Structural Spurious Regressions and A Hausman-Wu-type Cointegration Test*

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Abstract

Economic models often imply that certain variables are cointegrated. However, tests often fail to reject the null hypothesis of no cointegration for these variables. One possible explanation of these test results is that the error is unit root nonstationary due to a nonstationary measurement error in one variable. For example, currency held by domestic economic agents for legitimate transactions is very hard to measure due to currency held by foreign residents and black market transactions. Therefore, money may be measured with a nonstationary error. If the money demand function is stable in the long-run, we have a cointegrating regression when money is measured with a stationary measurement error but have a spurious regression when money is measured with a nonstationary measurement error. We can still recover structural parameters under certain conditions for the nonstationary measurement error. This paper proposes two estimators based on asymptotic theory to estimate structural parameters with spurious regressions involving unit root nonstationary variables. This approach motivates a Hausman-Wu-type test for the null hypothesis of cointegration for dynamic Ordinary Least Squares estimation using one of our estimators for spurious regressions. We show that this test can be used to test for cointegration even when the spurious regression is not structural under the alternative hypothesis.

Keywords: Spurious regression, GLS correction method, Dynamic regression, Test for cointegration.

JEL Classification Numbers: C10, C15

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1 Introduction

Economic models often imply that certain variables are cointegrated. However, tests often fail to reject the null hypothesis of no cointegration for these variables. One possible explanation of these test results is that the error is unit root nonstationary due to a nonstationary measurement error in one variable. A nonstationary error in one variable leads to a situation in which no linear combination of the variables is cointegrated, and thus the stochastic error of a regression of these variables is unit-root nonstationary. In the unit root literature, when the stochastic error of a regression is unit root nonstationary, the regression is technically called a spurious regression. This is because the standard t test tends to be spuriously significant even when the regressor is statistically independent of the regressand in Ordinary Least Squares. Monte Carlo simulations have often been used to show that the spurious regression phenomenon occurs with regressions involving unit root nonstationary variables (see, e.g., Granger and Newbold (1974), Nelson and Kang (1981, 1983)). Asymptotic properties of estimators and test statistics for regression coefficients of these spurious regressions have been studied by Phillips (1986, 1998) and Durlauf and Phillips (1988) among others. For example, currency held by domestic economic agents for legitimate transactions is very hard to measure due to currency held by foreign residents and black market transactions. Therefore, money may be measured with a nonstationary error. As shown by Stock and Watson (1993) among others, if the money demand function is stable in the long-run, we have a cointegrating regression when all variables are measured without error. If the variables are measured with stationary measurement errors, we still have a cointegrating regression. However, if money is measured with a nonstationary measurement error, we have a spurious regression. We can still recover structural parameters under certain conditions. The crucial assumption is that the nonstationary measurement error is not cointegrated with the regressors.

The purpose of this paper is twofold. First, we propose a new approach to estimating structural parameters with spurious regressions. When structural parameters can be recovered from spurious regressions, we call these structural spurious regressions. Second, we propose a Hausman-Wu-type test for the null hypothesis of cointegration. This test is naturally motivated by the structural regression approach. We also show that this test can be used to test for cointegration even when the spurious regression is not structural under the alternative hypothesis.

Our structural spurious regression approach is based on the Generalized Least Squares (GLS) solution of the spurious regression problem analyzed by Ogaki and Choi (2001), who use an exact small sample analysis based on the conditional probability version of the Gauss-Markov Theorem. We develop asymptotic theory for two estimators motivated by the GLS correction: GLS corrected dynamic regression and FGLS corrected dynamic regression estimators. These estimators will be shown to be consistent and asymptotically normally distributed in spurious regressions. When the error term is in fact stationary, and hence the variables are cointegrated, the GLS corrected estimator is not efficient, but the FGLS corrected estimator is superconsistent as the OLS estimator. Hence the FGLS estimation is a robust procedure with respect to the error specifications. The FGLS corrected estimator is asymptotically equivalent to the GLS corrected estimator in spurious regressions and it is asymptotically equivalent to the OLS estimator in cointegrating regressions.

In some applications, it is hard to determine whether or not the error in the regression is stationary or unit root nonstationary because test results are inconclusive. In such applications, the FGLS corrected estimator is attractive because it is consistent under both situations as long as the method of the dynamic regression removes the endogeneity problem.

This approach naturally motivates a Hausman-Wu-type test for the null hypothesis of cointegration against the alternative hypothesis of no cointegration (or a spurious regression) in the dynamic OLS framework. We construct this test by noting that while both the dynamic OLS and GLS corrected dynamic regression estimators are consistent in cointegration estimation, the dynamic OLS estimator is more efficient[1]. On the other hand, when the regression is spurious only the GLS corrected dynamic regression estimator is consistent. Hence, we could do a cointegration test based on the specifications on the error. We show that under the null hypothesis of cointegration the test statistics have a usual χ^2 limit distribution, while under the alternative hypothesis of a spurious regression, the test statistic diverges.

In some applications, the assumption that the spurious regression is structural under the alternative hypothesis is not very attractive. If the violation of cointegration arises from reasons other than a nonstationary measurement error, it is hard to believe that the resulting spurious regression is structural. For this reason, we relax the assumption that the spurious regression is structural and show that the Hausman-Wu-type cointegration test statistic still diverges under the alternative hypothesis.

Dynamic OLS is often used in many applications for cointegration. However, few tests for cointegration have been developed for dynamic OLS with an exception of Shin's (1994) test. As in Phillips and Ouliaris (1990), the popular Augmented Dickey-Fuller (ADF) test for the null hypothesis of no cointegration is originally designed to be applied to the residual from static OLS rather than the residual from dynamic OLS. Because the OLS and dynamic OLS estimates are often substantially different, it is desirable to have a test for cointegration applied to dynamic OLS. Another aspect of our Hausman type test is that it is for the null hypothesis of cointegration. Ogaki and Park (1998) argued that it is desirable to test the null hypothesis of cointegration rather than that of no cointegration in many applications when economic models imply cointegration.

Using Monte Carlo experiments, we compare the finite sample performance of the Hausman-Wu-type test with the test proposed by Shin (1994), which is a locally best invariant test for the null of zero variance of a random walk component in the disturbances. The experiment results show that up to sample size 300 the Hausman-Wu-type test is superior in both size and power. When the sample size increases, the Shin's test is better in power, but it suffers from a serious oversize problem.

In some applications, it is appropriate to consider the possibility that measurement error is I(1) and is not cointegrated with the regressors. For these applications, the ADF test is applicable under the null hypothesis of a structural spurious regression as shown in Hu and Phillips (2005). For such applications, we recommend that both the ADF test and Hausman-Wu-type test be applied because it is not clear which null hypothesis is more appropriate.

We applied our estimation and testing methods to four applications. In the first two applications, we estimated unknown structural parameters. The purpose of the last two applications was to test for cointegration with the Hausman-Wu-type cointegration test. We do not assume that the spurious regression under the alternative hypothesis is structural for these two applications.

In the first application of estimating the money demand function, the results suggest that the endogeneity correction of the dynamic regression works with a moderately large number of leads and lags for the GLS corrected dynamic regression estimator. The GLS corrected dynamic regression estimates are very low with low orders of leads and lags and then increase to more plausible values as the order of leads and lags increases. Dynamic OLS estimates are close to the GLS corrected dynamic regression estimates for a large enough order of leads and lags, and we find little evidence against cointegration with the Hausman-Wu-type cointegration test. The FGLS corrected dynamic regression estimates are very close to the GLS corrected dynamic regression and the dynamic OLS estimates for large enough order of leads and lags.

[1] After completing the first draft, it has come to our attention that the Hausman-Wu-type test was originally proposed by Fernández-Macho and Mariel (1994) for the static OLS cointegrating regression with strict exogeneity and without any serial correlation. The test probably has not been popular because these assumptions are hard to justify in applications and because the test was not developed for dynamic regressions.

In the second application of the long-run implications of the consumption-leisure choice, we found strong evidence against cointegration when nondurables are used as the measure of consumption with the Hausman-Wu-type cointegration test. We also found some evidence against cointegration when nondurables plus services are used as the measure of consumption. We estimate the RRA coefficient to be about one with both GLS corrected and FGLS corrected dynamic regression estimators.

Hence, in these first two applications, the FGLS corrected dynamic regression estimator worked well in the sense that it yielded estimates that are close to those of the estimator that seems to be correctly specified. This is consistent with our simulation results in Section 2 that the small sample efficiency loss from using the FGLS corrected dynamic regression estimator is negligible for reasonable sample sizes. Therefore, we recommend the robust FGLS corrected dynamic regression estimator when the researcher is unsure about whether or not the regression error is I(0) or I(1). This is important because it is difficult to detect a small random walk component in the error term when the error is actually I(1) and to detect a small deviation from a unit root when the dominant autoregressive root is very close to one when the error is actually I(0).

In the third application, we applied the Hausman-Wu-type cointegration test to log real output of pairs of countries to study output convergence across national economies. Our test results are consistent with the stylized fact of convergence clubs in that we reject the null hypothesis of cointegration between developing and developed countries while failing to reject the null hypothesis of cointegration between two developed countries. Finally, we applied the Hausman-Wu-type cointegration test to study long-run PPP. Our test results support the commodity-arbitrage view that long-run PPP holds for traded goods but not for non-traded goods.

The rest of the paper is organized as follows. Section 2 gives econometric analysis of the model, including asymptotic theories and finite sample simulation studies. Section 3 presents empirical results in four applications. Section 4 contains concluding remarks.

2 The econometric model

We consider the following data generating process for observations $\{x_t, y_t\}$,

$$y_t = \beta' x_t + \eta_t \tag{1}$$

where x_t is a m by 1 vector. The innovation processes are generated from

$$\Delta x_{it} = v_{it} \tag{2}$$

$$\eta_t = \sum_{i=1}^{m} \sum_{j=-k}^{k} \gamma_{i,j} v_{i,t-j} + e_t \tag{3}$$

$$e_t = \rho e_{t-1} + u_t \tag{4}$$

$v_t = (v_{1t}, \ldots, v_{mt})'$ and u_t are zero mean stationary processes with $E|v_{it}|^\alpha < \infty$, $E|u_t|^\alpha < \infty$ for some $\alpha > 2$ and strong mixing with size $-\alpha/(\alpha - 2)$. We also assume that the method of dynamic regression removes the endogeneity problem, that is, $E(u_t v_s) = \mathbf{0}$ for all t, s. We call this the strict exogeneity assumption for the dynamic regression.

The conditions on v_t and u_t ensure the invariance principles: for $r \in [0, 1]$, $n^{-1/2} \sum_{t=1}^{[nr]} v_t \to_d V(r)$, $n^{-1/2} \sum_{t=1}^{[nr]} u_t \to_d U(r)$, where $V(r)$ is m-vector Brownian motions with covariance $\sum_{j=-\infty}^{\infty} E(v_t v_{t-j}')$, and $U(r)$ is Brownian motion with variance $\sum_{j=-\infty}^{\infty} E(u_t u_{t-j})$. The functional central limit theorem holds for weaker assumptions than assumed here (De Jong and Davidson (2000)), but the conditions assumed above are general enough to include many stationary Gaussian or non-Gaussian ARMA processes that are commonly assumed in empirical modeling.

Let $\mathbf{v}_t = (\Delta x_{1,t-k}, \ldots, \Delta x_{1,t}, \ldots, \Delta x_{1,t+k}, \ldots, \Delta x_{m,t-k}, \ldots, \Delta x_{m,t}, \ldots, \Delta x_{m,t+k})'$, and $\boldsymbol{\gamma} = (\gamma_{1,-k}, \ldots, \gamma_{1,0}, \ldots, \gamma_{1,k}, \ldots, \gamma_{m,-k}, \ldots, \gamma_{m,0}, \ldots, \gamma_{m,k})'$, then to estimate the structure parameter β, we consider the regression

$$y_t = \beta' x_t + \boldsymbol{\gamma}' \mathbf{v}_t + e_t. \tag{5}$$

The inference procedure about β differs according to different assumptions on the error term e_t in (4). When $|\rho| < 1$, e_t is stationary, and hence regression (5) is a cointegration regression with serially correlated error. When $\rho = 1$, e_t is a unit root nonstationary process and the OLS regression is spurious. Both models are important in empirical studies in macroeconomics and finance.

In the next two sections, we will summarize the asymptotic properties of different estimation procedures under these two assumptions. Under the assumption that $\rho = 1$, OLS is not consistent while both GLS correction and feasible GLS correction will give consistent and asymptotically equivalent estimators. Under the assumption that $|\rho| < 1$, GLS corrected estimator is not efficient as it is $\sqrt{n}$ convergent, but the feasible GLS estimator is n convergent and asymptotically equivalent to the OLS estimator. Hence, FGLS is robust with respect to the error specifications ($\rho = 1$ or $|\rho| < 1$).

2.1 Regressions with I(1) error

In this section we consider the situation when the error term is I(1), i.e., $\rho = 1$ in (4). The estimation methods we consider are dynamic OLS, GLS correction, and FGLS correction.

2.1.1 The dynamic OLS spurious estimation

We start with dynamic OLS estimation of regression (5). Under the assumption $\rho = 1$, this regression is spurious since for any value of β the error term is always I(1). Let $\theta = (\beta', \boldsymbol{\gamma}')'$ and $z_t = (x_t', \mathbf{v}_t')'$, then the OLS estimator $\hat{\theta}_{\text{dols}}$ has the following limiting distribution

$$\hat{\theta}_{\text{dols}} - \theta_0 \to \begin{bmatrix} \int_0^1 V(r)V(r)'dr & \Pi_v \\ \mathbf{0} & \Gamma_{\mathbf{v},\mathbf{v}} \end{bmatrix}^{-1} \begin{bmatrix} \int_0^1 V(r)U(r)dr \\ \int_0^1 U(r)d\mathbf{V}(\mathbf{r}) \end{bmatrix} \equiv \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}. \tag{6}$$

where Π_v is a matrix with typical element computed from the sum of $\int V(r)dV(r)'$ and the autocovariance matrix of $\{v_t\}$ and $\Gamma_{\mathbf{v},\mathbf{v}}$ is the covariance matrix with typical element $E(v_{i,t-s}v_{j,t-l})$ for $i, j = 1, 2, \ldots, m$, $s, l = -k, \ldots, 0, \ldots, k$. For example, when $m = 1$ and $\eta_t = \gamma_{1,0}v_{1t} + e_t$, then

$$\hat{\theta}_{\text{dols}} - \theta_0 = \begin{bmatrix} \hat{\beta}_n - \beta_0 \\ \hat{\boldsymbol{\gamma}}_n - \boldsymbol{\gamma}_0 \end{bmatrix} \to \begin{bmatrix} \int V_1(r)^2 dr & \frac{1}{2}[V_1(r)^2 + \sigma_{1v}^2] \\ 0 & \sigma_{1v}^2 \end{bmatrix}^{-1} \begin{bmatrix} \int_0^1 V_1(r)U(r)dr \\ \int_0^1 U(r)dV_1(r) \end{bmatrix}.$$

As remarked in Phillips (1986, 1989), in spurious regressions the noise is as strong as the signal. Hence, uncertainty about β persists in the limiting distributions. Note that in dynamic OLS estimation of cointegration regressions, $\hat{\beta}_n$ and $\hat{\boldsymbol{\gamma}}_n$ are asymptotically independent. But in spurious regressions, the estimators are correlated and converge at the same rate.

