Contracts and Externalities: How Things Fall Apart

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Abstract

A single principal interacts with several agents, offering them contracts. The crucial assumption of this paper is that the outside-option payoffs of the agents depend *positively* on how many "free agents" there are (these are agents who are not under contract). We study how such a principal, unwelcome though he may be, approaches the problem of contract provision to agents when coordination failure among the latter group is explicitly ruled out. Two variants are studied. When the principal cannot re-approach agents, there is a unique equilibrium, in which contract provision is split up into two phases. In phase 1, *simultaneous* offers at good (though varying) terms are made to a number of agents. In phase 2, offers must be made *sequentially*, and their values are "discontinuously" lower: they are close to the very lowest of all the outside options. When the principal can repeatedly approach the same agent, there is a multiplicity of equilibria. In some of these, the agents have the power to force delay. They can hold off the principal's overtures temporarily, but they *must* succumb in finite time. Furthermore, even though the maximal delay does go to infinity as the discount factor approaches one, the (discount-normalized) payoff of the agents must stay below *and bounded away from* the fully free reservation payoff. It is in this sense that "things [eventually] fall apart" as far as the agents are concerned.

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1. Introduction

A single principal interacts with several agents, offering them contracts. Our specification of a contract is reduced-form in the extreme: it is the offer of a *payoff* c to the agent, with consequent payoff $\pi(c)$ to the principal, where π is some decreasing function. The crucial assumption of this paper is that the outside-option payoffs of the agents depend *positively* on how many "free agents" there are (these are agents who are not under contract). In short, the positive externalities imply that the agents are better off not having the principal around.

In this paper, we study how such a principal, unwelcome though he may be, approaches the problem of contract provision to agents. At the outset, however, we rule out two possible avenues along which the principal can make substantial inroads.

First, it is trivial to generate equilibria which rely on coordination failure among the agents, and which yield large profit to the principal. For instance, consider the value of the outside option when there is only a single free agent. This is the lowest possible value of the outside option. It is possible for the principal to simultaneously offer contracts to all the agents, yielding a payoff equal to the value of this option. If all agents believe that other agents will accept this offer, it is obviously a best response to accept. But such dire outcomes for the agents rely on an extreme form of coordination failure. They may happen, but in part because of the specific examples we have in mind, and in part because there is not much else to say about such equilibria, we are not interested in these situations. We explicitly refine such equilibria away by assuming that agents are always able to coordinate their actions.

Second, the principal may be able to offer "multilateral" contracts, the terms of which — as far as an individual is concerned — explicitly rely on the number of *other* individuals accepting contracts. It is easy enough to show that such contracts can effectively create prisoners' dilemmas among the agents, sliding them into the acceptance of low-payoff outcomes even in the absence of coordination failure. In keeping with a large literature on this subject (see below), we rule out such contracts. For reasons of enforceability, law, or custom, we assume that all contracts must be strictly bilateral.

The purpose of this paper is to show that even in the absence of these two avenues of potential domination by the principal, agents must "eventually" succumb to the contracts offered by the principal — and often at inferior terms — no matter which equilibrium we study (there may be more than one) and despite the existence of perfect coordination among the agents. What is of particular interest is the form this eventual takeover assumes. To describe it, we study two models.

In the first model, the principal can make contractual offers to agents, but cannot return to an agent who has refused him to make further offers. In the second model, the principal can return to an agent as often as he wishes. In both variants, time plays an explicit role; indeed, the dynamic nature of contractual offers is crucial to the results we obtain.

In the first specification, we show that there is a unique perfect equilibrium satisfying the agent-coordination criterion. In this equilibrium, the principal will generally split contract provision up into two phases. In phase 1, which we call the *temptation phase*, he makes *simultaneous offers* to a number of agents. The offers are not the same. Suppose that this initial group of agents are named m through n (where n is the total number of agents); then agent k in this group receives r_k, where r_k is the outside-option payoff when there are k free agents. In phase 2, which we call the *exploitation phase*, offers must be made *sequentially*, and their values are "discontinuously" lower than $(r_m, \ldots, r_n)$. In fact, the offer values are close to r_1, the very lowest of all the outside options. [The degree of this closeness depends on the discount factor.]

Thus — in model 1 — a community of agents is invaded in two phases: some agents are bought, the rest are "exploited", in the sense that at the time of their contract, their outside option is strictly higher than what they receive. Yet they cannot turn down the offer, because it is known by this stage that other agents must succumb to the sequential onslaught.

In the second model, the outcomes look very different. Now there is a multiplicity of equilibria. In some of these equilibria all agents succumb to the principal right away, and there are equilibria with immediate acquiescence that nevertheless involve very different payoff levels for the agents. But there are other equilibria in which the agents have the power to force *delay*. They can hold off the principal's overtures. But we show that they cannot hold the principal off forever; in every perfect equilibrium, agents *must* succumb in finite time.

At the same time, the maximal delay does go to infinity as the discount factor approaches one. This suggests that along such equilibria, the agents manage to approach their "fully free" payoff, which is the value of their outside option when all agents are free. However, our final result shows that this supposition is false: even though the maximal delay becomes arbitrarily long as the discount factor approaches unity, the (discount-normalized) payoff of the agents must stay below *and bounded away from* the fully free reservation payoff.

It is in this sense that "things fall apart".

Multilateral externalities are, of course, widespread in practice and have therefore received much attention in economics. In particular, the industrial organization literature has been much concerned with the analysis of network effects and exclusive agreements.[1] Such externalities have also received tremendous attention in the literature on economic development and macroeconomic analysis of recessions, though the focus there has almost exclusively been on issues of coordination failure, something we avoid here as noted above.[2]

The particular game that we study, in which the principal makes take-it-or-leave-it offers to the agents, has much in common with Hart and Tirole [1990], McAfee and Schwartz [1994], Laffont, Rey and Tirole [1996], and Segal [1999]. This is in contrast to bidding games in which the agents make simultaneous offers to the principal, as in models of common agency; see, e.g., Bernheim and Whinston [1986a,b, 1998], O'Brien and Shaffer [1997], and Martimort and Stole [1998, 2000]. Because we focus on externalities in the reservation payoffs, we find the principal-offer game more natural in our particular context.

However, in contrast to most of the literature, the dynamic aspects of this game are of crucial importance. In the bulk of the cited papers, the principal makes simultaneous offers to the agents in a static context.[3] But there is no reason why this should be the case. Might the principal not benefit from *not* approaching all agents at once? Indeed, as already discussed above, he might, *even when the agents can coordinate away inferior responses.*

Our general theme is closely related to Segal [1999, 2001], who also studies contracts between a single principal and several agents in the presence of multilateral externalities (indeed, the externalities he considers are more general than those studied here). Following Segal [1999] we too restrict attention to bilateral contracting. Moreover, we also follow him in assuming that the principal can discriminate by making different offers available to different agents. However, unlike Segal [1999], we do *not* assume that the principal can coordinate agents on his preferred equilibrium; indeed, we entertain the opposite presumption whenever there is the chance of

[1]See for instance Dybvig and Spatt [1983], Katz and Shapiro [1986a,b], Farrell and Saloner [1986], and Ellison and Fudenberg [2000] on network effects, and Aghion and Bolton [1987], Rasumsen, Rasmeyer and Wiley [1991] (corrected by Segal and Whinston [2000]), Innes and Sexton [1994], and Bernheim and Whinston [1998] on exclusive dealing.

[2]See, for example, Rosenstein-Rodan [1944], Cooper [1988], Murphy, Shleifer and Vishny [1989], Kranton [1996], Acemoglu and Zilibotti [1997], Adserá and Ray [1998], and Cooper [1999].

[3]However, papers such as Bizer and de Marzo [1992] and Kahn and Mookherjee [1998] do study specific dynamic bidding games when there are externalities across agents.

coordination failure. [However, Segal [2001] examines bilateral contracting with externalities when one or both of the above assumptions are reversed.]

To be sure, the fact that the offers discriminate between the agents does not come at a surprise as this is a recurring feature of the static version of this game.[4] But the real departure from this important work is that our model is explicitly dynamic, and needs to be dynamic in order to even frame our results. Our propositions concern dynamic shifts in behavior patterns, as described above, and the nature of equilibrium delays, and both of these are crucially relevant to understanding the way in which our principal operates. In this sense, the framework we consider is very different indeed.[5]

More generally, there is a large body of literature on bilateral contracting with externalities. Some of the papers do have an explicitly dynamic structure. For instance, Katz and Shapiro [1986b] study the competition between two technologies exhibiting network externalities for two generations of consumers. Rasumsen et al. [1991] (see also the correction by Segal and Whinston [2000]) consider a market in which a price monopolist incumbent can offer exclusive contracts to some customers in order to discourage a potential competitor to enter. They also describe the equilibrium if the principal had to approach the agents sequentially. Our model instead has a completely open horizon, in which the game can — in principle — last forever, while payoffs are received in real time. Moreover, the timing is endogenous: agents can be approached simultaneously or sequentially, or in any combination of the two. And as we see, the results display a mixed timing structure; endogenously so. We consider this dynamic (and the description of how it matters) to be the central methodological contribution of this paper.

The rest of the paper is organized as follows. Section 2 presents a series of examples, the kind that motivated this research. Section 3 then introduces the dynamic model. The next two sections, 4 and 5, analyze the equilibria of the two variants introduced above. Section 6 concludes. All proofs are relegated to Section 7.

2. Examples

There are, of course, several examples that fit into the general category of interest in this paper. We mention some.

2.1. **Bonded Labor.**

This example is in the spirit of Genicot [2002]. Period 1 and 2 are a lean and a peak season respectively. There are n risk-averse peasants who are endowed with one unit of labor each season, have no other source of income, and supply their labor inelastically. The labor market is competitive and wages are 0 in the lean season and $w > 0$ during the peak season.

There are two sources of credit: a risk-neutral landlord-moneylender, and a competitive fringe of moneylenders who lend on a short-term basis. The latter group must incur fixed operational costs in every period, if they are to be active in the market. There is heterogeneity in these costs so that the extent of entry in any period is determined by the potential mass of “free borrowers" in that period.

At each date, the landlord can either offer simple labor contracts to the peasants, leaving them free to borrow from her or someone else later, or offer them bonded labor contracts. Bonded labor contracts are modelled as an exclusive contract combining labor and credit over the whole period by specifying the amount of consumption the laborer is to receive in each season c_1 and c_2.

[4] See for instance Innes and Sexton [1994], Segal and Whinston [2000], Segal [2001] and Caillaud [2001].

[5] We also do not make the assumption that payoffs are transferable, a feature that is of some importance for the sorts of questions that Segal is interested in.

Clearly the more peasants sign a bonded labor contract the fewer the alternative credit sources that will be active in the region. This lowers the outside options of a free borrower, if we make the reasonable assumption that the terms of outside credit worsen as the number of fringe lenders goes down. Put another way, the reservation payoffs of the agents exhibit positive spillovers.

2.2. **Mutual Insurance.** Consider an economy with a risk-neutral employer, a landlord, and n identical risk averse laborers. There is a competitive daily labor market where agents can earn a wage of 1 when employed. This occurs with a probability p, but with probability $1-p$ the worker is sick and remains unemployed (or not productive in that period), in which case the payment is 0. Suppose that a worker union sets up a *mutual insurance* fund in which the total income of all employed members is divided equally among all members. As a result, a daily laborer's utility r depends on the number of daily laborers n

$$r_n = \sum_{i=0}^{n} p(i,n)u(i/n) \tag{1}$$

where $p(i,n)$ is just the probability of i members employed out of n draws.[6]

The landlord, being risk neutral, could benefit from offering to a laborer i a labor contract in which he is guaranteed employment at a fixed wage c. If i refuses, the landlord can always hire daily laborers, such that if i accepts the landlord's net expected profit from each such contract is $p-c$. As long as they haven't accepted a guaranteed employment contract from the principal, the agents are daily worker and belong to the union.

