Essays in Optimal Taxation

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Abstract

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This thesis consists of four essays that span a wide range of topics in, and perspectives on, the dominant modern framework for optimal taxation research.

The first analyzes a potential tax reform based on this framework: age-dependent taxation. Using modern dynamic optimal tax methods, I provide a comprehensive theoretical and quantitative examination of age dependence and compare it to two alternative policies: an age-independent policy and a dynamic optimal policy. Despite its simplicity, age dependence yields a large welfare gain equal to between one and three percent of aggregate annual consumption, and it captures a substantial portion of the gain from reform to the dynamic optimal policy.

The second incorporates a factor into this framework that is generally neglected: preference heterogeneity. I avoid technical difficulties that arise with preference heterogeneity in general by focusing on a specific but important class of
preferences: those against which individuals do not want to be insured. Prominent critics of redistributive taxation have long stressed the normative importance of this class, and I examine its effects on the optimal design of taxation. In addition, I present cross-country evidence that is consistent with a key prediction of the model, suggesting preference heterogeneity’s importance in a positive analysis of taxation.

The third applies this framework to a new issue: the optimal policy response to heterogeneity in parents’ altruism toward their children. I derive a normative criterion for optimality by modifying the standard framework to apply to an economy with parents and children. I argue that the social welfare function in this setting fully offsets the effects of heterogeneity in parental altruism on children, contrary to the conventional economic approach to optimal intergenerational policy.

The fourth challenges this framework by pointing out that its theory, combined with empirical evidence, recommends a tax credit for short taxpayers and a tax surcharge for tall ones. With my coauthor, N. Gregory Mankiw, I show that the optimal height tax in the United States is substantial. We argue that if society rejects this policy, we must revisit the standard framework for optimal taxation that recommended it.
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Preface

Taxation is society’s fundamental economic policy tool, and its analysis has always been a primary focus of economists, sociologists, and philosophers. Recent economic research on taxation centers on a model economy, introduced in Mirrlees (1971), that can be used to derive tax policy that is "optimal" in a normatively appealing sense. The research included below works with this framework to suggest policy reforms, extends this framework to address factors neglected in the literature, and raises fundamental questions about this framework’s use as the basis for optimal tax analysis.

This set of analyses may seem contradictory: how can one both use a framework to generate policy recommendations and challenge the use of that framework as a tool for policy analysis? The perspective embodied by this dissertation is that the Mirrleesian framework provides an important tool for both normative and positive analysis, rigorously capturing a key aspect of the tradeoff between efficiency and equality that has long played a central role in debates over taxation. Moreover, the framework’s flexibility suggests that continued research based on it will reveal additional insights with meaningful effects on policy. At the same time, the framework is subject to important critiques that, if left unaddressed, will limit its influence on both researchers and practitioners.

To realize the potential of our current approach to optimal taxation and to develop additional, complementary approaches, we must engage the full range of factors that matter for the formation and effects of tax policy. This dissertation is a small contribution to that challenging but important effort.
Chapter 1
The Surprising Power of Age-Dependent Taxes

1.1 Introduction

Individuals’ wages change over their lifecycle.\(^1\) Starting with Golosov, Kocherlakota, and Tsyvinski (2003), this fact has inspired a recent surge of research on optimal taxation in dynamic economies that builds on the classic, static Mirrlees (1971) framework. Unifying the important insights of this research is the principle that optimal taxation in dynamic economies depends, other than in special cases,\(^2\) on a taxpayer’s history. While this principle points the way toward potentially powerful reforms, its near-term impact may be limited by the complexity of the reforms to existing policies that it implies.

This paper studies a simple reform to tax policy that responds to changes in wages over the lifecycle: age-dependent labor income taxes. Though not often recognized as such, age dependence is a component of Mirrleesian dynamic optimal taxation and thus can be

\(^1\) I am grateful to Stefania Albanesi, Robert Barro, Michael Boskin, V.V. Chari, David Cutler, Emmanuel Farhi, Martin Feldstein, Caroline Hoxby, Oleg Itskhoki, Larry Jones, Emir Kamenica, Larry Katz, Patrick Kehoe, Narayana Kocherlakota, Erzo F.P. Luttmer, Greg Mankiw, Ellen McGrattan, Thomas Mertens, Brent Neiman, Chris Phelan, Jim Poterba, Robert Shiller, Larry Summers, Aleh Tsyvinski, and Ivan Werning for their comments and suggestions.

\(^2\) Saez (2001) is the leading recent study of the classic Mirrlees framework. In the dynamic Mirrlees framework, Albanesi and Sleet (2006) shows that history dependence can be replaced with dependence on current wealth if shocks to skills are i.i.d.. Golosov and Tsyvinski (2006) shows that asset-testing can replace history dependence if shocks are permanent: i.e., an absorbing state. For general shock processes, Kocherlakota (2005) shows that history dependence is required.
analyzed as a partial reform on the path toward optimal policy. Taking that approach, this paper makes three contributions.

First, its status as a partial reform allows me to provide a comprehensive theoretical characterization of age dependence using the tools of modern dynamic Mirrleesian optimal tax research. I derive theoretical results for age-dependent policy that connect to and extend results and intuition from two alternative Mirrleesian policies, an age-independent optimal policy and a dynamic optimal policy. In a baseline model with deterministic wage paths and no private saving or borrowing, I show that age-dependent policy avoids discouraging the labor supply of the top earner at each age while age-independent policy cannot, extending a classic result from static Mirrlees research. Then, I analytically characterize the intertemporal consumption margin, where age-dependent policy satisfies a condition that improves on age-independent policy but falls short of the full optimum. I characterize how age dependence affects these margins in economic environments with stochastic wages and private saving and borrowing, as well.

Second, I provide a detailed quantitative study of age dependence using individual wage data from the U.S. Panel Study of Income Dynamics to calibrate and simulate policy. This is in contrast to most of the dynamic optimal taxation literature, in which illustrative numerical simulations are the rule, and it resembles the realistic calibrations by Saez (2001) of the static Mirrlees model and Golosov and Tsyvinski (2006) of optimal disability insurance. The numerical simulations generate specific implications for policy design. Two results largely robust across environments are that age dependence: (1) lowers marginal taxes on average and especially on high-income young workers, and (2) lowers average tax
rates for young workers relative to older workers when private saving and borrowing are restricted. These results capture key features of the optimal dynamic policy.

Finally, the numerical simulations allow me to quantify and decompose the welfare gain from this partial reform. Age dependence yields a large welfare gain equivalent to between 1 and 3 percent of aggregate annual consumption, and this gain represents a substantial share of the potential gain from full reform to the dynamic optimal policy. A detailed decomposition reveals three main components of the gain. Efficiency and equity gains each account for a bit less than half of the total welfare gain, while intertemporal consumption smoothing accounts for approximately ten percent in the baseline model. Age dependence allows tax policy to be tailored to changes in the distribution of wages at each age and to transfer resources between age groups, avoiding inefficient distortions to labor effort and enabling more redistribution.\(^3\)

Age dependence is a compelling example of the potential value of partial reforms. The notion of partial reform was formalized by Guesnerie (1977) in response to concerns raised by Feldstein (1976) about the practicality of static optimal tax design. Partial reforms are most valuable when they yield substantial welfare gains while raising fewer real-world concerns than fully optimal policy. A particularly strong case can be made for partial reforms that yield Pareto improvements, a characteristic stressed by Guesnerie. Thus, to further motivate the practicality of this reform, I consider a model in which I add the constraint that age dependence must be ex post Pareto improving. This constrained reform

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\(^3\) A related policy is lifetime income taxation, where an individual pays taxes or receives refunds in each year based on their (currently) expected lifetime income. The age-dependent policy studied here subsumes that policy and adds the important ability to tailor marginal taxes by age. Moreover, lifetime income taxation also uses age as an argument in the tax function, so it provides little or no advantage in simplicity relative to fully age-dependent taxation.
generates nearly as large a welfare gain as does the Utilitarian-optimal policy, underscor-
ing the potential power of age dependence.

Though this paper is the first to give the partial reform of age dependence a compre-
hensive treatment in a modern dynamic Mirrleesian tax model, the idea of having taxes depend on age has been around for some time and was even mentioned in passing by Mir-

rlees (1971). Despite this long history, age dependence was first rigorously analyzed in the important work of Kremer (2002), who demonstrated that marginal income taxes not conditioned on age are unlikely to be optimal and suggested that they should be lower for young workers, consistent with the results below. Kremer’s analysis did not provide a full characterization or simulation of age dependent taxes and was limited to a static setting, limitations his paper acknowledges. Following on that work, Blomquist and Micheletto (2003) and Judd and Su (2006) provide results on age-dependent taxation in illustrative dy-

namic economies using theoretical and numerical approaches, respectively. The need for a more comprehensive analysis is underscored by the suggestion in the upcoming Mirrlees Review that age dependence is one of the most promising areas for the near-term reform of developed-country tax policy (see Banks, Diamond, and Mirrlees, 2007).

The literature on age dependence may be relatively sparse because it has been assumed that age is merely another "tag", following Akerlof (1978), and that standard tagging

---

4 Age dependence has been studied in the Ramsey tax framework. For example, Erosa and Gervais (2002) study the effect of private asset accumulation and changes in the elasticity of labor supply with age on optimal linear taxes. As a representative agent framework, the conventional Ramsey approach neglects the redistributonal role of taxation. I have simulated a Ramsey version of age-dependent taxation (i.e., age-specific linear taxes and lump-sum taxes or grants) in the heterogeneous-agent model economy discussed below, and the welfare gains from age-dependence are approximately one-tenth their size in the Mirrlees approach. The Ramsey approach cannot tailor marginal taxes to variation in the distribution of wages with age.

5 The Mirrlees Review is the modern counterpart to the Meade Report of 1978, the influential review of taxation.
analysis teaches us all we need to know about age dependence. That assumption is mis-
taken. A standard tag, such as gender, provides information on an individual’s expected
place in the distribution of lifetime income, but age reveals no information by itself. The
difference is that a tag divides the population into mutually exclusive groups, while the en-
tire population moves through all age groups. In other words, each age group has the same
distribution of lifetime incomes. To provide useful information to the tax authority, age
must be combined with data on an individual’s current income and how incomes at each
age relate to lifetime income. Therefore, age dependence cannot be fully understood with
intuition from conventional tagging analysis.

Consistent with the small academic literature on age dependence, current policy in
developed countries includes only uncoordinated and often unintentional age dependence. In the United States, for example, the main dependencies on age work at cross purposes. Social Security and Medicare payroll taxes are levied on all individuals but, given the pay-as-you-go structure of these programs, these taxes place a larger effective burden on the young than on the old (see Feldstein and Samwick, 1992). On the other hand, the existing disability system effectively places a larger effective burden on the old than on the young, since the latter would receive a longer string of benefits if disabled. Some transfer programs, such as the child tax credit or Earned-Income Tax Credit, are more likely to benefit the young than the old, but they also mean higher marginal rates on the young who earn enough to be in the "phase-out" region of these benefits. Finally, the deductibility of mortgage interest and charitable donations are more likely to benefit the middle-aged, for

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6 One exception is Singapore’s Provident Fund, a national retirement savings program that tailors personal contribution rates to age.
whom renting is less common and incomes are higher, than the young or old. Together, these largely unintentional age dependencies are unlikely to mimic, and may often work in the oppositional direction of, the optimal age-dependent policy as studied in this paper.

Therefore, the analysis of this paper contributes to several literatures. Most directly, it provides a new, empirically-driven application of the tools of the dynamic Mirrleesian optimal tax literature surveyed in Golosov, Tsyvinski, and Werning (2006). It also extends the literature on partial reform, coinciding with recent work of Farhi and Werning (2007) on a different partial reform based on dynamic optimal policy. The paper’s consideration of both deterministic and stochastic wages relates to the important ongoing debate over the determinants of lifecycle wage inequality and the appropriate policy responses to it: e.g., Storesletten, Telmer, and Yaron (2001) and Keane and Wolpin (1997). Finally, it adds to a recent literature that empirically estimates the value of conditioning taxes on personal characteristics, such as Alesina and Ichino (2007).

The paper proceeds as follows. Section 1.2 describes the social planner’s problem in three policy scenarios for a baseline economy, i.e., with deterministic wage paths and no private saving or borrowing. For each policy, I analytically characterize the intratemporal and intertemporal distortions on private behavior and numerically simulate the structure of taxes. I also quantify the welfare implications of reform from the static optimal policy to age-dependent policy and the dynamic optimal policy. Section 1.2 serves as the reference point for the following three sections, in which I vary the assumptions about the economy to test the robustness of the baseline results. Section 1.3 allows individuals to save and borrow, Section 1.4 incorporates stochastic wage paths, and Section 1.5 combines these
two variations. Throughout these extensions, the results from the baseline economy are largely robust. Section 1.6 discusses a range of specific topics that fall outside the main analysis of the paper, and Section 1.7 concludes. Appendix A, which gives further details related to this paper, can be found at the end of this dissertation.

1.2 Baseline economy

In this section I analyze age-dependent labor income taxes for a baseline economy characterized by two simplifying assumptions. First, individuals cannot transfer resources across periods: that is, they can neither save nor borrow. Second, each individual’s lifetime wage path is deterministic, so that each individual knows in advance the exact path of wages it will have over its lifetime (i.e., there are no stochastic shocks to wages). These simplifying assumptions allow for cleaner analytical results, but I generalize the model to relax them in later sections of the paper. One assumption made throughout this paper, and throughout the dynamic optimal tax literature in general, is that wage paths are exogenous to individuals; I discuss the potential implications of this assumption in Section 1.6. I start by setting up the economy and then specifying the social planner’s problem in three policy scenarios.
1.2 Baseline economy

1.2.1 Setup

All individuals live and work for \( T \) periods, indexed by \( t = \{1, 2, \ldots, T\} \), and are members of the same generation. Individuals are heterogeneous in their ability to earn income over their lifetimes. This ability comes in \( I \) types, indexed by \( i = \{1, 2, \ldots, I\} \), with probabilities \( \pi^i \) so that \( \sum_{i=1}^{I} \pi^i = 1 \). At each age \( t \), an individual of type \( i \) can earn a wage \( w^i_t \) for each unit of its labor effort, and each individual knows its full lifetime path of wages \( \{w^i_t\}_{t=1}^{T} \) at time \( t = 1 \). Wages are not publicly observable. I refer to the present value of wages for type \( i \) as type \( i \)'s lifetime income-earning ability, defined as \( \sum_{t=1}^{T} \frac{w^i_t}{R^{t-1}} \) where \( R \) is the exogenous gross rate of return. Types are sorted so that \( i = I \) refers to the type with the highest lifetime income-earning ability.

Production is structured as follows. Labor income \( y \) is the product of the wage and labor effort \( l \), so \( y = wl \). There is no capital in the economy.

All individuals have the same separable preferences over consumption \( c \) and labor effort \( l \), where \( l = \frac{y}{w} \). The utility \( U^i_t \) for individual \( i \) of age \( t \) is

\[
U^i_t (c, y) = u(c) - v \left( \frac{y}{w^i_t} \right),
\]

where I assume \( u' (\cdot) > 0 \), \( u'' (\cdot) < 0 \), \( v' (\cdot) > 0 \), \( v'' (\cdot) > 0 \). For an individual \( i \), lifetime utility \( V^i \) is the discounted sum of its utility at each age:

\[
V^i = \sum_{t=1}^{T} \beta^{t-1} U^i_t (c, y),
\]

I assume an exogenous date of entry into the labor market. This matches the quantitative analyses below, which consider age-dependent taxes designed to have minimal effects on incentives to obtain higher education or training. Age dependence could generate even larger welfare gains if it were designed to operate effectively on these incentives.

In the Appendix, I discuss how the analysis generalizes largely unchanged to a setting with overlapping generations.
where individuals discount utility flows with the factor $\beta$. Social welfare $W$ is a weighted sum of individual lifetime utilities:

$$W = \sum_{i=1}^{I} \pi^{i} \alpha^{i} V^{i},$$

(1.3)

where $\alpha^{i}$ indicates a scalar Pareto weight on individual $i$. In general, the form of (1.3) allows us to consider any point along the Pareto frontier. I assume a form for Pareto weights (specified in the numerical simulations) in which an individual’s weight is positive, bounded above by one, and not increasing in its lifetime income-earning ability.

### 1.2.2 Social planner’s problem in three policy scenarios

Now I derive optimal taxes in three policy scenarios using the techniques of modern dynamic optimal tax analysis. In this approach, the tax problem is recast as a problem for a fictitious social planner that uses a direct mechanism to allocate resources (see Golosov, Tsyvinski, and Werning, 2006 for a review).

The social planner maximizes social welfare (1.3) by offering a menu of income and consumption pairs to individuals. Individuals choose optimally from the menu, earn the assigned income, and receive the assigned consumption. Knowing this, the planner designs its menu of $\{c, y\}$ pairs intending each pair to be chosen by a specific individual. Because individuals differ in their lifetime income-earning ability and age, I write $c_{i}^{t}$ and $y_{i}^{t}$ for the pair intended for the individual of type $i$ and age $t$.

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9 I show the connection between these allocations and taxes in the Appendix: an appropriate nonlinear labor income tax system implements each policy scenario’s allocations. In the Static Mirrlees policy, the labor income tax is a direct function of income only; in the Partial Reform it is a direct function of income and the taxpayer’s current age; while in the Full Optimum it is a direct function of the lifetime path of incomes and the taxpayer’s current age.

10 In the language of the direct mechanism, each individual makes a report to the planner about its personal
The planner maximizes social welfare subject to two types of constraints: a feasibility constraint and incentive constraints. The feasibility constraint is:

\[ \sum_{i=1}^{I} \pi^i \sum_{t=1}^{T} R^{T-t} (y^i_t - c^i_t) = 0. \] (1.4)

which says that the lifetime paths of income must fund the lifetime paths of consumption across all types.\(^{11}\) The planner can transfer resources across time and earn or pay the gross rate \(R\). I assume \(\beta R = 1\) for simplicity.

Incentive constraints reflect that individuals choose from the planner’s menu of \(\{c, y\}\) pairs to maximize their utility. In this approach, these constraints take the form of inequalities ensuring that each individual chooses the allocation of \(c^i_t\) and \(y^i_t\) that the planner intended.\(^{12}\)

Importantly, variations in the set of incentive constraints allow us to succinctly distinguish the planner’s problem in three policy scenarios: Static Mirrlees, Partial Reform, and Full Optimum. I now state these planner’s problems formally and discuss the differences between them.

The Static Mirrlees planner in the baseline model solves the following problem:

**Problem 1: (Static Mirrlees: Age-Independent)**

\[ \max_{\{c,y\}} \sum_{i=1}^{I} \pi^i \alpha^i V^i, \]

characteristics. The planner then assigns to each report a consumption and income pair, \(\{c, y\}\).

\(^{11}\) Note that taxation is purely redistributive. A positive net revenue requirement would imply larger tax distortions on average, likely increasing the welfare gain from age dependence calculated below.

\(^{12}\) These incentive constraints reflect this approach’s application of the Revelation Principle, by which we can restrict attention to incentive-compatible direct mechanisms, i.e., where individuals reveal their true types to the planner.
subject to the feasibility constraint (1.4) and the incentive constraints

\[ \beta^{t-1} \left( u \left( c_i^t \right) - v \left( \frac{y_i^t}{w_i^t} \right) \right) \geq \beta^{t-1} \left( u \left( c_j^t \right) - v \left( \frac{y_j^s}{w_j^s} \right) \right) \]  

for all \( i, j \in \{1, 2, \ldots, I\} \) and \( t, s \in \{1, 2, \ldots, T\} \).

These incentive constraints mean that the Static Mirrlees planner must guarantee that each individual of type \( i \) and age \( t \) chooses the allocation intended for it over that intended for any other individual of type \( j \) and any age \( s \). To see this, note that each side of the inequality (1.5) equals period utility for an individual of type \( i \) and age \( t \). The left-hand side is the utility this individual obtains by earning \( y_i^t \) and consuming \( c_i^t \), while the right-hand side is the utility it obtains by earning \( y_j^s \) and consuming \( c_j^s \). The inequality guarantees that this individual weakly prefers its \( \{c, y\} \) allocation. I denote the multipliers on these constraints with \( \mu_{\delta|s,t}^{j|i} \), where \( \mu_{\delta|s,t}^{j|i} \) corresponds to the constraint preventing individual \( i \) of age \( t \) from preferring the allocation intended for individual \( j \) of age \( s \).

The incentive constraints in (1.5) capture the restriction on the Static Mirrlees policy that each individual can choose among the same menu of income and consumption pairs, regardless of age. This is different from the requirement that two individuals with the same wage but different ages receive the same allocations of \( c \) and \( y \). The latter is a stronger condition and restricts the Static Mirrlees planner more than is justified. Age may affect an individual’s optimal choice of income and consumption, even if its wage is unchanged, and the Static Mirrlees planner can take advantage of this without making taxes age-dependent.\(^\text{13}\)

\(^\text{13}\) In the language of the direct mechanism, individuals in the Static Mirrlees scenario report only their current wage, not their age. But, the planner can assign up to \( T \) different \( \{c, y\} \) pairs to each reported wage, knowing that individuals of different ages may choose different allocations. The stronger alternative would
The Partial Reform planner in the baseline model solves the following problem:

**Problem 2:** *(Partial Reform: Age-Dependent)*

$$\max_{\{c,y\}} \sum_{i=1}^{I} \pi^i \alpha^i V^i$$

subject to the feasibility constraint (1.4) and the incentive constraints

$$\beta^{t-1} \left( u(c^i_t) - v \left( \frac{y^i_t}{w^i_t} \right) \right) \geq \beta^{t-1} \left( u(c^j_t) - v \left( \frac{y^j_t}{w^j_t} \right) \right)$$ (1.6)

for all \( i, j \in \{1, 2, \ldots, I\} \) and \( t \in \{1, 2, \ldots, T\} \)

Because of the Partial Reform planner’s ability to restrict individuals to age-specific allocations, these incentive constraints say that the planner must guarantee only that each individual \( i \) of age \( t \) chooses the allocation intended for it over that intended for any other individual \( j \) of the same age \( t \). To see this, notice that the right-hand side of (1.6) depends on \( c^j_t \) and \( y^j_t \), so that both sides of the inequality are specific to age \( t \) (compare this to the Static Mirrlees planner, where the right-hand side depended on \( c^j_s \) and \( y^j_s \)). Formally, the set of constraints (1.6) is a subset of (1.5). This makes the set of incentive constraints in the Partial Reform planner’s problem weakly easier to satisfy than the set in the Static Mirrlees planner’s problem. I denote the multipliers on these constraints with \( \mu^{ij}_{t|t} \), where \( \mu^{ij}_{t|t} \) corresponds to the constraint preventing individual \( i \) of age \( t \) from preferring the allocation intended for individual \( j \) of age \( t \).

In practical terms, age dependence means that taxes can be tailored to the wage distribution at each age and that transfers can be made between age groups. As we will see in further constrain the Static Mirrlees problem, magnifying the results of the paper.
the numerical results below, these turn out to be valuable tools because the distribution of wages varies with age.

Finally, the Full Optimum planner in the baseline model solves the following problem:

**Problem 3:** *(Full Optimum: Age-Dependent and History-Dependent)*

\[
\max_{\{c,y\}} \sum_{i=1}^{I} \pi^i \alpha^i V(i)
\]

subject to the feasibility constraint (1.4) and the incentive constraints

\[
\sum_{t=1}^{T} \beta^{t-1} \left( u\left(c^i_t\right) - v\left(y^i_t \over w^i_t\right) \right) \geq \sum_{t=1}^{T} \beta^{t-1} \left( u\left(c^j_t\right) - v\left(y^j_t \over w^j_t\right) \right).
\]

for all \( i, j \in \{1, 2, ..., I\} \).

These incentive constraints reflect the Full Optimum planner’s ability to make, and commit to, history-dependent allocations.\(^{14}\) History dependence allows the planner to hold an individual to the lifetime path of allocations intended for a single type at all ages. Thus, the Full Optimum planner must guarantee only that each individual \( i \) chooses the *lifetime path* of allocations intended for it over that intended for any other type \( j \). This is apparent from (1.7) in that each side of the inequality is a discounted sum of period utilities over individual \( i \)’s lifetime. The left-hand side is \( i \)’s lifetime utility if it chooses its intended allocations \((c^i_t, y^i_t)\) at each age \( t \), while the right-hand side is \( i \)’s lifetime utility from claiming the allocations intended for type \( j \) at each age. I denote the multipliers on these

\(^{14}\) The assumption that the planner can commit to a path of allocations is standard in the dynamic optimal tax literature. Bisin and Rampini (2005) study the impacts of relaxing that assumption.
constraints with \( \{\mu^{|i|}j^{|j|}\}^{|i|} \), where \( \mu^{|i|}j^{|j|} \) corresponds to the constraint preventing individual \( i \) from preferring the lifetime allocation intended for individual \( j \).

Using history dependence to satisfy incentives on a lifetime basis can be a powerful tool for the planner. For example, suppose the planner wants to give individual \( i \) a generous allocation later in life in exchange for a "bad" allocation early in life. In the Full Optimum, the planner can offer that path of allocations to the individual because it can make later allocations dependent on earlier ones. In the Static Mirrlees or Partial Reform scenarios, such a path is not sustainable. In those scenarios, individuals know that the planner cannot reward early sacrifice because it cannot use history dependence, so they will not accept the bad allocation early in life.

These three sets of incentive constraints, and their corresponding policy scenarios, lie along a continuum of sophistication,

\[
\text{Least sophisticated} \quad \text{Most sophisticated}
\]

where the precise meaning of "sophistication" depends on the characteristics of the economy being considered. In this baseline model, where there is no private saving or borrowing, a policy’s sophistication depends on two features: age dependence and history dependence.\(^{15}\) The Static Mirrlees policy is neither age-dependent nor history-dependent. At the other extreme, in the tradition of recent work on optimal dynamic taxation, the Full Optimum policy is both age-dependent and history-dependent.\(^{16}\) Partial Reform policy strikes

\(^{15}\) This informal discussion of sophistication does not apply to the models of the economy (discussed later) that include private saving and borrowing. Rather than specifying a variant on sophistication for each model economy, we rely on the formal incentive constraints to clarify the distinctions.

\(^{16}\) History dependence cannot be avoided in this baseline economy by using wealth to encode past wages,
a middle ground between the Static Mirrlees and Full Optimum, being age-dependent but not history-dependent.

We can see this relationship between the three policies formally. The transition between the Static Mirrlees incentive constraints (1.5) and the Full Optimum incentive constraints (1.7) can be decomposed into two steps. First, the planner replaces all the $s$ subscripts in (1.5) with $t$ subscripts, limiting individuals to claiming only those allocations intended for individuals of the same age. This adds age dependence. Second, the planner adds the discounting and summations over $t$ that appear in (1.7), using its ability to commit to lifetime allocations. This adds history dependence. The Partial Reform policy takes only the first of these two steps.

Before analyzing these models in detail, I note that the Static Mirrlees policy is not designed to match the detailed structure of existing tax policy on labor income. Rather, it is the optimal policy constrained by two characteristics of existing tax policy: age independence and history independence. Our comparison of the Partial Reform to this Static Mirrlees policy rather than to existing tax policies allows us to isolate the potential for age dependence to generate welfare gains by itself, relative to an age-neutral benchmark.\textsuperscript{17}

1.2.3 Analytical results

Now, I compare the allocations chosen by the social planner in the Static Mirrlees, Partial Reform, and Full Optimum policies along two margins: the intratemporal margin between such as in Albanesi and Sleet (2006), because individuals cannot accumulate wealth.\textsuperscript{17} In addition to the simulations of optimal policy presented below, I have simulated a policy approximating the current U.S. tax system. Using the data and parameterization described in the paper, I apply the 1999 U.S. income tax schedule to labor earnings and simulate individuals’ behavior. The welfare gain from that policy to the Static Mirrlees policy is equivalent to a 6.5 percent increase in aggregate annual consumption.
consumption and leisure and the intertemporal margin between consumption in one period and the next. In particular, I derive theoretical results on these margins that connect to and extend classic results and intuition from static and dynamic Mirrleesian tax analysis. The importance assigned to these margins is due to their relation, conceptually and formally, to marginal taxes on labor income and taxes on capital income. I evaluate average taxes, a key dimension along which the three policy scenarios differ, in the section using numerical simulations. The Mirrlees model, and this extension of it, are not well-suited to discussing average taxes analytically.

**Intratemporal distortions**

First, I compare the distortions to individuals’ choices of how much income to earn: i.e., distortions on their marginal choices between consumption and leisure. I begin with a definition:

**Definition**  
The intratemporal distortion for an individual of type $i$ and age $t$ is denoted $\tau (i, t)$ and equals

\[
\tau (i, t) = 1 - \frac{v' \left( \frac{w_i}{w_t} \right)}{w_i' u' (c_t^i)}. \tag{1.8}
\]

Denote $\tau^{SM} (i, t)$, $\tau^{PR} (i, t)$, and $\tau^{FO} (i, t)$ as the intratemporal distortions for an individual of type $i$ and age $t$ in the solutions to the Static Mirrlees (SM), Partial Reform (PR), and Full Optimum (FO) planner’s problems.

In expression (1.8), positive $\tau (i, t)$ distorts the individual’s choice away from work (and consumption) and toward leisure. If $\tau (i, t) = 0$, the individual sets the marginal
utility from an extra unit of consumption equal to the marginal disutility of earning it, so there is no distortion on this margin.

With these preliminaries, we can now turn to the results. The following proposition serves as a benchmark:

**Proposition 1 (Intratemporal Benchmark)** Let the utility function for all \( i \in \{1, 2, ..., I\} \) and \( t \in \{1, 2, ..., T\} \) be defined by (1.1), and let \( \beta R = 1 \). If \( w_s^i = w_t^i \) for all \( i \in \{1, 2, ..., I\} \) and \( s, t \in \{1, 2, ..., T\} \), then

1. \( \tau^{SM}(i, s) = \tau^{SM}(i, t) \), \( \tau^{PR}(i, s) = \tau^{PR}(i, t) \), and \( \tau^{FO}(i, s) = \tau^{FO}(i, t) \) for all \( i \in \{1, 2, ..., I\} \) and \( s, t \in \{1, 2, ..., T\} \).

2. \( \tau^{SM}(i, t) = \tau^{PR}(i, t) = \tau^{FO}(i, t) \) for each \( i \in \{1, 2, ..., I\} \) and \( t \in \{1, 2, ..., T\} \).

**Proof.** In Appendix. ■

This proposition is about an economy in which each individual has a constant wage throughout its lifetime: \( w_s^i = w_t^i \) for all \( i \in \{1, 2, ..., I\} \) and \( s, t \in \{1, 2, ..., T\} \). This means that the distribution of wages in the population is the same at every age. In such an economy, each individual faces the same distortion at all ages in each policy.\(^{18}\) For example, in the Static Mirrlees policy, \( \tau^{SM}(i, s) = \tau^{SM}(i, t) \) for all \( i \in \{1, 2, ..., I\} \) and \( s, t \in \{1, 2, ..., T\} \). In addition, each individual faces the same distortion at a given age in all three policy scenarios, so \( \tau^{SM}(i, t) = \tau^{PR}(i, t) = \tau^{FO}(i, t) \) for each \( i \in \{1, 2, ..., I\} \) and \( t \in \{1, 2, ..., T\} \).

\(^{18}\) Werning (2007a) proves a similar result for the optimal dynamic policy.
Intuitively, an unchanging wage distribution means that each age is a replica of the others, so that the planner’s best solution for one age will be the best solution for all ages. This means constant intratemporal distortions for each individual over time. It also means that the three planners are solving equivalent problems, so they choose the same pattern of intratemporal distortions.

As this proposition implies, any differences between the three policy scenarios’ intratemporal distortions rely on individuals’ lifecycle wage paths not being constant. Before characterizing these distortions in general, I focus on a specific distortion: that at the top of the income distribution.

**The top marginal distortion**

In this subsection, I focus on a classic result from static optimal tax analysis: the top earner in the economy should face *no* intratemporal distortion.\(^\text{19}\) The intuition for the classic result is that an intratemporal distortion has both a cost and a benefit. The cost is that it causes individuals to work and consume differently than they would without taxes, leading to either lower utility or less efficiently-provided utility for these individuals. The benefit is that it enables the planner to collect more tax revenue from higher earners, increasing the extent of redistribution.\(^\text{20}\) At the top of income distribution, this benefit

---

\(^\text{19}\) Diamond (1998) and Saez (2002) show that this result depends on the shape of the wage distribution. With a bounded wage distribution such as that used throughout the paper, the zero top rate result always holds. In the Appendix, I show that the results presented here have analogues in a model with a wage distribution more similar to that which they use.

\(^\text{20}\) It does this by taking advantage of differences in the marginal disutility of income across individuals with different wages. A distortion on a given individual *i* harms *i* less than it harms a higher-skilled individual *j* claiming *i*’s allocations. While *i* suffers lower consumption due to the distortion, it also enjoys working less. For *j*, the loss in consumption exacerbates the main disadvantage to claiming *i*’s allocation, while the decrease in required income is of less value because *j* is already working much less than it would to earn its own allocation. Thus, the planner can assign a larger lumpsum transfer to *i* while discouraging *j* from
is zero (there are no higher earners from whom to collect more tax revenue). Thus, a
distortion on the top earner solely discourages effort, and it is avoided in the optimal policy.

The following proposition describes how this classic result applies to a dynamic econ-
omy under this paper’s three policy scenarios:

**Proposition 2 (Top Marginal Distortion)** *In the baseline economy,*

1. if \( w_i^t \geq w_j^t \) for all \( j \in \{1, 2, ..., I\} \), then \( \tau^{PR}(i, t) \leq 0 \) and \( \tau^{FO}(i, t) \leq 0 \),

2. if \( w_i^t \geq w_j^t \) for all \( j \in \{1, 2, ..., I\} \) and for all \( t \in \{1, 2, ..., T\} \), and if \( a_i^j \leq a_j^j \) for all
   \( j \in \{1, 2, ..., I\} \), then \( \tau^{PR}(i, t) = 0 \) and \( \tau^{FO}(i, t) = 0 \) for all \( t \in \{1, 2, ..., T\} \), and
   \( \tau^{SM}(i, t) = 0 \) for \( t \) such that \( w_i^t \geq w_s^t \) for all \( s \in \{1, 2, ..., T\} \).

**Proof.** In Appendix. ■

The first part of this proposition states that the highest wage earner at each age faces
a nonpositive intratemporal distortion whenever the social planner can condition taxes on
age: that is, in the Partial Reform and Full Optimum policy scenarios. In particular, the dis-
tortion on the top earner at each age could be negative in these scenarios. The second part
of this proposition states that an individual who is the highest wage earner at all ages faces
no intratemporal distortion at any age in the Partial Reform and Full Optimum scenarios,
but only at its peak-earnings age in the Static Mirrlees scenario.

The first part of this proposition is a surprising theoretical deviation from the classic
static optimal tax result because it allows for the possibility that, with age dependence,
the top earner at a given age may face a negative distortion.\footnote{The optimality of a negative top distortion has also been suggested by Judd and Su (2006), but for a different reason. In their model with multiple dimensions of heterogeneity, the interaction of wage and labor supply elasticity differences can justify negative top distortions. Judd and Su also perform an illustrative simulation of age-dependent taxation.} Intuitively, suppose an individual has the highest wage within its current age but also has a low lifetime income-earning potential. An example might be an entertainer or athlete whose earnings power is temporarily high at a young age. The planner wants to assign this individual both high income and high consumption: the former because it is a productive worker, and the latter because its welfare weight is relatively large. A negative marginal distortion makes this possible. With it, the planner can levy high average taxes on individuals who earn less at the current age but more over their lifetimes, for example consultants or lawyers, while using the negative distortion to reduce the tax burden on the top current earner. The lower earners will not be willing to earn enough income to qualify for the negative distortion, so the planner is able to target its resources.\footnote{In the language of the mechanism, a worker with a lower current wage but a higher lifetime income-earning potential may be tempted to claim the allocation assigned to this top current earner. To prevent this, the planner makes such a claim more costly for the lower-skilled worker by negatively distorting the top earner, increasing its pre-tax income. Following a similar logic, a negative distortion is, in principle, possible in the Static Mirrlees for the top earner in the entire economy across ages if that top earner is, at other ages, a low earner.}

The second part of the proposition highlights a key difference in intratemporal distortions between the Static Mirrlees and the two more sophisticated scenarios. To see why it holds, consider an individual $i^*$ with the highest wage at all ages. As mentioned above, the potential benefit of a positive marginal distortion on any individual is that it makes possible the collection of more tax revenue from higher earners. If there are no higher earners than $i^*$ at each age, positive distortions on $i^*$ have costs but no benefits, so they are avoided.
by the planners with access to age-dependent taxes. The Static Mirrlees planner, on the other hand, faces a more difficult problem. While no other individuals of the same age earn more than $i^*$, some individual of a different age (perhaps $i^*$ itself) earns more than $i^*$ for each age except the age at which $i^*$’s earnings peak. Thus, in order to collect revenue from the highest earners across all age groups, the planner will use distortions on $i^*$ at all but its peak-earnings age.\footnote{In the language of the direct mechanism, no other individual of the same age wants to mimic $i^*$. The only reason to distort an individual is to discourage mimicking by others, so the age-dependent policies leave $i^*$ undistorted. In the Static Mirrlees, some individual of a different age (perhaps $i^*$ itself) earns more than $i^*$ for each age except the one at which $i^*$’s earnings peak. Thus, $i^*$ will be a tempting target for mimicking, and the Static Mirrlees planner must distort it at all but its peak-earnings age.} A simple numerical illustration of this is provided in the Appendix for the interested reader.

Though the second part of the proposition is about a special topic, the top marginal distortion, it highlights the fundamental limitation of the Static Mirrlees policy relative to the Partial Reform and Full Optimum: the former’s inability to hold individuals to age-specific tax schedules. The effects of this limitation ripple throughout the wage distribution, allowing the more sophisticated planners to distort individuals’ labor efforts less than the Static Mirrlees planner. Later, we will quantify these effects and their impact on social welfare in the numerical simulations below. Next, however, we turn to characterizing intratemporal distortions other than at the top of the income distribution.

**General characterization**

In this subsection, I begin by giving formal expressions for the intratemporal distortions in each scenario. As in classical Mirrleesian analysis, characterizing intertemporal distortions in general is difficult due to their dependence on the details of the wage distribu-
1.2 Baseline economy

Thus, the formal expressions below are not in closed form. Nevertheless, we can use them to determine the key forces driving distortions and to address some natural questions about the pattern of distortions in each policy. To simplify the results, I assume disutility takes the isoelastic form

\[ v \left( \frac{y_t^i}{w_t^i} \right) = \frac{1}{\sigma} \left( \frac{y_t^i}{w_t^i} \right)^\sigma. \]  (1.9)

where the parameter \( \frac{1}{\sigma-1} \) gives the constant-consumption elasticity of labor supply.

In the Static Mirrlees scenario, the intratemporal distortion on worker of type \( i \) and age \( t \) is:

\[ \tau^{SM} (i, t) = \frac{\sum_{s=1}^T \sum_{j=1}^I \left( 1 - \left( \frac{w_t^i}{w_t^j} \right)^\sigma \right) \beta^{s-t} \mu_{ij}^{s,t}}{\alpha^{i\pi} + \sum_{s=1}^T \sum_{j=1}^I \mu_{ij}^{s,t} - \sum_{s=1}^T \sum_{j=1}^I \left( \frac{w_t^i}{w_t^j} \right)^\sigma \beta^{s-t} \mu_{ij}^{s,t}}. \]  (1.10)

where, as stated after (1.5), \( \mu_{ij}^{s,t} \) is the multiplier on the incentive constraint preventing individual \( i \) of age \( t \) from claiming the allocation of any other individual \( j \) of age \( s \).

In the Partial Reform scenario, it is:

\[ \tau^{PR} (i, t) = \frac{\sum_{j=1}^I \left( 1 - \left( \frac{w_t^i}{w_t^j} \right)^\sigma \right) \mu_t^{i|j}}{\alpha^{i\pi} + \sum_{j=1}^I \mu_t^{i|j} - \sum_{j=1}^I \left( \frac{w_t^i}{w_t^j} \right)^\sigma \mu_t^{i|j}}. \]  (1.11)

where \( \mu_t^{i|j} \) is the multiplier on the incentive constraint preventing individual \( i \) of age \( t \) from claiming the allocation of any other individual \( j \) of the same age \( t \).

In the Full Optimum scenario, it is:

\[ \tau^{FO} (i, t) = \frac{\sum_{j=1}^I \left( 1 - \left( \frac{w_t^i}{w_t^j} \right)^\sigma \right) \mu_t^{i|j}}{\alpha^{i\pi} + \sum_{j=1}^I \mu_t^{i|j} - \sum_{j=1}^I \left( \frac{w_t^i}{w_t^j} \right)^\sigma \mu_t^{i|j}}. \]

Similar conclusions to those below, as well as closed form results for intratemporal distortions, are obtainable for a continuous wage distribution from an analysis using the Hamiltonian methods familiar from static Mirrlees analysis such as Saez (2002) or Salanie (2003). Such an analysis for the baseline model can be found in the Appendix. The method does not extend to the dynamic settings with private saving considered later in this paper (Case 2 and Case 4), so I use the alternative method, conventional in the dynamic optimal tax literature, throughout the paper.
where $\mu^{ij}_t$ is the multiplier on the incentive constraint preventing individual $i$ from claiming the lifetime allocation of any other individual $j$.

One lesson we learn from these expressions is that the tradeoff familiar from static Mirrlees analysis remains relevant in this baseline dynamic model. As discussed after Proposition 2, increasing the distortion on individual $i$ of age $t$ has a cost and a benefit to the planner. Consider the expression for $\tau^{PR}(i, t)$, the intratemporal distortion on individual $i$ of age $t$ in the Partial Reform policy (similar analysis holds for the Static Mirrlees and Full Optimum). This distortion is increasing in $\mu^{ij}_t$ when $\mu^{ij}_t$ is positive for $j$ with a higher wage than $i$ at age $t$. Why? Recall that $\mu^{ij}_t$ is the cost in terms of social welfare to the Partial Reform planner of ensuring that $j$ prefers its allocation to $i$’s when of age $t$. The planner has two tools to use in satisfying $j$’s incentives: it can allocate resources to $j$ that it would prefer to allocate to those lower in the earnings distribution, or it can distort the allocations to lower types in order to make them less tempting to $j$. Because the planner has redistributive tastes (or, because utility is concave) and the latter tool allows for a more egalitarian distribution of resources, a distortion on $i$ can be beneficial despite its cost in reducing $i$’s labor effort. The more costly it is to satisfy $j$’s incentives (i.e., the larger is $\mu^{ij}_t$) the more the planner is willing to distort $i$ rather than transfer resources to $j$.

We can also use these expressions to consider more specific topics. Next, I address three natural questions about the structure of intratemporal distortions in an age-dependent tax policy.

First, does an individual face rising or falling intratemporal distortions over its lifecycle? In the Static Mirrlees scenario, the tax schedule is constant over the lifecycle, so the
answer to the question depends only on two factors: (1) the shape of that tax schedule, and (2) whether the individual moves up or down in the income distribution over its lifetime. Of course, these factors depend on the individual’s wage path, the distribution of all wages in the economy, and the planner’s preferences, so we can say very little about the lifecycle path of intratemporal distortions in general. The answer is yet more complicated in the Partial Reform scenario, because that planner sets policy based on an individual’s wage relative to its own age’s wage distribution. Thus, the path of distortions for an individual is sensitive not only to the individual’s wage path but also to the wage paths of all individuals of the same age, further complicating the general characterization of the lifecycle path of distortions. I provide a numerical illustration of this sensitivity in the Appendix.

Second, how would variation in the elasticity of labor supply with age affect the intratemporal distortions an individual faces? If an isolated individual is more elastic at one age than at all other ages, it is more responsive to distortions at that age. This affects the tradeoff facing planners that can use age dependence, who respond by lowering the intratemporal distortion on the individual at that age. In contrast, the Static Mirrlees planner is unable to respond because it cannot hold individuals to age-specific tax schedules. General results are difficult for broader differences in elasticity, such as when all individuals are more elastic at some ages than at others. Intuitively, if all individuals are more elastic, a larger distortion on some may be optimal to enable redistribution of income without discouraging effort among the high-skilled. Nevertheless, in Section 1.6 I find that the in-

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25 This ambiguity is in part due to the distinction between intratemporal distortions and marginal taxes (which include changes in lump-sum grants and taxes) in a model with a discrete wage distribution. As shown in the Appendix, age-dependent marginal rates are always inversely correlated with labor supply elasticities for a continuous wage distribution.
tuition from the individual case carries through for a calibrated numerical simulation with
elasticity differences by age, so that intratemporal distortions are lower at ages with more
elastic labor supply in the Partial Reform and Full Optimum scenarios.

Third, if two individuals earn the same current income at different ages, who should
face the higher distortion on that income? Intuitively, suppose a manual laborer earns
the same income when middle-aged as a professional earns when young. In the Static
Mirrlees, these workers are treated identically (i.e., they face the same distortion) because
they have the same position in the overall earnings distribution. In contrast, the Partial
Reform and Full Optimum planners are able to distinguish between these workers and
consider each worker’s place in its own age-specific distribution, so they need not face the
same distortion. It is ambiguous who will face the larger distortion, because the relative
benefit from distorting the two workers is ambiguous. Distorting the young professional
raises more tax revenue from a population of higher earners that is both smaller (implying
a smaller distortion) and richer in lifetime income (implying a larger distortion) than the
population of higher earners above the manual laborer. A numerical illustration of this is
provided in the Appendix.

As is standard in the Mirrleesian optimal tax literature, in Section 1.2.4 I turn to
calibrated simulations to further explore these questions and others. Before doing so,
however, I discuss a second set of analytical results, these on the intertemporal consumption
margin.
1.2 Baseline economy

**Intertemporal Distortions**

In this section, I analytically characterize the intertemporal consumption margin in an age-dependent policy and compare it to well-known results from the dynamic optimal tax literature. I show that age-dependent policy satisfies a condition that improves on age-independent policy but falls short of the full optimum. The recent development of a dynamic Mirrlees literature has highlighted this margin because optimal distortions to it, most prominently characterized by Golosov, Kocherlakota, and Tsyvinski (2003), have renewed interest in the taxation of capital after a long period during which the Chamley-Judd result of zero optimal capital taxation held sway.

Consider an individual’s problem of maximizing lifetime utility (1.2) given a wage path \( \{ w_i^t \}_{t=1}^T \) and the lifetime budget constraint \( \sum_{t=1}^T R^{T-t} (y_i^t - c_i^t) = 0 \). Continue to assume \( \beta R = 1 \). This individual’s optimal choice of consumption satisfies, for each \( (t, t+1) \) pair:

\[
 u'(c_i^t) = u'(c_{i+1}^t). \tag{1.12}
\]

This is the familiar intertemporal Euler condition that sets the marginal utility from consumption equal across periods. Expression (1.12) represents an undistorted intertemporal margin.

I analyze the extent to which the planners’ chosen allocations distort this intertemporal margin. Recall that individuals cannot save or borrow in the baseline economy. The following Proposition serves as a benchmark:
Proposition 3 (Intertemporal Benchmark) Let the lifetime utility function for all \( i \in \{1, 2, ..., I\} \) be defined by (1.2), and let \( \beta R = 1 \). If \( w_i^s = w_i^t \) for all \( i \in \{1, 2, ..., I\} \) and \( s, t \in \{1, 2, ..., T\} \), then \( u'(c_i^t) = u'(c_{i+1}^t) \) for all \( i \in \{1, 2, ..., I\} \) and \( t, t + 1 \in \{1, 2, ..., T\} \) in the Static Mirrlees, Partial Reform, and Full Optimum planner’s problems.

Proof. In Appendix. 

This proposition, a parallel to Proposition 1 above, states that no individual faces an intertemporal distortion in any of the three policy scenarios if each individual has a constant wage over its lifetime.\(^{26}\) Intuitively, if each age is a replica of the next, the allocations to each individual will be the same at each age.

While this proposition provides a useful benchmark, we are interested in more realistic settings, namely with changing wage distributions. Solving the planner’s problems as stated at the start of this section, we can obtain the following results.

The Full Optimum planner’s allocations satisfy, for all \( i \in \{1, 2, ..., I\} \) and \( t, t + 1 \in \{1, 2, ..., T\} \):

\[
u'(c_i^t) = u'(c_{i+1}^t),
\]

a classic Atkinson and Stiglitz (1976) result: i.e., the Full Optimum policy does not distort intertemporal allocations.\(^{27}\) This result depends on the Full Optimum planner’s ability to use history-dependent allocations, as is made clear by the contrast between expression (1.13) and the result for the Partial Reform planner to which I now turn.

\(^{26}\) Werning (2007a) proves a similar result for the optimal dynamic policy.

\(^{27}\) Note that wages are deterministic, so there is no reason for an intertemporal distortion along the lines of Rogerson (1985) or Golosov, Kocherlakota, and Tsyvinski (2003). In Sections 1.4 and 1.5, I analyze generalizations of the baseline model where wages are stochastic.
1.2 Baseline economy

The Partial Reform planner’s allocations satisfy, for individual \( i \) and ages \( t, t + 1 \):

\[
\begin{align*}
\left( \frac{\pi^i c^i_t}{\pi^i c^i_{t+1}} - \frac{\sum_{j=1}^{I} \mu^j_{t+1}}{\sum_{j=1}^{I} \mu^j_t} \right) \left( \frac{\sum_{j=1}^{I} \beta^j_{t+1}}{\sum_{j=1}^{I} \beta^j_t} \right) u'(c^i_{t+1}) = 1.
\end{align*}
\]

The ratio in parentheses in (1.14) is generally different from one, implying that the Partial Reform planner imposes a distortion on the intertemporal margin. To see this, recall that \( \mu^j_{t+1} \) and \( \mu^j_t \) are the multipliers on the incentive constraints preventing type \( i \) from claiming type \( j \)’s allocation at ages \( t \) and \( t + 1 \), respectively. Unless \( \mu^j_{t+1} = \mu^j_t \) for all \( i, j, \) and \( t \), the ratio in parentheses in (1.14) is not equal to one. Intuitively, whenever the incentive problems facing the planner differ across ages, the Partial Reform planner generally fails to satisfy the intertemporal Euler equation. The Partial Reform planner must satisfy incentives at each age, so changing wage distributions make the allocation for an individual differ across ages, violating the intertemporal Euler equation.

The ability to satisfy the standard Euler equation and provide smooth consumption is a substantial advantage for the Full Optimum planner relative to the Partial Reform (or Static Mirrlees\textsuperscript{28}) planner. Consider two workers who have similar lifetime incomes in present value: say, an engineer and a lawyer. The engineer has a relatively flat earnings profile over its lifetime, while the lawyer has low earnings when young but a steep earnings profile that eventually raises its earnings well above the engineer’s. Because these workers have similar lifetime incomes, the planner weighs them equally in the social welfare function. It would like to give them both smooth paths of consumption that have similar present values while concentrating their labor supply on the ages at which they are most

\textsuperscript{28} I omit the Static Mirrlees planner’s intertemporal result for brevity, but it is an intuitive modification of (1.14).
productive. The Full Optimum planner is able to achieve these goals, even though this includes giving the engineer the same consumption as the lawyer when young but having the engineer earn much more income. The Full Optimum planner achieves this by promising the engineer that the situation will be reversed when they are older: the engineer will have the same consumption as the lawyer but will have to earn much less.

The Partial Reform and Static Mirrlees planners cannot achieve the smoothed consumption paths obtained by the Full Optimum planner because they cannot make history-dependent allocations. In other words, they cannot promise to reward sacrifice at a later date. Suppose the Partial Reform planner tried to match the Full Optimum’s allocation just described for the engineer and lawyer. Knowing that the planner would not be able to reward her for earning high income and accepting low consumption, the young engineer will be tempted to work less and claim the lower average tax rates intended for the lawyer. Similarly, the lawyer will know that the planner cannot prevent him from working less and claiming the lower average tax rates intended for the engineer when old. Thus, the planner cannot tailor labor effort while smoothing consumption. Instead, it more closely aligns the lawyer’s consumption to its earnings path and skews the engineer’s consumption in the other direction. This encourages the lawyer to work more when old and the engineer to work more when young, solving the incentives problem. But, it requires intertemporal distortions. A simple numerical example of this two-type economy is provided in the Appendix.
While at a disadvantage to the Full Optimum, the Partial Reform allocations do satisfy what I will call the "Symmetric Inverse Euler Equation." The following proposition states the result.

**Proposition 4 (Symmetric Inverse Euler)** Let the lifetime utility function for all \( i \in \{1, 2, ..., I\} \) be defined by (1.2). Then, the solution to the Partial Reform planner's problem satisfies:

\[
\sum_{i=1}^{I} \pi^i u'(c(i,t)) = \sum_{i=1}^{I} \pi^i u'(c(i,t+1)),
\]

for any \( t, t + 1 \in \{1, 2, ..., T\} \).

**Proof.** In Appendix. □

This "Symmetric Inverse Euler Equation" guarantees that resources are being allocated efficiently between age groups, as it equalizes across ages the cost (in consumption) of increasing welfare. Though not as powerful a restriction as the intertemporal Euler equation, the Symmetric Inverse Euler Equation is nevertheless an achievement of Partial Reform that the Static Mirrlees planner cannot replicate. Because it cannot restrict individuals to age-specific tax schedules, the Static Mirrlees planner cannot make these efficient transfers across ages.

**Summary**

The analytical results above make clear that an age-dependent policy differs substantially from an age-independent one and resembles in many important ways a fully optimal policy. They show how theoretical results for age-dependent policy in a dynamic economy with lifecycle wage paths connect to and extend prominent results from the static and dy-
dynamic optimal tax literature. Moreover, they identify the factors at play in determining the pattern of both intratemporal and intertemporal distortions under age dependence.

As with most Mirrleesian analyses, however, these results can pin down only select characteristics of tax policy, and on many questions they provide only ambiguous theoretical guidance. To provide a more general characterization of policy and to answer many of these questions, I turn to numerical simulations.

### 1.2.4 Numerical results

The conventional, static Mirrlees optimal tax literature was linked to calibrated simulations of optimal policy from its beginnings. In contrast, the dynamic Mirrleesian literature has thus far used mostly abstract, illustrative simulations to reveal the effects of its recommendations.\(^{29}\) In this section, I calibrate the dynamic optimal tax problems specified above to detailed individual wage data from the U.S. PSID, simulate and characterize policy, and quantify the welfare impacts of reform. I begin by describing the data and parameter specification.

#### Data

The structure of the baseline model matches a well-known existing empirical literature on lifecycle income distributions and wage paths, beginning with Fullerton and Rogers (1993) and including recent work such as Altig, Auerbach, Kotlikoff, Smetters, and Wal-liser (2001) and Diamond and Tung (2006)\(^{30}\). My approach follows that literature.

---

\(^{29}\) An exception is Golosov and Tsyvinski (2006), who calibrate their disability model.

\(^{30}\) Thanks to John Diamond and Joyce Tung for providing helpful advice on the construction of the dataset.
The goal is to construct representative lifetime wage paths for $I$ groups of individuals classified according to lifetime income-earning potential. The construction of the required data can be divided into four steps. First, I limit the data to household heads from the U.S. PSID core sample for the years 1968-2001 and collect data on their income, hours worked, age, race, gender, and education for each year they are a head of household. Second, I calculate reported real wages for each observation by dividing reported labor income by reported hours (a potentially noisy measure of the wage but the best one available) and inflating or deflating the data with the CPI to put all wages in 1999 dollars. Third, I remove potentially problematic observations by eliminating all those for which reported annual hours were less than 500 or greater than 5,824, for which reported labor income was zero but hours were positive, or for which the nominal wage implied by earnings and hours was less than half the applicable minimum wage in that calendar year. After these adjustments, the dataset contains approximately 155,000 observations on just over 19,000 individuals with an average of 8.1 years observed per person. Fourth, I use these data to estimate a weighted (by the PSID sample weights) individual fixed-effects regression of the log wage on a quartic in age and interaction terms that multiply education, gender, and race with both age and age-squared. With the results of this estimation, I predict a wage path for each individual. The resulting dataset contains the predicted wage paths over a full lifecycle as well as the observed wage paths for the sample of household heads.

---

31 When using an empirical income distribution to infer the distribution of skills and simulate optimal taxes, it is important to back out the effects of the current tax system on income as in Saez (2001). With data on income and hours, it is possible to calculate wages directly, instead.

32 Controlling for time and cohort effects, as studied in Heathcote, Storesletten, and Violante (2005), does not materially affect the results. For time effects, I control for year dummies in the estimation of individual fixed effects and in the calculation of representative wage paths for each type. For cohort effects, I perform
In the baseline economy, individuals are classified into types $i = \{1, 2, ..., I\}$, where $i$ indexes lifetime income-earning potential. The natural empirical counterpart to this is the amount of income an individual can earn over its lifetime, given its lifecycle wage path. As Fullerton and Rogers (1993) express it,

$$\text{Potential Lifetime Income} = \sum_{t=1}^{T} \frac{\hat{w}_t \times 4,000}{(1 + r)^{t-1}}$$

where $T$ is set to 55 to include predicted income over the age range 21-75, $\hat{w}_t$ is the predicted wage rate at age $t$, the number 4,000 represents maximum feasible hours of work in a year (e.g., 80 hours per week for 50 weeks), and I set the gross rate of return $1 + r = 1.05$.\(^{33}\)

After calculating potential lifetime income for each individual, I divide the sample into $I$ quantiles and assign an index $i = \{1, ..., I\}$ to each individual. For the baseline model, I use $I = 10$.

The final step is to calculate a representative wage path (using observed, not predicted, wages) for each lifetime income group. I retain only those individuals who report a wage in at least four years to limit the influence of atypical observations. This eliminates approximately 13,000 observations from the data corresponding to approximately 7,500 infrequently observed individuals. Due to computational speed considerations, I calculate representative wage paths for each income group by grouping data into the three main decades of work life.\(^{34}\) Within each type $i$, I calculate the average real wage over

---

\(^{33}\) The specific choices of $T = 55$, a maximum of 4,000 hours, and $1 + r = 1.05$ are all of minor importance, in that very few individuals would be classified into different income deciles if they were modified.

\(^{34}\) Using decade-average income smooths out fluctuations in annual incomes that could make age dependence less powerful. This possibility is best examined in the stochastic wage paths extension below (Case 3), where wages are more free to fluctuate across ages. Simulations in that model suggest that this paper’s
the ranges 30-39, 40-49, and 50-59. These averages form the series of wages \( \{w_t^i\}_{i=1}^T \) for \( i = \{1, 2, \ldots, I\} \) and \( t = \{1, 2, 3\} \). The results are shown in Table 1.1 along with demographic data describing the income groups.

