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Foundations and Applications of Modern Nonparametric Statistics



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Outline

1 Robust Risk Management

Motivation. Market Risk

Adaptive univariate volatility estimation

Accounting for heavy tails

ICA: dimension reduction

Conclusion and Outlook

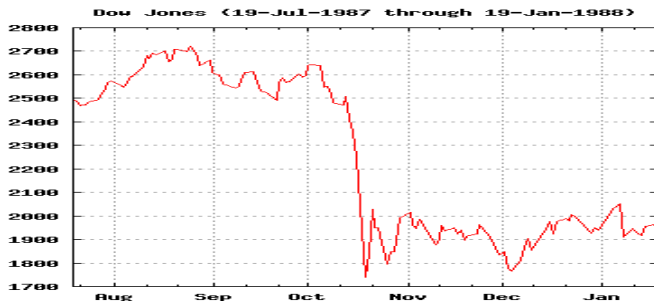
References

Outline

1. Motivation
2. Nonstationarity: adaptive volatility (AV) estimation, procedure and some theoretical properties
3. Heavy tails: generalized hyperbolic (GH) distribution and AV
4. Dimension reduction: Independent component analysis (ICA)
5. Conclusion

Stock market crash

October 19 1987 Dow Jones industrial dropped by over 500 points and the consequent economic depression



About market risk

Market risk: uncertainty due to changes in market prices and rates, the correlations among them and their levels of volatility, Jorion (2001).

- ▶ Regulatory: risk charge w.r.t. 1% risk level over the last 250 days.
 - ▶ ensure the adequacy of capital
 - ▶ restrict the happening of large losses
- ▶ Internal supervisory: measuring and controlling risk level of holding portfolios

Target: estimate distribution (quantile) of returns.

Measuring risk exposures

$$x_t = \Sigma_t^{1/2} \varepsilon_t$$

$x_t \in \mathbb{R}^d$: asset returns with $\text{Var}(x_t | \mathcal{F}_{t-1}) = \Sigma_t$, ε_t standardized stochastic innovations.

Target: estimate distribution (quantile) of returns.

Critical gaps

| | Standard assumptions | Stylized facts |
|-------------|--------------------------------------|------------------|
| vola. model | stationary (Black-Scholes/ GARCH) | nonstationary |
| innovations | Gaussian (e.g. RiskMetrics) | heavy tails |
| dimension | low-dimensional | high-dimensional |

Illustration of nonstationarity

The realized variances, the sum of squared returns sampled at 15 minutes tick-by-tick, of Dow Jones Euro StoXX 50 Index futures.

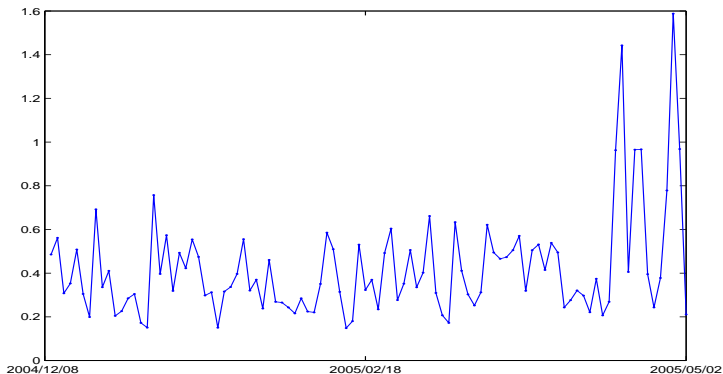
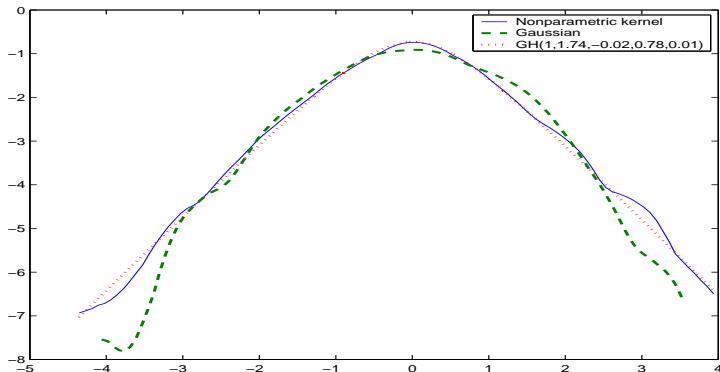


Illustration of heavy tails

Estimated (log) density of the daily devolatilized DEM/USD returns. Time interval: 1979/12/01 to 1994/04/01 (3719 observations).



