For this allocation, the feasibility constraint (4) is satisfied, and therefore (27) follows (one could also check (27) by plugging $\{x_1^u(I^*, R^*; \theta), x_2^u(I^*, R^*; \theta)\}_{\theta\in\{0,1\}}$ found above). Therefore, (I^*, R^*) lead to a feasible solution of (24), and the allocations obtained for (I^*, R^*) coincide with allocations that solve SP^1 and SP^2. Since SP^3 is a constrained version of these, then (I^*, R^*) is a solution to SP^3. This proves the theorem.

13.4 Proof of Proposition 2

The dual of the program of an intermediary when the interest rate is R^* and the liquidity requirement is i^* is

$$\max_{\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}} \pi\tilde{V}(\{c_1\left(\theta\right), c_2\left(\theta\right)\}_{\theta\in\{0,1\}}, R^*, 0) + (1-\pi)\rho\tilde{V}(\{c_1\left(\theta\right), c_2\left(\theta\right)\}_{\theta\in\{0,1\}}, R^*, 1)$$

s.t.

$$\pi c_1(0) + (1-\pi)c_1(1) \geq i^*, \tag{41}$$

$$c_1(0) + \frac{c_2(0)}{R^*} = c_1(1) + \frac{c_2(1)}{R^*}, \tag{42}$$

$$\pi\left(c_1(0) + \frac{c_2\left(0\right)}{\hat{R}}\right) + (1-\pi)\left[c_1(1) + \frac{c_2\left(1\right)}{\hat{R}}\right] \leq e. \tag{43}$$

Here, (42) stems from (18), since we assumed that the interest rate faced by intermediaries and consumers is R^*, while (43) is simply(20).

Since, as we proved, $1 < R^* \leq \rho\hat{R} < \hat{R}$, then for any menu of contracts $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$, each agent of type $\theta = 0$ will, after trading on the private market, end up with first period good only, and each agent of type $\theta = 1$ will end up with second period good only (under the condition that the market clears). To put it differently,

$$\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R^*, 0) = u\left(c_1(0) + \frac{c_2(0)}{R^*}\right)$$

(in the parentheses we have the wealth of agents of type $\theta = 0$) and

$$\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R^*, 1) = \rho u\left(R^*\left(c_1(1) + \frac{c_2(1)}{R^*}\right)\right).$$

Since markets clear, the intermediaries may as well offer contracts with $c_2(0) = c_1(1) = 0$, merely providing each consumer with what he would get as a result of the trade. We can therefore write the program as

$$\max_{c_1(0),c_2(1)} \pi u(c_1(0)) + (1-\pi)\rho u(c_2(1))$$

s.t.

$$\pi c_1(0) \geq i^*, \tag{44}$$

$$\pi c_1(0) + (1-\pi)\frac{c_2(1)}{\hat{R}} \leq e. \tag{45}$$

This problem is convex. Hence, if we prove that $c_1(0) = I^*$ and $c_2(1) = R^*I^*$ satisfies both the constraints and the first order conditions; this would imply that the solution to SP^3 is also the unique competitive equilibrium with the liquidity floor. It is easy to check that constraints are satisfied as equalities; indeed,

$$\pi c_1(0) = \pi I^* = i^*$$

by (30), and

$$\pi I^* + (1-\pi)\frac{R^*I^*}{\hat{R}} = I^*\left(\pi + (1-\pi)\frac{R^*}{\hat{R}}\right) = e,$$

because (I^*, R^*) satisfies (39). Because of complementary slackness conditions, we only need to verify that the Lagrange multipliers λ on the constraint (44) and μ on the constraint (45) are non-negative. The first-order conditions are

$$\pi u'(I^*) = \pi(-\lambda + \mu),$$

$$(1-\pi)u'(R^*I^*) = (1-\pi)\mu\frac{1}{\hat{R}}.$$

Rearranging and using the fact that $u'(I^*) = \rho\hat{R}u'(R^*I^*)$ (which holds since (I^*, R^*) satisfies (38)), we get

$$\mu = \hat{R}u'(R^* I^*) > 0,$$

$$\lambda = (1-\rho)\,\hat{R}u'(R^* I^*) > 0.$$

Moreover, no agent wants to borrow or lend at the interest rate $R^* > 1$, so the market indeed clears for such allocation. This completes the proof of the Proposition.

