# A Belief-Based Model of Investor Trading Behavior

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April 2018

**Abstract**

We explore whether irrational beliefs can predict a disposition effect. We propose a model of an overconfident investor whose beliefs can change over time. We find that such an investor exhibits a disposition effect. Moreover, we predict that the disposition effect may be driven by investors being most likely to hold onto an asset after the asset has experienced a small loss. We also find that our predictions match empirical evidence on trading behavior as a function of magnitude of return.

*I would like to profusely thanks Prof. Barberis for advising me. This essay would not be possible without his guidance and feedback. All errors are my own.

# 1 Introduction

One of the most widely observed facts about the behavior of individual investors is the disposition effect: a greater propensity to selling winning positions compared to losing ones. To date, rational models of trading behavior do little to explain this phenomenon. Odean (1998) shows that tax considerations, portfolio rebalancing, transaction costs and rationally driven mean reversion fail to explain the observed data. Odean (1998) also posits two possible behavioral theories behind this effect: prospect theory and an irrational belief in mean-reversion. In turn, a sizable literature examining behavioral explanations has emerged. The majority of this literature focuses on preference-based models given the intuitive similarities between the disposition effect and diminishing sensitivity to losses. Shefrin and Statman (1985) were the first to apply prospect theory to individual trading behavior. More recently, Barberis and Xiong (2009) provide a model that combines prospect theory and realization utility to predict a disposition effect. Similarly, Meng and Weng (2017) show that prospect theory with dynamic reference points also predicts a disposition effect.

However, Ben-David and Hirshleifer (2012) show that prospect theory models may not match the data. They estimate probability of sale as a function of profit and find that this function has a V-shape i.e. investors are more likely to realize gains and losses when they are higher in magnitude. Moreover, they find that the disposition effect could be driven by asymmetry within this V-shape. They suggest that this could be explained by investors updating their beliefs in line with Odean's (1998) second hypothesis surrounding an overconfident investor.

There is a large literature on overconfidence and its effect on trading behavior. Odean (1999) shows that overconfident investors trade too much and to their detriment. Barber and Odean (2001) predict and confirm that men should trade more frequently than women given that psychological research finds men to be more overconfident on average. Scheinkman and Xiong (2003) also show that speculation by overconfident investors can cause bubbles.

In this paper, we present a model of an overconfident investor and examine his trading behavior to see if he exhibits a disposition effect. The investor believes that the distribution of returns of a given stock is governed by one of two regimes. The first is a belief in short-term mean reversion, which is meant to capture his overconfidence. The second is a belief that the stock in question has a positive but low expected return such that the investor would prefer a risk free asset to the stock. This model aims to describe an underconfident investor who believes he is unable to properly evaluate the return distribution of this particular stock and therefore is wary of investing in it. The investor's beliefs evolve over time according to Bayes' rule and the observed return history, capturing the investor's varying confidence over time.

We find that the investor exhibits a disposition effect. This disposition effect tends to be stronger when the stock has lower expected returns and when there are fewer trading periods over the course of the investment's life cycle. We also examine how the investor's trading behavior changes with respect to the magnitude of the stock's return. Our results are largely consistent with those of Ben-David and Hirshleifer (2012) as the investor is less likely to hold the stock when it has performed very well or very poorly. We also find that the investor is least likely to reduce his position after the asset has achieved a small loss. This is a slight deviation from Ben-David and Hirshleifer (2012) who predict that the probability of sale is minimized at zero return. Our finding however, does not require an asymmetry in the probability of sale curve to generate a disposition effect.

The paper is structured as follows. Section 2 describes the disposition effect in greater detail along with the evidence supporting it. We will also highlight some of the psychological evidence behind overconfidence and its applications to finance. In Section 3, we formally define the model and solve for the investor's optimal share allocation. Section 4 explores the investor's trading behavior to see if he exhibits a disposition effect. Section 5 discusses the model in the context of related research along with suggesting possible adaptions of our model and extensions to other problems. Section 6 concludes the paper.

# 2 Background

## 2.1 Evidence and Explanations for the Disposition Effect

Perhaps the most comprehensive exposition of the disposition effect comes from Odean's (1998) study of the trading behavior of retail investors. Using data from 10,000 households' trading activity between 1987 and 1993, Odean (1998) finds a greater propensity to sell shares of stock that have achieved positive returns. He measures this propensity using the following methodology. For every investor, Odean examines all of the days on which the investor chooses to sell shares of at least one stock in his portfolio. On these days, he places every stock in the investor's portfolio into one of four categories. If the stock is sold on that day, he marks it as a "realized gain" if it is sold at a price higher than its average purchase price and as a "realized loss" otherwise. If the stock is not sold that day and the market price on that day exceeds its average purchase price the stock is marked as a "paper gain". Otherwise it is marked as a "paper loss". Odean (1998) then calculates the proportion of gains realized (PGR) and proportions of losses realized (PLR) by

$$\mathrm{PGR} = \frac{\text{Number of Realized Gains}}{\text{Number of Paper Gains} + \text{Number of Realized Gains}} \quad (1)$$

and

$$\mathrm{PLR} = \frac{\text{Number of Realized Losses}}{\text{Number of Paper Losses} + \text{Number of Realized Losses}} \quad (2)$$

He finds that PGR = 0.148 and PLR = 0.098 and thus concludes that the disposition effect exists.

Odean (1998) also examines potential rational explanations of the disposition effect and finds none that would predict a higher propensity to realize gains rather than losses. The most straightforward justification for a disposition effect is that investors are trading on good information i.e. they hold on to paper losses because they know the stock will rebound and they realize gains in advance of poor short-term performance. However, Odean finds that the average return of stocks on which investors have realized gains is 3.4 % higher than stocks on which investors retain paper losses. Thus, it is unlikely that the disposition effect is driven by investors trading on quality information.

Other possible explanations include tax considerations and portfolio rebalancing. Yet, neither is consistent with observed phenomenon. Tax considerations should actually encourage investors to realize losses as realized losses can be used to offset taxable gains in other parts of the investor's portfolio. To study the effect of rebalancing, Odean (1998) limits his sample to cases where the investor completely liquidates his position in an individual stock. This filtering follows from the fact that rebalancing is more likely to be achieved by a partial reduction in one's holding as opposed to a sale of the entire position. Yet, even after this filtering Odean (1998) still finds a disposition effect. Another failure in this explanation is that the disposition effect is stronger among less sophisticated investors. Yet, we would expect rebalancing to be more common among sophisticated investors if it is, in fact, the optimal approach to investing. Hence, there is little to suggest that the disposition effect is driven by rational behavior.

Finally, the disposition effect is not limited to the purchase of individual stocks. Genesove and Mayer (2001) find that homeowners are reluctant to sell their homes for less than the original purchase price. Meanwhile, Coval and Shumway (2005) find that futures traders who earned positive returns in the morning are less likely to take on risk in the afternoon.

