Central Bank Balance Sheet Concerns, Monetary and Fiscal Rules, and Macroeconomic Stability*

Feng Zhu
Department of Economics
Yale University
28 Hillhouse Avenue
New Haven, CT 06520
U.S.A.

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Abstract

We provide a fully articulated theoretical foundation for the view that a central bank's (CB) balance sheet concerns may hinder monetary policy "activism" needed to achieve macroeconomic stability. When the Taylor rule is constrained by the zero lower bound on the nominal interest rate, there emerges a second steady state equilibrium which has the properties of a liquidity trap. CB's balance sheet concerns makes it more likely for an economy to drift towards the liquidity trap equilibrium even under the Taylor Principle.

In a reserve-based monetary system based on net worth targeting and central bank independence, monetary activism embodied in the Taylor Principle cannot be applied to its full extent, monetary conservatism may lead to local indeterminacy and bifurcation and embeds in the economic system an inherent tendency towards structural instability. This can only be averted by an institutional reform ensuring better monetary-fiscal cooperation.

Our results have important implications on the institutional design for the European System of Central Banks, the Bank of Japan, and existing pseudo-currency-board regimes in Hong Kong and Singapore.

Keywords: balance sheet concerns, central bank independence, Hopf bifurcation, local indeterminacy, liquidity trap, net worth targeting and Taylor rule.

JEL Classification: E31, E42, E52, E58

1 Introduction

Disinflation programs set in motion in the 1980s have enjoyed great success across both industrial and developing economies. Price stability, defined as low and stable inflation, has become a widespread reality[1]. Central bank (CB) independence is now firmly enshrined in the legal framework of most industrial economies, and the credibility of central bankers engaged in rule-based policy-making has been greatly enhanced.

Hardly did the celebrations of the demise of the "Great Inflation" begin when the alarm of a deflationary spiral sounded and the fear of the economies hitting the zero lower bound (ZLB) on nominal interest rates[2] and of falling into the infamous "liquidity trap" began to trouble the minds of many central bankers around the world. The decade-long "Great Recession" of Japan and the recent economic slowdown in the US and in some major European economies, combined with very low and occasionally negative inflation rates, raised the specter of a new era of deflationary recessions and of a loss of monetary policy effectiveness in an environment of price stability[3].

In an important seminal paper, Sargent and Wallace (1975) first illustrated how price level indeterminacy arises under a pure interest rate peg in a rational-expectations flexible-price model. McCallum (1981, 1983) argued that a commitment to feedback from endogenous state variables renders the rational expectations equilibrium (REE) determinate. Taylor (1993) pro-

[1]In a recent speech, Bernanke (2003b) emphasized the importance of price stability: "Achieving and maintaining price stability is the bedrock principle of a sound monetary policy. Price stability promotes economic growth and welfare by increasing the efficiency of the market mechanism, facilitating long-term planning, and minimizing distortions created by the interaction of inflation and the tax code, accounting rules, financial contracts, and the like. Price stability also increases economic welfare by promoting stability in output and employment."

[2]The US federal funds rate has been sliding downwards and is now targeted to the lowest level of the last 40 years, currently standing at 1.25%. The Japanese economy has experienced sustained deflation and close-to-zero money market rate for the last few years.

On January 25, 2003, the overnight call rate between ABN Ambro, a Dutch bank, and two French banks, Société Générale and BNP Paribas, fell to minus 0.01%. Banks are obliged to lend surplus funds or deposit them with the Bank of Japan, and under the Bank's "quantitative easing policy", the amount of banks' deposits is limited. So banks are forced to lend even at negative rates.

[3]For an early warning against the possibility of a loss of monetary policy effectiveness when the ZLB on nominal interest rate becomes binding in an environment of price stability, see Summers (1989).

posed an interest rate rule that targets inflation rate and output gap. It has been shown that under the Taylor Principle which dictates that the CB raises interest rate by more than one-to-one in response to an increase in inflation, such a rule, when coupled with a passive fiscal policy that ensures fiscal solvency, guarantees local uniqueness of the REE and promotes macroeconomic stability.

In a recent paper, Benhabib, Schmitt-Grohé and Uribe (2001b) questioned the local determinacy result for an active Taylor rule. They demonstrated that such a result may be sensitive to specifications of preferences and technology, for instance, when money enters the production function, and when money and consumption are Edgeworth substitutes. Furthermore, when a Taylor rule is constrained by the ZLB on the nominal interest rate, Benhabib, Schmitt-Grohé and Uribe (2001a, 2002) uncovered a second, low-inflation equilibrium, besides the desirable target equilibrium. The low-inflation equilibrium has the characteristics of a liquidity trap. In this case, more complex dynamics emerge and the possibility of macroeconomic instability is greater. The focus on local determinacy is misleading when multiple steady state equilibria arise naturally, a global analysis may be imperative.

Extending the line of argument followed in Sargent and Wallace (1981), Leeper (1991), Sims (1994) and Woodford (1994, 1995, 1996, 2003) showed that fiscal policy is no less important than monetary policy in price level determination. In fact, local uniqueness under an interest rate rule may be restored by a reconsideration of fiscal policy and the government budget constraint. In a series of papers, Sims (1997, 1999, 2001, 2003) has been a tireless advocate of closer monetary-fiscal cooperation as the institutional setup better suited to weather situations of severe economic distress. For him, monetary policy rules without adequate fiscal backup are fundamentally flawed and may lead to greater macroeconomic instability. CB independence and credibility built up in the crusade against high and persistent inflation may hurt, rather than benefit an economy in a deflationary environment with nearly binding ZLB on nominal interest rates.

It has long been argued, albeit informally, that in the face of major stock and housing market crashes in the late 1980s, the Bank of Japan (BoJ) has done too little and acted too late to prevent the Japanese economy from sliding into a deflationary recession, and the BoJ's concern with its own balance sheet position has been repeatedly cited as the reason for its policy conservatism[4]. In a recent policy speech delivered to the Japan Society of Monetary Economics, Bernanke (2003a) called the BoJ's balance sheet concern a "barrier to more aggressive policies". Because of it, "not all the possible methods for easing monetary policy in Japan have been fully exploited". For Bernanke (2003a), "one possible approach to ending deflation in Japan would be greater cooperation, for a limited time, between the monetary and the fiscal authorities"[5].

In this paper, we provide a fully articulated theoretical foundation for the view that a central bank's balance sheet concern may hinder monetary policy "activism" needed for achieving macroeconomic stability, thereby opening the door for local indeterminacy and bifurcation. We draw a clear distinction between the tax-based monetary system where no balance sheet concerns arise, best exemplified by the US Federal Reserve System, and the reserve-based system, a pertinent characterization of the institutional frameworks of the European System of Central Banks (ESCB) and the BoJ. In a reserve-based system, the central bank and the fiscal authority are modelled as two distinct entities, each with its own budget constraint. The central bank follows an interest rate rule that reacts to deviations of its net worth[6] and inflation from target levels.

With net worth targeting and a strong form of central bank independence, monetary activism embodied the Taylor Principle cannot be applied to its full extent, the well-known result that an active interest rate rule combined with a passive fiscal policy ensures local uniqueness of the desired steady state equilibrium is partially reversed. Even a small dose of monetary conservatism, created by the CB's balance sheet concerns, may lead to local indeterminacy and bifurcation. The reserve-based monetary system has an inherent tendency towards structural instability, which can only be averted by an institutional reform ensuring better monetary-fiscal cooperation. Echoing Sims' (2001) and Bernanke's (2003a) view, we find that a

[4] Sims (2001) observes that "a bank with such (balance sheet) concerns could also hoard interest earnings and refrain from bold, risky open market purchases to sustain fiscal institutions or end the deflation."

[5] Sims (2001) expressed similar views earlier: "to the extent that the central bank has the power to make risky open market purchases to end the deflation, it requires an understanding that it will if necessary have fiscal backing."

[6] A central bank's net worth is the difference between its assets and liabilities. In a highly stylized model, we define the net worth as the difference between its reserve of real asset and real money balances in circulation.

tax-based monetary system, with *less central bank independence*, more and better monetary-fiscal collaboration and automatic fiscal backup for monetary policy, can greatly enhance monetary policy effectiveness in a situation of severe economic distress by reducing the probability of macroeconomic instability and by preventing the economy from falling into a liquidity trap in a timely manner.

The conclusions of this paper has immediate import on the institutional setup of the ESCB, BoJ and modern pseudo-currency boards in Hong Kong and Singapore. For the sake of macroeconomic stability, institutional reforms are necessary so as to abolish the reserve-based monetary system, curb the degree of "central bank independence"[7] and eliminate the CB's balance sheet concerns, without sacrificing the CB's credibility and credentials for maintaining price stability. In fact, solid and immediate fiscal backing for money is essential for the success of monetary policy making in a new era of price stability.

In Section II we provide a detailed discussion of the

2 Tax *versus* Reserve-Based Monetary Systems

Before venturing into the formal analysis, it is important to draw a clear distinction between the institutional frameworks currently in place in major industrial economies. After the collapse of the Bretton Woods System, all monetary systems are now based on purely fiat units of account, but the way how the value of money is supported in a fiat system differs. At one extreme, we have the US Federal Reserve System (FRB), which conducts its open market operations primarily on short-term homogeneous domestic nominal bonds against high-powered money. The return distributions for the Federal Reserve's assets and liabilities, which are seen as close substitute and quoted in the same unit of account, are quite uniform and almost perfectly hedged[8].

[7] Central bank independence from the fiscal authority is defined below in the main text. The need to *curb* CB independence sounds counter-intuitive. However, the containment of unfettered zeal for CB independence is most needed in unusual economic circumstances such as deflationary recessions. This will become clear as our analysis unfolds.

[8] In a situation of economic distress, when the FRB takes unconventional measures, such as the purchase of long-term government bonds, private bonds and securities, foreign currencies, the asset and liability sides of its balance sheet become mismatched but balance

Both government bonds and money are simply claims to a fraction of the stream of current and future government tax revenues net of spending, hence one consolidated government budget constraint suffices and fiscal backing for Fed policies is understood to be automatic and immediate[9]. In this *tax-based* monetary system[10], it is ultimately the fiscal power of taxation which backs up the value of money, there is no need to worry about the CB's balance sheet position. The CB hands over its surplus seigniorage revenues to the Treasury, while the latter replenishes the CB's account whenever the net worth of the CB becomes negative in a more persistent manner.

At the other extreme, we have a *reserve-based* monetary system where the CB maintains a large pool of diverse assets, the value of which is often out of the CB's direct control and fluctuates according to market conditions. The asset side of the CB balance sheet consists of assets which may have very different maturity structures and may be denominated in domestic or foreign currencies, and the return distribution of this heterogeneous portfolio does not match closely that of the CB's money liabilities. In a situation of significant deflationary risk and when unconventional monetary policy measures are most needed, the CB's balance sheet may easily run into trouble. Usually the CB is legally independent from the fiscal authority (FA), each having a separate budget constraint. Monetary policy is seen to be backed up by the CB reserve, and the CB makes all efforts to fend off possible interference from the FA and it does not expect rapid remedial actions by the FA once balance sheet problems emerge. Without fiscal backing, the CB is obliged to monitor its net worth closely so as to guarantee a minimal value for its currency and to prevent a possible collapse of confidence in the fiat system.

Most monetary systems in practice are hybrid in nature, but the reserve-based system can be seen as a fairly accurate characterization of the European System of Central Banks (ESCB), the Bank of Japan (BoJ) and certain pseudo-currency boards in Hong Kong and Singapore. The European Central Bank (ECB) is highly independent in its pursuit of price stability. Prohib-

sheet problem does not arise because of implicit fiscal backing.