2.1.2 GLS corrected estimation

When $\rho = 1$, we can filter all variables in regression (5) by taking full difference, and use OLS to estimate

$$\Delta y_t = \beta' \Delta x_t + \boldsymbol{\gamma}' \Delta \mathbf{v}_t + u_t. \tag{7}$$

This procedure can be viewed as a GLS corrected estimation[2]. Rewrite regression (7) as

$$\Delta y_t = \theta' \Delta z_t + u_t.$$

If we let $\tilde{\theta}_{\text{dgls}}$ denote the GLS corrected estimator, then we can show that

$$\sqrt{n}(\tilde{\theta}_{\text{dgls}} - \theta_0) \to_d N(\mathbf{0}, \Omega) \tag{8}$$

where $\Omega = Q^{-1}\Lambda Q^{-1}$ with $Q = E(z_t z_t')$ and Λ is the long run variance matrix of $z_t u_t$. We can see that β can be consistently estimated (jointly with $\boldsymbol{\gamma}$) and the estimators are asymptotically normal. In the special case when $m = 1$, $\{v_{1t}\}$ and $\{u_t\}$ are *i.i.d.* sequences and $\eta_t = e_t$, we simply have that

$$\sqrt{n}(\tilde{\theta}_{\text{dgls}} - \theta_0) \to_d N(0, \sigma_u^2/\sigma_{1v}^2),$$

where σ_u^2 and σ_{1v}^2 are variances of u_t and v_{1t}, respectively. Still with *i.i.d.* assumptions and $m = 1$, when $\eta_t = \gamma_{1,0} v_{1t} + e_t$,

$$\Omega = \frac{\sigma_u^2}{\sigma_{1v}^2} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}.$$

2.1.3 The Cochrane-Orcutt feasible GLS estimation

To use GLS to estimate a regression with serial correlation in empirical work, a Cochrane-Orcutt feasible GLS procedure is generally adopted. This procedure also works for spurious regressions as has been shown by Phillips and Hodgson (1994). They show that the FGLS estimator is asymptotically equivalent to that in the differenced regression when the error is unit root nonstationary. In the present paper, we will show that the FGLS correction to dynamic regression provides a consistent and robust estimator to structural spurious regressions.

Let the residual from OLS regression (5) be denoted by $\hat{e}_t$,

$$\hat{e}_t = y_t - \hat{\beta}_n' x_t - \hat{\boldsymbol{\gamma}}_n' \mathbf{v}_t.$$

To conduct the Cochrane-Orcutt GLS estimation, first we run an AR(1) regression of $\hat{e}_t$,

$$\hat{e}_t = \hat{\rho}_n \hat{e}_{t-1} + \hat{u}_t. \tag{9}$$

It can be shown that

$$\hat{\rho}_n - 1 = o_p(1) \quad \text{and} \quad n(\hat{\rho}_n - 1) = O_p(1).$$

Next, consider the following Cochrane-Orcutt transformation of the data:

$$\tilde{y}_t = y_t - \hat{\rho}_n y_{t-1}, \quad \tilde{x}_t = x_t - \hat{\rho}_n x_{t-1}, \quad \tilde{\mathbf{v}}_t = \mathbf{v}_t - \hat{\rho}_n \mathbf{v}_{t-1}. \tag{10}$$

Now consider OLS estimation of the regression

$$\tilde{y}_t = \beta' \tilde{x}_t + \boldsymbol{\gamma}' \tilde{\mathbf{v}}_t + error = \theta' \tilde{z}_t + error, \tag{11}$$

where $\tilde{z}_t = (\tilde{x}_t', \tilde{\mathbf{v}}_t')'$. The OLS estimator of θ in (11) is

$$\tilde{\theta}_{\text{fgls}} = \left[\sum_{t=1}^{n} \tilde{z}_t \tilde{z}_t' \right]^{-1} \left[\sum_{t=1}^{n} \tilde{z}_t \tilde{y}_t \right]. \tag{12}$$

[2]This is a conventional GLS procedure when u_t is *i.i.d.*. When u_t is serially correlated as in our approach, we name this procedure as GLS corrected dynamic estimation.

The limiting distribution of $\tilde{\theta}_{\text{fgls}}$ can be shown to be the same as in (8). Intuitively, since ρ_n approaches unity in the limit, $\tilde{z}_t$ behaves asymptotically equivalent to Δz_t.

In Appendix B, we discuss some extensions of the model. We show that if a constant is included in the data generating process, the GLS or FGLS corrected estimators are asymptotically equivalent to that given in (8).

2.2 Regressions with I(0) error

In this section we consider the asymptotic distributions of the three estimators (DOLS estimator, GLS corrected estimator and FGLS corrected estimator) under the assumption of cointegration, i.e., $|\rho| < 1$.

2.2.1 The dynamic OLS estimation

Under the assumption of cointegration, the DGP of y_t is

$$y_t = \beta' x_t + \boldsymbol{\gamma}' \mathbf{v}_t + e_t, \quad e_t = \rho e_{t-1} + u_t, \quad |\rho| < 1. \tag{13}$$

Applying the invariance principle, for $r \in [0,1]$, $n^{-1/2} \sum_{t=1}^{[nr]} e_t \rightarrow E(r)$, where $E(r)$ is a Brownian motion with variance $\sum_{j=-\infty}^{\infty} E(e_t e_{t-j})$. The limiting distribution of the OLS estimator of β, which is asymptotically independent of $\hat{\boldsymbol{\gamma}}_n$, can be shown to be

$$n(\hat{\beta}_{\text{dols}} - \beta) \rightarrow_d \left(\int_0^1 V(r)V(r)' dr \right)^{-1} \left(\int_0^1 V(r) dE(r) \right). \tag{14}$$

2.2.2 GLS corrected estimation

Now, if we take a full difference as we did in the I(1) case, the regression becomes

$$\Delta y_t = \beta' \Delta x_t + \boldsymbol{\gamma}' \Delta \mathbf{v}_t + e_t - e_{t-1} = \theta' z_t + e_t - e_{t-1}.$$

Note that we lose efficiency in this transformation as the estimator $\tilde{\beta}_{\text{dgls}}$ is now $\sqrt{n}$ convergent rather than n convergent as the OLS estimator. With some minor revision of equation (8), the limiting distribution of the estimator in this case can be written as

$$\sqrt{n}(\tilde{\theta}_{\text{dgls}} - \theta_0) \rightarrow_d N(\mathbf{0}, \Omega^*) \tag{15}$$

where $\Omega^* = Q^{-1} \Lambda^* Q^{-1}$. Q is defined as following equation (8) and Λ^* is the long run variance matrix of vector $z_t \Delta e_t$. In the special case when $m = 1$, $\{v_{1t}\}$ and $\{u_t\}$ are *i.i.d.* sequences and $\eta_t = e_t$, $\Omega^* = 2\sigma_e^2(1 - \psi_e)/\sigma_{1v}^2$, where ψ_e is the first order autocorrelation coefficient of $\{e_t\}$.

Compared with the dynamic OLS estimator in the level regression, the GLS correction or differencing is not efficient if the variables are in fact cointegrated.

2.2.3 The Cochrane-Orcutt feasible GLS estimation

Instead of taking full difference, if we estimate the autoregression coefficient in the error and use this estimator to filter all sequences, we will obtain an estimator that is asymptotically equivalent to the OLS estimator. Intuitively, in the case that the error $e_t = u_t$ is serially uncorrelated, then the AR(1) coefficient $\hat{\rho}_n$ will converge to zero, and hence the transformed regression will be asymptotically equivalent to the original regression. Or, if the error is stationary and serially correlated, then the

AR(1) coefficient will be less than unity, and, as has been shown in Phillips and Park (1988), the GLS estimator and the OLS estimator in a cointegration regression are asymptotically equivalent.

First, run OLS estimation of

$$y_t = \hat{\beta}_n' x_t + \hat{\boldsymbol{\gamma}}_n' \mathbf{v}_t + \hat{e}_t,$$

then run an AR(1) regression of $\hat{e}_t$,

$$\hat{e}_t = \hat{\rho}_n \hat{e}_{t-1} + error.$$

Now, consider the Cochrane-Orcutt transformation (10) and estimate

$$\tilde{y}_t = \tilde{\beta}_n' \tilde{x}_t + \tilde{\boldsymbol{\gamma}}_n' \tilde{\mathbf{v}}_t + error. \tag{16}$$

The limiting distribution of $\tilde{\beta}_n$ can be shown to be the same as the limit of the OLS estimator given in (14). In summary, the feasible GLS is not only valid in a spurious regression but also harmless to the estimator in the limit when the regression is cointegration.

2.3 Finite sample performance of the FGLS estimator

From the above analysis, we note that the FGLS estimator is a robust procedure with respect to error specifications. It is asymptotically equivalent to the GLS estimator in spurious regressions and it is asymptotically equivalent to the OLS estimator in cointegration regressions. In this section, we use simulations to study its finite sample performances compared to the other two estimators. In the simulation we consider the case when x_t is a scalar variable and we generate v_t and ϵ_t from two independent standard normal distributions and let $u_t = \epsilon_t + 0.5\epsilon_t$. The structural parameter is set to be $\beta = 2$ and $\boldsymbol{\gamma}'\mathbf{v}_t = 0.5v_t$. Figure 1 shows the finite sample distribution when the error term is unit root nonstationary ($e_t = \sum_{j=0}^{t} u_j$). The left figure plots the distribution of the GLS estimator and the right figure plots that of the feasible GLS estimator. We can see that FGLS estimators are consistent and they have slightly larger variance than GLS estimators. Figure 2 shows the finite sample distribution when the error term is I(0) ($e_t = u_t$). The left figure plots the distribution of the OLS estimators and the right figure plots that of the FGLS estimators. These two estimators are both superconsistent and they share almost the same distributions.

2.4 Hausman-Wu specification test for cointegration

2.4.1 The test statistic and its asymptotic properties

In this section we construct a Hausman-Wu-type cointegration test based on the difference of two estimators: an OLS estimator ($\hat{\beta}_{\text{dols}}$) and a GLS corrected estimator ($\tilde{\beta}_{\text{dgls}}$). This is equivalent to comparing estimators in a level regression and in a differenced regression. We use the Hausman-Wu-type test statistic to test the null hypothesis of cointegrating relationships against the alternative of a spurious regression:

$$H_0 : |\rho| < 1; \quad H_a : \rho = 1.$$

Our above discussions show that under the null of cointegration, both OLS and GLS are consistent but OLS estimator is more efficient. However under the alternative of spurious regression, only the GLS corrected estimator is consistent.

Let $\hat{V}_\beta$ denote a consistent estimator for the asymptotic variance of $\sqrt{n}(\tilde{\beta}_{\text{dgls}} - \beta)$. Under our assumptions, it converges to the corresponding submatrix of Ω^* under the null hypothesis and to the corresponding submatrix of Ω under the alternative. For example, when $m = 1$, $\{v_{1t}\}$ and $\{u_t\}$ are independent *i.i.d.* and $\eta_t = e_t$, then $\hat{V}_\beta = \left(\frac{1}{n}\sum_{t=1}^{n} \hat{w}_t^2\right) / \left(\frac{1}{n}\sum_{t=1}^{n} \Delta x_t^2\right)$, where $\hat{w}_t$ represents the

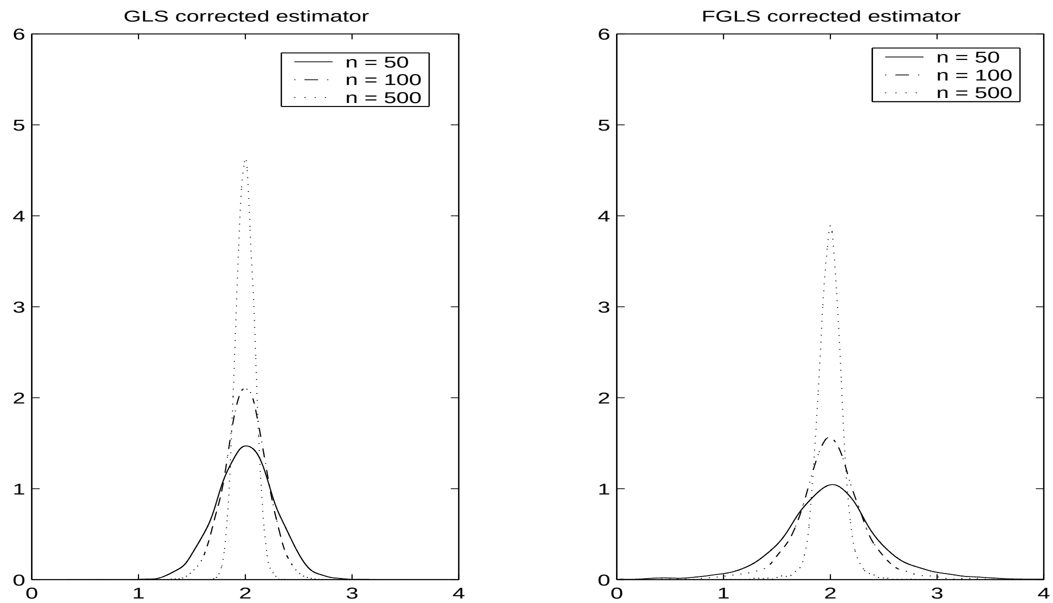

Figure 1: Distributions of GLS and FGLS estimators when the error is I(1)

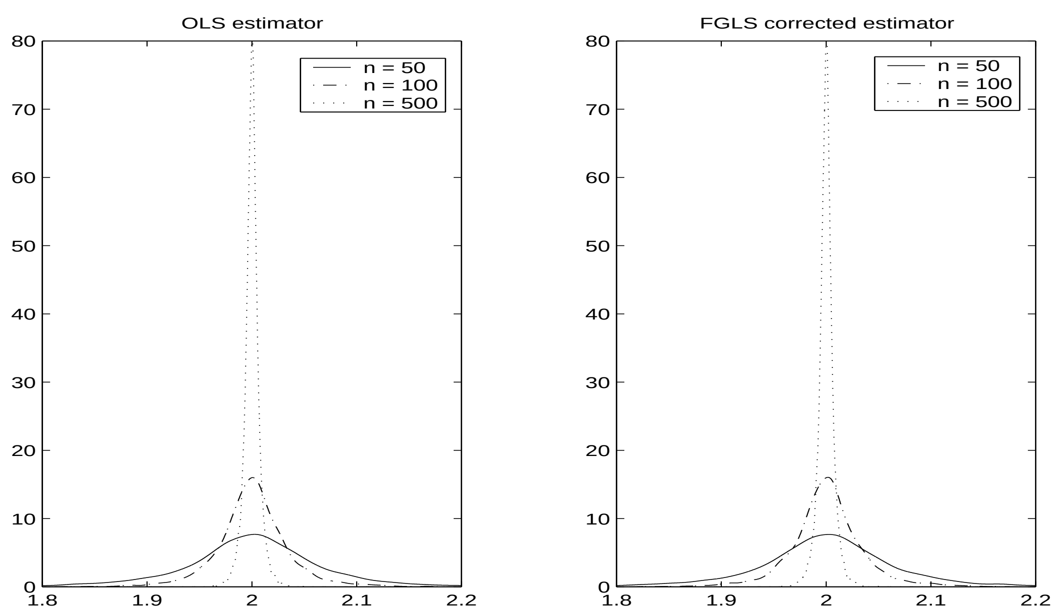

Figure 2: Distributions of OLS and FGLS estimators when the error is I(0)

residuals from OLS estimation of differenced regression. Under the null of cointegration $\hat{V}_\beta \to 2\sigma_e^2(1-\psi_e)/\sigma_{1v}^2$ and under the alternative of spurious regression $\hat{V}_\beta \to \sigma_u^2/\sigma_{1v}^2$.