## 2.2 Overconfidence and Trading Behavior

Before considering a model of an overconfident investor, it is important to review some of the psychological evidence for overconfidence among investors. A common form of overconfidence is overprecision i.e. excessive confidence in the accuracy of one's beliefs. A clear example of this is people's tendency to provide overly narrow confidence intervals when asked to estimate quantities. For example, Alpert and Raiffa (1982) find that 98% confidence intervals include the true value only 60% of the time.

A related behavior is belief perseverance. We find that individuals are reluctant to give up on their initial beliefs (Lord, Ross and Lepper, 1979). This reluctance has two sources. The first is an unwillingness to seek out evidence that does not agree with their initial hypothesis. The second is to treat contradictory evidence with too much skepticism.

In practice, we find substantial evidence for overconfidence among investors. One of the most surprising observations in the behavior of individual investors is the amount they trade. In theory, one should be very reluctant to trade due to fears of adverse selection i.e. one should be afraid of buying when someone else is eager to sell. Yet, in practice, we find very high trading volume in markets across the world. Barber and Odean (2000) show that investors, on average, underperform against standard benchmarks due to trading costs incurred from excessive trading. Overconfidence provides a simple explanation for this phenomenon as overconfident investors are more likely to believe that they have sufficiently strong information to justify a trade. Barber and Odean (2001) predict and confirm that this should lead men to trade more than women as psychological evidence suggests that men tend to be more overconfident in areas such as finance.

# 3 A Model of an Overconfident Investor

We consider a two asset setting over $T+1$ trading periods, $t = 0, 1, ..., , T$. For intuition, the trading periods are thought to be evenly spaced and the interval from $t = 0$ to $t = T$ is thought to be a year. The first asset is a risk free asset, which earns a return of $R_f \geq 1$ in each period. The second is a risky asset, which can be thought of as a single stock. Let $P_t$ denote the price of the stock at time $t$ and $R_{t,t+1}$ its return from period $t$ to $t+1$. The stock price then evolves as a binomial tree i.e.

$$R_{t,t+1} = \begin{cases} R_u > R_f \text{ with probability } \pi \\ R_d < R_f \text{ with probability } 1-\pi \end{cases} \tag{3}$$

and

$$P_t = P_0 \prod_{i=0}^{t-1} R_{i,i+1} \tag{4}$$

under the assumption that

$$\pi R_u + (1-\pi) R_d > R_f \tag{5}$$

The stock is i.i.d over all periods. Going forward, we will fix $\pi = \frac{1}{2}$ and instead alter $R_u$ and $R_d$ to describe the riskiness of the asset. Moreover, our imperfectly rational investor will always have correct information about $R_u$ and $R_d$ and will express any views on the risky asset through beliefs on the probabilities of the up and down state.

## 3.1 The Investor

The investor is given logarithmic preferences with the goal of maximizing his expected utility at time $T$. At time $t$, let $W_t$ be the investor's wealth and $x_t$ be the investor's allocation towards the risky asset. We can then formalize the investor's problem as

$$\max_{x_0, x_1, \ldots, x_{T-1}} \mathbb{E}[\log(W_T)] \tag{6}$$

with the budget constraint

$$\begin{aligned} W_t &= (W_{t-1} - x_{t-1}P_{t-1})R_f + x_{t-1}P_{t-1}R_{t-1,t} \\ &= W_{t-1}R_f + x_{t-1}P_{t-1}(R_{t-1,t} - R_f) \qquad \forall t \geq 1 \end{aligned} \tag{7}$$

and a nonnegativity of wealth constraint

$$W_T \geq 0 \tag{8}$$

In order to solve his allocation problem, the investor forms beliefs about the evolution of the risky asset. He correctly understands that the risky asset evolves according to a binomial tree and has accurate beliefs about the values of $R_u$ and $R_d$. However, he has different beliefs concerning the return distribution of the risky asset. More specifically, he believes that there are two possible return regimes: Model 1 and Model 2. In Model 1, the investor correctly believes that $\mathbb{P}(R_{0,1} = R_u) = \frac{1}{2}$. However, in each subsequent trading period, the investor believes that the return of the risky asset follows a mean-reverting Markov process given by $\mathbb{P}(R_{t,t+1} = R_{t-1,t}) = p < 0.5$. $R_{t,t+1}$ can then be described by the following transition matrix

$$\begin{array}{c c} & \begin{array}{cc} R_u & R_d \end{array} \\ \begin{array}{c} R_u \\ R_d \end{array} & \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix} \end{array}$$

In Model 2, the investor believes that the return distribution of the risky asset is i.i.d Bernoulli with $\mathbb{P}(R_{t,t+1} = R_u) = p^*$ such that

$$p^* R_u + (1 - p^*)R_d \geq R_f \tag{9}$$

and

$$p^* \log(R_u) + (1 - p^*)\log(R_d) \leq \log(R_f) \tag{10}$$

In words, the investor believes that the risky asset has an expected return higher than the risk free rate but weakly prefers the risk-free asset to the risky one. It is necessary that $p < p^*$ for these beliefs to be meaningful. To see why note that

$$\mathbb{P}(R_{t,t+1} = R_u \mid \text{Model 1}) \geq p \ \forall t \tag{11}$$

Thus, if $p \geq p^*$, the investor will simply associate all up states as evidence for Model 1 and all down states as evidence for Model 2.

Finally, the investor believes that the regime governing the return of the risky asset is determined ex ante i.e. the return regime does not change across periods. He sets a prior probability, $q$, that returns follow Model 1. In each period, prior to making his allocation, the investor updates in accordance with Bayes' Rule.

Before solving the investor's allocation problem, we would like to stop and motivate the two return models. The first model is meant to describe an overconfident investor. For intuition, consider an investor who has taken a long position in a stock as a result of research he performed. Since he has taken a position, we can infer that the investor strongly believes that he is being overcompensated for the risk that he is taking i.e. the stock is mispriced. Now, suppose that the stock experiences poor returns soon after the investor opens his position. Since the investor is confident in his analysis, he still believes that the perceived mispricing will eventually correct itself. Moreover, the more confident the investor is in his analysis, the less likely he is to believe that this mispricing will persist. Thus, an overconfident investor would assign higher probability to seeing positive returns in the subsequent periods. Now, suppose that the stock exhibits strong positive returns soon after the investor opens his position. If the stock exceeds the target return predicted by his research, an overconfident investor is more inclined to view the stock's strong performance as an overcorrection and would believe that the stock price is more likely to fall in the near future. Altogether, in the short-term an overconfident investor can behave very similarly to an investor who believes in mean reversion.