(Classical) Volatility estimation

Model: $R_t = \sqrt{\theta_t} \varepsilon_t$, where $R_t \in \mathbb{R}$, $\varepsilon_t | \mathcal{F}_{t-1} \sim \mathcal{N}(0, 1)$

Suppose $\theta_t \equiv \theta^*$ (**homogeneity**) for $t = 1, \dots, T$.

Maximum Likelihood Estimate (MLE):

$$\tilde{\theta} = \operatorname{argmax}_{\theta \in \Theta} L(\theta) = \operatorname{argmax}_{\theta \in \Theta} \sum_t \ell(R_t, \theta)$$

with $\ell(R_t, \theta) = -(1/2) \log(2\pi\theta) - R_t^2/(2\theta)$. Leads to

$$\tilde{\theta} = \frac{1}{T} \sum_t R_t^2.$$

Standard ways of accounting for nonstationarity

- ▷ Reestimate parameters using a **time varying window**, e.g.

$$\tilde{\theta}_t = \frac{1}{M} \sum_{i=1}^M R_{t-i}^2, \quad M = 250.$$

- ▷ **Assign weights** with decreasing importance to the historical observations, e.g.

$$\tilde{\theta}_t = \sum_{m=0}^{\infty} \eta^m R_{t-m-1}^2 / \sum_{m=0}^{\infty} \eta^m, \quad \eta = 0.94.$$

Goal: identify the weighting scheme for every time point t .

Idea: Identification of weighting scheme

At time point t , choose a weighting scheme from

$$\{W_t^{(1)}, W_t^{(2)}, \dots, W_t^{(K)}\}$$

which leads to the best possible accuracy of estimation.

“Oracle” choice $W_t^{(k^*)}$: the largest weighting scheme for which the approximation $\theta_t \approx \theta^*$ still holds.

Aim: mimic the “oracle” choice.

Local ML Estimation

Given a weighting scheme $W_t = \{w_{st}\}, s \leq t$, the weighted (local) maximum likelihood estimate (MLE) is:

$$\tilde{\theta}_t = \underset{\theta}{\operatorname{argmax}} L(W, \theta) = \frac{\sum_s w_{st} R_s^2}{\sum_s w_{st}} = \sum_s w_{st} R_s^2 / N_t.$$

Fitted local likelihood:

$$L(W, \tilde{\theta}) = \max_{\theta} L(W, \theta).$$

Accuracy of local est. under homogeneity

Define $L(W, \theta, \theta') = L(W, \theta) - L(W, \theta')$.

Theorem (Polzehl and Sp (2006))

It holds for any θ

$$L(W_t, \tilde{\theta}_t, \theta) = \max_{\theta'} L(W_t, \theta', \theta) = N_t \mathcal{K}(\tilde{\theta}_t, \theta)$$

where $N_t = \sum_s w_{st}$ and $\mathcal{K}(\theta, \theta') = -0.5 \{ \log(\theta/\theta') + 1 - \theta/\theta' \}$ is the Kullback-Leibler information.

Moreover, if $\theta_t \equiv \theta^$, then for any $\mathfrak{z} \geq 0$*

$$P_{\theta^*} (L(W, \tilde{\theta}, \theta^*) \geq \mathfrak{z}) \leq 2e^{-\mathfrak{z}}.$$

Risk bound and confidence set

In the local homogeneous case ($\theta_t \equiv \theta^*$ for $w_{st} > 0$),

- ▷ the estimation loss $L(W_t, \tilde{\theta}_t, \theta^*)$ is stochastically bounded:

$$\mathbb{E}_{\theta^*} |L(W_t, \tilde{\theta}_t, \theta^*)|^r \equiv \mathbb{E}_{\theta^*} |N_t \mathcal{K}(\tilde{\theta}_t, \theta^*)|^r \leq \mathfrak{r}_r,$$

where $\mathfrak{r}_r = 2r \int_{\mathfrak{z} \geq 0} \mathfrak{z}^{r-1} e^{-\mathfrak{z}} d\mathfrak{z} = 2r \Gamma(r)$;