[9]It is understood that whenever the FRB's balance sheet runs into trouble, be it the result of unexpected asset price movements, be it the consequence of not-so-conventional policy contingencies, the Treasury would lend its full support in an expedient manner.

[10]For an illuminating early discussion of the tax and reserve-based systems, see Sims (2001). In that paper, the tax and reserve-based monetary systems are termed Models F and E, respectively.

ited by law from holding interest-bearing government debt, the ECB holds a reserve of assets much of which are non-Euro-denominated and subject to exchange rate risks. Euro, the European currency, is seen as a claim to a fraction of the ESCB's reserve. The ECB conducts monetary operations through repurchase agreements, lending high-powered money against qualified assets as collateral. Even if only short-term government bonds are selected as collateral, since they are issued by the twelve independent FA's participating the European Monetary Union (EMU), these debt instruments involve different levels of sovereign default risk and yield necessarily different risk premia and rates of return. When private bonds and securities are chosen to serve as collateral, monetary operations acquire a fiscal dimension. In the event of severe economic distress such as a deflationary spiral, the ECB may shy away from taking necessary measures if these acquire a fiscal character and foreshadow balance sheet problems.

Therefore, monetary policy actions of the ECB are essentially short-term credit operations that involve certain default risk, which may represent a threat to the CB balance sheet in unfavorable economic situations with widespread debt default[11]. Even if the FA's are willing to help, the process of negotiations between sovereign states for a temporary fiscal pact that warrants an equitable share of political responsibilities and fiscal contributions may prove to be lengthy and difficult, and any agreement may be subject to discussions and approval of national parliaments and therefore to popular sentiment. The prospect of a rapid and automated resolution of possible ECB balance sheet problems is at best uncertain, unless a concurrent fiscal union or a conclusive fiscal settlement envisioning shared responsibilities for fiscal backing of money can be put in practice once and for all in the near future.

In the case of pseudo-currency boards, a reserve of foreign currencies and debt instruments is maintained to cover partially or fully money liabilities in circulation, money is perceived to be convertible into these assets. Albeit limited in scope, monetary authorities of pseudo currency boards do conduct their own policy, and the effectiveness of the policy actions greatly depends on changes in the balance sheet position, where the value of assets may vary dramatically according to the asset and exchange market conditions.

The BoJ has long been criticized for letting its financial concerns meddle

[11] The severe consequences of debt-deflation have been stressed and analyzed by Fisher (1933), Benanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997).

with proper policy-making when more drastic measures were in demand in the face of an unprecedented deflationary recession. Although the BoJ does not formally maintain a reserve as the ECB and the basic structure of its balance sheet is apparently quite similar to that of the FRB, it does show important differences. Unlike the composition of the FRB's assets, the BoJ's portfolio consists of heterogeneous assets with distinct maturity structure and risk and return characteristics. The BoJ earned its independence from the Japanese Ministry of Finance (MoF) only recently, after the Bank of Japan Law took effect in April 1998. Cautious in its approach to monetary policy, the BoJ has made the financial soundness of the Bank, seen as a safeguard of its independence from the MoF, its top priority.[12] Although one's best guess is that the MoF would be happy to provide support for the BoJ's balance sheet once the latter runs into financial trouble, the Bank does everything in its power to make sure that its finances are sound and it acts as if a reserve is maintained so as to guarantee a small but positive net worth. CB independence is the means to achieve credibility and price stability, but when the means becomes itself a rigorously pursued policy goal

[12]Excerpts from the Minutes of the (BoJ) Monetary Policy Meeting on April 7 and 8, 2003 (Italics added by the author) make the BoJ's "obsession" with its balance sheet transparent:

"Members agreed that, when examining the new scheme, the Bank should give due consideration to how it would secure the *soundness of its financial condition*. One member warned that if the Bank were not able to prove the soundness of its financial condition clearly to the public, its *credibility* and the *effectiveness of policy* could decline";

"A different member said that the following basic features of the Bank's capital should be fully understood. First, seigniorage, the profit generated by issuing banknotes, belonged to the public, and it should be used for the *conduct of fiscal policy* upon approval by the Diet as part of the budget. Second, the Bank kept part of the seigniorage in its capital basically as part of the *reserve for dealing with possible materialization of unexpected risk* when it exercised the lender of last resort function. And third, the Bank could allocate any remaining capital for the conduct of a particular policy. Based on this understanding, this member continued that the possibility that the Bank might have to increase its holding of JGBs (Japanese Government Bonds) depending on the future economic situation meant that the current amount of the Bank's capital could not be considered sufficient for the above purposes. In this situation, if the Bank were to take bold action to purchase risk assets, this might create market concern that the Bank would not be able to allocate sufficient capital to purchase other risk assets."

"Many members agreed that the Bank's proposed participation in fostering markets by taking on credit risk would inevitably cause the Bank to enter the *domain of fiscal policy*. One member said that it would therefore be important to cooperate with relevant government institutions in addition to market participants in fostering markets."

that directly impinges on the making of monetary policy, the consequences may be far-reaching.

3 A Flexible-Price Model

In a simple flexible-price representative agent model, we investigate possible macroeconomic consequences of the balance sheet concerns of a central bank in pursuit of price stability, taking into account a variety of different fiscal policy rules. We augment Benhabib, Schmitt-Grohé and Uribe's (2001a, 2002) model with an interest rate rule which reacts to changes in the CB's reserve-to-money coverage ratio besides inflation rate, under the assumption of separate budget constraints for the fiscal and monetary authorities, therefore postulating a reserve-backed money supply. We study how CB balance sheet concerns induce an element of conservatism in monetary policy-making and leads to greater macroeconomic instability.

To analyze the effects of net worth targeting in a reserve-based monetary system, we assume that the economy has a real asset (F^D) which can be held by both the household (F^H) and the central bank (F^{CB})

$$F^D = F^H + F^{CB}$$

The real asset serves as the CB's reserve and it yields a real rate of return r, which is exogenously fixed in the world market and determines the domestic real interest rate. Assume $F >> F^D$, so although the total amount of the real asset available to the model economy is assumed to be fixed at $F^D = \bar{F}$, the world supply F^W is elastic and large in comparison with F, so the real rate of return of the asset is kept constant. Holders of the real asset F receive a stream of interest payment $\{rF\}$.

3.1 Household

We postulate that money facilitates transactions and enters the model as an argument in the instantaneous utility function[13]. In the Brock-Sidrauski

[13] Money can be introduced in other ways, notably as an argument in the production function, or through a standard cash-in-advance constraint for consumption and/or investment goods, or through a shopping time constraint or a transaction cost function in the household's budget constraint. For a detailed account, see Sargent (1987), Walsh (2003) and Woodford (2003).

setup[14], the representative household maximizes lifetime utility,

$$U_0 = \int_0^\infty e^{-\beta t} u(c\,(t)\,, m\,(t))dt \tag{1}$$

subject to the budget constraint

$$c\,(t) + \frac{\dot{M}\,(t) + \dot{B}\,(t)}{P\,(t)} + \dot{F}^H\,(t) = y + \frac{iB\,(t)}{P\,(t)} + rF^H\,(t) - \tau\,(t)$$

where c is consumption, τ is the household's tax payment, P is the price level, $M \geq 0$ and B are the household's holdings of nominal money balances and government bonds, respectively, F^H is the household's holdings of the real asset F paying a fixed rate of return r to the asset holder, and i is the nominal rate of interest on government bonds. The same asset F also serves the central bank for the purpose of reserve build-up, which in turn backs up the value of money. The endowment income y is exogenous and fixed. Define $m = M/P$, $b = B/P$ and the economy's rate of inflation as $\pi = \dot{P}/P$, then

$$\dot{m} + \dot{b} + \dot{F}^H = (i - \pi)\left(m + b + F^H\right) - im - (i - \pi - r)\,F^H + y - c - \tau$$

Define the household's financial asset as $A^H = M + B + PF^H$ and the real financial asset holding as $a^H = A^H/P$. The household's budget constraint is

$$\dot{a}^H = (i - \pi)\,a^H - im - (i - \pi - r)\,F^H + y - c - \tau \tag{2}$$

Assumption A *Assume that (1) the utility function $u\,(\cdot,\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable in both arguments; (2) $u_c u_{mm} - u_m u_{cm} < 0$ and $u_c u_{cm} - u_m u_{cc} > 0$.*

To simplify the analysis, we assume that the instantaneous felicity function takes the Cobb-Douglas form[15]

$$u\,(c, m) = c^a m^{1-a} \tag{3}$$

where $a \in (0, 1)$ is the utility share of consumption goods.

These modelling devices are shortcut mechanisms to introduce monetary frictions in a simple manner, and many of these yield similar results as our money-in-utility specification.

[14] To simplify notation, we will drop the time argument t whenver the context is clear.

[15] Under this specification, consumption and money are Edgeworth complements ($u_{cm} > 0$).

The household's borrowing constraint is the conventional No-Ponzi Game (NPG) condition

$$\lim_{t\to\infty}\left\{\exp\left(-\int_0^t\left[i\left(s\right)-\pi\left(s\right)\right]ds\right)\left(m+b\right)+e^{-rt}F^H\right\}\geq 0 \tag{4}$$

Assuming an interior solution, the necessary first-order conditions for the household's maximization problem include

$$u_c\left(c,m\right) = \lambda \tag{5a}$$
$$u_m\left(c,m\right) = \lambda i \tag{5b}$$
$$\dot{\lambda} = \left(\pi+\beta-i\right)\lambda \tag{5c}$$
$$i = \pi+r \tag{5d}$$

where λ is the marginal utility of wealth and hence consumption. Notice the equation (5d) is the equilibrium Fisher relation.

The transversality at infinity condition (TVC) is derived from the No-Ponzi Game (NPG) condition (4)

$$\lim_{t\to\infty}\left\{\exp\left(-\int_0^t\left[i\left(s\right)-\pi\left(s\right)\right]ds\right)\left(m+b\right)+e^{-rt}F^H\right\}=0$$

which in equilibrium can be simplified as

$$\lim_{t\to\infty}e^{-rt}a^H=0 \tag{6}$$

From conditions (5a) and (5b), we obtain the usual liquidity preference function

$$m=\phi\left(i,c\right) \tag{7}$$

Since $u_{cc}, u_{mm} < 0$, the assumption that consumption and money are Edgeworth complements ($u_{cm} > 0$) is a sufficient condition for $\phi_i < 0$ and $\phi_c > 0$.

3.2 Government Sector

Assume that the fiat system is reserve-based. Departing from the existing literature, we model the government sector to consist of two separate and mutually independent entities, the Central Bank (CB) and the Fiscal Authority

(FA). Each entity has its own budget constraint and policy rules. Moreover, we incorporate the CB's concerns over its own balance sheet explicitly in its policy rule in the form of a net worth targeting. The fact that the CB's financial soundness may take priority over more fundamental policy objectives (price stability and economic growth) imposes serious constraints on monetary policy making and has important implications for local determinacy of equilibria and macroeconomic stability.