Table 1.1: Data for simulation of baseline model

<table>
<thead>
<tr>
<th>Wage paths</th>
<th>Age range</th>
<th>Bottom</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>age range</td>
<td>40-49</td>
<td>7.02</td>
<td>9.81</td>
<td>11.73</td>
<td>13.94</td>
<td>15.93</td>
<td>18.23</td>
<td>20.48</td>
<td>23.83</td>
<td>28.79</td>
<td>46.52</td>
</tr>
<tr>
<td>Descriptive data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion: race=white</td>
<td></td>
<td>0.35</td>
<td>0.46</td>
<td>0.49</td>
<td>0.62</td>
<td>0.65</td>
<td>0.67</td>
<td>0.72</td>
<td>0.79</td>
<td>0.80</td>
<td>0.91</td>
</tr>
<tr>
<td>Proportion with college</td>
<td></td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
<td>0.09</td>
<td>0.14</td>
<td>0.17</td>
<td>0.20</td>
<td>0.28</td>
<td>0.37</td>
<td>0.60</td>
</tr>
<tr>
<td>Proportion gender=male</td>
<td></td>
<td>0.50</td>
<td>0.64</td>
<td>0.72</td>
<td>0.79</td>
<td>0.84</td>
<td>0.86</td>
<td>0.90</td>
<td>0.93</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>Pareto weight (calculated)</td>
<td></td>
<td>1.00</td>
<td>0.99</td>
<td>0.98</td>
<td>0.97</td>
<td>0.96</td>
<td>0.94</td>
<td>0.93</td>
<td>0.92</td>
<td>0.90</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Figure 1.1 plots these wage paths.

---

results are robust to this concern.
The wage paths shown in Figure 1.1 are similar to those implied by the results of Fullerton and Rogers (1993), shown in their Table 4.11. Now I discuss the specification of the models’ parameter values.

**Parameter specification**

I assume the period utility function

\[ U(c, l) = \ln c - \frac{1}{\sigma} l^\sigma, \]

and set \( \sigma = 3 \), which implies a constant-consumption elasticity of labor supply of 0.5. The results described below are robust to alternative parameterizations, with the welfare gains from age dependence increasing when utility from consumption is more concave and the elasticity of labor supply is greater. I use an annual gross rate of return of five percent, which implies that \( R = (1.05)^{10} \) because I am using decade-long age ranges.
For the Pareto weights, I assume a form that is not increasing in an individual’s lifetime income-earning ability. Formally, the Pareto weight on individual $i$ is:

$$\alpha^i = \exp\left(-\rho \frac{\sum_{t=1}^{T} w_t^i R_t^{i-1} - \sum_{t=1}^{T} w_t R_t^{i-1}}{\sum_{i=1}^{I} \sum_{t=1}^{T} \pi^i w_t R_t^{i-1}}\right).$$

(1.16)

where $\rho \geq 0$. Note that $\alpha^i = 1$ for type $i = 1$, who has the lowest lifetime income-earning ability (defined above as the present value of wages).

The parameter $\rho$ allows us to vary the redistributive tastes of the planner: i.e., the extent to which Pareto weights decline with $i$. If $\rho$ equals zero, the planner is Utilitarian, and all Pareto weights equal one. For larger $\rho$, Pareto weights decline with $i$. My benchmark will be $\rho = 0.1$, implying moderate redistributive tastes for the planner. The weights corresponding to the data described above are shown in Table 1.1 above. They decline from 1.00 for the bottom income decile to 0.83 for the top decile.

With these data and parameters, I simulate the policy models. Now, I turn to the results of these simulations.

### Simulation results

I focus on four outputs from the numerical simulations: intratemporal distortions, average tax rates, intertemporal distortions, and welfare. For each, I compare the results under the Static Mirrlees, Partial Reform, and Full Optimum policy scenarios. Together, these four outputs allow me to provide a detailed characterization of optimal policy under each scenario and to quantify the welfare gains of reform from the Static Mirrlees to the more sophisticated policies.
1.2 Baseline economy

Table 1.2: Intratemporal distortions in baseline model simulation

Intratemporal distortion

<table>
<thead>
<tr>
<th>Age range</th>
<th>Bottom</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Top</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Mirrlees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>0.26</td>
<td>0.34</td>
<td>0.39</td>
<td>0.45</td>
<td>0.41</td>
<td>0.43</td>
<td>0.34</td>
<td>0.40</td>
<td>0.34</td>
<td>0.31</td>
<td>0.324</td>
</tr>
<tr>
<td>40-49</td>
<td>0.34</td>
<td>0.35</td>
<td>0.33</td>
<td>0.34</td>
<td>0.36</td>
<td>0.28</td>
<td>0.32</td>
<td>0.23</td>
<td>0.50</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>50-59</td>
<td>0.26</td>
<td>0.37</td>
<td>0.39</td>
<td>0.32</td>
<td>0.34</td>
<td>0.29</td>
<td>0.31</td>
<td>0.25</td>
<td>0.45</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Partial Reform</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>0.25</td>
<td>0.32</td>
<td>0.33</td>
<td>0.34</td>
<td>0.34</td>
<td>0.32</td>
<td>0.30</td>
<td>0.29</td>
<td>0.33</td>
<td>0.00</td>
<td>0.309</td>
</tr>
<tr>
<td>40-49</td>
<td>0.32</td>
<td>0.33</td>
<td>0.39</td>
<td>0.36</td>
<td>0.37</td>
<td>0.33</td>
<td>0.36</td>
<td>0.36</td>
<td>0.41</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>50-59</td>
<td>0.34</td>
<td>0.35</td>
<td>0.33</td>
<td>0.36</td>
<td>0.40</td>
<td>0.30</td>
<td>0.39</td>
<td>0.31</td>
<td>0.43</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Full Optimum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>0.27</td>
<td>0.33</td>
<td>0.35</td>
<td>0.36</td>
<td>0.35</td>
<td>0.34</td>
<td>0.32</td>
<td>0.31</td>
<td>0.36</td>
<td>0.00</td>
<td>0.303</td>
</tr>
<tr>
<td>40-49</td>
<td>0.30</td>
<td>0.32</td>
<td>0.37</td>
<td>0.35</td>
<td>0.36</td>
<td>0.32</td>
<td>0.34</td>
<td>0.33</td>
<td>0.39</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>50-59</td>
<td>0.32</td>
<td>0.34</td>
<td>0.32</td>
<td>0.35</td>
<td>0.38</td>
<td>0.29</td>
<td>0.36</td>
<td>0.29</td>
<td>0.40</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Intratemporal distortions

First, consider intratemporal distortions as defined in (1.8). Table 1.2 lists these distortions by age and lifetime income decile, while Figures 1.2a, 1.2b, and 1.2c plots them against annual income. The lines in the figures connect discrete points corresponding to the $I$ types at each age; the somewhat jagged pattern of distortions is a consequence of this discreteness.

The most striking difference between the scenarios is the treatment of the highest-income young (individuals in the 30-39 age range). The Static Mirrlees policy substantially distorts their intratemporal choice, while the Partial Reform and Full Optimum policies do

---

35 Income from the simulation results is converted to annual U.S. dollars as follows. The median annual hours worked in the data is 2,070 per year, while the Partial Reform planner has the corresponding worker exert 0.84 units of labor effort. This implies that a worker exerting one unit of labor effort per period in the model would work approximately 2,477 hours per year. I use this number as the benchmark for normal hours per year, and multiply the simulation results for income by it to obtain annual income as shown.
Figure 1.2a: Intratemporal Distortions in Static Mirrlees, Baseline Model

The graph shows the intratemporal distortions in a baseline economic model. The x-axis represents income, ranging from 0 to 160,000, while the y-axis represents intratemporal distortion, ranging from 0 to 0.60. Three distinct income groups are highlighted: 30-39, 40-49, and 50-59, each represented by different line styles.

- The blue line with markers indicates the 30-39 income group.
- The red dashed line represents the 40-49 income group.
- The black dashed line shows the 50-59 income group.

The graph illustrates how intratemporal distortions vary across different income levels.
Figure 1.2b: Intratemporal Distortions in Partial Reform, Baseline Model
Figure 1.2c: Intratemporal Distortions in Full Optimum, Baseline Model
not. This is the numerical counterpart to Proposition 2, in which we saw that the classic result of no marginal distortion at the top of the income distribution fails to extend to the Static Mirrlees policy within age groups. To repeat the intuition, a distortion allows the planner to collect more tax revenue from higher earners. Distortions are therefore valuable on all except the top earner across ages in the Static Mirrlees policy. But, they bring no benefits for age-dependent policies when levied on the top earners in each age group, because individuals are restricted to age-specific tax schedules.

More generally, the use of intratemporal distortions decreases as the sophistication of policy increases. This is most apparent for policy toward individuals when they are young, where Table 1.2 shows that distortions are everywhere lower under the Partial Reform policy than under the Static Mirrlees policy. The intuition for this is that a distortion at a given income when individuals are young raises less revenue from higher earners than does the same distortion when they are older, since wage disparities rise with age. Outside this example, the differences between the policies’ distortions are less systematic, in that distortions are lower for some individuals at some ages and higher for others as the sophistication of the policy increases. Nevertheless, a telling summary statistic is provided in the final column of Table 1.2: the unweighted average marginal distortion across all types and ages is 0.324 in the Static Mirrlees policy, 0.309 in the Partial Reform policy, and 0.303 in the Full Optimum policy. If we weight distortions by income, the income-weighted

36 The Static Mirrlees distortions differ across age groups even though its taxes are age-independent because the intratemporal distortion \( \tau (i, t) \) depends on an individual’s wage. Two individuals of different ages with different wages who choose the same income and consumption allocation will have different implied distortions.

37 In the language of the direct mechanism, when the highest earner among the young faces no marginal distortion, smaller distortions are needed to prevent the highest earner from mimicking lower earners. This chain reaction lowers distortions in general on the young.
average marginal distortion falls from 0.278 in the Static Mirrlees policy to 0.247 in the Partial Reform policy and 0.241 in the Full Optimum policy. Moreover, these differences do not reflect the improved pattern of distortions in the more sophisticated policies. The combination of lower average distortions and better-designed distortions encourages labor effort under the more sophisticated policies, raising total output and the efficiency of the economy. We will see the welfare impact of these efficiency gains below.

Average taxes

Average tax rates are substantially affected by age dependence, as well. Table 1.3 lists the average tax rates for each scenario.

Table 1.3: Average tax rates in baseline model simulation

<table>
<thead>
<tr>
<th>Average tax rate (in percent)</th>
<th>Lifetime income decile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age range</td>
<td>Bottom 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>Static Mirrlees</td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>-110.4 -68.3 -46.1 -33.7 -19.0 -10.1 0.0 4.1 11.4 27.3</td>
</tr>
<tr>
<td>40-49</td>
<td>-110.4 -54.9 -34.0 -19.0 -10.1 0.0 4.1 11.4 13.3 31.7</td>
</tr>
<tr>
<td>50-59</td>
<td>-110.4 -54.9 -34.0 -19.0 -10.1 0.0 4.1 11.4 13.3 30.7</td>
</tr>
<tr>
<td>Partial Reform</td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>-141.4 -89.1 -59.0 -40.5 -26.4 -15.1 -6.3 0.6 7.3 24.5</td>
</tr>
<tr>
<td>40-49</td>
<td>-113.3 -53.1 -34.9 -17.0 -7.7 1.5 6.3 13.2 20.0 38.3</td>
</tr>
<tr>
<td>50-59</td>
<td>-126.1 -51.8 -28.1 -17.9 -9.2 2.7 5.5 14.7 17.9 40.7</td>
</tr>
<tr>
<td>Full Optimum</td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>-130.9 -85.3 -56.4 -39.4 -25.1 -16.0 -5.2 -0.4 6.1 16.8</td>
</tr>
<tr>
<td>40-49</td>
<td>-122.6 -56.5 -39.0 -18.1 -8.4 1.3 5.7 12.9 21.7 43.0</td>
</tr>
<tr>
<td>50-59</td>
<td>-138.3 -55.8 -28.3 -19.6 -11.6 4.0 3.3 16.6 17.2 45.8</td>
</tr>
</tbody>
</table>

Figures 1.3a, 1.3b, and 1.3c plot average tax rates against annual income for each policy. In the Static Mirrlees, the average tax schedule is the same for all age groups. In

38 An individual’s average tax rate is defined as the ratio $\frac{y-c}{c}$. 

-141.4 -89.1 -59.0 -40.5 -26.4 -15.1 -6.3 0.6 7.3 24.5
Figure 1.3a: Average Tax Rates in Static Mirrlees, Baseline Model
Figure 1.3b: Average Tax Rates in Partial Reform, Baseline Model
Figure 1.3c: Average Tax Rates in Full Optimum, Baseline Model
the Partial Reform and Full Optimum, separate average tax schedules face workers in their thirties, forties, and fifties.

Workers face lower average taxes in their thirties and (to a lesser extent) fifties than in their forties under the more sophisticated policies. The magnitude of the difference across ages can be substantial. For example, in the Partial Reform policy, a middle-income worker earning a constant income over his lifetime faces an average tax rate in his thirties that is more than 7 percentage points lower than in his forties.

Why do the more sophisticated planners use lower average taxes on individuals when they are young? The data show wages rising from the thirties to the forties in all income groups. Individuals want to borrow against future wages to raise consumption when young, but in this baseline economy they cannot transfer resources across periods. Age-dependent tax policy can substitute for private borrowing by lowering average taxes when wages are low: i.e., in workers’ thirties. The Static Mirrlees planner cannot do so, because it cannot target lower average taxes at an age group.

**Intertemporal distortions**

Next, Table 1.4 shows the intertemporal distortions under each policy. Here, the main advantage of history dependence is made clear, as only the Full Optimum policy avoids these distortions entirely, providing fully smoothed consumption to all workers. While not smoothing for each worker, the Partial Reform policy smooths in aggregate across ages, as

---

39 The simulation results show that age dependence makes consumption less smooth for low earners. The reason for this was discussed in the section on intertemporal distortions: when smoothing high-earners’ consumption, the planner skews low-earners’ consumption in the opposite direction to most efficiently satisfy incentives.
shown formally in the Symmetric Inverse Euler Equation above, expression (1.15). This improves on the Static Mirrlees policy, which we can see in Table 1.4 by noting that the distortions to the intertemporal margin are smaller, on average, in the Partial Reform than in the Static Mirrlees policy.

**Welfare gain and decomposition**

Finally, I quantify and identify the sources of the welfare gain from reform. For each policy, Table 1.5 lists overall social welfare and lifetime utility by income decile, and Figure 1.4a plots social welfare.40

The main findings are that age dependence generates a large welfare gain in absolute size and that it captures nearly all of the gain from full reform to a more complex, optimal

---

40 I report social welfare and utility levels in consumption equivalent units. See the notes to Table 1.5 for specifics.
Table 1.5: Welfare in baseline model simulation

<table>
<thead>
<tr>
<th>Policy Scenario</th>
<th>Social Welfare*</th>
<th>Lifetime Utility (consumption equivalents**)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Static Mirrlees</td>
<td>Lifetime income decile</td>
</tr>
<tr>
<td></td>
<td>2.585</td>
<td>Bottom 2 3 4 5 6 7 8 9 Top</td>
</tr>
<tr>
<td>Static Mirrlees</td>
<td>2.585</td>
<td>2.29 2.36 2.42 2.48 2.53 2.60 2.66 2.75 2.87 3.13</td>
</tr>
<tr>
<td>Partial Reform</td>
<td>2.605</td>
<td>2.34 2.40 2.45 2.50 2.56 2.62 2.68 2.76 2.86 3.06</td>
</tr>
<tr>
<td>Full Optimum</td>
<td>2.606</td>
<td>2.34 2.40 2.45 2.50 2.56 2.62 2.68 2.76 2.86 3.06</td>
</tr>
</tbody>
</table>

* The value for consumption that, if provided freely to all workers at each age, would generate the same social welfare as the actual allocation.

** The value for consumption that, if provided freely to the worker at each age, would generate the same lifetime utility as the worker obtains with the actual allocation.

Figure 1.4a: Social Welfare Comparisons, Baseline Model
dynamic policy. Moreover, age dependence yields a more equal distribution of utility than is possible under Static Mirrlees policy.

First, the increase in welfare due to age dependence alone is large, equivalent to a 2.0 percent increase in aggregate consumption or roughly $200 billion in current U.S. dollars, annually. Specifically, if the Static Mirrlees planner received a windfall enabling it to increase each individual’s consumption by 2.0 percent while holding labor effort fixed, welfare in the Static Mirrlees policy would equal that in the Partial Reform policy. Below, I provide a detailed decomposition of this large welfare gain.

Second, the gain from this Partial Reform captures 96 percent of the gain from reform to the Full Optimum. Specifically, the additional welfare gain from history dependence is small, at 0.1 percent of aggregate consumption, so that the total gain due to reform from the Static Mirrlees to the Full Optimum is 2.1 percent of aggregate consumption. As discussed more below, the advantage of history dependence is minimal in this baseline model because many of the empirical wage paths shown in Figure 1.1 have similar shapes.

Finally, this welfare gain is particularly pronounced among the low-skilled, so that the Partial Reform and Full Optimum policies achieve more egalitarian (and nearly identical) distributions of lifetime utility across income groups. Table 1.5 and Figure 1.4b show the lifetime utility levels of each income group. The Static Mirrlees policy provides higher utility to only the top income group, while the more sophisticated policies substantially increase utility among individuals in the bottom half of the income distribution.

What drives this large and equitably-distributed welfare gain from age dependence? Figure 1.4c shows the results of a welfare gain decomposition that attributes the gain from
Figure 1.4b: Utility Comparisons, Baseline Model
Partial Reform to improvements in efficiency, equity (due to concave utility of consumption, convex disutility of labor, and the influence of Pareto weights), and consumption-smoothing. I now discuss each of the components in turn.41

**Figure 1.4c: Decomposition of Welfare Gains from Partial Reform**

Note: Components sum to 105%

- Efficiency: Higher output (47.9%)
- Equity: Convex disutility of labor (25.0%)
- Equity: Pareto weights (8.4%)
- Concave utility of consumption (15.6%)

Nearly half of the welfare gain from age dependence is because the economy is more efficient under age-dependent taxes. In Table 1.2, we saw that the average marginal distortion to labor effort is lowered by adding age dependence to a static Mirrleesian policy. This encourages more effort, so that total output is 2.5 percent higher under the Partial Reform policy than under the Static Mirrlees policy. This output must be produced as well as con-

41 All told, my estimated gains from these components sum to slightly more than the total welfare gain, so that I explain approximately 105 percent of the welfare gain. This overestimate is attributable to gaps between the estimates yielded by the experiments described below, which are necessarily imperfect, and the components’ true effects.
sumed, however, so its welfare impact is not equal to that of a resource windfall equalling 2.5 percent of income. Instead, I calculate the net benefit of this higher output using a simple thought experiment. Take the Static Mirrlees allocation of income and consumption and suppose that each individual were required to earn and allowed to consume 2.5 percent more at each age. The welfare gain of reform from this modified version of the Static Mirrlees policy to the Partial Reform policy is slightly more than 1.0 percent of aggregate consumption. Thus, the increased output due to efficiency gains account for approximately 48 percent of the total welfare gain from age dependence.

Most of the remaining welfare gain from age dependence is due to a more equitable distribution of resources than under an age-independent policy. In particular, the Partial Reform planner allocates consumption to individuals with higher marginal utilities of consumption and requires production from individuals with lower marginal disutilities of income than does the Static Mirrlees planner. I separately estimate the welfare impacts of each of these two factors.

To estimate the effect of the distribution of consumption, consider an experiment in which each individual’s consumption path under the Static Mirrlees policy is scaled to provide the same share of total consumption (in present value) as under the actual Partial Reform policy. This hypothetical allocation replicates the Partial Reform’s allocation of consumption across individuals while holding fixed the Static Mirrlees level of total consumption.\(^42\) Specifically, it raises the present value of consumption for the lowest income

---

\(^{42}\) Note that it also violates the incentive constraints on the Static Mirrlees problem, which is why the Static Mirrlees planner could not achieve this hypothetical allocation even though it satisfies the feasibility constraint.
group by almost 3 percent relative to the actual Static Mirrlees policy, an increase offset by lower present values of consumption for higher income groups. Because utility from consumption is concave, this hypothetical Static Mirrlees policy yields higher welfare than does the actual Static Mirrlees policy, and the consumption-equivalent welfare gain from this hypothetical Static Mirrlees to the Partial Reform is only 1.7 percent of Static Mirrlees output. Thus, the distribution of consumption in accordance with the Partial Reform policy accounts for approximately 15 percent of the welfare gain from age dependence (0.3 of the 2.0 percent of total consumption-equivalent gain).

To estimate the impact of allocating required income to those with lower marginal disutilities of labor effort, I consider an analogous experiment to that for consumption. I scale the income required from each individual in the Static Mirrlees to equal (in present value) the same share of total income as in the Partial Reform. Similar calculations to those for consumption imply that the distribution of required income in accordance with the Partial Reform policy accounts for approximately 25 percent of the welfare gain from age dependence.

The welfare impact of these more redistributive allocations of consumption and income is magnified by the assumption that Pareto weights decline as we move up the income distribution. To gauge the importance of this assumption, I calculate social welfare with uniform weights. The consumption-equivalent difference between the Static Mirrlees and Partial Reform scenarios implies that declining Pareto weights account for approximately 8 percent of the gain from age dependence. A nearly identical estimate is obtained when
I solve the planner’s problems in each policy scenario assuming uniform Pareto weights, i.e., a pure Utilitarian planner.43

Finally, age dependence allows for more efficient intertemporal allocations, i.e., more consumption-smoothing, than in the Static Mirrlees. Consider an experiment in which each individual’s present value of consumption under the Static Mirrlees policy is allocated across ages as it is in the actual Partial Reform policy. This hypothetical Static Mirrlees policy achieves higher social welfare than the true Static Mirrlees policy, implying that the Partial Reform’s increased consumption-smoothing accounts for about 8 percent of the gain from age dependence.

Why do these gains capture nearly all of the gain from reform to the Full Optimum? The Full Optimum’s advantage over Partial Reform is history dependence. History dependence is most valuable when wage paths cross or have substantially different slopes, as history-independent policies then have to address incentive problems that vary substantially by age. In contrast, the Full Optimum planner’s ability to track individuals allows it to target redistribution and smooth consumption despite differently-sloped wage paths. In the data used for the baseline simulation, as shown in Figure 1.1, most of the wage paths have similar shapes, thereby reducing the benefit from history dependence. In Section 1.4, I consider an extension of the baseline model that incorporates crossing wage paths, as guided by the data, by allowing wage paths to be stochastic rather than deterministic. As

43 Note that the welfare gains from age dependence do not depend on funding gains for the poor with losses by the rich. In Section 1.6, I show that age dependence generates only slightly smaller welfare gains when constrained to be Pareto-improving.
discussed there, the Partial Reform policy continues to capture nearly all of the gain from the Full Optimum in that extension.

The quantitative simulations have therefore revealed several key lessons about how age-dependent taxes differ from age-independent taxes that, as we will see, are largely robust to the extensions considered in the rest of the paper. First, age dependence lowers the marginal intratemporal distortion on the highest-earning young workers. This is a specific example of the benefits from being able to tailor marginal distortions to the wage distribution at each age and avoid cross-age incentive problems. Second, average taxes are lower for younger workers than older workers, by about seven percentage points across the income distribution in this simulation. In this baseline model, lower average taxes act as a substitute for private consumption-smoothing, a motivation that is absent when individuals can save and borrow privately. In fact, the optimal path of average taxes over the lifecycle will be indeterminate with private saving and borrowing, but that indeterminacy allows for a similar pattern of average taxes to be optimal. Third, the welfare gains from age dependence are large, here equal to about two percent of aggregate consumption. These gains are evenly divided between improvements in efficiency and equity. Finally, welfare gains from Partial Reform capture a substantial share (here, nearly all) of the potential gain from a reform to the Full Optimum, and they do so while providing a more egalitarian outcome.
1.2.5 Summary of baseline model

In this section, I theoretically and numerically characterized age-dependent taxation in a baseline model. I found that age dependence substantially improves on age-independent policy along the two margins that dominate theoretical Mirrleesian tax analysis: the intratemporal consumption-leisure margin and the intertemporal savings margin. Then, I used detailed individual wage data to show the effects of age dependence quantitatively. Age dependence affects the use of marginal distortions as well as the pattern of average taxes, deviating dramatically from the optimal age-independent policy and mimicking key features of the dynamic optimal policy. These improvements generate a large welfare gain, estimated at 2 percent of aggregate annual consumption, and capture nearly all of the gains from reform to the dynamic optimum. In the next several sections, I show that these results are largely robust to extensions of this baseline model to more complicated economic environments.

1.3 Case 2: Model with private saving and borrowing

In this section, I examine how the results from the baseline model are affected by allowing individuals to save and borrow across ages. Private saving and borrowing generate an important new set of incentive problems for policy, in that individuals may now subsidize consumption with after-tax income earned at a different age.44 This affects the marginal tradeoffs facing individuals at each age and, therefore, the optimal policy toward them.

44 A technical note: I assume that savings and debt are observable to the planner. The term "private" indicates private sector, not "hidden," which has a specific meaning in the optimal tax literature.
Before showing how the three policy scenarios respond to private transfers of resources across periods, it is important to clarify my assumptions on how the three policy scenarios can respond. In particular, I need to specify whether capital taxation is available to each policy and what forms it can take. While it is natural to assume that there are no restrictions on capital taxation for the Full Optimum policy, it is less clear what the appropriate assumption is for the Static Mirrlees and Partial Reform policies.

I assume that the Static Mirrlees and Partial Reform planners can neither tax nor subsidize private saving or borrowing in any way. This is a conservative assumption when gauging the power of age dependence, in that it maximizes both the potential for private saving and borrowing to undermine the baseline results and the relative power of the Full Optimum, which has unlimited flexibility in taxing and subsidizing intertemporal transfers. For example, if I allow the Partial Reform and Static Mirrlees policies to include a 15 percent tax rate on capital income (resembling the current U.S. system for capital gains and dividends), the absolute and relative sizes of the welfare gains from Partial Reform increase relative to the results below.

Thus, Partial Reform is defined consistently throughout the paper: it always means only that labor income taxes can depend on age. One interesting extension to this paper’s analysis would be to consider age-dependent linear capital taxation, which would increase the potential power of age dependence.45

Now, I follow the structure of Section 1.2 and specify the social planner’s problem in each policy scenario.

45 However, age-dependent linear capital taxes would sacrifice some of the practical advantage of simple age-dependence, and their effectiveness would likely be seriously undermined by avoidance behavior.
1.3 Case 2: Model with private saving and borrowing

1.3.1 Social planner’s problem in three policy scenarios

As in the baseline model, the social planner specifies a menu of bundles to maximize social welfare subject to feasibility and incentive constraints. With private saving and borrowing, however, these bundles are of pre-tax income and after-tax income, not consumption, because after-tax income in a given year may be used by individuals for consumption at any age. Thus, I distinguish between after-tax income, denoted $x^i_t$ for individual $i$ of age $t$, and consumption, denoted $c^i_t$ as before.

The planner assigns after-tax income to each pre-tax income to maximize social welfare, which is the same as in expression (1.3) from the baseline model. The feasibility constraint is similar to the baseline model’s, though I replace consumption with after-tax income:

$$\sum_{i=1}^{I} \prod_{t=1}^{T} R_{i}^{T-t} \left( y^i_t - x^i_t \right) = 0 \quad (1.17)$$

For the incentive constraints, we need notation that reflects the individual’s ability to claim a wage different from its true wage at any age, be assigned another individual’s allocations, and transfer resources across ages.\(^{46}\) Let $W^{j(s_t)}_{T} = \{w^{j_s_1}_1, w^{j_s_2}_2, \ldots, w^{j_s_T}_T\}$ denote a $T$-period path of wages corresponding to individuals of type $j_{s_t}$ and age $s_t$, where $s_t$ can vary across $t$. Thus, $W^i_T = \{w^i_1, w^i_2, \ldots, w^i_T\}$ denotes the true path of wages for individual $i$. Then, $\{y \left( w^{j_s}_s \right)\}_{t=1}^{T}$ and $\{x \left( w^{j_s}_s \right)\}_{t=1}^{T}$ are the sequence of pre-tax income and after-tax income allocations assigned to an individual who claims the wage sequence $W^{j(s_t)}_{T}$.

\(^{46}\) Recall that these planner’s problems are structured as direct mechanisms, in which an individual claims (or reports) a wage level and receives an allocations based on that claim.
Using this notation, the Static Mirrlees planner in Case 2 solves the following problem:

**Problem 4:**  \((\text{Case 2 Static Mirrlees: Age-Independent})\)

\[
\max_{\{c_t, y_t\}} \sum_{i=1}^{I} \pi^t \alpha^t V^i,
\]

subject to the feasibility constraint (1.17) and the incentive constraints

\[
\sum_{t=1}^{T} \beta^{t-1} \left( u\left( c_t \left( W^i_T\right) \right) \right) - v\left( \frac{y\left( w^i_t\right)}{w^i_t} \right) \geq \sum_{t=1}^{T} \beta^{t-1} \left( u\left( c_t \left( W^{j(s_t)}_T\right) \right) \right) - v\left( \frac{y\left( w^{j(s_t)}_{s_t}\right)}{w^i_t} \right)
\]

for all \( i, j \in \{1, 2, ..., I\} \) and all \( W^{j(s_t)}_T = \{w^{j(s_t)}_{s_1}, w^{j(s_t)}_{s_2}, ..., w^{j(s_t)}_{s_T}\} \), where

\[
\left\{ c_t \left( W^{j(s_t)}_T \right) \right\}_{t=1}^{T} = \arg \max_{\{c_t\}} \left\{ \sum_{t=1}^{T} \beta^{t-1} \left( u\left( c_t \left( W^{j(s_t)}_T \right) \right) \right) - v\left( \frac{y\left( w^{j(s_t)}_{s_t}\right)}{w^i_t} \right) \right\}
\]

s.t. \( \sum_{t=1}^{T} R^{T-t} \left( x\left( w^{j(s_t)}_{s_t}\right) - c_t \right) = 0 \)

is the consumption path individual \( i \) chooses when it claims the sequence of wage levels \( W^{j(s_t)}_T \).

Though more complicated, these incentive constraints are closely related to those from the baseline model. They reflect that individuals are free to choose any path of after-tax incomes, including those that are intended for individuals of different types and ages, and transfer them across periods using saving and borrowing in order to maximize their lifetime utility. The constraints ensure that each individual prefers its own path of wages \( W^{i}_{T} \) to any other path \( W^{j(s_t)}_T \).

\footnote{The summation over \( t \) in these incentive constraints does not imply that the Static Mirrlees planner is allowed to make history-dependent allocations. As in the baseline model, an individual’s choice of an allocation at age \( t \) will have no effect on the planner’s allocations to it at age \( t+1 \). The summation is needed because the individuals can independently link periods.}
As in the baseline model, the Partial Reform planner has the advantage of conditioning taxes on age. To express its problem, let \( W^{j(t)}_T = \{ w^{j_1}_t, w^{j_2}_t, \ldots, w^{j_T}_T \} \) denote a path of wages corresponding to individual \( j_t \) at each age \( t \). Note the notation \( j_t \) rather than \( j_{st} \) as in the Static Mirrlees policy. This indicates that the Partial Reform planner can restrict an individual to claiming only wages of others of the same age, not all ages. Then, \( \{ y(w^{j_t}_t) \}_{t=1}^T \) and \( \{ x(w^{j_t}_t) \}_{t=1}^T \) are the sequence of pre-tax income and after-tax incomes assigned to an individual who claims the wage sequence \( W^{j(t)}_T \).

Using this notation, the Partial Reform planner in Case 2 solves the following problem:

**Problem 5:** (Case 2 Partial Reform: Age-dependent)

\[
\max_{\{x,y\}} \sum_{i=1}^I \pi^i \alpha^i V^i,
\]

subject to the feasibility constraint: (1.17) and the incentive constraints

\[
\sum_t \beta^{t-1} \left( u(c_t(W^{j(t)}_T)) - v \left( \frac{y(w^{j_t}_t)}{w^{j_t}_t} \right) \right) \geq \sum_t \beta^{t-1} \left( u(c_t(W^{j(t)}_T)) - v \left( \frac{y(w^{j_t}_t)}{w^{j_t}_t} \right) \right)
\]

for all \( i, j_t \in \{1, 2, \ldots, I\} \) and all \( W^{j(t)}_T = \{ w^{j_1}_1, w^{j_2}_2, \ldots, w^{j_T}_T \} \), where

\[
\left\{ c_t \left( W^{j(t)}_T \right) \right\}_{t=1}^T = \arg \max_{\{c_t\}} \left\{ \sum_t \beta^{t-1} \left( u(c_t) - v \left( \frac{y(w^{j_t}_t)}{w^{j_t}_t} \right) \right) \right\}
\]

subject to \( \sum_t R^{T-t} (x(w^{j_t}_t) - c_t) = 0 \)

is the consumption path individual \( i \) chooses when it claims (i.e., reports) the sequence of wage levels \( W^{j(t)}_T \).

As in the baseline model, the Partial Reform planner’s incentive constraints are easier to satisfy than the Static Mirrlees planner’s because each individual must prefer its path of
wages \( W_i^t \) to only the set of wage paths composed of wages corresponding to individuals of the same age at each age \( t \). As in the baseline, this reflects the Partial Reform planner’s ability to restrict individuals to age-specific tax schedules.

Finally, the planner’s problem for the Full Optimum scenario is unchanged from the baseline model. Because it can link allocations across an individual’s lifetime, the Full Optimum planner spreads the after-tax income received by an individual over its lifetime optimally, leaving the individual’s optimal choice undistorted. Recall that this was shown formally in result (1.13) from the baseline model. Thus, consumption equals after-tax income at each age for each individual in the Full Optimum, so \( c_i^t = x_i^t \) for all \( i \in \{1, 2, \ldots, I\} \) and \( t \in \{1, 2, \ldots, T\} \). The individuals’ intertemporal optimization problem is irrelevant, and the planner’s problem remains one of specifying optimal consumption and income bundles.

Therefore, the Full Optimum planner in the Case 2 model solves the following problem:

**Problem 6:** *(Case 2 Full Optimum: Age-Dependent and History-Dependent)*

\[
\max_{\{c,y\}} \sum_{i=1}^{I} \pi^i \alpha^i V(i)
\]

subject to the feasibility constraint (1.4) and the incentive constraints (1.7).

As in the baseline model, the Full Optimum planner has the benefit of making allocations on a lifetime basis, so it can make promises or threats to individuals that encourage behavior at one age with consequences at another. The other two policy scenarios cannot
make these promises or threats because they lack history dependence: that is, they cannot keep track of individuals as they age.

The presence of private saving and borrowing complicates the incentives problems facing the Static Mirrlees and Partial Reform planners. I now turn to showing the effects of these more complicated incentive problems on the characteristics of taxes.

1.3.2 Analytical results

In the baseline model, I derived theoretical results connected to classic results on the intratemporal and intertemporal margins from the static and dynamic optimal tax literatures. In this model with private saving and borrowing, the second of these margins goes undistorted in all three policy scenarios.

Therefore, I focus my analysis in this section on the intratemporal margin.

The impact of private saving and borrowing on intratemporal distortions can best be seen in a simple example. Consider an economy with only two worker types, \( i = \{L, H\} \) for low and high skilled, and two ages \( t = \{1, 2\} \). Suppose that the high-skilled type \( H \) is always higher-skilled than the low-skilled type \( L \), so that \( w_H^1 > w_L^1 \) and \( w_H^2 > w_L^2 \).

Finally, assume that the utility function takes the following simple form:

\[
U(c, y) = \ln c - \frac{1}{\sigma} \left( \frac{y}{w} \right)^\sigma.
\]

The Full Optimum policy’s treatment of the high-skilled worker is unchanged by private saving and borrowing. We know from Proposition 2 (Top Marginal Distortion)

\[\text{[footnote]}\]

\[\text{[footnote]}\] The Static Mirrlees and Partial Reform policies cannot distort the intertemporal margin by assumption, as discussed above. The Full Optimum chooses not to, as proven in Section 1.2.
that, in the baseline model’s Full Optimum policy, the high-skilled worker in this example would face a zero intratemporal distortion at both ages. The Full Optimum planner’s problems are equivalent in Case 2 and the baseline model, so the high-skilled worker also faces zero distortions at both ages in the Full Optimum policy in Case 2.

In the Partial Reform planner’s solution to the this two-type example, the expression for the intratemporal distortion on the high-skilled worker when it is young \( (t = 1) \) is:

\[
\tau^{PR}(y_1^H) = \frac{\mu^{HL|HH}}{\pi^H \alpha^H + \mu^{LL|HH} + \mu^{HH|HH}} \left( \frac{c_1^{HH}}{c_1^{HL}} - 1 \right) \tag{1.18}
\]

where I use \( \mu^{ij|kk} \) to denote the Lagrange multiplier on the incentive constraint preventing type \( k \) from claiming the series of wages \( W_{ij} = \{ w_{1}^i, w_{2}^j \} \), and all other notation is as in the baseline model.

Intuitively, the distortion in (1.18) is positive if the young, high-skilled worker is tempted to save some of its after-tax income from the first period and use that savings to raise its consumption while working less and claiming the tax treatment of the low-skilled worker later in life. The Partial Reform planner uses this distortion to raise the marginal utility of consumption for the high-skilled worker when young, discouraging it from the strategy of oversaving and working less later in life. Formally, \( \mu^{HL|HH} > 0 \) because the incentive constraint preventing the high-skilled worker from claiming the \( \{ w_{1}^H, w_{2}^L \} \) wage path binds, and \( \frac{c_1^{HH}}{c_1^{HL}} > 1 \) because a high-skilled worker claiming the \( \{ w_{1}^H, w_{2}^L \} \) wage path can subsidize its consumption in the second period with savings from the first. These in turn imply, through (1.18), a positive intratemporal distortion on the young high-skilled worker. Analogous conditions for the high-skilled old worker and the low-skilled worker, as well as an illustrative numerical example, are provided in the Appendix.
There is an enlightening relationship between this distortion and the well-known Inverse Euler Equation in stochastic dynamic optimal tax models, first noted in Rogerson (1985) and prominently explored by Golosov, Kocherlakota, and Tsyvinski (2003). The Inverse Euler Equation suggests that policy ought to distort the after-tax return to saving (ex post, as shown in Kocherlakota, 2005) in order to counteract the temptation people face in the presence of skill shocks to oversave and falsely claim a low skill level, and thus lower taxes, in the future. The intratemporal distortion above works toward the same goal, even though wages are fully deterministic in this economy. The reason is that the Partial Reform planner, lacking history dependence, must act as if wages are stochastic even when they are not. Though, by assumption, it has no way to tax savings in the Partial Reform, it uses intratemporal distortions to try to achieve the same results.

Unlike the Full Optimum planner in a stochastic economy, the Partial Reform planner in this economy must worry about the possibility of overborrowing as well as oversaving. Because it cannot make future allocations contingent on past behavior, the Partial Reform planner cannot directly discourage individuals from working less when young and subsidizing consumption with borrowing. In fact, given the rising wages paths apparent in the data, overborrowing is a more prominent concern for the planner than is oversaving.

As illustrated in result (1.18), the use of intratemporal distortions to substitute for intertemporal distortions puts into jeopardy, in principle, the result from the baseline model that age dependence lowers marginal disortions on the highest-earning young workers. To check whether that result holds in Case 2, and to test the robustness of the other main lessons from the baseline model, I turn to numerical simulations.
1.3.3 Numerical results

For the numerical simulation of this model, I use the same data and approach as in the baseline economy simulation. The only difference in procedure is that computational considerations cause me to simplify the setting by reducing the number of types of individuals to \( I = 5 \). Therefore, I classify individuals into lifetime income quintiles rather than deciles.

Table 1.6 shows the wage paths for the five income groups, descriptive demographic data, and the calculated Pareto weights that range from 1.00 to 0.87.

### Table 1.6: Data for simulation of Case 2

<table>
<thead>
<tr>
<th>Wage paths</th>
<th>Age range</th>
<th>Bottom</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average wage by age range ($1999)</td>
<td>30-39</td>
<td>7.92</td>
<td>11.65</td>
<td>15.49</td>
<td>20.10</td>
<td>30.60</td>
</tr>
<tr>
<td>Proportion race=white</td>
<td></td>
<td>0.41</td>
<td>0.55</td>
<td>0.66</td>
<td>0.76</td>
<td>0.85</td>
</tr>
<tr>
<td>Proportion with college</td>
<td></td>
<td>0.04</td>
<td>0.08</td>
<td>0.15</td>
<td>0.24</td>
<td>0.49</td>
</tr>
<tr>
<td>Proportion gender=male</td>
<td></td>
<td>0.57</td>
<td>0.75</td>
<td>0.85</td>
<td>0.90</td>
<td>0.94</td>
</tr>
<tr>
<td>Pareto weight (calculated)</td>
<td></td>
<td>1.00</td>
<td>0.98</td>
<td>0.96</td>
<td>0.93</td>
<td>0.87</td>
</tr>
</tbody>
</table>

To preview the results presented below, marginal distortions on the labor supply of high-income young workers are lowered by age dependence, and Partial Reform yields a large welfare gain that captures a substantial (though somewhat smaller than in the baseline model) share of the potential gain from a fully optimal reform. While lower average taxes on the young remain optimal, average tax rates are no longer uniquely determined, so
alternative patterns of average rates are also optimal. Now, I will describe each of these results in more detail.

First, the intratemporal distortions are shown in Table 1.7 and plotted in Figures 1.5a, 1.5b, and 1.5c.

Table 1.7: Intratemporal distortions in Case 2 model simulation

<table>
<thead>
<tr>
<th>Intratemporal distortion</th>
<th>Lifetime income quintile</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Age range</td>
<td>Bottom</td>
</tr>
<tr>
<td>Static Mirrlees</td>
<td>30-39</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>40-49</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>50-59</td>
<td>0.27</td>
</tr>
<tr>
<td>Partial Reform</td>
<td>30-39</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>40-49</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>50-59</td>
<td>0.29</td>
</tr>
<tr>
<td>Full Optimum</td>
<td>30-39</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>40-49</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>50-59</td>
<td>0.29</td>
</tr>
</tbody>
</table>

As in the baseline model, the high-income young continue to face larger distortions to labor supply in the age-independent Static Mirrlees than in the more sophisticated policies. This is in spite of result (1.18), which showed that the Partial Reform policy uses intratemporal distortions to discourage deviation strategies in which individuals save or borrow to supplement consumption while working less to claim a more generous tax treatment at another age. Thus, the Partial Reform policy’s ability to identify the position of top earners within their age group, which we learned in the baseline model reduces the optimal distor-
Figure 1.5a: Intratemporal Distortions in Static Mirrlees, Case 2
Figure 1.5b: Intratemporal Distortions in Partial Reform, Case 2
Figure 1.5c: Intratemporal Distortions in Full Optimum, Case 2
tion on them, overwhelms the incentive problems raised by private saving and borrowing. Meanwhile, the Full Optimum planner can avoid such incentive problems directly by taxing intertemporal transfers, so it has no need to distort the consumption-leisure choice of the top earner in each age group, just as in the baseline economy.

Importantly, the use of intratemporal distortions in general decreases as the sophistication of policy increases, just as in the baseline model. The final column of Table 1.2 shows that the unweighted average marginal distortion is 0.316 in the Static Mirrlees policy, 0.272 in the Partial Reform policy, and 0.242 in the Full Optimum policy. The difference is greater when we weight distortions by income: the income-weighted average marginal distortion falls from 0.279 in the Static Mirrlees policy to 0.232 in the Partial Reform policy and 0.169 in the Full Optimum policy. As in the baseline, the combination of lower average distortions and a better-designed pattern of distortions encourages labor effort under the more sophisticated policies, raising total output and the efficiency of the economy. We will see the welfare impacts of these efficiency gains below.

Optimal average taxes in this model are, in contrast to the baseline model, indeterminate in the Partial Reform policy. As individuals can freely transfer resources across ages, any pattern of average taxes under this policy can be replaced with another that transfers resources lump-sum from one age to another without affecting any individual’s choices, therefore not affecting aggregate welfare, the allocation’s feasibility, or incentive constraints.

This implies that average taxes may, without loss of generality, take a form that resembles that from the baseline model. For instance, Table 1.8 shows one set of optimal
average tax rates in the Partial Reform model alongside the optimal rates for the Static Mirrlees and Full Optimum policies. The average tax rate for individual $i$ of age $t$ is $\frac{y_i - x_i}{x_i}$.

### Table 1.8: Average tax rates in Case 2 model simulation

<table>
<thead>
<tr>
<th></th>
<th>Lifetime income quintile</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Age range</td>
<td>Bottom</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>Static Mirrlees</strong></td>
<td>30-39</td>
<td>-87.3</td>
<td>-34.4</td>
<td>-11.7</td>
<td>3.9</td>
</tr>
<tr>
<td></td>
<td>40-49</td>
<td>-87.3</td>
<td>-31.3</td>
<td>-7.4</td>
<td>5.9</td>
</tr>
<tr>
<td></td>
<td>50-59</td>
<td>-87.3</td>
<td>-31.3</td>
<td>-7.4</td>
<td>5.9</td>
</tr>
<tr>
<td><strong>Partial Reform</strong></td>
<td>30-39</td>
<td>-147.4</td>
<td>-71.0</td>
<td>-34.1</td>
<td>-11.1</td>
</tr>
<tr>
<td></td>
<td>40-49</td>
<td>-54.0</td>
<td>-7.6</td>
<td>10.3</td>
<td>20.4</td>
</tr>
<tr>
<td></td>
<td>50-59</td>
<td>-70.8</td>
<td>-10.6</td>
<td>6.5</td>
<td>18.2</td>
</tr>
<tr>
<td><strong>Full Optimum</strong></td>
<td>30-39</td>
<td>-117.8</td>
<td>-52.9</td>
<td>-21.8</td>
<td>-2.5</td>
</tr>
<tr>
<td></td>
<td>40-49</td>
<td>-101.5</td>
<td>-32.3</td>
<td>-4.9</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>50-59</td>
<td>-114.6</td>
<td>-29.7</td>
<td>-5.8</td>
<td>10.0</td>
</tr>
</tbody>
</table>

These example average tax rates are plotted against lifetime income in Figures 1.6a, 1.6b, and 1.6c.

It is important to emphasize, however, that the Partial Reform schedules in Figure 1.6b are not the unique optimal average tax rate schedules. To see why, consider transferring one unit of after-tax income from each type of worker in its thirties to each type of worker in its forties under the Partial Reform policy. As individuals can transfer resources across time using the same technology as the planner, the paths of consumption corresponding to each path of labor effort are unchanged, so each worker’s choices are un-
1.3 Case 2: Model with private saving and borrowing

Figure 1.6a: Average Tax Rates in Static Mirrlees, Case 2
Figure 1.6b: Average Tax Rates in Partial Reform, Case 2
1.3 Case 2: Model with private saving and borrowing

Figure 1.6c: Average Tax Rates in Full Optimum, Case 2
affected by this transfer. Thus, a very different pattern of optimal taxes, such as with higher average taxes on the young than on the old, would also be optimal in this setting.\footnote{I am grateful to Ivan Werning for pointing out this result.}

Next, the Partial Reform continues to capture a large absolute welfare gain and a substantial share of the potential gains from more comprehensive reform. Table 1.9 shows social welfare and each income quintile’s lifetime utility under the three policies.

**Table 1.9: Welfare in Case 2 model simulation**

<table>
<thead>
<tr>
<th></th>
<th>Social Welfare*</th>
<th>Lifetime Utility (consumption equivalents**)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lifetime income quintile</td>
<td>Bottom</td>
</tr>
<tr>
<td><strong>Static Mirrlees</strong></td>
<td>2.60</td>
<td>2.35</td>
</tr>
<tr>
<td><strong>Partial Reform</strong></td>
<td>2.62</td>
<td>2.41</td>
</tr>
<tr>
<td><strong>Full Optimum</strong></td>
<td>2.63</td>
<td>2.44</td>
</tr>
</tbody>
</table>

* The value for consumption that, if provided freely to all workers at each age, would generate the same social welfare as the actual allocation.

** The value for consumption that, if provided freely to the worker at each age, would generate the same lifetime utility as the worker obtains with the actual allocation.

Figure 1.7a plots social welfare under the three policies. Reform from the Static Mirrlees policy to the Partial Reform policy yields a gain of 1.9 percent of aggregate consumption, nearly the same gain as in the baseline model. Because the Full Optimum planner is better able to respond to the new incentive problems introduced by private saving and borrowing, this gain makes up somewhat less of the gain from full reform. Nevertheless, it
1.3 Case 2: Model with private saving and borrowing

Figure 1.7a: Social Welfare Comparisons, Case 2

![Social Welfare Comparisons](image)

The Partial Reform policy produces a more egalitarian distribution of utility than does the Static Mirrlees, though not as egalitarian as the Full Optimum.

captures a substantial share, 67 percent, of that potential gain. More sophisticated capital taxation would magnify the power of age dependence. For instance, in an extension with a uniform 15 percent tax on capital income (not shown), the welfare gain from Partial Reform rises to 2.2 percent of aggregate consumption, comprising 72 percent of the potential gain from the Full Optimum.

Finally, as in the baseline model, the Partial Reform’s higher overall social welfare is also shared more equally among the individuals in the population. Table 1.9 and Figure 1.7b show the lifetime utility for each income group when individuals can save and borrow. The Partial Reform policy produces a more egalitarian distribution of utility than does the Static Mirrlees, though not as egalitarian as the Full Optimum.
Figure 1.7b: Utility Comparisons, Case 2
Thus, the results from the baseline model are largely robust to the addition of private saving and borrowing. Even when individuals are free to transfer after-tax income across periods, age dependence is a powerful reform. In the next section, I test the robustness of this result to relaxing a second assumption in the baseline model.

1.4 Case 3: Model with stochastic wage paths

In this section, I return to a setting in which individuals cannot transfer resources across time, but I explore a new variation on the baseline model by modeling wage paths as stochastic rather than deterministic. Stochastic wage paths generalize the baseline model in two important ways. First, they mean that individuals and the planner are uncertain about their future wages. As shown in Rogerson (1985) and Golosov, Kocherlakota, and Tsyvinski (2003), this uncertainty affects individuals’ labor supply and saving and borrowing behavior, with important implications for dynamic optimal policy. Second, they allow for substantially more heterogeneity in wage paths. In the baseline model, individuals were assigned to types by lifetime income-earning potential, and each type was assumed to have a single representative wage path. In this section and the next, such types play no role, and two individuals with similar lifetime incomes may have very different wage paths.

The extent to which individuals’ wage paths are determined over time due to stochastic shocks, rather than at the start of their working lives, is the subject of substantial recent research. Keane and Wolpin (1997) and Storesletten, Telmer, and Yaron (2001), provide evidence that stochastic shocks account for as little as 10 percent and as much as 40 percent of total variation in wage paths, respectively. Guvenen (2007) finds evidence that
individuals undergo substantial learning over time about the shape of their wage path. The larger the role of uncertainty in wage paths, the more important it is to understand how stochasticity affects the power of age dependence.

I model stochastic wages as a simple Markov process. At each age of working life, individuals are distributed among an age-specific set of discrete wage levels. A separate transition matrix links each age’s wage distribution to the next, so that a transition matrix between ages $t$ and $t + 1$ determines the distribution of all individuals with a given wage at age $t$ among the set of wage levels at age $t + 1$. This simple Markov approach yields a transparent and computationally tractable representation of the dynamic uncertainty and heterogeneity in wage paths from the data.

As with the baseline model, I begin with a theoretical analysis of the policy scenarios as social planners’ problems. The key result is Proposition 5 (Baseline and Case 3 Equivalence), which shows that the Static Mirrlees and Partial Reform planners’ problems in this Case 3 model are identical to their problems in an appropriately-specified baseline model from Section 1.2. Then, I use numerical simulations to show that the quantitative results from the baseline model carry through to this model with stochastic wage paths.

### 1.4.1 Social planner’s problem in three policy scenarios

As in the baseline model, I work with social planners’ problems in three policy scenarios: the Static Mirrlees, Partial Reform, and Full Optimum.

I begin by defining some notation. Denote an individual’s true path of wages as $W^{i(t)}_T = \{w^{i_1}_1, w^{i_2}_2, \ldots w^{i_t}_t, \ldots w^{i_T}_T\}$ and let $\pi^{i(t)}$ denote the population proportion represented
by this individual. Using these probabilities, let $\pi^j_t = \sum_{i(t) : w^{i(t)}_t = w^j_t} \pi^{i(t)}$ denote the probability of wage level $w^j_t$ at age $t$. It is the sum of the population proportions of the individuals whose wage paths equal $w^j_t$ at age $t$. Note that $\sum_{j=1}^I \pi^j_t = 1$ for all $t$. Denote the transition matrix between ages $t$ and $t + 1$ as $P_{t,t+1}$, whose element $(m,n)$ is:

$$P_{t,t+1}(m,n) = \Pr(w^n_{t+1} | w^m_t).$$

In words, $P_{t,t+1}(m,n)$ is the probability that an individual with wage $w^m_t$ will have wage $w^n_{t+1}$. Thus, the population proportion of an individual with wage path $W^{i(t)}_T$ can also be written $\pi^{i(t)} = \prod_{t=1}^{T-1} \pi^i_t P_{t,t+1}(i_t, i_{t+1})$.

The structure of each planner’s problem is the same as in the previous sections. To maximize social welfare, each planner offers a menu of income and consumption pairs to individuals. In the Static Mirrlees and Partial Reform policies, the planner offers a pair $\{c^j_t, y^j_t\}$ as the consumption and income intended to be chosen by an individual with wage $w^j_t$ at age $t$. In the Full Optimum policy, the allocations can be history-dependent.\textsuperscript{50}

Social welfare depends on the Pareto weights assigned to individuals with different wage paths. To determine these welfare weights, the planner uses its (complete) knowledge of the stochastic structure of wages. It calculates all possible lifetime income-earning potentials as determined by the truthful wage paths $W^{i(t)}_T$ and assign Pareto weights $\alpha\left(W^{i(t)}_T\right)$ to each of them, just as in the baseline model. Thus, $\alpha\left(W^{i(t)}_T\right)$ indicates a scalar Pareto weight on the individual with the wage path $W^{i(t)}_T$. Using these Pareto weights, define the

\textsuperscript{50} The Static Mirrlees and Partial Reform allocations may, in principle, be different for two individuals with different histories but the same current wage, in that these individuals could choose different $(c, y)$ pairs. In this Case 3 model, however, individual decisions depend only on their current wage, so this possibility is irrelevant.
as the expected Pareto weight on an individual of age $t$ with wage $j$. Expression (1.19) is the probability-weighted average of the Pareto weights on individuals with wage paths that include $w_{jt}$ when they are of age $t$. For instance, individuals with the first-period wage $w_{j1}$ will go on to have a variety of wage paths. The weight $\alpha_i^t$ captures the probability-weighted average of their eventual Pareto weights.

Using this notation, I now state the planner’s problems in the Static Mirrlees and Partial Reform policy scenarios. After discussing an important result on these policies, I then state the Full Optimum planner’s problem.

The Static Mirrlees planner in the Case 3 model solves the following problem:

**Problem 7: (Case 3 Static Mirrlees: Age-Independent)**

$$\max_{\{c,y\}} \sum_{j=1}^I \sum_{t=1}^T \beta^{t-1} \pi_i^t \alpha_i^t \left( u \left( c_i^t \right) - v \left( \frac{y_i^t}{w_i^t} \right) \right)$$ (1.20)

subject to feasibility

$$\sum_{j=1}^I \sum_{t=1}^T R^{T-t} \pi_i^t (y_i^t - c_i^t) = 0.$$ (1.21)

and incentive constraints:

$$\beta^{t-1} \left( u \left( c_i^t \right) - v \left( \frac{y_i^t}{w_i^t} \right) \right) \geq \beta^{t-1} \left( u \left( c_s^t \right) - v \left( \frac{y_s^t}{w_s^t} \right) \right).$$ (1.22)

for all $i_t, k \in \{1, 2, ..., I\}$ and $t, s \in \{1, 2, ..., T\}$

As in the baseline model, the Static Mirrlees incentive constraints reflect that each individual must prefer the allocation intended for its wage level and age to that intended for
any other wage level of any age. Formally, this is captured by the use of \( (c^i_t, y^i_t) \) on the left-hand side of (1.22) and \( (c^k_s, y^k_s) \) on the right-hand side.

Next, the Partial Reform planner in the Case 3 model solves the following problem:

**Problem 8: (Case 3 Partial Reform: Age-Dependent)**

\[
\max_{\{c,y\}} \sum_{j=1}^I \sum_{t=1}^T \beta^{t-1} \pi^j_i \alpha^j_i \left( u \left( c^j_t \right) - v \left( \frac{y^j_t}{w^j_t} \right) \right)
\]

subject to feasibility (1.21) and incentive constraints:

\[
\beta^{t-1} \left( u \left( c^j_t \right) - v \left( \frac{y^i_t}{w^i_t} \right) \right) \geq \beta^{t-1} \left( u \left( c^k_t \right) - v \left( \frac{y^k_t}{w^k_t} \right) \right).
\]

for all \( i, k \in \{1, 2, ..., I\} \) and \( t \in \{1, 2, ..., T\} \).

As in the baseline model, the Partial Reform planner’s incentive constraints are a subset of the Static Mirrlees planner’s and are, therefore, easier to satisfy. The Partial Reform planner need only satisfy incentives within age groups. Formally, the right-hand side of (1.24) depends on \( (c^k_s, y^k_s) \) rather than \( (c^k_s, y^k_s) \) as in the Static Mirrlees problem.

With these statements of the Static Mirrlees and Partial Reform planner’s problems in Case 3, I now establish an equivalence between these planners’ problems and those in an appropriately-chosen baseline economy with deterministic wage paths. The following proposition gives the result:

**Proposition 5 (Baseline and Case 3 Equivalence)** Consider a stochastic wage path \( W^{i(t)}_T \) and a deterministic wage path \( W^j_T \). If, for each \( i(t) \), there exists a \( j \) such that \( \pi^{i(t)} = \pi^j \) and \( w^i_t = w^j_t \) for all \( t \), where \( w^i_t \in W^{i(t)}_T \) and \( w^j_t \in W^j_T \), then:
• the solution to the Static Mirrlees planner’s problem in the baseline model is also
  the solution to the Case 3 Static Mirrlees planner’s problem,

• the solution to the Partial Reform planner’s problem in the baseline model is also
  the solution to the Case 3 Partial Reform planner’s problem.

Proof. In Appendix.

This proposition considers a deterministic economy that has the same set of individ-
ual wage paths as the stochastic economy will have, ex post. Its implications hold because,
in each of these policy scenarios, the objective function, feasibility constraint, and incen-
tive constraints are the same with stochastic or deterministic wage paths. Thus, the same
distributions of \( c \) and \( y \) solve the planner’s problems in each model.

The key to this result is that adding stochasticity in wages fails to change the problem
for either the planner or the individuals in the Static Mirrlees and Partial Reform scenarios.
For these planners, being restricted to history-independence plays the same role as wage
stochasticity. When a planner cannot track individuals across ages, it must satisfy incen-
tives age-by-age, rather than over a lifetime, so it is already setting policy as if wages were
stochastic. For individuals, the addition of stochasticity has no effect on their incentives
relative to the baseline model because, in the Case 3 model, individuals cannot transfer
resources between periods and utility is separable across periods. Each age involves an
isolated optimization for the individual, so the stochasticity of wages is irrelevant.