Now, Model 2 is meant to describe the opposite of an overconfident investor i.e. an investor who believes that he cannot properly evaluate the stock. For intuition, let $p'$ be the probability of the up state such that the investor is indifferent between the risky asset and the risk free asset. Then, we can think of Model 2 as the investor believing that $p^* = p' - \epsilon$. Since he does not believe he can accurate evaluate the stock, he has no reason to believe that the stock is underpriced. Thus, he believes the probability of an up return should be, at best, such that he is indifferent between this stock and the risk-free asset i.e $p'$. He then slightly discounts this probability by $\epsilon$, to account for the risk that he is further misjudging the distribution of the stock. At its core, this regime aims to capture the fact that investors do not find assets they do not understand very attractive. As a result, an investor who believes in Model 2 has very little incentive to make a large allocation towards the risky asset. There are other ways to model this behavior — for example setting the probabilities of each state such that the expected return of the stock is $R_f$. In practice, it makes nearly no difference.

Finally, together, the two models are meant to illustrate the changes in an investor's confidence over time. More specifically, we would expect a confident investor to maintain or increase his position in the face of short-term deviations from his predictions, due to his faith in his initial hypothesis. However, as an investor faces a larger sample of results that contradict his hypothesis he should begin losing faith in his research. This is modeled by the investor placing greater weight on Model 2. Concretely, after seeing a string of down states, we expect the investor to certainly stop 'doubling down' and eventually liquidate their position in the asset. This is in contrast to a purely overconfident investor who would never give up on his position, a behavior which, introspectively, does not seem likely. Moreover, this model should also predict that investors will liquidate their position after seeing a string of positive returns. While it is certainly hard to give up on a stock that has been a consistent winner, we believe that investors are fundamentally uncomfortable with investing in stocks whose returns they cannot explain and therefore would not trust the good returns to continue.

## 3.2 Optimal Strategy

Before solving for the investor's optimal strategy it is helpful to define some notation regarding the binomial tree. Note that since the investor's beliefs are path dependent, the tree does not recombine. Thus, at time $t$, there are $2^t$ nodes in our tree.[1] We will therefore represent each node at time $t$ by a vector, $\Phi$, of length of $t$ containing the observed return path up until time $t$ where $\Phi_i = R_{i,i+1}$. We will begin indexing all time-dependent variables by $t$ and $\Phi$. Finally, let

$$p^u_{t,\Phi} = \mathbb{P}(R_{t,t+1} = R_u \mid \Phi) \tag{12}$$

and

$$p^d_{t,\Phi} = \mathbb{P}(R_{t,t+1} = R_d \mid \Phi) \tag{13}$$

be the investor's updated beliefs about the probability of up and down returns at time $t$ and node $\Phi$

[1]Since the investor's beliefs are Markovian, it is possible to represent the tree in fewer nodes. We, however, believe that this notation causes more confusion than it resolves.

**Proposition 1.** *Given the investor's preferences and beliefs his optimal allocation to the risky asset is*

$$x_{t,\Phi} = \frac{-W_{t,\Phi} R_f \left[p^u_{t,\Phi}(R_u - R_f) + p^d_{t,\Phi}(R_d - R_f)\right]}{P_{t,\Phi}(R_u - R_f)(R_d - R_f)} \tag{14}$$

*Proof.* See Appendix 1 □

Note that the investor would choose the same optimal share allocation, $x_{t,\Phi}$, if he were trying to maximize his expected utility at time $t+1$ i.e. the investor's share allocations are intertemporally separate. This is not a general consequence of the investor's beliefs but rather a result of his logarithmic preferences. In practice, the assumption of logarithmic preferences allows for a tractable, and more importantly, analytic solution to the investor's allocation problem.

# 4 Results

To examine the results of the investor's trading strategy, we must set $R_u$ and $R_d$. Instead of choosing $R_u$ and $R_d$ directly, we choose an annualized expected return $\mu$ and standard deviation $\sigma$ of the risky asset. Having established the interval $t = 0$ to $t = T$ to be a year and $\pi = \frac{1}{2}$, we have

$$\mu = \left(\frac{R_u + R_d}{2}\right)^T \tag{15}$$

and

$$\mu^2 + \sigma^2 = \left(\frac{R_u^2 + R_d^2}{2}\right)^T \tag{16}$$

Together (15) and (16) imply

$$R_u = \mu^{\frac{1}{T}} + \sqrt{(\mu^2 + \sigma^2)^{\frac{1}{T}} - (\mu^2)^{\frac{1}{T}}} \tag{17}$$

and

$$R_u = \mu^{\frac{1}{T}} - \sqrt{(\mu^2 + \sigma^2)^{\frac{1}{T}} - (\mu^2)^{\frac{1}{T}}} \tag{18}$$

We will examine results for a range of values for $\mu$ and $T$ while fixing $p, q, r_f$ and $\sigma$ at $0.4, 0.5, 1$ and $0.3$ respectively. We will also choose $p^*$ to be the midpoint of the probability that make the investor indifferent between the two assets and the probability such that risky asset has an expected return of $R_f$. The significance and effect of these parameters will be discussed in Section 4.3. Finally, we will fix $W_0 = P_0 = 1$.

## 4.1 Disposition Effect

Before determining whether a disposition effect exists, we must first define a metric for quantifying the disposition effect. The method of Odean (1998) discussed in Section 2.1 is most appropriate for settings with multiple assets and many possible trading periods as it uses other assets in an investor's portfolio to determine periods in which a investor has any propensity to liquidate a portion of his portfolio. Moreover, it does not account for the possibility of shorting the asset. In this paper, we will test for a disposition effect by examinining the investor's allocation to the risky asset at time $T-1$ i.e his final position. Under our assumption that the probability of up and down returns are equal, we note that each of the $2^{T-1}$ return paths possible at time $T-1$ have equal probability of occurring. We can then compute the investor's average final share allocation at states where the risky asset has achieved a positive gross return and the analogous figure for a negative gross return. If the investor's average final share allocation given a negative gross return exceeds his average final share allocation given a positive gross return we will conclude that the model predicts a disposition effect. Finally, the results presented below will consider only even values of $T$ so that as of time $T-1$, the investor will have seen an odd number of returns and therefore clearly positive or negative gross returns. We present the results in Table I.