3.2.1 Monetary Authority

We assume that the CB holds a reserve of real assets ($F^{CB} \geq 0$) to back up the value of money. Furthermore, the CB is prohibited from possessing bonds issued by the Treasury ($B^{CB} = 0$).[16] The CB has its own separate budget constraint

$$\frac{\dot{M}}{P} + [r - \chi(\Phi)] F^{CB} = \dot{F}^{CB} \tag{8}$$

where the net worth Φ is defined as the difference between the CB's reserve of real asset (F^{CB}) and outstanding real balances (m): $\Phi = F^{CB} - m$, and $\chi(\Phi)$ is the rate at which the CB transfers its surplus net worth to the FA. In the case of $\chi(\Phi) < 0$ whenever $\Phi < 0$, there is (at least partial) fiscal backing for the value of money and there would be no CB balance sheet concerns if the transfer payment χF^{CB} are of adequate magnitude. We assume that the CB alone decides the rate $\chi(\cdot)$. In a reserve-based monetary system, CB independence from fiscal authority can be defined as below.

Definition 1 *A central bank is said to be independent from the fiscal authority if (1) the CB is not required to monetarize government bonds; (2) the CB alone decides $\chi(\cdot)$, the rate of transfer of the CB net worth to the FA, with $\chi(\Phi) \geq 0$, $\forall$ Φ; (3) the CB has its own budget constraint so that the FA does not consider itself responsible for outstanding liabilities and balance sheet deficits of the CB; (4) instead of the fiscal power of taxation, the CB relies on a positive net worth $\Phi > 0$ to back up its policy actions:*

Remark 1 *The reserve-backed monetary system is characterized by net worth targeting and CB independence from fiscal authority as defined above.*

[16] With $B^{CB} = 0$, the CB may conduct open market operations through repurchase agreements with trusted counterparts using qualified assets as collateral.

To simplify analysis, we assume the CB is not allowed to hold Treasury bonds ($B^{CB} = 0$) and it sets the transfer rate to zero irrespective of the level of net worth, *i.e.*, $\chi(\Phi) = 0$, $\forall\ \Phi$, then

$$\frac{\dot{M}}{P} + rF^{CB} = \dot{F}^{CB}$$

The law of motion of the CB's net worth Φ can be derived as follows

$$\dot{\Phi} = (i - \pi)\,\Phi + i\phi\,(i, \bar{c}) \tag{9}$$

Clearly, the evolution of Φ depends on the CB's interest rate policy, which itself reacts to fluctuations in Φ. With a separate budget constraint and by not holding Treasury bonds, the CB is effectively disconnected from the FA.

This definition of CB independence is probably more stringent than what has often been used in the "Rules *versus* Discretion" literature[17]. By our definition, the CB is "disconnected" and insulated from fiscal considerations, tax revenues can no longer guarantee the value of money and automated fiscal backing for monetary policy actions cannot be expected. This definition of central bank independence is a good characterization of the operational framework of the ESCB and BoJ. We show that CB independence and net worth targeting, both essential elements of the reserve-based monetary system, have important bearing on macroeconomic stability.

There is little doubt that central bankers are often concerned with their balance sheet, particularly in a situation of economic distress, be it deflationary or inflationary. In a deflationary environment, when the ZLB on nominal interest rate is attained or nearly so, money and bonds become close substitutes and conventional open market operations become meaningless. Alternative measures, including the purchase of long-term government bonds, private debt or equity issues, often imply increased credit risk and the possibility of large capital losses for the CB.[18] By taking these measures, the

[17] In fact, the definition of CB independence in the "Rules *versus* Discretion" literature is often murky and mostly informal.

[18] Until very recently, the BoJ has resisted pressure to purchase long-term government bonds in order to bring down the yield curve. Such an action, if successful in pulling the Japanese economy out of the deflationary recession, immediately implies a higher nominal interest rate on government bonds and lower bond prices. Accumulating a large amount of long-term government bonds on the CB balance sheet will then entail "unbearable" capital losses for the BoJ. The risk of outright purchase of asset-based securities (ABS), which has gained cautious approval of the BoJ, in an unfavorable economic environment, is even greater to the BoJ's balance sheet.

CB operations clearly acquire a fiscal dimension[19] and need a firm commitment from the FA that guarantees the financial viability of such monetary policy actions. When this commitment is not forthcoming, the CB, worried with its balance sheet, will refrain from taking these actions in a timely and quantitatively significant manner, or even works in the wrong direction by tightening monetary policy in an attempt to shore up its net worth and avoid any unpleasant fiscal implications of the necessary "easy money" policy on its hard-won independence.

The balance sheet concern is therefore transformed, often implicitly, into a more conservative monetary policy stance, the fact being captured in our model by assuming, as a "reduced form", an interest rule that also reacts to fluctuations in the net worth of the CB besides inflation rate. Suppose that the monetary policy takes the general form of an interest rate rule

$$i = \rho\left(\pi,\, \Phi\right) \tag{10}$$

which adjusts the nominal interest rate towards both a target rate of inflation π^* and a target level of net worth Φ^*. By inducing changes in the portfolio composition of the private sector, the CB adjusts the nominal interest rate and its policy rule implicitly determines money supply M. Now equation (9) becomes

$$\dot{\Phi} = \left[\rho\left(\pi, \Phi\right) - \pi\right]\Phi + \rho\left(\pi, \Phi\right)\phi\left(\rho\left(\pi, \Phi\right), \bar{c}\right) \tag{11}$$

[19] For instance, the purchase of private debt or private equities by the CB may be seen as government bailout or outright acquisition of control over private firms, respectively. These are fiscal actions that usually pertain to the fief of the Treasury.

More concretely, we specify the following log-linear rule which imposes a ZLB on the nominal interest rate[20]:

$$i = i^* \exp\left[\frac{\alpha_\pi}{i^*}(\pi - \pi^*) + \frac{\alpha_\Phi}{i^*}\left(\frac{\Phi - \Phi^*}{\Phi^*}\right)\right] \tag{12}$$

where $i, i^*, \alpha_\pi > 0$, $\alpha_\Phi \leq 0$. $\Phi^* > 0$ is the target level of CB's net worth. We assume that the CB adjust the interest rate symmetrically[21] around a target reserve-to-money coverage ratio slightly over 100%, so Φ^* is a relatively small positive number while Φ can take any value in $\Re$. The assumption that the CB intends to maintain a high reserve-to-money coverage ratio is in line with the common practice of many currency boards and it is also a reflection that the ECB's monetary operations are all fully covered by collateral assets.

Assumption B *Assume that (1) $\rho(\cdot,\cdot)$ is continuous in both arguments, $\rho(\cdot,\cdot) \geq 0$ and $\rho_\pi \geq 0$, $\rho_\Phi \leq 0$; (2) There exists at least one $\pi^* > -\beta$ such that $i^* = \rho(\pi^*, \Phi^*) = \pi^* + \beta$.*

Assumption B' *Assume that (1) $i, i^* > 0$, $\forall \pi$; (2) There exists at least one $\pi^* > -\beta$ such that $i^* = \pi^* + \beta$; (3) $\alpha_\pi > 0$ and $\alpha_\Phi \leq 0$.*

As in Leeper (1991), monetary policy is termed *active* if $\alpha_\pi > 1$ and *passive* if $\alpha_\pi < 1$. The assumption that $\alpha_\Phi < 0$ implies that, *ceteris paribus*,

[20] Given the interest rate rule

$$\log i = \log i^* + \frac{\alpha_\pi}{i^*}(\pi - \pi^*) + \frac{\alpha_\Phi}{i^*}\left(\frac{\Phi - \Phi^*}{\Phi^*}\right)$$

We have

$$\left.\frac{\partial i}{\partial \pi}\right|_{i=i^*} = \alpha_\pi \frac{i}{i^*} \qquad \left.\frac{\partial i}{\partial \Phi}\right|_{i=i^*} = \frac{\alpha_\Phi}{\Phi^*}\frac{i}{i^*}$$

which in equilibrium becomes

$$\frac{\partial i}{\partial \pi} = \alpha_\pi \qquad \frac{\partial i}{\partial \Phi} = \frac{\alpha_\Phi}{\Phi^*}$$

and

$$\frac{\partial \log i}{\partial \pi} = \frac{\alpha_\pi}{i^*} \qquad \frac{\partial \log i}{\partial \Phi} = \frac{\alpha_\Phi}{i^* \Phi^*}$$

[21] In reality, the CB might be more willing to maintain or even accumulate a positive level of net worth and loath to allow negative deviations to unfold. There may be some asymmetry and nonlinearity in the reaction function.

the CB lowers the nominal interest rate and loosens the monetary policy by injecting liquidity into the economy when its net worth is above the target level. And *vice versa*, the CB reduces lending and raises the nominal interest rate when its net worth is low relative to the target level, so the public is induced to reduce their money holdings, and the CB's balance sheet problem can be instantly alleviated. We differentiate the degree of monetary conservatism according to the magnitude of the net worth targeting policy parameter α_Φ.

Definition 2 *Let*[22]

$$\bar{\alpha}_\pi = \frac{\pi^* + \beta}{\pi^* + (2-a)\,\beta} \in \left(\frac{\pi^* + \beta}{\pi^* + 2\beta}, 1 \right) \tag{13}$$

For any fixed $\alpha_\pi > \bar{\alpha}_\pi$, define

$$\bar{\alpha}_\Phi = \frac{\pi^* + \beta}{1-a} - \frac{\pi^* + (2-a)\,\beta}{1-a}\alpha_\pi < 0 \tag{14}$$

Monetary policy is ultra-conservative if $\alpha_\Phi \leq \bar{\alpha}_\Phi$, it is conservative if $\alpha_\Phi \in (\bar{\alpha}_\Phi, 0)$, and it is liberal if $\alpha_\Phi = 0$.

We require that $\alpha_\pi > \bar{\alpha}_\pi$ so as to guarantee that $\bar{\alpha}_\Phi < 0$. Notice that the value of $\bar{\alpha}_\Phi$ varies with the specific value of policy parameter α_π selected by the monetary authority, the more active the CB is (larger α_π), the larger $\bar{\alpha}_\Phi$ will be in terms of absolute value ($|\bar{\alpha}_\Phi|$). When the monetary policy is passive ($\alpha_\pi < 1$), a small $|\bar{\alpha}_\Phi|$ is needed to classify the CB as ultra-conservative. If the monetary policy is active ($\alpha_\pi > 1$), a relatively large value of $\bar{\alpha}_\Phi$ is needed to classify the CB as ultra-conservative. Intuitively, monetary activism interpreted as the observance of the Taylor Principle contains elements that work against excessive net worth targeting and it necessarily entails a larger $|\bar{\alpha}_\Phi|$ for a CB to be classified as ultra-conservative. In short, net worth targeting injects a degree of passivism into monetary policy-making.

We have assumed that $B^{CB} = 0$, so the CB cannot issue money to directly finance the FA's fiscal spending. Instead of conducting open market operations by purchasing and selling government bonds, the CB may provide or

[22] Taking the annual target inflation rate as $\pi^* = 2\%$ and the annual discount rate as $\beta = 0.97$, then the value of $\bar{\alpha}_\pi$ varies empirically between 0.505 and 1. Notice that $\bar{\alpha}_\pi$ varies inversely with a. If $a = 0.5$, then $\bar{\alpha}_\pi = 0.671$.

withdraws liquidity through short-term repurchase agreements with selected counterparties, with qualified government or private securities serving as collateral[23]. From the policy rule (12), it is easy to see that net worth targeting potentially undermines monetary policy activism. The mechanism through which net worth targeting may jeopardizes monetary policy objectives can be exemplified as follows.

