Arbitrage in the European Soccer Betting Market

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Abstract

This paper will explore the European soccer betting market and its fundamental ties to underlying economic theory. By studying the odds of English Premier League soccer games posted on seven different betting websites, I was able to discover significant arbitrage opportunities that have only become available in recent years. Additionally, I provide a theory as to why this arbitrage exists and why it is a relatively recent phenomenon. I suggest that competition between different betting websites has caused the initial odds offered by the sites to get close enough to fair game values that random drift can cause the odds on different sites to vary by an amount great enough to allow for arbitrage. I also conclude that the arbitrage available in the sports betting market should be considered limited arbitrage based on Shleifer's definition, which states that arbitrage opportunities with perfect substitutes can still be limited in the real world if an arbitrageur must worry about maintaining the position.[1]

Part 1 - Introduction

The size of the online gambling market is projected to surpass forty-five billion dollars in 2016, which would represent an eighty-five percent increase since 2009.[2] Additionally, it is estimated that sixty-five percent of global sports betting occurs on soccer matches.[3] Despite the massive size and growth of this market, there has been very little academic research into the nature of online betting markets. In this paper, I examine the online European soccer betting market by studying all games played in the English Premier League, the world's most commercially successful soccer league, from the 2000/01 to the 2014/15 season. I tested the efficiency of the online betting market for the English Premier League by examining the odds on seven different betting websites (Bet365, Bet&Win, Interwetten, Ladbrokes, Stan James, William Hill, and VC Bet) and studying whether arbitrage opportunities existed. I found that it was possible to find significant arbitrage opportunities by combining the best odds from different sites on an individual game. The exact definition and nature of this arbitrage will be discussed later in the paper. To start, a game was considered to offer an arbitrage opportunity if a series of bets could be placed that would lead to a profit in every possible outcome.

The textbook definition of arbitrage is, "the simultaneous purchase and sale of the same, or essentially similar, securities in two different markets at advantageously different prices."[4] Typically, this is applied to financial markets, where, in the most basic example, an arbitrageur could buy some security in Market A for some price P_1 and simultaneously sell that security in Market B for some price P_2, where $P_1<P_2$. Without taking on any risk, the arbitrageur would collect the price difference. With betting on soccer matches through online betting sites, arbitrage becomes more complicated because it is impossible to "sell" the security, and there are three possible outcomes: a home win, a draw, and an away win. Therefore, to capture arbitrage three simultaneous bets must be placed (one on each result) that would each pay out more than the total amount wagered.

In Europe, odds are typically given in decimal form, where the amount paid out in the event of a win is the amount wagered multiplied by the decimal odds. For example, if the odds of the home team winning are 2.5 and an individual wagers ten dollars on that outcome, the individual would receive twenty-five dollars if the home team were to win. In this study, to test whether or not the odds on the various sites offered an arbitrage opportunity for a given game, I first identified the best odds for each outcome and then calculated if arbitrage was available using a simple equation. To calculate whether arbitrage is possible given a set of three odds, we can use equation (1)[5]:

$$1/H + 1/D + 1/A = X$$

In this formula, H represents the stated odds that the home team will win, D represents the stated odds that the game will end in a draw, and A represents the stated odds that the away team will win. The reason that this equation is able to detect arbitrage given a certain set of odds is that one divided by the stated odds is equivalent to the implied probability of a given event. For example, if the stated odds of an outcome are 2, then the implied probability of that occurring is 1/2, or fifty percent. Therefore, if "fair odds" are given, X should be one, because as there are three possible outcomes, the total probability of one of the three outcomes occurring is one. The concept of fair odds will be developed later in the paper as analogous to a "fair game," where the expected profit is 0. If X is greater than one, the stated odds imply that the probability of one of three states occurring is greater than one, which is not possible. As the cost of a bet can be thought of as the probability of the event occurring, if X is greater than one then the combined basked of odds is overly expensive, and there are no arbitrage opportunities. Using the same logic, if X is less than one there is an arbitrage opportunity. The risk-free profit that can be generated from the opportunity can be calculated using equation (2)[6]:

$$P = (S/X) – S$$

This equation simply states that profit (P) is equal to the total amount invested (S) divided by X minus the amount invested (S). The key to arbitrage betting is that the proper amount of money must be wagered on each outcome, so the result is the same profit in all possible scenarios. Therefore, the arbitrageur should be ambivalent to which state actually ends up occurring. The amount to bet on each game can be found by using equation (3)[7]:

B(H,D,A) = (S*(1/(H,D,A))/X

In this equation, B stands for the bet size on a specific state (home win, draw, or away win) and S stands for the combined stake invested in all three bets. So, the equation states that the proper bet size on a given state is equal to the total investment multiplied by the implied probability of that state divided by the implied probability of all states. It is useful to look at an example in order to better quantify these ideas. In this example I will use two fictitious bookmakers, which both provide odds on all three possible outcomes of one game. The odds for each outcome vary slightly across bookmakers. I shall assume a total investment of $1000.

Bookmaker	Home Team Win	Draw	Away Team Win
Y	2	4	3.5
Z	1.5	3	5

In this simple state, neither bookmaker is offering arbitrage by betting on all three results on one site, but by combining the best odds for each possible outcome it is possible to unlock arbitrage. To do this, an arbitrageur would bet on the home team to win on bookmaker Y, a draw on bookmaker Y, and the away team to win on bookmaker Z. Using equation (1), this would give us an implied probability of:

X = (1/2) + (1/4) + (1/5) = 0.95

As X is less than one, we know that we have an opportunity for riskless profit. Using equation (2), we can discover how much risk-free profit is in this opportunity given a $1000 investment.

$$P = \$1000/0.95 - \$1000 = \$52.63$$

Therefore, we know that by taking advantage of this arbitrage opportunity we will earn $52.63 (5.3%) in riskless profit. To decide how much to bet on each state we use equation (3):

$$B(H) = (\$1000/2)/.95 = \$526.32$$

$$B(D) = (\$1000/4)/.95 = \$263.16$$

$$B(A) = (\$1000/5)/.95 = \$210.52$$

The total investment is merely B(H)+B(D)+B(A), which is $1000. Independent of the actual result, we know we will receive $1,052.36, for $52.36 of risk-free profit.

Using the equations above, I tested for arbitrage in every game of the English Premier League from the 2000/01 season to the 2014/15 season. Unlike any study known to the author, I was able to find significant patterns of arbitrage in English Premier League games by using odds from different sites. The first game to offer an arbitrage opportunity occurred in the 2007/08 season, and the number of games offering arbitrage has steadily increased each year since. In addition to the number of arbitrage opportunities increasing over time, the average expected return for each betting site declined in every year in the study. The average expected return for a betting site was calculated based on equation (1), with the return to the betting site being equal to X minus one. This is a legitimate approximation for the return for betting sites assuming that the sites have a balanced book, where the bookmaker makes the same profit independent of the outcome. This pattern leads me to postulate that increasing competition between betting sites has caused the average odds bookmakers initially give to continually move closer to “fair value” odds. Fair value odds occur when the implied probability of all outcomes (X) is equal to one. As each bookmaker then rationally adjusts their own odds to attempt to balance their book, discrepancies between bookmakers can become large enough to allow for an arbitrage opportunity.

In order to properly establish the connections between arbitrage in soccer betting markets and fundamental economic theory, the remainder of the paper will be structured as follows: Part 2 will establish the theoretical history and framework of the Efficient Market Hypothesis and will explore the connections between the Efficient Market Hypothesis and arbitrage. Part 3 will introduce the theoretical history and framework of refutations of the Efficient Market Hypothesis, further discuss the role of arbitrage in the debate of efficient markets, provide various definitions for arbitrage, and discuss examples of arbitrage that have occurred in the real world. Part 4 will briefly discuss the few pieces of literature that have previously examined arbitrage in sports betting markets. Part 5 will introduce the data and results found in this paper and will introduce my theory that competition and noise, which are represented by shrinking profits and random drift, has led to the increase in arbitrage opportunities seen in the data. Part 6 will introduce a simple model where I will attempt to show that the results found in this paper can be explained by my theory of competition and noise, and that arbitrage can occur even as bookmakers and the individuals betting all act rationally. Part 7 will discuss the nature of the arbitrage offered in the soccer betting market. Part 8 will conclude.