Upon examining Table I, we notice two immediate trends: the disposition effect is stronger when the risky asset has a lower expected return and when there are fewer trading periods. Both trends are a result of the speed at which the investor updates his beliefs. For the first trend, note that the probability of an up return in Model 2 is determined semi-endogenously. More specifically, in Model 2, the probability of an up-state, $p^*$, is set so that the expected return of the asset is slightly greater than $R_f$. Therefore, as $\mu$ rises, $p^*$ must fall to keep the expected return of the risky asset low. Since $p^*$ is higher for smaller $\mu$, the investor is more likely to associate a string of positive returns with Model 2 when $\mu$ is small. Thus, for small $\mu$, the investor more quickly updates towards Model 2 after a string of positive returns, which leads him to liquidate his position faster. Meanwhile, when

**Table I**
**Analysis of the Disposition Effect by Average Final Share Allocation**

Each $(\mu, T)$ pair denotes the average share allocation at $t = T - 1$ at states where the risky asset has achieved a negative gross return followed by the average share allocation at $t = T - 1$ at states where the risky asset has achieved a positive gross return. If the average share allocation given a negative gross return exceeds the average share allocation given a positive gross return, we observe a disposition effect

| Expected Return | Number of Trading Periods per Year | | | | |
|---|---|---|---|---|---|
| $\mu$ | T=4 | T=6 | T=8 | T=10 | T=12 |
| 1.05 | 0.87/0.05 | 0.84/0.09 | 0.83/0.1 | 0.82/0.11 | 0.81/0.11 |
| 1.06 | 0.92/0.11 | 0.89/0.14 | 0.88/0.16 | 0.87/0.17 | 0.87/0.17 |
| 1.07 | 0.97/0.17 | 0.94/0.2 | 0.93/0.22 | 0.92/0.23 | 0.92/0.23 |
| 1.08 | 1.02/0.23 | 0.99/0.27 | 0.98/0.29 | 0.97/0.29 | 0.97/0.3 |
| 1.09 | 1.08/0.3 | 1.04/0.34 | 1.03/0.36 | 1.02/0.37 | 1.02/0.37 |
| 1.1 | 1.13/0.38 | 1.09/0.42 | 1.08/0.44 | 1.08/0.44 | 1.07/0.45 |
| 1.11 | 1.18/0.47 | 1.15/0.51 | 1.14/0.52 | 1.13/0.53 | 1.12/0.53 |
| 1.12 | 1.24/0.56 | 1.2/0.6 | 1.19/0.61 | 1.18/0.62 | 1.18/0.63 |
| 1.13 | 1.3/0.66 | 1.26/0.7 | 1.25/0.72 | 1.24/0.73 | 1.24/0.73 |

facing a string of negative returns, Model 2 is not as convincing when $\mu$ is small. Thus, the investor is slower to react to a string of negative returns and holds his position longer.

The trend along the time axis is weaker and follows from the irrationality of the investor. While both Model 1 and Model 2 are inaccurate characterizations of the risky asset, Model 2 assumes probabilities closer to $\frac{1}{2}$ as $p < p^* < \frac{1}{2}$. Thus, over larger samples of returns, Model 2 will, on average, do a better job explaining the return of the risky asset. Since the investor will have seen a larger return history when there are more trading periods, he will place relatively more weight on Model 2. Because Model 2 assumes that returns are i.i.d investors who believe in Model 2 will exhibit the same trading behavior regardless of whether or not the asset has exhibited a positive or negative gross return. Thus, if the investor is more likely to believe in Model 2, we should observe a weaker disposition effect.

## 4.2 Trading Behavior by Return Magnitude

Having established a disposition effect at a high-level, we now want to explore its nuances. We are particularly interested in how the observed gross return affects trading behavior. We begin by noting that although the investor's beliefs are path-dependent the gross return of the risky asset is not. Thus, at $t = T - 1$ there are $T$ possible gross returns, each corresponding to the number of up returns the asset achieved. For each possible return, we compute average share allocations. The results are presented in Table II for $T = 8$. Results for other choices for $T$ are similar and are included in Appendix 2.

From Table II, we immediately notice that the investor, for the most part, decreases his average position size as the magnitude of the gross loss or gain rises. This phenomenon corresponds to Ben-David and Hirshleifer's (2012) observation that the probability of selling as a function of profit is V-shaped. This follows fairly intuitively from the investor's beliefs. Given the binomial structure of the risky asset, larger magnitude returns are the result of runs of consecutive returns of the same type. These runs are unlikely under Model 1 which predicts mean reversion. Thus, the investor updates towards Model 2. Under Model 2, the risky asset is not particularly attractive so the investor prefers to take a small position.

There are two exceptions to this claim. The first is fairly minor. For low $\mu$, the investor begins taking small short positions in the risky asset after many up returns reaching a maximum, in terms of magnitude, at 6 up returns. In practice, this behavior should be interpreted as the investor effectively closing out his position altogether.

The second exception is that we see a small increase in average share allocation in states where the asset has taken a relatively small gross loss. For example, we see that the investor's average allocation in states with 2 up returns jumps to 1.07 from 0.97 in states with 3 up returns. This is depicted in Figure 1 which plots the average share allocations for $\mu = 1.08$. From Table II we see that this phenomenon exists for other $\mu$ as well. We will use the example of $\mu = 1.08$ to explain why this happens. As noted earlier Model 1 appears less likely after larger magnitude losses or gains. Therefore, we would correctly expect the average probability of Model 1 across all states with 3 up returns to be higher than that of all states with 2 up returns. For states with 3 up returns, the average probability of Model 1 is 0.5 compared to 0.41 for states with 2 up returns. Alone, this should lead to a smaller

**Table II**
**Average Final Share Allocation by Return Magnitude**

Each $(\mu, n)$ pair denotes the average share allocation at $t = T - 1$ at states where the risky asset has achieved exactly n up returns

| | | Number of Up Periods Over the Year | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\mu$ | Initial Allocation | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1.05 | 0.43 | 0.51 | 0.8 | 0.98 | 0.75 | 0.25 | -0.07 | -0.13 | -0.07 |
| 1.06 | 0.5 | 0.5 | 0.8 | 1.01 | 0.82 | 0.33 | -0.02 | -0.12 | -0.07 |
| 1.07 | 0.57 | 0.49 | 0.79 | 1.04 | 0.9 | 0.41 | 0.03 | -0.1 | -0.07 |
| 1.08 | 0.64 | 0.48 | 0.79 | 1.07 | 0.97 | 0.49 | 0.08 | -0.08 | -0.07 |
| 1.09 | 0.71 | 0.47 | 0.78 | 1.1 | 1.05 | 0.58 | 0.14 | -0.05 | -0.07 |
| 1.1 | 0.78 | 0.45 | 0.78 | 1.13 | 1.13 | 0.68 | 0.21 | -0.02 | -0.06 |
| 1.11 | 0.86 | 0.44 | 0.77 | 1.16 | 1.21 | 0.78 | 0.29 | 0.02 | -0.05 |
| 1.12 | 0.94 | 0.43 | 0.76 | 1.19 | 1.3 | 0.89 | 0.37 | 0.06 | -0.03 |
| 1.13 | 1.02 | 0.42 | 0.76 | 1.22 | 1.39 | 1.01 | 0.47 | 0.11 | -0.02 |

average share allocation in states with 2 up returns. However, the crucial distinction is that of the 28 states with 2 up returns, 21 have a down return as their most recent return i.e. $R_{T-2,T-1} = R_d$. This is compared to 49 out of 70 possible 3 up return states. Given that the investor is still somewhat confident in Model 1, he is likely to take a relatively large long position whenever the most recent return was negative. Thus, the greater likelihood of recent negative returns and the investor's aggressive response to recent losses outweigh the conservatism that comes from a slightly stronger belief in Model 2. This logic also implies that this effect is more pronounced for lower $\mu$ as the investor is slower to adopt Model 2 for small $\mu$.