- ▷ leads to the **confidence set**:

$$\mathcal{E}_t(\mathfrak{z}) = \{\theta : N_t \mathcal{K}(\tilde{\theta}_t, \theta) \leq \mathfrak{z}\}$$

in the sense that

$$\mathbb{P}_{\theta^*} (\mathcal{E}_t(\mathfrak{z}) \not\ni \theta^*) \leq \alpha.$$

“Small modeling bias” (SMB) condition

Local parametric assumption (LPA): $\theta_t \approx \theta$.

Applying a **parametric** assumption $\theta_s \equiv \theta$ in the **nonparametric** situation leads to **modeling bias** measured by

$$\Delta(W_t, \theta) = \sum_s \mathcal{K}(\theta_s, \theta) \mathbf{1}(w_{st} > 0).$$

“SMB” \Leftrightarrow “ $\Delta(W_t, \theta)$ is small for some θ ”

(with a high probability).

Accuracy of local estimation under SMB

Theorem

Let θ be such that

$$\mathbb{E} \Delta(W_t, \theta) \leq \Delta$$

for some $\Delta \geq 0$ “(SMB)”. Then for any $r > 0$

$$\mathbb{E} \log \left(1 + \frac{|N_t \mathcal{K}(\tilde{\theta}_t, \theta)|^r}{\tau_r} \right) \leq 1 + \Delta$$

Inference under SMB

Under (local) homogeneity $\theta_t \equiv \theta^*$, the fitted log-likelihood $L(W_t, \tilde{\theta}_t, \theta^*) = N_t \mathcal{K}(\tilde{\theta}_t, \theta^*)$ has bounded first moment and yields the confidence set $\mathcal{E}_t(\mathfrak{z}) = \{\theta : N_t \mathcal{K}(\tilde{\theta}_t, \theta) \leq \mathfrak{z}\}$.

Under SMB $E \Delta(W, \theta) \leq \Delta$:

- ▶ The “loss” $L(W_t, \tilde{\theta}_t, \theta) = N_t \mathcal{K}(\tilde{\theta}_t, \theta)$ is stochastically bounded, yielding the accuracy $\tilde{\theta}_t - \theta \asymp 1/\sqrt{N_t}$.
- ▶ the “parametric” confidence set $\mathcal{E}_t(\mathfrak{z})$ still applies with a slightly larger covering error.

“Oracle” choice: the largest scheme for which the SMB holds.

Setup of adaptive local exponential smoothing

Given a finite set $\{\eta_k, k = 1, \dots, K\}$, define $w_{st}^{(k)} = \eta_k^{t-s}$ for $s \leq t$:

$$\begin{array}{cccc}
 \eta_1 = 0.60 & \eta_2 = 0.68 & \dots & \eta_K = 0.98 \\
 \downarrow & \downarrow & \downarrow & \downarrow \\
 \tilde{\theta}_t^{(1)} & \tilde{\theta}_t^{(2)} & \dots & \tilde{\theta}_t^{(K)} \\
 N_1 = 2.48 \leq & N_2 = 3.09 \leq & \dots & N_K = 56.28
 \end{array}$$

where the local MLEs for $k = 1, \dots, K$ are:

$$\tilde{\theta}_t^{(k)} = N_k^{-1} \sum_{m \leq 0} \eta_k^m R_{t-m-1}^2$$

Spatial stagewise aggregation procedure

- ▶ Initialization: $\widehat{\theta}_t^{(1)} = \widetilde{\theta}_t^{(1)}$.
- ▶ Loop: for $k \geq 2$

$$\widehat{\theta}_t^{(k)} = \left(\frac{\gamma_k}{\widetilde{\theta}_t^{(k)}} + \frac{1 - \gamma_k}{\widehat{\theta}_t^{(k-1)}} \right)^{-1}$$

where the aggregating parameter γ_k is computed as:

$$\gamma_k = K_{\text{ag}}(N_k \mathcal{K}(\widetilde{\theta}_t^{(k)}, \widehat{\theta}_t^{(k-1)}) / 3k-1)$$

If $\gamma_k = 0$ then **terminate** with $\widehat{\theta}_t^{(k)} = \dots = \widehat{\theta}_t^{(K)} = \widehat{\theta}_t^{(k-1)}$.