Part 2 – A Brief Synopsis of the Efficient Market Hypothesis

The definition of an efficient market according to Eugene Fama's famous 1970 paper, "Efficient Capital Markets: A Review of Theory and Empirical Work," is that an efficient market always fully reflects all available information, which suggests that it is impossible for an investor to generate above average returns. [8] This idea of efficient markets is inherently tied to arbitrage. In fact, according to Andrei Shleifer, the efficiency of markets is actually dependent upon arbitrage being able to function in the real world as it does in efficient market models such as those put forward by Fama and Friedman.[9] However, the importance of the connection between arbitrage and efficiency is not necessarily intuitive. It stems from the idea that the basic theoretical case for the Efficient Market Hypothesis relies on three core assumptions.[10] The first is that investors are assumed to be rational, which means that they value securities rationally.[11] If all investors are rational, then markets are efficient by definition. The second is that even if some investors are not rational, as long as irrational investors deviate from rationality in random ways the irrational investors will cancel each other out and markets will remain efficient.[12] The third is that even if irrational investors deviate from rationality in non-random ways, arbitrage can act as a force to ensure markets remain efficient.[13] Therefore, if these assumptions hold then markets must be efficient. In fact, much of the theoretical attack on the Efficient Market Hypothesis stems from disagreements with the core of these arguments.[14] In order to understand how these assumptions form the heart of the Efficient Market Hypothesis, it is necessary to first discuss the history of ideas and empirical evidence that led to the conclusion that markets are indeed efficient. The Efficient Market Hypothesis itself could not have been formulated without the prior theoretical work on the randomness of price movements in markets, the type of randomness that price movements exhibit, speculation as a fair game, and the empirical work that defended these ideas.

One of the initial concepts that seemed to support the idea that markets are efficient was that prices seemed to move randomly. The first theoretical study of this stemmed from the French mathematician Louis Bachelier in his PhD thesis, “The Theory of Speculation.”[15] Published in 1900, Bachelier’s work is considered truly ground breaking to the economists of today, as it is credited as the first paper to use advanced mathematics in the study of finance. Perhaps the most famous concept of the thesis revolved around Bachelier’s use of Brownian motion, a continuous-time stochastic process, in an effort to evaluate the price of stock options.[16] Another of Bachelier’s concepts that would prove to be vital to future theories on markets is that he posited that the fundamental principle for the behavior of stock market prices was that speculation should be a fair game, where the expected profits to the speculator would be zero.[17]

To ponder the question of the efficiency of markets, one must have both a theoretical framework to explain the efficiency and empirical research to solidify the theoretical claims. While Bachelier’s theories were groundbreaking, he was not able to generate a rigorous enough empirical dataset or theoretical framework that could have connected the ideas of random price movements and speculation as a fair game to the more complex distinction of an efficient market. One of the first great leaps forward in the collection of empirical data to test the changes of prices in markets came in 1953, when Maurice Kendall published his pivotal paper, “The Analysis of Economic Time-Series-Part1: Prices.”[18] In the paper, Kendall examined the behavior of weekly changes in nineteen indices of the British industrial share price as well as the spot price for cotton in New York and wheat in Chicago.[19] His conclusion, which was later restated in Eugene Fama’s most famous work, “Efficient Capital Markets: A Review of Theory and Empirical Work," stated: "The series looks like a wandering one, almost as if once a week the Demon of Chance drew a number from a symmetrical population of fixed dispersion and added it to the current price to determine next week's price."[20]

Kendall was not the only researcher to discover what seemed to be randomness in the movement of prices in markets. In 1959, M.F.M. Osborne published, "Brownian Motion in the Stock Market," which discussed the movement of the logarithms of common-stock prices.[21] In the paper, Osborne provided empirical evidence that the ensemble of logarithms of prices seems to follow Brownian motion, in that the prices seem to vary with the ensemble of coordinates of a large number of molecules.[22]

The depth of empirical support for randomness continued to grow beyond the work of Kendall and Osborne, as Sidney Alexander and Paul Cootner also published significant studies. Alexander's 1961 paper, "Price Movements in Speculative Markets: Trends or Random Walks," went beyond the idea of randomness and efficiency, and also tried to postulate about the stochastic nature of the random price movements of securities. [23] He concluded that markets follow a random walk, whereby the next move of the speculative price is independent of all past moves or events, and that at any time the change to be expected in prices can be represented by the tossing of a coin. [24] Alexander's suggestion that prices follow a random walk was really a special case of an efficient market. While important, Alexander's paper generated high levels of criticism, which led him to reevaluate his work and write a new paper in 1964, in which he concluded that the S&P Industrials average did not follow a random walk as he found evidence of profitability for persistence-type technical formulas.[25] Alexander's work in 1961 still holds significance as it represents the shift in efficient market thinking from attempting to merely show that markets are efficient to actually attempting to explain how efficient markets act.

Cootner's 1962 paper, "Stock Prices: Random vs. Systematic Change," followed this trend of attempting to explain how markets operated, but his conclusions differed greatly from Alexander.[26] Cootner's research suggested that markets were not perfectly efficient and did not follow a random walk. Instead, he concluded that markets were close to efficient. He postulated that market prices change as uninformed investors move prices around in essentially random ways, which may move prices away from their true value. When this occurs, intelligent investors bring prices back into balance, but they wait until market prices deviate far enough in order to generate an attractive return. Therefore, Cootner claimed that while markets were generally efficient, prices could move within a small band around the expected price. As prices deviate from true market value, random price movements would still occur, but changes would be more likely to revert the market price towards the expected price rather than further away. [27]

The empirical evidence explained above created an impetus for the development of a unifying efficient market theory.[28] Two of the first minds to attempt to develop a unifying theory by rigorously studying the ideas of fair games, efficient markets, and the stochastic processes that displayed how prices move were Paul Samuelson and Benoit Mandelbrot. Samuelson's 1965 paper, "Proof That Properly Anticipated Prices Fluctuate Randomly," is considered to be one of the most important papers ever written regarding the Efficient Market Hypothesis. Not only was Samuelson one of the first to thoroughly provide a detailed theoretical framework for the efficiency of markets, but he also identified the randomness associated with price movements in financial markets, which continued the trend begun by Alexander and Cootner.[29] Unlike Alexander's 1961 paper which described the randomness in price movements as a random walk, Samuelson instead explained them as a martingale.[30] The martingale is a stochastic variable X_t, which has the property that given a certain information set there is no way for an investor to use that information set to profit beyond the level which is consistent with the risk inherent in a security. [31] This differs importantly from the random walk, which assumes that all price changes are independent of one another.[32] Mandelbrot's contribution, through his 1966 paper, "Forecasts of Future Prices, Unbiased Markets, and Martingale Models," was to display some of the first theories showing how, in a competitive market with rational, risk-neutral investors, returns are unpredictable.[33]

At the same time as Samuelson and Mandelbrot were publishing the important works described above, Eugene Fama was publishing his 1965 paper, "The Behavior of Stock Market Prices." In this paper, Fama defined an efficient market for the first time, and claimed that markets follow a random walk.[34] While Fama's 1965 paper is considered to be one of the most important in the development of the efficient market hypothesis, it is largely overshadowed by his seminal work, "Efficient Capital Markets: A Review of Theory and Empirical Work," which was published in 1970. In the paper, Fama defines an efficient market as one that fully reflects all available information, which implies that it is impossible for an investor to generate above average returns once risk is accounted for.[35] Therefore, speculation is a fair game, and the expected return to the speculator is zero. Importantly, Fama further stated that there are three possible forms of the Efficient Market Hypothesis: weak form, semi-strong form, and strong form. The critical idea underpinning the theory is that a set of information cannot be used to generate superior risk-adjusted returns. The only difference in the three forms was which subset of information was not advantageous. The weak form of the hypothesis holds that the information set is all historical information of the stock.[36] The semi-strong form holds that the set of information is all publicly available information.[37] The strong form of the hypothesis included the possibility that a subset of investors could have monopolistic access to some information relevant to price formation.[38] Fama claimed to find strong evidence for the weak and semi-strong form of the hypothesis and very little evidence to refute the strong form.

In stating an efficient market in this way, Fama relied upon the idea that equilibrium prices on securities were generated as in the two-parameter Sharpe-Litner world. Therefore, just as Bachelier posited, it was a fair game that could be represented using the equation[39]:

$$E(P_{j,t+1}|\Phi_t) = (1+ E(R_{j,t+1}|\ \Phi_t)P_{j,t}$$

This equation states that the expected price of security j in the next period given some information set is equal to one plus the expected percentage return of security j in the next period given that same information set multiplied by the price of the security in the last period.

In addition to his discussion of market efficiency, just as Samuelson and Alexander before him, Fama was interested in being able to describe an extension of the fair game whereby he could explain the stochastic process that generates returns. Fama discussed two possible special cases of the fair game, the submartingale and the random walk. Fama concluded that the random walk best fit the empirical evidence. The random walk model is based on two core assumptions.[40] The first is that the current price of a security fully reflects all available information, which implies that successive price changes are independent. The second core assumption is that successive changes are identically distributed. Mathematically, Fama represented the random walk using the equation[41]:

$$F(R_{j,t+1} | \Phi_t) = F(R_{j,t+1})$$

This equation simply states that the conditional and marginal probability distributions of an independent variable are identical.