To be more specific, suppose that, at a time of high and persistent inflation ($\pi > \pi^*$), the CB tightens monetary policy by reducing its lending, therefore raising the nominal interest rate ($\alpha_\pi > 0$). *Ceteris paribus*, reduction in outstanding CB liabilities induced by a withdrawal of money from circulation necessarily increases the CB's net worth Φ and automatically leads to a reduction of the nominal interest rate through (12), when the CB is conservative ($\alpha_\Phi < 0$). The contrary occurs when $\pi < \pi^*$. The effect of monetary policy is immediately attenuated by actions entailed by the CB's desire to maintain a target level of net worth Φ^*, and the CB's balance sheet concerns as reflected in net worth targeting become a major impediment to implementing the policy rule in the desired direction to the full extent. Even under the Taylor Principle ($\alpha_\pi > 1$), local indeterminacy is a distinct possibility because the CB is conservative and unduly concerned with its balance sheet position ($\alpha_\Phi < 0$). In fact, net worth targeting builds in an "automatic destabilizer" into the monetary transmission mechanism, and monetary conservatism may jeopardize the CB's capability of achieving its policy goals.

3.2.2 Fiscal Authority

The fiscal authority (FA) has its own separate budget constraint

$$iB = \dot{B} + P\tau \tag{15}$$

The government deficit, which consists of interest payments on its outstanding debt, must be financed by tax revenues and by the issue of new debt.

[23] The ESCB (ECB and national central banks) are legally forbidden to hold bonds issued by national governments participating the EMU. However, monetary operations of the ESCB inject liquidity into the EMU economies by lending under short-term repurchase agreements using government bonds and other qualified assets as collateral.

The collateral assets are not part of the CB's reserve and are therefore out of the management control of the CB. But the quality of collateral assets and fluctuations of their market value may have a large impact on the CB's balance sheet under certain circumstances. As the economy expands and the CB's monetary operations increase in magnitude, more risky assets may be selected as collateral.

The assumption of separate budget constraints and the lack of any transfer payments from the FA to the CB between the CB and FA imply that taxes cannot be used to back up the value of money. In fact, from the representative household's perspective, the value of government liabilities issued by the CB and FA can only be supported by assets held by each entity. The value of money, liabilities of the CB, is backed up by the real asset reserve, while bonds issued by the Treasury are supported by the stream of future tax revenues. In our setup, the FA cannot tap on the CB's seigniorage revenues to supplement it's budget and the CB does not expect any form of fiscal backing. Separate budget constraints imply that the CB and Treasury are insulated from each other and can go bankrupt independent of the balance sheet position of the other entity. The fact has important implications for macroeconomic stability.

We simplify the government's fiscal policy making by assuming that the Treasury follows certain tax rules, which can be either non-Ricardian,

$$\tau^{NR} = \bar{\tau} \tag{16}$$

or Ricardian, say

$$\begin{aligned}
\tau_1^R &= \delta_0 + \delta_1 b && \text{(17a)}\\
\tau_2^R &= \delta_0 + \delta_2 (m + b) && \text{(17b)}\\
\tau_3^R &= \delta_0 + \delta_3 \left(m + b + F^H\right) \\
&= \delta_0 + \delta_3 \bar{F} + \delta_3 (b - \Phi) && \text{(17c)}
\end{aligned}$$

where $\delta_0 < 0$ and $\delta_1, \delta_2, \delta_3 \in (0, 1)$.

Definition 3 *A Ricardian fiscal rule is said to be active if* $r - \delta_j > 0$ *and it is passive otherwise. A non-Ricardian rule is always active.*

With an active tax rule, the Treasury does not simply act as a follower in the fiscal-monetary game, it does not tax all the way to balance the consolidated government budget at all times. With the non-Ricardian fiscal policy (16), the fiscal budget constraint becomes

$$\dot{b} = (i - \pi) b - \bar{\tau} \tag{18}$$

Under the proposed Ricardian rules (17a)-(17c), we have

$$\begin{aligned}
\dot{b} &= (i - \pi - \delta_1) b - \delta_0 && \text{(19a)}\\
\dot{b} &= (i - \pi - \delta_2) b - \delta_2 m - \delta_0 && \text{(19b)}\\
\dot{b} &= (i - \pi - \delta_3) b + \delta_3 \Phi - \left(\delta_0 + \delta_3 \bar{F}\right) && \text{(19c)}
\end{aligned}$$

From the representative household's view, only the consolidated government budget constraint matters for its optimization problem. This takes the following form

$$\frac{\dot{M}+\dot{B}}{P}+\tau+rF^{CB}=\dot{F}^{CB}+i\frac{B}{P}$$

The left-hand side is the sum of new issues of government liabilities plus the FA's tax and the CB's seigniorage revenues, whereas the right-hand side consists of changes in the CB's reserve position and interest payment on Treasury bonds. Combined with the household's budget constraint, we obtain the social resource constraint

$$c=r\bar{F}+y\equiv\bar{c} \tag{20}$$

In equilibrium, the consolidated government budget constraint is equivalent to

$$\dot{b}-\dot{\Phi}=r\left(b-\Phi\right)-im-\tau \tag{21}$$

4 Solving the Model: Local Analysis

From equations (5c) and (11) we obtain

$$\dot{\pi}=\beta\frac{\lambda}{\rho_\pi\lambda_i}-\left[\frac{\lambda}{\rho_\pi\lambda_i}+\frac{\rho_\Phi}{\rho_\pi}\Phi\right]\left[\rho\left(\pi,\Phi\right)-\pi\right]-\frac{\rho_\Phi}{\rho_\pi}\rho\left(\pi,\Phi\right)\phi \tag{22}$$

Now, $\lambda=\lambda\left(i\right)=\lambda\left(\pi,\Phi\right)>0$, and $\lambda_i\left(i\right)=u_{cm}\left(\bar{c},m\right)\phi_i\left(i,\bar{c}\right)$. Assuming that consumption and money are Edgeworth complements ($u_{cm}>0$), then $\lambda_i'\left(i\right)<0$ and

$$-\frac{\lambda\left(\rho\left(\pi,\Phi\right)\right)}{\lambda_i\left(\rho\left(\pi,\Phi\right)\right)}>0$$

The TVC can now be re-written as

$$\lim_{t\to\infty}e^{-rt}\left(b-\Phi\right)=0 \tag{23}$$

In equilibrium and under alternative fiscal policy rules, the household budget constraint can be re-written as

$$\begin{aligned}
\dot{b}-\dot{\Phi} &= r\left(b-\Phi\right)-im-\bar{\tau}\\
\dot{b}-\dot{\Phi} &= \left(r-\delta_1\right)\left(b-\Phi\right)-\delta_1\Phi-im-\delta_0\\
\dot{b}-\dot{\Phi} &= \left(r-\delta_2\right)\left(b-\Phi\right)-\delta_2\Phi-\left(i+\delta_2\right)m-\delta_0\\
\dot{b}-\dot{\Phi} &= \left(r-\delta_3\bar{F}\right)\left(b-\Phi\right)-im-\delta_0
\end{aligned}$$

From (16), (17a)-(17c) and (21), we can see that under Ricardian fiscal rules, the TVC (23) is satisfied and is therefore redundant for the definition of equilibrium. Under the non-Ricardian tax rule $\tau = \bar{\tau}$, the composite variable $b - \Phi$ grows at the rate $r = \beta$, violating the transversality condition, so (23) continues to be part of the equilibrium definition. However, by restricting attention to local analysis and to a linear approximation of the dynamical system (11), (22) and (18), the TVC (23) is still satisfied in a small bounded neighborhood of the steady state. It is therefore valid to analyze the dynamic properties of the flexible-price model close to the steady state equilibria.

Define the perfect foresight equilibrium for the flexible price model as follows:

Definition 4 *A flexible-price perfect foresight equilibrium (PFE) is defined as functions of time* $\{c, m, b, \Phi\}_{t=0}^{\infty}$ *and* $P(0)$ *that: (1) solve the household's constrained utility maximization problem, taking as given initial conditions* $A(0) = M(0) + B(0) + F^H(0)$*, price* P *and government policies* $\{\tau,\ i\}$*; (2) have all markets clear*[24]*, i.e.,* $c = y + r\bar{F}$ *and* $F^H + F^{CB} = \bar{F}$*.*

Alternatively, a more operational definition of PFE for the flexible-price model is

Definition 5 *A flexible-price perfect foresight equilibrium (PFE) is defined as functions of time* $\{c, m, b, \Phi\}_{t=0}^{\infty}$ *and* $P(0)$ *that satisfy the dynamical system (11), (22) and (18) or (19) and the TVC (23).*

4.1 Equilibria Converging to the Steady State

With our log-linear interest rate rule (12), the ZLB on nominal interest rate i never binds ($i > 0$) and there are two different steady state equilibria as described in detail in Benhabib, Schmitt-Grohé and Uribe (2001a, 2002): the higher-inflation steady state equilibrium $\left(\pi^H, \Phi^H, b^H\right)$ where the monetary policy is active ($\alpha_\pi > 1$); and the low-inflation steady state equilibrium $\left(\pi^L, \Phi^L, b^L\right)$ where the monetary policy is passive ($\alpha_\pi < 1$). The low-inflation steady state equilibrium has the characteristics of a "liquidity trap". Henceforth we refer to the higher-inflation steady state equilibrium $\left(\pi^H, \Phi^H, b^H\right)$ as the "target equilibrium" and the low-inflation steady state equilibrium

[24] In equilibrium, households' net demand of financial assets must equal the government's net supply of liabilities. Money and bond markets clear as a consequence of our notation.

(π^L, Φ^L, b^L) as the "liquidity trap" equilibrium. We use (π^*, Φ^*, b^*) to represent generically both the high and low-inflation steady state equilibria, where $\dot{\pi} = \dot{\Phi} = \dot{b} = 0$ and $\rho(\pi^*, \Phi^*) = \pi^* + \beta = \pi^* + r$.

We analyze the local dynamics of the model economy by linearizing around the steady state (π^*, Φ^*, b^*). We obtain the following linear system

$$\begin{bmatrix} \dot{\pi} \\ \dot{\Phi} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & 0 \\ J_{21} & J_{22} & 0 \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \begin{bmatrix} \pi - \pi^* \\ \Phi - \Phi^* \\ b - b^* \end{bmatrix} \tag{24}$$

where the non-zero entries of the Jacobian matrix J are evaluated at the steady state. Denote as J^S the 2×2 submatrix consisting of $\{J_{11}, J_{12}, J_{21}, J_{22}\}$, which is independent of the fiscal policy. It is easy to see that one eigenvalue of J is J_{33}, and the remaining two eigenvalues of J are the eigenvalues of J^S. Under Cobb-Douglas preferences (3) and a log-linear interest rate rule (12), the elements of J^S can be simplified as follows (see Appendix A for derivation)[25]

$$J_{11} = \left(1 - \frac{1}{\alpha_\pi}\right)\left(\frac{\pi^* + \beta}{1-a} - \alpha_\Phi\right) \tag{25a}$$

$$J_{12} = \frac{\alpha_\Phi}{\alpha_\pi \Phi^*}\left(\frac{\pi^* + a\beta}{1-a} - \alpha_\Phi\right) \tag{25b}$$

$$J_{21} = (\alpha_\pi - 1)\,\Phi^* \tag{25c}$$

$$J_{22} = \alpha_\Phi + \beta \tag{25d}$$

Observe that $J_{12} < 0$ and $sgn(J_{11}) = sgn(\alpha_\pi - 1)$. In particular, $J_{11}, J_{21} > 0$ if the monetary policy is active ($\alpha_\pi > 1$) and $J_{11}, J_{21} < 0$ if the monetary policy is passive ($\alpha_\pi < 1$). In the case of no CB balance sheet concerns ($\alpha_\Phi = 0$), $J_{12} = 0$.