- ▶ Final estimate: $\widehat{\theta}_t = \widehat{\theta}_t^{(K)}$.

Parameters

- ▶ $K_{\text{ag}}(u)$ aggregation kernel, default choice
 $K_{\text{ag}}(u) = \{1 - (u - 1/6)_+\}_+$
- ▶ ES localizing schemes $W_t^{(k)}$ for $\eta_1 < \eta_2 < \dots < \eta_K$. Default $\eta_1 = 0.60$, $\eta_{k+1} = 1.25 * \eta_k$, $\eta_K = 0.98$ providing a smoothing window of length below 500.

Choice of critical values by “propagation” condition

In the **parametric** case $\theta_t \equiv \theta^*$: $\mathbb{E}_{\theta^*} |N_k \mathcal{K}(\tilde{\theta}_t^{(k)}, \theta^*)|^r \leq \mathfrak{r}_r$.

$$\sup_{\theta^* \in \Theta} \mathbb{E}_{\theta^*} |N_K \mathcal{K}(\tilde{\theta}_t^{(K)}, \hat{\theta}_t^{(K)})|^r \leq \rho \mathfrak{r}_r$$

$$\sup_{\theta^* \in \Theta} \mathbb{E}_{\theta^*} |N_k \mathcal{K}(\tilde{\theta}_t^{(k)}, \hat{\theta}_t^{(k)})|^r \leq \frac{(k-1)\rho \mathfrak{r}_r}{K-1}, \quad k = 2, \dots, K.$$

- ▷ α is similar to testing level, default choice **0.5**;
- ▷ r is the power of polynomial losses, default choice **1/2**;

Sequential choice of critical values

- ▷ Choice of \mathfrak{z}_1 leading to the aggregated estimate $\widehat{\theta}_t^{(k)}(\mathfrak{z}_1)$ by setting $\mathfrak{z}_2 = \dots = \mathfrak{z}_{K-1} = \infty$:

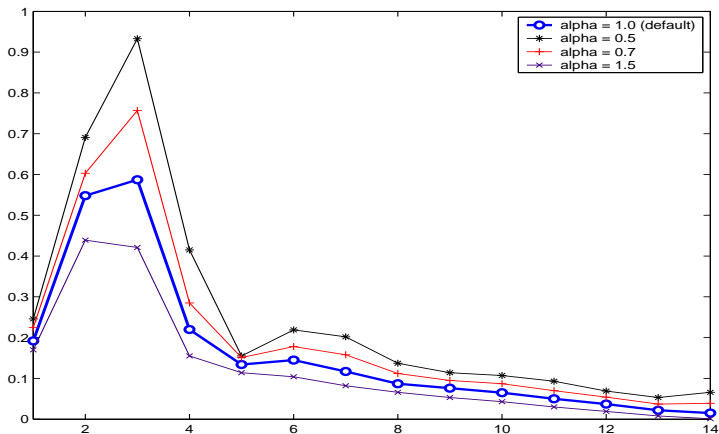
$$\mathbb{E}_{\theta^*} |N_k \mathcal{K}(\widetilde{\theta}_t^{(k)}, \widehat{\theta}_t^{(k)}(\mathfrak{z}_1))|^r \leq \frac{\rho \mathbf{r}_r}{K-1}, \quad k = 2, \dots, K.$$

- ▷ Choice of \mathfrak{z}_2 leading to the aggregated estimate $\widehat{\theta}_t^{(k)}(\mathfrak{z}_1, \mathfrak{z}_2)$ by setting $\mathfrak{z}_3 = \dots = \mathfrak{z}_{K-1} = \infty$:

$$\mathbb{E}_{\theta^*} |N_k \mathcal{K}(\widetilde{\theta}_t^{(k)}, \widehat{\theta}_t^{(k)}(\mathfrak{z}_1, \mathfrak{z}_2))|^r \leq \frac{2\rho \mathbf{r}_r}{K-1}, \quad k = 3, \dots, K.$$