While Fama's explanation of the stochastic process generating returns as a random walk received some pushback, his explanatory model showing how markets are efficient was widely celebrated.[42] The ideas that markets were a fair game, that the expected return of any security was only a function of that security's risk, and that prices moved randomly in some stochastic process (random walk or martingale) were widely heralded. Markets were deemed to be efficient, and a flood of empirical evidence came to support this idea. This included Jensen's 1968 study, "The Performance of Mutual Funds in the Period 1945-64," in which Jensen detailed the failure of professional investors to beat or even match the index in which they traded in.[43] It also included the first ever event study, undertaken by Fama, Fisher, Jensen, and Roll in 1969, which analyzed the price movement of stocks after stock splits, and found that stock prices did not adjust irrationally.[44] The evidence was so overwhelming that it led Jenson to famously state in 1978 that, "There is no other proposition in economics which has a more solid empirical evidence supporting it than the Efficient Market Hypothesis."[45]

Aside from his explanation of the three forms of possible market efficiency, one of Fama's most important insights was an idea that had been first discussed by Milton Friedman in his 1953 book, "Essays in Positive Economics."[46] Prior to Friedman's work, the defense of the efficient market had relied on two foundational statements.[47] The first stated that it is likely that all investors are rational. If all investors are rational markets are efficient by definition, as no investor would every pay more for a security than what it was worth.[48] The second was that even if the first condition didn't hold and some investors were irrational, these investors would deviate from rationality in random ways and therefore markets would remain efficient. The irrational investors would trade among themselves and would not affect the integrity of the market. [49] Friedman's seminal insight provided the third foundational statement, which stated that even if irrational investors somehow deviated non-randomly, there would always be arbitrageurs in the market who would take advantage of these irrational trades and keep the market in equilibrium.[50] Fama also espoused Friedman's belief, and arbitrage became a prominent component of the Efficient Market Hypothesis. Arbitrage was employed by Samuelson, Merton, and Scholes to close their respective models of security pricing.[51] It was also vital to Sharpe's development of the Capital Asset Pricing Model and Ross's development of Arbitrage Pricing Theory.[52]

In summary, the Efficient Market Hypothesis stemmed from the ideas that speculation should be a fair game where the expected profit of the speculator is zero and that prices seem to move randomly in markets. Empirical evidence seemed to support both the random movement in prices and the idea that markets were a fair game. Empirical and theoretical arguments were developed to support the hypothesis as further studies attempted to define the stochastic process through which prices moved. The efficient market hypothesis was deemed to be irrefutable, not only because of the empirical evidence that supported it, but also because of the three foundational statements that seemed to ensure that markets must always remain efficient: investors are rational, if investors are not rational they will deviate from rationality in random ways, and the insight of Fama and Friedman that claimed that if investors deviate from rationality in non-random ways arbitrageurs would rapidly and efficiently ensure that market prices remained efficient. The idea of the efficient market hypothesis both fit the data and had a firm theoretical framework.

Part 3 – The Role of Arbitrage in Refutations of the Efficient Market Hypothesis

Despite the overwhelming support and prevalence of the Efficient Market Hypothesis, eventually challenges began to arise to some of the most deeply entrenched beliefs about efficient markets. These empirical and theoretical challenges attacked all three of the core assumptions behind the basic arguments of efficiency: rationality, randomness in any deviations from rationality, and the efficacy of arbitrage in cases of non-random deviations from rationality. Empirically, two of the most important challenges to the efficiency of markets came from Robert Shiller. In 1981, Shiller published, “Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?” The idea behind this paper was that rational investors should assume that prices are equal to the net present value of future dividends. Shiller found that stock prices varied too greatly to merely be explained by subsequent changes in dividends, which meant that stock prices were too volatile than could be justified by rationality. [53] Shiller also attacked the rationality of investors in his book, “Irrational Exuberance,” which was published in 2000 at the height of the dot-com boom.[54] The book argued that stock markets at the time were so overpriced that only irrationality could have led investors to bid prices up to that level. [55] He was given anecdotal support when the market crashed merely months after the book's publication.[56] In addition to the empirical evidence provided by Shiller, many important theoretical claims against efficiency were also proposed.

Perhaps two of the most important pieces of literature to attack the theories behind the Efficient Market Hypothesis came from Amos Tversky and Daniel Kahneman. The first, published in 1979, discussed what the authors called Prospect Theory, in which they continued to build off of a paper they wrote in 1973 that highlighted the fact that people seem to have Non-Bayesian expectation formation.[57] In their 1979 work, the authors identified two primary effects on the psychology of investors that caused them to behave irrationally.[58] The first was called the Certainty Effect, and it described how a reduction in the probability of a reward creates a psychological effect of displeasure, which leads to a perception of loss, which favors a risk-averse decision. [59] However, the same reduction results in larger psychological effects when it is done from certainty rather than from uncertainty. This means that individuals have an irrational tendency to be less willing to gamble with profits than with losses.[60] The second was the Isolation Effect, which showed that people often disregard components that alternatives share and focus on what distinguishes them; this can lead to inconsistent preferences. [61] Combined, these effects also meant that individual investors use reference points, look at relative gains and losses, and assign value to gains and losses rather than final asset allocation.[62] In the simplest of terms, Tversky and Kahneman had provided significant evidence that not all investors were rational.

Further convincing evidence that investors are not always rational came from Fischer and Black in 1986, when they discussed the idea that investors don't merely trade on information, but that they also trade on noise, which can be thought of as any trade not based upon rationality. The authors argued that these noise trades are inherently irrational and are what make market observations imperfect.[63] They also stated that it is this noise trading that leads to the majority of trading of individual shares. The logic of their argument was that without noise trading people would hold individual assets, but rarely trade them, and even then only usually to change exposure to broad market risks by trading mutual funds or portfolios. [64] This is due to the fact that only someone with privileged insight to an individual security would want to trade, but that individual would realize that the only way someone else would trade that individual security with them would be if that person believed that they were the one that held privileged insight. From the point of view of a neutral observer, one of these beliefs must be wrong. [65] Therefore, it is this noise trading that provides the essential aspect of liquidity to markets, and the more noise trading in a market the more liquid a market will be.[66]

The rebuttals referenced in the three preceding paragraphs strongly refuted the first argument of the efficient market hypothesis that all investors behave rationally in valuing securities. This shifted the argument for the efficiency of markets to the second set of beliefs, that irrational investors will deviate from rationality in random ways, which means that the effects of irrationality will cancel out and markets will remain efficient.

One of the most convincing arguments that refuted the theory of random deviation from rationality by irrational investors came from Tversky and Kahneman in 1986 with their paper, “Rational Choices and the Framing of Decisions.”[67] The authors showed that irrational actors deviate in the same or similar ways, which means that irrational traders likely aren’t trading with each other. The authors were able to show systematic deviations in rationality in the four normative rules from which expected utility theory is derived: cancellation, transitivity, dominance, and invariance.[68] While other authors had previously refuted cancellation and transitivity, subsequent studies had merely weakened normative theory so that it could still remain a descriptive model.[69] The refutation of dominance and invariance left this position untenable. Therefore, just as Tversky and Kahneman had laid the foundation for the refutation of the idea that markets could be efficient because all investors were rational, they also were able to convincingly provide evidence that markets could not be proven to be rational based on the idea that irrational investors will deviate from rationality randomly. Therefore, as Shleifer penned in his book, "Inefficient Markets: An Introduction to Behavioral Finance," the only remaining tenant holding up the efficiency of markets was the idea espoused by Friedman and Fama that arbitrageurs would always be able to ensure that markets remained efficient by correcting the deviations caused by irrational actors.[70]

Before delving into the intricate details of arbitrage, it is useful to reexamine the textbook definition of arbitrage, which states that arbitrage is, "the simultaneous purchase and sale of the same, or essentially similar, securities in two different markets at advantageously different prices."[71] Pure arbitrage requires no capital and entails no risk; it is a path towards sure, riskless profit. It is also useful to discuss the way that arbitrage functions in the efficient market models of Fama, the Capital Asset Pricing Model, and Arbitrage Pricing Theory. In these models, arbitrage is undertaken by a very large number of arbitrageurs each taking an infinitesimally small position against the mispricing in a variety of markets.[72] Because positions are so small, capital constraints are not binding and arbitrageurs are risk-neutral towards each other. Additionally, arbitrage is so efficient that it appears in such small amounts and disappears so rapidly that not even arbitrageurs are able to continually earn superior risk-adjusted returns when accounting for risk.[73]

In 1997, Andrei Shleifer and Robert Vishny published, "The Limits of Arbitrage," in which they discussed how in the real world arbitrage was limited in its ability to perform the role that Fama and Friedman suggested it would.[74] Shleifer and Vishny argue that in the real world arbitrage requires capital and entails some risk. The authors explain that in reality professional arbitrage is conducted by a relatively small number of highly specialized investors; there are not millions of little traders as discussed in efficient market models.[75] Additionally, the specialized investors are not using their own money to trade, but instead have investors give them money so that they can put to use their expertise in a certain market. This creates an agency problem, where in an arbitrage situation short-term price movements against the position can lead investors to withdraw their money from arbitrage specialists, who then may need to close out positions or put up more collateral just as the arbitrage opportunity is actually increasing its expected return.[76] The authors go on to suggest that due to these issues, arbitrage will occur less in more volatile markets where values are unable to be known with as much precision, which leads to less arbitrage in stocks than in bonds or foreign exchange. The authors posit that these limits to arbitrage can then lead to anomalies that break with the Efficient Market Hypothesis.[77]