4.1.1 Without Net Worth Targeting ($\alpha_\Phi = 0$)

We first investigate the issue of local determinacy in a tax-based monetary system where the CB has no balance sheet concerns and does not practise net

[25] The entries of J^S take exactly the same simplified form if we assume a globally linear Taylor rule

$$i = i^* + \alpha_\pi(\pi - \pi^*) + \alpha_\Phi\left(\frac{\Phi - \Phi^*}{\Phi^*}\right)$$

where $i^* = \pi^* + \beta$. This is shown in Appendix A.

worth targeting ($\alpha_\Phi = 0$). Under our non-Ricardian fiscal policy $\tau = \bar{\tau}$, the transversality condition (23) is violated although the equilibrium solutions of the model satisfy (11), (22) and (18). In this case it is still valid to study local dynamic properties of the model by focussing on linear approximations around the steady state equilibria. Linearizing (18), we obtain

$$J_{31} = (\alpha_\pi - 1)\, b^* \qquad J_{32} = \alpha_\Phi b^* / \Phi^* \qquad J_{33} = r$$

Under alternative Ricardian fiscal rules, the TVC (23) is satisfied and the equilibrium solutions of the model satisfy the dynamical system composed of (11), (22) and (18). Linearizing around the steady state,

$$\begin{aligned} J_{31}^1 &= (\alpha_\pi - 1)\, b^* & J_{32}^1 &= \frac{\alpha_\Phi}{\Phi^*} b^* & J_{33}^1 &= r - \delta_1 \\ J_{31}^2 &= (\alpha_\pi - 1)\, b^* - \delta_2 \alpha_\pi \phi_i & J_{32}^2 &= \frac{\alpha_\Phi}{\Phi^*} (b^* - \delta_2 \phi_i) & J_{33}^2 &= r - \delta_2 \\ J_{31}^3 &= (\alpha_\pi - 1)\, b^* & J_{32}^3 &= \frac{\alpha_\Phi}{\Phi^*} b^* + \delta_3 & J_{33}^3 &= r - \delta_3 \end{aligned}$$

The determinacy and stability results differ from the non-Ricardian case ($\tau = \bar{\tau}$), because for $j = 1, 2, 3$, the third eigenvalue J_{33}^j ($= r - \delta_j$) of the Jacobian matrix J can now take non-positive values, *i.e.*, $r - \delta_j \leq 0$. We only consider the case of either active or passive fiscal rules, where $r - \delta \neq 0$, since it is unlikely that the Treasury happens to follow a knife-edge "neutral" rule with $r - \delta = 0$. In the case of $r - \delta_j < 0$, the fiscal authority passively adjusts taxes to balance the consolidated government budget.

Local determinacy results for a CB without balance sheet concerns (no net worth targeting) conform largely to those obtained in Leeper (1991). Under the non-Ricardian fiscal rule (16) and active Ricardian rules (17a)-(17c) where $r - \delta > 0$, the target steady state equilibrium is locally indeterminate with an active monetary policy, while the low-inflation liquidity trap equilibrium is locally unique with a passive monetary policy. Therefore, although we obtain local uniqueness under the policy mix of active fiscal and passive monetary rules, the focus equilibrium of a liquidity trap is undesirable.

Under passive Ricardian rules ($r - \delta < 0$) which ensures fiscal solvency, it is well-known that an interest rate rule that obeys the Taylor Principle ($\alpha_\pi > 1$) implies that the target equilibrium ($\pi = \pi^H$) is locally determinate. Local uniqueness of such a fiscal-monetary policy mix has been shown by Clarida, Galí and Gertler (1997), Leeper (1991), Sims (1994), Woodford (1994, 2003). On the contrary, if the monetary policy is passive ($\alpha_\pi < 1$), the liquidity trap equilibrium with $\pi = \pi^L$ is locally indeterminate and a passive monetary policy stance may destabilize the economy by inducing expectations-driven fluctuations.

The possibility that a passive monetary policy combined with an active fiscal rule leads to a locally unique liquidity trap is disturbing, monetary activism in the form of Taylor Principle is justified as one way to prevent the economy from drifting towards the undesirable deflationary trap. Furthermore, monetary activism has the power to stabilize the real economy by ensuring local uniqueness of the desired equilibrium. These results, proved in our model setup, are summarized in the following two propositions.

Proposition 1 (Non-Ricardian Fiscal Rule) *Assume that* $\alpha_\Phi = 0$*, then under the non-Ricardian fiscal rule: (1) If the monetary policy is active (*$\alpha_\pi > 1$*), the target equilibrium where* $\pi = \pi^H$ *is locally indeterminate; (2) If the monetary policy is passive (*$\alpha_\pi < 1$*), the liquidity trap equilibrium where* $\pi = \pi^L$ *is locally determinate.*

Proof. See the Appendix B. ■

Proposition 2 (Ricardian Fiscal Rules) *Assume that* $\alpha_\Phi = 0$*. Under active Ricardian rules (*$r - \delta > 0$*), the local determinacy results remain unchanged from those under the non-Ricardian fiscal rule* $\tau = \bar{\tau}$*; Under a passive Ricardian rule (*$r - \delta < 0$*), (1) If the monetary policy is active (*$\alpha_\pi > 1$*), the target equilibrium where* $\pi = \pi^H$ *is locally determinate; (2) If the monetary policy is passive (*$\alpha_\pi < 1$*), the liquidity trap equilibrium where* $\pi = \pi^L$ *is locally indeterminate.*

Proof. See the Appendix B.

Remark 2 *Notice that although the FA may treat real CB liabilities (m) as part of its own liabilities and backes these up with taxation, local determinacy results are determined by whether the fiscal policy is active or passive.*

■

4.1.2 With Net Worth Targeting ($\alpha_\Phi < 0$)

We now study local determinacy issues of the steady state equilibria in a reserve-based monetary system, under the assumption that the CB is concerned with its balance sheet position and follows an interest rate rule that reacts to deviations from the target levels of both inflation and the CB net worth ($\alpha_\Phi < 0$). The monetary policy is active at the "target equilibrium" and passive at the liquidity trap equilibrium. The following two results show that CB balance sheet concerns and net worth targeting have a profound impact on macroeconomic stability and may indeed be responsible for much of the BoJ's policy conservatism and prolonged deflationary recession.

Proposition 3 (Non-Ricardian Fiscal Rule) *Assume that* $\alpha_\Phi < 0$. *(1) If the monetary policy is passive (*$\alpha_\pi \in (\bar{\alpha}_\pi, 1)$*), then independent of the degree of CB conservatism (*α_Φ*), the low-inflation equilibrium is locally determinate; (2) If the monetary policy is active (*$\alpha_\pi > 1$*) and if the CB is ultra-conservative (*$\alpha_\Phi < \bar{\alpha}_\Phi$*), then the target equilibrium is locally indeterminate; (3) If the monetary policy is active (*$\alpha_\pi > 1$*) and if the CB is conservative (*$\alpha_\Phi > \bar{\alpha}_\Phi$*), then no stable equilibrium solution exists.*

Proof. See the Appendix B. ■

The CB balance sheet concerns have a major impact on the existence and local uniqueness of equilibrium solutions to the model. Proposition 3 and 4 indicate that under a non-Ricardian fiscal rule and active Ricardian policies, monetary conservatism coupled with passive interest rate rule renders the liquidity trap equilibrium locally unique, therefore producing an undesirable focal point for the formation and coordination of expectations. Monetary passivism ($\alpha_\pi < 1$) should therefore be avoided at all costs. When the Taylor Principle is adhered to but the CB is too conservative ($\alpha_\Phi < \bar{\alpha}_\Phi$), the desired steady state equilibrium becomes locally indeterminate; and if the CB is only mildly conservative ($\alpha_\Phi > \bar{\alpha}_\Phi$), there exist no equilibrium solutions which are non-explosive. Monetary conservatism ($\alpha_\Phi < 0$), be it weak or strong, ties the hands of central bankers and may destabilize the economy.

Proposition 4 (Ricardian Fiscal Rules) *Assume that* $\alpha_\Phi < 0$. *Under an active Ricardian rule (*$r - \delta > 0$*), the local determinacy results remain unchanged from those under the non-Ricardian fiscal rule* $\tau = \bar{\tau}$*; Under a passive Ricardian rule (*$r - \delta < 0$*), (1) If the monetary policy is passive (*$\alpha_\pi \in (\bar{\alpha}_\pi, 1)$*), then independent of the degree of CB conservatism (*α_Φ*), the liquidity trap equilibrium is locally indeterminate; (2) If the monetary policy is active (*$\alpha_\pi > 1$*) and if the CB is ultra-conservative (*$\alpha_\Phi < \bar{\alpha}_\Phi$*), then the target equilibrium is locally indeterminate; (3) If the monetary policy is active*

($\alpha_\pi > 1$) and if the CB is conservative ($\alpha_\Phi > \bar{\alpha}_\Phi$), then the target equilibrium is locally determinate.

Proof. See the Appendix B. ∎

When the Ricardian rules are passive ($r - \delta < 0$) and the CB is conservative, a passive monetary policy stance ($\alpha_\pi \in (\bar{\alpha}_\pi, 1)$) leads to local indeterminacy of the low-inflation steady state equilibrium. Under monetary activism ($\alpha_\pi > 1$), the desired target equilibrium is also rendered locally indeterminate if the CB is too concerned about its balance sheet ($\alpha_\Phi < \bar{\alpha}_\Phi$). When monetary activism is combined with a mild dose of CB conservatism ($\alpha_\Phi \in (\bar{\alpha}_\Phi, 0)$), we have the fortunate result that the the target equilibrium is still locally determinate. Therefore the important result that an active Taylor rule along with fiscal passivism would ensure local determinacy of the desired target equilibrium and hence macroeconomic stability is maintained only if the degree of monetary conservatism is relatively mild. When CB balance sheet concerns are sufficiently pronounced ($\alpha_\Phi > \bar{\alpha}_\Phi$), the stability result for an active Taylor rule is completely reversed. The formula of monetary activism and fiscal passivism no long suffice to guarantee stabilization of the real economy when the CB is conservative enough. When the CB is sufficiently constrained by concerns over its own financial soundness, the desirable target equilibrium becomes locally indeterminate and the economy is more likely to be driven into a deflationary recession of the type observed in Japan, even if the fiscal-monetary policy mix follows the recommended rules. For this success formula to work, institutional reform that ensures fiscal backing for monetary policy is necessary.

4.2 Equilibria Converging to a Deterministic Cycle

In this section, we study the conditions under which there exist perfect foresight equilibria in which (π, Φ, b), instead of converging to a steady state equilibrium (π^*, Φ^*, b^*), converge to a deterministic periodic cycle. Here we entertain local cyclical equilibrium dynamics that are bounded in a small neighborhood around the steady state equilibrium, but they do not converge asymptotically to the steady state equilibrium. Fundamental to the dynamics around a deterministic cycle is the existence of a local Hopf bifurcation at some critical value $\bar{\alpha}_\Phi$ of the bifurcation parameter $\alpha_\Phi < 0$, which in our model represents the CB's balance sheet concerns. At the ascertained bifurcation value, a unique limit cycle emerges when the steady state equilibria change their stability.[26] In its turn, the existence of a Hopf bifurcation implies the existence of a family of cycles for values of α_Φ in a small neighborhood to the left or right of $\bar{\alpha}_\Phi$. The existence of a stable limit cycle implies cyclical fluctuations in (π, Φ, b) which still satisfy equilibrium conditions.

4.2.1 With Net Worth Targeting

We first prove the existence of a Hopf bifurcation when the CB is concerned with its balance sheet and follows an interest rate rule that targets both inflation and the CB net worth. We focus on the bifurcation parameter α_Φ.