- ▷ Choice of \mathfrak{z}_k leading to the aggregated estimate $\widehat{\theta}_t^{(\ell)}(\mathfrak{z}_1, \dots, \mathfrak{z}_k)$ by setting $\mathfrak{z}_{k+1} = \dots = \mathfrak{z}_{K-1} = \infty$:

$$\mathbb{E}_{\theta^*} |N_\ell \mathcal{K}(\widetilde{\theta}_t^{(\ell)}, \widehat{\theta}_t^{(\ell)}(\mathfrak{z}_1, \dots, \mathfrak{z}_k))|^r \leq \frac{k\rho \mathbf{r}_r}{K-1}, \quad \ell = k+1, \dots, K.$$

Critical values w.r.t. different ρ 

“Oracle” result

Theorem

Let $\max_{k \leq k^\circ} \mathbb{E} \Delta_t^{(k)} \leq \Delta$ for some k° , θ and Δ . Then

$$\mathbb{E} \log \left(1 + \frac{N_{k^\circ}^{1/2} \mathcal{K}^{1/2}(\hat{\theta}_t, \theta)}{\alpha \mathbf{r}_{1/2}} \right) \leq \log \left(1 + c_u \mathbf{r}_{1/2}^{-1} \sqrt{\mathfrak{z}_{k^\circ}} \right) + \Delta + \alpha + 1$$

where c_u is constant.



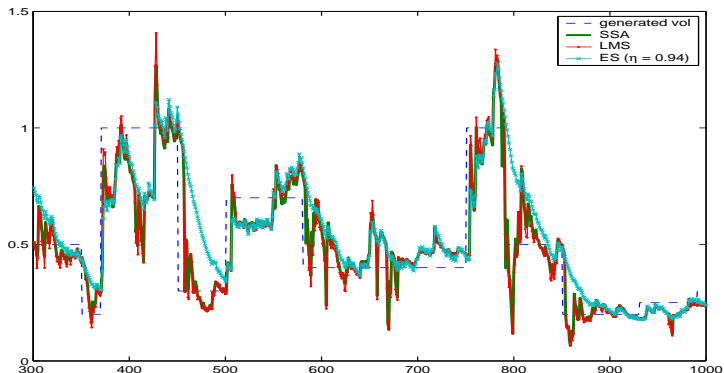
Propagation under SMB

$$\tilde{\theta}_t^{(k)} \approx \hat{\theta}_t^{(k)}$$

Stability

$$N_{k^\circ} \mathcal{K}(\hat{\theta}_t^{(k)}, \hat{\theta}_t^{(k^\circ)}) \leq \mathfrak{z}_{k^\circ}$$

Simulation with Gaussian innovations

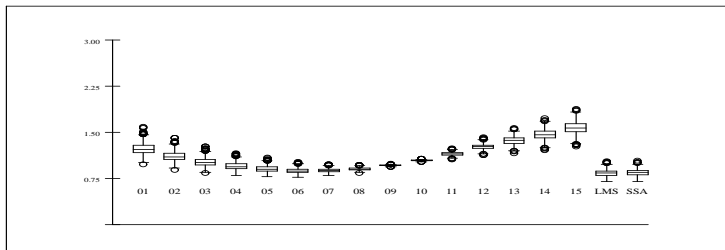


Estimated volatility process based on one realized simulation data with $\varepsilon_t \sim \mathcal{N}(0, 1)$.

Simulation with Gaussian innovations

Let $\tilde{\theta}_t^{1/2}$ is the local MLE with $\eta = 0.94$ and

$$\text{RAE} = \left(\sum_{t=301}^T |\hat{\theta}_t^{1/2} - \theta_t^{1/2}| \right) / \left(\sum_{t=301}^T |\tilde{\theta}_t^{1/2} - \theta_t^{1/2}| \right) = 0.84$$



Results are based on 1000 simulated data.

Outline

1. Motivation ✓
2. Nonstationarity: adaptive volatility (AV) estimation ✓
3. Heavy tails: generalized hyperbolic (GH) distribution and AV
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CH model with heavy tailed innovations

Conditional heteroscedasticity model:

$$y_t = \theta_t \varepsilon_t, \quad \text{with} \quad \mathbb{E}(\varepsilon_t^2 | \mathcal{F}_{t-1}) = 1.$$

Local homogeneity assumption: $\theta_t \approx \theta$ for $t \in I$.