Once again, reservation payoffs exhibit positive externalities.

2.3. **Working Hours.** Consider a region with a monopsonist employer (a large landlord or firm), and n identical potential workers endowed one unit of time each. Assume that the leisure times enjoyed by each resident in the region are complements. There are several reasons why one might expect this. Not only do many people prefer spending their time off with their family and friends, but in addition, the availability of leisure activities (goods and services that are leisure-oriented) generally depends on the size of the market for it. [Consider, for instance, the decidedly poor (or perhaps one should say, *even* poorer) quality of TV programs during weekdays, or the smaller number of plays or concerts during the work week. See Makowski [1980] for a model of the economy with an endogenous commodity space.] In brief, it is reasonable to suppose a reduced-form in which leisure time is less pleasurable when everybody else is working.

Let the utility of an agent i be

$$u(c_i, \ell) = u(c_i) + v(\ell_i, \ell_{-i})$$

where c_i and ℓ_i are i's consumption and leisure, and ℓ_{-i} is the vector of leisure times of all other agents. To capture the above complementarities, we assume (apart from the usual conditions) that $\partial^2 v_i / \partial \ell_i \partial \ell_j > 0 \; \forall \ell_i > 0$.

Now suppose that workers can either work full-time for the principal (in which case we assume that $\ell_i = 0$) or sets $\ell_i = 1$ (this could represent part-time work), or work part-time and have access to a subsistence level of consumption $\underline{c}$. If a worker accepts a full-time offer at wage w it generates a profit $\pi(w)$ for the principal. In this example again the reservation utilities of the agents are positively interdependent. If i expects $n-1$ other agents to be working part time and enjoying leisure,

$$r_n = u(\underline{c}) + v(1, n-1)$$

represents reservation payoffs in that situation.

[6] That is, $p(i,n) = \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i}$.

2.4. **Technological Externalities.** Suppose that n potential users endowed with a dollar each could buy into a software system L that costs $1, and in which the return r_m increases with the number of users m. The software example is meant to be provocative — any investment opportunity with complementarities will do. In general, the situation could represent the users of a productive technology exhibiting healthy network externalities.

A monopolist — we shall safely call him M —is trying to lure investors to (his) operating system W: one that generates a constant, low return per unit installed. If a unit were to cost a dollar, this return may be way below the full potential of sector L, but is unfortunately above the return to sector L when very few are invested in it. The monopolist can, however, give the system away for a lower cost. The point is that that the more individuals he locks in, the lower the return of system L.

2.5. **Bribing a Committee.** Consider a group of n agents, a "committee" or the "voters", that has to make a binary decision by majority voting. These voters are potentially under the influence of a lobbyist. This is the example studied in Bó [2003]. Assume that three members of a committee would each suffer a utility loss of size $\theta > 0$ if a certain law that they have to vote on was to pass. The committee members are to vote, simultaneously, for or against the law project. The criterion is simple majority: if two or more members vote "yes", the project is accepted. Otherwise, it is rejected. Given the members' preferences, one can expect them to vote against it. Now imagine that, before voting takes place, a lobbyist attempts to bribe the committee members to get the project to pass. To be sure, two members accepting the bribe has a negative externality on the remaining agent and thereby reduces his reservation utility.

2.6. **Housing Zones.** Consider a situation in which a real estate developer plans to buy a green area to build houses. This green space is collectively owned by several individuals. The developer plans to convert each plot into an overcrowded housing complex, an outcome that none of the individuals prefers. Therefore, the larger the number of plots acquired by the developer, the lower is the reservation value of those individuals who still own their plots.

2.7. **Takeovers.** A raider attempts to take over a company. Grossman and Hart [1981] suggest that, after a takeover, a dilution of post-takeover value of each non-tendered share would occur. Examples of such dilution consists in large salary or option agreements by the raider, or sale of the target's assets or output to another company owned by the bidder. Such dilution may reduce the value of non-tendered shares to below its original value in the absence of takeover. The reservation utilities of the agents correspond to the original value of the shares as long less than 50% of the shares have been tendered to the raider. As soon as the raider has 50% of the shares, and thereby takes over the firm, the values of the non-tendered shares drops to its diluted level.

3. Contracts with Externalities

A principal makes binding, bilateral, contractual offers to some or all of n agents. We look at reduced-form versions of contracts: an offer of a *payoff* c is made to an agent, and this payoff is received every period for life. The payoff to the principal from a contract c is $\pi(c)$, where π is a continuous, decreasing function. A contract vector, or simply an *offer*, is just a list of agent payoffs $\mathbf{c}$ listed in nondecreasing order: $c_k \leq c_{k+1}$. As for which agents are actually receiving the contracts, the context should make this amply clear.

At any date, an agent is either "contracted" (by the principal) or "free". In the latter case, she has either turned down all offers from the principal, or has never received one. A "free agent" receives a one-period payoff r_k, where k is the number of free agents (counting herself) at that date. The r_k's are parameters of the model, representing (one-shot) reservation payoffs when there are k free agents. Our basic assumption, maintained throughout the paper, is

[A.1] Positive Spillovers. $r_k < r_{k+1}$ for all $k = 1, \ldots, n-1$.

This is not to assert that a failure of [A.1] is to be branded unrealistic — far from it — but only to say that we focus on the positive-spillovers case in this paper.

At any date t, a *t-history* — call it h_t — is a complete list of all that has transpired up to (but not including) date t: agents approached, offers made, whether such offers were accepted, etc. [Use the convention that $h(0)$ is some arbitrary singleton.] Let $N(h_t)$ be a set of *available agents* given history h_t. These are free agents who are available to receive an offer; more on this specification below.[7]

The game proceeds as follows. At each date t and for each history h_t, the principal selects a subset $S_t \subseteq N(h_t)$ of agents and announces a personalized or anonymous offer (vector) $\mathbf{c}$ to them. [Note: S_t may be empty, so that not making any offer is permitted.] All agents in S_t simultaneously and publicly accept or reject their component of the offer. Then the t-history is appropriately updated and so is the set of available agents. The process repeats itself at date $t+1$, and is only declared to end when the set of available agents is empty.

Lifetime payoffs are received as the sum of discounted one-shot payoffs. A contracted agent with offer c simply receives this every period for life, while a free agent obtains the (possibly changing) reservation payoffs. The principal makes no money from free agents, and $\pi(c)$ every period from an agent contracted at c. We assume that the agents and the principal discount the future using a common discount factor $\delta \in (0, 1)$. We shall always normalize lifetime payoffs by multiplying by $1 - \delta$: this permits us to vary the discount factor in some of the propositions.

By formally specifying strategies for the principal and agents, we may defines subgame-perfect equilibrium in the usual way. In what follows, we study a subset of such equilibria (see the restriction [A.2] below).

Observe that r_n may be viewed as the "fully free" payoff, obtained when the principal has not been able to contract with any of the agents. On the other hand, r_1 may be viewed as the "fully bonded" payoff: it is the outside option available to a free agent when there are no other free agents around.

Now, it is always possible for the principal to implement the fully bonded contract if we allow for coordination failure across agents: all she does is make the offer $(r_1, r_1, \ldots, r_1)$ to all the agents at date 0. It is an equilibrium to accept. Of course, it is an equilibrium which the agents could have coordinated their way out of: saying "no" is also sustainable as a best response. In this paper, we not only allow for such coordination, we insist on them. [In any case, if one wishes to seriously entertain coordination failure, its consequences are obvious.] Especially given the small or rural communities we have in mind, we would like to imagine communication as taking place easily among the agents. An agent who is about to accept an offer has no incentive to state otherwise, while an agent who plans on rejecting has all the interest in revealing his intention. Therefore, we impose throughout the following assumption:

[A.2] Agent Coordination. Only perfect equilibria satisfying the following restriction are considered: there is no date and no subset of agents who have received offers at that date, who can change their responses and *all* be strictly better off, with the additional property that the changed responses are *also* individual best responses, given the equilibrium continuation from that point on.[8]

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[7]Of course, $N(h(0))$ is the entire set of agents.

[8]In this specific model, agent coordination may be viewed as a coalition-proofness requirement applied to the set of agents. Because of the specific structure, the nested deviations embodied in the definition of coalition-proof equilibrium

Just in case it is not obvious, we stress that [A.2] only permits agents to coordinate their moves at particular stages. In particular, [A.2] in no way selects — among possibly many equilibria — the equilibrium preferred by the agents. *Which* equilibrium gets to be played does not simply depend on the agents. The principal's strategy matters too.

It should be noted that agent coordination carries little bite if the principal is permitted to offer contracts that are contingent to the *simultaneous* acceptance-rejection decisions of other players. If such contracts are permitted, the principal can create "prisoners' dilemmas" for the agents and get them all to accept the lowest possible price r_1.[9] Thus an important implicit assumption is that we rule out such contracts, which is in line with most of the literature. We view the present exercise as a test of the efficacy of agent coordination in bilateral contracting.

In what follows, we study two leading cases. In the first case, discussed in Section 4, each agent can receive at most a single offer. Formally, this means that the set of available agents satisfies the recursion $N(h_{t+1}) = N(h_t) \backslash S_t$. In the second leading case, studied in Section 5, it is assumed that all the free agents at any date are up for grabs: $N(h_{t+1})$ is just $N(h_t)$ less the set of agents who accepted an offer in period t.

4. The Single-Offer Model

In this section, it is assumed that the principal cannot make more than one offer to any agent. It will be useful to begin with a simple example.

Example 1. Suppose, to start with, that there are just two agents. Let $\delta = 0.9$, $r_1 = 10$ and $r_2 = 20$, and suppose that $\pi(r) = 25 - r$. We solve this game "backwards". Suppose that there is only one available agent. If the other agent is free (he must have rejected a previous offer), then the principal will offer the available agent 20, and if the other agent is contracted, then the available agent will receive 10. Now return to the stage in which both agents are free and available.

Define $\widetilde{r}_2 \equiv (1-\delta)r_2 + \delta r_1 = 11$, which is the stationary payoff that's equivalent to enjoying r_2 in one period and r_1 ever after. Given the calculations in the previous paragraph, it is easy to see that the principal will make an offer of 11 to a single agent, which must be accepted. Tacking on the second stage from the previous paragraph, the remaining agent will receive 10 in the second period.

Notice that the principal cannot benefit from making simultaneous offers in the first period (even though he would like to do so because he discounts future payoffs). By the agent coordination criterion, the only payoff vector which he can implement is $(20, 10)$, yielding him a (normalized) payoff of 20. In contrast, the staggered sequence in the previous paragraph yields him a payoff of 27.5 ($14 + 0.9 \times 15$).

Now add a third agent. Let r_1 and r_2 be as before, and set $r_3 = 28$. Once again, we solve this problem backwards. We already know what will happen if one of the agents is contracted and the other two are available; this is just the case we studied above. If, on the other hand, there are two agents available but the unavailable agent is free (he refused a previous offer), this induces a two agent problem with reservation values $(r_1', r_2') = (r_2, r_3) = (20, 28)$.

This subproblem is different from the one we studied before. If a single agent refuses an offer, *the principal will not make an offer to the one remaining agent*, because the agent's reservation value

do not need to be invoked. Because a satisfactory definition of coalition-proofness does not exist for infinite games, we do not feel it worthwhile to state this connection more formally.

[9] Consider the case of two agents for instance. The principal could simultaneously offer them r_2 if only one of them accepts the offer but r_1 if both accept. If when indifferent the agents accept the offer, they would both accept the offer. Otherwise the principal would just increase slightly the previous offer to guarantee their acceptance. In the case of n agents, it can be checked that if the principal offers r_{n-s+1} (or slightly more than this amount) to the agents conditional on s agents accepting her offer, then she gets all the agents to sign for her for just about r_1.

stands at 28 and there are negative profits to be made. Knowing this, if any *single* agent receives an offer, he will hold out for 28, and the remaining agent will be paid 20. Now the principal will prefer to make a simultaneous offer with payoffs $(28, 20)$, which is accepted in equilibrium.