From this two related questions emerge: why does this logic not extend to states where there has only been one up period and why don't we see a similar effect for positive gross returns? The first question can be explained by the investor's faith in Model 1. For $\mu = 1.08$, when the asset has only exhibited one period of positive returns, the investor, on average, believes that there is a 27% chance of Model 1. This represents a sharper decline in the average probability of Model 1 than in the previous example, which cannot be outweighed by the greater likelihood of recent negative returns. The second question can be answered by examining the investor's behavior

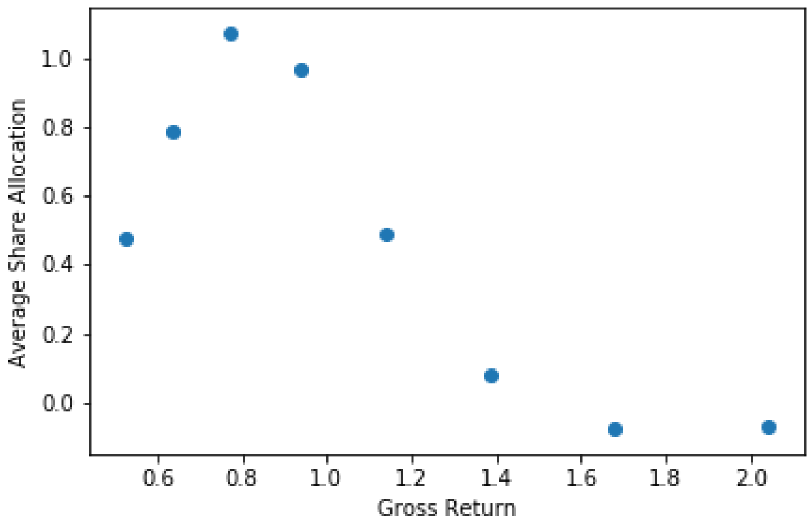


Figure 1: Average Final Share Allocations by Return for $\mu = 1.08$

after seeing a positive return. While his belief in mean-reversion suggests that the risky asset is unlikely to rise again, he is not as eager to short the risky asset as he is to take a relatively large long position after a down return. This is because $R_u > R_d$ i.e. it has a positive expected return. As a result, the investor has a more tempered reaction to the recent positive returns and we do not see this blip in the V-shape.

Figure 1 provides another interesting insight. Ben-David and Hirshleifer (2012) also estimate the probability of selling as a function of profit and find that their estimated functions are steeper for gains than losses. They argue that this phenomenon drives the disposition effect and suggest that an overconfident investor will drive this effect. While the structure of our risky asset is not sufficiently granular to confirm this result[2], there does not appear to be a significant difference in slopes on either side of the peak in Figure 1.

Instead, the surprising observation is the location of the peak itself. Our model predicts that the investor has an implicit threshold loss and only starts

[2]More specifically, due to the binomial nature of the asset and the limited number of trading periods, there is not enough return data to accurately calculate slope.

to scale down his position if he exceeds that loss. When the investor is in the region between the peak and 0, he likely has lower confidence in his forecast than before opening his initial position. However, due to the poor performance of the asset, if his forecast is, in fact, correct trusting it would become very profitable. Thus, he sticks with his long position until the losses exceed this threshold. This prediction is in contrast to Ben-David and Hirshleifer (2012) who estimate that the return at which the lowest probability of sale is zero. When the probability of sale curve attains its minimum at 0, asymmetric slopes are necessary to predict a disposition effect. However, they are not required if the probability of sale curve reaches a minimum to the left of 0, as even a symmetric probability of sale curve will exhibit a disposition effect when translated to the left.

## 4.3 Robustness

In order to capture the investor's changing confidence, this model requires a number of parameters. In this section, we will examine the 3 parameters that are unique to this model: $p, q$ and $p^*$. We will also briefly discuss the behavior of the model at extreme choices for $\mu$ and $T$

We begin by considering $q$. We can simply interpret $q$ as the investor's ex ante confidence in his forecast or more colloquially, his stubbornness. In practice $q$ controls the speed at which the investor loses confidence in his model. For smaller $q$, the investor is quicker to give up on his forecast. While the results of the previous section used $q = 0.5$ there is certainly an argument for choosing $q > 0.5$. For example an overconfident investor is likely to be fairly stubborn. That being said, the choice of $q$ makes little difference on the model's predictions.

We find that for all $0 < q < 1$, this model predicts a disposition effect. Varying $q$ only affects the strength of the disposition effect i.e. the magnitude of the difference in final share allocations. For larger $q$, we see a stronger disposition effect. This follows from the same logic that drives the trend along the time axis in Table I: low $q$ places greater weight on Model 2 which predicts that future returns are independent of the past. The model also predicts a V- shaped sale probability function for all $0 < q < 1$. Higher values of $q$ lead to a more pronounced V-shape sale probability function. This is largely because the investor's additional faith in Model 1 lead him to take on more leverage. Finally, for all $0 < q < 1$, we continue to observe that the peak average final share allocation occurs to the left of 0 suggesting a probability of sale curve that is translated to the left.

We will next examine $p^*$. In constructing the model we assert that $p^*$ is set so that the risky asset has a higher expected return than the risk free asset but the investor weakly prefers the risk free asset to the risky one. Varying $p^*$ within this range affects how conservative the investor becomes when he loses faith in Model 1. In practice, it has little affect on the model's predictions. All the phenomena reported in the previous two sections hold for all $p^*$ in this range.

For the results presented in the previous sections, we opted for a more conservative value for $p$ to bring the investor's beliefs closer to the true distribution. Given that $p$ is bounded above by $p^*$, we could not choose $p > 0.41$ while still using this model for larger $\mu$. In general, varying $p$ affects the aggressiveness of the investor. For low values of $p$, the investor is eager to take on large amounts leverage. This is, in part, due to the the binomial structure of the risky asset which sets a maximum possible loss in any one period. As a result, for smaller $p$, we find a stronger disposition effect and a much steeper average final allocation curve. For extreme $p$ the V-shape probability of sale curve begins to break down for positive returns. This because the investor tends to take on large short positions in states where the asset has exhibited small positive returns. However, for larger positive returns, the investor begins to update towards Model 2 and reduces his position size.