In the next two propositions, we collect the existence results for an active interest rate rule under both non-Ricardian and Ricardian fiscal rules. We discovered that an adherence to the Taylor Principle when the CB is conservative in its policy-making leads to local Hopf bifurcation and the emergence of a unique deterministic cycle at the desirable target steady state equilibrium $\left(\pi^H, \Phi^H, b^H\right)$, with $\alpha_\Phi = \bar{\alpha}_\Phi$ as the bifurcation value. This result is independent of the magnitude of monetary activism ($\alpha_\pi > 1$) and the degree of CB balance sheet concerns ($\alpha_\Phi < 0$). On one side of the bifurcation value $\bar{\alpha}_\Phi$, there exists a unique equilibrium in which (π, Φ, b) converge to the desired steady state $\left(\pi^H, \Phi^H, b^H\right)$, and on the other side of $\bar{\alpha}_\Phi$, a continuum of equilibria exist in which (π, Φ, b) converge to the desired steady state $\left(\pi^H, \Phi^H, b^H\right)$.

At the bifurcation value $\alpha_\Phi = \bar{\alpha}_\Phi$ and the non-hyperbolic steady state equilibrium $\left(\pi^H, \Phi^H, b^H\right)$, the Jacobian matrix J has a simple pair of pure imaginary eigenvalues and the other eigenvalues has no zero real part. Then for each α_Φ near the bifurcation value $\bar{\alpha}_\Phi$ there corresponds a unique equilibrium $\left(\pi^\Phi, \Phi^\Phi, b^\Phi\right)$ near the steady state equilibrium $\left(\pi^H, \Phi^H, b^H\right)$. When the Jacobian matrix J crosses the imaginary axis at $\bar{\alpha}_\Phi$, the dimensions of the stable and unstable manifolds of this unique equilibrium point changes, and a periodic orbit or limit cycle is created as the stability properties of this equilibrium point $\left(\pi^\Phi, \Phi^\Phi, b^\Phi\right)$ change. The extreme sensitivity of the local stability properties of the steady state equilibrium $\left(\pi^H, \Phi^H, b^H\right)$ to a small change in the degree of the CB conservatism around the bifurcation value $\bar{\alpha}_\Phi$ is disturbing, and an institutional framework that eliminates CB balance sheet concerns and the very cause of monetary conservatism by pro-

[26] Whether the Hopf bifurcation is supercritical or subcritical, *i.e.*, whether the steady state equilibrium generates a stable or unstable limit cycle as α_Φ passes through the bifurcation value $\bar{\alpha}_\Phi$, depends on the sign of the Liapunov number or coefficient σ.

moting close fiscal-monetary cooperation and refraining from unfettered CB independence is important for ensuring macroeconomic stability.

Proposition 5 (Non-Ricardian Fiscal Rule) *Assume that $\alpha_\Phi < 0$, then under active monetary policy ($\alpha_\pi > 1$), the Hopf Bifurcation exists at the target steady state equilibrium $\left(\pi^H, \Phi^H, b^H\right)$ with $\alpha_\Phi = \bar{\alpha}_\Phi$ as the bifurcation value.*

Proof. See the Appendix B. ∎

Proposition 6 (Ricardian Fiscal Rules) *Assume that $\alpha_\Phi < 0$ and $r - \delta \neq 0$, then under active monetary policy ($\alpha_\pi > 1$), the Hopf Bifurcation exists at the steady state equilibrium $\left(\pi^H, \Phi^H, b^H\right)$ with $\alpha_\Phi = \bar{\alpha}_\Phi$ as the bifurcation value.*

Proof. See the Appendix B. ∎

4.2.2 Without Net Worth Targeting

Without net worth targeting, we focus on the other policy parameter α_π as the bifurcation parameter. By inspection, the eigenvalues of the submatrix J' are purely imaginary only if $tr\left(J'\right) = 0$. But this implies that $\det\left(J'\right) = -\beta^2$ so no simple pair of purely imaginary eigenvalues could exist at $\alpha_\pi = \bar{\alpha}_\pi < 1$. Therefore the Hopf bifurcation cannot occur in this case.

Proposition 7 (Non-Ricardian Fiscal Rule) *Assume that $\alpha_\Phi = 0$, and take α_π as the free parameter, then the Hopf Bifurcation does not occur at the steady state equilibrium $\left(\pi^L, \Phi^L, b^L\right)$ with $\alpha_\pi = \bar{\alpha}_\pi < 1$.*

Proof. See the Appendix B. ∎

This result differs from that contained in Proposition 7 in Benhabib, Schmitt-Grohé and Uribe (2001b). Their result relies on the specification of the functional form of the interest rate rule at the target steady state equilibrium.

Proposition 8 (Ricardian Fiscal Rules) *Assume that $\alpha_\Phi = 0$ and $r - \delta \neq 0$. Taking α_π as the free parameter, then the Hopf Bifurcation does not occur at the steady state equilibrium $\left(\pi^L, \Phi^L, b^L\right)$ with $\alpha_\pi = \bar{\alpha}_\pi < 1$.*

Proof. See the Appendix B. ∎

5 Conclusion

In this paper we provide a fully articulated theoretical foundation for the view that a CB's balance sheet concerns may hinder monetary policy "activism" needed for achieving and sustaining macroeconomic stability, thereby opening the door for local indeterminacy and bifurcation. As in Benhabib, Schmitt-Grohé and Uribe's (2002a, b), we study the situation where an interest rate rule is constrained by the zero lower bound on the nominal interest rate, which implies the emergence of a second steady state equilibrium that has the properties of a liquidity trap. We draw a clear distinction between the tax and the reserve-based monetary systems. We model the CB and the FA in a reserve-based monetary system as two distinct entities, each with its own budget constraint. The central bank follows an interest rate rule that reacts to deviations from the target levels of both inflation rate and its net worth.

We demonstrate that the introduction of CB's balance sheet concerns into a simple flexible-price model reverses the now conventional wisdom that the combination of a passive fiscal rule that guarantees fiscal solvency and an active Taylor rule would ensure local determinacy of the desired target steady state equilibrium and therefore promotes macroeconomic stability. With net worth targeting and a strong form of CB independence, monetary activism embodied the Taylor Principle cannot be applied to its full extent. CB financial conservatism, caused by balance sheet concerns, may lead to local indeterminacy of the desired target equilibrium and to bifurcation, therefore embeds in the economic system an inherent tendency towards structural instability. This can only be averted by an institutional reform that ensures better monetary-fiscal cooperation and eliminates or at least alleviates the CB's balance sheet concerns.

Our results indicate that when the CB is sufficiently conservative, a combination of passive fiscal rules and an active monetary policy rule yields the outcome that the desirable equilibrium becomes locally indeterminate. Furthermore, compared with the situation of no CB balance sheet concerns, monetary conservatism creates a local Hopf bifurcation and a deterministic cycle when the interest rate rule observes the Taylor Principle. Locally in a small neighborhood around the bifurcation value $\bar{\alpha}_\Phi$ of the policy parameter α_Φ, the stability properties of the equilibrium solutions of our model changes abruptly. These unpleasant local (in)determinacy results and the extreme sensitivity of the structural stability of the model to a small variation of the policy parameter α_Φ are unsettling.

To achieve macroeconomic stability that is suggested for the policy mix of a passive fiscal policy and an active Taylor rule, institutional reforms are necessary to eliminate the CB's balance sheet concerns and establish a tax-based monetary system that is exemplified by the US Federal Reserve System, where fiscal backing is automatic, immediate and readily understood by the public. In a new era of price stability, monetary policy-making faces new difficults such as the risk of deflationary spiral and the ZLB on nominal interest rates. Unquestioned cult of CB independence may have unpleasant economic implications and needs to be restrained for the benefit of macroeconomic stability. Our results have important import for the institutional design for the European System of Central Banks, the Bank of Japan, and existing pseudo-currency-board regimes in Hong Kong and Singapore.

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Appendix

A Deriving J_{ij}'s

Linearizing around the steady state equilibria (π^*, Φ^*, b^*), we obtain

$$
\begin{aligned}
J_{11} &= \frac{\beta\left(\rho_\pi \lambda_i\right)^2 - \beta\lambda\left(\rho_{\pi\pi}\lambda_i + \rho_\pi^2 \lambda_{ii}\right)}{\left(\rho_\pi \lambda_i\right)^2} - \\
&\quad \beta\left[\frac{\left(\rho_\pi \lambda_i\right)^2 - \lambda\left(\rho_{\pi\pi}\lambda_i + \rho_\pi^2 \lambda_{ii}\right)}{\left(\rho_\pi \lambda_i\right)^2} + \frac{\rho_{\Phi\pi}\rho_\pi \Phi - \rho_\Phi \rho_{\pi\pi}\Phi}{\rho_\pi^2}\right] - \\
&\quad \left(\frac{\lambda}{\rho_\pi \lambda_i} + \frac{\rho_\Phi}{\rho_\pi}\Phi\right)\left(\rho_\pi - 1\right) - \frac{\rho_\pi\left(\rho_{\Phi\pi} i\phi + \rho_\Phi \rho_\pi \phi + \rho_\Phi \rho_\pi i\phi_i\right) - \rho_\Phi i\phi\rho_{\pi\pi}}{\rho_\pi^2} \\
&= -\frac{\rho_\pi \rho_{\Phi\pi} - \rho_\Phi \rho_{\pi\pi}}{\rho_\pi^2}\left(\beta\Phi + i\phi\right) \\
&\quad - \left(\frac{\lambda}{\lambda_i} + \rho_\Phi \Phi\right)\left(1 - \frac{1}{\rho_\pi}\right) + \rho_\Phi\left(\phi + i\phi_i\right) \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
J_{12} &= \frac{\beta\left(\lambda_i\right)^2 \rho_\Phi \rho_\pi - \beta\lambda\left(\rho_{\pi_\Phi}\lambda_i + \rho_\Phi \rho_\pi \lambda_{ii}\right)}{\left(\rho_\pi \lambda_i\right)^2} - \\
&\quad \beta\left[\frac{\left(\lambda_i\right)^2 \rho_\Phi \rho_\pi - \lambda\left(\rho_{\pi_\Phi}\lambda_i + \rho_\Phi \rho_\pi \lambda_{ii}\right)}{\left(\rho_\pi \lambda_i\right)^2} + \frac{\left(\rho_{\Phi\Phi}\Phi + \rho_\Phi\right)\rho_\pi - \rho_\Phi \rho_{\pi_\Phi}\Phi}{\rho_\pi^2}\right] - \\
&\quad \left(\frac{\lambda}{\rho_\pi \lambda_i} + \frac{\rho_\Phi}{\rho_\pi}\Phi\right)\rho_\Phi - \frac{\rho_\pi\left(\rho_{\Phi\Phi} i\phi + \rho_\Phi^2 \phi + \rho_\Phi^2 i\phi_i\right) - \rho_\Phi i\phi\rho_{\pi\Phi}}{\rho_\pi^2} \\
&= -\frac{\rho_\pi \rho_{\Phi\Phi} - \rho_\Phi \rho_{\pi_\Phi}}{\rho_\pi^2}\left(\beta\Phi + i\phi\right) - \frac{\rho_\Phi}{\rho_\pi}\left(\frac{\lambda}{\lambda_i} + \rho_\Phi \Phi + \beta\right) - \frac{\rho_\Phi^2}{\rho_\pi}\left(\phi + i\phi_i\right) \qquad (27)
\end{aligned}
$$

and

$$
\begin{aligned}
J_{21} &= \left(\rho_\pi - 1\right)\Phi + \rho_\pi\left(\phi + i^*\phi_i\right) \qquad (28) \\
J_{22} &= i - \pi + \rho_\Phi \Phi + \rho_\Phi\left(\phi + i\phi_i\right) \qquad (29)
\end{aligned}
$$

where J_{11}, J_{12}, J_{21} and J_{22} are evaluated at the steady state equilibria.