It follows that the very first of our three agents can — through refusal — obtain a payoff of $\widetilde{r}_3 \equiv (1-\delta)r_3 + \delta r_1 = 11.8$. By making fully sequential offers, the principal implements the payoffs $(11.8, 11, 10)$ for the three agents. Once again, the coordination criterion assures us that he can do no better making a simultaneous offer.

Matters are different, however, if we change the value of r_3 to 31 and leave everything else unchanged. Now revisit the subproblem in which there are two available agents, and the third agent is free (but no longer available). It is now easy to see that the principal will make no offers at all. For the induced two-agent problem has reservation values $(r_1', r_2') = (r_2, r_3) = (20, 31)$, and there is no way the principal can turn a positive profit in this subgame. *The third agent is now pivotal.* By refusing, he can ensure that the principal goes no further, and therefore he obtains a payoff of r_3.

The only way for the principal to proceed in this case is to make an initial offer of $r_3 = 31$ to a single agent, which will be accepted. With this agent contracted and out of the way, the principal can now proceed to give the remaining agents 11 and 10 (sequentially). There is no harm in making the offer of 11 up front as well but the offer of 10 must wait a further period. This yields a payoff of $-6 + 14 + \delta \times 15 = 21.5$. Again, it can be checked, using the coordination criterion, that no simultaneous offer will do better.

To summarize the discussion so far, we see that

(1) Either the third agent is "pivotal" in that his refusal sparks a market collapse, in which case he obtains "discontinuously more" than the other two agents; or

(2) The third agent's actions has no bearing on what happens next — future agents are always contracted — in which case he also receives a low payoff.

The phrases "discontinuously more" and "low payoff" are best appreciated when the discount factor is close to one. The "low payoff" is then essentially r_1, while a "discontinuously higher" payoff entails receipt of some r_k, for $k > 1$.

If the example so far has been absorbed, consider the addition of a fourth player. We will study the pivotal three-player subcase: hence retain $(r_1, r_2, r_3) = (10, 20, 31)$. Let us suppose that $r_4 = 35$. Consider again an offer to a single agent (call him the fourth agent). If he refuses the offer, we induce the three-agent game with reservation values $(r_1', r_2', r_3') = (r_2, r_3, r_4) = (20, 31, 35)$. This game is way too costly for the principal, and no further offer will be made. The fourth player is therefore pivotal: his refusal will shut the game down. On the other hand, his acquiescence induces the three-agent game in which agent 3 is pivotal as well. It follows that both the third and fourth players need to be bought out at the high reservation values. This can be done sequentially but given discounting, the right way to do it is to make the discontinuously high *simultaneous* offer $(r_3, r_4) = (31, 35)$ to any two of the players, and then follow through with sequential contracting using the low offers $(11, 10)$. Once again, the sequential process can begin at date 0 as well. It is easy to check, using the coordination criterion, that no other offer sequence is better, and once again we have solved for the equilibrium. The principal's payoff is $-10 - 6 + 14 + \delta \times 15 = 11.5$.

The four-agent game yields a couple of fresh insights. Notice that *if the third agent is pivotal conditional on the fourth agent's initial acquiescence,the fourth agent must be pivotal as well.* The reason is simple: the third player's pivotality (conditional on the fourth agent's prior acquiescence) simply means that (r_3, r_2) is an unprofitable 2-agent environment for the principal. Now if the fourth agent were to refuse an initial offer, this would induce the three-agent environment (r_4, r_3, r_2). If the principal cannot turn a profit in the environment (r_3, r_2), he *cannot* do so under

(r_4, r_3, r_2), which is even worse from his point of view. Therefore the fourth agent must be pivotal as well.

The example suggests, therefore, that

[3] The pivotality of a player in an environment in which all “previous" agents have accepted their offers *implies* the pivotality of these “earlier" agents in an analogous environment.

But more is suggested. If all these pivotal players need to be bought at their reservation values, there is no sense in postponing the inevitable (recall that that the overall profit of the principal is positive for there to be any play at all, and there is discounting). Therefore

[4] Pivotal agents are all made simultaneous (but unequal) offers at the high reservation values.

In contrast,

[5] The remaining non-pivotal players are offered low payoffs (approximately r_1 when $\delta \simeq 1$) and *must be approached sequentially* so as to avoid the use of the coordination criterion.

[Remember, in reading this, that all players are identical, so that by “pivotal players" we really mean pivotal player *indices*.]

In what follows, we show that the insights of Example 1 can be formalized into a general proposition. This is a characterization of unique equilibrium, which obtains under the following convention: the principal will immediately make an offer and an agent will immediately accept an offer if they are indifferent between doing so and not doing so.

Proposition **1.** *There is $\hat{\delta} \in (0,1)$ such that for every $\delta \in (\hat{\delta}, 1)$, there exists an equilibrium satisfying [A.2], which is unique up to a permutation of the agents. Either the principal makes no offers at all,*[10] *or the following is true: there is $m(\delta) \in \{1, \ldots, n\}$ such that for agent j with $j \geq m(\delta)$, an offer of precisely r_j is made in the very first period, date 0. All these offers are accepted. Thereafter, agent $j < m(\delta)$ is made an offer $r'_j \in [r_1, r_1(\delta)]$ at date $m(\delta) - j - 1$, and these offers are accepted as well.*

The index $m(\delta)$ is nondecreasing in δ, and $r_1(\delta) \to r_1$ as $\delta \to 1$.

The proposition shows that there is a unique equilibrium path, which exhibits two distinct phases. Because all agents are identical, the uniqueness assertion is obviously subject to arbitrary renaming of the agents. Think of the first phase as the *temptation phase*. Agents “selected" for the this phase must be approached simultaneously. They receive relatively high offers (agent j receives the reservation value when there are j free agents, r_j). They also must receive *different* offers: after all, $r_j < r_{j+1}$. In contrast, in the second phase, which we may call the *exploitation* phase, agents must be approached sequentially. They are also made offers that are hardly different from the lowest possible payoff, r_1.

As Example 1 shows, the indices above and below $m(\delta)$ correspond to very different agent roles. The former indices are pivotal in the sense explained in the example: if these indices reject an offer, then in the resulting continuation subgame, the principal cannot make offers that are *both* acceptable and profitable. These indices must therefore be bought outright. By the agent coordination criterion, to prevent a subset of size s of pivotal agents from rejecting her offer, the principal needs to offer at least $r_{m(\delta)+s}$ to one of these agents. This reasoning applied to subsets of all sizes between 1 and $n - m(\delta) + 1$ determines the vector of offers. These offers could have been made sequentially, but because of discounting there is a strict loss in doing so. This explains the simultaneous nature of the temptation phase.

The exploitation phase corresponds to the sequential “acquisition" of the remaining $m(\delta) - 1$ agents. A careful reading of the proposition shows that this phase starts at date 0 as well. These agents are paid “low" amounts, but cannot refuse them in equilibrium. The payoffs exceed r_1,

[10] Equivalently, he makes offers that are unacceptable.

but converge to r_1 as δ goes to one (these are analogous to the payoffs of 10, 11, and 11.8 in Example 1). This procedure *must* be sequential even though the principal is eager to complete the acquisitions (by discounting). Any form of simultaneity at this stage would be blocked by the agent coordination criterion: the agents would benefit from a joint refusal.

As in the temptation phase, the exploitation phase is also characterized by differentiated offers. For instance, the first person in that phase receives strictly more than r_1 (though close to it, as noted), while the last person receives precisely r_1. However, the differentiation of offers, while also an essential feature of this phase, is not as striking or important as in the temptation phase.

Note well the discontinuity between the two phases. Agent $m(\delta)$ receives $r_{m(\delta)}$. Agent $m(\delta)-1$ receives something close to r_1. This result is closely related to Rasumsen et al. [1991] as corrected by Segal and Whinston [2000]. These two papers study a market in which a monopolist incumbent can offer exclusive dealing contracts to customers in order to discourage a potential competitor to enter. A fixed cost of entry but a lower marginal cost for the entrant means that the reservation utility of the the agents is a step function jumping from 0 to a higher value (non-profitable for the incumbent) at the minimum market size needed for the entrant to enter. There is a specific number of agents that the monopolist needs to sign up in the first period in order to discourage entry. When considering a fully sequential approach, they find a similar concept of pivotality such that some agents needs to receive the high value while the others receive 0. Our results show is that these type of discontinuity in the offers received by the agents is actually a general feature of *all* models with positive externalities in the reservation utility, even if the reservation utility increases smoothly with the number of agents. Moreover, by fully endogenizing the approach of the principal we show that the optimal approach is *not* fully sequential: some offers are made simultaneously while others are made sequentially.

Our equilibrium is illustrated in Figure 1 for the simple case in which $\pi(r) = Y - r$ and $\delta = 1$. The solid line represents the reservation utility of the agents r_m, and the dotted line represents the equilibrium offers. The striped areas represents the principal's profit which is negative in the temptation phase (to the right of $m(\delta)$) but positive in the exploitation phase (to the left of $m(\delta)$).

A contract may be deemed *exploitative* when a party uses its power to restrain the set of alternatives available to another party, so as the latter has no better choice than to agree upon a contract very advantageous to the first party (see Basu [1986], Hirshleifer [1991], Bardhan [1991] and Genicot [2002]). Here, the reason the principal is willing to incur losses in the temptation phase is because, by doing so, she lowers the reservation utility of the other agents and put them in a situation in which their best option is to accept very low offers. This is the very idea that characterizes the exploitation phase.

Might one or the other phase be empty? Certainly, as long as $\pi(r_2) \geq 0$, it is easy to see that the equilibrium *must* exhibit an exploitation phase (provided that the principal makes acceptable offers and the discount factor is large enough). On the other hand, the temptation phase may be empty, as our example shows.

Indeed, the logic of our proof suggests that the temptation phase *will* be empty if the number of agents is large enough, and if the principal makes positive profits from the entire enterprise. To formulate this observation more precisely, suppose that the outside option payoff always lies between a maximum of $\bar{r}$ (when all agents are free) and a minimum of $\underline{r}$ (when there is no other free agent). Suppose, further, that as we increase the total number of agents, we keep this range unchanged but only lower the size of the downward jump in reservation payoffs as the number of free agents is decreased by one.[11] Then if the equilibrium payoff from all n agents if positive, the equilibrium payoff from $n-1$ agents (conditional on the first agent's refusal) should be positive

[11]For instance, one could use the formulation $r_k = \underline{r} + \frac{k-1}{n}[\bar{r} - \underline{r}]$.