Refutations of the Efficient Market Hypothesis state that real world arbitrage is risky and therefore limited. They state that arbitrage relies on the availability of close substitutes for securities affected by noise trading, and that a lack of obvious substitutes makes it harder for the prices of stocks and bonds to remain rational. In his book, "Inefficient Markets: An

Introduction to Behavioral Finance," Shleifer penned descriptions of two types of arbitrage that exist in the real world. The first, risk arbitrage, focuses on situations where there is a high statistical likelihood of convergence of the price of two securities, but not certainty.[78] An example of risk arbitrage that Shleifer provides is that if an arbitrageur believes that Ford is overvalued when compared to GM, the investor can buy GM and short Ford. Much of the risk is laid off, such as market and sector risk, but idiosyncratic risk remains. [79] The second type of arbitrage Shleifer discusses is limited arbitrage. This occurs when even in the case of perfect substitutes arbitrageurs must worry about financing and maintaining their position. [80] This can occur in a trade with convergence probability of one if the trade goes in the opposite direction before convergence occurs due to issues that the Shleifer and Vishny paper discussed such as capital constraints and agency problems.[81]

In order to more fully examine the ideas of arbitrage put forward by proponents and detractors of the Efficient Markets Hypothesis, it is useful to look into examples of arbitrage that have occurred in financial markets in the real world. One such example occurred during the dot-com boom in early 2000 when 3Com Corporation spun off a subsidiary, Palm Pilot, by floating five percent of Palm Pilot shares to the public.[82] The other ninety-five percent of the shares were still held on 3Com's balance sheet. As Palm Pilot trading began, the shares went up to such an extent that the ninety-five percent of shares on the 3Com balance sheet had a market value considerably more than the entire market capitalization of 3Com, suggesting that the rest of the business had a negative value. [83] This example can be viewed in two lights. The first, through the lens of a detractor of the Efficient Market Hypothesis, would suggest that the fact that this obvious arbitrage opportunity existed and that equilibrium was not quickly restored by arbitrageurs suggests that markets can be inefficient. On the other hand, Burton Malkiel, a noted proponent of the Efficient Market Hypothesis who worked closely with Fama and penned the famous book, "A Random Walk Down Wall Street," suggests that this example provides the opposite lesson. Malkiel claims that this was merely an idiosyncratic event that occurred because there were not enough shares of Palm Pilot floated to sufficiently short and hedge a position. His interpretation is supported by the fact that once the number of publicly floated shares of Palm Pilot went up the arbitrage opportunity disappeared.[84]

Another example of real world arbitrage that occurred in financial markets was described in the paper, "The TIPS-Treasury Bond Puzzle," which was written by Fleckenstein, Francis, and Lustig in 2014.[85] The authors showed that the price of a Treasury Bond and an inflation-swapped Treasury Inflation-Protected Security that exactly replicates the cash flow of the Treasury Bond can differ by up to twenty dollars per one hundred dollars notional. [86] The authors looked at whether the mispricing was due to a mispricing of inflation swaps, transaction costs, differential taxation, credit risk, institutional and foreign ownership, collateralization, the ability to short Treasury Bonds, market liquidity, and other factors, and none were able to fully account for the existence of the mispricing.[87] The authors then asked the question as to whether the arbitrage was "risk-free" arbitrage in the classic sense. They claimed that it was. The authors subsequently asked if the situation was a risky leveraged strategy that could result in losses for an arbitrageur in some states of the world. They also claimed yes. These seemingly contradictory answers were explained by the fact that even in a classic arbitrage situation with a convergence probability of one, real world factors can pose risks.[88]

In the previous two sections, I have discussed the histories of the literature that both support and refute the idea that markets are efficient. I have also delved into the centrality of arbitrage to this argument, different definitions of arbitrage, and different factors that may affect the ability to capture arbitrage. Arbitrage opportunities that consistently offer an opportunity to generate above average risk adjusted returns pose an interesting problem to the way arbitrage is viewed in both the efficient market models and the models that refute efficiency. In efficient market models arbitrage should exist, but it should disappear rapidly. In models that deny efficiency, arbitrage can exist in the real world, but it should almost always be risky arbitrage or limited arbitrage. The latter models claim that issues such as a lack of perfect substitutes, capital constraints, and other risk factors will almost always turn even apparently "risk-free" arbitrage into limited or risky arbitrage.

Part 4 – Other Literature on Arbitrage in Soccer Betting Markets

Following the previous discussion of the history of efficient market and arbitrage theory, I will now attempt to briefly summarize the history of the formal study of arbitrage in betting markets. While there is a dearth of literature regarding arbitrage in online betting markets, I will discuss and cite a few significant papers before I delve into my own data and results.

One of the first significant studies that examined possible arbitrage opportunities in sports betting markets by looking at odds on different bookmakers was performed by Haush and Ziemba in 1990. The authors explored the potential for risk-free arbitrage profits in cross-track betting on US racetracks, and found that there are significant differences in prices on the same race from one track to another. They believed that this was due to the existence of different betting pools across tracks.[89] The year before, in 1989, one of the earliest studies that looked at the English soccer betting market was performed by Pope and Peel. The authors looked at the odds for English soccer games during the 1981/82 season at four different odds makers, and they found one opportunity for arbitrage after all costs of bets were accounted for.[90] In 2004, Dixon and Pope reanalyzed the English soccer betting market using three bookmakers from the UK from 1993 to 1996. They concluded that there were no violations of the no-arbitrage condition. When they compared this to the result of Pope and Peel (1989), they concluded that the decrease in arbitrage opportunities was likely due to the increased efficiency of odds makers.[91]

One of the first studies to find multiple cases of arbitrage in the European soccer betting market was written in 2009 by Vlastakis, Dotsis, and Markellos. The paper, "How Efficient Is the European Football Betting Market? Evidence from Arbitrage and Trading Strategies," examined six major bookmakers across Europe from 2002 to 2004. The authors looked at games in 26 countries and events, and found that in about one in every two hundred games an arbitrage opportunity was available.[92] The authors also found that when arbitrage was available it was highly profitable, offering an average return of 21.78%.[93] However, when the authors looked only at online betting sites the number of arbitrage games reduced to one in one thousand, and they found that the number of arbitrage games declined in each period of study. This seemed to lend credence to the idea of Dixon and Pope that betting markets were getting more efficient over time and offering fewer opportunities for arbitrage.[94]

The most similar study performed to the one I undertook was done by Franck, Verbeek, and Nuesch in 2013, "Inter-market Arbitrage in Betting." This analysis examined arbitrage opportunities using both online bookmakers and Betfair, which is a bet exchange where individuals place wagers against each other. The authors examined the top five soccer leagues in Europe from 2004 to 2011 (England, France, Germany, Italy, and Spain) and found that 0.8% of games had an arbitrage opportunity with an average return of 0.9%.[95] Consistent with efficient market theory, the authors argued that by picking soccer, the most popular European sport and the one with the most available information, they were picking the market with the least likelihood of arbitrage opportunities, and they therefore claim that they may be understating arbitrage in sports betting as a whole.[96] In the end, the authors conclude that they believe that arbitrage arises as some betting websites set inefficient odds to try to lure customers with a winning bet because once a customer picks a betting site they are very unlikely to change. Therefore, over time the customer that is gained will have a positive value to the company even if the company loses on the initial wager.[97]

Interestingly, I find evidence that contradicts many of the author's conclusions in the previous studies discussed in this section. For one, I find that as markets become more transparent and increase in size, arbitrage seems to become more prevalent, which derails the claim that betting markets with less information will have more arbitrage opportunities. Additionally, I find the claim by Franck, Verbeek, and Nuesch that arbitrage occurs because betting websites purposely set inefficient odds to lure customers with winning bets to be insufficient for a few reasons. It seems unlikely that it would be profitable for bookmakers to purposely set inefficient odds so often that twenty-one percent of games in the 2014/15 English Premier League season could have offered arbitrage. This would also require a strong correlation among bookmakers choosing the same game to attempt to lure new customers.

Additionally, the purposeful inefficiency explanation doesn’t properly account for why average profits per game for bookmakers has steadily declined since 2001, or why the number of arbitrage games has steadily risen since 2008. Finally, I believe that there is a more compelling explanation that fits with the data. Namely, competition has driven initial bookmaker odds closer to their true or fair values, which, when combined with random noise, causes arbitrage games to occur. Later I will provide a simple model that will detail how this could occur in a simplified version of the real world.

In summary, while some previous studies have looked at arbitrage opportunities in sports betting, this is the first study known to the author that looks at English Premier League data spanning fifteen years. Additionally, this is the first study to note the increase in arbitrage games and the decrease in expected profits for bookmakers in recent years. The significance of the arbitrage opportunity discovered is also unprecedented in other literature. Finally, the conclusions I draw, which include that larger more transparent markets have more arbitrage opportunities and that competition and declining profit margins for bookmakers have played a key role in creating arbitrage opportunities have not been noted in any paper known to the author. The vast differences in this paper to past literature can likely be explained by the rapidly changing nature of the online betting market and the fact that other published studies did not have access to more recent data.