A.1 Alternative Interest Rate Rules

Under the log-linear Taylor rule (12), we have

$$
\begin{aligned}
\rho_\pi &= \alpha_\pi \frac{i}{i^*} \qquad \rho_\Phi = \frac{\alpha_\Phi}{\Phi^*}\frac{i}{i^*} \\
\rho_{\pi\pi} &= \alpha_\pi^2 \frac{i}{i^{*2}} \qquad \rho_{\Phi\Phi} = \frac{\alpha_\Phi^2}{\Phi^{*2}}\frac{i}{i^{*2}} \\
\rho_{\pi\Phi} &= \rho_{\Phi\pi} = \frac{\alpha_\pi \alpha_\Phi}{\Phi^*}\frac{i}{i^{*2}}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\rho_\pi \rho_{\Phi\pi} - \rho_\Phi \rho_{\pi\pi} &= 0 \\
\rho_\pi \rho_{\Phi\Phi} - \rho_\Phi \rho_{\pi_\Phi} &= 0 \\
\frac{\rho_\Phi}{\rho_\pi} &= \frac{\alpha_\Phi}{\alpha_\pi \Phi^*}
\end{aligned}
$$

Hence, the elements of the submatrix J^S can be simplified as[27]

$$
\begin{aligned}
J_{11} &= \frac{\alpha_\Phi}{\Phi^*}\left(\phi + i^* \phi_i\right) - \left(\frac{\lambda}{\lambda_i} + \alpha_\Phi\right)\left(1 - \frac{1}{\alpha_\pi}\right) && \text{(30a)} \\
J_{12} &= -\frac{\alpha_\Phi}{\alpha_\pi \Phi^*}\left(\frac{\lambda}{\lambda_i} + \alpha_\Phi + \beta\right) - \frac{\alpha_\Phi^2}{\alpha_\pi \Phi^{*2}}\left(\phi + i^* \phi_i\right) && \text{(30b)} \\
J_{21} &= \left(\alpha_\pi - 1\right)\Phi^* + \alpha_\pi \left(\phi + i^* \phi_i\right) && \text{(30c)} \\
J_{22} &= \alpha_\Phi + \beta + \frac{\alpha_\Phi}{\Phi^*}\left(\phi + i^* \phi_i\right) && \text{(30d)}
\end{aligned}
$$

If we assume a globally linear Taylor rule instead of the nonlinear rule

$$
i = i^* + \alpha_\pi \left(\pi - \pi^*\right) + \alpha_\Phi \left(\frac{\Phi - \Phi^*}{\Phi^*}\right)
$$

where $i^* = \pi^* + \beta$, we have

$$
\begin{aligned}
\rho_\pi &= \alpha_\pi \qquad \rho_\Phi = \frac{\alpha_\Phi}{\Phi^*} \\
\frac{\rho_\Phi}{\rho_\pi} &= \frac{\alpha_\Phi}{\alpha_\pi \Phi^*} \\
\rho_{\pi\pi} &= \rho_{\pi\Phi} = \rho_{\Phi\Phi} = 0
\end{aligned}
$$

[27] At the steady state equilibrium (π^*, Φ^*, b^*)

$$
i = i^*
$$

All J_{ij}'s are evaluated at the SS.

Therefore, the elements of the submatrix J^S take the same form as in (30a)-(30d).

A.2 Non-Separable Preferences

To further simplify the expressions for J_{11} and J_{12}, we assume the instantaneous utility function takes the constant elasticity of money demand (CES) form with non-separable preferences:

$$u(c,m) = \left[ac^{1-b} + (1-a)\, m^{1-b}\right]^{\frac{1}{1-b}} \tag{31}$$

with $a \in (0,1)$ and $b > 0$, $b \neq 1$. Then,

$$m = \phi(i) = \left(\frac{1-a}{a}\right)^{1/b} c i^{-1/b}$$

Substitute $\phi(i)$ into $\lambda = u_c(c,m)$, we obtain

$$\begin{aligned} \frac{\lambda}{\lambda_i} &= -\left(\frac{a}{1-a}\right)^{\frac{1}{b}} a i^{\frac{1}{b}} - i \\ \frac{\lambda\lambda_{ii}}{(\lambda_i)^2} &= \frac{2-b}{b} + \frac{a^{1+\frac{1}{b}}}{b\,(1-a)^{\frac{1}{b}}} i^{\frac{1}{b}-1} \end{aligned}$$

Also

$$\phi + i\phi_i = \left(1 - \frac{1}{b}\right)\phi \tag{32}$$

As $b \to 1$, the CES specification assumes the Cobb-Douglas form

$$u(c,m) = c^a m^{1-a} \tag{33}$$

We have

$$m = \phi(i) = \frac{1-a}{a}\frac{\bar{c}}{i}$$

Substitute $\phi(i)$ into $u_c(c,m)$,

$$\begin{aligned} \frac{\lambda}{\lambda_i} &= -\frac{i}{1-a} \\ \frac{\lambda\lambda_{ii}}{(\lambda_i)^2} &= \frac{2-a}{1-a} \end{aligned}$$

Also

$$\phi + i\phi_i = 0 \tag{34}$$

Therefore[28],

$$J_{11} = \left(1 - \frac{1}{\alpha_\pi}\right)\left(\frac{\pi^* + \beta}{1 - a} - \alpha_\Phi\right) \tag{35a}$$

$$J_{12} = \frac{\alpha_\Phi}{\alpha_\pi \Phi^*}\left(\frac{\pi^* + a\beta}{1 - \alpha} - \alpha_\Phi\right) \tag{35b}$$

$$J_{21} = (\alpha_\pi - 1)\,\Phi^* \tag{35c}$$

$$J_{22} = \alpha_\Phi + \beta \tag{35d}$$

B Proofs of Main Results for Local Analysis

The dynamical system (11), (22) and (18) has two jump variables (π, Φ) which are forward-looking and can move discontinuously and respond instantaneously when new information arrives, and one predetermined variable which is tied to the past and constrained to move continuously (b). Although the total financial assets held by the household cannot jump because the household's budget constraint has to hold continuously, $\Phi = F^{CB} - M/P$ and $m = M/P$ are forward-looking and can jump according to the household's portfolio decisions. Similarly, by the

[28]Notice that the interest elaticity for money demand $m = \phi(i, c)$ is

$$\varepsilon_i = -\frac{\partial m/m}{\partial i/i} = -\frac{\partial m}{\partial i}\frac{i}{m} = -\frac{i\phi_i}{\phi} > 0$$

and the consumption elaticity for money demand $m = \phi(i, c)$ is

$$\varepsilon_c = \frac{\partial m/m}{\partial c/c} = \frac{\partial m}{\partial c}\frac{c}{m} = \frac{c\phi_c}{\phi} > 0$$

Therefore

$$\phi + i\phi_i = \phi(1 - \varepsilon_i)$$

For coefficients J_{11} and J_{12}, assuming a constant ε_i and a simple money demand function would be sufficient.

assumption of a separate fiscal budget constraint, $b = B/P$ is backward-looking and can only adjust continuously. In fact, M can jump at any given time, making the household's portfolio choice forward-looking. The economy's rate of inflation π is forward-looking and it depends on the monetary policy rule $\rho(\pi, \Phi)$, which reacts to current inflation and net worth.

For the Jacobian matrix J,

$$\begin{aligned} \det(J) &= (J_{11}J_{22} - J_{12}J_{21}) J_{33} \\ tr(J) &= J_{11} + J_{22} + J_{33} \end{aligned}$$

For the submatrix J^S of J,

$$\begin{aligned} \det\left(J^S\right) &= J_{11}J_{22} - J_{12}J_{21} \\ &= \frac{\beta(\pi^* + \beta)}{1-a}\left(1 - \frac{1}{\alpha_\pi}\right) \\ tr\left(J^S\right) &= J_{11} + J_{22} \\ &= \left(1 - \frac{1}{\alpha_\pi}\right)\left(\frac{\pi^* + \beta}{1-a}\right) + \frac{\alpha_\Phi}{\alpha_\pi} + \beta \end{aligned}$$

Assuming $\pi^* > -\beta$, then the sign of $\det\left(J^S\right)$ is the same as that of $1 - \alpha_\pi^{-1}$. That is, $\det\left(J^S\right) > 0$ if the monetary policy is active ($\alpha_\pi > 1$), and $\det\left(J^S\right) < 0$ if $\alpha_\pi < 1$. However, the sign of $tr\left(J^S\right)$ depends on both α_Φ and α_π. Specifically, $tr\left(J^S\right) > 0$ ($tr\left(J^S\right) < 0$) if $\alpha_\Phi > \bar{\alpha}_\Phi$ ($\alpha_\Phi < \bar{\alpha}_\Phi$), respectively.

Define

$$U = tr\left(J^S\right) = \left(1 - \frac{1}{\alpha_\pi}\right)\left(\frac{\pi^* + \beta}{1-a}\right) + \frac{\alpha_\Phi}{\alpha_\pi} + \beta \tag{36a}$$

$$V = 4\det\left(J^S\right) = \frac{4\beta(\pi^* + \beta)}{1-a}\left(1 - \frac{1}{\alpha_\pi}\right) \lesseqgtr 0 \tag{36b}$$

Hence, $sgn(U) = sgn(\alpha_\Phi - \bar{\alpha}_\Phi)$ and $sgn(V) = sgn(\alpha_\pi - 1)$. The the eigenvalues E_1 and E_2 of matrix J^S are,

$$\begin{aligned} E_{1,2} &= \frac{tr\left(J^S\right) \pm \sqrt{tr\left(J^S\right)^2 - 4\det\left(J^S\right)}}{2} \\ &= \frac{1}{2}U \pm \frac{1}{2}\sqrt{U^2 - V} \end{aligned}$$

B.1 Equilibria Converging to the Steady States

In this section, we provide proofs for the local determinacy results, in the cases both with and without CB balance sheet concerns.

B.1.1 Without Net Worth Targeting

Under the non-Ricardian fiscal rule, linearizing (18), we obtain

$$J_{31} = (\alpha_\pi - 1)\, b^* \qquad J_{32} = \frac{\alpha_\Phi}{\Phi^*} b^* \qquad J_{33} = r$$

Under alternative Ricardian fiscal rules, linearizing around the steady state,

$$\begin{aligned} J_{31}^1 &= J_{31}^3 = (\alpha_\pi - 1)\, b^* \qquad J_{31}^2 = (\alpha_\pi - 1)\, b^* - \delta_2 \alpha_\pi \phi_i \\ J_{32}^1 &= \frac{\alpha_\Phi}{\Phi^*} b^* \qquad J_{32}^2 = \frac{\alpha_\Phi}{\Phi^*} (b^* - \delta_2 \phi_i) \qquad J_{32}^3 = \frac{\alpha_\Phi}{\Phi^*} b^* + \delta_3 \\ J_{33}^j &= r - \delta_j \qquad \text{for } j = 1,2,3 \end{aligned}$$

The system has two jump variables (π, Φ) and one predetermined variable (b). Local determinacy results differ for non-Ricardian, active and passive Ricardian fiscal policy rules, because for $j = 1,2,3$, the third eigenvalue J_{33} of the Jacobian matrix J can now take non-positive values, *i.e.*, $r - \delta_j \leq 0$.