Problem: big returns can be caused by heavy tails rather than by the changes in the volatility parameters.

Generalized Hyperbolic (GH) distribution

$X \sim GH$ with density:

$$f_{GH}(\varepsilon; \lambda, \alpha, \beta, \delta, \mu) = \frac{(\iota/\delta)^\lambda}{\sqrt{2\pi}K_\lambda(\delta\iota)} \frac{K_{\lambda-1/2} \left\{ \alpha \sqrt{\delta^2 + (\varepsilon - \mu)^2} \right\}}{\left\{ \sqrt{\delta^2 + (\varepsilon - \mu)^2} / \alpha \right\}^{1/2-\lambda}} \cdot e^{\beta(\varepsilon - \mu)}$$

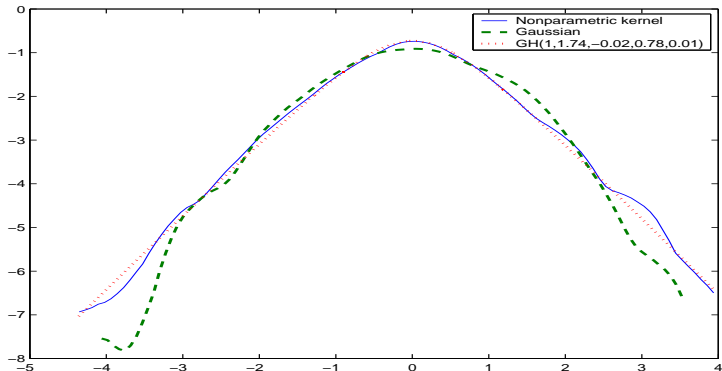
Where $\iota^2 = \alpha^2 - \beta^2$, $K_\lambda(\cdot)$ is the modified Bessel function of the third kind with index λ : $K_\lambda(\varepsilon) = \frac{1}{2} \int_0^\infty y^{\lambda-1} \exp\{-\frac{\varepsilon}{2}(y + y^{-1})\} dy$

Furthermore, the following conditions must be fulfilled:

- ▷ $\delta \geq 0$, $|\beta| < \alpha$ if $\lambda > 0$
- ▷ $\delta > 0$, $|\beta| < \alpha$ if $\lambda = 0$
- ▷ $\delta > 0$, $|\beta| \leq \alpha$ if $\lambda < 0$

Performance of GH distribution

Estimated (log) density of the daily devolatilized DEM/USD returns. Time interval: 1979/12/01 to 1994/04/01 (3719 observations).



Subclass of GH distribution

The parameters $(\mu, \delta, \beta, \alpha)^\top$ can be interpreted as trend, riskiness, asymmetry and the likeliness of extreme events.

Normal-inverse Gaussian (NIG) distribution: $\lambda = -1/2$,

$$f_{NIG}(\varepsilon; \alpha, \beta, \delta, \mu) = \frac{\alpha\delta}{\pi} \frac{K_1 \left\{ \alpha \sqrt{\delta^2 + (\varepsilon - \mu)^2} \right\}}{\sqrt{\delta^2 + (\varepsilon - \mu)^2}} e^{\{\delta\varepsilon + \beta(\varepsilon - \mu)\}}.$$

where $\varepsilon, \mu \in \mathbb{R}$, $0 < \delta$ and $|\beta| \leq \alpha$.

Adaptive estimation with NIG innovations

Model: $R_t = \sqrt{\theta_t} \varepsilon_t$, where $\varepsilon_t | \mathcal{F}_{t-1} \sim \text{NIG}$.

Theoretical problem: exponential moment $\mathbb{E}\{\exp(\lambda \varepsilon_t^2)\}$ does not exist. Therefore the risk function $N_t \mathcal{K}(\tilde{\theta}_t, \theta^*)$ does not have exponential moments.