13.5 Proof of Lemma 3

Part 1. If preferences are homothetic, then

$$\frac{I x^u_{2,I}(I,R;\theta)}{x^u_2(I,R;\theta)} = 1. \tag{46}$$

This implies, first, that date 2 consumption is a normal good, so that $x^u_{2,I}(I,R;\theta) > 0$. Second, it implies that $\mathrm{Cov}\left\{x^u_{2,I}(I,R;\theta), \frac{x^u_2(I,R;\theta)}{R^2}\right\} \geq 0$, which in turn implies assumption 2.

Part 2. Let K be a compact set containing the optimal values of (I,R) for $0 \leq \gamma \leq 1$. Then there exists M such that for all $(I,R) \in K$

$$\left|\mathrm{Cov}\left\{x^u_{2,I}(I,R;\theta), \frac{x^u_2(I,R;\theta)}{R^2}\right\}\right| < M\sigma^2_{F^\gamma}, \tag{47}$$

meaning that for γ sufficiently small, the Lemma holds.

13.6 Proof of Theorem 3

We first note that the solution to problem (32) is equivalent to the solution of

$$\max_{I,R} \int_\Theta V(I,R;\theta) dF(\theta) \tag{48}$$

subject to

$$\int_\Theta \left\{x^u_1(I,R;\theta) + \frac{x^u_2(I,R;\theta)}{\hat{R}}\right\} dF(\theta) \leq e. \tag{49}$$

We now analyze two key first order conditions that characterize problem (48). Consider the first order condition of this program with respect to income I:

$$\int_\Theta \left[V_I(I,R;\theta) - \lambda\left\{x^u_{1,I}(I,R;\theta) + \frac{x^u_{2,I}(I,R;\theta)}{\hat{R}}\right\}\right] dF_\theta = 0, \tag{50}$$

and the first order condition with respect to interest rate R :

$$\int_\Theta \left\{V_R(I,R;\theta) - \lambda\left[x^u_{1,R}(I,R;\theta) + \frac{x^u_{2,R}(I,R;\theta)}{\hat{R}}\right]\right\} dF_\theta = 0, \tag{51}$$

where we denote by λ a multiplier on (49), by $x^u_{1,I}$ and $x^u_{2,I}$ the derivatives of the uncompensated demands with respect to I, and by $x^u_{1,R}$ and $x^u_{2,R}$ derivatives of uncompensated demands with respect to R.

We manipulate these conditions to obtain a characterization of the optimal wedge between the interest rate on the private market and the return on savings. As these manipulations are purely algebraic and use basic properties of the indirect utility functions, we refer the interested reader to the working paper version of the paper. Specifically, let I^* and R^* be solutions to the problem (48). Let $x^c_2(I,R;\theta)$ be compensated demand in the problem (24), and $x^c_{2,R}(I,R;\theta)$ denote its derivative with respect to R. Then R^* satisfies

$$\frac{1}{\hat{R}}-\frac{1}{R^*}=\frac{\frac{1}{\lambda}\operatorname{Cov}\left\{V_I(I^*,R^*;\theta),\frac{x^u_2(I^*,R^*;\theta)}{R^{*2}}\right\}}{\int x^c_{2,R}(V\left(I^*,R^*,\theta\right),R^*,\theta)dF_\theta+\operatorname{Cov}\left\{x^u_{2,I}(I^*,R^*;\theta),\frac{x^u_2(I^*,R^*;\theta)}{R^{*2}}\right\}}. \tag{52}$$

Formula (52) characterizes the optimal wedge between the interest rate R^* and $\hat{R}$ in terms of easily interpretable parameters such as indirect utility functions, uncompensated and compensated demands, and the properties of the distribution of shocks.