Our Hausman-Wu-type test statistic is defined as:

$$h_n = n(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}})'\hat{V}_\beta^{-1}(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}}). \tag{17}$$

We show in the appendix that $h_n \to \chi^2(m)$ under the null of cointegration. Under the alternative of spurious regressions, $\hat{\beta}_{\text{dols}}$ dominates and $h_n = O_p(n)$.

We can extend the test to allow endogeneity under the alternative. Consider the following DGP:

$$y_t = \beta' x_t + \boldsymbol{\gamma}'\mathbf{v}_t + \phi q_t + e_t, \tag{18}$$

$$e_t = \rho e_{t-1} + u_t, \tag{19}$$

where $\{q_t\}$ satisfies the same conditions as u_t and v_t, but it is correlated with $\{v_t\}$. All other variables are defined as before. The hypotheses are:

$$H_0' : |\rho| < 1 \quad \text{and} \quad \phi = 0;$$
$$H_a' : \rho = 1 \quad \text{and} \quad \phi \neq 0.$$

The asymptotics of h_n under the null H_0' are the same as that under H_0. Under the alternative H_a', we show in the appendix that the DOLS estimator has the same asymptotic limit distribution as that under H_a and $h_n = O_p(n)$. Therefore, this Hausman-Wu-type test is consistent for the null hypothesis of cointegration against the alternative of spurious regression, regardless of whether the exogeneity assumption holds under the alternative.

2.4.2 Finite sample properties of the Hausman-Wu-type cointegration test

Before applying the Hausman-Wu-type cointegration test to empirical issues, it will be instructive to examine its finite sample properties in comparison with other comparable tests under the same null hypothesis. To this end, we conduct a small simulation experiment based on the following dynamic regression model,

$$y_t = \gamma_1 \Delta x_{t+1} + \beta x_t + \gamma_2 \Delta x_{t-1} + e_t, \tag{20}$$

$$e_t = \rho e_{t-1} + u_t, \tag{21}$$

where $\gamma_1 = 0.3$, $\beta = 2$, $\gamma_2 = -0.5$, and setting $\rho = 0.9$ for the size performance and $\rho = 1$ for the power performance. We consider sample sizes of $n \in \{50, 100, 200, 300, 500\}$ that are commonly encountered in empirical analysis. In the simulations pseudo random numbers are generated using the GAUSS (version 6.0) RNDNS procedures. Each simulation run is carried out with 5,000 replications. At each replication $100 + n$ random numbers are generated of which the first 100 observations are discarded to avoid a start-up effect.

Table 1 reports selected finite sample properties of the Hausman-Wu-type cointegration test together with a residual based test under the null of cointegration due to Shin (1994, hereafter Shin's test) who extended the KPSS test in the parametrically corrected cointegrating regression. In the simulations the lengths of lead and lag terms for DOLS and DGLS are chosen by the BIC rule.[3] The nonparametric estimation method of long run variance is employed using the QS kernel with the bandwidth

[3]It is an interesting research topic to investigate the performance of various lag length selection rules, but would be beyond the scope of this paper.

of 'integer[$8(n/100)^{1/4}$]'. The results in Table 1 illustrate two points. First, the empirical size of the Hausman-Wu-type test is close to the nominal size, in particular when sample size is relatively large, whereas Shin's test suffers from serious oversize problem. Second, in terms of power, the Hausman test dominates Shin's test for moderate sample sizes which are very likely to be encountered empirically. Shin's test seems more powerful when n is relatively large, but only at the cost of severe size distortions. Overall, our simulation results provide evidence in favor of the Hausman-Wu-type test over those tests available in the literature.

3 Empirical applications

In this section we apply the GLS-type correction methods and the Hausman-Wu-type cointegration test to analyze four macroeconomic issues: (i) long-run money demand in the U.S.; (ii) long-run implications of the consumption-leisure choice; (iii) output convergence among industrial and developing countries; (iv) Purchasing Power Parity (PPP) for traded and non-traded goods. The main purpose of the first two applications is to illustrate the spurious regression approach to estimating unknown structural parameters. Identification of the structural parameters in these two applications are based on the following idea of nonstationary measurement error. Let y_t^0 be the true value of y_t, and assume that

$$y_t^0 = \beta' x_t + \boldsymbol{\gamma}^{0\prime}\mathbf{v}_t + e_t^0 \tag{22}$$

is a dynamic cointegrating regression that satisfies the strict exogeneity assumption. Let y_t be the measured value of y_t^0, and assume that measurement error satisfies

$$y_t - y_t^0 = \boldsymbol{\gamma}^{m\prime}\mathbf{v}_t + e_t^m, \tag{23}$$

where e_t^m is I(1) and its first difference is uncorrelated with $\mathbf{v}_s$ for any s. Here, the crucial assumption for identification is that measurement error is not cointegrated with x_t. Then

$$y_t = \beta' x_t + \boldsymbol{\gamma}'\mathbf{v}_t + e_t \tag{24}$$

where $\boldsymbol{\gamma} = \boldsymbol{\gamma}^0 + \boldsymbol{\gamma}^m$[4], and e_t is I(1) that satisfies the strict exogeneity assumption.

3.1 U.S. money demand

The long-run money demand function has often been estimated under the cointegrating restriction among real balances, real income, and the interest rate. The restriction is legitimate if the money demand function is stable in the long-run and if all variables are measured without nonstationary error. Indeed, Stock and Watson (1993) found supportive evidence of stable long-run M1 demand by estimating cointegrating vectors. However, if money is measured with a nonstationary measurement error, we have a spurious regression and the estimation results based on a cointegration regression become questionable.

To be specific, we follow Stock and Watson (1993) and assume that the dynamic regression error is stationary and the strict exogeneity assumption holds for the dynamic regression error when money is correctly measured. We then assume that money is measured with a multiplicative measurement error. We assume that the log measurement error is unit root nonstationary, and that the residuals of the projection of the log measurement error on the leads and lags of the regressors in the dynamic regression

[4] Here, we assume that the dimensions of $\boldsymbol{\gamma}^0$ and $\boldsymbol{\gamma}^m$ are the same without loss of generality, because we can add zeros as elements of $\boldsymbol{\gamma}^0$ and $\boldsymbol{\gamma}^m$ as needed.

satisfy the strict exogeneity assumption. Given that the large component of the measurement error is arguably currency held by foreign residents and black market participants, the log measurement error is likely to be very persistent. Therefore, the assumption that the log measurement error is unit root nonstationary may be at least a good approximation. The strict exogeneity assumption is plausible because we allow the log measurement error to be correlated with leads and lags of the first difference of the regressors.

We apply our GLS correction methods to estimate long-run income and interest elasticities of M1 demand. To this end, the regression equations are set up with the real money balance ($\frac{M}{P}$) as regressand and income (y) and interest (i) as regressors. Following Stock and Watson (1993), the annual time series for M1 deflated by the net national product price deflator is used for $\frac{M}{P}$, the real net national product for y and the six month commercial paper rate in percentage for i. $\frac{M}{P}$ and y are in logarithms while three different regression equations are considered depending on the measures of interest. We have tried the following three functional forms. Equation 1 has been studied by Stock and Watson (1993).

$$\begin{aligned}
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma i_t + e_t, \quad \text{(equation 1)} \\
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma \ln(i_t) + e_t, \quad \text{(equation 2)} \\
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma \ln\left[\frac{1+i_t}{i_t}\right] + e_t. \quad \text{(equation 3)}
\end{aligned}$$

It is worth noting that the liquidity trap is possible for the latter two functional forms. When the data contain periods with very low nominal interest rates, the latter two functional forms may be more appropriate.

Table 2 presents the point estimates for β (income elasticity of money demand) and γ based on the three estimators under scrutiny: dynamic OLS, GLS corrected dynamic regression estimator, and FGLS corrected dynamic regression estimator.[5] Several features emerge from the table. First, all the estimated coefficients have theoretically 'correct' signs: positive signs for income elasticities and negative signs for γ for the first two functional forms and positive signs for γ for the third functional form. Second, GLS corrected regression estimates of the income elasticity are implausibly low for all three functional forms for low values of k and increase to more plausible values near one as k increases. The fact that the results become more plausible as k increases suggests that the endogeneity correction of dynamic regressions works in this application for moderately large values of k such as 3 and 4. The results for low values of k are consistent with those of low income elasticity estimates of first differenced regressions before researchers started to apply the cointegration methods to estimate money demand. Therefore, the estimators in the old literature of first differenced regressions are likely to be downward biased because of the endogeneity problem. Third, all point estimates of the three estimators are very similar, and the Hausman test fails to reject the null hypothesis of cointegration for large enough values of k. Hence, there is little evidence against cointegration. However, it should be noted that a small random walk component is very hard to detect by any test for cointegration. Therefore, it is assuring to know that all three estimators are similar for large enough values of k, and the estimates are robust with respect to whether the regression error is I(0) or I(1).

We report the value of k chosen by the Bayesian Information Criterion (BIC) rule throughout our empirical applications in order to give some guidance in interpreting results. It is beyond the scope of

[5]For the FGLS corrected dynamic regression estimator, the serial correlation coefficient of the error term is estimated before being applied to the Cochrane-Orcutt transformation while it is assumed to be unity in GLS corrected dynamic regression estimator which is equivalent to regressing the first difference of variables without constant term.

this paper to study detailed analysis of how k should be chosen because this issue has not been settled in the literature of dynamic cointegrating regressions.

Table 2 also reports ADF test results when the test is applied to the residual of OLS. These test results typically show evidence against the null hypothesis of structural spurious regressions. This is consistent with the Hausman-Wu-type test results with the null hypothesis of cointegration.

3.2 Long-run implications of the consumption-leisure choice

Consider a simplified version of Cooley and Ogaki's (1996) model of consumption and leisure in which the representative household maximizes

$$U = E_0[\sum_{t=0}^{\infty} \delta^t e(t)]$$

where E_t denotes the expectation conditioned on the information available at t. We adopt a simple intraperiod utility function that is assumed to be time and state-separable and separable in nondurable consumption, durable consumption, and leisure

$$e(t) = \frac{C(t)^{1-\beta} - 1}{1-\beta} + v(l(t))$$

where $v(\cdot)$ represents a continuously differentiable concave function, $C(t)$ is nondurable consumption, and $l(t)$ is leisure.

The usual first order condition for a household that equates the real wage rate with the marginal rate of substitution between leisure and consumption is given as:

$$W(t) = \frac{v'(l(t))}{C(t)^{-\beta}}$$

where $W(t)$ is the real wage rate. We assume that the stochastic process of leisure is (strictly) stationary in the equilibrium as in Eichenbaum, Hansen, and Singleton (1988). An implication of the first order condition is that $\ln(W(t)) - \beta \ln(C(t)) = \ln(v'(l(t)))$ is stationary. When we assume that the log of consumption is difference stationary, this implies that the log of the real wage rate and the log of consumption are cointegrated with a cointegrating vector $(1, -\beta)'$.

We now assume that $(W(t))$ is measured with a multiplicative measurement error, so that $\ln(W(t))$ is measured with a measurement error $\epsilon(t)$. We assume that this measurement error is unit root nonstationary. One component of the measurement error arises because it is difficult to measure fringe benefits. Therefore, we expect the measurement error to be very persistent.

Consider a regression

$$\ln(W^m(t)) = a + \beta \ln(C(t)) + e(t), \tag{25}$$

where $W^m(t)$ is the measured real wage rate, and $e(t) = -\epsilon(t) + \ln(v'(l(t))) - a$. If $\epsilon(t)$ is stationary, then $e(t)$ is stationary, and Regression (25) is a cointegrating regression as in Cooley and Ogaki. In this simple version, the preference parameter β is the Relative Risk Aversion (RRA) coefficient, which is equal to the reciprocal of the intertemporal elasticity of substitution (IES). Cooley and Ogaki show that the same regression can be used to estimate the reciprocal of the long-run IES when preferences for consumption are subjected to time nonseparability such as habit formation. For simplicity, we interpret β as the RRA coefficient in this paper.

If $\epsilon(t)$ is unit root nonstationary, then Regression (25) is a spurious regression because $e(t)$ is nonstationary in this case. Hence, the standard methods for cointegrating regressions cannot be used.

However, the preference parameter β can still be estimated by the spurious regression method as long as the strict exogeneity assumption holds in the dynamic regression. The exogeneity assumption holds as long as the endogeneity problem is removed by including leads and lags of the first difference of the regressors.

Table 3 presents the estimation results for the RRA coefficient (β) based on various estimators. We used the same data set that Cooley and Ogaki used.[6] The results in Table 3 illustrate several points. First, all point estimates for β have the theoretically correct positive sign. Second, for nondurables (ND), GLS-corrected dynamic regression estimates of β are much lower than Dynamic OLS estimates for all values of k. As a result, the Hausman-Wu-type cointegration test rejects the null hypothesis of cointegration for all values of k at the 1 percent level. Therefore, the evidence supports the view that Regression (25) is a spurious regression, and the true value of the RRA coefficient is likely to be much lower than the dynamic OLS estimates. Both the GLS corrected and the robust FGLS corrected dynamic regression estimation results are consistent with the view that the RRA coefficient is about one for the value of k chosen by BIC. For nondurables plus services (NDS), GLS corrected dynamic regression estimates of β are much lower than Dynamic OLS estimates for small values of k. As a result, the Hausman-Wu-type cointegration test rejects the null hypothesis of cointegration at the 5 percent level when k is 0, 1, and 2. It still rejects the null hypothesis of cointegration at the 5 percent level when k is 3. It does not reject the null hypothesis when k is 4 or 5. According to the BIC rule, k is chosen to be 3, and there is some evidence against cointegration. However, because the GLS corrected dynamic regression estimates get closer to dynamic OLS estimates as k increases, the evidence is not very strong. It is likely that a small random walk component exists for the error term of the regression for NDS making it a spurious regression. The robust FGLS corrected dynamic regression estimates are close to both GLS corrected dynamic regression estimates and dynamic OLS estimates as long as k is 3 or greater.