Finally, for large $(\mu, T)$ pairs this model fails to predict a disposition effect. Moreover, the largest average final allocation occurs to the right of 0. We expect the largest average final allocations in states that have experienced slightly more down returns than up returns. For reasonable $\mu$, the gross return in these states is negative. However, for large $\mu$ and $T$ it is possible for the asset to experience more down returns than up returns and still exhibit a positive gross return. Thus, the average final share allocation given positive gross returns become artificially inflated while the analogous figure for negative gross returns is deflated. Larger $\mu$ also leads the investor to take on more leverage. Since the investor is almost always long the risky asset, he becomes quite wealthy in states with many up returns. Thus, even if he is contributing a smaller share of his wealth towards the risky asset, he can still hold more shares of his risky asset.

# 5 Discussion and Extensions of the Model

This paper is most closely related to Ben-David and Hirshleifer (2012) who examine probability of sale as a function of profit and suggest that the disposition effect could be driven by beliefs rather preferences. Our analysis supports this hypothesis as we have shown that an overconfident investor with rational preferences is likely to exhibit a disposition effect. Moreover our results predict a V-shaped probability of sale function i.e. increasing in the magnitude of return. Our model does depart from Ben-David and Hirshleifer's (2012) suggestion that the probability of sale is minimized at 0 return, as we predict that the investor is most likely to hold a position after experiencing a small gross loss. This, however, is not necessarily a contradiction. When reporting selling probability schedules, Ben-David and Hirshleifer (2012) report a small dip to the left of the origin. Moreover their estimated probability of sale functions are derived from a probit model that is inherently monotonic. Since return magnitude is bounded below by 0 and the estimated function is increasing, the best fit will always suggest a minimum at 0. Finally, our prediction that the probability of sale is minimized to the left of 0 has the added benefit of not requiring that the V-shape is asymmetric.

Our model can be extended to other disposition effects. For example Coval and Shumway (2005) show that futures traders are less likely to take risk in the afternoon if they have accrued profits that morning.

More generally, our model can be applied to larger classes of portfolio allocation problems. One particularly relevant problem is excessive trading. Barber and Odean (2000) show that retail investors trade quite frequently and underperform relative to the market. Much of this underperformance is the result of trading costs. However, Odean (1999) also shows evidence for poor asset selection. While there exist many overconfidence based explanations for excessive trading in general, a model like the one presented here could potentially explain poor stock selection.

For example, consider an adaptation of this model where we replace Model 2 with a return continuation model i.e the opposite of Model 1. In this case we interpret Model 2 as the introduction of new information that will cause the stock to trend in a certain direction as a it percolates through the population of investors. We would then model overconfidence by setting $q > 0.5$. In practice, this version of the model should lead to similar results as the one presented in this paper. We would expect the investor to update faster towards Model 2 after seeing a string of similar returns. However, this will be balanced by the the higher initial probability of Model 1. Thus, our current prediction that investors will allocate more to small losers would likely hold and provide some explanation for why overconfident investors' poor stock selection. The version of the model used for this paper was chosen to more explicitly indicate when the investor has 'no belief' on how to evaluate asset. However, this adapted model is also a perfectly valid approach for trying to explain the disposition effect. It is perhaps a more elegant approach to the problem as the return regimes are symmetric.

# 6 Conclusion

This paper examines whether irrational beliefs can predict a disposition effect. We consider an overconfident investor who believes in mean reversion and whose beliefs vary over time. We find that our hypothetical investor will exhibit a disposition effect. Moreover, we predict that the investor is more likely to exit his position when the asset has exhibited returns that are large in magnitude. Surprisingly, we find that the investor is most likely to hold onto his position after the asset has achieved a small gross loss. This suggests that the disposition effect is not necessarily dependent on asymmetry between the probability of sale with respect to losses and gains.

# Appendix 1

*Proof of Proposition 1.* We begin with a Lemma.

**Lemma 1.** *Suppose that at time $t$ and node $\Phi$ the investor's allocation to the risky asset follows*

$$x_{t_0,\Phi^0} = \frac{-W_{t_0,\Phi^0} R_f \left[ p^u_{t_0,\Phi^0}(R_u - R_f) + p^d_{t_0,\Phi^0}(R_d - R_f) \right]}{P_{t_0,\Phi^0}(R_u - R_f)(R_d - R_f)}$$

*for all $t_0 > t$ and appropriate paths $\Phi^0$. Then for any path of returns of length $T$, $\Phi'$, such that $\Phi'_i = \Phi_i$ where $i < t$*

$$W_{T,\Phi'} = k W_{t+1,\Phi^*} \tag{19}$$

*where $\Phi^* = (\Phi_0, ..., \Phi_{t-1}, \Phi'_t)$ and $k$ does not depend on $x_{t_0,\Phi_0}$ for any $t_0$ and appropriate $\Phi_0$*

*Proof.* We proceed by backwards induction on $t$. The base case $t = T - 1$ holds trivially For the inductive case, we seek to show that the result holds for $t - 1$ i.e. wealth at time $T$ can be expressed as a multiple of wealth at time $t$. Let $\Phi^1 = (\Phi_0, ..., \Phi_{t-1}, \Phi'_t)$ and $\Phi^2 = (\Phi_0, ..., \Phi_{t-1}, \Phi'_t, \Phi'_{t+1})$. Then, by equation (7)

$$\begin{aligned}
W_{t+1,\Phi^2} &= W_{t,\Phi^1} R_f + x_{t,\Phi^1} P_{t,\Phi^1} (R_{t,t+1} - R_f) \\
&= W_{t,\Phi^1} R_f + x_{t,\Phi^1} P_{t,\Phi^1} (\Phi'_t - R_f) \\
&= W_{t,\Phi^1} R_f \\
&- \frac{W_{t,\Phi^1} R_f \left[ p^u_{t,\Phi^1} (R_u - R_f) + p^d_{t,\Phi^1} (R_d - R_f) \right]}{P_{t,\Phi^1} (R_u - R_f)(R_d - R_f)} \times P_{t,\Phi^1} (\Phi'_t - R_f)
\end{aligned}$$

Thus, we have

$$W_{t+1,\Phi^2} = k_1 W_{t,\Phi^1}$$

where

$$k_1 = R_f \left[ 1 - \frac{\left[ p^u_{t,\Phi^1} (R_u - R_f) + p^d_{t,\Phi^1} (R_d - R_f)(\Phi'_t - R_f) \right]}{(R_u - R_f)(R_d - R_f)} \right]$$