In contrast, the Full Optimum planner’s problem is substantially affected by stochas-
ticity. Stochastic wages multiply the number of possible wage paths, each of which is as-
1.4 Case 3: Model with stochastic wage paths

signed a history-dependent allocation at each age. In particular, let \( \{ c_t^i (W_{t-1}^{j(t)}) , y_t^i (W_{t-1}^{j(t)}) \} \) be the allocation of consumption and pre-tax income intended for an individual of age \( t \) who has reported the (possibly false) wage path \( W_{t-1}^{j(t)} = \{ w_1^{j_1}, w_2^{j_2}, \ldots, w_{t-1}^{j_{t-1}} \} \) and who reports the current wage \( w_t^i \). The Full Optimum planner’s incentive constraints must guarantee that individuals would rather reveal their true wage path \( W_t^{j(t)} \), age by age, rather than any other path, taking into account that individuals know the true transition matrices.

The Full Optimum planner in the Case 3 model solves the following problem:

**Problem 9:** \( (\text{Case 3 Full Optimum: Age-Dependent and History-Dependent}) \)

\[
\begin{align*}
\max_{\{c,y\}} & \left\{ \sum_{i(t)} \pi^{i(t)} \alpha \left( W_t^{j(t)} \right) \sum_t \beta^{t-1} \left( u \left( c_t^i \left( W_{t-1}^{j(t)} \right) \right) - v \left( \frac{y_t^i \left( W_{t-1}^{j(t)} \right)}{w_t^i} \right) \right) \right\},
\end{align*}
\]

subject to feasibility

\[
\sum_{i(t)} \pi^{i(t)} \sum_{t=1}^T R^{T-t} \left( y_t^i \left( W_{t-1}^{j(t)} \right) - c_t^i \left( W_{t-1}^{j(t)} \right) \right) = 0.
\]

and incentive constraints, which are defined recursively. First, for the last working age, \( T \), and for all \( i, j \):

\[
\begin{align*}
\left( c_T^{j_T} \left( W_T^{j_T} \right) \right) - v \left( \frac{y_T^{j_T} \left( W_T^{j_T} \right)}{w_T^{j_T}} \right) & \geq \left( c_T^{i_T} \left( W_T^{j_T} \right) \right) - v \left( \frac{y_T^{i_T} \left( W_T^{j_T} \right)}{w_T^{i_T}} \right).
\end{align*}
\]

Next, for all \( i, j \) and \( t < T \):

\[
\begin{align*}
\left[ U \left( W_{t-1}^{j(t)} , w_t^i \right) + \beta \sum_{i_t+1} P_{t,t+1} (i, i_t+1) U \left( W_{t-1}^{j(t)} , w_t^i , w_{t+1}^{i_{t+1}} \right) \right] \geq U \left( W_{t-1}^{j(t)} , w_t^i \right) + \beta \sum_{i_t+1} P_{t,t+1} (i, i_t+1) U \left( W_{t-1}^{j(t)} , w_t^i , w_{t+1}^{i_{t+1}} \right)
\end{align*}
\]
where \( U \left( W_{t-1}^{j(t)}, w_t^{i(t)} \right) \) is the period utility at age \( t \) of an individual reporting the sequence of wages defined by \( W_{t-1}^{j(t)}, w_t^{i(t)} \), so

\[
U \left( W_{t-1}^{j(t)}, w_t^{i(t)} \right) = u \left( c_t^{i(t)} \left( W_{t-1}^{j(t)} \right) \right) - v \left( \frac{y_t^{i(t)} \left( W_{t-1}^{j(t)} \right)}{w_t^{i(t)}} \right)
\]

\[
U \left( W_{t-1}^{j(t)}, w_t^{i(t)}, w_{t+1}^{i(t+1)} \right) = u \left( c_{t+1}^{i(t+1)} \left( W_{t-1}^{j(t)}, w_t^{i(t)} \right) \right) - v \left( \frac{y_{t+1}^{i(t+1)} \left( W_{t-1}^{j(t)}, w_t^{i(t)} \right)}{w_{t+1}^{i(t+1)}} \right)
\]

This planner’s problem is the stochastic analogue of the Full Optimum problems in the baseline and Case 2 models. Intuitively, the incentive constraints have two components. First, they ensure that, no matter the previous path of wage claims, individuals want to reveal their true wage in the last period of working life, age \( T \). Second, they ensure that truth-telling is optimal at each age \( t \) prior to the final working period, no matter the previous path of claims, given that truth-telling is optimal at the next age \( t + 1 \). These two steps guarantee that truth-telling is optimal at all ages for all individuals.

Now, I work with the planner’s problems specified above to characterize policy analytically and numerically.

### 1.4.2 Analytical results

Proposition 5 (Baseline and Case 3 Equivalence) showed that the Static Mirrlees and Partial Reform planners’ problems in a stochastic-wage economy with no private saving or borrowing are equivalent to their problems in an appropriately-chosen deterministic economy. As such, the analysis of their allocations is a straightforward extension of the analysis of the
baseline model. The Full Optimum planner’s problem does not yield simple conditions on optimal intratemporal distortions, but it does yield an important result on the intertemporal margin that I compare to the less sophisticated models.

**Intratemporal distortions**

The Partial Reform and Static Mirrlees intratemporal distortions are directly related to those in the baseline model. Using the notation defined above and, to make the results cleaner, the assumption that the disutility of labor takes the isoelastic form of expression (1.9), the Static Mirrlees policy’s intratemporal distortion on an individual with wage $w_{it}$ is:

$$
\tau^{SM} (i_t, t) = \frac{\sum_{s=1}^{T} \sum_{j=1}^{I} \left( 1 - \left( \frac{w_{it}^{j}}{w_{it}^{s}} \right)^{\sigma} \right) \beta^{s-t} \mu_{ti|s}^{ij} \pi^{j(t)} \alpha \left( W_{T}^{j(t)} \right) + \sum_{s=1}^{T} \sum_{j=1}^{I} \lambda_{j|it}^{j|st} - \sum_{s=1}^{T} \sum_{j=1}^{I} \left( \frac{w_{it}^{j}}{w_{it}^{s}} \right)^{\sigma} \beta^{s-t} \mu_{ti|s}^{ij} \right)}{\sum_{j(t):w_{jt}=w_{it}} \pi^{j(t)} \alpha \left( W_{T}^{j(t)} \right) + \sum_{j=1}^{I} \lambda_{j|it}^{j|it} - \sum_{j=1}^{I} \left( \frac{w_{it}^{j}}{w_{it}^{s}} \right)^{\sigma} \mu_{ti}^{ij}}
$$

where $\mu_{ti|s}^{ij}$ is the multiplier on the incentive constraint preventing an individual of age $t$ with wage $w_{it}^{j}$ from claiming the wage $w_{it}^{j}$ of an individual of age $s$, and all other notation is as specified above. The Partial Reform policy’s intratemporal distortion on an individual with wage $w_{it}^{j}$ is:

$$
\tau^{PR} (i_t, t) = \frac{\sum_{j=1}^{I} \left( 1 - \left( \frac{w_{it}^{j}}{w_{it}^{s}} \right)^{\sigma} \right) \mu_{ti}^{ij} \pi^{j(t)} \alpha \left( W_{T}^{j(t)} \right) + \sum_{j=1}^{I} \lambda_{j|it}^{j|it} - \sum_{j=1}^{I} \left( \frac{w_{it}^{j}}{w_{it}^{s}} \right)^{\sigma} \mu_{ti}^{ij}}{\sum_{j(t):w_{jt}=w_{it}} \pi^{j(t)} \alpha \left( W_{T}^{j(t)} \right) + \sum_{j=1}^{I} \lambda_{j|it}^{j|it} - \sum_{j=1}^{I} \left( \frac{w_{it}^{j}}{w_{it}^{s}} \right)^{\sigma} \mu_{ti}^{ij}}.
$$

where $\mu_{ti}^{ij}$ is the multiplier on the incentive constraint preventing an individual of age $t$ with wage $w_{it}^{j}$ from claiming the wage $w_{it}^{j}$ of an individual of the same age, and all other notation is as specified above.
These expressions are identical\textsuperscript{51} to the baseline model results (1.10) and (1.11), so wage stochasticity has little effect on the theoretical characterization of intratemporal distortions in these policies. The Partial Reform planner retains its advantage over the Static Mirrlees planner in limiting individuals to age-specific tax schedules.

Next, I characterize distortions to the intertemporal consumption margin.

**Intertemporal distortions**

Unlike in the baseline economy, the Full Optimum policy distorts the intertemporal margin in this model. Echoing the well-known result shown by Rogerson (1985) and Golosov, Kocherlakota, and Tsyvinski (2003), the Full Optimum allocation is described by an Inverse Euler Equation. For individual $i$ of age $t$, this expression is:

$$
\frac{1}{u'(c^i_t(W^i_{t-1}))} = \sum_{j=1}^I P_{t,t+1}(i,j) \frac{u'(c^j_{t+1}(W^j_{t+1}))}{u'(c^j_{t+1}(W^j_{t+1}))},
$$

(1.25)

The Inverse Euler Equation sets the consumption cost to the planner of providing marginal utility for an individual at age $t$ equal to the expected consumption cost of providing marginal utility for the same individual at age $t + 1$. If wage paths were deterministic, the expectation would be degenerate and the allocations would satisfy the standard Euler equation. With stochastic wage paths, the most efficient way for the planner to satisfy incentives is to allocate consumption according to (1.25).

The Partial Reform policy fails to satisfy the Inverse Euler Equation because it lacks history dependence. Rather, its satisfies what I called in Section 1.2 the Symmetric Inverse

\textsuperscript{51} The Pareto weight terms seem to differ, but they are the expected weights given the currently-observed wage. A deterministic model with the same set of ex post wage paths as the stochastic model would have the same expected Pareto weights given a currently-observed wage.
Euler Equation:

\[
\sum_{i=1}^{I} \frac{\pi^i_t}{u'(c^i_t)} = \sum_{i=1}^{I} \frac{\pi^i_{t+1}}{u'(c^i_{t+1})},
\]  

(1.26)

for any \(t, t + 1 \in \{1, 2, ..., T\}\). Recall that this condition means the planner has equalized across ages the consumption cost of increasing welfare. As with the intratemporal distortion, stochasticity does not affect the intertemporal distortions in the Partial Reform (or Static Mirrlees) policies because, for these planners, being restricted to history-independence plays the same role as wage stochasticity.

As in the baseline model, the efficiency of intertemporal allocations in Case 3 increases with the policy’s sophistication. The Full Optimum planner’s ability to satisfy the Inverse Euler Equation (1.25) is an advantage over the Partial Reform planner, while the latter’s ability to satisfy the Symmetric Inverse Euler Equation improves on the Static Mirrlees policy. The Static Mirrlees planner cannot target resources to specific ages, so it cannot equalize the cost of raising social welfare across ages.

Now, I turn to numerical simulations to test the robustness of the baseline model’s quantitative results.

### 1.4.3 Numerical results

In this section, I simulate the Case 3 planners’ problems. First, I discuss the construction of the required data and the parameter specification. Then, I describe the results of the simulations.
1.4 Case 3: Model with stochastic wage paths

Data and Parameters

As stated earlier, I model stochastic wage paths as a simple Markov process in which individuals are distributed among age-specific sets of discrete wage levels and move between these wage levels over time according to transition matrices linking each age to the next. As with the Case 2 model, computational considerations cause me to limit the size of the simulation. I set the number of wage levels at each age to \( I = 4 \) and continue to use \( T = 3 \) to represent the three decades of working life.

The simulations require a wage distribution for each age and transition matrices between ages. For the wage distributions, I use the data on household heads from the PSID core sample as described in Section 1.2, though I restrict the sample to individuals who were observed at least twice in each age range (i.e., 30-39, 40-49, and 50-59 years of age). This eliminates approximately 23,000 observations representing 2,000 sparsely-observed individuals, leaving us with 120,000 observations representing over 9,500 individuals with an average of over twelve observations each. With this sample I calculate the 5th, 35th, 65th, and 95th percentile wages within each age range and use these as the four wage levels among which individuals stochastically move. These wage levels are shown in Table 1.10.

Transition matrices are calculated from the data as follows. I assign percentile rankings to individual wages at each age, and for each age range (i.e., 30-39, 40-49, 50-59), I average each individual’s percentile rank for the observed years. Within each age range, I then sort individuals according to this average rank and group them into wage quartiles.
This assigns each individual to a wage quartile in each age range, allowing me to calculate empirical transition probabilities that populate the transition matrices in Table 1.10.

**Table 1.10: Data for simulation of Case 3**

<table>
<thead>
<tr>
<th>Wage levels</th>
<th>Wage for specified percentile and age range ($1999)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age range 30-39</td>
<td>5th</td>
</tr>
<tr>
<td>30-39</td>
<td>5.46</td>
</tr>
<tr>
<td>40-49</td>
<td>5.60</td>
</tr>
<tr>
<td>50-59</td>
<td>5.02</td>
</tr>
<tr>
<td>Initial pbb</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transition matrices</th>
<th>Wage quartile in 40-49 age range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>Bottom</td>
</tr>
<tr>
<td>2</td>
<td>0.71</td>
</tr>
<tr>
<td>3</td>
<td>0.18</td>
</tr>
<tr>
<td>Top</td>
<td>0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wage quartile in 50-59 age range</th>
<th>Bottom</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.66</td>
</tr>
<tr>
<td>3</td>
<td>0.13</td>
</tr>
<tr>
<td>Top</td>
<td>0.02</td>
</tr>
</tbody>
</table>

While the most common movement is no movement across quartiles, about 40 percent of moderate wage earners and between 20 and 30 percent of low and high wage earners switch quartiles in each transition.

The simulations also require parameterization. Other than for the Pareto weights, I use the parameters specified in Section 1.2. Pareto weights are assigned to each (ex post) lifetime path of wages using the expression (1.16) from the baseline model simulation, yielding weights similar to the baseline case. The maximum weight is 1.00 and applies to
the individual receiving the lowest wage level at each age. The minimum weight is 0.83 and applies to the individual receiving the highest wage level at each age.

**Simulation Results**

The simulation results for the Case 3 planner’s problems reinforce the lessons of the baseline and Case 2 simulations. As in the baseline model, I examine intratemporal distortions, average tax rates, intertemporal distortions, and social welfare.

First, consider intratemporal distortions. The Static Mirrlees and Partial Reform distortions are shown in Table 1.11.

**Table 1.11: Intratemporal distortions in Case 3**

Intratemporal distortion

<table>
<thead>
<tr>
<th>Wage quartile in each age range</th>
<th>Age range</th>
<th>Bottom</th>
<th>2</th>
<th>3</th>
<th>Top</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Mirrlees</td>
<td>30-39</td>
<td>0.35</td>
<td>0.30</td>
<td>0.27</td>
<td>0.26</td>
<td>0.276</td>
</tr>
<tr>
<td></td>
<td>40-49</td>
<td>0.40</td>
<td>0.43</td>
<td>0.47</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50-59</td>
<td>0.16</td>
<td>0.27</td>
<td>0.39</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Partial Reform</td>
<td>30-39</td>
<td>0.29</td>
<td>0.28</td>
<td>0.28</td>
<td>0.00</td>
<td>0.248</td>
</tr>
<tr>
<td></td>
<td>40-49</td>
<td>0.34</td>
<td>0.34</td>
<td>0.35</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50-59</td>
<td>0.35</td>
<td>0.35</td>
<td>0.38</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Figures 1.8a and 1.8b plot these distortions against lifetime income. The striking disparity in the treatment of the high-income young that we saw in previous models (Figures 1.2a and 1.2b or Figures 1.5a and 1.5b) is apparent here as well, so that high-skilled

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52 To avoid confusion, the Full Optimum distortions are not shown in the table because they are not readily comparable to the other two scenarios, as they are not functions of current income only.
1.4 Case 3: Model with stochastic wage paths

Figure 1.8a: Intratemporal Distortions in Static Mirrlees, Case 3
Figure 1.8b: Intratemporal Distortions in Partial Reform, Case 3
young workers are inefficiently discouraged from working by an age-independent tax system. This is consistent with Proposition 5 (Baseline and Case 3 Equivalence), which implied that the characteristics of the Partial Reform and Static Mirrlees policies in the baseline model were likely to carry over to the Case 3 model.

Moreover, the Partial Reform policy again uses marginal distortions less overall than does the Static Mirrlees policy. The unweighted average distortion is 0.276 in the Static Mirrlees, compared to 0.248 in the Partial Reform policy. The difference in the average distortion increases when weighted by income, from 0.186 in the Static Mirrlees to 0.128 under Partial Reform.

The results on average tax rates also resemble those from the baseline model. The average tax rates for the Static Mirrlees and Partial Reform results are given in Table 1.12.

### Table 1.12: Average Tax Rates in Case 3

<table>
<thead>
<tr>
<th>Average tax</th>
<th>Wage quartile in each age range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age range</td>
<td>Bottom</td>
</tr>
<tr>
<td>Static Mirrlees</td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>-293.7</td>
</tr>
<tr>
<td>40-49</td>
<td>-293.7</td>
</tr>
<tr>
<td>50-59</td>
<td>-293.7</td>
</tr>
<tr>
<td>Partial Reform</td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>-346.3</td>
</tr>
<tr>
<td>40-49</td>
<td>-303.0</td>
</tr>
<tr>
<td>50-59</td>
<td>-363.3</td>
</tr>
</tbody>
</table>

These average tax rates are plotted against lifetime income in Figures 1.9a and 1.9b. As in the baseline model, the Partial Reform policy lowers average tax rates on workers in their thirties. The size of the gap in rates in the middle of the income distribution between
Figure 1.9a: Average Tax Rates in Static Mirrlees, Case 3
Figure 1.9b: Average Tax Rates in Partial Reform, Case 3
the young and peak earners resembles that in the baseline case, and the intuition is the same. In Case 3, individuals cannot borrow against their higher expected future wages, so tax policy can substitute for private borrowing.

I also compare the intertemporal distortions under each scenario. Table 1.13 shows the ratio

\[ \frac{c_i^t}{\sum_{j=1}^I P_{t+1} (i, j) c_{t+1}^{j} (W_{t}^i)}, \]

which rearranges the Inverse Euler Equation in expression (1.25) when utility of consumption is logarithmic, for the Partial Reform and Static Mirrlees policies.

### Table 1.13: Intertemporal distortions in Case 3

<table>
<thead>
<tr>
<th>Wage quartile in each age range</th>
<th>Age range</th>
<th>Bottom</th>
<th>2</th>
<th>3</th>
<th>Top</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Static Mirrlees</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>0.94</td>
<td>0.95</td>
<td>0.89</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>40-49</td>
<td>0.93</td>
<td>0.95</td>
<td>0.84</td>
<td>1.07</td>
<td></td>
</tr>
<tr>
<td>50-59</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td></td>
</tr>
<tr>
<td><strong>Partial Reform</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>1.01</td>
<td>1.01</td>
<td>0.95</td>
<td>1.04</td>
<td></td>
</tr>
<tr>
<td>40-49</td>
<td>0.97</td>
<td>0.99</td>
<td>0.91</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>50-59</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td></td>
</tr>
</tbody>
</table>

Though not shown in the table, this ratio is equal to one in the Full Optimum for each individual (aside from some numerical noise in the extremely unlikely paths). Apparent from the table is that the deviations of this ratio from one are larger for the Static Mirrlees planner than for the Partial Reform planner. Note, in particular, the substantial distortions on the high-skilled young by the Static Mirrlees planner.
Finally, Partial Reform continues to capture a large absolute and relative welfare gain. Table 1.14 shows overall social welfare and lifetime utility for individuals with four representative wage paths under the three policies.

**Table 1.14: Welfare in Case 3**

<table>
<thead>
<tr>
<th>Wage paths</th>
<th>Lifetime Utility**</th>
<th>Social Welfare*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Always lowest</td>
<td>2,2,2 3,3,3</td>
<td>2.466 2.47 2.54 2.68 2.96</td>
</tr>
<tr>
<td>Always highest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Static Mirrlees</td>
<td>2.47 2.54 2.68 2.96</td>
<td></td>
</tr>
<tr>
<td>Partial Reform</td>
<td>2.53 2.58 2.70 2.91</td>
<td></td>
</tr>
<tr>
<td>Full Optimum</td>
<td>2.53 2.58 2.70 2.90</td>
<td></td>
</tr>
</tbody>
</table>

* The value for consumption that, if provided freely to all workers at each age, would generate the same social welfare as the actual allocation.

** The value for consumption that, if provided freely to the worker at each age, would generate the same lifetime utility as the worker obtains with the actual allocation.

Figure 1.10a plots social welfare under the three policies. Age dependence yields a welfare gain equivalent to a 2.5 percent increase in aggregate consumption relative to the Static Mirrlees policy in Case 3. Moreover, this gain captures 95 percent of the welfare gain from reform to the Full Optimum policy. Both of these results are similar in magnitude to the results from the baseline and Case 2 models.

Again, the utility gains are especially substantial for the lower-income workers in this model. In Table 1.14 and Figure 1.10b, I show the lifetime utilities of four individuals with wage paths that stay in the same quartile for all three periods. As in previous models,
age dependence makes the distribution of lifetime utility more equal while raising overall welfare.

As mentioned at the start of Section 1.4, an important reason to allow for stochastic wage paths is that they capture heterogeneity in wage paths conditional on lifetime income. The same topic was raised in Section 1.2 when discussing why the Partial Reform captures such a large share of the gain from the Full Optimum policy. Why don’t stochastic wage paths undermine the relative case for Partial Reform? The result relies on two factors. First, the stochastic nature of these heterogeneous wage paths weakens the Full Optimum policy, because it now has to satisfy incentives repeatedly rather than only once. This narrows the gap between the Full Optimum and the history-independent policies that were already forced to satisfy incentives at each age. Second, the empirical magnitude of
Figure 1.10b: Utility Comparisons, Case 3
wage path heterogeneity (i.e., wage path crossing) is not large enough to change the main results. At each age, current income is a powerful enough predictor of lifetime income-earning ability that the Partial Reform policy can still redistribute on a lifetime basis by redistributing within ages.\textsuperscript{53}

Thus, the results from the baseline model are robust to the inclusion of wage stochasticity, at least to the extent implied by the data used in these analyses and for a parsimonious specification of the stochastic process. In the next section, I again relax the assumption that individuals cannot save or borrow, but now in the context of stochastic wages.

\textbf{1.5 Case 4: Model with stochastic wage paths and private saving and borrowing}

In this section, I combine the variations on the baseline model examined separately in the previous two sections and consider a model with both stochastic wages and private saving and borrowing. I make the same modifications to the model in Case 3 as I did to the baseline model when specifying the model in Case 2. In particular, I retain the assumption from Case 2 that the Static Mirrlees and Partial Reform planners cannot tax intertemporal transfers by individuals.

As in the previous models, I consider a social planning problem for each policy, where a planner maximizes social welfare subject to feasibility and incentive compatibility

\textsuperscript{53} This result does not rely on the use of incomes averaged over decade-long age ranges. Though these averages smooth incomes, the correlations between income at each age and lifetime income-earning ability in these data are nearly as large as those between the decade-long average incomes and lifetime income-earning ability. Moreover, I have simulated a version of the Case 3 model in which I use five-year age ranges. The welfare gain from Partial Reform in fact increases relative to the main results, as the wider differences between age groups raise the value of age-dependent marginal distortions and transfers across age groups.
constraints. When individuals can transfer resources across periods, the planners in the Static Mirrlees and Partial Reform scenarios do not control consumption directly. Thus, these planners specify pre-tax income and after-tax income bundles in Case 4, just as in Case 2. The objective function for these two policies is

$$\max_{\{x, y\}} \left\{ \sum_{i(t)} \pi_i^{i(t)} c \left( W_T^i(t) \right) \sum_{t=1}^T \beta^{t-1} \left( u \left( c_t^i \right) - v \left( \frac{y_t^i}{w_t^i} \right) \right) \right\},$$

(1.27)

and the feasibility constraint is:

$$\sum_{i(t)} \pi_i^{i(t)} \sum_{t=1}^T R_t^{T-t} \left( y_t^i - x_t^i \right) = 0.$$  

(1.28)

Both of these expressions include, as in Case 2, after-tax income $x$, and all other notation is as before.\(^{54}\)

As usual, variations in the incentive constraints allow us to distinguish between policy scenarios. The combination of private access to capital markets and stochasticity makes these incentive constraints quite complicated, however. Therefore, I relegate them to the Appendix.

In words, the incentive constraints for the Static Mirrlees and Partial Reform scenarios reflect that an individual can choose a separate deviation strategy, including saving and borrowing, for each possible true path of wages. So, the Static Mirrlees incentive constraints must ensure that each individual prefers its allocation to any of the other allocation streams it might claim. If there are $T$ periods and $I$ wage levels, the number of these other

---

\(^{54}\) The Static Mirrlees and Partial Reform allocations may, in principle, be different for two individuals with different histories but the same current wage, in that these individuals could choose different $(x, y)$ pairs. Computational considerations prevent me from allowing for this, however, and instead I restrict these policies to allocations that are identical across two such individuals. The impact of this restriction is likely to be minimal, as economic efficiency and incentive constraints require allocations to these individuals to be similar. Moreover, this restriction has no effect on the Full Optimum policy and primarily handicaps the Partial Reform policy, causing me to, if anything, underestimate the relative gain from age dependence.
streams is \( (IT)^{1+I(T-1)} - 1 \). That is, in the first period, an individual can claim any of \( IT \) wages, including his own. When planning for the second period, for each of the \( I \) possible second period wages, he can claim any of the \( IT \) wages again. The Partial Reform incentive constraints are, as usual, a subset of the Static Mirrlees scenario’s because the planner can make age-dependent allocations. Each individual in the Partial Reform must be prevented from claiming allocation streams other than her own that number only \( I^{1+I(T-1)} - 1 \), as she can claim \( I \), not \( IT \), wages for each wage level she receives.

In contrast, the Full Optimum planner’s problem is unchanged from the problem without private saving, just as its Case 2 problem was unchanged from its baseline problem. Thus, the Full Optimum planner’s problem in Case 4 is identical to the Case 3 Full Optimum planner’s problem.

The complexity of the Static Mirrlees and Partial Reform problems makes it most convenient to study their solutions numerically, so I turn to the quantitative simulations now.

### 1.5.1 Numerical results

The computational demands of the Case 4 model are substantial, so I further limit the size of the economy. I condense the lifecycle into two age periods \( T = 2 \) covering the same range as before, so that \( t = 1 \) for ages 30-44 and \( t = 2 \) for ages 45-59. I also limit the number of wage levels at each age to three \( (I = 3) \).

The construction of the data is the same as in Case 3, though I now choose the 5th, 50th, and 95th percentiles for each age range as the representative wage levels and classify
individuals into three wage quantiles. Table 1.15 gives the wage levels at each age and the transition matrix between the two age ranges.

<table>
<thead>
<tr>
<th>Wage levels</th>
<th>Percentile of wage distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age range 30-44</td>
<td>5th</td>
</tr>
<tr>
<td>Wage for specified percentile and age range ($1999)</td>
<td>30-44</td>
</tr>
<tr>
<td></td>
<td>45-59</td>
</tr>
<tr>
<td>Initial pbb</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transition matrix</th>
<th>Wage quantile in 44-59 age range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>Middle</td>
</tr>
<tr>
<td>Wage quantile in 30-44 age range</td>
<td>0.72</td>
</tr>
<tr>
<td>Bottom</td>
<td>0.15</td>
</tr>
<tr>
<td>Middle</td>
<td>0.01</td>
</tr>
<tr>
<td>Top</td>
<td></td>
</tr>
</tbody>
</table>

The parameterization of the models is the same as in the previous cases, and the Pareto weights follow a similar pattern as before, with a maximum weight of 1.00 and a minimum weight of 0.83.

I examine results on intratemporal distortions, average tax rates, and social welfare.

First, consider intratemporal distortions. The distortions in the Static Mirrlees and Partial Reform policies are listed in Table 1.16.
Table 1.16: Intratemporal distortions in Case 4

<table>
<thead>
<tr>
<th>Intratemporal distortion</th>
<th>Wage quantile in 45-59 age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Age range</td>
</tr>
<tr>
<td>Static Mirrlees</td>
<td>30-44</td>
</tr>
<tr>
<td></td>
<td>45-59</td>
</tr>
<tr>
<td>Partial Reform</td>
<td>30-44</td>
</tr>
<tr>
<td></td>
<td>45-59</td>
</tr>
</tbody>
</table>

Figures 1.11a and 1.11b plot these intratemporal distortions against lifetime income.

Figure 1.11a: Intratemporal Distortions in Static Mirrlees, Case 4
As in all previous models, the high-skilled young workers are inefficiently discouraged from working by (here, slightly) higher intratemporal distortions under an age-independent tax system.

Moreover, the use of marginal distortions is lower in the Partial Reform policy than in the Static Mirrlees policy in general, with the unweighted average distortion falling from 0.256 to 0.250 between the policies. If the distortions are weighted by income, the gap is larger, with the average distortion falling from 0.176 in the Static Mirrlees to 0.161 in the
Partial Reform policy. The smaller magnitude of these results in Case 4 than in the other models is likely due, at least in part, to the compression of the data into two age groups. This compression limits the extent of cross-age incentives problems that cause the Static Mirrlees planner to use more distortionary taxation than the Partial Reform planner.

As in Case 2, optimal average tax rates are indeterminate in this setting for the Partial Reform policy. Lump-sum transfers across ages, which affect calculated average tax rates, can be reallocated across time by individuals using the same technology as the planner. This allows for the possibility that average tax rates in the Partial Reform policy follow a similar pattern as in the baseline and Case 3 models, where average tax rates are lower on young workers. An example of such a policy is shown in Table 1.17.

### Table 1.17: Average Tax Rates in Case 4

<table>
<thead>
<tr>
<th>Average tax (in percent)</th>
<th>Wage quantile in 45-59 age range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age range</td>
<td>Bottom</td>
</tr>
<tr>
<td>Static Mirrlees</td>
<td></td>
</tr>
<tr>
<td>30-44</td>
<td>-352.0</td>
</tr>
<tr>
<td>45-59</td>
<td>-352.0</td>
</tr>
<tr>
<td>Partial Reform</td>
<td></td>
</tr>
<tr>
<td>30-44</td>
<td>-370.9</td>
</tr>
<tr>
<td>45-59</td>
<td>-353.4</td>
</tr>
</tbody>
</table>

Figures 1.12a and 1.12b plot these example average tax rates against lifetime income. It is important to emphasize that the average tax schedules shown in Figure 1.12b are not the unique optimal schedules, and there exist schedules that include higher average rates for young workers that yield the same aggregate welfare. Nevertheless, a policy placing
Figure 1.12a: Average Tax Rates in Static Mirrlees, Case 4
Figure 1.12b: Average Tax Rates in Partial Reform, Case 4
lower average taxes on the young may be preferable, given its advantages when private saving and borrowing are restricted as in the baseline and Case 3 settings.

Finally, age dependence continues to yield a large welfare gain and capture a substantial share of the welfare gain from reform to the Full Optimum policy. Table 1.18 shows social welfare and lifetime utility for individuals with three representative wage paths under the three policies. Figure 1.13a plots social welfare under these policies. Partial Reform

Table 1.18: Welfare in Case 4

<table>
<thead>
<tr>
<th>Wage paths</th>
<th>Social Welfare*</th>
<th>Always lowest wage</th>
<th>Always middle wage</th>
<th>Always highest wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Mirrlees</td>
<td>3.629</td>
<td>2.03</td>
<td>2.06</td>
<td>2.22</td>
</tr>
<tr>
<td>Partial Reform</td>
<td>3.643</td>
<td>2.04</td>
<td>2.07</td>
<td>2.21</td>
</tr>
<tr>
<td>Full Optimum</td>
<td>3.662</td>
<td>2.06</td>
<td>2.09</td>
<td>2.19</td>
</tr>
</tbody>
</table>

* The value for consumption that, if provided freely to all workers at each age, would generate the same social welfare as the actual allocation.
** The value for consumption that, if provided freely to the worker at each age, would generate the same lifetime utility as the worker obtains with the actual allocation.

generates a welfare gain equivalent to 1.0 percent of aggregate consumption in the Static Mirrlees. This captures 41 percent of the welfare gain from reform to the Full Optimum. As in previous models, the low-skilled particularly benefit from reform: both the Partial Reform and Full Optimum policies provide more redistribution than the Static Mirrlees policy.
1.6 Discussion and Special Topics

The preceding sections show that the partial reform of age dependence yields large absolute and relative welfare gains by systematically altering optimal labor income taxation. Moreover, these effects are robust to fundamental variations in the assumed economic environment. In this section, I discuss additional topics of interest that were not addressed directly in these analyses.

1.6.1 Endogeneity of wage paths

As stated at the beginning of the baseline model setup, I assume throughout this paper that wage paths are exogenous to individuals. This assumption is standard in the optimal tax literature, but it is not necessarily innocuous.
If, as this paper’s analysis recommends, tax schedules were to differ by age, individuals would have an incentive to tailor their career choice and employment relationships to minimize their tax bill. This could reduce the variation in wage distributions with age that gives age-dependent taxes their power and introduce additional distortions to the economy. For instance, lower average tax rates on young workers would encourage people to take jobs with flatter income profiles and to bargain with their employers to shift the timing of income.55

The specific results of this paper therefore require that a substantial portion of the variation of wages with age is inelastic to taxes. A few considerations suggest that this requirement’s effects on the paper’s results may be limited.

First, this paper’s focus on age-dependent taxes between the ages of 30 and 60 limits concerns about distorting individuals’ career choices. In their late teens and twenties, individuals have substantial opportunities to shift the timing of higher education and job training to respond to taxes or other incentives.56 By age thirty, however, nearly all have completed their education and begun careers, so any distortions to career choice would apply to only those individuals who were substantially forward-looking and for whom the distortion itself had relatively small costs.

Second, the temporary nature of most employer-employee relationships provides a natural barrier to shifting income across ages in response to age dependence, because shift-

55 It is important to be clear that while wages are assumed to be exogenous, this paper’s analysis allows income to respond to taxes because individuals choose their level of labor effort.

56 Age dependence during this younger age range would be a more treacherous reform to design, though properly-designed age-dependent taxes during this range would potentially add significantly to the welfare gains calculated in this paper.
ing income is risky without long-term contracts that tie employees to employers. For instance, the taxes recommended by this paper’s analysis would imply shifting income forward, so that some of a worker’s earnings in her forties would be pre-paid to her in her thirties. Of course, an employer will be hesitant to do this unless the worker can commit to remaining at the firm through her forties. Such commitments are rare in modern labor markets. More generally, different theories of wage determination suggest different sensitivities of income’s timing to taxation. The key for this paper is that wages rise over the lifecycle because the passage of time is, for whatever reason, required for a given worker’s effort to be worth more to their employers.

Finally, one piece of evidence suggests that the sensitivity to taxes of both career choice and the timing of income is limited. Currently, wage paths rise sharply with age, especially at high incomes. These upward-sloping paths exist in the context of progressive taxation that ought to encourage flat wage profiles. If wage paths were highly elastic to tax incentives, we would expect to see smoother wage profiles than we do.

Despite the potential importance of wage path endogeneity, characterizing optimal dynamic taxation (and age-dependent taxation) with endogenous wage paths is beyond the scope of this paper and is an important task for future work. Doing so will require a careful treatment of career choice and the timing of income, as discussed above. It will also require the inclusion of a model of human capital investment through education and

---

57 For instance, if learning about the quality of matches or on-the-job training is important, individuals are unlikely to be able to shift income to earlier in their careers, while if the acquisition (but not the timing) of outside training is important, it might be shifted to an age at which taxes on income are higher.
work experience, a factor that may increase the welfare gains from age dependence as its added flexibility could be used to encourage human capital accumulation.

### 1.6.2 Elasticity of labor supply by age

The analyses of the preceding sections, other than a brief discussion in Section 1.2, have ignored one of the most direct reasons for the differentiation of taxation by age or any other personal characteristic: variation in the elasticity of labor supply across subgroups (see, for instance, Alesina and Ichino (2007) on gender). Standard optimal tax theory implies that less elastic subgroups should face larger tax distortions, all else the same, as revenue can be raised more efficiently from them. Therefore, a potentially important determinant of age-dependent taxes absent from this paper’s results is variation in the elasticity of labor supply across ages.

Unfortunately, empirical evidence on variation in the elasticity of labor supply with age is limited. Kremer (2002) argues that "The limited available evidence suggests that younger workers have more elastic labor supply than prime-age workers," citing Clark and Summers (1981), who show more variation in employment rates with the business cycle for young workers. French (2005) estimates that "labour supply elasticities rise from 0.3 at age 40 to 1.1 at age 60," but estimates for other ages are not given. Lacking more robust evidence, I have made the conservative assumption that the elasticity of labor supply is uniform across age.

If labor supply elasticity varies in the directions suggested by this limited evidence, the recommendations of this paper are strengthened. To illustrate this, I consider a para-
meterization that includes a simple difference in elasticities by age. Consider the baseline model from Section 1.2 where there are $I = 10$ types of individuals living for $T = 3$ periods. Suppose that $\sigma = 3$ for the second age group (workers in their forties) while $\sigma = 2$ for the workers in their thirties and fifties. Given the isoelastic disutility function (1.9), the constant-consumption elasticity of labor supply is $\frac{1}{\sigma - 1}$, so these values imply an elasticity of 1 for the youngest and oldest groups and an elasticity of $\frac{1}{2}$ for the workers in their forties.

The results of this experiment magnify those of the main analyses. Intratemporal distortions remain too high for the high-earning young and are used more in general by the Static Mirrlees policy than by the policies with age dependence. Average tax rates for workers in their thirties are even lower relative to older workers in this Partial Reform policy than in the model with uniform elasticities. The welfare gain from Partial Reform is increased from 2.0 percent of aggregate income in the baseline model to 2.4 percent in this model with varying elasticities, as it can target distortions at the inelastic age groups better than the Static Mirrlees. Age dependence continues to capture nearly all of the potential gain from the Full Optimum.

### 1.6.3 Extensive margin

One reason that we may intuitively think the elasticity of labor supply is higher for the young and old is not captured by the previous discussion. Young and old workers may be elastic along the extensive labor supply margin (the choice whether to work or not) rather
than the intensive margin (the choice of how much to work). How would an extensive margin affect this paper’s results?

To add an extensive margin to the analysis, I modify the baseline model of Section 1.2 to include an eleventh type of individual, type $i = 0$, who never works. A worker with type $i > 0$ who chooses not to work is operating on the extensive margin. Note that, because the Partial Reform planner cannot make history-dependent allocations, workers can move across the extensive margin in a single period or any combination of periods.

To properly model this extensive margin, I must make not working qualitatively different from working less. To do so, I add a fixed cost of working, $\phi$. Formally, the incentive constraints in the Partial Reform planner’s problem for individual $i$ of age $t$ have two parts: first,

$$
\beta^{t-1} \left( u(c^i_t) - v \left( \frac{y^i_t}{w^i_t} \right) - \phi \right) \geq \beta^{t-1} u(c^0_t),
$$

(1.29)

for all $i \in \{1, 2, ..., I\}$ which prevents $i > 0$ from preferring not to work; and second,

$$
\beta^{t-1} \left( u(c^i_t) - v \left( \frac{y^j_t}{w^j_t} \right) - \phi \right) \geq \beta^{t-1} \left( u(c^i_t) - v \left( \frac{y^j_t}{w^j_t} \right) - \phi \right),
$$

(1.30)

for all $i, j \in \{1, 2, ..., I\}$ and all $t$, which simplify to the same conditions as (1.6) from the baseline model because $\phi$ cancels on both sides. I simulate the model with $\phi = \ln (1.712)$, representing a fixed cost equal to 25 percent of the average wage for type $i = 1$.

The lessons from this paper’s main analyses are unchanged by adding an extensive margin, though the optimal policies do respond to the extensive margin. One response of policy is that, while average tax rates have the same shape as in the baseline model, they are increased throughout the income distribution by the addition of an extensive margin. Intuitively, consumption is being provided to the $i = 0$ individuals who do not work, so av-
average taxes on all other types must increase. A second, more subtle response is consistent with the analysis of Saez (2002). In the simulation of policy with an extensive margin, allocations mimic the U.S. Earned Income Tax Credit, whereby low earners receive a subsidy (i.e., a negative marginal tax rate) to encourage them to work rather than claim the $i = 0$ allocation.

### 1.6.4 Pareto-improving age dependence

The main analysis in this paper assumes that social planner’s problem is to maximize a Utilitarian social welfare function. This is a restrictive though standard assumption, and concerns about it have inspired research on Pareto efficient taxation such as Stiglitz (1987) and, more recently, Werning (2007b). In a similar vein, the original partial reform approach of Guesnerie (1977) stressed incremental Pareto improvements to tax policy, not incremental Utilitarian improvements. For those uncomfortable with reforms that sacrifice the welfare of some individuals for greater gains by others, the key question is whether age dependence is a Pareto-improving partial reform: that is, a reform that can raise social welfare without harming any individuals.\(^{58}\)

Pareto-improving age dependence would also be more likely to succeed as a policy proposal. In particular, concerns about the impact of moving to an age-dependent system can be mitigated by using some of the surplus value generated by the Pareto improvement to compensate those who would otherwise lose in the transition.\(^{59}\)

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\(^{58}\) Blomquist and Micheletto (2003) illustrate the theoretical potential for age dependence to be Pareto-improving.

\(^{59}\) Another option to avoid transition concerns is to make age dependence apply only to generations born after the date of the policy being approved.
To test whether age dependence is a Pareto-improving partial reform, I simulate the baseline model with the additional restriction that no individual can be worse off under the age-dependent policy than under the Static Mirrlees policy. As in the Utilitarian model, marginal distortions on high-income young workers and average taxes on all young workers are lower under the Pareto-improving age dependent tax policy than under the Static Mirrlees. More important, the welfare gain from Partial Reform is nearly as large as in the baseline, equivalent to 1.8% of aggregate consumption in the Static Mirrlees. The Pareto-improvement restriction ensures that the highest earners are left with their utility levels from the Static Mirrlees policy, while reform generates a substantial increase in welfare for lower earners. This result suggests that age dependence is a reform capable of attracting broad-based support.

1.7 Conclusion

In this paper, I studied a partial reform of tax policy: age-dependent labor income taxes. To do so, I used modern dynamic Mirrleesian optimal tax methods to contrast three policy scenarios: a Static Mirrlees policy restricted to age-independent taxes, a Partial Reform policy in which labor income taxes can be age-dependent, and a Full Optimum policy in which only private information constrains the design of taxes. In a baseline model, I showed how classic theoretical results on the intratemporal and intertemporal policy margins apply to recall that there is no mortality risk in the model economy. In reality, individuals with shorter lives may be relatively disadvantaged, as taxes are likely to be lower on the old. As the welfare benefits calculated above are based solely on the ages between 30 and 59, however, the affected population is small.
age-dependent policy. I examined how age dependence affects these margins in economic environments with stochastic wages and private saving and borrowing, as well.

Then, I used data from the U.S. Panel Study of Income Dynamics to calibrate and simulate the three policy scenarios. This quantitative analysis yielded two specific policy recommendations that were largely robust across settings. First, marginal income taxes ought to be lower for high-earning young workers in an optimal age-dependent policy than in an age-independent policy. These individuals are near the top of their age-specific wage distribution, so the efficiency costs of distorting their labor effort are substantial. In an age-dependent tax system, the benefit from such a distortion (increasing tax revenue from higher earners) is relatively small, whereas the benefit appears much larger in an age-independent system that cannot recognize the position of these individuals within their age’s distribution. This specific example illustrates a more general finding that age dependence avoids using marginal distortions that age-independent policy cannot, raising the efficiency of the tax system. Second, younger workers ought to face a lower average tax schedule than middle-aged workers if private saving and borrowing are restricted, as differential average taxes by age substitute for private borrowing in the presence of rising wage paths. In models with private saving and borrowing, a variety of average tax schedules can implement the optimum, including policies that have lower average taxes on the young.

Finally, the calibrated policy simulations allowed me to quantify the welfare gain from age dependence and understand its components. Age dependence yields a large welfare gain equal to between one and three percent of aggregate annual consumption. Moreover it captures a substantial portion of the gain from reform to the optimal dynamic
policy, ranging from above 40 percent to approximately 95 percent of the potential gain depending on assumptions about the economic environment. Age dependence provides especially large welfare gains for the low-skilled, but most people obtain higher utility than they would under an age-independent policy. In fact, a simulation with the added constraint that age dependence be Pareto-improving yields nearly as large a social welfare gain as does the standard, Utilitarian-optimal age-dependent policy.

These findings show that that age dependence, which requires only a simple change to current tax policies, is nevertheless a potentially powerful reform. Future work on age dependence ought to extend this analysis in a few directions outside the scope of this paper.

First, the quantitative analysis of this paper focuses on individuals between 30 and 60 years of age. Some of the largest gains to age dependence may come from individuals outside this range, as wage distributions for people in their twenties and sixties are substantially different from those in the range studied here. This paper neglected those age ranges to avoid large uncertainties about how to treat distortions to the acquisition of human capital early in life (i.e., education) and to the retirement margin in the presence of Social Security, and the age-dependent taxes derived above would have little effect on individuals’ choices on these margins. If these margins were properly modeled, however, the benefits of age dependence may be substantially increased.

Second, as mentioned in Section 1.6, this paper has assumed that the elasticity of labor supply is constant across age groups. While this assumption is almost certainly false, solid evidence on variation in labor supply elasticity with age is surprisingly rare.
Any variation in this elasticity will raise the value of age dependence, so identifying it should be a high priority for future work.

Finally, an important next step toward taking advantage of this policy opportunity is to use the results of this paper to design and study specific changes to existing taxes. The findings of this paper suggest that such an exercise would identify relatively simple ways to increase the efficiency and equity of current tax policy, yielding substantial welfare gains.
Chapter 2
Incorporating Preference Heterogeneity into Optimal Tax Models

2.1 Introduction

The leading modern model of optimal taxation, initiated by Mirrlees (1971), focuses on a form of heterogeneity against which individuals would like to insure: heterogeneity in productivity. This paper is about a form of heterogeneity assumed away in the conventional Mirrlees model but prominent in debates over taxation: heterogeneity in preferences across which individuals do not want to insure. I propose a method for incorporating these preferences into Mirrleesian optimal tax models and derive the implications of this heterogeneity for optimal policy toward intratemporal and intertemporal private decisions. I also provide a suggestive empirical test of the model, showing that international data on preferences and policies are consistent with the model’s results. Therefore, preference heterogeneity is potentially important for both the normative and positive understanding of optimal taxation.

Heterogeneity in preferences was assumed away in the Mirrlees model because of long-standing concerns over how to translate ordinal preferences into the cardinal utility functions necessary for using von Neumann-Morgenstern expected utility theory as the basis for the objective function of the ex ante individual (or, equivalently, the social planner).

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61 I am grateful to Robert Barro, Caroline Hoxby, Greg Mankiw, and Aleh Tsyvinski for helpful comments and suggestions.
The few previous treatments of preference heterogeneity in optimal tax models have clarified but not overcome these concerns, resulting in a persistent ambiguity about whether preference heterogeneity yields more or less progressive taxes than the standard model. This ambiguity has limited the impact of preference heterogeneity on standard analyses of optimal taxation and has made it difficult to test whether preference heterogeneity is important empirically.

The first contribution of this paper is to show that we can avoid ambiguity over the effects of preference heterogeneity by restricting attention to the class of preferences across which individuals do not want to be insured. This restriction pins down a specific representation of preferences in the individual utility function. While sacrificing generality and therefore not addressing all forms of preference heterogeneity, this restriction on preferences allows us to make progress on an important component of the broader issue.

Despite its seemingly narrow definition, this class of preferences is of substantial relevance for optimal tax design. In fact, these preferences form the basis of a longstanding and prominent critique of redistributive taxation. Consider the following passage from the philosopher Robert Nozick’s influential 1974 book, *Anarchy, State, and Utopia*:

"Why should we treat the man whose happiness requires certain material goods or services differently from the man whose preferences and desires make such goods unnecessary for his happiness?"

Or, consider how Milton Friedman (1962) put the same point:

"Given individuals whom we are prepared to regard as alike in ability and initial resources, if some have a greater taste for leisure and others for marketable goods,

\[\text{Sandmo 1993 is an important early contribution; Kaplow 2007 is a more recent analysis. Judd and Su (2006) use a numerical approach to incorporate several types of heterogeneity into optimal tax models, including in preferences, though they do not discuss the normative problem that is the focus of this paper.}\]
inequality of return through the market is necessary to achieve equality of total return or equality of treatment...If both we paid equally in money, their incomes in a more fundamental sense would be unequal."

This paper formally incorporates these political philosophy arguments into the optimal tax problem through preferences that, on their own, do not merit redistribution across individuals.

The second contribution of this paper is to show how this type of preference heterogeneity challenges conventional optimal tax analyses. While heterogeneity in preferences and productive ability generate observationally equivalent behavior, the former does not justify redistribution while the latter is the basis of redistribution in workhorse optimal tax models. Thus, to the extent that variation in income is due to preference differences rather than productivity differences, the optimal extent of redistribution is lower, and the neglect of preference heterogeneity biases the results of conventional optimal tax analyses in favor of redistribution of income.63 Using data from the U.S. PSID on wages and variation in hours worked, I provide a rough but suggestive example of the potential effects of preference heterogeneity on optimal taxation.

Third, this paper shows that incorporating this form of preference heterogeneity has descriptive value as well as normative implications. One prediction of the model is that countries with more internal heterogeneity in preferences ought to have less redistributive policies. I show that international data on redistribution and preferences for work and leisure are consistent with this prediction, implying that the forces identified in this pa-

63 Judd and Su (2006) numerically examine optimal taxation in the presence of several dimensions of heterogeneity and identify some that reduce the optimal level of redistribution for a similar reason.
per may be important for understanding policy differences across countries. Moreover, this finding suggests an alternative interpretation of the evidence in Alesina and Angeletos (2005) that countries with less redistributive policies are those in which most individuals believe effort, rather than luck, is the main determinant of personal income. In countries with more heterogeneous preferences, and therefore less redistributive optimal taxation, a larger share of high earners are those willing to exert substantial effort.

Finally, this paper examines preference heterogeneity’s implications for optimal taxation in a dynamic setting, a currently active area of research surveyed in Golosov, Tsyvinski, and Werning (2006). The main result is that the inverse Euler equation examined in Golosov, Kocherlakota, and Tsyvinski (2003) no longer describes the optimal intertemporal allocation of consumption; with preference heterogeneity, the optimal allocation drives a smaller wedge into the individual’s private optimum. Intuitively, agents with a high preference for consumption relative to leisure will earn more and save more throughout their lives, in part so as to self-insure against the possibility of lower future ability (or disability). Thus, they will accumulate assets and use those assets to supplement consumption if their productive ability falls. If these preferences are not a valid basis for redistribution, optimal dynamic policies such as disability insurance and capital taxation ought to take this reason for asset accumulation into account.46 A similar result holds if individuals differ in their

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46 Conventional dynamic optimal tax policy discourages asset accumulation because it is part of the strategy of "cheating the system" in which individuals oversave, work less in the future, claim the generous tax treatment provided to low-income workers, and supplement consumption out of savings. Kocherlakota (2006) shows that a policy solution to this is to have capital taxes increase as labor income decreases, and Golosov and Tsyvinski (2006) show that optimal disability insurance is "asset-tested" so that individuals can claim disability benefits only if their assets fall below a certain threshold. But, if this behavior is driven by preference differences, such policies will inefficiently (and inequitably) discourage saving by those with high preferences for consumption relative to leisure.
discounting of future utility. As with the Nozick and Friedman statements about justifiable income taxes, these results are consistent with normative discussions of capital taxation in Mankiw (2003), who questions the justifiability of taxing "savers" more than "spenders."

Appendix B, which gives further details related to this paper, can be found at the end of this dissertation.

2.2 Modeling preference heterogeneity

First, I review the history of preference heterogeneity in the study of optimal taxation. The now-standard Mirrlees (1971) approach to optimal taxation starts with the observation that people differ in their productive ability, so that producing a given income requires more labor effort from some than from others. Optimal taxation would redistribute from high-ability to low-ability types, but if ability is unobservable, the tax authority (or planner) must pursue a "second-best" approach of taxing income as a proxy for ability. Taxing income discourages effort by high-ability workers who can avoid taxation by imitating lower-ability workers, creating the classic tradeoff between efficiency and equity.

As Mirrlees acknowledged in his original paper, this formulation of the optimal tax problem makes some strong assumptions. This paper will focus on his second assumption in particular: that all individuals have the same preferences. As Mirrlees puts it:

"Differences in tastes, in family size and composition, and in voluntary transfers, are ignored. These raise rather different kinds of problems, and it is natural to assume them away."

Mirrlees was not alone in endorsing this simplification. Pigou (1952) wrote, in a classic text:
"Of course, in so far as tastes and temperaments differ, allowance ought, in strictness, to be made for this fact...But, since it is impossible in practice to take account of variations between different people’s capacity for enjoyment, this consideration must be ignored, and the assumption made, for want of a better, that temperamentally all taxpayers are alike."

These two statements argue that the proper way to deal with the difficulty of observing taste differences is to pretend as though they do not exist, and this approach has been dominant ever since, with prominent examples being Tuomala (1990), and Saez (2001).

### 2.2.1 The problem with heterogeneous preferences

Mirrlees and Pigou were eager to assume away preference heterogeneity because it complicates, and even fundamentally challenges, the Utilitarian approach to optimal taxation. The Utilitarian approach represents each individual’s preferences with a *cardinal* utility function and then sums these utilities across all individuals in the society. The problem with heterogeneous preferences is that preferences can be represented by a wide variety of utility functions. If all individuals have the same preferences, then the specific functional representation is largely immaterial to the Utilitarian approach. If preferences are heterogeneous, however, the specific representation can be a matter of great importance; in this section, I show how observationally equivalent representations of preferences can yield dramatically different optimal tax policies for the same economy.

The observational equivalence of different representations of preferences is a particular problem for Mirrlees’ formulation of the optimal taxation problem. The driving force

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65 More complex aggregation is possible, such as social welfare functions that are concave in individual utilities.

66 Kaplow (2007) makes a similar observation.
behind redistribution in Mirrlees’ model is heterogeneity in unobservable productive ability, but that heterogeneity is itself observationally equivalent to preference heterogeneity.\textsuperscript{67} Thus, the ambiguity of how to represent preferences in a cardinal utility function threatens the basis of optimal taxation in the Mirrlees framework, and assuming it away is not innocuous.\textsuperscript{68}

To make this point more tangible, consider a specific optimal tax problem in which a social planner’s first-best (full-information) problem is to choose feasible consumption and income allocations to maximize the sum of utilities across heterogeneous individuals. Individual preferences take a separable form with logarithmic utility of consumption $c$ and quadratic disutility from labor $l$, where labor is income $y$ divided by the uniform wage $w$. Individuals differ in their relative taste for consumption and leisure, parameterized by $\theta_i$. For simplicity, suppose that there are only two preference types, $\theta_i \in \{\theta_a, \theta_b\}$, held by equal proportions of the population. I assume $\theta_a > \theta_b$, meaning that the marginal rate of substitution of leisure for consumption is higher for $\theta_a$ and $\theta_b$. In other words, a "type a" individual (i.e., with preference $\theta_a$) is willing to work harder in exchange for consumption than is a "type b" individual at any given level of consumption and labor effort.

One version of the social planner’s problem in this setting is:

\textbf{Problem 10:} First-best planner’s problem with "consumption preferences:"

\textsuperscript{67} In an insightful early paper, Sandmo (1993) points out this problem.

\textsuperscript{68} An important corollary to this issue, not pursued in this paper, is that simulations of optimal policy that extract a productivity distribution from the income distribution under the assumption that preferences are homogeneous are potentially misleading.
2.2 Modeling preference heterogeneity

\[
\max_{c,y} \frac{1}{2} \left[ \theta_a \ln (c_a) - \frac{1}{2} \left( \frac{y_a}{w} \right)^2 \right] + \frac{1}{2} \left[ \theta_b \ln (c_b) - \frac{1}{2} \left( \frac{y_b}{w} \right)^2 \right]
\]  

(2.1)

subject to the feasibility constraint:

\[
\frac{1}{2} (y_a - c_a) + \frac{1}{2} (y_b - c_b) \geq 0
\]  

(2.2)

Note that the parameter \(\theta_i\) directly affects the cardinal value of utility from consumption in (2.1), corresponding to the label "consumption preferences."

An alternative version of the social planner’s problem is:

**Problem 11:** First-best planner’s problem with "labor effort preferences:"

\[
\max_{c,y} \frac{1}{2} \left[ \ln (c_a) - \frac{1}{2} \left( \frac{y_a}{w} \right)^2 \right] + \frac{1}{2} \left[ \ln (c_b) - \frac{1}{2} \left( \frac{y_b}{w} \right)^2 \right],
\]  

(2.3)

subject to the same feasibility constraint (2.2).

Note that the individual utility function in (2.3) interprets the taste parameter as "labor effort preferences", affecting the disutility of earning income. Importantly, this utility function is the utility function from the "consumption preferences" problem multiplied through by \((\theta_i)^{-1}\), so they are observationally equivalent representations. In other words, examining individuals’ consumption and income choices cannot distinguish between these two representations of preferences.

The first-best allocations from these observationally equivalent planner problems differ dramatically, as shown in this chart:
2.2 Modeling preference heterogeneity

<table>
<thead>
<tr>
<th>Preference type</th>
<th>$\frac{c_a}{c_b}$</th>
<th>$\frac{l_a}{l_b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption preferences</td>
<td>$\frac{\theta_a}{\theta_b}$</td>
<td>1</td>
</tr>
<tr>
<td>Labor effort preferences (standard Mirrlees)</td>
<td>1</td>
<td>$\frac{\theta_a}{\theta_b}$</td>
</tr>
</tbody>
</table>

In terms of both consumption and labor effort, shifting from "consumption preferences" to "labor effort preferences" favors type $b$. In particular, the consumption preferences policy gives $a$ more consumption than $b$ but requires no more effort from $a$ than from $b$, while the labor effort preferences policy gives equal consumption to both types but requires $a$ to exert more effort than $b$. The direction of income redistribution reverses when we use a different cardinal representation of the same preferences.

These optimal allocations differ because the objective functions in (2.1) and (2.3) cardinalize preferences differently, leading the planner to interpret heterogeneity in fundamentally different ways across the two problems. In the former, type $a$ is better, in a relative and absolute sense, at producing utility from consumption than is type $b$, while the types avoid disutility from earning income equally well. In the latter, both types generate utility from consumption equally well, but type $a$ is better at avoiding disutility from earning income.

Note that the labor effort preferences form is subtitled "standard Mirrlees" in the chart. To see why, note that if we were to move $\theta_i$ inside the parentheses of the second term of (2.3), raise it to the $\frac{1}{\sigma}$ power, and relabel it $\frac{w^i}{w}$, we would have preserved each individual’s preferences but we would also have converted the problem into the conventional Mirrlees setting in which individuals have heterogeneous unobservable abilities, not unob-
servable preferences. If both preferences and ability are unobservable, then consumption preferences, labor effort preferences, and productive ability are all observationally equivalent representations of the same choice behavior. This suggests that preference heterogeneity presents a potentially serious challenge for the Mirrleesian analysis of optimal taxation.