Table 3 also reports ADF test results when the test is applied to the residual of dynamic OLS. These test results are sensitive to the choice of the order of lags, but show little evidence against the null hypothesis of structural spurious regressions for ND with large enough lags. The evidence is a little more mixed for NDS.

Thus, we have fairly strong evidence that we have a spurious regression for ND and some evidence that we have spurious regression for NDS. The true value of RRA is likely to be about one for both ND and NDS.

3.3 Output convergence across national economies

In this section, we apply the techniques to reexamine a long standing issue of macroeconomics, the hypothesis of output convergence. For this application and the next application, our main purpose is not to estimate unknown structural parameters but to test the null hypothesis of cointegration with the Hausman-Wu-type test. For this purpose, we do not need the strict exogeneity assumption under the alternative hypothesis of no cointegration (or a spurious regression). As a key proposition of the neoclassical growth model, the hypothesis has been one of the popular subjects in macroeconomics and has attracted considerable attention in the empirical field, particularly during the last decade. Besides its important policy implications, the convergence hypothesis has been used as a criterion to discern the two main growth theories, the exogenous growth theory and the endogenous growth theory. However, it remains the subject of continuing debate mainly because the empirical evidence supporting the hypothesis is mixed. Nevertheless, the established literature based on popular international dataset

[6]See Cooley and Ogaki (1996, page 127) for the detailed description of the data.

such as the Summers-Heston (1991) suggests a stylized fact in output convergence among various national economies: convergence among industrialized countries but not among developing countries and not between industrialized and developing countries.

Given that a mean stationary stochastic process of output disparities between two economies is interpreted as supportive evidence of stochastic convergence, unit-root or cointegration testing procedures are often used by empirical researchers to evaluate the convergence hypothesis. In this vein, our techniques proposed here fit in the study of output convergence. We consider four developing countries (Columbia, Ecuador, Egypt, and Pakistan) along with four industrial countries (Germany, Luxemburg, New Zealand, Switzerland). The raw data are extracted from the *Penn World Tables* of Summers-Heston (1991) and consist of annual real GDP per capita (RGDPCH) over the period of 1950-1992. The following two regression equations are considered with regard to the cointegration relation.

$$y_t^D = \alpha + \beta y_t^I + e_t, \tag{26}$$

$$y_t^I = \alpha + \beta y_t^I + e_t, \tag{27}$$

where y_t^D and y_t^I denote log real GDP per capita for developing and industrial countries, respectively.

Tables 4-1 and 4-2 report the results which exhibit a large variation in estimated coefficients. Recall that our interest in this application lies in the cointegration test based on the Hausman-Wu-type test. As can be seen from Table 4-1, irrespective of country combinations, the null hypothesis of cointegration can be rejected when developing countries are regressed onto industrial countries, indicating that there is little evidence of output convergence between developing countries and industrial countries. The picture changes dramatically when industrial countries are regressed onto industrial countries as in (27). Table 4-2 displays that the Hausman test fail to reject the null of cointegration in all cases considered. Our finding is therefore consistent with the so-called *convergence clubs*.

3.4 PPP for traded and non-traded goods

As a major building block for many models of exchange rate determination, PPP has been one of the most heavily studied subjects in international macroeconomics. Despite extensive research, however, the empirical evidence on PPP remains inconclusive, largely due to econometric challenges involved in determining its validity. As is generally agreed, most real exchange rates show very slow convergence which makes estimating long-run relationships difficult with existing statistical tools. The literature suggests a number of potential explanations for the very slow adjustment of relative price: volatility of the nominal exchange-rate, market frictions such as trade barriers and transportation costs, imperfect competition in product markets, and the presence of non-traded goods in the price basket. According to the commodity-arbitrage view of PPP, the law of one price holds only for traded goods and the departures from PPP are primarily attributed to the large weight placed on nontraded goods in the CPI. This view has obtained support from many empirical studies based on disaggregated price indices. They tend to provide ample evidence that prices for non-traded goods are much more dispersed than for their traded counterpart and consequently non-traded goods exhibit far larger deviations from PPP than traded goods. Given that general price indices involve a mix of both traded and non-traded goods, highly persistent deviations of non-traded goods from PPP can lead to the lack of conclusive evidence on the long run PPP relationship. As in the previous application, our main purpose for this application is not to estimate unknown structural parameters but to test the null hypothesis of cointegration with the Hausman-Wu-type test.

Let p_t and p_t^* denote the logarithms of the consumer price indices in the base country and foreign country, respectively, and s_t be the logarithm of the price of the foreign country's currency in terms of the base country's currency. Long-run PPP requires that a linear combination of these three variables be stationary. To be more specific, long-run PPP is said to hold if $f_t = s_t + p_t^*$ is cointegrated with p_t such that $e_t \sim I(0)$ in

$$f_t^T = \alpha + \beta p_t^T + e_t,$$
$$f_t^N = \alpha + \beta p_t^N + e_t,$$

where the superscripts T and N denote the price levels of traded goods and non-traded goods, respectively.

Following the method of Stockman and Tesar (1995), Kim (2004) recently used the real exchange rate for total consumption using the general price deflator and the real exchange rate for traded and non-traded goods using implicit deflators for non-service consumption and service consumption, respectively.[7] We use Kim's dataset to apply our techniques to the linear combination of sectorally decomposed variables. Table 5 presents the results using quarterly price and exchange rate data for six countries, Canada, France, Italy, Japan, U.K., and U.S. for the period of 1974 Q1 through 1998 Q4. With the Canadian dollar used as numeraire, Table 5 presents the estimates for β which should be close to unity according to long-run PPP. For traded goods, estimates are above unity in most cases, but the variation across estimators does not seem substantial, resulting in non-rejection of the null of cointegration in all cases considered. By sharp contrast, the Hausman-Wu-type cointegration test rejects the null hypothesis in every country when the price for non-traded goods is used. It is noteworthy that there exists considerable difference between GLS-corrected estimates for β and their DOLS and FGLS counterparts which are far greater than unity. That is, supportive evidence of PPP is found for traded goods but not for non-traded goods, congruent with the general intuition as well as the findings by other studies in the literature such as Kakkar and Ogaki (1999) and Kim (2004).[8]

4 Concluding remarks and future work

In this paper, we developed two estimators to estimate structural parameters in spurious regressions: GLS corrected dynamic regression and FGLS corrected dynamic regression estimators. A GLS corrected dynamic regression estimator is a first differenced version of a dynamic OLS regression estimator. The asymptotic theory showed that, under some regularity conditions, the endogeneity correction of the dynamic regression works for the first differenced regressions for both cointegrating and spurious regressions. This result is useful because it is not intuitively clear that the endogeneity correction works even in regressions with stationary first differenced variables.

For the purpose of estimation of structural parameters when the possibility of nonstationary measurement error cannot be ruled out, we recommend the FGLS corrected dynamic regression estimators. They are consistent both when the error is I(0) and I(1). They are asymptotically as efficient as dynamic OLS when the error is I(0) and as GLS corrected dynamic regression when the error is I(1). This feature may be especially attractive when the FGLS corrected dynamic estimator is extended to a panel data setting when some regression errors are I(0) and the others are I(1). This extension is studied by Hu (2005).

We also developed the Hausman-Wu-type cointegration test by comparing the dynamic OLS regression and GLS corrected dynamic regression estimators. As noted in the Introduction, this task is

[7]For details, see the Appendix for the description of the data. We thank JB Kim for sharing the dataset.

[8]Engel (1999) finds little evidence for long-run PPP for traded goods with his variance decomposition method. However, it should be noted that his method is designed to study variations of real exchange rates over relatively shorter periods compared with cointegration-type methods that are designed to study long-run relationships.

important because few tests for cointegration have been developed for dynamic OLS and because tests for the null hypothesis of cointegration are useful in many applications. For this test, the spurious regression obtained under the alternative hypothesis does not have to be structural.

In future work, it is important to study the choice of k, the number of leads and lags in the endogeneity correction. Another aspect that will be useful in empirical estimations is to study possible deterministic time trend and seasonal effects in the model.

Appendix

Appendix A: Proof of results in section 2.1

To show the distribution of the OLS estimator in regression (5), define

$$D_n = \begin{bmatrix} I_m n^2 & \mathbf{0} \\ \mathbf{0} & I_{m(2k+1)} n \end{bmatrix}.$$

The OLS estimator for β and $\boldsymbol{\gamma}$ can be written as

$$\begin{aligned}
\begin{bmatrix} \hat{\beta}_n - \beta_0 \\ \hat{\boldsymbol{\gamma}}_n - \boldsymbol{\gamma}_0 \end{bmatrix} &= \left[\begin{bmatrix} \sum_{t=1}^n x_t x_t' & \sum_{t=1}^n x_t \mathbf{v}_t' \\ \sum_{t=1}^n \mathbf{v}_t x_t' & \sum_{t=1}^n \mathbf{v}_t \mathbf{v}_t' \end{bmatrix} D_n^{-1} \right]^{-1} D_n^{-1} \begin{bmatrix} \sum_{t=1}^n x_t e_t \\ \sum_{t=1}^n \mathbf{v}_t e_t \end{bmatrix} \\
&= \begin{bmatrix} n^{-2} \sum_{t=1}^n x_t x_t' & n^{-1} \sum_{t=1}^n x_t \mathbf{v}_t' \\ n^{-2} \sum_{t=1}^n \mathbf{v}_t x_t' & n^{-1} \sum_{t=1}^n \mathbf{v}_t \mathbf{v}_t' \end{bmatrix}^{-1} \begin{bmatrix} n^{-2} \sum_{t=1}^n x_t e_t \\ n^{-1} \sum_{t=1}^n \mathbf{v}_t e_t \end{bmatrix} \\
&\to \begin{bmatrix} \int_0^1 V(r) V(r)' dr & \Pi_v \\ \mathbf{0} & \Gamma_{\mathbf{v},\mathbf{v}} \end{bmatrix}^{-1} \begin{bmatrix} \int_0^1 V(r) U(r) dr \\ \int_0^1 U(r) d\mathbf{V}(\mathbf{r}) \end{bmatrix},
\end{aligned}$$

which gives equation (6).

To show the limit distribution of the GLS corrected estimator in regression (7), write

$$\sqrt{n}(\tilde{\theta}_{\text{dgls}} - \theta_0) = \left[n^{-1} \sum_{t=1}^n \Delta z_t \Delta z_t' \right]^{-1} \left[n^{-1/2} \sum_{t=1}^n \Delta z_t u_t \right]. \tag{28}$$

For the denominator,

$$n^{-1} \sum_{t=1}^n \Delta z_t \Delta z_t' = \begin{bmatrix} n^{-1} \sum_{t=1}^n v_t v_t' & n^{-1} \sum_{t=1}^n v_t \Delta \mathbf{v}_t' \\ n^{-1} \sum_{t=1}^n \Delta \mathbf{v}_t v_t' & n^{-1} \sum_{t=1}^n \Delta \mathbf{v}_t \Delta \mathbf{v}_t' \end{bmatrix} \to \begin{bmatrix} \Sigma_v & \Gamma_{v,\Delta \mathbf{v}}' \\ \Gamma_{v,\Delta \mathbf{v}} & \Gamma_{\Delta \mathbf{v},\Delta \mathbf{v}} \end{bmatrix} \equiv Q. \tag{29}$$

where Σ_v is the variance matrix of $\{v_t\}$ and Γ is a vector or matrix with elements computed from the autocovariances of $\{v_t\}$. In the special case when $m = 1$, $\eta_t = \gamma_{1,0} v_{1t} + e_t$,

$$n^{-1} \sum_{t=1}^n \Delta z_t \Delta z_t' = \begin{bmatrix} n^{-1} \sum_{t=1}^n v_{1t}^2 & n^{-1} \sum_{t=1}^n v_{1t}(v_{1t} - v_{1,t-1}) \\ n^{-1} \sum_{t=1}^n v_{1t}(v_{1t} - v_{1,t-1}) & n^{-1} \sum_{t=1}^n (v_{1t} - v_{1,t-1})^2 \end{bmatrix} \to \sigma_{1v}^2 \begin{bmatrix} 1 & 1 - \psi_{1v} \\ 1 - \psi_{1v} & 2(1 - \psi_{1v}) \end{bmatrix}.$$

where σ_{1v}^2 is the variance of v_{1t} and ψ_{1v} is its first order autocorrelation coefficient. If we further assume that v_{1t} is *i.i.d.*, the limit matrix becomes:

$$Q = \sigma_{1v}^2 \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}.$$

For the numerator, the assumptions on the innovation processes ensure that CLT holds:

$$n^{-1/2} \sum_{t=1}^n \Delta z_t u_t = \begin{bmatrix} n^{-1/2} \sum_{t=1}^n v_t u_t \\ n^{-1/2} \sum_{t=1}^n \Delta \mathbf{v}_t u_t \end{bmatrix} \to N(0, \Lambda) \tag{30}$$

where Λ is the long run covariance matrix of the vector $\Delta z_t u_t$:

$$\Lambda = \begin{bmatrix} \sum_{j=-\infty}^{\infty} E(v_t v_{t-j}' u_t u_{t-j}) & \sum_{j=-\infty}^{\infty} E(v_t \Delta \mathbf{v}_{t-j}' u_t u_{t-j}) \\ \sum_{j=-\infty}^{\infty} E(\Delta \mathbf{v}_t v_{t-j}' u_t u_{t-j}) & \sum_{j=-\infty}^{\infty} E(\Delta \mathbf{v}_t \Delta \mathbf{v}_{t-j}' u_t u_{t-j}) \end{bmatrix}.$$

When $m = 1$, v_{1t}, u_t are both *i.i.d.* and $\eta_t = \gamma_{1,0} v_{1t} + e_t$,

$$\Lambda = \sigma_{1v}^2 \sigma_u^2 \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}.$$

Hence, for the quantity defined in (28), we have the limit distribution given in (8):

$$\sqrt{n}(\tilde{\theta}_{\text{dgls}} - \theta_0) \to N(\mathbf{0}, \Omega)$$

where $\Omega = Q^{-1} \Lambda Q^{-1}$.