Note that none of the terms in $k_1$, depend on $x_{t_0,\Phi^0}$ for any $t_0$ and appropriate $\Phi^0$. Finally, by the inductive hypothesis, we have that $W_{t+1,\Phi_2} = k_2 W_{T,\Phi'}$ where $k_2$ does not depend on $x_{t_0,\Phi^0}$ for any $t_0$ and appropriate $\Phi^0$. Thus,

$$W_{t,\Phi^1} = \frac{k_2}{k_1} W_{T,\Phi'}$$

and the inductive case holds. $\square$

Now, to the main result. We once again proceed by backwards induction. For the base case, we consider $t = T - 1$ and node $\Phi$. For ease of notation, let $\Phi^u = (\Phi_0, ..., \Phi_{T-1}, R_u)$ and $\Phi^d = (\Phi_0, ..., \Phi_{T-1}, R_d)$ Then, the investor seeks to maximize

$$\begin{aligned}
\mathbb{E} \left[ \log(W_T) \right] &= p^u_{T-1,\Phi} \log(W_{T,\Phi^u}) + p^d_{T-1,\Phi} \log(W_{T,\Phi^d}) \\
&= p^u_{T-1,\Phi} \log \left( W_{T-1,\Phi} R_f + x_{T-1,\Phi} P_{T-1,\Phi} (R_u - R_f) \right) \\
&+ p^d_{T-1,\Phi} \log \left( W_{T-1,\Phi} R_f + x_{T-1,\Phi} P_{T-1,\Phi} (R_d - R_f) \right)
\end{aligned}$$

which yields the following first order condition with respect to $x_{T-1,\Phi}$

$$\frac{p^u_{T-1,\phi}(R_u - R_f)}{W_{T-1,\Phi}R_f + x_{T-1,\Phi}P_{T-1,\Phi}(R_u - R_f)} = \frac{-p^d_{T-1,\phi}(R_d - R_f)}{W_{T-1,\Phi}R_f + x_{T-1,\Phi}P_{T-1,\Phi}(R_d - R_f)}$$

which is solved by

$$x_{T-1,\Phi} = \frac{-W_{T-1,\Phi}R_f\left[p^u_{T-1,\Phi}(R_u - R_f) + p^d_{T-1,\Phi}(R_d - R_f)\right]}{P_{T-1,\Phi}(R_u - R_f)(R_d - R_f)}$$

and the base case therefore holds.

Now, for the inductive case. By the law of total expectation, we have

$$\begin{aligned}
\mathbb{E}[\log(W_T)] &= \sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1}} \mathbb{P}(\Phi'|\Phi)\log(W_{T,\Phi'}) \\
&= \sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_u}} \mathbb{P}(\Phi'|\Phi)\log(W_{T,\Phi'}) + \sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_d}} \mathbb{P}(\Phi'|\Phi)\log(W_{T,\Phi'})
\end{aligned}$$

Now, let $\Phi^u = (\Phi_0, ..., \Phi_{t-1}, R_u)$ and $\Phi^d = (\Phi_0, ..., \Phi_{t-1}, R_d)$. By the inductive hypothesis, we have

$$x_{t_0,\Phi^0} = \frac{-W_{t_0,\Phi^0}R_f\left[p^u_{t_0,\Phi^0}(R_u - R_f) + p^d_{t_0,\Phi^0}(R_d - R_f)\right]}{P_{t_0,\Phi^0}(R_u - R_f)(R_d - R_f)}$$

for all $t_0 > t$ and appropriate paths $\Phi^0$. Thus, we can apply Lemma 1 to find

$$\begin{aligned}
\sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_u}} \mathbb{P}(\Phi'|\Phi)\log(W_{T,\Phi'}) &= \sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_u}} \mathbb{P}(\Phi'|\Phi)\log(k'W_{t+1,\Phi^u}) \\
&= \sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_u}} \mathbb{P}(\Phi'|\Phi)\left[\log(W_{t+1,\Phi^u}) + \log(k')\right] \\
&= \sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_u}} \mathbb{P}(\Phi'|\Phi)\log(W_{t+1,\Phi^u}) + K^u
\end{aligned}$$

where

$$K^u = \sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_u}} \mathbb{P}(\Phi'|\Phi) \log(k')$$

By similar logic, we have

$$\sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_d}} \mathbb{P}(\Phi'|\Phi) \log(W_{T,\Phi'}) = \sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_d}} \mathbb{P}(\Phi'|\Phi) \log(W_{t+1,\Phi^d}) + K^d$$

where

$$K^d = \sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_d}} \mathbb{P}(\Phi'|\Phi) \log(k')$$

Now, from Lemma 1, for all $\Phi'$, $k'$ is not a function of $x_{t-1,\Phi}$. Thus, $K^u$ and $K^d$ will drop out of our first order condition and maximizing $\mathbb{E}\log(W_T)$ is equivalent to maximizing

$$\sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_u}} \mathbb{P}(\Phi'|\Phi) \log(W_{t+1,\Phi^u}) + \sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1 \\ \Phi'_t = R_d}} \mathbb{P}(\Phi'|\Phi) \log(W_{t+1,\Phi^d})$$

Now, note that since $\Phi'|\Phi^u$ is a distribution

$$\sum_{\substack{\Phi' \\ \Phi'_i = \Phi_i \forall i < t-1}} \mathbb{P}(\Phi'|\Phi^u) = 1$$

Thus,

$$\begin{aligned}
\sum_{\substack{\Phi' \\ \Phi'_i=\Phi_i \forall i<t-1 \\ \Phi'_t=R_u}} \mathbb{P}(\Phi'|\Phi) &= \sum_{\substack{\Phi' \\ \Phi'_i=\Phi_i \forall i<t-1}} \mathbb{P}(\Phi' \cap R_{t,t+1}|\Phi) \\
&= \sum_{\substack{\Phi' \\ \Phi'_i=\Phi_i \forall i<t-1}} \mathbb{P}(\Phi'|R_{t,t+1}=R_u \cap \Phi)\mathbb{P}(R_{t,t+1}=R_u|\Phi) \\
&= \sum_{\substack{\Phi' \\ \Phi'_i=\Phi_i \forall i<t-1}} \mathbb{P}(\Phi'|\Phi^u)p^u_{t,\Phi} \\
&= p^u_{t,\Phi} \sum_{\substack{\Phi' \\ \Phi'_i=\Phi_i \forall i<t-1}} \mathbb{P}(\Phi'|\Phi^u) \\
&= p^u_{t,\Phi}
\end{aligned}$$

Similarly, we have

$$\sum_{\substack{\Phi' \\ \Phi'_i=\Phi_i \forall i<t-1 \\ \Phi'_t=R_d}} \mathbb{P}(\Phi'|\Phi) = p^d_{t,\Phi}$$

Thus, our objective function can be simplified to

$$\sum_{\substack{\Phi' \\ \Phi'_i=\Phi_i \forall i<t-1 \\ \Phi'_t=R_u}} \mathbb{P}(\Phi'|\Phi)\log(W_{t+1,\Phi^u}) + \sum_{\substack{\Phi' \\ \Phi'_i=\Phi_i \forall i<t-1 \\ \Phi'_t=R_d}} \mathbb{P}(\Phi'|\Phi)\log(W_{t+1,\Phi^d})$$

$$=$$

$$\log(W_{t+1,\Phi^u}) \sum_{\substack{\Phi' \\ \Phi'_i=\Phi_i \forall i<t-1 \\ \Phi'_t=R_u}} \mathbb{P}(\Phi'|\Phi) + \log(W_{t+1,\Phi^d}) \sum_{\substack{\Phi' \\ \Phi'_i=\Phi_i \forall i<t-1 \\ \Phi'_t=R_d}} \mathbb{P}(\Phi'|\Phi)$$

$$=$$

$$p^u_{t,\Phi}\log(W_{t+1,\Phi^u}) + p^d_{t,\Phi}\log(W_{t+1,\Phi^d})$$

$$=$$

$$p^u_{t,\Phi}\log(W_{t,\Phi}R_f + x_{t,\Phi}P_{t,\Phi}(R_u - R_f)) + \log(W_{t,\Phi}R_f + x_{t,\Phi}P_{t,\Phi}(R_d - R_f))$$