As this analysis shows, without pinning down a cardinal representation of preferences the implications of preference heterogeneity for optimal taxation are so ambiguous as to be uninformative. Following on Sandmo (1993), a few additional analyses on this point are Fleurbaey and Maniquet (1999), Boadway, Marchand, Pestieau, and Racionero (2000), Tarkiainen and Tuomala (2004), and Kaplow (2007a). These projects have largely stated the conditions under which different allocations would obtain, making little progress on determining which is the appropriate result for understanding the role of preference heterogeneity in optimal taxation.

This paper argues that the previous analyses have aimed for too general a treatment of the problem, and that the Mirrlees approach can be made robust to heterogeneity in the important class of preferences that Nozick and Friedman emphasized in the quotations above. As suggested by the previous section’s analysis, the key advantage of this class of preferences is that they require a particular cardinalization in the utility function, eliminating the ambiguity of the general case. I call this class of preferences pure preferences.
2.2 Modeling preference heterogeneity

2.2.2 "Pure preferences"

Following Nozick and Friedman, pure preferences are those across which redistribution is not justified. To state this more formally, suppose an individual with preference parameter \( \theta_i \) and wage \( w \) solves the following utility maximization problem:

**Problem 12:** \( Laissez-faire \) problem:

\[
\max_{c,y} u \left( c, \frac{y}{w}, \theta_i \right)
\]

subject to:

\[
y - c = 0
\]

The \( Laissez-faire \) problem yields an individual’s choice for consumption and labor income given its wage and preference parameter and subject to its private budget constraint. The solution to this problem will be important for the remainder of the paper, so I highlight it with the following definition:

**Definition** Suppose an individual with preference parameter \( \theta_i \) and wage \( w \) maximizes Problem 12 (Laissez-faire). The solution to this problem is the "laissez-faire allocation" for this individual, and it is denoted \( \{c_i^*, y_i^*\} \).

To define pure preferences, consider optimal taxes in this environment when only preferences are heterogeneous. The social planner’s problem in that case is:
2.2 Modeling preference heterogeneity

Problem 13: First-best planner’s problem with uniform wages

\[
\max_{\{c_i, y_i\}} W = \sum_{i=a,b} \pi_i u \left( c_i, \frac{y_i}{w}, \theta_i \right),
\]

subject to the economy-wide feasibility constraint:

\[
\sum_{i=h,l} \pi_i (y_i - c_i) = 0.
\]

The solutions to these two problems yields the formal definition of pure preferences.

Definition If and only if preferences are "pure preferences," then the solution to Problem 13 (First-best planner’s problem with uniform wages) is the laissez-faire allocation for each individual.

Intuitively, pure preferences are preferences that do not, in isolation, justify redistribution, so they prevent the social planner from altering the private, untaxed choices of individuals with the same wage.

Another way to describe pure preferences relates to the insurance role of redistributive taxation. In the classic thought experiments of of Vickrey (1945) and Harsanyi (1953 and 1955), and as developed by Rawls (1974), individuals are assumed to be ignorant their personal characteristics when defining the social welfare function that will guide tax policy. Because they face uncertainty, individuals may want to commit to future taxes and transfers between them to provide insurance. In this context, pure preferences can be defined as those preferences across which individuals would not want to make any such commit-
2.3 Implications for labor income taxation

In this section, I focus on the implications for optimal labor income taxation of pure preferences as defined in the previous section. I start by showing how pure preferences, unlike preferences in general, imply a particular cardinalization for use in optimal tax analyses. Then, I derive the implications of this cardinalization for optimal policy in a first-best setting, showing how the standard Mirrleesian model occupies a polar position in favor of redistribution. Finally, I use data from the U.S. Panel Study of Income Dynamics to provide suggestive results on optimal policy when preference heterogeneity is taken into account.

2.3.1 The cardinalization of pure preferences

The definition from the previous section implies a restriction on the representation of pure preferences that allows us to unambiguously incorporate them into the optimal tax problem. Using the notation from above, the following lemma is a consequence of our definition of pure preferences:

**Lemma 1** Define social welfare according to (2.4). If and only if \[ \{\theta_i\}_{i=1}^T \] are “pure

---

69 A similar way to think about these preferences is to imagine an individual whose preference over two goods is not yet formed but who has to specify whether to transfer resources from herself under one realization of those preferences to herself under another realization. Pure preferences are the class of preferences for which an individual in this situation maximizing expected utility would choose zero transfers. I am grateful to Robert Barro for suggesting this example.
preferences," then

\[ W_i \left( \{u(c_i^*, y_i^*, w, \theta_i)\}_{i=1}^I \right) = W_j \left( \{u(c_i^*, y_i^*, w, \theta_i)\}_{i=1}^I \right) \]

where \( W_i \) is the marginal increase in social welfare from allocating a unit of resources to an individual with preference \( \theta_i \), starting from that individual’s "laissez-faire allocation."

**Proof.** Follows directly from the definition of pure preferences.

In words, this lemma says that, in a society with only preference heterogeneity, the marginal benefit of allocating resources to two individuals with different *pure preferences* must be the same when these individuals are earning and consuming their laissez-faire allocations \( \{c_i^*, y_i^*\} \). Intuitively, we defined pure preference differences as those that do not justify redistributive taxes or transfers from one individual to another with the same wage.

Thus, when two such individuals are at their laissez-faire allocations, each consuming what he or she earns, society is indifferent between allocating either of them a marginal unit of resources. If not, it would want to redistribute between them, violating the definition of pure preferences.

Next, I show that Lemma 1 is enough to pin down the cardinalization of pure preferences in the social welfare function. The lemma suggests a method for finding this cardinalization: it must equalize the marginal Utilitarian social welfare value of allocating resources to any two individuals with different preferences, starting at their laissez-faire allocations.

Suppose we represent individual preferences with the utility function:

\[ u_i = \ln (c_i) - \frac{1}{2\theta_i} \left( \frac{y_i}{w} \right)^2, \]  \hspace{1cm} (2.5)
We could begin with any other representation of these preferences without affecting the results.\footnote{The Appendix executes a similar procedure for a more general utility function to derive the corresponding pure preferences cardinalization.}

Following Lemma 1, the first step in finding the pure preference cardinalization is to solve for each individual’s laissez-faire allocation. If an individual with the preference parameter $\theta_i$ and wage $w$ solves Problem 12 (Laissez-faire) with preferences represented by (2.5) it chooses the laissez-faire allocation:

$$c_i = w \left( \theta_i \right)^{\frac{1}{2}}, \quad y_i = w \left( \theta_i \right)^{\frac{1}{2}}$$

(2.6)

Note that an individual consumes more and earns more in the laissez-faire allocation if it has a higher preference parameter, as suggested by Nozick and Friedman.

The second step is to suppose that the planner offers a lumpsum grant $\Delta$ to each individual solving Problem 12 (Laissez-faire). Each individual would then face a new budget constraint:

$$y_i + \Delta = c_i.$$  

(2.7)

where labor income plus the lump-sum grant can be used to fund consumption. The individual’s optimal choices are then:

$$c_i = \frac{\Delta + \left( \Delta^2 + 4\theta_i \left( w \right)^2 \right)^{\frac{1}{2}}}{2}, \quad y_i = \frac{-\Delta + \left( \Delta^2 + 4\theta_i \left( w \right)^2 \right)^{\frac{1}{2}}}{2}.$$  

(2.8)

These show that the individual has used the lumpsum grant to both raise consumption and lower the income it is required to earn. As we reduce the lumpsum grant toward zero, the quadratic terms in $\Delta$ drop out and (2.8) reduces to:

$$c_i = \frac{\Delta}{2} + w \left( \theta_i \right)^{\frac{1}{2}}, \quad y_i = \frac{-\Delta}{2} + w \left( \theta_i \right)^{\frac{1}{2}}.$$
These results imply the following changes in consumption and labor due to a small lump-sum grant:

\[ dc_i = \frac{\Delta}{2}, \quad dy_i = -\frac{\Delta}{2}. \quad (2.9) \]

Note that these changes take such simple forms because of the utility function assumed in (2.5).

The third step in the course suggested by Lemma 1 is to find a social welfare function that yields equal marginal social welfare when these changes in consumption and earnings result from a grant to any individual. For simplicity, I will restrict attention to social welfare functions that are affine transformations of individual utility as represented by (2.5), so that:

\[ W = \sum_{i=1}^{I} \pi_i \left( \alpha_i + \frac{1}{\psi_i} \left( \ln (c_i) - \frac{1}{2\theta_i} \left( \frac{y_i}{w} \right)^2 \right) \right), \quad (2.10) \]

where \( \alpha_i \) and \( \psi_i \) are constants to be derived for each individual. Within this form, the following result holds:

**Proposition 6 (Pure preferences Social Welfare Function)** Suppose the social welfare takes the general form (2.10). If \( \theta_i \) are pure preferences, then the social welfare function is:

\[ W = \sum_{i=1}^{I} \pi_i \left( \alpha_i + (\theta_i)^{\frac{1}{2}} \ln (c_i) - (\theta_i)^{-\frac{1}{2}} \frac{1}{2} \left( \frac{y_i}{w} \right)^2 \right), \quad (2.11) \]

where the values for \( \alpha_i \) are unrestricted.

**Proof.** The total derivative of (2.10) is:

\[ dW = \sum_{i=1}^{I} \pi_i \left( \frac{1}{\psi_i c_i} dc_i - \frac{1}{\psi_i w \theta_i} \left( \frac{y_i}{w} \right) dy_i \right). \]
Using the results \((2.6)\) and \((2.9)\),
\[
W_i = \frac{1}{w\psi_i (\theta_i)^{\frac{1}{2}}} \Delta.
\]

Next, choose
\[
\psi_i = (\theta_i)^{-\frac{1}{2}},
\]
which yields
\[
W_i = \frac{1}{w} \Delta.
\]
which is constant across individuals, satisfying Proposition 6. 

Thus, \((2.11)\) gives the social welfare function for optimal tax analysis that incorporates pure preference heterogeneity. With this social welfare function, we can now solve for optimal allocations with pure preferences and compare them to optimal allocations with alternative preference cardinalizations.

### 2.3.2 Optimal policy implications

In this section, I apply the findings from the previous subsection to derive the implications of pure preference heterogeneity for optimal labor income taxation when both wages and preferences are heterogeneous. In particular, I compare the optimal policy when preferences are cardinalized as pure preferences, as in expression \((2.11)\), to the optimal policy when all heterogeneity is assumed to be in productive ability or, equivalently, of the labor effort preferences type from Section 2.2.

I focus on the first-best policy, where all types are publicly observable. Thus, there are no incentive constraints in the social planner’s problem. This assumption is particularly helpful in isolating the role of preference heterogeneity, whose main impact on the
optimal tax problem is on the form of the planner’s objective function, not its constraints. Moreover, omitting the incentive constraints clarifies our results by avoiding the technical complexities of multiple screening problems discussed at length by Mirrlees (1986), Tarkianen and Tuomala (2004) and Judd and Su (2006).\textsuperscript{71}

Using the form of the social welfare function derived in Proposition 6, but allowing for wage heterogeneity, the social planner’s problem with pure preferences is:

**Problem 14:** First-best planner’s problem with pure preferences

\[
W = \sum_{m=1}^{M} \sum_{i=1}^{I} \left( (\theta_i)^{\frac{1}{2}} \ln \left( c_{m,i} \right) - (\theta_i)^{-\frac{1}{2}} \frac{1}{2} \left( \frac{y_{m,i}}{w_m} \right)^2 \right),
\]

where I set \( \alpha_{m,i} = 0 \) for all \( m, i \) without loss of generality, subject to the economy-wide feasibility constraint:

\[
\sum_{m=1}^{M} \sum_{i=1}^{I} (y_{m,i} - c_{m,i}) = 0.
\]

In contrast, consider the social planner’s problem when preferences are assumed to affect the cardinal disutility of labor: i.e., labor effort preferences. Note that this is equivalent to assigning all heterogeneity to productive ability, or wages, as in the standard Mirrleesian model. The social planner’s problem in this case is:

\textsuperscript{71} The "multiple screening problem" refers to the case in which more than one characteristic of the individual is private information. Here, both skill and preferences would be private information in a second-best problem. Unfortunately, an analytical approach to these problems that yields clean results has not been discovered.
Problem 15:  *First-best planner’s problem with labor effort preferences (standard Mirrleesian model)*

\[
W = \sum_{m=1}^{M} \sum_{i=1}^{I} \left( \ln (c_{m,i}) - \frac{1}{2\theta_i} \left( \frac{y_{m,i}}{w_m} \right)^2 \right),
\]

(2.13)

where I set \( \alpha_{m,i} = 0 \) for all \( m, i \) without loss of generality, subject to the economy-wide feasibility constraint:

\[
\sum_{m=1}^{M} \sum_{i=1}^{I} (y_{m,i} - c_{m,i}) = 0.
\]

The solutions to these planner’s problems are shown in the following chart:

<table>
<thead>
<tr>
<th>Preference type</th>
<th>( \frac{c_{m,i}}{c_{n,j}} )</th>
<th>( \frac{l_{m,i}}{l_{n,j}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure preferences</td>
<td>( \left( \frac{\theta_i}{\theta_j} \right)^{\frac{1}{2}} )</td>
<td>( \left( \frac{\theta_i}{\theta_j} \right)^{\frac{1}{2}} \frac{w_m}{w_n} )</td>
</tr>
<tr>
<td>Labor effort preferences</td>
<td>1</td>
<td>( \frac{\theta_i}{\theta_j} \frac{w_m}{w_n} )</td>
</tr>
<tr>
<td>(standard Mirrlees)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A comparison of these solutions shows how pure preferences have the potential to fundamentally alter optimal taxation.

The pure preferences policy has high-wage and high-preference individuals enjoy more consumption and exert more effort than low-wage and low-preference individuals. It recognizes that high-preference individuals are willing to give up leisure for consumption and allows them to do so, but it does not try to place a lighter or heavier burden on them than on low-preference individuals.
In particular, note that two individuals with the same wage but different preferences pay the same average tax rate in this example:

\[
\frac{y_{m,i} - c_{m,i}}{y_{m,i}} = \left(\frac{\theta_i}{\theta_j}\right)^{\frac{1}{2}} \frac{y_{m,j} - c_{m,j}}{y_{m,j}} = \frac{y_{m,j} - c_{m,j}}{y_{m,j}}
\]

Thus, the cardinalization of pure preferences that yielded no redistribution across preference types when wages were homogeneous yields equal average tax rates across preference types for all individuals with the same wage when wages are heterogeneous. As shown in the Appendix, this result is more general. In fact, it holds for any calibration of the commonly-used utility function:

\[
u_i = \theta_i (c_i)^{1-\gamma} - \frac{1}{\sigma} \left(\frac{y_i}{w}\right)^\sigma,
\]
given the correct cardinalization of pure preferences.

In contrast, the labor effort preferences (standard Mirrlees) policy grants high-wage and high-preference individuals no more consumption but requires them to exert more effort than low-preference individuals. This cardinalization treats preferences as it would treat wages. For example, two individuals with the same wage but different values of \(\theta\) will face different average tax rates, and the higher preference type, who would choose to earn more income in the laissez-faire scenario, bears the larger average tax burden. Thus, the standard Mirrleesian optimal tax framework occupies an extreme position in favor of redistributive taxation.

These results show that once preference heterogeneity is taken into account, the optimal degree of progressive income redistribution is less than the conventional analysis implies. In the next subsection, I explore this finding quantitatively with U.S. data.
2.3.3 Suggestive simulation results

In this subsection, I provide a rough but data-driven illustration of preference heterogeneity’s potential effect on optimal taxation. I use PSID data to calibrate a social planner’s problem with unobserved wage and pure preference heterogeneity. In particular, I simulate Problem 14 and Problem 15 from the previous subsection, augmented by incentive constraints of the following form:

\[ \ln(c_{m,i}) - \frac{1}{2\theta_i} \left( \frac{y_{m,i}}{w_m} \right)^{\frac{1}{2}} \geq \ln(c_{n,j}) - \frac{1}{2\theta_i} \left( \frac{y_{n,j}}{w_m} \right)^{\frac{1}{2}}, \]

for all \( m, n \in \{1, 2, ..., M\} \) and all \( i, j \in \{1, 2, ..., I\} \). Incorporating these incentive constraints reflects that the planner can observe neither wages nor preferences, realism that we avoided in the analytical section for clarity. These constrained planner’s problems are the conventional problems in the Mirrleesian optimal tax literature.

Calibrating this constrained planner’s problem requires estimating individuals’ wages and preferences. In this paper, I take a rough approach to this calibration, leaving to future work a more detailed treatment. To estimate wages and preferences, I rely on an assumption that labor effort is accurately measured by reported work hours in the PSID.\(^{72}\) Given this assumption, I calculate wages as an individual’s labor income divided by his or her reported hours worked. Then, controlling for wages and observable variables likely to affect demand for an individual’s labor, I attribute all variation in incomes to preference heterogeneity.

\(^{72}\) This is clearly an imperfect assumption, in that an hour of work need not elicit the same effort from different workers in different occupations at different times. Nevertheless, there is surely some substantial correlation between hours and true effort, so it serves as a natural starting point for the analysis.
Formally, I observe income $y_{m,i}$, calculate consumption $c_{m,i}$ from the relevant tax schedule, and calculate the wage $w_m$ using observed effort $l_{m,i} = \frac{y_{m,i}}{w_m}$. Then, I use the utility function

$$U_{m,i} = \ln \left( c_{m,i} \right) - \frac{1}{2\theta_i} \left( \frac{y_{m,i}}{w_m} \right)^2$$

to calculate a value of $\theta_i$ for each individual.

Unobservable factors may cause this procedure to overestimate or underestimate the degree of preference heterogeneity in the population. On one hand, there are likely to be personal characteristics for which I cannot control that affect work effort but that are not best modeled as "preferences." On the other hand, restrictions on hours worked (e.g., to 40 hour work weeks) are likely to reduce variation in reported hours relative to what individuals would desire, thus limiting the preference heterogeneity my procedure is able to identify.

**Data and calibration**

I begin with household heads from the U.S. PSID core sample for the years 1968-2001 and collect data on their income, hours worked, age, race, gender, and education for each year they are a head of household. The reported real wage for each observation is reported labor income divided by reported hours, inflated or deflated with the CPI to be in 1999 dollars. I remove potentially problematic observations by eliminating all those for which reported annual hours were less than 500 or greater than 5,824, for which reported labor income was zero but hours were positive, or for which the nominal wage implied by earnings and hours was less than half the applicable minimum wage in that calendar year.
2.3 Implications for labor income taxation

After these adjustments, the dataset contains approximately 155,000 observations on just over 19,000 individuals with an average of 8.1 years observed per person.

To estimate wage and preference heterogeneity, I focus on a subset of individuals with similar demographic characteristics. This subset is likely to face similar demand conditions and have less variation in unobserved characteristics than the population as a whole. In particular, I use observations from 1996 through 1999 on single white men between the ages of 35 and 50 whose calculated wages are between $23.00 and $31.00 per hour and whose incomes put them in the 28 percent marginal tax rate bracket that extended from $34,550 to $89,150 in 1999. This subset consists of 79 observations on 64 individuals.

To calculate preferences, I use the utility function

\[ U_{m,i} = \ln c_{m,i} - \frac{1}{2\theta_i} \left( \frac{y_{m,i}}{w_m} \right)^2. \]

I observe \( y_{m,i} \) and calculate \( w_m \) as \( y_{m,i} \) divided by reported hours. To calculate \( c_{m,i} \) I use the 1999 U.S. tax schedule for heads of households. For this subset of men, the equation

\[ c_{m,i} = y_{m,i} - 0.28(y_{m,i} - 34550) - 5812.50 \]

calculates consumption as after-tax income. With \( y_{m,i}, w_m, \) and \( c_{m,i} \), I calculate \( \theta_i \) for each observation in the subsample.

---

73 This wage range is chosen so that workers who earn these wages and work between 1500 and 3000 hours per year fall within the 28% marginal tax rate income range. To the extent that some with these wages earn income outside this range, I underestimate the amount of preference heterogeneity.

74 Some men are observed multiple times in the sample. Though variation in preferences over time is conceptually distinct from variation across individuals, it has the same effect on optimal policy in the planner’s problems specified here.

75 This assumes that capital income, saving and borrowing is negligible for these men.
Table 2.1 shows the estimated joint distribution of wages and preferences for this subset. For ease of computation and interpretation, I have discretized the distribution of wages and the preference parameter $\theta$. Recall that a high value for $\theta$ indicates a worker who puts a relative large weight on consumption compared to leisure. Thus, the lower right corner of the table gives the type for whom earning income is least costly along both dimensions of heterogeneity.

<table>
<thead>
<tr>
<th>Wage ($1999$)</th>
<th>Preferences (larger values=more weight on consumption)</th>
<th>Percent</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.00</td>
<td>1.3 1.3 13.9 8.9 6.3 1.3</td>
<td>32.9</td>
<td>26</td>
</tr>
<tr>
<td>26.00</td>
<td>-- 6.3 8.9 6.3 6.3 --</td>
<td>27.8</td>
<td>22</td>
</tr>
<tr>
<td>28.00</td>
<td>-- 2.5 10.1 5.1 3.8 --</td>
<td>21.5</td>
<td>17</td>
</tr>
<tr>
<td>30.00</td>
<td>-- -- 6.3 7.6 3.8 --</td>
<td>17.7</td>
<td>14</td>
</tr>
</tbody>
</table>

The joint distribution of wages and preferences reveals two interesting patterns. First, there is substantial variation in estimated preferences at all wage levels. This is especially striking given the narrow subset of men on which we have focused. Second, the calculated correlation between wages and preferences is essentially zero, at -0.04.

**Optimal policy simulation**

Using the data described above, I simulate the constrained optimal policies when preferences are cardinalized as pure preferences and as labor-effort preferences. The latter is equivalent to the standard Mirrlees model in which heterogeneity is assumed to be in productive ability.
The most striking difference between the pure preferences policy and the standard Mirrleesian policy is in the shape of the average tax rate schedule. Figure 2.1 shows average tax rates at annual income levels for these workers in the optimal policies.

Figure 2.1: Optimal Average Tax Rates

Figure 2.1 shows that the pure preferences policy is substantially less redistributive than the standard Mirrleesian policy. Individuals at the top of the income distribution pay much lower average tax rates under the pure preferences policy than under the standard Mirrleesian policy, while those at the bottom pay higher rates.\(^{76}\) Note that the highest earner in the pure preferences policy faces a negative marginal tax rate: that is, average tax rates fall at the top of the income distribution. This reflects the planner’s knowledge that this worker has a low wage but has preferences that value consumption highly, a result also

\(^{76}\) Taxes are only for redistribution in this model, so the average tax rate on all income is zero.
found in the hypothetical examples of Judd and Su (2006). In particular, the highest earner under both policies is the individual in the upper-right corner of Table 2.1, whose wage is the lowest in this population but whose preferences put the most weight on consumption relative to leisure.

The lesson of this section is that heterogeneous pure preferences make a social planner more hesitant to use redistributive taxation because high incomes may be due to effort as well as ability. In the next section, I take from this observation a testable prediction and examine it empirically.

### 2.4 International data on preferences and policy

In this section, I look to data for evidence on the existence and importance of pure preference heterogeneity. As implied by the simulations above, income is a worse signal of ability when preferences vary. Given that optimal policy redistributes across ability but not preferences, a greater role for preference heterogeneity implies less redistribution. Therefore, a testable prediction of the model is that the diversity of preferences for leisure in a country should be negatively correlated to the extent of redistribution in that country’s tax policy.

To test whether this prediction is reflected in existing policies, I use measures of redistribution through the tax and transfer system along with evidence from an international survey that asks individuals about their preferences for leisure relative to work. This test implicitly assumes that existing policy is motivated, at least in part, by the core concerns of the Mirrleesian optimal tax model. All analysis is cross-sectional.
I use two measures of redistribution: (1) the size of social transfers as a share of GDP in the mid-1990s from Lindert (2004), and (2) the highest rate of personal income tax in 2000 from the OECD tax database.

For preferences, I use data from the World Values Survey, a prominent source of data on attitudes around the world. Question C008, asked between 1995 and 2001, is:

"Which point on this scale [1 through 5] most clearly describes how much weight you place on work (including housework and schoolwork), as compared with leisure or recreation?"

1. It’s leisure that makes life worth living, not work
...
5. Work is what makes life worth living, not leisure

The extremes of this scale indicate quite different attitudes toward the value of leisure. Though an ideal question would have substituted the words "consumption" or "material possessions" for "work," this question nevertheless asks individuals to compare the value of leisure to the value of work, the return to which is (in economists’ models) consumption.

Figures 2.2 and 2.3 plot the measures of redistribution on the vertical axis and the standard deviation of responses to the preference question on the horizontal axis. As predicted by the model of Section 2.4, there is a negative relationship between these variables: countries with wide variation in preferences for consumption relative to leisure tend to have less redistributive policies.\(^77\)

\(^77\) Other factors undoubtedly contribute to differences in redistributive policies across countries. Unfortunately, we have only a small number of observations from these survey questions, making regressions that control for other influences on policy infeasible.
Figure 2.2: Social transfers and diversity of preferences (WVS C008)

Standard deviation of responses to WVS C008: the relative value of work and leisure in life
Figure 2.3: Top personal income tax rates and diversity of preferences (WVS C008)

The diagram shows a scatter plot with the highest personal income tax rates on the y-axis and the standard deviation of responses to WVS C008 on the x-axis. The countries represented include Sweden, Finland, Germany, Japan, Australia, NZ, Switz, Spain, US, Can., Czech, Korea, Mex., Slovakia, and Hungary. The plot illustrates the relationship between tax rates and the diversity of preferences across these countries.
Simple OLS regressions of the redistribution variables on the standard deviation of preferences give negative and significant (at the 5 percent level or better) coefficients in both cases, despite the small number of observations.

Though only suggestive, these data provide evidence that preference heterogeneity ought to be included in our optimal tax models and that it may (at least in part) explain differences in policy across countries.

### 2.4.1 Preferences as an alternative to beliefs

The evidence above may affect how we interpret international differences in redistributive policy. One prominent explanation of these differences relies on heterogeneity in beliefs about the workings of the economy. Alesina and Angeletos (2005) hypothesize that people’s beliefs on whether luck or effort determines economic success can lead to multiple equilibria that resemble, at least qualitatively, the "European" and "American" redistributive policies. In particular, if individuals believe that effort is more important than luck to success, they exert more effort, and therefore effort is indeed important in determining income. The costs of redistribution in such a society are large, because it discourages effort, so optimal policy is less redistributive. If the alternative belief is held, individuals exert less effort, so luck turns out to be more important to success, the costs of redistribution are less, and thus more redistribution is pursued. Alesina and Angeletos present evidence on beliefs and policies that is consistent with this hypothesis.

The findings of this paper suggest an alternative way to interpret this evidence on the role of beliefs in luck or effort. Figures 2.2 and 2.3 show that preferences for leisure
are more heterogeneous in some countries than in others, and that the more heterogeneous countries have less redistributive policies. In heterogeneous countries, the share of individuals who earn high incomes due to effort rather than natural ability is larger than in homogeneous-preference countries. Thus, in more heterogeneous countries, policy is less redistributive and effort is indeed more important to individual success.

This alternative interpretation is not merely a semantic shift. If variation in the role of effort versus luck across different countries is due to self-fulfilling beliefs, a given country may be stuck in an equilibrium from which most citizens would like to escape. If, instead, the extent of preference heterogeneity explains the role of effort versus luck, the constrained efficient equilibrium might be reached in every country and yet be quite different across countries.

This interpretation also provides a possible explanation for why entho-linguistic fractionalization inhibits redistribution, a finding of Alesina, Glaeser, and Sacerdote (1999). While this finding has usually been attributed to racism or so-called "identity politics," preference heterogeneity offers a less troubling explanation. If preferences for leisure relative to consumption vary across ethnic groups, more diverse societies will tend to have less redistribution than homogeneous societies.

2.5 Implications of pure preference heterogeneity for capital taxes and social insurance

This paper focuses on Mirrlees’ second assumption. The first assumption of Mirrlees (1971) was that dynamic issues were to be set aside, and relaxing this assumption has
been the basis of a steadily-growing, recent literature. As with the static Mirrlees literature, the dynamic Mirrlees literature (or the New Dynamic Public Finance, as it is called by Golosov, Tsyvinski, and Werning, 2006) assumes preference homogeneity. In this section, I modify that literature by incorporating pure preference heterogeneity.

I derive results in two specific contexts for which the dynamic Mirrlees approach has proven particularly useful. First, I extend the consumption-leisure preference heterogeneity scenario from above to a dynamic disability insurance model similar to Golosov and Tsyvinski (2006). Second, I consider a different a form of pure preference heterogeneity—heterogeneous intertemporal discounting—and examine its effects on optimal capital taxation. This illustrates the potentially broad applicability of the preference heterogeneity model to optimal policy analysis as well as modifying recent conclusions about optimal intertemporal distortions.

### 2.5.1 Disability insurance and means-testing with preference heterogeneity

Golosov and Tsyvinski (2006) use a Mirrlees approach to the optimal design of disability insurance, concluding that means-testing can play a key role in the system. In particular, they find that the optimal policy can be implemented by paying benefits only to those with assets below a threshold level. Intuitively, if benefits are not means-tested, individuals have an incentive to build up assets and then fake disability so as to increase their leisure

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78 Some analyses, such as Atkeson and Lucas (1992) and Albanesi and Sleet (2006), consider preference differences of the form shown in the "labor effort preferences" or "consumption preferences" models above. These do not capture the variation in pure preferences on which this paper focuses.

79 I do not examine the implications of heterogeneous intertemporal discounting for optimal labor income taxes. This topic has been analyzed in a commodity tax setting by Saez (2002).
while funding consumption with disability benefits and their income from assets. I add heterogeneity in preferences between consumption and leisure to this framework.

The model is as follows. Individuals live for two periods and differ in their pure preferences for consumption relative to leisure, parameterized by $\theta_i$. Let $C$ and $Y$ denote first-period consumption and income and $c$ and $y$ denote second-period consumption and income. All agents discount second-period utility with the factor $\beta < 1$, and $R$ is the rate of return on savings. An individual’s preferences are constant over the two periods, and for simplicity I assume that preferences take on one of two values, $\theta_i \in \{\theta_a, \theta_b\}$ with probabilities $\pi_a$ and $\pi_b$. Individuals all have the same unit wage in the first period but are subject to uncertainty in their second-period wage. For simplicity, the second-period wage is either $w_h$ or $w_l$ with probabilities $p_h$ and $p_l$. I assume a separable utility function of the log-quadratic form used in Section 2.3 and apply the cardinalization of preferences derived there.

I solve analytically for the characteristics of second-best policy in this case, unlike in the static analysis above where I restricted attention to the first-best policy to avoid a multiple screening problem. The dynamic problem avoids multiple screening because the two dimensions of private information are realized by agents sequentially, one dimension per period.\(^80\)

Using the cardinalization of pure preferences from Section 2.3, the planner’s problem is

\(^{80}\) In reality, wage heterogeneity in the first period complicates the analysis. This analysis is designed to show the impact of preferences on the disability analysis of Golosov and Tsyvinski (2006), however, who also have homogeneous wages in the first period.
Problem 16:  Optimal disability insurance:

\[
\max_{c,y} \sum_{i=a,b} \pi_i \left( (\theta_i)^{-\frac{1}{2}} u(C_i) - (\theta_i)^{-\frac{1}{2}} v(Y_i) + \beta \sum_{m=h, l} p_m \left( (\theta_i)^{-\frac{1}{2}} u(c_{m,i}) - (\theta_i)^{-\frac{1}{2}} v\left(\frac{y_{m,i}}{w_m}\right)\right) \right),
\]

subject to feasibility

\[
\sum_{i=1}^I \pi_i \left( Y_i - C_i \right) + \sum_{m=h, l} p_m (y_{m,i} - c_{m,i}) \geq 0,
\]

incentive compatibility constraints that require truthtelling in the second period to be optimal for high-wage individuals regardless of their report in the first period:

\[
\begin{align*}
(\theta_a u(c_{h,a}) - v\left(\frac{y_{h,a}}{w_h}\right)) &\geq (\theta_a u(c_{l,a}) - v\left(\frac{y_{l,a}}{w_h}\right)), \\
(\theta_b u(c_{h,b}) - v\left(\frac{y_{h,b}}{w_h}\right)) &\geq (\theta_b u(c_{l,b}) - v\left(\frac{y_{l,b}}{w_h}\right)), \\
(\theta_a u(c_{h,b}) - v\left(\frac{y_{h,b}}{w_h}\right)) &\geq (\theta_a u(c_{l,b}) - v\left(\frac{y_{l,b}}{w_h}\right)), \\
(\theta_b u(c_{h,a}) - v\left(\frac{y_{h,a}}{w_h}\right)) &\geq (\theta_b u(c_{l,a}) - v\left(\frac{y_{l,a}}{w_h}\right)),
\end{align*}
\]

and that require that both preference types prefer their allocation to the other’s in the first period, knowing that they will tell the truth in the second period:

\[
\begin{align*}
\left[\theta_a u(C_a) - v(Y_a) + \beta \sum_m p_m (\theta_a u(c_{m,a}) - v\left(\frac{y_{m,a}}{w_m}\right))\right] \\
\geq \theta_a u(C_a) - v(Y_a) + \beta \sum_m p_m (\theta_a u(c_{m,a}) - v\left(\frac{y_{m,a}}{w_m}\right)) \\
\left[\theta_b u(C_b) - v(Y_b) + \beta \sum_m p_m (\theta_b u(c_{m,b}) - v\left(\frac{y_{m,b}}{w_m}\right))\right] \\
\geq \theta_b u(C_b) - v(Y_b) + \beta \sum_m p_m (\theta_b u(c_{m,b}) - v\left(\frac{y_{m,b}}{w_m}\right))
\end{align*}
\]

Note that I have retained the general expressions \(u(c)\) and \(v(l)\) for ease of interpretation, though the cardinalization assumes \(u(c) = \ln c\) and \(v(l) = \frac{1}{\sigma} l^\sigma\). In the Appendix, I show that this planner’s problem yields the following condition on intertemporal alloca-
2.5 Implications for capital taxes and social insurance

\[ \beta R = \frac{1}{\frac{1}{u'(C_a)} + \frac{1}{u'(C_b)}} \left( \frac{1}{u'(c_{h,a})} + \frac{1}{u'(c_{l,a})} + \frac{1}{u'(c_{h,b})} + \frac{1}{u'(c_{l,b})} \right). \]  

(2.14)

Compare condition (2.14) to the condition on intertemporal allocations that would result in a model without preference heterogeneity in the first period:

\[ \beta R = u'(C) \frac{1}{2} \left( \frac{1}{u'(c_h)} + \frac{1}{u'(c_l)} \right), \]  

(2.15)

the so-called "inverse Euler equation" derived in Rogerson (1985) and studied in-depth by Golosov, Kocherlakota, and Tsyvinski (2003). Finally, compare both (2.14) and (2.15) to the full information intertemporal Euler equation given homogeneous preferences:

\[ \beta R = u'(C) \frac{1}{2} \left( \frac{1}{u'(c_h) + u'(c_l)} \right). \]  

(2.16)

Comparing conditions (2.14) through (2.16) shows how private information on disability and heterogeneous preferences affect the planner’s optimal intertemporal allocations.

The difference between (2.15) and (2.16) is due to private information on disability, as derived and discussed in Golosov and Tsyvinski (2006). By Jensen’s inequality, the average of the inverse of the marginal utilities in the second period (on the right-hand-side of condition 2.15) is greater than the inverse of the average marginal utilities in the second period (on the right-hand-side of condition 2.16). This means that the marginal utility of consumption in the first period must be smaller in (2.15) than in (2.16). Intuitively, agents save against the possibility of being disabled, but having saved, they find claiming disability more attractive even if they are not truly disabled because consumption can be supplemented with their accumulated assets. The planner who cannot observe ability wants...
to prevent agents from saving and then pretending to be disabled, so it discourages saving in the first period, pushing up consumption $C$.

Condition (2.14) differs from both (2.15) and (2.16) due to preference heterogeneity. The formal difference is that (2.14) uses the inverse of the average of the inverse of the marginal utility of consumption in the first period, while (2.15) and (2.16) use simply the marginal utility of consumption in the first period, where all agents are the same. This offsets the impact of private information on disability, because by Jensen’s inequality,

$$
\frac{1}{2} \left( \frac{1}{u'(c_a)} + \frac{1}{u'(c_b)} \right)
$$

is less than the marginal utility of first-period consumption $u'(C)$ for the same average first-period consumption. Thus, the planner can satisfy its optimal condition with a smaller distortion to intertemporal decisions.

Why would the planner distort intertemporal decisions less due to heterogeneous preferences for consumption relative to labor? The intuition is that preference heterogeneity causes some agents to enter the second period with substantial savings not because they are tempted to fake disability but because they prefer consumption to leisure. For these agents, the risk of disability is worse than for more leisure-focused agents, because the "gain" in leisure due to disability is worth less to them. To insure against this risk, these agents accumulate more assets in the first period. Assuming that these are pure preferences as defined above, the planner would choose not to tax or subsidize agents based on their preferences. In contrast, imposing an intertemporal wedge as in the inverse Euler equation distorts these agents more than those who prefer leisure relative to consumption, because they are the ones who have accumulated more savings. Thus, the optimal policy reduces the wedge on intertemporal allocations.
An interesting extension to this analysis would be to examine whether the optimal policy can be implemented with asset-testing, as in Golosov and Tsyvinski, but with asset tests differing across preference types. The asset tests might be conditioned on first-period income, as that is a perfect indicator (in this simple model) of preferences across which redistribution is unjustified. Specifically, a person with higher first-period income might be granted a higher cutoff level of assets below which she is allowed to claim disability benefits.

### 2.5.2 Capital taxation with heterogeneity in discounting

In this section, I consider another form of heterogeneity in pure preferences and its implications for dynamic optimal Mirrleesian tax policy. Specifically, I focus on heterogeneity in intertemporal discount factors. As with consumption-leisure pure preferences, this heterogeneity is viewed by agents as not providing justification for redistribution.

This form of pure preference heterogeneity has also been mentioned in commentaries on taxation. Opponents of capital taxation often point out that it is borne disproportionately by those who value thrift or saving for their own or their children’s futures. Mankiw (2003) gives a particularly vivid example:

"Consider the story of twin brothers – Spendthrift Sam and Frugal Frank. Each starts a dot-com after college and sells the business a few years later, accumulating a $10 million nest egg. Sam then lives the high life, enjoying expensive vacations and throwing lavish parties. Frank, meanwhile, lives more modestly. He keeps his fortune invested in the economy, where it finances capital accumulation, new technologies, and economic growth. He wants to leave most of his money to his children, grandchildren, nephews, and nieces. Now ask yourself: Which millionaire should pay higher taxes?... What principle of social justice says that Frank should be
penalized for his frugality? None that I know of."

This section takes Mankiw’s argument seriously, though without any judgment on whether lavish parties are valued more by society than bequests—Frank should neither be penalized nor rewarded for his frugality. The point is that differences in when individuals want to spend their money may partially explain differences in asset accumulation and should therefore be taken into account when designing optimal policy.

The model is as follows. I consider a two period model for simplicity, where all agents are alive in both periods and no new agents are born in the second period. In each period, agents consume and supply labor effort to earn income, where income is the product of wages and effort. As with the disability model above, I exploit the two-period structure of the model to incorporate two dimensions of heterogeneity in a way that avoids the multiple screening problem. In the first period, all agents have the same unit wage but may differ in discounting, i.e., in the relative weight they place on first-period utility relative to second-period utility. In the second period, this discounting has been revealed by first-period decisions but agents may now differ in their wage, which can take one of two values $w_h$ or $w_l$ and which have an average value of one. I use the same log-quadratic utility function as above to solve a tractable example.

The first step in solving for optimal allocations is to find the social welfare function that satisfies an analogue to Proposition 6 with this new form of preference heterogeneity. An individual with discount factor $\beta_i$ has preferences that can be represented by the utility
function:

\[ u_{m,i} = \left( \ln (C_i) - \frac{1}{2} (Y_i)^2 \right) + \beta_i \sum_{m=1}^{M} p_m \left( \ln (c_{m,i}) - \frac{1}{2} \left( \frac{y_{m,i}}{w_m} \right)^2 \right) \]  

(2.17)

where \( C \) and \( Y \) are consumption and income in the first period and \( c \) and \( y \) are consumption and income in the second period. The discount factor \( \beta_i \) can take the values \( \beta_a \) or \( \beta_b \), where \( \beta_a > \beta_b \).

I use a procedure similar to that for consumption-leisure preferences in Section 2.4 to determine the form of the social welfare function that cardinalizes (2.17) so that \( \beta_i \) is treated as a pure preference. The laissez-faire choices are a result of individuals maximizing (2.17) subject to feasibility:

\[ (Y_i - C_i) R + (y_{h,i} - c_{h,i}) \geq 0 \]

and

\[ (Y_i - C_i) R + (y_{l,i} - c_{l,i}) \geq 0 \]

where the two feasibility constraints reflect the absence of a market for transfers across wage-states in the second period. This problem does not have a clean analytical simplification in its current form. I further simplify by finding the deterministic laissez faire allocation when agents are certain to receive the average second-period wage, \( E[w_m] \). This simplifies the laissez faire problem to:

\[
\max_{c,y} \left[ \left( \ln (C_i) - \frac{1}{2} (Y_i)^2 \right) + \beta_i \left( \ln (c_i) - \frac{1}{2} \left( \frac{y_i}{E[w_m]} \right)^2 \right) \right]
\]

s.t.

\[ (Y_i - C_i) R + [y_i - c_i] \geq 0. \]
Note that this simplification is adopted only for determining the appropriate cardinalization of preferences, not for solving for the optimal allocations (where uncertainty in the wage is reintroduced).

The Appendix derives the cardinalization which equalizes the social marginal value of allocating resources to different individuals. If \( E[w_m] = 1 \), so that average wages are constant across the two periods, this cardinalization factor is

\[
\psi_i = \sqrt{\frac{1 + \beta_i}{1 + \beta_i R^2}},
\]

and the social welfare function takes the form:

\[
W = \sum_i \pi_i \left( \frac{1 + \beta_i R^2}{1 + \beta_i} \right)^{1/2} \left( \frac{1}{2} (\beta_i)^{-1/2} \left( \ln C_i - \frac{1}{2} (Y_i)^2 \right) + \beta_i \frac{1}{2} \sum_{m=1}^M p_m \left( \ln c_{m,i} - \frac{1}{2} \left( \frac{y_{m,i}}{w_m} \right)^2 \right) \right).
\]

Now, I derive optimal allocations taking into account heterogeneous discounting. I use the general notation \( u(c) \) and \( v(l) \) to denote utility from consumption and disutility of labor so as to make the theoretical results more easily interpreted, though the cardinalization of preferences used the log-quadratic form assumed above. Then, the planner’s problem is:

**Problem 17:** *Optimal intertemporal distortion*

\[
\max_{c,y} \sum_i \pi_i \left( \frac{1 + \beta_i R^2}{1 + \beta_i} \right)^{1/2} \left( \frac{1}{2} (\beta_i)^{-1/2} \left( \ln C_i - \frac{1}{2} (Y_i)^2 \right) + \beta_i \frac{1}{2} \sum_{m=1}^M p_m \left( \ln c_{m,i} - \frac{1}{2} \left( \frac{y_{m,i}}{w_m} \right)^2 \right) \right),
\]

subject to feasibility:

\[
\sum_i \pi_i \left( (Y_i - C_i) R + \sum_{m=1}^M p_m (y_{m,i} - c_{m,i}) \right) \geq 0,
\]
incentive compatibility constraints that require truthtelling in the second period to be optimal for each high-wage individual regardless of its report in the first period:

\[
\begin{align*}
  u(c_{h,a}) - v\left(\frac{y_{h,a}}{w_h}\right) & \geq u(c_{l,a}) - v\left(\frac{y_{l,a}}{w_h}\right), \\
  u(c_{h,b}) - v\left(\frac{y_{h,b}}{w_h}\right) & \geq u(c_{l,b}) - v\left(\frac{y_{l,b}}{w_h}\right),
\end{align*}
\]

and that require individuals of either preference type to prefer their allocation in the first period to that of the other type, knowing that they will tell the truth about their wage in the second period:

\[
\begin{align*}
  u(C_a) - v(Y_a) + \beta_a \sum_{m=1}^M p_m \left( u(c_{m,a}) - v\left(\frac{y_{m,a}}{w_m}\right) \right) & \geq \\
  u(C_b) - v(Y_b) + \beta_b \sum_{m=1}^M p_m \left( u(c_{m,b}) - v\left(\frac{y_{m,b}}{w_m}\right) \right) \\
  u(C_b) - v(Y_b) + \beta_b \sum_{m=1}^M p_m \left( u(c_{m,b}) - v\left(\frac{y_{m,b}}{w_m}\right) \right) & \geq \\
  u(C_a) - v(Y_a) + \beta_a \sum_{m=1}^M p_m \left( u(c_{m,a}) - v\left(\frac{y_{m,a}}{w_m}\right) \right)
\end{align*}
\]

In the Appendix, I show that this planner’s problem yields the following condition on intertemporal allocations:

\[
\hat{\beta} R = \frac{1}{2} \left( \frac{1}{w'(C_a)} + \frac{1}{w'(C_b)} \right) \left( \frac{1}{w'(c_{h,a})} + \frac{1}{w'(c_{l,a})} + \frac{1}{w'(c_{h,b})} + \frac{1}{w'(c_{l,b})} \right),
\]

where

\[
\hat{\beta} = \frac{(1+\beta_a R^2)^{\frac{1}{2}} (\beta_a)^{\frac{1}{2}} + (1+\beta_b R^2)^{\frac{1}{2}} (\beta_b)^{\frac{1}{2}}}{(1+\beta_a R^2)^{\frac{1}{2}} (\beta_a)^{-\frac{1}{2}} + (1+\beta_b R^2)^{\frac{1}{2}} (\beta_b)^{-\frac{1}{2}}}
\]

is used for simplicity of presentation. Condition (2.18) closely resembles condition (2.14) from the disability section. As in that analysis, preference heterogeneity across individuals leads to a smaller intertemporal wedge than when preferences are homogeneous. Intuitively, agents with high saving are now potentially agents with high patience, a characteristic that the planner is not interested in taxing.
The formal similarity between (2.18) and (2.14) indicates that pure preference heterogeneity generally provides a countervailing force to wage heterogeneity in determining optimal intertemporal distortions.

2.6 Conclusion

The argument that differences in preferences for consumption relative to leisure drive some of the heterogeneity of income across individuals has a long history in economic and philosophical debates over redistribution, but it is absent from the leading modern economic model of optimal taxation. This paper introduces a particular form of this preference heterogeneity into that model: pure preferences across which redistribution from the laissez-faire allocation is not justified.

Preference heterogeneity of this sort has clearly identifiable implications for both the normative and positive analysis of optimal taxation. For static optimal income taxation, recognizing preference heterogeneity makes the optimal extent of income redistribution less than would be recommended by a conventional model that attributes all heterogeneity to productive ability, and an illustrative simulation using U.S. data on prime-age single men suggests that this effect may be substantial. In a dynamic context, optimal disability insurance and capital taxation that take into account preference heterogeneity will distort intertemporal distortions less than what recent research assuming homogeneous preferences recommends. From a positive perspective, an examination of cross-country data on preferences and policy shows that countries with more heterogeneous preferences have less redistributive policies, a pattern consistent with the predictions of optimal tax analysis that
takes preference heterogeneity into account. This provides an alternative way to interpret the sources of policy differences across countries, and it suggests that a proper understanding of the role of preference heterogeneity improves our ability not only to design taxation but to understand existing tax policies.

A final lesson of this paper is that rigorous attempts to estimate preference heterogeneity both on the intratemporal and intertemporal margins, though challenging, would yield large payoffs in both the normative and positive analysis of taxation.
Chapter 3
Optimal Policy with Heterogeneous Parental Altruism

3.1 Introduction

The appropriate allocation of resources across generations is of long-standing interest to economists, but a key dimension of it has gone unstudied thus far. Most research on intergenerational allocations has focused on the extent to which one generation discounts, and ought to discount, future generations’ well-being. This paper addresses a different question: how should policy respond to heterogeneity in the extent to which the members of one generation discount the well-being of specific members of future generations over whom they have substantial control?

In particular, this paper focuses on heterogeneity in parental altruism toward children: that is, in the willingness of parents to make choices that have negative direct consequences for themselves but that benefit their children. Differences in parental altruism represent a large and undiversifiable risk for every child. Parents control much of the environment of early childhood and adolescence, when children acquire the personality traits and education that are critical for economic success (e.g., Mischel et al. 1989; Heckman

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81 Thanks to Manuel Amador, Robert Barro, David Cutler, Caroline Hoxby, David Laibson, Greg Mankiw, and Aleh Tsyvinski for helpful discussions and comments on earlier versions of this paper.

82 Some research (see Harris, 1998) questions the role of parents relative to peers and genes in determining children’s traits and abilities. While recognizing this possibility, I assume some truth to the conventional wisdom that parents’ decisions matter for their children’s future (perhaps through their influence on the choice of peer group).
In some cases, parents’ preferences stand in for children’s preferences, such as in education systems with school choice. Most directly, parents often choose whether to assist in financing large investments in human capital, especially higher education. To the extent that parents’ choices such as these affect their children’s development, children whose parents are relatively less willing to sacrifice for their benefit suffer a substantial negative endowment shock. John Rawls (1974) stated the problem as follows: "...the principle of fair opportunity can be only imperfectly carried out, at least as long as the institution of the family exists...Even the willingness to make an effort, to try, and so to be deserving in the ordinary sense is itself dependent upon happy family and social circumstances." Or, as James Heckman has recently said: "The family is the major source of human inequality in American society." 

The main normative conclusion of this paper is that unconstrained policy should treat all children equally, no matter their parents’ degree of altruism. Intuitively, the idea is that children do not choose their parents, just as adults do not choose, for example, their skin color, and therefore society should no more want to treat children differently because of their parents than it wants to countenance racial discrimination.

I derive this conclusion by generalizing the normative framework behind modern optimal policy analysis to include heterogeneous parental altruism. That framework was pioneered by Vickrey (1945) and Harsanyi (1953, 1955) and developed more fully by Rawls (1971), and it lies at the heart of Mirrlees’ (1971) foundational work on optimal taxation. In it, individuals engage in a "thought experiment" in which they imagine themselves in

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83 Though not stressed in this paper, the approach taken below also applies to optimal policy toward parental heterogeneity along other dimensions, such as productive ability. The Appendix provides an illustration.
an original position from which they specify the social welfare function that guides policy. In the original position, each individual has no knowledge of which position in society he or she will take outside the original position. In Rawls’ terminology, knowledge in the original position is limited by a "veil of ignorance."

Generalizing this normative framework to include heterogeneous parental altruism is the main contribution of this paper, and it requires two steps.

First, to reflect the existence of families in society, the original position thought experiment is undertaken by individuals in families rather than by isolated individuals. In particular, I consider two-person families consisting of one parent and one child. When each parent imagines herself in the original position, she retains her concern for her child. This is consistent with Rawls’ interpretation of the original position, and it applies the standard economic model of parental altruism, such as in Barro (1974), behind the veil of ignorance.

Second, in the original position thought experiment, each individual imagines having no knowledge of which position in society it or any other individual, including its child, takes outside the original position. In other words, positions in society are assigned individually, rather than to families. This is consistent with Harsanyi’s description of his thought experiment, in that the each individual’s position in society is unknown. It also echoes critiques within the political philosophy literature of Rawls’ original statement of the framework, in which the family was treated as a unit throughout the thought experiment.
These generalizations of the Vickrey-Harsanyi-Rawls normative framework yields a surprising implication: a parent in the original position must confront the possibility that she would not be the one to raise her child outside the original position. As a consequence, parents behind the veil of ignorance specify a social welfare function (SWF) that treats all children equally; that is, they choose to fully offset heterogeneity in their altruism. I call this the "Equal Weights SWF."

The Equal Weights SWF contradicts the conventional approach in optimal policy analysis, as in Atkeson and Lucas (1992), where parents’ preferences determine society’s preferences toward intrafamilial resource allocation. In the context of heterogeneous parental altruism, this conventional approach yields a social welfare function in which children of different parents are weighed differently. I call this the "Unequal Weights SWF."

This paper’s conclusion that children ought to be treated equally, without regard to their parent’s altruism, sounds natural to some and unnatural to others. For those who think the idea natural, this paper provides a formal justification of that unconventional position using the standard normative framework of optimal policy design. For those who find it unnatural, this paper provides a clear derivation against which objections can be raised.

One commonly-raised objection argues that the proper SWF will put weight on both parents’ and childrens’ preferences. This, the argument goes, would balance the former’s heterogeneous altruism with the latter’s desire for equal treatment but not overrule the parents’ preferences. This is the approach taken by Phelan (2006) and Farhi and Werning (2005), and it reduces the disparity in social weights across children in the social welfare function but does not eliminate them. I call this the "Diluted Weights SWF."
As will be shown below, the Diluted Weights SWF can be decomposed into two parts: (1) the conventional Unequal Weights SWF; and (2) an additional, uniform social weight on all children’s preferences. To accept the first component, one must endorse the conventional normative linkage between parents and children, rejecting at least one of the two generalizations discussed above that yield the Equal Weights SWF. In contrast, the second component is complementary, not contradictory, to this paper’s approach. Placing a direct weight on children’s welfare can be combined with the Equal Weights SWF to yield a social welfare function that modifies the Diluted Weights SWF policy in much the same way as the Equal Weights SWF policy modifies the Unequal Weights SWF policy. I call this the "Combination SWF."

A second common objection to this paper’s approach is that parents, in reality, do not face uncertainty as to whether they raise the child about whom they care, so they have no reason to want to offset their heterogeneity. This objection is usually the result of confusion about the original position thought experiment. The purpose of the original position thought experiment is to separate individuals from their realized social positions. In the context of families, this paper argues that the social position of children includes the type of parents by which they are raised. Thus, the assignment of children to parents cannot be known in the original position. Of course, real-world policy functions outside the original position, and it must take into account that parents care for the children they

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84 It can also result from rejecting the second generalization. Again, if the second generalization is rejected, it must be that children are treated not as individuals by the thought experiment but as appendages to their parents.
raise, not all children equally, but that is captured in restrictions on policy rather than in the objective that policy pursues.

The second contribution of this paper is to demonstrate the implications of the Equal Weights SWF in a modern private information optimal policy model. I find that the Equal Weights SWF substantially alters the optimal intergenerational allocation of resources relative to the standard approach, imposing weaker linkages between a child’s welfare and her parents’ preferences. More formally, it introduces new wedges into the intertemporal Euler equations that would otherwise hold within families, narrowing the gap between the intertemporal tradeoffs made by heterogeneous parents. Such wedges can be interpreted as having implications for optimal estate taxation, but the wedges apply more generally to any choice parents make between their own current consumption and investment in their children’s welfare. Similar results apply for the Combination SWF, which mitigates the inequality yielded by maximization of the Diluted Weights SWF.

The paper is structured as follows. Section 3.2 reviews the normative framework at the heart of the analysis and formalizes it in an economy of individuals. Section 3.3 generalizes this framework to an economy with families and derives the Equal Weights social welfare function. For comparison, this section also describes the social welfare functions used in previous analyses. Section 3.4 derives the implications of this paper’s model for optimal allocations in a simple two-period model and compares them to what the standard approach would yield. Section 3.5 extends the model to map out the effects of a one-time shock on the infinite path of allocations, and Section 3.6 concludes. Appendix C, which gives further details related to this paper, can be found at the end of this dissertation.
3.2 The Original Position

Vickrey (1945) and Harsanyi (1953, 1955) pioneered the rigorous approach to specifying a social welfare function that could guide social policy despite interpersonal incomparability of utility. The same approach is at the heart of the political philosophy of Rawls (1971), who introduced the vivid images of the "original position" and a "veil of ignorance" to describe it. This framework’s key step is to separate what I will call "individuals" from "positions." Individuals design society from an imagined original position behind a veil of ignorance that prevents them from knowing the positions that they take in that society.

Rawls describes this as follows:

"Among the essential features of this situation is that no one knows his place in society, his class position or social status, nor does any one know his fortune in the distribution of natural assets and abilities, his intelligence, strength, and the like. I shall even assume that the parties do not know their conceptions of the good or their own psychological propensities. The principles of justice are chosen behind a veil of ignorance." (p. 12)

The power of the Vickrey-Harsanyi-Rawls approach is that it transforms the problem of comparing utilities across individuals into one of a single individual facing a choice involving uncertainty. Harsanyi (1953) writes that choice made behind the veil of ignorance may be "interpreted as an expression of what sort of society one would prefer if one had an equal chance of being ‘put in the place of’ any particular member of the society, so that cardinal utility ‘maximized’ in value judgments concerning social welfare and the cardinal utility maximized in choices involving risk may be regarded as fundamentally based upon

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85 Rawls cites Harsanyi (1953) on page 137, though not Vickrey (1945). As Arrow (1973) points out, Vickrey’s contribution was long overlooked.

the same principle." (p. 435) This equivalence implies that standard methods for analyzing individual choice under uncertainty—specifically the von Neumann-Morgenstern expected utility function—can be applied to questions of social welfare.

A related consequence of applying the veil of ignorance is that it makes all individuals identical ex ante, yielding unanimity in decision-making. As Rawls writes: "Therefore, we can view the choice in the original position from the standpoint of one person selected at random. If anyone after due reflection prefers a conception of justice to another, then they all do, and a unanimous agreement can be reached." (p. 139). Unanimity allows us to avoid consideration of the political mechanism by which competing interests are reconciled when specifying a social welfare function.

Ever since this approach was first proposed, philosophers and economists have debated the form of the SWF that would be chosen ex ante. Rawls argued that individuals in this position would maximize the minimum welfare of an individual in society: the maximin criterion. Economists have generally disagreed, arguing that individuals behind the veil of ignorance would maximize their ex ante expected welfare and that the maximin criterion is a special case in which risk aversion is infinite. Arrow (1973) gives a particularly clear statement of the economists’ perspective, which I will maintain. Thus, I assume that individuals behind the veil of ignorance direct the planner to maximize a SWF equal to their ex ante expected welfare.

This paper takes as a starting point that the SWF equals ex ante expected welfare and focuses on specifying that SWF in an economy with families. First, however, I formalize this problem in an economy of individuals to provide a basis for the later analysis.
3.2.1 The original position thought experiment with individuals

Individuals are indexed in the economy by $f \in \{1, 2, \ldots, F\}$, and each individual has a privately-observed type $i \in \{1, 2, \ldots, I\}$, where $I \leq F$ and the population proportion of type $i$ is $\pi(i)$. Type might represent skill, a dimension of preferences, or any other personal characteristic. Individuals report a type to a social planner, who maximizes a social welfare function by allocating an exogenous flow of resources among individuals based on their reported types. Individuals derive utility from consuming these allocations.

Formally, the utility function for individual $f$ of type $i$ is written:

$$U(f, i) = \ln(c(f, i))$$

where $c(f, i)$ is the consumption allocated to individual $f$ who reports type $i$. Logarithmic utility of consumption will be used throughout this paper, as it simplifies many of the analytical results. The analysis can readily be adapted to more general utility functions, as outlined in the Appendix.

The allocations of consumption satisfy the following feasibility constraint:

$$\sum_f \sum_i \pi(i)c(f, i) = Y,$$

where $Y$ is the exogenous resource flow received by the social planner.