To derive the limit distribution for the FGLS estimator, we first derive the limit distribution for $\hat{\rho}_n$ in regression (9). Write the process of $\hat{e}_t$ as

$$\begin{aligned} \hat{e}_t &= y_t - \hat{\theta}_{\text{dols}}' z_t \\ &= e_t + (\theta_0 - \hat{\theta}_{\text{dols}})' z_t \\ &= e_{t-1} + (\theta_0 - \hat{\theta}_{\text{dols}})' z_t + u_t \\ &= \hat{e}_{t-1} + (\theta_0 - \hat{\theta}_{\text{dgls}})' \Delta z_t + u_t \\ &= \hat{e}_{t-1} + g_t, \quad \textit{say}. \end{aligned}$$

From this expression, we can see that $\hat{e}_t$ is a unit root process with serially correlated error g_t. The OLS estimator $\hat{\rho}_n$ can be written as

$$\hat{\rho}_n - 1 = \frac{\sum_{t=1}^n \hat{e}_{t-1} g_t}{\sum_{t=1}^n \hat{e}_{t-1}^2} \tag{31}$$

If we let $b = (1, (\beta_0 - \hat{\beta}_n)')'$, $a_t = (e_t, x_t')'$, we can write

$$\begin{aligned} \hat{e}_t &= e_t + (\beta_0 - \hat{\beta}_n)' x_t + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)' \mathbf{v}_t \\ &= b' a_t + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)' \mathbf{v}_t. \end{aligned} \tag{32}$$

We can write the denominator in (31) as

$$n^{-2} \sum_{t=1}^{n} \hat{e}_t^2 \quad = b' n^{-2} \textstyle\sum_{t=1}^n a_t a_t' b + o_p(1) \to \alpha' \int A(r) A(r)' dr \ \alpha$$

where we let $\alpha = (1, -h_1')'$ and $A(r) = (U(r), V(r)')'$.

The numerator in (31) can be written as

$$\hat{e}_{t-1} g_t \quad = \quad [b' a_t + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)' \mathbf{v}_t][(\beta_0 - \hat{\beta}_n)' v_t + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)'(\mathbf{v}_t - \mathbf{v}_{t-1}) + u_t].$$

The sum of all the terms of products in this expression converges when normed with n^{-1}. We omit the details here since we will not make use of the distribution form of $\hat{\rho}_n$. We let c denote this limit, i.e., $n^{-1} \sum_{t=1}^n \hat{e}_{t-1} g_t \to c$.

Then,

$$n(\hat{\rho}_n - 1) = \frac{n^{-1} \sum_{t=1}^n \hat{e}_{t-1} g_t}{n^{-2} \sum_{t=1}^n \hat{e}_{t-1}^2} \to_d c \left(\alpha' \int A(r) A(r)' dr \ \alpha \right)^{-1}. \tag{33}$$

In fact, in our following computations, we only make use of the fact that

$$\hat{\rho}_n - 1 = o_p(1) \quad \text{and} \quad n(\hat{\rho}_n - 1) = O_p(1).$$

Below we show how to derive the limit distribution for $\tilde{\theta}_{\text{fgls}}$. For the sequence of $\tilde{y}_t$, we can write it as

$$\begin{aligned}
\tilde{y}_t &= y_t - \hat{\rho}_n y_{t-1} \\
&= \theta_0' z_t + e_t - \hat{\rho}_n(\theta_0' z_{t-1} + e_{t-1}) \\
&= \theta_0'(z_t - \hat{\rho}_n z_{t-1}) + (e_t - e_{t-1}) + (1-\hat{\rho}_n)e_{t-1} \\
&= \theta_0' \tilde{z}_t + u_t + (1-\hat{\rho}_n)e_{t-1}
\end{aligned}$$

Now, we can write

$$\hat{\theta}_{\text{fgls}} - \theta_0 = \left[\sum_{t=1}^{n} \tilde{z}_t \tilde{z}_t'\right]^{-1} \left[\sum_{t=1}^{n} \tilde{z}_t [u_t + (1-\hat{\rho}_n)e_{t-1}]\right]. \tag{34}$$

The denominator can be written as

$$\sum_{t=1}^{n} \tilde{z}_t \tilde{z}_t' = \begin{bmatrix} \sum_{t=1}^{n} \tilde{x}_t \tilde{x}_t' & \sum_{t=1}^{n} \tilde{x}_t \tilde{\mathbf{v}}_t' \\ \sum_{t=1}^{n} \tilde{\mathbf{v}}_t \tilde{x}_t' & \sum_{t=1}^{n} \tilde{\mathbf{v}}_t \tilde{\mathbf{v}}_t' \end{bmatrix}$$

First,

$$\begin{aligned}
\sum_{t=1}^{n} \tilde{x}_t \tilde{x}_t' &= \sum_{t=1}^{n} (x_t - \hat{\rho}_n x_{t-1})(x_t - \hat{\rho}_n x_{t-1})' \\
&= \sum_{t=1}^{n} [(1-\hat{\rho}_n)x_{t-1} + v_t][(1-\hat{\rho}_n)x_{t-1} + v_t]' \\
&= (1-\hat{\rho}_n)^2 \sum_{t=1}^{n} x_{t-1}x_{t-1}' + (1-\hat{\rho}_n)\sum_{t=1}^{n}[x_{t-1}v_t' + v_t x_{t-1}'] + \sum_{t=1}^{n} v_t v_t'.
\end{aligned}$$

Hence,

$$\begin{aligned}
& n^{-1}\sum_{t=1}^{n} \tilde{x}_t^2 \\
= \;& n(1-\hat{\rho}_n)^2 \left(n^{-2}\sum_{t=1}^{n} x_{t-1}x_{t-1}'\right) + (1-\hat{\rho}_n)\left(n^{-1}\sum_{t=1}^{n} x_{t-1}v_t' + n^{-1}\sum_{t=1}^{n} v_t x_{t-1}'\right) + n^{-1}\sum_{t=1}^{n} v_t v_t' \\
= \;& n^{-1}\sum_{t=1}^{n} v_t v_t' + o_p(1) \to \Sigma_v
\end{aligned}$$

Similarly,

$$\begin{aligned}
n^{-1}\sum \tilde{x}_t \tilde{\mathbf{v}}_t' &= n^{-1}\sum_{t=1}^{n} v_t(\mathbf{v}_t - \mathbf{v}_{t-1})' + (1-\hat{\rho}_n)n^{-1}\sum_{t=1}^{n} v_t \mathbf{v}_{t-1}' \\
&\quad + (1-\hat{\rho}_n)n^{-1}\sum_{t=1}^{n} x_{t-1}(\mathbf{v}_t - \mathbf{v}_{t-1})' + (1-\hat{\rho}_n)^2 n^{-1}\sum_{t=1}^{n} x_{t-1}\mathbf{v}_{t-1}' \\
&= n^{-1}\sum_{t=1}^{n} v_t(\mathbf{v}_t - \mathbf{v}_{t-1})' + o_p(1) \to \Gamma_{v,\Delta\mathbf{v}}
\end{aligned}$$

Finally, $n^{-1}\sum_{t=1}^{n}\tilde{\mathbf{v}}_t\tilde{\mathbf{v}}_t' = \frac{1}{n}\sum_{t=1}^{n}\Delta\mathbf{v}_t\Delta\mathbf{v}_t' + o_p(1) \to \Gamma_{\Delta\mathbf{v},\Delta\mathbf{v}}$. Hence,

$$n^{-1}\sum_{t=1}^{n}\tilde{z}_t\tilde{z}_t' = n^{-1}\sum_{t=1}^{n}\Delta z_t\Delta z_t' + o_p(1) \to_p Q. \tag{35}$$

Next, consider the numerator in (34)

$$\sum_{t=1}^{n}\tilde{z}_t[u_t + (1-\hat{\rho}_n)e_{t-1}] = \begin{bmatrix} \sum_{t=1}^{n}\tilde{x}_t[u_t + (1-\hat{\rho}_n)e_{t-1}] \\ \sum_{t=1}^{n}\tilde{\mathbf{v}}_t[u_t + (1-\hat{\rho}_n)e_{t-1}] \end{bmatrix}.$$

It is not hard to see that $n^{-1}\sum_{t=1}^{n}\tilde{z}_t[u_t+(1-\hat{\rho}_n)e_{t-1}] \to_p 0$. Intuitively, $\tilde{z}_t$ behaves asymptotically like the differenced regressors $(v_t', \Delta\mathbf{v}_t')'$, and u and v are uncorrelated by assumption. Our remaining task is to show that

$$n^{-1/2}\sum_{t=1}^{n}\tilde{z}_t[u_t + (1-\hat{\rho}_n)e_{t-1}] = n^{-1/2}\sum_{t=1}^{n}\Delta z_t u_t + o_p(1) \to N(\mathbf{0}, \Lambda). \tag{36}$$

This can be shown using similar arguments as before. Combining (36) with (35), we obtain the limit distribution for $\tilde{\theta}_{\text{dgls}}$ as given in (12).

Appendix B: Some extensions

So far we have assumed that there is no constant term or deterministic time trends in the DGP of y_t. If there is a constant term, e.g.

$$y_t = \delta + \beta' x_t + \boldsymbol{\gamma}'\mathbf{v}_t + e_t = \kappa b_t + e_t,$$

where $\kappa = (\delta, \beta', \boldsymbol{\gamma}')'$ and $b_t = (1, x_t', \mathbf{v}_t')'$. Define

$$P_n = \begin{bmatrix} n^{-3/2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I_m n^{-2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_{m(2k+1)}n^{-1} \end{bmatrix}.$$

and

$$G_n = \begin{bmatrix} n^{1/2} & \mathbf{0} \\ \mathbf{0} & I_{m(2k+2)} \end{bmatrix}.$$

Then the OLS estimators for κ are

$$G_n(\hat{\kappa}_n - \kappa_0) = \left[G_n^{-1}\sum_{t=1}^{n} b_t b_t' P_n^{-1}\right]^{-1} P_n^{-1}\sum_{t=1}^{n} b_t e_t.$$

Therefore,

$$\begin{aligned} \begin{bmatrix} n^{-1/2}(\hat{\delta}_n - \delta) \\ \hat{\beta}_n - \beta_0 \\ \hat{\boldsymbol{\gamma}}_n - \boldsymbol{\gamma}_0 \end{bmatrix} &= \begin{bmatrix} 1 & n^{-3/2}\sum_{t=1}^{n} x_t' & n^{-1/2}\sum_{t=1}^{n}\mathbf{v}_t' \\ n^{-3/2}\sum_{t=1}^{n} x_t & n^{-2}\sum_{t=1}^{n} x_t x_t' & n^{-1}\sum_{t=1}^{n} x_t\mathbf{v}_t' \\ n^{-3/2}\sum_{t=1}^{n}\mathbf{v}_t & n^{-2}\sum_{t=1}^{n}\mathbf{v}_t x_t' & n^{-1}\sum_{t=1}^{n}\mathbf{v}_t\mathbf{v}_t' \end{bmatrix}^{-1} \begin{bmatrix} n^{-3/2}\sum_{t=1}^{n} e_t \\ n^{-2}\sum_{t=1}^{n} x_t e_t \\ n^{-1}\sum_{t=1}^{n}\mathbf{v}_t e_t \end{bmatrix} \\ &\to \begin{bmatrix} 1 & \int V(r)dr & \mathbf{V(1)} \\ \int V(r)dr & \int V(r)V(r)'dr & \Pi_v \\ \mathbf{0} & \mathbf{0} & \Gamma_{\mathbf{v},\mathbf{v}} \end{bmatrix}^{-1} \begin{bmatrix} \int U(r)dr \\ \int V(r)U(r)dr \\ \int U(r)d\mathbf{V(r)} \end{bmatrix}. \end{aligned}$$

Therefore, the OLS estimator of the intercept diverges in the spurious regression. If we do GLS correction or the differenced regression, the constant is canceled so we could have the same limit result as given by (8). Finally, consider the Cochrane-Orcutt feasible GLS estimation. Let $\hat{e}_t$ denote the OLS residual

$$\hat{e}_t = y_t - \hat{\delta}_n - \hat{\beta}_n' x_t - \hat{\boldsymbol{\gamma}}_n' \mathbf{v}_t.$$

Then do another OLS estimation of

$$\hat{e}_t = \hat{\rho}_n \hat{e}_{t-1} + \hat{u}_t.$$

Write

$$\begin{aligned} \hat{e}_t &= y_t - \hat{\kappa}_n' b_t \\ &= e_{t-1} + (\kappa_0 - \hat{\kappa}_n)' b_t + u_t \\ &= \hat{e}_{t-1} + [(\beta_0 - \hat{\beta}_n)' v_t + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)'(\mathbf{v}_t - \mathbf{v}_{t-1}) + u_t] \\ &= \hat{e}_{t-1} + g_t \end{aligned}$$

which takes the same form as in the previous section where no constant is included. Hence we still have

$$\hat{\rho}_n - 1 = \frac{\sum_{t=1}^n \hat{e}_{t-1} g_t}{\sum_{t=1}^n \hat{e}_{t-1}^2}.$$

Write the process of $\hat{e}_t$ as:

$$\hat{e}_t = y_t - \hat{\delta}_n - \hat{\beta}_n' x_t - \hat{\boldsymbol{\gamma}}_n' \mathbf{v}_t = (\beta_0 - \hat{\beta}_n)'(x_t - \bar{x}) + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)'(\mathbf{v}_t - \bar{\mathbf{v}}) + (e_t - \bar{e}). \tag{37}$$

Comparing equation (37) with (32), the only difference is that all terms in (37) are subtracted by their sample means. This leads to demeaned Brownian motions instead of standard Brownian motions in the limit of the distribution of $\hat{\rho}_n$. Using similar methods as in the previous section, we can show that

$$\hat{\rho}_n - 1 = o_p(1) \quad \text{and} \quad n(\hat{\rho}_n - 1) = O_p(1).$$

Next, conduct the Cochrane-Orcutt transformation as in (10), and consider the OLS estimator in the regression

$$\tilde{y}_t = \tilde{\beta}_n' \tilde{x}_t + \tilde{\boldsymbol{\gamma}}_n' \tilde{\mathbf{v}}_t + error.$$

Define $\tilde{z}_t = (\tilde{x}_t, \tilde{\mathbf{v}}_t)'$ and $\theta = (\beta', \boldsymbol{\gamma}')'$, then

$$\tilde{\theta}_{\text{fgls}} = \left[\sum_{t=1}^n \tilde{z}_t \tilde{z}_t'\right]^{-1} \left[\sum_{t=1}^n \tilde{z}_t \tilde{y}_t\right].$$

For $\tilde{y}_t$, write

$$\begin{aligned} \tilde{y}_t &= y_t - \hat{\rho}_n y_{t-1} \\ &= (1 - \hat{\rho}_n)\delta + \beta_0' \tilde{x}_t + \boldsymbol{\gamma}_0' \tilde{\mathbf{v}}_t + e_t + (1 - \hat{\rho}_n) e_{t-1} \\ &= \theta_0 z_t' + (1 - \hat{\rho}_n)\delta + e_t + (1 - \hat{\rho}_n) e_{t-1} \end{aligned}$$

Hence we can write

$$\hat{\theta}_{\text{fgls}} - \theta_0 = \left[\sum_{t=1}^n \tilde{z}_t \tilde{z}_t'\right]^{-1} \left[\sum_{t=1}^n \tilde{z}_t [(1 - \hat{\rho}_n)\delta + e_t + (1 - \hat{\rho}_n) e_{t-1}]\right]. \tag{38}$$

The only difference between (38) and (34) is that (38) has one additional term $(1-\hat{\rho}_n)\delta$. However, since δ is a constant and $1-\hat{\rho}_n = O_p(n^{-1})$, this term vanishes in the limit. Therefore, using the Cochrane-Orcutt transformation, the limit distribution of the estimators are the same regardless of whether we have a constant in the data generating process of the data. In either case, we have the same result as given by (8).