Now we turn our attention to the denominator of (52). It is clear that $x^c_{2,R}(I,R;\theta)>0$; this is a standard property of compensated demand functions. However, the sign of the denominator is a priori ambiguous, even under the assumption that $V_I(I,R;\theta)>0$ or $V_{I,\theta}(I,R;\theta)<0$. Using Lemma 3 we are able to prove that it is positive under certain assumptions

$$\int_\Theta x^c_{2,R}(I,R;\theta)dF_\theta+\operatorname{Cov}\left\{x^u_{2,I}(I,R;\theta),\frac{x^u_2(I,R;\theta)}{R^2}\right\}>0. \tag{53}$$

Assumption 1 ensures that $\frac{\partial}{\partial\theta}\left(\frac{x^u_2(I,R;\theta)}{R^2}\right)>0$. Then we conclude that if $V_{I,\theta}(I,R^*;\theta)>0$ for all θ, then $\operatorname{Cov}\left\{V_I(I^*,R^*;\theta),\frac{x^u_2(I^*,R^*;\theta)}{R^{*2}}\right\}>0$; if $V_{I,\theta}(I,R^*;\theta)<0$ for all θ, then $\operatorname{Cov}\left\{V_I(I^*,R^*;\theta),\frac{x^u_2(I^*,R^*;\theta)}{R^{*2}}\right\}<0$.

We now present a lemma that determines how $V_I(I,R^*;\theta)$ depends on preferences (below, $V_{I,\theta}$ denotes the second partial derivative with respect to I and θ).

Lemma 4 *$V_{I,\theta}(I,R^*;\theta)>0$ for all θ, if preferences are discount factor shocks as in example* 1. *$V_{I,\theta}(I,R^*;\theta)<0$ for all θ, if preferences are liquidity shocks as in example* 2*; if preferences are valuation-neutral shocks as in example* 3*, $V_{I,\theta}(I,R^*;\theta)=0$ for all θ.*

Proof. We have

$$V(I,R;\theta)=\max_{x_1,x_2}u(x_1,x_2;\theta)$$

subject to

$$x_1+\frac{x_2}{R}\leq I.$$

Suppose first that preferences are given by $u(x_1)+\theta u(x_2)$. Then, substituting x_1 using the budget constraint, we can rewrite this problem as

$$V(I, R; \theta) = \max_{x_2} \hat{u}\left(I - \frac{x_2}{R}\right) + \theta \hat{u}(x_2).$$

By the Envelope theorem, we have

$$V_\theta(I, R; \theta) = \hat{u}(x_2^u(I, R, \theta)).$$

Hence,

$$V_{I,\theta} = \hat{u}'(x_2^u(I, R, \theta))x_{2,I}^u(I, R, \theta) > 0.$$

Suppose now that preferences are given by $\frac{1}{\theta}u(x_1) + u(x_2)$. By the Envelope theorem, we have

$$V_\theta(I, R; \theta) = -\frac{1}{\theta^2}\hat{u}(x_1^u(I, R, \theta)).$$

Hence

$$V_{I,\theta} = \frac{-1}{\theta^2}\hat{u}'(x_2^u(I, R, \theta))x_{1,I}^u(I, R, \theta) < 0.$$

For the value neutral preferences $V_{I,\theta} = 0$.

This proves the Lemma.

Now apply Lemma 4 to formula (52) to get the results on the optimal interest rates. Let I^* and R^* be the solutions to (48) and let

$$i^* = \int_\Theta x_1^u(I^*, R^*; \theta) dF(\theta)$$

When $R^* < \hat{R}$, the intermediaries want to invest as much as possible in the long asset and let agents borrow on the private market against future payments by the intermediary, to take advantage of the interest rate difference. In this case, liquidity floor fixed at i^* would be binding for them. A similar reasoning applies when $R^* > \hat{R}$: in that case, one would need to impose a liquidity cap. Combined with the resource constraint faced by each intermediary, this is enough to prove that they will offer exactly the allocation prescribed by (48). ■

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