Now, I state two axioms that describe the original position and resulting social welfare function. These axioms also serve as the starting point for the modifications of the next section, where families are introduced into the thought experiment.

**Axiom 1:** The social welfare function is equal to the sum across $f$ of expected utilities.
for each individual $f \in \{1, 2, \ldots, F\}$ while in the original position.

The specification of individual $f$ in Axiom 1 is without loss of generality, as all individual are identical in the original position. In particular, the next axiom clarifies the individual’s status in the original position.

**Axiom 2:** In the "original position," each individual is ignorant of its type. Formally, let $\rho(x|y)$ denote the conditional probability of outcome $x$, given information $y$, as perceived in the original position. Then, $\rho(i|f)$ is the conditional probability of individual $f$ becoming type $i$, as perceived in the original position. In the original position, the following condition holds for all $f$ and $i$:

$$\rho(i|f) = \pi(i).$$

These axioms imply that the social welfare function is:

$$W = \sum_f \sum_i \pi(i) \ln (c(f,i))$$  \hspace{1cm} (3.1)

It is important to understand the role of the individual index $f \in \{1, 2, \ldots, F\}$ in this derivation. In particular, each individual remembers its $f$ when imagining itself in the original position, even though it forgets its type $i$. This formally captures the implicit notion of the "self" in the Vickrey-Harsanyi-Rawls framework, in that individuals retain a concern for their own well-being when they undertake the original position thought experiment.\(^87\)

\(^87\) This has been an area of considerable controversy in the political philosophy literature, with critics arguing that the original position is nonsensical because once all identifying characteristics of individuals are obscured, no individual remains who can be expected to make choices. While the existence of the self is an
In the next section, an extension of this concept of the self to families will play a similar role. I now turn to that extension.

### 3.3 The Original Position with families

In this section, I generalize the Vickrey-Harsanyi-Rawls normative framework to include heterogeneous parental altruism and derive the social welfare function specified from the original position.

It is important to emphasize that my approach follows Rawls in imagining the original position as a thought experiment undertaken by real people. To defend this approach, Rawls wrote:

"I have emphasized that this original position is purely hypothetical. It is natural to ask why, if this agreement is never actually entered into, we should take any interest in these principles, moral or otherwise. The answer is that the conditions embodied in the description of the original position are the ones that we do in fact accept. Or if we do not, then perhaps we can be persuaded to do so by philosophical reflection."

This is a particularly important argument to keep in mind when applying the framework to a new question, as in this paper with intrafamilial allocations. The original position thought experiment asks real-world persons to imagine themselves in a particular situation that elicits their fundamental moral judgments. Therefore, I begin by describing how the economy works in reality. Then, I consider the thought experiment in which all members of society imagine themselves in the original position, behind the veil of ignorance. From
this original position, they specify the social welfare function that is to be maximized by the social planner.

### 3.3.1 The economy

As in the previous subsection, all individuals in the economy derive utility from consumption. The economy has no production and funds consumption out of a fixed annual endowment flow.

Each individual is a member of one of two generations indexed by $t \in \{1, 2\}$. Members of generation $t = 1$ are called parents and members of $t = 2$ are called children. A family consists of one parent and one child, and families are indexed with $f = \{1, 2, \ldots, F\}$. The parent in each family is altruistic toward the child in that family, but children are not altruistic toward parents.

Note that I use the same notation to index families as I used to index individuals in the previous section. The use of $f$ in both cases highlights this paper’s formal model of parental altruism. In words, altruism extends the notion of the "self" to include another: in this case, the parent’s "self" about whom it cares includes the parent’s child.

Each individual is also a member of a household, which consists of one parent and one child. The household is the main economic unit of society. Within a household, the parent receives resources for consumption from the social planner and divides those resources between itself and the child.
In the real-world economy, the family is identical to the household, but this need not be the case in principle. In fact, the hypothetical disconnect between households and families is the key step in our analysis below.

Parents and children both have unobservable types. Each parent has an unobservable type that takes one of two values $\beta_i$ where $i \in \{G, S\}$ and $1 > \beta^G > \beta^S > 0$. The population proportion of each type is $\pi(i)$. Parental types represent generous ($G$) and selfish ($S$), where generous parents put relatively more emphasis on the utility of the children in their family (they are more altruistic) than do selfish parents. Each child has an unobservable type equal to the parent’s type in its household, i.e., the type of the parent who controls its consumption allocation.

Formally, in the real economy (outside the thought experiment) welfare for the parent in family $f$ with type $i$ is:

$$W(f, i) = \frac{1}{1 + \beta^i} \ln (c_1(f, i)) + \frac{\beta^i}{1 + \beta^i} \ln (c_2(f, i)).$$

(3.2)

where $c_1(f, i)$ denotes this parent’s consumption and $c_2(f, i)$ denotes the consumption of the child in family $f$ and in a household with a parent of type $i$. The parent’s relative weight on the child’s utility from consumption depends positively on $\beta^i$, so that higher $\beta^i$ indicates a more altruistic parent.

---

88 The concept of family is not intended to be biological. It merely represents the relationship between parents and children prior to the introduction of the original position thought experiment. In fact, the existence of adoptive households in the real world argues against one common critique of this paper’s approach: that a genetic connection ensures that families and households are identical in the original position thought experiment.

89 The use of "generous" and "selfish" implies that type $G$ is morally preferred to type $S$. A more neutral perspective but also more cumbersome description might stress that type $G$ parents believe in financially supporting their children, while type $S$ believe in fostering financial independence in their children.
It is important to note that in expression (3.2) parental altruism applies within the *family* while each child’s type depends on the altruism of the parent in its *household*. When families and households are identical, this distinction is irrelevant and the \((f, i)\) notation is redundant. Next, however, I apply the original position thought experiment to this economy, and this distinction becomes important.

### 3.3.2 The original position thought experiment with families

Now, I apply the original position thought experiment to the economy with heterogeneous parental altruism specified above. In particular, I state two axioms that generalize those from Section 3.2, incorporating families into the framework. Notationally, let \(f\) indicate a family, \(j_1\) indicate a parent’s type, and \(j_2\) indicate a child’s type (i.e., the preference type of the parent in the household by which the child is raised).

The first generalization modifies Axiom 1 by having the parents imagine their *families* in the original position.

**Axiom 3:** *The social welfare function is equal to the sum across \(f\) of expected utilities for the parents of each family \(f \in \{1, 2, \ldots, F\}\) while in the original position.*

Having parents retain their concern for their families in the original position matches Rawls’ (1974) description of the thought experiment. Importantly, it interprets parental altruism as an extension of self-interest to include another individual. Formally, this is captured by using \(f\), which represented the individual in Axiom 1, to represent the family in Axiom 3.
Consider a more concrete example. In the model with only individuals, an individual $f$ may have been a tall person before imagining herself in the original position, but her concern while in the original position is not for whoever ends up as a tall person but for herself, individual $f$. Axiom 3 applies a similar principle to family members linked by parental altruism. When a parent in family $f$ imagines her family in the original position, she retains her concern for her child’s well-being, not transferring her concern to whichever child may be assigned to her household outside the original position.\footnote{Divorce provides an additional illustration of this idea. Imagine an economy in which divorces were a random, exogenous occurrence, and sole custody was the rule in all divorces. A parent who differs in altruism from her partner would like to insure her child against the consequences of being assigned to that partner in case of divorce.}

The second generalization modifies Axiom 2 by applying the original position’s information restriction to all information about types, whether parents’ or children’s. The same notation is used as in Axiom 2.

**Axiom 4:** *In the "original position," each parent is ignorant of its type and its child’s type. Formally, in the original position, the following conditions hold for all $f$, $j_1$, and $j_2$:

\[
\rho(j_1|f) = \pi(j_1),
\]

\[
\rho(j_2|f) = \pi(j_2),
\]

\[
\rho(j_1|f,j_2) = \pi(j_1),
\]

\[
\rho(j_2|f,j_1) = \pi(j_2).
\]
In other words, each individual has no knowledge of which position in society she or any other individual, including her child, takes outside the original position. The realization of types for the parent and child within a family are independent.

Axiom 4 ensures that the original position treats children as individuals, not as appendages to their parents. Aside from the normative appeal of treating all persons as independent individuals, regardless of their generation, Axiom 4 also seems consistent with Harsanyi’s description of the normative framework he helped develop. Though he did not formalize a model with parents and children, Harsanyi (1953) wrote: "Now, a value judgment on the distribution of income would show the required impersonality to the highest degree if the person who made this judgment had to choose a particular income distribution in complete ignorance of what his own relative position (and the position of those near to his heart) would be within the system chosen." Harsanyi’s inclusion of the expression "the position of those near to his heart" indicates that he thought of positions as allocated to individuals, not family units, just as in Axiom 4.

These two generalizations of the original position thought experiment imply the social welfare function:

$$W = \sum_{f} E_{j_1} E_{j_2} \left[ \frac{1}{1 + \beta^j_1} \ln \left( c_1 \left( f, j_1 \right) \right) + \frac{\beta^j_1}{1 + \beta^j_1} \ln \left( c_2 \left( f, j_2 \right) \right) \right]. \tag{3.3}$$

where $E_x$ is the mathematical expectation taken over the values of the variable $x$.

In words, expression (3.3) formalizes that the expected welfare of the parent from family $f$ in the original position depends on the consumption of the child from her family, $c_2 \left( f, j_2 \right)$, not the consumption of the child assigned to her household, which would be
for some $g$ not necessarily equal to $f$. When parents differ in ways that affect their children, this feature of the social welfare function can play an important role.\footnote{As shown in the Appendix, this paper’s approach can be applied to a variety of kinds of parental heterogeneity, such as in productive ability.}

### 3.3.3 The Equal Weights Social Welfare Function

Next, I work with (3.3) to derive a simpler expression for ex ante expected welfare of the representative parent in the original position. Observe that the expectation over $E_{j_2}$ applies only to the second half of the expression in brackets, so we can write:

$$ W = \sum_f E_{j_1} \left[ \frac{1}{1 + \beta^{j_1}} \ln (c_1 (f, j_1)) + \frac{\beta^{j_1}}{1 + \beta^{j_1}} E_{j_2} \left[ \ln (c_2 (f, j_2)) \right] \right]. \tag{3.4} $$

Then, we can use the notation described above to rewrite (3.4) and obtain:

$$ W = \sum_f \sum_{j_1 = S, G} \pi (j_1) \left( \frac{1}{1 + \beta^{j_1}} \ln (c_1 (f, j_1)) + \frac{\beta^{j_1}}{1 + \beta^{j_1}} \sum_{j_2 = S, G} \pi (j_2) \ln (c_2 (f, j_2)) \right), \tag{3.5} $$

which is the probability-weighted average of the parent’s ex post welfare. Define the average welfare weight on children in society as:

$$ \frac{\delta}{1 + \delta} = \sum_{j_1 = S, G} \pi (j_1) \frac{\beta^{j_1}}{1 + \beta^{j_1}}. $$

Substitution into expression (3.5) yields the **Equal Weights Social Welfare Function**:

$$ W = \sum_f E_{j_1} \left[ \frac{1}{1 + \beta^{j_1}} \ln (c_1 (f, j_1)) + \frac{\delta}{1 + \delta} \ln (c_2 (f, j_1)) \right], \tag{3.6} $$

which applies the same welfare weight to each child.

Parents in the original position direct the social planner to use the Equal Weights SWF (3.6) as society’s objective function. Before considering the impact of this SWF...
3.3 The Original Position with families

on the allocation of resources, I consider two prominent alternative SWFs drawn from the optimal policy literature.

3.3.4 Alternative approaches

The Equal Weights SWF is a fundamental departure from standard models of optimal intergenerational allocations. Here, I consider two of these approaches: the influential study of Atkeson and Lucas (1992), and modern variations on that study by Phelan (2006) and Farhi and Werning (2005). All of these alternatives yield social welfare weights that differ across children. I also show how the Equal Weights SWF can be modified to include the most important component of the Phelan (2006) and Farhi and Werning (2005) approach, yielding a "Combination SWF" that applies a uniform social welfare weight to all children that is larger than that used in the Equal Weights SWF.


Though Atkeson and Lucas (1992) focused on a different form of preference heterogeneity than this paper, their approach to optimal policy with intergenerational altruism is a natural benchmark for our analysis. The key component of their approach for our purposes is its implicit assumption that parents in the original position are certain of their child’s position in society: i.e., the household to which the child is assigned. In particular, each parent is certain that her child will be assigned to her household outside the original position.

This alternative model was, in fact, implied by Rawls’ (1971) original characterization of the framework, but that characterization came under heavy criticism in the political
philosophy literature and was later modified by Rawls. Nevertheless, Nussbaum (2001) argues that even Rawls’ modifications did not fully address the issue of intrafamilial justice precisely because he continued to grant parents unlimited authority over the treatment of their children. She writes: "He [Rawls] now insists that the family is not a private realm immune from justice... Yet he still gives parents a very large measure of unqualified control over the upbringing of their children..."

Thus, the Atkeson-Lucas departs from this paper’s second generalization of the original position, Axiom 4, by assuming knowledge that this paper argues ought to be obscured in the original position. Using the same notation as above, we can formalize the Atkeson-Lucas approach by assuming the following alternative to Axiom 4:

**Axiom 5: (Atkeson-Lucas)** In the "Atkeson-Lucas original position," each parent is ignorant of its type but knows that its child’s type is identical to its type. Formally, in the Atkeson-Lucas original position the following conditions hold:

\[
\begin{align*}
\rho(j_1|f) &= \pi(j_1) \\
\rho(j_2|f) &= \pi(j_2) \\
\rho(j_1|f, j_2) &= \begin{cases} 1 & \text{if } j_1 = j_2 \\ 0 & \text{if } j_1 \neq j_2 \end{cases} \\
\rho(j_2|f, j_1) &= \begin{cases} 1 & \text{if } j_1 = j_2 \\ 0 & \text{if } j_1 \neq j_2 \end{cases}
\end{align*}
\]

for all \(f, j_1\), and \(j_2\).

---

92 Rawls originally described the parties behind the veil of ignorance as "heads of families" who were "representing continuing lines of claims," granting these heads of households jurisdiction over the internal workings of the family. This raised concerns first about justice toward women, such as in Okin (1989): "since those in the original position are the heads or representatives of families, they are not in a position to determine questions of justice within families." (pp. 94-5). Munoz-Darde (1998) offers a related critique.
This axiom implies the following **Unequal Weights Social Welfare Function**:

\[
W = \sum_f E_{j_1} \left[ \frac{1}{1 + \beta_{j_1}} \ln \left( c_1 (f, j_1) \right) + \frac{\beta_{j_1}}{1 + \beta_{j_1}} \ln \left( c_2 (f, j_1) \right) \right],
\]  \hspace{1cm} (3.7)

which applies different weights to different ex post children. In particular, a parent’s preferences determine the social welfare weight on each child, in contrast to the Equal Weights SWF.


Recent work by Phelan (2006) has introduced a modification of the Atkeson-Lucas approach that reduces the linkage between parent’s characteristics and their children’s welfare. Phelan’s modification is to assign a direct social welfare weight to all generations in addition to any altruistic weights placed on them by preceding generations.\(^{93}\) Farhi and Werning (2005) generalize Phelan’s modification to include different weights on different generations.

The approach of Phelan (2006) and Farhi and Werning (2005) has two components. First, it retains the Atkeson-Lucas assumption that each parent knows its child will be assigned to its household outside the original position. As a consequence, it does not directly offset heterogeneity in parental altruism, contradicting this paper’s approach. Second, it applies an additional uniform social welfare weight on all children, *diluting* the hetero-

---

\(^{93}\) One way to formalize this modification is to assume that when individuals imagine themselves in the original position, they are uncertain as to which generation they belong. There is some support for generational uncertainty in the original writings of Rawls (1971), see page 137: "[the parties]...must choose principles the consequences of which they are prepared to live with whatever generation they turn out to belong to."
geneity of altruism across families. This innovative component is complementary to this paper’s argument and can be combined with the Equal Weights SWF, as shown below.

To formalize the Phelan (2006) and Farhi and Werning (2005) approach, I follow the latter’s notation and suppose that the ex ante individuals direct the social planner to put direct weight $\alpha^{t-1}$ on generation $t \in \{1, 2\}$, where $\alpha \in [0, 1]$. Note that $\alpha = 1$ is the case considered by Phelan and $\alpha = 0$ is the case implicitly considered by Atkeson and Lucas (1992). Applying this modification to the Atkeson-Lucas approach yields the **Diluted Weights Social Welfare Function:**

$$W = \sum_f E_{j_1} \left[ \frac{\pi^{j_1}}{1 + \alpha} \left( \frac{1}{1 + \beta^{j_1}} \ln(c_1(f, j_1)) + \left( \frac{\beta^{j_1}}{1 + \beta^{j_1}} + \alpha \right) \ln(c_2(f, j_1)) \right) \right]$$

(3.8)

The ratio $\frac{1}{1+\alpha}$ normalizes the sum of social welfare weights to one for each family, as in the other social welfare functions discussed above. Note that only as $\alpha \to \infty$, an implausible case, do the social welfare weights on children raised in different households equalize.

The result (3.8) gives a social welfare function that is weighted toward the future relative to the preferences of currently-alive adults. Whether such weighting is appropriate has been extensively debated in the literature on intergenerational justice. Regardless of whether one adopts it, however, it is important to realize that this approach fails to yield equal social welfare weights on children across families.\(^{94}\)

---

\(^{94}\) Farhi and Werning have recently developed an alternative motivation for their results that relies on aversion to inequality among future generations rather than on exogenous weights on future generations’ welfare. See Farhi and Werning (2006). The advantage of that modification is that it directly models discomfort with inequality created by parent-child linkages. Its disadvantage is that the resulting mitigation of intergenerational transmission of welfare may be quite limited. This widens the gap between their approach and this paper’s.
3.3.5 Combination of Equal and Diluted Weights

The key innovation of the Phelan (2006) and Farhi and Werning (2005) approach can be combined with the Equal Weights SWF. Specifically, the planner may both place direct social welfare weight on future generations beyond that implied by current parental altruism and use a uniform welfare weight on all children following the arguments that yielded the Equal Weights SWF. The result is the **Combination Social Welfare Function**:

\[
W = \sum_f E_{j_1} \left[ \frac{\pi^{j_1}}{1 + \alpha} \left( \frac{1}{1 + \beta^{j_1}} \ln (c_1 (f, j_1)) \right) + \left( \frac{\delta}{1 + \delta} + \alpha \right) \ln (c_2 (f, j_1)) \right] \quad (3.9)
\]

which shows that the weight on children is the same for all families and that this weight includes a uniform, extra weight \(\alpha\).

Now that we have derived four social welfare functions, I turn to characterizing their implications for optimal policy.

3.4 Optimal Policy

In this section, I compare the effects of using the Unequal Weights SWF and the Equal Weights SWF to determine the optimal allocation of resources in an economy where parents’ preferences are private information.\(^{95}\) The economy is as specified above, where preference types take two values: \(\beta^i \in \{\beta^S, \beta^G\}\) where \(\beta^S < \beta^G\). This policy problem is well-suited to the techniques of the dynamic Mirrleesian optimal tax literature (see Golosov, Tsyvinski, and Werning 2006), in which a social planner seeks to maximize social welfare subject to the unobservability of individual heterogeneity.

---

\(^{95}\) Similar analysis could compare the results of using the Diluted Weights SWF and the Combination SWF. In the next section, I provide a numerical illustration comparing these policies.
Naturally, using the Equal Weights SWF as policy’s objective function rather than the Unequal Weights SWF reduces the optimal extent of inequality among children. More formally, the Equal Weights SWF introduces new wedges into the intertemporal Euler conditions within families relative to those yielded by the standard Atkeson-Lucas approach.\footnote{Similar results apply to using the Combination SWF rather than the Diluted Weights SWF, as shown in the next section’s numerical results.}

The social planner’s problem has the same structure regardless of the social welfare function it uses as its objective. The planner maximizes social welfare, expressed in either (3.6) or (3.7) subject to two types of constraints: feasibility and incentive compatibility. I omit the family indexing \( f \), as symmetry among families makes it irrelevant to allocations.

The feasibility constraint is that an annual endowment flow \( (Y = 1) \) must fund consumption:

\[
\frac{1}{1+\delta} \left( 1 - \sum_{j_1=S,G} \pi^{j_1} c_1^{j_1} \right) + \frac{\delta}{1+\delta} \left[ 1 - \sum_{j_2=S,G} \pi^{j_2} c_2^{j_2} \right] \geq 0; \tag{3.10}
\]

where the intertemporal prices \( \frac{1}{1+\delta} \) and \( \frac{\delta}{1+\delta} \) reflect the exogenous rate of return to saving by the planner.

Incentive compatibility constraints reflect that policy is implemented in the real world rather than in the original position. Outside the original position, families and households are identical in this economy.\footnote{As noted above, this does not imply that families or households are biological in basis. Adoptive families are equally well-suited to this analysis.} Thus, policy must be implemented through parents who have heterogeneous altruism. Namely, when a parent is allocated resources, she will not allocate them according to the social welfare weights expressed in the social welfare function but rather according to her own personal preferences.
Following the optimal taxation literature (see Golosov, Kocherlakota, and Tsyvinski 2003), we can express these incentive constraints as conditions that ensure each type of parent prefers the allocation intended for its type to that intended for any other type. For example, the constraint preventing the parent type $G$ from preferring the allocation intended for type $S$ is:

\[
\frac{1}{1 + \beta^G} \ln \left( c_1^G \right) + \frac{\beta^G}{1 + \beta^G} \ln \left( c_2^G \right) \geq \frac{1}{1 + \beta^S} \ln \left( c_1^S \right) + \frac{\beta^G}{1 + \beta^S} \ln \left( c_2^S \right).
\] (3.11)

These incentive constraints prevent the final allocation of resources from being equal across children even if that were the unconstrained maximum. Intuitively, if generous parents expect that their generosity will be futile because the government will spread their altruism among all children, they will pretend to be selfish parents in order to receive a higher allocation for themselves (and the same for their children as if they had revealed their true type). This is analogous to the tradeoff between efficiency and equality in the optimal tax literature pioneered by Mirrlees (1971).

### 3.4.1 Optimal inequality across children

In this section I derive expressions for the optimal degree of inequality between the two types of children when the planner maximizes the Equal Weights and Unequal Weights social welfare functions. I denote the solution to each problem by the name of the social welfare function used to derive it.

The Equal Weights solution, the constrained maximization of (3.6), satisfies:
where $\mu^G$ is the Lagrange multiplier on condition (3.11), the incentive constraint for generous parents, and all other notation is as before. The incentive compatibility constraint on generous parents always binds in this problem, so $\mu^G > 0$, while the constraint preventing selfish parents from pretending to be generous parents never binds.\footnote{Proofs of these results are in the Appendix.}

Result (3.12) defines a wedge between the consumption allocations of the children of generous parents and the children of selfish parents. This wedge is greater than one, given that $\mu^G > 0$, so more consumption is given to the children of generous parents than to those of selfish parents. To better understand this wedge, we can compare it to the analogous wedge that results from the Unequal Weights SWF, instead.

The Unequal Weights solution, the constrained maximization of (3.7), satisfies:

$$c_2^G = \left( \frac{\delta}{1+\delta} + \frac{1}{\pi^G 1+\beta^G \mu^G} \right) c_2^S, \quad (3.12)$$

Two differences separate (3.12) from (3.13).

First, note that (3.12) has $\frac{\delta}{1+\delta}$ in the first term of the wedge’s numerator and denominator, while (3.13) has the parents’ discount factors. Given that $\beta^G > \delta > \beta^S$, this difference reduces the wedge in (3.12) relative to (3.13). This reduces the inequality between the children of generous and selfish parents, reflecting that the Equal Weights SWF values ameliorating the natural disadvantage faced by the children of selfish parents. Because parents in the original position instruct the planner to treat all children identically,
the planner applies a uniform weight rather than the disparate weights that heterogeneous parents apply.

The second difference relies on the positive multiplier $\mu^G$, and it partially offsets the first difference by enlarging the wedge in (3.12) relative to (3.13). Intuitively, $\mu^G$ is the shadow value (in terms of social welfare) of relaxing the generous parents’ incentive constraint, as doing so would allow the planner to give more equal allocations to each child, furthering its objective. The size of $\mu^G$ reflects how far the optimal incentive-compatible allocation is from full insurance; when it is larger, we know that the planner has been forced to accept more disparity between children.

Manipulating these conditions, we can show the following natural result:

**Proposition 7**  The ratio of the consumption of the children of generous parents to the consumption of the children of selfish parents is smaller when the planner maximizes the Equal Weights social welfare function rather than the Unequal Weights social welfare function.

**Proof.** See Appendix

Thus, the Equal Weights SWF yields less inequality among children then does the conventional approach, using the Unequal Weights SWF. Next, I derive a related result that more direct translates to policy toward intergenerational transfers.

### 3.4.2 Optimal intergenerational allocations

Here, I derive the conditions describing the optimal allocation of consumption between parents and their children. I compare the Equal Weights solution to two benchmark intertemporal Euler equations: (1) the Unequal Weights solution, and (2) the solution for a planner
maximizing the Equal Weights SWF with full information (i.e., who observes types). I focus on the conditions for the generous parent’s family, with the Appendix containing a similar analysis for the selfish parent’s family.

The Equal Weights solution satisfies:

\[
c^G_2 = \frac{\pi^G + \frac{1+\delta}{\delta} \beta^G \mu^G}{\pi^G + \mu^G} \frac{1 + \beta^G}{1 + \delta} c^G_1. \tag{3.14}
\]

The Unequal Weights solution satisfies

\[
c^G_2 = \frac{\beta^G}{\delta} c^G_1. \tag{3.15}
\]

Finally, with full information, the planner maximizing the Equal Weights SWF (3.6) subject only to (3.10) would choose

\[
c^G_2 = \frac{1 + \beta^G}{1 + \delta} c^G_1 \tag{3.16}
\]

Comparing these results shows that the intertemporal margin in the Equal Weights planner’s problem is distorted relative to these two benchmark cases. The following proposition holds:

**Proposition 8** The ratio of the consumption of the children of generous parents to the consumption of their parents is smaller if the planner maximizes the Equal Weights social welfare function rather than the Unequal Weights social welfare function, but it is not as small as if the planner had full information. That is,

\[
\frac{\beta^G}{\delta} \frac{\pi^G + \frac{1+\delta}{\delta} \beta^G \mu^G}{\pi^G + \mu^G} \frac{1 + \beta^G}{1 + \delta} > \frac{1 + \beta^G}{1 + \delta}, \tag{3.17}
\]

**Proof.** See Appendix.  ■
The first inequality in condition (3.17) implies that the planner’s optimal policy when using the Equal Weights SWF is to choose a "less generous" allocation for the generous parents than in the standard, Unequal Weights, approach. On the other hand, the planner’s ability to distort allocations in this way is limited by the requirement that it satisfy generous parents’ incentive compatibility constraints. The second inequality shows that the planner’s optimal feasible and incentive compatible policy implements an allocation for generous parents that is "more generous" than the planner would most prefer. An analogous result holds for selfish parents, as shown in the Appendix.

In sum, the planner’s optimal policy using Equal Weights SWF yields intertemporal allocations that reflect a compromise between the parents’ desire to insure children when in the original position and the planner’s need to satisfy parents’ incentive compatibility constraints outside the original position.

3.5 Infinite horizon model with a one-time preference shock

This section extends the analysis above to an infinite-horizon economy and provides a numerical illustration of the optimal policies under each social welfare function derived in Section 3.3.

Consider an economy made up of an infinite sequence of generations with heterogeneity in altruism among the first generation only. Assume this heterogeneity takes the same form as above, where half of the first generation uses the discount factor \( \beta^G \) and half uses \( \beta^S \) where \( \beta^G > \beta^S \). After this "one-time shock," all members of future generations place the same relative weights on their own and their descendants’ utility, using the para-
As before, the planner chooses feasible allocations that convince first-generation members to reveal their types. To abstract from the optimal allocation of resources between generations in the aggregate, I use period-specific feasibility constraints.

Extending the model in this way allows us to examine the long-run distribution of consumption. Following Atkeson and Lucas (1992), the Unequal Weights policy has optimal consumption allocations differ across families in the long run. The approach of this paper mitigates these differences, which this section illustrates analytically and with a numerical example.

This section also shows how this paper’s approach modifies the results of Phelan (2006) and Farhi and Werning (2005). Those authors showed that a planner who puts direct weight on children’s preferences (yielding what I called the Diluted Weights SWF) chooses to have all consumption paths revert to the mean in the long run, so that any early advantage gained by a family fades over time and eventually disappears. As discussed above, we can combine the key suggestion of these authors with the Equal Weights SWF, yielding the Combination SWF (3.9). Maximization of that objective reduces disparities across families across all generations relative to the Diluted Weights SWF, much as the Equal Weights SWF yielded less inequality than the Unequal Weights SWF.

When specifying and solving the infinite-horizon versions of the three models of interest, I utilize the useful technique employed by Farhi and Werning (2005) in which the planner’s problem is divided into two parts. A full derivation of the following results is available in an Appendix from the author.
3.5.1 The Equal Weights SWF policy

The planner’s problem is to maximize the Equal Weights social welfare function by choosing, for each type of individual in generation $t = 1$, a consumption allocation $c_1$ and a promised utility $v_2$. I suppress the family notation $f$ for simplicity. The planner solves:

$$
\max_{c_1, v_2} E_i \left[ \frac{1}{1 + \beta^*} \ln (c^i_1) - \frac{1}{1 + \delta} \lambda_1 c^i_1 + \frac{\delta}{1 + \delta} \kappa (v^i_2) \right] \quad (3.18)
$$

where $\lambda_1$ is the multiplier on a first-period feasibility constraint, $v_2$ is promised utility, $\kappa (v_2)$ is the continuation value function evaluated at the promised utility, and the $E_i$ operator indicates that the planner maximizes the expectation over the population of first-generation types.

The continuation value function for generation $t$ is described by the Bellman equation

$$
\kappa (v^i_t) = \max_{c_t, v_{t+1}} \left[ \frac{1}{1 + \delta} \ln (c^i_t) - \frac{1}{1 + \delta} \lambda_t c^i_t + \frac{\delta}{1 + \delta} \kappa (v^{i+1}_{t+1}) \right],
$$

where $\lambda_t$ is the multiplier on the period-$t$ feasibility constraint.

Maximization is subject to the recursive incentive compatibility constraint on generous parents:

$$
\frac{1}{1 + \beta^G} \ln (c^G_1) + \beta^G \frac{\beta^G v^G_2}{1 + \beta^G} \geq \frac{1}{1 + \beta^G} \ln (c^S_1) + \frac{\beta^G v^S_2}{1 + \beta^G},
$$

where promised utility is:

$$
v^i_t = \frac{1}{1 + \delta} \ln (c^i_t) + \frac{\delta}{1 + \delta} v^i_{t+1}
$$

for $t > 1$. Note that all generations after the first have homogeneous preferences.
We can derive optimal allocations from this model for each period. For the first generation,

\[ c_1^G = \frac{1 + \delta}{1 + \beta^G} (1 + 2\mu^G), \]
\[ c_1^S = \frac{1 + \delta}{1 + \beta^S} \left( 1 - 2\mu^G \frac{1 + \beta^S}{1 + \beta^G} \right), \]

where \( \mu^G \) is the multiplier on the generous members of the first generation, as in Section 3.4.

For the second generation and beyond, an important intermediate result is that consumption is constant for each type after the first generation:

\[ c_{i+1}^i = c_i^i \text{ for all } t > 1. \]

Using this, we can show that for all generations after the first generation,

\[ c_t^G = \left( 1 + 2\mu^G \frac{\beta^G}{1 + \beta^G} \frac{1 + \delta}{\delta} \right), \]
\[ c_t^S = \left( 1 - 2\mu^G \frac{\beta^G}{1 + \beta^G} \frac{1 + \delta}{\delta} \right). \]

These expressions for descendants’ consumption show how the planner maximizing the Equal Weights SWF wants to fully offset parental heterogeneity but is prevented from doing so by its need to satisfy the first generation’s incentive compatibility constraints. Specifically, if these constraints were non-binding, \( \mu^G = 0 \), and the results above would yield \( c_t^G = c_t^S \), or full equality of opportunity. Instead, with \( \mu^G > 0 \), the descendants of generous parents are permitted to retain some of their advantages.
3.5.2 The Unequal Weights SWF policy

The Atkeson-Lucas approach uses the Unequal Weights social welfare function instead. Its planner’s problem is to solve:

$$\max \tilde{c}^G_1, \tilde{c}^S_1 E \left[ \frac{1}{1 + \beta^i} u \left( \tilde{c}^i_1 \right) - \frac{1}{1 + \delta} \tilde{\lambda}_1 \tilde{c}^i_1 + \frac{\beta^i}{1 + \beta^i} \kappa \left( \tilde{v}^i_2 \right) \right]$$

(3.19)

where $\tilde{\lambda}_1$ is the multiplier on a first-period feasibility constraint, $\tilde{v}_2$ is promised utility, and $\kappa \left( \tilde{v}_2 \right)$ is a continuation value function evaluated at the promised utility. This value function is described by the Bellman equation

$$\kappa \left( \tilde{v}^i_t \right) = \max_{\tilde{c}^i_t, \tilde{v}^i_{t+1}} \left[ \frac{1}{1 + \delta} u \left( \tilde{c}^i_t \right) - \left( \frac{\delta}{1 + \delta} \frac{1 + \beta^i}{\beta^i} \right) \frac{1}{1 + \delta} \tilde{\lambda}_t \tilde{c}^i_t + \left( \frac{\delta}{1 + \delta} \right) \kappa \left( \tilde{v}^i_{t+1} \right) \right]$$

where $\tilde{\lambda}_t$ is the multiplier on the period-$t$ feasibility constraint.\(^{99}\)

The maximization is subject to the incentive constraint:

$$\frac{1}{1 + \beta^G} u \left( \tilde{c}^G_1 \right) + \frac{\beta^G}{1 + \beta^G} \tilde{v}^G_2 \geq \frac{1}{1 + \beta^G} u \left( \tilde{c}^S_1 \right) + \frac{\beta^G}{1 + \beta^G} \tilde{v}^S_2.$$

where promised utility is:

$$\tilde{v}^i_t = \frac{1}{1 + \delta} u \left( \tilde{c}^i_t \right) + \frac{\delta}{1 + \delta} \tilde{v}^i_{t+1}$$

for $t > 1$.

We can derive optimal allocations from this model for each period. For the first generation,

$$\tilde{c}^G_1 = \frac{1 + \delta}{1 + \beta^G}$$

$$\tilde{c}^S_1 = \frac{1 + \delta}{1 + \beta^S}$$

\(^{99}\) Note that I multiply this embedded feasibility constraint by the factor $\frac{\delta}{1 + \delta} \frac{1 + \beta^i}{\beta^i}$, which adjusts the intertemporal price of consumption in year $t$ to be the same for both types of $i$, given that $\frac{\beta^i}{1 + \beta^i}$ multiplies $\kappa \left( \tilde{v}^i_2 \right)$ in the objective function.
For the second generation and beyond, as in the Equal Weights policy, consumption is constant for each type after the first generation:

\[ \tilde{c}_{t+1}^i = \tilde{c}_t^i \text{ for all } t > 1, \]

so that for \( t > 1 \),

\[ \tilde{c}_t^G = \frac{\beta^G}{1 + \beta^G} \frac{1 + \delta}{\delta}, \]

\[ \tilde{c}_t^S = \frac{\beta^S}{1 + \beta^S} \frac{1 + \delta}{\delta}. \]

### 3.5.3 The Diluted Weights SWF policy

Next, consider the policy chosen to maximize a Diluted Weights social welfare function, as in Phelan (2006) and Farhi and Werning (2005). The social planner puts direct weight \( \alpha^t \) on generation \( t \)'s objective function. The social welfare weights, prior to normalization across families, are thus:

\[ \Omega_t^i = \begin{cases} 
\frac{1}{\sum_{t=0}^{\infty} \alpha^t} \frac{1}{1 + \beta^t} & \text{for } t = 1 \\
\frac{1}{\sum_{t=0}^{\infty} \alpha^t} \frac{1}{1 + \beta^t} \left[ \sum_{s=2}^{t} \left( \alpha^{s-1} \left( \frac{\delta}{1 + \delta} \right)^{t-s} \right) + \frac{\beta^t}{1 + \beta^t} \left( \frac{\delta}{1 + \delta} \right)^{t-2} \right] & \text{for } t > 1 
\end{cases} \]

I normalize these to obtain type-specific welfare weights:

\[ \omega_t^i = \frac{\Omega_t^i}{\sum_i \sum_{t=1}^{\infty} \pi^i \Omega_t^i}, \]

such that:

\[ \sum_i \sum_{t=1}^{\infty} \pi^i \omega_t^i = 1. \]

I take the expectation of these to obtain the uniform welfare weight \( \omega_t \):

\[ \omega_t = \pi^G \omega_t^G + \pi^S \omega_t^S. \]
Using this notation, the planner solves:

\[
\max_{c_1, c_2} E \left[ \omega_1^i u (c_1^i) - \omega_1 \lambda_1 \bar{c}_1^i + \kappa (\bar{v}_2^i) \right]
\]  \hspace{1cm} (3.20)

where carets distinguish this model’s variables.\(^{100}\)

The continuation value is described by the Bellman equation

\[
\kappa (v_i^i) = \max_{c_1, c_2} \left[ \omega_1^i u (c_1^i) - \omega_1 \lambda c_1^i + \kappa (v_{i+1}^i) \right].
\]

The maximization is subject to the recursive incentive compatibility constraint on generous parents:

\[
\frac{1}{1 + \beta^G} u (\bar{c}_1^G) + \frac{\beta^G}{1 + \beta^G} \bar{v}_2^G \geq \frac{1}{1 + \beta^G} u (\bar{c}_1^S) + \frac{\beta^G}{1 + \beta^G} \bar{v}_2^S.
\]

where promised utility is:

\[
\bar{v}_i^i = \frac{1}{1 + \delta} u (\bar{c}_i^i) + \left( \frac{\delta}{1 + \delta} \right) \bar{v}_{i+1}^i.
\]

We can derive optimal allocations from this model for each period. For the first generation,

\[
\bar{c}_1^G = \left( \frac{\omega_1^G}{\omega_1} + 2 \mu^G \frac{1}{(1 + \beta^G) \omega_1} \right),
\]

\[
\bar{c}_1^S = \left( \frac{\omega_1^S}{\omega_1} - 2 \mu^G \frac{1}{(1 + \beta^G) \omega_1} \right).
\]

Unlike the previous models, the allocations are not constant after the second generation in this model. Instead, second generation allocations are:

\[
c_2^G = \frac{1}{\omega_2} \left( \omega_2^G + 2 \frac{1}{1 + \delta} \frac{\beta^G}{1 + \beta^G} \mu^G \right),
\]

\(^{100}\) Note that there is no apparent discounting of the future value \(\kappa (\bar{v}_2^i)\) in (3.20) because the discounting is contained within the \(\omega\) weights in the future.
3.5 Infinite horizon model with a one-time preference shock

\[ c^S_2 = \frac{1}{\omega_2} \left( \omega^S_2 - 2 \frac{1}{1 + \delta} \frac{\beta^G}{1 + \beta^G \mu^G} \right), \]

and future allocations are governed by the process:

\[ c^i_{t+1} = \left( \frac{\omega^i_{t+1} - \frac{\delta}{1 + \delta} \omega^i_{t}}{\omega_{t+1}} + \frac{\delta}{1 + \delta} \frac{\omega^i_t}{\omega_{t+1}} c^i_t \right), \]

for \( t > 1 \). We can use this equation to solve for the steady state allocations in the model, denoted by \( c^i_T \):

\[ c^i_T = \frac{\omega^i_T - \frac{\delta}{1 + \delta} \omega^i_{T-1}}{\omega_T - \frac{\delta}{1 + \delta} \omega_{T-1}}. \]

If the welfare weights \( \omega^i_T \) converge as \( T \to \infty \), then this result simplifies to

\[ c^i_T = 1, \]

so the long run features no disparity between descendants of heterogeneous parents.\(^{101}\)

3.5.4 The Combination SWF policy

As discussed above, the Equal Weights SWF can be augmented to include a uniform weight on children, as in the Diluted Weights SWF, yielding the Combination SWF. In that case, the raw welfare weights are:

\[ \Omega^i_t = \frac{1}{\sum_{t=0}^{\infty} \alpha^t} \frac{1}{1 + \delta} \left[ \sum_{s=1}^{t} \left( \frac{\delta}{1 + \delta} \right) \left( \omega^S_t \right)^{t-s} \right] \]

which are equal across families. Therefore \( \omega^G_t = \omega^S_t \) for \( t > 1 \), so

\[ c^G_2 = \frac{1}{\omega_2} \left( \omega_2 - 2 \frac{1}{1 + \delta} \frac{\beta^G}{1 + \beta^G \mu^G} \right), \]

\[ c^S_2 = \frac{1}{\omega_2} \left( \omega_2 - 2 \frac{1}{1 + \delta} \frac{\beta^G}{1 + \beta^G \mu^G} \right). \]

\(^{101}\) The Appendix shows that the welfare weights may not converge if \( \alpha \) is not large enough. Whether convergence is complete is not of primary importance to this paper.
For future generations,
\[
c^i_{t+1} = \left( \frac{\omega_{t+1} - \frac{\delta}{1+\gamma} \omega_t}{\omega_{t+1}} + \frac{\delta}{1+\gamma} \omega_t c^i_t \right)
\]
governs the consumption path. This implies a process of mean reversion as in the Diluted Weights SWF policy above, so that
\[
c^i_T = 1
\]
for all types \( i \).

### 3.5.5 Numerical illustration and discussion

In this subsection, I specify parameter values for the planning problems just discussed and compute the optimal consumption paths as derived above. I set \( G = 0.95 \) and \( S = 0.80 \), so that a generous parent puts a weight \( \left( \frac{\beta^G}{1+\beta^G} \right) \) on its child that is approximately 10 percent larger than a selfish parent’s weight on its child.

Figure 3.1 plots the constrained optimal consumption paths for families with generous and selfish parents according to the Equal Weights SWF and the Unequal Weights SWF. The horizontal axis indexes generations, starting with the heterogeneous first generation, and the vertical axis measures consumption.
As derived above, consumption is constant beginning in the second period for all types in these two policies. Both policies give the descendants of generous parents more than they give to the descendants of selfish parents. However, this inequality across families is substantially less in the Equal Weights policy than in the Unequal Weights policy. The 10 percent gap in parental altruism implied by setting $\beta^G = 0.95$ and $\beta^S = 0.80$ yields a nearly 10 percent gap in consumption between children in the Unequal Weights policy. In the Equal Weights policy, this gap is less than 5 percent.

Figure 3.2 plots the results of the other two policies analyzed in this paper: the Diluted Weights and Combination policies. In these policies, the planner puts $\alpha^{t-1}$ weight on generation $t$’s preferences, and I set $\alpha = 0.50$. 
Inequality across families is less in these policies than in the policies that considered only the first generation’s preferences, reflecting the "dilution" effect of positive $\alpha$. As derived above, both of the policies shown in Figure 3.2 generate mean-reversion in consumption, so that the gaps between Generous and Selfish parents’ families narrow as the generations pass. Nevertheless, the Combination SWF yields substantially less inequality than the Diluted Weights SWF. The percentage gap in consumption for each generation is approximately half as large in the Combination policy as in the Diluted Weights policy.

As these figures show, this paper’s approach leads to substantially less inequality among descendants of heterogeneous parents, even when constrained by unobservability of preferences. The effects hold whether the approach modifies the Unequal Weights
3.6 Conclusion

This paper examines how policy should respond to heterogeneity in parental altruism when society’s objective function is chosen by parents in an "original position" as in the normative framework pioneered by William Vickrey, John Harsanyi, and John Rawls. The key conceptual innovation is to apply the original position thought experiment to parents and children linked in families. Assuming that the original position obscures all information about both parents’ and children’s positions in society, it is impossible for a parent in the original position to know the type of parent by which his or her child would be raised outside the original position.

This generalization of the original position to an economy with families stands in stark contrast to the conventional approach to optimal intergenerational allocations. Following Atkeson and Lucas (1992), the conventional approach implicitly allows parents to pierce the "veil of ignorance" and be certain that they will raise their own children. That yields what I call the Unequal Weights social welfare function, placing weights on children that depend on the preferences of their parents. In contrast, this paper’s approach yields a social welfare function that places equal social welfare weights on all children despite heterogeneous parental altruism. I call this the Equal Weights social welfare function. A similar modification applies to more recent models of optimal intergenerational allocations, such as Phelan (2006) and Farhi and Werning (2005).
This paper’s results have potentially substantial implications for a wide variety of policies such as education finance, medical insurance, and estate or inheritance taxation. In this paper, I focus on the simple example of the allocation of consumption across generations when parents’ altruism is private information. In that setting, the use of the Equal Weights social welfare function as the policy objective yields an optimal allocation of resources that offsets disparities between children of heterogeneous parents. Formally, this can be expressed as policy driving wedges into the intertemporal Euler conditions within families. A numerical illustration suggests that, even for relatively small differences in preferences, this paper’s approach may have quantitatively large implications for optimal policy.
Chapter 4
The Optimal Taxation of Height: A Case Study of Utilitarian Income Redistribution
(Joint with N. Gregory Mankiw)

4.1 Introduction

This paper can be interpreted in one of two ways. Some readers can take it as a small, quirky contribution aimed to clarify the literature on optimal income taxation. Others can take it as a broader effort to challenge that entire literature. In particular, our results can be seen as raising a fundamental question about the framework for optimal taxation for which William Vickrey and James Mirrlees won the Nobel Prize and which remains a centerpiece of modern public finance.

More than century ago, Edgeworth (1897) pointed out that a Utilitarian social planner with full information will be completely egalitarian. More specifically, the planner will equalize the marginal utility of all members of society; if everyone has the same separable preferences, equalizing marginal utility requires equalizing after-tax incomes as well. Those endowed with greater than average productivity are fully taxed on the excess, and those endowed with lower than average productivity get subsidies to bring them up to average.

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102 We are grateful to Ruchir Agarwal for excellent research assistance and to Robert Barro, Raj Chetty, Emmanuel Farhi, Ed Glaeser, Louis Kaplow, Andrew Postlewaite, David Romer, Julio Rotemberg, Alex Tabarrok, Aleh Tsyvinski, and Ivan Werning for helpful comments and discussions.
Vickrey (1945) and Mirrlees (1971) emphasized a key practical difficulty with Edgeworth’s solution: The government does not observe innate productivity. Instead, it observes income, which is a function of productivity and effort. The social planner with such imperfect information has to limit his Utilitarian desire for the egalitarian outcome, recognizing that too much redistribution will blunt incentives to supply effort. The Vickrey-Mirrlees approach to optimal nonlinear taxation is now standard; for some recent examples of its application, see Saez (2002), Golosov, Kocherlakota, and Tsyvinski (2003), Albanesi and Sleet (2006), and Kocherlakota (2006), and for an overview of this growing literature, see Golosov, Tsyvinski, and Werning (2006).

Although Vickrey and Mirrlees assumed that income was the only piece of data the government could observe about an individual, that assumption is far from true. In practice, a person’s income tax liability is a function of many variables beyond income, such as mortgage interest payments, charitable contributions, health expenditures, number of children, and so on. Following Akerlof (1978), these variables can be considered "tags" that identify individuals whom society deems worthy of special support. This support is usually called a "categorical transfer" in the substantial literature on optimal tagging (e.g., Mirrlees 1986, Kanbur et al. 1994, Immonen et al. 1998, Viard 2001a, Viard 2001b, Kaplow 2007b). In this paper, we use the Vickrey-Mirrlees framework to explore the potential role of another variable: the taxpayer’s height.

The inquiry is supported by two legs—one theoretical and one empirical. The theoretical leg is that, according to the theory of optimal taxation, any exogenous variable correlated with productivity should be a useful indicator for the government to use in de-
termining the optimal tax liability (e.g., Saez 2001, Kaplow 2007b). The empirical leg is that a person’s height is strongly correlated with his or her income. Judge and Cable (2004) report that “an individual who is 72 in. tall could be expected to earn $5,525 [in 2002 dollars] more per year than someone who is 65 in. tall, even after controlling for gender, weight, and age.” Persico, Postlewaite, and Silverman (2004) find similar results and report that "among adult white men in the United States, every additional inch of height as an adult is associated with a 1.8 percent increase in wages." Case and Paxson (2006) write that "For both men and women...an additional inch of height [is] associated with a one to two percent increase in earnings." This fact, together with the canonical approach to optimal taxation, suggests that a person’s tax liability should be a function of his height. That is, a tall person of a given income should pay more in taxes than a short person of the same income. The policy simulation presented below confirms this implication and establishes that the optimal tax on height is substantial.

Many readers will find the idea of a height tax absurd, whereas some will find it merely highly unconventional. The purpose of this paper is to ask why the idea of taxing height elicits such a response even though it follows ineluctably from a well-documented empirical regularity and the dominant modern approach to optimal income taxation. If the policy is viewed as absurd, defenders of this approach are bound to offer an explanation that leaves their framework intact; otherwise economists ought to reconsider this standard approach to policy design.

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103 Such a correlation is sufficient but not necessary: even if the average level of productivity is not affected by the variable, effects on the distribution of productivity can influence the optimal tax schedule for each tagged subgroup.
Before proceeding, a note about our own (the authors’) interpretation of the results. One of us takes from this *reductio ad absurdum* the lesson that the modern approach to optimal taxation, such as the Vickrey-Mirrlees model, poorly matches people’s intuitive notions of fairness in taxation and should be reconsidered or replaced. The other sees it as clarifying the scope of the framework, which nevertheless remains valuable for the most important questions it was originally designed to address. The paper presents both interpretations and invites readers to make their own judgments.

The remainder of the paper proceeds as follows. In Section 4.2 we review the Vickrey-Mirrlees approach to optimal income taxation and focus it on the issue at hand—optimal taxation when earnings vary by height. In Section 4.3 we examine the empirical relationship between height and earnings, and we combine theory and data to reach a first-pass judgment about what an optimal height tax would look like for white males in the United States. We also discuss how the case for a height tax extends beyond the Vickrey-Mirrlees model to a broader set of tax policy frameworks. In Section 4.4 we consider some of the reasons that economists might be squeamish about advocating such a tax. Section 4.5 concludes.

### 4.2 The model

We begin by introducing a general theoretical framework, keeping in mind that our goal is to implement the framework using empirical wage distributions.
4.2 The model

4.2.1 A general framework

We divide the population into $H$ height groups indexed by $h$, with population proportions $p_h$. Individuals within each group are differentiated by their exogenous wages, which in all height groups can take one of $I$ possible values. The distribution of wages in each height group is given by $\pi_h = \{\pi_{h,i}\}_{i=1}^I$, where $\sum_i \pi_{h,i} = 1$ for all $h$, so that the proportion $\pi_{h,i}$ of each height group $h$ has wage $w_i$. Individual income $y_{h,i}$ is the product of the wage and labor effort $l_{h,i}$:

$$y_{h,i} = w_i l_{h,i}.$$ 

An individual’s wage and labor effort are both private information; only income and height are observable by the government.

Individual utility is a function of consumption $c_{h,i}$ and labor effort:

$$U_{h,i} = u(c_{h,i}, l_{h,i}),$$

and utility is assumed to be increasing and concave in consumption and decreasing and convex in labor effort. Consumption is equal to after-tax income, where taxes can be a function of income and height. Note that we are assuming preferences are not a function of height.

The social planner’s objective is to choose consumption and income bundles to maximize a Utilitarian social welfare function which is uniform and linear in individual util-

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104 Throughout the paper, we focus our discussion on the Utilitarian social welfare function because of its prominence in the optimal tax literature. The Vickrey-Mirrlees framework allows one to consider any Pareto-efficient policy, but nearly all implementations of this framework have used Utilitarian or more egalitarian social welfare weights. See Werning (2007) for an exception. Our analysis would easily generalize to any social welfare function that is concave in individual utilities. That is, a height tax would naturally arise as optimal with a broader class of “welfarist” social welfare functions.
ities. The planner is constrained in its maximization by feasibility—taxes are purely redistributive\(^{105}\)—and by the unobservability of wages and labor effort. Following the standard approach, the unobservability of wages and effort leads to an application of the Revelation Principle, by which the planner’s optimal policy will be to design the set of bundles that induce each individual to reveal his true wage and effort level when choosing his optimal bundle. This requirement can be incorporated into the formal problem with incentive compatibility constraints.

The formal statement of the planner’s problem is:

\[
\max_{c,y} \sum_{h} H \sum_{i} I \pi_{h,i} u \left( c_{h,i}, \frac{y_{h,i}}{w_i} \right), \tag{4.1}
\]

subject to the feasibility constraint that total tax revenue is non-negative:

\[
\sum_{h} H \sum_{i} I \pi_{h,i} (y_{h,i} - c_{h,i}) \geq 0, \tag{4.2}
\]

and individuals’ incentive compatibility constraints:

\[
u \left( c_{h,i}, \frac{y_{h,i}}{w_i} \right) \geq u \left( c_{h,j}, \frac{y_{h,j}}{w_i} \right) \tag{4.3}
\]

for all \( j \) for each individual of height \( h \) with wage \( w_i \), where \( c_{h,j} \) and \( y_{h,j} \) are the allocations the planner intends to be chosen by an individual of height \( h \) with wage \( w_j \).

As shown by Immonen et al. (1998), Viard (2001a, 2001b), and others, we can decompose the planner’s problem in (4.1) through (4.3) into two separate problems: setting optimal taxes within types and setting optimal aggregate transfers between types. Denote the transfer paid by each group \( h \) with \( \{R_h\}_{h=1}^{H} \). Then, we can restate the planner’s prob-

\(^{105}\) We have performed simulations in which taxes also fund an exogenous level of government expenditure. The welfare gain from conditioning taxes on height increases.
4.2 The model

LEM as:

$$\max_{\{c,y,R\}} \sum_{h} p_{h} \sum_{i} \pi_{h,i} u\left(\frac{c_{h,i}}{w_{i}}, \frac{y_{h,i}}{w_{i}}\right),$$  \hspace{1cm} (4.4)

subject to $H$ height-specific feasibility constraints:

$$\sum_{i} \pi_{h,i} (y_{h,i} - c_{h,i}) \geq R_{h};$$  \hspace{1cm} (4.5)

an aggregate budget constraint that the sum of transfers is non-negative:

$$\sum_{h} R_{h} \geq 0;$$  \hspace{1cm} (4.6)

and a full set of incentive compatibility constraints from (4.3). Let the multipliers on the $H$ conditions in (4.5) be $\{\lambda_{h}\}_{h=1}^{H}$.

One advantage of using this two-part approach is that, when we take first-order conditions with respect to the transfers $R_{h}$ we obtain

$$\lambda_{h} = \lambda_{h'}$$

for all height groups $h, h'$. This condition states that the marginal social cost of increased tax revenue (i.e., income less consumption) is equated across types. Note that this equalization is possible only because height is observable to the planner.

Throughout the paper, we will also consider a "benchmark" model for comparison with this optimal model. In the benchmark model, the planner fails to use the information on height in designing taxes. Formally, this can be captured by rewriting the set of incentive constraints in (4.3) to be

$$u\left(\frac{c_{h,i}}{w_{i}}, \frac{y_{h,i}}{w_{i}}\right) \geq u\left(\frac{c_{g,j}}{w_{i}}, \frac{y_{g,j}}{w_{i}}\right)$$  \hspace{1cm} (4.7)

for all $g$ and all $j$ for each individual of height $h$ with wage $w_{i}$. Constraints (4.7) require that each individual prefer his intended bundle to not merely the bundles of other individu-
4.2 The model

als in his height group but to the bundles of all other individuals in the population. Given
that (4.7) is a more restrictive condition than (4.3), the planner solving the optimal prob-
lem could always choose the tax policy chosen by the benchmark planner, but it may also
improve on the benchmark solution. To measure the gains from taking height into ac-
count, we will use a standard technique in the literature and calculate the windfall that the
benchmark planner would have to receive in order to be able to achieve the same aggregate
welfare as the optimal planner.

The models outlined above yield results on the optimal allocations of consumption
and income from the planner’s perspective, and these allocations may differ from what in-
dividuals would choose in a private equilibrium. After deriving the optimal allocations,
we next consider how a social planner could implement these allocations. That is, fol-
lowing standard practice in the optimal taxation literature, we use these results to infer the
tax system that would distort individuals’ private choices so as to make them coincide with
the planner’s choice. When we refer to "marginal taxes" or "average taxes" below, we are
describing that inferred tax system.

4.2.2 Analytical results for a simple example

To provide some intuitive analytical results, we consider a version of the model above
in which utility is additively separable between consumption and labor, exhibits constant
relative risk aversion in consumption, and is isoelastic in labor:

\[
 u(c_{h,i}, \frac{y_{h,i}}{w_i}) = \left(\frac{c_{h,i}}{1 - \gamma} - \frac{\alpha}{\sigma} \left(\frac{y_{h,i}}{w_i}\right)^\sigma \right) .
\]
The parameter $\gamma$ determines the concavity of utility from consumption, $\alpha$ sets the relative weight of consumption and leisure in the utility function, and $\sigma$ determines the elasticity of labor supply. In particular, the compensated (constant-consumption) labor supply elasticity is $\frac{1}{\sigma - 1}$.

The planner’s problem, using the two-part approach from above, can be written:

$$\max_{\{c, y, R\}} \sum_{h=1}^{H} p_h \sum_{i} \pi_{h,i} \left[ (c_{h,i})^{1-\gamma} - 1 \right] - \frac{\alpha}{\sigma} \left( \frac{y_{h,i}}{w_i} \right)^{\sigma} \right], \quad (4.8)$$

subject to $H$ feasibility constraints

$$\sum_{i} \pi_{h,i} (y_{h,i} - c_{h,i}) \geq R_h; \quad (4.9)$$

an aggregate budget constraint that the sum of transfers is zero:

$$\sum_{h=1}^{H} R_h = 0; \quad (4.10)$$

and incentive constraints for each individual:

$$\frac{(c_{h,i})^{1-\gamma} - 1}{1 - \gamma} - \frac{\alpha}{\sigma} \left( \frac{y_{h,i}}{w_i} \right)^{\sigma} \geq \frac{(c_{h,j})^{1-\gamma} - 1}{1 - \gamma} - \frac{\alpha}{\sigma} \left( \frac{y_{h,j}}{w_i} \right)^{\sigma}. \quad (4.11)$$

We can learn a few key characteristics of an optimal height tax from this simplified example.

First, the first-order conditions for consumption and income imply that the classic result from Mirrlees (1971) of no marginal taxation on the top earner holds for the top earners in all height groups. Specifically, the optimal allocations satisfy:

$$(c_{h,I})^{-\gamma} = \frac{\alpha}{w_I} \left( \frac{y_{h,I}}{w_I} \right)^{\sigma - 1} \quad (4.12)$$

for the highest wage earner $I$ in each height group $h$. 