Appendix C: Proof of results in section 2.2

To show the limit distribution of the dynamic OLS estimator in the cointegration define

$$H_n = \begin{bmatrix} I_m n & 0 \\ 0 & I_{m(2k+1)} n^{1/2} \end{bmatrix}. \tag{39}$$

We can write

$$H_n(\hat{\theta}_{\text{dols}} - \theta_0) = \begin{bmatrix} n(\hat{\beta}_n - \beta_0) \\ n^{1/2}(\hat{\boldsymbol{\gamma}}_n - \boldsymbol{\gamma}_0) \end{bmatrix} = \begin{bmatrix} n^{-2}\sum_{t=1}^n x_t x_t' & n^{-3/2}\sum_{t=1}^n x_t \mathbf{v}_t' \\ n^{-3/2}\sum_{t=1}^n \mathbf{v}_t x_t' & n^{-1}\sum_{t=1}^n \mathbf{v}_t \mathbf{v}_t' \end{bmatrix}^{-1} \begin{bmatrix} n^{-1}\sum_{t=1}^n x_t e_t \\ n^{-1/2}\sum_{t=1}^n \mathbf{v}_t e_t \end{bmatrix}$$

For the first term on the right hand side,

$$\begin{bmatrix} n^{-2}\sum_{t=1}^n x_t x_t' & n^{-3/2}\sum_{t=1}^n x_t \mathbf{v}_t' \\ n^{-3/2}\sum_{t=1}^n \mathbf{v}_t x_t' & n^{-1}\sum_{t=1}^n \mathbf{v}_t \mathbf{v}_t' \end{bmatrix} \to \begin{bmatrix} \int_0^1 V(r)V(r)'dr & 0 \\ 0 & \Gamma_{\mathbf{v},\mathbf{v}} \end{bmatrix}. \tag{40}$$

Thus, the estimator of the I(1) and I(0) components are asymptotically independent. For the second term on the right hand side,

$$\begin{bmatrix} n^{-1}\sum_{t=1}^n x_t e_t \\ n^{-1/2}\sum_{t=1}^n \mathbf{v}_t e_t \end{bmatrix} \to_d \begin{bmatrix} \int_0^1 V(r)dE(r) \\ N(0,\Lambda_{\mathbf{v},e}) \end{bmatrix}, \tag{41}$$

where $\Lambda_{\mathbf{v},e}$ is the long run variance of $\mathbf{v}_t e_t$. Equation (14) then follows.

To show the limit distribution for FGLS estimator in regression (16), write

$$\begin{aligned} & n^{-1}\sum_{t=1}^n \hat{e}_t^2 \\ = \; & n^{-1}\sum_{t=1}^n e_t^2 + 2n^{-1}\left(\sum_{t=1}^n e_t z_t' H_n^{-1}\right) H_n(\theta_0 - \hat{\theta}_{\text{dols}}) + n^{-1}H_n(\theta_0 - \hat{\theta}_{\text{dols}})'\left(H_n^{-1}\sum_{t=1}^n z_t z_t' H_n^{-1}\right) H_n(\theta_0 - \hat{\theta}_{\text{dols}}) \\ = \; & n^{-1}\sum_{t=1}^n e_t^2 + o_p(1) \to \sigma_e^2. \end{aligned}$$

Similarly, we can show that

$$n^{-1}\sum_{t=1}^n \hat{e}_t \hat{e}_{t-1} = n^{-1}\sum_{t=1}^n e_t e_{t-1} + o_p(1) \to \psi_e \sigma_e^2,$$

where ψ_e is the first order autocorrelation coefficient of $\{e_t\}$. Then the OLS estimator

$$\hat{\rho}_n = \frac{n^{-1}\sum_{t=1}^n \hat{e}_t e_{t-1}}{n^{-1}\sum_{t=1}^n \hat{e}_t^2} \to_p \psi_e$$

Conduct the Cochrane-Orcutt transformation (10) and estimate

$$\tilde{y}_t = \beta'\tilde{x}_t + \boldsymbol{\gamma}'\tilde{\mathbf{v}}_t + error.$$

For the sequence of $\tilde{y}_t$, we can write it as

$$\tilde{y}_t = \beta_0' \tilde{x}_t + \boldsymbol{\gamma}_0' \tilde{\mathbf{v}}_t + \tilde{e}_t$$

where $\tilde{e}_t = e_t - \hat{\rho}_n e_{t-1}$. Using the same weight matrix H_n, write

$$\begin{bmatrix} n(\tilde{\beta}_n - \beta_0) \\ n^{1/2}(\tilde{\boldsymbol{\gamma}}_n - \boldsymbol{\gamma}_0) \end{bmatrix} = \left[H_n^{-1} \begin{bmatrix} \sum_{t=1}^n \tilde{x}_t \tilde{x}_t' & \sum_{t=1}^n \tilde{x}_t \tilde{\mathbf{v}}_t' \\ \sum_{t=1}^n \tilde{\mathbf{v}}_t \tilde{x}_t' & \sum_{t=1}^n \tilde{\mathbf{v}}_t \tilde{\mathbf{v}}_t' \end{bmatrix} H_n^{-1} \right]^{-1} \begin{bmatrix} n^{-1} \sum_{t=1}^n \tilde{x}_t \tilde{e}_t \\ n^{-1/2} \sum_{t=1}^n \tilde{\mathbf{v}}_t \tilde{e}_t \end{bmatrix} \tag{42}$$

Define that $\tilde{E}(r) = (1 - \psi_e)E(r)$ and $\tilde{V}(r) = (1 - \psi_e)V(r)$. By Lemma 2.1 in Phillips and Ouliaris (1990), $n^{-1/2}\tilde{x}_{[nr]} \to \tilde{V}(r)$ and $n^{-1/2}\sum_{t=1}^{[nr]} \tilde{e}_t \to \tilde{E}(r)$. Therefore we can show that

$$\begin{aligned} n^{-2} \sum_{t=1}^n \tilde{x}_t \tilde{x}_t' &\to \int_0^1 \tilde{V}(r)\tilde{V}(r)' dr \\ n^{-3/2} \sum_{t=1}^n \tilde{x}_t \tilde{\mathbf{v}}_t' &\to 0 \\ n^{-1} \sum_{t=1}^n \tilde{\mathbf{v}}_t \tilde{\mathbf{v}}_t' &\to P, \quad say. \end{aligned}$$

Hence, the limit of the denominator in (42) is

$$H_n^{-1} \left[\sum_{t=1}^n z_t z_t' \right] H_n^{-1} \to \begin{bmatrix} (1-\psi_e)^2 \int_0^1 V(r)V(r)' dr & 0 \\ 0 & P \end{bmatrix}.$$

Next, consider the numerator in (42). In fact, we are only interested in the first element,

$$n^{-1} \sum_{t=1}^n \tilde{x}_t \tilde{e}_t \to \int_0^1 \tilde{V}(r) d\tilde{E}(r).$$

Therefore, we obtain the limit distribution for $\tilde{\beta}_{\text{fgls}}$,

$$\begin{aligned} n(\tilde{\beta}_{\text{fgls}} - \beta_0) &\to \left(\int_0^1 \tilde{V}(r)\tilde{V}(r)' dr \right)^{-1} \left(\int_0^1 \tilde{V}(r) d\tilde{E}(r) \right) \\ &= \left(\int_0^1 V(r)V(r)' dr \right)^{-1} \left(\int_0^1 V(r) dE(r) \right), \end{aligned}$$

which is the smae as the limit of $\hat{\beta}_{\text{dols}}$. In the last equation, we cancled the term $(1 - \psi_e)^2$.

Appendix D: Proof of results in section 2.4

Under the null of cointegration,

$$\begin{aligned} & \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}}) \\ = & \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \beta_0) - \sqrt{n}(\hat{\beta}_{\text{dols}} - \beta_0) \\ = & \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \beta_0) + o_p(1) \\ \to & N(0, V_\beta) \end{aligned}$$

where V_β is the asymptotic variance of $\tilde{\beta}_{\text{dgls}}$ under the assumption of cointegration. Therefore,

$$h_n = n(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}})'(\hat{V}_\beta)^{-1}(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}}) \to \chi^2(m),$$

where $\hat{V}_\beta$ is a consistent estimator for V_β.

Under the alternative of spurious regressions,

$$\begin{aligned}
& \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}}) \\
= \; & \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \beta_0) - \sqrt{n}(\hat{\beta}_{\text{dols}} - \beta_0) \\
= \; & \sqrt{n}(\hat{\beta}_{\text{dols}} - \beta_0) + O_p(1) \\
= \; & O_p(\sqrt{n}).
\end{aligned}$$

In this case, the HAC estimator, $\hat{V}_\beta$, will converge to the asymptotic variance of $\tilde{\beta}_{\text{dgls}}$ under the assumption of I(1) errors. Hence, $h_n = O_p(n)$ under the alternative.

Next, in the extended test, where we allow endogeneity under the alternative, the regression becomes:

$$y_t = \beta' x_t + \boldsymbol{\gamma}' v_t + \phi q_t + e_t.$$

Similar as in Appendix A, the OLS estimators are

$$\begin{aligned}
\begin{bmatrix} \hat{\beta}_n - \beta_0 \\ \hat{\boldsymbol{\gamma}}_n - \boldsymbol{\gamma}_0 \end{bmatrix} &= \begin{bmatrix} n^{-2} \sum x_t x_t' & n^{-3/2} \sum x_t \mathbf{v}_t' \\ n^{-2} \sum \mathbf{v}_t x_t' & n^{-1} \sum \mathbf{v}_t \mathbf{v}_t' \end{bmatrix}^{-1} \begin{bmatrix} n^{-2} \sum_{t=1}^n x_t(\phi q_t + e_t) \\ n^{-1} \sum_{t=1}^n \mathbf{v}_t(\phi q_t + e_t) \end{bmatrix} \\
&\to \begin{bmatrix} \int_0^1 V(r)V(r)' dr & \Pi_v \\ \mathbf{0} & \Gamma_{\mathbf{v},\mathbf{v}} \end{bmatrix}^{-1} \begin{bmatrix} \int_0^1 V(r)U(r)dr \\ \int_0^1 U(r) d\mathbf{V}(\mathbf{r}) + \phi E(\mathbf{v}_t q_t) \end{bmatrix},
\end{aligned}$$

Due to endogeneity, the estimator in the differenced regression is not consistent either. The estimators $(\tilde{\beta}'_{\text{dgls}} - \beta_0', \tilde{\boldsymbol{\gamma}}'_{\text{dgls}} - \boldsymbol{\gamma}_0')' \to Q^{-1}\phi(E(v_t \Delta q_t), E(\Delta \mathbf{v}_t \Delta q_t)')'$. Let $\bar{\beta}$ denote the limit of $\tilde{\beta}_{\text{dgls}}$, then

$$\begin{aligned}
& \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}}) \\
= \; & \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \bar{\beta}) - \sqrt{n}(\hat{\beta}_{\text{dols}} - \bar{\beta}) \\
= \; & \sqrt{n}(\hat{\beta}_{\text{dols}} - \bar{\beta}) + O_p(1) \\
= \; & O_p(\sqrt{n}).
\end{aligned}$$

Finally, in the differenced regression, the variance estimate still converges. Therefore, the Hausman-test statistics is of order n under the alternative of spurious regressions no matter whether exogeneity holds or not.

Appendix E: Data descriptions

In the first two empirical analysis we use the same data set as in Stock and Watson (1993, page 817) for the U.S. money demand, and the data set of Cooley and Ogaki (1996, page 127) for the long-run intertemporal elasticity of substitution. Readers are referred to the original work for further details on data.

Per capita output series are extracted from the *Penn World Tables* of Robert Summers and Alan Heston (1991). They are annual data on real GDP per capita (RGDPCH) for four developing countries (Columbia, Ecuador, Egypt, and Pakistan) along with four industrial countries (Germany, Luxemburg, New Zealand, and Switzerland) over the period of 1950-1992.

In the PPP application we borrow the dataset from Kim (2004) who constructed the real exchange rate for total consumption using the general price deflator and the real exchange rate for traded and non-traded goods using implicit deflators for non-service consumption and service consumption, respectively.

Data are quarterly observations spanning from 1974 Q1 to 1998 Q4. The exchange rates for Canada, France, Italy, Japan, the United Kingdom, and the United States are taken from the International Financial Statistics (IFS) CD-ROM, and bilateral real exchange rates of traded and non-traded goods classified by type and total consumption deflators from the Quarterly National Accounts and Data Stream are studied.

Table 1: Finite sample performance of the Hausman cointegration test

	Hausman Test		Shin's Test	
T	power	size (5%)	power	size (5%)
50	0.621	0.114	0.249	0.141
100	0.688	0.072	0.402	0.171
200	0.754	0.050	0.652	0.199
300	0.783	0.039	0.775	0.184
500	0.816	0.040	0.882	0.181

Note: Hausman test represents the Hausman-Wu-type cointegration test as stipulated in section 2.4. Nonparametric estimator of long run variance is used based on the QS kernel with the bandwidth of 'integer$[8(T/100)^{1/4}]$'.