Power transformation: $y_{t,p} = (R_t^2)^p$, $0 \leq p < 1/2$:

$$\mathbb{E}\{y_{t,p} \mid \mathcal{F}_{t-1}\} = \theta_t^p \mathbb{E}|\varepsilon_t|^{2p} = \theta_t^p C_p = \vartheta_{t,p}$$

Approach: apply the adaptive estimation after the power transformation to estimate $\vartheta_{t,p}$.

Procedure

1. Do power transformation to the squared returns $Y_t = R_t^2$:
 $Y_{t,p} = Y_t^p$.
2. Compute the estimate $\hat{\vartheta}_{t,p}$ of the parameter $\vartheta_{t,p}$ from $Y_{t,p}$ applying the critical values z_k obtained for the Gaussian case.
3. Estimate the value C_p s.t. the innovations are standardized.
4. Compute the estimates $\hat{\theta}_t = (\hat{\vartheta}_{t,p}/C_p)^{1/p}$ and identify the NIG distributional parameters from $\tilde{\varepsilon}_t = R_t \hat{\theta}_t^{-1/2}$.
5. (Optional) Calculate critical values z_k with the identified NIG parameters using Monte Carlo simulation. Repeat the above procedure to estimate θ_t .

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1. Motivation ✓
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High-dimensional analysis

GHICA: (Chen, Haerdle and Spokoiny 2009)

$$\begin{aligned}r_t &= b_t^\top x_t = b_t^\top W^{-1} y_t \\ &= b_t^\top W^{-1} D_t^{1/2} \varepsilon_t\end{aligned}$$

where r_t is the portfolio return, $x_t \in \mathbb{R}^d$ are asset returns, b_t is trading strategy, $y_t \in \mathbb{R}^d$ is an **independent** vector, D_t is the covariance matrix of y_t and W a nonsingular matrix.

How to find ICs? - Minimize mutual information

$$\begin{aligned} I(W, y) &= \sum_{j=1}^d H(y_j) - H(y) \\ &= \sum_{j=1}^d H(y_j) - H(x) - \log |\det(W)| \\ \min \sum_{j=1}^d H(y_j) &\geq \sum_{j=1}^d \min H(y_j) \end{aligned}$$

where $H(\cdot)$ is the entropy, see Hyvaerinen, Karhunen and Oja (2001)

Procedure: GHICA

1. Implement ICA to get ICs.
2. Estimate variance of each IC by using the local exponential smoothing approach
3. Identify GH distributional parameters of the innovations of each IC
4. Estimate the density of portfolio returns using the FFT technique
5. Calculate risk measures

Foreign exchange rate portfolio

- ▶ Data: 7 FX rate 1997/01/02 to 2006/01/05 (2332 observations).
- ▶ Dynamic trading strategies: $b^{(3)}(t) = \frac{x(t-1)}{\sum_{j=1}^d x_j(t-1)}$, where $x(t) = \{x_1(t), \dots, x_d(t)\}^\top$. EUR/USD and EUR/SGD rates are most correlated with the coefficient 0.6745
- ▶ **Goal: GHICA versus DCCN** (DCC with the Gaussian distributional assumption)

Risk analysis of the dynamic exchange rate portfolio

The best results to fulfill the regulatory requirement are marked by r . The recommended method to the investor is marked by i . For the internal supervisory, we recommend the method marked by s .

| | GHICA | | | DCCN | | |
|------|--------------------|---------------------|---------------------|------------|--------|---------------------|
| pr | $\hat{p}r$ | RC | ES | $\hat{p}r$ | RC | ES |
| 1% | 1.28% ^s | 0.0453 ^r | 0.0778 | 1.59% | 0.0494 | 0.0254 ⁱ |
| 0.5% | 0.59% ^s | 0.0493 | 0.1944 ⁱ | 0.94% | 0.0547 | 0.0289 |
| 1% | 1.53% ^s | 0.0806 ^r | 0.2630 ⁱ | 4.17% | 0.0993 | 0.1735 |
| 0.5% | 0.79% ^s | 0.1092 | 0.2801 ⁱ | 3.44% | 0.1100 | 0.1389 |

Conclusion

- ▶ Propose approach to account for nonstationarity and heavy tails and deal with high dimensional financial data
- ▶ Provide realistic and fast risk management methods which outperform standard methods

Outlook

- ▶ Time-varying ICA
- ▶ Online estimation based on high-frequency data

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