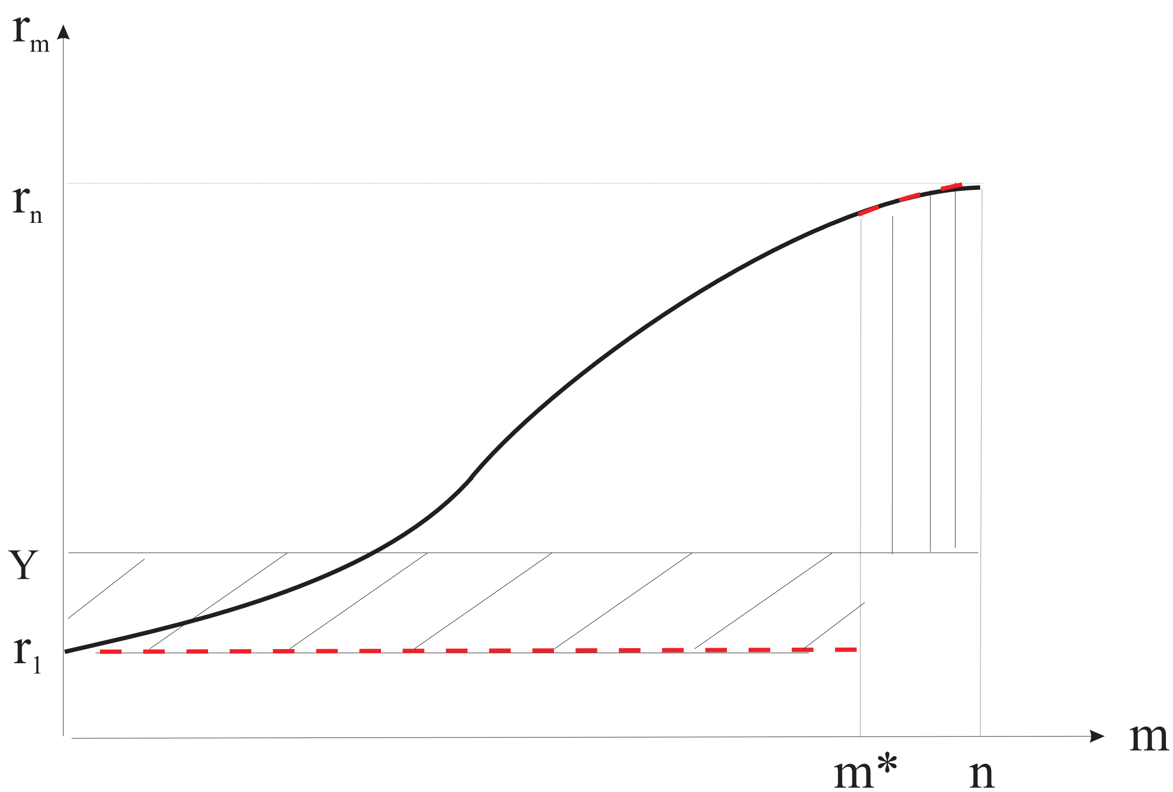

Figure 1. An Illustration of Proposition 1.

as well, provided that n is large. But this implies that no agent is pivotal, so that the exploitation phase should commence right away.

One may question our implicit focus on a high discount factor. Clearly, if δ were very small, the principal would hardly value the future and therefore would either make simultaneous offers to all agents in the first period or not make any offers at all. However, since the principal has all the interest in approaching the agents as rapidly as possible, it is only natural to consider very short periods and therefore values of δ close to one.

Finally, notice that Proposition 1 assures us that a raising of the discount factor can never wipe out an exploitation phase, if one exists to begin with.

Note that our resolution of indifference does matter in ruling out multiple equilibria (with distinct payoffs). As an example, take a two-agent game in which $\pi(r_1) > 0$ and $\pi(r_2) = 0$. Assume that agent 2 gets the first offer and agent 1 the second. If agent 1 accepts offers when indifferent, agent 2 is non-pivotal and will therefore accept an offer in $(r_1, r_+\epsilon)$. In contrast, if agent 1 rejects offers when indifferent, then 2 is pivotal and won't accept offers of less than r_2. This equilibrium exhibits a lower payoff for the principal. In a three-agent game, then, we can support payoffs to agent 3 that are lower than r_3 but larger than $r_+\epsilon$ simply by having agent 1 threaten the principal by resolving indifference in different ways, depending on history. This is a substantial departure, and would make a complete characterization of the equilibria hopeless.

5. The Model With Multiple Offers

In this section, we drop the assumption that the principal is only able to make a single offer to an agent. It turns out that our notion of what constitutes "resistance to invasion" changes significantly. Instead of a unique equilibrium outcome, there are now multiple equilibria. Nevertheless, there is a strong sense in which we can say that "exploitation" occurs in every equilibrium.

Consider the following scenario. At the start of any period, the principal makes contractual offers to some or all of the free agents at that date. Agents who accept an offer become "unfree" or "contracted" and must hold that contract for life. [Later, we comment on contracts which can be ended after some finite duration. As far as our results are concerned, not much of substance changes.] After these decisions are made, payoffs are received for that period. Contracted agents receive their contract payoffs. Each free agent receives an identical payoff r_k, where k is the number of free agents at the end of the decision-making process. The very same agents constitute the starting set of free agents in the next period, and the process repeats itself. No more offers are made once the set of free agents is empty.

Throughout this section, the following assumption will be in force:

[A.1] $\sum_{i=1}^{n} \pi(r_i) > 0$.

It will be clear from what follows that if this assumption was violated, no offer would be made. Therefore, we focus on situation where $[A.1]$ holds. To establish our main results, it will be useful to study the worst and best equilibrium payoffs to the principal (and in addition, these observations may be of some intrinsic interest). First, some notation. Suppose that at any date a set of m free agents are arrayed as $\{1, \ldots, m\}$. We shall refer to these indices as the *names* of agents, and note explicitly that in some of our equilibrium constructions agent-renaming will be involved. Next, look at the special package $\mathbf{r}$ made to all m free agents, where $\mathbf{r} = (r_1, \ldots, r_m)$. We shall call this the *standard offer* (to m free agents), where it is understood that the agent with name i receives the offer r_i.

Next, define for any m, $\hat{r}_m \equiv (1-\delta)r_m + \delta r_1$. As interpretation, this would be the (normalized) lifetime payoff to a free agent when he spends one period of "freedom" with $m-1$ other free agents, and is then the *only* free agent for ever after. Now look at the special package $\hat{\mathbf{r}}$ made to all m free agents, where $\hat{\mathbf{r}} = (\hat{r}_1, \ldots, \hat{r}_m)$. We shall call this the *low offer* (to m free agents). Once again, it is understood that the agent with name i receives the offer $\hat{r}_i$.

The following proposition pins down worst and best (discount-normalized) equilibrium payoffs to the principal.

Proposition **2.** *The principal's worst equilibrium payoff is precisely his payoff from the standard offer made to — and accepted by — all n agents:*

$$\underline{P}(n) = \sum_{i=1}^{n} \pi(r_i), \tag{2}$$

while his best equilibrium payoff arises from the low offer made to — and accepted by — all n agents:

$$\overline{P}(n) = \sum_{i=1}^{n} \pi(\hat{r}_i). \tag{3}$$

The standard offer is similar to the divide-and-conquer offer that appears in the static context (see for instance Innes and Sexton [1994], Segal and Whinston [2000], Segal [2001] and Jullien [2001]). The difficulty is to show that this can actually be an equilibrium outcome in a dynamic model.

To provide some intuition for this proposition, first consider the assertion regarding worst equilibrium payoffs. It is obvious that in no equilibrium can the principal's payoff fall below the bound described in (2). For suppose, on the contrary, that this were indeed the case at some equilibrium; then the principal could make an offer $\mathbf{c}$ with each component c_i exceeding the corresponding component r_i of the standard offer by a tiny amount ϵ_i. Clearly agent n cannot resist such an offer (for in no equilibrium can she get strictly more than r_n). But then nor can agent $n-1$ resist her component of the offer, and by an iterative argument all agents must accept the offer $\mathbf{c}$. If the ϵ_i's are small enough, this leads to a contradiction.

So it must be that the principal can guarantee at least $\sum_{i=1}^{n} \pi(r_i)$ in any equilibrium. But what guarantees that the principal's equilibrium payoff can be pushed all the way down to this level? It is conceivable that the principal might be able to deviate from the standard offer (or its payoff-equivalent contractual offer) in ways that are beneficial to himself. This possibility pushes the equilibrium construction in a somewhat complicated direction. Various phases need to be constructed to support the original equilibrium payoff, and at each date several incentive constraints need to be checked. A brief description follows, which the reader may skip without loss of continuity.

To construct the equilibrium, think of three phases: a *normal phase*, which starts the process, a collection of *evaluation phases*, in which agents decide how to react to a principal's "deviation offer", and a *punishment phase*, in which a consideration of further offers is deliberately delayed by the agents. To be sure, all three phases may be revisited in appropriate subgames.

In the normal phase, the principal makes the standard offer to all free agents, and all agents must accept. Agent deviations are followed by a suitable renaming of agents (deviators being assigned the "lowest" names), with the normal phase started up again in the ensuing subgames. If the *principal* deviates at any time and makes an offer $\mathbf{c}$, we enter an evaluation phase in which the agents respond to this offer. To describe this phase, first rename the agents such that their offers in $\mathbf{c}$ are nondecreasing. If for any positive integer K the first K agents in the list would prefer to reject their new offers in return for the standard offer (to exactly K free agents) in the next period, the equilibrium prescribes that they do reject their components of the offer $\mathbf{c}$. Otherwise set $K = 0$.

Consider, now, the remaining $n - K$ agents. There are two possibilities. If

$$\sum_{i=K+1}^{n} \pi(c_i) \leq \sum_{i=K+1}^{n} \pi(r_i)$$

then agents with indices larger than K must accept the offer. In the following period, proceed to the *normal phase* with the first K agents if any. Otherwise, if

$$\sum_{i=K+1}^{n} \pi(c_i) > \sum_{i=K+1}^{n} \pi(r_i),$$

then only the agents who should accept c_i by iterated deletion of dominated strategies accept. The others reject and we proceed to a punishment phase starting in the following period.

In the punishment phase, this second group of free agents, as well as the original group of K rejectors (if any) force the principal to wait for T periods without accepting any offer at all. After that, the K agents must be made the standard offer (for K), while the remaining group of rejectors are re-issued the very same offer that they had earlier rejected, which they must now accept.

Several incentive constraints (including possible reversions to normal and evaluation phases) need to be specified here to ensure that all this is indeed possible. In particular, some of these constraints tell us what the length of T must be. In the end, all of the paraphernelia is employed to ensure that the principal does not deviate from the normal phase in the first place, so that we indeed have an equilibrium.

We refer to this equilibrium the *standard-offer equilibrium*. [The proof of the proposition contains a full description of the equilibrium.]

Once the existence of a standard-offer equilibrium is established, it can be used to shore up other equilibria. In particular, we can use it to calculate the *best* equilibrium payoff to the principal. First, make the low offer $\hat{\mathbf{r}}$ to all free agents (under some naming). If any subgroup of m agents rejects its component of this offer, rename the deviators so that the rejector with the "highest" name is now given the "lowest" name. Now start up the standard-offer equilibrium with this set of free agents. This ensures that the highest-named deviator (agent m) earns no more than

$$(1-\delta)r_m + \delta r_1$$

from her deviation. But she was offered $\hat{r}_m = (1-\delta)r_m + \delta r_1$ to start with, anyway. She therefore has no incentive to go along with this group deviation. We therefore have an equilibrium, which we shall refer to as the *low-offer equilibrium*.

Notice how our construction ensures that the low-offer equilibrium is immune to the agent-coordination criterion, which we impose throughout.[12] Indeed, it is precisely because we impose this criterion that even better payoffs are unavailable to the principal, and why $\overline{\mathrm{P}}$ as described in (3) is truly the best equilibrium payoff. To see this, it suffices to recall that r_1 is the worst possible continuation utility an agent can ever receive. Hence, to sign on any agent when n agents are free, the principal has to make *at least one offer* that's better than $\hat{r}_n$ for the agent. Otherwise the agents would profitably coordinate their way out of such an offer. Given this fact, the principal *must* make an offer of $\hat{r}_{n-1}$ or better to sign up another agent, whether in the current period or later. By repeating the argument for all remaining agents, we may conclude that the principal can earn no more than a payoff of $\overline{\mathrm{P}}(n) = \sum_{i=1}^{n} \pi(\hat{r}_i)$, and this completes our intuitive discussion of Proposition 2.

The profile of equilibrium offers in the standard- and low-offer equilibria are illustrated in Figure 2 for the simple case in which $\pi(r) = Y - r$. The dotted curve on top represents the standard offer while the one below represents the low offer. The striped areas represents the principal's profit (agent by agent) in the standard-offer equilibrium, which is negative for some agents and positive for others.

We turn now to the main topic of interest: the extent to which the agents can hold on to their "best payoff" of r_n against an "invasion" by the principal. To be sure, *some* agent might get r_n (see, for instance, the standard-offer equilibrium). But we are interested in average agent payoffs, not the best payoff to some agent. To this question one might attempt to invoke the standard-offer equilibrium to provide a quick answer. Surely, if this is the worst equilibrium for the principal, it yields the best average payoff for the agents. Because this average falls short of r_n, we must conclude that the agents cannot resist invasion, in the sense that their equilibrium payoffs are bounded below and away from r_n.