Part 5 – Data and Results

The data used in this paper was collected by football-data.co.uk, a soccer betting portal that provides historical results and odds of soccer games to “help football betting enthusiasts analyze many years of data quickly and efficiently."[98] The odds cited on the site were recorded on Friday afternoons for weekend matches and Tuesday afternoons for midweek matches.[99] The site has recorded odds for thirteen different betting sites, but only seven sites were used in this study in order to maintain consistency between the different years studied. The seven betting websites used for this paper were: Bet 365, Bet&Win, Interwetten, Ladbrokes, VC Bet, Stan James, and William Hill. (In my initial tests of the data additional sites were used. The precise reason for the narrowing of the universe of sites used will be expanded upon below.)

The detailed method of how I tested for arbitrage was explained in Part 1. In general, I first determined the odds with the highest payout to the bettor for each outcome of every English Premier League game from the 2000/01 season through the 2014/15 season by comparing the different odds offered by the different betting websites. Next, using the best odds for each outcome, I determined if an arbitrage opportunity was available in a given game by using equation (1). For each game I also calculated the expected winnings if arbitrage was used using equation (2).

The number of games that provided an arbitrage opportunity is shown by year in Table 1 below. The year given in the table refers to the year that the season ended. For example, 2015 refers to the 2014/15 season (This method of referring to individual seasons will be used for the remainder of the paper). The "Number of Arbitrage Games" column denotes the number of games that possessed an arbitrage opportunity in the stated season. The "Percentage of Arbitrage Games" column states the percentage of games that arbitrage was found (380 games per season).

Table 1

Year	Number of Arbitrage Games	Percentage of Arbitrage Games
2015	81	21.3%
2014	68	17.9%
2013	77	20.3%
2012	7	1.8%
2011	13	3.4%
2010	2	0.5%
2009	1	0.3%
2008	3	0.8%
2007	0	0.0%
2006	0	0.0%
2005	0	0.0%
2004	0	0.0%
2003	0	0.0%
2002	0	0.0%
2001	0	0.0%

As can clearly be seen in the data, by using the odds of the sites in the study, no games in the English Premier League offered an arbitrage opportunity until the 2008 season. This may explain why previous studies, which were not able to use more recent data, obtained such different results. Additionally, it is interesting to note that arbitrage was extremely rare until a small jump during the 2011 season, which was followed by an extremely large jump during the 2013 season.

After the discovery that arbitrage was fairly prevalent in recent years, I looked at the average return and the maximum return to the bettor for a single arbitrage game in each of the years that had at least one instance of arbitrage in the study. They are displayed below in Table 2.

Table 2

Year	Average Arbitrage Profit (%)	Maximum Arbitrage Profit (%)
2015	0.71%	3.28%
2014	0.65%	3.15%
2013	0.57%	4.62%
2012	0.44%	0.99%
2011	0.63%	1.97%
2010	0.24%	0.43%
2009	0.16%	0.16%
2008	0.18%	0.29%

After discovering that arbitrage occurred at a relatively high rate and that the average arbitrage profit was meaningful, I undertook an analysis to discover how profitable the arbitrage strategy would have been in each year that arbitrage opportunities were available. I assumed that there were no costs to set up the account on each site as this is not how betting sites earn their revenue. I then assumed that an individual could wager on one game each day there was a match. This makes sense as an arbitrageur would want to use all capital on the game that promised the greatest profit. Therefore, for each day that had a match, I calculated which game provided the highest expected profit. I then ran two different analyses. The first (Case 1) assumed that the individual had to bet on each day there was a match. Therefore, if no betting opportunities offered arbitrage on a given match day, the individual would still bet on the game that had the highest expected return. In these cases, that return would just be negative. The second analysis (Case 2) allowed the individual to only bet on match days that offered an arbitrage opportunity. The winnings from each match were reinvested in the strategy for the entirety of the season.

Table 3

Year	Case 1 Return	Case 2 Return
2015	17.01%	54.97%
2014	5.25%	44.08%
2013	1.75%	43.18%
2012	-72.54%	3.12%
2011	-76.59%	7.45%
2010	-90.16%	0.35%
2009	-87.65%	0.12%

2008	-90.22%	0.20%

As can be seen in the data, starting in 2013, a massive profit opportunity appeared that seemed to allow an arbitrageur to earn above a 40% return by betting on an arbitrage game each day that one was available during the English Premier League season. This massive return brought up a few questions. The first was whether or not the data was robust. The second was whether or not this was truly “risk-free” arbitrage. The first question will be discussed below. The second question will be discussed in Part 7.

In order to check for robustness, I looked to see if there was ever an arbitrage betting opportunity by only using one site’s odds for a given game. This would suggest there were problems with the data, as it would be irrational for a betting site to offer arbitrage using only its own odds. I found that this was never the case. I then calculated how many times each site’s odds were used for arbitrage during the 2015, 2014, and 2013 seasons to ensure that no one site was skewing the data in the years that clearly had the most arbitrage. If two sites both supplied the best odds for a particular outcome they both were considered to have been used in the arbitrage. The data is summarized in the table below.

Site	2013	2014	2015
Bet 365	171	356	354
Bet &Win	54	50	164
Interwetten	121	136	118
Ladbrokes	95	130	174

Pinnacle Sports	554	494	461
William Hill	234	214	220
Stan James	157	99	9
VC Bet	303	360	330
Mean	211	230	229
SD	149	146	136

In 2013, all but 2 sites fell within one standard deviation of the mean and all but one site within two standard deviations. In both 2014 and 2015, all sites fell within two standard deviations of the mean, and only one fell outside one standard deviation. Pinnacle Sports had the highest number of arbitrage games in each year from 2013 to 2015, and it seemed like it could be an outlier. Therefore, it was important to check how much the data would change if Pinnacle Sports was excluded. I found that the number of arbitrage games would drop from eighty-one to forty-seven in the 2015 season, from sixty-eight to thirty in the 2014 season, and from seventy-seven to thirty-two in the 2013 season. As these numbers were substantial, I decided to restate the previous tables without Pinnacle Sports. The tables above that included Pinnacle Sports also included odds from all of the sites that I had data from for the seasons 2008 through 2012 (ten available sites). Therefore, in the tables above, the seasons 2008 through 2012 actually have data from more sites than the seasons 2013 through 2015. In the following tables, seasons 2008 through 2015 all only use the same seven sites: Bet 365, Bet&Win, Interwetten, Ladbrokes, VC Bet, Stan James, and William Hill. For seasons before

2008 there was no arbitrage opportunity, so limiting the number of sites that odds could be collected from had no effect.

Table 1.1

Year	Number of Arbitrage Games	Percentage of Arbitrage Games
2015	47	12.4%
2014	30	7.9%
2013	32	8.4%
2012	7	1.8%
2011	13	3.4%
2010	1	0.3%
2009	1	0.3%
2008	1	0.3%
2007	0	0.0%
2006	0	0.0%
2005	0	0.0%
2004	0	0.0%
2003	0	0.0%
2002	0	0.0%
2001	0	0.0%

Table 2.2

Year	Average Arbitrage Profit (%)	Maximum Arbitrage Profit (%)
2015	0.77%	2.90%
2014	0.71%	1.97%
2013	0.52%	4.62%
2012	0.44%	0.99%
2011	0.63%	1.97%
2010	0.05%	0.05%
2009	0.16%	0.16%
2008	0.24%	0.24%

Table 3.2

Year	Case 1 Return	Case 2 Return
2015	-22.86%	32.87%
2014	-38.34%	21.93%
2013	-43.22%	17.36%
2012	-73.29%	3.12%
2011	-76.96%	7.45%
2010	-91.34%	0.05%

2009	-89.85%	0.16%
2008	-93.63%	0.24%

While it is clear that Pinnacle Sports had a large effect on results, the results obtained without Pinnacle Sports are still remarkable. They suggest that an investor could have obtained a “risk-free” 32.87% return in the 2015 season, a 21.93% return in the 2014 season, and a 17.36% return in the 2013 season. Due to the facts that the adapted data set had a very small impact on the years before 2013 and that Pinnacle Sports seemed to have an outsized impact on years 2013, 2014, and 2015, the new adjusted data set will be used for the remainder of the paper.

Following my analysis of the number of arbitrage games, maximum and average profit of arbitrage games, and the return that could be expected from using arbitrage throughout the course of a season, I next began to look for explanations as to why this massive arbitrage opportunity existed in the market. Therefore, I looked at the average expected return per game for each individual site in the sample and for a theoretical bookmaker that offered the best odds for each outcome of each game. I did this using equations (1) and (2). The average expected return per game for an individual site was calculated as the return an arbitrageur would have earned using the arbitrage betting method only on the odds supplied by that individual site multiplied by negative one. The average expected return per game for the theoretical bookmaker offering the best odds was calculated as the return an arbitrageur would have earned using the arbitrage method betting on the best odds for each game multiplied by negative one. This is a logical approximation of the expected return to the betting site, because bookmakers attempt to balance their book for each game, which means that they attempt to ensure that the amount of money they will have to pay out is independent of the result.[100] This ensures that the profit to the bookmaker will be independent of the outcome of the event. When this is the case, the profit for the bookmaker is entirely dependent upon how far away from a fair game the odds are. The results shown below in Table 4 can be thought of as the expected return to the bookmaker assuming that the bookmaker had a balanced book for each game in that season.