Table 2: Application to Long Run U.S. Money Demand

Estimator	k	Equation 1		Equation 2		Equation 3	
		$\tilde{\beta}$	$\hat{\gamma}$	$\tilde{\beta}$	$\hat{\gamma}$	$\tilde{\beta}$	$\hat{\gamma}$
DOLS	0	0.944 (0.054)	-0.090 (0.015)	0.889 (0.057)	-0.308 (0.058)	0.850 (0.085)	0.906 (0.280)
	1	0.958 (0.048)	-0.096 (0.014)	0.884 (0.046)	-0.313 (0.045)	0.843 (0.066)	0.915 (0.216)
	2	0.970 (0.051)	-0.101 (0.014)	0.879 (0.044)	-0.320 (0.043)	0.837 (0.072)	0.941 (0.233)
	3	0.975 (0.055)	-0.104 (0.015)	0.871 (0.036)	-0.328 (0.035)	0.832 (0.062)	0.975 (0.205)
	4	0.967 (0.054)	-0.108 (0.015)	0.855 (0.029)	-0.334 (0.028)	0.824 (0.065)	0.995 (0.215)
	BIC						
	[lag]	[3]		[5]		[5]	
GLS-corrected	0	0.407 (0.081)	-0.014 (0.004)	0.419 (0.079)	-0.086 (0.022)	0.388 (0.078)	0.300 (0.082)
	1	0.654 (0.119)	-0.025 (0.010)	0.685 (0.115)	-0.177 (0.046)	0.643 (0.115)	0.506 (0.148)
	2	0.837 (0.134)	-0.050 (0.013)	0.848 (0.130)	-0.248 (0.053)	0.787 (0.133)	0.620 (0.161)
	3	0.856 (0.145)	-0.067 (0.017)	0.884 (0.140)	-0.289 (0.061)	0.816 (0.146)	0.725 (0.185)
	4	0.962 (0.161)	-0.086 (0.022)	0.898 (0.151)	-0.283 (0.067)	0.811 (0.153)	0.654 (0.195)
	BIC						
	[lag]	[2]		[2]		[5]	
FGLS-corrected AR(1)	0	0.942 (0.052)	-0.083 (0.023)	0.893 (0.049)	-0.290 (0.079)	0.858 (0.071)	0.850 (0.435)
	1	0.888 (0.040)	-0.065 (0.009)	0.872 (0.035)	-0.278 (0.030)	0.815 (0.045)	0.744 (0.115)
	2	0.940 (0.045)	-0.081 (0.010)	0.901 (0.036)	-0.309 (0.031)	0.840 (0.054)	0.797 (0.128)
	3	0.980 (0.050)	-0.096 (0.011)	0.905 (0.029)	-0.330 (0.026)	0.851 (0.046)	0.912 (0.124)
	4	1.010 (0.045)	-0.108 (0.011)	0.886 (0.025)	-0.333 (0.023)	0.833 (0.051)	0.895 (0.133)
	BIC						
	[lag]	[4]		[5]		[5]	
FGLS-corrected AR(2)	0	0.942 (0.052)	-0.083 (0.023)	0.893 (0.049)	-0.290 (0.079)	0.858 (0.071)	0.850 (0.435)
	1	0.900 (0.039)	-0.069 (0.009)	0.872 (0.038)	-0.276 (0.031)	0.809 (0.049)	0.722 (0.118)
	2	0.948 (0.042)	-0.086 (0.010)	0.894 (0.033)	-0.312 (0.029)	0.839 (0.049)	0.830 (0.131)
	3	0.991 (0.044)	-0.100 (0.011)	0.907 (0.029)	-0.332 (0.026)	0.853 (0.050)	0.903 (0.128)
	4	1.012 (0.042)	-0.109 (0.010)	0.889 (0.026)	-0.335 (0.023)	0.827 (0.061)	0.856 (0.142)
	BIC						
	[lag]	[3]		[5]		[5]	
Hausman-Test	0	4.147		2.643		2.270	
	1	0.691		0.041		0.113	
	2	0.549		0.007		0.012	
	3	0.001		0.092		0.010	
	4	0.485		0.246		0.188	
ADF-based Test	0	-3.768‡		-3.128‡		-2.726†	

Note:

$$
\begin{aligned}
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma i_t + e_t, \quad \text{(equation 1)} \\
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma \ln(i_t) + e_t, \quad \text{(equation 2)} \\
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma \ln\left[\frac{1+i_t}{i_t}\right] + e_t. \quad \text{(equation 3)}
\end{aligned}
$$

'GLS-corrected (FGLS-corrected)' denotes the GLS (FGLS) corrected dynamic regression estimator. Figures in the parenthesis represent standard errors. 'k' denotes the maximum length of leads and lags. In FGLS corrected estimation, the serial correlation coefficient in error term is estimated before being applied to the Cochrane-Orcutt transformation, whereas it is assumed to be unity in GLS corrected estimation which is analogous to regressing the first difference of variables without a constant term. Hausman test represents the Hausman-Wu-type cointegration test as stipulated in section 2.4. The test statistic is constructed as $(\hat{\Gamma}_{dgls} - \tilde{\Gamma}_{dols})\Sigma(\hat{\Gamma}_{dgls} - \tilde{\Gamma}_{dols})' \to \chi^2(2)$ where $\Gamma = [\beta, \gamma]$ and $\Sigma = \begin{bmatrix} var(\tilde{\beta}_{dgls} & cov(\tilde{\beta}_{dgls}, \tilde{\gamma}_{dgls}) \\ cov(\tilde{\beta}_{dgls}, \tilde{\gamma}_{dgls}) & var(\tilde{\gamma}_{dgls}) \end{bmatrix}$. The critical values of $\chi^2(2)$ are 4.61, 5.99 and 9.21 for 10%, 5%, and 1% significance levels. The critical values of the ADF-based tests are -2.88 and -2.57 for 5% and 10% significance levels. ‡(†) represents that the null hypothesis can be rejected at 5% (10%).

Table 3: Application to Preference Parameter (β) Estimation

Estimator	k	DOLS	GLS-corrected	FGLS (AR(1))	FGLS (AR(2))	Hausman test	ADF-based test
	0	1.865 (0.218)	0.222 (0.061)	1.874 (0.151)	1.874 (0.151)	140.35‡	-2.302
	1	1.865 (0.192)	0.628 (0.091)	0.983 (0.087)	0.983 (0.087)	102.46‡	-2.861†
	2	1.870 (0.181)	0.720 (0.102)	1.160 (0.089)	1.160 (0.089)	60.38‡	-2.700†
	3	1.873 (0.193)	0.850 (0.110)	1.199 (0.097)	1.199 (0.097)	41.53‡	-1.866
ND	4	1.877 (0.204)	0.963 (0.117)	1.207 (0.107)	1.207 (0.107)	37.01‡	-1.984
	5	1.880 (0.196)	1.041 (0.123)	1.391 (0.104)	1.391 (0.104)	34.68‡	-1.937
	6	1.888 (0.202)	1.129 (0.125)	1.437 (0.109)	1.488 (0.109)	32.63‡	-1.969
	7	1.891 (0.200)	1.185 (0.129)	1.533 (0.111)	1.558 (0.114)	35.05‡	-2.043
	BIC [lag]	[0]	[3]	[4]	[4]		
	0	1.102 (0.052)	0.480 (0.066)	1.106 (0.095)	1.106 (0.095)	15.93‡	-2.867†
	1	1.103 (0.052)	0.796 (0.080)	0.912 (0.041)	0.932 (0.035)	8.22‡	-1.596
	2	1.102 (0.042)	0.855 (0.084)	0.952 (0.034)	0.911 (0.038)	3.88†	-1.996
	3	1.100 (0.041)	0.924 (0.085)	0.978 (0.032)	0.975 (0.031)	1.60	-2.177
NDS	4	1.099 (0.036)	0.967 (0.087)	0.998 (0.030)	1.000 (0.029)	0.98	-2.226
	5	1.095 (0.033)	0.995 (0.088)	1.018 (0.028)	1.009 (0.029)	0.58	-2.374
	6	1.091 (0.030)	1.019 (0.089)	1.031 (0.025)	1.025 (0.025)	0.44	-2.804†
	7	1.088 (0.029)	1.031 (0.091)	1.040 (0.023)	1.042 (0.024)	0.77	-3.485‡
	BIC [lag]	[0]	[3]	[3]	[3]		

Note: Results for $W(t) = \frac{v'(l(t))}{C(t)^{-\beta}}$. 'GLS-corrected (FGLS-corrected)' denotes the GLS (FGLS) corrected dynamic regression estimator. Figures in parenthesis represent standard errors. 'k' denotes the maximum length of leads and lags. In FGLS corrected estimation, the serial correlation coefficient in the error term is estimated before being applied to the Cochrane-Orcutt transformation, whereas it is assumed to be unity in GLS corrected estimation which is analogous to regressing the first difference of variables without constant term. Hausman test represents the Hausman-Wu-type cointegration test as stipulated in section 2.4. The test statistic is constructed as $\frac{(\hat{\beta}_{dgls}-\tilde{\beta}_{dols})^2)}{Var(\hat{\beta}_{dgls})} \to \chi^2(1)$. The critical values of $\chi^2(1)$ are 2.71, 3.84 and 6.63 for ten, five, and one percent significance levels. The critical values of the ADF-based tests are -2.88 and -2.57 for 5% and 10% significance levels. ‡(†) represents that the null hypothesis can be rejected at 5% (10%) significance level.

Table 4-1: Application to Output Convergence (Regressand: developing countries; Regressor: industrial countries)

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
COL	GER	0	0.916 (0.108)	0.372 (0.133)	0.851 (0.009)	3.668†
		1	1.011 (0.145)	0.550 (0.143)	1.224 (0.094)	9.261‡
		2	1.125 (0.256)	0.663 (0.165)	1.278 (0.140)	11.294‡
		3	1.160 (0.342)	0.691 (0.192)	1.263 (0.143)	13.996‡
		4	1.233 (0.449)	0.767 (0.223)	1.227 (0.151)	11.654‡
		BIC	[1]	[1]	[1]	
	LUX	0	1.228 (0.270)	0.059 (0.136)	1.203 (0.056)	4.715‡
		1	1.313 (0.362)	0.568 (0.198)	0.596 (0.199)	6.544‡
		2	1.390 (0.141)	0.611 (0.229)	1.239 (0.107)	4.494‡
		3	1.436 (0.144)	0.831 (0.264)	1.280 (0.122)	3.094†
		4	1.476 (0.100)	1.025 (0.287)	1.396 (0.091)	3.012†
		BIC	[5]	[1]	[1]	
	NZL	0	1.674 (0.317)	0.299 (0.155)	1.658 (0.115)	13.819‡
		1	1.737 (0.301)	0.829 (0.257)	1.666 (0.260)	9.067‡
		2	1.794 (0.388)	1.090 (0.298)	1.653 (0.331)	3.104†
		3	1.837 (0.456)	1.355 (0.324)	1.940 (0.412)	2.205
		4	1.852 (0.594)	1.473 (0.359)	2.719 (0.497)	0.109
		BIC	[3]	[1]	[4]	
	SWI	0	1.344 (0.243)	0.388 (0.169)	1.311 (0.075)	4.219‡
		1	1.422 (0.266)	0.666 (0.229)	1.464 (0.246)	3.127†
		2	1.498 (0.387)	0.991 (0.235)	1.571 (0.372)	4.359‡
		3	1.537 (0.483)	1.080 (0.268)	1.636 (0.462)	6.130‡
		4	1.756 (0.453)	1.176 (0.297)	2.005 (0.254)	5.398‡
		BIC	[2]	[2]	[2]	
ECU	GER	0	0.746 (0.112)	0.328 (0.155)	0.696 (0.007)	5.410‡
		1	0.858 (0.144)	0.454 (0.177)	0.930 (0.096)	3.464†
		2	0.981 (0.193)	0.526 (0.213)	0.994 (0.103)	4.720‡
		3	1.020 (0.269)	0.727 (0.241)	1.066 (0.131)	5.272‡
		4	0.999 (0.339)	0.771 (0.283)	0.985 (0.157)	3.305†
		BIC	[0]	[0]	[1]	
	LUX	0	0.980 (0.233)	0.267 (0.148)	0.972 (0.031)	8.272‡
		1	1.014 (0.257)	0.315 (0.245)	0.649 (0.188)	11.803‡
		2	1.055 (0.167)	0.330 (0.282)	0.935 (0.127)	4.465‡
		3	1.059 (0.196)	0.617 (0.321)	0.932 (0.164)	3.543†
		4	1.038 (0.169)	0.809 (0.355)	0.937 (0.140)	1.487
		BIC	[0]	[0]	[1]	
	NZL	0	1.354 (0.318)	0.294 (0.177)	1.347 (0.079)	5.338‡
		1	1.377 (0.268)	0.675 (0.314)	1.392 (0.258)	5.694‡
		2	1.382 (0.273)	0.598 (0.381)	1.453 (0.251)	3.645†
		3	1.447 (0.311)	0.936 (0.409)	1.454 (0.303)	3.419†
		4	1.538 (0.582)	1.409 (0.384)	1.701 (0.633)	0.065
		BIC	[4]	[0]	[4]	
	SWI	0	1.121 (0.185)	0.557 (0.182)	1.088 (0.048)	10.777‡
		1	1.177 (0.176)	0.655 (0.260)	1.320 (0.172)	8.719‡
		2	1.254 (0.169)	0.722 (0.304)	1.413 (0.143)	5.337‡
		3	1.333 (0.202)	1.036 (0.312)	1.579 (0.170)	4.840‡
		4	1.542 (0.170)	1.208 (0.331)	1.692 (0.133)	0.492
		BIC	[0]	[0]	[3]	

Note: See the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). Regression equation is $ln(y_{DEV}) = \alpha + \beta ln(y_{IND}) + e$.