However, the answer is not that simple, because the standard-offer equilibrium for the principal may *not* correspond to the one with the best average payoffs for the agents. An example to this effect will be provided below.

For now, taking this assertion on faith, we approach the problem of agent-resistance in two different ways. First, notice from the example that delays are generally a good thing from the

[12]The reader may be uncomfortable that agent m is just indifferent to the deviation, while the others may strictly prefer the deviation. In that case, $\overline{\mathrm{P}}$ as described in (3) may be thought out as the *supremum* equilibrium payoff, where the open-set nature of the weaker agent-coordination criterion prevents the supremum from being attained. Either interpretation is fine with us.

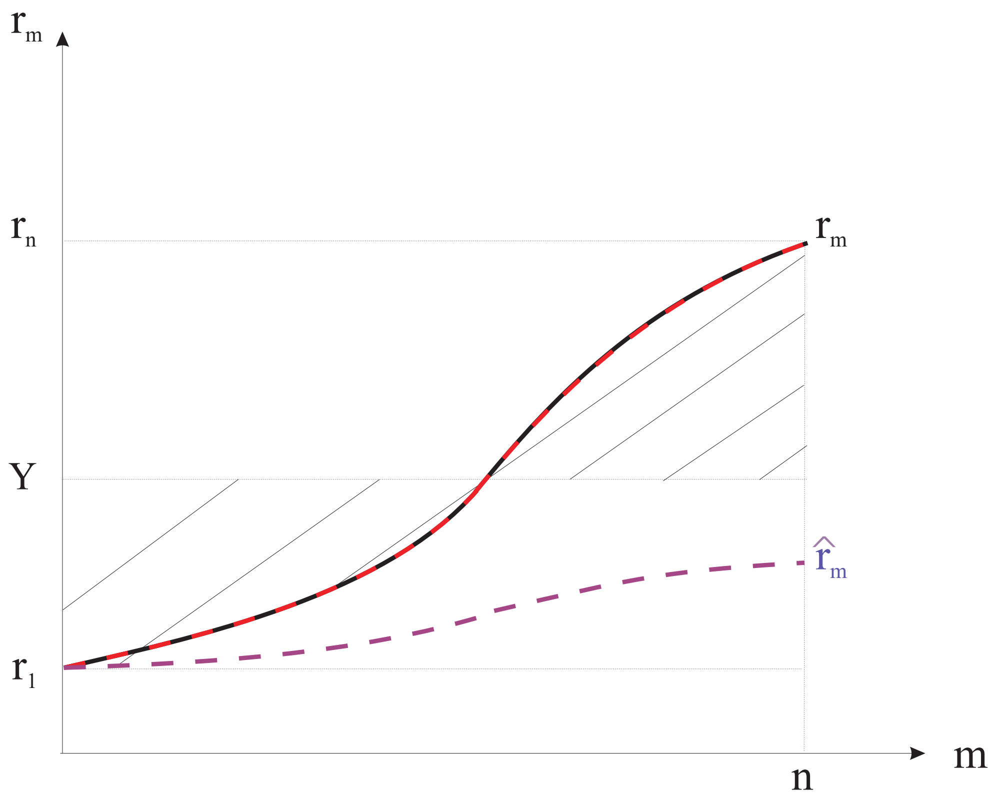

Figure 2. The Standard Offer and the Low Offer.

point of view of the agents: each agent obtains the full payoff r_n for each period in which no offers are made or accepted. So one measure of resistance may be the length of a *maximal delay* equilibrium: the amount of time for no offers are made (or if one is made, it isn't accepted). Proposition 3 below gives us a bound as an easy consequence of our worst and best equilibrium characterizations. [13]

Define the *delay* of an equilibrium to be the number of periods that elapse before a first offer is made and accepted.

Proposition **3.** *An equilibrium with delay T exists if and only if*

$$T \leq \frac{\ln(\underline{P}(n)) - \ln(\overline{P}(n))}{\ln \delta} \tag{4}$$

The arguments underlying this proposition are quite simple. To support the longest possible delay, the principal must receive her *highest* possible payoff ($\overline{\text{P}}(n)$) at the end of the delay, while she must receive the *worst* possible payoff if she deviates. Hence, if T is the size of the delay, we must have $\delta^t \overline{\text{P}}(n) \geq \underline{\text{P}}(n)$, which yields (4).

[13]Cai [1999] studies delays arising in a model with one principal and many agents but in a very different setting. The complementarities arise within the contract not in the reservation utility: in their model, the principal needs to sign *all* agents in order to get a revenue. These externalities do matter because both agents have some bargaining power. When the principal males take-or-leave it offers and in the absence of asymmetry of information, only externalities in the reservation utility do matter.

Notice that less discounting is amenable to the creation of greater delay. This makes sense: the principal must be induced to accept the delay, instead of bailing out — say, with the use of the standard offer. Indeed, if δ is small enough no delay is possible, because $\underline{\mathrm{P}}$ and $\overline{\mathrm{P}}$ come arbitrarily close to each other, and in addition the principal is in a hurry to close deals as soon as possible.[14] On the other hand, as δ approaches unity, it is easy to see from (4) that arbitrarily long delays can be sustained.

Proposition 3 can easily be used to construct an example in which agent average payoffs exceed those under the standard-offer equilibrium. This happens precisely when there is delay.

Example 2. Suppose that $\pi(r) = Y - r$ for some $Y > 0$. We will show that if $\pi(r_n) = Y - r_n < 0$, then the best average payoff to an agent exceeds the average under the standard-offer equilibrium, which is $(\sum r_i)/n$.

Along the lines suggested by Proposition 3, construct an equilibrium in which nothing happens for the first T periods, after which the principal initiates the low-offer equilibrium. Any deviations by the principal during this initial "quiet phase" is punished by reversion to the standard-offer equilibrium, so that the largest possible such T is the largest integer that satisfies

$$\delta^T \geq \frac{\underline{\mathrm{P}}}{\overline{\mathrm{P}}} = \frac{Y - (\sum r_i)/n}{Y - (\sum \hat{r}_i)/n}.$$

For the purposes of this example, we shall choose δ so that equality holds in the inequality above (i.e., so that integer problems in the choice of T can be neglected):

$$\delta^T = \frac{\underline{\mathrm{P}}}{\overline{\mathrm{P}}} = \frac{Y - (\sum r_i)/n}{Y - (\sum \hat{r}_i)/n}. \tag{5}$$

Such an equilibrium would yield an average agent payoff of

$$a_T = (1 - \delta^T) r_n + \delta^T (\sum \hat{r}_i)/n$$

Recalling that $\hat{r}_i = (1-\delta) r_i + \delta r_1$, it is possible to show that $a_T > \sum_i r_i/n$ if (and only if)

$$\frac{1 - \delta^T}{\delta^{T+1}} > \frac{(\sum_i r_i)/n - r_1}{r_n - (\sum_i r_i)/n}. \tag{6}$$

On the other hand, (5) and the fact that $(\sum r_i)/n - (\sum \hat{r}_i)/n = \delta[(\sum r_i)/n - r_1]$ together imply that

$$\frac{1 - \delta^T}{\delta^{T+1}} = \frac{(\sum r_i)/n - r_1}{Y - (\sum r_i)/n}$$

Combining this equality with (6), the required condition becomes

$$r_n - (\sum r_i)/n > Y - (\sum r_i)/n,$$

which is clearly the case when $Y < r_n$.

The example shows that the question of average agent payoffs cannot be resolved by simply studying the standard-offer equilibrium. Indeed, equilibria with delay appear to yield higher agent payoffs, and what is more, we've seen that as all agents approach infinite patience, equilibrium delay can grow without bound. Does this mean that agent payoffs approach r_n (after discount and normalization)?

Let us, then, turn to a direct examination of agent average payoffs. Clearly, the answer to the question raised in the previous paragraph must turn on how quickly the delay approaches infinity as the discount factor tends to one. This is because an increase in the discount factor

[14] A sufficient condition for there to be no delay in equilibrium is $\frac{\delta^2}{1-\delta} \geq \frac{\sum_{i=1}^n \pi(r_i)}{\sum_{i=1}^n [\pi(r_1) - \pi(r_i)]}$.

forces agent payoffs to depend more sensitively on the far future (when the agents will finally be contracted by the principal). At the same time, this far future is growing ever more distant, by Proposition 3. In addition, there are other concerns. Proposition 3 only discussed the initial delay, but there may be equilibria involving various *sets* of delays, interspersed with accepted offers. Proposition 4 tells us, however, that all these effects can be brought together.

Proposition **4.** *There exists $\eta > 0$ such that the supremum of average agent payoffs over all equilibria and all discount factors cannot exceed $r_n - \eta$.*

This proposition establishes conclusively that, despite the fact that delays can become arbitrarily large (and are indeed unbounded in the discount factor), normalized average payoffs for the agent stay below and bounded away from the "fully free" payoff, which is r_n. The principal's overtures can be resisted to some extent, but not fully.

The proof of Proposition 4 employs a recursive argument. Along any equilibrium, whenever there are m free agents left at any stage, Proposition 3 provides a bound on the maximal delay permissible at that stage. Moreover, when some agents are contracted after the delay, there are bounds on what they can receive from the principal. The proof proceeds by taking a simple average of all these (delayed) payoffs, and then completes the argument by noting that while the delay T may be going to infinity as δ goes to one, Proposition 3 pins down the *combined* term δ^T. This permits us to conclude that the present value of the bounds are uniform in the discount factor.[15]

We end this section with a comment on contracts that last only for a finite duration. One may wonder if our results are robust to such a variation on the model. To be sure, new equilibria might appear, but the bounds that we have identified remain unchanged. The reasoning is straightforward, so we only provide a brief outline. The first step is to to note that the standard **r**-offer is immune to agents leaving a contract, even if they can do so (once again, an argument based on iterated deletion of dominated strategies will apply). It follows that the standard-offer equilibrium continues to remain an equilibrium. Given this, our main source of agent punishment remains unaffected: it is possible to deter a group of agents who would leave the principal by a standard-offer equilibrium with the lowest offer going to the group member who had the highest offer to start with. In particular, the low-offer equilibrium is unaffected by the consideration of contracts with finite lifetimes. So is the fact that this is the best equilibrium available to the principal. The main propositions now go through just as before.

6. Conclusion

A principal interacts with several agents by offering them contracts. We assume that all contracts are bilateral, and that the principal's payoff from a package of contracts is just the sum of payoffs from the individual contracts. Our specification of a contract is reduced-form: it stipulates a a net payoff pair to the principal and agent. The crucial assumption of this paper is that the outside-option payoffs of the agents depend positively on how many uncontracted or "free" agents there are. Indeed, such positive externalities imply that the agents are better off not having the principal around in the first place.

To be sure, the principal can still make substantial profits by relying on coordination failure among the agents. However, we explicitly rule out problems of coordination. This paper shows, nevertheless, that in a dynamic framework, agents must "eventually" succumb to the contracts offered by the principal — and often at inferior terms.

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[15]This final step appears to rely on the fact that the agents and the principal have the same discount factor. It remains to be seen how this result would look for the case of different discount factors.

This result is obtained in two distinct versions of the model. In the first specification, the principal cannot return to an agent who has earlier refused him. In the second specification, the principal can return to an agent as often as he wishes. In both these variants, time plays an explicit role in the description of the equilibria of interest. In variant 1, there is a unique perfect equilibrium, in which contract provision is divided into two phases. In the first phase, *simultaneous* and relatively attractive offers are made to a number of agents, though the offers must generally vary among the recipients. In the second, "exploitative" phase, offers are made (and accepted)*sequentially*, and their values are "discontinuously" lower than those of the first phase. In fact, the payoffs received by agents in this phase are close to the very lowest of all the outside options. So, in this model, a community of agents is invaded in two phases: some agents are bought, the rest are "exploited", in the sense that at the time of their contract, their outside option is strictly higher than what they receive.