Table 4

Year	2001	2002	2003	2004	2005	2006	2007
Average	11.69%	10.93%	10.46%	9.96%	9.43%	9.07%	9.16%
Average (Adjusted Data Set)	11.69%	10.93%	10.46%	9.96%	9.43%	9.07%	9.16%
Best Odds	6.99%	6.49%	6.19%	5.61%	4.85%	4.34%	4.78%
Best Odds (Adjusted Data Set)	6.99%	6.49%	6.19%	5.61%	4.85%	4.34%	4.78%
# of Arb Games	0	0	0	0	0	0	0

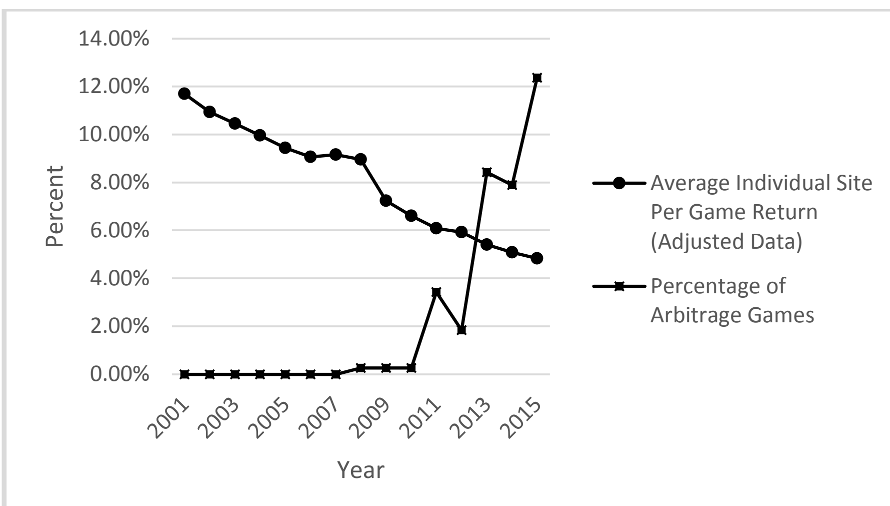
Year	2008	2009	2010	2011	2012	2013	2014	2015
Average	8.79%	7.26%	6.92%	6.58%	6.21%	5.20%	4.70%	4.47%
Average (Adjusted Data Set)	8.96%	7.23%	6.61%	6.09%	5.92%	5.40%	5.09%	4.83%
Best Odds	3.50%	2.83%	2.88%	2.02%	1.92%	0.61%	0.63%	0.55%
Best Odds (Adjusted Data Set)	3.80%	3.12%	3.00%	2.04%	1.97%	1.32%	1.22%	1.01%
# of Arb Games	3	1	2	13	7	77	68	81

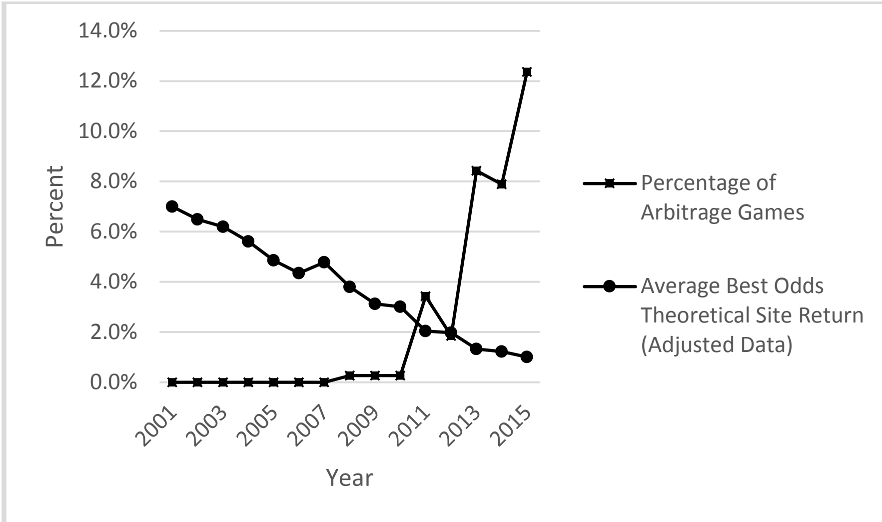
In the data displayed above, “Average” represents the average expected return for all individual bookmakers and “Best Odds” represents the average expected return for the theoretical bookmaker that set the best possible odds for each outcome. The rows denoted by “Adjusted Data Set” represent the results after the betting sites used were adjusted as in for tables 1.2, 2.2, and 3.2. The results above seem to show a pattern of decreasing expected return to the bookmaker in each succeeding year and an increase in the number of arbitrage games as the expected return to the bookmaker decreases. This can be displayed by looking at the correlation between the number of arbitrage games and year, and the correlations between each of the four expected profit percentages and arbitrage games. The correlation between the number of arbitrage games and the year (assuming 2001 was year 0) is 0.75, which seems to be quite high. The correlation between the various percentages and arbitrage games is displayed in Table 5 below:

Table 5

	# or Arb Games
Average	-0.78
Average (Adjusted Data Set)	-0.76
Best Odds	-0.76
Best Odds (Adjusted Data Set)	-0.76

As we can see, the correlations between the expected return to the bookmaker and the number of arbitrage games have a very strong negative relationship. The correlation is also clearly displayed in the following graphs:





An important property of the expected return to the bookmaker (or arbitrageur) is that if betting was a fair game as described by Bachelier, the expected profit for both the bookmaker and the arbitrageur would be zero. In a fair game, betting sites would offer fair odds where the implied probability of one of the three outcomes occurring would be one, which would mean that the expected profit would be zero. This can best be explained by an example. Suppose there is a match where it is believed that the true odds of a home win, draw, or away win occurring are equal. This would mean that the odds of each outcome is 1/3. (The total implied probability would therefore be 1/3 + 1/3+ 1/3 = 1.) As we know that decimal odds are given by one divided by the implied probability, the decimal odds in this case would be 3 for each result. As the odds are equal, an arbitrageur would attempt to bet an equal amount on each possible result. Therefore, suppose an arbitrageur has ninety-nine dollars to wager. The arbitrageur would wager thirty-three dollars on each possible outcome. No matter the result, the arbitrageur would earn three times the thirty-three dollar bet on the winning outcome, which would earn the arbitrageur ninety-nine dollars, the exact amount that was invested. This connects to efficient markets in that the market would be efficient if it was impossible to use some subset of information to earn a larger expected profit (when accounting for risk) than this outcome of zero expected return. If this was the case, the implied probability of the odds would always equal one. Any deviation of the odds for an outcome would be random, and the odds would all adjust so that the total implied probability remained one.

Betting markets are actually slightly different than a fair game. Betting sites earn money not by charging a price to bet, but instead by giving slightly imperfect odds so that they win money on average.[101] In order to do this, the betting sites attempt to balance their book. In order for a betting site to balance their book, if one outcome is seeing more bets, the site will adjust their odds to encourage more people to bet on the other outcomes so that the amount of money the bookmaker will have to pay out is independent of the outcome.[102] If a bookmaker has a perfectly balanced book, the bookmaker will earn the average expected profit, which stems from the slightly inefficient odds that the bookmaker offers. Consider again our example of an event with three equally likely outcomes. While the fair odds would be 3 for each outcome, the bookmaker would set the odds for each outcome on their site at some number slightly below three, such as 2.9. Therefore, if an even amount of money was placed on each outcome as in the last example, the bookmaker would take in ninety-nine dollars, but only have to pay out 2.9 times thirty-three dollars, which is $95.70. In this way, the bookmaker would make a risk-free $3.30, which would be a 3.3% return, as long as the book was balanced. Consider however, if while the bookmaker believes the odds are equal for each outcome, the universe of individuals who want to bet on the site believe that one outcome is more likely than another, such as the home team being more likely to win than the away team. This would mean that more bets would be placed on the home team winning, and the bookmaker would be exposed to losing money if the home team indeed won. Therefore, if the bookmaker detects that the universe of individuals betting on the site believes that the odds are different than the ones posted, the betting site will continue to adjust the odds until the amount to be paid out in the event of each outcome is equivalent.

The knowledge of the way that betting sites adjust their odds and the strong negative correlation between the number of arbitrage games and the average expected profit of both individual bookmakers and a theoretical bookmaker offering the best odds for each game leads to the theory espoused in this paper as to why arbitrage games have recently become so prevalent. I suggest that increased transparency and competition between bookmakers has lead them to continually begin to give initial odds that are closer and closer to a fair game. This is supported by the fact that the average expected return for the individual sites (full data set and adjusted data set) and the average expected return for the best odds using a combination of sites (full data set and adjusted data set) have steadily declined each year that I studied. Moreover, the largest year-over-year changes between the average expected profit percentages for individual sites occurred in 2009 and 2013, while the largest percentage changes between average expected returns using the best available odds occurred in 2013 and 2008. These dates are significant because 2008 was the first year to have an arbitrage opportunity, and 2013 was the year that arbitrage first became a relatively common occurrence (seven instances of arbitrage in 2012 versus thirty-two in 2013).