Condition (4.12) states that the optimal allocations equate the marginal utility of consumption to the marginal disutility of producing income for all highest-skilled individuals, regardless of height. Individuals’ private choices would also satisfy (4.12), so optimal taxes do not distort the choices of the highest-skilled. As we will see below, the highest-skilled individuals of different heights will earn different incomes under optimal policy. Nonetheless, they all will face zero marginal tax rates. This extension of the classic "no marginal tax at the top" result is due to the observability of height, which prevents individuals from being able to claim allocations meant for shorter height groups. Therefore, the planner need not manipulate incentives by distorting shorter highest-skilled individuals’ private decisions, as it would if it were not allowed to condition allocations on height.\textsuperscript{106}

Second, the average cost of increasing social welfare is equalized across height groups:

\[
\sum_{i}^{I} \pi_{h,i} (c_{h,i})^{\gamma} = \sum_{i}^{I} \pi_{g,i} (c_{g,i})^{\gamma}
\]  

(4.13)

for all height groups \(g, h\). The term \((c_{h,i})^{\gamma}\) is the cost, in units of consumption, of a marginal increase in the utility of individual \(h,i\). The planner’s allocations satisfy condition (4.13) because, if the average cost of increasing welfare were not equal across height groups, the planner could raise social welfare by transferring resources to the height group for which this cost was relatively low. Note that in the special case of logarithmic utility, where \(\gamma = 1\), condition (4.13) implies that average consumption is equalized across height groups.

\textsuperscript{106} This result does not depend on the highest wage \(w_I\) being the same across groups.
Readers familiar with recent research in dynamic optimal taxation (e.g., Golosov, Kocherlakota, and Tsyvinski, 2003) may recognize that (4.13) is a static analogue to that literature’s so-called Inverse Euler Equation, a condition originally derived by Rogerson (1985) in his study of repeated moral hazard. What is the connection between these results? In a dynamic optimal tax model, the incentive problem stems from individuals receiving shocks to their wages between one period and the next that are not observable by the planner, who allocates resources across individuals and periods to maximize social welfare. If the planner could observe shocks, it would allocate resources to an individual over time just as the individual would choose on his own, thus satisfying the traditional Euler equation that relates an individual’s marginal utilities across periods (e.g., Atkinson and Stiglitz, 1976). Because the planner cannot observe shocks, however, an attempt to satisfy the traditional Euler equation for each individual will tempt those who receive a high wage shock to feign a low shock in order to receive smoothed consumption with less labor effort. In that situation, the best a planner can do is to equalize across periods the expected cost (across shock values) of raising an individual’s utility. The resulting allocation is described by the Inverse Euler Equation, which relates an individual’s expected inverse of marginal utilities across periods.

Height groups play a role in our static setting similar to that played by time periods in the dynamic setting. Across height groups, just as across periods, the planner may have information on the distribution of wages. However, within height groups, just as within periods, the planner cannot observe individuals’ abilities. As in the dynamic model, the planner must settle for equalizing across groups the cost of raising utility. This
implies equalizing across height groups the expected inverse of marginal utility, or condition (4.13).

In the next section, we continue this example with numerical simulations to learn more about the optimal tax policy taking height into account.

4.3 Calculations based on the empirical distribution

In this section, we use data from the National Longitudinal Survey of Youth and the methods described above to calculate the optimal tax schedule for the United States, taking height into account. The data are the same as that used in Persico, Postlewaite, and Silverman (2004), and we thank those authors for making their data available for our use.

4.3.1 The data

The main empirical task is to construct wage distributions by height group. For simplicity, we focus only on adult white males. This allows us to abstract from potential interactions between height and race or gender in determining wages. Though interesting, such interactions are not the focus of this paper. We also limit the sample to men between the ages of 32 and 39 in 1996. This limits the extent to which, if height were trending over time, height might be acting as an indicator of age. The latest date for which we have height is 1985, when the individuals were between 21 and 28 years of age. After these screens, we are left with 1,738 observations.\footnote{It is unclear whether a broader sample would increase or decrease the gains from the height tax. For example, adding women to the sample is likely to increase the value of a height tax, as men are systematically taller than women and, as the large literature on the gender pay gap documents, earn more on average. In this case, a height tax would serve as a proxy for gender-based taxes (see Alesina and Ichino, 2007). Our use}
4.3 Calculations based on the empirical distribution

Table 4.1 shows the distribution by height of our sample of white males in the United States. Median height is 71 inches, and there is a clear concentration of heights around the median. We split the population into three groups: "short" for less than 70 inches, "medium" for between 70 and 72 inches, and "tall" for more than 72 inches. In principle, one could divide the population into any number of distinct height groups, but a small number makes the analysis more intuitive and simpler to calculate and summarize. Moreover, to obtain reliable estimates with a finer division would require more observations.

of a limited sample focuses attention on height itself as a key variable.
We calculate wages\textsuperscript{108} by dividing reported 1996 wage and salary income by reported work hours for 1996.\textsuperscript{109} We consider only full-time workers, which we define (following Persico, Postlewaite, and Silverman 2004) as those working at least 1,000 hours. Table 4.2 gives summary statistics on the distribution of wages and hours across our sample.

\begin{table}
\centering
\caption{Height distribution of adult white males in the U.S.}
\begin{tabular}{|c|c|c|}
\hline
Height in inches & Percent of population & Cumulative percent of population \\
\hline
60 & 0.1\% & 0.1\% \\
61 & 0.1\% & 0.2\% \\
62 & 0.3\% & 0.6\% \\
63 & 0.5\% & 1.1\% \\
64 & 1.0\% & 2.1\% \\
65 & 2.0\% & 4.1\% \\
66 & 3.2\% & 7.2\% \\
67 & 4.8\% & 12.1\% \\
68 & 8.5\% & 20.5\% \\
69 & 10.1\% & 30.7\% \\
70 & 14.8\% & 45.5\% \\
71 & 12.9\% & 58.4\% \\
72 & 17.0\% & 75.4\% \\
73 & 9.8\% & 85.3\% \\
74 & 8.3\% & 93.6\% \\
75 & 3.0\% & 96.5\% \\
76 & 2.6\% & 99.1\% \\
77 & 0.5\% & 99.6\% \\
78 & 0.2\% & 99.8\% \\
79 & 0.1\% & 99.9\% \\
80 & 0.1\% & 100.0\% \\
\hline
\end{tabular}
\end{table}

\textsuperscript{108} Note that since we observe hours, we can calculate wages even though the social planner cannot. An alternative approach is to use the distribution of income and the existing tax system to infer a wage distribution, as in Saez (2001).

\textsuperscript{109} There is top-coding of income in the NLSY for confidentiality protection. This should have little effect on our results, as most of these workers are in our top wage bin and thus are already assigned the average wage among their wage group.
We group wages into 18 wage bins, as shown in the first three columns of Table 4.3, and use the average wage across all workers within a wage bin as the wage for all individuals who fall within that bin’s wage range.

<table>
<thead>
<tr>
<th>Summary statistics</th>
<th>Percentiles</th>
<th>Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>16.29</td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>10.85</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1,738</td>
<td></td>
</tr>
<tr>
<td>Min</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td>90.01</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Summary statistics</th>
<th>Percentiles</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2,436</td>
<td>1,125</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>665</td>
<td>1,540</td>
</tr>
<tr>
<td>Observations</td>
<td>1,738</td>
<td>1,820</td>
</tr>
<tr>
<td>Min</td>
<td>1,000</td>
<td>2,080</td>
</tr>
<tr>
<td>Max</td>
<td>6,680</td>
<td>2,313</td>
</tr>
</tbody>
</table>

Source: National Longitudinal Survey of Youth, Authors' calculations
### Table 4.3: Wage distribution of adult white males in the U.S. by height

<table>
<thead>
<tr>
<th>Bin</th>
<th>Min wage in bin</th>
<th>Max wage in bin</th>
<th>Average wage in bin</th>
<th>Number of observations in each height group</th>
<th>Proportion of each height group in each wage range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pop. Avg</td>
<td>Short</td>
<td>Medium</td>
<td>Tall</td>
<td>Short</td>
</tr>
<tr>
<td>1</td>
<td>4.50</td>
<td>2.88</td>
<td>23</td>
<td>29</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>6.25</td>
<td>5.51</td>
<td>40</td>
<td>33</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>8.25</td>
<td>7.24</td>
<td>57</td>
<td>63</td>
<td>29</td>
</tr>
<tr>
<td>4</td>
<td>10.00</td>
<td>9.17</td>
<td>58</td>
<td>67</td>
<td>39</td>
</tr>
<tr>
<td>5</td>
<td>12.25</td>
<td>10.91</td>
<td>67</td>
<td>94</td>
<td>48</td>
</tr>
<tr>
<td>6</td>
<td>14.50</td>
<td>12.98</td>
<td>60</td>
<td>102</td>
<td>53</td>
</tr>
<tr>
<td>7</td>
<td>16.91</td>
<td>14.98</td>
<td>56</td>
<td>68</td>
<td>44</td>
</tr>
<tr>
<td>8</td>
<td>19.25</td>
<td>16.91</td>
<td>38</td>
<td>57</td>
<td>33</td>
</tr>
<tr>
<td>9</td>
<td>21.50</td>
<td>18.95</td>
<td>32</td>
<td>54</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td>23.75</td>
<td>20.91</td>
<td>24</td>
<td>46</td>
<td>25</td>
</tr>
<tr>
<td>11</td>
<td>26.00</td>
<td>22.83</td>
<td>22</td>
<td>38</td>
<td>21</td>
</tr>
<tr>
<td>12</td>
<td>28.25</td>
<td>25.26</td>
<td>15</td>
<td>50</td>
<td>15</td>
</tr>
<tr>
<td>13</td>
<td>30.50</td>
<td>29.55</td>
<td>14</td>
<td>24</td>
<td>25</td>
</tr>
<tr>
<td>14</td>
<td>32.75</td>
<td>33.18</td>
<td>9</td>
<td>19</td>
<td>12</td>
</tr>
<tr>
<td>15</td>
<td>35.00</td>
<td>47.19</td>
<td>9</td>
<td>19</td>
<td>7</td>
</tr>
<tr>
<td>16</td>
<td>37.25</td>
<td>54.55</td>
<td>5</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>17</td>
<td>39.50</td>
<td>63.53</td>
<td>4</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>18</td>
<td>41.75</td>
<td>n/a</td>
<td>81.52</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

**Total observations**: 533, 778, 427

**Average wage by height group, using average wage in each height group**: 14.84, 16.74, 17.28

Source: National Longitudinal Survey of Youth, Authors’ calculations

The distribution of wages for tall people yields a higher mean wage than does the distribution for short people. This can be seen in the final three columns of Table 4.3, which shows the distribution of wages by height group. Figure 4.1 plots the data shown in Table 4.3.
As the figure illustrates, the distributions are similar around the most common wages but are noticeably different toward the tails. Many more tall white males have wages toward the top of the distribution and many fewer have wages toward the bottom than short white males. This causes the mean wage for the tall to be $17.28 compared to $16.74 for the medium and $14.84 for the short. The tall therefore have an average wage 16 percent higher than the short in our data. Given that the mean height among the tall is 74 inches compared with 67 inches among the short, this suggests that each inch of height adds just over two percent to wages (if the effect is linear)—quite close to Persico et al.’s estimate of 1.8 percent.
4.3.2 What explains the height premium?

We have just seen that each inch of height adds about two percent to a young man’s income in the United States, on average. Two recent papers have provided quite different explanations for this fact.

Persico, Postlewaite, and Silverman (2005) attribute the height premium to the effect of adolescent height on individuals’ development of characteristics later rewarded by the labor market, such as self-esteem. They write: "We can think of this characteristic as a form of human capital, a set of skills that is accumulated at earlier stages of development." By exploiting the same data used in this paper, they find that "the preponderance of the disadvantage experienced by shorter adults in the labor market can be explained by the fact that, on average, these adults were also shorter at age 16." They control for family socioeconomic characteristics and height at younger ages and find that the effect of adolescent height remains strong. Finally, using evidence on adolescents’ height and participation in activities, they conclude that "social effects during adolescence, rather than contemporaneous labor market discrimination or correlation with productive attributes, may be at the root of the disparity in wages across heights."

In direct contrast, Case and Paxson (2006) argue that the evidence points to a "correlation with productive attributes," namely cognitive ability, as the explanation for the adult height premium. They show that height as early as three years old is correlated with measures of cognitive ability, and that once these measures are included in wage regressions the height premium substantially declines. Moreover, adolescent heights are no more predictive of their wages than adult heights, contradicting Persico et al.’s proposed explanation.
Case and Paxson argue that both height and cognitive ability are affected by prenatal, in utero, and early childhood nutrition and care, and that the resulting positive correlation between the two explains the height premium among adults.

Thus, the two most recent, careful econometric studies of the adult height premium reach very different conclusions about its source. How would a resolution to this debate affect the conclusions of this paper? Is the optimal height tax dependent upon the root cause of the height premium?

Fortunately, we can be agnostic as to the source of the height premium when discussing optimal height taxes. What matters for optimal height taxation is the consistent statistical relationship between height and income, not the reason for that relationship. Of course, if taxes could be targeted at the source of the height premium, then a height tax would be redundant, no matter the source. Depending on the true explanation for the height premium, taxing the source of it may be appropriate: for example, Case and Paxson’s analysis would suggest early childhood investment by the state in order to offset poor conditions for some children. To the extent that these policies reduced the height premium, the optimal height tax would be reduced as well. However, so long as a height premium exists, the case for an optimal height tax remains.

4.3.3 Baseline results

To simulate the optimal tax schedule, we need to specify functional forms and parameters. We will use the same utility function that we analyzed in Section 4.2.2:

\[ u(c_{h,i}, l_{h,i}) = \frac{(c_{h,i})^{1-\gamma} - 1}{1 - \gamma} - \frac{\alpha}{\sigma} \left( \frac{y_{h,i}}{w_i} \right)^\sigma, \]
where $\gamma$ determines the curvature of the utility from consumption, $\alpha$ is a taste parameter, and $\sigma$ makes the compensated (constant-consumption) elasticity of labor supply equal to $\frac{1}{\sigma - 1}$. Our baseline values for these parameters are $\gamma = 1.5$, $\alpha = 2.55$, and $\sigma = 3$. We vary $\gamma$ and $\sigma$ below to explore their effects on the optimal policy, while an appropriate value for $\alpha$ is calibrated from the data. We determined the baseline choices of $\sigma$ and $\alpha$ as follows.

Economists differ widely in their preferred value for the elasticity of labor supply. A survey by Fuchs, Krueger, and Poterba (1998) found that the median labor economist believes the traditional compensated elasticity of labor supply is 0.18 for men and 0.43 for women. By contrast, macroeconomists working in the real business cycle literature often choose parameterizations that imply larger values: for example, Prescott (2004) estimates a (constant-consumption) compensated elasticity of labor supply around 3. Kimball and Shapiro (2003) give an extensive discussion of labor supply elasticities, and they show that the constant-consumption elasticity is generally larger than the traditional compensated elasticity. Taking all of this into account, we use $\frac{1}{\sigma - 1} = 0.5$ in our baseline estimates to be conservative. In the sensitivity results shown below, we see that the size of the optimal height tax is positively related to the elasticity of labor supply.

In our sample, the mean hours worked in 1996 was 2,435.5 hours per full-time worker. This is approximately 42 percent of total feasible work hours, where we assume eight hours per day of sleeping, eating, etc., and five days of illness per year. We choose $\alpha$ so that the population-weighted average of work hours divided by feasible hours in the benchmark (no height tax) allocation is approximately 42 percent: this yields $\alpha = 2.55$. The results on the optimal height tax are not sensitive to the choice of $\alpha$. 
With the wage distributions from Table 4.3 and the specification of the model just described, we can solve the planner’s problem to obtain the optimal tax policy. For comparison, we also calculate optimal taxes under the benchmark model in which the planner ignores height when setting taxes. Figure 4.2 plots the average tax rate schedules for short, medium, and tall individuals in the optimal model as well as the average tax rate schedule in the benchmark model (the two lowest wage groups are not shown because their average tax rates are large and negative, making the rest of the graph hard to see). Figure 4.3 plots the marginal tax rate schedules. We calculate marginal rates as the implicit wedge that the optimal allocation inserts into the individual’s private equilibrium consumption-leisure tradeoff. Using our assumed functional forms, the first order conditions for consumption and leisure imply that the marginal tax rate can be calculated as:

\[
T'(y_{h,i}, h) = 1 + \frac{u_y (c_{h,i}, \frac{y_{h,i}}{w_i})}{w_i u_c (c_{h,i}, \frac{y_{h,i}}{w_i})} = 1 - \frac{\alpha \left(\frac{y_{h,i}}{w_i}\right)^{\sigma-1}}{w_i \left(c_{h,i}\right)^{-\gamma}}
\]

where \(T'(y_{h,i}, h)\) is the height-specific marginal tax rate at the income level \(y_{h,i}\). Table 4.4 (shown in three parts) lists the corresponding income, consumption, labor, and utility levels as well as tax payments, average tax rates, and marginal tax rates at each wage level for the height groups in the optimal model. Table 4.5 shows these same variables for the benchmark model (with no height tax).
Figure 4.2: Average Tax Rates

Note: the two lowest income groups are not shown
Figure 4.3: Marginal Tax Rates

- Tall
- Med
- Short
- Bmk
Table 4.4 (part 1): Optimal Allocations in the Baseline Case

<table>
<thead>
<tr>
<th>Wage bin</th>
<th>Wage</th>
<th>Pop. Avg</th>
<th>Wage Short</th>
<th>Med</th>
<th>Tall</th>
<th>Short</th>
<th>Med</th>
<th>Tall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.88</td>
<td>4,086</td>
<td>4,104</td>
<td>4,107</td>
<td>27,434</td>
<td>25,332</td>
<td>24,913</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5.51</td>
<td>10,588</td>
<td>10,181</td>
<td>10,629</td>
<td>29,306</td>
<td>26,784</td>
<td>26,548</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7.24</td>
<td>15,174</td>
<td>15,386</td>
<td>15,004</td>
<td>31,178</td>
<td>28,624</td>
<td>28,064</td>
<td></td>
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Maximum work hours per year | 5,760

Alpha | 2.55
 Sigma | 3.00
 Gamma | 1.50

Avg. transfer paid(+) or received(-) as pct. of per capita income:

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<td>0.44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>54.55 0.38</td>
<td>0.24</td>
<td>0.33</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>63.53 0.37</td>
<td>0.18</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>81.52 0.39</td>
<td>0.42</td>
<td>0.43</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exp. Values (0.62) (0.34) (0.28) 0.39 0.42 0.43

The graphical tax schedules provide several useful insights about the optimal solution. First, notice the relative positions of the average tax schedules in Figure 4.2. The average tax rate for tall individuals is always above that for short individuals, and usually above that for the medium group, with the gap due to the lump-sum transfers between groups. The benchmark model’s average tax schedule lies in between the optimal tall and short schedules and near the optimal medium schedule. Other than their levels, however, the tax schedules are quite similar and fit with the conclusions of previous simulations (see Saez, 2001 and Tuomala, 1990) that optimal average tax rates rise quickly at low income
### Table 4.5: Benchmark Case

<table>
<thead>
<tr>
<th>Wage bin</th>
<th>Wage</th>
<th>Annual income</th>
<th>Annual consumption</th>
<th>Fraction of time working</th>
<th>Utility</th>
<th>Annual tax</th>
<th>Average Tax Rate</th>
<th>Marginal Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.88</td>
<td>4,106</td>
<td>25,799</td>
<td>0.25</td>
<td>1.04</td>
<td>-21,693</td>
<td>-5.28</td>
<td>0.49</td>
</tr>
<tr>
<td>2</td>
<td>5.51</td>
<td>10,479</td>
<td>27,443</td>
<td>0.33</td>
<td>1.05</td>
<td>-16,964</td>
<td>-1.62</td>
<td>0.48</td>
</tr>
<tr>
<td>3</td>
<td>7.24</td>
<td>15,251</td>
<td>29,206</td>
<td>0.37</td>
<td>1.07</td>
<td>-13,955</td>
<td>-0.91</td>
<td>0.46</td>
</tr>
<tr>
<td>4</td>
<td>9.17</td>
<td>20,926</td>
<td>31,461</td>
<td>0.40</td>
<td>1.09</td>
<td>-10,535</td>
<td>-0.50</td>
<td>0.44</td>
</tr>
<tr>
<td>5</td>
<td>10.91</td>
<td>26,281</td>
<td>33,850</td>
<td>0.42</td>
<td>1.11</td>
<td>-7,569</td>
<td>-0.29</td>
<td>0.42</td>
</tr>
<tr>
<td>6</td>
<td>12.98</td>
<td>32,962</td>
<td>37,004</td>
<td>0.44</td>
<td>1.14</td>
<td>-4,041</td>
<td>-0.12</td>
<td>0.38</td>
</tr>
<tr>
<td>7</td>
<td>14.98</td>
<td>38,327</td>
<td>39,686</td>
<td>0.44</td>
<td>1.16</td>
<td>-1,359</td>
<td>-0.04</td>
<td>0.39</td>
</tr>
<tr>
<td>8</td>
<td>16.91</td>
<td>42,837</td>
<td>41,913</td>
<td>0.44</td>
<td>1.19</td>
<td>924</td>
<td>0.02</td>
<td>0.43</td>
</tr>
<tr>
<td>9</td>
<td>18.95</td>
<td>49,585</td>
<td>45,305</td>
<td>0.45</td>
<td>1.21</td>
<td>4,280</td>
<td>0.09</td>
<td>0.39</td>
</tr>
<tr>
<td>10</td>
<td>20.91</td>
<td>55,518</td>
<td>48,507</td>
<td>0.46</td>
<td>1.23</td>
<td>7,012</td>
<td>0.13</td>
<td>0.37</td>
</tr>
<tr>
<td>11</td>
<td>22.83</td>
<td>59,718</td>
<td>50,787</td>
<td>0.45</td>
<td>1.25</td>
<td>8,931</td>
<td>0.15</td>
<td>0.40</td>
</tr>
<tr>
<td>12</td>
<td>25.26</td>
<td>64,720</td>
<td>53,296</td>
<td>0.44</td>
<td>1.27</td>
<td>11,424</td>
<td>0.18</td>
<td>0.44</td>
</tr>
<tr>
<td>13</td>
<td>29.55</td>
<td>73,290</td>
<td>56,895</td>
<td>0.43</td>
<td>1.30</td>
<td>16,394</td>
<td>0.22</td>
<td>0.50</td>
</tr>
<tr>
<td>14</td>
<td>37.18</td>
<td>92,058</td>
<td>63,385</td>
<td>0.43</td>
<td>1.33</td>
<td>28,673</td>
<td>0.31</td>
<td>0.54</td>
</tr>
<tr>
<td>15</td>
<td>47.19</td>
<td>135,042</td>
<td>81,508</td>
<td>0.50</td>
<td>1.36</td>
<td>53,535</td>
<td>0.40</td>
<td>0.29</td>
</tr>
<tr>
<td>16</td>
<td>54.55</td>
<td>153,574</td>
<td>92,198</td>
<td>0.49</td>
<td>1.40</td>
<td>61,376</td>
<td>0.40</td>
<td>0.28</td>
</tr>
<tr>
<td>17</td>
<td>63.53</td>
<td>182,763</td>
<td>110,400</td>
<td>0.50</td>
<td>1.44</td>
<td>72,363</td>
<td>0.40</td>
<td>0.16</td>
</tr>
<tr>
<td>18</td>
<td>81.52</td>
<td>236,347</td>
<td>145,040</td>
<td>0.50</td>
<td>1.49</td>
<td>91,307</td>
<td>0.39</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Expected Values**

<table>
<thead>
<tr>
<th>Wage bin</th>
<th>Wage</th>
<th>Annual income</th>
<th>Annual consumption</th>
<th>Fraction of time working</th>
<th>Utility</th>
<th>Annual tax</th>
<th>Average Tax Rate</th>
<th>Marginal Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>41,345</td>
<td>41,345</td>
<td>0.42</td>
<td>1.164</td>
<td>0</td>
<td>-0.40</td>
<td>0.42</td>
</tr>
</tbody>
</table>

**Maximum work hours per year**

- Alpha= 2.55
- Sigma= 3
- Gamma= 1.5

**Windfall for benchmark to obtain optimal, as pct of aggregate income:**

- 0.19%

**Avg. transfer paid(+) or received(-) as pct of per capita income:**

<table>
<thead>
<tr>
<th>Short</th>
<th>Medium</th>
<th>Tall</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.71%</td>
<td>1.59%</td>
<td>3.23%</td>
</tr>
</tbody>
</table>
levels and then level off as income gets large. Finally, in Figure 4.3, we can see an approximately flat marginal tax rate for most incomes and then a sharp drop to zero marginal rates for the highest wage earners in each group. The drop at the top of the income distribution reflects the extension of the classic zero top marginal rate result to a model with observable height.

Turning to the data in Tables 4.4, 4.5 and 4.6, we can learn more detail about the optimal policy. Table 4.4 shows that the average tax on the tall is about 7.1 percent of the average tall income, while the average tax on the medium is about 3.8 percent of average medium income. These taxes pay for an average transfer to the short of more than 13 percent of average short income. Note that Table 4.5 shows that the planner also transfers resources to the short population in the benchmark Mirrlees model. Importantly, this is not an explicit transfer. Rather, it reflects the differences in the distributions of the height groups across wages. Due to the progressive taxes of the benchmark model, the tall and medium end up paying more tax on average than the short even when taxes are not conditioned on height. The resulting implicit transfers are in the same direction as the average transfers in Table 4.4, though substantially smaller.

Table 4.4 also shows that the optimal tax policy usually gives lower utility to taller individuals of a given wage than to shorter individuals of the same wage. This translates into lower expected utility for the tall population as a whole than for shorter populations, as shown at the bottom of Table 4.4. Intuitively, this is because the planner wants to equalize the marginal utility of consumption and the marginal disutility of income across all individuals, not their levels of utility. To see why this results in lower expected utility
for the tall, suppose that wages were perfectly correlated with height, so that the planner had complete information. Then, the planner would equalize consumption across height groups, but it would not equalize labor effort across height groups. Starting from equal levels of labor effort, the marginal disutility of income will be lower for taller populations because they are higher-skilled. Thus, the planner will require more labor effort from taller individuals, lowering their utility. Another way to think of this is that a lump-sum tax on taller individuals doesn’t affect their optimal consumption-labor tradeoff but lowers their consumption for a given level of labor effort. Thus, they work more to satisfy their optimal tradeoff and obtain a lower level of utility.

We make the optimal tax policy more concrete by using the results from Table 4.4 to generate a tax schedule that resembles those used by U.S. taxpayers each year—this schedule is shown as Table 4.6.
### Table 4.6: Example Tax Table

<table>
<thead>
<tr>
<th>Income (in thousands)</th>
<th>Short</th>
<th>Medium</th>
<th>Tall</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,000</td>
<td>-22,697</td>
<td>-20,546</td>
<td>-20,137</td>
</tr>
<tr>
<td>10,000</td>
<td>-19,136</td>
<td>-16,741</td>
<td>-16,391</td>
</tr>
<tr>
<td>15,000</td>
<td>-16,107</td>
<td>-13,488</td>
<td>-13,062</td>
</tr>
<tr>
<td>20,000</td>
<td>-13,248</td>
<td>-10,413</td>
<td>-9,962</td>
</tr>
<tr>
<td>25,000</td>
<td>-10,581</td>
<td>-7,563</td>
<td>-7,061</td>
</tr>
<tr>
<td>30,000</td>
<td>-7,992</td>
<td>-4,882</td>
<td>-4,319</td>
</tr>
<tr>
<td>35,000</td>
<td>-5,549</td>
<td>-2,274</td>
<td>-1,671</td>
</tr>
<tr>
<td>40,000</td>
<td>-3,201</td>
<td>327</td>
<td>860</td>
</tr>
<tr>
<td>45,000</td>
<td>-882</td>
<td>2,920</td>
<td>3,420</td>
</tr>
<tr>
<td>50,000</td>
<td>1,411</td>
<td>5,444</td>
<td>5,976</td>
</tr>
<tr>
<td>55,000</td>
<td>3,599</td>
<td>7,746</td>
<td>8,368</td>
</tr>
<tr>
<td>60,000</td>
<td>5,810</td>
<td>10,044</td>
<td>10,788</td>
</tr>
<tr>
<td>65,000</td>
<td>8,867</td>
<td>12,350</td>
<td>13,766</td>
</tr>
<tr>
<td>70,000</td>
<td>11,931</td>
<td>14,828</td>
<td>16,744</td>
</tr>
<tr>
<td>75,000</td>
<td>15,264</td>
<td>18,151</td>
<td>19,722</td>
</tr>
<tr>
<td>80,000</td>
<td>18,622</td>
<td>21,506</td>
<td>22,715</td>
</tr>
<tr>
<td>85,000</td>
<td>21,979</td>
<td>24,861</td>
<td>25,819</td>
</tr>
<tr>
<td>90,000</td>
<td>25,211</td>
<td>28,216</td>
<td>28,922</td>
</tr>
<tr>
<td>95,000</td>
<td>28,123</td>
<td>31,349</td>
<td>32,028</td>
</tr>
<tr>
<td>100,000</td>
<td>31,035</td>
<td>34,134</td>
<td>35,154</td>
</tr>
<tr>
<td>105,000</td>
<td>33,947</td>
<td>36,919</td>
<td>38,280</td>
</tr>
<tr>
<td>110,000</td>
<td>36,859</td>
<td>39,704</td>
<td>41,406</td>
</tr>
<tr>
<td>115,000</td>
<td>39,771</td>
<td>42,488</td>
<td>44,532</td>
</tr>
<tr>
<td>120,000</td>
<td>42,682</td>
<td>45,273</td>
<td>47,658</td>
</tr>
<tr>
<td>125,000</td>
<td>45,594</td>
<td>48,058</td>
<td>50,784</td>
</tr>
<tr>
<td>130,000</td>
<td>48,506</td>
<td>50,843</td>
<td>53,559</td>
</tr>
<tr>
<td>135,000</td>
<td>51,289</td>
<td>53,628</td>
<td>55,930</td>
</tr>
<tr>
<td>140,000</td>
<td>53,290</td>
<td>56,244</td>
<td>58,300</td>
</tr>
<tr>
<td>145,000</td>
<td>55,291</td>
<td>58,344</td>
<td>60,671</td>
</tr>
<tr>
<td>150,000</td>
<td>57,292</td>
<td>60,444</td>
<td>63,041</td>
</tr>
<tr>
<td>155,000</td>
<td>59,204</td>
<td>62,481</td>
<td>65,412</td>
</tr>
<tr>
<td>160,000</td>
<td>60,694</td>
<td>64,500</td>
<td>67,615</td>
</tr>
<tr>
<td>165,000</td>
<td>62,184</td>
<td>66,519</td>
<td>69,658</td>
</tr>
<tr>
<td>170,000</td>
<td>63,674</td>
<td>68,538</td>
<td>71,701</td>
</tr>
<tr>
<td>175,000</td>
<td>65,163</td>
<td>70,556</td>
<td>73,743</td>
</tr>
<tr>
<td>180,000</td>
<td>66,653</td>
<td>72,575</td>
<td>75,778</td>
</tr>
<tr>
<td>185,000</td>
<td>68,143</td>
<td>74,594</td>
<td>77,722</td>
</tr>
<tr>
<td>190,000</td>
<td>n/a</td>
<td>76,613</td>
<td>79,665</td>
</tr>
<tr>
<td>195,000</td>
<td>n/a</td>
<td>78,632</td>
<td>81,609</td>
</tr>
<tr>
<td>200,000</td>
<td>n/a</td>
<td>80,651</td>
<td>83,552</td>
</tr>
</tbody>
</table>

Note: Taxes calculated by interpolating between the 18 optimal tax levels calculated for each height group.

Whereas a typical U.S. tax schedule has the taxpayer look across the columns to find his or her family status (single, married, etc.), our optimal schedule has height groups...
4.3 Calculations based on the empirical distribution

across the columns. As the numbers show, taller individuals pay substantially more taxes than shorter individuals for most income levels. For example, a tall person with income of $50,000 pays about $4,500 more in taxes than a short person of the same income.

Finally, we can use the results of the benchmark model to calculate a money-metric welfare gain from the height tax by finding the windfall revenue that would allow the benchmark planner to reach the same level of social welfare as the planner that uses a height tax. Table 4.5 shows that the windfall required is about 0.19 percent of aggregate income in our baseline parameter case. In 2007, when the national income of the U.S. economy is about $12 trillion, a height tax would yield an annual welfare gain worth about $23 billion.

4.3.4 Sensitivity to parameters

Here, we explore the effects on optimal taxes of varying our assumed parameters. In particular, we consider a range of values for risk aversion and the elasticity of labor supply. To summarize the effects of each parameter, we focus on two statistics: the average transfer to the short as a percent of average short income and the windfall required by the benchmark planner to achieve the aggregate welfare obtained by the optimal planner. Table 4.7 shows these two statistics when we vary the risk aversion parameter $\gamma$, and Table 4.8 shows them when we vary the elasticity of labor supply $\frac{1}{\sigma-1}$. In both cases, when either $\gamma$ or $\sigma$ is changed, the parameter $\alpha$ must also be adjusted so as retain an empirically plausible level of hours worked. We adjust $\alpha$ to match the empirical evidence as in the baseline analysis.

Increased risk aversion (higher $\gamma$) increases the average transfer to the short and the gain to aggregate welfare obtained by conditioning taxes on height. For example, rais-
### Table 4.7: Varying risk aversion

<table>
<thead>
<tr>
<th>Risk aversion parameter gamma (γ)</th>
<th>0.75</th>
<th>1.00: ( u(c) = \ln(c) )</th>
<th>1.50</th>
<th>2.50</th>
<th>3.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average transfer to short group, as percent of per capita short income:</td>
<td>12.81%</td>
<td>13.05%</td>
<td>13.38%</td>
<td>13.75%</td>
<td>13.97%</td>
</tr>
<tr>
<td>Windfall needed for benchmark planner to obtain optimal planner's social welfare, as percent of aggregate income</td>
<td>0.119%</td>
<td>0.146%</td>
<td>0.187%</td>
<td>0.242%</td>
<td>0.275%</td>
</tr>
</tbody>
</table>

Gamma=1.50 is the baseline level assumed throughout paper

Note: Maintains \( \sigma = 3.00 \) as in the baseline; adjusts \( \alpha \) to match hours worked:

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>12.50</th>
<th>7.50</th>
<th>2.55</th>
<th>0.30</th>
<th>0.04</th>
</tr>
</thead>
</table>

### Table 4.8: Varying labor supply elasticity

<table>
<thead>
<tr>
<th>Constant-consumption elasticity of labor supply</th>
<th>0.20</th>
<th>0.30</th>
<th>0.50</th>
<th>1.00</th>
<th>3.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value for parameter sigma (( \sigma ))</td>
<td>6.0</td>
<td>4.33</td>
<td>3.00</td>
<td>2.00</td>
<td>1.33</td>
</tr>
<tr>
<td>Average transfer to short group, as percent of per capita short income:</td>
<td>11.21%</td>
<td>11.93%</td>
<td>13.38%</td>
<td>17.06%</td>
<td>31.73%</td>
</tr>
<tr>
<td>Windfall needed for benchmark planner to obtain optimal planner's social welfare, as percent of aggregate income</td>
<td>0.097%</td>
<td>0.134%</td>
<td>0.187%</td>
<td>0.274%</td>
<td>0.493%</td>
</tr>
</tbody>
</table>

Sigma=3.00 is the baseline level assumed throughout paper

Note: Maintains \( \gamma = 1.50 \) as in the baseline; adjusts \( \alpha \) to match hours worked:

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>30.00</th>
<th>8.00</th>
<th>2.55</th>
<th>1.15</th>
<th>0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha/\sigma )</td>
<td>5.00</td>
<td>1.85</td>
<td>0.85</td>
<td>0.58</td>
<td>0.49</td>
</tr>
</tbody>
</table>
ing risk \( \gamma \) from 1.50 to 3.50 increases the average transfer to the short from 13.38 percent to 13.97 percent of average short income and increases the windfall equivalent to the welfare gain from 0.19 percent of aggregate income to 0.28 percent. Intuitively, more concave utility makes the Utilitarian planner more eager to redistribute income and smooth consumption across types. The transfer across height groups is a blunt redistributive tool, as it taxes some low-skilled tall to give to some high-skilled short, but it is on balance a redistributive tool because the tall have higher incomes than the short on average. Thus, as risk aversion rises, the average transfer to the short increases in size and in its power to increase aggregate welfare.

Increased elasticity of labor supply (lower \( \sigma \)) has a more dramatic effect on the optimal height tax. For example, raising the constant-consumption elasticity of labor supply from 0.5 to 3.0 increases the average transfer to the short from 13.38 percent to 31.73 percent of average short income and increases the windfall equivalent to the welfare gain from 0.19 percent of aggregate income to 0.49 percent. Intuitively, a higher elasticity of labor supply makes redistributing within height groups more distortionary, so the planner relies on the transfer across height groups for more of its redistribution toward the short, low-skilled. As with increased risk aversion, increased elasticity of labor supply makes the average taxes and transfers across height groups larger and gives the height tax more power to increase welfare.
4.3.5 The taxation of height in other approaches to optimal taxation

The analysis above has focused on the Vickrey-Mirrlees framework for optimal taxation, both because it is the dominant and least restrictive modern approach and because its focus on individual-specific lump-sum taxation directly invites the use of height as a tag. The case for a height tax extends well beyond that specific framework, however. In fact, any Utilitarian model of income redistribution will recommend conditioning taxes on an inelastic characteristic correlated with an individual’s ability to earn income.

**Ramsey model**

For example, consider the model of optimal linear taxation based on the work of Frank Ramsey (1928). In the Ramsey approach, lump-sum taxes are prohibited by assumption, and the goal of taxation is to fund government expenditure using distortionary linear taxes with the minimum welfare cost to a representative household. Just as in the model above, when the Ramsey model’s planner sets taxes as a function of an endogenous variable (namely, income), the elastic response of taxpayers has efficiency costs. Conditioning taxes on any exogenous variable correlated with income, such as height, makes it possible for the Ramsey planner to maintain a higher level of social welfare while funding government expenditure.\(^{110}\)

\(^{110}\) We have simulated a Ramsey model with two types of individuals, short and tall, who differ only in their wage. For a wide variety of parameterizations, the optimal Ramsey policy sets a higher tax rate on the (higher-wage) tall than on the short.
Pareto efficiency

Some readers have asked whether this paper’s analysis is a critique of Pareto efficiency. The answer depends on how one chooses to apply the Pareto criterion.

One approach is to consider the set of tax policies that place the economy on the Pareto frontier—that is, the frontier on which it is impossible to increase the welfare of one person without decreasing the welfare of another. This set of policies can be derived within the Mirrless approach by changing the weights attached to the different individuals in the economy.\(^\text{111}\) (By contrast, throughout the paper, we use a Utilitarian social welfare function with equal weight on each person’s utility.) Nearly every specification of these social welfare weights, except perhaps a knife-edge case, has taxes conditioned on height. Thus, most Pareto efficient allocations include height-dependent taxes.

A related, but slightly different, question is whether height-dependent taxes are a Pareto improvement starting from a position without such taxes. In principle, they can be. Consider the extreme case in which height is perfectly correlated with ability. Then, income taxes could be replaced with lump-sum height taxes specific to each individual’s height. By removing marginal distortions without raising tax burdens, the lump-sum taxes make all individuals better off.\(^\text{112}\) In general, the tighter the connection between height and wages and the greater the distortionary effects of marginal income taxes, the larger is the Pareto improvement provided by a height tax.

\(^{111}\) Ivan Werning (2007) uses this approach to study the conditions under which taxes are Pareto efficient, including in the context of observable traits.

\(^{112}\) Louis Kaplow suggested this example.
4.3 Calculations based on the empirical distribution

In practice, however, such Pareto improvements are so small as to be uninteresting. We have calculated the height tax that provides a Pareto improvement to the height-independent benchmark tax system derived above. We solve an augmented planner’s problem that adds to the set of equations (4.1) through (4.3) new constraints guaranteeing that no individual’s utility falls below what it received in the benchmark allocation, i.e., the solution to the problem described by equations (4.1), (4.2), and (4.7). Given the data and our benchmark parameter assumptions described above, it turns out that only an extremely small Pareto-improving height tax is available to the planner. The planner seeking a Pareto-improving height tax levies a very small (approximately $4.15 annual) lump-sum tax on the middle height group to fund lump-sum subsidies to the short ($2.90) and tall ($2.37) groups. Not surprisingly, in light of how small the Pareto-improving height tax is, the changes in utility from the policy are trivial in size.

Nevertheless, if a nontrivial Pareto-improving height tax were possible, and if people both understood and were convinced of that possibility, it is our sense that most people would be comfortable with such a policy. In contrast, we believe most people would be uncomfortable with the Utilitarian-optimal height tax that we derived above. The difference is that the Utilitarian-optimal height tax implies substantial costs to some and gains for others relative to a height-independent policy designed according to the same welfare weights. Therefore, this paper’s critique concerns the intuitive discomfort people feel toward height taxes that sacrifice the utility of the tall for the short, not Pareto improvements that come through unconventional means such as a tax on height.
4.4 Perspectives from political philosophy

So far, this paper has made the case for the optimal taxation of height using the dominant modern approach to Utilitarian policy design, namely the Vickrey-Mirrlees framework, and has calculated the details of this optimal height tax using the empirical earnings distribution for thirty-something white males in the United States. Nothing in the preceding analysis is unconventional for the optimal tax literature, except for the focus on height rather than on an unobserved characteristic, such as "ability," that affects individuals’ wages.

There are various ways to react to the idea of a height tax. One option is to accept a height tax once the preceding logic and evidence have been presented. While a height tax may seem unnatural at first, one purpose of economic analysis is to produce results that are not obvious. Perhaps it is our intuition that needs to change, not the analysis.

Most of our readers, we suspect, are both accustomed to thinking about optimal taxation from a Utilitarian perspective and instinctively uncomfortable with a tax on height. What explains this cognitive dissonance, and how can it to be resolved? If one does not accept a height tax, then is that because of something particular to height or have we stumbled onto a more fundamental problem with the modern framework for optimal taxation? Here we consider three notable responses in increasing order of the extent to which they question the fundamental approach.
4.4 Perspectives from political philosophy

4.4.1 Political economy constraints

This response acknowledges that a height tax would be optimal in a first-best political system but argues that political constraints make a height tax undesirable or infeasible in practice.

Perhaps a height tax would act as a "gateway" tax for a government, making taxes based on demographic characteristics seem natural and dangerously expanding the scope for government information collection and policy personalization. For instance, much the same analysis as we performed above could, in principle, be applied to characteristics such as skin color, gender, and physical attractiveness, each of which is a (relatively) inelastic characteristic that has been shown to affect economic outcomes. Even those who are comfortable with a height tax would likely be uncomfortable with a system of taxes tailored to so many personal characteristics. No matter how compelling the theory, the administrative burden and invasiveness of such a system may be too great. Moreover, democratic societies may have an interest in avoiding the taxation of specific groups as a matter of course to counter the majority’s temptation to tax minority groups.\(^\text{113}\)

A counterargument to this concern is that modern tax systems already condition on a great deal of personal information, such as number of children, marital status, and personal disabilities, without conditioning on many others. To argue that a height tax would lead to an over-reaching tax policy while these conditional taxes do not, one would have to believe that a height tax would trigger a descent down a slippery slope for tax policy. It seems

\(^{113}\) Ed Glaeser suggested this point.
more natural to think that a height tax could be endorsed on its own merits while taxes based on gender, for instance, could be resisted for the reasons currently applicable.

### 4.4.2 Costs missing from the conventional model

The next set of concerns sets aside political economy, but argues that a height tax is objectionable because it would have costs that are not reflected in the conventional optimal tax model.

One prominent example is stigma. Perhaps government transfers to the short, based on evidence that the short are less skilled on average, would lower short persons’ self-respect, an unmodeled component of welfare. Amartya Sen (1995) discusses this cost, among others, of transfers based on observable characteristics. Sen writes: "there are also direct costs and losses involved in feeling–and being–stigmatized. Since this kind of issue is often taken to be of rather marginal interest (a matter, allegedly, of fine detail), I would take the liberty of referring to John Rawls’s argument that self-respect is ‘perhaps the most important primary good’ on which a theory of justice as fairness has to concentrate..."

This cost may be particularly relevant for height, given that one explanation for the height wage premium relies upon the advantage it gives individuals in developing self-esteem (see Persico et al. 2005). Moreover, if height is a characteristic engendering discrimination, a height tax risks "institutionalizing" differential treatment based on height and thus perpetuating costly stigma. In fact, a colleague of ours who is shorter than average remarked that he would not want to receive a height transfer because it would be degrading.
The interesting question raised by this critique–that a transfer to short individuals would lower their self-esteem–is whether the same problem arises with transfers based on unobserved "ability." In fact, when Sen (1995) writes that "Any system of subsidy...that is seen as a special benefaction for those who cannot fend for themselves would tend to have some effects on their self-respect ....," it seems likely that a transfer designed for those who are low in general ability to "fend for themselves" would be particularly damaging to a recipient’s self-respect, perhaps even more so than a transfer based on a relatively narrow physical characteristic such as height. While stigma has been analyzed for some transfer programs such as the United States’ welfare program for poor families, it is rare to encounter an argument that taxpayers toward the bottom of the schedule of tax rates feel stigmatized by the implicit subsidy they receive from those at the top.

4.4.3 Critiques of the basic framework

Finally, we turn to the response that a height tax is not desirable because Utilitarianism is the wrong philosophical framework for determining optimal tax policy.

Utilitarianism is "the paradigm case of consequentialism," in that it relies solely on the consequences of an action–or a policy–to determine its desirability (Sinnott-Armstrong, 2006). For example, the means by which a policy achieves its ends or the motives of policy-makers are irrelevant to the desirability of a policy. Moreover, it is also the most prominent case of the "welfarist" subset of consequentialist philosophies, in that it is "motivated by the idea that what is of primary moral importance is the level of welfare of people" rather than, for instance, equality or liberty (Lamont, 2007). In this subsection, we discuss two
4.4 Perspectives from political philosophy

critiques of a height tax that can also be understood as critiques of the welfarist-Utilitarian framework in general: the Libertarian critique and the horizontal equity critique.

**Libertarianism**

Libertarians emphasize individual liberty and rights as the sole determinants of whether a policy is justified. In particular, any transfer of resources by policies that infringe upon individuals’ rights is deemed unjust from a Libertarian perspective. Hausman and McPherson (1996) discuss the views of Robert Nozick, a prominent Libertarian, by writing: "According to Nozick’s entitlement theory of justice, an outcome is just if it arises from just acquisition of what was unowned or by voluntary transfer of what was justly owned...Only remedying or preventing injustices justifies redistribution..." If the existing distribution of resources was generated by voluntary transfers between individuals, a Libertarian views that distribution as just and, therefore, any redistributive taxation as unjust.

Libertarians are skeptical of the redistribution of income or wealth because they believe that individuals are entitled to the returns on their justly-acquired endowments. Is height a "justly-acquired endowment?" On the one hand, height may seem to be an ideal example, given that it is assigned by nature. Thus, if individuals are entitled to the returns to their endowments, a height premium is a just source of inequality and the government ought not try to offset it with redistribution. It might be argued, however, that height is acquired in a more complicated way that is less obviously just. The mating decisions and health of past generations affects modern individuals’ heights, so if one’s ancestors unjustly
acquired the resources that generated one’s height today, height taxation could potentially be justified even within a Libertarian framework.

Whether one agrees with the Libertarian critique is of fundamental importance for tax policy. Unlike critiques that accept Utilitarianism, which are essentially quarrels with details about the height tax as a policy, the Libertarian critique questions the very basis of the dominant modern model of optimal taxation. It argues that differences in ability are not appropriate targets for redistribution so long as they are generated in a just manner. Even though these differences may mean suffering for some, the Libertarian critique argues that it is no other individual’s responsibility to remedy that suffering unless it has been generated by the violation of someone’s rights. These differences in ability are, in contrast, the basis of tax policy in the Utilitarian framework. While Utilitarian policy would recommend steeply progressive taxes if ability were observable, a Libertarian policy would not. For example, the prominent Libertarian Milton Friedman (1962) writes: "I find it hard, as a liberal, to see any justification for graduated taxation solely to redistribute income. This seems a clear case of using coercion to take from some in order to give to others."

At the root of the difference between the Libertarian and welfarist-Utilitarian conception of optimal tax policy is the relationship of the individual to the state. The welfarist-Utilitarian model sees the state as an entity outside the individuals who compose it, in that the government puts in place policies that are optimal according to its own social welfare function. This function is dependent upon the individuals’ welfare, but by combining them in a particular way the state assumes an authority to force individuals to act in ways with which they may disagree. In contrast, a Libertarian model sees the state as merely a
collection of individuals who agree to cooperate only insofar as it serves their individual interests. Thus, all contributions by individuals to the state’s activities must be voluntary, and the state has authority over individuals only insomuch as they wish to grant it. Once framed in these terms, it becomes clear why legal scholars (e.g., Hasen, 2006) have identified much the same tension between classically liberal theories of society and modern optimal tax theory as we have in this paper.

Though these perspectives seem to have little philosophical connection, one way that economists often frame them is to follow Harsanyi (1953, 1955) in thinking of the Utilitarian model as an *ex ante* model in which individuals set up society’s rules prior to knowing their position in society (in this case, their height) while the Libertarian model is an *ex post* model in which existing individuals cooperate to form a society with full knowledge of their endowments. Given this distinction, it is not surprising that these models yield starkly different recommendations.

**Horizontal Equity**

A second critique of the Utilitarian approach to taxation that has particular relevance for a height tax is based on the principle of horizontal equity. Traditionally, horizontal equity requires that people with a similar ability to pay taxes should pay similar taxes. Feldstein (1976) suggests a slightly different variant: "If two individuals would be equally well off (have the same utility level) in the absence of taxation, they should also be equally well off if there is a tax." Using either definition, the violation of horizontal equity by a height tax is glaring. In particular, return to the simulation from the previous section and
consider the taxes shown in Table 4.4. For any given wage, the amount of tax and the average tax rates rise substantially with height.

The conflict between horizontal equity and maximization of a Utilitarian social welfare function is not unique to a height tax. When ability is unobservable, as in the Vickrey-Mirrlees model, respecting horizontal equity means neglecting information on how any exogenous personal characteristic is related to ability. This information can make redistribution more efficient, as we have seen in the analysis above. In other words, as Kaplow (2001) points out, horizontal equity gives priority to a dimension of heterogeneity across individuals—ability—and focuses on equal treatment within the groups defined by that heterogeneity. He argues that it is difficult to think of a reason why that approach, rather than one which aims to maximize the well-being of individuals across all groups, is an appealing one. Why would society sacrifice potentially large gains for its members in order to preserve equal treatment of individuals within an arbitrarily-defined group?

Nevertheless, it is likely that concerns about horizontal equity limit the political viability of a height tax. As Auerbach and Hassett (1999) write, "...there is virtual unanimity that horizontal equity – the extent to which equals are treated equally – is a worthy goal of any tax system." For instance, it may be difficult to explain to a tall person that he has to pay more in taxes than a short person with the same earnings capacity because, as a tall person, he had a better chance of earning more.
4.5 Conclusion

The problem addressed in this paper is a classic one: the optimal redistribution of income. A Utilitarian social planner would like to transfer resources from high-ability individuals to low-ability individuals, but he is constrained by the fact that he cannot directly observe ability. In conventional analysis, the planner observes only income, which depends on ability and effort, and is deterred from the fully egalitarian outcome because taxing income discourages effort. If the planner’s problem is made more realistic by allowing him to observe other variables correlated with ability, such as height, he should use those other variables in addition to income for setting optimal policy. Our calculations show that a Utilitarian social planner should levy a sizeable tax on height. A tall person making $50,000 should pay about $4,500 more in taxes than a short person making the same income.

Height is, of course, only one of many possible personal characteristics that are correlated with a person’s opportunities to produce income. In this paper, we have avoided these other variables, such as race and gender, because they are intertwined with a long history of discrimination. In light of this history, any discussion of using these variables in tax policy would raise various political and philosophical issues that go beyond the scope of this paper. But if a height tax is deemed acceptable, tax analysts should entertain the possibility of using other such “tags” as well.

Many readers, however, will not so quickly embrace the idea of levying higher taxes on tall taxpayers. Indeed, when first hearing the proposal, most people recoil from it or are amused by it. And that reaction is precisely what makes the policy so intriguing. A tax on height follows inexorably from a well-established empirical regularity and the standard
approach to the optimal design of tax policy. If the conclusion is rejected, the assumptions must be reconsidered.

Our results, therefore, leave readers with a menu of conclusions. You must either advocate a tax on height, or you must reject, or at least significantly amend, the conventional Utilitarian approach to optimal taxation. The choice is yours, but the choice cannot be avoided.
Appendix A
Appendix to Chapter 1

This Appendix contains a variety of material, grouped into six sections:

- Implementation of the optimal allocations with nonlinear taxes

- Multiple generations

- The Hamiltonian approach to deriving intratemporal distortions in the baseline model with a continuous wage distribution
  - Results on high-income marginal distortions; relation to results of Saez (2001)

- Proofs of Propositions

- Specific expressions omitted from the main text

- Numerical illustrations omitted from the main text

A.1 Implementation of the optimal allocations

In this section I show how the allocations in the three baseline policy scenarios discussed in the main paper can be implemented with nonlinear labor income taxes. Then, I show how the Partial Reform (age-dependent) optimal allocations in Case 2 can be implemented with nonlinear labor income taxes. Similar reasoning applies to Cases 3 and 4.


A.1 Implementation of the optimal allocations

A.1.1 Baseline case

The tax authority sets labor income taxes $T(\cdot)$ to maximize social welfare. I consider three tax policies: Static Mirrlees, Partial Reform, and Full Optimum. In the Static Mirrlees policy, the labor income tax is a direct function of income only; in the Partial Reform it is a direct function of income and the taxpayer’s current age; while in the Full Optimum it is a direct function of the lifetime path of incomes and the taxpayer’s current age. Formally, write $T(y)$ for the Static Mirrlees policy, $T(y, t)$ for the Partial Reform policy, and $T\left(\{y(\cdot)\}_{t=1}^{T}, t\right)$ for the Full Optimum policy. Note that variation in the wage with age means that the tax $T(y)$ depends indirectly on age through income, even though the Static Mirrlees tax policy cannot directly depend on age. This is why the allocations $c^i_t$ and $y^i_t$ depend on age in the Static Mirrlees policy in the main paper.

Individuals maximize utility subject to these tax policies. The maximization problem of an individual of type $i$ is

\[
\max_{\{c,y\}_{t=1}^{T}} \sum_{t=1}^{T} \beta^{t-1} \left( u(c^i_t) - v \left( \frac{y^i_t}{w^i_t} \right) \right) \\
\text{subject to the individual’s budget constraint for each age } t \in \{1, 2, ..., T\} : \\
\]

\[
c^i_t = y^i_t - T(\cdot),
\]

where the arguments of the tax function depend on the policy scenario.

What we want to show is that the solutions to the Static Mirrlees, Partial Reform, and Full Optimum planner’s problems from the paper’s baseline model will be endoge-
nously chosen by individuals solving the above maximization subject to the appropriate tax function $T(\cdot)$ for each policy.

The following proposition states this formally for the Static Mirrlees policy:

**Proposition 9** For the solution $\{c^{s_i}_t, y^{s_i}_t\}_{i=1,t=1}^{I,T}$ to the Static Mirrlees planner’s problem from the baseline model, there exists a function of labor income $T^*(y)$ such that individuals solving the Individual Maximization problem subject to $T^*(y)$ choose $\{c^i_t, y^i_t\}_{i=1,t=1}^{I,T}$, where $c^i_t = c^{s_i}_t$ and $y^i_t = y^{s_i}_t$ for all $i$ and all $t$.

This proof resembles the approach in Kocherlakota (2005). Define

$$T^*(y) = \begin{cases} y^{s_i}_t - c^{s_i}_t, & \text{if } y = y^{s_i}_t \text{ for some } i, t \\ 2y, & \text{if } y \neq y^{s_i}_t \text{ for any } i, t \end{cases}$$

for each labor income $y$. Now, consider the individual of type $i$ and age $t$ who earns wage $w^i_t$ and solves the Individual Maximization problem above. For any $y$ not necessarily equal to $y^{s_i}_t$, it will consume $c^i_t = y - T^*(y)$, with $T^*(y)$ defined as above. Any $y$ not equal to some $y^{s_i}_t$ yields negative consumption and, therefore, negative infinite utility, so it is not chosen. Note that these punishing taxes on off-equilibrium incomes would be unnecessary if the income distribution were continuous—the discreteness of the wage distribution is required for computational purposes. If $y = y^{s_i}_t$ for some $i, t$, then the individual consumes $c^{s_i}_t$ because $u'(c)$ is always positive and the individual cannot save or borrow across periods. All that remains is to ensure that the individual with wage $w^i_t$ prefers to earn $y^{s_i}_t$ and consume $c^{s_i}_t$ than to pay some other tax $T^*(y^{s_j}_s)$, earn $y^{s_j}_s$ and consume $c^{s_j}_s$. This is ensured by the incentive constraints in the Static Mirrlees problem, so $T^*(y)$ implements the optimal allocation. QED
Now we turn to the Partial Reform policy.

**Proposition 10**  For the solution $\{c_t^{*i}, y_t^{*i}\}_{i=1,t=1}^{I,T}$ to the Partial Reform planner’s problem from the baseline model, there exists a function of labor income and age $T^*(y,t)$ such that individuals solving the Individual Maximization problem subject to $T^*(y,t)$ choose $\{c_t^i, y_t^i\}_{i=1,t=1}^{I,T}$, where $c_t^i = c_t^{*i}$ and $y_t^i = y_t^{*i}$ for all $i$ and all $t$.

For this policy, define

$$T^*(y,t) = \begin{cases} y_t^{*i} - c_t^{*i}, & \text{if } y = y_t^{*i} \text{ for some } i, \text{ given } t \\ 2y, & \text{if } y \neq y_t^{*i} \text{ for any } i, \text{ given } t \end{cases}$$

for each labor income $y$ and age $t$. Note that the function now conditions on age, so that two individuals earning a given income may pay different taxes if they are of different ages. Also, an individual earning an income not in the set of possible incomes for its age pays a high tax. Now, consider the individual of type $i$ and age $t$ who earns wage $w_t^i$ and solves the Individual Maximization problem above. Any $y$ not equal to some $y_t^{*i}$, given its age $t$, yields negative consumption and, therefore, negative infinite utility, so it is not chosen. If $y = y_t^{*i}$ for some $i$ given $t$, then the individual consumes $c_t^{*i}$ because $u'(c)$ is always positive and the individual cannot save or borrow across periods. The incentive constraints in the $(PR)$ problem ensure that the individual with wage $w_t^i$ at age $t$ prefers to earn $y_t^{*i}$ and consume $c_t^{*i}$ than to pay some other tax $T^*(y_t^{*j})$, earn $y_t^{*j}$ and consume $c_t^{*j}$. QED

Finally we turn to the Full Optimum policy. Let $y_t^i = (y_1, y_2, ... y_t)$ denote a labor income history.

**Proposition 11**  For the solution $\{c_t^{*i}, y_t^{*i}\}_{i=1,t=1}^{I,T}$ to the Full Optimum planner’s problem from the baseline model, there exists a function of the individual history of labor incomes...
and the current age $T^*(y^t, t)$ such that individuals solving the Individual Maximization problem subject to $T^*(y^t, t)$ choose $\{c^i_t, y^i_t\}_{i=1, t=1}^{1, T}$, where $c^i_t = c^*_t$ and $y^i_t = y^{*_i}_t$ for all $i$ and all $t$.