Table 4-1: Continued–

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
EGT	GER	0	0.916 (0.108)	0.372 (0.133)	0.851 (0.009)	5.032‡
		1	1.011 (0.145)	0.550 (0.143)	1.224 (0.094)	6.349‡
		2	1.125 (0.256)	0.663 (0.165)	1.278 (0.140)	6.886‡
		3	1.160 (0.342)	0.691 (0.192)	1.263 (0.143)	7.081‡
		4	1.233 (0.449)	0.767 (0.223)	1.227 (0.151)	3.663†
		BIC	[1]	[1]	[1]	
	LUX	0	1.228 (0.270)	0.059 (0.136)	1.203 (0.056)	16.382‡
		1	1.313 (0.362)	0.568 (0.198)	0.596 (0.199)	15.242‡
		2	1.390 (0.141)	0.611 (0.229)	1.239 (0.107)	5.224‡
		3	1.436 (0.144)	0.831 (0.264)	1.280 (0.122)	3.104†
		4	1.476 (0.100)	1.025 (0.287)	1.396 (0.091)	0.951
		BIC	[5]	[1]	[1]	
	NZL	0	1.674 (0.317)	0.299 (0.155)	1.658 (0.115)	11.892‡
		1	1.737 (0.301)	0.829 (0.257)	1.666 (0.260)	6.955‡
		2	1.794 (0.388)	1.090 (0.298)	1.653 (0.331)	2.523
		3	1.837 (0.456)	1.355 (0.324)	1.940 (0.412)	2.061
		4	1.852 (0.594)	1.473 (0.359)	2.719 (0.497)	0.203
		BIC	[3]	[1]	[4]	
	SWI	0	1.344 (0.243)	0.388 (0.169)	1.311 (0.075)	13.502‡
		1	1.422 (0.266)	0.666 (0.229)	1.464 (0.246)	5.930‡
		2	1.498 (0.387)	0.991 (0.235)	1.571 (0.372)	3.723†
		3	1.537 (0.483)	1.080 (0.268)	1.636 (0.462)	5.455‡
		4	1.756 (0.453)	1.176 (0.297)	2.005 (0.254)	4.184‡
		BIC	[2]	[2]	[2]	
PAK	GER	0	0.746 (0.112)	0.328 (0.155)	0.696 (0.007)	4.188‡
		1	0.858 (0.144)	0.454 (0.177)	0.930 (0.096)	6.657†
		2	0.981 (0.193)	0.526 (0.213)	0.994 (0.103)	2.638
		3	1.020 (0.269)	0.727 (0.241)	1.066 (0.131)	1.212
		4	0.999 (0.339)	0.771 (0.283)	0.985 (0.157)	0.127
		BIC	[0]	[0]	[1]	
	LUX	0	0.980 (0.233)	0.267 (0.148)	0.972 (0.031)	7.201‡
		1	1.014 (0.257)	0.315 (0.245)	0.649 (0.188)	7.329‡
		2	1.055 (0.167)	0.330 (0.282)	0.935 (0.127)	4.463‡
		3	1.059 (0.196)	0.617 (0.321)	0.932 (0.164)	0.864
		4	1.038 (0.169)	0.809 (0.355)	0.937 (0.140)	0.001
		BIC	[0]	[0]	[1]	
	NZL	0	1.354 (0.318)	0.294 (0.177)	1.347 (0.079)	2.775†
		1	1.377 (0.268)	0.675 (0.314)	1.392 (0.258)	2.642
		2	1.382 (0.273)	0.598 (0.381)	1.453 (0.251)	2.090
		3	1.447 (0.311)	0.936 (0.409)	1.454 (0.303)	0.161
		4	1.538 (0.582)	1.409 (0.384)	1.701 (0.633)	1.559
		BIC	[4]	[0]	[4]	
	SWI	0	1.121 (0.185)	0.557 (0.182)	1.088 (0.048)	3.709†
		1	1.177 (0.176)	0.655 (0.260)	1.320 (0.172)	3.034†
		2	1.254 (0.169)	0.722 (0.304)	1.413 (0.143)	0.767
		3	1.333 (0.202)	1.036 (0.312)	1.579 (0.170)	1.055
		4	1.542 (0.170)	1.208 (0.331)	1.692 (0.133)	0.982
		BIC	[0]	[0]	[3]	

Note: Refer to the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). Regression equation is $ln(y_{DEV}) = \alpha + \beta ln(y_{IND}) + e$.

Table 4-2: Application to Output Convergence (Regressand: industrial countries; Regressor: industrial countries)

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
		0	1.291 (0.246)	0.623 (0.112)	1.330 (0.044)	2.970†
		1	1.302 (0.207)	1.018 (0.162)	0.719 (0.079)	1.219
	LUX	2	1.312 (0.174)	1.094 (0.174)	0.849 (0.077)	2.314
		3	1.294 (0.222)	1.040 (0.184)	0.706 (0.101)	2.046
		4	1.293 (0.262)	1.074 (0.178)	0.694 (0.119)	0.224
		BIC	[4]	[4]	[1]	
		0	1.829 (0.125)	0.494 (0.159)	1.889 (0.078)	8.602‡
		1	1.802 (0.165)	1.089 (0.240)	1.465 (0.165)	4.908‡
GER	NZL	2	1.775 (0.105)	1.268 (0.285)	1.658 (0.096)	3.161†
		3	1.760 (0.060)	1.480 (0.301)	1.643 (0.056)	0.029
		4	1.692 (0.076)	1.671 (0.174)	1.746 (0.079)	0.318
		BIC	[5]	[5]	[5]	
		0	1.494 (0.087)	0.811 (0.147)	1.531 (0.064)	2.006
		1	1.482 (0.127)	1.260 (0.150)	1.305 (0.121)	0.631
	SWI	2	1.452 (0.127)	1.326 (0.169)	1.379 (0.107)	0.223
		3	1.413 (0.118)	1.339 (0.171)	1.331 (0.090)	0.023
		4	1.369 (0.184)	1.347 (0.148)	1.421 (0.134)	0.329
		BIC	[4]	[4]	[4]	
		0	0.726 (0.239)	0.702 (0.126)	0.678 (0.045)	1.914
		1	0.800 (0.168)	0.581 (0.136)	0.909 (0.092)	4.105‡
	GER	2	0.925 (0.191)	0.536 (0.163)	0.938 (0.088)	2.793†
		3	1.030 (0.265)	0.723 (0.185)	1.039 (0.111)	1.971
		4	1.104 (0.328)	0.838 (0.209)	1.000 (0.133)	3.667†
		BIC	[0]	[1]	[1]	
		0	1.289 (0.633)	0.412 (0.176)	1.290 (0.040)	3.015†
		1	1.256 (0.852)	0.796 (0.294)	0.445 (0.447)	1.272
LUX	NZL	2	1.227 (0.555)	0.797 (0.354)	1.231 (0.503)	0.201
		3	1.255 (0.886)	1.098 (0.362)	0.464 (0.814)	0.067
		4	1.188 (0.676)	1.277 (0.383)	2.311 (0.608)	0.094
		BIC	[3]	[1]	[1]	
		0	1.040 (0.499)	0.603 (0.184)	1.035 (0.007)	0.219
		1	1.016 (0.642)	0.898 (0.240)	0.932 (0.480)	0.617
	SWI	2	1.006 (0.461)	0.771 (0.273)	1.443 (0.469)	0.126
		3	0.984 (0.616)	0.875 (0.309)	1.578 (0.626)	0.009
		4	0.932 (0.900)	0.967 (0.336)	1.766 (0.845)	0.157
		BIC	[1]	[1]	[1]	

Note: Refer to the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). Regression equation is $ln(y_{IND}) = \alpha + \beta ln(y_{IND}) + e$.

Table 4-2: Continued–

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
		0	0.509 (0.044)	0.395 (0.127)	0.501 (0.046)	0.130
		1	0.498 (0.060)	0.449 (0.149)	0.507 (0.047)	0.053
	GER	2	0.498 (0.075)	0.459 (0.175)	0.488 (0.055)	2.174
		3	0.520 (0.096)	0.285 (0.177)	0.511 (0.060)	2.115
		4	0.517 (0.142)	0.315 (0.170)	0.452 (0.081)	0.106
		BIC	[5]	[5]	[5]	
		0	0.676 (0.232)	0.292 (0.125)	0.671 (0.044)	0.605
		1	0.703 (0.233)	0.552 (0.202)	0.314 (0.161)	0.948
NZL	LUX	2	0.731 (0.133)	0.576 (0.224)	0.485 (0.105)	2.393
		3	0.734 (0.209)	0.483 (0.206)	0.496 (0.164)	0.389
		4	0.745 (0.184)	0.628 (0.228)	0.597 (0.144)	0.014
		BIC	[5]	[5]	[5]	
		0	0.797 (0.043)	0.613 (0.145)	0.783 (0.013)	0.155
		1	0.801 (0.050)	0.742 (0.201)	0.791 (0.050)	0.060
	SWI	2	0.778 (0.047)	0.817 (0.224)	0.769 (0.046)	0.203
		3	0.763 (0.057)	0.681 (0.227)	0.789 (0.055)	0.039
		4	0.844 (0.080)	0.812 (0.238)	0.832 (0.080)	0.068
		BIC	[0]	[0]	[3]	
		0	0.634 (0.045)	0.531 (0.097)	0.639 (0.045)	0.077
		1	0.649 (0.079)	0.623 (0.100)	0.653 (0.063)	0.042
	GER	2	0.671 (0.119)	0.650 (0.114)	0.665 (0.078)	0.406
		3	0.676 (0.179)	0.572 (0.130)	0.672 (0.091)	0.078
		4	0.709 (0.278)	0.672 (0.149)	0.696 (0.122)	4.580
		BIC	[1]	[1]	[1]	
		0	0.831 (0.340)	0.350 (0.107)	0.848 (0.022)	0.934
		1	0.850 (0.244)	0.724 (0.162)	0.421 (0.142)	0.626
SWI	LUX	2	0.857 (0.214)	0.746 (0.183)	0.469 (0.137)	0.606
		3	0.845 (0.250)	0.702 (0.214)	0.423 (0.169)	1.101
		4	0.824 (0.268)	0.611 (0.238)	0.442 (0.182)	0.133
		BIC	[0]	[1]	[1]	
		0	1.206 (0.073)	0.503 (0.119)	1.233 (0.014)	1.700
		1	1.219 (0.098)	0.991 (0.187)	1.148 (0.106)	1.219
	NZL	2	1.224 (0.095)	1.005 (0.232)	1.171 (0.095)	0.634
		3	1.233 (0.088)	1.056 (0.260)	1.180 (0.091)	0.036
		4	1.234 (0.116)	1.194 (0.272)	1.263 (0.124)	0.051
		BIC	[1]	[1]	[1]	

Note: Refer to the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). Regression equation is $ln(y_{IND}) = \alpha + \beta ln(y_{IND}) + e$.

Table 5: Application to PPP for traded and non-traded goods

Estimator	k	Traded Goods					Non-traded Goods				
		FRA	ITA	JPN	U.K.	U.S.	FRA	ITA	JPN	U.K.	U.S.
	0	1.149	1.379	1.558	1.306	1.053	1.872	2.142	2.357	2.059	1.711
		(0.312)	(0.165)	(0.326)	(0.201)	(0.198)	(0.165)	(0.241)	(0.299)	(0.278)	(0.443)
	1	1.179	1.456	1.485	1.439	1.078	1.887	2.175	2.376	2.066	1.728
		(0.424)	(0.217)	(0.459)	(0.262)	(0.276)	(0.151)	(0.222)	(0.299)	(0.290)	(0.441)
	2	1.195	1.511	1.442	1.533	1.092	1.898	2.216	2.399	2.067	1.748
DOLS		(0.515)	(0.277)	(0.646)	(0.342)	(0.328)	(0.161)	(0.237)	(0.331)	(0.272)	(0.473)
	3	1.186	1.531	1.390	1.571	1.102	1.888	2.250	2.397	2.054	1.762
		(0.524)	(0.308)	(0.561)	(0.392)	(0.381)	(0.165)	(0.247)	(0.346)	(0.261)	(0.511)
	4	1.195	1.553	1.388	1.613	1.109	1.871	2.287	2.402	2.042	1.763
		(0.502)	(0.353)	(0.471)	(0.412)	(0.401)	(0.159)	(0.252)	(0.343)	(0.233)	(0.484)
	BIC	[0]	[0]	[0]	[2]	[0]	[0]	[0]	[5]	[1]	[0]
	0	0.833	1.114	1.086	1.030	0.919	0.375	0.448	0.372	0.351	0.159
		(0.393)	(0.381)	(0.411)	(0.365)	(0.140)	(0.178)	(0.176)	(0.198)	(0.171)	(0.080)
	1	0.984	1.086	1.221	1.324	1.027	0.864	0.900	0.888	0.988	0.397
		(0.454)	(0.436)	(0.477)	(0.419)	(0.161)	(0.315)	(0.309)	(0.352)	(0.295)	(0.142)
	2	1.259	1.333	1.415	1.516	1.103	1.127	1.104	1.046	1.001	0.569
GLS-		(0.469)	(0.454)	(0.477)	(0.430)	(0.167)	(0.369)	(0.368)	(0.408)	(0.355)	(0.166)
corrected	3	1.374	1.391	1.246	1.560	1.158	1.255	1.171	1.192	1.088	0.751
		(0.501)	(0.485)	(0.504)	(0.460)	(0.176)	(0.422)	(0.422)	(0.456)	(0.407)	(0.185)
	4	1.549	1.661	1.248	1.600	1.184	1.577	1.411	1.379	1.166	0.785
		(0.520)	(0.511)	(0.535)	(0.492)	(0.188)	(0.456)	(0.468)	(0.503)	(0.457)	(0.208)
	BIC	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[1]	[0]
	0	1.156	1.358	1.605	1.269	1.049	1.868	2.129	2.354	2.063	1.712
		(0.259)	(1.163)	(0.783)	(0.125)	(0.054)	(0.232)	(1.223)	(0.760)	(0.156)	(0.073)
	1	1.229	1.456	1.607	1.248	0.766	1.909	2.005	1.947	1.671	0.291
		(0.329)	(0.149)	(0.339)	(0.162)	(0.242)	(0.141)	(0.201)	(0.257)	(0.226)	(0.159)
	2	1.178	1.487	1.717	1.323	0.802	1.932	2.102	2.095	1.864	0.357
FGLS-		(0.355)	(0.181)	(0.396)	(0.217)	(0.266)	(0.152)	(0.218)	(0.286)	(0.217)	(0.191)
corrected	3	1.152	1.463	1.625	1.376	0.822	1.920	2.158	2.173	1.983	0.392
		(0.339)	(0.204)	(0.318)	(0.250)	(0.301)	(0.157)	(0.233)	(0.311)	(0.214)	(0.216)
	4	1.252	1.603	1.506	1.423	0.826	1.922	2.285	2.202	2.052	0.631
		(0.333)	(0.245)	(0.277)	(0.272)	(0.284)	(0.153)	(0.240)	(0.310)	(0.189)	(0.234)
	BIC	[1]	[1]	[2]	[1]	[1]	[2]	[1]	[5]	[1]	[3]
	0	0.182	0.883	0.256	0.080	0.132	12.667‡	20.200‡	16.524‡	13.972‡	18.459‡
Hausman	1	0.022	0.209	0.002	0.002	0.006	5.097‡	10.603‡	9.597‡	8.118‡	17.424‡
Test	2	0.171	0.109	0.051	0.001	0.134	2.317	6.604‡	6.494‡	4.912‡	16.549‡
	3	0.553	0.058	0.039	0.001	0.222	0.424	3.363†	3.738†	3.019†	14.485‡
	4	0.173	0.008	0.016	0.011	0.007	0.096	1.934	0.874	1.813	12.988‡

Note: Results are for $f_t^T = \alpha + \beta p_t^T + e_t$ and $f_t^N = \alpha + \beta p_t^N + e_t$ using Canada as a base country. Figures in parenthesis represent standard errors. 'k' denotes the maximum length of leads and lags. Hausman-Wu test represents the Hausman-Wu-type cointegration test as stipulated in section 2.4. The test statistic is constructed as $\frac{(\hat{\beta}_{dgls} - \tilde{\beta}_{dols})^2)}{Var(\tilde{\beta}_{dgls})} \to \chi^2(1)$. The critical values of $\chi^2(1)$ are 2.71, 3.84 and 6.63 for ten, five, and one percent significance level. ‡(†) represents that the null hypothesis of $\hat{\beta}_{dgls} = \tilde{\beta}_{dols}$ can be rejected at 5% (10%) significance level.

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