As initial odds have come closer to being a fair game, I propose that random noise trading can differ from site to site, and that as this random noise trading is occurring, sites adjust their offered odds in an effort to balance their books. As sites attempt to maintain a balanced book, the odds given on different sites can drift far enough apart that arbitrage opportunities become possible. Therefore, I suggest that while this random drift occurred before arbitrage was available, the initial odds that were set were too far away from a fair game to drift to the extent that arbitrage was possible. Now that increased competition has brought odds closer to a fair game, arbitrage has become more likely. A result of this theory would be that contrary to the beliefs of previous authors in the literature, markets that have more information, are more transparent, and have more individuals betting are actually more likely to have arbitrage opportunities because the increased amount of money being wagered in the market increases the amount of competition, which brings the initial odds closer to fair value.

In order to test this theory, I examined the English Championship, which is the second highest division of English soccer. I chose to examine the English Championship because I wanted to find a league that had much in common with the English Premier League. The teams in both leagues are all based in the same country, and there is even a promotion and relegation system where the bottom three teams in the Premier League at the end of each season are relegated to playing in the Championship for the next year while three teams in Championship are promoted to the Premier League. This means that there would be teams that were both in my Premier League and Championship data sets. The main differences between the leagues (aside from quality of play) are the amount of money involved (both in terms of the value of the league and the amount of money bet on games) and the amount and ease of collection of information about the league. Premier League games are shown all around the world and are discussed by individuals everywhere, while the importance of the Championship is generally confined to England. Another reason to use the Championship was that the same betting websites that gave odds on the Premier League also gave odds on the Championship. The data was again collected from football-data.co.uk. The results I found are displayed below in Table 6. (Note: There are 552 games in the Championship each year versus 380 games in the Premier League season.)

Table 6

Year	# of Arbitrage Games	% of Arbitrage Games	Best Odds Expected Return (For Betting Sites)
2015	17	3.20%	1.48%
2014	21	3.80%	1.52%
2013	16	2.90%	1.78%
2012	4	1.05%	2.27%
2011	6	1.09%	2.51%
2010	0	0.00%	4.32%
2009	0	0.00%	4.14%
2008	0	0.00%	5.37%

As can be seen, the data from the Championship supports the theories put forward in this paper in numerous ways. For one, instances of arbitrage are rarer in the Championship than the Premier League, which provides empirical support for the idea that arbitrage will occur more often in markets that are more transparent and have more information. Additionally, the

“Best Odds” expected return for betting sites is higher in the Championship, and it also is highly negatively correlated with the percentage of arbitrage games, with a correlation of -0.89.

The idea of more transparent markets having more instances of arbitrage can seem counterintuitive at first, but it actually makes theoretical sense if the theory that odds being initially set closer to a fair game is what leads to arbitrage holds. The logic can be described as follows. The easier true information is to acquire, the easier it is for a market to be efficient. Therefore, if information can be found more easily, a market should trend towards greater efficiency. As an efficient market is one that is a fair game, a market that has more information will trend closer toward a fair game. Therefore, if it is true that odds being set closer to fair game values leads to arbitrage, it is also true that a betting market that has more information will have more arbitrage. While this logic is able to explain how arbitrage has occurred in the soccer betting market, it doesn’t explain how it has been able to persist. If each individual site is viewed as its own market, it is clear to see how arbitrage could occur even as these sites behave totally rationally. However, this doesn’t explain why arbitrageurs have not reacted to this opportunity to bet across different sites, which would prevent the arbitrage opportunity from continuing to deliver such fantastic potential returns. In efficient market models, these arbitrage opportunities would quickly be taken advantage of so that there was no opportunity for long-term profit above the inherent risk in the security.[103] In theories that reject that Efficient Market Hypothesis, the arbitrage in a market could persist if it is not true “risk-free” arbitrage, but is instead limited or risky in some way.[104] The exact nature of the arbitrage found in these results will be discussed further in Part 7.

In summary, I found that arbitrage opportunities were first available in my data set in 2008, and that the number of opportunities has increased dramatically since that date. I found that during the 2015 English Premier League season it would have been possible to earn a "risk-free" return of 32.87% by betting on an arbitrage game each day that there was a match that offered arbitrage. Additionally, I found very significant correlation between the average expected return per game of both individual bookmakers and the best combined odds offered and the number of games that offered an arbitrage opportunity during that season. The knowledge that bookmakers attempt to balance their books to ensure equal profit independent of the outcome and the correlation between the average expected return per game for bookmakers and the number of games with an arbitrage opportunity in a given season lead me to propose a new theory as to why arbitrage opportunities have recently become available. I suggest that as the odds initially offered by bookmakers becomes closer to the odds that would be offered in a fair game, random drift that differs between betting sites is more likely to be significant enough to lead to an arbitrage opportunity. This theory differs greatly from other literature in the field such as the paper by Franck, Verbeek, and Nuesch discussed in Part 4, which suggested that odds makers purposely set inefficient odds for some games in order to attract new customers. Next, I will attempt to provide more evidence as to how rational actions by individual bookmakers and by individuals placing bets can lead to arbitrage opportunities across betting sites by introducing a simplified model of how the betting markets work.

Part 6 – Simplified Model

In order to discuss how my theory could theoretically function and lead to arbitrage, I created a simple model that mimics how the betting markets work. In the model, I assumed that there were two rational betting sites, Site X and Site Y, that provide odds on a game that has two possible outcomes, A and B. The betting sites both believe that there is an equal chance of either outcome occurring. As there is believed to be a fifty percent chance of either outcome occurring, in a fair game the betting sites would initially offer decimal odds of 2 on both outcomes. However, as betting sites are an adjusted fair game, the model assumes that the betting sites adjust their odds to earn an expected return of five percent. The five percent return is consistent with the data found in the real world. In addition to the betting sites, the model assumes that there are two sets of one hundred individual bettors. The sets of one hundred individuals are identical, except that one set will either bet on Site X or not bet at all, and the other set will either bet on Site Y or not bet at all. The belief probabilities in both sets of individuals is evenly distributed in their belief probabilities of outcome A occurring. Their belief in outcome B occurring is one minus their belief in outcome A. The individuals in each set were uniformly distributed in the following fashion:

Probability of Outcome A	0.1	0.2	0.3	0.4	0.5	0/6	0.7	0.8	0.9
Number of Individuals	11	11	11	11	12	11	11	11	11

In the model, two sets of random numbers from one to one hundred were generated. The first set of randomly generated numbers determined the order that the first set of individuals would look at Site X. The second set of randomly generated numbers determined the order that the second set of individuals would look at Site Y. The individual would place a bet on either outcome A or B if the individual's perceived probabilities of the outcome occurring was greater than the probability implied by the odds that the bookmaker supplied. In this way, the individual acted rationally and only bet if they believed the expected value of that bet was positive. After each individual bet, the bookmaker would adjust the odds to account for the fact that a bet had been placed while ensuring that their expected return on the next bet remained five percent. In the model, the bookmakers determine their odds for outcome A using the following equation:

$$P_{(a,t(x))} = P_{(a,t(0))} – B_{(t(x))} * \alpha$$

The equation above states that the cited probability of outcome A at time X is equal to the probability of outcome A at time 0 minus the Bet Direction multiplied by alpha. Bet direction is a variable which tracks the number of bets placed on outcome A versus outcome B. If a bet is placed on outcome A, the variable bet direction goes up by one, and if a bet is placed on outcome B, the variable bet direction goes down by one. Alpha is a constant used to describe to what extent the variable bet direction affects prices. Once the odds for outcome A are set, the bookmaker calculates the new odds for outcome B by using the following equation:

$$P_{(b,(t(x))} = 1/(I – (1/P_{(a,t(x))})$$

This equation states that the odds of outcome B at time x is equal to 1 divided by the implied probability of outcome A or B minus one divided by the probability of outcome A. The implied probability of outcome A or B, denoted by I, represents the implied probability of the two outcomes assuming that the bookmaker will get a five percent return. In a fair game, I would be one and the expected return would be zero. To calculate the value of I that will lead to a given percent return, we can use the following equation:

$$I = -(1/(R-1))$$

In the equation above, R represents return for the bookmaker and I represents the implied probability that either outcome A or B will occur. In order to get a five percent return, the bookmaker will set the implied probability at approximately 1.05263. As was explained in Part 1, the fact that this number is above one means that the odds are expensive, and the bookmaker is expected to earn some profit. By using the correct implied probability, the probability of outcome B in the model adjusts based on the probability of outcome A so that the expected return is always five percent for the bookmaker.