For this policy, define

$$T^*(y^t, t) = \begin{cases} y^i_t - c^i_t, & \text{if there is an } i : y^i_s \in y^t : y^i_s = y^{*_i}_s \text{ for all } s \in [1, 2, \ldots t] \\ 2y, & \text{if there is no such } i \end{cases}$$

for each labor income history $y^t$ and age $t$. Consider the individual of type $i$ and age $t$ who earns wage $w^i_t$ and solves the Individual Maximization problem above. When $t = 1$, any income not equal to $y^{*_i}_1$ for some $i$ yields negative consumption and, therefore, negative infinite utility, so it is not chosen. Suppose the individual chooses the income corresponding to some $i'$. Then, for $t > 1$, any income not equal to $y^{*_i'}_t$ for that $i'$ also yields negative consumption, so the individual chooses the age-specific income path corresponding to a single $i'$ throughout its life. The only remaining step is to ensure that the individual prefers the path assigned to its true type than any other type, which is ensured by the incentive constraints in the Full Optimum planner’s problem. QED

A.1.2 Case 2 Partial Reform

Now I show how the Case 2 Partial Reform allocations can be implemented with nonlinear labor income taxation. In principle, the private saving and borrowing that differentiates Case 2 from the baseline could complicate the translation from optimal allocations to taxes. It turns out, however, that the implementation is quite similar to that shown above.
The key result of this section is that the intratemporal distortions derived in the main paper are, in fact, the marginal taxes faced by individuals choosing among bundles of pre-tax and after-tax incomes \((y \text{ and } x)\), not pre-tax income and consumption \((y \text{ and } c)\) as in the baseline. To show this, we consider the following tax system:

\[
T^*(y, t) = \begin{cases} 
    y^{*i}_t - x^{*i}_t, & \text{if } y = y^{*i}_t \text{ for some } i, \text{ given } t \\
    T \cdot y, & \text{if } y \neq y^{*i}_t \text{ for any } i, \text{ given } t
\end{cases}
\]

By the same arguments as in the baseline case, individuals choosing subject to \(T^*(y, t)\) will choose the optimal allocations as derived in the main paper. The nonlinearity of the tax system, which severely punishes choices other than those within the set of optimal allocations (the size of the punishment can be made arbitrarily large to ensure that only optimal allocations are chosen). The proper choices within that set are guaranteed by the incentive compatibility constraints from the direct mechanism problem specified in the main paper.

Unlike in the baseline case, it is not immediate that the intratemporal distortions which are implied by the optimal \(\{x^{*i}_t, y^{*i}_t\}\) allocations are equal to marginal taxes on income that would implement these allocations. To find the marginal taxes that implement these allocations, we rewrite this tax system so that choosing each bundle \(\{x^{*i}_t, y^{*i}_t\}\) is equivalent to choosing a lump-sum grant \(\Omega^{*i}_t\) and a constant marginal tax on income \(y^{*i}_t\), namely \(\tau^{*i}_t\). Formally, \(y^{*i}_t - x^{*i}_t = y^{*i}_t \tau^{*i}_t - \Omega^{*i}_t\). We then show that, in fact, the marginal tax rates \(\tau^{*i}_t\) that implement the optimal allocations are the same as the intratemporal distortions derived in the main paper.

To see that this is true, suppose an individual of type \(i\) when age \(t\) considers earning one extra unit of \(y\), starting at \(\{x^{*i}_t, y^{*i}_t\}\). Given the "lump-sum/linear" tax system we
specified, this raises its after-tax income by $dx = (1 - \tau^*_{it})$. Because the individual can save and borrow, it spreads $dx$ across its lifetime. The resulting change in consumption for period $s \in \{1, 2, \ldots, T\}$ is $R^{s-1} \frac{dx}{T}$, which changes utility by $(\beta R)^{s-1} \frac{dx}{T} u'(c^*_{it})$, where $c^*_{it}$ is the optimal (constant) consumption level. Letting $\beta R = 1$ as in the main paper, the total utility change across all $T$ periods is $dxu'(c^*_{it})$. For the individual to choose $\{x^*_i, y^*_i\}$, individual maximization must therefore set $dxu'(c^*_{it}) - v'\left(\frac{y^*_i}{w^*_i}\right) \frac{1}{w^*_i} = 0$, i.e. the change in utility from earning an extra unit of $y$ must be zero at $\{x^*_i, y^*_i\}$. This simplifies to $dx = v'\left(\frac{y^*_i}{w^*_i}\right) \frac{1}{w^*_i u'(c^*_{it})}$, or using the expression for $dx$, we obtain $\tau^*_{it} = 1 - \frac{v'\left(\frac{y^*_i}{w^*_i}\right)}{w^*_i u'(c^*_{it})}$. Thus, the marginal tax rate that implements the optimal allocation is the same as the intratemporal distortion derived in the main paper. (We define $\Omega^*_{it} = x^*_{it} - \tau^*_{it} x^*_{it}$ to ensure that the rate $\tau^*_{it}$ applied to $y^*_{it}$ yields $x^*_{it}$ in after-tax income).

### A.2 Multiple generations

In this section, I discuss how the analysis of age dependence would change if there were multiple generations.

In the most direct extension of the analysis to multiple generations, little changes from the baseline model. Index generations with $g \in \{1, 2, \ldots, G\}$. Assume wages may depend on the generation and denote $\{w(i, g, t)\}_{t=1, g=0}^{T,G}$ as the wage for individual $i$ born in year $g$ and currently of age $t$. Denote the corresponding allocations of consumption and income $\{c(i, g, t)\}_{t=1, g=0}^{T,G}$ and $\{y(i, g, t)\}_{t=1, g=0}^{T,G}$. The planner’s problem is:
I retain the generation-specific feasibility constraints, so for each \( g \):

\[
\sum_{i=1}^{I} \sum_{t=1}^{T} R^{T-t} [y(i, g + t, t) - c(i, g + t, t)] = 0
\]

Generation-specific feasibility constraints capture a plausible moral restriction on policy. Relaxing them, and allowing for net transfers between generations, would introduce an additional purpose for tax policy that all three policy scenarios, if generation-dependent, could pursue. Depending on the importance of these transfers to social welfare, the Static Mirrlees may perform better relative to the Partial Reform and Full Optimum policies than in the analysis without intergenerational transfers (though the welfare ordering of the policies would be unchanged).

The incentive constraints reflect our assumption that the FO and PR planners can make taxes generation-dependent as well as age-dependent. In the Partial Reform, the incentive constraints are, for each \( g,t,i,i' \)

\[
\left( u(c(i, g + t, t)) - v\left( \frac{y(i, g + t, t)}{w(i, t)} \right) \right) \geq \left( u(c(i', g + t, t)) - v\left( \frac{y(i', g + t, t)}{w(i, t)} \right) \right),
\]

which means that each individual prefers its allocation to any other individual’s within its generation at each age.

For the Full Optimum, the incentive constraints are, for each \( g,i,i' \)

\[
\left[ \sum_{t=1}^{T} \beta^{t-1} \left( u(c(i, g + t, t)) - v\left( \frac{y(i, g + t, t)}{w(i, t)} \right) \right) \right] \geq \left[ \sum_{t=1}^{T} \beta^{t-1} \left( u(c(i', g + t, t)) - v\left( \frac{y(i', g + t, t)}{w(i, t)} \right) \right) \right],
\]
which means that each individual prefers its lifetime allocation to any other individual’s within its generation.

For the Static Mirrlees policy problem, there is a question of whether the planner ought to be allowed to condition on generation or not. If it cannot, generational conditioning widens the gap between the Static Mirrlees and Partial Reform policies relative to a single generation case. If it can, the incentive constraints are, for each \( g, t, t', i, i' \)

\[
\left( u(c(i, g + t, t)) - v \left( \frac{y(i, g + t, t)}{w(i, t)} \right) \right) \geq \left( u(c(i', g + t', t')) - v \left( \frac{y(i', g + t', t')}{w(i, t)} \right) \right),
\]

so that individuals must prefer their own allocation to any other individual’s of any age within its generation.

As is clear from these formal problems, generation dependence combined with generation-specific feasibility constraints imply that these policy problems are separable into generation-specific problems that are identical (up to changes in wage paths across generations) to the single generation problem considered in the main paper. Thus, the results of the main analysis carry through unchanged, robust to the inclusion of multiple generations.

### A.2.1 Calendar-year dependence

One interesting issue that arises once multiple generations are introduced is whether tax policy can be calendar-year dependent. In the Static Mirrlees policy, the combination of generation dependence and calendar year dependence yields age dependence, so to remain age-independent, the Static Mirrlees policy cannot be both generation-dependent and year-dependent. As shown above, the generation-dependent model closely resembles the
A.2 Multiple generations

How would assuming calendar-year dependence instead of generation dependence affect the results of our Static Mirrlees analysis?

**No real wage growth**

I begin with the case of no real wage growth. Then, we can show that calendar-year dependence is no different from generation dependence for the Static Mirrlees policy. To see this, note that the wage distributions will be the same for each generation, so the currently old have the same wage distribution as the currently young will have when old. Formally, the Static Mirrlees planner’s problem with calendar-year dependence would be identical to its problem with generation dependence except for the incentive constraints. In the year-dependent problem, the incentive constraints would be, for each $g, t, t', i, i'$:

$$
\left( u(c(i,g+t,t)) - v \left( \frac{y(i,g+t,t)}{w(i,t)} \right) \right) \geq \left( u(c(i',g+t,t')) - v \left( \frac{y(i',g+t,t')}{w(i',t)} \right) \right).
$$

I want to show that the solution to the Static Mirrlees problem with generation dependence is the same as that for the problem with calendar-year dependence when the age-dependent wage distributions are the same for all generations.

The outline of the proof is as follows. Start with the generation-dependent problem. The problem can be decomposed into separate optimizations for each generation: the objective and feasibility readily separate by generation and the incentive constraints are generation-specific by definition. However, each of these component problems is identical, so they have the same solutions. If not, I could choose the welfare maximizing solution (there must be one, since the objective is concave and the feasibility convex) and apply it to all generations, raising welfare. Thus, allocations are constant across generations in the
A.2 Multiple generations

generation-dependent problem with no real wage growth. These allocations also satisfy the incentive constraints of the year-dependent problem. To see this, take the ICs from the generation-dependent problem:

\[
\begin{align*}
(u(c(i, g + t, t)) - v \left( \frac{y(i, g + t, t)}{w(i, t)} \right)) & \geq (u(c(i', g + t', t')) - v \left( \frac{y(i', g + t', t')}{w(i, t)} \right)),
\end{align*}
\]

and use the result that allocations are constant across generations, so that \(c(i', g + t', t') = c(i, g + t, t')\) and \(y(i', g + t', t') = y(i, g + t, t')\) to substitute for the right-hand side and obtain:

\[
\begin{align*}
(u(c(i, g + t, t)) - v \left( \frac{y(i, g + t, t)}{w(i, t)} \right)) & \geq (u(c(i', g + t, t')) - v \left( \frac{y(i', g + t, t')}{w(i, t)} \right)),
\end{align*}
\]

which is the IC constraint for the year-dependent problem. Thus, the solution to the generation-dependent problem solves the year-dependent problem. Next, I show the reverse.

Start with the year-dependent Static Mirrlees problem, and recall that I assumed the age-dependent wage distributions are the same for all generations. Now, I will show that constant allocations across calendar years are optimal for this problem. Suppose not. Then, incentive-compatible allocations

\[
\{ c(i, g + s, t), y(i, g + s, t) \}_{i=1, t=1}^{T}
\]

and

\[
\{ c(i, g + s', t), y(i, g + s', t) \}_{i=1, t=1}^{T}
\]

differ for some \(s, s'\). Assume, without loss of generality, that \(s' > s\). Then, consider the generations that are of age \(T\) in year \(s\) and \(s'\), so that the first of these generations does not live to receive the allocations in year \(s'\). These generations receive different lifetime
paths of allocations. Because each generation enters symmetrically into the planner’s objective function and feasibility constraint, this cannot be optimal (i.e., if replace one generation’s allocations with the other’s, feasibility must hold because it is by generation, but social welfare must either increase or decrease, which is not optimal). Thus, the solution to the year-dependent problem is constant allocations across calendar years, or $c(i', g + t', t') = c(i', g + t, t')$ and $y(i', g + t', t') = y(i', g + t, t')$ for all $t, t'$. This is equivalent to constant allocations across generations, so we have shown the equivalence of the problems. In words, when the age-dependent wage distributions are the same for all generations, calendar-year dependence is equivalent to generation-dependence for the Static Mirrlees policy.

**Real wage growth**

The analysis is more complicated if real wage growth is positive. To take an extreme example, suppose that the wage distribution is the same across ages within a calendar year, so that real wage growth is the sole source of wage growth during a lifetime. In this situation, age- and generation-dependent policy allows for higher average taxes at older ages, transferring consumption to earlier ages and smoothing it over the lifetime (much as in the baseline analysis within a single generation), while taking advantage of higher earnings power at later ages. In contrast, the Static Mirrlees policy with calendar-year dependence cannot make such transfers, so it inefficiently allocates consumption and income across ages for each generation. This problem is worse than in the generation-dependent case because the wage distributions of the young and old in a calendar year now entirely overlap.
At the same time, this overlap of the wage distributions within a calendar year has a benefit to the Static Mirrlees planner, because it may be able to avoid some of the distortions it otherwise is forced to use. For example, if real wage growth is the sole source of wage growth, then the maximum wage for the young will also be the maximum wage for the mature in each calendar year, and neither would need to be distorted. This is a benefit of calendar-year dependence for the Static Mirrlees planner.

Generally, calendar-year dependence means that the Static Mirrlees policy acts on the cross-sectional distribution of wages rather than on the distribution of wages for a generation over time. This can generate both benefits (in terms of lower distortions) and costs (in terms of worse intertemporal allocations) for the Static Mirrlees planner. Illustrative simulations suggest that the combination of these effects can either increase or decrease the power of the Static Mirrlees policy relative to when it is generation-dependent.

A.3 Hamiltonian approach to baseline model

This section works through the Partial Reform policy in the baseline model using the classic approach to static Mirrlees models. To more easily map the approach to the paper’s analysis, I use discrete notation.

I simplify the analysis by assuming quasilinear utility, as in Diamond (1998):

\[ u(c, l) = c - v(l) \]

The social planner specifies a tax function \( T_t(wl) \), such that \( c = wl - T_t(wl) \), to maximize social welfare:
\[ W = \sum_i \pi^i \alpha^i \left( \sum_t \beta^{t-1} U(w^i_t) \right) \]

where all notation is as in the main paper, and period utility for individual \( i \) of age \( t \) is:

\[ U(w^i_t) = u \left( w^i_t l^i_t - T_t \left( w^i_t l^i_t \right) \right) - v \left( l^i_t \right). \]

Note that the tax function \( T_t(\cdot) \) can be age-specific (\( t \)-specific) in the Partial Reform policy.

Using quasilinearity, we rewrite period utility as:

\[ U(w^i_t) = w^i_t l^i_t - T_t \left( w^i_t l^i_t \right) - v \left( l^i_t \right). \]

The social planner’s maximization is subject to two sets of constraints, feasibility and incentive constraints.

The feasibility constraint is that taxes are purely redistributive and that the (single) generation of individuals must fund its consumption with income. Given quasilinear utility, and \( c = w l - T_t(w l) \), we can write the feasibility constraint as

\[ \sum_t \sum_i \pi^i R^{T-t} \left[ w^i_t l^i_t \left( w^i_t \right) - U \left( w^i_t \right) - v \left( l^i_t \left( w^i_t \right) \right) \right] = 0. \]

The incentive constraints are that each individual chooses labor supply to maximize utility, given the tax function. The individual solves:

\[ \max_{\{l^i_t\}_t} \left\{ \sum_t \beta^{t-1} \left( u \left( w^i_t l^i_t - T_t \left( w^i_t l^i_t \right) \right) - v \left( l^i_t \left( w^i_t \right) \right) \right) \right\} \]

which yields \( T \) first-order conditions of the form:

\[ FOC_{l^i_t} : u' \left( w^i_t l^i_t - T_t \left( w^i_t l^i_t \right) \right) w^i_t \left( 1 - T_t' \left( w^i_t l^i_t \right) \right) - v' \left( l^i_t \right) = 0. \]
Given quasilinear utility, these simplify to:

\[ FOC_{it} : w_t^i \left( 1 - T'_t \left( w_t^i l_t^i \right) \right) - v' \left( l_t^i \right) = 0 \]

for each \( i \) and each \( t \). As is conventional in what is called the "first order approach," I will replace the incentive constraints with these first order conditions when solving the planner’s problem. This approach relies on the Spence-Mirrlees condition and a second order condition that income rises with wage, both of which I assume hold (the second does hold in my numerical simulations).

For the analysis below, we use these first-order conditions to simplify differential constraints on the state variables of the planner’s problem. Those constraints are merely the envelope conditions of the planner’s objective with respect to \( w_t^i \), which are:

\[ \frac{\partial U(w_t^i)}{\partial w_t^i} = u' \left( w_t^i l_t^i - T_t \left( w_t^i l_t^i \right) \right) l_t^i \left( 1 - T'_t \left( w_t^i l_t^i \right) \right) . \]

Given quasilinear utility, these simplify to

\[ \frac{\partial U(w_t^i)}{\partial w_t^i} = l_t^i \left( 1 - T'_t \left( w_t^i l_t^i \right) \right) . \]

Substituting in the first-order conditions from above, we obtain:

\[ \frac{\partial U(w_t^i)}{\partial w_t^i} = l_t^i \frac{v' \left( l_t^i \right)}{w_t^i} . \]

Following Mirrles’ (1971) original approach, I represent the Partial Reform planner’s optimization problem in the baseline model with a Hamiltonian that is a function of utilities and labor effort only. Though this is an approach designed for a static model, it can be applied to this dynamic model because of time-separability, history-independence, no private saving, and Pareto-weights. In particular, these characteristics cause the planner’s
problem to separate into \( T \) age-specific problems, so I can write a set of Hamiltonians, one for each age \( t \), and then combine them. For each \( t \), I write the Hamiltonian:

\[
H_t = \alpha^i \frac{\pi^i}{dw_i^t} \beta^{t-1} U (w_i^t) + \lambda \frac{\pi^i}{dw_i^t} R^{T-t} \left[ w_i^t l_i^t (w_i^t) - U (w_i^t) - v \left( l_i^i (w_i^t) \right) \right] + \mu_t \frac{\partial U (w_i^t)}{\partial w_i^t},
\]

or

\[
H_t = \alpha^i \frac{\pi^i}{dw_i^t} \beta^{t-1} U (w_i^t) + \lambda \frac{\pi^i}{dw_i^t} R^{T-t} \left[ w_i^t l_i^t (w_i^t) - U (w_i^t) - v \left( l_i^i (w_i^t) \right) \right] + \mu_t l_i^t \frac{v' (l_i^t)}{w_i^t}.
\]

where \( dw_i^t \) is the distance between the wage level \( i \) and the next-highest wage. The multipliers on the feasibility constraint and the differential constraint are denoted \( \lambda \) and \( \mu \). To go from the first expression to the second, I have substituted in for the differential constraints with the first-order conditions as mentioned above. Each of these Hamiltonians is independent of any other \( t \), so we can combine them to obtain:

\[
H = \left\{ \begin{array}{l}
\sum_t \alpha^i \frac{\pi^i}{dw_i^t} \beta^{t-1} U (w_i^t) \\
+ \lambda \sum_t \frac{\pi^i}{dw_i^t} R^{T-t} \left[ w_i^t l_i^t (w_i^t) - U (w_i^t) - v \left( l_i^i (w_i^t) \right) \right] \\
+ \sum_t \mu_t l_i^t \frac{v' (l_i^t)}{w_i^t}
\end{array} \right\}
\]

The key result is derived from the planner’s first-order conditions with respect to labor effort, which are:

\[
FOC_{l_i^t} : \frac{\partial H}{\partial l_i^t} = \left[ \lambda \frac{\pi^i}{dw_i^t} \frac{dw_i^t}{dw_i^{t-1}} R^{T-t} \left( w_i^{t-1} - v' \left( l_i^{t-1} (w_i^{t-1}) \right) \right) + \mu_t \left( \frac{v' (l_i^t) + l_i^t v'' (l_i^t)}{w_i^t} \right) \right] = 0
\]

Now, we want to simplify these to be in terms of the tax function only. This involves using first-order conditions and conditions on the multipliers. As for the multipliers, note that Pontryagin’s Maximum Principle implies \( \mu_t (w_i^t) = -\frac{\partial H}{\partial U_i} \). Using the Hamiltonian, we can say:

\[
\mu_t (w_i^t) = \frac{\pi^i}{dw_i^t} \beta^{t-1} (\lambda R^{T-1} - \alpha^i),
\]
The transversality conditions are

\[ \mu(0) = \lim_{w_t \to \infty} \mu(w_t) = 0 \]

Now, integrate each of these multipliers’ FOCs from \( w \) to \( 1 \) and use the tranversality at the maximum wage to get:

\[ \left[ \mu_t(\infty) - \mu_t(w^i_t) \right] = -\mu_t(w^i_t), \]

\[ -\mu_t(w^i_t) = \sum_{w^j_t = w^i_t}^{w_t = \infty} \pi^j \beta^{t-1} \left( \lambda R^{T-1} - \alpha^j \right), \]

which yields

\[ \mu_t(w^i_t) = \sum_{w^j_t = w^i_t}^{w_t = \infty} \pi^j \beta^{t-1} \left( \alpha^j - \lambda R^{T-1} \right). \]

Next, use these expressions and both transversality conditions to get

\[ \left[ \mu_t(\infty) - \mu_t(0) \right] = 0 = \sum_{w^j_t = 0}^{w^i_t = \infty} \pi^j \beta^{t-1} \left( \alpha^j - \lambda R^{T-1} \right), \]

so

\[ \lambda = \frac{\sum_{w^j_t = 0}^{w^i_t = \infty} \pi^j \beta^{t-1} \alpha^j}{\sum_{w^j_t = 0}^{w^i_t = \infty} \pi^j \beta^{t-1} R^{T-1}} = \frac{1}{R^{T-1}} \sum_{w^j_t = 0}^{w^i_t = \infty} \pi^j \alpha^j, \]

which is constant across ages.

Now, define

\[ \Pi_t(w^i_t) = \sum_{w^j_t = 0}^{w^i_t} \pi^j, \]

as the proportion of \( t \)-aged individuals who have wages below \( w^i_t \). Also, define

\[ D_t(w^i_t) = \frac{1}{1 - \Pi_t(w^i_t)} \sum_{w^j_t = w^i_t}^{w^i_t = \infty} \pi^j \alpha^j. \]
as the average social marginal welfare weight of \( t \)-aged individuals who have wages above \( w_t^i \) (recall that utility is quasilinear, so \( u'(c) = 1 \)).

Thus,

\[
\lambda = \frac{1}{R^{T-1}} (1 - \Pi_t (0)) D_t (0) = \frac{1}{R^{T-1}} D_t (0) \quad \text{for all } t.
\]

Also, we can simplify the expression for \( \mu_t (w_t^i) \) to obtain:

\[
\mu_t (w_t^i) = \beta^{t-1} (1 - \Pi_t (w_t^i)) \left( D_t (w_t^i) - D_t (0) \right).
\]

Then, define the labor supply elasticity for person \( i \) of age \( t \) as:

\[
\varepsilon_t^i = \frac{w_t^i [1 - T'_t (w_t^i l_t^i)]}{l_t^i v''(l_t^i)} = \frac{v' (l_t^i)}{l_t^i v''(l_t^i)}.
\]

Finally, recall the individual FOCs:

\[
FOC_{l_t^i} : w_t^i (1 - T'_t (w_t^i l_t^i)) - v' (l_t^i) = 0
\]

Now simplify the planner’s FOCs using the conditions on \( \lambda \) and \( \mu_t \), the expression for the labor supply elasticity, and the individual FOCs. After some simplification, we obtain

\[
FOC_{l_t^i} : \frac{\partial H}{\partial l_t^i} = 0
\]

or,

\[
\left\{ D_t (0) \frac{\pi_t^i}{w_t^i} w_t^i T'_t (w_t^i l_t^i) + (1 - \Pi_t (w_t^i)) (D_t (w_t^i) - D_t (0)) \left( 1 + \frac{1}{\varepsilon_t^i} \right) (1 - T'_t (w_t^i l_t^i)) \right\} = 0
\]

which implies

\[
\frac{T'_t (w_t^i l_t^i)}{1 - T'_t (w_t^i l_t^i)} = \left( 1 - \frac{\Pi_t (w_t^i)}{\pi_t^i} \right) \frac{dw_t^i}{w_t^i} \left( 1 - \frac{D_t (w_t^i)}{D_t (0)} \right) \left( 1 + \frac{1}{\varepsilon_t^i} \right)
\]

This condition characterizes marginal labor income taxes in the Partial Reform model. It is identical to the condition derived in Kremer (2002), though the dynamic setting of this
derivation provides more detail on the determinants of the distributional terms $D_t(w^*_t)$ than Kremer’s static setting could. As derived in Diamond (1998), an analogous condition holds for the Static Mirrlees policy, but without the age subscripts. This seemingly small difference drives the disparities in intratemporal distortions identified in the main text.

A.3.1 Results on high-income marginal distortions; relation to Saez (2001)

In a footnote to the main text, I mentioned that the numerical simulation’s suggestion of lower optimal distortions on the high-income young under age dependence is not limited to the top earner, though the discrete model has trouble showing as much. Here, I use the same data as in the baseline numerical simulation to estimate the key components of the results from the Hamiltonian method above, which allows for the use of a much finer wage distribution.

Assuming constant elasticities across age (which is conservative, given that the young are likely to be more elastic), the key terms are $\left(\frac{1-\Pi_t(w^*_t)}{w^*_t\pi^*_t}\right)$ and $\left(1 - \frac{D_t(w^*_t)}{D_t(0)}\right)$. I call these the "hazard term" and "distribution term," respectively.

In Figures 1.A1, 1.A2, and 1.A3, I plot these terms for my baseline calibration against income for each age group. I take a moving average for the hazard term over a five dollar wage range to smooth out noise. In Figures 1.A1 and 1.A2, we can see that the two work in opposite directions—the hazard term is lower for the young and the distribution term is higher.

Figure 1.A3 shows that the hazard term is the more powerful factor and that the product of the two terms, and thus the recommended marginal tax, is substantially lower.
A.3 Hamiltonian approach to baseline model

Figure 1.A1: "Hazard Term"
Moving average over five wages
Figure 1.A2: "Distribution Term"
Figure 1.A3: Product of "Hazard" and "Distribution" Terms
for the young over a relatively wide range of high incomes. This is the source of the footnote in the main paper.

Saez (2002) stressed that skills appear to follow a Pareto distribution. For the purposes of this paper, the shape of the distribution is less important than that the product of the key terms is lower for the young than the old at high incomes, meaning lower distortions on the high-income young. Also note that our conclusion does not depend on quasilinearity. Diminishing marginal utility from consumption would increase the power of the distributional term, but the decline of the hazard term would continue to drive distortions lower for the young. Moreover, the highest consumption is likely to be at older ages, reinforcing our conclusion.

A.4 Proofs of Propositions

A.4.1 Proposition 1 (Intratemporal Benchmark)

We want to show that the Full Optimum (FO) planner chooses constant allocations. If that holds, then each other planner will do so as well, if feasible, because they can do no better than the FO.

Consider the first-order conditions of the Full Optimum planner. The intratemporal distortion derived from these FOCs is

\[
1 - \frac{u'(l_i)}{w_i^c r_i} = \frac{\sum_j \left(1 - \left(\frac{w_j}{w_i^c}\right)^a\right) \mu^{ij}}{\alpha^2 \pi^i + \sum_j \mu^{ij} - \sum_j \left(\frac{w_j}{w_i^c}\right)^a \mu^{ij}}.
\]
If the wage distribution is fixed across periods, all variables on the RHS are constant over time, so the distortion is identical across periods. Now consider the intertemporal first-order condition for consumption:

\[ u'(c^i_t) = u'(c^i_{t+1}) \]

which is the classic Atkinson-Stiglitz result for a deterministic economy. Thus, the FO planner chooses a constant consumption allocation for each individual \( i \). With the intratemporal distortion, this implies a constant income allocation, as well. Thus, the Full Optimum planner assigns a constant \( \{c, y\} \) allocation for each \( i \) over \( t \).

Both the Partial Reform (PR) and Static Mirrlees (SM) planner can do no better than the FO planner, as their constraint sets are subsets of its constraint set. Thus, if the FO allocation is feasible and incentive compatible, they will choose it. It is feasible for both because all three scenarios use the same feasibility condition. It is incentive compatible for the PR planner because the FO planner’s ICs with constant allocations can be rewritten as:

\[
(1 + \beta + ... \beta^{T-1}) \left( u(c^i) - v\left(\frac{y^i}{w^i}\right)\right) \geq (1 + \beta + ... \beta^{T-1}) \left( u(c^j) - v\left(\frac{y^j}{w^j}\right)\right).
\]

or

\[
\left( u(c^i) - v\left(\frac{y^i}{w^i}\right)\right) \geq \left( u(c^j) - v\left(\frac{y^j}{w^j}\right)\right).
\]

which holds, therefore, for each \( t \) and thus for the PR’s set of IC constraints. For the SM, cross-age deviations are replicas of intra-age deviations, given constant allocations and wages, so its IC set is identical to the PR’s, and the constant allocations are incentive compatible for it as well. Thus, all three scenarios choose constant allocations.
A.4.2 Proposition 2 (Top Marginal Distortion)

We begin with the first implication. Consider the expression for the Partial Reform (PR) distortion:

\[
1 - \frac{u'(l_i^*)}{w_i^*u'i(c_t^i)} = \frac{\sum_j \left(1 - \left(\frac{w_j^i}{w_t^i}\right)^\sigma\right) \mu_t^{ij}}{\alpha^\pi^i + \sum_j \mu_t^{ji} - \sum_j \left(\frac{w_j^i}{w_t^i}\right)^\sigma \mu_t^{ij}}.
\]

By the supposition, \(\frac{w_j^i}{w_t^i} > 1\) for all \(j\). We show that if \(\mu_t^{ij} > 0\) for any \(j\), the distortion must be negative. If, instead, \(\mu_t^{ij} = 0\) for all \(j\), the distortion is zero (the classic "no distortion on the top" case). The IC constraint for \(\mu_t^{ij}\) is

\[
\beta^{t-1} \left(u(c_t^i) - v\left(\frac{u_j^i}{w_t^i}\right)\right) \geq \beta^{t-1} \left(u(c_t^i) - v\left(\frac{u_j^i}{w_t^i}\right)\right).
\]

We can construct examples in which this IC binds, namely if redistributive preferences of the planner raise \(c_t^i\) high enough to offset higher \(y_t^i\). Thus, it is possible that \(\mu_t^{ij} > 0\) for some \(j\). If that is the case, then the numerator in the PR intratemporal distortion expression is negative. To prove the implication, the denominator must be positive. Any \(\mu_t^{ji} > 0\) will make this more likely, so suppose that \(\mu_t^{ji} = 0\) for all \(j\) to be conservative. Thus, if \(\alpha^\pi^i > \sum_j \left(\frac{w_j^i}{w_t^i}\right)^\sigma \mu_t^{ij}\), we have proved the result. The first-order condition for the PR planner with respect to \(l_i^*\) is:

\[
v'(l_i^*) \beta^{t-1} \left(\alpha^\pi^i - \sum_j \left(\frac{w_j^i}{w_t^i}\right)^\sigma \mu_t^{ij}\right) = R^{T-t}\lambda_{\pi^i}
\]

The multiplier on the feasibility constraint must be positive, i.e., \(\lambda > 0\), since marginal utility is always positive. We assume that disutility of labor is such that \(v'(l_i^*) > 0\). Thus, the first-order condition implies \(\alpha^\pi^i > \sum_j \left(\frac{w_j^i}{w_t^i}\right)^\sigma \mu_t^{ij}\), and we have proved the result. This proves that top distortions within each age are nonpositive for the PR scenario.
A similar proof holds for the FO scenario, where the intratemporal distortion is
\[ 1 - \frac{v'(l^i_t)}{w^i_t u'(c^i_t)} = \sum_j \left( 1 - \left( \frac{w^j_t}{w^i_t} \right)^\sigma \right) \mu^{ij} \]
and the relevant IC is
\[ \sum_t \beta^{t-1} \left( u(c^i_t) - v \left( \frac{y^j_t}{w^j_t} \right) \right) \geq \sum_t \beta^{t-1} \left( u(c^i_t) - v \left( \frac{y^j_t}{w^j_t} \right) \right). \]
If \( \mu^{ij} > 0 \) for some \( j \) and \( \frac{w^j_t}{w^i_t} > 1 \) for all \( j \) at \( t \) (by assumption) then the numerator of the FO distortion expression is negative. Again, the relevant FOC is
\[ \frac{1}{w^i_t} v' \left( \frac{y^j_t}{w^j_t} \right) \beta^{t-1} \left( \alpha^i \pi^i + \sum_j \mu^{ij} - \sum_j \left( \frac{w^j_t}{w^i_t} \right)^\sigma \mu^{ij} \right) = R^{T-t} \pi^i \lambda \]
so that the denominator must be positive and the distortion negative when \( \mu^{ij} > 0 \) for some \( j \). If \( \mu^{ij} = 0 \) for all \( j \), the distortion is zero. This completes the proof.

We now consider the second implication.

We want to show that no IC constraint corresponding to the multipliers \( \mu^{ij} \) will bind. I use variational arguments assuming non-increasing Pareto weights. I show the case for \( I = 2 \), but the same arguments can be used when \( I > 2 \). Suppose, contrary to the proposition, that \( \mu^{ij} > 0 \). The reverse IC (multiplied by \( \mu^{ij} \)) does not bind because \( \frac{w^j_t}{w^i_t} > 1 \). The incentive constraints imply that if \( \mu^{ij} > 0 \), then either (1) \( c^i_t > c^j_t \) or (2) \( y^i_t < y^j_t \) or (3) both \( c^i_t > c^j_t \) and \( y^i_t < y^j_t \). First, if \( c^i_t > c^j_t \) and \( y^i_t \leq y^j_t \), transfer \( \varepsilon \) units of consumption from \( i \) to \( j \) at age \( t \). These transfers are feasible and raise social welfare due to diminishing marginal utility of consumption and non-increasing Pareto weights. If these transfers proceed until \( c^i_t = c^j_t \), then \( \mu^{ij} = 0 \) because \( y^i_t \geq y^j_t \) and \( \frac{w^j_t}{w^i_t} > 1 \). If, however, these transfers cause \( \mu^{ij} > 0 \) with \( c^i_t > c^j_t \), then \( \mu^{ij} = 0 \) because \( i \) and \( j \) cannot
both be indifferent between the two allocations when \( \frac{w_i^t}{w_j^t} > 1 \). Second, if \( c_i^t \leq c_j^t \) and \( y_i^t < y_j^t \), increase \( y_i^t \) and decrease \( y_j^t \) by \( \delta \). This is feasible and raises social welfare due to increasing marginal disutility of labor (note \( l_i^t < l_j^t \)) and non-increasing Pareto weights. If this adjustment proceeds until \( \frac{1}{w_i^t} y_i^t = \frac{1}{w_j^t} y_j^t \), which implies \( y_i^t > y_j^t \), then \( \mu_{i,j}^{t} = 0 \) because \( c_i^t \leq c_j^t \). If this adjustment causes \( \mu_{i,j}^{t} > 0 \) with \( l_i^t < l_j^t \), then \( \mu_{i,j}^{t} = 0 \) because \( i \) and \( j \) cannot both be indifferent between the two allocations when \( \frac{w_i^t}{w_j^t} > 1 \). Finally, if \( c_i^t > c_j^t \) and \( y_i^t < y_j^t \), then use both the transfers of consumption and adjustments of income. If both proceed until \( c_i^t = c_j^t \) and \( l_i^t = l_j^t \), then \( \mu_{i,j}^{t} = 0 \) because \( y_i^t > y_j^t \). If not, then \( \mu_{i,j}^{t} > 0 \), implying \( \mu_{i,j}^{t} = 0 \). This shows that the incentive constraint preventing \( j \) from claiming \( i \)'s allocations cannot bind at the optimal policy. If \( I > 2 \), different \( j \)'s may be relevant at different ages. As long as \( \frac{w_i^t}{w_j^t} > 1 \) for all \( j \) and all \( t \), as assumed in the proposition, the same procedures as above can be applied.

### A.4.3 Proposition 3 (Intertemporal Benchmark)

This proposition is a direct consequence of the proof of Proposition 1, as it implies that all individuals face constant allocations over time.

### A.4.4 Proposition 4 (Symmetric Inverse Euler)

The proof manipulates the first order conditions from the planner’s problem in the baseline economy, Partial Reform scenario. These FOCs are, for consumption in periods \( t \) and \( t + 1 \) for individual \( i \):

\[
u' (c_i^t) \beta^{t-1} \left( \pi^i \alpha^i + \sum_j \mu_{i,j}^t - \sum_j \mu_{j,i}^t \right) = \lambda R^{T-t} \pi^i,
\]
\[ u' \left( c_{i+1}^t \right) \beta^t \left( \pi^i \alpha^i + \sum_j \mu^j_{t+1} - \sum_j \mu^j_{t+1} \right) = \lambda R^{T-t-1} \pi^i, \]

implying
\[
\beta^{t-1} \left( \pi^i \alpha^i + \sum_j \mu^j_t - \sum_j \mu^j_t \right) = \frac{\lambda R^{T-t} \pi^i}{u' \left( c_t^i \right)},
\]
\[
\beta^t \left( \pi^i \alpha^i + \sum_j \mu^j_{t+1} - \sum_j \mu^j_{t+1} \right) = \frac{\lambda R^{T-t-1} \pi^i}{u' \left( c_{i+1}^t \right)},
\]

Now, sum across \( i \) on each side:
\[
\beta^{t-1} \sum_{i=1}^I \left[ \pi^i \alpha^i + \sum_j \mu^j_t - \sum_j \mu^j_t \right] = \sum_{i=1}^I \left[ \frac{\lambda R^{T-t} \pi^i}{u' \left( c_t^i \right)} \right],
\]
\[
\beta^t \sum_{i=1}^I \left[ \pi^i \alpha^i + \sum_j \mu^j_{t+1} - \sum_j \mu^j_{t+1} \right] = \sum_{i=1}^I \left[ \frac{\lambda R^{T-t-1} \pi^i}{u' \left( c_{i+1}^t \right)} \right],
\]

which simplify (due to constraint multipliers that cancel and \( \beta R = 1 \)) to:
\[
\beta^{T-1} \sum_{i=1}^I \pi^i \alpha^i = \sum_{i=1}^I \frac{\lambda \pi^i}{u' \left( c_t^i \right)},
\]
\[
\beta^{T-1} \sum_{i=1}^I \pi^i \alpha^i = \sum_{i=1}^I \frac{\lambda \pi^i}{u' \left( c_{i+1}^t \right)},
\]

This implies the symmetric IEE
\[
\sum_{i=1}^I \frac{\pi^i}{u' \left( c_{i+1}^t \right)} = \sum_{i=1}^I \frac{\pi^i}{u' \left( c_t^i \right)},
\]

The SIEE can also be proven through a variational argument. In the Partial Reform policy, baseline scenario, lower utility for each individual at age \( t \) by \( \delta \) through providing lower consumption \( c \). This preserves incentive compatibility within age \( t \) and raises "revenue" of \( \frac{\delta}{u' \left( c_t^i \right)} \) from each individual, for a total of \( \sum_{i=1}^I \frac{\pi^i \delta}{u' \left( c_t^i \right)} \). Then, raise utility for each individual at age \( t + 1 \) by the same \( \delta \) through \( c \). This leaves total utility same for each individual, since \( \beta R = 1 \). It also preserves incentive compatibility within age \( t + 1 \). The "cost" is \( \frac{\delta}{u' \left( c_t^i \right)} \) for each individual, for a total of \( \sum_{i=1}^I \frac{\pi^i \delta}{u' \left( c_{i+1}^t \right)} \). If "revenue" exceeds
"cost", total utility can be increased with these operations. Since that’s not optimal, it must be that the optimal allocation satisfies: \( \sum_{i=1}^{I} \frac{u'(c_i)}{u'(c_{i+1})} = \sum_{i=1}^{I} \frac{u'(c_i)}{u'(c_{i+1})} \), the symmetric inverse Euler equation.

A.4.5 Proposition 5 (Baseline and Case 3 Equivalence)

The proof of this proposition is to show that the objective function, feasibility constraint, and incentive compatibility constraints are the same in each model for the Static Mirrlees and Partial Reform policies. Intuitively, the same paths of wages exist in each model ex post, so I assign each ex post wage path in the Case 3 model to a deterministic path in the baseline model. Because neither the planner nor the individuals can link one age to another in these two models, the problems are therefore identical.

I show the proof for the Static Mirrlees; the same approach applies to the Partial Reform.

The Case 3 feasibility condition is:

\[
\sum_{j=1}^{I} \sum_{t=1}^{T} R^{T-t} \pi_j^t \left( y_j^t - c_j^t \right) = 0.
\]

Replace \( j \) with \( k \) to avoid confusion

\[
\sum_{k=1}^{I} \sum_{t=1}^{T} R^{T-t} \pi_k^t \left( y_k^t - c_k^t \right) = 0.
\]

Then, use \( \pi_i^t = \sum_{i(t):w_i^t = w_i^k} \pi^{i(t)} \) to rewrite this as:

\[
\sum_{k=1}^{I} \sum_{t=1}^{T} R^{T-t} \sum_{i(t):w_i^t = w_i^k} \pi^{i(t)} \left( y_i^k - c_i^k \right) = 0.
\]
or, noting the summation over \( k \),

\[
\sum_{t=1}^{T} R_{T-t}^{T} \sum_{i(t)} \pi^{i(t)} (y_{t}^{i} - c_{t}^{i}) = 0.
\]

Now, use the proposition’s condition that there is a unique \( j \) such that \( w_{t}^{i} = w_{t}^{j} \) for all \( t \) and \( \pi^{i(t)} = \pi^{j} \).

\[
\sum_{t=1}^{T} R_{T-t}^{T} \sum_{j} \pi^{j} (y_{t}^{j} - c_{t}^{j}) = 0.
\]

which is the same as the baseline feasibility constraint, up to a change from \( j \) to \( i \) :

\[
\sum_{i=1}^{l} \sum_{t=1}^{T} R_{T-t}^{T} (y_{t}^{i} - c_{t}^{i}) = 0.
\]

For the incentive constraints, the Case 3 model’s constraints are of the form:

\[
\beta^{t-1} \left( u \left( c_{t}^{i} \right) - v \left( \frac{y_{t}^{i} - w_{t}^{i}}{w_{t}^{i}} \right) \right) \geq \beta^{t-1} \left( u \left( c_{t}^{k} \right) - v \left( \frac{y_{t}^{k} - w_{t}^{k}}{w_{t}^{k}} \right) \right),
\]

The proposition’s conditions allow us to replace each \( i_{t} \) in the Case 3 constraint with \( j_{t} \), yielding

\[
\beta^{t-1} \left( u \left( c_{t}^{j} \right) - v \left( \frac{y_{t}^{j} - w_{t}^{j}}{w_{t}^{j}} \right) \right) \geq \beta^{t-1} \left( u \left( c_{t}^{i} \right) - v \left( \frac{y_{t}^{i} - w_{t}^{i}}{w_{t}^{i}} \right) \right),
\]

exactly the baseline condition, up to a change from \( j \) to \( i \) and \( k \) to \( j \) :

\[
\beta^{t-1} \left( u \left( c_{t}^{j} \right) - v \left( \frac{y_{t}^{j} - w_{t}^{j}}{w_{t}^{j}} \right) \right) \geq \beta^{t-1} \left( u \left( c_{t}^{i} \right) - v \left( \frac{y_{t}^{i} - w_{t}^{i}}{w_{t}^{i}} \right) \right)
\]

Finally, for the objective function, the Case 3 objective is

\[
\max_{\{c,y\}} \sum_{j=1}^{l} \sum_{t=1}^{T} \beta^{t-1} \pi^{j} \alpha^{j} \left( u \left( c_{t}^{j} \right) - v \left( \frac{y_{t}^{j} - w_{t}^{j}}{w_{t}^{j}} \right) \right).
\]

Use \( \pi^{j}_{t} = \sum_{i(t):w_{t}^{i} = w_{t}^{j}} \pi^{i(t)} \) and \( \alpha^{j}_{t} = \frac{\sum_{i(t):w_{t}^{i} = w_{t}^{j}} \pi^{i(t)} (W_{t}^{i(t)})}{\sum_{i(t):w_{t}^{i} = w_{t}^{j}} \pi^{i(t)}} \) to rewrite the Case 3 objective as

\[
\max_{\{c,y\}} \sum_{j} \sum_{t} \beta^{t-1} \sum_{i(t):w_{t}^{i} = w_{t}^{j}} \pi^{i(t)} \alpha \left( W_{t}^{i(t)} \right) \left( u \left( c_{t}^{j} \right) - v \left( \frac{y_{t}^{j} - w_{t}^{j}}{w_{t}^{j}} \right) \right),
\]
Now, to avoid confusion, change $j$ to $k$,

$$\max_{\{c,y\}} \sum_k \sum_t \beta^{t-1} \sum_{i(t):w^i_t = w^k_t} \pi^{i(t)} \alpha \left( W_{T}^{i(t)} \right) \left( u \left( c_t^k \right) - v \left( \frac{y_t^k}{w_t^k} \right) \right).$$

Noting the summation over $k$, this is also

$$\max_{\{c,y\}} \sum_t \beta^{t-1} \sum_{i(t)} \pi^{i(t)} \left( W_{T}^{i(t)} \right) \left( u \left( c_t^i \right) - v \left( \frac{y_t^i}{w_t^i} \right) \right).$$

Then apply the proposition’s condition that there is a unique $j$ in the baseline model such that $w_t^i = w_t^j$ for all $t$ and $\pi^{i(t)} = \pi^j$, and note that $\alpha \left( W_{T}^{j} \right) = \alpha^j$ in the baseline (deterministic) model, so that this condition can be written

$$\max_{\{c,y\}} \sum_t \beta^{t-1} \sum_j \pi^j \alpha^j \left( u \left( c_t^j \right) - v \left( \frac{y_t^j}{w_t^j} \right) \right).$$

which is the same as the baseline objective function, up to a reordering of the summations and a change from $j$ to $i$:

$$\max_{\{c,y\}} \sum_{i=1}^{I} \pi^i \alpha^i \sum_{t=1}^{T} \beta^{t-1} \left( u \left( c_t^i \right) - v \left( \frac{y_t^i}{w_t^i} \right) \right).$$

Thus, the Static Mirrlees planner solves the same problems in the baseline and Case 3 models. The same procedure can be used to prove the result for the Partial Reform planner.

### A.5 Specific expressions omitted from text

#### A.5.1 Case 2: Intratemporal distortions for other types in two-type example

The condition for the intratemporal distortion on the high-skilled old worker is analogous:

$$1 - \frac{u'(\ell^H_2)}{w^H_2 u'(c^{HH}_2)} = \frac{\mu^{LH|HH}}{\alpha^H \pi^H + \mu^{HL|HH} + \mu^{LL|HH}} \cdot \left( \frac{c^{HH}_2}{c^L_2} - 1 \right)$$
so that if the worker is tempted to borrow and mimic the low-skilled worker when young, the planner levies a positive intratemporal distortion on the older worker to raise its marginal utility of consumption and discourage that deviation. Note that it is possible to have positive distortions on the high-skilled worker in both periods.

We can derive similar conditions for the marginal distortion on the low-skilled worker. For example, the intratemporal distortion in the first period on this worker is

\[1 - \frac{u'(I^L_i)}{w^L_i u'(c_1^{LL})} = \frac{1 - \left( \frac{w^L_i}{w^L_i} \right) \mu^{LL}_{HH} + \left( \frac{c^{LL}}{c^{LL}_i} - \frac{w^L_i}{w^L_i} \right) \mu^{LH}_{HH}}{\pi^L \alpha^L - \left( \frac{w^L_i}{w^L_i} \right) \mu^{LL}_{HH} - \left( \frac{w^L_i}{w^L_i} \right) \mu^{LH}_{HH}}.\]

**A.5.2 Case 4: SM and PR planners’ problems**

As stated in the main text, the objective function is:

\[
\max_{\{x,y\}} \left\{ \sum_i \pi^{i(t)} \alpha (W_{hi}(t)) \beta^{t-1} \left( u(c_t^i) - v \left( \frac{y_t^i}{w_t^i} \right) \right) \right\},
\]

and the feasibility constraint is:

\[
\sum_i \pi^{i(t)} \sum_t R^{T-t} (y_t^i - x_t^i) = 0.
\]

The incentive constraints in Case 4 for the Static Mirrlees and Partial Reform scenarios were described in words in the text because of their complexity.

For the Static Mirrlees, let

\[W_{hi}^{t(s)} (W_{hi}) = \{ w_{s_1}^{j_1} (w_1^i), w_{s_2}^{j_2} (w_2^i), ..., w_{s_t}^{j_t} (w_t^i), ..., w_{s_T}^{j_T} (w_T^i) \}\]

be a (potentially false) claimed path of wages that depends on the true path \(W_{hi}^t\). Using this notation, the incentive constraints for the Static Mirrlees planner are that, for all \(i, j \in \ldots\)
\{1, 2, ...I\} and all \(W_T^{j(s)} (W_T^i)\), the expression
\[
\left[ u \left( c_1 \left( W_T^i \right) \right) - v \left( \frac{y (w_{t+1}^i)}{w_t^i} \right) \right] + \sum_{t=1}^{T} \beta_{t+1} P_t, t+1 (i_t, i_{t+1}) \left( u \left( c_{t+1} \left( W_T^i \right) \right) - v \left( \frac{y (w_{t+1}^{j(s)}) (w_{t+1}^i)}{w_{t+1}^i} \right) \right)
\]

must be greater than or equal to
\[
\left[ u \left( c_1 \left( W_T^{j(s)} (W_T^i) \right) \right) - v \left( \frac{y (w_{s+1}^{j(s)} (w_1^i))}{w_t^i} \right) \right] + \sum_{t=1}^{T} \beta_{t+1} P_t, t+1 (i_t, i_{t+1}) \left( u \left( c_{t+1} \left( W_T^{j(s)} (W_T^i) \right) \right) - v \left( \frac{y (w_{s+1}^{j(s)} (w_{t+1}^i))}{w_{t+1}^i} \right) \right)
\]

where
\[
\left\{ c_t \left( W_T^{j(s)} (W_T^i) \right) \right\}_{t=1}^{T} = \arg \max_{\{c_t\}} \left\{ \text{expression (A.1)} \right\} \quad \text{s.t.} \sum_t R^{T-t} \left[ x (w_{s+1}^{j(s)} (w_t^i)) - c_t \right] = 0
\]

Note that the consumption path chosen by the individual is subject to its own feasibility constraint, as in Case 2, but that stochasticity means consumption may not be smooth. In fact, individuals will choose to satisfy a version of the intertemporal Euler equation that takes into account stochasticity:

\[
u' \left( c_t \left( W_T^{j(s)} (W_T^i) \right) \right) = \sum_{i+1} P_t, t+1 (i_t, i_{t+1}) \left[ u' \left( c_{t+1} \left( W_T^{j(s)} (W_T^i) \right) \right) \right].
\]

This Euler equation implies that consumption may not be smooth because, for example, an individual who saves in order to insure against a low future skill shock may have extra resources available late in life if it receives high skill shocks instead.

For the PR scenario, let \(W_T^{j(t)} (W_T^i) = \{w_1^j (w_1^i), w_2^j (w_2^i), ..., w_T^j (w_T^i)\}\) be a (potentially false) claimed path of wages that depends on the true path \(W_T^i\). Note that, in contrast to the Static Mirrlees, each wage claim must be of the same age as the true wage,
reflecting the planner’s ability to make age-dependent allocations. Using this notation, the incentive constraints for the Static Mirrlees planner are that, for all $i,j \in \{1,2,...,I\}$ and all $W^j_t (W^i_T)$, the expression

$$
\left[ u\left( c_1 \left( W^i_T \right) \right) - v \left( \frac{y \left( w^i_T \right)}{w^i_T} \right) \right] \\
+ \sum_{t=1}^{T} \beta^{t-1} \sum_{i_{t+1}} P_{t,t+1} (i_t, i_{t+1}) \left( u\left( c_{t+1} \left( W^i_T \right) \right) - v \left( \frac{y \left( w^{i+1}_{t+1} \right)}{w^{i+1}_{t+1}} \right) \right)
$$

must be greater than or equal to

$$
\left[ u\left( c_1 \left( W^j(t) \left( W^i_T \right) \right) \right) - v \left( \frac{y \left( w^j(t) \left( w^i_T \right) \right)}{w^j(t) \left( w^i_T \right)} \right) \right] \\
+ \sum_{t=1}^{T} \beta^{t-1} \sum_{i_{t+1}} P_{t,t+1} (i_t, i_{t+1}) \left( u\left( c_{t+1} \left( W^j(t) \left( W^i_T \right) \right) \right) - v \left( \frac{y \left( w^{j+1}_{t+1} \left( w^{i+1}_{t+1} \right) \right)}{w^{j+1}_{t+1} \left( w^{i+1}_{t+1} \right)} \right) \right)
$$

where

$$
\left\{ c_t \left( W^j(t) \left( W^i_T \right) \right) \right\}_{t=1}^{T} = \arg \max_{c_t} \left\{ \text{expression (A.2)} \right\} \text{ s.t. } \sum_{t} R^{T-t} \left( x \left( w^j_t \left( w^i_t \right) \right) - c_t \right) = 0
$$

Again, individuals will choose to satisfy a version of the intertemporal Euler equation that takes into account stochasticity:

$$
u' \left( c_t \left( W^j(t) \left( W^i_T \right) \right) \right) = \sum_{i_{t+1}} P_{t,t+1} (i_t, i_{t+1}) \left[ u' \left( c_{t+1} \left( W^j(t) \left( W^i_T \right) \right) \right) \right].$$

A.6 Numerical illustrations omitted from the main text
A.6 Numerical illustrations omitted from the main text

A.6.1 Baseline economy: Proposition 2 (Top Marginal Distortion)

A simple numerical example may help build intuition for the differences between scenarios. Consider an economy with only two individuals and two time periods, so that \( i = \{t, s\} \) and \( t = \{1, 2\} \). Consider the following wage paths in this two-by-two economy:

\[
\begin{bmatrix}
(1) & 8 & 12 \\
(2) & 12 & 16 
\end{bmatrix}
\]

Assuming the main paper’s standard parameterization, the intratemporal distortions in the policy scenarios are:

- **Static Mirrlees**:
  \[
  \begin{bmatrix}
  (t) & (s) \\
  (1) & 0.27 & 0.10 \\
  (2) & 0.10 & 0.00 
  \end{bmatrix}
  \]

- **Partial Reform**: 
  \[
  \begin{bmatrix}
  (t) & (s) \\
  (1) & 0.17 & 0.00 \\
  (2) & 0.15 & 0.00 
  \end{bmatrix}
  \]

- **Full Optimum**: 
  \[
  \begin{bmatrix}
  (t) & (s) \\
  (1) & 0.18 & 0.00 \\
  (2) & 0.15 & 0.00 
  \end{bmatrix}
  \]

The Static Mirrlees distortions differ dramatically from the Partial Reform. The Static Mirrlees distorts the young high earner, unlike either of the more sophisticated models, because it must prevent the older, low earner from mimicking that type. This new distortion forces the planner to distort the young low earner more as well, ensuring that the young high type will not mimic it. Finally, the old low earner faces a larger distortion than in the more sophisticated models because that is the only way for the planner to direct resources to it without tempting the old high earner.

A.6.2 Baseline economy: Lifecycle path of intratemporal distortions

We use a simple example to illustrate the ambiguity. Consider an economy with only two individuals and two time periods, so that \( i = \{t, s\} \) and \( t = \{1, 2\} \), and the following sets
of wage paths:

\[
\begin{bmatrix}
\text{Economy 1} \\
(t) & (s) \\
(1) & 8 & 12 \\
(2) & 12 & 16 \\
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
\text{Economy 2} \\
(t) & (s) \\
(1) & 8 & 12 \\
(2) & 12 & 18 \\
\end{bmatrix}.
\]

The only difference between these economies is that type \(s\)’s wage rises less in Economy 1 than in Economy 2 (the former was our example economy in the previous illustration).

The intratemporal distortions chosen by the Partial Reform planner in these cases are:

\[
\begin{bmatrix}
\text{Economy 1} \\
(t) & (s) \\
(1) & 0.17 & 0.00 \\
(2) & 0.15 & 0.00 \\
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
\text{Economy 2} \\
(t) & (s) \\
(1) & 0.18 & 0.00 \\
(2) & 0.24 & 0.00 \\
\end{bmatrix}.
\]

Type \(t\)’s wage has risen in each case, but its intratemporal distortions are higher in period \(t = 1\) in Economy 1 and in period \(t = 2\) in Economy 2. The switch is due to the difference in the path of \(s\)’s wages. In the second economy, the planner is willing to distort \(t\)’s choice in period \(t = 2\) more because doing so levies an inframarginal tax on a relatively higher-earning \(s\), discourages \(s\) from mimicking \(t\)’s allocation and allowing the planner to provide \(s\) with a less generous allocation.

### A.6.3 Baseline economy: Individuals with the same wage but different ages

To see this ambiguity for the Partial Reform planner in a simple example, consider an economy in which we add a third individual to our previous examples. With three individuals \(i = \{t, s, c\}\) and two periods \(t = \{1, 2\}\), we examine the following set of wage paths:

\[
\begin{bmatrix}
\text{Economy 1} \\
\tilde{w}_t^i (t) & (s) & (c) \\
(1) & 8 & 12 & 16 \\
(2) & 12 & 16 & 20 \\
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
\text{Economy 2} \\
\tilde{w}_t^i (t) & (s) & (c) \\
(1) & 8 & 12 & 18 \\
(2) & 12 & 16 & 20 \\
\end{bmatrix}.
\]
where $w_2^t = w_1^s$ in both economies, and the only difference between them is that $c$ earns more in $t = 1$ in Economy 2. The intratemporal distortions chosen by the Partial Reform planner are:

\[
\begin{bmatrix}
w(t)_1 & w(s)_1 & c(t)_1 \\
(1) & 0.21 & 0.16 & 0.00 \\
(2) & 0.20 & 0.14 & 0.00 \\
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
w(t)_2 & w(s)_2 & c(t)_2 \\
(1) & 0.23 & 0.23 & 0.00 \\
(2) & 0.20 & 0.15 & 0.00 \\
\end{bmatrix},
\]

Notice that a different individual earning $w = 12$ faces a larger intratemporal distortion in each economy. Intuitively, the wider gap at the top of the wage distribution for $t = 1$ in Economy 2 necessitates greater distortions on type $s$, flipping the relative size of the distortions on the two workers earning $w = 12$. As with the question on the lifecycle path of distortions, this example shows how difficult it is to make general predictions in these more sophisticated policy scenarios.