Proof of Proposition 1. First, notice that the dynamical system has two jump variables (π, Φ) and one predetermined variable (b). Since $\alpha_\Phi = 0$, $J_{12} = 0$. In this case, the submatrix J^S has two eigenvalues J_{11} and $J_{22} = \beta$. Now, $sgn\,(J_{11}) = sgn\,(\alpha_\pi - 1)$. In particular, $J_{11} > 0$ if the monetary policy is active $(\alpha_\pi > 1)$ and $J_{11} < 0$ if the monetary policy is passive $(\alpha_\pi < 1)$. The third eigenvalue of J is just $E_3 = J_{33} = \beta$.

Therefore of the three eigenvalues of the Jacobian matrix J, two take real positive value $E_2 = E_3 = \beta$. The first eigenvalue $E_1 = J_{11}$ is positive if $\alpha_\pi > 1$ and it is negative otherwise. Since we have two jump variables and one predetermined variable, the target equilibrium where $\pi = \pi^H$ is locally indeterminate, and the liquidity trap equilibrium is locally determinate. ■

Proof of Proposition 2. Under an active Ricardian fiscal rule, $r - \delta > 0$, the third eigenvalue of the Jacobian matrix is again positive and the submatrix J^S remains unchanged, so all determinacy results remain the same as those under the non-Ricardian fiscal rule $\tau = \bar{\tau}$.

Under a passive Ricardian fiscal rule, the third eigenvalue $E_3 = r - \delta$ is negative. Since $\alpha_\Phi = 0$, $J_{12} = 0$. In this case, the submatrix J^S has two eigenvalues J_{11} and $J_{22} = \beta$. Now, $J_{11} > 0$ if the monetary policy is active ($\alpha_\pi > 1$) and $J_{11} < 0$ if the monetary policy is passive ($\alpha_\pi < 1$).

Since the dynamical system has two jump variables and one predetermined variable, with active monetary policy, the target equilibrium is locally determinate. Under passive monetary policy, the liquidity trap equilibrium is locally indeterminate, *i.e.*, there is an infinite number of stable equilibrium solutions for the model. ■

B.1.2 With Net Worth Targeting

Assume that $\alpha_\Phi < 0$ so that the CB adjusts interest rate to target its net worth besides the inflation rate π. Again, $sgn(U) = sgn(\alpha_\Phi - \bar{\alpha}_\Phi)$ and $sgn(V) = sgn(\alpha_\pi - 1)$, *i.e.*, for $\bar{\alpha}_\Phi$ defined in (13), $U > 0$ if $\alpha_\Phi > \bar{\alpha}_\Phi$, and $U < 0$ otherwise. When the monetary policy is active ($\alpha_\pi > 1$), $V > 0$. A passive monetary policy ($\alpha_\pi < 1$) implies $V < 0$.

Proof of Proposition 3. First, notice that $J_{33} = \beta$. Consider the case of $\alpha_\Phi = \bar{\alpha}_\Phi$, which implies $U = 0$. Then if $\alpha_\pi > 1$, $U^2 - V = -V < 0$ and the submatrix J^S has one simple pair of purely imaginary eigenvalues. If $\alpha_\pi \in (\bar{\alpha}_\pi, 1)$, $U^2 - V > 0$ and the submatrix J^S has one real positive and one real negative eigenvalues. The low-inflation equilibrium is locally determinate.

When $\alpha_\Phi \neq \bar{\alpha}_\Phi$, $U \neq 0$. Since $V \neq 0$, the real part of the eigenvalues of the submatrix J^S is non-zero. If $\alpha_\pi < 1$, $V < 0$ and $U^2 - V > 0$ and $\sqrt{U^2 - V} > |U|$. Then if $\alpha_\Phi > \bar{\alpha}_\Phi$, $U > 0$, the submatrix J^S has one positive and one negative eigenvalues. The same is true for the case of $\alpha_\Phi < \bar{\alpha}_\Phi$ and $U < 0$. So the liquidity trap equilibrium is locally unique independent of the degree of monetary conservatism.

If $\alpha_\pi > 1$, $V > 0$ and $\sqrt{U^2 - V} < |U|$. Then if $\alpha_\Phi > \bar{\alpha}_\Phi$, $U > 0$, the submatrix J^S has either two positive eigenvalues (if $U^2 - V \geq 0$), or two complex eigenvalues with equal real positive parts (if $U^2 - V < 0$). In this case no stable equilibrium solution exists. If $\alpha_\Phi < \bar{\alpha}_\Phi$ and $U < 0$, the submatrix J^S has either two negative eigenvalues (if $U^2 - V \geq 0$), or two complex eigenvalues with equal real negative parts (if $U^2 - V < 0$). Then the target equilibrium is locally indeterminate. ■

Proof of Proposition 4. Under an active Ricardian fiscal rule, $r - \delta > 0$, the third eigenvalue of the Jacobian matrix is again positive. Since the submatrix J^S remains unchanged, so all determinacy results remain the same as those under the non-Ricardian fiscal rule $\tau = \bar{\tau}$.

Under a passive Ricardian fiscal rule, $r - \delta < 0$, the third eigenvalue J_{33} is negative. In the case of $\alpha_\Phi = \bar{\alpha}_\Phi$, if $\alpha_\pi > 1$, the submatrix J^S has one simple pair of purely imaginary eigenvalues[29]. If $\alpha_\pi \in (\bar{\alpha}_\pi, 1)$, $U^2 - V > 0$ and the submatrix J^S has one real positive and one real negative eigenvalues. Therefore the liquidity trap equilibrium is locally indeterminate and there is an infinite number of stable equilibrium solutions.

When $\alpha_\Phi \neq \bar{\alpha}_\Phi$, since $U \neq 0$ and $V \neq 0$, the real part of the eigenvalues of the submatrix J^S is non-zero. If $\alpha_\pi < 1$, then $V < 0$. When $\alpha_\Phi > \bar{\alpha}_\Phi$, $U > 0$, the submatrix J^S has one positive and one negative eigenvalues. The same is true for the case of $\alpha_\Phi < \bar{\alpha}_\Phi$ and $U < 0$. So the liquidity trap equilibrium is locally indeterminate independent of the degree of monetary conservatism.

If $\alpha_\pi > 1$, then $V > 0$. When $\alpha_\Phi > \bar{\alpha}_\Phi$, the submatrix J^S has either two positive eigenvalues (if $U^2 - V \geq 0$), or two complex eigenvalues with equal real positive parts (if $U^2 - V < 0$). In this case the target equilibrium is locally determinate. If $\alpha_\Phi < \bar{\alpha}_\Phi$, the submatrix J^S has either two negative eigenvalues (if $U^2 - V \geq 0$), or two complex eigenvalues with equal real negative parts (if $U^2 - V < 0$). Then the target equilibrium is locally indeterminate. ■

B.2 Equilibria Converging to a Deterministic Cycle: Local Bifurcation

We prove local bifurcation results in this section, taking α_Φ as the bifurcation parameter. The critical bifurcation value $\bar{\alpha}_\Phi$ is defined as in (14).

B.2.1 With Net Worth Targeting

When the CB has concerns over its own balance sheet and targets its own net worth ($\alpha_\Phi < 0$), a Hopf bifurcation is proved to occur under the Taylor Principle ($\alpha_\pi > 1$) independently of the type of fiscal rules under consideration.

Proof of Proposition 5. The dynamical system is continuous and differentiable. Assume $U^2 \leq V$ so the submatrix J^S has a pair of complex conjugate eigenvalues. Notice that at $\alpha_\Phi = \bar{\alpha}_\Phi$, the real part of the complex eigenvalues of J^S vanishes and the dynamical system has a simple pair of pure imaginary eigenvalues and

[29]In the case of $\alpha_\Phi = \bar{\alpha}_\Phi$ and $\alpha_\pi > 1$, if $r - \delta = 0$, there are one simple pair of imaginary eigenvalues and one third eigenvalue that is zero. In this case, we will have a Fold-Hopf or Garvrilov-Guckenheimer Bifurcation of codimension two. For details, see Kuznetsov (1995), p257.

no other eigenvalues with zero real part. So there is a smooth curve of equilibrium points $(\pi(\alpha_\Phi), \Phi(\alpha_\Phi), b(\alpha_\Phi))$, with $(\pi(\bar{\alpha}_\Phi), \Phi(\bar{\alpha}_\Phi), b(\bar{\alpha}_\Phi)) = (\pi^H, \Phi^H, b^H)$. The eigenvalues of the submatrix J^S, $(E_1(\alpha_\Phi), E_2(\alpha_\Phi))$, which are pure imaginary at $\bar{\alpha}_\Phi$, vary smoothly with α_Φ. Taking derivative of the real part of this pair of complex conjugate eigenvalues with respect to α_Φ and evaluate it at $\bar{\alpha}_\Phi$, we obtain

$$\frac{d}{d\alpha_\Phi}\left[\operatorname{Re} E(\alpha_\Phi)\right]_{\alpha_\Phi=\bar{\alpha}_\Phi} = \frac{1}{2\alpha_\pi} > 0$$

This is known as the Transverality Condition in the Bifurcation Theory. It says that the pair of complex conjugate eigenvalues of the submatrix J^S, $E_{1,2}(\alpha_\Phi)$, crosses the imaginary axis with non-zero speed.

Using the Poincaré-Andronov-Hopf Theorem (see Guckenheimer and Holmes (1983), p. 151-152, or Perko (2000), p. 353-354), there is a unique two-dimensional center manifold passing through the steady state equilibrium (π^H, Φ^H, b^H) at the bifurcation value $\alpha_\Phi = \bar{\alpha}_\Phi$. A Hopf bifurcation exists at this point. Specifically, the steady state equilibrium is asymptotically stable for $\alpha_\Phi < \bar{\alpha}_\Phi$ and it is unstable for $\alpha_\Phi > \bar{\alpha}_\Phi$ (see Wiggins 1990, p. 275). ■

Proof of Proposition 6. First, notice that at $\alpha_\Phi = \bar{\alpha}_\Phi$, since $r - \delta \neq 0$, the dynamical system has a simple pair of pure imaginary eigenvalues and no other eigenvalues with zero real part. The rest of the proof is similar to the case of a non-Ricardian fiscal rule. ■

B.2.2 Without Net Worth Targeting

When CB balance sheet concerns are eliminated ($\alpha_\Phi = 0$) but the CB continues to hold the reserve asset, no Hopf bifurcation exists when we take α_π as the bifurcation parameter.

Proof of Proposition 7. The proof is simple. In the case of $\alpha_\Phi = 0$,

$$U = \left(1 - \frac{1}{\alpha_\pi}\right)\left(\frac{\pi^* + \beta}{1 - a}\right) + \beta$$

If $\alpha_\pi > 1$, we have both $U > 0$ and $V > 0$. If $\alpha_\pi \in (\bar{\alpha}_\pi, 1)$, then $U > 0$ and $V < 0$. If $\alpha_\pi < \bar{\alpha}_\pi$, $U < 0$ and $V < 0$. There are no simple pair of complex eigenvalues in these cases. In particular, with $\alpha_\Phi = 0$, $U = 0$ if and only if $\alpha_\pi = \bar{\alpha}_\pi$, in which case

$$-V = 4\beta^2 > 0$$

so the Jacobian matrix J has eigenvalues $E_1 = E_3 = \beta$ and $E_2 = -\beta$, there are no simple pair of complex eigenvalues in this case. ■

Proof of Proposition 8. Since $r - \delta \neq 0$, the third eigenvalue of the Jacobian matrix J has a nonzero real part. The rest of the proof is similar to the case of a non-Ricardian fiscal rule. ■