This function is analogous to the objective function from the base case and therefore yields a first order condition of the same form giving us the desired solution

$$x_{t,\Phi} = \frac{-W_{t,\Phi} R_f \left[ p^u_{t,\Phi}(R_u - R_f) + p^d_{t,\Phi}(R_d - R_f) \right]}{P_{t,\Phi}(R_u - R_f)(R_d - R_f)}$$

Thus, the inductive step holds and the proof is complete.

□

# Appendix 2

**Table A2.1**
**Average Final Share Allocation by Return Magnitude for T = 6**

Each $(\mu, n)$ pair denotes the average share allocation at $t = T - 1$ at states where the risky asset has achieved exactly n up returns

| | | Number of Up Periods Over the Year | | | | | |
|---|---|---|---|---|---|---|---|
| $\mu$ | Initial Allocation | 0 | 1 | 2 | 3 | 4 | 5 |
| 1.05 | 0.43 | 0.67 | 0.96 | 0.79 | 0.2 | -0.11 | -0.11 |
| 1.06 | 0.5 | 0.66 | 0.99 | 0.86 | 0.28 | -0.07 | -0.1 |
| 1.07 | 0.57 | 0.65 | 1.01 | 0.93 | 0.35 | -0.03 | -0.09 |
| 1.08 | 0.64 | 0.64 | 1.02 | 1.01 | 0.44 | 0.01 | -0.08 |
| 1.09 | 0.71 | 0.64 | 1.04 | 1.08 | 0.52 | 0.06 | -0.07 |
| 1.1 | 0.78 | 0.63 | 1.06 | 1.16 | 0.62 | 0.12 | -0.05 |
| 1.11 | 0.86 | 0.62 | 1.08 | 1.24 | 0.72 | 0.18 | -0.02 |
| 1.12 | 0.94 | 0.61 | 1.09 | 1.32 | 0.83 | 0.26 | 0.01 |
| 1.13 | 1.02 | 0.6 | 1.11 | 1.4 | 0.95 | 0.34 | 0.04 |

**Table A2.2**
**Average Final Share Allocation by Return Magnitude for T = 10**

Each $(\mu, n)$ pair denotes the average share allocation at $t = T - 1$ at states where the risky asset has achieved exactly n up returns

| | | Number of Up Periods Over the Year | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mu$ | Initial | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1.05 | 0.43 | 0.4 | 0.62 | 0.88 | 0.97 | 0.72 | 0.28 | -0.03 | -0.13 | -0.1 | -0.04 |
| 1.06 | 0.5 | 0.39 | 0.61 | 0.88 | 1.01 | 0.79 | 0.36 | 0.02 | -0.11 | -0.1 | -0.04 |
| 1.07 | 0.57 | 0.38 | 0.59 | 0.88 | 1.05 | 0.87 | 0.44 | 0.08 | -0.09 | -0.09 | -0.04 |
| 1.08 | 0.64 | 0.37 | 0.58 | 0.89 | 1.09 | 0.95 | 0.53 | 0.14 | -0.06 | -0.09 | -0.05 |
| 1.09 | 0.71 | 0.36 | 0.57 | 0.89 | 1.13 | 1.03 | 0.62 | 0.21 | -0.02 | -0.08 | -0.05 |
| 1.1 | 0.79 | 0.35 | 0.56 | 0.89 | 1.17 | 1.11 | 0.71 | 0.28 | 0.02 | -0.06 | -0.05 |
| 1.11 | 0.86 | 0.34 | 0.55 | 0.89 | 1.2 | 1.2 | 0.82 | 0.36 | 0.07 | -0.05 | -0.05 |
| 1.12 | 0.94 | 0.33 | 0.54 | 0.89 | 1.24 | 1.28 | 0.93 | 0.46 | 0.12 | -0.02 | -0.04 |
| 1.13 | 1.02 | 0.32 | 0.53 | 0.89 | 1.28 | 1.37 | 1.05 | 0.56 | 0.19 | 0.01 | -0.03 |

# References


Alpert, M., and H. Raiffa (1982), "A Progress Report on the Training of Probability Assessors," in D. Kahneman, P. Slovic and A. Tversky, eds., *Judgment Under Uncertainty: Heuristics and Biases*, Cambridge: Cambridge University Press.

Barber, B., and T. Odean (2000), "Trading Is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors," *The Journal of Finance*, 55, 773-806.

Barber, B., and T. Odean (2001), "Boys will be Boys: Gender, Overconfidence, and Common Stock Investment," *The Quarterly Journal of Economics* 116, 261–292.

Barberis, N., and W. Xiong (2009), "What Drives the Disposition Effect? An Analysis of a Long-Standing Preference-Based Explanation," *The Journal of Finance* 64, 751-784.

Ben-David I., and D. Hirshleifer (2012), "Are Investors Really Reluctant to Realize Their Losses? Trading Responses to Past Returns and Disposition Effect," *The Review of Financial Studies* 25, 2485-2532.

Coval, J., and T. Shumway (2005), "Do Behavioral Biases Affect Prices?" *Journal of Finance* 60, 1-34.

Genesove, D., and C. Mayer (2001), "Loss Aversion and Seller Behavior: Evidence from the Housing Market," *Quarterly Journal of Economics* 116, 1233–60.

Lord, C., Ross, L., and M. Lepper (1979), "Biased Assimilation and Attitude Polarization: The Effects of Prior Theories on Subsequently Considered Evidence," *Journal of Personality and Social Psychology* 37, 2098-2109.

Meng, J., and X. Weng (2017) "Can Prospect Theory Explain the Disposition Effect? A New Perspective on Reference Points," *Management Science*.

Odean, T. (1998) "Are Investors Reluctant to Realize Their Losses?" *The Journal of Finance* 53, 1775-1798.

Odean, T. (1999) "Do Investors Trade Too Much?" *American Economic Review* 89, 1279-1298.

Scheinkman, J., and W. Xiong (2003), "Overconfidence and Speculative Bubbles," *Journal of Political Economy* 111, 1183-1219

Shefrin, H., and M. Statman (1985), "The Disposition to Sell Winners Too Early and Ride Losers Too Long: Theory and Evidence," *The Journal of Finance* 40, 777-790