In the second version, there is a multiplicity of equilibria. In some of these outcomes every agent succumbs to the principal immediately, though with different payoff consequences depending on the equilibrium in question. But there are other equilibria in which the agents have the power to force delay. Nevertheless, we show that they cannot hold the principal off forever; in every perfect equilibrium, agents *must* succumb in finite time. Indeed, even if the delay becomes unboundedly large as the the discount factor approaches one, our final result shows that the payoff of the agents must stay below *and bounded away from* the fully free reservation payoff.

These results have implications for the way we think about markets and equilibrium in the context of externalities. They also allow us to define a notion of "exploitation" which may be useful, at least in certain development contexts. Development economists have long noted that while informal contractual arrangements may be extremely unequal, it is hard to think of them as being exploitative as long as outside opportunities are respected. In this paper, outside opportunities *are* respected but they are endogenous, and are affected by the principal's past dealings. It is in this sense — in the deliberate altering of outside options — that the principal's actions may be viewed as "exploitative".[16]

7. Proofs

7.1. **The Single-Offer Model.**

In this section, we look at the case in which offers to a particular agent cannot be made more than once.

Proof of Proposition 1. First we choose $\hat{\delta}$. Pick any $\epsilon \in (0, \min_i\{r_{i+1} - r_i\})$. Now choose $\hat{\delta} \in (0,1)$ such that for all $\delta \in (\hat{\delta}, 1)$,

$$(1-\delta^n)r_n + \delta^n r_i \le r_i + \epsilon \tag{7}$$

and

$$(\delta^{n-k} + \cdots + \delta^n)\pi(r_i+\epsilon) > \sum_{j=i}^{i+k} \pi(r_j) \text{ for any } k \in \{1, \ldots, n-1\}, \tag{8}$$

for all $i = 1, \ldots, n-1$. Because $r_{i+1} > r_i$ and $\pi(r)$ is strictly decreasing, it is always possible to choose $\hat{\delta}$ such that both these requirements are met.

Throughout this proof, we assume $\delta \ge \hat{\delta}$.

By a *minigame* we will refer to a collection $(m, \mathbf{z})$, where $1 \le m \le n$, and $\mathbf{z} = (z_1, \ldots, z_m)$ is a vector of reservation payoffs for the m agents arranged in increasing order. Throughout, a minigame will have no more than n agents, and $\mathbf{z}$ will always be drawn from a "connected string" of the r_i's in the original game. To illustrate, $(z_1, \ldots, z_m)$ could be $(r_1, \ldots, r_m)$, or $(r_3, \ldots, r_{m+2})$.

[16] The work of Basu (1986) is related to this view, though the channels he studies are different.

Imagine that one agent accepts an offer at this minigame. This induces a fresh minigame $(m-1, \mathbf{z})$, where it is understood that $\mathbf{z}$ now refers to the vector $(z_1, \ldots, z_{m-1})$. We then say that $(m, \mathbf{z})$ *positively induces* $(m-1, \mathbf{z})$.

Now imagine that a single agent refuses an offer (and only one offer is made) at $(m, \mathbf{z})$. This induces a fresh minigame $(m-1, \mathbf{z}') = (m-1, \psi(\mathbf{z}))$, where it is understood that $\mathbf{z}' = \psi(\mathbf{z})$ now refers to the vector $(z_2, \ldots, z_m)$. We then say that $(m, \mathbf{z})$ *negatively induces* $(m-1, \psi(\mathbf{z}))$.

The proof proceeds by induction on the number of agents in any minigame. We shall suppose that the properties listed below hold for all stages of the form $(m, \mathbf{z})$, where $2 \leq m \leq M$ and $\mathbf{z}$ is some arbitrary vector of reservation payoffs (but a substring of $\mathbf{r}$ as discussed). We shall then establish these properties for all stages of the form $(M+1, \mathbf{z})$.

Induction Hypothesis.

[A] For all stages $(m, \mathbf{z})$ with $1 \leq m \leq M$, there is a unique equilibrium. Either no acceptable offers are made, or all agents accept offers in the equilibrium, and the principal makes nonnegative profits.

Before proceeding further, some definitions. Let $P(m, \mathbf{z})$ denote the principal's profit at any such minigame $(m, \mathbf{z})$. For $m \geq 2$ but no bigger than $M+1$, say that a minigame $(m, \mathbf{z})$ is *pivotal* if it negatively induces the minigame $(m-1, \mathbf{z}')$, *and* the principal makes no acceptable offers in that minigame. Otherwise $(m, \mathbf{z})$ is *not pivotal*. (Note that a definition of pivotality is included for stages of the form $(M+1, \mathbf{z})$.)

We now continue with the description of the induction hypothesis.

[B] If for $2 \leq m \leq M$, $(m, \mathbf{z})$ is a pivotal minigame, then look at the *minimal* $k \leq m$ such that $(k, \mathbf{z})$ is pivotal.[17] Then *either* the principal makes no acceptable offers at that minigame, *or* — if the principal's payoff under the description that follows is nonnegative — the principal makes simultaneous offers to $m-k+1$ agents, and begins the play of the minigame $(k-1, \mathbf{z})$ at the same time. The simultaneous offers to the $m-k+1$ agents — call them agents $k, \ldots, m$ — satisfy the property that agent j (in this group) receives the offer z_j.

[C] If for $2 \leq m \leq M$, $(m, \mathbf{z})$ is not a pivotal minigame, the principal makes a single offer to each agent, one period at a time, which are all accepted. Each agent's payoff lies in the range $[z_1, z_1 + \epsilon]$, where ϵ was chosen at the start of this proof (see (7) and (8)).

The following lemmas will be needed.

Lemma **1.** *Suppose that* $2 \leq m \leq M$*, and that [A] of the induction hypothesis holds.* [1] *If the principal makes no acceptable offers during the minigame* $(m, \mathbf{z})$*, then at its negatively induced minigame* $(m-1, \psi(\mathbf{z}))$*, the principal makes no acceptable offers as well.* [2] *If the principal makes equilibrium offers at the minigame* $(m, \mathbf{z})$*, then at its positively induced minigame* $(m-1, \mathbf{z})$*, he does so as well.*

Proof.

[1] Suppose not, so that the principal makes acceptable equilibrium offers at the minigame $(m, \psi(\mathbf{z}))$. Consider the strategy followed by the principal at this minigame, and follow exactly this strategy for the minigame $(m, \mathbf{z})$, ignoring one of the agents completely. It must be the case that all $m-1$ agents behave exactly as they did in the minigame $(m-1, \psi(\mathbf{z}))$. By [A] and our convention that offers are made when profits are nonnegative, all agents are thereby contracted. Finally, offer the ignored agent z_1; he will accept. The principal's total return from this feasible strategy is therefore $P(m-1, \mathbf{z}') + \delta^T \pi(z_1) > 0$, where T is the time it takes to contract the $m-1$ agents. [This is because $P(m-1, \mathbf{z}') \geq 0$ and so $\pi(z_1) > 0$.] But $P(m, \mathbf{z}) \geq P(m-1, \mathbf{z}') + \delta^T \pi(z_1)$, which implies that $P(m, \mathbf{z}) > 0$, a contradiction.

[17] Recall that $\mathbf{z}$ is now to be interpreted as the old vector of reservation payoffs up to the first k terms.

[2] Consider the strategy followed by the principal at $(m, \mathbf{z})$, and follow exactly this strategy for the minigame $(m-1, \mathbf{z})$, simply deleting the highest offer made. It is optimal for the $m-1$ agents to stick to exactly the same strategy that they used before. Therefore this strategy yields the principal a possible payoff of $P(m, \mathbf{z}) - \delta^T \pi(z^*)$, where z^* was the highest offer made and T the date at which it was made. Notice that

$$P(m-1, \mathbf{z}) \geq P(m, \mathbf{z}) - \delta^T \pi(z^*). \tag{9}$$

Now the right-hand side of (9) is clearly nonnegative if $\pi(z^*) \leq 0$. But it is also nonnegative when $\pi(z^*) \geq 0$, because the amount $\delta^T \pi(z^*)$ is already included in $P(m, \mathbf{z})$, and the remainder, consisting of lower offers to the agents, must yield nonnegative payoff to the principal. ∎

Lemma **2.** *Assume* [A]*, and let* $2 \leq m \leq M$*. Then if* $(m+1, \mathbf{z})$ *is not pivotal, its positively induced minigame* $(m, \mathbf{z})$ *cannot be pivotal either.*

Proof. If $(m+1, \mathbf{z})$ is not pivotal, this means that its negatively induced minigame $(m, \psi(\mathbf{z})$ has the principal making equilibrium offers. By [2] of Lemma 1, the positively induced minigame from $(m, \psi(\mathbf{z}))$, which is $(m-1, \psi(\mathbf{z}))$, has the principal making equilibrium offers as well. But $(m-1, \psi(\mathbf{z}))$ is also the negatively induced minigame of $(m, \mathbf{z})$. This means that $(m, \mathbf{z})$ cannot be pivotal. ∎

Proof of the Induction Step. Our goal is to establish [A]–[C] for minigames of the form $(M+1, \mathbf{z})$. Suppose, first, that $(M+1, \mathbf{z})$ is a pivotal minigame. We claim that if any acceptable offers are made at all, then at least *one* agent must be offered z_{M+1} or more. To see this, let s agents be made offers at the first date. If no agent is offered z_{M+1} or more, their equilibrium strategy is to reject. For this will induce the minigame $(M+1-s, \psi^s(\mathbf{z}))$, where ψ^s is just the s-fold composition of ψ. Remember that $(M+1, \mathbf{z})$ is pivotal, so that in $(M, \psi(\mathbf{z}))$ no acceptable offers can be made. By applying Lemma 1, part [1], repeatedly, we must conclude that no acceptable offers can be made at the minigame $(M+1-s, \psi^s(\mathbf{z}))$. Because two or more offers cannot be made, this means that all agents enjoy z_{M+1}, their highest possible payoff. But now we have shown that no acceptable offers are made at all, which is a contradiction.

So the claim is true: if any acceptable offers are made, then at least *one* agent must be offered z_{M+1} or more. It is obvious that an offer of *exactly* z_{M+1} need be made in equilibrium. Suppose this offer is indeed made. Notice that there is positive profit to be made from the remaining M agents.[18] By discounting, the minigame $(M, \mathbf{z})$ must be started as soon as possible (now!).

Now look at this minigame $(M, \mathbf{z})$, to which the induction hypothesis applies. If it is not pivotal, then by Lemma 2, no positively induced minigame of it can be pivotal as well. In this case the minimal pivotal k such that $(k, \mathbf{z})$ is pivotal is exactly $M+1$, and [A] and [B] have been established.[19]

If, on the other hand, the minigame $(M, \mathbf{z})$ is pivotal, then, too, [A] and [B] have been established, because we have shown that the principal must immediately start this minigame along with the offer of z_{M+1} to the first agent.

Now we establish [A] and [C] in the case where $(M+1, \mathbf{z})$ is not a pivotal minigame. First suppose that the principal makes a single offer, which is refused. By nonpivotality, all agents will be contracted in the negatively induced minigame $(M, \psi(\mathbf{z}))$. The payoff to the single agent who refused, therefore, *cannot* exceed

$$(1-\delta^n)z_n + \delta^n z_1 \leq (1-\delta^n)r_n + \delta^n z_1,$$

[18] If $\pi(z_{M+1}) \geq 0$ this is certainly true. But if $\pi(z_{M+1}) < 0$ this is also true, because no remaining agent will be offered more than z_M, which is strictly less than z_{M+1}.