This model allows for the randomness of the order of individuals that look at the given market, which represents random noise, to influence the odds the bookmaker provides. This initial influence of odds has further knock on effects, as later individuals will be less likely to bet on a certain outcome if the implied probability has already gone up and the odds offered, or payout, has gone down. However, if the right set of individuals check Site X and Site Y in the appropriate order, arbitrage can occur.

Arbitrage was considered to be possible in the model if after the final individual of each set was offered the opportunity to bet, the odds had been changed significantly enough so that by betting on one outcome on one site and the other outcome on the other site arbitrage was possible. We only considered arbitrage to occur if it existed after the final individuals because the data used in the analysis of this paper only had the odds at one particular moment.[105] I ran the model two hundred times. For the first one hundred runs, alpha was set at 0.03, and arbitrage ended up being possible in seven cases. Alpha being set at 0.03 means that for every bet on outcome A the odds offered on outcome A drop by 0.03 for the next bettor. For example, since the initial odds are set at 1.9 for both outcome A and B (This is done because odds of 1.9 for each outcome lead to an expected return of five percent for the bookmaker), if the first individual bets on outcome A, the second individual will see odds for outcome A of 1.87. This seven percent rate of arbitrage compares similarly to the rates found in 2013 and 2014 in the adjusted data set. For the second hundred runs, alpha was set to 0.04. In this case, arbitrage ended up being available in twenty-six cases, which shows how sensitive instances of arbitrage can be to small changes.

The model assumes a simple world, where rational individuals place bets if they believe that the expected value of that bet is greater than zero and rational betting sites constantly adjust their odds in an effort to balance their book to maintain a constant profit. The model then allows for random noise, which is simulated by the random order that individuals choose to bet, to cause the betting sites to change their given odds. The most important conclusion that can be drawn from this model is that given a certain set of assumptions, betting sites do not have to act irrationally for arbitrage to occur.

Part 7 – The Nature of the Arbitrage

Now that I have provided an empirical and theoretical defense of the theory of why arbitrage could be possible in the European soccer betting market, I will attempt to explain the nature of this arbitrage and how it is possible for this arbitrage to continue to exist. The Efficient Market Hypothesis holds that where arbitrage is available in an efficient market it is sure to be taken advantage of quickly so that it does not persist. In this sense, arbitrage is considered to be based on its textbook definition, “the simultaneous purchase and sale of the same, or essentially similar, securities in two different markets at advantageously different prices."[106] This type of arbitrage is risk-free, but it requires some difficult conditions. The first is perfect substitutes.[107] In financial markets, the ability to find perfect substitutes can sometimes be difficult. In the case of soccer betting, finding perfect substitutes is not an issue. There are three possible outcomes, and if an individual places a bet on each of those three outcomes, the probability that one of those bets will win is one. In that way, the arbitrage available in sports betting seems to be risk-free. However, arbitrage as described by Shleifer and Vishny can generally take two forms in the real world. The first is known as risk arbitrage, and it involves cases that rely on the statistical probability, but not the statistical certainty, of the convergence of the prices of the securities used in the arbitrage.[108] Risk arbitrage clearly doesn't describe the market for sports betting. The second type of arbitrage that Shleifer describes is limited arbitrage. Limited arbitrage can occur with perfect substitutes when the arbitrageur must worry about financing and maintaining the positions. [109] The example that Vishny and Shleifer cite is that in financial markets positions can go against the arbitrageur in the short run before convergence, which can require that the arbitrageur posts more collateral or abandon the position altogether.[110] In sports betting, there is no risk of being required to post additional collateral. However, the betting sites have certain regulations that seem to suggest that limited arbitrage describes the soccer betting market perfectly.

Two of the betting websites used in this study, William Hill and Bet 365, explicitly have stated that they will limit the stakes of an individual whenever they identify the individual as an arbitrageur.[111] Additionally, all seven of the betting sites used in the analysis maintain the right to cancel all or part of a bet at any time. This can be done on the basis of technical problems, suspicion of fraud, suspicion of arbitrage, or for no stated reason.[112] The fact that an arbitrageur could have a bet on a site for a certain outcome canceled means that the arbitrageur must worry about maintaining the position in a similar fashion to Shleifer's description of limited arbitrage. In fact, arbitrage in the soccer betting market seems to most closely resemble the arbitrage found between Treasury Bonds and TIPS discussed in Part 3. The authors of that paper, Fleckenstein, Francis, and Lustig, note that the arbitrage they found seemed to both fit the classic textbook definition of arbitrage while simultaneously being a risky strategy that could result in losses for an arbitrageur in some states of the world.[113] I would argue that the arbitrage found in this paper mirrors that definition. It is interesting to note that the only site that doesn't maintain the right to cancel a bet on suspicion of arbitrage is Pinnacle Sports.[114] Pinnacle Sports explicitly states on their website that they are the only site that will not alter a bet even if they are able to detect an arbitrageur, because they strive to offer better odds than their competitors, which they then try to make up for in volume. [115] This policy of Pinnacle Sports likely explains why their site could have been used for arbitrage opportunities more often than any other in the initial data set.

Those who espouse that the market for sports betting must be efficient would claim that the returns of the arbitrage strategy must only be so high because of the risk of being caught arbitraging and having a position canceled. However, this explanation would not be able to account for the correlation between the number of games with an arbitrage opportunity and the averaged expected returns for bookmakers. This explanation would also not be able to account for the fact that more information heavy markets such as the English Premier League seem to have both lower expected profits for bookmakers and a higher number of arbitrage opportunities when compared to other leagues such as the English Championship. Finally, the theory that the return to arbitraging stems from the risk of having a position canceled would not explain why there were zero arbitrage opportunities in the English Premier League before 2008.

Therefore, I hypothesize that the recent occurrence of arbitrage opportunities in the English Premier League is primarily due to increased competition between bookmakers leading to decreases in the average expected return as the betting sites offer initial odds closer to that of a fair game. Odds have now become close enough that random drift, which can sometimes differ from site to site, can lead to great enough differences between sites that arbitrage is sometimes possible. The idea that this random drift could vary from site to site would make sense in efficient market models as investors would deviate from rationality randomly, but it would be harder to support in models that refute efficiency. The critical piece of evidence that suggests that random drift could vary from site to site stems from the previously noted unwillingness of most bettors to change sites once they begin betting on one particular site.[116] This would suggest that the universe of bettors on each site is likely different in some way. Before the average expected return was close to the odds that would be offered in a fair game, the random drift was never significant enough to lead to arbitrage opportunities, which explains why arbitrage was first found in the study in 2008. While this provides a good theory as to why arbitrage has recently become possible and explains that it can occur without betting sites acting irrationally, it doesn't fully answer the question of why the arbitrage opportunity hasn't yet been sufficiently exploited by bettors that it no longer exists. To understand why this may be the case, we must consider that the arbitrage here is limited in some ways, notably by the regulations of the bookmakers themselves. The fact that part or all of a bet can be canceled for any reason including suspicion of arbitrage may provide part of the answer. It may sufficiently discourage enough capital from attempting arbitrage that it cannot offset the amount of random noise in the odds. Another possibility is that limits on bet size prevent arbitrage from fully correcting odds. The maximum winnings on the sites studied vary from nine thousand pounds per bet (Interwetten, Bet&Win) to two million pounds per day (William Hill).[117] The idea that limits on bet sizes play a role in sustaining the possibility of arbitrage would make more sense if Shleifer's description of arbitrage, with only a few professional investors attempting to invest in arbitrage situations, is more consistent with the soccer betting market than Fama's description of millions of small traders.

Part 8 - Conclusion

This paper began by discussing how to calculate whether an arbitrage opportunity exists for an event with three possible outcomes and how to exploit the opportunity if it did. It went on to discuss the formation of the Efficient Market Hypothesis and the importance of ideas such as a fair game, randomness in stock price movement, and arbitrage. Next, this paper explained the empirical and theoretical evidence against the Efficient Market Hypothesis, the differences between arbitrage in efficient and inefficient markets, and examples of arbitrage that have occurred in the real world. The paper then discussed past literature on arbitrage in soccer betting in Europe and explained why past explanations for this arbitrage were insufficient. Subsequently, I revealed that arbitrage opportunities have become available in the English Premier League, and I introduced my theory as to why they have started to exist. The theory states that competition has driven the initial odds offered by bookmakers closer to that of fair games odds, which, when coupled with random noise trading, sometimes causes odds on different betting websites to vary substantially enough to offer arbitrage opportunities. I then provided a model displaying how in a simple world of two rational bookmakers arbitrage opportunities are possible. Finally, I discussed the nature of the arbitrage that can be found in the sports betting market and concluded that it most closely resembled what Shleifer describes as limited arbitrage. The main purpose of this paper was to display that arbitrage opportunities do exist in the sports betting market and to discuss how those opportunities relate to fundamental economic theories regarding the efficiency of markets and the role of arbitrage. This paper begins to fill a critical void in the lack of research on arbitrage in sports betting markets, and will hopefully spur additional research in the area. The analysis done in this paper on how arbitrage opportunities have emerged suggests that future research should focus on attempting to better understand what has allowed these opportunities to persist.

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