# ASYMPTOTIC INFERENCE IN NONLINEAR COINTEGRATED TIME SERIES WITH FRACTIONALLY INTEGRATED AND POSSIBLY HEAVY TAILED ERRORS

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In this workshop we shall treat some aspects of asymptotic inference problems in the context of a nonlinear cointegrated time series model. These will consist of extensions and generalizations (and possibly some simplifications) of the recent work "Nonlinear regressions with integrated time series" by J.Y.Park and P.C.B.Phillips (*Econometrica*, 69, pp 117-161, 2001).

Specifically, we shall consider two-dimensional observations $(X_k, Y_k), k = 1, \ldots, n$, generated by the nonlinear model

$$\begin{aligned} Y_k &= f(X_k, \beta) + u_k, \\ X_k &= \gamma X_{k-1} + v_k \end{aligned}$$

where the errors $u_k$ and $v_k$ are stationary and fractionally integrated, that is

$$\begin{aligned} (1-L)^{\alpha_1} u_k &= \varepsilon_k \\ (1-L)^{\alpha_2} v_k &= \eta_k \end{aligned}$$

where $L$ is the back-shift operator and $(\varepsilon_k, \eta_k)$ are i.i.d. (or more generally an approximately stationary linear process with i.i.d. innovations such as a stationary ARMA process). In addition to the usual situation where the components of the preceding i.i.d. innovations will have finite variances, the situation where they will have heavy tailed distributions (that is, belonging to the domain of attraction of strictly stable laws with indexes of stability belonging to the interval $(0, 2]$) will also be considered. The parameters of the model are $\beta$, $\gamma$, as well as those associated with the error process $(u_k, v_k)$ such as $\alpha_1$ and $\alpha_2$. Here $\gamma$ will be assumed to be in an *unknown* vicinity of unity 1 (so that $X_k$ becomes a nonstationary integrated process).

When $\gamma = 1$ and $\alpha_1 = 0 = \alpha_2$, and when the i.i.d. innovations have finite variances, the above model reduces to that considered in Park and Phillips (2001). In this case the quantity $\sum_{k=1}^{n} f^2(X_k, \beta_0)$ serves the analogue of the Fisher information or the amount of information of the parameter $\beta$ when $\beta_0$ is the true parameter. An interesting and unusual feature isolated and its statistical implications analyzed in detail in this work is that $\frac{1}{\sqrt{n}}\sum_{k=1}^{n} f^2(X_k, \beta_0)$ converges in distribution to a proper random quantity known as the 'local time' of the Brownian motion, when $\int f^2(x, \beta_0)\,dx < \infty$.

The essence of this feature is the nonlinearity of the model in the regression variables $X_k$, as is seen in the special case $f(X_k, \beta) = \beta f(X_k)$ in which the model is linear in the parameter $\beta$ but nonlinear in the regressions $X_k$.

Despite this, it is shown there that the asymptotic distribution of for instance the Wald test statistic is the usual Chi-squared distribution. In other words, it is implicit in Park and Phillips (2001) that the approximate structure of the likelihood with respect to the parameter $\beta$ takes the form of what is known as the Locally Asymptotically Mixed Normal (LAMN) likelihood structure. It will in particular be seen that this structural feature remains true in the present general context also, independently of the approximating structures of the remaining parameters. (In place of the local time of the Brownian motion, one will now have the local time of the Fractional Brownian Motion or more generally what is known as the Linear Fractional Stable Motion. ) Specifically, it will be seen that the approximate structure of the likelihood decomposes into three mutually independent parts, one corresponding to the parameter $\beta$, the one corresponding to $\gamma$ and the remaining corresponding to $\alpha_1, \alpha_2$ and other parameters associated with the process $(\varepsilon_k, \eta_k)$.

Answers to the asymptotic inference problems will be deduced from these approximating structural features in the usual way. Asymptotic distributions of the usual estimators, such as Conditional Sum of Squares estimators and the approximate MLE, as well as those of the associated test statistics, will also be considered.

The main emphasize of the workshop in general will be on the underlying ideas and methods, and on the significance of the above indicated approximating structural features in the construction and in deducing the properties of the inference procedures. (Detailed statements and proofs and other technical details will be made available in the form of manuscripts.) In the first lecture we shall mostly deal with the specific model of Park and Phillips (2001) (specializing further to the case $f(X_k, \beta) = \beta f(X_k)$ whenever it helps) with the main aim of introducing various approximating likelihood structural features. The second lecture will deal with the present more general model. The results and conclusions will also be compared with those of the linear model, that is with the situation where $f(X_k, \beta) = \beta X_k$.