[19] Note: the uniqueness makes use of our convention that indifference in all cases results in positive action, whether in the making or in the acceptance of offers.

where n, it will be recalled, is the grand total of all agents. Using (7) and remembering that $z_1 = r_i$ for some i, we must conclude that our single agent must accept *some* offer not exceeding $z_1 + \epsilon$. So the principal, if he so wishes, can generate a payoff of at least $\pi(z_1+\epsilon) + \delta P(M, \mathbf{z})$. But by Lemma 2, if $(M+1, \mathbf{z})$ is not pivotal, its positively induced minigame $(M, \mathbf{z})$ cannot be pivotal either. By the induction hypothesis applied to this minigame (see part [C]), we may conclude that the principal makes a single accepted offer to each agent, one period at a time, and that each agent's offer lies in the range $[z_1, z_1+\epsilon]$. So the principal's payoff at the minigame $(M+1, \mathbf{z})$ is bounded below by

(10) $$\pi(z_1+\epsilon)(1+\delta+\cdots+\delta^{M+1}).$$

What are the principal's other alternatives? He could attempt, now, to make an offer to some set of agents instead, say of size $s > 1$. If $s' \leq s$ of these agents refuse the offers, they would be assured of a long-term payoff of at least z'_s. Using the agent coordination criterion, it follows that to get these s agents to accept, the principal would therefore have to offer at least the vector $(z_1, \ldots, z_s)$. The minigame that remains is just $(M+1-s, \mathbf{z})$, which continues to be nonpivotal by Lemma 2. By applying the induction hypothesis, the additional payoff to the principal here is

$$\pi(z_1+\epsilon)(1+\delta+\cdots+\delta^{M+1-s}).$$

so the *total* payoff under this alternative is bounded above by

(11) $$\sum_{j=1}^{s} \pi(z_j) + \pi(z_1+\epsilon)(1+\delta+\cdots+\delta^{M+1-s}).$$

Compare (10) and (11). The former is larger if

$$\pi(z_1+\epsilon)(\delta^{M+2-s}+\cdots+\delta^{M+1}) > \sum_{j=1}^{s} \pi(z_j).$$

Recalling that $\mathbf{z}$ is always drawn as a "substring" of $\mathbf{r}$, and invoking (8) (by setting $k = s-1$ and noting that $n \geq M+1$), we see that this inequality must be true. So the alternative is worse.

Finally, there is an obvious loss to the principal in making offers to agents and deliberately have them refuse. So all options are exhausted, and [A] and [C] of the induction step are established.

Starting Point. All that is left to do is to establish the validity of the induction step when $m = 2$, and for any vector (z_1, z_2) (substrings of $\mathbf{r}$, of course). Using (7) and (8) once again, this is a matter of simple computation.

The last step needed to complete the proof of the proposition consists in proving that $m(\delta)$ is non-decreasing in δ. This is equivalent to proving the following lemma.

Lemma **3.** *For any minigame $(m, \mathbf{z})$, if $(m, \mathbf{z})$ is non-pivotal under δ then it is non-pivotal under $\delta' > \delta$.*

Proof. Let's use an inductive argument. It is straightforward to check that if $(2, \mathbf{z})$ is non-pivotal under δ, then it is non-pivotal under any $\delta' > \delta$.

Assuming the lemma is true for all stages of the form $(m, \mathbf{z})$ for $2 \leq m \leq M$ and all $\mathbf{z}$, we need to prove that this property holds for all stages $(M+1, \mathbf{z})$. Suppose not, so that $(M+1, \mathbf{z})$ is non-pivotal under δ but pivotal under $\delta' > \delta$. That is $P_\delta(M, \psi(\mathbf{z})) \geq 0$ $P_{\delta'}(M, \psi(\mathbf{z})) < 0$, where the subscript indicates the dependence on the discount rate. Denote as $k^*(\delta)$ the *minimal* $k \leq M$ such that $(k, \psi(\mathbf{z}))$ is pivotal given δ. Since Lemma 3 is true for all $2 \leq m \leq M$ and $\mathbf{z}$, it must be that $k^*(\delta) < k^*(\delta')$. Using this and using the first part of the proposition to compute the principal profit it is clear that $P_{\delta'}(M, \psi(\mathbf{z})) \geq P_\delta(M, \psi(\mathbf{z}))$, which is a contradiction.[20] ■

Now the proof of the proposition is complete. ■

7.2. **The Model With Multiple Offers.** We now turn to the case in which the principal can repeatedly approach agents.

Proof of Proposition 2. It is obvious that in no equilibrium can the principal's payoff fall below the bound described in (2). For suppose, on the contrary, that this were indeed the case at some equilibrium; then the principal could make an offer $\mathbf{c}$ with each component c_i exceeding the corresponding component r_i of the standard offer by a $\epsilon_i > 0$. Clearly agent n cannot resist such an offer (for in no equilibrium can she get strictly more than r_n). But then nor can agent $n-1$ resist her component of the offer, and by an iterative argument all agents must accept the offer $\mathbf{c}$. If the ϵ_i's are small enough, this leads to a contradiction.

So it must be that the principal can guarantee at least $\sum_{i=1}^n \pi(r_i)$ in any equilibrium. It remains to show that there are equilibria in which the payoff of $\sum_{i=1}^n \pi(r_i)$ is attained.

Notice that an agent must reject any offer of strictly less than r_1. We can, without loss of generality, treat a no-offer as equivalent to an offer of less than r_1. In what follows, this will eases notation: we can think, when convenient, of the principal as always making an offer to every free agent in every subgame.

Our proof proceeds by describing three kinds of phases.

Normal Phase. If there are m free agents, the principal makes them the standard offer. All agents must accept.

The following deviations from the normal phase are possible:

[A] Some agents reject. In that case, "reverse-name" the rejectors so that the rejector with the highest label is now agent 1, the rejecting agent with the next-highest label is now 2, and so on. Restart with normal phase with these free agents.

[B] The principal deviates with a different offer $\mathbf{c}$, where we arrange the components so that $c_1 \leq c_2 \leq \cdots \leq c_n$. Rename the agents so that component i is given to agent i, and proceed to the *evaluation phase*; see below.

$\mathbf{c}$-Evaluation Phase. Given an offer of $\mathbf{c}$ made to n agents, this phase proceeds as follows. Define K to be the largest integer $k \in \{1, ...n\}$ such that $c_i < (1-\delta)r_k + \delta r_i$ for all $1 \leq i \leq k$, and if this condition cannot be satisfied for any $k \geq 1$, set $K = 0$. All the agents between 1 and K *must* reject their offers. If, in addition,

$$\sum_{i=K+1}^{n} \pi(c_i) \leq \sum_{i=K+1}^{n} \pi(r_i), \tag{12}$$

the agents with indices larger than K must accept the offer. Proceed to the normal phase with the K agents, if any. If, on the other hand,

$$\sum_{i=K+1}^{n} \pi(c_i) > \sum_{i=K+1}^{n} \pi(r_i), \tag{13}$$

[20] Note that the first part of the proposition applies since, if ϵ satisfies (7) and (8) under δ then it satisfies them under δ' too.

then define L as the smallest integer $\ell \in \{1, ...n-1\}$ such that $c_i > r_i$ for all $i > \ell$, and if this condition cannot be satisfied for any $\ell \geq 1$, set $L = n$.[21] *All* agents 1 to L must reject the offer while agents with indices larger than L must accept the offer. Let $\widetilde{\mathbf{c}} \equiv \{c_1, ...c_L\}$. Now proceed to the $\widetilde{\mathbf{c}}$ *-punishment phase*; see below.

The following deviations from the evaluation phase are possible:

[A] Some agents accept when asked to reject; and / or some reject when asked to accept. If there are $m \geq 1$ rejectors following these deviations, rename agents from 1 to m respecting the original order of their names; go to the normal phase.

$\widetilde{\mathbf{c}}$-Punishment Phase. Recall that an offer of $\widetilde{\mathbf{c}} \in \mathbb{R}^L$ is on the table and that agents are named as in the evaluation phase. Define $T \geq 0$ to be the largest integer such that

$$\delta^{T+1}\left[\sum_{i=1}^{K}\pi(r_i) + \sum_{i=K+1}^{L}\pi(c_i)\right] \leq \sum_{i=1}^{L}\pi(r_i), \tag{14}$$

while at the same time,

$$\delta^{T}\left[\sum_{i=1}^{K}\pi(r_i) + \sum_{i=K+1}^{L}\pi(c_i)\right] \geq \sum_{i=1}^{L}\pi(r_i). \tag{15}$$

The principal must now wait for T periods, making no offer at all (or offer of strictly less than r_1). Following the T periods he makes the offer $(r_1, \ldots, r_K, c_{K+1}, \ldots, c_L)$, which is to be unanimously accepted.

The following deviations from the $\widetilde{\mathbf{c}}$-punishment phase are possible:

[A] If any agents reject the final offer, “reverse-name" the rejectors so that the rejector with the highest label is now agent 1, the rejecting agent with the next-highest label is now 2, and so on. Proceed to the normal phase with these rejecting agents as the free agents.

[B] If the principal deviates in any way, by making an offer of $\mathbf{c}' \in \mathbb{R}^L$, start a $\mathbf{c}'$-evaluation phase.

We now prove that this description constitutes an equilibrium which is, moreover, immune to coordinated deviations by the agents.

Begin with deviations in the normal phase. Suppose that a group of $s \geq 1$ agents reject their components of the standard offer; let j be the agent with the highest rank in that group. Given the prescription of play, he will be offered (and will accept) r_1 next period. His overall return is, therefore,

$$(1-\delta)r_s + \delta r_1,$$

where is the number of rejectors. Because $s \leq j$, we see that $r_s \leq r_s$. Of course, $r_1 \leq r_s$. Consequently, the agent’s payoff is no higher than r_s, which is what he is offered to start with. Therefore no agent deviation, coordinated or otherwise, can improve the well-being of every deviating agent.

Now suppose that the principal deviates with offer $\mathbf{c}$. The prescription then takes us to the evaluation phase. If (12) applies, then agents with indices larger than K accept, and the remaining agents accept in the normal phase one period after that. Consequently, the principal’s payoff is given by

$$\sum_{i=K+1}^{n}\pi(c_i) + \delta\sum_{i=1}^{K}\pi(r_i) \quad\leq\quad \sum_{i=K+1}^{n}\pi(c_i) + \sum_{i=1}^{K}\pi(r_i) \quad\leq\quad \sum_{i=1}^{n}\pi(r_i),$$

[21] Note that $L > K$ for otherwise (13) is contradicted.

where the first inequality follows from (A.1) and the fact that π is decreasing (so that $\sum_{i=1}^{K} \pi(r_i) \geq 0$), and the second inequality is a direct consequence of (12). So this deviation is not profitable.

If (13) applies in the evaluation phase, then only agents with indices larger than L, if any, accept. The remaining agents must reject and we proceed thereafter with these agents to a further wait of T periods (where T is defined by (14) and (15)), followed by an offer of $(r_1, \ldots, r_K, c_{K+1}, \ldots, c_L)$, which they accept. Therefore, the principal's payoff is given by

$$\delta^{T+1}\left[\sum_{i=1}^{K} \pi(r_i) + \sum_{i=K+1}^{L} \pi(c_i)\right] + \sum_{i=L+1}^{n} \pi(c_i) \leq \sum_{i=1}^{N} \pi(r_i),$$

by (14) and the definition of L. Once again, the deviation is not profitable. This completes our verification in the normal phase.

Turn now to the evaluation phase. Suppose, first, that some agent accepts an offer when he has been asked to reject. Let i and j be the deviating agent with the *lowest* and *largest* index respectively. First, consider the case in which $i \leq K$. His return from the deviation is c_i. In contrast, if all deviators were to stick to the prescribed path, agent i would receive *at least*

$$(1-\delta)r_K + \delta r_i$$

(it would be even more if (13) were to hold). By the definition of K, $c_i < (1-\delta)r_K + \delta r_i$. It follows that there is always *some* agent in the set of K agents who would not be better off by participating in any deviation, coordinated or unilateral.