### A.6.4 Baseline economy: Intertemporal distortions example

Consider an economy with two worker types $i = \{t, s\}$, two ages $t = \{1, 2\}$, and wage paths as follows:

\[
\begin{bmatrix}
w^i(t) & w(s) \\
(1) & 8 & 12 \\
(2) & 16 & 12 \\
\end{bmatrix}.
\]

Note that the planner has little desire to redistribute income across $t$ and $s$, as they have roughly the same lifetime income-earning potential. The Full Optimum planner chooses the following allocations of pre-tax and after-tax income (consumption):

\[
\begin{bmatrix}
y^i(t) & y(s) \\
(1) & 5 & 12 \\
(2) & 21 & 12 \\
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
c^i(t) & c(s) \\
(1) & 12 & 12 \\
(2) & 12 & 12 \\
\end{bmatrix}.
\]
Note that the Full Optimum planner smooths consumption while tailoring \( t \)'s income to its wage path.

Suppose the Partial Reform planner tried to offer the Full Optimum allocation. Then, type \( s \) would prefer type \( t \)'s allocation in \( t = 1 \). While the Full Optimum planner can use the promise of a better future allocation to convince \( s \) to choose its own \( t = 1 \) allocation, the Partial Reform planner must satisfy incentives age-by-age. It cannot prevent \( s \) from mimicking \( t \) when \( t = 1 \) and then reverting to its true type when \( t = 2 \). In response, the Partial Reform planner’s allocations are:

\[
\begin{array}{c|cc}
\text{Partial Reform} & y^i_t (t) & y^i_t (s) \\
(1) & 6 & 11 \\
(2) & 19 & 13 \\
\end{array}
\quad \text{and} \quad
\begin{array}{c|cc}
\text{Partial Reform} & c^i_t (t) & c^i_t (s) \\
(1) & 10 & 14 \\
(2) & 14 & 10 \\
\end{array}
\]

Thus, the Partial Reform planner more closely aligns type \( t \)'s consumption to its wage path and skews \( s \)'s consumption in the other direction. In other words, it must use intertemporal distortions.

### A.6.5 Case 2: Discussion and illustrative example of intratemporal distortions

The cost of private saving to the Partial Reform planner is that it magnifies the incentive problems of redistribution to low-income individuals. Recall that any redistribution of after-tax income to low-income individuals within an age group will tempt high-earners of the same age to falsely claim the lower-income allocation. This temptation is worse with private saving and borrowing, because these high-earners can now transfer resources across...
ages to maintain an elevated level of consumption when they falsely claim a lower-income allocation.

At the same time, private saving and borrowing has a potentially substantial benefit for the planner. Recall, from the baseline model, that the Partial Reform planner cannot offer smooth consumption paths to individuals because it cannot keep track of their identities across ages. Private saving and borrowing allows agents to accomplish that smoothing on their own.

To make the planner’s choices more concrete, consider a two-by-two example economy, where the wage paths are:

\[
\begin{bmatrix}
  w^i_t & (t) & (s) \\
  (1) & 10 & 12 \\
  (2) & 12 & 18 \\
\end{bmatrix},
\]

The intratemporal distortions chosen by the Partial Reform planner in the baseline model and in the Case 2 model with private saving and borrowing are:

\[
\begin{bmatrix}
  \text{Baseline} \\
  (t) & (s) \\
  (1) & 0.03 & 0.00 \\
  (2) & 0.14 & 0.00 \\
\end{bmatrix}, \quad \text{and} \quad \begin{bmatrix}
  \text{Case 2} \\
  (t) & (s) \\
  (1) & 0.02 & 0.04 \\
  (2) & 0.09 & 0.003 \\
\end{bmatrix}
\]

The planner in Case 2, where individuals can privately save and borrow, distorts the high-earner in both periods. This combats the temptation for type \( s \) to oversave or overborrow. In this example, as in the empirical data, wage paths are rising, so the risk of overborrowing is especially great.
Appendix B
Appendix to Chapter 2

B.1 General utility function analysis

The analysis of the main paper extends readily to a general utility function. Start by representing preferences with the consumption preferences utility function:

$$u(c, \frac{y}{w}, \theta) = \theta u(c) - v\left(\frac{y}{w}\right).$$

Laissez-faire optimization is subject to the budget constraint

$$w_m l_{m,i} = c_{m,i},$$

because there are no transfers between individuals in the laissez-faire scenario. This optimization yields the results:

$$u'(c_{m,i}) = \frac{1}{\theta_i w_m} v'\left(\frac{y_{m,i}}{w_m}\right)$$

which we can combine with the budget constraint to derive an implicit expression for income as a function of parameters:

$$\frac{u'(y_{m,i})}{v'\left(\frac{y_{m,i}}{w_m}\right)} = \frac{1}{\theta_i w_m}$$

Now, suppose the planner offers a lumpsum grant $\Delta$ to each $i$. Then, the new budget constraint is

$$w_m l_{m,i} + \Delta = c_{m,i}.$$
so i’s chosen bundle is then
\[
\frac{u'(y_{m,i} + \Delta)}{v'(\frac{y_{m,i}}{w_m})} = \frac{1}{\theta_i w_m}
\]

Now, applying the implicit function theorem, we can say that
\[
u'(c_{m,i}) - \frac{1}{\theta_i w_m} v'(l_{m,i}) = 0
\]
implies
\[
d_{c_{m,i}} = \frac{1}{\theta_i w_m} v''(l_{m,i}) d_{l_{m,i}}.
\]
We can combine this with the budget constraint that
\[
w_m d_{l_{m,i}} + \Delta = d_{c_{m,i}}
\]
to get
\[
dl_{m,i} = \frac{-u''(c_{i,j})}{w_m u''(c_{i,j}) - \frac{1}{\theta_i w_m} v''(l_{m,i})} \Delta.
\]
so
\[
d_{c_{m,i}} = \Delta \left( 1 - \frac{w_m}{w_m u''(c_{i,j}) - \frac{1}{\theta_i w_m} v''(l_{m,i})} \frac{u'(c_{i,j})}{u''(c_{i,j}) - \frac{1}{\theta_i w_m} v''(l_{m,i})} \right)
\]
\[
d_{c_{m,i}} = \frac{-\frac{1}{\theta_i w_m} v''(l_{m,i})}{w_m u''(c_{i,j}) - \frac{1}{\theta_i w_m} v''(l_{m,i})} \Delta
\]
Now, an SWF that would choose the same bundle for \(m, i\) if choosing on behalf of \(m, i\) can be written:
\[
W = \sum_{m=1}^{M} p_m \sum_{i=1}^{I} \pi_i \frac{1}{\psi_{m,i}^{i,j}} \left( \theta_i u'(c_{m,i}) - v'\left(\frac{y_{m,i}}{w_m}\right) \right).
\]
where \(\psi_{m,i}^{i,j}\) is a value to be derived. The assumption made above was that the marginal social welfare of allocating \(\Delta\) to any individual should be the same, conditional on her wage. Marginal social welfare is:
\[
W_{m,i} = p_m \pi_i \frac{1}{\psi_{m,i}^{i,j}} \left( \theta_i u'(c_{m,i}) d_{c_{m,i}} - v'\left(\frac{y_{m,i}}{w_m}\right) d \left(\frac{y_{m,i}}{w_m}\right) \right).
\]
into which we can substitute the results from above and simplify to obtain

\[
W_{m,i} = p_m \pi_i \frac{1}{\psi_{m,i}} \left( \frac{\theta_i u'(c_{m,i})}{\frac{1}{w_m u''(c^{1,j})} - \frac{1}{\theta_i w_m u''(l_{m,i})} \Delta} \right)
\]

or

\[
W_{m,i} = p_m \pi_i \frac{1}{\psi_{m,i}} \frac{1}{w_m} \left( \frac{w_m u''(c^{1,j}) v' \left( \frac{y_{m,i}}{w_m} \right)}{w_m u''(c^{1,j}) - \frac{1}{\theta_i w_m} v''(l_{m,i})} \right) \Delta,
\]

using the laissez-faire allocation,

\[
u'(c_{m,i}) = \frac{1}{\theta_i w_m} v' \left( \frac{y_{m,i}}{w_m} \right)
\]

this is

\[
W_{m,i} = p_m \pi_i \frac{1}{\psi_{m,i}} \frac{1}{w_m} \left( \frac{w_m u''(c^{1,j}) v' \left( \frac{y_{m,i}}{w_m} \right)}{w_m u''(c^{1,j}) - \frac{1}{\theta_i w_m} v''(l_{m,i})} \right) \Delta,
\]

To make marginal social welfare constant across individuals with the same wage, and again using laissez-faire allocations to simplify, we can choose

\[
\psi_{m,i} = \theta_i w_m u' \left( c_{m,i}^* \right).
\]

Note that the factors \( \psi_{m,i} \) may depend on both taste and skill.

This implies an SWF of the form

\[
W = \sum_{m=1}^{M} p_m \sum_{i=1}^{I} \pi_i \left( \frac{1}{w_m u'(c_{m,i}^*)} u'(c_{m,i}) - \frac{1}{\theta_i w_m u' \left( c_{m,i}^* \right)} v' \left( \frac{y_{m,i}}{w_m} \right) \right).
\]

Note that \( u' \left( c_{m,i}^* \right) = \left( w_m \left( \frac{\theta_i}{2} \right) \right)^{-1} \) for the simple model above, so result (24) reduces to the SWF from the main paper for a log-quadratic utility function, as expected.
B.1 General utility function analysis

B.1.1 Log-isoelastic example

For instance, suppose the utility function is a slight generalization of the log-quadratic function used in the main paper:

\[ u\left( c_i, \frac{y_i}{w}, \theta \right) = \theta_i \ln (c_i^*) - \frac{1}{\sigma} \left( \frac{y_i^*}{w} \right)^\sigma. \]

Then, the FOCs and the private budget constraint imply the laissez-faire allocation:

\[ c_i^* = y_i^* = \theta_i^{\frac{1}{\sigma}} w \]

and the SWF is

\[ W = \sum_{m=1}^M p_m \sum_{i=1}^I \pi_i \left( \theta_i^{\frac{1}{\sigma}} \ln (c_{m,i}) - \frac{1}{\sigma\theta_i^{\frac{\sigma-1}{\sigma}}} \left( \frac{y_{m,i}}{w_m} \right)^\sigma \right). \]

Here, average tax rates are constant across wage types for a given preference type in the first-best planner’s problem, just as in the main text section 3.2. To see this, note that the optimal allocations satisfy:

\[ c_{m,i} = \left( \frac{\theta_i}{\theta_j} \right)^{\frac{1}{\sigma}} c_{m,j} \]
\[ y_{m,i} = \left( \frac{\theta_i}{\theta_j} \right)^{\frac{1}{\sigma}} y_{m,j} \]

so

\[ \frac{y_{m,i} - c_{m,i}}{y_{m,i}} = \left( \frac{\theta_i}{\theta_j} \right)^{\frac{1}{\sigma}} \frac{y_{m,j} - c_{m,j}}{y_{m,j}} = \left( \frac{\theta_i}{\theta_j} \right)^{\frac{1}{\sigma}} \left( \frac{\theta_i}{\theta_j} \right)^{\frac{1}{\sigma}} = \left( \frac{\theta_i}{\theta_j} \right)^{\frac{2}{\sigma}} \left( \frac{\theta_i}{\theta_j} \right)^{\frac{1}{\sigma}} = \frac{y_{m,j} - c_{m,j}}{y_{m,j}}. \]

B.1.2 CRRA-isoelastic example

Or, to generalize further, suppose the utility function is

\[ u\left( c, \frac{y}{w}, \theta \right) = \theta (c^*)^{1-\gamma} - \frac{1}{\sigma} \left( \frac{y^*}{w} \right)^\sigma. \]
Then, the FOCs and the private budget constraint imply the laissez-faire allocation:

\[ c_i^* = y_i^* = (\theta_i)^{1/(\sigma + \gamma - 1)} \, w_i^{\sigma/(\sigma + \gamma - 1)} \]

and the SWF is

\[ W = \sum_{m=1}^{M} \sum_{i=1}^{I} \pi_i \left( \left( \frac{(1-\gamma)(1-\sigma)}{\sigma+\gamma-1} \right) \theta_i^{\sigma/(\sigma+\gamma-1)} \left( c_{m,i} \right)^{1-\gamma} - \frac{1}{\sigma \theta_i^{\sigma/(\sigma+\gamma-1)}} \left( y_{m,i} \right)^{\sigma} \right) . \]

As in the previous examples, average tax rates are the same across preferences given the wage for this utility function. To see why, note that the optimal allocations satisfy:

\[ c_{m,i} = \left( \frac{\theta_i}{\theta_j} \right)^{1/(\sigma + \gamma - 1)} c_{m,j} \]

\[ y_{m,i} = \left( \frac{\theta_i}{\theta_j} \right)^{1/(\sigma + \gamma - 1)} y_{m,j} \]

so

\[ \frac{y_{m,i} - c_{m,i}}{y_{m,i}} = \left( \frac{\theta_i}{\theta_j} \right)^{1/(\sigma + \gamma - 1)} \frac{y_{m,i} - c_{m,i}}{y_{m,i}} = \frac{y_{m,j} - c_{m,j}}{y_{m,j}} \]

just as before.

### B.2 Intertemporal applications

#### B.2.1 Solving the disability model

The planner’s problem is

\[
\max_{c,y} \sum_{i=1}^{I} \pi_i \left( \left( \theta_i \right)^{\frac{1}{2}} u \left( C_i \right) - \left( \theta_i \right)^{-\frac{1}{2}} v \left( Y_i \right) + \beta \sum_{m=1}^{M} p_m \left( \left( \theta_i \right)^{\frac{1}{2}} u \left( c_{m,i} \right) - \left( \theta_i \right)^{-\frac{1}{2}} v \left( y_{m,i} \right) \right) \right) ,
\]

s.t.

\[
\sum_{i=1}^{I} \pi_i \left( (Y_i - C_i) R + \sum_{m=1}^{M} p_m (y_{m,i} - c_{m,i}) \right) \geq 0 ,
\]
and

\[
\begin{bmatrix}
(\theta_h)^{\frac{1}{2}} u(C_h) - (\theta_h)^{-\frac{1}{2}} v(Y_h) \\
+ \beta \sum_{m=1}^{M} p_m \left( (\theta_h)^{\frac{1}{2}} u(c_{m,h}) - (\theta_h)^{-\frac{1}{2}} v \left( \frac{y_{m,h}}{y_{m,t}} \right) \right) \\
(\theta_t)^{\frac{1}{2}} u(C_t) - (\theta_t)^{-\frac{1}{2}} v(Y_t) \\
+ \beta \sum_{m=1}^{M} p_m \left( (\theta_t)^{\frac{1}{2}} u(c_{m,t}) - (\theta_t)^{-\frac{1}{2}} v \left( \frac{y_{m,t}}{y_{m,t}} \right) \right) \\
\geq \\
(\theta_h)^{\frac{1}{2}} u(c_{a,h}) - (\theta_h)^{-\frac{1}{2}} v \left( \frac{y_{a,h}}{w_a} \right) \\
(\theta_t)^{\frac{1}{2}} u(c_{a,t}) - (\theta_t)^{-\frac{1}{2}} v \left( \frac{y_{a,t}}{w_a} \right) \\
(\theta_h)^{\frac{1}{2}} u(c_{d,h}) - (\theta_h)^{-\frac{1}{2}} v \left( \frac{y_{d,h}}{w_a} \right) \\
(\theta_t)^{\frac{1}{2}} u(c_{d,t}) - (\theta_t)^{-\frac{1}{2}} v \left( \frac{y_{d,t}}{w_a} \right)
\end{bmatrix},
\]

The multipliers on these constraints are \(\lambda, \rho_h, \rho_t, \mu_h, \mu_t, \mu_{h|t}\) and \(\mu_{t|h}\).

Solving the planner's problem yields the FOCs:

\[
(\theta_h)^{\frac{1}{2}} u'(C_h) \left( \frac{1}{2} + \rho_h - \left( \frac{\theta_t}{\theta_h} \right)^{\frac{1}{2}} \rho_t \right) = \frac{1}{2} \lambda R
\]

\[
(\theta_t)^{\frac{1}{2}} u'(C_t) \left( \frac{1}{2} - \left( \frac{\theta_h}{\theta_t} \right)^{\frac{1}{2}} \rho_h + \rho_t \right) = \frac{1}{2} \lambda R
\]

\[
u' (c_{a,h}) \left( \beta \frac{1}{4} (\theta_h)^{\frac{1}{2}} + \beta (\theta_h)^{\frac{1}{2}} \mu_h + (\theta_t)^{\frac{1}{2}} \mu_{h|t} \right) = \frac{1}{4} \lambda
\]

\[
u' (c_{d,h}) \left( \beta \frac{1}{4} (\theta_h)^{\frac{1}{2}} - \beta (\theta_h)^{\frac{1}{2}} \mu_h - (\theta_t)^{\frac{1}{2}} \mu_{h|t} \right) = \frac{1}{4} \lambda
\]

\[
u' (c_{a,t}) \left( \beta \frac{1}{4} (\theta_t)^{\frac{1}{2}} + \beta (\theta_t)^{\frac{1}{2}} \mu_t + (\theta_h)^{\frac{1}{2}} \mu_{h|t} \right) = \frac{1}{4} \lambda
\]

\[
u' (c_{d,t}) \left( \beta \frac{1}{4} (\theta_t)^{\frac{1}{2}} - \beta (\theta_t)^{\frac{1}{2}} \mu_t - (\theta_h)^{\frac{1}{2}} \mu_{h|t} \right) = \frac{1}{4} \lambda
\]

These yield:

\[
\left( (\theta_h)^{\frac{1}{2}} + (\theta_t)^{\frac{1}{2}} \right) = \lambda R \left( \frac{1}{\nu'(C_h)} + \frac{1}{\nu'(C_t)} \right)
\]
and
\[
\frac{1}{2} \left( (\theta_h)^{\frac{1}{2}} + (\theta_l)^{\frac{1}{2}} \right) = \frac{1}{4\beta} \lambda \left( \frac{1}{u'(c_{a,h})} + \frac{1}{u'(c_{d,h})} + \frac{1}{u'(c_{a,l})} + \frac{1}{u'(c_{d,l})} \right)
\]
which simplifies to:
\[
\beta R = \frac{1}{\left( \frac{1}{u'(C_h)} + \frac{1}{u'(C_l)} \right)^2} \left( \frac{1}{u'(c_{a,h})} + \frac{1}{u'(c_{d,h})} + \frac{1}{u'(c_{a,l})} + \frac{1}{u'(c_{d,l})} \right)
\]

Preference-specific Euler equations would be:
\[
\frac{1}{u'(C_h)} = \frac{1}{\beta R} \left( \frac{1}{2} + \rho_h - \left( \frac{\theta_l}{\theta_h} \right)^{\frac{1}{2}} \rho_l \right) \left( \frac{1}{u'(c_{a,h})} + \frac{1}{u'(c_{d,h})} \right)
\]
\[
\frac{1}{u'(C_l)} = \frac{1}{\beta R} \left( \frac{1}{2} - \left( \frac{\theta_h}{\theta_l} \right)^{\frac{1}{2}} \rho_h + \rho_l \right) \left( \frac{1}{u'(c_{a,l})} + \frac{1}{u'(c_{d,l})} \right)
\]

### B.2.2 The SWF for capital taxes

The laissez-faire allocations solve:
\[
\max_{c,y} \left( \ln (C_i) - \frac{1}{2} (Y_i)^2 + \beta_i \left[ \ln (c_i) - \frac{1}{2} \left( \frac{y_i}{E[w_m]} \right)^2 \right] \right)
\]
s.t.
\[(Y_i - C_i) R + [y_i - c_i] \geq 0.\]

FOCs are:
\[C_i = \frac{1}{\lambda R}, \quad Y_i = \lambda R \]
\[c_i = \frac{1}{\lambda} \beta_i, \quad y_i = \frac{(E[w_m])^2}{\beta_i} \lambda, l_i = \frac{E[w_m]}{\beta_i} \lambda\]

so, in the b.c.
\[
\left( \lambda R - \frac{1}{\lambda R} \right) R + \frac{(E[w_m])^2}{\beta_i} \lambda - \frac{1}{\lambda} \beta_i \geq 0
\]
\[
\lambda = \frac{\sqrt{\beta_i (1 + \beta_i)}}{\sqrt{\beta_i R^2 + (E[w_m])^2}}
\]
so

\[ C_i = \frac{\sqrt{\beta_i R^2 + (E[w_m])^2}}{R \sqrt{\beta_i (1 + \beta_i)}} \]

\[ Y_{m,i} = \frac{\sqrt{\beta_i (1 + \beta_i)}}{\sqrt{\beta_i R^2 + (E[w_m])^2}} R \]

\[ c_i = \frac{\sqrt{\beta_i R^2 + (E[w_m])^2}}{\beta_i} \]

\[ y_i = \frac{(E[w_m])^2 \sqrt{\beta_i (1 + \beta_i)}}{\beta_i \sqrt{\beta_i R^2 + (E[w_m])^2}} \]

\[ l_i = \frac{E[w_m]}{\beta_i} \frac{\sqrt{\beta_i (1 + \beta_i)}}{\sqrt{\beta_i R^2 + (E[w_m])^2}} \]

While if there were a lumpsum \( \Delta \) granted to individual \( i \), plugging into the b.c. would give:

\[ \left( \lambda R - \frac{1}{\lambda R} + \Delta \right) R + \frac{(E[w_m])^2}{\beta_i} \lambda - \frac{1}{\lambda} \beta_i \geq 0 \]

\[ \lambda^2 \left( \frac{\beta_i R^2 + (E[w_m])^2}{\beta_i} \right) + \lambda \Delta R - (1 + \beta_i) = 0 \]

\[ \lambda = \frac{-\Delta R + \sqrt{\Delta^2 R^2 + 4 \frac{\beta_i R^2 + (E[w_m])^2}{\beta_i} (1 + \beta_i)}}{2 \left( \frac{\beta_i R^2 + (E[w_m])^2}{\beta_i} \right)} \]

which, as \( \Delta \to 0 \), is just

\[ \lambda = \frac{-\Delta \beta_i R + 2 \sqrt{\beta_i R^2 + (E[w_m])^2} \sqrt{\beta_i (1 + \beta_i)}}{2 \left( \beta_i R^2 + (E[w_m])^2 \right)} \]

\[ \lambda = \frac{-\Delta R \beta_i}{2 \left( \beta_i R^2 + (E[w_m])^2 \right)} + \frac{\sqrt{\beta_i (1 + \beta_i)}}{\sqrt{\beta_i R^2 + (E[w_m])^2}} \]

so

\[ C_i = \frac{2 \left( \beta_i R^2 + (E[w_m])^2 \right)}{-\Delta \beta_i R^2 + 2 \sqrt{\beta_i R^2 + (E[w_m])^2} \sqrt{\beta_i (1 + \beta_i)} R} \]

\[ Y_i = \frac{-\Delta \beta_i R^2}{2 \left( \beta_i R^2 + (E[w_m])^2 \right)} + \frac{\sqrt{\beta_i (1 + \beta_i)}}{\sqrt{\beta_i R^2 + (E[w_m])^2}} R \]
\[ c_i = \frac{2(\beta_i R^2 + (E[w_m])^2)}{-\Delta \beta_i R + 2\sqrt{\beta_i R^2 + (E[w_m])^2} \sqrt{\beta_i (1 + \beta_i)}} - \beta_i, \]
\[ y_i = \frac{(E[w_m])^2 \left( \frac{-\Delta \beta_i R}{2(\beta_i R^2 + (E[w_m])^2)} + \frac{\sqrt{\beta_i (1 + \beta_i)}}{\sqrt{\beta_i R^2 + (E[w_m])^2}} \right)}{\beta_i} \]
\[ l_i = \frac{E[w_m] - \Delta \beta_i R + 2\sqrt{\beta_i R^2 + (E[w_m])^2} \sqrt{\beta_i (1 + \beta_i)}}{2(\beta_i R^2 + (E[w_m])^2)} - \frac{\beta_i R^2 + (E[w_m])^2}{\sqrt{\beta_i R^2 + (w_m)^2}} \]

So, with

\[ C_i = \frac{\sqrt{\beta_i R^2 + (E[w_m])^2}}{R \sqrt{\beta_i (1 + \beta_i)}}, \quad Y_i = \frac{\sqrt{\beta_i (1 + \beta_i)}}{\sqrt{\beta_i R^2 + (w_m)^2}} \]

we know that

\[ dC_i = \frac{2(\beta_i R^2 + (E[w_m])^2)}{-\Delta \beta_i R^2 + 2\sqrt{\beta_i R^2 + (E[w_m])^2} \sqrt{\beta_i (1 + \beta_i)} R} - \frac{\beta_i R^2 + (E[w_m])^2}{R \sqrt{\beta_i (1 + \beta_i)}} \]
\[ dC_i = \frac{\Delta \beta_i R \sqrt{\beta_i R^2 + (w_m)^2}}{-\Delta \beta_i R^2 + 2\sqrt{\beta_i R^2 + (w_m)^2} \sqrt{\beta_i (1 + \beta_i)} R} \sqrt{\beta_i (1 + \beta_i)} \]
\[ dY_i = \frac{-\Delta \beta_i R^2}{2(\beta_i R^2 + (E[w_m])^2)} \]
\[ dc_i = \frac{(\beta_i)^2 R \sqrt{\beta_i R^2 + (E[w_m])^2}}{-\Delta \beta_i R^2 + 2\sqrt{\beta_i (1 + \beta_i)} \sqrt{\beta_i R^2 + (E[w_m])^2}} \sqrt{\beta_i (1 + \beta_i)} \]
B.2 Intertemporal applications

\[
d y_i = \frac{\beta_i R}{2 \left( \beta_i R^2 + (E[w_m])^2 \right)} \Delta
\]

\[
d l_i = \frac{-E[w_m] R}{2 \left( \beta_i R^2 + (E[w_m])^2 \right)} \Delta
\]

so

\[
\frac{1}{c_i} dC_i = \frac{\beta_i R}{\left( -\Delta \beta_i R + 2 \sqrt{\beta_i (1 + \beta_i)} \sqrt{\beta_i R^2 + (E[w_m])^2} \right)} \Delta
\]

\[
Y_i dY_i = \frac{\sqrt{\beta_i (1 + \beta_i)}}{\beta_i \sqrt{\beta_i R^2 + (E[w_m])^2}} \frac{-\Delta \beta_i R^3}{2 \left( \beta_i R^2 + (E[w_m])^2 \right)} \Delta
\]

\[
\frac{1}{c_i} dC_i = \frac{\beta_i R}{\left( -\Delta \beta_i R + 2 \sqrt{\beta_i (1 + \beta_i)} \sqrt{\beta_i R^2 + (E[w_m])^2} \right)} \Delta
\]

\[
l_i dl_i = \frac{\sqrt{\beta_i (1 + \beta_i)}}{\beta_i \sqrt{\beta_i R^2 + (E[w_m])^2}} \frac{-\left( E[w_m] \right)^2 R}{2 \left( \beta_i R^2 + (E[w_m])^2 \right)} \Delta
\]

So, if the SWF is

\[
W = \frac{1}{\psi_i} \left( \left[ \left( \ln (C_i) - \frac{1}{2} (Y_i)^2 \right) \right] + \beta_i \left[ \ln (c_i) - \frac{1}{2} (l_i)^2 \right] \right)
\]

then marginal SW is

\[
dW = \frac{1}{\psi_i} \left( \left[ \left( \frac{\beta_i R}{\left( -\Delta \beta_i R + 2 \sqrt{\beta_i (1 + \beta_i)} \sqrt{\beta_i R^2 + (E[w_m])^2} \right)} \right) \Delta \right] + \beta_i \left[ \left( \frac{\left( 1 + \beta_i \right)}{2(1 + \beta_i) \sqrt{\beta_i R^2 + (E[w_m])^2} - \Delta R \sqrt{\beta_i (1 + \beta_i)}} \right) \right] \right)
\]

\[
dW = \frac{\sqrt{\beta_i (1 + \beta_i)} R \Delta}{\psi_i} \left( \left( \frac{1 + \beta_i}{2(1 + \beta_i) \sqrt{\beta_i R^2 + (E[w_m])^2} - \Delta R \sqrt{\beta_i (1 + \beta_i)}} \right) \right)
\]

\[
dW = \left[ \frac{R \sqrt{\beta_i (1 + \beta_i)}}{\psi_i \sqrt{\beta_i R^2 + (E[w_m])^2} - \Delta R \sqrt{\beta_i (1 + \beta_i)}} \right] \Delta
\]
which, as $\Delta \to 0$, becomes

\[
dW = \frac{R \sqrt{\beta_i (1 + \beta_i)}}{\psi_i \sqrt{\beta_i R^2 + (E[w_m])^2}} \Delta
\]

### B.2.3 Capital tax Eulers

\[
\max_{c,y} \sum_i \pi_i \left(1 + \beta_i R^2 \right) \frac{1}{1 + \beta_i} \left( \left( \beta_i \right)^{-\frac{1}{2}} (u(C_i) - v(Y_i)) + (\beta_i)^{-\frac{1}{2}} \sum_{m=1}^{M} P_m \left(u(c_{m,i}) - v\left(\frac{y_{m,i}}{w_m}\right)\right) \right).
\]

subject to:

\[
\sum_i \pi_i \left( Y_i - C_i \right) R + \sum_{m=1}^{M} P_m \left(y_{m,i} - c_{m,i}\right) \geq 0,
\]

\[
\left\{ u(C_h) - v(Y_h) + \beta_h \sum_{m=1}^{M} P_m \left(u(c_{m,h}) - v\left(\frac{y_{m,h}}{w_m}\right)\right) \right\} \\
\left\{ u(C_i) - v(Y_i) + \beta_i \sum_{m=1}^{M} P_m \left(u(c_{m,i}) - v\left(\frac{y_{m,i}}{w_m}\right)\right) \right\},
\]

\[
u(C_h) - v(Y_h) + \beta_h \sum_{m=1}^{M} P_m \left(u(c_{h,m}) - v\left(\frac{y_{h,m}}{w_m}\right)\right)
\]

\[
u(C_i) - v(Y_i) + \beta_i \sum_{m=1}^{M} P_m \left(u(c_{i,m}) - v\left(\frac{y_{i,m}}{w_m}\right)\right)
\]

The multipliers on these constraints are $\lambda$ for feasibility, $\rho_h$ and $\rho_i$ for first-period incentives, and $\mu_h$ and $\mu_l$ for second-period constraints. This yields the FOCs

\[
u'(C_h) \left[ \left( \frac{1}{2} \left( u'(c_{h,m}) - v\left(\frac{y_{h,m}}{w_m}\right) \right) \right) = \frac{1}{2} \lambda R
\]

\[
u'(C_i) \left[ \left( \frac{1}{2} \left( u'(c_{i,m}) - v\left(\frac{y_{i,m}}{w_m}\right) \right) \right) = \frac{1}{2} \lambda R
\]

\[
u'(c_{a,h}) \left[ \left( \frac{1}{4} \left( u'(c_{a,h}) - v\left(\frac{y_{a,h}}{w_a}\right) \right) \right) = \frac{1}{4} \lambda
\]

\[
u'(c_{a,1}) \left[ \left( \frac{1}{4} \left( u'(c_{a,1}) - v\left(\frac{y_{a,1}}{w_a}\right) \right) \right) = \frac{1}{4} \lambda
\]
where we've substituted four FOCs, and assuming zero parental multipliers. Namely, we get these two conditions:

\[
\left( \frac{1 + \beta_h R^2}{1 + \beta_h} \right)^{\frac{1}{2}} (\beta_h)^{\frac{1}{2}} + \left( \frac{1 + \beta_t R^2}{1 + \beta_t} \right)^{\frac{1}{2}} (\beta_t)^{\frac{1}{2}} = \lambda R \left( \frac{1}{u'(C_h)} + \frac{1}{u'(C_t)} \right)
\]

\[
\left( \frac{1 + \beta_h R^2}{1 + \beta_h} \right)^{\frac{1}{2}} (\beta_h)^{\frac{1}{2}} + \left( \frac{1 + \beta_t R^2}{1 + \beta_t} \right)^{\frac{1}{2}} (\beta_t)^{\frac{1}{2}} = \frac{1}{2} \lambda \left( \frac{1}{u'(c_{a,h})} + \frac{1}{u'(c_{d,h})} + \frac{1}{u'(c_{a,t})} + \frac{1}{u'(c_{d,t})} \right)
\]

which combine to yield

\[
\hat{\beta} R = \frac{1}{2} \left( \frac{1}{u'(C_h)} + \frac{1}{u'(C_t)} \right) \left( \frac{1}{u'(c_{a,h})} + \frac{1}{u'(c_{d,h})} + \frac{1}{u'(c_{a,t})} + \frac{1}{u'(c_{d,t})} \right)
\]

where we’ve substituted

\[
\hat{\beta} = \left( \frac{1 + \beta_h R^2}{1 + \beta_h} \right)^{\frac{1}{2}} (\beta_h)^{\frac{1}{2}} + \left( \frac{1 + \beta_t R^2}{1 + \beta_t} \right)^{\frac{1}{2}} (\beta_t)^{\frac{1}{2}}
\]

\[
\left( \frac{1 + \beta_h R^2}{1 + \beta_h} \right)^{\frac{1}{2}} (\beta_h)^{-\frac{1}{2}} + \left( \frac{1 + \beta_t R^2}{1 + \beta_t} \right)^{\frac{1}{2}} (\beta_t)^{-\frac{1}{2}}
\]

as a stand-in discount factor.

The FOCs also yield preference-specific Inverse Euler Equations:

\[
\frac{1}{2} \left( \frac{1 + \beta_h R^2}{1 + \beta_h} \right)^{\frac{1}{2}} (\beta_h)^{-\frac{1}{2}} + \rho_h - \frac{\beta_t}{\beta_h} \rho_t - 2\beta_h R = u'(C_h) \frac{1}{2} \left( \frac{1}{u'(c_{a,h})} + \frac{1}{u'(c_{d,h})} \right)
\]

\[
\frac{1}{2} \left( \frac{1 + \beta_h R^2}{1 + \beta_h} \right)^{\frac{1}{2}} (\beta_h)^{-\frac{1}{2}} + \rho_h - \rho_t
\]

\[
\frac{1}{2} \left( \frac{1 + \beta_t R^2}{1 + \beta_t} \right)^{\frac{1}{2}} (\beta_t)^{-\frac{1}{2}} - \frac{\beta_t}{\beta_h} \rho_h + \rho_t - 2\beta_t R = u'(C_t) \frac{1}{2} \left( \frac{1}{u'(c_{a,t})} + \frac{1}{u'(c_{d,t})} \right)
\]

\[
\frac{1}{2} \left( \frac{1 + \beta_t R^2}{1 + \beta_t} \right)^{\frac{1}{2}} (\beta_t)^{-\frac{1}{2}} - \rho_h + \rho_t
\]
Appendix C
Appendix to Chapter 3

This Appendix contains several sections. First, I show how to apply the approach of the main paper to a second dimension of parental heterogeneity: different productive abilities as in the Mirrleesian optimal income tax model. Second, I show how to extend the main paper’s analysis to non-logarithmic utility from consumption. Third, I give proofs of the paper’s propositions. A derivation of the infinite horizon results summarized in the paper is available upon request.

C.1 Application of approach to ability heterogeneity

The approach taken in the main paper may be applied to parental heterogeneity along other dimensions. For example, suppose parents are heterogeneous in their productive ability, \( w(j_1) \), while children are homogeneous. Each individual’s utility function is composed of utility from consumption \( c \) and disutility from labor effort, which is income \( y \) divided by ability. In this setting, a conventional social welfare function that is concave in parents’ utilities would be:

\[
W = \sum_f \sum_{j_1} \pi(j_1) \left[ u(c_1(f, j_1)) - v\left(\frac{y_1(f, j_1)}{w_1(j_1)}\right) + \beta \left( u(c_2(f, j_1)) - v\left(\frac{y_2(f, j_1)}{w_2(j_1)}\right)\right) \right]^{\rho}
\]

where \( f \) indexes families, subscripts denote generation, and \( \rho \) controls the concavity of social welfare. This social welfare function is the analogue to the Unequal Weights SWF...
from the main paper, in that a child’s allocation in the first-best policy depends on its
parent’s ability.

Applying the original position thought experiment as in the main paper, the con-
nection between a child’s type and its parent’s type is severed. A parent of family \( f \) is
uncertain of its type \( j_1 \) and it is uncertain of the type of parent \( j_2 \) by whom its child (i.e.,
the child of family \( f \)) will be raised ex post. The resulting social welfare function is:

\[
W = \sum_{f} \sum_{j_1} \pi(j_1) \left[ u(c_1(f, j_1)) - v\left(\frac{y_1(f, j_1)}{w_1(j_1)}\right) + \beta \sum_{j_2} \pi(j_2) \left( u(c_2(f, j_2)) - v\left(\frac{y_1(f, j_2)}{w_2(j_2)}\right)\right) \right]^\rho
\]

the analogue to the Equal Weights SWF from the main paper. Note that, while the conven-
tional social welfare function links a child’s allocations to the type of its parent, this social
welfare function treats all children equally, at least in the first best. As in the main paper,
unobservability of type implies that the constrained policy will treat children of different
parents differently, but the disparities will be less than in conventional analysis.

C.2 More general utility functions

In the main paper, I assume logarithmic utility. This simplifies the analysis, but the analysis
can be extended to a more general functional form. Here, I describe one reason why
logarithmic utility is particularly convenient. Then I discuss how to generalize the analysis
to more general utility functions.
C.2 More general utility functions

C.2.1 Logarithmic utility

Suppose that parent type $i$ has preferences described in general by:

$$W^i_1 = u(c^i_1) + \beta^i u(c^i_2)$$

and the planner starts by allocating an equal amount $e$ to both $i$ parents. If the planner allocates $e = 1$ to both types, and constrains them so that

$$c^i_1 + \frac{1}{R}c^i_2 = 1$$

for both $i$, where $R = \frac{1}{R}$ as in the main paper, then we know that each type will set

$$u'(c^i_1) = \beta^i Ru'(c^i_2)$$

which together implicitly define

$$c^i_1 = c_1(\beta^i, R).$$

Suppose that the planner wanted to specify a social welfare function such that the marginal social welfare from allocating another unit of resources to each parent is equal. To derive this social welfare function, note that with log utility,

$$\frac{1}{\beta^i R} c^i_2 = \frac{1}{\beta^i R} (R - Rc^i_1) = \frac{1}{\beta^i} (1 - c^i_1) = c^i_1$$

so,

$$c^i_1 = \frac{1}{1 + \beta^i}, \ c^i_2 = \frac{\beta^i R}{1 + \beta^i}.$$

If the planner were to increase the allocation to each type by $\Delta$, their private choices would imply:
C.2 More general utility functions

\[ dc_1^i = \frac{1}{1 + \beta^i} \Delta, \]
\[ dc_2^i = \frac{\beta^i R}{1 + \beta^i} \Delta. \]

Now, suppose the planner considers a social welfare function of the form:

\[ W = \sum_i \pi^i \frac{1}{\psi^i} \left( \ln (c_1^i) + \beta^i \ln (c_2^i) \right) \]

where \( \psi^i \) is the normalization factor that will set the marginal social value of resources equal across \( i \). Taking the total derivative of social welfare,

\[ dW = \sum_i \pi^i \frac{1}{\psi^i} \left( \frac{1}{c_1^i} dc_1^i + \beta^i \frac{1}{c_2^i} dc_2^i \right) \]

Plugging in for the values above, this is

\[ dW = \sum_i \pi^i \frac{1}{\psi^i} \left( (1 + \beta^i) \frac{1}{1 + \beta^i} \Delta + \beta^i \frac{1 + \beta^i}{\beta^i R} \frac{\beta^i R}{1 + \beta^i} \Delta \right), \]

or

\[ dW = \sum_i \pi^i \frac{1}{\psi^i} (1 + \beta^i) \Delta. \]

Thus,

\[ \psi^i = (1 + \beta^i) \]

is the implied normalization factor, yielding the SWF:

\[ W = \sum_i \pi^i \left( \frac{1}{1 + \beta^i} u(c_1^i) + \frac{\beta^i}{1 + \beta^i} u(c_2^i) \right) \]

which is the expression used in the main paper. The simplicity of this normalization of altruism relies on logarithmic utility. The approach, however, can be applied in a more general setting, as I now show.
C.2 More general utility functions

More general utility

In the previous section, we derived a social welfare function that makes the marginal social value of additional resources equal across types, starting from equality of resources, for logarithmic utility. The same method can be applied for more general utility functions, yielding a normalized SWF that can be used for analysis. Here, I illustrate this method for CRRA utility:

\[ u(c) = \frac{1}{1 - \gamma} c^{(1-\gamma)} \]

where \( \gamma \) determines the concavity of utility.

We want to renormalize the SWF so that the marginal social welfare from allocating resources is equal across types. Start with

\[ W_i^i = \frac{1}{\psi(1 - \gamma)} \left( (c_1^i)^{(1-\gamma)} + \beta^i (c_2^i)^{(1-\gamma)} \right) . \]

Private optimization yields

\[ c_1^i = (\beta^i R)^{\frac{1}{\gamma}} c_2^i \]

so,

\[ c_1^i = \frac{(\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma - 1}{\gamma}}}{1 + (\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma - 1}{\gamma}}} \]

and

\[ c_2^i = \frac{R^{\frac{\gamma - 1}{\gamma}}}{1 + (\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma - 1}{\gamma}}} \]

Allocating additional resources to each type would yield:

\[ dc_1^i = \frac{(\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma - 1}{\gamma}}}{1 + (\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma - 1}{\gamma}}} \Delta \]
and
\[ dc_2^i = \frac{R^{\frac{\gamma-1}{\gamma}}}{1 + (\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma-1}{\gamma}}} \Delta. \]

Thus, given that the total derivative of \( W_i \) is
\[ dW_i = \frac{1}{\psi} \left( (c_1^i)^{-\gamma} dc_1^i + \beta^i (c_2^i)^{-\gamma} dc_2^i \right), \]
we can plug in these values to obtain
\[
dW_i = \frac{1}{\psi} \left[ \left( \frac{(\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma-1}{\gamma}}}{1+(\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma-1}{\gamma}}} \right)^{-\gamma} \frac{R^{\frac{\gamma-1}{\gamma}}}{1+(\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma-1}{\gamma}}} \Delta \\
+ \beta^i \left( \frac{R^{\frac{\gamma-1}{\gamma}}}{1+(\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma-1}{\gamma}}} \right)^{-\gamma} \frac{R^{\frac{\gamma-1}{\gamma}}}{1+(\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma-1}{\gamma}}} \Delta \right] ,
\]
or
\[ dW_i = \frac{1}{\psi} \left( \beta^i + (\beta^i)^{\frac{2}{\gamma}} \right) \left( \frac{R^{\frac{\gamma-1}{\gamma}}}{1+(\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma-1}{\gamma}}} \right)^{1-\gamma} \Delta. \]

This implies that
\[ \psi = \left( \beta^i + (\beta^i)^{\frac{2}{\gamma}} \right) \left( \frac{R^{\frac{\gamma-1}{\gamma}}}{1+(\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma-1}{\gamma}}} \right)^{1-\gamma} \]
is the normalizing factor, and
\[
W = \sum_i \pi^i \left( \beta^i + (\beta^i)^{\frac{2}{\gamma}} \right) \left( \frac{R^{\frac{\gamma-1}{\gamma}}}{1+(\beta^i)^{\frac{1}{\gamma}} R^{\frac{\gamma-1}{\gamma}}} \right)^{1-\gamma} \left( (c_1^i)^{1-\gamma} + \beta^i (c_2^i)^{1-\gamma} \right),
\]
is the social welfare function. As this makes clear, working with non-logarithmic utility is substantially more cumbersome.

C.3 Proofs
C.3 Proofs

C.3.1 Proof that $\mu^G > 0$ in Equal Weights policy

First, note that it is not possible for both $\mu^G$ and $\mu^S$ to be positive if the allocations differ across types. To see why, note that the generous parent’s IC implies that

$$\frac{\ln (c^S_1) - \ln (c^G_1)}{\ln (c^S_2) - \ln (c^G_2)} \leq \beta^G,$$

while the selfish parent’s IC implies that

$$\frac{\ln (c^S_1) - \ln (c^G_1)}{\ln (c^S_2) - \ln (c^G_2)} \geq \beta^S.$$

Now,

$$\beta^S < \beta^G$$

so the ratio $\frac{\ln (c^S_1) - \ln (c^G_1)}{\ln (c^S_2) - \ln (c^G_2)}$ can be equal to at most one of the discount factors, $\beta^S$ and $\beta^G$, implying that at most one of the ICs can bind at the optimal constrained solution.

Furthermore, the optimal allocations cannot be the same across types. If they were, suppose first that the uniform consumption level of children is greater than that of the parents, so that $c_2 > c_1$. Then, transfer $\varepsilon$ utils from the child of the selfish parent to give $\delta \varepsilon$ utils to the selfish parent. This generates net revenue of $\left(\frac{\delta \varepsilon}{c_2} - \frac{\delta \varepsilon}{c_1}\right)$, which is positive, so it is feasible to the planner. It also satisfies the IC constraints, because the selfish parent’s value of truth-telling increases ($\delta > \beta^s$), and the generous parent’s value of cheating decreases ($\beta^G > \delta$). Suppose, instead, that $c_2 < c_1$. Then, transfer $\varepsilon$ utils from the generous parent to give $\delta \varepsilon$ utils to the child of the generous parent and follow the reverse argument as for the previous case.
Second, we prove that $\mu^S = 0$, i.e., that the selfish parent’s IC doesn’t bind. To do so, note that the selfish parent’s IC implies that

$$\frac{\ln (c^S_1) - \ln (c^G_1)}{\ln (c^S_2) - \ln (c^G_2)} \geq \beta^S,$$

where the equality is if it binds and $\mu^S > 0$.

The planner’s unconstrained optimal policy sets:

$$\frac{1}{\beta} \frac{1 + \delta}{1 + \beta^G} = c^G_1, \quad \frac{1}{\beta} \frac{1 + \delta}{1 + \beta^S} = c^S_1$$

$$\frac{1}{\phi} = c^G_2, \quad \frac{1}{\phi} = c^S_2$$

where $\phi$ is the multiplier on the feasibility constraint. Thus, the planner’s optimal policy sets:

$$\frac{\ln (c^S_1) - \ln (c^G_1)}{\ln (c^S_2) - \ln (c^G_2)} = \frac{\ln \left( \frac{1 + \delta}{1 + \beta^S} \right) - \ln \left( \frac{1 + \delta}{1 + \beta^G} \right)}{\ln (1) - \ln (1)}.$$

or, simplifying,

$$\frac{\ln (c^S_1) - \ln (c^G_1)}{\ln (c^S_2) - \ln (c^G_2)} = \frac{\ln \left( \frac{1 + \beta^G}{1 + \beta^S} \right)}{\ln (1)} = \infty.$$

So, the unconstrained ratio is necessarily bigger than $\beta^S$,

$$\beta^S < \infty.$$

This implies that the selfish IC cannot bind. To see why, suppose that $\mu^S > 0$, so that the selfish parent’s IC binds. This implies that either $c^S_1 < c^G_1$, or $c^S_2 < c^G_2$, or both. If $c^S_2 < c^G_2$, then lower $c^G_2$ and raise $c^S_2$ by $\varepsilon > 0$ units, choosing $\varepsilon$ so that both ICs are satisfied. Some such $\varepsilon > 0$ must exist, given that the generous parent’s IC does not bind. This is feasible, and it raises social welfare, since the Equal Weights SWF puts an equal weight on the utility of each child. Thus, $c^S_2 < c^G_2$ cannot hold. Suppose, then, that $c^S_1 < c^G_1$. Again, lower $c^G_1$ and raise $c^S_1$ by $\varepsilon > 0$ units, choosing $\varepsilon$ so that both ICs are satisfied. Some such
\( \varepsilon > 0 \) must exist, given that the generous parent’s IC does not bind. This is feasible, and it raises social welfare, since the Equal Weights SWF puts a higher weight on the utility of the selfish parent. Thus, \( c_1^S < c_1^G \) cannot hold. If both inequalities hold, either of the transfers above will be feasible and raise social welfare. Thus, \( \mu^S \) cannot be positive.

Given that \( \mu^S = 0 \), we now need to show that \( \mu^G > 0 \). Suppose not. Then \( \mu^G = 0 \).

Consider the planner’s first order conditions, which yield

\[
\begin{align*}
c_1^G &= \frac{\frac{1}{1+\beta^G} + \frac{1}{1+\beta^G} \mu^G}{\frac{1}{1+\beta^S} - \frac{1}{1+\beta^G} \mu^G} c_1^S, \\
c_2^G &= \frac{\frac{\delta}{1+\delta} + \frac{\beta^G}{1+\beta^G} \mu^G}{\frac{\delta}{1+\delta} - \frac{\beta^G}{1+\beta^G} \mu^G} c_2^S, \\
\end{align*}
\]

Now, if \( \mu^G = 0 \), then both \( c_1^S > c_1^G \) and \( c_2^S = c_2^G \), so selfish parents get more consumption than generous parents and the children of selfish parents get the same as the children of generous parents. This violates the generous parent’s IC constraint, so \( \mu^G > 0 \).

**C.3.2 Proof that incentive constraints do not bind in Unequal Weights policy**

Here we show that \( \mu^G = 0 \) when maximizing the Unequal Weights SWF (the same proof applies for the multiplier \( \mu^S \)). Intuitively, this is true because the government and generous parents agree on discounting.

The generous parent’s IC implies that

\[
\frac{\ln (c_1^S) - \ln (c_1^G)}{\ln (c_2^G) - \ln (c_2^S)} \leq \beta^G.
\]
while the selfish parent’s IC implies that
\[
\frac{\ln (c_1^S) - \ln (c_1^G)}{\ln (c_2^G) - \ln (c_2^S)} \geq \beta^S.
\]

Now,
\[
\beta^S < \beta^G
\]
so the ratio \(\frac{\ln (c_1^S) - \ln (c_1^G)}{\ln (c_2^G) - \ln (c_2^S)}\) can be equal to at most one of the discount factors, \(\beta^S\) and \(\beta^G\).

Moreover, we know that neither IC binds if the optimal unconstrained choice of the government (i.e. when both incentive constraints are absent, so \(\mu^G = \mu^S = 0\)) produces a value for this ratio between \(\beta^S\) and \(\beta^G\).

The government’s optimal policy sets:
\[
\frac{\pi^G}{\phi \pi^G} \frac{1 + \delta}{1 + \beta^G} = c_1^G, \quad \frac{\pi^S}{\phi \pi^S} \frac{1 + \delta}{1 + \beta^S} = c_1^S
\]
\[
\frac{\pi^G}{\phi \pi^G} \frac{\beta^G}{1 + \beta^G} \frac{1 + \delta}{1 + \delta} = c_2^G, \quad \frac{\pi^S}{\phi \pi^S} \frac{\beta^S}{1 + \beta^S} \frac{1 + \delta}{1 + \delta} = c_2^S
\]

where \(\phi\) is the multiplier on the feasibility constraint. Thus, the government’s optimal ratio is:
\[
\frac{\ln (c_1^S) - \ln (c_1^G)}{\ln (c_2^G) - \ln (c_2^S)} = \frac{\ln \left( \frac{1}{\phi (1 + \beta^S)} \right) - \ln \left( \frac{1 + \delta}{\phi (1 + \beta^S)} \right)}{\ln \left( \frac{1 + \beta^G}{\phi (1 + \beta^G)} \right) - \ln \left( \frac{1 + \beta^S}{\phi (1 + \beta^S)} \right)}.
\]

or, simplifying,
\[
\frac{\ln (c_1^S) - \ln (c_1^G)}{\ln (c_2^G) - \ln (c_2^S)} = \frac{\ln \left( \frac{1 + \beta^G}{1 + \beta^S} \right)}{\ln \left( \frac{\beta^G}{1 + \beta^G} \frac{1 + \beta^S}{\beta^S} \right)}.
\]
C.3 Proofs

holds, so the government’s unconstrained optimum implies that neither IC binds.

C.3.3 Proof of Proposition 7

Proposition 7 holds if

\[
\frac{\left( \frac{\delta}{1 + \delta} + \frac{1}{\pi^G} \frac{\beta^G}{1 + \beta^G} \mu^G \right)}{\left( \frac{\delta}{1 + \delta} - \frac{1}{\pi^S} \frac{\beta^G}{1 + \beta^G} \mu^G \right)} < \frac{\beta^G}{1 + \beta^G}
\]

or

\[
\frac{1}{1 + \beta^G} + \frac{1}{\pi^G} \frac{\beta^G}{1 + \beta^G} \mu^G \left( \frac{\beta^G}{1 + \beta^G} - \frac{\beta^S}{1 + \beta^S} \right)
\]

Solving for the multiplier with the parents’ first-order conditions from the Equal Weights policy,

\[
c_1^G = \frac{\left( \frac{\delta}{1 + \delta} + \frac{1}{\pi^G} \frac{\beta^G}{1 + \beta^G} \mu^G \right)}{\left( \frac{\delta}{1 + \delta} - \frac{1}{\pi^S} \frac{\beta^G}{1 + \beta^G} \mu^G \right)} c_1^S
\]

\[
c_2^G = \frac{\left( \frac{\delta}{1 + \delta} + \frac{1}{\pi^G} \frac{\beta^G}{1 + \beta^G} \mu^G \right)}{\left( \frac{\delta}{1 + \delta} - \frac{1}{\pi^S} \frac{\beta^G}{1 + \beta^G} \mu^G \right)} c_2^S,
\]

we can obtain:

\[
\mu^G = \pi^S \pi^G \frac{c_1^S}{\frac{1}{\pi^S} \frac{1}{c_1^S} + \pi^G \frac{1}{c_1^G} \left( \frac{1}{1 + \beta^G} - \frac{c_1^S}{c_1^G} \right)}
\]

To prove that the Equal Weights SWF reduces disparities across children, consider the incentive constraint for selfish parents:

\[
\frac{1}{1 + \beta^S} \ln \left( c_1^S \right) + \frac{\beta^S}{1 + \beta^S} \ln \left( c_2^S \right) \geq \frac{1}{1 + \beta^G} \ln \left( c_1^G \right) + \frac{\beta^S}{1 + \beta^S} \ln \left( c_2^G \right).
\]

With the planner’s FOCs and \( \mu^G > 0 \), we know that \( c_2^G > c_2^S \), so more consumption is allocated to the children of generous parents, which in turn implies that \( u \left( c_2^S \right) < u \left( c_2^G \right) \). In combination with the incentive constraint, it must be that \( u \left( c_1^S \right) > u \left( c_1^G \right) \) and thus \( u' \left( c_1^S \right) < u' \left( c_1^G \right) \), so \( c_1^S > c_1^G \). This implies that \( \frac{1}{\pi^S} \frac{c_1^S}{\frac{1}{c_1^S} + \pi^G \frac{1}{c_1^G}} < 1 \) and \( \frac{c_1^S}{c_1^G} > 1 \), so that we
can conclude:
\[
\mu^G < \pi^S \pi^G \left( \frac{\beta^G}{(1 + \beta^S)} - \frac{\beta^S}{(1 + \beta^S)} \right)
\]
Recall that we want to show:
\[
\mu^G < \pi^S \pi^G \frac{1 + \beta^G}{\beta^G} \left( \frac{\beta^G}{(1 + \beta^G)} - \frac{\beta^S}{(1 + \beta^S)} \right)
\]
so we’ve proven our result if
\[
\frac{\beta^G}{(1 + \beta^S)} - \frac{\beta^S}{(1 + \beta^S)} < \frac{1 + \beta^G}{\beta^G} \left( \frac{\beta^G}{(1 + \beta^G)} - \frac{\beta^S}{(1 + \beta^S)} \right)
\]
This reduces to whether
\[
(\beta^G - \beta^S) \beta^G < \beta^G - \beta^S
\]
which holds, because \((\beta^G - \beta^S) > 0\), so this is just:
\[
\beta^G < 1
\]
which we assumed in the paper.

C.3.4 Proof of Proposition 8

We want to prove the set of inequalities
\[
\frac{\beta^G}{\delta} > \frac{\pi^G + \frac{1 + \delta}{\frac{\beta^G}{\delta}} \cdot \mu^G}{\pi^G + \mu^G} > \frac{1 + \beta^G}{1 + \delta}
\]
Both inequalities holds as long as \(\beta^G > \delta\), which is true by definition.

Proof of Proposition 8, for selfish parents

Here, we show an analogous result to Proposition 8. In particular, we want to prove that the ratio of the consumption of the children of selfish parents to their parents is larger if the planner maximizes the Equal Weights SWF rather than the Unequal Weights SWF, but
not as large as if it had full information. Formally, we want to prove the set of inequalities

\[
\frac{\beta^S}{\delta} < \frac{\mu^G}{\beta^S} \frac{1}{1 + \beta^S} - \frac{\beta^G}{\beta^S} \frac{1}{1 + \beta^S} \frac{1 + \beta^S}{\delta} < \frac{1 + \beta^S}{1 + \delta}.
\]

Doing so requires showing that the shadow price of satisfying generous parent’s incentives is not too large. For the first inequality, the key condition is that:

\[
\mu^G < \pi^S \pi^G \frac{(\beta^G - \beta^S) (1 + \beta^S)}{(1 + \beta^S) (\beta^G - \beta^S)}
\]

while for the second inequality the key condition is:

\[
\frac{\beta^G}{1 + \beta^S} > \frac{\delta}{1 + \delta}
\]

which we know is true because $\beta^S < \delta < \beta^G$. Now, to show that

\[
\mu^G < \pi^S \pi^G \frac{(\beta^G - \beta^S) (1 + \beta^S)}{(1 + \beta^S) (\beta^G - \beta^S)}
\]

refer to the proof of Proposition 7. There, we showed that

\[
\mu^G < \pi^S \pi^G \frac{1 + \beta^G}{\beta^G} \left( \frac{\beta^G}{1 + \beta^S} - \frac{\beta^S}{1 + \beta^S} \right)
\]

Notice that

\[
\pi^S \pi^G \frac{1 + \beta^G}{\beta^G} \left( \frac{\beta^G}{1 + \beta^S} - \frac{\beta^S}{1 + \beta^S} \right) = \pi^S \pi^G \left( \frac{\beta^G - \beta^S (1 + \beta^G)}{1 + \beta^S} \right)
\]

since this reduces to

\[
\frac{1}{\beta^G} \left( \frac{\beta^G}{1 + \beta^S} - \frac{\beta^S (1 + \beta^G)}{1 + \beta^S} \right) < \frac{(\beta^G - \beta^S (1 + \beta^G))}{(1 + \beta^S) (\beta^G - \beta^S)}
\]

Now, since

\[
\beta^G - \frac{\beta^S (1 + \beta^G)}{1 + \beta^S} > 0
\]

\[
\beta^G (1 + \beta^S) > \beta^S (1 + \beta^G)
\]
this is just

\[ \frac{1}{\beta^G} < \frac{1}{(\beta^G - \beta^S)} \]

\[ \beta^S > 0 \]

which is always true.
References


Federal Reserve Bank of Minneapolis (2005), "Interview with James J. Heckman," The Region, June.