Next, consider the case in which (13) holds and an agent i with $K < i \leq L$ accepts the offer. Along the prescribed play, agent j's payoff is

$$(1-\delta^{T+1})r_L + \delta^T c_j.$$

which is at least as much as c_j, his return from the deviation. Indeed, otherwise, if $r_L < c_j$, then $r_L < c_L$ which contradicts the definition of L.

Now, assume that a set S of m agents reject offers when they were supposed to accept them. Pick the deviating agent with the *largest* index, say j. Note that, if $j > L$, this is clearly not a profitable deviation since $c_j > r_j$. So we need *only* consider deviations with $j \leq L$, and therefore situations in which (12) applies. Let's index the agents who rejects the offer from 1 to m respecting the original order of their names, and let $\mu(i) \in \{1, ...m\}$ be the new index for an agent with original index i. By deviating, agent $i \in S$ receives $(1-\delta)r_m + \delta r_{\mu(i)}$. Note that for at least one $i \in S$, it must be that

$$(1-\delta)r_m + \delta r_{\mu(i)} \quad \geq \quad c_i$$

otherwise this would contradict the fact that $i > K$. It follows that there is always *some* agent in the set of S agents who would not be better off by participating in this deviation.

Finally, consider a **c**-punishment phase. Suppose that a group of agents rejects the final offer. Let k be the highest index in that group. Recalling the subsequent prescription, this individual receives, by rejecting, *no more than* $(1-\delta)r_k + \delta r_1$ (and may receive less if some agent below k does accept). If $k \leq K$, this agent receives r_k in the event of no deviation, which cannot be lower. If $k > K$, he is supposed to receive c_k in the event that no one deviates. We claim that

$$c_k \geq (1-\delta)r_k + \delta r_1.$$

Suppose not, then $c_k < (1-\delta)r_k + \delta r_1 \leq (1-\delta)r_k + \delta r_i$ for all $i \leq k$. This contradicts the fact that $k > K$, so the claim is proved. We have therefore shown that there is always some participating agent who is not made better off by a deviation, coordinated or not.

Now suppose the principal deviates at any stage of the punishment phase with M periods of waiting left, where $0 \leq M \leq T$. Suppose she offers $\mathbf{c}'$. Now the evaluation phase is invoked.

Denote by K' the corresponding construction of K for $\mathbf{c}'$. If (12) holds (for $\mathbf{c}'$), then the principal's return is

$$\begin{aligned}
\sum_{i=K'+1}^{L} \pi(c_i') + \delta \sum_{i=1}^{K'} \pi(r_i) &\leq \sum_{i=K'+1}^{L} \pi(c_i') + \sum_{i=1}^{K'} \pi(r_i) \\
&\leq \sum_{i=1}^{L} \pi(r_i) \\
&\leq \delta^T \left[\sum_{i=1}^{K} \pi(r_i) + \sum_{i=K+1}^{L} \pi(c_i) \right] \\
&\leq \delta^M \left[\sum_{i=1}^{K} \pi(r_i) + \sum_{i=K+1}^{L} \pi(c_i) \right], \qquad (16)
\end{aligned}$$

where the first inequality follows from (A.1), as before, the second from (12), the third from (15), and the last from the fact that $M \leq T$.[22] Because the final expression is the payoff from not deviating, we see there is no profitable deviation in this case.

Alternatively, (13) holds for $\mathbf{c}'$. Denote by L' the corresponding construction of L for $\mathbf{c}'$. Then agents $L'+1$ to L accept the offer while all other offers must be rejected. Let T' be the *further* wait time as a new punishment phase starts up. Applying the prescription, the principal's payoff is

$$\begin{aligned}
\sum_{i=L'+1}^{L} \pi(c_i') + \delta^{T'+1} \left[\sum_{i=1}^{K'} \pi(r_i) + \sum_{i=K'+1}^{L'} \pi(c_i') \right] &\leq \sum_{i=1}^{L} \pi(r_i) \\
&\leq \delta^T \left[\sum_{i=1}^{K} \pi(r_i) + \sum_{i=K+1}^{L} \pi(c_i) \right] \\
&\leq \delta^M \left[\sum_{i=1}^{K} \pi(r_i) + \sum_{i=K+1}^{L} \pi(c_i) \right],
\end{aligned}$$

where the first inequality follows from (14) applied to T' and the definition of L', and the remaining inequalities follow exactly the same way as in (16). Here, too, no deviation is profitable.

Thus far we have established the first part of the proposition; see (2). Now we turn to (3).

First, we show that an equilibrium exists in which the principal's payoff is $\overline{\mathrm{P}} = \sum_{i=1}^{n} \pi(\hat{r}_i)$. It suffices to show that an equilibrium exists in which the low offer is made and accepted. It is supported by two phases, the low-offer phase and the standard-offer phase, and the following prescriptions are in force:

[1] Begin with the low-offer phase. The low offer is made to all agents and should be accepted. If the principal conforms but there are any rejections, proceed to the standard-offer phase with a suitable renaming of agents (see below). If the principal makes a different offer, move to the standard-offer phase with no renaming of the free agents.

[2] In the low-offer phase, if m agents receive an offer $\mathbf{c}$, where we arrange the components so that $c_1 \leq c_2 \leq \ldots \leq c_m$, then let $j \in \{0, \ldots m\}$ be the lowest index such that $c_i > \hat{r}_i$ for all $i > j$ or 0 if no such index exists. All agents with index $i > j$, if any, accept the offer; all others reject.

[22] To perform this final step, we also need to reassure ourselves that the last expression is positive, but that is automatically guaranteed by the presence of the third expression in (16), and (A.1).

Move to the standard-offer phase with suitable renaming (see below) if some agent who is meant to accept rejects the offer.

[3] In any subgame where the standard-offer phase is called for, and renaming is required, identify the rejectors who were meant to accept their offers. "Reverse-name" the rejectors so that the rejector with the highest label is now agent 1, the rejecting agent with the next-highest label is now 2, and so on. If there are any other free agents, give them higher labels. If no renaming is required, ignore above instructions. Now play a standard-offer equilibrium with all these agents.

To examine whether this constitutes an equilibrium, consider agent strategies first. It suffices to consider only the low-offer phase. All agents have been made an offer. If k agents jointly reject, the subsequent payoff to the agent with the highest offer is r_k today (because there are exactly k free agents after the rejection), followed by r_1 in the next period (by virtue of item [3]). This means that such an agent would accept any offer that exceeds $\hat{r}_k = (1-\delta)r_k + \delta r_1$. Hence, by iterated deletion of dominated strategies the agents would all accept the offers $(\hat{r}_1, ...\hat{r}_n)$.

As for the principal, consider again the low-offer phase. Offering more than $(\hat{r}_1, ...\hat{r}_n)$ in any component is not a profitable deviation. Making acceptable offers to less than the full set of free agents just precipitates the standard offer equilibrium thereafter, which certainly makes him worse off. Finally, making unacceptable offers to a set of k agents again triggers the standard-offer phase and a continuation profit of $\underline{\mathrm{P}}(k)$ which is the lowest possible continuation profit on k agents. Clearly, the principal does not have incentive to deviate from his prescribed actions.

Finally, to see that this is the highest payoff that the principal could ever receive, it suffices to recall that r_1 is the worst possible continuation utility an agent can ever receive. Hence, to sign on any agent when n agents are free, the principal has to make at least one offer of $\hat{r}_n$. Given this, whether in this period or later, the principal cannot offer anything less than $\hat{r}_{n-1}$ in order to sign up another agent. By repeating the argument for all remaining agents, and recalling that delays are costly, we see that $\overline{\mathrm{P}}(n) = \sum_{i=1}^{n} \pi(\hat{r}_i)$. ■

Proof of Proposition 3. To support the longest possible delay, the principal must receive her *highest* possible payoff ($\overline{\mathrm{P}}(n)$) at the end of the delay, while she must receive the *worst* possible payoff if she deviates. Hence, the T-equilibrium that exhibits the longest delay is supported by the following strategies:

[1] The principal does not extend any offer for periods $0, \ldots, T-1$, where T is the largest integer t such that $\delta^t \overline{\mathrm{P}}(n) \geq \underline{\mathrm{P}}(n)$.

[2] If there are no deviations from the prescription in [1], the low-offer equilibrium is implemented at date T.

[3] If the principal deviates at any time during [1], a standard-offer equilibrium is implemented immediately thereafter.

It is easy to see that specifications [1]–[3] constitute an equilibrium. The proposition now follows from the definition of T. ■

Proof of Proposition 4. Proposition 3 tells us that if there are m free agents left in the game, the longest delay that can be endured is given by the largest integer $T(m)$ satisfying the inequality

$$\delta^{T(m)} \geq \underline{\mathrm{P}}(m)/\overline{\mathrm{P}}(m). \tag{17}$$

Moreover, by an iterative argument, it is obvious that if a package $\mathbf{c}$ of k offers are made and accepted at the end of this period, where $k \leq m$, then

$$c_i \leq r_{m-k+i}, \tag{18}$$

where the c_i's have been arranged in nondecreasing order.

With these two observations in mind, consider any equilibrium, in which at dates $\tau_1, \tau_2, \ldots, \tau_S$, accepted offers are made. Let n_j be the number of such accepted offers at date τ_j; then, because no equilibrium permits infinite delay, we know that $\sum_{j=1}^{S} n_j = n$. Define $m_1 \equiv n$ and recursively, $m_{j+1} \equiv m_j - n_j$ for $j = 1, \ldots, S-1$; this is then the number of free agents left at the start of "stage j".

Set $\tau_0 \equiv 0$. Notice that for every stage $j \geq 1$, that $\tau_j - \tau_{j-1} \leq T(m_j)$, so that by repeated use of (17), we may conclude that

$$\delta^{\tau_j - \tau_{j-1}} \geq \delta^{T(m_j)} \geq \underline{\mathrm{P}}(m_j)/\overline{\mathrm{P}}(m_j) \tag{19}$$

for all $j \geq 1$. Expanding (19), we may conclude that for all j,

$$\delta^{\tau_j} \geq \prod_{k=1}^{j} [\underline{\mathrm{P}}(m_k)/\overline{\mathrm{P}}(m_k)]. \tag{20}$$

Now denote by a^j the *average* equilibrium payoffs to agents who close a deal at stage j (date τ_j). Then, using (18), we must conclude that

$$a^j \leq (1 - \delta^{\tau_j}) r_n + \delta^{\tau_j} \frac{1}{n_j} \sum_{i=1}^{n_j} r_{m_j - n_j + i}, \tag{21}$$

so that the *overall* average$a$ — satisfies the inequality

$$\begin{aligned} a &\leq \frac{1}{n} \sum_{j=1}^{S} \left[(1 - \delta^{\tau_j}) r_n + \delta^{\tau_j} \frac{1}{n_j} \sum_{i=1}^{n_j} r_{m_j - n_j + i} \right] n_j \\ &= r_n - \frac{1}{n} \sum_{j=1}^{S} \delta^{\tau_j} \left[r_n - \frac{1}{n_j} \sum_{i=1}^{n_j} r_{m_j - n_j + i} \right] n_j \\ &\leq r_n - \frac{1}{n} \sum_{j=1}^{S} \prod_{k=1}^{j} [\underline{\mathrm{P}}(m_k)/\overline{\mathrm{P}}(m_k)] \left[r_n - \frac{1}{n_j} \sum_{i=1}^{n_j} r_{m_j - n_j + i} \right] n_j, \end{aligned} \tag{22}$$

where the last inequality invokes (20).

Now all that remains to be seen is that the expression

$$\frac{1}{n} \sum_{j=1}^{S} \prod_{k=1}^{j} [\underline{\mathrm{P}}(m_k)/\overline{\mathrm{P}}(m_k)] \left[r_n - \frac{1}{n_j} \sum_{i=1}^{n_j} r_{m_j - n_j + i} \right] n_j$$

is strictly positive everywhere and bounded away from zero uniformly in δ. The result follows. ■

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